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An approach to the solution of a conflict situation with n participants

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AN APPROACH TO THE SOLUTION OF A CONFLICT SITUATION WITH n PARTICIPANTS

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1. INTRODUCTION

Various papers have been devoted to the problem of finding suitable solution of a conflict situation with n participants, where $n \geqq 3$ (see e.g. [2], [3]). The authtors usually try to find a deterministic preference group order, which is reasonable or acceptable from a certain point of view (e.g. it satisfies some of the five Arrow's axioms described in [2], [3]). Besides the deterministic preference order, the probabilistic preference order theory has been described as well (see e.g. [2]). In this article an attempt is made to find in a sense reasonable probabilistic preference group order. The criterion of reasonability or acceptability of the group ordering selected is the value of a real function f (the so called function of discontent) defined on a set of feasible group decision rules, each of which determines a probabilistic preference ordering on a given set of alternatives. The value $f(P)$, where P is a decision rule, characterizes numerically the discontent of the group of n participants in the conflict situation with the decision rule P. The decision rules which minimize the value of the function f on the set of feasible group decision rules are supposed to be reasonable for the whole group and can be recommended to a leader of the group as a "reasonable" dictate". The situation considered is that when a member of the given group of n persons must be appointed leader. The members of the group who tend to minimize the value of the discontent function f, if selected, are called reasonable dictators (the set $\mathscr{D}_r(f)$ in the text of the article). It is supposed that $\mathscr{D}_r(f) \neq \emptyset$. The problem of choosing the "most suitable reasonable dictator" (in a certain sense described further) is also considered. A small numerical example is solved. The approach suggested can be interpreted, using Arrow's terminology, as a kind of "reasonable dictatorship principle".

The corresponding theoretical basis from [1] is given for completeness in Appendix.

2. AN APPROACH TO THE SOLUTION OF A CONFLICT SITUATION — MATHEMATICAL DESCRIPTION

Suppose M is a set of alternatives which is common to a given set $N = \{1, 2, \ldots, n\}$ of n persons — participants of a conflict situation (the concept of the conflict situation will be defined later). Let X be the set of all binary relations which are weak orders on M, and let $\mathscr{P}(X)$ be the set of all simple probability measures defined on X (see Appendix, Definition 4A). Suppose $\mathscr{T}$ is a topology on $\mathscr{P}(X)$ such that $(\mathscr{P}(X), \mathscr{T})$ is a Hausdorff topological space. Let $\prec_i$ $(i = 1, 2, \ldots, n)$ be a binary relation defined on the space $(\mathscr{P}(X), \mathscr{T})$, which satisfies the following four conditions:

$$\prec_i \text{ is a weak order on } \mathscr{P}(X); \tag{1}$$

$$(P \prec_i Q, 0 < \alpha < 1) \Rightarrow \alpha P + (1 - \alpha) R \prec_i \alpha Q + (1 - \alpha) R \tag{2}$$

for every $P, Q, R \in \mathscr{P}(X)$;

$$(P \prec_i Q, Q \prec_i R) \Rightarrow \alpha P + (1 - \alpha) R \prec_i Q , \quad Q \prec_i \alpha P + (1 - \alpha) R \tag{3}$$

for every $P, Q, R \in \mathscr{P}(X)$ and for some $\alpha, \beta \in (0, 1)$;

$$\{P \mid P \in \mathscr{P}(X),\ P \prec_i Q\} \in \mathscr{T} , \quad \{P \mid P \in \mathscr{P}(X),\ Q \prec_i P\} \in \mathscr{T} , \tag{4}$$

for every $Q \in \mathscr{P}(X)$.

Theorems 3A, 4A and Remark 1A from Appendix imply that for each $i \in N$ under the conditions (1)–(4) there exists a real. valued continuous function u_i defined on $\mathscr{P}(X)$ such that

$$P \prec_i Q \quad \text{if and only if} \quad u_i(P) < u_i(Q) \tag{5}$$

for every $P, Q \in \mathscr{P}(X)$.

Suppose now that $\mathscr{P}_1(X)$ is a compact subset in the space $(\mathscr{P}(X), \mathscr{T})$. We define for all $i \in N$ the sets $\mathscr{B}_i$, $\mathscr{B}_i \subseteq \mathscr{P}_1(X)$ as follows:

$$\mathscr{B}_i = \{Q \mid u_i(Q) = \max_{P \in \mathscr{P}_1(X)} u_i(P)\} .$$

Because of continuity of u_i and compactness of $\mathscr{P}_1(X)$ it is $\mathscr{B}_i \neq \emptyset$ for for all $i \in N$. Suppose that $P_i \in \mathscr{B}_i$ is a fixed element, chosen by the i-th person.

Let us suppose that the real-valued function f is defined on $\mathscr{P}(X)$ as follows:

$$f(P) = \max_{i \in N} \alpha_i |u_i(P) - u_i(P_i)| , \tag{6}$$

where α_i are non-negative real numbers. The function f will be called „function of discontent.".

Suppose that $\mathscr{P}_2(X)$ is a compact subset in the space $(\mathscr{P}(X), \mathscr{T})$. Since the function f is continuous on $\mathscr{P}_2(X)$, the set

$$R(\mathscr{P}_2(X), f) = \{Q \mid f(Q) = \min_{P \in \mathscr{P}_2(X)} f(P)\}$$

is a non-empty compact subset of $\mathscr{P}_2(X)$.

We shall assume further that for each $i \in N$ a mapping F_i from $\mathscr{B}_i$ into $\mathscr{P}_2(X)$ is given. We shall call this mapping "election mapping".[1]) Let $\mathscr{D}_r(f)$ be a set defined as follows:

(7)
$$\mathscr{D}_r(f) = \{i \mid i \in N,\ F_i(\mathscr{B}_i) = \mathscr{R}(\mathscr{P}_2(X), f)\} .$$

We shall assume further that $\mathscr{R}_r(f) \neq \emptyset$.
Let φ be the following real-valued function defined on the set N:

(8)
$$\varphi(i) = \min_{P \in \mathscr{R}(\mathscr{P}_2(X), f)} \alpha_i |u_i(P) - u_i(P_i)|$$

for every $i \in N$.
Suppose

$$\mathscr{D}_r(\mathrm{f}, \varphi) = \{\mathrm{j} \mid \varphi(\mathrm{j}) = \min_{i \in N \cap \mathscr{D}_r(f)} \varphi(\mathrm{i})\} .$$

Now we are ready to introduce the following definitions:

Definition 1. *The* $(3n + 6)$*-tuple* $\mathscr{S} = \{\mathrm{M}, \mathscr{P} \prec_1, \ldots, \prec_n, u_1, \ldots, u_n, F_1, \ldots, F_n, \mathscr{P}_1(X), \mathscr{P}_2(X), f, \varphi\}$ *is called the conflict situation with n participants (persons).*

Definition 2. *Suppose that* $\mathscr{S}$ *is a given conflict situation with n participants. The elements of* $\mathscr{P}(X)$ *are called decision rules, the set* $\mathscr{P}_1(X)$ *is called the set of feasible decision rules for an individual (single person), the set* $\mathscr{P}_2(X)$ *is called the set of feasible decision rules for the group N.*

Definition 3. *Let* $\mathscr{S}$ *be a given conflict situation with n participants. The elements of the set* $\mathscr{R}(\mathscr{P}_2(X), f)$ *are called reasonable decision rules. An arbitrary element of the set* $\mathscr{D}_r(f)$ *is called a reasonable dictator, and an arbitrary element of the set* $D_r(f, \varphi)$ *is called a suitable reasonable dictator.*

Definition 4. *Suppose that* $\mathscr{S}$ *is a given conflict situation with n participants. The Cartesian product* $\mathscr{R}(\mathscr{P}_2(X), f) \times \mathscr{D}_r(f, \varphi)$ *is called the solution set of the conflict situation* $\mathscr{S}$*; the elements of the solution set are called solutions of the conflict situation* $\mathscr{S}$*.*

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[1]) The election mapping F_i can be interpreted as a mapping which reflects the change in the behaviour of the i-th person, if this person is appointed leader of the given group of n persons. A more detailed discussion is given in paragraph 3. We could write also $F_i(\mathscr{B}_i) \subset \mathscr{R}(\mathscr{P}_2(X), f)$ in (7). A slight change in the further text would then be needed.

Remark 1. The solution of the conflict situation $\mathscr{S}$ is thus a pair $(P; j)$, where $P \in \mathscr{R}(\mathscr{P}_2(X), f)$, $j \in \mathscr{D}_r(f, \varphi)$. Therefore, the solution $(P; j)$ shows us a reasonable decision rule for the whole group of n participants ("a resonable dictate") and a person who can choose this reasonable decision rule, if appointed leader of the group.

Theorem 1. *Let $\mathscr{S}$ be an arbitrary conflict situation with n participants in the sense of Definition* 1. *Suppose $\mathscr{D}_r(f) \neq \emptyset$. Then there exists a solution of thesconflict situation $\mathscr{S}$ in the sense of Definition* 4.

Proof. The conflict situation $\mathscr{P}$ is defined in such a way that conditions (1), (2), (3), (4) are satisfied and $\mathscr{P}_1(X)$, $\mathscr{P}_2(X)$ are compact sets in the space $(\mathscr{P}(X), \mathscr{T})$. Thus there exist continuous real-valued functions u_i satisfying (5) (this holds according to Theorem 3A, Remark 1A and Theorem 4A from Appendix). The function f defined by (6) is then also a continuous real-valued function on $(\mathscr{P}(X), \mathscr{T})$ and hence the set $\mathscr{R}(\mathscr{P}_2(X), f)$ is a non-empty (compact) set in $(\mathscr{P}(X), \mathscr{T})$. We have assumed $\mathscr{D}_r(f) \neq \emptyset$. Thus the set $\mathscr{D}_r(f, \varphi)$ is non-empty, too. Therefore $\mathscr{R}(\mathscr{P}_2(X), f) \times \mathscr{D}_r(f, \varphi) \neq \emptyset$ and each element of this set is a solution of the conflict situation $\mathscr{S}$ in the sense of Definition 4. So we have proved that under the conditions of the theorem there exists at least one solution of the conflict situation $\mathscr{S}$.

3. INTERPRETATION OF THE MATHEMATICAL DESCRIPTION

The element of M. The elements of the set M can be interpreted as alternatives. The set M is supposed to be common to all participants of the conflict situation under consideration.

The elements of X. The elements of X are binary relations on M. These elements are supposed to be weak orders on M and can thus be interpreted as various preference relations on M. If $B \in X$, the $(x, y) \in B$ for some $(x, y) \in M \times M$ is written also as xBy and it is read "x is better than y".

The elements of $\mathscr{P}(X)$. The elements of $\mathscr{P}(X)$ can be interpreted as probabilistic decision rules for "judging" the alternatives from the set M. If $P \in \mathscr{P}(X)$, $B \in X$, then the number $P(B)$ is equal to the probability with which the altetnatives from M will be judged in accordance with the preference order B. If $x, y \in M$, then the probability of the event "x is preferred to y" under the decision rule $P \in \mathscr{P}(X)$ is equal to the number

$$\sum_{B \in \mathscr{L}} P(B) , \quad \text{where} \quad \mathscr{L} = \{B \mid B \in X, xBy\} .$$

To choose an element $P \in \mathscr{P}(X)$ is thus equivalent to the choice of a probabilistic preference order (in the sense of [2]). Let us notice that the number of non-zero elements in the sum mentioned above is finite (Theorem 1A in Appendix). The interpretation of the elements of $\mathscr{P}_1(X)$ and $\mathscr{P}_2(X)$ is clear from Definition 2. It follows from this definition that in most cases of conflict situations it will be probably $\mathscr{P}_1(X) \subseteq \mathscr{P}_2(X)$ (the leader has at his disposal at least all individual feasible decision rules). The element $P_i \in \mathscr{B}_i$ is one of the best decision rules of the i-th participant.

The function of discontent f. The number $f(P)$ can be interpreted as a numerical measure of discontent of the whole group N with the decision rule P. The number α_i are various "weights" of the participants of the conflict situation. If all but one α_i's are equal to zero, we obtain the pure dictatorship (in Arrow's sense, see [2]).

The election mapping F_i. We assume that the conflict situation is solved in such a way that one of the n participants must be appointed leader of the group N. The mapping F_i is supposed to describe how the decision making of the i-th person in the group N is influenced by the election of this person to a leading position.

The set $\mathscr{D}_r(f)$. The set $\mathscr{D}_r(f)$ is interpreted as a set of "reasonable dictators", i.e. the set of persons who tend to minimize the value of f, if appointed. In the leaders theory described in Paragraph 2 we supposed that $\mathscr{D}_r(f) \neq \emptyset$.

The function φ. The number $\varphi(i)$ enables us to measure the inevitable discontent (or "indignation") of the i-th person ($i \in \mathscr{D}_r(f)$) with the duty to choose a reasonable decision rule (in the sense of Definition 3).

The set $\mathscr{D}_r(f, \varphi)$. The set $\mathscr{D}_r(f, \varphi)$ is the set of reasonable dictators that have the least inevitable discontent (measured by the function φ). Let us notice that if we suppose $\mathscr{D}_r(f) \neq \emptyset$, then it is always $\mathscr{D}_r(f, \varphi) \neq \emptyset$.

4. A NUMERICAL EXAMPLE

Suppose a zero-sum-three-person game is given and S_i is the set of pure strategies of the i-th person ($i = 1, 2, 3$). Suppose $S_1 = \{1, 2\}$, $S_2 = \{1, 2, 3\}$, $S_3 = \{1, 2\}$. Then the set of alternatives M will be $M = S_1 \times S_2 \times S_3$. Thus the set M is finite and has 12 elements. Let f_i be a pay-off function of the i-th person ($i = 1, 2, 3$). We shall suppose that the functions f_i are given by the following table (notice that for each $(j, k, h) \in M$ it is $\sum_{i=1}^{3} f_i(j, k, h) = 0$):

(j, k, h)	(1, 1, 1)	(1, 1, 2)	(1, 2, 1)	(1, 2, 2)	(1, 3, 1)	(1, 3, 2)	(2, 1, 1)	(2, 1, 2)	(2, 2, 1)	(2, 2, 2)	(2, 3, 1)	(2, 3, 2)
f_1	2	1	1	−5	0	2	−1	3	4	1	1	1
f_2	−2	0	2	2	−1	−1	5	−3	−2	−1	1	−2
f_3	0	−1	−3	3	1	1	−4	0	−2	0	−2	1

Let $\mathscr{T}$ be the discrete topology on $\mathscr{P}(X)$ and $\mathscr{P}_1(X) = \{P_1, P_2, P_3\}$, where $P_i(B_i) = 1$, $P_i(B) = 0$ for $B \neq B_i$ for $i = 1, 2, 3$. The relations $B_i \in X$ satisfy the condition

$$xB_iy \quad \text{if and only if} \quad f_i(x) > f_i(y)$$

for every $x, y \in M$.
Let $x_k \in M$ be an element satisfying the relation

$$f_k(x_k) = \max_{x \in M} f_k(x)$$

for $k = 1, 2, 3$.

Then $x_k B_k y$ for all $y \in M$. Suppose the utility functions u_i, $i = 1, 2, 3$, are defined on $\mathscr{P}_2(X) \equiv \{P_1, P_2, P_3, \frac{1}{2}P_1 + \frac{1}{2}P_2, \frac{1}{2}P_2 + \frac{1}{2}P_3, \frac{1}{2}P_3 + \frac{1}{2}P_1\}$ as follows:

$$u_i(P_k) = f_i(x_k) \quad \text{for} \quad i, k = 1, 2, 3$$
$$u_i(\tfrac{1}{2}P_k + \tfrac{1}{2}P_h) = \tfrac{1}{2}u_i(P_k) + \tfrac{1}{2}u_i(P_h) \quad \text{for} \quad i, h, k = 1, 2, 3.$$[2]

The values of the functions u_i in P_1, P_2, P_3 are thus given by the following table:

	P_1	P_2	P_3
u_1	4	−1	−5
u_2	−2	5	2
u_3	−2	−4	3

Hence $\mathscr{B}_i = \{P_i\}$ for $i = 1, 2, 3$. Define the relations $\prec_i$ as follows:

$$P \prec_i Q \quad \text{if and only if} \quad u_i(P) < u_i(Q)$$

for all $P, Q \in \mathscr{P}(X)$.

Then the condition (5) is satisfied.

We have supposed that $\mathscr{P}_2(X) = \{P_1, P_2, P_3, \frac{1}{2}P_1 + \frac{1}{2}P_2, \frac{1}{2}P_2 + \frac{1}{2}P_3, \frac{1}{2}P_3 + \frac{1}{2}P_1\}$. Let the function of discontent f be given by the formula (6) with $\alpha_1 = 1$, $\alpha_2 = \frac{1}{2}$, $\alpha_3 = 1$. Then the values of f on th e set $\mathscr{P}_2(X)$ are given by the following table:

	P_1	P_2	P_3	$\frac{1}{2}P_1 + \frac{1}{2}P_2$	$\frac{1}{2}P_2 + \frac{1}{2}P_3$	$\frac{1}{2}P_3 + \frac{1}{2}P_1$
$f(P)$	5	7	9	6	7	9/2

Thus we have

$$\mathscr{R}(\mathscr{P}_2(X), f) = \{\tfrac{1}{2}P_3 + \tfrac{1}{2}P_1\}.$$

[2]) It is not important for further considerations how the functions u_i are defined outside the set $\mathscr{P}_2(X)$. We do not bring therefore the values of u_i on the whole $\mathscr{P}(X)$.

Suppose that the election mapping F_i satisfies for all i the condition

$$F_i(\{P_i\}) = \{\tfrac{1}{2}P_3 + \tfrac{1}{2}P_1\} .$$

Then

$$\mathscr{D}_r(f) = \{1, 2, 3\} .$$

Suppose that the function φ is defined by (8) with $\alpha_1 = \alpha_3 = 1$, $\alpha_2 = \frac{1}{2}$. Then the values of φ are given by the following table:

i	1	2	3
$\varphi(i)$	9/2	5/2	5/2

So we have

$$\mathscr{D}_r(f, \varphi) = \{2, 3\} .$$

Thus the solution set of the conflict situation under consideration is

$$<\{\tfrac{1}{2}P_3 + \tfrac{1}{2}P_1\}; \{2, 3\}> .$$

5. SOME CONCLUDING REMARKS

Topology $\mathscr{T}$. In the example of the preceding paragraph we chose the discrete topology on $\mathscr{P}(X)$. There are of course other possibilities of defining a topology on the set $\mathscr{P}(X)$, e.g. the topology can be defined with the aid of the metrics ϱ or ϱ_k, where $\varrho(P, Q) = \max_{B \in X} |P(B) - Q(B)|$, $\varrho_k(P, Q) = \sqrt[k]{\sum_{B \in X} |P(B) - Q(B)|^k}$ and so on. If the set M is finite and the metrics ϱ ot ϱ_k is introduced on $\mathscr{P}(X)$, then $\mathscr{P}(X)$ with the topology given by this metrics is a compact topological Hausdorff space. There exists in this case a continuous one-one mapping between the set $\mathscr{P}(X)$ and the bounded closed subset $\{(y_1, y_2, \ldots, y_s) \mid y_i \geqq 0, \sum_{i=1}^{s} y_i = 1\}$ of the s-dimensional Euclidean space, where s is the number of elements in X. Thus in this case we can have also $\mathscr{P}_1(X) = \mathscr{P}(X)$ or $\mathscr{P}_2(X) = \mathscr{P}(X)$.

The functions f and φ. It is possible to choose other expressions for defining the function of discontent f. The theory described above is valid when the function f is continuous on $\mathscr{P}(X)$. Some possibilities of defining the function f are: $f(P) = \sqrt[k]{(\sum_{i=1}^{n} \alpha_i |u_i(P) - u_i(P_i)|^k)}$ or $f(P) = \max_{i \in N} \varrho(P, P_i)$, where ϱ is a metrics on $\mathscr{P}(X)$ and so on. Analogously the function φ can be defined in a way different from that in (8). Various topologies $\mathscr{T}$ and various forms of the functions f, φ lead to various mathematical models of conflict situations.

The condition $\mathscr{D}_r(f) \neq \emptyset$. Some other (not so idealized) assumptions can be made. For instance, we can suppose

$$\{i \mid i \in N, F_i(\mathscr{B}_i) \subseteq \beta_i \mathscr{R}(\mathscr{P}_2(X), f) + (1 - \beta_i) \mathscr{B}_i, \beta_i \in [\varepsilon_i, 1], \varepsilon_i > 0\} \neq \emptyset .$$

The possible generalization. The model can be easily extended to the case when each person $i \in N$ has its own set of feasible decision rules, $\mathscr{P}_k(X)$, which is compact in the topology $\mathscr{T}$.

Deterministic group preference order. In a special case the set $\mathscr{R}(\mathscr{P}_2(X), f)$ can consist only of the elements from $\mathscr{P}_2(X)$, which correspond to deterministic preference orders on M. Such a situation will occur if for instance each $P \in \mathscr{P}_2(X)$ is in all but one elements of X equal to zero. In the numerical example of the previous paragraph this will occur for instance if $\mathscr{P}_2(X) = \mathscr{P}_1(X)$.

APPENDIX

Definition 1A. ([1], p. 10.) A binary relation $\prec$ on a set X is called

(a) reflective if $x \prec x$ for every $x \in X$;
(b) symmetric if $x \prec y \Rightarrow y \prec x$ for every $x, y \in X$;
(c) asymmetric if $x \prec y \Rightarrow$ not $y \prec x$ for every $x, y \in X$;
(d) transitive if $(x \prec y, y \prec z) \Rightarrow x \prec z$ for every $x, y, z \in X$;
(e) negatively transitive if (not $x \prec y$, not $y \prec z$) $\Rightarrow$ not $x \prec z$ for every $x, y, z \in X$;
(f) complete if $x \prec y$ or $y \prec x$ (possibly both) for every $x, y \in X$.

Definition 2A. ([1], p. 11.) A binary relation $\prec$ on a set X is a weak order on X if $\prec$ is asymmetric and negatively transitive.

Definition 3A. ([1], p. 12–13.) Binary relations $\sim$, $\lesssim$ are defined as follows:

$$x \sim y \Leftrightarrow (\text{not } x \prec y, \text{ not } y \prec x) \quad \text{for every} \quad x, y \in X$$
$$x \lesssim y \Leftrightarrow (x \prec y \text{ or } x \sim y) \quad \text{for every} \quad x, y \in X .$$

Theorem 1A. ([1], p. 13.) Suppose $\prec$ on X is a weak order. Then

(a) exactly one of $x \prec y$, $y \prec x$, $x \sim y$ holds for each $x, y \in X$;
(b) $\prec$ is transitive;
(c) $\sim$ is an equivalece (i.e. reflexive, symmetric and transitive);
(d) $(x \prec y, y \sim z) \Rightarrow x \prec z$, and $(x \sim y, y \prec z) \Rightarrow x \prec z$;
(e) $\lesssim$ is transitive and complete.

Definition 4A. ([1], p. 105.) A simple probability measure on a given set X is a real-valued function P defined on the set of all subsets of X such that

1. $P(A) \geqq 0$ for every $A \subseteq X$;
2. $P(X) = 1$;
3. $P(A \cup B) = P(A) + P(B)$ when $A, B \subseteq X$ and $A \cap B = \emptyset$;
4. $P(A) = 1$ for some finite $A \subseteq X$.

Theorem 2A. ([1], p. 106.) Suppose P is a simple probability measure on X. Then $P(x) = 0$[3]) for all but a finite number of $x \in X$ and, for all $A \subseteq X$,

$$P(A) = \sum_{x \in A} P(x) .$$

The set of all simple probability measures on X will be denoted by $\mathscr{P}(X)$. If we define the multiplication of elements from $\mathscr{P}(X)$ by a real number and the addition of a finite number of elements from $\mathscr{P}(X)$ in the usual way, the following implication will hold:

$$(P, Q \in \mathscr{P}(X), \alpha \in \langle 0, 1 \rangle) \Rightarrow \alpha P + (1 - \alpha) Q \in \mathscr{P}(X) .$$

If P is a simple probability measure on X and g is a real-valued function on X, then the so-called expected value $E(g, P)$ with respect to P is defined by

$$E(g, P) = \sum_{x \in X} g(x) P(x) .$$

It holds

$$E(g, \alpha P + (1 - \alpha) Q) = \alpha E(g, P) + (1 - \alpha) E(g, Q)$$

for every $\alpha \in \langle 0, 1 \rangle$ and every $P, Q \in \mathscr{P}(X)$.

Theorem 3A. ([1], p. 107.) Suppose that $\mathscr{P}(X)$ is the set of all simple probability measures on X and $\prec$ is a binary relation on $\mathscr{P}(X)$. Then there is a real-valued function u on X that satisfies

$$P \prec Q \Leftrightarrow E(u, P) < E(u, Q) \quad \text{for all} \quad P, Q \in \mathscr{P}(X)$$

if and only if, for all $P, Q, R \in \mathscr{P}(X)$,

1. $\prec$ on $\mathscr{P}(X)$ is a weak order,
2. $(P \prec Q, 0 < \alpha < 1) \Rightarrow \alpha P + (1 - \alpha) R \prec \alpha Q + (1 - \alpha) R$,
3. $(P \prec Q, Q \prec R) \Rightarrow \alpha P + (1 - \alpha) R \prec Q$ and $Q \prec \beta P + (1 - \beta) R$ for some $\alpha, \beta \in (0, 1)$.

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[3]) We make use of the notation $P(\{x\}) = P(x)$ for all $x \in X$.

Moreover, the function u is unique up to a positive linear transformation.

Remark 1A. The function u from Theorem 3A can be extended to $\mathscr{P}(X)$ by defining $u(P) = E(u, P)$. Then $P \prec Q \Leftrightarrow u(P) < u(Q)$ for every P, $Q \in \mathscr{P}(X)$. Let us note that if v on $\mathscr{P}(X)$ is any order-preserving (not necessarily linear) transformation u, then we have $P \prec Q \Leftrightarrow v(P) < v(Q)$, Given such a v that satisfies the condition $P \prec Q \Leftrightarrow v(P) < v(Q)$, we can define v on X by $v(x) = v(P)$, when $P(x) = 1$. However, if v is not a linear transformation of u then $v(P) \neq E(v, P)$.

Theorem 4A. ([1], p. 36.) Suppose a binary relation $\prec$ on Y is a weak order. If (Y is a topological space ($\mathscr{T}$ is a topology) and there is a real-valued function u on Y such that

$$(*) \qquad x \prec y \Leftrightarrow u(x) < u(y) \quad \text{for every} \quad x, y \in Y,$$

then there is a real-valued function on Y satisfying (*) and continuous in the topology $\mathscr{T}$ if and only if

$$\{x \mid x \in Y, x \prec y\} \in \mathscr{T} \quad \text{and} \quad \{x \mid x \in Y, y \prec x\} \in \mathscr{T}$$

for every $y \in Y$.[4])

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Souhrn

JEDEN PŘÍSTUP K ŘEŠENÍ KONFLIKTNÍ SITUACE S n ÚČASTNÍKY

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V literatuře bylo již dosti prací věnováno problému řešení konfliktních situací s více než dvěma účastníky. Autoři těchto prací se většinou snaží nalézt deterministické preferenční uspořádání alternativ, které je společné pro všechny účastníky konfliktní situace a které je v jistém smyslu rozumné nebo přijatelné (např. vyhovuje některým z pěti axiomů uvedených v práci [2] nebo [3]). V tomto článku je uveden

[4]) Some other conditions for the existence of continuous utility function can also be found in [1].

pokus o nalezení vhodného pravděpodobnostního preferenčního uspořádání alternativ (ve smyslu pravděpodobnostní teorie užitku popsané např. v práci [2]). Kriteriem „vhodnosti" je přitom hodnota jisté reálné funkce definované na množině všech rozhodovacích pravidel. Je-li P rozhodovací pravidlo, lze hodnotu této funkce v bodě P považovat za jistou míru nespokojenosti dané skupiny n účastníků konfliktní situace s volbou rozhodovacího pravida P jako závazného předpisu pro celou skupinu při posuzování jednotlivých alternativ. Za nejvhodnější se považují ta rozhodovací pravidla, která minimalizují hodnotu této funkce na množině přípustných rozhodovacích pravidel. Každé rozhodovací pravidlo přitom jednoznačně určuje jisté pravděpodobnostní preferenční uspořádání alternativ. Uvádí se malý numerický příklad. Věty a definice převzaté z knihy [1] jsou pro úplnost uvedeny v dodatku (Appendix).

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