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ABSTRACT

This paper is an attempt to assess the effect of capital gains taxation on non-Austrian assets, such as claims to profits of continuing enterprises. As compared to taxation on an accrual basis, the capital gains tax discourages sales of appreciated assets. This is the "lock-in" effect. Because assets subject to capital gains taxation are generally held a long time, conventional estimates suggest that the effective rate of capital gains taxation is low. We contend that conventional estimates could seriously underestimate the effective rate of capital gains taxation because they ignore uncertainty. We construct a model which allows us to calculate the value of being able to actively manage a portfolio and use this model to calculate the effective rate of capital gains taxation. For several plausible parameter values the effective rate is significantly higher than estimates under certainty. We also discuss some of the ways in which the lock-in effect may distort the allocation of investment funds and the efficient workings of the capital market.

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I. Introduction

Capital gains taxes are different from most other taxes on capital. Because they are levied on a realization basis, how much capital gains tax is paid depends on when an asset is sold not when the capital gain actually occurred. If you buy an asset for price $P(t_o)$ at time t_o and you sell it at T, your tax (payable at T) is $\tau\,[P(T) - P(t_o)]$ where τ is the statutory rate of capital gains tax. This is true regardless of when the increase in value took place. That is, the tax depends only on $P(t_o)$ and $P(T)$; it is independent of the rest of the price path $P(t)$. If assets appreciate, the longer you hold an asset the lower the discounted value of taxes paid on increases in value which took place just after you acquired the asset. In the United States, if sale of an asset can be put off until death no capital gains taxes need be paid. Accrual taxation of capital gains would tax capital gains as they accrue. If capital gains are taxed on an accrual basis, the effect is to lower the after tax rate of return by a factor equal to the tax rate. That is, if an asset grows according to

$$P(t) = P(t_o) \exp \int_{t_o}^{t} \alpha(s)ds,$$

under accrual taxation at rate λ, its rate of return at time s will be $\alpha(s)(1-\lambda)$. Since tax obligations are incurred as capital gains are earned, if investors pay tax at the same rate, tax burdens are independent of changes in ownership.

As compared to taxation on an accrual basis, the capital gains tax discourages sales of appreciated assets. This is called the "lock-in" effect. Investors who hold assets which have increased in value are locked into these assets. They can sell them only on pain of paying taxes which could be deferred or avoided.

Economists have documented that the lock-in effect is real. Feldstein and Yitzhaki (1978), Feldstein and Slemrod (1978), and Feldstein, Slemrod and Yitzhaki (1980) have demonstrated that the holding period for assets subject to the capital gains tax is sensitive to tax considerations. Bailey (1969) and more recently Protopapadakis (1983) have found, after examining tax returns, that on average assets on which capital gains taxes are levied are held for a long time. Protopapadakis estimates that the average holding period is between 24 and 31 years.

In a previous paper (Kovenock and Rothschild, 1983) we analyzed the effect of capital gains taxation on "Austrian" assets, that is, on assets whose real value depends on when they are harvested or consumed. The standard examples are wine and trees. We discussed the impact of capital gains taxation on the harvest time of Austrian assets and on the allocation of investment between Austrian and non-Austrian assets. In our model, capital gains taxation decreased the harvest times of some Austrian assets and left unchanged the harvest times of other Austrian assets. We did, however, find a financial lock-in effect. The owner of an Austrian asset would never find it profitable to sell the asset to another investor before the asset was harvested or consumed.

Most assets subject to capital gains taxation are not of the Austrian type. They represent claims to the profits of continuing enterprises. For such assets the economic consequences of capital gains taxation are different and less apparent. In general, taxes on capital can have two kinds of effects on economic efficiency. First, by reducing the attractiveness of investing, the capital gains tax can distort the savings consumption decision. Second, by making some investments more attractive than others or by interfering with the efficient operation of the capital market, taxes on capital can cause an innefficient allocation of the funds which are available for investment.[1]

Conventional wisdom suggests the first effect is small. Because appreciated assets are held for a long time, the effective rate of the capital gains tax is held to be quite low. Protopapadakis puts the effective rate of capital gains taxation between 3.4% and 6.6% in the period from 1960 to 1978. This is much less than the statutory rate. For the same period Protopapadakis reports that the statutory rate which the average dollar of reported capital gains income faced varied from 18.1% to 27.1%. We will reexamine this assumption in the next section. There, we contend that conventional estimates could seriously underestimate the effective rate of capital gains taxation because they ignore uncertainty.

In the remainder of this section we will discuss the consequences of the lock-in effect on the efficient allocation of capital. We conclude that there are ways in which the lock-in effect can lead to an inefficient allocation of capital. However, we believe, without knowing how we would document our belief, that these effects are small if not insignificant.

We find this problem easiest to think about within the following framework: Suppose that society has a number of potential investment projects, projects which can be ranked in terms of their social desirability. Ignoring uncertainty and capital market imperfections, this ranking is equivalent to an ordering in terms of the rate of return of each project. Capital market misallocations occur when projects with a high return are deferred while projects with a low return are undertaken. Thus, our questions are: can the capital gains tax cause such mistakes and are they likely to be large?

The capital gains tax <u>can</u> cause such misallocations. The tax may make a project with high social returns unattractive to the person who will decide whether the project will be undertaken. Consider, for example, a company which is largely owned by its founder. Suppose that most of the founder's wealth is in the stock of the company which shows large (unrealized) capital gains. As an investor, the founder is anxious to diversify his holdings. He can do this by selling shares of the company's stock, paying capital gains taxes, and using the proceeds to buy a diversified portfolio. He can avoid the capital gains tax by having his company buy the same diversified portfolio. In an extreme case he can turn his company into a mutual fund. In a less extreme case he can choose to have his company undertake projects which diminish the risk of his own portfolio. If there are investment projects which this company alone can pursue, then there is a potential social loss if this company does not develop them. Society can diversify the risk; the company's founder cannot. For this reason, valuable projects are shelved. It is clear that this problem, which we have labeled the "Steve Jobs problem," probably does occur. We doubt that it is very significant. However, we have no idea how we would demonstrate this.

We note another possible effect of the capital gains tax on market efficiency. Since the capital gains tax is a transactions tax, it diminishes trading. If investors get information from asset prices, reduced trading may entail investors getting less information and thus making less good decisions. We have worked out an example (not reported here) which illustrates this point. It is difficult to assess the importance of this aspect of the lock-in effect but we doubt it is very significant.

Other theories of how the capital gains tax can distort the allocation of investment are less credible. Most stories of how the tax system distorts investment allocation depend on the favorable (relative to dividends) tax treatment of retained earnings rather than the lock-in effect. Because capital gains are taxed at a lower rate than dividends, companies have an incentive to retain earnings rather than to pay dividends. The difficulty this causes is that companies must do something with their retained earnings. If they invest them in their own projects, and if their own projects have lower rates of return than the projects of companies which have no retained earnings, then it may happen that worthy projects go begging while relatively unprofitable projects receive funding. Again, for this mechanism to cause misallocations of investment it is necessary that projects be tied specifically to firms. If all firms can invest in all projects then the financing of investments by retained earnings cannot by itself cause harm.

Even with this assumption, it is difficult to understand how financing projects out of retained earnings can lead firms to invest in projects with below market rates of return. Suppose there is a competitive bond market which provides funds to companies for investment at a constant cost. If the rate of interest on bonds is B, then all companies will undertake investments which promise a rate of return of B or more. Furthermore, if all companies finance some projects with bonds then the market will operate efficiently. Marginal projects are financed out of bonds; if all companies are in the bond market then all are using the same cutoff rate to determine the marginal investment project. A company with so much retained earnings that the marginal project can be financed out of retained earnings has a rate of return of B. To obtain this rate of return, it is not necessary to invest in the bonds of other companies. An alternative is to buy its own bonds on the open market or to call outstanding bonds. Thus a competitive bond market ensures that rational companies will, even when using retained earnings as a primary source of financing, invest in projects which earn B or more and not invest in projects which earn B or less. While the capital gains tax may encourage companies to retain earnings rather than to pay dividends, this should not lead to inefficient allocations of investment.

This model is oversimplified. Companies do not all pay the same price for bonds. Differences in interest rates charged companies have many causes. An important one is the debt equity ratio. The extent to which companies use the bond market may increase the rate of interest which they must pay to raise new funds. The lock-in effect of the capital gains tax may exacerbate differences in interest rates charged to different companies. The lock-in effect reduces the incentive to sell the shares of companies with retained earnings. This may keep the prices of their shares artificially high relative to what they would be in a world of accrual taxation. That is, the lock-in effect may cause share prices to reflect past earnings as well as future expected earnings. This will raise the value of equity of firms with past (retained) earnings and lower the cost of bond capital to them. Again, it is hard to know how important are such deviations from the perfectly competitive model (of the bond market); we doubt they are significant.

II. The Effective Rate of Capital Gains Taxation

Suppose you had a million dollars which you didn't need and you decided to give it to your children on your death which you knew would be in T years. Rather than letting the money lie idle, you decide to invest it. Mindful of the tax law, you invest only in assets which pay neither dividends nor interest but instead reward their owners with capital gains when they are sold. You do not plan to sell any of the assets which you have put in the portfolio to be turned over to your children. On your death, the portfolio will be liquidated and its proceeds will be distributed to your children. In the United States at least, the capital gains in the portfolio will escape taxation. What then is the effective rate of capital gains tax which you will pay on the investments you have undertaken on behalf of your children?

At first glance the answer to this question seems clearly to be: "Zero." This is certainly the answer which is consistent with the method which most economists use to calculate the effective rate of capital gains taxation. The standard source for such calculations is Bailey (1969) whose procedure is described below.

"Zero" seems a perfectly reasonable answer if capital gains accrue with certainty. However, the values of most assets which produce capital gains evolve in a manner which is highly uncertain. For such assets the effective rate of capital gains taxation is not zero. Here's why. Consider again the million dollars you plan to invest to leave to your children. To avoid paying capital gains taxes, you must not sell any assets which have increased in value. Suppose, to take an extreme case, that because of inflation all assets always show a paper gain. Then to avoid paying capital gains taxes, you must never sell any asset; in other words, you must follow a buy and hold policy. If there were no capital gains tax (and no other transaction costs) then you would not follow a buy and hold policy. You would continually rebalance your portfolio to maintain the optimal composition of assets. If assets followed a stationary distribution, you would often want to keep the share of each asset in the entire portfolio constant. To do this you would have to sell the assets which had increased in value (relative to the average) and buy those which had decreased in value. This is the opposite of the rule: "Sell losers and hold winners" which those who wish to avoid the capital gains tax are urged to follow. To avoid the capital gains tax, you must give up the privilege of managing your portfolio. Tax avoidance has a price and this price should be calculated as part of the effective rate of taxation. With strong assumptions we can calculate this price and thus the effective rate of capital gains taxation.

In the remainder of this section we build a model which will allow us to calculate the value of being able to actively manage a portfolio. We then use this model to calculate the effective rate of capital gains taxation. We find that for some parameter values the effective rate is quite high -- much higher than the statutory rate. For others it is very low. The parameters that seem to matter most are those which describe the level of diversifiable risk. However, the degree of risk aversion is also important.

Not all economists will find the model and the assumptions we have used to calculate the effective rate of taxation compelling. We discuss some of the more serious objections to our procedure in the final part of this section.

A. A Model for Calculating the Value of Portfolio Management

Suppose an investor has the opportunity of dividing his wealth among N assets. Asset returns are correlated; each asset's value grows according to

(1) $$d\,P_i = P_i\,(\alpha\,d\,t + \delta\,d\,Z_i + \sigma\,d\,Z_o) \quad i = 1,\ldots,N.$$

Where Z_o, $Z_1,\ldots,Z_N$ are independent Wiener processes. As we will see σ^2 represents a common component of undiversifiable variance and δ^2 represents uncertainty which can be diversified away. A risk averse investor facing investment opportunities given by (1) will split his wealth evenly among the N assets. Suppose he follows a buy and hold policy. After T years his wealth will be W(T), a random variable whose distribution we now calculate. Let $b_i(T) = \log P_i(T)$ and $b(T) = (b_1(T),\ldots,b_N(T))$. It is straightforward (for details see Arnold 1974: 141-44) to show that b(T) has a multivariate normal distribution with mean

(2) $$\mu(T) = T\,(\alpha - (\sigma^2 + \delta^2)/2)\iota$$

where ι is a vector of N ones, and variance covariance matrix

(3) $$\Sigma(T) = T \left\{ \begin{matrix} \delta^2+\sigma^2 & \sigma^2 & \sigma^2 & \ldots & \sigma^2 \\ \sigma^2 & \delta^2+\sigma^2 & \sigma^2 & \ldots & \sigma^2 \\ \vdots & \vdots & & & \vdots \\ \sigma^2 & \sigma^2 & \ldots & & \delta^2+\sigma^2 \end{matrix} \right\}.$$

It follows that $P(T) = (P_1(T),\ldots,P_N(T))$ has a multivariate log normal distribution (Johnson and Kotz 1972: 20) with mean vector

(4) $$E(P(T)) = e^{\alpha T}\iota$$

and variance covariance matrix Ω (T) with elements

$$(5) \qquad \Omega(T)_{ij} = \begin{cases} e^{2\alpha T}\left(e^{T(\delta^2+\sigma^2)}-1\right) & i = j \\ e^{2\alpha T}\left(e^{T\sigma^2}-1\right) & i \neq j \end{cases}$$

Suppose $W(0) = 1$ is the amount initially invested in the portfolio (Under the assumption, soon to be made, of constant relative risk aversion this is a normalization). Then $W(T) = N^{-1} \sum_{i=1}^{N} P_i(T)$, and the mean and variance of a buy and hold portfolio are

$$(6) \qquad E[W(T)] = e^{\alpha T}$$

and

$$(7) \qquad V[W(T)] = e^{2\alpha T}[N^{-1}e^{T(\delta^2+\sigma^2)} + (1-N^{-1})e^{T\sigma^2}-1].$$

If the proceeds of this portfolio are taxed on a realization basis at rate τ, after tax returns are

$$(8) \qquad W(T,\tau) = W(T)\ (1-\tau) + \tau$$

with mean and variance

$$(9) \qquad E[W(T,\tau)] = e^{\alpha T}(1-\tau) + \tau$$

and

$$(10) \qquad V[W(T,\tau)] = (1-\tau)^2\ e^{2\alpha T}[N^{-1}\ e^{T(\delta^2+\sigma^2)} + (1-N^{-1})e^{T\sigma^2}-1].$$

Now consider how the wealth of the same investor evolves if before tax asset returns continue to evolve according to (1) but (i) the investor continually rebalances his portfolio and (ii) assets are taxed on an accrual basis at rate λ. Let $Y(T,\lambda)$ be the value of such a portfolio. Let $S_i(t)$ denote the number of shares which the investor holds in asset i; then

$$Y(T,\lambda) = \sum_{i=1}^{N} S_i\ (T)\ P_i\ (T)$$

and, as Merton (1971: 378-9) shows,

$$(11) \qquad dY = \sum_i S_i\ d\ P_i$$

describes the evolution of the portfolio's value. If the investor is risk averse, then he will manage his portfolio so as to keep an equal share of his wealth invested in each asset. Let

$w_i(t) = \frac{S_i(t)\,P_i(t)}{Y(t,\lambda)}$. In the optimal portfolio $w_i(t) = N^{-1}$.

Thus, (11) becomes

$$dY = \sum_i Yw_i P_i^{-1} dP_i = YN^{-1} \sum_i P_i^{-1} dP_i. \tag{12}$$

Accrual taxation changes asset dynamics from (1) to

$$dP_i = (1-\lambda)\, P_i\, (\alpha dt + \delta dZ_i + \sigma dZ_o). \tag{13}$$

To see this suppose that at t, the vector of holdings of assets is P(t). Let $\rho(t,t+h) = P(t+h) - P(t)$; $\rho(t,t+h)$ is a random vector which depends on t, h, and the events which have occurred up to, and including, time t. The meaning of (1) is essentially that for h small the expected value of $\rho(t,t+h)$ is approximately equal to $\alpha hP(T)$ and the variance covariance matrix of $\rho(t,t+h)$ is approximately equal to $h\, \hat{P}(T)\, \Sigma\, \hat{P}(T)$ where $\hat{P}(T)$ is a diagonal matrix with P(T) on the diagonal and Σ is a matrix with $\sigma^2 + \delta^2$ on the diagonal and σ^2 elsewhere.

Now suppose there is accrual taxation at rate λ; we can approximate the effect of accrual taxation by considering realization taxation over short time intervals. If there is a before tax gain of $\rho(t,t+h)$, then after tax gains have means approximately equal to $P(T)\, \alpha(1-\lambda)h$ and variance covariance matrix approximately equal to $h(1-\lambda)^2\, \hat{P}(T)\, \Sigma\, \hat{P}(T)$. But this is the meaning of (13).

Substituting (13) in (12) we see that

$$dY = Y(1-\lambda)\, (\alpha dt + \sigma dZ_o + N^{-1}\delta \sum_i dZ_i). \tag{14}$$

Assuming that $Y(0,\lambda) = 1$, the solution to this stochastic differential equation is (Arnold 1974: 138)

$$(15) \qquad Y(T,\lambda) = \exp[(\alpha(1-\lambda) - (\frac{\delta^2}{2N} + \frac{\sigma^2}{2})(1-\lambda)^2)T + (1-\lambda)(\frac{\delta^2}{N} + \sigma^2)^{\frac{1}{2}}T^{\frac{1}{2}}X]$$

where X is a standard normal random variable; that is $Y(T,\lambda)$ is lognormally distributed with mean

$$(16) \qquad E[Y(T,\lambda)] = e^{\alpha(1-\lambda)T}$$

and variance

$$(17) \qquad V[Y(T,\lambda)] = e^{2\alpha(1-\lambda)T}[\exp((T)\ (1-\lambda)^2(\sigma^2 + \delta^2/N))-1].$$

As is clear from the formulae (9), (10), (16), and (17) taxation changes the distribution of returns of the two portfolios we have examined. Taxes reduce both the mean and variance of returns; however the relative effects of taxation on the two portfolios are different. Specifically suppose $\lambda = \tau$ and compare $E[W(T,\tau)]$ with $E[Y(T,\tau)]$. It is straightforward to calculate that $E[W(T,\tau)] \geq E[Y(T,\tau)]$ for all $\tau \varepsilon$ [0,1] with equality holding only if $\tau = 0$ or $\tau = 1$. Similarly $V(W(T,\tau)) \geq V(Y(T,\tau))$ for $\tau \varepsilon$ [0,1] equality holding if $\tau = 1$ or if $N = 1$ and $\tau = 0$. Figures 1, 2 and 3 show the mean and variance of these portfolios as the level of taxation varies for various values of the parameters. In all the figures $\alpha = .1$, $\sigma^2 = .039$, and $N = T = 10$; in Figure 2, $\delta^2 = .038$ and in Figure 3, $\delta^2 = .39$. The reasons for choosing these parameter values are discussed in subsection C below. As is clear from the figures, taxes reduce mean and variance in each case. Figure 4 shows the parametric curves giving the means and standard deviations of the two portfolios as τ varies from 0 to 1. In both Figure 4a and 4b, $T = 10$, $\alpha = .1$, $\sigma^2 = .039$, and $\delta^2 = .39$; in Figure 4a, $N = 1$ while in Figure 4b, $N = 10$. Realization taxation induces a linear trade off between mean and standard deviation; as τ increases the decrease in the standard deviation is proportional to the decrease in the mean. Under accrual taxation the standard deviation decreases with the mean at a decreasing rate; standard deviation is a convex function of the mean.[2] For N = 1 the endpoints of both curves coincide. At low levels of τ accrual taxation provides a more favorable trade off of mean for decreases in standard deviation, while at high rates of taxation accrual taxation provides a less favorable trade off of mean for decreases in standard deviation. For N = 10 the right endpoint of the curve for accrual taxation is below the right endpoint for realization taxation. This reflects the fact that with a zero tax rate both portfolios have the same mean, while the continually managed portfolio is less risky.

B. The Effective Rate of Taxation

In the context of certainty, Bailey (1969) defined the effective rate of capital gains taxation as the rate of accrual taxation, λ, which would ensure equality between the following two things: The after tax profits on an investment subject to capital gains tax on a realization basis at rate τ and the after tax profits on the same investment taxed on an accrual basis at rate λ. With this definition, the effective rate of taxation depends on the characteristics of the investment, the rate of capital gains taxation and the length of time the investment is held. If an investment has an instantaneous rate of return of $\alpha(s)$ and it is held T years, its after tax return is

$$C = [\exp \int_0^T \alpha(s)ds]\ (1-\tau) + \tau. \tag{18}$$

Taxes levied on an accrual basis reduce the rate of return from $\alpha(s)$ to $\alpha(s)\ (1-\lambda)$. Thus the equivalent accrual rate is the solution to the equation

$$C = \exp \int_0^T \alpha(s)\ (1-\lambda)ds. \tag{19}$$

Therefore,

$$\lambda(\tau,T) = 1 - \frac{\log\left[\left(\exp \int_0^T \alpha(s)ds\right)(1-\tau) + \tau\right]}{\int_0^T \alpha(s)ds} \tag{20}$$

is the formula for the effective rate of taxation as a function of τ and T. Clearly, $\lambda(0,T) = 0$ and a simple application of L'Hospital's rule gives $\lambda(\tau,0) = \tau$. It is also straightforward to show that $\lambda_\tau > 0$. The effective rate of capital gains taxation declines with the holding period. To see that this is so, note that sign λ_T = - sign f $(\int_0^T \alpha(s)ds)$ where

$$f(x) = \frac{xe^x(1-\tau)}{e^x(1-\tau) + \tau} - \log(e^x(1-\tau) + \tau).$$

Note that $f(0) = 0$ and calculate that $f'(x) > 0$ to conclude that $\lambda_T < 0$. Figure 5 graphs the effective rate of taxation as a function of the holding period T under the assumption that $\alpha(s) = \alpha = .1$ and $\tau = .2$.

We extend this definition in a straightforward way to uncertainty: the effective rate of taxation is that rate of accrual taxation which makes an investor indifferent between holding a buy and hold portfolio which is taxed at rate τ and holding a continually managed portfolio which is taxed at accrual rate λ. A definition in terms of indifference is a definition which depends on preferences. If the investor maximizes expected utility and has utility function $U(\cdot)$ then the effective rate of taxation is the solution, $\lambda(\tau,T)$, to

$$E[U(W(T,\tau))] = E[U(Y(T,\lambda))]. \tag{21}$$

Having the effective rate of taxation depend on the utility function is unfortunate but unavoidable. No rate of accrual taxation can make the distributions of the two random variables $W(T,\tau)$ and $Y(T,\lambda)$ equal in any sense. For our calculations we choose a constant relative risk aversion utility function. We do this both because it makes some calculations tractable and because it makes the effective rate of taxation independent of the amount invested.

Let $(1-\gamma)$ be the coefficient of relative risk aversion. Then $U(X) = \gamma^{-1} X^{\gamma}$. Let $A(T,\lambda) = E[U(Y(T,\lambda))] = E\,\gamma^{-1}Y(T,\lambda)^{\gamma}$. This is, because of (15), the moment generating function of a normal random variable. Thus,

$$A(T,\lambda) = \gamma^{-1} \exp[\gamma T(\alpha(1-\lambda) - (\frac{\delta^2}{2N} + \frac{\sigma^2}{2})(1-\lambda)^2) + \gamma^2(1-\lambda)^2(\frac{\delta^2}{2N} + \frac{\sigma^2}{2})T]. \tag{22}$$

Similarly let $R(T,\tau) = E[U(W(T,\tau))] = \frac{1}{\gamma} E[W(T,\tau)^{\gamma}]$. Unfortunately $R(T,\tau)$ does not have such a neat closed form expression. However the value of $R(T,\tau)$ for given parameters can be approximated using Monte Carlo methods. That is let

$$S(T,\tau;Q) = Q^{-1}\gamma^{-1} \sum_{q=1}^{Q} (N^{-1} \sum_{i=1}^{N} (\exp (b_{iq}(T))(1-\tau) + \tau))^{\gamma}$$

where the $b_q(T) = (b_{1q}(T),\ldots,b_{Nq}(T))$ are independent realizations of a random vector with mean $\mu(T)$ and variance covariance matrix $\Sigma(T)$ as given in equations (2) and (3). Then

$$\lim_{Q\to\infty} S(T,\tau;Q) = R(T,\tau)$$

and we can estimate λ by $\hat{\lambda}(T,\tau;Q)$ the solution of

(23) $$S(T,\tau;Q) = A(T,\hat{\lambda}(T,\tau;Q)).$$

Because $S(T,\tau;Q)$ is a sample mean, the central limit theorem states that it is approximately normal for large values of Q. We estimate the variance of $S(T,\tau,Q)$ by

(24) $$\hat{\sigma}^2(Q) = \frac{\sum_{q=1}^{Q}\left[\gamma^{-1}(N^{-1}\sum_{i=1}^{N} \exp(b_{iq}(T))(1-\tau) + \tau)^{\gamma} - S(T,\tau,Q)\right]^2}{Q^2}$$

Thus, we can compute an asymptotic 95% confidence interval for λ as $[\underline{\lambda},\bar{\lambda}]$ where $\underline{\lambda}$ and $\bar{\lambda}$ are the solutions of

$$R(T,\tau) + 2\ \hat{\sigma}\ (Q) = A(T,\underline{\lambda})$$

$$R(T,\tau) - 2\ \hat{\sigma}\ (Q) = A(T,\bar{\lambda})$$

respectively.

C. Parameters

In this subsection we discuss the parameters we used to estimate the effective rate of taxation.

1. α.

We set α, the mean rate of return, equal to .1. Ibbotson and Sinquefield (1982) report an average rate of return (from 1926 to 1981) of .091 for the Standard and Poor's 500 and .121 for stocks of small companies - the smallest (in value) fifth of the companies listed on the New York Stock Exchange.

2. σ^2.

The annual variance of a portfolio which is completely diversified ($N = \infty$) is

$$V_\infty = e^{2\alpha}[e^{\sigma^2}-1].$$

Arguably this is the annual variance of the market portfolio. Ibbotson and Sinquefield report $V_\infty = .048$ from 1926 to 1981. (This is actually the variance of the value weighted Standard and Poor's 500 and as such it may understate the variance of returns on a more widely diversified portfolio. Ibbotson and Sinquefield report a much higher variance (.14) for the returns of the equities of the smallest one fifth of the companies held on the New York Stock Exchange in the same period.) A value of $V_\infty = .048$ implies $\sigma^2 = .039$.

3. δ^2.

The parameter δ^2 measures the gains from diversification. It is easiest to express this as follows. If a portfolio of 10 stocks is held for a year (and not managed), the variance of returns is,

$$V_{10} = e^{2\alpha}\,(.1\ e^{\sigma^2 + \delta^2} + .9\ e^{\sigma^2} - 1).$$

Let $h = (V_{10}/V_\infty)^{\frac{1}{2}} - 1$, h is a measure of the efficacy of diversification or of the rate at which the benefits of diversification are achieved as more assets are added to a portfolio; h x 100 is the percentage by which the standard deviation of a ten stock portfolio exceeds the standard deviation of the optimally diversified portfolio. If h = .1 then a 10 stock portfolio is only 10% more variable than the optimally diversified portfolio. Clearly h determines δ^2. It is easy to calculate that

$$\delta^2 = \log[10[e^{-\sigma^2} - .9 + (h + 1)^2(1 - e^{-\sigma^2})]].$$

We are unsure what value of h best describes the opportunities available to investors; setting h = .05 (δ^2 = .038) and .5 (δ^2 = .39) seems to encompass the range of alternatives.[3]

4. γ.

$1-\gamma$ is the coefficient of relative risk aversion. The theoretical literature identifies $\gamma = 0$ as a pivotal case; the empirical literature produces estimates of γ ranging from .95 to -1000. See Choi and Menzes (1984) for a compendium of such estimates. With this guidance we felt free to choose γ to suit our purposes. Low values of γ correspond to high values of relative risk aversion. It would seem that high degrees of risk aversion would make the reduction of risk which can be achieved through active portfolio management more valuable. This should raise the effective rate of taxation as we have defined it.

However, high values of relative risk aversion have other consequences for the calculation of the effective rate of taxation. Our procedure assumes full loss offsets; the government shares in both gains and losses of investment. As figures 1 through 4 make clear increasing taxes decreases both the mean and the variance of returns. If an investor is sufficiently risk averse then his expected utility can be an increasing function of the rate of taxation. To see this note that the sign of the effect of an increase in taxation on a continuously rebalanced portfolio is determined by

$$\text{(25)} \qquad \text{sign } A_{\lambda} = \text{sign } [(\sigma^2 + \frac{\delta^2}{N}) \ (1-\lambda)(1-\gamma)-\alpha]$$

thus for high values of relative risk aversion expected utility increases as taxes increase.

Figures 1 through 4 suggest that the sign of R_{τ} (which cannot be calculated directly) is also likely to be positive if γ is small. While the possibility that increasing rates of taxation can increase utility raises many tantalizing issues, it does confuse the meaning of the phrase "effective rate of taxation." Thus we have used relatively high values of γ (low values of relative risk aversion) in our calculations: In particular we have chosen $\gamma = .8$ and $\gamma = -.5$; in the sequel the first case is described as a low degree of risk aversion, the second as a high degree of risk aversion.

5. T.

We have chosen values of T ranging from 1 to 30. Protopapadakis (1983) estimates that assets on which capital gains taxes are assessed are held on the average between 24 and 31 years.

6. N.

As explained above N and δ^2 together calibrate the gains from diversification and portfolio rebalance. The larger N is, the smaller is the variance of returns; however, for small N, as N increases so do the benefits of portfolio management. In the calculations we have chosen N sufficiently large that $A_\lambda < 0$. It is easy to calculate using (25) that this implies $N \geq 1$ for $\gamma = .8$ and $h = .05$. If both diversifiable risk and risk aversion are high ($\gamma = -.5$ and $h = .5$), then we must have $N \geq 15$.

7. Q.

Q determines the accuracy of $\hat{\lambda}$ as an estimate of the effective rate of taxation. However, the accuracy of the estimate is measured by the size of the interval $[\underline{\lambda},\bar{\lambda}]$ which is reported along with $\hat{\lambda}$. For most of our estimates we set $Q = 5000$.

D. Results

The results of our attempts to estimate the effective rate of taxation are reported in Tables 1 through 8. From these we draw the following conclusions:

1. The effective rate depends on the parameters; it is increasing in diversifiable risk, risk aversion, and, by and large, the statutory rate. For low values of these variables the effective rate is negligible. The entries in Table 1 are not significantly different from zero.[4] The effective rates reported in Table 2 do not differ significantly from effective rates under certainty. For high values of these variables, the effective rate can be very high. The rates reported in Tables 7 and 8 are much higher than the statutory rate. Such very high rates indicate that a buy and hold policy is not optimal; if transaction costs other than capital gains taxes are low, then the investor always has the option of trading often. His effective rate cannot exceed the statutory rate. Still the very high rates of Tables 7 and 8 illustrate dramatically that buy and hold policies have costs when investment results are uncertain.

2. If the statutory tax rate is zero and the level of diversifiable risk is high, the effective tax rate is greater than zero for most asset portfolios held 10 years or longer. This is shown in Table 5. The effective rate is especially high for portfolios with five assets. As Figure 4 demonstrates, realization taxation offers a different, and on the whole, less favorable trade off between risk and return than does accrual taxation. For a one asset portfolio subject to a zero statutory tax rate there is no difference between buying and holding and continually rebalancing. As the number of assets in an untaxed portfolio gets large the portfolio variance under a buy and hold policy approaches that of a policy of continuous management. This indicates that the effective rate of taxation will be greatest for

portfolios only partially diversified. This is reflected in the high effective tax rates for large T and N = 5 in Table 5. It is also evident in Table 3 where diversifiable risk is low but risk aversion is high. Although most entries in Table 3 are close to zero, for $T \geq 20$ and $N = 5$ the effective tax rate is significantly higher.

If the statutory tax rate is 20% and either the level of diversifiable risk or the level of risk aversion is high, the effective rate of taxation is higher than in the certainty case for most asset portfolios held 10 years or longer. In these cases the effective rate does not decline rapidly with the holding period as it does under certainty. This is demonstrated in Tables 4, 6 and 8. In Table 4, the effective rate does not change much as the holding period changes. In Table 6, the same appears to be true, except perhaps for N = 5, where the effective tax rate may increase. In Table 8 the effective rate increases as the holding period lengthens.

3. In some cases the effective rate of taxation can decrease as the statutory rate increases. Compare the entries for $T \geq 5$ in the high diversifiable risk - high risk aversion cases of Tables 7 and 8. This is again a reflection of the fact that increasing taxes may increase utility by reducing risk. The effective rate of taxation, $\lambda(\tau)$, is defined as the solution to the equation

$$R(T,\tau) - A(T,\lambda) = 0.$$

Thus,

$$\frac{d\lambda}{d\tau} = \frac{R_\tau}{A_\lambda}$$

We choose parameters so that $A_\lambda < 0$. Thus, sign $\frac{d\lambda}{d\tau}$ = - sign R_τ. Thus a finding that the effective rate of taxation decreases as the statutory tax rate increases indicates that $R_\tau > 0 > A_\lambda$.

E. Conclusions and Caveats

The calculations reported above suggest that the effective rate of capital gains taxation under uncertainty is higher than conventional estimates which take no account of uncertainty would suggest. Three strong arguments weaken the force of the case we have made. First as Stiglitz (1983) and Constantinides (1983) have argued, only fools pay capital gains taxes.[5] If tax liabilities only are considered, the optimal portfolio policy is obviously to sell losers and keep winners. Stiglitz shows that for every transaction which violates this rule there is another transaction with the same real and financial consequences which adheres to the rule. The equivalent tax free transaction involves options, short sales, or other relatively complex transactions which we feel many investors are unwilling to engage in. We do not question the logic of the Constantinides - Stiglitz position that capital gains taxes can be avoided. We believe that many people pay capital gains taxes even if they could avoid them. The reported responsiveness of sales of appreciated assets to tax rates is evidence for this position. For people who could but do not avoid taxes the effective rate of taxation is the effective rate on the taxes they pay, not on those they could avoid. We believe that for many people the effective tax rate on capital gains is positive.

A second, and related, argument is that the buy and hold policy which we have compared to a policy of continual management in order to compute the effective rate is obviously not the optimal portfolio policy even if options and short sales are ruled out. If there are no transactions costs other than capital gains taxes, then the investor has the option of continually managing his portfolio and paying an effective rate of taxation of 20%. Every entry in Tables 1 to 8 with a higher effective rate than 20% reflects the return on a policy which is dominated by such a policy. However, while the buy and hold policy is not the optimal policy we haven't the faintest idea what the optimal portfolio policy in the presence of capital gains taxes (and possibly other transaction costs) is. The study of optimal portfolio policy with transactions costs is in its infancy. Almost the only problem which has been studied is the simple one of dividing one's portfolio between a risky and a riskless asset in the presence of linear transaction costs. For this standard problem Constantinides (1984b) has done calculations which show that losses from transactions costs may be very small. It is not clear that the relatively few results on this problem will carry over to the different problem we have considered. One problem that we have investigated is the problem of dividing one's portfolio between a riskless asset and a mutual fund consisting of $\frac{1}{N}$ shares of each of N assets with returns generated by (13). We assume that the riskless asset is subject to the same accrual tax rate as the mutual fund. If P_o is the price of the riskless asset

$$dP_o = r(1-\lambda)P_o dt.$$

While continuous rebalancing between the riskless asset and the mutual fund is allowed rebalancing of asset weights within the mutual fund is not allowed. Using Ibbotson and Sinquefield's (1982) estimate of the average return on short term U.S. Treasury bills as the riskless rate of return ($r = .03$) we can estimate the optimal portfolio shares. With $\alpha = .1$, $\sigma^2 = .039$, $N = 10$, and $\lambda = .2$ three out of the four possible combinations of values for δ^2 and γ provided in part C yield portfolio shares for the mutual fund which are greater than one. The exception is the case of high diversifiable risk and high risk aversion. In this case .75 of wealth is held in the mutual fund. If we extend our analysis and assume that rebalancing between the mutual fund and the riskless asset can only take place periodically, the results of Goldman (1979) can be used to show that, as the period between rebalancing opportunities increases, all of the portfolios examined plunge into the mutual fund of risky assets. These results seem to indicate that the problem addressed here and that analyzed by Constantinides are sufficiently different that there is no reason to suppose that his unimportance result should carry over. For a sampling of recent work on the transactions costs problem see Constantinides (1979, 1984b); Kandel and Ross (1983); Taksar, Klass and Assaf (1983).

Third, we have discussed portfolio choice in a partial equilibrium context. General equilibrium considerations might make it either unnecessary or impossible for people to continually rebalance their portfolios. Suppose for example that the Capital Asset Pricing Model (CAPM) held. Then all investors would hold the same, market, portfolio. No one would ever want to revise his portfolio. An investor who followed a buy and hold policy would hold an optimally diversified portfolio. Our only response to this is that we don't believe that anything like the CAPM describes the asset markets of the United States. Investors have diverse holdings; many do not hold the market portfolio. For these investors the need to rebalance their holdings through active management is probably real. Furthermore, our findings that the effective rate was high depended on the fact that buy and hold portfolios taxed on a realization basis are riskier than portfolios taxed on an accrual basis as well as on the advantages of portfolio management.

Thus, despite these arguments, we believe our results are at least weak evidence for the proposition that the effective rate of capital gains taxation is higher than the 3 to 7% which, for example, Protopapadakis (1983) reports.

Footnotes

[1]Students of reswitching will note that this is an artificial distinction; it is nonetheless a useful one and we will adopt it.

[2]The proof of this is tedious and is omitted. It involves inverting equation (16) to obtain λ as a function of the mean, inserting this function in place of λ in the square root of the right hand side of equation (17), and twice differentiating with respect to the mean. The resulting expression gives the second derivative of the standard deviation of a portfolio with respect to the mean, and can be shown to be positive for the relevant range of values of the mean.

[3]Evans and Archer (1968) examine how the variability of a portfolio of randomly selected stocks declines as the number of (equally weighted) assets held in the portfolio increases. They conclude that their results "raise doubts concerning the economic justification of increasing portfolio sizes beyond 10 or so securities." Since Evans and Archer's estimate of undiversifiable risk differs from ours, it is difficult to compare their measure of relative variability to the one employed here.

[4]One of the entries in Table 1 is significantly different from zero at the .95 level. Our discussion here focuses on the economic significance and not the statistical significance of our results.

[5]This conclusion rests on the assumption, made throughout this paper, that short and long term capital gains are taxed at the same rate. For an analysis of optimal trading policies with differential taxation of short and long term gains and losses see Stiglitz (1983) and Constantinides (1984a).

References

Lugwig Arnold 1974, Stochastic Differential Equations (Wiley).

Martin J. Bailey 1969, "Capital Gains and Income Taxation." In Arnold C. Harberger and Martin J. Bailey, editors, The Taxation of Income from Capital (Brookings).

E. K. Choi and C. F. Menzes 1984, "On the Magnitude of Relative Risk Aversion," mimeo.

George M. Constantinides 1979, "Multiperiod Consumption and Investment Behavior with Convex Transactions Costs," Management Science 25, 1127-37.

George M. Constantinides 1983, "Capital Market Equilibrium with Personal Tax," Econometrica 51, 611-636.

George M. Constantinides 1984a, "Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns," Journal of Financial Economics 13, 65-89.

George M. Constantinides 1984b, "Capital Market Equilibrium with Transactions Costs," mimeo.

John L. Evans and Stephen H. Archer 1968, "Diversification and the Reduction of Dispersion: An Empirical Analysis," Journal of Finance 5: 761-767.

Martin Feldstein and Joel Slemrod 1978, "The Lock-in Effect of the Capital Gains Tax: Some Time Series Evidence," Tax Notes, 134-135.

Martin Feldstein, Joel Slemrod and Shlomo Yitzhaki 1980, "The Effects of Taxation on the Selling of Corporate Stock and the Realization of Capital Gains," Quarterly Journal of Economics 94, 777-791.

Martin Feldstein and Shlomo Yitzhaki 1978, "The Effects of the Capital Gains Tax on the Selling and Switching of Common Stock," Journal of Public Economics 9, 17-36.

M. Barry Goldman 1979, "Anti-Diversification or Optimal Programmes for Infrequently Revised Portfolios," Journal of Finance 34, 505-516.

Roger G. Ibbotson and Rex A. Sinquefield 1982, Stocks, Bonds, Bills and Inflation: The Past and the Future (Financial Analysts Research Foundation, Monograph No. 15, Charlottesville, VA).

Norman L. Johnson and Samuel Kotz 1972, Distributions in Statistics: Continuous Multivariate Distributions (Wiley: New York).

Shmuel Kandel and Stephen A. Ross 1983, "Some Intertemporal Models of Portfolio Selection with Transactions Costs," mimeo.

Daniel J. Kovenock and Michael Rothschild 1983, "Capital Gains Taxation in an Economy with an 'Austrian Sector,'" Journal of Public Economics 21, 215-256.

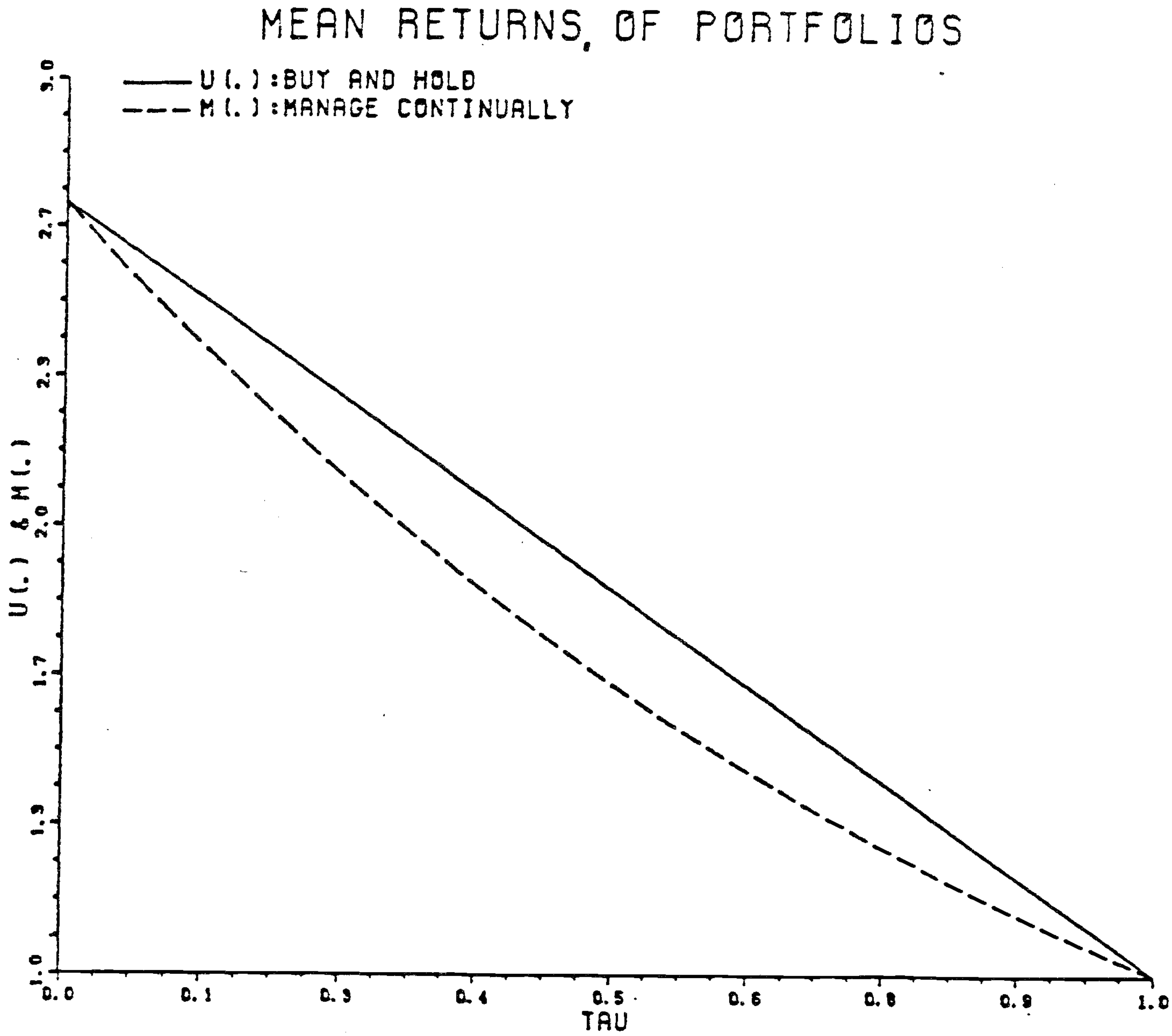
Robert C. Merton 1971, "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," Journal of Economic Theory 3, 373-413.

Aris Protopapadakis 1983, "Some Indirect Evidence on Effective Capital Gains Tax Rates," Journal of Business 56, 127-138.

Joseph E. Stiglitz 1983, "Some Aspects of the Taxation of Capital Gains," Journal of Public Economics 21, 257-294.

Michael Taksar, Michael J. Klass and David Assaf 1983, "A Diffusion Model for Optimal Portfolio Selection in the Presence of Brokerage Fees," Technical Report Number 4 (Department of Operations Research, Stanford University).

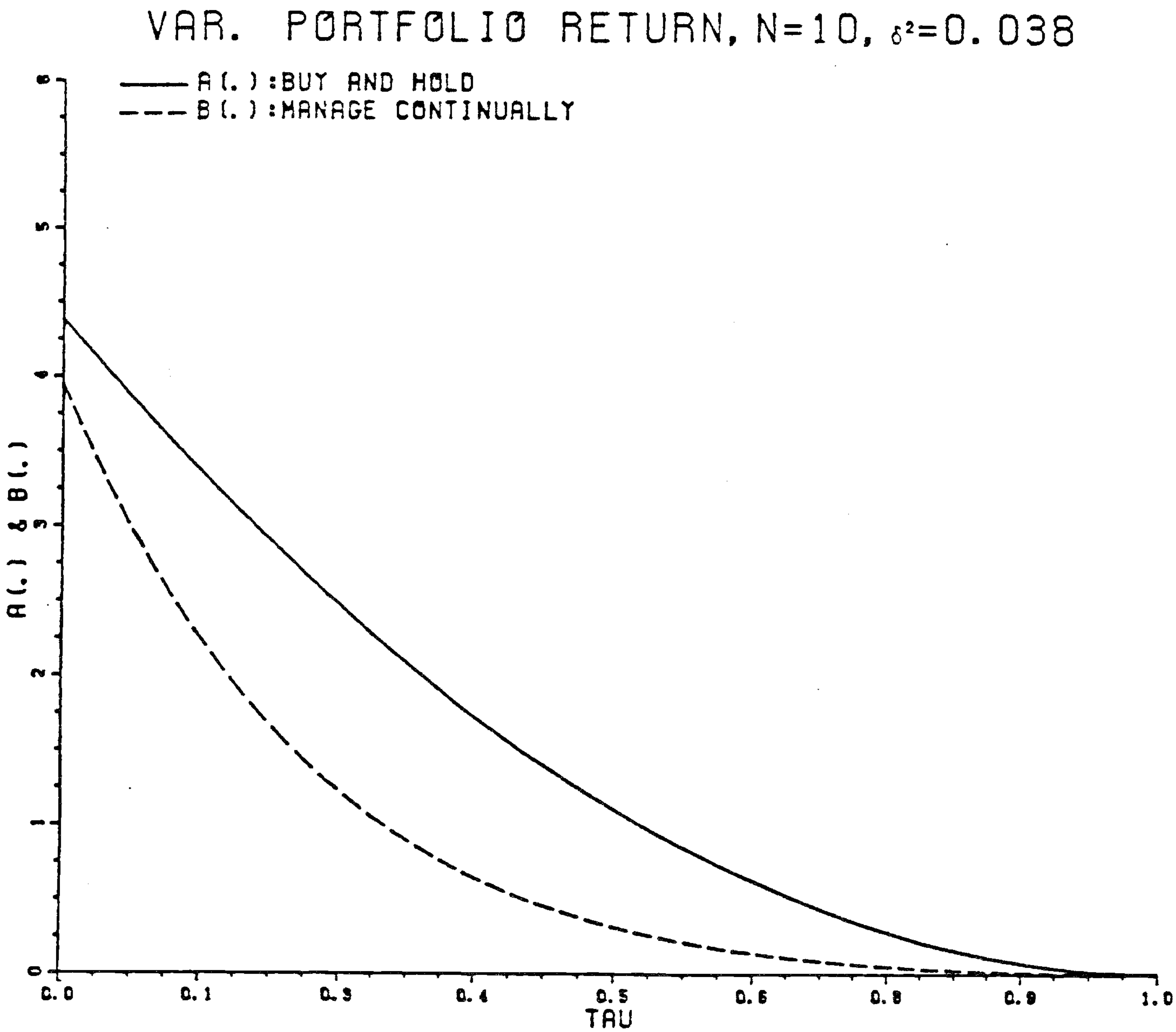
FIGURE 1



Parameters:

$\alpha = .1$
$N = 10$
$T = 10$

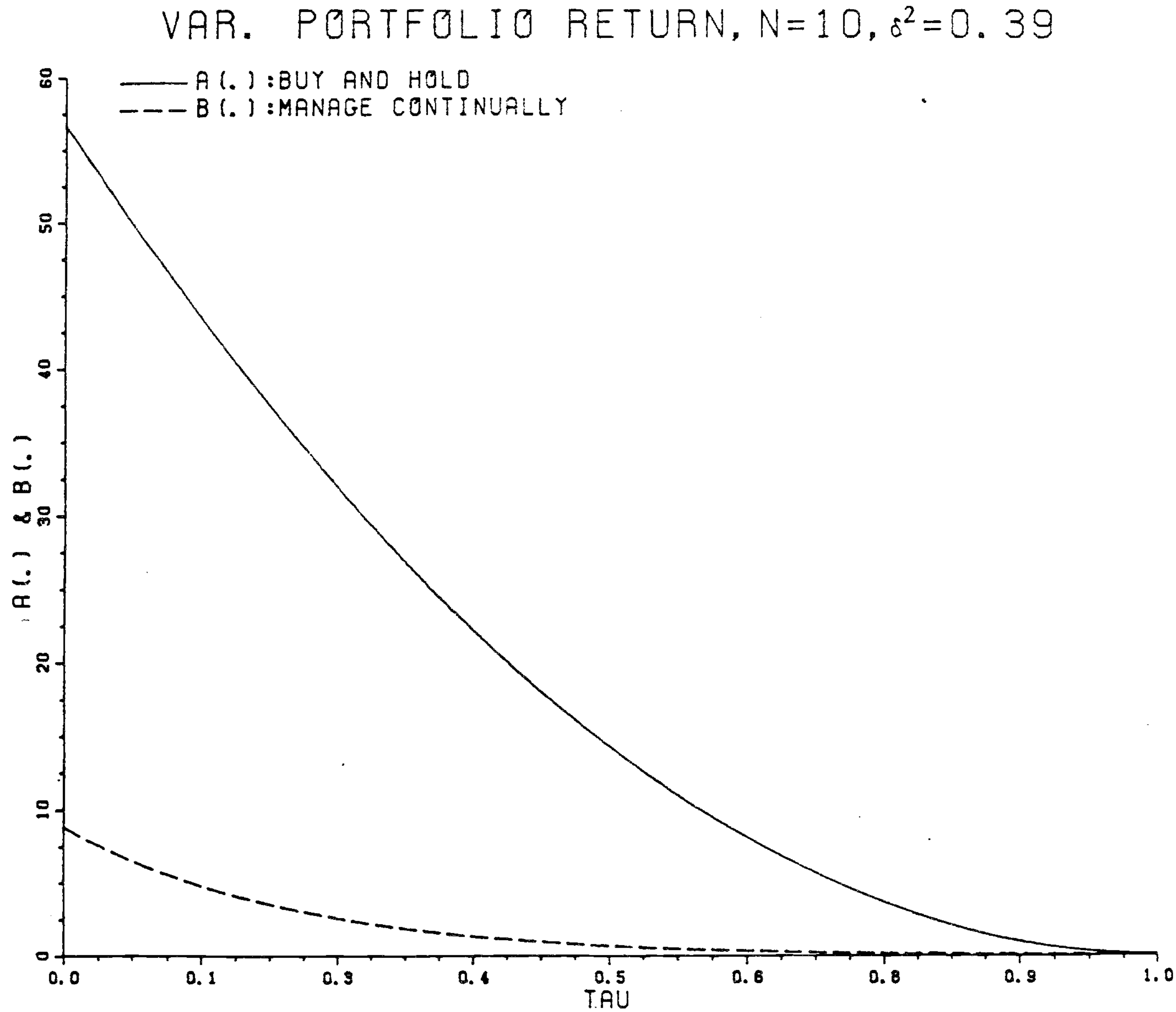
FIGURE 2



Parameters:

α = .1
σ^2 = .039
T = 10

FIGURE 3



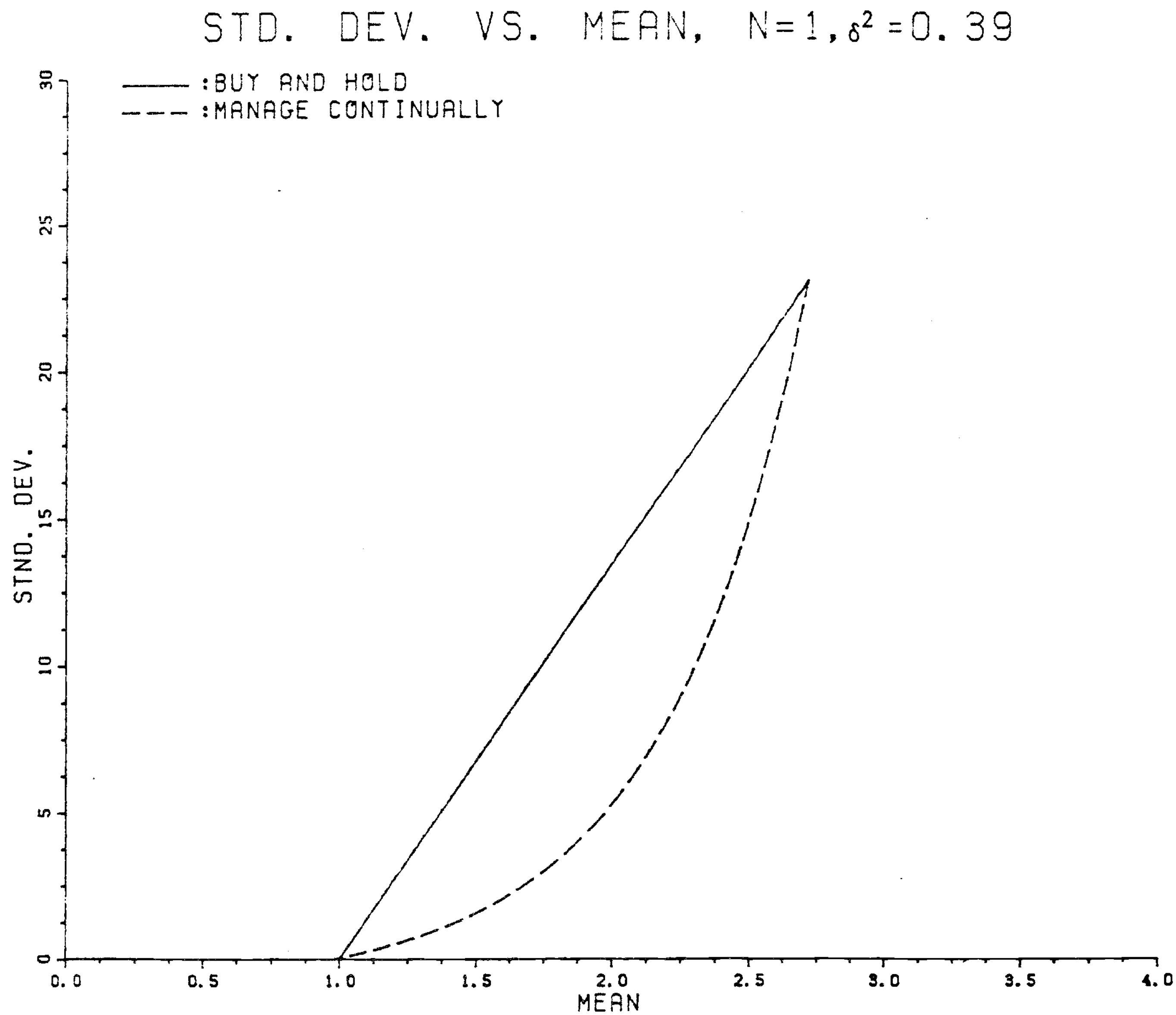
Parameters:

α .1

σ^2 = .039

T = 10

FIGURE 4a



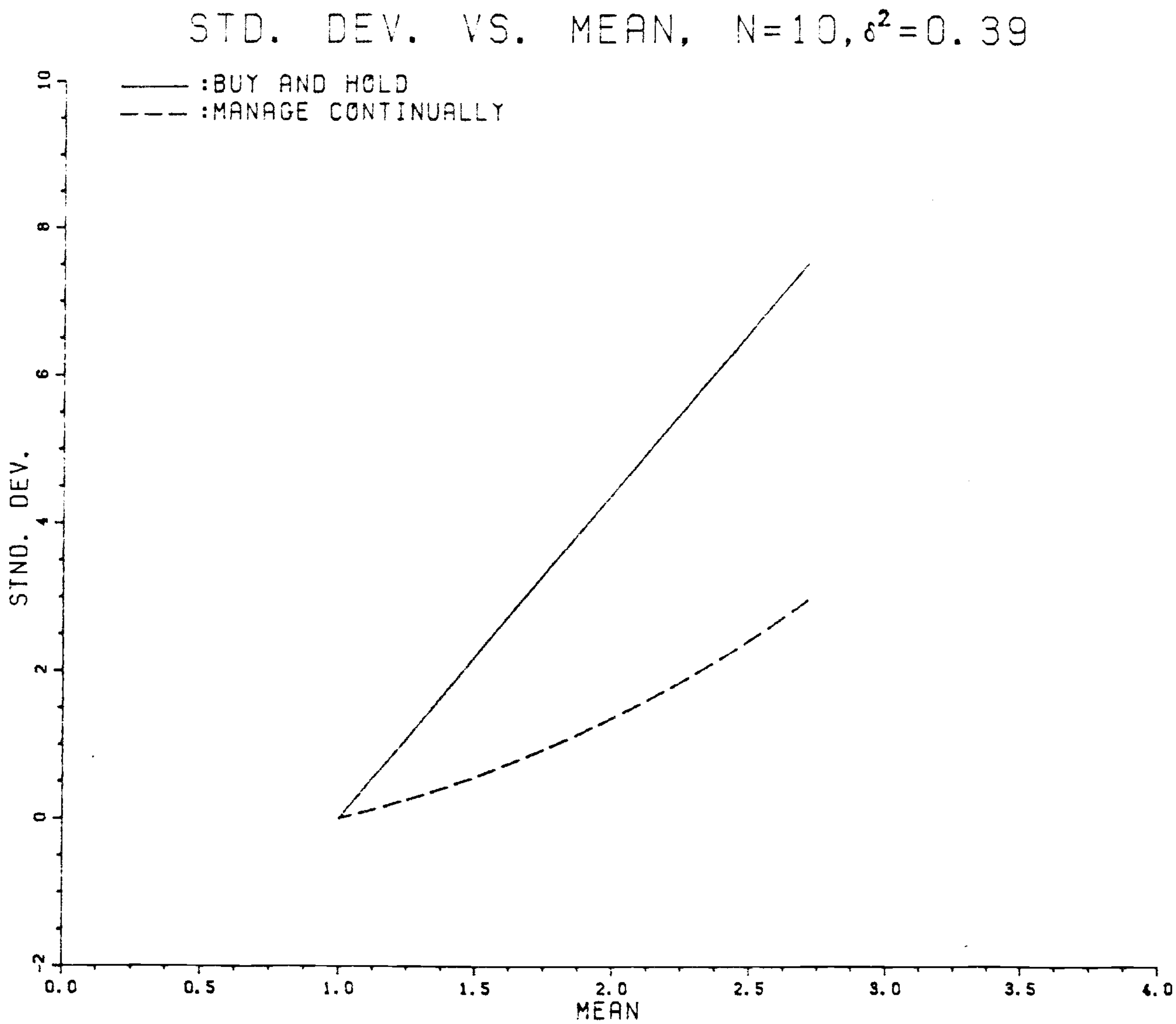
Parameters:

α .1

σ^2 = .039

T = 10

FIGURE 4b



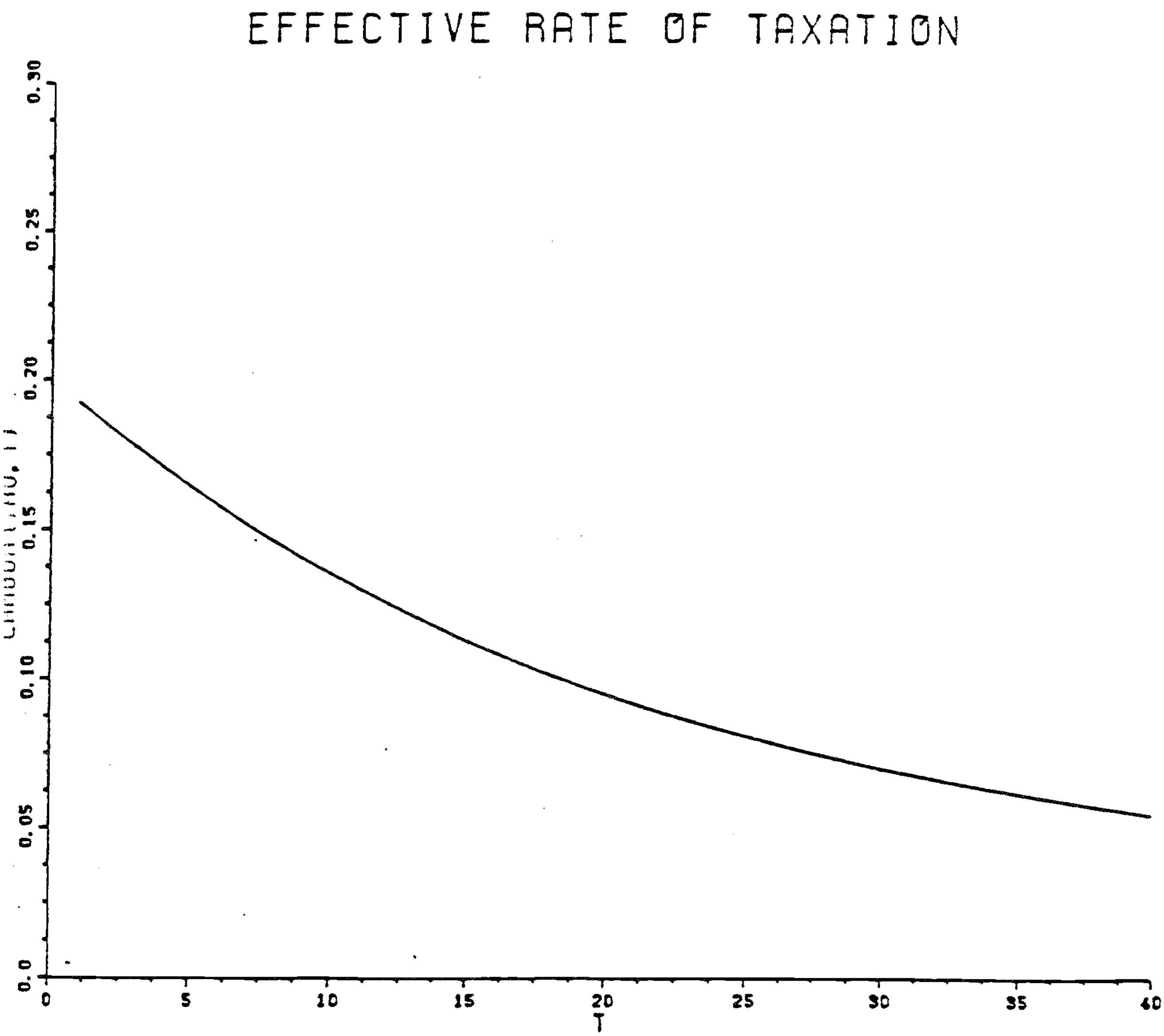
Parameters:

$\alpha \quad .1$

$\sigma^2 = .039$

$T = 10$

FIGURE 5



EFFECTIVE RATE OF TAXATION
0.30
0.25
0.20
0.15
0.10
0.05
0.0
0
5
10
15
20
25
30
35
40
T

Estimates of the Effective Rate of Taxation

TABLE 1

Diversifiable Risk -- Low

Risk Aversion -- Low

Statutory Tax Rate -- 0

N/T	1	3	5	10	15	20	25	30
5	0.059	0.013	-0.018	-0.001	0.006	0.004	-0.006	0.001
	(+0.068)	(+0.040)	(+0.032)	(+0.024)	(+0.020)	(+0.019)	(+0.022)	(+0.017)
	(-0.068)	(-0.040)	(-0.031)	(-0.024)	(-0.019)	(-0.018)	(-0.021)	(-0.016)
10	0.022	-0.006	0.003	0.005	0.009	0.003	-0.003	-0.007
	(+0.066)	(+0.038)	(+0.030)	(+0.022)	(+0.019)	(+0.018)	(+0.017)	(+0.016)
	(-0.066)	(-0.038)	(-0.030)	(-0.022)	(-0.019)	(-0.018)	(-0.017)	(-0.015)
15	0.036	0.020	-0.006	-0.003	-0.003	0.004	-0.008	0.005
	(+0.063)	(+0.037)	(+0.029)	(+0.022)	(+0.018)	(+0.017)	(+0.015)	(+0.014)
	(-0.063)	(-0.037)	(-0.029)	(-0.022)	(-0.018)	(-0.016)	(-0.015)	(-0.013)
20	-0.002	0.030	0.009	0.002	0.021	-0.016	0.000	-0.015
	(+0.063)	(+0.037)	(+0.029)	(+0.021)	(+0.017)	(+0.017)	(+0.015)	(+0.015)
	(-0.063)	(-0.037)	(-0.028)	(-0.021)	(-0.017)	(-0.017)	(-0.015)	(-0.015)

Confidence intervals are at the 95% level.

TABLE 2

Diversifiable Risk -- Low

Risk Aversion -- Low

Statutory Tax Rate -- 20%

N/T	1	3	5	10	15	20	25	30
5	0.161	0.175	0.143	0.134	0.124	0.110	0.090	0.095
	(+0.055)	(+0.033)	(+0.027)	(+0.022)	(+0.018)	(+0.017)	(+0.018)	(+0.016)
	(-0.055)	(-0.033)	(-0.027)	(-0.022)	(-0.018)	(-0.017)	(-0.017)	(-0.015)
10	0.201	0.186	0.150	0.135	0.124	0.107	0.084	0.075
	(+0.051)	(+0.031)	(+0.025)	(+0.021)	(+0.017)	(+0.016)	(+0.016)	(+0.016)
	(-0.051)	(-0.031)	(-0.025)	(-0.021)	(-0.017)	(-0.016)	(-0.015)	(-0.016)
15	0.196	0.153	0.181	0.169	0.113	0.104	0.086	0.078
	(+0.051)	(+0.032)	(+0.025)	(+0.019)	(+0.017)	(+0.016)	(+0.015)	(+0.015)
	(-0.051)	(-0.032)	(-0.025)	(-0.019)	(-0.017)	(-0.015)	(-0.014)	(-0.015)
20	0.221	0.218	0.164	0.158	0.131	0.103	0.098	0.076
	(+0.050)	(+0.030)	(+0.024)	(+0.019)	(+0.016)	(+0.015)	(+0.014)	(+0.014)
	(-0.050)	(-0.030)	(-0.024)	(-0.019)	(-0.016)	(-0.015)	(-0.014)	(-0.014)

Effective Rate Under Certainty

.192	.177	.164	.135	.113	.095	.081	.070

TABLE 3

Diversifiable Risk -- Low

Rish Aversion -- High

Statutory Tax Rate -- 0

N/T	3	5	10	15	20	25	30
5	0.039	0.044	0.039	0.009	0.075	0.068	0.076
	(+0.099)	(+0.076)	(+0.058)	(+0.051)	(+0.040)	(+0.038)	(+0.033)
	(-0.127)	(-0.092)	(-0.067)	(-0.059)	(-0.044)	(-0.042)	(-0.036)
10	0.039	0.043	0.035	0.012	0.000	0.029	0.045
	(+0.084)	(+0.066)	(+0.049)	(+0.041)	(+0.037)	(+0.033)	(+0.029)
	(-0.099)	(-0.074)	(-0.054)	(-0.045)	(-0.040)	(-0.035)	(-0.031)
15	-0.103	0.018	0.014	0.022	0.007	0.033	0.023
	(+0.097)	(+0.064)	(+0.047)	(+0.039)	(+0.036)	(+0.031)	(+0.029)
	(-0.122)	(-0.072)	(-0.052)	(-0.042)	(-0.038)	(-0.033)	(-0.031)
20	-0.004	-0.013	-0.016	-0.006	-0.024	-0.009	0.011
	(+0.081)	(+0.065)	(+0.047)	(+0.038)	(+0.035)	(+0.031)	(+0.028)
	(-0.093)	(-0.073)	(-0.052)	(-0.041)	(-0.037)	(-0.033)	(-0.029)

TABLE 4

Diversifiable Risk -- Low

Risk Aversion -- High

Statutory Tax Rate -- 20%

N/T	1	3	5	10	15	20	25	30
5	0.163	0.222	0.165	0.199	0.194	0.164	0.172	0.168
	(+0.109)	(+0.061)	(+0.052)	(+0.037)	(+0.032)	(+0.030)	(+0.027)	(+0.026)
	(-0.135)	(-0.068)	(-0.057)	(-0.039)	(-0.034)	(-0.032)	(-0.029)	(-0.028)
10	0.177	0.179	0.153	0.201	0.187	0.132	0.172	0.150
	(+0.093)	(+0.056)	(+0.047)	(+0.032)	(+0.028)	(+0.027)	(+0.023)	(+0.023)
	(-0.107)	(-0.060)	(-0.051)	(-0.034)	(-0.030)	(-0.029)	(-0.024)	(-0.024)
15	0.177	0.185	0.180	0.158	0.146	0.148	0.136	0.152
	(+0.091)	(+0.054)	(+0.044)	(+0.033)	(+0.028)	(+0.026)	(+0.023)	(+0.022)
	(-0.103)	(-0.058)	(-0.047)	(-0.034)	(-0.029)	(-0.027)	(-0.025)	(-0.023)
20	0.258	0.183	0.163	0.161	0.156	0.145	0.142	0.165
	(+0.081)	(+0.054)	(+0.043)	(+0.032)	(+0.028)	(+0.025)	(+0.023)	(+0.020)
	(-0.090)	(-0.058)	(-0.045)	(-0.034)	(-0.029)	(-0.026)	(-0.024)	(-0.021)

Effective Rate Under Certainty

.192	.177	.164	.135	.113	.095	.081	.070

TABLE 5

Diversifiable Risk -- High

Risk Aversion -- Low

Statutory Tax Rate -- 0

N/T	1	3	5	10	15	20	25	30
5	-0.041	0.150	-0.028	0.149	0.080	0.182	0.314	0.250
	(+0.137)	(+0.087)	(+0.085)	(+0.067)	(+0.124)	(+0.103)	(+0.091)	(+0.215)
	(-0.141)	(-0.088)	(-0.085)	(-0.065)	(-0.115)	(-0.094)	(-0.081)	(-0.157)
10	-0.019	0.049	0.056	0.008	0.159	0.147	0.128	0.188
	(+0.101)	(+0.068)	(+0.057)	(+0.063)	(+0.059)	(+0.081)	(+0.164)	(+0.145)
	(-0.103)	(-0.068)	(-0.056)	(-0.061)	(-0.056)	(-0.074)	(-0.131)	(-0.113)
15	0.020	-0.012	0.038	0.100	0.138	0.198	0.241	0.183
	(+0.086)	(+0.058)	(+0.051)	(+0.049)	(+0.050)	(+0.059)	(+0.067)	(+0.088)
	(-0.087)	(-0.058)	(-0.050)	(-0.048)	(-0.048)	(-0.055)	(-0.060)	(-0.075)
20	-0.022	0.016	0.036	0.073	0.112	0.108	0.185	0.202
	(+0.080)	(+0.052)	(+0.045)	(+0.043)	(+0.053)	(+0.079)	(+0.082)	(+0.091)
	(-0.081)	(-0.052)	(-0.044)	(-0.042)	(-0.051)	(-0.072)	(-0.072)	(-0.076)

TABLE 6

Diversifiable Risk -- High

Risk Aversion -- Low

Statutory Tax Rate -- 20%

N/T	1	3	5	10	15	20	25	30
5	0.155	0.143	0.262	0.202	0.236	-0.040	0.265	0.319
	(+0.104)	(+0.077)	(+0.061)	(+0.071)	(+0.082)	(+0.799)	(+0.106)	(+0.085)
	(-0.107)	(-0.077)	(-0.060)	(-0.069)	(-0.078)	(-0.452)	(-0.093)	(-0.074)
10	0.236	0.236	0.208	0.296	0.288	0.310	0.246	0.283
	(+0.077)	(+0.052)	(+0.049)	(+0.045)	(+0.059)	(+0.050)	(+0.136)	(+0.072)
	(-0.078)	(-0.052)	(-0.049)	(-0.044)	(-0.056)	(-0.047)	(-0.112)	(-0.063)
15	0.187	0.186	0.207	0.193	0.191	0.254	0.211	0.107
	(+0.070)	(+0.046)	(+0.043)	(+0.047)	(+0.096)	(+0.059)	(+0.145)	(+0.607)
	(-0.070)	(-0.046)	(-0.043)	(-0.046)	(-0.088)	(-0.055)	(-0.117)	(-0.266)
20	0.150	0.181	0.164	0.188	0.190	0.252	0.181	0.185
	(+0.066)	(+0.042)	(+0.038)	(+0.040)	(+0.050)	(+0.051)	(+0.101)	(+0.372)
	(-0.066)	(-0.042)	(-0.038)	(-0.039)	(-0.048)	(-0.048)	(-0.087)	(-0.208)

Effective Rate Under Certainty

.192	.177	.164	.135	.113	.095	.081	.070

TABLE 7

Diversifiable Risk -- High

Risk Aversion -- High

Statutory Tax Rate -- 0

N/T	3	5	10	15	20	25	30
15	0.436	0.571	0.814	0.971	1.062	1.123	1.189
	(+0.093)	(+0.061)	(+0.034)	(+0.026)	(+0.021)	(+0.018)	(+0.017)
	(-0.119)	(-0.069)	(-0.036)	(-0.027)	(-0.022)	(-0.019)	(-0.018)
20	0.333	0.479	0.731	0.875	0.972	1.026	1.104
	(+0.096)	(+0.064)	(+0.036)	(+0.027)	(+0.021)	(+0.019)	(+0.017)
	(-0.123)	(-0.072)	(-0.038)	(-0.028)	(-0.022)	(-0.020)	(-0.018)

TABLE 8

Diversifiable Risk -- High

Risk Aversion -- High

Statutory Tax Rate -- 20%

N/T	3	5	10	15	20	25	30
15	0.453	0.521	0.745	0.817	0.913	0.969	1.000
	(+0.075)	(+0.053)	(+0.029)	(+0.021)	(+0.016)	(+0.013)	(+0.011)
	(-0.089)	(-0.059)	(-0.030)	(-0.022)	(-0.017)	(-0.013)	(-0.011)
20	0.367	0.407	0.662	0.773	0.848	0.897	0.954
	(+0.074)	(+0.056)	(+0.029)	(+0.021)	(+0.017)	(+0.014)	(+0.011)
	(-0.087)	(-0.063)	(-0.030)	(-0.022)	(-0.017)	(-0.014)	(-0.012)

Effective Rate Under Certainty

.177	.164	.135	.113	.095	.081	.070