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Public Spending on Education and the Incentives for Student Achievement¹

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Abstract

We build a model where homogeneous workers can accumulate human capital by investing in education. Schools combine public resources and individual effort to generate productive skills. If skills are imperfectly compensated, then in equilibrium students may under-invest in effort. We examine the effect on human capital accumulation of three basic education finance policies. Increased tuition subsidies may not be beneficial because they increase enrollment but they may lower the incentives for student achievement, hence the skill level. Policies directed at enhancing the productivity of education or making degrees more informative are more successful at improving educational outcomes.

JEL Codes: I2; E6; H52. Keywords: Public spending and policy, education, student achievement, human capital

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1 Introduction

A vast economic literature has explored the role of schooling in increasing human capital and this productive function of education has traditionally motivated the government's involvement in school financing. However, a prominent issue in the current U.S. debate on education reform is an apparently weak connection between public education expenditures and educational outcomes (e.g. Hanushek, 1986, 2003-a,b). In short, public money spent on education does not appear to necessarily result in increased human capital.

The rationalizations for this phenomenon depend on how one perceives the connection between schooling and human capital creation. If one takes the view that students are to a large extent passive beneficiaries of the schooling process, then poor educational outcomes simply reflect a misallocation of educational resources.¹ This limits *per se* the students' possible attainment. However, if active student involvement is necessary to make education a productive endeavor, then poor educational outcomes might also stem from inadequate incentives for academic achievement.² This discourages student effort, hence attainment.

This paper develops a model that incorporates these complementary views to study different education financing policies. It contributes to the education debate by building intuition as to why public spending on education should be guided by considerations about students' motivation to perform. To do so we augment a standard model where education has a productive role (for example, as in Becker, 1964, or Ben-Porath, 1967) by introducing an explicit role for student effort and incentives for educational achievement. This is accomplished by drawing from recent theoretical research that has developed insights into the links between students' motivation to succeed and the equilibrium distribution of human capital (e.g. Blankenau and Camera, 2006, Sahin, 2003). In these models, students may have disparate attainment ambi-

tions not simply due to innate differences (in ability or motivation) but rather because of the expected benefit from augmenting their own skill.

We build a general equilibrium model with finitely-lived homogeneous workers who can raise their productivity by investing in schooling. Agents can have different motivations for earning a degree since—due to imperfect information—the less productive graduates can be overcompensated at the expense of the more productive. When skills are not perfectly compensated, not every student will make education a productive endeavor. Thus, different education finance policies affect not only school enrollment but also the students’ incentives to perform.

We study the effects of government education spending on three key measures of policy performance: enrollment, the skill level of the workforce, and welfare. Since resources can be used in different ways we consider three basic types of policies. The first involves lessening the private cost of education, for example via tuition subsidies. The second involves school funding directed at raising the productivity of education, for example facilitating the process of learning by hiring more specialized teachers, buying better equipment or reducing class sizes. The third policy involves using resources to enhance the informativeness of academic certificates, for example by developing better testing procedures, or fighting ‘grade inflation.’

The analysis progresses in three steps. We show how each policy affects the student’s incentives for academic achievement. Then, we contrast each policy’s impact on equilibrium enrollment and skill level when incentives are weak, and when they are strong. Finally, we discuss welfare implications of each policy.

Our analysis shows that fostering human capital accumulation is not simply a matter of spending public resources to raise enrollment. In fact, when incentives for student performance are weak some policies that are successful in raising enrollment may have negative consequences

on educational outcomes and aggregate productivity. An example is relying too heavily on subsidizing the private cost of education at the expense of enhancing the process of learning. If student's motivation to achievement is weak, such policies sustain equilibria where it is individually optimal to earn a degree while choosing to accomplish little. This wastes resources and it also degrades the overall level of educational outcomes. Improving the quality of education is a more effective policy, because it raises the expected return from schooling.

These findings add to the debate on education reform by calling attention to the dangers of ignoring the role of incentives for educational attainment. If education is thought to be necessarily productive—as it is in the standard model of human capital accumulation—any type of government spending that can successfully encourage enrollment will also be effective in fostering human capital accumulation. Policy outcomes can be very different if the productivity of education hinges also on student effort. In this case it is desirable to avoid financing education in ways that lessen the students' motivation to perform.

2 The Model

We use an overlapping generations model where ex-ante homogenous young agents can enhance their future productivity via education financed by borrowing (e.g., as in Fender and Wang, 2003). In this environment any heterogeneity in educational attainment or productivity must necessarily stem from optimal behavior of individuals as opposed to innate differences. This feature allows us to more clearly pin down the link between various economic incentives and the emergence and extent of equilibrium heterogeneity in human capital accumulation. We later demonstrate that key results are robust to several forms of initial agent heterogeneity.

At each date a unit mass of two-period lived agents is born and endowed with one unit of

unskilled labor. Young agents can either enjoy leisure, yielding zero utility, or can undertake a one-period educational opportunity (go to school) that is costly (tuition) and requires effort (study). Old agents inelastically supply labor to one of many competitive firms (each of which produces an identical consumption good) and consume. A student's lifetime utility is

$$U = -e + \beta E \ln c$$

where $e > 0$ is disutility from effort, $0 < \beta \leq 1$ is the discount factor, c is consumption, and E is the expectation operator (we omit time subscripts for simplicity). Tuition $T > 0$ is financed by borrowing at gross rate R . Letting I be second period net income, the agent's second period budget constraint is $c + TR = I$.³ Thus, an agent who does not go to school has utility $\beta E \ln c$, where $c = I$.

The school transforms student effort e into a degree and $z(e)$ productive skills where

$$z(e) = \begin{cases} z & \text{if } e = e_s + e_d \\ 0 & \text{if } e = e_d, \end{cases}$$

and we normalize $z \geq 1$. That is, if a student selects high effort, $e_s + e_d$, she earns a degree and z units of *skilled* labor. By selecting low effort, e_d , a student earns a degree, but no skill. So, an old can be in one of three states: *skilled* (with degree and skills), *schooled* (degree but no skills), or *unschooled* (no degree and no skills), denoted by s , d , and u .⁴

As in Blankenau and Camera (2006) firms observe degrees, but recognize a worker's productivity with probability $\theta \in [0, 1]$. The parameter θ can be thought of as gauging the extent to which grades, letters of recommendation, and other supplemental information succeed in communicating the productivity of school graduates. Finally, to introduce policy we assume a government that finances education expenditure via income taxation. The government is

an inanimate entity that interacts with agents for the sole purpose of implementing taxes and transfers—a typical abstraction in the macroeconomic literature (e.g., see the discussion in Boel and Camera, 2006). The model proposed is thus equally appropriate for considering the final years of K-12 education or college, because in both cases educational choices are influenced by resource and effort costs associated with them.⁵

3 Stationary Symmetric Equilibria

We focus on Nash equilibria where strategies are time-invariant and symmetric. We start by studying the agents' choices. Define ω_u and ω_s as the (endogenously determined) market values of a unit of unskilled and skilled labor, and let ω_k be the wage paid to a worker whose skill is unrecognized. For the purpose of analytical tractability, we also assume that tuition T is proportional to the skilled wage and takes the form $T = \rho\omega_s$, with $\rho > 0$.⁶

The government taxes at rate τ all income net of education expenditures. Therefore uneducated workers have disposable income $(1 - \tau)\omega_u$. If productivity is observed, schooled workers receive the same income as unskilled workers. Thus, after education expenses, a recognized schooled worker has disposable income $(1 - \tau)(\omega_u - R\rho\omega_s)$. A skilled worker of recognized skill has net income $(1 - \tau)(z\omega_s - R\rho\omega_s)$.

Consumption must be positive and is the lowest for schooled workers who have been recognized. Thus $c > 0$ requires $\tau < 1$ and $R\rho\omega_s < \omega_u$. We show in the appendix that both conditions hold if z is sufficiently large. In the remainder of the paper we assume

$$z > \underline{z}, \tag{1}$$

where \underline{z} is defined in the appendix.

Any worker whose productivity is unrecognized receives $(1 - \tau)(\omega_k - R\rho\omega_s)$. Given this,

the expected lifetime utility of being skilled, schooled, or unschooled satisfies:

$$\begin{aligned}
V_s &= -(e_s + e_d) + \beta\{\theta \ln[(1 - \tau)\omega_s(z - R\rho)] + (1 - \theta) \ln[(1 - \tau)(\omega_k - R\rho\omega_s)]\} \\
V_d &= -e_d + \beta\{\theta \ln[(1 - \tau)(\omega_u - R\rho\omega_s)] + (1 - \theta) \ln[(1 - \tau)(\omega_k - R\rho\omega_s)]\} \\
V_u &= \beta \ln [(1 - \tau)\omega_u].
\end{aligned} \tag{2}$$

The first line in (2) shows that a skilled agent suffers effort disutility $-(e_s + e_d)$ when young. The agent's income as a worker is uncertain, since he is recognized and paid as a skilled worker only with probability θ , and compensated as an unrecognized worker otherwise. This feature allows us to effectively capture the empirically relevant notion that there are costs to correctly assess a worker's productivity.⁷ So, wage uncertainty arises from stochastic compensation for ability rather than stochastic ability (as in, for example, Eckwert and Zilcha, 2007).

The expression for V_d shows that a schooled worker suffers small effort disutility when young, $-e_d$. If the worker goes unrecognized, with probability $1 - \theta$, then he receives the same income as an unrecognized skilled worker. If recognized, he receives the same income as an unskilled worker. By virtue of having no degree, firms recognize unschooled agents as being unskilled, hence their lifetime utility is V_u .

The agent's education strategy is a pair $(\delta', \sigma') \in [0, 1]^2$, i.e., the probabilities of going to school and of exerting high effort while in school (to acquire skill). Optimal schooling and effort choices are made taking as given the choices (σ, δ) of others, and satisfy

$$\delta' = \arg \max_{\varphi \in [0,1]} \varphi[\max\{V_s, V_d\} - V_u], \tag{3}$$

$$\sigma' = \arg \max_{\varphi \in [0,1]} \varphi(V_s - V_d). \tag{4}$$

An agent attends school only if this improves his expected lifetime utility. So, there is indifference to schooling if $V_u = \max\{V_s, V_d\}$, in which case $\delta' = [0, 1]$. Once in school it is individually optimal to become skilled if $V_s > V_d$, there is indifference when $V_s = V_d$, or else the student exerts low effort ($V_s < V_d$). Education and skill choices are symmetric when

$$(\sigma', \delta') = (\sigma, \delta). \quad (5)$$

In that case, $\delta\sigma$ represents the fraction of the workforce that is skilled and educated, $\delta(1 - \sigma)$ represents the fraction of the workforce that is educated but unskilled (i.e. the schooled agents), and $1 - \delta$ is the fraction of the workforce that is uneducated and unskilled.

3.1 The Firm's Problem

A firm is a technology that creates y output according to the CES function

$$y = [\alpha n_s^\phi + (1 - \alpha)n_u^\phi]^{\frac{1}{\phi}}.$$

Here, n_s is the effective units of skilled labor employed by the firm, n_u is units of unskilled labor, $\alpha \in (0, 1)$ and $\phi \leq 1$. This function displays constant returns to scale, so we consider a representative firm that hires all labor. In this case, n_s and n_u are the aggregate quantities of skilled and unskilled labor inputs and y is total output.

The firm's objective is to maximize expected profits by hiring workers of three types. Let ℓ_j denote the demand for workers of type $j = u, s, k$, i.e., workers known to be unskilled, known to be skilled, and of unknown skill level. In an outcome where (5) holds, the variable σ is the probability that an educated worker of unrecognized ability has skill. Workers with skill provide z units of skilled labor while all others provide one unit of unskilled labor. Hence,

$$n_s = z\ell_s + z\sigma\ell_k \quad \text{and} \quad n_u = \ell_u + (1 - \sigma)\ell_k. \quad (6)$$

The firm's problem is thus

$$\max_{\ell_s, \ell_u, \ell_k} \{[\alpha n_s^\phi + (1 - \alpha)n_u^\phi]^{\frac{1}{\phi}} - (\omega_s z \ell_s + \omega_u \ell_u + \omega_k \ell_k)\},$$

and the first order conditions give

$$\omega_u = \frac{(1-\alpha)y/n_u}{\alpha\left(\frac{n_s}{n_u}\right)^\phi + 1 - \alpha}, \quad \omega_s = \frac{\alpha y/n_s}{\alpha + (1-\alpha)\left(\frac{n_u}{n_s}\right)^\phi}, \quad \text{and } \omega_k = \sigma z \omega_s + (1 - \sigma)\omega_u. \quad (7)$$

So, if a worker's productivity is observed his wage equals his marginal product; otherwise, it equals the expected marginal product.

Due to market clearing ℓ_j must equal the supply of type j workers. In each period $\delta\sigma\theta$ workers are recognized as skilled, $\delta\sigma(1 - \theta)$ are skilled but unrecognized, $\delta(1 - \sigma)(1 - \theta)$ are unskilled and unrecognized. The remainder are known to be unskilled. Hence,

$$\ell_s = \delta\sigma\theta, \quad \ell_k = \delta(1 - \theta) \quad \text{and} \quad \ell_u = 1 - \ell_s - \ell_k. \quad (8)$$

From (6) and (8) then the optimal quantities of skilled and unskilled labor employed are

$$n_s = z\delta\sigma \quad \text{and} \quad n_u = 1 - \delta\sigma. \quad (9)$$

Given (7) and (9), the choices of a worker's education cohort affect the return to schooling for the representative individual, as aggregate choices affect equilibrium wages.⁸ This is clearly seen if, for example, we let $\phi \rightarrow 0$ so the production function is Cobb-Douglas, and

$$\omega_u = (1 - \alpha) \left(\frac{z\delta\sigma}{1 - \delta\sigma}\right)^\alpha, \quad \omega_s = \omega_u \frac{\alpha(1 - \delta\sigma)}{z\delta\sigma(1 - \alpha)}, \quad \omega_k = \sigma z \omega_s + (1 - \sigma)\omega_u. \quad (10)$$

3.2 Government Policy and Definition of Equilibrium

Government education expenditure per student is assumed proportional to the wage of a skilled worker, being $\gamma\omega_s$ per student or $\delta\gamma\omega_s$ in total.⁹ Here $\gamma > 0$ is the policy instrument

that determines the amount of education spending. Recall that income net of education expenditures is taxed at rate τ . Since aggregate income is y and $\delta R\rho\omega_s$ are total resources spent on education, the government balanced budget rule is

$$\tau(y - \delta R\rho\omega_s) = \delta\gamma\omega_s, \quad (11)$$

which pins down the tax rate τ as a function of the parameter γ and of endogenous variables.

Definition. *Given the policy parameter γ , a symmetric stationary Nash equilibrium is a time-invariant list of education strategies $\{\delta, \sigma\}$, labor demands and wages $\{\ell_j, \omega_j\}_{j=n,s,u}$ that satisfy (2) through (8) and of government taxes τ that satisfy (11).*

We can identify two broad classes of outcomes, according to the values taken by (σ, δ) . The first class displays equilibrium homogeneity of workers, i.e. $\sigma\delta = 0, 1$. Here, either no one goes to school, $\delta = 0$, or no student acquires skills, $\sigma = 0$, or everyone goes to school and earns skill, $\sigma\delta = 1$. The second class of outcomes, instead, displays heterogeneity in skill or in educational attainment, i.e. $\sigma\delta \in (0, 1)$. Three possible types of equilibria belong to this class. A first possibility is $(\sigma, \delta) = (1, \Phi)$, where $\Phi \in (0, 1)$, in which case we say that there are strong incentives for student achievement. Here, not all workers go to school, since $\delta < 1$, but all those who do go raise their productivity, since $\sigma = 1$. Alternatively, when the incentives to achieve are weak, there can be two types of outcomes characterized by $\sigma = \Phi$. Some agents (or everyone) may go to school, but not every student chooses to improve his skill; that is $(\sigma, \delta) = (\Phi, 1)$ or $(\sigma, \delta) = (\Phi, \Phi)$ (meaning that $\sigma, \delta \in (0, 1)$).

Similar to Blankenau and Camera (2006) one can prove that existence of outcomes of the first and second class hinges on the parameter values. What especially matters is the extent of informational frictions. Because our objective is to study the impact of education

policy in economies characterized by workers' ex-post heterogeneity, we will focus on studying equilibria of the second class, i.e., where $\delta\sigma \in (0, 1)$. We proceed as follows. First, we study existence and characterization of equilibria, specializing the analysis to the Cobb-Douglas production function case, which allows a clean analysis, hence generates clear intuition. So, all propositions that follow involve environments that satisfy the following:

Assumption 1. Let $\phi \rightarrow 0$.

We then follow up with a comprehensive numerical study that builds on the analytical findings, and validates the intuition developed in the propositions.

3.3 Existence of Equilibria with Heterogeneity

We start by proving that an equilibrium exists in which workers are heterogeneous in skill or educational attainment. Proofs and definitions of critical values are in the Appendix.

Proposition 1. *If θ is sufficiently large then an equilibrium exists and it is unique. Specifically there exist two critical values $0 < \underline{e} < \bar{e}$ such that: (i) If $e_d \geq \bar{e}$ then $(\sigma, \delta) = (1, \Phi)$; (ii) If $\underline{e} < e_d < \bar{e}$ then $(\sigma, \delta) = (\Phi, \Phi)$, and (iii) If $e_d \leq \underline{e}$ then $(\sigma, \delta) = (\Phi, 1)$.*

There is always equilibrium skill heterogeneity because, when Assumption 1 holds, (10) implies $\lim_{\sigma\delta \rightarrow 0} \omega_s = \infty$ and $\lim_{\sigma\delta \rightarrow 1} \omega_u = \infty$. In short, both skilled and unskilled workers are necessary for production. So, equilibrium wages for workers of skill j become unbounded as the proportion of these workers vanishes. When information problems are not too severe, an equilibrium of one of these three types always exists, and it is unique. Since the production function is continuous in ϕ , an equilibrium should also exist for values of ϕ close to zero. Our numerical analysis confirms this (next section).

We discuss the role of θ and e_d in sustaining equilibrium, separately. If θ is close to zero,

then the more productive school graduates are almost always under-compensated and the less productive are almost always over-compensated. Thus, there is little incentive to earn skill and only a small fraction of students chooses to make education productive. This implies low output, hence low income. In short, there are not enough resources in the economy to support an equilibrium with education financed by private and public funds. Thus, to sustain equilibrium we need a sufficiently large proportion of skilled workers, hence a sufficiently large θ (see the proof of Proposition 1).

To understand the role played by model parameters it is useful to report (from equation (18) in the Appendix) that $\sigma \in (0, 1]$ whenever

$$\ln \left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s} \right) \geq \frac{e_s}{\theta\beta}. \quad (12)$$

A strong inequality reflects the presence of strong incentives for academic achievement and implies $\sigma = 1$. Otherwise, the incentives are weak and $\sigma = \Phi$. These incentives depend on the expected compensation of skill in an intuitive way; they rise when the market either expects higher skilled wages (higher ω_s) or a more accurate compensation of productivity (higher θ).

To see it, recall that—due to the model’s imperfect information—the less productive graduates are on average over-compensated at the expense of the more productive. Thus, consider the left hand side of (12). It reports a ratio reflecting the net benefit from education to skilled and unskilled school graduates, when correctly compensated (i.e. when their productivity is recognized). We have $\sigma = 1$ only when the inequality is strict, which is when there is a sufficiently large relative benefit from earning skill. Of course this size requirement hinges on the frequency of incorrect compensation, which is why θ appears on the right hand side of (12). For example, skills are often under-paid when θ is small in which case $\sigma = 1$ only if

skilled workers earn a lot when their productivity is recognized.

This discussion helps us understand the role played by the effort parameter e_d . When $e_d \geq \bar{e}$ students must make a considerable effort *just* to earn a degree so that the return from schooling must be sufficiently high. In equilibrium this is possible only if schooling is undertaken with the objective to raise own productivity, i.e. $\sigma = 1$. Here, the expected compensation of an unskilled graduate does not justify the cost and effort that goes into earning a degree.¹⁰ The temptation to graduate without skill is also minimized because when $\sigma = 1$ productivity is always correctly compensated as ownership of a degree indicates skill. Of course, since unskilled labor is a necessary input in the production function, equilibrium wages adjust in such a way to provide incentives for some agents to avoid schooling altogether. This explains why although $\sigma = 1$ we also have $\delta = \Phi$.

As e_d falls below \bar{e} , education is not always productive because there is a stronger temptation to earn a degree *only* as a means to falsely suggest higher productivity. Clearly, as e_d falls it takes less effort to free-ride off the skills of others by earning a degree. In addition, as e_d falls there is higher enrollment (see (17) in the appendix), which lowers the relative expected wage of the more productive graduates, all else equal. These two effects reduce the students' incentives to be high-achievers. Thus, when $\underline{e} < e_d < \bar{e}$ we have $(\sigma, \delta) = (\Phi, \Phi)$, so that a class of educated yet low-productive workers emerges. The key message is that the effect of student incentives on human capital accumulation depends on the relative impact of two opposing effects on schooling and effort choices. To be precise, when it is harder to earn a degree, fewer will earn it (a negative extensive margin effect), but those who do are more likely to be skilled (a positive intensive margin effect). If the intensive margin dominates, then policies that increase e_d also motivate students to be higher achievers. This message is

consistent with theoretical and empirical results suggesting that high graduation standards and exit exams tend to lower enrollment but raise achievement.¹¹

When e_d falls below \underline{e} the schooling process is so effortless that everyone enrolls in school and $(\sigma, \delta) = (\Phi, 1)$. Here, workers are all educated but unequally productive, an outcome which brings to mind the education signaling literature (e.g. Arrow, 1973, Spence, 1973, and Stiglitz, 1975). As in that literature, in our model agents may optimally choose to acquire an academic certificate even if doing so does not raise their productivity. But this emerges for different reasons than in signaling models. There, agents with higher innate ability spend on unproductive education to *manifest* their higher productivity. This motive is absent from our model where ex-ante homogeneous agents buy degrees only to *obscure* their low productivity, much as low-productive agents do in a pooling equilibrium of signaling models.

4 Policy implications

We now study the effects of government spending on education, focussing on three key measures of policy performance: enrollment δ , the workforce's average skill level $\sigma\delta z$, and welfare. We model changes in education spending as changes in the exogenous policy parameter γ . Since resources can be used in different ways we consider three types of policies.

The first policy is to subsidize the private cost of education. Here, an increase in γ finances tuition subsidies that lower the private cost of education parameter ρ , while leaving school funding unchanged. The second policy is to improve the productivity of education. Here a higher γ is associated with higher overall school funding, leaving unchanged the private cost of education. These resources can be spent to make it possible to learn a wider set of productive skills¹² (modeled by assuming z increases in γ) or to facilitate learning a fixed set

of skills more easily (modeled by assuming e_s falls in γ). Examples are hiring more specialized teachers, reducing class sizes or buying better equipment. The third policy is to use resources to improve the informativeness of academic certificates. This corresponds, for example, to implementing better testing procedures, and is modeled by assuming a positive association between θ and γ .

The main message of the analysis (reported below), is that fostering human capital accumulation is not simply a matter of spending public resources to raise enrollment. The reason is that policy outcomes depend significantly on the existence (or lack) of incentives for student performance. Consequently, we find that policies designed to raise the workforce's productivity by focusing only on raising enrollment, may have unintended consequences when incentives for student achievement are weak.

The analysis proceeds as follows. We first provide intuition on how each policy affects the incentives for academic achievement. Then, we contrast the policy's impact on equilibrium enrollment and skill level when incentives are strong as opposed to when they are weak. Finally, we discuss welfare implications. At each step we provide intuition for an analytically tractable economy (Assumption 1 holds), and then numerically study general economies.

4.1 On the Incentives to Earn Skill

Proposition 1 has clarified that the incentives for academic achievement are strong only if $e_d \geq \bar{e}$, as only in this case is education always productive ($\sigma = 1$). For this reason, here we examine the impact of the parameters (ρ, z, e_s, θ) on the critical value \bar{e} . We say that a policy improves the incentives for academic achievement if it can lower \bar{e} . In this case a larger set of e_d values can sustain the equilibrium where $\sigma = 1$. We have the following result:

Proposition 2. *In equilibrium $\frac{\partial \bar{e}}{\partial \rho} < 0$, $\frac{\partial \bar{e}}{\partial z} > 0$ and $\frac{\partial \bar{e}}{\partial \theta} < 0$. The sign of $\frac{\partial \bar{e}}{\partial e_s}$ is ambiguous.*

The model indicates that improving the informativeness of academic certificates is the most effective way to encourage academic achievement. In particular, public money spent simply on tuition subsidization can go in the *opposite* direction, since $\frac{\partial \bar{e}}{\partial \rho} < 0$.

To understand why, consider that by lowering ρ the private cost of schooling falls. This has two effects, the first of which works its way through the labor market. As ρ falls enrollment grows (conversely, $\frac{\partial \delta}{\partial \rho} < 0$ as shown below) which in equilibrium lowers skilled relative to unskilled wages. This lowers the student's motivation to perform. Also, as ρ falls the net payoff from schooling rises for every student. However, this beneficial effect is stronger for the less productive graduates, which further weakens the motivation to earn skill. Technically, the left hand side of (12) falls as ρ falls.

Interestingly, we have a similar result when education is more productive, since $\frac{\partial \bar{e}}{\partial z} > 0$. The reason is that higher productivity raises the incentive to enroll in school, $\frac{\partial \delta}{\partial z} > 0$. In general equilibrium, the skilled wage falls relative to the unskilled wage when z or δ increase. Therefore, when z grows the return from studying to earn skill falls relative to the return expected from going to school to simply earn a degree. Technically, the left hand side of (12) falls as z increases. As explained earlier, this lowers the students' motivation to perform.

The opposite occurs when public resources are used to increase the information content of a degree, since $\frac{\partial \bar{e}}{\partial \theta} < 0$. This is because as θ rises workers are more frequently compensated correctly. This raises the attractiveness of earning skill (technically, the right hand side of (12) falls with θ). We show below that a higher θ contributes to increased enrollment so in equilibrium we have $\frac{\partial \delta}{\partial \theta} > 0$. The skilled wage falls as a result, but this can be proved to have

a weaker effect on the incentive to earn skill.

Finally, improvements in the learning process (lowering e_s) have an ambiguous effect on \bar{e} . On one hand earning skill is more attractive when this requires less effort (the right hand side of (12) falls). However, as more agents acquire skill, wages adjust as discussed above. This reduces the incentives to earn skill (the left hand side of (12) falls).

4.2 On Human Capital Accumulation

Having seen how different policies affect the incentives to achieve skill, we examine their effect on the equilibrium average skill level, $\delta\sigma z$. To do so we will complement the analytical results with a series of numerical experiments on more general economies. Specifically we consider both the case where $\phi = 0$ and $\phi = .25$. The baseline parameters of these economies will be as follows: $R = 1$, $\beta = 1$, $\alpha = \theta = .5$, $e_d = .05$, $e_s = \rho = .2$, and $z = 5$.

4.2.1 Tuition Subsidies

A first result of our analysis is a clear warning for policymakers. Lowering the private cost of education by means of public subsidies can raise the skill level of the workforce only if students are motivated to perform well. Otherwise, the effect can be exactly the opposite.

Proposition 3. *In equilibrium $\frac{\partial\delta}{\partial\rho} < 0$ when $\delta < 1$. However, if $\sigma = \Phi$ then $\frac{\partial\delta\sigma}{\partial\rho} > 0$.*

Figure 1 reports the equilibrium share of the population with skill for a Cobb-Douglas function (denoted $\phi = 0$) and for $\phi = .25$ (darker lines) and welfare for the case where $\phi = 0$ (thin line) in economies with different degrees of subsidization of the cost of education. Moving left to right we trace economies with an increasingly higher private cost of schooling (implying lower subsidies). Recall from Proposition 1 that ρ affects the critical values for the areas of existence of the different types of equilibria. This is why $(\sigma, \delta) = (\Phi, 1)$ if ρ is low, and as ρ

grows we obtain $(\sigma, \delta) = (\Phi, \Phi)$ and subsequently $(\sigma, \delta) = (1, \Phi)$. This result seems consistent with the U.S. experience where enrollment is near one hundred percent in $K - 12$ (where the private cost of education is low) while enrollment falls substantially in college (where the private cost of education is higher).¹³

Proposition 3 shows that subsidies can be effective in increasing schooling rates, a finding that is consistent with a great deal of empirical work (e.g., see Dynarski 2000, 2003). What is interesting is that despite providing incentive for greater schooling, subsidies can nonetheless backfire on the human capital accumulation dimension. That is to say, they can lead to a reduction in the aggregate supply of skills, hence a decrease in aggregate productivity. To see why, notice that in our model, for both values of ϕ the equilibrium fraction of skilled workers $\delta\sigma$ is hump-shaped (while z is a constant). When incentives are strong (occurring when $\sigma = 1$ for $\rho > .5$, in this example) a reduction in ρ increases the skill level. As in models that ignore incentives for achievement, more subsidies are associated with more graduates, which in turn raises the average skill level. However, when there are weak incentives for attainment (which occurs when $\sigma = \Phi$, for $\rho < .5$), our model reveals the weak link in this chain of events: more graduates are associated with *less* skill. Of course, as ρ falls enrollment still rises. However, a smaller share of students makes education a productive endeavor, so the skill level falls. Intuitively, cheaper education creates a stronger temptation to earn a degree with the least possible effort, in order to benefit from compensation imperfections.

Figure 1 shows that the qualitative relationship between ρ and $\delta\sigma$ is preserved for $\phi > 0$. The key change is an upward shift of the $\delta\sigma$ curve, as skilled labor and unskilled labor become more substitutable (the curve shifts downward if $\phi < 0$).

[Figure 1 approximately here]

This link between schooling costs, student incentives, and educational outcomes brings to mind a remark of Milton Friedman (1968) who lamented: “Our state colleges and universities are burdened with youngsters who value the schooling they are receiving at what they pay for it—namely zero.” Recent analyses support this view. Sahin (2003) finds that subsidizing tuition boosts enrollment but reduces student effort, hence human capital accumulation. The theoretical analysis in Blankenau and Camera (2006) suggests that when a worker’s productivity is imperfectly recognized, low cost education might support lower skill accumulation.

4.2.2 Productivity of Education

A second finding is that spending directed at increasing the productivity (or quality) of education has beneficial effects on average human capital accumulation *even if* students are not strongly motivated to perform well.

Proposition 4. *In equilibrium $\frac{\partial \delta}{\partial z} > 0$ when $\delta < 1$ and $\frac{\partial \delta \sigma z}{\partial z} > 0$ always. However, if $\sigma = \Phi$ then $\frac{\partial \delta \sigma}{\partial z} < 0$. Finally, $\frac{\partial \delta \sigma}{\partial e_s} < 0$ always.*

Consider spending that increases z , so students can now learn a greater set of skills. This raises enrollment—as indicated in Proposition 2—since the payoff to skill and schooling both increase in z . However, it does not always result in a higher fraction of skilled population $\delta \sigma$. When there are strong incentives for academic achievement, $\sigma = 1$, we have $\frac{\partial \delta \sigma}{\partial z} > 0$. When incentives are weak, the relationship is reversed and $\frac{\partial \delta \sigma}{\partial z} < 0$. Figure 2 illustrates this finding.

[Figures 2 and 3 approximately here]

This result is perhaps surprising. Higher z raises a skilled worker’s income (hence the

return to skill), so one might expect more agents will choose skill, even if incentives are weak. Two effects work to counter this intuition. General equilibrium adjustments cause ω_s to fall in z , so the return to skill increases, but not very much. Second, average compensation of the less productive graduates grows in z , so there is a greater temptation to earn a degree but not skill. With $\phi = 0$ this second effect is dominant, so fewer agents earn skill when z rises. This does not imply that the average skill level falls. In fact, the positive intensive effect of greater z (higher per-capita productivity) always dominates the negative extensive effects (smaller skilled population). Hence, $\delta\sigma z$ and z are positively associated. This holds also when $\phi = .25$ (Figure 2). Indeed, when skilled and unskilled labor are more substitutable (ϕ is larger) changes in z have a smaller negative effect on ω_s . The increase in income for a skilled worker is sufficient to induce more skill, even with weak incentives. Hence, raising the productivity of education is more effective when skilled and unskilled labor are more substitutable.

Now, suppose that the quality of education is improved by reducing the effort required to achieve the skill level z . That is, resources are spent to lower e_s . Proposition 2 and Figure 3 indicate that this policy is effective in raising the economy's skill level, as it lowers the opportunity cost of human capital investment. This induces both higher enrollment and higher incidence of skill achievement among students. With $\phi = .25$, the relationship is similar.

Our findings contribute to a literature concerned with how the allocation of education resources can impact teachers and administrators' incentives to effective instruction (Hanushek, 1994). Introducing school choice, performance contracting, or merit pay can all be seen as ways to improve the quality of education, which in our model loosely corresponds to lower e_s and higher z . Our analysis shows that an additional benefit of education quality-enhancing policies could be improved student effort.

4.2.3 Improved Testing

Now, consider spending directed at improving the informativeness of academic certificates.

Proposition 5. *In equilibrium $\frac{\partial \delta \sigma}{\partial \theta} \geq 0$ always.*

If the incentives for academic achievement are weak, then spending that improves testing is always beneficial in raising students' attainment, hence the workforce's skill level. Clearly, if students' skills are more easily recognized, then there is lower incidence of both under- and over-compensation in those equilibria where $\sigma = \Phi$. This policy raises the incentive to earn skill hence $\frac{\partial \delta \sigma}{\partial \theta} \geq 0$. Figure 4 demonstrates that this holds true even when ϕ is positive.

[Figure 4 approximately here]

The information content of degrees might be increased in a number of ways. For example, it is reasonable to presume that if grade inflation¹⁴ can be lessened, then there should be an improvement in the usefulness of grades in differentiating the ability of graduates. This can be seen as corresponding to an increase in θ . The current focus on standardized testing in the national education debate (Hanushek, 1994) can also be interpreted as attempt to raise θ in our model. Much of the literature on testing focuses on the need to identify productive teachers, schools and administrators. Our model highlights that improved testing can have an additional beneficial effect by increasing student effort.

4.3 On Welfare

We evaluate social welfare using the standard measure of ex-ante utility

$$W = \delta \sigma V_s + \delta(1 - \sigma)V_d + (1 - \delta)V_u.$$

Unfortunately, there is no clear-cut answer as to how welfare responds to different education

finance policies. The reason is that spending on education is financed via income taxation. This does not distort labor decisions (labor is inelastically supplied), but it does reduce disposable income, hence consumption. Thus, whether a given policy is successful at raising welfare depends on the impact of public spending on average productivity.

To see it, let $\phi \rightarrow 0$ and focus on the equilibrium in which only some agents go to school, i.e., $\delta = \Phi$. Here we have either $V_u = V_s = V_d$ and $\sigma = \Phi$, or $V_u = V_s$ and $\sigma = 1$. Either way $W = V_u$, so (2) and (10) imply

$$W = \beta(1 - \tau)(1 - \alpha) \left(\frac{z\delta\sigma}{1 - \delta\sigma} \right)^\alpha. \quad (13)$$

In short, in every equilibrium with $\delta = \Phi$ welfare grows in the average skill level $z\delta\sigma$. Our prior results, then, indicate that for a given τ , welfare grows in z and θ grow and falls in e_s . The problem is that desirable changes in these variables require greater public spending, i.e., a greater tax rate τ . This lowers disposable income, so the effect of increased education spending on welfare hinges on how greater spending affects the parameters z , θ , and e_s .

For example, welfare would not respond positively to greater spending on education that has limited impact on z and e_s . Indeed, some observers noted that educational outcomes do not improve with greater government expenditures simply because the productivity and quality of education is unresponsive to increments in funding. In contrast, much of the current discussion focuses on ways to improve efficiency (increase z or decrease e_s) at current funding levels, often through more market-based approaches to education (Hanushek and Jorgenson, 1996). Success along these lines would be unambiguously welfare improving.¹⁵

Welfare gains are perhaps better achieved via policies that raise the “informativeness” of academic certificates, which could be cheaply implemented. For example, reducing grade infla-

tion may require little or no funding but may significantly increase the information conveyed by degrees. In our model, this corresponds to a significant increase in θ , which would clearly raise welfare. Numerical experiments for the case $\delta = 1$ suggest similar trade-offs.

Next consider a policy of tuition subsidies, which can be welfare-improving when incentives for student achievement are strong (since $z\delta\sigma$ falls in ρ). However, when incentives are weak, subsidies not only increase the tax burden but also lower $z\delta\sigma$. To demonstrate the point, suppose that private and public spending are linked by the relationship $\gamma(\rho) = S - \rho$ where S represents a fixed level of public education expenditures. Figure 1 reports (a monotone transform of) W for $S = 2$ as a function of ρ . Moving right to left, there are initial welfare gains as subsidies increase, because more degrees mean more productive workers and this gain dominates the loss from the increased tax burden. When $(\sigma, \delta) = (\Phi, \Phi)$ more subsidies are harmful as both effects work against welfare. Welfare continues to fall as we move into the equilibrium $(\sigma, \delta) = (\Phi, 1)$ as the average skill level continues to fall.

5 Robustness

The results above are obtained within the context of a model that contains several abstractions. In this section we establish that our findings are robust to assuming more general, and perhaps realistic, settings. We extend our model in two directions. First, we introduce ex-ante heterogeneity in innate ability or in wealth. Then, we assume workers' skills can be recognized. Analysis of these generalizations can be summed up as follows: the findings associated with the basic stylized model emerge also from studying more sophisticated settings that preserve some key features (discussed below).

5.1 Ex-ante Heterogeneity

We introduce two alternative modes of ex-ante heterogeneity. On the one hand, we wish to consider agents who differ in their innate ability, i.e., they must exert different effort in order to acquire the same skill level. Heterogeneity of this sort captures the observation that the return to education is higher for more able people (e.g., Carneiro and Heckman, 2004). On the other hand, we also wish to consider economies where agents face unequal constraints in financing of their education. To do so, one could assume borrowing constraints or differences in family wealth. We follow this second modeling avenue, for two main reasons. It allows us to retain the model's basic structure, and yet introduce heterogeneity in constraints on educational expenditures. Second, differences in family wealth are often seen as an empirically more relevant form of heterogeneity than credit constraints.¹⁶

To formalize these sources of heterogeneity, we modify the model as follows. Considering heterogeneity in innate abilities, denote individual ability level by $i \in \mathcal{I}$, with \mathcal{I} an ordered set. Assume agents with ability $i' < i$ must exert more effort than agents i to attain the same skill level z . Formally, let $0 < x_i < x_{i'} < \infty$ for all $i' < i$. Then, for agent of ability i define

$$z(e_i) = \begin{cases} z & \text{if } e_i = x_i(e_s + e_d), \\ 0 & \text{if } e_i = x_i e_d, \end{cases}$$

Given ability i , the agent's expected utility is thus $U_i = -e_i + \beta E \ln c_i$.

Now consider wealth heterogeneity. Denote family wealth by $j \in J$, where J is also an ordered set, and assume that wealthier families finance a larger portion of their children's education. Formally, if wealth is $j' > j$, then we assume $\rho_{j'} < \rho_j$. Suppose that the cdf $G(i)$ and $F(j)$ identify, respectively, the distribution of abilities and wealth.

Given the above, an agent's initial state is identified by a pair $(i, j) \in \mathcal{I} \times J$ and so the

value functions V_s and V_d in (2) are modified as

$$V_s(i, j) = -x_i(e_s + e_d) + \beta\{\theta \ln[(1 - \tau)(z\omega_s - R\rho_j\omega_s)] \\ + (1 - \theta) \ln[(1 - \tau)(\omega_k - R\rho_j\omega_s)]\},$$

$$V_d(i, j) = -x_i e_d + \beta\{\theta \ln[(1 - \tau)(\omega_u - R\rho_j\omega_s)] + (1 - \theta) \ln[(1 - \tau)(\omega_k - R\rho_j\omega_s)]\},$$

while V_u is the same as in (2) because it is independent of type. In this setting, the individual choice of education and skill are denoted $\delta'(i, j)$ and $\sigma'(i, j)$, and must satisfy

$$\delta'(i, j) = \arg \max_{\varphi \in [0, 1]} \varphi[\max\{V_s(i, j), V_d(i, j)\} - V_u]$$

$$\sigma'(i, j) = \arg \max_{\varphi \in [0, 1]} \varphi[V_s(i, j) - V_d(i, j)].$$

We will consider alternatively, innate ability and wealth heterogeneity, by letting either the set J or \mathcal{I} be a singleton. In these cases, in equilibrium most agents will not be indifferent to schooling and to earning skill, but there will be some threshold agent type that will. For this reason we proceed as follows.

Conjecture an equilibrium in which some agent type(s) is indifferent to going to school and (possibly other type(s)) to earning skill. Define by $p \in (0, 1)$ the proportion of the population that goes to school, and let $p_s \in (0, p)$ denote the population proportion of those skilled. So $1 - p$ are uneducated workers. Clearly, the supply of skilled and unskilled labor must satisfy

$$n_s = zp_s \quad \text{and} \quad n_u = 1 - p_s,$$

while ω_s and ω_u still satisfy (7). Someone with a degree but unrecognized skill level earns

$$\omega_k = \frac{p_s}{p} z\omega_s + \frac{p - p_s}{p} \omega_u.$$

Here $\frac{p_s}{p} z$ is expected skill level of unrecognized educated agents, while $\frac{p - p_s}{p}$ is the probability that an unrecognized educated agent has no skill.

Differences in innate abilities. Assume that agents are heterogeneous in innate abilities. Recall that we are considering an equilibrium in which not everyone goes to school and not all that do go to school earn skill. Conjecture that in equilibrium all agents $i' < i \in \mathcal{I}$ prefer to not go to school and all agents of type $i' < \hat{i} \in \mathcal{I}$ do not earn skill. So, we have $p = 1 - G(i)$ and $p_s = 1 - G(\hat{i})$. Under these conjectures, we must have $V_s(i, \cdot) = V_d(\hat{i}, \cdot)$ and $V_d(i, \cdot) = V_u$ for some agent types i and \hat{i} , where $i = \hat{i}$ is also possible. Since $\rho_j = \rho$ for all $j \in J$, the government budget constraint becomes

$$\tau (y - pR\omega_s\rho) = p\gamma\omega_s.$$

For numerical implementation we must specify a distribution of ability, and a mapping from ability i to the disutility scalar x_i . So, assume ability is uniformly distributed over the unit interval and $x_i = a(1 - i^2)$. Figure 5 shows the skill level as a function of ρ , for baseline parameters $a = 10$, $\phi = 0$, $R = 1$, $\beta = 1$, $\alpha = \theta = .5$, $e_d = .05$, $e_s = \rho = .2$, and $z = 1$.

[Figure 5 approximately here]

This figure is analogous to the dark line in Figure 1. The key finding is that despite introducing heterogeneity into the model, the policy implications of the analytically tractable model remain. Moving from left to right, an increase in ρ (decrease in subsidy) increases the amount of skill in the economy so long as incentives to achieve are weak ($\sigma < 1$). As in the earlier case, enrollment falls with ρ but σ rises and the net effect is positive. When the private cost of education is sufficiently high, incentives to achieve are strong and $\sigma = 1$. Beyond this level, increases in ρ further reduce enrollment and with σ constant at 1, the skill level falls.

Differences in family wealth. Assume agents are heterogeneous in family wealth, in the manner specified above. Conjecture that in equilibrium all agents $j' < j \in J$ prefer to not

go to school, and all agents of type $j' < \hat{j} \in J$ do not earn skill. So, we have $p = 1 - F(j)$ and $p_s = 1 - F(\hat{j})$. Under these conjectures we must have $V_s(\cdot, \hat{j}) = V_d(\cdot, \hat{j})$ and $V_d(\cdot, j) = V_u$ for some types j and \hat{j} , where $j = \hat{j}$ is also possible. As in this case $x_i = x$ for all $i \in \mathcal{I}$, normalize $x = 1$. The two equalities above give rise to expressions similar to those earlier obtained.

The government budget constraint must now satisfy

$$\tau \left(y - Rp\omega_s \int_{t \geq \hat{j}} \rho_t dF(t) \right) = p\gamma\omega_s .$$

To complete the model assume wealth is uniformly distributed over the unit interval, and specify $\rho_j = \rho(1 - j)^b$ so that ρ scales ρ_j . The baseline parameters are $b = 2$, $\phi = 0$, $R = 1$, $\beta = 1$, $\alpha = \theta = .5$, $e_d = .05$, $e_s = .2$, and $z = 5$. Figure 6 shows that greater subsidies (lower ρ) increase the skill level only in the case where incentives to achieve are strong. The findings associated to Figures 2-4 also hold for both types of heterogeneity (though not presented here).

[Figure 6 approximately here]

5.2 Observable skills

We now return to the case with homogeneous agents and relax the assumption on informational frictions by allowing employers to correctly gauge a worker's skill. To do so we modify the original model by assuming that (i) agents live for four (instead of two) periods and (ii) employers can observe workers' past productivity, hence their skills. So, a worker's true skill level is uncertain only in his first period of employment. Here the return to effort may be delayed but eventually is realized later in life, which is something one observes also in practice (e.g. see Owen, 1995, p. 41).

In this setting agents must decide on allocating consumption across more periods. The first two periods are as before. In the third period, an agent inelastically supplies a unit of time on a competitive market, which is now privy to the agents' skill, having observed his earlier productivity. Thus the agent earns $z\omega_s$ in period three if he is skilled, and gets ω_u otherwise. In period four, the agent is retired and funds consumption through savings.

Lifetime utility is now

$$U = -e + E \sum_{j=1}^3 \beta^j \ln c_j,$$

where j denotes a period, while the lifetime budget constraint is

$$\sum_{j=1}^3 \frac{c_j}{R^{j-1}} + TR = \sum_{j=1}^2 \frac{I_j}{R^{j-1}},$$

where $T = e = 0$ if the agent does not go to school.

Equilibrium wages are still given by (7). Workers whose productivity is immediately recognized receive the same wage in each period. Hence, their lifetime income is scaled by $(1 + \frac{1}{R})$ relative to the two-period model. Those whose skills are not immediately recognized receive a lifetime income that is a linear combination of ω_k and ω_s (or ω_u). Thus, as in the two-period model, those whose productivity cannot be assessed in the second period are still underpaid if skilled and overpaid if unskilled. So, educated but less productive workers can temporarily extract ability rents, which affects the levels of σ and δ as in the two-period model.

It can be shown that in this setting value functions reduce to

$$\begin{aligned}
V_s &= -(e_s + e_d) + \tilde{\beta}\{\theta \ln [(1 - \tau) \omega_s (z (1 + \frac{1}{R}) - R\rho)] \\
&\quad + (1 - \theta) \ln [(1 - \tau) (\omega_k + \frac{z\omega_s}{R} - R\rho\omega_s)]\} + b(\beta), \\
V_d &= -e_d + \tilde{\beta}\{\theta \ln [(1 - \tau) (\omega_u (1 + \frac{1}{R}) - R\rho\omega_s)] \\
&\quad + (1 - \theta) \ln [(1 - \tau) (\omega_k + \frac{\omega_u}{R} - R\rho\omega_s)]\} + b(\beta), \\
V_u &= \tilde{\beta} \ln [(1 - \tau) (\omega_u (1 + \frac{1}{R}))] + b(\beta).
\end{aligned}$$

which is analogous to equation (2). The longer life spans are reflected in the addition of a constant, $b(\beta)$, different discounting, $\tilde{\beta}$, and additional items reflecting period three wages:

$$\tilde{\beta} \equiv \beta + \beta^2 + \beta^3 \quad \text{and} \quad b(\beta) \equiv \beta[\tilde{\beta} \ln \tilde{\beta}^{-1} + (\beta + 2\beta^2) \ln(R\beta)].$$

For the case $\phi \rightarrow 0$, the propositions and proofs go through with minor changes, though there is a greater amount of algebraic manipulations.

6 Conclusion

An ample literature in economics has studied why and how public funding for education matters. Our contribution is to provide intuition as to why incentives for student achievement matter and to provide insights into the best uses of public resources devoted to education.

The analysis has demonstrated that if student effort is a necessary input to acquire human capital—but skills are imperfectly compensated—then incentives might exist for students to be under-achievers. In this case, public spending on education may alter these incentives in both favorable or unfavorable ways, depending on the use made of these resources. In particular, we have shown why greater human capital accumulation does not generally follow from policies that merely focus on raising enrollment.

The above lessons emerge from a model that—to reduce analytical complexity—abstracts from ex-ante heterogeneity elements and assumes away physical capital use in the human capital as well as the consumption goods production processes. However, we have shown that our findings are robust to different specifications that preserve the model’s core features. For instance, the results are qualitatively robust to introduction of heterogeneity in agents’ innate abilities or in wealth, and to eventual resolution of ability uncertainty. More generally, the model’s findings should be qualitatively preserved in environments that display the following features. First, a worker’s human capital is an increasing function of his effort while in school. Second, own productivity is imperfectly observed by firms, at least in the early stages of the worker’s career. Third, workers *known* to be more productive are better compensated, i.e. there is a skill premium in equilibrium. We surmise that—much as in our model—in such environments the key to improve the economy’s skill level is to enhance the productive aspects of education and the informativeness of academic certificates.

Footnotes

1. E.g., see Hoxby (2000), Rouse (1998), Card and Krueger (1992), Hanushek (1986), and the papers in Hanushek (1994) and Hanushek and Jorgenson (1996).
2. Angrist and Lavy (2002) provide evidence that students respond to achievement incentives. In a field experiment, success-contingent payments raised matriculation rates by 7 percentage points. A reasonable interpretation is that effort increased with incentives, since effort is a relevant control variable for students. Keane and Wolpin (2000) provide qualitatively similar evidence.
3. The goods cost to education is as in Galor and Moav (2000). The opportunity cost of time may matter in evaluating the impact of different taxes since the time input is not taxed (e.g., Milesi-Ferretti and Roubini, 1998). This distinction is not key here since labor is inelastically supplied, so it is the size rather than the nature of the cost that matters.
4. The notion that some students do not put forth enough effort to fully exploit the ‘productive’ benefits of education is a natural one. The National Survey of Student Engagement (2006) reports that student engagement is found to be positively correlated with grades and the Digest of Education Statistics (1998) reports that students who spent more time studying math outside of class had higher scores on standardized exams.
5. Enrollment requirements and rates for the final years of K-12 education can be found in Blankenau et al. (2007). Unlike that work, which studies all of K-12 education and takes this as mandatory, the present paper studies only the later years, which can more

easily be considered optional.

6. This assumption reflects the notion that the education sector employs skilled labor (e.g. teachers, superintendents). E.g., see Galor and Moav (2000) or Blankenau (2005).
7. E.g., Bishop (1987) argues that “information about a worker’s effort and productivity is often costly to obtain, and the information asymmetries that these costs create often make it optimal to limit the adjustment of the wage rates to productivity.”
8. This differs from the “peer effects” studied in the literature exploring links between peer group characteristics and human capital accumulation (e.g., Benabou, 1996, Caucutt, 2001, 2002, or Epple and Romano, 1998), i.e., high-ability classmates directly facilitate a student’s human capital accumulation.
9. This removes feedback effects from tax changes on equilibrium skills and education levels, since δ and σ are independent of τ .
10. Enrollment falls in e_d when $\sigma = 1$ (see (17) in the Appendix). This lowers average compensation of unproductive school graduates, relative to those with skill. So, the incentive to earn a degree and no skill falls.
11. E.g., Betts and Grogger (2003) show that higher standards for high school graduation lower attainment for some groups but increase overall test scores. Bishop (1988) shows that curriculum based exit exams can lead to better achievement outcomes.
12. This makes government education spending a direct input into the production of human capital, as in Ben-Porath (1967). Examples include Glomm and Ravikumar (1992) and Eckstein and Zilcha (1994).

13. See Blankenau et al. (2007) for summary statistics and a discussion.
14. E.g., average high-school GPA grew by 1/3 of a letter grade since 1990, but the share of students proficient in reading fell to 73% from 80% (National Assessment of Educational Progress, 2007).
15. Acemoglu, Kremer, and Mian (2003) warn that under some circumstances, market incentives within education can lead to increased wasteful signals of performance, rather than improved performance.
16. Keane and Wolpin (2001) show that wealthier families make larger college-contingent transfers, and family transfers have a strong influence on the choice to attend college. Instead, Keane and Wolpin (1991), Cameron and Taber (2000), and Carneiro and Heckman (2004), indicate that credit constraints are not so important in determining college enrollment. See also De Fraja (2002), where the education level is a non-decreasing function of household income and extreme borrowing constraints exist.

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Appendix

Proof of Proposition 1. Let $\phi \rightarrow 0$. Here wages are given by (10), and so $y = \frac{n_s \omega_s}{\alpha}$. Since $n_s = z\delta\sigma$ from (9), then from (11) we have

$$\tau = \frac{\alpha\gamma}{z\sigma - R\rho\alpha}. \quad (14)$$

The proof has three parts corresponding to the three types of equilibria. We begin by defining the variables

$$E_d = \exp\left(\frac{e_d}{\beta}\right); \quad E_s = \exp\left(\frac{e_s}{\beta}\right) \quad (15)$$

which are monotonic transformations of e_d and e_s , each of which is greater than one. We also define the variables

$$\bar{E} = E_s^{-1} \frac{E_s^{\frac{1}{\theta}} \left(1 - \frac{R\rho}{z}\right)}{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}; \quad \underline{E} = \left(\frac{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + 1}{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + E_s^{\frac{1}{\theta}}} \right)^{1-\theta} \times \bar{E}. \quad (16)$$

If $z > R\rho$ then both are positive. Thus we need $z > R\rho$.

Since $E_s^{\frac{1}{\theta}} > 1$, the coefficient on \bar{E} in the E_d^L expression is less than one. We use these terms to define the critical values referred to in Proposition 1:

$$\bar{e} = \beta \ln \bar{E}$$

$$\underline{e} = \beta \ln \underline{E}.$$

It is important to note that $e_d \leq \bar{e}$ when $E_d \leq \bar{E}$. For example, $E_d < \bar{E}$ assures $\beta \ln E_d < \beta \ln \bar{E}$. Using the above definitions this is $e_d < \bar{e}$. Similarly, $e_d \geq \underline{e}$ when $E_d \geq \underline{E}$. Using this relationship, we state the following proofs in terms of the relationship of E_d to \bar{E} and \underline{E} rather than in terms of the relationship of e_d to \bar{e} and \underline{e} .

Part 1. We first show that $(\sigma, \delta) = (1, \Phi)$ for $E_d \geq \bar{E}$ (equivalently $e_d > \bar{e}$). We must show that under the conjecture $(\sigma, \delta) = (1, \Phi)$ then $(\sigma', \delta') = (1, \Phi)$ satisfies individual optimality. This means we need $V_s > V_d$, from (4), and we need $V_u = V_s$, from (3). When $\sigma = 1$ then $\omega_k = z\omega_s$. Hence, the equality $V_u = V_s$ can be simplified to

$$\ln \left(\frac{z\omega_s - R\rho\omega_s}{\omega_u} \right) = \frac{e_s + e_d}{\beta}.$$

Multiplying both sides by $\frac{\omega_s}{\omega_u}$, exponentiating and substituting in for E_s and E_d from (15) gives

$$\frac{z - R\rho}{\frac{\omega_u}{\omega_s}} = E_s E_d \Rightarrow \frac{z - R\rho}{\frac{z(1-\alpha)}{\alpha} \frac{\delta}{1-\delta}} = E_s E_d$$

since $\frac{\omega_u}{\omega_s} = \frac{(1-\alpha)}{\alpha} \frac{z\delta\sigma}{1-\delta\sigma}$ from (10). We can thus solve for the equilibrium δ :

$$\delta = \delta_1 = \frac{1}{1 + \frac{(1-\alpha)E_s E_d}{\alpha(1-\frac{R\rho}{z})}} \in (0, 1). \quad (17)$$

Thus, given $\sigma = 1$, $\delta' = \delta = \delta_1$ is the unique fixed point of the correspondence (3).

Now consider $V_s \geq V_d$ rearranged as

$$\ln \left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s} \right) \geq \frac{e_s}{\theta\beta} \Rightarrow \frac{1 - \frac{R\rho}{z}}{\frac{(1-\alpha)}{\alpha} \frac{\delta}{1-\delta} - \frac{R\rho}{z}} \geq E_s^{\frac{1}{\theta}}. \quad (18)$$

Using (17), it is straightforward to show that numerator and denominator are both positive if $z > R\rho(E_s E_d + 1)$. Thus we need $z > R\rho(E_s E_d + 1)$. Under the conjecture that $\delta = \delta_1$, this amounts to $E_d \geq \bar{E}$ or $e_d \geq \bar{e}$, equivalently. Thus, given $\delta = \delta_1$, $\sigma' = \sigma = 1$ is the unique fixed point of the correspondence (4) when $e_d \geq \bar{e}$. Hence, if $e_d \geq \bar{e}$ then $(\sigma, \delta) = (1, \delta_1)$ is the unique equilibrium.

Part 2. We now show that $(\sigma, \delta) = (\Phi, \Phi)$ for $\underline{E} < E_d < \bar{E}$ (equivalently $\underline{e} < e_d < \bar{e}$).

We must show that if $\underline{e} < e_d < \bar{e}$, then there is a unique fixed point $(\sigma, \delta) \in (0, 1)^2$ to the

correspondences (3) and (4). This implies $(\sigma, \delta) \in (0, 1)^2$ must satisfy $V_s = V_d = V_u$. Thus, conjecture $(\sigma, \delta) \in (0, 1)^2$.

The equality $V_s = V_d$ is

$$\theta \ln \left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s} \right) = \frac{e_s}{\beta}.$$

Exponentiating each side, dividing the top and bottom of the left hand side by ω_s and substituting in for E_s gives

$$\frac{z - R\rho}{\frac{\omega_u}{\omega_s} - R\rho} = E_s^{\frac{1}{\theta}}. \quad (19)$$

Equation (10) gives $\frac{\omega_u}{\omega_s} = \frac{(1-\alpha)}{\alpha} \frac{\delta\sigma}{1-\delta\sigma}$ so that

$$\frac{z - R\rho}{\frac{(1-\alpha)}{\alpha} \frac{\delta\sigma}{1-\delta\sigma} - R\rho} = E_s^{\frac{1}{\theta}}.$$

Solving for δ gives

$$\delta = \frac{1}{\sigma} \times \frac{\alpha + \alpha \frac{R\rho}{z} [E_s^{\frac{1}{\theta}} - 1]}{E_s^{\frac{1}{\theta}} - \alpha(1 - \frac{R\rho}{z}) [E_s^{\frac{1}{\theta}} - 1]} \quad (20)$$

Since $\omega_k = \sigma z \omega_s + (1 - \sigma) \omega_u$, we can write the equality $V_u = V_d$ as

$$\left(\frac{\frac{\omega_u}{\omega_s} - R\rho}{\frac{\omega_u}{\omega_s}} \right)^\theta \left(\frac{\frac{\omega_u}{\omega_s} + \sigma \left(z - \frac{\omega_u}{\omega_s} \right) - R\rho}{\frac{\omega_u}{\omega_s}} \right)^{(1-\theta)} = E_d.$$

Solving for σ gives

$$\sigma = \frac{1}{\left(z - \frac{\omega_u}{\omega_s} \right) \left(\frac{\frac{\omega_u}{\omega_s} - R\rho}{\frac{\omega_u}{\omega_s}} \right)^{\frac{\theta}{(1-\theta)}} - \frac{\left(\frac{\omega_u}{\omega_s} - R\rho \right)}{\left(z - \frac{\omega_u}{\omega_s} \right)}}. \quad (21)$$

Equation (19) implies

$$\frac{\omega_u}{\omega_s} = \frac{z - R\rho + E_s^{\frac{1}{\theta}} R\rho}{E_s^{\frac{1}{\theta}}}.$$

Using this in (21) and rearranging gives

$$\sigma = \sigma_2 = \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 + \frac{R\rho}{z} (E_s^{\frac{1}{\theta}} - 1)}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right). \quad (22)$$

We next find conditions such that $(\sigma, \delta) \in (0, 1)^2$. Since $E_d, E_s > 1$, then $\sigma_2 > 0$ is immediate from (22). When $\sigma = \sigma_2$ then let δ_2 denote the δ that solves (20). In that case we see that $\delta_2 > 0$. Straightforward calculations show $\sigma_2 < 1$ if and only if

$$E_d < \bar{E} = E_s^{\frac{1-\theta}{\theta}} \frac{\left(1 - \frac{R\rho}{z}\right)}{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)} \quad (23)$$

while $\delta_2 < 1$ if

$$\frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} < \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

Note from the definition of \bar{E} that

$$\frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} = \frac{E_s^{\frac{1-\theta}{\theta}}}{\bar{E}}$$

so that the requirement becomes

$$\frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} < \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{E_s^{\frac{1-\theta}{\theta}}}{\bar{E}} \right)^{\frac{1}{1-\theta}} - 1 \right). \quad (24)$$

After some simplification, this can be shown to require

$$E_d > \underline{E} = \left(\frac{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + 1}{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + E_s^{\frac{1}{\theta}}} \right)^{1-\theta} \bar{E}.$$

Finally, since $\tau = \frac{\alpha\gamma}{z\sigma - R\rho\alpha}$, then $\tau \in (0, 1)$ if $z\sigma > \alpha\gamma + R\rho\alpha$. This inequality holds if

$$\frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\frac{\frac{\alpha}{(1-\alpha)} \left(z + R\rho \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + z}{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) + E_s^{\frac{1}{\theta}}} E_s^{\frac{1}{\theta}} - 1 \right) > \alpha\gamma + R\rho\alpha$$

which is obtained by substituting $E_d = \underline{E}$ into (22) and simplifying. The left-hand side is increasing in z . Let z_1 be the value of z which equates the left and right-hand sides. Then if

$\underline{z} > \max(z_1, R\rho(E_s E_d + 1))$ we have $\tau \in (0, 1)$ by (1). Thus $(\sigma, \delta) = (\sigma_2, \delta_2) \in (0, 1)^2$ is the unique equilibrium with $\underline{e} < e_d < \bar{e}$.

Part 3. Finally we show that $(\sigma, \delta) = (\Phi, 1)$ for $E_d \leq \underline{E}$ (equivalently $e_d \leq \underline{e}$). In order to be a fixed point of the correspondences (3) and (4) it must satisfy $V_s = V_d \geq V_u$. Under the conjecture $\delta = 1$, (20) gives the expression for σ , call this value σ_1 . Inspection reveals $0 < \sigma_1 < 1$. Next, since $\omega_d = \sigma\omega_s + (1 - \sigma)\omega_u$, the inequality $V_d \geq V_u$ can be written as

$$\left(\frac{\frac{\omega_u - \frac{R\rho}{z}}{\omega_s}}{\frac{\omega_u}{\omega_s}} \right)^\theta \left(\frac{\frac{\omega_u + \sigma \left(1 - \frac{\omega_u}{\omega_s}\right) - \frac{R\rho}{z}}{\omega_s}}{\frac{\omega_u}{\omega_s}} \right)^{(1-\theta)} \geq E_d$$

which implies

$$\sigma \geq \frac{1}{\left(1 - \frac{\omega_u}{\omega_s}\right) \frac{E_d^{1-\theta} \frac{\omega_u}{\omega_s}}{\left(\frac{\omega_u - \frac{R\rho}{z}}{\omega_s}\right)^{\frac{\theta}{(1-\theta)}}} - \frac{\left(\frac{\omega_u - \frac{R\rho}{z}}{\omega_s}\right)}{\left(1 - \frac{\omega_u}{\omega_s}\right)}}.$$

Equation (19) implies $\frac{\omega_u}{\omega_s} = \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right) E_s^{-\frac{1}{\theta}}$. Using this in the above equation yields

$$\sigma \geq \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(\frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{E_d \frac{1 - \frac{R\rho}{z}}{1 - \frac{R\rho}{z}}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

Next substitute in for σ_1 using (20) to get

$$\frac{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} \geq \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(\frac{1 - \frac{R\rho}{z} + \frac{R\rho}{z} E_s^{\frac{1}{\theta}}}{E_d \frac{1 - \frac{R\rho}{z}}{1 - \frac{R\rho}{z}}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

This is the same as (24) with the inequality reversed so $E_d \leq \underline{E}$ is required to satisfy this inequality.

Finally, as in Part 2 of the proof, we need $z\sigma > \alpha\gamma + R\rho\alpha$. Using (20) yields the requirement that

$$\frac{z\alpha + R\rho[E_s^{\frac{1}{\theta}} - 1]}{E_s^{\frac{1}{\theta}} - \alpha\left(1 - \frac{R\rho}{z}\right)[E_s^{\frac{1}{\theta}} - 1]} > \alpha\gamma + R\rho\alpha.$$

The left-hand side is increasing in z . Let z_2 be the value of z which equates the left and right-hand sides. Then if we let

$$z \equiv \max \{R\rho(E_s E_d + 1), z_1, z_2\} \quad (25)$$

we have $\tau \in (0, 1)$ under (1). Thus if $e_d \leq \underline{e}$, then $(\sigma, \delta) = (\sigma_1, 1)$ is the unique equilibrium.

Proof of Proposition 2

Let $\phi \rightarrow 0$. To verify that $\frac{\partial \bar{e}}{\partial \rho} < 0$, $\frac{\partial \bar{e}}{\partial z} > 0$, and $\frac{\partial \bar{e}}{\partial \theta} < 0$ while the sign of $\frac{\partial \bar{e}}{\partial e_s}$ is ambiguous, it is equivalent to show $\frac{\partial \bar{E}}{\partial \rho} < 0$, $\frac{\partial \bar{E}}{\partial z} > 0$, and $\frac{\partial \bar{E}}{\partial \theta} < 0$ while the sign of $\frac{\partial \bar{E}}{\partial e_s}$ is ambiguous. The first two items are obvious from (16). To see the third and fourth, verify that $E_s^{\frac{1}{\theta}} \left(1 - \frac{R\rho}{z}\right) \left[1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right]^{-1}$ is increasing in $E_s^{\frac{1}{\theta}}$. Since $E_s^{\frac{1}{\theta}}$ is decreasing in θ , $\frac{\partial \bar{E}}{\partial \theta} < 0$ holds. Also $E_s^{\frac{1}{\theta}}$ is increasing in e_s . However the first term in \bar{E} , E_s^{-1} , is decreasing in e_s and the sign of $\frac{\partial \bar{E}}{\partial e_s}$ is ambiguous.

Proof of Proposition 3

Let $\phi \rightarrow 0$. We start by showing that $\frac{\partial \delta}{\partial \rho} < 0$ when $\delta < 1$. When $\sigma = 1$, this relationship follows directly from (17). When $\sigma = \Phi$, use (22) into (20) and rearrange to obtain

$$\delta = \frac{\left(E_s^{\frac{1}{\theta}} - 1\right) \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right)}{\left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right) \left(\left(E_d \frac{\left[1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)\right]}{1 - \frac{R\rho}{z}}\right)^{\frac{1}{1-\theta}} - 1\right)}. \quad (26)$$

If we define $q \equiv 1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1\right)$ and $x \equiv 1 - \frac{R\rho}{z}$ then

$$\delta = \frac{\left(E_s^{\frac{1}{\theta}} - 1\right) q}{\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right) \left(\left(E_d \frac{q}{x}\right)^{\frac{1}{1-\theta}} - 1\right)}. \quad (27)$$

Note $\frac{\partial \delta}{\partial \rho} = \frac{\partial \delta}{\partial q} \frac{\partial q}{\partial \rho} + \frac{\partial \delta}{\partial x} \frac{\partial x}{\partial \rho}$. It is straightforward to show that $\frac{\partial \delta}{\partial x} > 0$ and $\frac{\partial x}{\partial \rho} < 0$ so $\frac{\partial \delta}{\partial x} \frac{\partial x}{\partial \rho} < 0$.

Also, since $\frac{\partial q}{\partial \rho} > 0$ we need $\frac{\partial \delta}{\partial \rho} < 0$ so long as $\frac{\partial \delta}{\partial q} < 0$. Using (27) we find that $\frac{\partial \delta}{\partial q} < 0$ requires

$$\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}} \right) \left((E_d \frac{q}{x})^{\frac{1}{1-\theta}} - 1 \right) < q \left[\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}} \right) \frac{1}{1-\theta} \left(\frac{E_d}{x} \right)^{\frac{1}{1-\theta}} q^{\frac{\theta}{1-\theta}} + \left(\left(\frac{E_d q}{x} \right)^{\frac{1}{1-\theta}} - 1 \right) \right].$$

Since $q > 1$, it is sufficient that the term in brackets on the right-hand side exceeds the left-hand side. For this it is sufficient that $\frac{1}{1-\theta} \left(\frac{E_d}{x} \right)^{\frac{1}{1-\theta}} q^{\frac{\theta}{1-\theta}} > (E_d \frac{q}{x})^{\frac{1}{1-\theta}} - 1$ and for this it is sufficient that $\frac{1}{1-\theta} q^{\frac{\theta}{1-\theta}} > 1$, which holds since $\theta \in (0, 1)$.

When $\delta = 1$, further increases in ρ cannot yield further increases in δ and $\frac{\partial \delta}{\partial \rho} = 0$. With $\sigma = \Phi$, $\frac{\partial \delta \sigma}{\partial \rho} > 0$ follows directly from (20).

Proof of Proposition 4

Let $\phi \rightarrow 0$. To show that $\frac{\partial \delta}{\partial z} > 0$, when $\delta < 1$ note that ρ and z enter expressions (17) and (26) only through the expression $\frac{R\rho}{z}$. Thus $\frac{\partial \delta}{\partial \rho}$ and $\frac{\partial \delta}{\partial z}$ will always be of opposite sign. When $\delta = 1$, further decreases in z cannot yield further increases in δ and $\frac{\partial \delta}{\partial \rho} = 0$.

With $\sigma = \Phi$, $\frac{\partial \delta \sigma}{\partial z} < 0$ follows directly from (20). Also, (20) can be rewritten as

$$z\sigma\delta = \frac{z\alpha + \alpha R\rho[E_s^{\frac{1}{\theta}} - 1]}{E_s^{\frac{1}{\theta}} - \alpha(1 - \frac{R\rho}{z})[E_s^{\frac{1}{\theta}} - 1]}$$

that clearly shows how $\frac{\partial \delta \sigma z}{\partial z} > 0$ always.

Now consider $\frac{\partial \delta \sigma}{\partial e_s}$. With $\sigma = 1$, $\frac{\partial \delta \sigma}{\partial e_s} < 0$ requires $\frac{\partial \delta}{\partial E_s} < 0$ in (17). This is clear from inspection. With $\sigma = \Phi$, $\frac{\partial \delta \sigma}{\partial e_s} < 0$ requires $\frac{\partial \delta \sigma}{\partial E_s} < 0$ in (20) or that $\frac{\partial \delta \sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$. Using (20) to find $\frac{\partial \delta \sigma}{\partial E_s^{\frac{1}{\theta}}}$, we find that $\frac{\partial \delta \sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$ requires

$$\left(E_s^{\frac{1}{\theta}} - \alpha(1 - \frac{R\rho}{z}) \left(E_s^{\frac{1}{\theta}} - 1 \right) \right) \frac{R\rho}{z} < \left(1 + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1 \right) \right) \left(1 - \alpha(1 - \frac{R\rho}{z}) \right)$$

or

$$\left(\left(E_s^{\frac{1}{\theta}} - 1 \right) \left(1 - \alpha \left(1 - \frac{R\rho}{z} \right) \right) + 1 \right) \frac{R\rho}{z} < \left(1 - \alpha \left(1 - \frac{R\rho}{z} \right) \right) + \frac{R\rho}{z} \left(E_s^{\frac{1}{\theta}} - 1 \right) \left(1 - \alpha \left(1 - \frac{R\rho}{z} \right) \right).$$

This requires $\frac{R\rho}{z} < 1 - \alpha + \frac{\alpha R\rho}{z}$ which holds if $\frac{R\rho}{z} < 1$, as assumed in (1).

Proof of Proposition 5

Let $\phi \rightarrow 0$. Consider $\frac{\partial \delta \sigma}{\partial \theta}$. For the case where $\sigma = 1$, it is clear from (17) that θ has no effect on $\delta \sigma$. Next consider $\sigma = \Phi$. Since $E_s^{\frac{1}{\theta}}$ is decreasing in θ and θ enters (20) only through this expression, $\frac{\partial \delta \sigma}{\partial \theta} > 0$ whenever $\frac{\partial \delta \sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$. This has been shown to hold in the proof of Proposition 4. ■

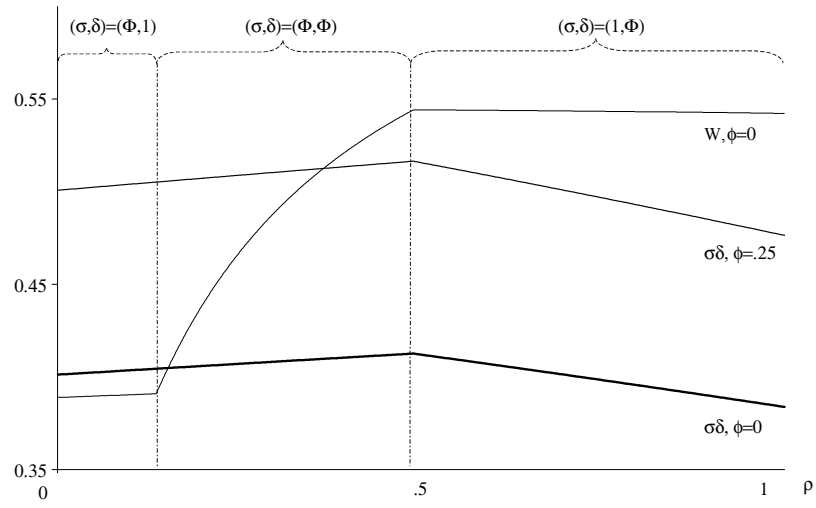


Figure 1

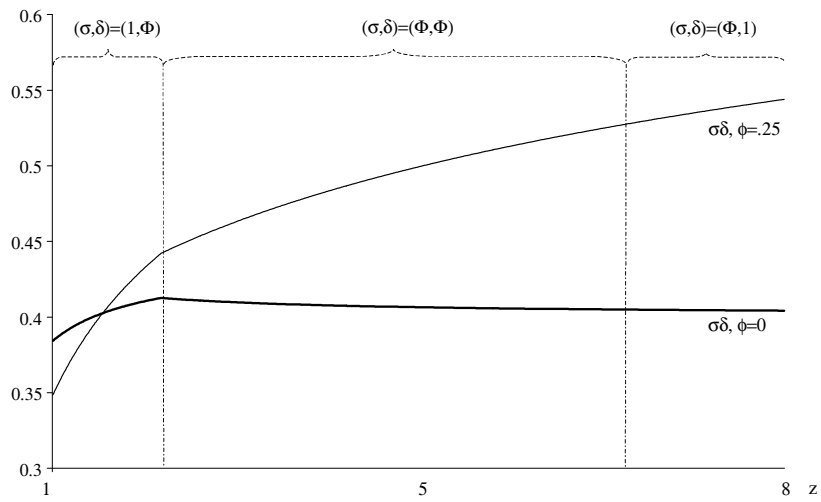


Figure 2

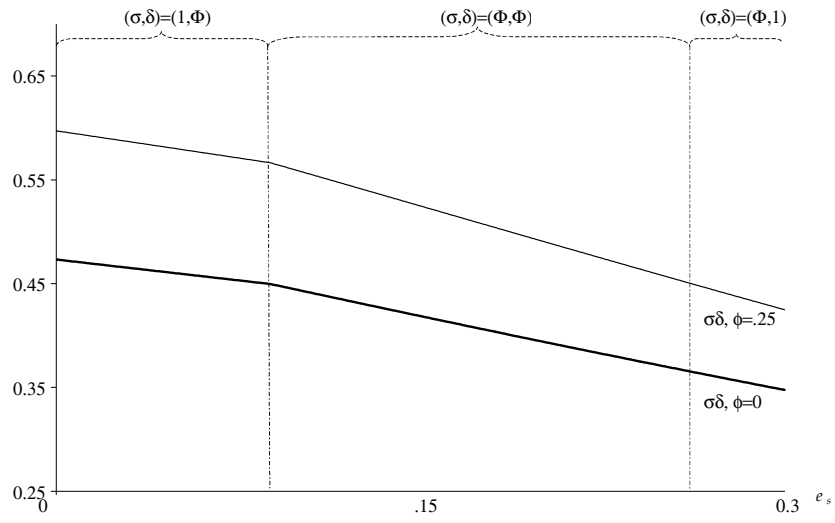


Figure 3

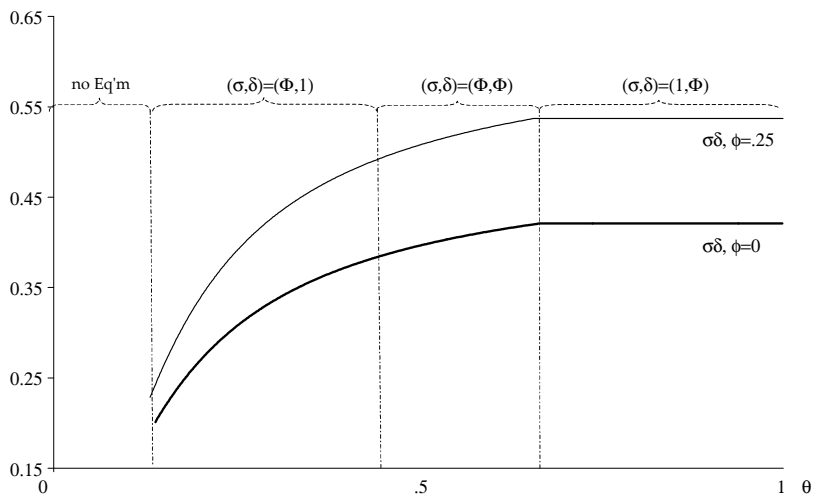


Figure 4

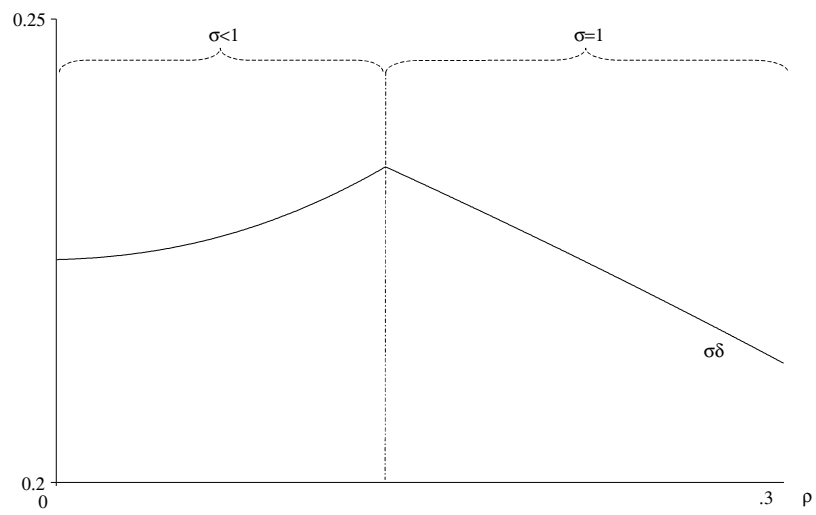


Figure 5

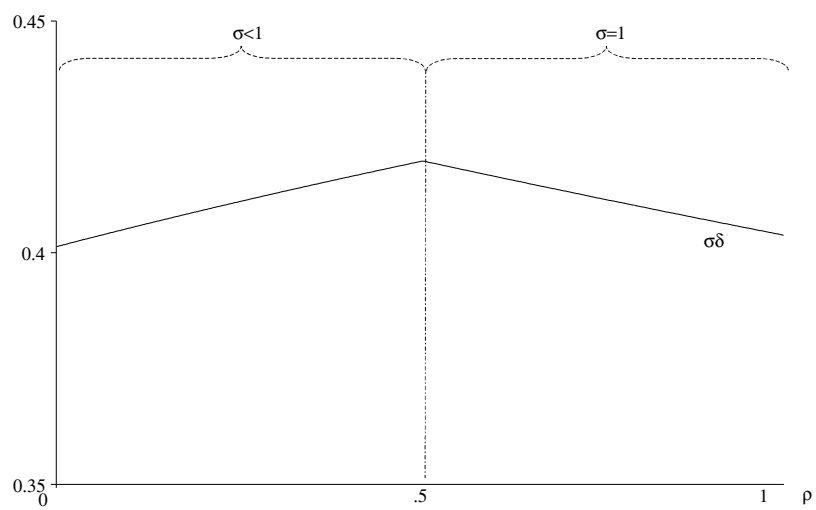


Figure 6