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Toward solving the cosmological constant problem by embedding

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Abstract

The typical scalar field theory has a cosmological constant problem. We propose a generic mechanism by which this problem is avoided at tree level by embedding the theory into a larger theory. The metric and the scalar field coupling constants in the original theory do not need to be fine-tuned, while the extra scalar field parameters and the metric associated with the extended theory are fine-tuned dynamically. Hence, no fine-tuning of parameters in the full Lagrangian is needed for the vacuum energy in the new physical system to vanish at tree level. The cosmological constant problem can be solved if the method can be extended to quantum loops.

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I. INTRODUCTION

There is no symmetry to prevent a term $\Lambda g_{\mu\nu}$ from being added to the Einstein equation, where Λ is called the cosmological constant and $g_{\mu\nu}$ is the metric tensor. If this is done in a cosmological context, the natural scale for the cosmological constant is $\Lambda \sim m_{\mathrm{P}}^4$, where m_{P} is the Planck scale, which disagrees with observation by approximately 120 orders of magnitude. This disagreement is the cosmological constant problem and it has been with us for some time now (for reviews see for example Refs. [1–5]). Ideally we would like a cosmology where the cosmological constant is zero to first approximation, but corrected by some process to the small value observed today.

There have been many attempts to solve the cosmological constant problem in classical and quantum field theory. Early examples include the use of extra dimensions, where it was conjectured in Ref. [6] that theories with $\Lambda \sim 0$ can be picked out by quantum corrections. Application of the anthropic principle [7, 8] and backreaction arguments [9] have also been used to zero Λ. It was argued that if wormholes exist then Λ can vanish [10]. This led to a long, sometimes controversial, discussion in the literature [11–13]. A technically similar, but physically different solution was presented in Ref. [14], where it was argued that $\Lambda \simeq 0$ dominates the euclidean path integral. It was argued in Ref. [15] that a stochastic model of vacuum energy fluctuations treated as a non-equilibrium process gives a natural explanation for the smallness of Λ. Higher spin models have been introduced to solve the cosmological constant problem [16], an approach that was challenged [17], but more recently the objection has been circumvented [18]. An interesting interpretation of cosmological constant problem given in Ref. [19] allows a large Λ that can be made compatible with observation. Somewhat closer to the spirit of the present paper are the works on k-essence [20–28]. Finally, the relaxation of boundary and hermiticity constraints on quantum fields has been shown to have implications for the cosmological constant problem [29].

Much work has also been done on the cosmological constant problem in string and M-theory. In Ref. [30] it was shown that the cosmological constant can be neutralized by multiple 4-fluxes in M-theory, braneworld solutions have been given in Refs. [31–33], and more recently, a string theory landscape solution to the cosmological constant problem [34] has generated a considerable amount of interest. In addition, various self-tuning mechanisms, with and without extra dimensions, have been considered [35–40]. Finally, a variety of applications of quantum gravity and modifications of general relativity have been used to address the cosmological constant problem [41–44].

While this brief and incomplete summary does not cover all the ideas put forward for solving the cosmological constant problem, we hope it at least gives a flavor for the ingenuity being expended toward finding a compelling solution.

The work we will present here, on extensions of renormalizable particle physics models by embedding in larger theories, makes technical progress that sheds light on the nature of the cosmological constant problem. What remains is to find how these extensions arise naturally in a more fundamental theory.

We should point out that the method we propose here does not contradict Weinberg's no-go theorem [1]. The correct counterpart to Weinberg's equation (6.3) is our equation $(\partial\mathcal{L}/\partial\varphi')_* = 0$, which is satisfied in our method. However, a related equation $(\partial\mathcal{L}'/\partial\varphi')_* = 0$, due to different physical interpretations of $\mathcal{L}$ and $\mathcal{L}'$, does not need to be satisfied. Similar comments apply to Weinberg's equation (6.2). See below for full details.

II. METHOD

For a generic scalar field theory, one reasonably expects that an equilibrium field configuration is a solution of the equations of motion. It is likely that the energy of such a configuration is at its minimum, in which case the solution corresponds to a vacuum state of the theory. The energy-momentum tensor for such a solution is proportional to the metric since such a relation is the only one possible given the symmetry of the vacuum state. The coefficient of proportionality is the cosmological constant. In order for the cosmological constant to be zero, parameters of a typical standard field Lagrangian have to be fine-tuned. For a generic Lagrangian, such fine-tuning depends on the equilibrium configuration, which makes the occurrence of such a situation physically highly unlikely.

One possible way to ameliorate this problem is to introduce extra fields (whose properties will be elucidated below) in addition to fields of the type seen in the standard model of particle physics. These standard fields are chosen to be in a vacuum state. For each configuration of the standard field, configurations of the extra fields must be fine-tuned in such a way as to achieve the vanishing cosmological constant without fine-tuning of the Lagrangian. If this fine tuning can be made technically natural, e.g., via dynamics, then we have solved the cosmological problem. Since the introduction of the extra fields changes the energy-momentum tensor, the Einstein equations require that the space-time metric changes as well. The original theory will be replaced by an embedding theory with extra fields, and we require that a suitable projection of the embedding theory gives the original theory. Clearly, for a given physical theory, there may be an infinite number of embedding theories. In view of this, it might be instructive to select certain classes of theories according to the presence of specific properties, and to choose, based on certain criteria, the minimal embedding theory. We now give a mathematical formulation of the method outlined above.

Let M and N be manifolds, G the space of smooth metrics on M, and Φ the space of smooth maps from M to N. We choose a Lagrangian $\mathcal{L} : G \times \Phi \to \mathbb{R}$. These objects define a theory S. Let T be the energy-momentum tensor and $E = 0$ the equation of motion for S.

Suppose $E(g, \varphi) = 0$ has a solution $(g, \varphi) = (g_*, \varphi_*)$, where $g_* \in G$, $\varphi_* \in \Phi$. For an arbitrary quantity $Q(g, \varphi)$, we define $Q_* = Q(g_*, \varphi_*)$. Of special interest are solutions for which (M, g_*) is an Einstein manifold, and we consider only such solutions. It follows that $(T_*)_{ij} = \Lambda_* (g_*)_{ij}$, where $\Lambda_* = \mathrm{const}$, and we say that (g_*, φ_*) is a vacuum solution.

The quantity Λ_* plays the role of the cosmological constant. For a generic $\mathcal{L}(g, \varphi)$, the requirement $\Lambda_* = 0$ leads to the dependence of $\mathcal{L}$ on (g_*, φ_*). In such cases, the theory S has the cosmological constant problem. The same condition $\Lambda_* = 0$ also implies that (M, g_*) is a Ricci-flat manifold.

Consider the case $N = N' \times N''$, where N' and N'' are two manifolds. Let Φ' and Φ'' be the spaces of smooth maps from M to N' and M to N''. For an arbitrary quantity Q defined on $\Phi = \Phi' \times \Phi''$, let $Q' = Q|_{\Phi' \times \{0\}}$ and $Q'' = Q|_{\{0\} \times \Phi''}$ be the restrictions of Q to $\Phi' \times \{0\}$ and $\{0\} \times \Phi''$, where $\{0\}$ is the space of zero functions. As a result, we have the restricted Lagrangians $\mathcal{L}' : G \times \Phi' \to \mathbb{R}$ and $\mathcal{L}'' : G \times \Phi'' \to \mathbb{R}$ and theories S' and S''. Let T' and T'' be the energy-momentum tensors and $E' = 0$ and $E'' = 0$ the equations of motion for S' and S''. We say that S' and S'' are the sub-theories of S and that S is the super-theory of S' and S''.

Suppose (g_*, φ_*) is a vacuum solution of $E = 0$. We seek vacuum solutions (g'_*, φ'_*) and (g''_*, φ''_*) of $E' = 0$ and $E'' = 0$ such that φ'_* and φ''_* are the restrictions of φ_*. The quantities g'_* are g''_* are obtained by solving the equations of motion, not by restricting g_* as the notation may suggest.

We require $\Lambda_* = 0$. Solving the resulting equation $T_* = 0$, we find that $\mathcal{L}$ depends on (g_*, φ_*). In general, $\mathcal{L}'$ depends on (g'_*, φ'_*) and $\mathcal{L}''$ depends on (g''_*, φ''_*). If it is possible to arrange for $\Lambda_* = 0$ for any vacuum solution of $E = 0$ in such a way that $\mathcal{L}'$ does not depend on (g'_*, φ'_*) and $\mathcal{L}''$ does not depend on (g''_*, φ''_*), then we say that the cosmological constant problems for S' and S'' are solved by the super-theory S.

We will show that the cosmological constant problem for a given sub-theory S' can always be solved by choosing an appropriate super-theory. Among all possible super-theories, it might be desirable to choose a certain super-theory, which is the closest to the given sub-theory according to some criteria. We call this a minimal super-theory.

III. SOLUTION

It is instructive to turn to a specific theory and show how the general construction described in Section II proceeds. We specify S by setting $N' = \mathbb{R}$ and $N'' = \mathbb{R}$, and choosing $\mathcal{L}$ to be an arbitrary $\mathbb{R}$-valued function of φ', φ'', X', X'', and Y, where φ' and φ'' are real scalar singlet fields and

$$X' = \nabla_i\varphi'\nabla^i\varphi', \quad X'' = \nabla_i\varphi''\nabla^i\varphi'', \quad Y = \nabla_i\varphi'\nabla^i\varphi''. \tag{1}$$

The Euler-Lagrange equations are

$$(\partial\mathcal{L}/\partial\varphi') - \nabla_i\big(2(\partial\mathcal{L}/\partial X')\nabla^i\varphi' + (\partial\mathcal{L}/\partial Y)\nabla^i\varphi''\big) = 0, \tag{2}$$

$$(\partial\mathcal{L}/\partial\varphi'') - \nabla_i\big((\partial\mathcal{L}/\partial Y)\nabla^i\varphi' + 2(\partial\mathcal{L}/\partial X'')\nabla^i\varphi''\big) = 0 \tag{3}$$

and the energy-momentum tensor is

$$T_{ij} = -\mathcal{L}g_{ij} + 2(\partial\mathcal{L}/\partial X')\nabla_i\varphi'\nabla_j\varphi' + 2(\partial\mathcal{L}/\partial X'')\nabla_i\varphi''\nabla_j\varphi'' + 2(\partial\mathcal{L}/\partial Y)\nabla_i\varphi'\nabla_j\varphi''. \tag{4}$$

We seek the solution (g_*, φ_*) for which (M, g_*) is a Ricci-flat manifold and $\varphi'_* = \text{const}$. Equations (2), (3), (4) give

$$\mathcal{L}_* = 0, \quad (\partial\mathcal{L}/\partial\varphi')_* = 0, \quad (\partial\mathcal{L}/\partial\varphi'')_* = 0, \quad (\partial\mathcal{L}/\partial X'')_* = 0, \quad (\partial\mathcal{L}/\partial Y)_* = 0. \tag{5}$$

We choose $\mathcal{L}'$ to be an arbitrary $\mathbb{R}$-valued function of φ' and X', and $\mathcal{L}''$ to be an arbitrary $\mathbb{R}$-valued function of φ'' and X''. The Euler-Lagrange equation are

$$(\partial\mathcal{L}'/\partial\varphi') - \nabla_i\big(2(\partial\mathcal{L}'/\partial X')\nabla^i\varphi'\big) = 0, \tag{6}$$

$$(\partial\mathcal{L}''/\partial\varphi'') - \nabla_i\big(2(\partial\mathcal{L}''/\partial X'')\nabla^i\varphi''\big) = 0, \tag{7}$$

and the energy-momentum tensors are

$$T'_{ij} = -\mathcal{L}'g_{ij} + 2(\partial\mathcal{L}'/\partial X')\nabla_i\varphi'\nabla_j\varphi', \tag{8}$$

$$T''_{ij} = -\mathcal{L}''g_{ij} + 2(\partial\mathcal{L}''/\partial X'')\nabla_i\varphi''\nabla_j\varphi''. \tag{9}$$

If (g'_*, φ'_*) and (g''_*, φ''_*) are vacuum solutions of $E' = 0$ and $E'' = 0$, then

$$\mathcal{L}'_* = (2m)^{-1}(m-2)R'_*, \quad (\partial\mathcal{L}'/\partial\varphi')_* = 0, \tag{10}$$

$$\mathcal{L}''_* = (2m)^{-1}(m-2)R''_*, \quad (\partial\mathcal{L}''/\partial\varphi'')_* = 0, \tag{11}$$

where $m = \dim M$, and R'_* and R''_* are the scalar curvatures of (M, g'_*) and (M, g''_*). If S' and S'' are sub-theories of S, then

$$\mathcal{L}' = \mathcal{L}|_{\varphi''=0,\, X''=0,\, Y=0}, \quad \mathcal{L}'' = \mathcal{L}|_{\varphi'=0,\, X'=0,\, Y=0}. \tag{12}$$

Without loss of generality, we assume that the functions $\mathcal{L}$, $\mathcal{L}'$, $\mathcal{L}''$ can be expanded in power series around the point $(\varphi', \varphi'', X', X'', Y) = (0, 0, 0, 0, 0)$. Equations (12) imply

$$\mathcal{L}(\varphi', \varphi'', X', X'', Y) = \mathcal{L}'(\varphi', X') + \sum_{\substack{p\geq 0,\, q\geq 0,\, r\geq 0 \\ p+q+r\geq 1}} F'_{p,q,r}(\varphi', X')\varphi''^p X''^q Y^r, \tag{13}$$

$$\mathcal{L}(\varphi', \varphi'', X', X'', Y) = \mathcal{L}''(\varphi'', X'') + \sum_{\substack{p\geq 0,\, q\geq 0,\, r\geq 0 \\ p+q+r\geq 1}} F''_{p,q,r}(\varphi'', X'')\varphi'^p X'^q Y^r, \tag{14}$$

where $\{F'_{p,q,r}\}$ and $\{F''_{p,q,r}\}$ are arbitrary functions. Substituting equation (13) into equation (5), we find

$$\mathcal{L}'_* + \sum_{\substack{p\geq 0,\, q\geq 0 \\ p+q\geq 1}} (F'_{p,q,0})_* \varphi''^p_* X''^q_* = 0, \tag{15}$$

$$\sum_{\substack{p\geq 0,\, q\geq 0 \\ p+q\geq 1}} (\partial F'_{p,q,0}/\partial\varphi')_* \varphi''^p_* X''^q_* = 0, \tag{16}$$

$$\sum_{\substack{p\geq 0,\, q\geq 0 \\ p+q\geq 1}} (F'_{p,q,0})_* p\varphi''^{p-1}_* X''^q_* = 0, \tag{17}$$

$$\sum_{\substack{p\geq 0,\, q\geq 0 \\ p+q\geq 1}} (F'_{p,q,0})_* \varphi''^p_* qX''^{q-1}_* = 0, \tag{18}$$

$$\sum_{p\geq 0,\, q\geq 0} (F'_{p,q,1})_* \varphi''^{p}_* X''^{q}_* = 0. \tag{19}$$

Functions $\{F'_{p,q,0}\}$ are constrained by equations (15), (16), (17), (18), functions $\{F'_{p,q,1}\}$ are constrained by equation (19), and functions $\{F'_{p,q,r}\}$ for $r \geq 2$ are not constrained by these equations. In general, we assume that $\mathcal{L}'_* \neq 0$, $\varphi''_* \neq 0$, $X''_* \neq 0$.

Let k_r be the number of nonzero functions among $\{F'_{p,q,r}\}$ for each $r \geq 0$. Equations (15), (17), (18) give $(k_0)_* \geq 2$ and equation (19) gives $(k_1)_* = 0$ or $(k_1)_* \geq 2$. Since $k_0 \geq (k_0)_*$ and $k_1 \geq (k_1)_*$, it follows that $k_0 \geq 2$ and $k_1 \geq 0$.

Let $k_0 = 2$, which implies $(k_0)_* = 2$. There are three cases to consider.

In the first case, $(F'_{p_1,q_1,0}, F'_{p_2,q_2,0}) \neq (0,0)$, for some fixed (p_1, q_1, p_2, q_2) such that $p_1 \geq 1$, $p_2 \geq 1$, $q_1 \geq 1$, $q_2 \geq 1$, $(p_1, q_1) \neq (p_2, q_2)$, so that

$$\mathcal{L} = \mathcal{L}' + F'_{p_1,q_1,0} \varphi''^{p_1} X''^{q_1} + F'_{p_2,q_2,0} \varphi''^{p_2} X''^{q_2} + \sum_{p\geq 0,\, q\geq 0,\, r\geq 1} F'_{p,q,r} \varphi''^{p} X''^{q} Y^{r}. \tag{20}$$

From equations (15), (17), (18), we find $p_1 q_2 = p_2 q_1$ and

$$(F'_{p_1,q_1,0})_* = s_2 (s_1 - s_2)^{-1} \mathcal{L}'_* \varphi''^{-p_1}_* X''^{-q_1}_*, \tag{21}$$

$$(F'_{p_2,q_2,0})_* = s_1 (s_2 - s_1)^{-1} \mathcal{L}'_* \varphi''^{-p_2}_* X''^{-q_2}_*, \tag{22}$$

where either $(s_1, s_2) = (p_1, p_2)$ or $(s_1, s_2) = (q_1, q_2)$. Equation (16) becomes

$$s_1^{-1} (F'_{p_1,q_1,0})^{-1}_* (\partial F'_{p_1,q_1,0}/\partial\varphi')_* - s_2^{-1} (F'_{p_2,q_2,0})^{-1}_* (\partial F'_{p_2,q_2,0}/\partial\varphi')_* = 0. \tag{23}$$

Since φ'_* is an arbitrary constant which satisfies only the condition $(\partial\mathcal{L}'/\partial\varphi')_* = 0$, equation (23) implies

$$(\partial F'_{p_1,q_1,0}/\partial\varphi')_* = 0, \quad (\partial F'_{p_2,q_2,0}/\partial\varphi')_* = 0. \tag{24}$$

In the second case, $(F'_{p_1,0,0}, F'_{p_2,0,0}) \neq (0,0)$, for some fixed (p_1, p_2) such that $p_1 \geq 1$, $p_2 \geq 1$, $p_1 \neq p_2$. The corresponding expressions are obtained from equations (21), (22), (23), (24) by setting $(q_1, q_2) = (0,0)$ and $(s_1, s_2) = (p_1, p_2)$.

In the third case, $(F'_{0,q_1,0}, F'_{0,q_2,0}) \neq (0,0)$, for some fixed (q_1, q_2) such that $q_1 \geq 1$, $q_2 \geq 1$, $q_1 \neq q_2$. The corresponding expressions are obtained from equations (21), (22), (23), (24) by setting $(p_1, p_2) = (0,0)$ and $(s_1, s_2) = (q_1, q_2)$.

It is straightforward to proceed with a similar analysis for $k_0 \geq 3$.

We require that φ'' is a dynamical field and that φ' and φ'' are coupled. If $k_r = 0$ for all $r \geq 1$, these conditions imply $(q_1, q_2) \neq (0,0)$ and $(F'_{p_1,q_1,0}, F'_{p_2,q_2,0}) \neq (\mathrm{const}, \mathrm{const})$.

IV. EXAMPLES

In this section, we restrict our attention to four-dimensional space-time manifolds, i.e., $m = 4$. We define dimensions

$$d(\varphi') = 1, \quad d(\varphi'') = 1, \quad d(X') = 4, \quad d(X'') = 4, \quad d(Y) = 4 \tag{25}$$

and the corresponding dimension $d(Q)$ of an arbitrary polynomial $Q(\varphi', \varphi'', X', X'', Y)$ as the maximal dimension of its monomials. As a criterion for a minimal super-theory S, we choose a requirement that $d(\mathcal{L})$ takes its least possible value. For equation (13), we find

$$d(\mathcal{L}) = \max\{d(\mathcal{L}'), d'\}, \tag{26}$$

$$d' = \max\{d(F'_{p,q,r}) + p + 4(q+r) : p \geq 0,\ q \geq 0,\ r \geq 0,\ p+q+r \geq 1,\ F'_{p,q,r} \neq 0\}. \tag{27}$$

We assume that $d(\mathcal{L}')$ is fixed and thus we need to find the least possible value for d'.

If $k_0 = 2$, $k_r = 0$, $r \geq 1$, then the least possible value for d' is achieved for

$$(p_1, q_1) = (0, 1), \quad (p_2, q_2) = (0, 2), \tag{28}$$

$$0 \leq d(F'_{0,1,0}) \leq 4, \quad d(F'_{0,2,0}) = 0, \tag{29}$$

$$\mathcal{L} = \mathcal{L}' + F'_{0,1,0} X'' + F'_{0,2,0} X''^2, \tag{30}$$

$$(F'_{0,1,0})_* = -2\mathcal{L}'_* X''^{-1}_*, \quad (F'_{0,2,0})_* = \mathcal{L}'_* X''^{-2}_*, \tag{31}$$

$$d(\mathcal{L}) = \max\{d(\mathcal{L}'), 8\}. \tag{32}$$

If $d(\mathcal{L}') > 4$, then there does not exist $F'_{0,1,0}$ which satisfies equations (29) and (31). If $0 \leq d(\mathcal{L}') \leq 4$, then

$$F'_{0,1,0} = -2(\mathcal{L}' + C'_1 X') X''^{-1}_*, \quad F'_{0,2,0} = C'_2, \tag{33}$$

where C'_1 and C'_2 are arbitrary constants. In Table I, we have listed examples which give few smallest values for $\min\{d(\mathcal{L} - \mathcal{L}')\}$ for the case $k_0 = 2$, $k_r = 0$, $r \geq 1$.

If $k_0 = 2$, $k_1 = 1$, $k_r = 0$, $r \geq 2$, then the least possible value for d' is achieved for

$$1 \leq p_1 < p_2 \leq 4, \quad (q_1, q_2) = (0, 0), \tag{34}$$

$$0 \leq d(F'_{p_1,0,0}) \leq 8 - p_1, \quad 0 \leq d(F'_{p_2,0,0}) = 8 - p_2, \quad d(F'_{0,0,1}) = 4, \tag{35}$$

$$\mathcal{L} = \mathcal{L}' + F'_{p_1,0,0} \varphi''^{p_1} + F'_{p_2,0,0} \varphi''^{p_2} + F'_{0,0,1} Y \tag{36}$$

TABLE I. Choices of (p_1, q_1, p_2, q_2) which give few smallest values for $\min\{d(\mathcal{L}-\mathcal{L}')\}$ for the case $k_0 = 2$, $k_r = 0$, $r \geq 1$. For each choice, $(F'_{p_1,q_1,0}, F'_{p_2,q_2,0}) \neq (\mathrm{const}, \mathrm{const})$.

p_1	q_1	p_2	q_2	$\min\{d(\mathcal{L}-\mathcal{L}')\}$	$\mathcal{L}-\mathcal{L}'$	$(F'_{p_1,q_1,0})_*$	$(F'_{p_2,q_2,0})_*$
0	1	0	2	8	$F'_{0,1,0}X'' + F'_{0,2,0}X''^2$	$-2\mathcal{L}'_* X''^{-1}_*$	$\mathcal{L}'_* X''^{-2}_*$
1	1	2	2	10	$F'_{1,1,0}\varphi'' X'' + F'_{2,2,0}\varphi''^2 X''^2$	$-2\mathcal{L}'_*\varphi''^{-1}_* X''^{-1}_*$	$\mathcal{L}'_*\varphi''^{-2}_* X''^{-2}_*$
0	1	0	3	12	$F'_{0,1,0}X'' + F'_{0,3,0}X''^3$	$-\frac{3}{2}\mathcal{L}'_* X''^{-1}_*$	$\frac{1}{2}\mathcal{L}'_* X''^{-3}_*$
0	2	0	3	12	$F'_{0,2,0}X''^2 + F'_{0,3,0}X''^3$	$-3\mathcal{L}'_* X''^{-2}_*$	$2\mathcal{L}'_* X''^{-3}_*$
2	1	4	2	12	$F'_{2,1,0}\varphi''^2 X'' + F'_{4,2,0}\varphi''^4 X''^2$	$-2\mathcal{L}'_*\varphi''^{-2}_* X''^{-1}_*$	$\mathcal{L}'_*\varphi''^{-4}_* X''^{-2}_*$

$$(F'_{p_1,0,0})_* = p_2(p_1-p_2)^{-1}\mathcal{L}'_*\varphi''^{-p_1}_*, \quad (F'_{p_2,0,0})_* = p_1(p_2-p_1)^{-1}\mathcal{L}'_*\varphi''^{-p_2}_*, \tag{37}$$

$$d(\mathcal{L}) = \max\{d(\mathcal{L}'), 8\}. \tag{38}$$

We find

$$F'_{p_1,0,0} = p_2(p_1-p_2)^{-1}\big(\mathcal{L}' + C'_1(\varphi')X'\big)\varphi''^{-p_1}_*, \tag{39}$$

$$F'_{p_2,0,0} = p_1(p_2-p_1)^{-1}\big(\mathcal{L}' + C'_2(\varphi')X'\big)\varphi''^{-p_2}_*, \tag{40}$$

where C'_1 and C'_2 are arbitrary polynomials of φ' such that

$$0 \leq d(C'_1) \leq 4 - p_1, \quad 0 \leq d(C'_2) \leq 4 - p_2. \tag{41}$$

It is straightforward to proceed with a similar analysis for different values of $\{k_r\}_{r\geq 0}$. The above computations give explicit construction of the minimal super-theory for a given sub-theory. It is easy to generalize these computations to more complicated cases such as, for example, higher dimensional space-times, multiple scalar fields, or scalar fields in a representation of a gauge group.

V. DISCUSSION AND CONCLUSION

Here we focus on simple examples of obtaining a zero cosmological constant. If we have a scalar field theory of the type found as a component of the standard model of particle physics, where the scalar field is renormalizable, then we are dealing with the case

$$\mathcal{L}'(\varphi', X') = \tfrac{1}{2}X' - V'(\varphi'), \tag{42}$$

where $V'(\varphi')$ is a polynomial potential of at most dimension 4. We call these φ' standard scalar fields. Note that $\mathcal{L}'$ is the most general renormalizable Lagrangian for a single scalar field. Unless fine tuned, such theories always have a cosmological constant problem. Solutions to the cosmological constant problem involve fields with nonstandard properties, which we collectively call exotic scalar fields. The class of exotics includes ghost fields with wrong sign kinetic energy terms, k-essence fields with kinetic and potential parts appearing in the form $X'^{m}V'(\varphi')$, phantom fields, auxiliary fields, and all other types of scalar fields that do not fit the standard scalar field classification. The solutions we have found are models that have a limiting case where only standard fields are present and where exotic fields must be included to solve the cosmological constant problem.

For example, $\mathcal{L}'(\varphi', X')$ can be supplemented with an extra field φ'', the quantity $X'' = \nabla_i\varphi''\nabla^i\varphi''$, and the new Lagrangian $\mathcal{L}(\varphi', \varphi'', X', X'')$ such that there is no cosmological constant problem for a solution $(\varphi'_*, X'_*) = (\text{const}, 0)$ with an appropriate choice of (φ''_*, X''_*). In the limit $(\varphi'', X'') \to (0, 0)$, we recover $\mathcal{L}'(\varphi', X')$, and the cosmological constant problem. Note that the field φ' itself becomes exotic because of the way φ'' has to be added to the theory.

Assuming no contributions from the kinetic cross term Y, it turns out that Lagrangians linear in X'' are insufficient, but there exists an infinite class of Lagrangians quadratic in X'' which allow satisfactory solutions. Many of these solutions are in the spirit of a generalized k-essence in the sense that the potential $V'(\varphi')$ couples to X''. (In k-essence, $V'(\varphi')$ couples to X'.)

Generalizations to models with multiple fields, higher order terms in X'' and Y, or more complicated φ'' terms are straightforward.

There is no solution of the cosmological constant problem with standard fields alone. Any generic standard scalar field Lagrangian is plagued with a cosmological constant problem and exotic fields are required to avoid it. We can express this in a concise way since the results of Sections III and IV establish that the cosmological constant problem in a standard field Lagrangian can only be avoided in a technically natural way by incorporating exotic fields of the type introduced above. There are a large variety of exotic field properties, including non-polynomial potentials, non quadratic kinetic terms, mixed kinetic-potential terms, etc.

Our method is generic in the sense that for any standard field Lagrangian there are infinitely many choices for the Lagrangian of the full system. Since we have a large class of models without a cosmological constant problem, it is not unrealistic to hope that some members of the class may arise naturally in a more fundamental context, like string or M-theory. Since the way the exotic fields enter can vary greatly, our results provide a large parameter space of new models to explore.

Let us consider two simple explicit examples for the new Lagrangian. In the first example,

$$\mathcal{L}(\varphi', \varphi'', X', X'') = \tfrac{1}{2}X' - V'(\varphi')(1 - M^{-4}X'')^2, \tag{43}$$

and in the second example,

$$\mathcal{L}(\varphi', \varphi'', X', X'') = \tfrac{1}{2}X' - \left(V'(\varphi')^{1/2} - M^{-2}X''\right)^2, \tag{44}$$

where M is a quantity with the dimension of mass and $V'(\varphi') \geq 0$. Note that in neither example do we have symmetry or renormalizability to restrict the form of the extended Lagrangian. While these examples do solve the cosmological constant problem, for their forms to arise in a natural way we need them to be embeddable in an overarching theory (e.g., string theory) to make that specification. Hence our results should be considered as technical progress toward a solution of the cosmological constant problem until an all-encompassing theory can be found where our examples can reside.

The solution $\varphi'_* = \text{const}$, $X'_* = 0$, $X''_* = M^4$ in the first example and $\varphi'_* = \text{const}$, $X'_* = 0$, $X''_* = M^2 V'(\varphi'_*)^{1/2}$ in the second example solve the Euler-Lagrange equations for the new Lagrangian. In both examples we see there is no cosmological constant problem as the value of the energy-momentum tensor vanishes at the extremum. In the limit $(\varphi'', X'') \to (0, 0)$, we recover $\mathcal{L}'$ of equation (42) from $\mathcal{L}$. (We note that in order for the Lagrangian in the second example to agree with the approach in Section IV, we need to assume that the polynomial $V'(\varphi')$ is the square of a second order polynomial of φ'.)

If $\mathcal{L}'$ is renormalizable and contains only operators of dimension not exceeding 4, then the solutions of the cosmological constant problem we have found in the form of $\mathcal{L}$ are all non-renormalizable with operators of at least dimension 8. If V' has dimension 4, then $\mathcal{L}$ has dimension 12 in the first example and 8 in the second example. We assume M is a high scale, say M_{GUT} or M_{Planck}, and that the potential in $\mathcal{L}'$ contains a lower scale m, say the electroweak scale in the form of a mass term in $V'(\varphi') = C - m^2\varphi'^2 + \lambda\varphi'^4$, where $\lambda \lesssim O(1)$.

At temperatures below, say 1 TeV, the φ' potential becomes

$$V'(\varphi', T) = C + (-m^2 + \tfrac{1}{2}\lambda T^2)\varphi'^2 + \lambda\varphi'^4, \tag{45}$$

In the first example $X''_*(T) = M^4$ is approximately constant since $M \gg 1$ TeV and so the φ'' field is "frozen" at $T \sim 1$ TeV, while in the second example,

$$X''_*(T) = M^2\big(C - (4\lambda)^{-1}(-m^2 + \tfrac{1}{2}\lambda T^2)^2\big)^{1/2}. \tag{46}$$

so φ'' is still running with temperature. One concludes that the solutions to the cosmological problem derived by our methods can have dramatically different phenomenologies.

As we have pointed out, operators of dimension greater than 4 are not surprising from the perspective of string theory, in fact they are ubiquitous. Hence it would be expected that the high energy completion of a standard model type Lagrangian involve such operators. As we have shown, any renormalizable models with a cosmological constant problem has an infinite class of extensions that solve this problem. Thus it is quite conceivable that some of these extremal solutions coincide with members of the vast landscape of string vacua. To find such a solution we need not explore the entire string theory landscape statistically, rather we only need to search for string compactification with the properties specified above.

Hence we have provided a scenario by which a renormalizable quantum field theory may be extended to solve the cosmological constant problem. While it seems unlikely that this solution can withstand all possible scrutiny, we do believe we have made progress in finding a deeper understanding of the problem and hope our work will spark further discussion. Assuming there exists a viable UV completion of the standard model, one could be lead to the extreme point of view that the observational lack of a Planck size cosmological constant is phenomenological evidence for such a UV completion.

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