Aplikace matematiky

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Aplikace matematiky, Vol. 34 (1989), No. 3, 197–212

Persistent URL: `http://dml.cz/dmlcz/104348`

34 (1989) APLIKACE MATEMATIKY No. 3, 197—212

ON CAUCHY PROBLEM FOR THE EQUATIONS OF REACTOR KINETICS

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(Received May 18, 1987)

Summary. In this paper, the initial value problem for the equations of reactor kinetics is solved and the temperature feedback is taken into account. The space where the problem is solved is chosen in such a way that it may correspond best of all to the mathematical properties of the cross-section models. The local solution is found by the method of iterations, its uniqueness is proved and it is shown also that existence of global solution is ensured in the most cases. Finally, the problem of mild solution is discussed.

Keywords: Initial value problem, reactor kinetics, analytical solution, neutron flux, temperature feedback, local, global and mild solution.

INTRODUCTION

Let us consider a nuclear reactor and study the behaviour of the neutron field and the changes of the material composition of such an equipment in time. In a good approach, the differential flux φ and the material density N_i $(i = 1, 2, \ldots, n)$ are described by the equations

$$\frac{\partial \varphi}{\partial t} + \sqrt{(2E)}\, \omega \nabla \varphi = \mathbf{A}_1(N, T, x, E, t)\, \varphi + \mathbf{A}_2(\varphi, N, T, x, E, \omega, t) + \\ + \mathbf{A}_3(N, T, x, E, t) + \sqrt{(2E)}\, S_0(x, E, \omega, t) \tag{1a}$$

and

$$\frac{\partial N_i}{\partial t} = \sum_{j=1}^{n} \mathbf{B}_{ij}(\varphi, N, T, x, t) + S_i(x, t) \quad (i = 1, 2, \ldots, n)\,. \tag{2a}$$

Here $N \equiv (N_1, N_2, \ldots, N_n)$ means the vector of material density and the following notation is used:

$$\mathbf{A}_1 \equiv -\sqrt{(2E)} \sum_{i=1}^{n} N_i(x, t)\, (\sigma_{si}(E, T) + \sigma_{ai}(E, T));$$

$$\mathbf{A}_2 \equiv \sqrt{(2E)} \sum_{i=1}^{n} N_i(x, t) \int_0^\infty \mathrm{d}E' \int_\Omega \mathrm{d}\omega'\, \varphi(x, E', \omega', t)\, [\sigma_{si}(E' \to E, \omega' \to \omega, T) + \\ + \alpha_i\, \chi_i(E)\, \nu_i(E', T)\, \sigma_{fi}(E', T)];$$

$$\boldsymbol{A}_3 \equiv \sqrt{(2E)} \sum_{i=1}^{n} N_i(\boldsymbol{x}, t)\, \lambda_i\, \chi_i(E);$$

$$\boldsymbol{B}_{ij} \equiv N_j(\boldsymbol{x}, t) \{ \int_0^\infty \mathrm{d}E \int_{\boldsymbol{\Omega}} \mathrm{d}\boldsymbol{\omega}\, \varphi(\boldsymbol{x}, E, \boldsymbol{\omega}, t) [a_{ij}\sigma_{aj}(E, T) + b_{ij}\sigma_{fj}(E, T) + \\ + c_{ij}\nu_j(E, T)\, \sigma_{fj}(E, T)] + d_{ij} \};$$

$\boldsymbol{x}, E, \boldsymbol{\omega}, t$ ... coordinates of location, kinetic energy, direction and time, respectively;
$\boldsymbol{\Omega}$ ... surface area of the unit sphere;
T ... absolute temperature;
$\sigma_{si}(E, T), \sigma_{ai}, \sigma_{fi}$ ... microscopic effective cross-sections for scattering, absorption and fission, respectively;
$\sigma_{si}(E' \to E, \boldsymbol{\omega}' \to \boldsymbol{\omega}, T)$ ... microscopic differential cross-section for scattering;
$\nu_i(E', T), \chi_i(E)$ ... the number of neutrons created in the process of fission and the fission spectrum;
S_0, S_i ... external source terms;
n ... the number of different nuclei;
α_i, λ_i ... nonnegative constants;
$a_{ij}, b_{ij}, c_{ij}, d_{ij}$ ... constants;

Equations (1a) and (2a) together with the initial conditions

$$\varphi(\boldsymbol{x}, E, \boldsymbol{\omega}, 0) = \varphi_0(\boldsymbol{x}, E, \boldsymbol{\omega}) \tag{1b}$$

and

$$N_i(\boldsymbol{x}, 0) = N_{0i}(\boldsymbol{x}), \quad (i = 1, 2, \ldots, n) \tag{2b}$$

express mathematically the problem of reactor kinetics. Temperature T can be understood either as a given quantity or as a function of the neutron flux and of the material density which is governed in general by some equations (the temperature feedback effect). The problem in the above formulation has not yet been solved in general and only some particular cases were analyzed.

For instance, the case of multigroup transport approximation in plane geometry was studied in the paper [1]. The delayed neutrons were taken into account but changes of material properties were not considered in the equation for the neutron flux. Next, in the book [2], the temperature feedback effect was studied both for the one-speed transport approximation and for the energy-dependent equation. Furthermore, delayed neutrons and xenon poisoning were considered. On the other hand, it was assumed there that

$$\{E, T\} \in [0, E_{\max}] \times [T_{\min}, T_{\max}]$$

where $E_{\max}$, $T_{\min}$ and $T_{\max}$ are finite positive numbers, and the material densities $N_i(\boldsymbol{x}, t)$ $(i = 1, 2, \ldots, n)$ were assumed to be bounded functions. Moreover, the dependence of the integral term in Eq. (1a) on the time changes of the material densities was not considered there. Discussion of the problem for the case of one-group diffusion approximation can be found in the book [3]. In this paper we will deal with the initial value problem having the form (1) and (2). First, the basic physical properties of the medium will be stated in four generalizing suppositions (which are satisfied in all known real cases, of course). Then a space of functions will be chosen appropriately with respect to these properties. Next, considering the temperature feedback effect, we will investigate the question of existence and uniqueness of a solution to this problem. Finally, we will show conditions under which the global solution exists and we will also look for the so called mild solution to this problem.

BASIC ASSUMPTIONS

Supposition 1. For any element $i = 1, 2, \ldots, n$ the following assertions hold:

a) The effective cross-sections σ_{si}, σ_{ai} and σ_{fi} are real nonnegative functions of energy $E \in (0, \infty)$ and of temperature $T \in (0, \infty)$. These quantities together with the functions

$$\frac{\partial}{\partial T}\sigma_{si}(E, T)\,, \quad \frac{\partial}{\partial T}\sigma_{ai}(E, T) \quad \text{and} \quad \frac{\partial}{\partial T}\sigma_{fi}(E, T)$$

are finite, continuous in the variable T for any E and sectionally continuous in the variable E for any T.

b) There exist continuous functions $a(T)$, $b(T)$: $(0, \infty) \to (0, \infty)$ such that

$$\sigma_{ti}(E, T) \equiv \sigma_{si}(E, T) + \sigma_{ai}(E, T) \leqq \frac{a(T)}{\sqrt{E}} + b(T)\,.$$

Supposition 2. For any element $i = 1, 2, \ldots, n$ the differential effective cross-section $\sigma_{si}(E' \to E,\ \omega' \to \omega, T): (0, \infty) \times (0, \infty) \times \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times (0, \infty) \to (0, \infty]$ has the form

$$\begin{aligned}\sigma_{si}(E' \to E,\ \omega' \to \omega, T) = {} & a_i^1 \sigma_{si}(E'\, E, \omega', \omega, T) + \\ & + a_i^2 \sum_{j=1}^{J_i} \sigma_{si}^{2j}(E', E, \omega', \omega, T)\, \delta(E - E' + E_j) + \\ & + a_i^3 \sum_{j=1}^{Ji} \sigma_{si}^{3j}(E', E, \omega', \omega, T)\, \delta(E' - E)\, \delta(\omega\omega' - g_j(E))\,.\end{aligned}$$

Here J_i is a positive integer, E_j and a_i^k $(j = 1, 2, \ldots, J_i;\ k = 1, 2, 3)$ are nonnegative constants, $g_j(E)$: $(0, \infty) \to [-1, 1]$ are sectionally continuous functions and $\delta(E' - E)$ is the Dirac function. The functions σ_{si}^1, σ_{si}^{2j} and σ_{si}^{3j} $(j = 1, 2, \ldots, J_i)$ are nonnegative.

Let $f(E)$ be a positive function of energy. For brevity, we denote

$$F_i^1(f, E', \omega', E, \omega, T) \equiv \sqrt{(2E)}\frac{f(E')}{f(E)}\, \sigma_{si}(E' \to E,\ \omega' \to \omega, T)\,,$$

$$F_i^2(f, E', \omega', E, \omega, T) \equiv \sqrt{(2E)}\, \chi_i(E)\, \sigma_{fi}(E', T)\, \nu_i(E', T) \frac{f(E')}{f(E)}\,,$$

$$F_i^3(f, E', \omega', E, \omega, T) \equiv f(E')\, \sigma_{ai}(E', T)\,,$$

$$F_i^4(f, E', \omega', E, \omega, T) \equiv f(E')\, \sigma_{si}(E', T)\,, \quad i = 1, 2, \ldots, n\,.$$

Supposition 3. There exist functions f_0 and f_1: $(0, \infty) \to (0, \infty)$ with the following properties:

a) They are bounded, sectionally continuous and such that

$$a(1 + \sqrt{E}) f_0(E) \leqq f_1(E)$$

where $a > 0$ is a finite constant.

b) For any given $T \in (0, \infty)$, the functions

$$\int_0^\infty \mathrm{d}E' \int_{\boldsymbol{\Omega}} \mathrm{d}\omega'\, F_i^j(f_k, E', \omega', E, \omega, T) \quad \text{and} \quad \frac{\partial}{\partial T} \int_0^\infty \mathrm{d}E' \int_{\boldsymbol{\Omega}} \mathrm{d}\omega'\, F_i^j(f_0, E', \omega', E, \omega, T)$$

$(i = 1, 2, \ldots, n;\ j = 1, \ldots, 4;\ k = 0, 1)$ are bounded a.e.

c) For any $T_0 \in (0, \infty)$ there exist finite positive constants δ_1 and C_1 such that the inequalities

$$\int_0^\infty \mathrm{d}E' \int_{\boldsymbol{\Omega}} \mathrm{d}\omega' |F_i^j(f_k, E', \omega', E, \omega, T) - F_i^j(f_k, E', \omega', E, \omega, T')| \leqq C_1 |T - T'|\,,$$

$$\int_0^\infty \mathrm{d}E' \int_{\boldsymbol{\Omega}} \mathrm{d}\omega' \left| \frac{\partial}{\partial T} F_i^j(f_0, E', \omega', E, \omega, T) - \frac{\partial}{\partial T'} F_i^j(f_0, E', \omega', E, \omega, T') \right| \leqq C_1 |T - T'|$$

$(i = 1, 2, \ldots, n;\ j = 1, 2;\ k = 0, 1)$,

$$|F_i^j(f_k, E', \omega', E, \omega, T) - F_i^j(f_k, E', \omega', E, \omega, T')| \leqq$$
$$\leqq C_1 |T - T'|\, F_i^j(f_k, E', \omega', E, \omega, T_0)$$

and

$$\left| \frac{\partial}{\partial T} F_i^j(f_k, E', \omega', E, \omega, T) - \frac{\partial}{\partial T'} F_i^j(f_k, E', \omega', E, \omega, T') \right| \leqq$$
$$\leqq C_1 |T - T'|\, F_i^j(f_k, E', \omega', E, \omega, T_0)$$

$(i = 1, 2, \ldots, n; j = 3, 4;\ k = 0, 1)$ hold for any $T, T' \in (T_0 - \delta_1, T + \delta_1)$.

It can be shown that Suppositions 1, 2 and 3 are satisfied for all models of scattering and fission cross-sections usually employed (see e.g. [4]).

Consider a number $\tau > 0$, a function $f(E)$: $(0, \infty) \to (0, \infty)$, and denote by $\boldsymbol{M}_\tau$ and $\boldsymbol{M}^\tau$ the sets $\mathbf{R}_3 \times (0, \infty) \times \boldsymbol{\Omega} \times [0, \tau]$ and $\mathbf{R}_3 \times [0, \tau]$, respectively.

Definition. *We will say that a function $\Phi(x, E, \omega, t)$: $\boldsymbol{M}_\tau \to \mathbf{R}_1$ belongs to the linear space $\mathbf{m}(f, \tau)$ if:*

i) *For almost all pairs* $\{E, \omega\}$, *the function* Φ *is continuous in the variables* $\boldsymbol{x}$ *and* t *while the functions* $(\partial/\partial x_l)\,\Phi$ $(l = 1, 2, 3)$ *and* $(\partial/\partial t)\,\Phi$ *are finite in these variables a.e.*

ii) *The number*

$$\operatorname*{vraimax}_{M_\tau} \left|\frac{\Phi}{f}\right|$$

is finite. Similarly, the function Ψ *is said to belong to the linear space* $\mathbf{m}(\tau)$ *if*:

j) Ψ *is continuous on* $\mathbf{M}^\tau$, *the function* $(\partial/\partial x_l)\,\Psi$ $(l = 1, 2, 3)$ *and* $(\partial/\partial t)\,\Psi$ *being finite a.e.*

jj) *The number*

$$\operatorname*{vraimax}_{M^\tau} |\Psi|$$

is finite. The norms in the spaces $\mathbf{m}(f, \tau)$ *and* $\mathbf{m}(\tau)$ *are defined as* $\|\Phi\|_{f,\tau} \equiv \equiv \operatorname*{vraimax}_{M_\tau} |\Phi/f|$ *and* $\|\Psi\|_\tau \equiv \operatorname*{vraimax}_{M^\tau} |\Psi|$, *respectively.*

In what follows, the symbols $\|\Phi\|_{f,\tau}$ and $\|\Psi\|_\tau$ will mean the values $\operatorname*{vraimax}_{M_\tau} |\Phi/f|$ and $\operatorname*{vraimax}_{M^\tau} |\Psi|$ but not necessarily $\Phi \in \mathbf{m}(f, \tau)$ and $\Psi \in \mathbf{m}(\tau)$, respectively.

For any $\varphi \in \mathbf{m}(f, \tau)$ and $N_i \in \mathbf{m}(\tau)$ $(i = 1, 2, \ldots, n)$, let us denote

$$\varphi^0(\boldsymbol{x}, E, \omega, t) \equiv \varphi(\boldsymbol{x}, E, \omega, 0)\,,$$

$$N_i^0(\boldsymbol{x}, t) \equiv N_i(\boldsymbol{x}, 0) \quad \text{and} \quad \|\boldsymbol{N}\|_\tau \equiv \sum_{i=1}^{n} \|N_i\|_\tau\,.$$

In the majority of practical cases, the following assumption can be expected to be satisfied (see e.g. [5]):

Supposition 4. The temperature is a positive quantity, $T = T(\boldsymbol{x}, t, \varphi, \boldsymbol{N})$. There exist positive numbers T^0, δ_0, δ, a_j $(j = 1, \ldots, 5)$ such that, if we denote

$$\mathbf{M} \equiv \mathscr{E}\{\boldsymbol{x}, t, \tilde{\varphi}, \tilde{\boldsymbol{N}};\ \boldsymbol{x} \in \mathbf{R}_3,\ t \in [0, \delta_0]\,,\quad \tilde{\varphi} \in \mathbf{m}(f_0, \delta_0)\,,\quad \tilde{N}_i \in \mathbf{m}(\delta_0)\,,$$
$$i = 1, 2, \ldots, n\,;\quad \|\tilde{\varphi} - \tilde{\varphi}^0\|_{f_0,\delta_0} + \|\tilde{\boldsymbol{N}} - \tilde{\boldsymbol{N}}^0\|_{\delta_0} \leqq \delta\}$$

then the following assertions hold:

a) $T(\boldsymbol{x}, 0, \varphi^0, \boldsymbol{N}^0) \geqq T^0$ and $T(\boldsymbol{x}, t, \varphi, \boldsymbol{N}) \leqq a_1$ on $\mathbf{M}$.

b) The function $T(\boldsymbol{x}, t, \varphi, \boldsymbol{N})$ is continuous in the variables $\boldsymbol{x}$ and t for any pair $\{\varphi, \boldsymbol{N}\}$, $\{\boldsymbol{x}, t, \varphi, \boldsymbol{N}\} \in \mathbf{M}$ and

$$|T(\boldsymbol{x}, t, \varphi', \boldsymbol{N}') - T(\boldsymbol{x}, t, \varphi'', \boldsymbol{N}'')| \leqq$$
$$\leqq a_2(\|\varphi' - \varphi''\|_{f_1,t} + \|\varphi'^0 - \varphi''^0\|_{f_1,t} + \|\boldsymbol{N}' - \boldsymbol{N}''\|_t + \|\boldsymbol{N}'^0 - \boldsymbol{N}''^0\|_t)$$

for any quadruplets $\{\boldsymbol{x}, t, \varphi', \boldsymbol{N}'\}$, $\{\boldsymbol{x}, t, \varphi'', \boldsymbol{N}''\} \in \mathbf{M}$.

c)

$$\left|\frac{\partial}{\partial x_l} T(x, t, \varphi, N)\right| \leqq a_3\left(\|\varphi\|_{f_1,t} + \frac{1}{\sqrt{t}}\|\varphi^0\|_{f_1,t} + \left\|\frac{\partial}{\partial x_l}\varphi\right\|_{f_1,t} + \left\|\frac{\partial}{\partial x_l}\varphi^0\right\|_{f_1,t} + \right.$$

$$\left. + \|N\|_t + \frac{1}{\sqrt{t}}\|N^0\|_t + \left\|\frac{\partial}{\partial x_l}N\right\|_t + \left\|\frac{\partial}{\partial x_l}N^0\right\|_t\right) + a_4\,, \quad (l = 1, 2, 3)$$

on the set $\boldsymbol{M}$.

d) For any quadruplets $\{x, t, \varphi, N\}$, $\{x, t, \varphi', N'\} \in \boldsymbol{M}$ we have

$$\left|\frac{\partial}{\partial x_l}(T(x, t, \varphi, N) - T(x, t, \varphi', N'))\right| \leqq a_5\left(\|\varphi - \varphi'\|_{f_1,t} + \left\|\frac{\partial}{\partial x_l}(\varphi - \varphi')\right\|_{f_1,t} + \right.$$

$$+ \frac{1}{\sqrt{t}}\|\varphi^0 - \varphi'^0\|_{f_1,t} + \left\|\frac{\partial}{\partial x_l}(\varphi^0 - \varphi'^0)\right\|_{f_1,t} + \|N - N'\|_t + \left\|\frac{\partial}{\partial x_l}(N - N')\right\|_t +$$

$$\left. + \frac{1}{\sqrt{t}}\|N^0 - N'^0\|_t + \left\|\frac{\partial}{\partial x_l}(N^0 - N'^0)\right\|_t\right), \quad (l = 1, 2, 3)\,.$$

SOLUTION TO THE CAUCHY PROBLEM

In what follows, we will assume that the basic suppositions 1–4 are satisfied. Keeping the above notation we will suppose that the constant $\delta > 0$ in the definition of the set $\boldsymbol{M}$ is chosen in such a way that Supposition 3c) is also satisfied.

Theorem 1. *Let the following assumptions be satisfied:*

a) *The functions φ_0 and N_{0i} $(i = 1, 2, \ldots, n)$ are finite, nonnegative, and the functions*

$$\frac{\partial}{\partial x_l}\left(\frac{\varphi_0}{f_1}\right) \quad and \quad \frac{\partial}{\partial x_l} N_{0i} \quad (l = 1, 2, 3)$$

are bounded. Furthermore,

$$\operatorname*{vraimax}_{\boldsymbol{R}_3 \times (0,\infty) \times \boldsymbol{\Omega}} \frac{\varphi_0}{f_0} \leqq \frac{\delta}{6}\,.$$

b) *There exists a number $t_1 > 0$ such that $\sqrt{(2E)}\, S_0(x, E, \omega, t) \in \mathbf{m}(f_0, t_1)$,* $S_0 \geqq 0$ and $S_i(x, t) \in \mathbf{m}(t_1)$ $(i = 1, 2, \ldots, n)$ *while the functions*

$$\frac{\partial}{\partial x_l}\left(\frac{\sqrt{(2E)}\, S_0}{f_1}\right) \quad and \quad \frac{\partial S_i}{\partial x_l} \quad (l = 1, 2, 3)$$

are bounded on the sets $\boldsymbol{M}_{t_1}$ and $\boldsymbol{M}^{t_1}$, respectively.

c) *In the expression for $\mathbf{B}_{ij}$, the constants a_{ij}, b_{ij}, c_{ij} and d_{ij} are nonnegative (nonpositive) in case $i \neq j$ $(i = j)$ $(i, j = 1, 2, \ldots, n)$.*

Then there exists a number $t_2 > 0$ such that the Cauchy problem (1) *and* (2) *has a unique solution φ and N_i $(i = 1, 2, \ldots, n)$ which belongs to the space $\mathbf{m}(f_0, t_2)$* and $\mathbf{m}(t_2)$, *respectively.*

Proof. Let us confine ourselves to the set $\boldsymbol{M}$ (see Supposition 4). It is seen that

$$\|\cdot\|_{f_1,\tau} \leqq \frac{1}{a} \|\cdot\|_{f_0,\tau}, \quad \tau \in (0, \infty).$$

Then, by virtue of Suppositions 1b), 3c) and 4b) we have

$$(3a) \qquad |\mathbf{A}_1(N, T, x, E, t)| \leqq \sum_{i=1}^{n} (N_{0i} + \delta) \sqrt{(2E)}\, \sigma_{ti}(E, T_0) (1 + C_1 |T - T_0|) \leqq$$
$$\leqq \sum_{i=1}^{n} \sqrt{(2E)} \left(a(T_0) \frac{1}{\sqrt{E}} + b(T_0) \right) (N_{0i} + \delta)\, M_1(\delta),$$

$$(3b) \qquad |\mathbf{A}_2(\varphi, N, T, x, E, \omega, t)| \leqq f_0(E)\, M_2(\delta)$$

and, similarly,

$$(4a) \qquad |\mathbf{A}_3(N, T, x, E, t)| \leqq f_0(E)\, M_3(\delta),$$

$$(4b) \qquad |\mathbf{B}_{ij}(\varphi, N, T, x, E, t)| \leqq M_4(\delta) \quad (i, j = 1, 2, \ldots, n)$$

where $M_l(\delta)$ $(l = 1, 2, 3, 4)$ are finite constants. From the relations (3b) and (4) it follows that there exists a number $\delta_0^1 > 0$ such that the inequalities

$$(5a) \qquad \int_0^t \mathrm{d}s (M_2(\delta) + M_3(\delta)) \leqq \frac{\delta}{6}$$

and

$$(5b) \qquad \int_0^t \mathrm{d}s\, M_4(\delta) \leqq \frac{\delta}{6n}$$

are fulfilled for any $t \in [0, \delta_0^1]$. Next, by assumption a) of Theorem 1 and by the estimates (3a) and (5b) there exists a number $\delta_0^2 > 0$ such that

$$(6a) \qquad |\varphi_0(x, E, \omega) - \varphi_0(x - \sqrt{(2E)}\, \omega t, E, \omega)\, .$$
$$. \exp\left\{ \int_0^t \mathrm{d}s\, \mathbf{A}_1(N, T, x - \sqrt{(2E)}\, \omega(t - s), E, s) \right\} \Bigg| \leqq \delta \frac{f_0(E)}{6},$$

$$(6b) \qquad |N_{0i}(x)| \left| 1 - \exp\left\{ \int_0^t \mathrm{d}s \frac{1}{N_i(x, s)} \mathbf{B}_{ii}(\varphi, N, T, x, s) \right\} \right| \leqq \frac{\delta}{6n} \quad (i = 1, 2, \ldots, n)$$

hold for any $t \in [0, \delta_0^2]$. Finally, by assumption b) of the theorem, there exists a number $\delta_0^3 > 0$ such that the inequalities

$$(6c) \qquad \int_0^t \mathrm{d}s \sqrt{(2E)}\, S_0(x - \sqrt{(2E)}\, \omega(t - s), E, \omega, s) \leqq f_0(E) \frac{\delta}{6}$$

and

$$\text{(6d)} \qquad \int_0^t \mathrm{d}s\, |S_i(x, s)| \leqq \frac{\delta}{6} \quad (i = 1, 2, \ldots, n)$$

hold for any $t \in [0, \delta_0^3]$. Now we see that the constant δ_0 which occurs in Supposition 4 can be chosen in such a way that, for any quadruplet $\{x, t, \varphi, N\} \in \mathbf{M}$, the relations (3)–(6) are fulfilled at the same time. In what follows we will assume this property.

Consider $t \in [0, t_1]$, where without loss of generality we put $t_1 = \delta_0$, and examine the following iterative process:

$$\varphi^{(0)}(x, E, \omega, t) = \varphi_0(x, E, \omega)\,,$$

$$N^0(x, t) = N_0(x) = (N_{01}(x), \ldots, N_{0n}(x))\,,$$

$$T^{(0)}(x, t, \varphi, N) = T(x, t, \varphi_0, N_0)$$

$$\text{(7a)} \quad \varphi^{(k)}(x, E, \omega, t) = \textstyle\int_0^t \mathrm{d}s\, \mathbf{P}^{(k-1)}(x, E, \omega, t, s)\, \{\sqrt{(2E)}\, S_0(y(s), E, \omega, s) + \mathbf{A}_2^{(k-1)}(y(s), E, \omega, s) + \mathbf{A}_3^{(k-1)}(y(s), E, s)\} + \varphi_0(y(0), E, \omega)\, \mathbf{P}^{(k-1)}(x, E, \omega, t, 0)\,,$$

$$\text{(7b)} \quad N_i^{(k)}(x, t) = \textstyle\int_0^t \mathrm{d}s\, \mathbf{Q}_i^{(k-1)}(x, t, s)\, \{\sum\limits_{j \neq i}^{n} \mathbf{B}_{ij}(\varphi^{(k-1)}, N^{(k-1)}, T^{(k-1)}, x, s) + S_i(x, s)\} + N_{0i}(x)\, \mathbf{Q}_i^{(k-1)}(x, t, 0)$$

$$(i = 1, 2, \ldots, n)$$

and

$$T^{(k)}(x, t, \varphi, N) = T(x, t, \varphi^{(k)}, N^{(k)}) \quad (k = 1, 2, \ldots)\,.$$

Here we have put

$$y(s) = x - \sqrt{(2E)}\, \omega(t - s)\,, \quad s \in [0, t)\,,$$

$$\mathbf{A}_i^{(k)}(y(s), E, s) = \mathbf{A}_i(N^{(k)}, T^{(k)}, y(s), E, s) \quad (i = 1, 3)\,,$$

$$\mathbf{A}_2^{(k)}(y(s), E, \omega, s) = \mathbf{A}_2(\varphi^{(k)}, N^{(k)}, T^{(k)}, y(s), E, \omega, s) \quad (k = 1, 2, \ldots)\,.$$

$$\mathbf{P}(x, E, \omega, t, s) = \exp\{\textstyle\int_s^t \mathrm{d}s_1\, \mathbf{A}_1(N, T, y(s_1), E, s_1)\}\,,$$

$$\mathbf{P}^{(k)}(x, E, \omega, t, s) = \exp\{\textstyle\int_s^t \mathrm{d}s_1\, \mathbf{A}_1^{(k)}(y(s_1), E, s_1)\}\,,$$

$$\mathbf{Q}_i(x, t, s) = \exp\{\textstyle\int_s^t \mathrm{d}s_1 (N_i(x, s_1))^{-1}\, \mathbf{B}_{ii}(\varphi, N, T, x, s_1)\} \qquad \text{and}$$

$$\mathbf{Q}_i^{(k)}(x, t, s) = \exp\{\textstyle\int_s^t \mathrm{d}s_1 (N_i^{(k)}(x, s_1))^{-1}\, \mathbf{B}_{ii}(\varphi^{(k)}, N^{(k)}, T^{(k)}, x, s_1)\}$$

for brevity. Obviously, for any $\{x, t\} \in \mathbf{M}^{\delta_0}$, the quadruplet $\{x, t, \varphi^{(0)}, N^{(0)}\}$ belongs to the set $\mathbf{M}$. Using the estimates (5a), (6a) and (6c) we find by (7a)

$$|\varphi^{(1)}(x, E, \omega, t) - \varphi^{(0)}(x, E, \omega, t)| \leqq \frac{\delta}{2} f_0(E)$$

while by (7b), using (5b), (6b) and (6d) we have

$$|N_i^{(1)}(x, t) - N_i^{(0)}(x\ t)| \leqq \frac{\delta}{2n} \quad (i = 1, 2, \dots, n).$$

Therefore

$$\|\varphi^{(1)} - \varphi^{(0)}\|_{f_0,\delta_0} + \|N^{(1)} - N^{(0)}\|_{\delta_0} \leqq \delta$$

so that again $\{x, t, \varphi^{(1)}, N^{(1)}\} \in \mathbf{M}$ for any pair $\{x, t\} \in \mathbf{M}^{\delta_0}$. Recurrently, using (7) together with the estimates (5) and (6) we obtain

$$\|\varphi^{(k)} - \varphi^{(0)}\|_{f_0,\delta_0} + \|N^{(k)} - N^{(0)}\|_{\delta_0} \leqq \delta \tag{8}$$

so that the quadruplet $\{x, t, \varphi^{(k)}, N^{(k)}\}$ belongs to $\mathbf{M}$ for any $\{x, t\} \in \mathbf{M}^{\delta_0}$ and any $k = 0, 1, \dots$.

Now, we will examine the convergence of the iteratives. Using (7) we find the following recurrent integral formulas:

$$\begin{aligned} \text{(9a)} \qquad & \varphi^{(k+1)}(x, E, \omega, t) - \varphi^{(k)}(x, E, \omega, t) = \\ &= \int_0^t ds\{[\mathbf{P}^{(k)}(x, E, \omega, t, s) - \mathbf{P}^{(k-1)}(x, E, \omega, t, s)]\,[\mathbf{A}_2^{(k)}(y(s), E, \omega, s) + \\ &\quad + \mathbf{A}_3^{(k)}(y(s), E, s) + \sqrt{(2E)}\, S_0(y(s), E, \omega, s)] + \\ &\quad + \mathbf{P}^{(k-1)}(x, E, \omega, t, s)\,[\mathbf{A}_2^{(k)}(y(s), E, \omega, s) + \\ &\quad + \mathbf{A}_3^{(k)}(y(s), E, s) - \mathbf{A}_2^{(k-1)}(y(s), E, \omega, s) - \mathbf{A}_3^{(k-1)}(y(s), E, s)]\} + \\ &\quad + \varphi_0(y(0), E, \omega)\,[\mathbf{P}^{(k)}(x, E, \omega, t, 0) - \mathbf{P}^{(k-1)}(x, E, \omega, t, 0)]\,, \end{aligned}$$

$$\begin{aligned} \text{(9b)} \qquad & N_i^{(k+1)}(x, t) - N_i^{(k)}(x, t) = \int_0^t ds\{[\mathbf{Q}_i^{(k)}(x, t, s) - \mathbf{Q}_i^{(k-1)}(x, t, s)]\,. \\ &\quad .\,[\sum_{j \neq i}^{n} \mathbf{B}_{ij}(\varphi^{(k)}, N^{(k)}, T^{(k)}, x, s) + S_i(x, s)] + \\ &+ \mathbf{Q}_i^{(k-1)}(x, t, s) \sum_{j \neq i}^{n} [\mathbf{B}_{ij}(\varphi^{(k)}, N^{(k)}, T^{(k)}, x, s) - \mathbf{B}_{ij}(\varphi^{(k-1)} N^{(k-1)}, T^{(k-1)}, x, s)]\} + \\ &+ N_{0i}(x)\,[\mathbf{Q}_i^{(k)}(x, t, 0) - \mathbf{Q}_i^{(k-1)}(x, t, 0)] \quad (i = 1, 2, \dots, n;\ k = 1, 2, \dots). \end{aligned}$$

Suppositions 3c), 4b) and relations (8) imply

$$\begin{aligned} \text{(10)} \qquad & |\mathbf{A}_2^{(k)}(y(t), E, \omega, t) - \mathbf{A}_2^{(k-1)}(y(t), E, \omega, t)| \leqq \\ &\leqq C_1 f_1(E) \{\|\varphi^{(k)} - \varphi^{(k-1)}\|_{f_1,t} + \|N^{(k)} - N^{(k-1)}\|_t\} + \\ &+ C_2 f_0 |T^{(k)} - T^{(k-1)}| \leqq C_3 f_1(E) \{\|\varphi^{(k)} - \varphi^{(k-1)}\|_{f_1,t} + \|N^{(k)} - N^{(k-1)}\|_t\} \end{aligned}$$

for any $k = 1, 2, \dots$ and any $t \in [0, \delta_0]$. Here C_1, C_2 and C_3 are positive finite constants. Similarly, we find

$$\begin{aligned} \text{(11)} \qquad & |\mathbf{A}_3^{(k)}(y(t), E, t) - \mathbf{A}_3^{(k-1)}(y(t), E, t)| \leqq \\ &\leqq C_4 f_1(E) \{\|\varphi^{(k)} - \varphi^{(k-1)}\|_{f_1,t} + \|N^{(k)} - N^{(k-1)}\|_t\}\,, \end{aligned}$$

$$|\mathbf{A}_1^{(k)}(y(t), E, t) - \mathbf{A}_1^{(k-1)}(y(t), E, t)| \leqq$$
$$\leqq C_5 a(1 + \sqrt{E}) \{\|\varphi^{(k)} - \varphi^{(k-1)}\|_{f_1,t} + \|N^{(k)} - N^{(k-1)}\|_t\},$$
$$|\mathbf{B}_{ij}(\varphi^{(k)}, N^{(k)}, T^{(k)}, x, t) - \mathbf{B}_{ij}(\varphi^{(k-1)}, N^{(k-1)}, T^{(k-1)}, x, t)| \leqq$$
$$\leqq C_6\{\|\varphi^{(k)} - \varphi^{(k-1)}\|_{f_1,t} + \|N^{(k)} - N^{(k-1)}\|_t\}$$

$(i, j = 1, 2, \ldots, n)$ where C_4, C_5 and C_6 are finite positive constants and $k \approx 1, 2, \ldots$ Define functions

$$F^{(k+1)}(t) = \|\varphi^{(k+1)} - \varphi^{(k)}\|_{f_1,t} \quad \text{and} \quad G^{(k+1)}(t) = \|N^{(k+1)} - N^{(k)}\|_t,$$
$$t \in [0, \delta_0], \quad k = 0, 1, \ldots.$$

Using the stimates (10), (11) and the relation

$$|e^{-x} - e^{-y}| \leqq |x - y|, \quad x, y \geqq 0$$

we get from (9) the recurrent inequalities

$$F^{(k+1)}(t) \leqq A \int_0^t \mathrm{d}s(F^{(k)}(s) + G^{(k)}(s)),$$
$$G^{(k+1)}(t) \leqq A \int_0^t \mathrm{d}s(F^{(k)}(s) + G^{(k)}(s)) \tag{12}$$

where $A < \infty$ is a constant. Clearly,

$$F^{(1)}(t) + G^{(1)}(t) \leqq C$$

where $C < \infty$ is a constant and, therefore,

$$F^{(k+1)}(t) + G^{(k+1)}(t) \leqq C \frac{(2At)^k}{k!}, \quad k = 0, 1, 2, \ldots. \tag{13}$$

Taking into account this inequality we see that, for any $E \in (0, \infty)$ the iteration process (7) is uniformly convergent in the variables x, ω, t on the set $\mathbf{R}_3 \times \boldsymbol{\Omega} \times [0, \delta_0]$ a.e. By inequalities (8), the numbers $\|\varphi^{(k)}\|_{f_0,\delta_0}$ and $\|N_i^{(k)}\|_{\delta_0}$ $(i = 1, 2, \ldots, n)$ are uniformly bounded with respect to $k = 1, 2, \ldots$. Next, by virtue of the basic assumptions, for any k, the functions $N_i^{(k)}$ and $\varphi^{(k)}$ $(i = 1, \ldots, n)$ are continuous in the variables x and t (for almost all pairs $\{E, \omega\}$ in the case of the function φ). Therefore, the limit functions

$$N_i(x, t) = \lim_{k \to \infty} N_i^{(k)}(x, t) \quad \text{and} \quad \varphi(x, E, \omega, t) = \lim_{k \to \infty} \varphi^{(k)}(x, E, \omega, t)$$

have these properties, too. Obviously, they satisfy the Eqs.

$$\varphi(x, E, \omega, t) = \int_0^t \mathrm{d}s \exp\{\int_s^t \mathrm{d}s_1\, \mathbf{A}_1(N, T, y(s_1), E, s_1)\}\,. \tag{14a}$$
$$.\,[\mathbf{A}_2(\varphi, N, T, y(s), E, \omega, s) + \mathbf{A}_3(N, T, y(s), E, s) +$$
$$+ \sqrt{(2E)}\, S_0(y(s), E, \omega, s)] + \varphi_0(y(0), E, \omega) \exp\{\int_0^t \mathrm{d}s_1\, \mathbf{A}_1(N, T, y(s_1), E, s_1)\}$$

where $y(s) = x - \sqrt{(2E)}\, \omega(t - s)$, $T = T(y(s), s, \varphi, N)$, and

(14b) $$N_i(x, t) = \int_0^t ds\{\sum_{j \neq i}^{n} \mathbf{B}_{ij}(\varphi, N, T, x, s) + S_i(x, t)\} \mathbf{Q}_i(x, t, s) + \\ + N_{0i}(x) \mathbf{Q}_i(x, t, 0) \quad (i = 1, 2, \ldots, n)$$

where $T = T(x, s, \varphi, N)$.

Now, let us examine the functions $\partial\varphi^{(k)}/\partial x_l$ and $\partial N_i^{(k)}/\partial x_l$ ($i = 1, \ldots, n$; $l = 1, 2, 3$; $k = 0, 1, \ldots$). The necessary recurrent relations can be obtained by differentiating Eqs. (7) with respect to the spatial variables. Clearly, the right hand sides of these formulas will be linear with respect to the functions $\partial\varphi^{(k-1)}/\partial x_l$, $\partial N_i^{(k-1)}/\partial x_l$ and to the first spatial derivatives of the effective cross-sections, external sources and of initial values. Set

$$\tilde{f}^{(k)}(t) = \max_l \left\| \frac{\partial\varphi^{(k)}}{\partial x_l} \right\|_{f_1, t} \quad \text{and} \quad \tilde{g}^{(k)}(t) = \max_l \left\| \frac{\partial N^{(k)}}{\partial x_l} \right\|_t, \quad t \in [0, \delta_0]$$

and recall that the effective cross-sections depend on the spatial variables only via the temperature dependence. Considering the estimate (8), Suppositions 3b), 4c) and assumptions a) and b) of the theorem we find

(15) $$\tilde{f}^{(k)}(t) \leqq A \int_0^t ds(\tilde{f}^{(k-1)}(s) + \tilde{g}^{(k-1)}(s)) + B,$$
$$\tilde{g}^{(k)}(t) \leqq A \int_0^t ds(\tilde{f}^{(k-1)}(s) + \tilde{g}^{(k-1)}(s)) + B \quad (k = 1, 2, \ldots)$$

where A and B are finite positive constants. Therefore

(16) $$\tilde{f}^{(k)}(t) + \tilde{g}^{(k)}(t) \leqq 2Be^{2At}, \quad t \in [0, \delta_0]$$

for all $k = 0, 1, \ldots$. Finally, let us set

$$\tilde{F}^{(k+1)}(t) = \max_l \left\| \frac{\partial\varphi^{(k+1)}}{\partial x_l} - \frac{\partial\varphi^{(k)}}{\partial x_l} \right\|_{f_1, t} \quad \text{and} \quad \tilde{G}^{(k+1)}(t) = \max_l \left\| \frac{\partial N^{(k+1)}}{\partial x_l} - \frac{\partial N^{(k)}}{\partial x_l} \right\|_t$$
$$(k = 0, 1, \ldots).$$

Appropriate recurrent formulas for these functions will be obtained by differentiating (9) with respect to the spatial variables. If the estimates (8), (13) and (16) are applied together with Suppositions 3 and 4, we get

$$\tilde{F}^{(k+1)}(t) \leqq A \int_0^t ds(\tilde{F}^{(k)}(s) + \tilde{G}^{(k)}(s)) + \frac{(At)^k}{k!},$$
$$\tilde{G}^{(k+1)}(t) \leqq A \int_0^t ds(\tilde{F}^{(k)}(s) + \tilde{G}^{(k)}(s)) + \frac{(At)^k}{k!} \quad (k = 1, 2, \ldots)$$

where $A \in (0, \infty)$ is a constant. Therefore,

(17) $$\tilde{F}^{(k+1)}(t) + \tilde{G}^{(k+1)}(t) \leqq \frac{(2At)^k}{(k-1)!}, \quad t \in [0, \delta_0].$$

So, we have come to the following conclusion: Sequences

$$\left\{\frac{\partial \varphi^{(k)}}{\partial x_l}\right\}_{k=0}^{\infty} \quad \text{and} \quad \left\{\frac{\partial N_i^{(k)}}{\partial x_l}\right\}_{k=0}^{\infty} \quad (l = 1, 2, 3;\ i = 1, \ldots, n)$$

are uniformly convergent in the corresponding variables a.e. and their limit values are the functions $\partial\varphi/\partial x_l$ and $\partial N_i/\partial x_l$, respectively. Further, $\varphi \in \boldsymbol{m}(f_0, \delta_0)$ and $N_i \in \boldsymbol{m}(\delta_0)$ $(i = 1, 2, \ldots, n)$.

Now, applying the operator $\partial/\partial t$ to Eqs. (14), we obtain Eqs. (1) and (2). Conversely, let φ and N_i $(i = 1, 2, \ldots, n)$ be solutions to problem (1) and (2), $\varphi \in \boldsymbol{m}(f_0, \delta_0)$, $N_i \in \boldsymbol{m}(\delta_0)$. Let us substitute $t \to s$, $\boldsymbol{x} \to \boldsymbol{x} - \sqrt{(2E)}\,\boldsymbol{\omega}(t - s)$ in Eq. (1a) and multiply this equation by the factor $\exp\{\int_s^t \mathrm{d}s_1\, A(N, T, \boldsymbol{x} - \sqrt{(2E)}\,\boldsymbol{\omega}(t - s_1), E, s_1)\}$.

Finally, integrate the whole expression with respect to the variable s over the interval $[0, t]$, $t \in [0, \delta_0]$. In this manner we obtain Eq. (14a). In Eq. (2a) the variable $\boldsymbol{x}$ has the meaning of a parameter. Integrating (2a) over the interval $[0, t]$, $t \in [0, \delta_0]$ with respect to the time variable, we obtain Eq. (14b). By virtue of the basic assumptions, all operations just mentioned are justified.

It remains to prove uniqueness of the solution. Suppose there exists another solution $\varphi_1 \in \boldsymbol{m}(f_0, t_2)$ and $N_i^1 \in \boldsymbol{m}(t_2)$ to problem (1) and (2). Denote

$$\varphi_2 = \varphi - \varphi_1\,,$$
$$N_i^2 = N_i - N_i^1 \quad (i = 1, 2, \ldots, n)\,,$$
$$F(t) = \|\varphi_2\|_{f_1,t} \quad \text{and} \quad G(t) = \|\boldsymbol{N}^2\|_t\,, \quad t \in [0, t_2]\,.$$

Using (14) we obtain equations of the form (9) for the functions φ_2 and N_i^2. Then, applying Suppositions 3 and 4, we find

$$F(t) + G(t) \leqq A \int_0^t \mathrm{d}t_1(F(t_1) + G(t_1))\,, \quad t \in [0, t_2] \tag{18}$$

where $A < \infty$ is a constant (the derivation of this inequality is analogous to the derivation of (12) from Eqs. (9)). Obviously, we have

$$F(t) + G(t) \leqq C\,, \quad t \in [0, t_2]$$

where $C \in (0, \infty)$ is a constant (the first estimate). Using (18) recurrently, we get

$$F(t) + G(t) \leqq C\,\frac{(At)^{k-1}}{(k-1)!}$$

for the k-th estimate. Therefore,

$$F(t) + G(t) = 0$$

on the set $[0, t_2]$ and the theorem is proved.

Theorem 1 ensures existence and uniqueness of the solution to the Cauchy problem (1) and (2) only for a certain finite time interval. Here we will show that, under very general assumptions, the solution can be extended to the whole interval $[0, \infty)$.

Supposition 5. a) There exists a constant $B \in (0, \infty)$ such that

$$0 \leqq N_i(x, t) \leqq B \quad (i = 1, 2, \dots, n)$$

everywhere on the set $\boldsymbol{M}^\infty$.

b) There exist constant B_1 and $B_2 \in (0, \infty)$ such that $T \in [B_1, B_2]$ for any pair $\{x, t\} \in \boldsymbol{M}^\infty$ and Supposition 3c) holds for any $T, T' \in [B_1, B_2]$.

c) $\sqrt{(2E)}\, S_0 \in \boldsymbol{m}(f_0, \infty)$ and $S_i \in \boldsymbol{m}(\infty)$ while the functions

$$\frac{\sqrt{(2E)}}{f_0(E)} \frac{\partial S_0}{\partial x_l} \quad \text{and} \quad \frac{\partial S_i}{\partial x_l} \quad (i = 1, 2, \dots, n;\ l = 1, 2, 3)$$

are bounded on the sets $\boldsymbol{M}_\infty$ and $\boldsymbol{M}^\infty$, respectively. Furthermore, Supposition 4 holds for arbitrary option of the time scale origin and arbitrary $\delta > 0$.

As for Supposition 5a), it is fulfilled automatically in the case of fission elements [3]. In all practical cases, kinetic equations (2) have the property that, independently of the particular form of the neutron flux φ, they do not permit infinite increase of material density in finite time. The validity of assumption 5a) can be ensured also by a suitable choice of sources S_i $(i = 1, 2, \dots, n)$ which in practice corresponds to the exchange of burnup fuel or the removal of superfluous material. By manipulating the control rods and the coolant system properly we can ensure the validity of assumption 5b).

Now, consider the Cauchy problem (1) and (2) and assume that Suppositions 1–5 are satisfied. Take $\tau \in (0, t_2)$ (see Theorem 1) and put

$$\tilde{\varphi}_0(x, E, \omega) = \varphi(x, E, \omega, \tau),$$
$$\tilde{N}_{0i}(x) = N_i(x, \tau) \quad (i = 1, 2, \dots, n)$$

where φ and N_i are solutions to the problem in the spaces $\boldsymbol{m}(f_0, t_2)$ and $\boldsymbol{m}(t_2)$, respectively. Clearly,

$$|\tilde{\varphi}_0| \leqq \tfrac{7}{6}\delta f_0(E).$$

But, according to Supposition 5, the value of the constant δ in Supposition 4 can be replaced by the value $\tilde{\delta} = 7\delta$. Then the functions $\tilde{\varphi}_0$ and $\tilde{N}_{0i}$ $(i = 1, 2, \dots, n)$ as new initial values will satisfy the requirements of Theorem 1 in which the value of the constant δ is replaced by the value 7δ. Therefore, there exists an interval $[\tau, t_3]$ where a solution to the Cauchy problem can be found. In principle, repeating this procedure for the interval $[\tau, t_3]$, etc., we can extend the solution to some maximum interval $[0, t_m)$. Suppose $t_m < \infty$. By virtue of Supposition 5 we get from (14a) the inequality

$$\|\varphi\|_{f_0,t} \leqq D\{\textstyle\int_0^t \mathrm{d}s(1 + \|\varphi\|_{f_0,s}) + 1\}, \quad t \in [0, t_m),$$

hence

$$\|\varphi\|_{f_0,t_m} \leqq (D + 1)\, e^{Dt_m} - 1$$

where $D < \infty$ is a constant. Next, the constant A in the estimate (16) depends on the values $\|\varphi\|_{f_0,t_m}$ and $\|N_i\|_{t_m}$ $(i = 1, 2, \ldots, n)$, and if these values are finite it is finite, too. So, we have

$$\left\|\frac{\partial\varphi}{\partial x_l}\right\|_{f_1,t_m} < \infty \quad \text{and} \quad \left\|\frac{\partial N_i}{\partial x_l}\right\|_{t_m} < \infty \quad (i = 1, \ldots, n;\ l = 1, 2, 3)\,.$$

Therefore, the functions $\varphi(\boldsymbol{x}, E, \boldsymbol{\omega}, t_m)$ and $N_i(\boldsymbol{x}, t_m)$ again satisfy the requirements of Theorem 1 which are set on the initial values. Then the solution to the Cauchy problem can be extended to an interval $[0, t^*)$, $t^* > t_m$. This is a contradiction and, therefore, $t_m = \infty$.

It can be easily shown that, under Suppositions 1–5, Theorem 1 remains true if the condition

$$\underset{\boldsymbol{R}_3\times(0,\infty)\times\boldsymbol{\Omega}}{\operatorname{vraimax}} \left(\frac{\varphi_0}{f_0}\right) < \frac{\delta}{6}$$

of the theorem is replaced by the condition

$$\underset{\boldsymbol{R}_3\times(0,\infty)\times\boldsymbol{\Omega}}{\operatorname{vraimax}} \left(\frac{\varphi_0}{f_0}\right) < \infty\,.$$

Remarks. 1. In practice, we must be very cautious when applying the theory mentioned above to the description of a working nuclear reactor. The spatial structure of such an equipment is strongly heterogeneous and, clearly, the assumptions of Theorem 1 are not satisfied. Then there is no other possibility than to model this heterogeneous structure by a homogeneous one. However, sometimes it is necessary to stick to the heterogeneous model. Then we see that the jump changes of material properties, the movement of control rods and the insertion or removal of an external neutron source imply that source terms which have the form of the Dirac δ-function will appear in the integro-differential description (1) and (2). In such a case, it is better to start from the integral form (14). We know that if Theorem 1 holds then the both formulations of the Cauchy problem are equivalent but it is not so in general.

Let τ be a real positive number. Denote by $\boldsymbol{m}'(f_0, \tau)$ the linear space of measurable functions $\Phi(\boldsymbol{x}, E, \boldsymbol{\omega}, t)$: $\boldsymbol{M}_\tau \to \boldsymbol{R}_1$ which have a finite norm $\|\Phi\|_{\boldsymbol{m}'(f_0,\tau)} \equiv \|\Phi\|_{f_0,\tau}$ and by $\boldsymbol{m}'(\tau)$ the linear space of measurable functions $N(\boldsymbol{x}, t)$; $\boldsymbol{M}^\tau \to \boldsymbol{R}_1$ which have a finite norm $\|N\|_{\boldsymbol{m}'(\tau)} \equiv \|N\|_\tau$. It is seen that the spacees $\boldsymbol{m}'(f_0, \tau)$ and $\boldsymbol{m}'(\tau)$ have sufficiently general properties which comply with the practical requirements mentioned above.

Theorem 2. *Let the following conditions be fulfilled*:

a) *The functions φ_0 and N_{0i} $(i = 1, \ldots, n)$ are nonnegative, bounded and*

$$\underset{\boldsymbol{R}_3\times(0,\infty)\times\boldsymbol{\Omega}}{\operatorname{vraimax}} \left(\frac{\varphi_0}{f_0}\right) \leqq \frac{\delta}{6}$$

δ being the constant which occurs in Supposition 4.

b) *There exists a number* $t_1 > 0$ *such that* $\sqrt{(2E)}\, S_0 \in \mathbf{m}'(f_0, t_1)$, $S_0 \geqq 0$ ***and*** $S_i \in \mathbf{m}'(t_1)$ $(i = 1, \ldots, n)$.

c) *In the expression for* $\mathbf{B}_{ij}$, *the constants* a_{ij}, b_{ij}, c_{ij} *and* d_{ij} *are nonnegative (nonpositive) in case* $i \neq j$ $(i = j)$ $(i, j = 1, \ldots, n)$. *Then there exists a number* $t_2 \in (0, t_1]$ *such that Eqs.* (14) *have unique solutions* φ *and* N_i $(i = 1, \ldots, n)$ *which belong to the spaces* $\mathbf{m}'(f_0, t_2)$ *and* $\mathbf{m}'(t_2)$, *respectively.*

This theorem can be proved by repeating the first part of the proof of Theorem 1. The solutions φ and N_i $(i = 1, \ldots, n)$ of Eqs. (14) are called mild solutions to the Cauchy problem for the kinetic equations.

2. Let us replace Eqs. (14b) by the equations

$$N_i(\boldsymbol{x}, t) = \int_0^t \mathrm{d}s \{ \sum_{j \neq i}^{n} \mathbf{B}_{ij}(\varphi, N, T, \boldsymbol{y}_1, s) + S_i(\boldsymbol{y}_1, s) \} \, \mathbf{Q}_i(\boldsymbol{y}_1, t, s) + N_{0i}(\boldsymbol{y}) \, \mathbf{Q}_i(\boldsymbol{y}, t, 0) \quad (i = 1, \ldots, n) \tag{19}$$

where

$$\boldsymbol{y}_1 = \{x_1, x_2, x_3 - \int_s^t \mathrm{d}t_2 \, v(x_1, x_2, t_2)\} \quad \text{and} \quad \boldsymbol{y} = \{x_1, x_2, x_3 - \int_0^t \mathrm{d}t_2 \, v(x_1, x_2, t_2)\}$$

and $v: \mathbf{R}_2 \times [0, \infty) \to \mathbf{R}_1$ is a bounded function. Of course, Eqs. (14a) and (19) describe the situation better if the movement of the control rods or of the fuel ones is taken into account. We can construct an iteration process which is based on these equations (see (7)). Obviously, the estimating relations (8) will hold again so that boundedness of iteratives will be ensured in some time interval $[0, t_3]$, $t_3 > 0$. Similarly, the estimate (13) for the differences of iteratives can be obtained. Therefore, Theorem 2 holds also for the problem (14a) and (19).

Acknowledgement. The author is grateful to Dr. S. Míka, CSc. for stimulating discussions.

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Souhrn

O CAUCHYOVĚ ÚLOZE PRO ROVNICE KINETIKY REAKTORU

JAN KYNCL

V článku je řešena úloha s počáteční podmínkou pro rovnice kinetiky reaktoru, přičemž se uvažuje efekt tepelné zpětné vazby. Obor řešení je vybrán tak, aby pokud možno nejlépe odpovídal vlastnostem modelů účinných průřezů. Metodou iterací je nalezeno lokální řešení, dokázána jeho jednoznačnost a ukázáno, že ve většině případů je zaručena také existence globálního řešení úlohy. Nakonec se diskutuje problém zobecněného řešení.

Резюме

О ЗАДАЧЕ КОШИ ДЛЯ УРАВНЕНИЙ КИНЕТИКИ РЕАКТОРА

JAN KYNCL

Работа касается аналитического решения задачи Коши для уравнений кинетики реактора с учетом тепловой обратной связи. Пространство функций для решения проблемы выбрано таким способом, чтобы по возможности отвечало свойствам используемых моделей эффективных сечений. Методом итераций найдено решение задачи на ограниченном промежутке времени, доказана его однозначность и показано, что в большинстве случаев существует решение в целом. Изучается также обобщенное решение задачи.

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