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Abstract

We study equilibrium prices and trade volume in a market with several identical buyers and a seller who commits to an inventory and then offers goods sequentially. Prices are determined by a strategic costly bargaining process with a random sequence of proponents. A unique subgame perfect equilibrium exists, characterized by no costly delays and heterogeneous sale prices. In equilibrium constraining capacity is a bargaining tactic the seller uses to improve a weak bargaining position. With capacity constraints, sale prices approach the outcome of an auction as bargaining costs vanish. The framework provides a building block for price formation in models of equilibrium search with multilateral matching, and offers a rationale for the adoption of single-unit auctions with fixed reservation price.

Keywords: Commitment, Inefficiency, Peripheral players, Price heterogeneity, Noncooperative bargaining
JEL: C78, D0

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1 Introduction

This paper studies equilibrium in a market where several identical buyers desire to purchase one indivisible good from a seller with a fixed homogeneous inventory. The seller chooses and commits to the inventory before sequentially serving buyers. Prices are determined via costly negotiations, ex-post. Primarily, we are interested in determining equilibrium volume of trade and sequence of sale prices. We also characterize equilibrium in terms of the model's parameters and study its efficiency.

The application we have in mind is trading in models with directed search or random urn-ball matching, where capacity-constrained sellers may meet more than one buyer in a period.[2] These models have been used to study labor and product markets, monetary economies, and, though price posting is typical, recent works limit price commitments and introduce ex-post competition to study price dispersion and trading efficiency; see for instance the use of auctions in [2, 17, 20, 32] and the possibility of ex-post (re)negotiations in [9, 27]. Our work studies price formation in multilateral matches and offers a rationale for the existence of equilibrium capacity constraints and for the use of auctions.

We develop a strategic bargaining game between a central player (seller) and n peripheral players (buyers) each of whom desires a single indivisible object (a good or a job). The game is of complete information and is sequential, with two stages. In the first stage the seller commits to supply at most $c \leq n$ homogeneous objects, a choice that we call 'capacity.' Capacity is costlessly created, so holdup problems are excluded, and production is costless. In the second stage a discrete-time alternating offers game takes place, in the tradition of [31]. The key features are a sequential sale constraint (the seller can offer only one good per period), there is a cost to bargaining due to discounting, and players cannot exclude others from negotiations since proposers are randomly selected.

[2] In directed search models multiple buyers can visit, at no cost, one of several capacity-constrained sellers; e.g., see [4, 28, 30]. Because buyers' choices are simultaneous and independent, sellers may meet more than one buyer in a period and, with countable players, symmetric equilibrium demand is Poisson-distributed parameterized by the buyers-sellers ratio, as it happens in urn-ball matching models; e.g., see [5, 32].

The main findings are as follows. For any given choice of capacity c the multi-player bargaining game has a unique subgame perfect equilibrium, which is symmetric and stationary. Bargaining equilibrium is efficient since discounting eliminates delays.

Second, equilibrium offers are decreasing functions of capacity. Proposers are randomly selected, so anyone who renounces an option to buy today must compete with others tomorrow. It follows that if there are capacity constraints, $c < n$, then there is consumption risk; the seller's inventory may run out before the buyer can negotiate a better price, since the seller may accept other offers in the meantime. This risk falls with capacity, and so do equilibrium offers. This means that the seller faces a trade-off between extensive and intensive margins of trade, which leads to a third finding.

Capacity constraints arise in equilibrium if the seller is sufficiently inept at bargaining. Choosing $c < n$ creates consumption risk, and so forces buyers to compete raising their offers. This bargaining tactic improves the seller's surplus share in each trade but lowers trade volume. Hence, in equilibrium $c < n$ only if the seller is a sufficiently weak negotiator. Clearly, the resulting deadweight loss implies equilibrium inefficiency.

Fourth, the sequence of equilibrium sale prices is heterogeneous and generally non-monotonic. Heterogeneity stems from discounting but it is affected by the choice c. Without capacity constraints prices monotonically fall in the order of sale because there is no consumption risk. Competition among buyers falls as goods are sold and early buyers pay a premium simply due to discounting. Instead, if $c < n$, then consumption risk increases as items are sold. Consequently, the sale price sequence can be U-shaped or even monotonically increasing if the capacity constraint is sufficiently tight. Clearly, because proponents are randomly selected, equilibrium prices in a period differ depending on who gets to make the offer, seller or buyer. Interestingly, the buyer's equilibrium offer follows a very simple and intuitive rule. The buyer applies a fixed (round-invariant) discount to the seller's offer and such a reduction depends only upon time-discounting and the seller's bargaining skill, but neither on the number of competitors nor the seller's remaining stock.

Finally, as discounting vanishes equilibrium offers converge to a constant that depends on capacity constraints and players' negotiation skills. Absent constraints, players earn fixed surplus shares based on their relative bargaining skills. Otherwise, buyers earn zero surplus because they bid their reservation value in an attempt to avoid being rationed away. Intuitively, as bargaining becomes costless buyers participate in a scheme resembling an auction with a fixed reservation price. Hence, our model provides a microfoundation for the use of this type of trading mechanism in search models of multi-player matches such as [2, 17, 20, 32]. Because these models display random equilibrium demand, we also conduct an analysis for an economy with Poisson-distributed demand to confirm our basic results on capacity choices. We find that if the seller anticipates meeting few buyers who are skilled negotiators, then he will commit to serve only a fixed small number of buyers and may even choose to sell just one good. Additionally, we show that this results holds in an extended version of the model where multiple sellers can compete in capacity.

These findings fit into several literatures. They broaden the study of bargaining tactics used to strengthen a player's bargaining leverage, such as in [12, 14, 23, 29] for instance.[3] We present conditions under which a seller with weak bargaining position can improve his payoff by committing to serve only a fraction of the demand. In this case, disparities in bargaining skills can affect allocative efficiency but not bargaining efficiency, because equilibrium exhibits no bargaining delays. More generally, our work fits into the literature on multi-player bargaining under complete information. To put our contribution into perspective, recall that our analysis relies on a noncooperative sequential bargaining game in which a central player negotiates with $n \geq 2$ peripheral players over $c \leq n$ objects, given randomly alternating offers and unbounded bargaining rounds. Thus, we depart from the typical setting in the literature where negotiations are over shares of a single "pie" (e.g., see [19]), because the seller chooses how many homogeneous pies to offer and then negotiates over one pie at a time with multiple buyers. This implies that acceptances affect the continuation game, due to inventory changes, though rejections neither create costs nor impose restrictions on future offers (for a model where this is not the case,

[3] In [12] taking actions that enlarge the set of settlements preferred to disagreement may hurt the probability of disagreement; in [14] a firm may benefit by choosing high debt levels before bargaining with workers; and in [29, 23] making initial commitments that are costly to revoke provides bargaining leverage.

see [18]). Moreover, the framework adopted imposes neither a predetermined bargaining order (as, for instance, happens in [35]), nor a specific queuing order following a rejection (for examples see [6, 7]), nor an exogenous deadline by which bargaining has to end (as in [10], for instance); indeed, players in our model can respond to offers only with a random lag and offers cannot be made to all the players simultaneously (as in [21], for example). Finally, unlike papers in noncooperative coalitional sequential bargaining games, players in our model neither have veto power over allocation proposals, nor can impose one dictatorially; see the recent works in [16, 25, 26] for example.[4]

Our work is also related to a literature on durable goods monopolies and the Coase conjecture. This conjecture, put forward in [11], states that a monopolist who cannot commit to price/quantity sequences faces a classic time-inconsistency problem that can eliminate the monopoly distortion. Basically, buyers may postpone demand if they expect that prices will drop sufficiently rapidly towards the marginal cost, as the seller incrementally serves demand. So, an uncommitted monopolist will end up offering (close to) competitive prices from the get go. The Coase conjecture is not borne out in our model for two reasons. First, the seller can costlessly commit to serve only a fraction of the demand—which creates competition for goods. Second, the seller can costlessly limit period capacity to a minimum by serving impatient buyers sequentially—which slows down sales. Seen in this light, our work is especially related to the recent studies in [3, 15, 24] on the impact of commitment on monopoly distortions.[5]

[4] In the stochastic models in [25, 26], a player is randomly chosen to propose a division of a cake, while the other players sequentially respond by accepting/rejecting the proposal. Unanimity is necessary for the proposal to pass. The work in [16] combines stochastic components with general agreement rules.

[5] In [24] capacity can be costly augmented in each period and monopoly distortions emerge because, due to costly adjustments, the monopolist can credibly commit to future sales restrictions, which are key because they slow down sales. The work in [3] studies the link between commitment and strategic demand delays with time-varying demand. If demand increases, then the monopolist quickly increases its price (so demand is not postponed), and if demand falls, then prices slowly fall (so there is some demand posticipation). Commitment with varying demand is also in [15], which studies demand anticipation; consumers who expect higher future prices can pay a fixed fee to store goods. Here commitment may lower distortions since the monopolist can adjust prices intertemporally and eliminate wasteful storage.

In terms of applicability, the model of inventory selection and price formation that we propose is suitable for directed search and urn-ball random matching models. This study offers a rationale for the typical assumption of capacity-constrained markets; e.g., see [4, 28, 30], or [1] for a model with costly buyer search. The costly bargaining results are relevant to studies that consider the possibility of (re)negotiation in matches between many buyers and a seller with unit inventory, e.g., see [9, 27]. The analysis of the limiting case of costless bargaining, instead, offers a rationale for assuming single-unit auctions, e.g., as in [2, 17, 20, 32], when sellers face a short queue of customers who have substantial bargaining leverage. Finally, our framework can also find applicability in a literature devoted to study how intermediaries' choices of inventories help mitigate trading frictions in search and matching markets, as, for instance, in [8, 34, 36].[6]

We proceed as follows. Section 2 describes the model and studies the bargaining game. Section 3 studies the choice of capacity, Section 4 considers extensions to random matching and multiple sellers. Section 5 concludes.

2 Model and equilibrium concept

We study a game of complete information between a seller and $n \geq 1$ identical buyers each of whom desires to consume a single good. There are two stages. In the first stage the seller costlessly chooses capacity $c = 1, ..., n$. This allows the seller to produce, at no cost, up to c units of an indivisible homogeneous good. Hence, c is the seller's inventory and there are capacity constraints when there is excess demand, $n-c > 0$. In the second stage, goods are offered for sale one at a time, and trading takes place by means of a bargaining mechanism described below. Consumption utility is one for buyers and zero for the seller and since utility is transferable there are gains from trade. We adopt subgame perfection as the equilibrium concept, moving backward in our analysis. First, we study bargaining equilibrium in the second stage of the game, given c. Then, we study the choice c.

[6] [8] studies the emergence of dealers of differentiated goods as a function of cost of inventories, frictions, and negotiation leverage; [34] studies how intermediaries' choice of inventories impacts the frequency of random exchange, and so does [36] in a model with directed search markets.

2.1 The bargaining game

Consider the second stage of the game, i.e., the bargaining game. Suppose the seller has $c = 1, ..., n$ indivisible goods available. Every player observes c and n and then a trading process starts, which is based on a noncooperative sequential bargaining game of complete information, in the tradition of [31]. Negotiations take place in rounds indexed $t = 1, 2, ...$

In each round t players bargain over the sale of a single good as follows. First, a random selection device chooses a buyer with equal probability among all buyers present. This means buyers are not in a specific queue waiting their turn to negotiate with the seller. Second, the random selection device either picks the seller or the (randomly selected) buyer to propose an offer $q \in [0, 1]$. With probability $\gamma \in (0, 1)$ the seller is picked—so the responder is the buyer—and the converse occurs with probability $1 - \gamma$, in which case the buyer is picked and the seller is the responder. So, in each round any of the players, buyers or seller, face a random opportunity to make or respond to an offer. We will interpret the parameter γ as capturing the seller's negotiation skill.

Denote the elements of the responder's action set by 'accept' or 'reject.' There is disagreement in a round t if the responder rejects the offer q. In that case, the seller keeps the good and all players earn zero utility for the round. If there is agreement, instead, trade occurs so the payoff in round t is $1 - q$ for the buyer—who then leaves the game—and q for the seller. The remaining $n - 1$ buyers receive zero payoff in that round. At the end of round t, if the seller has no more goods to offer then the game stops, otherwise it continues. It is assumed all players discount future payoffs by $\beta \in (0, 1)$. This means that bargaining delays are costly to seller and buyers. Since in each round the seller makes or responds to one buyer's offer, as the game progresses the seller's capacity falls at most by one unit. Since goods are homogeneous, without loss in generality we let $i = 1, ..., c$ denote the good offered for sale in round $t \geq i$.

2.2 Bargaining: the main result

Fix $c = 1, ..., n$. The main result is a full characterization of the subgame perfect equilibrium (SPE) offers and realized payoffs.

Theorem 1 *The bargaining game between $n \geq 1$ buyers and a seller with $c = 1, ..., n$ goods has a unique subgame perfect equilibrium that is characterized as follows. The seller offers good i at price $q_i^s = q_i(c,n)$ with*

$$q_i(c,n) = 1 - \tfrac{\beta-\alpha}{n-i+1} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \tag{1}$$

and accepts any offer $q \geq \alpha q_i(c,n)$, where

$$\alpha = \tfrac{\beta\gamma}{\beta\gamma+1-\beta}. \tag{2}$$

Each buyer offers to buy good i at price $q_i^b = \alpha q_i(c,n)$ and accepts any offer $q \leq q_i(c,n)$.

In equilibrium there are no bargaining delays and a good is sold in each bargaining round, until the inventory is depleted. In addition, equilibrium sale prices are heterogeneous; they depend on the order of the transaction in the sale sequence and on the identity of the proposer, buyer or seller. (The limiting case of no discounting is studied in the next section)

In what follows we prove this theorem via a sequence of lemmas, starting by conjecturing the existence of a subgame perfect equilibrium that satisfies basic properties, namely, offers are stationary and are accepted without delay. Then we will calculate such offers and verify that they are indeed subgame perfect. Finally we show that our conjecture is the unique SPE by demonstrating that all SPE of this game satisfy the properties above.

Consider an equilibrium characterized by two properties. There is no delay, i.e., in equilibrium any offer is accepted in the same round in which it is made. Equilibrium offers are stationary, i.e., players do not modify their offers for a good unsold in the previous round. Suppose the game has reached some round $t \geq i$ and that the seller is offering the i^{th} good. Denote by A_i the set of buyers who desire to purchase the good. We have $|A_i| = n-i+1$ since in previous rounds $i-1$ buyers have traded with the seller, consumed and left. Given stationarity and a pair (c,n), let $q_{k,i}^b(c,n)$ denote the equilibrium offer of buyer k to the seller and let $q_{k,i}^s(c,n)$ denote the equilibrium offer to buyer $k \in A_i$. (We will omit c and n, when they are understood.)

Given no delay in accepting offers, let $\pi_i(c,n)$ denote the seller's expected earnings from bargaining over the i^{th} good, for a pair (c,n). It is defined as

$$\pi_i(c,n) = \sum_{k\in A_i} \frac{\gamma q^s_{k,i} + (1-\gamma) q^b_{k,i}}{n-i+1}.$$

With probability γ the seller gets to make the offer and buyer $k \in A_i$ is selected to receive it with probability $\frac{1}{n-i+1}$. When the offer is accepted without delay, the seller gets $q^s_{k,i}$ utility and the buyer $1-q^s_{k,i}$. Similarly with probability $\frac{1-\gamma}{n-i+1}$ some buyer $k \in A_i$ gets to make an offer, in which case the seller's utility is $q^b_{k,i}$ and the buyer is $1-q^b_{k,i}$.

The seller's payoff in the bargaining game is simply the expected utility from selling at most c goods. When there is no delay, we denote it by $\pi(c,n)$ with

$$\pi(c,n) = \sum_{i=1}^{c} \beta^{i-1}\pi_i.$$

Now consider buyer k. Let $u_{k,i}(c,n)$ denote his expected utility at the start of some trading round $t \geq i$ in which good i is offered for sale, given initial demand n and capacity c. When offers are immediately accepted we have

$$u_{k,i}(c,n) = \gamma \frac{1-q^s_{k,i}}{n-i+1} + (1-\gamma)\frac{1-q^b_{k,i}}{n-i+1} + \frac{n-i}{n-i+1}\beta u_{k,i+1}(c,n).$$

The first two terms on the right hand side refer to the case when buyer k is selected to receive or make an offer. The third term represents a continuation payoff. Due to random selection, $\frac{n-i}{n-i+1}$ is the probability that the buyer is excluded from this bargaining round. Exclusion is costly for the buyer because, even if the seller has goods left in inventory, future payoffs are discounted by β. The notation $u_{k,i+1} \geq 0$ denotes the expected utility from continuing the game, with $u_{k,c+1}(c,n) = u_{c+1}(c,n) = 0$ for all $k \in A_c$ (and note that $A_c \neq \varnothing$ since $c \leq n$). The buyer's payoff in the bargaining game is therefore $u_{k,1}$, which can be obtained by backward iteration.

Now consider best responses. Players choose offers on $[0,1]$ to maximize their payoffs. There is an incentive to quickly reach agreement. Indeed, suppose that round $t = i$ results in disagreement between the seller and a buyer $k \in A_i$. Given stationarity and absence of future delays, the seller's continuation payoff is $\beta\pi_i$ and the buyer's is $\beta u_{k,i}$. Therefore any player accepts an offer giving him utility greater than his continuation payoff, is indifferent if the offer corresponds to his continuation payoff, and rejects it, otherwise.

Note that $u_{k,i}$ is linearly decreasing in $q^b_{k,i}$ and π_i is linearly increasing in q^s_i. So, an offer is individually optimal only if it gives the opponent exactly his continuation payoff, i.e. if it leaves him indifferent. Hence, for each good i and each buyer k the expressions

$$q^b_{k,i} = \beta\pi_i \quad \text{and} \quad 1 - q^s_{k,i} = \beta u_{k,i}, \tag{3}$$

identify the best responses of buyer k and of the seller.

Lemma 2 *Fix a pair (c, n). Equilibrium offers must be symmetric. That is, $q^b_{k,i} = q^b_i$ and $q^s_{k,i} = q^s_i$ for each $i = 1, ..., c$ and all $k \in A_i$. In particular,*

$$q^b_i = \alpha q^s_i \tag{4}$$

$$q^s_i = \tfrac{n-i+1-\beta}{n-i+1-\alpha} - \tfrac{\beta^2(n-i)}{n-i+1-\alpha} u_{i+1} \tag{5}$$

where α is as in (2). It follows that in equilibrium:

$$\pi_i = \tfrac{\alpha}{\beta} q^s_i \tag{6}$$

$$u_i = \tfrac{1-\frac{\alpha}{\beta}q^s_i}{n-i+1} + \tfrac{\beta(n-i)}{n-i+1} u_{i+1}. \tag{7}$$

All proofs are in Appendix A. Conjecturing no delay and stationarity, the seller makes identical offers to any buyer k and every buyer k makes the same offer for good i. Intuitively, everyone is treated identically because buyers are not queued in any specific line and are homogeneous. Interestingly, the buyer's equilibrium offer follows a clean-cut rule. The buyer applies a fixed discount $1 - \alpha$ to the seller's offer. The portion α is constant across bargaining rounds, and it neither depends on i nor on the initial number of competitors n; it depends only upon time-discounting and the seller's bargaining skill, being an increasing function of γ and β.

We can now obtain an expression for the buyer's payoff and the equilibrium offer as functions of parameters, so we get π_i and q^b_i as functions of parameters, also.

Lemma 3 *Fix a pair* (c,n)*. In equilibrium we have*

$$u_i\left(c,n\right)=\tfrac{\Phi_i(c,n)}{n-i+1} \tag{8}$$

and $q_i^s(c,n)=q_i(c,n)$ *with*

$$q_i\left(c,n\right)=1-\tfrac{\beta\Phi_i(c,n)}{n-i+1} \tag{9}$$

$$\Phi_i\left(c,n\right)=\tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=i}^{c}\beta^{j-i}\displaystyle\prod_{m=i}^{j}\tfrac{n-m+1}{n-m+1-\alpha} \tag{10}$$

for all $i=1,...,c$*. In particular,* $q_i\left(c,n\right)\in(0,1)$ *for all* i*.*

Intuitively, Φ_i denotes expected future surplus for a buyer when goods $j=i,\ldots,c$ remain. Thus the buyer's expected utility u_i is Φ_i divided by $n-i+1$, the number of remaining buyers, and the buyer's (ex-ante) payoff is $u(c,n)=u_1(c,n)$, i.e., the expected utility at the beginning of the game. At any stage i of the game the seller's equilibrium offer $q_i\left(c,n\right)$ leaves the buyer indifferent to purchasing good i.

Clearly there is a unique pair (q_i^s,q_i^b) for each i, thus there is a unique SPE satisfying the two properties: stationarity and no-delay. It is easy to check that the strategies described in the Theorem are subgame perfect. The only thing left to demonstrate is that this is also the *unique* SPE of this game. To do so we must show that every SPE must satisfy stationarity and no-delay, which is a tedious but straightforward process.

Lemma 4 *The subgame perfect equilibrium described in Theorem 1 is the unique subgame perfect equilibrium of this game.*

To sum up, multi-player bargaining equilibrium is stationary, symmetric and unique. Since there are no delays, it is also efficient. If the seller makes the offer, then good i is sold at price q_i, and otherwise the sale price is lower, αq_i. Thus, the sale price sequence is generally heterogeneous not only due to the order of sale, but also because buyer and seller's offers differ. The section that follows characterizes equilibrium sale prices in detail.

2.3 Characterization of equilibrium offers

We show how offers respond to changes in capacity c and demand n.

Lemma 5 *Equilibrium offers are increasing functions of $n - c$. For each $i = 1, ..., c$ with $c \leq n$, the sequence $\{q_i(\tilde{c}, n)\}_{\tilde{c}=i}^{n}$ is strictly decreasing, $\{q_i(c, \tilde{n})\}_{\tilde{n}=c}^{\infty}$ is strictly increasing, and $q_{i+j}(c + j, n + j) = q_i(c, n)$ for $j = 1, 2, ...$*

Sale prices respond naturally to demand pressure, i.e., they increase with $n - c$. So, the seller may wish to strategically constrain capacity to induce buyers to make and accept higher offers. If $c < n$, then customers face *consumption risk*, which grows in $n - c$. Here disagreement is costly to a buyer (apart from discounting) since it carries the risk of not consuming at all; the seller may be sold out by the time they agree on an offer, since the seller may get (and accept) other offers in the meantime. Clearly, the seller does not face this type of risk. Intuitively, tighter capacity constraints make goods scarcer and induce buyers to pay a higher price. Hence, the seller can strengthen his bargaining position or, equivalently, raise his bargaining leverage, by restricting capacity. (In later Sections we present conditions for existence of equilibrium capacity constraints).

Lemma 5 also tells us that what matters for price determination is the number of goods left in inventory. Since larger capacity and greater demand have opposite effects, good i sold by a seller that has excess demand $n - c$ is the same as the price of good $i + j$ sold by a seller whose capacity and number of customers is also increased by j. (This is used to prove another result). Now, instead, we establish how equilibrium offers respond to changes in β and γ, given some (c, n) pair.

Lemma 6 *Fix a pair (c, n). We have $\frac{\partial q_i}{\partial \gamma} > 0$ for every $i = 1, ..., c$.*

Every equilibrium offer increases with the probability γ, which makes sense because this corresponds to an improvement in the strength of the seller's bargaining position. Now we characterize the price sequence.

Lemma 7 *Fix a pair (c, n). The sequence $\{q_i(c,n)\}_{i=1}^{c}$ is: (i) monotonically decreasing if $\frac{c}{n}$ is sufficiently close to one, (ii) monotonically increasing if $\frac{c}{n}$ is sufficiently close to zero, and (iii) U-shaped, otherwise.*

Whether early buyers pay less or more than late buyers depends on the severity of capacity constraints, i.e., $\frac{c}{n}$. The price sequence can be non-monotonic, falling and then rising when few items are left for sale. Intuitively, two opposing effects influence the shape of the price sequence. One the one hand, with $\beta < 1$ buyers wish to buy as soon as possible even if there is no shortage of goods. However, when good i is offered for sale, the probability of being selected to trade is $\frac{1}{n-i+1}$ and it *increases* with i since buyers leave after purchasing. So, competition for goods *falls* as goods are sold. On the other hand, when $c < n$ some buyers will not consume at all and this consumption risk increases with i. So, competition for goods *increases* as goods are sold.[7] The strengths of these opposing effects varies with i, and the second dominates if capacity constraints are sufficiently tight.

An illustration. Figure 1 plots equilibrium sale prices in three economies with different capacity levels, $c = 5, 14, 15$, against the order of sale $i = 1, 2, ..c$. The parameters are $n = 15$, $\beta = 0.9$ and $\gamma = 0.1$.

Figure 1 approximately here

There is no consumption risk for $c = 15$ but only a cost due to discounting. So, the sequence of equilibrium sale prices is monotonically decreasing in i. For small excess demand, $c = 14$, consumption risk is initially small. So, prices first fall and start to rise after the tenth item is sold. When capacity constraints are very tight, $c = 5$, the sequence of prices rises monotonically because consumption risk is always dominant.$\square$

We conclude the analysis of bargaining equilibrium by discussing the case of costless negotiations, $\beta \to 1$. We think of this as a situation in which the time interval between

[7] Suppose $i-1$ goods have been sold. $\prod_{j=i}^{c} \frac{n-j}{n-j+1} = \frac{n-c}{n-i+1}$ is the probability a buyer is unable to trade before the inventory runs out; $\frac{n-j}{n-j+1}$ is the probability that some other buyer is selected to trade over good $j = i, .., c$. Clearly $\frac{n-c}{n-i+1}$ increases with i.

offers shrinks to zero, i.e., every buyer gets to make an offer to the seller so the seller is not committed to deal with one buyer at a time.

Theorem 8 *Fix a pair* (c,n)*;* $\lim_{\beta\to 1} q_i^s(c,n) = q_i^b(c,n) = q(c,n)$ *for all* $i = 1,\ldots,c$ *with*

$$q(c,n) = \begin{cases} \gamma & \text{if } c = n, \\ 1 & \text{if } c < n. \end{cases}$$

As $\beta \to 1$, the outcome of the bargaining procedure we have proposed is comparable to the outcome of an auction with fixed reservation price γ. Intuitively, when players do not discount future payoffs each equilibrium sale price converges to a constant $q(c,n)$ the value of which depends on whether capacity constraints exist or not. Without capacity constraints every player earns a surplus share corresponding to the probability of making the offer, γ for the seller and $1-\gamma$ for each buyer. Instead, with capacity constraints buyers behave as if they were participating in an auction, bidding their reservation value (one) in order to effectively compete with other buyers for scarce goods. This finding is interesting because it offers a microfoundation for the use of single-unit auctions with fixed reservation values in search models of multilateral matching; e.g., see [2, 17, 20, 32] among others. We will develop this link further in Section 4, where we present a condition ensuring that $c = 1$ is a seller's optimal choice when demand n is random, and $\beta \to 1$.

3 Endogenous Capacity

The previous section has established that bargaining equilibrium is efficient since there are no wasteful delays. We now endogenize the choice of c and present a condition sufficient for the existence of equilibrium capacity constraints given a known demand n.

Consider the first stage of the game. The seller chooses $c \in \{1,...,n\}$ to maximize $\pi(c,n)$ given that offers are selected optimally in the second stage of the game, i.e., offers satisfy Theorem 1. We have the following.

Theorem 9 *Let* $\widetilde{c}(n)$ *denote the set of maximizers of* $\pi(c,n)$*, i.e.,*

$$\widetilde{c}(n) = \{c : c \in \{1,...,n\} \text{ and } \pi(c,n) \geq \pi(x,n) \text{ for all } x = 1,...,n\},$$

and let $\widehat{c} \in \{2, ..., n-1\}$ denote a generic interior capacity choice. We have that

$$\widetilde{c}(n) = \begin{cases} \{1\} & \text{if } \frac{\beta}{\beta-\alpha} < \varphi(2,n) \\ \{\widehat{c}-1,\ \widehat{c}\} & \text{if } \frac{\beta}{\beta-\alpha} = \varphi(\widehat{c},n) \\ \{\widehat{c}\} & \text{if } \frac{\beta}{\beta-\alpha} \in (\varphi(\widehat{c},n), \varphi(\widehat{c}+1,n)) \\ \{n\} & \text{if } \frac{\beta}{\beta-\alpha} > \varphi(n,n) \end{cases} \tag{11}$$

with

$$\varphi(c,n) = \prod_{m=1}^{c} \tfrac{n-m+1}{n-m+1-\alpha}. \tag{12}$$

Corollary 10 *If $\frac{\beta}{\beta-\alpha} \leq \varphi(n,n)$, then the equilibrium outcome is inefficient.*

The theorem characterizes the set of optimal capacities $\widetilde{c}(n)$ in terms of the parameters of the model. This allows us to establish that the seller generally selects a unique capacity, although knife edge cases exist in which he might be indifferent between two adjacent choices. Intuitively, the seller is a monopolist who faces a set number of consumers. So he can trade off extensive margin losses against intensive margin gains (as demonstrated in Lemma 7) to maximize his payoff. This suggests that (i) multiplicity can arise due to the discreteness of the choice set and (ii) full capacity $c = n$ is not generally optimal. The seller will constrain capacity only if his bargaining position is sufficiently weak. To see this, recall that γ captures the seller's relative bargaining strength and equilibrium offers grow in γ (Lemma 6). If γ is sufficiently low, then the seller can set $c < n$ to increase his bargaining leverage. Of course, the seller suffers a loss from $n - c$ unrealized trades. One can show that parameters exist such that $\frac{\beta}{\beta-\alpha} < \varphi(n,n)$ for γ close to zero, i.e., the intensive margin effect is dominant. Here, inefficiency is a straightforward consequence of the deadweight loss. Since this inefficiency depends on the distribution of bargaining skills, a planner would grant the seller enough bargaining power to ensure that $c = n$ is selected. The theorem is proved in two steps.

Lemma 11 *Given $n \geq 2$ and prices as in Theorem 1, the seller's equilibrium payoff satisfies*

$$\pi(c,n) = \textstyle\sum_{j=1}^{c} \beta^{j-1} - \Phi_1(c,n), \tag{13}$$

or, equivalently,

$$\pi\left(c,n\right)=\pi\left(c-1,n\right)+\beta^{c-1}[1-\tfrac{\beta-\alpha}{\beta}\varphi\left(c,n\right)]. \tag{14}$$

The buyer's equilibrium payoff satisfies

$$u\left(c,n\right)=\tfrac{\Phi_1(c,n)}{n}. \tag{15}$$

The Lemma derives the seller's and buyers' equilibrium payoffs, $\pi(c,n)$ and $u(c,n)$, in terms of the model's parameters. This also allows us to establish a measure of ex-ante welfare, i.e., the sum of players' equilibrium payoffs, which we denote by $W(c,n)$, where

$$W(c,n)=\pi(c,n)+nu(c,n)=\textstyle\sum_{j=1}^{c}\beta^{j-1}. \tag{16}$$

The last equality has been obtained using (13) and (15). Clearly, because players are risk-neutral and divide one unit of surplus any time they trade, ex-ante equilibrium welfare is simply the present discounted sum of the value created by trading c goods in an uninterrupted sequence of c periods.

Lemma 11 also establishes that the seller's payoff is a step function on $\{1,...,n\}$. So, now we characterize the seller's payoff change from a unit increment in capacity.

Lemma 12 *Define* $\Delta\left(c,n\right)=\pi\left(c,n\right)-\pi\left(c-1,n\right)$. *We have*

$$\Delta\left(c,n\right)=\beta^{c-1}[1-\tfrac{\beta-\alpha}{\beta}\varphi\left(c,n\right)],\quad \textit{for } c=1,...,n. \tag{17}$$

Therefore, for all $c=1,..,n-1$ *we have*

$$\Delta\left(c,n\right)\geq 0 \;\;\Leftrightarrow\;\; \tfrac{\beta}{\beta-\alpha}\geq\varphi\left(c,n\right), \tag{18}$$

with

$$\Delta\left(c,n\right)>\Delta\left(c+1,n\right). \tag{19}$$

For every $n\geq 2$ the change in payoff strictly falls with each step and due to (18), the expression (11) defines the set of maximizer of $\pi(c,n)$.

An illustration. Figure 2 plots the choice of capacity $\widetilde{c}(n)$ for $n = 2$ as β and γ vary. The seller constrains capacity if his bargaining skill is sufficiently low, $\gamma < \frac{3\beta-2}{2\beta}$ in the bottom-right of the figure. The seller is indifferent between offering one or two goods when $\gamma = \frac{3\beta-2}{2\beta}$, and otherwise always offers two goods. Intuitively, fix a low value of γ and vary β. When β is small constraining capacity does not raise prices very much since buyers make low offers simply due to the heavy discounting. Hence, $\widetilde{c} = 2$. The opposite occurs when β is high, hence $\widetilde{c} = 1$. Now fix β close to one and notice that the seller restricts capacity only if γ (his bargaining skill) is sufficiently small.

Figure 2 approximately here

Figure 3 shows how efficiency varies with market size and distribution of bargaining skill, for $\beta = 0.9$. Efficiency is measured as ex-ante equilibrium welfare relative to ex-ante welfare when $c = n$ is imposed. From (16), this ratio is $\frac{1-\beta^c}{1-\beta^n}$.

Figure 3 approximately here

The horizontal axis is demand n and the seller's bargaining skill can be either extremely limited, $\gamma = 0.01$, or substantial, $\gamma = 0.7$. As demand increases, the ratio $\frac{1-\beta^c}{1-\beta^n}$ converges to one because $c(n)$ grows unbounded as $n \to \infty$.[8] There is generally a greater efficiency loss when the seller has less bargaining skill, which is when he chooses to constrain capacity to improve his payoff. However, because efficiency displays an overall increasing trend in n, the efficiency losses associated to the two γ values differ markedly only for small n. For example, when $n = 2$, efficiency is 100% for $\gamma = 0.7$ but it is only about 53% for $\gamma = 0.01$. When $n > 100$ there is no discernible efficiency differential.$\square$

To conclude the analysis in this section consider the case $\beta \to 1$. Theorem 8 implies that if the seller's bargaining skill γ is sufficiently low, then $c = n - 1$, or else the seller

[8] The non-monotonic saw-tooth pattern arises because as n increases $\widetilde{c}(n)$ can remain constant initially, and then rise (due to the discreteness of the seller's choice set). For example, when $\gamma = 0.7$, we have $\widetilde{c}(2) = 2$, $\widetilde{c}(3) = 3$, $\widetilde{c}(4) = 4$, $\widetilde{c}(5) = 4$ and $\widetilde{c}(6) = 5$. So, efficiency equals 1 for $n = 2, 3, 4$, drops to about 0.84 for $n = 5$, rises to about 0.87 for $n = 6$, and so on. For γ small, this behavior is even more marked because $c < n$ even for small n, so the fluctuations in efficency die out more slowly as n grows large.

will serve the entire demand. To see this, note that in equilibrium the seller trades in each round i at price $q_i(c,n)$ until all goods are sold, earning a payoff $\pi(c,n) = \sum_{i=1}^{c} \beta^{i-1} q_i(c,n)$ for $c \leq n$. As $\beta \to 1$, we have $q_i(c,n) \to 1$ if $c < n$ and $q_i(c,n) \to \gamma$ if $c = n$, for all i. Hence, the seller's payoff is c if $c < n$, and otherwise is γn. Clearly only $c \geq n-1$ can be optimal—intuitively, slicing off just one unit of demand is sufficient to induce Bertrand competition among buyers—and so $c = n-1$ when $\gamma n < n-1$.

4 Extensions: stochastic demand and multiple sellers

We have seen that with costless bargaining the seller serves every customer but, at most, one. This is in contrast to the standard unit-inventory assumption in the multilateral matching literature where there is no ex-ante commitment to prices and demand is random; e.g., see [2, 9, 17, 20, 27, 32]. Would unit-capacity be chosen in equilibrium if we augmented our model with random demand?

To provide an answer, let c be chosen before a random value $n \in \mathbb{N} = \{0, 1, \ldots\}$ is observed. Let n be distributed as a Poisson with parameter $\lambda \in \mathbb{R}_+$, i.e., $m_n := \frac{e^{-\lambda}\lambda^n}{n!}$ is the probability that the seller is matched to n buyers, so $\lambda = \sum_{n=1}^{\infty} m_n n$ is expected demand. This is the typical distribution function in symmetric directed search equilibrium and in urn-ball matching with countable players. Given equilibrium bargained prices and costless bargaining, $\beta \to 1$, the payoff (expected profit) to a seller who picks $c \in \mathbb{N}$ is

$$\pi(c) = \gamma\lambda + \sum_{n=c+1}^{\infty} m_n(c - n\gamma), \tag{20}$$

i.e., in any trade the seller always gets the reservation price γ from at most c buyers, and grabs the remaining surplus from exactly c buyers only if he realizes excess demand.[9] Letting $c = \infty$ denote the case of no capacity constraints, we have the following.

Lemma 13 *Let demand be randomly distributed as a Poisson(λ) and let $\beta \to 1$. If*

$$\gamma\lambda < \frac{1-m_0-m_1}{1-m_0}, \tag{21}$$

then $c < \infty$ is optimal. In addition, if $\lambda \leq 1$ and $\gamma = 0$, then $c = 1$ is optimal.

[9] We have $\pi(c) = \sum_{n=1}^{c} m_n \pi(n,n) + \sum_{n=c+1}^{\infty} m_n \pi(c,n) = \gamma \sum_{n=1}^{c} m_n n + c \sum_{n=c+1}^{\infty} m_n$, because $\pi(c,n) = c$ for $c < n$ and γn otherwise, as $\beta \to 1$. Use the definition of λ to get (20).

The Lemma presents sufficient conditions for two results. Condition (21) tells us that the seller will constrain capacity whenever the payoff from committing to serve *any* demand, $\gamma\lambda$, is smaller than the payoff from offering just one good, $\gamma\lambda m_0 + (1 - m_0 - m_1)$. This holds when a seller of weak bargaining skill does not anticipate high demand, i.e., $\gamma\lambda$ is small. By restricting capacity the seller improves his bargaining position but only if extensive margin losses are unlikely, i.e., λ is not large. This leads to the second result, $\gamma = 0$ and $\lambda \leq 1$ is sufficient for $c = 1$ to be optimal. By continuity, this holds for some $\gamma > 0$ and $\lambda > 1$ (and for even larger sets of parameters, if capacity were not costless). For instance, fixing $\gamma = 0.01$, we find that the values $\lambda = 1.5, 2.5, 3.5$ are associated to optimal capacities $c = 1, 2, 5$, respectively.

Intuitively, when buyers make take-it-or-leave-it offers, a seller who anticipates meeting at most one buyer will simply store one good, hoping to get all surplus in case of multiple visits. This second finding is especially interesting because some directed search literature assumes sellers auction a single good at a reservation price equal to the price posted in a pre-matching stage (which is zero in large markets, see [2, 20]). Hence, the analysis offers a rationale for adopting single-unit auctions with a fixed reservation price when sellers compete in posted prices but not in capacity .

What if sellers could compete in capacities? To explore this scenario consider a directed search model with two buyers and two sellers, and three stages. Sellers simultaneously announce and commit to a capacity, then buyers see capacities, simultaneously visit a seller, and finally bargaining occurs.[10] Consider symmetric subgame perfect equilibria, when seller $s = 1, 2$ chooses $c_s = c \in \{1, 2\}$ and indifferent buyers identically randomize.

Fix $\mathbf{c} = (c_1, c_2)$. In the third stage, (13) and (15) define the payoffs $u(c_s, n)$ and $\pi(c_s, n)$ when seller s offers c_s goods to n buyers.[11] In the second stage, let U_s be expected utility

[10] Related work is the directed search models in [13], which studies how asymmetric information about sellers' initial inventory decisions affects price competition among sellers, and in [22], where firms compete for workers by choosing to advertise up to two vacancies before workers move.

[11] Earlier payoffs are for $c \leq n$ but now we can have $c > n$. Excess capacity does not affect payoffs in a match because buyers desire only one good. So we have $u(c, n) = u(n, n)$ and $\pi(c, n) = \pi(n, n)$ for $c \geq n$.

to a buyer who meets seller s, when the other buyer visits seller 1 with probability v. So

$$U_1 = (1-v)\,u\,(c_1,1) + vu\,(c_1,2)$$
$$U_2 = vu(c_2,1) + (1-v)\,u(c_2,2).$$

Consider U_1. With probability $1-v$ the buyer is alone with seller 1 and gets $u\,(c_1,1)$; else, he is not alone and he gets $u\,(c_1,2)$ (U_2 is similarly interpreted).

Under symmetry, indifferent buyers visits seller 1 with probability $v = v(\mathbf{c})$, where

$$v\,(1,2) = (1-\beta)v(2,1) < v(1,1) = v(2,2) = \tfrac{1}{2} < v(2,1) = \tfrac{1}{2-\beta}. \tag{22}$$

If capacities are identical, then buyers simply flip a coin, or else are less likely to visit the low-capacity seller (the probability is zero only if $\beta \to 1$). Indeed, the high-capacity seller can serve all buyers but one must always wait in line, which implies a loss if $\beta < 1$.[12]

Consider stage one. Given c_2 and $v = v(\mathbf{c})$, the payoff to seller 1 from setting c_1 is

$$W(\mathbf{c}) = 2v\,(1-v)\,\pi\,(c_1,1) + v^2\pi\,(c_1,2)\,. \tag{23}$$

He earns $\pi\,(c_1,1)$ with probability $2v(1-v)$ (when one buyer arrives), and earns $\pi\,(c_1,2)$ otherwise. The payoff to seller 2 is symmetric, so we have the following.

Lemma 14 *For the game above, there are two pure strategy symmetric SPE where*

$$c_1 = c_2 = \begin{cases} 2 & \textit{for all } \gamma \in [0,1], \\ 1 & \textit{if } \gamma < \bar{\gamma} \in (0,1). \end{cases}$$

If $\gamma < \bar{\gamma}$, then $W\,(1,1) > W\,(2,2)$ and a mixed strategy equilibrium also exists.

To understand the result consider two extremes. Sellers who make take-it-or-leave it offers ($\gamma = 1$) have no desire to limit capacity, as this cannot improve their bargaining position. Hence, their dominant strategy is to fully exploit extensive margin gains and set maximum capacity. Instead, when buyers make take-it-or-leave it offers ($\gamma = 0$), restricting capacity is strategically meaningful, so low-capacity equilibria also arise. Because the

[12] Symmetry and buyers' indifference requires $U_1 = U_2$. From (15) we have $u\,(2,1) = u\,(1,1) = \frac{\beta-\alpha}{\beta(1-\alpha)}$, $u\,(1,2) = \frac{\beta-\alpha}{\beta(2-\alpha)}$, and $u\,(2,2) = u\,(1,2)\,\frac{1-\alpha+\beta}{(1-\alpha)}$, which we use to get (22).

desire to limit capacity grows as γ falls, this equilibrium appears if γ is sufficiently small. In this case, we have equilibrium multiplicity because the seller who deviates from high capacity raises his expected price but loses demand to his competitor; for this game, such an extensive-margin loss always dominates the intensive-margin gain. The low-capacity outcome yields the highest payoff to sellers because expected demand is invariant to any symmetric capacity choice c, but the price is higher when capacity is restricted. Numerical experiments suggest that the basic findings hold in larger games.

5 Conclusion

We have studied equilibrium prices and trade volume in a market where n identical buyers bargain with a seller who offers goods sequentially after choosing an inventory. Prices emerge from a strategic process of multilateral bargaining that involves random alternating offers. Bargaining equilibrium is unique, symmetric, stationary and efficient. However, since the choice of capacity affects offers, equilibrium inefficiency may result because the seller may choose to restrict capacity to obtain more favorable terms of trade. Hence, disparities in bargaining leverage that penalize the seller give rise to a deadweight loss. With infinitely quick bargaining rounds the equilibrium bargained prices converge to the outcome of an auction with fixed reservation value γ. Hence, our model can be seen as providing a tractable microfoundation for the use of auctions with fixed reservation values in the equilibrium search literature of multilateral matches.

Appendix A

Proof of Lemma 2. Consider bargaining over good $i = 1, ..., c$. The right hand side of (3) is not a function of k and so $q^b_{k,i} = q^b_i$ for all $k \in A_i$. This result jointly with the definition of $u_{k,i}$ and (3) imply:

$$q^s_{k,i} = 1 - \tfrac{\beta(1-\gamma)(1-q^b_i)}{n-i+1-\beta\gamma} - \tfrac{\beta^2(n-i)}{n-i+1-\beta\gamma} u_{k,i+1} \tag{24}$$

Use backward induction on i. Start with $i = c$ in which case $u_{k,c+1} = u_{c+1} = 0$ by definition. Thus, we have $q^s_{k,c} = q^s_c = 1 - \frac{\beta(1-\gamma)(1-q^b_c)}{n-c+1-\beta\gamma}$ for all k. For the induction step, suppose $q^s_{k,i+1} = q^s_{i+1}$ for some $i < c-1$. Then, (3) implies $u_{k,i+1} = u_{i+1}$ for all $k \in A_{i+1}$. Therefore, using (24) we have $q^s_{i,k} = q^s_i$ for all $k \in A_i$.

Having established that offers are symmetric, we have $\pi_i = \gamma q^s_i + (1-\gamma) q^b_i$. Thus, we can use (3) to obtain (4) and (2). From (4) and symmetry, expression (24) gives us (5). Finally, use (4)-(5) and the definitions of π_i and $u_{k,i}$ to obtain (6)-(7).■

Proof of Lemma 3. Start by defining

$$\Phi_i\left(c, n\right) = \textstyle\sum_{j=i}^{c} \beta^{j-i} \left[1 - \frac{\alpha}{\beta} q^s_j\right] \tag{25}$$

Clearly, we have

$$\begin{aligned} \Phi_i(c,n) &= 1 - \tfrac{\alpha}{\beta} q^s_i + \beta \textstyle\sum_{j=i+1}^{c} \beta^{j-(i+1)} \left[1 - \frac{\alpha}{\beta} q^s_j\right] \\ &= 1 - \tfrac{\alpha}{\beta} q^s_i + \beta \Phi_{i+1}(c,n). \end{aligned} \tag{26}$$

We will omit the arguments, when understood.

From (6) recall that $\frac{\alpha}{\beta} q^s_j$ is the seller's equilibrium expected surplus in round $j > i$ of bargaining. Thus $1 - \frac{\alpha}{\beta} q^s_i$ is the expected surplus to the buyer of good i and Φ_{i+1} is the expected future surplus to buyers, from the sales of goods $i + 1$ through c.

To get (8) use backward induction on i. Let $i = c$. From (7) and $u_{c+1} = 0$ we have

$$u_c = \tfrac{1 - \frac{\alpha}{\beta} q_c}{n-c+1} = \tfrac{\Phi_c}{n-c+1}.$$

For the induction step suppose $u_{i+1} = \frac{\Phi_{i+1}}{n-i}$ holds for some $i < c-1$. Inserting u_{i+1} into (7), we obtain

$$u_i = \frac{1-\frac{\alpha}{\beta}q_i^s+\beta\Phi_{i+1}}{n-i+1} = \frac{\Phi_i}{n-i+1}$$

because of (26). This gives us (8).

To find an expression of $\Phi_i(c,n)$ in terms of the parameters, we use backward induction on i. Let $i=c$. Then, (25)

$$\Phi_c(c,n) = 1 - \frac{\alpha}{\beta}q_c^s = \frac{\beta-\alpha}{\beta}\frac{n-c+1}{n-c+1-\alpha},$$

where we have substituted (5) with $u_{c+1}=0$ for q_c^s. For the inductive step suppose that for some $i < c-1$ we have

$$\Phi_{i+1}(c,n) = \frac{\beta-\alpha}{\beta}\sum_{j=i+1}^{c}\beta^{j-(i+1)}\prod_{m=i+1}^{j}\frac{n-m+1}{n-m+1-\alpha}, \tag{27}$$

where we notice that $(\beta-\alpha)\in(0,1)$. From (26) we get

$$\begin{aligned}\Phi_i(c,n) &= 1-\frac{\alpha}{\beta}q_i^s+\beta\Phi_{i+1}(c,n)\\ &= 1-\frac{\alpha}{\beta}[\frac{n-i+1-\beta}{n-i+1-\alpha}-\frac{\beta^2(n-i)}{n-i+1-\alpha}u_{i+1}]+\beta\Phi_{i+1}(c,n)\\ &= 1-\frac{\alpha(n-i+1-\beta)}{\beta(n-i+1-\alpha)}+\frac{\alpha\beta\Phi_{i+1}}{n-i+1-\alpha}+\beta\Phi_{i+1}(c,n)\\ &= \frac{n-i+1}{n-i+1-\alpha}[\frac{\beta-\alpha}{\beta}+\beta\Phi_{i+1}(c,n)].\end{aligned} \tag{28}$$

where in the second line we have used (5) and in the third we used (8). Inserting Φ_{i+1} from (27) we obtain

$$\Phi_i(c,n) = \frac{\beta-\alpha}{\beta}[\frac{n-i+1}{n-i+1-\alpha}+\frac{n-i+1}{n-i+1-\alpha}\beta\sum_{j=i+1}^{c}\beta^{j-(i+1)}\prod_{m=i+1}^{j}\frac{n-m+1}{n-m+1-\alpha}],$$

which gives us (10).

To get q_i^s in terms of the parameters, plug (8) into (3), under symmetry. Note that $\frac{\beta\Phi_i}{n-i+1}\in(0,1)$ for each i, since $0<\beta-\alpha<1$. Rearranging (9) and (10) we obtain (1).■

Proof of Lemma 4

To prove uniqueness one should demonstrate that all SPE of this game must satisfy stationarity and no-delay. This involves three steps following the method by [33], i.e., showing that the supremum and infimum of the set of SPE payoffs coincide. Since the proof is straightforward, but it is lengthy, it is included in a separate additional appendix.

Proof of Lemma 5. Consider (10). For all $i \leq c < c' \leq n$ we have

$$\begin{aligned}\Phi_i(c', n) &= \tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=i}^{c'} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &= \tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} + \tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=c+1}^{c'} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &> \Phi_i(c, n)\end{aligned}$$

In the second line we have used the definition of $\Phi_i(c, n)$. From (9) we have $q_i(c, n) = 1 - \frac{\beta\Phi_i(c,n)}{n-i+1}$. This implies that $q_i(c, n) > q_i(c', n)$ for all $i \leq c < c' \leq n$.

Now consider the effect of n. Let $n' > n \geq c$, then $\frac{n'-i+1}{n'-i+1-\alpha} < \frac{n-i+1}{n-i+1-\alpha}$ for each $i \leq c$. Thus (10) implies $\Phi_i(c, n') < \Phi_i(c, n)$. From (9) we have $q_i(c, n') > q_i(c, n)$ for all $i = 1, ..., c$.

Finally, let $i' = i + j$, $n' = n + j$ and $c' = c + j$ with $j = 0, 1, ...$ Note that $\frac{n'-i'+1}{n'-i'+1-\alpha} = \frac{n-i+1}{n-i+1-\alpha}$, $n' - i' = n - i$, and $\Phi_{i'}(c', n') = \Phi_i(c, n)$, so (9) implies $q_{i'}(c', n') = q_i(c, n)$.■

Proof of Lemma 6. We start by demonstrating that the function $q_i(c, n)$ is strictly increasing in γ for every $i = 1, ..., c$. Use backward induction on i. Consider $\gamma \in (0, 1)$. Let $i = c$ and demonstrate that $\frac{\partial q_c(c,n)}{\partial \gamma} > 0$. In equilibrium we have $q_i^s = q_i(c, n)$ from Lemmas 2 and 3. Hence, use (24) with $u_{c+1} = 0$ to get

$$q_c(c, n) = \tfrac{n-c+1-\beta}{n-c+1-\alpha}.$$

From (2) we have $\frac{\partial \alpha}{\partial \gamma} > 0$, so $\frac{\partial q_c(c,n)}{\partial \gamma} > 0$. For the inductive step suppose $\frac{\partial q_{i+1}(c,n)}{\partial \gamma} > 0$ for some $i < c$, and demonstrate that $\frac{\partial q_i(c,n)}{\partial \gamma} > 0$. Using (3) and (24) we can write

$$q_i(c, n) = \tfrac{(n-i+1)(1-\beta)}{n-i+1-\alpha} + \tfrac{\beta(n-i)q_{i+1}(c,n)}{n-i+1-\alpha}. \tag{29}$$

The first term increases with α, and so with γ. Using the inductive step, we see that the second term increases with γ as well, thus the result. Since $q_i^b = \alpha q_i^s$ then every equilibrium offer increases in γ.■

Proof of Lemma 7. Let $c = 2, .., n$. From the proof of Lemma 5 we see that $\{\Phi_i(c,n)\}_{c=i}^{n}$ is a monotonically increasing positive sequence for all $i = 1, .., c$. From (10) we also see that $\{\Phi_i(c,n)\}_{i=1}^{c}$ is monotonically decreasing.

Define $d_{i+1} = \frac{\beta\Phi_{i+1}}{n-(i+1)+1} - \frac{\beta\Phi_i}{n-i+1}$ and notice that $q_{i+1} - q_i = -d_{i+1}$. Using (28) in the proof of Lemma 3 we get

$$d_{i+1} = -\tfrac{\beta-\alpha}{\beta(n-i)} + \Phi_i(1 - \beta + \tfrac{1-\alpha}{n-i}),$$

noting that $n \geq c \geq i+1$ since $c = 2$.

Notice that $\left\{\frac{1}{n-i}\right\}_{i=1}^{c}$ is increasing and $\{\Phi_i\}_{i=1}^{c}$ is decreasing. Therefore $\{d_{i+1}\}_{i=1}^{c-1}$ is decreasing. Thus, if $d_{i+1} < 0$ then we have $d_j < 0$ for $j > i$. That is to say, if $q_{i+1} > q_i$ then we have $q_{j+1} > q_j$ for all $j \geq i$.

Most importantly, if $d_2 < 0$ then $d_{i+1} < 0$ for all $i = 1, ..., c-1$. It is easy to find conditions such that $d_2 < 0$ (i.e., that $q_2 > q_1$). Substituting for Φ_i from (10) we get

$$d_{i+1} = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-i} + (1 - \beta + \tfrac{1-\alpha}{n-i})\textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha}].$$

Recall that $\alpha \in (0,1)$ is independent of i and c, so d_{i+1} increases with c. Thus, consider $c = 2$ in which case we obtain

$$d_2 = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-1} + (1 - \beta + \tfrac{1-\alpha}{n-1})\tfrac{n}{n-\alpha}(1 + \tfrac{\beta(n-1)}{n-1-\alpha})].$$

It is easy to see that $\lim_{n\to\infty} d_2 < 0$ since the second term in the square brackets converges to a positive constant, as n grows large. Therefore, if n is sufficiently large and c is sufficiently small, we have $d_2 < 0$ and therefore $d_{i+1} < 0$ for all $i = 1, ..., c-1$, because d_{i+1} falls with i. That is, we need $\frac{c}{n}$ sufficiently close to zero.

Now, consider $i + 1 = c$, which is when d_{i+1} is the smallest, so we have

$$d_c = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-c+1} + (1 - \beta + \tfrac{1-\alpha}{n-c+1})\textstyle\sum_{j=c-1}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha}].$$

We have established earlier that, for each i, d_{i+1} increases as c grows from $i+1$ to n. Therefore consider $c = n$, so we have

$$d_n = \tfrac{\beta-\alpha}{\beta}[-1 + (2 - \beta - \alpha)\tfrac{2}{2-\alpha}(1 + \tfrac{\beta}{1-\alpha})] = \tfrac{\beta-\alpha}{\beta}[1 + \tfrac{2\beta(1-\beta)}{(2-\alpha)(1-\alpha)}] > 0.$$

Since $\{d_{i+1}\}_{i=1}^{c-1}$ is a decreasing sequence, and d_{i+1} increases in c, it follows that $d_{i+1} > 0$ for all $i = 1, .., c-1$, when c is sufficiently close to n. Therefore, we must have $\{q_i\}_{i=1}^{c}$ is a monotonically decreasing sequence when c is sufficiently close to n. That is to say, we need $\frac{c}{n}$ sufficiently close to one.

In between these extreme cases, there is a case when $d_2 > 0$ but, since d_{i+1} falls with i, $d_c < 0$. In this case, we have $q_i < 0$ for i small and $q_i > 0$ for i large. Our prior discussion indicates that this will occur when $\frac{c}{n}$ is between zero and one.■

Proof of Theorem 8. From (1) we have

$$q_i(c,n) = 1 - \tfrac{\beta-\alpha}{n-i+1} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha},$$

where $\beta - \alpha = \frac{\beta(1-\gamma)(1-\beta)}{1-\beta(1-\gamma)}$ since $\alpha = \frac{\beta\gamma}{\beta\gamma+1-\beta}$. Define

$$\begin{aligned} f(\beta) \quad &:= \tfrac{\beta-\alpha}{n-i+1} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &= (\beta-\alpha) \textstyle\sum_{j=i}^{c} \tfrac{\beta^{j-i}}{n-i+1} \left[\tfrac{n-i+1}{n-i+1-\alpha} \times \tfrac{n-(i+1)+1}{n-(i+1)+1-\alpha} \times \cdots \times \tfrac{n-j+1}{n-j+1-\alpha}\right]. \end{aligned}$$

1) Fix $c < n$. In $f(\beta)$ we have $n > j$ for all $j \le c$ since $c < n$ by assumption, so

$$\lim_{\beta\to 1} \tfrac{n-m+1}{n-m+1-\alpha} = \tfrac{n-m+1}{n-m} < \infty \text{ for } m = i, \ldots, j.$$

Therefore we have

$$\lim_{\beta\to 1} \textstyle\sum_{j=i}^{c} \beta^{j-i} \tfrac{1}{n-i+1} \left[\tfrac{n-i+1}{n-i+1-\alpha} \times \tfrac{n-i}{n-(i+1)+1-\alpha} \times \cdots \times \tfrac{n-j+1}{n-j+1-\alpha}\right] = \textstyle\sum_{j=i}^{c} \tfrac{1}{n-j} < 1$$

Note that $\lim\limits_{\beta\to 1} (\beta-\alpha) = 0$. So, $\lim\limits_{\beta\to 1} q_i(c,n) = 1 - \lim\limits_{\beta\to 1} f(\beta) = 1 - 0 \times \sum_{j=i}^{c} \frac{1}{n-j} = 1.$

2) Now fix $c = n$. Hence, $n = j$ when $j = c$. So we write

$$\begin{aligned} f(\beta) = \quad & \textstyle\sum_{j=i}^{c-1} \tfrac{(\beta-\alpha)\beta^{j-i}}{n-i+1} \left[\tfrac{n-i+1}{n-i+1-\alpha} \times \tfrac{n-(i+1)+1}{n-(i+1)+1-\alpha} \times \cdots \times \tfrac{n-j+1}{n-j+1-\alpha}\right] \\ & + \tfrac{(\beta-\alpha)\beta^{c-i}}{n-i+1} \left[\tfrac{n-i+1}{n-i+1-\alpha} \times \tfrac{n-(i+1)+1}{n-(i+1)+1-\alpha} \times \cdots \times \tfrac{n-c+1}{n-c+1-\alpha}\right]. \end{aligned} \tag{30}$$

Consider the limit as $\beta \to 1$ of the right hand side in (30). As demonstrated above, the first term converges to zero since $n > c-1$. The limit of the second term, instead, is

$$\begin{aligned} & \lim_{\beta\to 1} \tfrac{\beta-\alpha}{1-\alpha} \times \lim_{\beta\to 1} \tfrac{\beta^{c-i}}{n-i+1} \left[\tfrac{n-i+1}{n-i+1-\alpha} \times \tfrac{n-i}{n-(i+1)+1-\alpha} \times \cdots \times \tfrac{n-(n-1)+1}{n-(c-1)+1-\alpha} \times (n-c+1)\right] \\ = \quad & \lim_{\beta\to 1} \tfrac{\beta-\alpha}{1-\alpha} \times 1 = \lim_{\beta\to 1} \beta(1-\gamma) = 1-\gamma \end{aligned}$$

since the product of the limits

$$\tfrac{1}{n-i+1} \times \lim_{\beta \to 1} \tfrac{n-i+1}{n-i+1-\alpha} \times \cdots \times \lim_{\beta \to 1} \tfrac{n-(n-1)+1}{n-(c-1)+1-\alpha} \times (n-c+1)$$

telescopes to 1 (the denominator of each fraction is equal to the numerator of the following fraction). It follows that

$$\lim_{\beta \to 1} q_i(c,n) = 1 - \lim_{\beta \to 1} \tfrac{\beta-\alpha}{1-\alpha} = \gamma \text{ for all } i = 1, \ldots, c.$$

■

Proof of Lemma 11. Using Lemma 2 and expression (6), we have

$$\pi(c,n) = \tfrac{\alpha}{\beta} \textstyle\sum_{j=1}^{c} \beta^{j-1} q_j(c,n).$$

Consider (25) in the proof of Lemma 3. For $i = 1$ we have

$$\Phi_1(c,n) = \textstyle\sum_{j=1}^{c} \beta^{j-1} \left[1 - \tfrac{\alpha}{\beta} q_j\right].$$

Therefore, we obtain (13). Now, for $c = 1, 2, ..., n$, define the function

$$\varphi(c,n) = \prod_{m=1}^{c} \tfrac{n-m+1}{n-m+1-\alpha}.$$

Note that $\varphi(c,n)$ increases in c and falls in n since for all $n \geq 2$ and $c = 2, ..., n$ we have

$$\varphi(c,n) > \varphi(c-1,n) \quad \text{and} \quad \varphi(c,n) > \varphi(c,n+1). \tag{31}$$

The first inequality is proved by noticing that

$$\varphi(c,n) = \varphi(c-1,n) \tfrac{n-c+1}{n-c+1-\alpha} > \varphi(c-1,n).$$

The second inequality is obtained from observing that

$$\varphi(c,n) = \varphi(c,n+1) \tfrac{(n-c+1)(n+1-\alpha)}{(n-c+1-\alpha)(n+1)} > \varphi(c,n+1).$$

From (13) we have

$$\pi(c-1,n) = \textstyle\sum_{j=1}^{c-1} \beta^{j-1} - \Phi_1(c-1,n). \tag{32}$$

Using (10) with $i = 1$, we have

$$\begin{aligned}
\Phi_1(c-1,n) &= \tfrac{\beta-\alpha}{\beta}\sum_{j=1}^{c-1}\beta^{j-1}\prod_{m=1}^{j}\tfrac{n-m+1}{n-m+1-\alpha} \\
&= \tfrac{\beta-\alpha}{\beta}\sum_{j=1}^{c}\beta^{j-1}\prod_{m=1}^{j}\tfrac{n-m+1}{n-m+1-\alpha} - \tfrac{\beta-\alpha}{\beta}\beta^{c-1}\prod_{m=1}^{c}\tfrac{n-m+1}{n-m+1-\alpha} \\
&= \Phi_1(c,n) - \tfrac{\beta-\alpha}{\beta}\beta^{c-1}\varphi(c,n)\,.
\end{aligned}$$

In the last step we have used (10) and $\varphi(c,n)$ from (12). Inserting this into (32) gives

$$\begin{aligned}
\pi(c-1,n) &= \textstyle\sum_{j=1}^{c}\beta^{j-1} - \Phi_1(c,n) - \beta^{c-1} + \tfrac{\beta-\alpha}{\beta}\beta^{c-1}\varphi(c,n) \\
&= \pi(c,n) - \beta^{c-1}\left[1 - \tfrac{\beta-\alpha}{\beta}\varphi(c,n)\right]
\end{aligned}$$

In the second line we have used (13). This gives (14).

Recalling that $u(c,n) = u_1(c,n)$ is the payoff to a buyer, we use (8) to obtain (15).■

Proof of Lemma 12. From (14) and the definition of $\Delta(c,n)$ we obtain (17). Clearly, $\Delta(1,n) = \pi(1,n) > 0$ since $\pi(0,n) = 0$ and (18) is obvious.

To prove $\Delta(c,n)$ is strictly decreasing in c note that β^{c-1} falls in c. From (31) in the proof of Lemma 11 we have that $\varphi(c,n)$ grows with c, thus $[1 - \frac{\beta-\alpha}{\beta}\varphi(c,n)]$ falls in c. Thus $\Delta(c,n) > \Delta(c+1,n)$ for $c = 1,...,n-1$. We use (17)-(18) to prove that (11) describes the set of maxima.

- First line of (11). If $\frac{\beta}{\beta-\alpha} < \varphi(2,n)$, then $\Delta(2,n) < 0$ from (18). So, $\Delta(c,n) < \Delta(2,n) < 0$ for all $c > 2$, from (19). So, $c = 1$ is the unique maximizer of $\pi(c,n)$.

- Second line of (11). If $\frac{\beta}{\beta-\alpha} = \varphi(\widehat{c},n)$ for some $\widehat{c} = 2,...,n-1$, then $\Delta(\widehat{c},n) = 0$ from (18). So, (19) implies that $\Delta(c,n) > 0$ for all $c < \widehat{c}$ and $\Delta(c,n) < 0$ for all $c > \widehat{c}$. Since $\pi(\widehat{c},n) = \pi(\widehat{c}-1,n)$ then there are two maximizers, $\{\widehat{c}-1,\widehat{c}\}$.

- Third line of (11). If $\varphi(\widehat{c},n) < \frac{\beta}{\beta-\alpha} < \varphi(\widehat{c}+1,n)$, then $\Delta(\widehat{c}+1,n) < 0 < \Delta(\widehat{c},n)$, from (18). Again, (19) implies that $\Delta(c,n) > 0$ for all $c < \widehat{c}$ and $\Delta(c,n) < 0$ for all $c > \widehat{c}+1$. Therefore, $c = \widehat{c}$ is the unique maximizer of $\pi(c,n)$.

- Fourth line of (11). If $\frac{\beta}{\beta-\alpha} > \varphi(n,n)$, then $\Delta(c,n) > \Delta(n,n) > 0$ for all $1 \leq c < n$. Therefore, $c = n$ is the unique maximizer of $\pi(c,n)$.■

Proof of Lemma 13. The profit for an unconstrained seller is $\lim_{c\to\infty}\pi(c) = \gamma\lambda$. This is because $c\sum_{n=c+1}^{\infty} m_n < \sum_{n=c+1}^{\infty} m_n n = \lambda - \sum_{n=1}^{c} m_n n$ and $\lim_{c\to\infty}\sum_{n=c+1}^{\infty} m_n n = 0$. So $\sum_{n=c+1}^{\infty} m_n c$ and $\sum_{n=c+1}^{\infty} m_n n\gamma$ vanish as $c \to \infty$.

Now, we prove that if (21) holds, then we have equilibrium capacity constraints. Let (21) hold and, by means of contradiction, suppose the seller serves every buyer, for any n, i.e., $c = \infty$. From (20), this means that we must have $\sum_{n=c+1}^{\infty} m_n(c - n\gamma) \leq 0$ for all c. To derive a contradiction let $c = 1$, so $c\sum_{n=c+1}^{\infty} m_n = 1 - m_0 - m_1 = 1 - m_0(1+\lambda) > 0$ and $\gamma\sum_{n=c+1}^{\infty} m_n n = \gamma(\lambda - m_1) = \gamma\lambda(1 - m_0) > 0$, so that $\sum_{n=c+1}^{\infty} m_n(c - n\gamma) \leq 0$ is rearranged as $\gamma\lambda \geq \frac{1-m_0-m_1}{1-m_0}$, which gives the desired contradiction. There is a nonempty set of γ values that satisfies (21) for all $\lambda > 0$, because $\frac{1-m_0-m_1}{1-m_0} \in (0,1)$ for all $\lambda > 0$.

To prove that $c = 1$ is optimal for $\lambda \leq 1$ and $\gamma = 0$ it is sufficient to prove that $\pi(c+1) < \pi(c)$ for all $c = 1, 2, \ldots$, given $\lambda \leq 1$ and $\gamma = 0$. Fix $\gamma = 0$. So $\pi(c) = c\sum_{n=c+1} m_n$, hence $\pi(c+1) < \pi(c)$ corresponds to $\frac{1}{c}\sum_{n=c+2}\frac{m_n}{m_{c+1}} < 1$. Rearrange it as $\sum_{n=1}^{\infty}\frac{\lambda^n}{c\prod_{i=1}^{n}(c+1+i)} < 1$ and denote the LHS by A, which increases in λ and decreases in c. So for $\lambda \leq 1$ we have

$$A \leq \sum_{n=1}^{\infty}\frac{1}{c\prod_{i=1}^{n}(c+1+i)} \leq \sum_{n=1}^{\infty}\frac{1}{\prod_{i=1}^{n}(2+i)},$$

where in the first inequality we used $\lambda = 1$, and $c = 1$ in the second.

Now notice that because $e = \sum_{n=0}^{\infty}\frac{1}{n!}$ we can write

$$\sum_{n=1}^{\infty}\frac{1}{\prod_{i=1}^{n}(2+i)} = \frac{1}{3} + \frac{1}{4\cdot 3} + \ldots = 2\sum_{n=3}^{\infty}\frac{1}{n!} = 2\left[\sum_{n=0}^{\infty}\frac{1}{n!} - \left(\frac{1}{0!} + 1 + \frac{1}{2}\right)\right] = 2e - 5 \in (0,1).$$

Therefore, $A \leq 2e - 5 < 1$, which completes the proof.■

Proof of Lemma 14. For seller 1 we have $\pi(1,1) = \pi(2,1) = \frac{\alpha(1-\beta)}{\beta(1-\alpha)}$, $\pi(1,2) = \frac{\alpha(2-\beta)}{\beta(2-\alpha)}$ and $\pi(2,2) = \pi(2,1)\,\frac{\beta(3-\alpha)+2(1-\alpha)}{2-\alpha}$. Given (2), (22), and (23) we get

$$W(1,1) = \frac{\gamma(2\beta\gamma-5\beta+6)}{4(2-2\beta+\beta\gamma)},\ W(2,2) = \frac{\gamma[2\beta\gamma(1+\beta)+3(2+\beta)(1-\beta)]}{4(2-2\beta+\beta\gamma)},$$

$$W(1,2) = (1-\beta)\, W(2,1) < W(2,1) = \frac{\gamma\left(6-7\beta+2\beta\gamma+\beta^2\right)}{(2-\beta)^2(2-2\beta+\beta\gamma)}$$

The payoff to seller 2 is symmetric. So we have the two by two game

	$c_2 = 1$	$c_2 = 2$
$c_1 = 1$	$W(1,1), W(1,1)$	$W(1,2), W(2,1)$
$c_1 = 2$	$W(2,1), W(1,2)$	$W(2,2), W(2,2)$

We have $W(1,2) \leq W(2,2)$ for all β and γ. Hence, $c_1 = c_2 = 2$ is always a Nash equilibrium. Also, $W(1,1) > W(2,1)$ if and only if $\gamma < \bar{\gamma}$, where $\bar{\gamma} := \frac{22\beta-5\beta^2-16}{2\beta(4-\beta)}$ uniquely solves $W(1,1) = W(2,1)$. Thus if $\gamma < \bar{\gamma}$, then $c_1 = c_2 = 1$ is also an equilibrium, hence two pure strategy equilibria coexist, but sellers prefer the low-capacity one because $W(1,1) > W(2,2)$ when $\gamma < \bar{\gamma}$. Clearly, a mixed strategy equilibrium also exists; e.g., with $\beta \to 1$, if η is the equilibrium probability to play $c = 1$, then $\eta = \frac{\gamma}{\frac{1}{2}(\frac{1}{2}-\gamma)}$ solves $\eta W(1,1)+(1-\eta)W(1,2) = \eta W(2,1)+(1-\eta)W(2,2)$. Clearly, $\eta \in (0,1)$ when $\gamma < \bar{\gamma} = \frac{1}{6}$. $\blacksquare$

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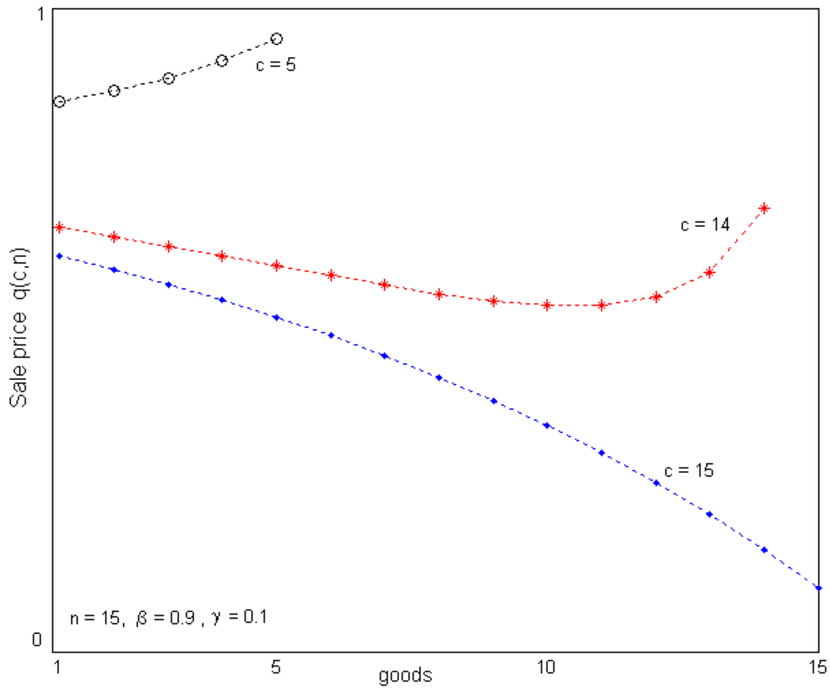

Figure 1 : Sale prices under different inventories

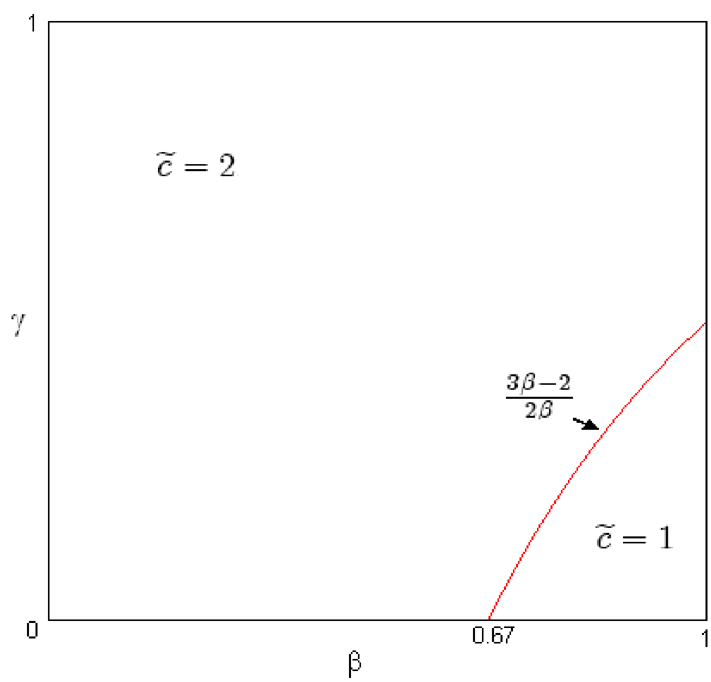

Figure 2 : Optimal capacity when n=2

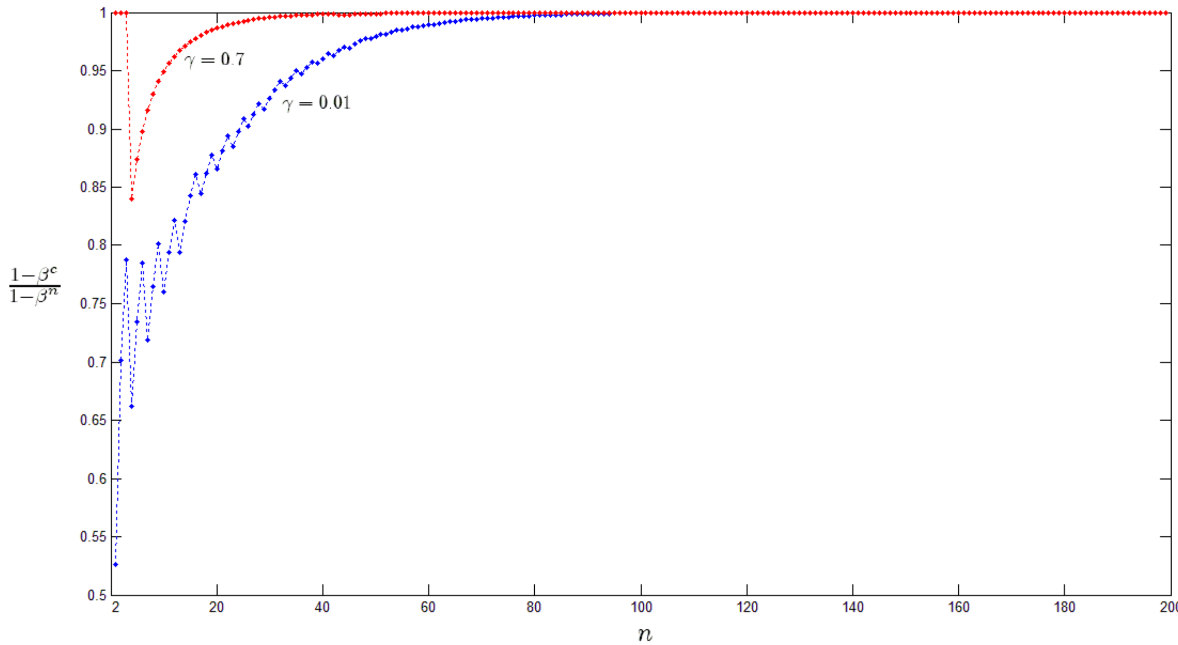

Figure 3: Trading efficiency