2011

I Have a Solution to Share: Learning through Equitable Participation in a Mathematics Classroom

Mary Q. Foote
Queens College, City University of New York

Rachel Lambert
Chapman University, lambertr@chapman.edu

Recommended Citation
Foote, M. Q., and R. Lambert. (2011). I Have a Solution to Share: Learning through Equitable Participation in a Mathematics Classroom. *Canadian Journal of Science, Mathematics and Technology Education, 11*(3), 247-260. doi: 10.1080/14926156.2011.595882

I Have a Solution to Share: Learning through Equitable Participation in a Mathematics Classroom

Comments
This is an Accepted Manuscript of an article published in *Canadian Journal of Science, Mathematics and Technology Education*, volume 11, issue 3, on September 2, 2011, available online at DOI: 10.1080/14926156.2011.595882

I Have a Solution to Share:
Learning Through Equitable Participation in a Mathematics Classroom

by

Mary Q. Foote and Rachel Lambert

ABSTRACT: *Student participation is an issue of equity. Without participation there can be no learning. This study focuses on the participation (and therefore learning) of struggling students (those with Individual Instructional Plans (IEPs)) during the implementation of a relational thinking routine in a third-grade inclusion classroom. Students with IEPs often initially used direct modeling with linking cubes as a resource for presenting their thinking. In this way, they were able to demonstrate their ability to think relationally. As the year progressed, these students, who had earlier been reluctant to share, and had done so only by using several of the resources that the participation structure of the routine provided, often showed a growth in their abilities to explain their thinking verbally.*

Student participation is an issue of equity and achievement; students who participate more generally learn more from the lesson, and low rates of participation can predict low achievement in the early grades (Cohen, 1984; Finn & Cox, 1992). Learning can be conceptualized as evolving participation in a set of practices (Lave & Wenger, 1991). Equity, although defined in many different ways, is fundamentally about fairness or justness (Gutiérrez, 2002); equitable participation in mathematics classrooms results in learning and therefore achievement. Gutiérrez posits that we could consider that equity has been achieved when we can no longer predict patterns such as achievement and participation "solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language" (p. 153). In this paper, we focus on student participation patterns during an algebraic (relational thinking) routine in a third-grade inclusion classroom. We examine how students took up opportunities to participate and therefore learn within a classroom environment that promoted the equitable participation of *all* students, including those students with Individual Education Plans (IEPs).

Empson (2003) contends that there are few studies that document the successful participation and learning of struggling students in discussion-based classrooms. We are particularly concerned with the inclusion of low-achieving students in conceptual mathematical practices such as solving relational thinking problems. Research has documented the unequal participation of low achieving students in discussion-based classrooms, including lack of participation and fewer mathematical contributions than their higher performing peers (Baxter, Woodward & Olsen, 2001). After days, months, and years of limited participation, low achieving students are less likely to take up opportunities to learn, such as making presentations in class. This study analyzes how students in this classroom were able to interrupt this cycle by using the resources available in the participation structure (Erickson, 1982) of the classroom routine.

Theoretical Perspectives

In examining the participation and learning of students in this study (particularly those with IEPs), we situate the discussion of mathematical content and strategies within a larger sociocultural framework. Within this framework, we follow Lave and Wenger (1991) in arguing that participation IS learning. More equitable participation then means more equitable learning outcomes. Yackel and Cobb (1996) have demonstrated how the co-constructed norms of a classroom community shape the learning of the students. Established norms such as the necessity of (a) explaining one's thinking, and (b) carefully attending to the presentations of classmates, help to create an atmosphere in the classroom where everyone can engage in substantive mathematical thinking and everyone is respected for his or her contribution. In our case, student participation occurs within the context of such a classroom community. Erickson (1982) points to the participation structure of the enacted task. Resources available to students within a given participation structure (in this case a classroom routine designed to develop relational thinking) also shape participation and therefore learning. We analyze how resources available to students were taken up in the classroom by students with IEPs, resulting in increased participation and therefore learning on their part.

The classroom routine used in this study is based on the work of Carpenter, Franke, and Levi (2003) around relational thinking. It examines the use of number sentences (equations) designed to develop concepts of equality and relational thinking. These include two types of number sentences. One type is complete number sentences about which students must ascertain the truth or falseness. Examples of these are: 2=2; 3+0=3; 5=1+4. These number sentences are used to challenge children's notions of the meaning of the equal sign. As Carpenter and colleagues note, children often reject the previous examples (and others) as being equal because respectively (a) there is no operation; (b) adding zero is not really adding anything so it isn't allowed; and (c) the order is wrong. A second type of number sentence used to develop relational thinking is one in which an unknown is present such as 3+10=□+7. Children who believe the equal sign indicates that the answer comes next will predict that 13 is the correct response for the unknown (Falkner, Levi, & Carpenter, 1999). Challenging children's emergent understanding of the equal sign is one aspect of supporting them in developing relational (or algebraic) reasoning.

A second aspect is supporting children in developing the capacity to use relational strategies instead of computational strategies when solving for an unknown (Carpenter et al., 2003). Through working on series of number sentences containing unknowns and carefully selected values for the given numbers, children begin to see that it can be easier NOT to compute to find an unknown value. For example, in the case of judging the truth or falseness of the number sentence 27+37= 25+39, instead of computing to find that the value for each side of the equation is 64, children will begin to use relational strategies to determine whether the expression is true. They will, for example decompose 27 into 25+2 and 39 into 2+37 resulting in an expression that they now see as clearly equal (25+2+37=25+2+37) since both sides of the equation contain identical numbers.

In order to understand the representations that students used to solve problems like the ones described above, we draw on earlier work of Carpenter and colleagues (Carpenter, Fennema, Franke, Levi, & Empson, 1999). We extend a framework (direct modeling, counting, and numeric strategies) previously used to analyze solution paths to contextualized problems, to this work done around relational thinking. Direct modeling involves modeling each value in a problem with concrete materials. The original framework supports us in understanding, for example, how a child might use direct modeling with cubes or other manipulatives to solve the following problem: I have three pencils. I pick up four more pencils from the classroom floor. How many pencils do I now have? Following the action of the problem, the child might lay out three cubes, representing the initial number of pencils, add four more cubes to represent the pencils found on the floor, and then count the entire number of cubes to arrive at the result of seven pencils. A child using a counting strategy might solve by counting on from four, saying 4, 5, 6, 7 to arrive at the solution. A child using a numeric strategy might know the math fact 4+3=7 or might derive the fact thinking: I know 3+3=6 and so one more will be 7. This framework supports our analysis that children with poor numeric strategies (often in our case, children with IEPs) were able to engage in relational thinking through the use of concrete materials.

For a final note, we are taking a social constructivist view of competence and disability in this paper. Within this study we are considering the label of *students with IEPs*, not as an inherent and static determinant of individual ability, but as a school-based designation which reflects and recreates differential ability within the classroom (Dudley-Marling, 2004; McDermott, Goldman & Varenne, 2006). Because of the importance of the designation of *students with IEPs* in the culture of schools, we choose to use this classification to focus attention on how *all* children were able to participate (and therefore learn) successfully in a discussion-based classroom, in this way achieving equity.

Research Question

In this paper we examine student participation during a classroom routine focused on the development of relational thinking (Carpenter et al., 2003) with particular attention to the participation of students with IEPs. We ask: How do students with IEPs take up opportunities to participate (and therefore learn) in an inclusion classroom during a routine focusing on relational thinking?

Methods

During the course of one school year, a teacher in a third grade inclusion classroom employed a weekly routine focused on developing children's competency with relational thinking (Carpenter et al., 2003). Once a week for approximately 30-45 minutes, the teacher presented the class with number sentences to solve. These were either (a) complete number sentences to be judged true or false, or (b) number sentences that had to be solved for an unknown. The students worked independently for 20-30 minutes (with the teacher, student teacher, and participant/researcher circulating to assist students). Several students then volunteered to present their thinking to the class. These presentations were video-taped. The data for this article are drawn from the presentation portion of the routine (the problem solving portion done independently was not recorded and is not therefore available for analysis). A total of 25 weekly sessions were videotaped and comprise the data set.

Background of the Study

Video clips of the classroom routine reported on in this paper were used in a component of a professional development seminar (PD). This seminar met monthly for one school year. The focus of the PD was a dual one examining both (a) children's relational thinking, and (b) the socio-political factors that might impact their performance and participation. (For a discussion of both foci of the PD, see Battey, Foote, Spencer, Taylor, & Wager, 2007; Foote, 2005; Foote, Loomis, Slaughter & Wager, 2005; Wager et al., 2010). The classroom sessions that comprise the data set for this paper were taped in order to provide video clips for the portion of the PD focusing on relational thinking. One of the teacher participants in the PD was approached, on the recommendation of a member of the district mathematics resource staff, to determine if she would be willing to have her sessions on relational thinking videotaped during that school year. The teacher agreed and taping of her class routine ensued. Each month, the research team reviewed the video corpus for the previous month and selected a short clip to present at the PD. For this paper, the entire video corpus was reviewed.

Participants

There were 14 participants, eight boys and six girls. Nine of the students were Black, five were White. Four students had IEPs (three were classified as learning disabled, one as behaviorally disabled); all four of these (three boys and one girl) were Black. These 14 participants are the students who were enrolled in this class for the majority of the school year and who participated in the routine at least once; there were two students with developmental delays (e.g. Downs Syndrome) who were also in the class for the entire year but did not participate in the routine. There were six other students who were class members for a smaller portion of the school year (several weeks or several months). Five of these students left the class before half the year had passed and one arrived late the second half of the year, making analysis of their participation incommensurable with those students who attended for the majority of the school year. Of the 14 participants then, 28.6 % were students with IEPs. It is the three participants who were classified as learning disabled (Caleb, Janice, and Marcus)[1] who will figure prominently in the results as we examine their trajectories of participation.

Classroom Context

An atmosphere of respect for all permeated the classroom. The teacher worked hard to establish a classroom community that encompassed much more than the mathematics periods. Throughout the year, but particularly at the beginning of the year, she held class meetings to discuss interpersonal issues such as name calling that arose inside the classroom and in other school spaces such as the cafeteria and playground. In addition to this work on social issues, she fostered an academic atmosphere in the classroom where students were supported in expressing their thinking (whether in mathematics or in other content areas). The classroom norms were enforced through gentle reminders by the teacher as well as explicit noticing of appropriate behaviors. Students were expected (a) to listen respectfully, and (b) to comment on other students' work affirmatively, not pejoratively. This is not to say that disagreement was discouraged, simply that any commentary needed to be made in a supportive manner.

Specifically with regard to mathematics, the teacher created a supportive environment for all students to learn complex concepts. She had high expectations that all students would move from using computational to using relational thinking strategies. She engaged the students in the co-construction of classroom norms that supported student success. For example, every student was expected to explain his/her thinking out loud at the board using a combination of verbal and written explanation; every student was expected to attend carefully to the presentations, and encouraged to compare the strategies that s/he used with the strategies of the presenter; attention was focused on thinking, not on correctness of answer. There was a co-constructed understanding regarding the resources that were available for students to use both for solving problems and presenting solutions. Students were encouraged to use manipulatives to solve problems without loss of status (without being seen as less competent mathematically) (Cohen, 1984; Cohen & Lotan, 1995; 1997), and then to present their solutions using these manipulatives as a resource. Students were encouraged to use the notebooks in which they had written their solutions as a resource in their presentation without loss of status. When a student struggled to present his/her ideas, the teacher asked questions and re-voiced statements, but only after allowing considerable wait time, often at several points throughout the student's presentation, thus allowing herself to be used by students as a resource, yet leaving the student in charge of the presentation.

Data Analysis

For analysis, the video was segmented by student presentation so that one student presenting his/her solution path to a given problem on a given day constituted one unit. For each unit, a detailed narrative description of the student solution path was constructed. These narratives included a detailing of all available resources (notebooks, manipulatives, the teacher) used in the presentation. The solution paths were then coded for relational or computational thinking (Carpenter et al., 2003). Following this initial coding, a second pass was taken through the data. At this time the solution paths were coded as to what type of representation was used: direct modeling, counting, or numeric (Carpenter et al., 1999). In addition we catalogued which participants presented their thinking at each session so as to examine to what extent all students (particularly those with IEPs) participated in this portion of the activity.

Results

We begin the results with an overview of participation in the relational thinking routine. This includes an overview of how both regular education students and those with IEPs used the resources (notebooks, manipulatives, the teacher) made available in the participation structure of the relational thinking routine. We then turn to more extended examples of the work of the three student with IEPs. Particular attention is paid to the ways in which these students used available resources to support their presentations.

Overview. At the beginning of the year regular education students were the ones who presented their thinking most often. Out of a total number of 62 presentations made in the first half of the year, four were made by students with IEPs. That is to say 6.5% of the presentations were made by students with IEPs. In the second half of the year, students with IEPs presented much more often. Of the 46 presentations made in the second half of the year, 11 (23.9%) were made by students with IEPs. This is nearly equivalent to their percentage (28.6%) in the classroom.

Notebooks and manipulatives as resources. On the majority of occasions, both students with and without IEPs used their notebook as a resource when presenting their thinking. There are only a few instances in which students did not go to the board to present with notebook in hand. These instances were generally when a student was commenting on the solution path of another student, rather than initiating the explanation of his/her own strategy. No regular education students used direct modeling with manipulatives when solving or presenting a solution. In six of the 15 instances of presentations by students with IEPs (or 40% of the time), however, students directly modeled their solution with linking cubes or other manipulative materials. In addition, no regular education student used counting as a solution strategy. In the case of students with IEPs, however, one solution path included the use of a counting strategy. Whereas regular education students presented numeric strategies for solving 100% of the time, students with IEPs did so 53% of the time.

The teacher as a resource. Students with IEPs, compared to peers without IEPs, drew heavily on the teacher as a resource for explaining their thinking. Rarely in the cases of students without IEPs, did the teacher join the student at the board in order to act as a resource for them in presenting their thinking. Even in the case of students with IEPs, the teacher did not offer herself as a resource until it appeared that the student needed some scaffolding questions in order to explain their solutions (most often when the student fell completely silent). To put it another way, students with IEPs had been effective problem solvers (as evidenced by the work in their notebooks to which the teacher referred when acting as a resource for them), but often had some difficulty in expressing verbally what they had accomplished, or in being effective solution reporters (Empson, 2003). Of the 15 presentations made by students with IEPs during the course of the year, in ten of these presentations (67%) students drew on the teacher as a resource.

The case of Caleb. Caleb made presentations to the class a total of six times (see Table 1). He was a student who was willing to present his thinking beginning relatively early in the school year.

Table 1

Caleb's Presentations

Date	Number Sentence	Solution type	Strategy	Resources Used
10/13	7+8=15+1	Computational	Numeric	Notebook Teacher
12/08	19+3=□+9+3	Relational	Direct modeling	Notebook Teacher Manipulatives
01/20	□+20=10+10+7	Relational	Numeric	Notebook Teacher
02/03	7+7=8+6	Relational	Direct modeling	Notebook Teacher Manipulatives
04/20	4/3=1/3+1/3+1/6+1/6	Computational	Direct modeling	Notebook Teacher Manipulatives
04/27	5+1+1=8	Computational	Numeric	None

Although initially he had been reluctant to present, with encouragement from the teacher he agreed to go to the board. In this instance of presenting his solution to the number sentence 7+8=15+1, True or False, he relied heavily on the teacher as a resource. Caleb went to the board and wrote 15 under 8+7, then after a long pause wrote 16 under 15+1, and circled false. He then stood silently. In what follows we see how Caleb was supported in drawing on the teacher as a resource to explain his thinking verbally.

Teacher: Tell us in words what you were thinking please?

Caleb: (No response, looks down)

Teacher: How did you know that this was a false equation, a false number sentence?

Caleb: (Looks down)

Teacher: Look at the board please.

Caleb: (He turns and looks at the board.)

Teacher: What is 8+7 equal to?

Caleb: 15

Teacher: What is 15 + 1 equal to?

Caleb: 16

Teacher: Is 15 equal to 16?

Caleb: (Under his breath, he repeats the question to himself.) No.

Teacher: No, so false is the correct answer.

On two subsequent occasions (12/08 and 2/03) Caleb used and presented a direct modeling strategy to support a relational thinking solution. The explanations were similar in both cases so we present only the solution to the number sentence 19+3=□+9+3, presented in the first instance. Caleb brought connecting cubes to the board and began his presentation by meticulously arranging his cubes into stacks of 10, 9, 3 and then 9 and 3. After a significant lapse of time during which Caleb remained silent, the following exchange occurred:

Teacher: What number of cubes do you have, Caleb?

Caleb: (He doesn't answer, but continues arranging his cubes.)

Teacher: Look. Here you've got 10 and 9 and 3. And here I see 9 and 3. (Teacher looks into his notebook). Well what did you do on this side, Caleb? (Indicating the 10 and 9 and 3.)

Caleb: I made 19 and 3.

Teacher: You started with the 10 . . .

Caleb: And then I put the 9.

Teacher: And that made how much?

Caleb: 19

Teacher: You've got the 19+3 on this side (showing his cube representation). What about this side of the equal sign?

Caleb: On this side I had 9 and 3.

Teacher: So what was missing here?

Caleb: The ten (showing his separated stack of ten cubes).

Although Caleb still relies on the teacher as a resource, his own explanation has grown beyond the one word responses we saw in the first example.

Much later, in the spring Caleb made his last presentation. Although his first and last presentations used computational thinking and numeric strategies, there were large differences between the two presentations. In this case he presented his thinking completely verbally and from his seat without drawing on any of the available resources to support his presentation. In responding to the question of whether the number sentence 5+1+1=8 is true or false, he not only answered that the equation was false, but added to his explanation what would be necessary to make it true: "This [indicating the 5] would have to be a 6."

As we see, Caleb drew on the teacher as a resource in explaining his thinking on occasions examined in this section and on others. In addition, he used the resource of manipulative materials to support his relational thinking solution on three occasions. By the end of the school year, however, Caleb provided a completely verbal explanation for his thinking, unsupported by the use of any resource.

The case of Janice. Janice made four presentations to the class (See Table 2). She was initially reluctant to be a solution presenter[2].

Table 2

Janice's Presentations

Date	**Number Sentence**	**Solution type**	**Strategy**	**Resources Used**
11/03	59+6=59+7	Computational/ Relational	Direct modeling	Notebook Teacher Manipulatives
02/17	2x8=8+8=9+□	Relational	Direct modeling	Notebook Teacher Manipulatives
03/17	16-3-7=16-7-3	Relational	Numeric	Notebook Teacher[a]

[a] She uses the teacher minimally.

Two months into the school year she made her first presentation (59+6=59+7, True or False). She used a direct modeling strategy to present a solution that is partially computational and partially relational. She went to the board with her notebook and linking cubes, but then was very reluctant to speak. She looked back and forth from her notebook to the board for nearly a minute until the teacher asked if she would like some help. She employed single words and nods to answer the teacher's questions as to how she had solved the problem. Using the teacher as a resource, she indicated that she had modeled both sides of the equation with cubes, and then added 59+6 for a result of 65. The teacher supported her in writing that result on the board under 59+6. The teacher then asked her how she thought about the numbers to the right of the equal sign. Janice said, "The same." When the teacher asked what was the same, Janice pointed to the two 59s. The teacher took a marker and circled both 59s saying, "You're saying these are the same on both sides. And you have six over here." Janice joined in saying, "And seven is there (indicating the right side of the equation)." The teacher then asked if the expressions on the both sides of the equal sign were equal and Janice said, "No." When the teacher asked why, Janice paused for several seconds. The teacher then asked which was bigger and Janice said, "The seven."

During her second presentation, Janice marched to the board with her notebook and linking cubes and confidently wrote seven in the blank in the number sentence 2x8=8+8=9+□. She then stood silently and did not proceed with an explanation until the teacher made herself available to be used as a resource, so she could explain that she had made two stacks of eight cubes, modeling 2x8, and another two stacks modeling 8+8, thus showing them to be equal. Then using two of the stacks of eight cubes, she demonstrated how she had taken one cube from one of the stacks of eight and added it to the other stack of eight so that she had nine cubes in one stack and seven in the other. In this instance any verbalizations by Janice were nearly whispers.

In her third presentation, Janice used a numeric strategy to present a relational solution to the problem 16-3-7=16-7-3. Although initially she ran up to the board smiling with notebook in hand, she then stood there silently, raising her notebook to cover her face. In this case she used the teacher as a resource but only in a minimal way. The teacher approached Janice, lowered her notebook so her face could be seen and stood silently beside her for support. Janice read from her notebook, “Since you are taking seven and three away from 16 on both sides, they are equal.”

The case of Marcus. Marcus made four presentations throughout the year. As we see in Table 3, Marcus did not use the teacher as a resource in his early presentations. It is worth noting, however, that he did not make his initial presentation until the second half of the year.

Table 3

Marcus's Presentations

Date	**Number Sentence**	**Solution type**	**Strategy**	**Resources Used**
02/10	6+6=4x4	Computational	Direct modeling	Notebook Manipulatives
02/17	16+39=38+17	Relational	Numeric	Notebook
02/24	38+46=37+47	Relational	Numeric	Notebook Teacher[a]
04/27	34-7=32-9	Computational	Counting	Notebook Teacher[b]

[a] He uses the teacher minimally and because initially he misspeaks (saying the number sentence is false instead of true) and becomes flustered when he realizes he misspoke.

[b] He uses the teacher only as support for representing his thinking on the board.

In this case too then, we see a reluctant participant who waited a significant time to begin to make presentations to the class. In his first presentation he used a combination of direct modeling and numeric strategies to present a computational solution to the number sentence 6+6=4x4, True or False. He added 6+6 for a result of 12. He then directly modeled 4x4, drawing four circles and four tally marks within each circle, finding a result of 16 for this portion of the equation and correctly determining that the number sentence was false. In the next two instances of determining the truth or falseness of number sentences (16+39=38+17 and 38+46=37+47) he used numeric strategies to present relational solutions, arguing in the first case that if he took one from the 39 and "gave" it to the 16, he would have 17+38 on both sides of the equal sign. He used similar thinking to solve the second number sentence. In his final presentation for the number sentence 34-7=32-9, however, he used a counting strategy to solve the problem computationally, counting back seven from 34 to arrive at 27 and back nine from 32 to arrive at 23. In this case, Marcus used the teacher as a resource. He came to the board prepared to explain his thinking verbally, but drew on the available resource of the teacher in order to present a written explanation.

Discussion

Mathematics classrooms are too often focused on a single ability: executing procedures correctly (Boaler, 2006). In this classroom we see a focus on student thinking, not merely correct answers. This focus supported the development of relational thinking even for students with IEPs. Through the analysis of Caleb, Janice, and Marcus's participation in the classroom routine, we can see a movement of algebraic thinking from computational to relational. The trajectories in the movement from computational to relational explanations, however, do not follow a uni-directional path. Neither is the trajectory from direct modeling or counting strategies to numeric strategies uni-directional. Not surprisingly, the choice of a solution path appears at least in some cases to be dependent on the particular number sentence. Marcus, for example, in the case of his final presentation (34-7=32-9), may have chosen a computational solution path since subtraction number sentences had been used much less frequently than addition ones throughout the year. Because of this he had less exposure to and experience with them. In addition, subtraction offers particular challenges to thinking relationally that addition (and multiplication) do not due to the fact that they are commutative. In the case of Caleb's final presentation (5+1+1=8), he may have used a computational solution path because the numbers were so small that computation was easy. As Carpenter and colleagues (2003) remind us, carefully selected values for the numbers in equations support (or not) students in thinking relationally.

Another critical aspect of the participation structure that supported equity was the license to use manipulatives as a resource and direct modeling as a solution path. The equal status that direct modeling held with numerical solutions meant that more students could be successful problem solvers and solution presenters and thereby through increased participation learn more. In his second presentation, Caleb needed fewer prompts to explain his thinking than he had in the first exchange. The manipulatives supported both his thinking, and the presentation of his solution. The cubes allowed him to demonstrate the decomposition of 19 into 10 and 9, which was central to his thinking relationally about the problem. This thinking was scaffolded by his use of direct modeling to solve the problems. In addition, we can see the development of his participation over time, from a student who was reluctant to speak (first presentation), to a student who confidently engaged in the discussion from his seat (final presentation).

A participation structure such as the one described here offers a context in which students (even those who struggle to present their mathematical thinking verbally) can become full participants in a discussion-based classroom. It takes time, however, for these presentations to evolve. Caleb's first presentation on the number sentence 7+8=15+1 lasted seven minutes; his second presentation on the number sentence 19+3=□+9+3 lasted 11 minutes. Both included several lengthy pauses. In addition, as we can see from the data, it took months for Caleb, Janice, and Marcus to become confident presenters. Again, it takes time, as well as resources, for students to develop their ability to participate in challenging mathematics.

As the results demonstrate, Caleb and Janice, who struggled significantly with computation and with verbal explanations, were nonetheless able to think relationally and demonstrate this thinking using direct modeling. In addition, by the end of the year, they were able to present their thinking more independently. Caleb's final presentation was done from his seat without drawing on any of the three resources. He answered quickly, confidently, and completely verbally in just a few seconds. Janice was able to read out of her notebook drawing on the teacher as a resource only minimally. It was the first time she had used more than one or two word utterances in presenting a solution to the class. Marcus had more developed verbal presentation skills, but became easily flustered when presenting. Being able to use the teacher as a resources meant that he was able to be a successful solution reporter.

In this classroom, all students were ultimately problem solvers and solution reporters (Empson, 2003). Students with IEPs presented their solutions to problems in the relational thinking routine, just as did regular education students. The role of solution reporter in particular, was not immediately taken on by Caleb or Janice or Marcus. They were initially reluctant participants in the reporting portion of the routine. But the participation structure of the relational thinking routine that provided for the use of the multiple resources of notebooks, manipulatives, and the teacher, supported these students in becoming fuller participants in the classroom routine, thus indicating learning on their parts.

Conclusion

In our work with teachers and students, we are frequently asked how to include all students in high-level mathematical thinking. Teachers, faced with low-achieving students who may not often participate in whole group discussion, assume that this kind of instruction is not for "them." As Empson (2003) suggests, teachers want to help students save face, and not to embarrass those who are struggling. This classroom presents an equitable resolution to this conflict. Through high expectations of participation, and available resources to support students as problem solvers and solution presenters, all students were able to present relational thinking to their classmates. We believe that the results of this study are significant, as they demonstrate that *all* students can be successful with a highly conceptual approach like relational thinking. Following Gutiérrez (2002) we note that in the second half of the year, it was not possible to predict based on the academic status of the student (regular education student or student with IEP) whether s/he would be a full participant during the relational thinking routine. In this way we move closer to achieving equity.

Implications

More work needs to be done to document the successful engagement and learning of students who have been traditionally underserved by schools (poor students, students of color, English learners). Little is written about the successful inclusion of students with IEPs (often also poor students, students of color, and English learners) in conceptually challenging mathematics. The results of this study demonstrate that this successful inclusion is certainly possible.

The resources available to students within the participation structure of the relational thinking routine described in this paper offer ideas for teachers as to effective ways to structure a classroom routine to support all students that they teach. If we are to support the growth and development of all students, not just those who start the year with good numeric skills and with good verbal presentation skills, then we must expand the resources we offer students in order to increase participation and therefore learning.

ACKNOWLEDGEMENT: This research was supported in part by a grant to the Diversity in Mathematics Education Center for Learning and Teaching (DiME) from the National Science Foundation (ESI9911679). The opinions expressed in this paper do not necessarily reflect the position, policy, or endorsement of the NSF.

References

Boaler, J. (2006) How a detracked mathematics approach promoted respect, responsibility, and high achievement. *Theory into Practice*, *45*(1), 40-46.

Battey, D., Foote, M. Q., Spencer, J., Taylor, E.V., & Wager, A. A. (2007, March). *Professional development at the intersection of mathematics and equity.* Symposium conducted at the annual research pre-session of the National Council of Teachers of Mathematics, Atlanta.

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of Reform-Based Mathematics Instruction on Low Achievers in Five Third-Grade Classrooms. *Elementary School Journal*, *101*(5), 529-547.

Carpenter, T. P., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Franke, M., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Cohen, E. (1984). Talking and working together: Status, interaction and learning. In P. L. Peterson, L. C. Wilkinson & M. Hallinan (Eds.), *Instructional groups in the classroom: Organization and processes* (pp. 171-187). Orlando, FL: Academic.

Cohen, E., & Lotan, R. (1995). Producing equal-status interaction in the heterogeneous classroom. *American Educational Research Journal, 32*(1), 99-120.

Cohen, E., & Lotan, R. (1997). Raising expectations for competence: The effectiveness of status interventions. In E. Cohen & R. Lotan (Eds.), *Working for equity in heterogeneous classroooms* (pp. 77-91). New York: Teachers College Press.

Dudley-Marling, C. (2004). The social construction of learning disabilities. *Journal of Learning Disabilities*, *37*(6), 482-489.

Empson, S. B. (2003). Low-performing students and teaching fractions for understanding: An interactional analysis. *Journal for Research in Mathematics Education*, 305–343.

Erickson, F. (1982). Classroom discourse as improvisation: Relationships between academic task structure and social participation structure in lessons. In L. C. Wilkinson (Ed.), *Communicating in the classroom* (pp. 153-181). New York: Academic Press.

Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics, 6*(4), 232-236.

Finn, J. D., & Cox, D. (1992). Participation and withdrawal among fourth-grade pupils. *American Educational Research Journal, 29*, 141-162.

Foote, M. Q. (2005, February). *Teachers' understanding of the influence of socio-cultural factors on children's mathematical thinking, performance, participation.* Poster session presented at the annual meeting of the National Science Foundation Centers for Learning and Teaching, Washington D.C.

Foote, M. Q., Loomis, T., Slaughter, M., & Wager, A. A. (2005, April). *The Influence of Socio-cultural Factors on Mathematics Learning*. Poster session presented at the Research Pre-session of the National Council of Teachers of Mathematics Annual Meeting, Anaheim, CA.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2/3).

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

McDermott, R., Goldman, S., & Varenne, H. (2006). The cultural work of learning disabilities. *Educational Researcher*, *35*(6), 12-17.

Wager, A. A., Foote, M. Q., Edwards, A. R., Pitts Bannister, V., Ho, K. M., & Quindel, J. (2010, April). *Professional development integrating equity and mathematics*. Symposium conducted at the research pre-session of the annual meeting of the National Council of Teachers of Mathematics, San Diego, CA.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, arguentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

---

[1] The names of all participants are pseudonyms.

[2] Janice was a student with poor attendance who was also persistently tardy. This meant that she often arrived at school after the relational thinking routine was completed. This impacted her opportunities to participate both in problem solving and in presenting her thinking. Nonetheless, Janice took up the role of solution presenter three times during the year, more often than two of the regular education students in the class.