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Math Is More Than Numbers: Beginning Bilingual Teachers' Mathematics Teaching Practices and Their Opportunities to Learn

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In this article, the author provides results from a 3-year, longitudinal study that examined two novice bilingual teachers' mathematics teaching practices and their professional opportunities to learn to teach. Primary data sources included videotaped mathematics lessons, teacher interviews, and field notes of their teacher preparation methods courses. Findings revealed that the teachers were oriented toward differing views of learning that shaped how they organized students' learning of language and mathematics during classroom instruction. While both teachers used similar teaching strategies to support students' development of mathematics specific literacies, there were variances in how the learners were positioned within the classroom community and how and which repertoires of language practices were available and used during mathematics instruction. The teachers' differing orientations toward learning are traced to their own professional opportunities to learn to teach. The significance of recognizing both the acquisition and participation metaphors of learning and the development of linguistically and culturally relevant teacher education are discussed.

KEYWORDS: bilingual education, mathematics education, novice teachers

Emergent bilinguals comprise the fastest growing demographic group and represent a fourth of all urban public school students in the United States (García, Kleifgen, & Falchi, 2008; Menken, Kleyn, & Chae 2012). As the population of emergent bilinguals grows across the country, the prevailing view about bilingual education has become increasingly more negative and debates around language policy more contentious (see, e.g., Flores & Bale, 2017; Menken et al., 2012; Ovando, 2000; Sleeter, Neal, & Kumashiro, 2015). Despite 40 years of research on the benefits of bilingual education, the mathematics learning experiences of emergent bilinguals have mirrored U.S. language politics of xenophobia and asimiliation (Aquino-Sterling, Rodríguez-Valls, & Zahner, 2016; Khisty & Willey, 2008; Valdés, 2015; Valenzuela, 2002). Emergent bilinguals educated in Western schooling practices have learned to perform mathematics using algorithms that are not their own, in a

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language different than their native tongue, and solving mathematics problems irrelevant to their interests and experiences (Aquino-Sterling et al.; García et al., 2008; Moschkovich, 2012).

In the current educational backlash against public education, and bilingual education in particular, political and financial groups such as the English-Only movement, with a primary goal to maintain Euroamerican dominance, are growing (see, e.g., Aquino-Sterling et al., 2016; García et al., 2008; Khisty & Willey, 2008; Valdés, 2015; Valenzuela, 2002). Nonetheless, there exists a growing counter-insurgence of bilingual teachers (Achinstein & Aguirre, 2008; Achinstein & Ogawa, 2011; Athanases, Banes, & Wong, 2015; Sleeter et al., 2015; Villegas & Irvine, 2010). The very nature of bilingual education represents a resistance toward cultural and linguistic hegemony.

In the research project reported here, I examine the teaching and learning experiences of two socially conscious[1] novice bilingual teachers serving in the same urban[2] elementary school in a state where English-only policies have been codifed into law. Specifically, the purpose of this 3-year longitudinal study follows two bilingual teachers to gain an understanding of their mathematics teaching practices and their own opportunities to learn to teach. The emphasis of the work presented is not only to understand their mathematics teaching in bilingual settings but also to understand how their teaching practices are negotiated and grounded by lived experiences and the perspectives developed over time and through their everyday teaching. Studies that examine beginning teachers' experiences, especially with bilingual mathematics curricula, are scarce (Ingersoll & Strong, 2011; Sleeter et al., 2015) and important in advancing understanding of the processes in which teacher preparation programs can increase their potential to prepare critical bilingual educators.

Literature Review

Conceptually, I build the project beginning with the idea that different ways of understanding learning contributes to our knowledge of the relationship between language and mathematics in bilingual classrooms. Therefore, I consider both cog-

[1] The term *socially conscious* refers to the two teachers' expressed commitment to social justice and issues of equity. These teachers entered the profession with the goal to challenge English hegemony in schools and to leverage the cultural and linguistic resources of children and their communities.

[2] The term *urban* takes on multiple meanings in public discourse and in educational research. Here,urban refers to a place with high population density, and *urban schools* take on two characteristics. First, urban schools serve a diversity of students across racial, linguistic, and socioeconomic backgrounds. Second, urban schools are part of a large school district characterized with bureaucratic leadership structures, emphasis on standardized testing, and high teacher turnovers.

nitive and sociocultural theories and pedagogical understandings on bilingualism in mathematics education (e.g., Cummins, 1984; Gutiérrez, 2002; Gutiérrez, Sengupta-Irving, & Dieckmann, 2010; Halliday, 1978; Moschkovich, 2007; 2012; O'Halloran, 2005; Téllez, Moschkovich, & Civil, 2011) and in the critiques and theories on the politics of language in the United States (Cummins, 2000; Khisty & Willey, 2008; Setati, 2008; Valenzuela, 2002). In the following section, I provide a historical account of bilingualism in mathematics education as well as research on beginning bilingual teachers of mathematics to provide context for the tensions, contradictions, and different motives that shape language and mathematics learning in bilingual classrooms.

Language and Mathematics Learning in Bilingual Classrooms

The relationship between language and mathematics began in the field of psychology and examined the cognitive functioning of bilingual learners during arithmetic computations and the solving of word problems (see Moschkovich, 2007 for a review of early research). These studies brought to the fore the importance of language in mathematics education. Learning simultaneously a second language and mathematics is cognitively demanding and can slow down the process of mathematics learning. Language demands, and even the absence of a common language, can generate tensions and impact who is (and is not) participating in classroom interactions (Cummins, 1984; Gutiérrez et al., 2010; Halliday, 1978; Pimm, 1987; Spanos, Rhodes, Dale, & Crandall, 1988; Setati, 2008). Contrary to popular beliefs, mathematics is more than just numbers; school mathematics is linguistically complex, with multiple language modalities—reading, writing, listening, speaking, and representing—in play (Aguirre & Bunch, 2012; Cummins, 1984; Dutro & Moran, 2003; Halliday, 1978; Kress, 2010; O'Halloran, 2005).

Over the last 3 decades, there has been growing attention in the mathematics education research community on bilingual and multilingual learners. The majority of past and current studies have examined the effects of student's language proficiency on academic performance and how teacher's facilitation and scaffolding can support students' acquisition of language and mathematics content (e.g., Cuevas, 1984; Cummins, 1984; Dutro & Moran 2003; Halliday, 1978; O'Halloran, 2005; Pimm, 1987; Quinn, Less, & Valdés, 2012; Spanos, el al., 1988). However, language is only one of many semiotic resources (e.g., physical control of space, gesture, and gaze) at play in bilingual/multilingual classrooms (Aguino-Sterling et al., 2016; DaSilva Iddings, 2005; Domínguez, 2005; Gutiérrez et al., 2010; Moschkovich, 2016; Turner et al., 2013; Zahner & Moschkovich, 2011).

A growing number of studies demonstrate that language use goes beyond language proficiency. In a study of student interactions within a reform-based calculus class, Gutiérrez (2002) found bilinguals fluent in English spoke in Spanish not as a necessity but as a way to bond with others. Moschkovich's (2007) study of code

switching in mathematical conversations similarly builds on language hybridity. Her analysis of student discussions found that code switching between English and Spanish, using everyday colloquialisms and school-based discourse, was most prevalent when students were negotiating ideas and engaging in mathematical arguments. These studies suggest that language and choice of language, more than reflecting proficiency, are integral to the identity work students engage as they learn. Bilingual speakers use language not only on the basis of proficiency but also as an expression of identity (Cummins, 2000; Khisty & Willey, 2008; Moschkovich, 2016; Setati, 2008; Valenzuela, 2002; White et al., 2016). Validation and maintenance of students' linguistic identities are intricately linked to academic performance. Bilinguals who can read, write, and communicate in their home language are more likely to enroll in advanced mathematics courses and continue to higher education (García, et al., 2008; Khisty & Willey, 2008). Instruction in a student's home language and its validation plays significant roles in achievement for bilingual learners (Aquino-Sterling et al., 2016; Celedon-Pattichis & Ramirez, 2012).

The literature reviewed offers important contributions to this study on develpoing principled instruction for bilingual mathematics learners. First, teachers must facilitate students' acquisition of both the mathematical knowledge and language skills needed to engage in meaningful disciplinary discourse practices (Aguirre & Bunch, 2012; Aquino-Sterling et al., 2016; Celedon-Pattichis & Ramirez, 2012; Cummins 1984; Moschkovich, 2012; O'Hallaran, 2005; Pimm, 1987; Quinn et al., 2012). Second, mathematics education research shows that patterns of classroom interaction as it relates to student agency, status, and positioning affect students' access and achievement in the classroom (Gutiérrez, 2002; 2012; Khisty & Willey, 2008; Turner et al., 2013; Zahner & Moschkovich, 2011). Third, bilingual teachers need to leverage the varied linguistic repertoires bilingual students bring into classrooms and can employ to learn and do mathematics (Celedon-Pattichis & Ramirez, 2012; Domínguez, 2005; Gutiérrez et al, 2010; Téllez et al., 2011; Zahner & Moschkovich, 2011). This body of work, encompassing over 3 decades of research, demonstrates the advantages of bilingual education; bilingual education still serves as the most divisive approach to teaching the 12 million emergent bilingual students in the United States.

The Politics of Bilingual Education in the United States

The question arises, why is there so much controversy over bilingual education? Tension in bilingual education involves complex issues of power, identity, and social status tied to an American history of xenophobia (Arce, 2004; García et al., 2008; Ovando, 2000; Valencia, 2002). As Arce (2004) argues, anti-bilingual movements are grounded on racialized discourse that, "to be 'American' means using only one language English and accepting the dominant culture's norms and values" (p. 230). It is no concidence that federal government and state efforts toward

banning bilingual education in California (1998), Arizona (2000), and Massachusetts (2002) occurred during the largest wave of non-English speaking immigration in American history.

The very nature of bilingual education, with the embracing of diverse cultures and recognition of other languages as significant to American mainstream institutions, serves as counter-resistance to Euroamerican and English hegemony. Schools are transmitters for capitalist interests as well as locations for opposition to the economic and political interests of the dominant (Giroux, 2001; McLaren & Jaramillo, 2007; Monzó & McLaren, 2014). Bilingual educators have the potential to break the power structures in schooling by challenging hierarchical mandates and leveraging the cultural and linguistic resources of children and their communities.

Bilingual Teachers of Mathematics

The need for well-prepared bilingual teachers is critical, which helps to explain why a growing body of research has focused on teacher recruitment and the practices of experienced bilingual teachers. Studies spanning over 2 decades indicate that bilingual teachers often enter and remain in teaching driven by a humanistic commitment to give back to their own communities and to redress inequities in schooling for students who share their cultural backgrounds (Achinstein & Aguirre, 2008; Achinstein & Ogawa, 2011; Cavazos, 2009; Gomez et al., 2008; Sleeter et al., 2015; Villegas & Irvine, 2010). A growing body of scholarship suggest that bilingual teachers may be more capable to support language learning, to tap into the cultural resources in themselves and their students, and to position bilingual learners as capable contributors to the classroom learning community (e.g., Cavazos, 2009; Celedón-Pattichis & Musanti, 2010; Gutiérrez, 2002; Musanti et al., 2009; Remillard & Cahnmann, 2005; Sleeter et al., 2015; Turner et al., 2013; Zahner & Moschkovich, 2011). While these studies justify teacher diversity initiatives, the larger research community currently knows little about the experiences of novice bilingual teachers, particularly in the area of mathematics, and what actually happens when graduates of teacher preparation programs begin professional teaching (Ingersoll & Strong, 2011; Sleeter et al., 2015).

What is known, however, is that the current orientation toward language and the structured sets of expectations and accepted teaching practices that dominate U.S. schools and teacher education programs have largely perpetuated language policies of assimilation (Gomez et al., 2008; Khisty & Gomez, 2008; López et al., 2013; Ovando, 2000; Sleeter et al., 2015; Téllez et al., 2011; Valenzuela, 2002; White et al., 2016). The majority of U.S. teachers, including bilingual teachers, received the majority, if not all, of their own K–12 schooling and teacher preparation in English with a focus on supporting students to "acquire English" rather than maintaining bilingualism. Mathematics is still often seen as consisting of *just numbers*; rarely do bilingual teachers have professional development opportunities cen-

tered on discussions of language and mathematics (Aquino-Sterling et al., 2016; Celedón-Pattichis & Musanti, 2010; Musanti et al., 2009).

Sleeter and colleagues (2015) contend that the teacher-education research community has concentrated on examining teacher preparation when more attention should be placed on what actually happens when graduates of teacher preparation programs begin to teach. A close examination of novice bilingual teaching and in relation to their teacher preparation is critical for school leaders and teacher educators to consider in our work to better support our teacher workforce and our students.

Methods

Purpose and Research Questions

In the 3-year, longitudinal study reported here, I explore the teaching and learning experiences of Laura and Elise (pseudonyms, as are all proper names throughout), two socially conscious novice bilingual teachers in a state where English-only policies were codifed into law and with stricter accountability measures. Both teachers attended a post-baccalaureate teacher education program at a West Coast public university and were purposefully selected (Patton, 2014) because of their expressed commitment to bilingual education and equity-oriented teaching. After completion of the credential program, both teachers were hired to work at the same dual language program (Spanish and English) in Valadez Elementary, an urban elementary school in which they also student taught during teacher preparation. Their shared commitments and teaching context offer a unique opportunity to examine their teaching practices and how their teaching practices are negotiated and grounded by their experiences and orientations developed over time. Studies that examine beginning teachers' experiences, especially with bilingual mathematics curricula, are scarce (Ingersoll & Strong, 2011; Sleeter, et al, 2015) and important in advancing understanding of the processes in which teacher preparation programs can increase their potential to prepare critical bilingual educators. Specifically, I explore two research questions:

1. How do two novice bilingual elementary teachers organize mathematics learning for their emergent bilingual students?
2. What opportunities arise for the teachers to learn to teach mathematics in bilingual settings during teacher preparation and in the first 2 years of professional teaching?

Data Sources

In the study, I take a situated lens and view mathematics teaching, neither as static nor fixed, but rather as evolving as teachers learn to participate in and across communities (Wenger, 1998). As such, this study was longitudinal in nature, and ethnographic methods (Patton, 2014) were used to collect data over a 3-year period encompassing one year in the teacher preparation program and the teachers' first 2 years of professional teaching after graduation. To examine their teacher preparation, I attended and took field notes of the two-quarter (10 weeks per quarter) elementary mathematics methods course. In addition to the field notes, all teaching artifacts from the mathematics methods course (course syllabi, PowerPoint slides, handouts, assignments) were gathered. Additional publically available programmatic data (e.g., program design, course sequence, and syllabi of courses that may have emphasized pedagogies for working with emergent bilinguals: child development, educational equity, and theories and methods in English language development) were collected.

As I sought to investigate their mathematics teaching after graduation, the main source of data gathered during the teachers' first 2 years of professional teaching were classroom observations. During the classroom visits, I observed their 50- to 75-minute mathematics lessons and followed each lesson observation with a 30- to 60-minute post-lesson interview, during which I interviewed the teacher about the mathematics lesson, their teaching, and the ways they saw themselves developing as teachers. All classroom lessons were videotaped and later transcribed and interviews were audiorecorded. Classroom artifacts (e.g., lesson plans, students assignments, and class generated work), e-mail exchanges, informal participant interviews with teachers and other school personnel were gathered and served as secondary data sources.

Data Analysis

Data analyses occurred in two phases. The first phase examined how the teachers organized mathematics learning for their emergent bilingual students, drawing on prior literature and the three dimensions of principled instruction that framed the study: types of language supports, authority of knowledge, and repertoires of language practices.

The software program Dedoose[3] was used to review video data repeatedly for close attention to the details in classroom interactions. Dedoose allows for microanalysis of video interactions and provides a time mark and key code deduction so that the data can be organized into themes. Data were coded according to the aforementioned three dimensions of focus. Each videotaped lesson was analyzed systematically, identifying aspects of the lesson and specific teaching practices related to types of language supports, authority of knowledge, and repertoires of lan-

[3] For details about Dedoose see http://www.dedoose.com.

guage practices (Strauss & Corbin, 1998). Data was also coded for the type of semiotic resources deployed by the teacher and student during whole classroom interactions. Multimodal analysis (Kress, 2010; Jewett, 2009) provides insight to the manner in which verbal language is enacted as well as the nonverbal in social interactions. This type of analysis is particularly helpful in research examining bilingual/multilingual learners as students often rely upon semiotic resources other than spoken language to participate (Cummins, 2000; DaSilva, 2005; Domínguez, 2005; Turner et al., 2013). Appendix A provides the practices found in the data at this stage.

Analysis was conducted on all videotaped classroom data. A research assistant and I reviewed the data independently and met regularly to discuss emerging themes, coding categories, and interpretations. When divergent opinions existed, data was reviewed multiple times until agreement was reached (Miles, Huberman, & Saldaña, 2014). Themes and interpretations were analyzed and compared across data sources: videotaped lessons, interview transcripts, and field notes (Fielding & Fielding, 1986). While the videotaped mathematics lessons allowed us to examine the classroom interactional architecture, the interviews and field notes provided a window into the intentions and reasons behind the teachers' actions.

Teacher's socialization into the teaching profession begins as students in K–12 classrooms, throughout teacher preparation, and continues as they encounter the reality of schools as they begin their professional lives (Lortie, 1975; Feiman-Nemser, 1983; Zeichner & Gore, 2010). The goal of the second phase of analysis was to examine the professional opportunities that arose for the teachers during teacher preparation and in professional developing settings; however, their earlier K–12 schooling experiences are included as these experiences also shape their perceptions of teaching.

The second phase builds from earlier analysis of the interview data. The interview data, field notes of observations at the school site and the mathematics methods courses, and programmatic data (e.g. teacher education program design, learning outcomes, and course syllabi) were reviewed to examine learning opportunities that arose for the teachers during their K–12 setting, teacher preparation, and professional development settings related to bilingual mathematics teaching and learning. Analyis consisted of identifying the *teaching repertoire* privileged in the data sources. The notion of repertoire is taken from Bernstein (1996) and refers to the set of symbolic and material resources selected and configured to shape classroom practice. In this case, I examined the specific instructional materials, pedagogic resources and strategies, and arrangement of task sequence privileged in the professional learning settings as identified in the interviews or in the programmatic data (Strauss & Corbin, 1998). Teacher portraits were written for each teacher highlighting their learning experiences in K–12 schools, within teacher preparation, and after graduation (Miles et al., 2014). To verify and confirm interpretations of the data, I

triangulated the various data sources (i.e., field notes, interview transcripts, field notes, and teaching artifacts) and teacher portraits were shared with each teacher to ensure accurate reflection of their experiences (Lincoln & Guba, 1986).

Findings

In this section, I present the findings in two parts. The first part presents findings from the study organized in three episodes to demonstrate the array of interactional encounters the students navigated and the role of the teacher in shaping the classroom interactions at three timepoints during their second year of professional teaching. While their teaching repertoires were relatively consistent, both Elise and Laura felt more confident in their mathematics teaching by their second year. In the second part, teacher portraits that connect their teaching practices to their opportunities to learn are shared.

Organization of Mathematics Learning

Both Elise and Laura utilized similiar pedagogical strategies that leveraged students' cultural and linguistic identities, but how these strategies were used revealed differing ideologies about learning. These differing ideologies are discussed in relation to the three dimensions of principled instruction that guided the study: types of instructional supports, authority of knowledge, and repertoires of language practices.

Types of instructional supports. Laura and Elise created a classroom environment where talking about language was as important as the mathematics lesson itself. In their lessons, Laura and Elise structured instructional time for language to serve as the topic of classroom discussion. In their practices, they used various strategies from research. They encouraged students' use of their native or hybrid language practices, modeled the target language in the lesson, and connected language with mathematical representations (e.g., pictures, words, numbers, gestures) (De Jong & Harper, 2008; Dutro & Moran, 2003; García et al., 2008; Moschkovich, 2002; 2007; 2012. O'Hallaran, 2005).

While both teachers used similar strategies, there were variations in the manner in which these strategies were enacted. An analysis of typical classroom interactions is illustrative. In Laura's class, mathematics instruction followed a consistent routine. The class began with a math talk activity in which students explored mathematical concepts or number patterns and relationships. The class then engaged in a problem-solving activity with students working independently or in pairs. A public sharing of student solutions followed.

Figure 1 and the discussion to follow are excerpted from the public sharing of a lesson taught in Laura's second year. Here, her first-grade students were introduced to a join change-unknown word problem (see Figure 1).

Nostros tenemos __ fotos en el altar. (Los estudiantes ponen algunas fotos más.) Ahora hay ___ fotos in el altar. ¿Cuántas fotos ponen los estudiantes?
[We have ____ photos on the altar. (The students put more photos on the altar.) Now, there are ___ photos on the altar. How many photos did the students put on?]

3/10 13/20 83/100

Figure 1. Join change-unknown word problem.

Excerpt 1[4] Laura [November, Year 2]:

(*Sammy walks up and stands next to Laura as she writes his name below the problem*)

T: Sammy, EXPLICANOS esta idea. ¿Como contaste?↑ / Sammy, EXPLAIN the idea to us. How did you count?↑

S: Yo conté↑ en mi mente↑ (*points to forehead*) que 3 Y 1, 2, 3, 4, 5, 6, 7 más↑ y 3, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. (*counts with fingers*) / I counted↑ in my mind↑ 3 AND 1, 2, 3, 4, 5, 6, 7 plus↑ 3, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

T: ¿Oh Sammy, entonces empezaste con el número TRES?↑ / Sammy, so you started with the number THREE?↑

S: Si (*Laura writes 3 below Sammy's name*) / Yes

T: ¿Por qué↑ empezaste con el número TRES?↑ / Why↑ did you start with the number THREE?↑

S: Porque tenemos TRES↑ (*points to the 3 written in the word problem displayed*) y los estudiantes pone algunos mas↑ (*points to the problem again*). / Because we have THREE↑ and the students put some more↑.

T: ¿Oh tenemos TRES fotos (*finger circling the 3 in the word problem*) en EL ALTAR↑ y los estudiantes PONEN ALGUNAS MAS?↑ / Oh we have THREE photos on THE ALTAR↑ and the students PUT SOME MORE?↑

[4] The following transcription symbols are used in all transcript excerpts / English translation follows
( ) Description of nonverbal communication such as gestures, gaze, movement, and so forth
Capital emphatic stress
↑ Indicates rising tone

T: Dígale↑ a su amigo lo que recién dijo Sammy. / Tell↑ your friend what Sammy just said. (*Walks around listening to student partner talks.*)

T: ¿POR QUE Sammy empezó↑ con el número TRES?↑ ¿Quién nos puede decir? / WHY do you think Sammy started↑ with the number THREE?↑ Who can tell us?

S1: Porque los otros tienen tres fotos y los estudiantes PONEN ALGUNOS MAS↑, don't know how much (*Lea points to "ponen algunas fotos más" in the word problem*). Pero, al final es 10 FOTOS / Because we have three photos and the students PUT SOME MORE↑, don't know how much more. But at the end, it's 10 PHOTOS.

T: ¿OH AL FINAL ES DIEZ?↑ Okay, entonces voy a escribir diez (*writes 10 on the same line as the 3*) Y sabemos que ahora hay diez fotos. ¿Cómo llegaste de TRES a DIEZ?↑ ¿Qué hiciste?↑ / SO AT THE END THERE IS TEN?↑ Okay. I'm going to write ten. We know that now there are ten photos. How did you get from THREE to TEN?↑ What did you do?↑

The vignette above documents the starting conversation around Sammy's strategy. Figure 2 provides an image of Laura's documentation of the student's strategy on chart paper as the class examines his problem solving together. The whole class discussion around Sammy's strategy lasted for over five minutes. Multiple students explained Sammy's strategy and connected it to the problem context. By doing so, students were required to explain Sammy's sense-making and the underlying characteristics of a change-unknown problem type. Exemplified here is Laura's integrated approach to learning language and the math content. Students arrived at meanings and definitions as they engaged in analysis of each other's reasoning and problem solving.

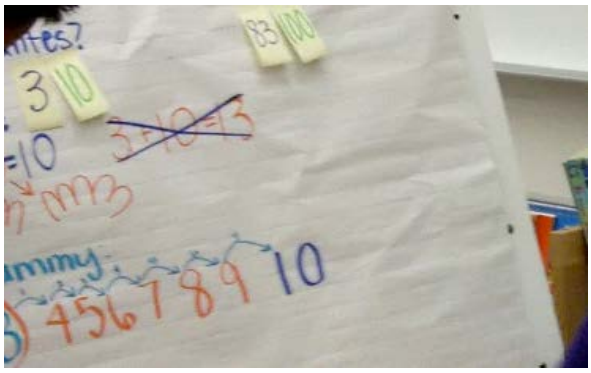

Figure 2. Documentation of Sammy's strategy.

Elise's classroom interactions followed a different pattern. Elise set a specific time before the math content instruction to teach language. Dutro and Moran (2003) term the teaching of vocabulary prior to content instruction "frontloading vocabulary." The excerpt that follows captures the beginning of math instruction from a lesson observed during a similar time to Laura's; this lesson occurred during Elise's second year of teaching. Here, Elise's second-grade students are learning to tell time:

> (*Students are standing and holding one arm up. Elise is standing in front of the class with her right arm extended out.*)
>
> **T**: VAMOS↑ a aprender acerca de LAS HORAS↑ del reloj. / We are going to learn about THE HOURS on the clock.
>
> **SS**: Vamos a aprender acerca de las horas del reloj. / We are going to learn about THE HOURS↑ of the clock.
>
> **T**: Punto↑ / point↑
>
> **SS**: Punto↑ / point↑
>
> **T**: Quince↑ / fifteen↑ (*left hand is extended straight out to represent the minute hand at the 15-minute mark on a clock*)
>
> **SS**: Quince↑ / fifteen↑ (*models the teacher with left hand extended straight out to represent the minute hand at the 15-minute mark on a clock*)
>
> **T**: Treinta↑ / thirty↑ (*left hand moves clockwise to the position of the 30-minute mark on a clock*)
>
> **SS**: Treinta↑ / thirty↑ (*follows after the teacher with left hand moves clockwise to represent the 30-minute mark on a clock*)
>
> **T**: Cuarenta-cinco↑ / forty-five↑ (*left hand moves clockwise to the position of the 45-minute mark on a clock*)
>
> **SS**: Cuarenta-cinco↑ / forty-five↑ (*left hand moves clockwise to the 45-minute mark on a clock*)

Elise organized instruction into two parts. Instruction began with students engaging in a vocabulary activity using total physical response—in this case, using their hands to represent the minute hand and identifying the parts of an analog clock. The clock vocabulary activity lasted for 10 minutes in which students engaged with mathematical terms across representations (e.g., gestural, visual, graphical, and oral). After the song, Elise and the students read from the vocabulary word bank posted on the wall with students echoing back in choral response as Elise

pointed to and read the written math vocabulary word or phrase. In other activities, students created a human number line (choral response of double digit numbers), searched for geometric shapes in the classroom (identification of geometric structures), and recited the place value chart (naming the value based on its place, or position, on the chart). Elise's goal for her vocabulary activity was to familiarize students with the vocabulary for the lesson. This format of front-loading the vocabulary followed by engagement with the mathematical content occurred in the majority of the math lessons observed in her class.

Authority of knowledge. Students in all classrooms traverse an array of interactional practices where power and status impacts students' access to learn and to demonstrate their learning (Cazden, 2001; DaSilva Iddings, 2005). Analysis of classroom interactional patterns provides insight to issues of agency and positioning in the classroom (Hand, 2012; Jewitt, 2009; Kress, 2010). Another difference identified during analysis was the positioning of competence in the two classrooms.

Laura attended closely to the nature of the classroom culture and tried to center instruction on inviting students to become a part of the classroom community. In the episode shown earlier, Laura positioned herself as the facilitator as well as a learner within the classroom community. As shown in the vignette above, Laura often adopted a posture of uncertainty ("*¿POR QUE Sammy empezó↑ con el número TRES?↑ ¿Quién nos puede decir? / WHY do you think Sammy started↑ with the number THREE?↑ Who can tell us?*") to open up space for students to take on the expert role ("*Dígale↑ a su amigo lo que dijo Sammy ahora. / Tell↑ your friend what Sammy just said."*).

In a later component of the public solution share, Laura called on a third student, Jesus, to explain his strategy. Jesus was a quiet student, who had never spoken out during class discussions on previous visits. What follows is the whole class discussion after Jesus shares his strategy with the class:

> **T**: EXPLICALE a su amigo↑ lo que hizo Jesus. / EXPLAIN to your friend↑ what Jesus did. (*The students talk amongst themselves, then Laura claps them to attention.*)
>
> **T**: Okay. Alguien puede explicar porque Jesus EMPEZO↑ con veinte y terminó con trece?↑ Por que hizo ESTO?↑ Jesus, escoge a otra persona que pueda explicar lo que hiciste. / Who can explain why Jesus STARTED↑ with 20 and ended with 13?↑ Why did he do THIS?↑ Jesus, pick another person who can explain what you did.
> (*Jesus points to a student sitting near the back, who comes and stands next to Jesus*)
>
> **S3**: Habían trece al principio↑ (*uses finger to circle the thirteen crossed out dots that represent a visual of Jesus's strategy shown on chart paper*) y

LUEGO↑ habían veinte y ESTOS↑ son los que ponen / There were thirteen in the beginning↑ and THEN there were twenty and THOSE↑ are the ones they put there.

T: Hmm, okay. (*puts finger on cheek pensively*) Alguien. / Hmm, okay. Anyone…

S4: Porque Jesus did it BACKWARDS a Sammy… so it's twenty take away thirteen is seven. / But Jesus did it BACKWARDS a Sammy… so it's twenty take away thirteen is seven.

T: Veinte quita trece es igual a siete y tú dijiste él lo hizo al revés↑ a Sammy. Qué quieres decir?↑ (*writes 20–13=7 under the dot representation*) / Twenty minus thirteen is equal to seven. You said that he did the reverse↑ to Sammy. What did you mean?↑

S5: Sammy counted up.↑ Sustracción es opposite. / Sammy counted up.↑ Subtraction is opposite.

In the scenario above, Laura demonstrated some of the instructional moves used to distribute the knowledge authority between herself and among the students. One way is her consistent questioning, displaying a sense of wondering ("*Alguien puede explicar porque Jesus empezó con veinte y terminó con trece? Por que hizo esto?" / Who can explain why Jesus started with 20 and ended with 13? Why did he do this?*"). Laura regularly asked the class to explain the reasoning behind a particular strategy. This practice was a common move used by Laura to open the floor to peer mediation, putting the onus of explanation and analysis on the students. Students' share-outs were not always fully articulated ideas, partial explanations were common, and students built upon each other's explanations. During lesson reflections, Laura regularly discussed the intentional selection and sequencing of specific solution strategies as well as strategies to reconfigure the classroom social order for more meaningful and respectful dialogue.

In another example, I highlight a lesson that took place during a similar period to illustrate differences between the two classrooms. During March of Elise's second year, the class was on day two of working on regrouping in subtraction. The day prior, the class used concrete models to represent the process of regrouping: working with place-value blocks and place value charts to connect the procedural skill of regrouping (the decomposing of a ten unit for ten ones) to its conceptual basis (each place value position is related to the next by a constant multiplier of ten). Each student had on their desks a set of base-ten blocks and the place-value chart. The following dialogue is from the start of the lesson during which Elise demonstrated the process of regrouping from concrete to representational (see

Figure 3) —connecting the written recording of each step of the procedure to the modeling with the base-ten blocks:

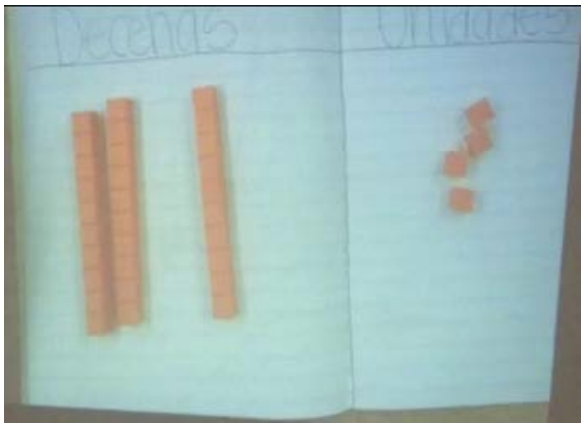

Figure 3. Teacher modeling of the problem on the overhead.

T: Con los bloques de diez, les voy a dar un cuento sobre algo, así que tienen que escuchar calladitos. ¿Por ejemplo, si digo Diego tiene TREINTA-CUATRO pájaros↑ (*Elise places the three base ten rods in the tens place and four units in the ones place of the place value chart, Figure 3*), cuantas DECENAS voy a poner?↑ / With the blocks of tens, I'm going to give you a story. So, you all must listen quietly. For example, if I say Diego has THIRTY-FOUR birds↑, how many TENS am I going to have?↑

Ss: Tres (*choral response*) / Three.

T: ¿Cuantas me faltan para ser TREINTA-CUATRO?↑ (*Elise points to the four ones units on the ones side of the place value chart.*) / How many more do I need to make THIRTY-FOUR?↑

Ss: Cuatro (*choral response)* / Four

T: (*A few students have the base ten blocks in hand.*) No es necesario tocar nada en este momento. (*Elise looks around the room and waits for students to put down the base ten blocks.*) Pero su hermanito, Allen, le abrió la jaula, y se le fueron VEINTICINCO. Se fueron VEINTICINCO pájaros↑ ¿Que voy hacer? Se fueron VEINTICINCO pájaros↑. ¿Qué vamos hacer?↑ / You don't need to touch anything right now. But his little brother, Allen, opened the cage, and TWENTY-FIVE left. TWENTY-FIVE birds left↑. What are we going to do?↑

Ss: Vamos a quitar UNA DECENA. (*choral response*) / We are going to take away A TEN.

T: No es necesario tocar NADA en este momento. (*Elise waits for students to put down the base ten blocks in their hands*.) / You don't need to touch ANYTHING right now.

T: ¿Si, vamos a quitar una decena y que se va hacer? / Yes, we are going to take away a ten and what is that going to make?

Ss: Pone unas unidades/ Put ones. (*Elise takes away a ten rod from the ten place-value column.*)

T: ¿Las pongo en unidades↑? (*Elise places the ten rod into the ones place value column.*)
/ I put them with the ones↑?

Ss: No. (*Choral response*)
T: ¿las voy a poner aquí↑? ¿Qué va a pasar↑? (*Elise lifts the placed ten rod from the ones place-value column and holds it up for students to see.*) / Am I going to put them here↑? What is going to happen↑?

S1: Vas a quitar dos decenas/ You are going to remove two tens

T: ¿DOS decenas?↑ (*Elise holds up the ten rod in hand and shakes it.*) / TWO tens?↑

S1: No

Ss: Una decena/ One ten.

T: Una decena. (*Elise holds up the ten rod in hand and shakes it.)* ¿Dónde se va a ir esta decena?↑ / One ten. Where is this ten going to go?↑

Ss: En las unidades/ In the ones. (*Elise places ten individual units in the ones column*.)

T: ¿Y vas a cambiarlas PORQUE?↑ / And WHY are you going to change them?↑

Ss: por unidades, se fueron / for ones, they left

T: ¿Veinticinco se fueron entonces ya puedo quitar veinticinco? / Twenty-five left and now can I take twenty-five?

Ss: Si / Yes (*Choral response*)

This vignette illustrates Elise's developmental approach to linking a procedural skill to a conceptual base. Elise was building on the concrete experience of the day prior addressing the representational stage—linking each step of regrouping to the written recording. She represented the minuend with the place value blocks and made trades (i.e., exchanging 1 for 10 in the position to the right) before subtracting. This process of "trading all" before subtracting has been shown to help prevent errors in subtraction (Van de Walle, et al., 2010) and demonstrated Elise's attention to students' mathematical development.

Elise regularly used physical and visual models to connect procedures and concepts; however, she also determined which models were used as well as the process for problem solving. At one point, a student appeared to suggest a different way to subtract ("*Vas a quitar dos decenas/ You remove two tens*."). It's possible that the student may have wanted to subtract by place value—taking the two tens of 25 from 34 first. Elise, however, redirected the students' attention back to the ten rod in hand as she wanted students to regroup before subtracting. Elise, unfortunately, often started lessons modeling a solution path for students. By providing a solution path, Elise unintentionally reinforced a hierarchy and dichotomy between the teacher and student and knower and learner.

Repertoire of language practices. The last theme in the examples is related to the previous and documents the repertoire of language practices utilized. Both teachers created lessons that encouraged multi-modal and hybrid language practices; however, there were variances in how the students' language practices were leveraged during instruction.

The following scenario illustrates how Laura built upon students' language practices over time. I discuss a visit made two months after the photo-problem lesson described in the earlier vignettes. The class regularly engaged in student-strategy share. In prior lessons, solution strategies were labeled with the students' name (e.g., Rosa's strategy). In this visit, the lesson focus was "estrategias para juntar /strategies for adding" to link each student's addition strategy to its mathematical term. However, the definition was not given by Laura but unpacked by the students as they analyzed the strategy's features:

> **T**: Cada vez↑ que hacemos UN PROBLEMA↑ hay personas que comparten su IDEA, verdad?↑ / Every time↑ we do A PROBLEM↑, there are other people who share the same IDEA as you, right?↑
>
> **SS**: Si. / Yes
>
> **T**: Si↑, cada vez que lo hacemos. Pero estaba pensando↑ que sus ideas no tienen nombre↑. Entonces si yo le digo, as la idea de Caleb↑, NO SE de qué estoy hablando. Entonces HOY↑ quería hacer UNA LISTA↑ de las ideas que tenemos. Entonces llamaré↑ esta hoja estrategias para juntar. ¿Que es una es-

trategia↑? ¿Quien ha escuchado esta palabra↑? (*Laura points to the words "Estrategias para juntar/strategies for adding" on the chart paper. Another sheet of chart paper covers this one. Laura lowers the top paper to reveal the first strategy "Modelar de uno en uno* (*Modeling one by one*)*." The strategy title and the visual example are shown.*) / Yes↑, whenever we do it. But what I was thinking↑ that your ideas do not have a name↑. Therefore, if I tell you all to do Caleb's idea↑, I DON'T KNOW what I am talking about. TODAY↑, I would like to do A LIST↑ of the ideas that we have. Therefore, I call this page strategies for adding. What is a strategy↑? Who has heard this word↑?

T: La primera estrategia↑ es de Ella. Su estraegia↑ el nombre de su estrategia es modelar de UNO EN UNO. Esta es una manera que MUCHOS↑ de ustedes están USANDO↑ modelar de uno en uno. Y que están observando de la estrategia de Ella?↑ Que ha hecho. Hmmm. (*Laura puts her hand to her face pensively. Many students in the class follow the same pose.*) / The first strategy↑ is Ella's. Her strategy's↑ name is modeling ONE BY ONE. This is one strategy that MANY of you USE. What do you observe about Ella's strategy?↑ What has she done?↑ Hmmm.

T: TODOS piensa su estrategia. ¿Que están observando?↑ ¿Que están observando de la idea de Ella, Alex? / EVERYONE think about her strategy. What do you observe?↑ What do you observe about Ella's idea, Alex?

S1: (*Alex walks up to the poster and points at Ella's strategy*) um, tiene quince y catorce-trece the same one as this (*Alex walks over to the class math wall*) because quince y trece and then right there↑ (*points to strategy on a poster on the math wall where the strategy shown is also a modeling all strategy*) he put quince y trece mas↑, but then she counts each one↑ – one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, (*touches each drawn unit and counts out loud*) and then↑ one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, and then you also could do this.↑ /um, there's 15, and 14, 13, the same one as this because 15 y 13 and then right there↑ (*points to strategy on another poster where the student is also modeling all*) he put 15 and 13 more, but then she counts each one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, and then↑ one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen.

T: Oh, Entonces↑ estas diciendo, el está diciendo que en la idea de Ella cuenta TODAS LAS COSAS↑. Entonces Alex dijo que cuenta↑ cada palo↑, ¿verdad?↑ Cuenta de uno en uno↑, voy a escribir eso. (*She writes "cuenta todas las cosas / counts one by one" under the modeling one by one representa-*

tion.) / Okay, then↑ what you are saying, he is saying that Ella's idea is to COUNT EVERY THING↑. Then, Alex said to count↑ each of the lines↑, right?↑ Count one by one↑, I am going to write this.

T: Algo más↑ que observan de su idea de Ella?↑ Ana?↑ / Is there anything else↑ you observe about Ella's idea?↑ Ana?↑

S2: Una manera fácil también↑ is to take quince FIRST↑ and add uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve los demás más↑ los demás le va hacer fácil solo hacer números. / An easy way to also↑ do this is to FIRST↑ take the first fifteen plus the other one, two, three, four, five, six, seven, eight, nine and then the rest of them plus↑ the rest of them. It will be easier to just use those numbers.

T: Estas diciendo que sería más fácil usar números?↑ / You are saying that it would easier to use numbers?↑

S2: Si↑ porque te vas a hacer CANSADA↑ de escribir muchas↑ muchas↑ líneas. / Yes↑ because you will get TIRED↑ of drawing so many↑, many↑ lines.

T: En esa tienes que escribir MUCHAS↑ cosas.↑ Voy a escribir eso también, (*Laura writes "escribes muchas cosas / writes down a lot" on the chart paper.*) Addy algo más?↑ / In this one, you must write MANY↑ things.↑ I am going to write this one too. Addy, anything else?↑

The interaction above highlights some of the ways Laura drew on students' language practices. First, she used students' ways of thinking and reasoning about the strategies as the focus of discussion. Laura's classroom walls were filled with posters of student strategies and their representations (e.g., drawings, words, and equations) gathered from lessons past. As in the vignette above, these posters were often referenced by Laura and the students during instruction.

The students also moved in and out of a range of language practices (Spanish, English, and code switching between Spanish and English). This flow of language practices was most observed when students were trying to communicate their reasoning and sense-making. It appeared Laura's students often articulated the academic language of mathematical concepts and numbers using the school-based discourse heard in class and would use their everyday, native language (Spanish or English) to provide illustrations ("*una manera fácil también is to take quince first and add uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve los demás más los demás le va hacer fácil solo hacer números*").

Notice in the vignette above, Laura accepted Alex's response but rephrased a more complete response in Spanish so the students could hear the formal Spanish

discourse (“*El está diciendo que en la idea de Ela cuenta todas las cosas. Entonces Alex dijo que cuenta cada palo, ¿verdad? Cuenta de uno en uno voy a escribir eso. / Okay, what you are saying, he is saying that Ella’s idea is to count. Alex said to count each of the lines, right? Count one by one, I am going to write this.*)

Semiotic resources other than spoken or written language, including gestures and physical movement, were often used. In the example above, Alex pointed to another student example of a modeling one-to-one strategy to explain how the strategy required students to “count one by one.” By revoicing his explanation in Spanish using the mathematical terms and writing down his observation on the poster sheet, Laura showed that all forms of communication—verbal or nonverbal, English or Spanish—were valued.

In Elise’s classroom, students also moved between language practices in their student-to-student interactions and during recess; however, hybrid and multimodal practices were seen less frequently than in Laura’s case during classroom discussions. I contrast here Laura’s use of students’ language practices during strategy shares with a vignette of Elise’s class discussion of a strategy for subtraction with regrouping. The strategy is not the traditional U.S. subtraction wherein the algorithm subtracts from right to left, one place-value column at a time, regrouping as necessary. In this strategy, the students are subtracting numbers in expanded form, the subtrahend and minuend are written in expanded form in vertical order with the same place value arranged in columns and differences found for each place value:

> **T:** (*points to the number 427 written in the problem 427–182.*) ¿Okay, que son las escenas de 427? ↑/ Ok, what is the expanded form for 427?↑
>
> **S1**: Cuatro / Four
>
> **T**: ¿Cuatro, y como vamos hacerlo?↑ / Four, how are we going to do it?↑
>
> **S2**: Cuatrocientos /400
>
> **T**: Cuatrocientos/ 400. (*writes down 400, 20, 7 on the worksheet projected on the overhead, then points to the 20 and 7*). y que es ESTO?↑ veinte y siete y ciento ochenta y dos (*points to the 182*) / And what is THIS?↑ 20 and 7. And 182.
>
> **SS**: cien (*writes down 100 below the 400 in the expanded form of 427*) / 100
>
> **SS**: 80 and 2 (*writes down 100, 80, 2 in columns by place value below the 400, 20, and 7*)
>
> **T**: ochenta y dos. ¿Qué ES siete menos dos?↑ / eighty and two. What IS seven minus two?↑
>
> **SS**: Cinco (*writes 5 down and points to “20-80”*) / Five

S3: sesenta / sixty

T: espérese↑ ¿Cómo se puede restar 20 de 80? ¿Qué le vas a decir? / Hold on↑, how can you subtract twenty from eighty? What are you going to say?

S4: Puedes prestar de los 400 / You can borrow from the 400.

T: ¿Le PRESTAMOS 400↑ y ese se convierte↑¿ (*points at the 400*) / We BORROW from 400↑ and what does this one convert into↑?

S4: Este se convierte en 300. (*Elise crosses out the 400 and writes 300 above.*) / That one turns into 300

T: ¿Por qué?↑ / Why?↑

S4: Porque le presaste uno prestado uno al 20. / Because you lend one to the 20.

T: ¿Por qué↑ le prestas?↑ / Why↑ do you lend it?↑

S5: Porque 80 es más que 20/ 80 is more than 20.

In the interaction above, the class used verbal and nonverbal communication (gestures and the expanded form) to make sense of the regrouping process in subtraction. What follows is a brief episode of the discussion for regrouping in the next problem:

(*The class already decomposed 639 and 256 into place value in expanded form together and this is shown on the paper on the overhead.*)

T: ¿Entonces↑ que estamos haciendo con nueve menos seis?↑ / So↑ what are we doing with 9-6?↑

S1: Tres / Three

T: ¿Jay↑, que vamos hacer con treinta y cincuenta?↑ / Jay↑, what are we doing with thirty and fifty?↑

S6: Puedes PRESTAR de seiscientos y poner quinientos/You BORROW from six hundred and you have five hundred

T: Quinientos?↑ (*Elise crosses out the 600 and writes 500 above*) / Five hundred?↑

S6: Quinientos / five hundred

T: ¿Pero porque↑ le vas a dar una centena al treinta?↑ / But why↑ are you giving one hundred to the thirty?↑

S6: Porque cincuenta es MAST↑ que treinta↑ / Because 50 is MORE↑ than 30.↑

The vignettes provide a window into the classroom discourse around regrouping for two problems, however, the same discursive pattern continued for the remaining 30 minutes of the lesson. As seen above, very little language hybridity occurred during the strategy shares. The class discussion was only in Spanish. This use can be expected, as students in Elise's classroom were a grade above Laura's and had more opportunities to develop their Spanish proficiency. However, there was also less variance in what was said. Note how student responses for both problems for the regrouping process ("*Porque le prestaste uno al 20." / "Because you lend one to the 20."; "Puedes prestar de seiscientos." / "You borrow from 600.*"), and the reason why regrouping was needed (*"Porque 80 es más que 20." / "80 is more than 20."; "Porque cincuenta es más que treinta" / Because 50 is more than 30.*) were very similar. In Elise's class, Spanish, mathematical representations, and gestures were frequently used, but she modeled the representations and gestures first. Student responses often consisted of a repetition of her words and phrases ("*you borrow one"; "more than*").

Examining Teachers' Opportunities to Learn

It is important to connect Laura's and Elise's teaching practices to their opportunities to learn to teach. What follows are teacher portraits for each teacher's K–12 schooling, motivation for bilingual teaching, and their professional learning opportunities during teacher preparation and after graduation.

Laura. Laura grew up in a bilingual household with a stepmother who only spoke Spanish at home, while her father only spoke in English. Laura took Spanish in middle and high school, and majored in Spanish in college. After college graduation, Laura did not plan to pursue a career in teaching but struggled for a year to find work. With both parents as teachers, Laura saw teaching, particularly a career as a bilingual teacher, as a stable career option.

Laura attended a teacher preparation program located in a city with a growing emergent bilingual student population. The program is designed to prepare teachers for the complexities faced in their teaching placements. The credential program focuses on developing pre-service teachers' competencies in four areas: (a) developing an inquiry stance; (b) supporting second-language learners; (c) collaborating with faculty, peers, and mentors to continually improve practice; and (d) appreciating unique resources students bring to the classroom.

During teacher preparation, Laura took courses on language acquisition, multicultural education, child development, and mathematics methods. Course readings and assignments focused on strengths-based approaches to learning, but the context of teaching language learners explored theories and teaching practices for English language acquisition only. Theoretical underpinnings, practical applications of bilingual education, and critique of English hegemony in U.S. schools were not included in the syllabi reviewed.

As a teacher candidate working toward a bilingual authorization, Laura was one of four students in a cohort of over 60 preparing to teach in bilingual settings. During the last interview of each year of the study, Laura was asked to reflect upon the extent to which she relied on materials learned from her university courses. Laura stated that she relied very little on her teacher preparation experiences. Her coursework felt unrelated to "the realities of teaching Spanish and English to students" and learning to teach emergant bilingual students for her was rooted in "professional developments outside of and after the credential program."

Laura student-taught in a Spanish/English bilingual kindergarten classroom at Valadez Elementary School when the bilingual education program was in its second year. At the time of student teaching, bilingual classes were only offered in kindergarten and first grade and only a small fraction of the school staff were bilingual. The student demographics were also linguistically diverse. The school had a predominantly Latin@ student population (over 85%); half of the children in school were native Spanish-speakers and the other half native English-speakers. Instruction at her grade level was a 90/10 model, with curriculum instructed 90% in Spanish and 10% in English. She described her student-teaching experience as vital in her development as a bilingual teacher:

> I spoke Spanish, but the academic language needed to teach English, mathematics, and science required words I never used before. I never had to speak Spanish all day; my own Spanish improved so much during that time. Then having to consider students' Spanish and English language development while planning lessons every week…

Laura attributed the learning of academic vocabulary, analysis of the basic elements of Spanish and English language structures, and instructional strategies for dual-language development to being situated at a field site where both Spanish and English were used for instruction. While the school offered a strong bilingual literacy program, Laura noticed that the mathematics instruction at Valadez consisted of "just numbers and procedures" and varied greatly from the inquiry-based instruction of the mathematics methods course. During the summer after graduation, Laura asked her university field supervisor, Julia, for additional professional development opportunities and attended a math professional development course and made site

visits to observe Cognitively Guided Instruction.[5]

At the time of Laura's hire, the state had just adopted new state standards, and the principal at Valadez requested additional professional training for the staff. A district math coach, who happened to be Julia, Laura's university field supervisor, came weekly for a year to work with the Valadez teachers. Laura worked with Julia biweekly for the entire year. By the third year of the study, Laura had developed a reputation for her strength in mathematics teaching and was selected to develop mathematics curricula for the district. Teachers from other schools and districts interested in bilingual mathematics instruction responsive to students' thinking frequently observed Laura's classroom.

Elise. Elise wanted to be a teacher since kindergarten and was in a dual-language program herself until California's passage of Proposition 227. The passage of the proposition led to the elimination of bilingual classes and Elise's transition to English-only settings in second grade. Elise noted that she entered teaching with the intent to share critical knowledge with her Latin@ students. Deeply committed to supporting students' self-identity, Elise was actively involved in a community-mentoring program in which she received mentoring as a teenager and subsequently mentored Latina youths.

Elise attended the same teacher preparation program as Laura but a year later. Simlar to Laura, Elise was one of six bilingual teacher candidates in a cohort of over 60 preparing to teach in bilingual settings. When asked to reflect upon her teacher preparation, Elise talked about the utility of the course readings from the university math and literacy methods courses and how these resources were still referenced occassionally, but she also described her loneliness and marginalization in the program:

> It's hard. All the materials I have in the classroom (now) are translations of English textbooks to Spanish. The Spanish is off and the stories don't connect to my kids. It (was) the same in the program. Everything was about preparing us to work with students in English. How (a)bout for those of us that are not White and teach in English and Spanish?

Elise noted that the prospective teachers and professoriate in the teacher education program were predominantly White and monolingual (in English); only one of her course instructor was bilingual and a person of color. Both Elise and Laura had the same instructors for multicultural education, English language acquisition, child development, and the mathematics methods courses.

Preparation for teaching language was limited to one course devoted to "Eng-

[5] Cognitively Guided Instruction, often abbreviated as CGI, is an approach to mathematics teaching that builds on children's problem-solving strategies (see Carpenter, Fennema, Franke, Levi, & Empson, 1999).

lish language acquisition" in which she learned about specifically designed academic instruction for English (SDAIE) strategies. These language methodologies explicitly target vocabulary development through teachers' active modeling and teaching of conversational and academic language through the use of visuals, demonstrations, and hands-on learning (Dutro & Moran, 2003; Krashen, 1989). Elise's frontloading of language in which she emphasized key vocabulary and made the learning experience multi-sensory is representative.

Elise student taught in a second-grade classroom at Valadez, where the dual language program was in its third year. With her master teacher, Claudia, Elise taught the first Spanish immersion second-grade classroom at Valadez. Instruction in second grade was an 80/20 model, in which Spanish was used 80% and English 20%. Mathematics was taught in Spanish, and English as a second language class was offered to students in the afternoon.

After graduation, Elise was hired to teach in the same grade, and Claudia became her grade-level colleague. Claudia and Elise were the only bilingual second grade teachers at the school site, and Valadez was the only school in the district to offer a dual language English and Spanish immersion program. Elise worked closely with Claudia throughout the 3 years of the study. Elise discussed curricular decisions with Claudia daily and stated that she "basically did the same things" as Claudia. Claudia regularly shared lessons and student worksheets with her. Given that there were limited instructional materials available in Spanish, Elise found the resources extremely helpful.

During the 3 years of the study, Elise attended two professional development workshops on CGI offered by the district and made three site visits to observe CGI at a local school. The school also received a 1-week workshop on their new math textbook during Elise's second year of professional teaching. Elise regularly attended professional development workshops centered on discussion of language and literacy. However, none of these workshops combined discussions of language with mathematics subject-matter knowledge.

Elise talked about the school site as a place of struggle. The dominant presence of English diminished the place of Spanish in the school-wide context. Spanish was rarely heard on the playground or in the lunchroom. Even in the classroom, students often spoke in English during group work. Elise felt a responsibility to orient students to the importance of Spanish. An avid proponent of students' linguistic development, Elise intentionally structured time at the start of math instruction to work on students' disciplinary literacies in Spanish.

Discussion

The first part of the findings examined Laura and Elise's mathematics teaching and the second examined the teachers' opportunities to learn. Study findings confirm the importance of cultivating a linguistically diverse teaching force noted in previous research (e.g., Achinstein & Ogawa, 2011; Sleeter et al., 2015; Villegas & Irvine, 2010). Both Laura and Elise valued and honored students' linguistic, cultural, and experiential knowledge. Their mathematics lessons directly attended to the role of language in mathematics learning, promoted multimodal communication, and encouraged students' use of their native language. While both teachers used similar instructional strategies, how the strategies were used revealed differing orientations toward learning: learning as acquisition and learning as participation.

A widely employed conceptual lens for understanding learning is the acquisition model, which views learning as the acquisition of specific knowledge and concepts. Both teachers led lessons with the goal of supporting students to "acquire" knowledge of specific concepts (e.g., part-whole relations) and skills (e.g., subtraction regrouping). Within this model, new knowledge is acquired and accumulates through a developmental progression, building from one concept to the next, through teacher facilitation and scaffolding. Conceiving of learning as a developmental trajectory provides a useful model for building students' Cognitive Academic Language Proficiency (CALP), or formal academic language, from Basic Interpersonal Communication Skills (BICS), or day-to-day language (Cummins, 1984). The acquisition model encourages a trajectory from concrete, representational, to abstract stages for mathematical concepts. Research on CGI demonstrates the benefits of teachers' understanding of learning trajectories and pedagogical considerations responsive to students' development (see, e.g., Carpenter et al., 1999).

As Sfard (1989) has argued, however, an overemphasis on one metaphor for learning can be dangerous. Conceiving learning exclusively within the acquisition model can lead to treatment of concepts as objects of learning and separate from the context in which they are learnt. A widely endorsed teaching practice is the frontloading of vocabulary, demonstrated by Elise, in which language is taught independently of its application. The idea of concepts building upon each other has also led to viewing development as linear, to notions that certain vocabulary or mathematics concepts must be taught before students' engagement in rich mathematical talk or more challenging tasks, and to the idea that knowledge must transfer from teacher to students. This linear transfer metaphor is widespread within Western education, shapes curriculum and pedagogy, and shaped Elise's mathematics instruction. This linearity can be seen in how Elise followed a lesson structure in which she gradually decreased the amount of "scaffolding" provided through repetition until students were able to replicate the learnt action on their own.

Just as dangerous is to view learning only as participation. While the acquisition model is entrenched in what happens in individual minds, the participation model views learning as situational, indexically bound to social context (e.g.,

Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991). The participation model can embed learning so completely within a given context that the individual is lost (Sfard, 1998).

Therefore, a balanced model of learning is needed that considers both acquisition and participation in which learning emerges through interaction between the individual and the environment (Engestrom, 2001). Laura's classroom reflects this balanced approach. Laura strategically designed mathematical tasks and selected student strategies to support the acquisition of specific concepts and skills based on her knowledge of students' development. Classroom interactions supported individual and collective understandings as Laura and her students embraced and built upon the repertoire of practices available. By valuing diverse voices and perspectives, Laura created an egalitarian space for learning in which students saw themselves in what was being taught. Students who might have been dismissed as less capable in more traditional settings (e.g., developing in school-based discourse, solving problems using direct modeling, and having only partial answers) were recognized by others and saw themselves as integral to the learning community.

While both models, learning as participation and learning as acquisition, are recognized in research and practice, the acquisition model is dominant. School-based learning often follows the assumption that students should receive and learn what is clearly communicated and explicitly taught by the teacher (Freire, 1970/2000; Valenzuela, 2002). A balanced model reminds us that students' learning as well as the teachers' are bound by the elements available in the ecological system. This boundness brings to light the significance of Elise and Laura's own learning to teach in bilingual settings.

The study findings, like those by Achinstein and Ogawa (2011), challenge the cultural match assumption. As Achinstein and Ogawa (2011) argue, "just being a person of color does not guarantee effectiveness in teaching students of color, nor do new teachers of color necessarily have knowledge of pedagogy for diverse students" (p. 15). The cultural match assumption downplays the significance of teacher preparation and of the cultural, structural, and institutional racism that shape K–12 schooling.

With few exceptions, most states do not require teacher preparation programs to offer coursework specific to teaching emergent bilinguals (López, Martin, Scanlan, & Gundrum, 2013), and most teacher education programs require only one "diversity" course (Aquino-Sterling et al., 2016; Sleeter et al., 2015). The teacher preparation program attended by Elise and Laura provided more: a program mission for high-quality teaching in culturally and linguistically diverse communities, two mathematics methods courses, and a few "diversity" courses. Still, their preparation was inadequate to classroom and programmatic needs.

Analysis of the program revealed that their teacher preparation was not designed to build upon the strengths and needs of bilingual teachers. The two-

semester mathematics methods course focused on developing high-leverage instructional practices to support students' development of central mathematical concepts; however, there was little emphasis on preparing teachers to support students' development of language skills to engage meaningfully in disciplinary discourse. As well, there was an underlying assumption that these teaching practices were the "right" teaching methods and strategies for all students and all teachers. This one-size-fits-all model ignores the Western, assimilationist perspective that dominates U.S. schools (Gutiérrez, 2002; Khisty & Willey, 2008; Ovando, 2000; Sleeter, 2001) and fails to leverage the different assets and needs bilingual prospective teachers bring to teacher preparation (Arce, 2004; Gomez et al., 2009; Musanti et al., 2009; Sleeter et al., 2015).

Language is an important medium through which equity and inequities are structured and sustained in K–12 classrooms and in teacher education (Aguino-Sterling et al., 2016; Gomez et al., 2008; Khisty & Willey, 2008; Sleeter et al., 2015). The study findings, as well as others (e.g., Moschkovich, 2012; Remillard & Cahnmann, 2005; Setati, 2008; White et al., 2016), create an imperative for the development of linguistically and culturally relevant mathematics teacher education.

This study is premised on the idea that bilingual teachers are "learners and innovators with cultural and linguistic repositories ... to challenge and inspire students toward academic achievement" (Gist et al., 2015, p. 29). However, the study findings serve as a reminder that teacher educators (and researchers) must be careful not to romanticize bilingual teachers nor presume that bilingual fluency or ethnic identity alone are sufficient to challenge the subtractive underpinnings that permeates schools serving bilingual communities (Cummins, 2000; García et al., 2008; Khisty & Willey, 2008; Valencia, 2002). Laura's experience, in which a teacher educator worked with her to incorporate discursively rich mathematics instruction in English and Spanish, is not the norm. Much more attention in teacher education and the mathematics education research community must be placed on the learning opportunities of bilingual students and bilingual teachers. Those who prepare bilingual teachers for the classroom must also model teaching practices that honor prospective teachers' language and cultural knowledge because how they are taught paves the way for the possibilities they see to reimagine mathematics learning in their future classrooms.

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Appendix A

Codes – Organized Mathematics Learning for Emergent Bilingual Students

I. Language Support
    a. Teacher direct modeling (e.g, repeated use of word, emphazing pronunciation, sophisticated use of language, simplifying the language)
    b. Revoicing
    c. Student created products (e.g. word bank, definition, strategy)
    d. Gestures
        i. Student
        ii. Teacher
    e. Student analysis (e.g. compare words, strategies, word problem context)
    f. Encouragement of L1 usage
    g. Visual aids (e.g., word banks, student strategies, concept maps)
    h. Games
    i. Use of real-life context
    j. Modify existing math textbook
    k. Focus on meaning making, not just students' production of "correct" Spanish or English
    l. Ask students to analyze (e.g. compare words, concepts or meanings)

II. Knowledge Authority
    a. Teacher presents solution method
    b. Teacher provides specific tool for problem solving
    c. Teacher has final word about correct answer/solution
    d. Multiple forms of student mathematical contributions are encouraged and valued
        i. Incomplete ideas
        ii. Incorrect answers
        iii. Multiple approaches to problem solving
        iv. Multiple representations
    e. Teacher encourages students to have final word about correct answer/solution
    f. Teacher intentional selection of student contribution to minimize status among students (and specific subgroups)
    g. Position students to use one another as mathematical resource

III. Language Practices
    a. Spoken words
        i. Spanish

   ii. English
   iii. Hybridity
b. Written communication (e.g. writing on white boards, student written strategy, poster of student strategy)
c. Symbolic/Equations
d. Physical
   i. math tools,
   ii. manipulatives
   iii. realia
   iv. objects
e. Gestures
   i. Teacher
   ii. student
f. Contextual
   i. Teacher-generated
   ii. Student-generated