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Price Dispersion with Directed Search[1]

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Abstract

We present a model that generates empirically plausible price distributions in directed search equilibrium. There are many identical buyers and many identical capacity-constrained sellers who post prices. These prices can be renegotiated to some degree and the outcome depends on the number of buyers who want to purchase the good. In equilibrium all sellers post the same price, demand is randomly distributed, and there is sale price dispersion. Prices and distributions depend on market tightness and on the properties of renegotiation outcomes. In a labor market context, the model generates a strong empirical prediction. If workers can renegotiate the posted wage, then the model predicts a positively skewed and realistic-looking density function of realized wages when the mean number of job-seekers per vacancy is large.

Keywords: Advertising, Directed search, Price commitments, Frictions, Wage dispersion.

JEL: C780, D390, D490, E390

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1 Introduction

This paper studies the theoretical underpinnings of price dispersion in markets for a homogeneous item. Such a phenomenon is empirically well-documented not only in product markets (see Baye, Morgan and Scholten 2006) but also in labor markets where, as noted in Burdett and Mortensen (1998, p.257), "inter-industry and cross-employer wage differentials exist, are stable, and cannot be explained by observable differences in worker or job characteristics that might require compensation."

The theoretical literature has rationalized price dispersion using models that rely on ex-ante heterogeneity, informational asymmetries and, especially, exogenous trading frictions.[2] Our approach ties into the trading frictions literature, which typically assumes it is costly to get information about prices either due to a random process of sequential encounters or search with fixed sample size. Instead, frictions arise endogenously in our model, which follows a directed search approach. Identical sellers costlessly advertise a list price; identical buyers see all prices and then choose to costlessly meet and trade with one seller. The key to price dispersion is that sellers are capacity-constrained and cannot fully commit to the posted price, while buyers are mobility-constrained and cannot coordinate their search decisions. These constraints give rise to trading frictions since in equilibrium

[2] Examples include the information heterogeneity in Baye, Kovenock and De Vries (1992), Varian (1980); the belief heterogeneity in Rauh (2001); costly search as in Carlson and McAfee (1983), Daughety (1992); the firm heterogeneity in Reinganum (1979); random search that limits price information as in Burdett and Judd (1983), Butters (1977), or as in Burdett and Mortensen (1998) and Camera and Corbae (1999); the predetermined shopping order in Arbatskaya (2007).

buyers *choose* to visit sellers at random. Price dispersion emerges because renegotiated prices depend on the (random) number of buyers who want to purchase the good.

The proposed approach follows in the footsteps of Peters (1984) and recent works such as Albrecht, Gautier and Vroman (2006), Burdett, Shi, and Wright (2001), Dana (2001), Julien, Kennes, and King (2000), Michelacci and Suarez (2006), Montgomery (1991), Shimer (2005). Unlike the majority of these works, our model draws a distinction between list and sale price to account for the observation that transactions in some markets are settled at a price that differs from the list price, depending on demand conditions (e.g., think of real estate, or automobiles). This is accomplished by relaxing the price-commitment assumption of the ordinary directed search model. Typically, sellers who realize excess demand are simply assumed to charge the list price to a randomly chosen buyer; see Burdett, Shi, and Wright (2001). Instead, we let realized prices depend on the number of buyers who want to purchase. This renegotiation idea is present in Albrecht, Gautier, and Vroman (2006) and Julien, Kennes, and King (2000), also. The difference between these studies and ours lies in the equilibrium price distributions, as we next explain.

A prominent feature in Albrecht, Gautier, and Vroman (2006) and Julien, Kennes, and King (2000) is the prediction of a two-point distribution of sale prices.[3] The reason

[3] These models consider labor markets, so one can think of sellers as job-seekers, buyers as vacancies, and prices as wages. We emphasize that the empirical relevance of the paper does not depend on this interpretation and the results hold when the players' roles are reversed. Some related works are the random matching model in Masters and Muthoo (2008), with renegotiation based on match-specific heterogeneity; the directed search model in Coles and Eeckhout (2003), where sellers do not renegotiate but compete

is that if a capacity-constrained seller meets more than one buyer, then these buyers compete by bidding the posted price up to their reservation value, as in an auction. Hence, sellers trade at the high price if they meet two or more buyers, or else sell at the low price. The high price is thus insensitive to realized demand for values above two. It is invariant to market composition, also. Our study offers a simple method to generate more comprehensive, analytically tractable, equilibrium distributions of sale prices. All we need is that renegotiated prices are an increasing function of demand.

What do we find? First, there is a unique equilibrium posted price. As in the ordinary model, it rises in expected demand. It differs from the ordinary model depending on how renegotiation redistributes surplus, on average. If renegotiation favors buyers, then sellers post a price above the full-commitment price (conversely, below). Second, there is a unique equilibrium distribution of sale prices parameterized by expected demand. Sale prices respond naturally to local and aggregate demand conditions. In particular, mean sale prices rise and price dispersion tends to fall in expected demand. Third, we extend the basic model to endogenize (instead of assuming) renegotiation outcomes as well as incidence of renegotiation. The analysis identifies market conditions under which fixed-price trading emerges in equilibrium, even if list prices are non-binding.

The above findings lead to empirically relevant predictions on market behavior. If

in direct mechanisms ex-ante, i.e., they post two prices contingent on realized demand (one, or above one); a two-point price distribution emerges also in Daughety (1992), due to costly search and incomplete information on production costs (see also Daughety and Reinganum 1991). Wage dispersion in directed search occurs with heterogeneous firms in Menzio (2005) and in Montgomery (1991), and with random application opportunities in Delacroix and Shi (2006).

we interpret our model as one of identical workers and firms, then perhaps the strongest prediction concerns wages paid across firms. The proposed theory is consistent with the empirical observations of cross-employer wage differentials for (apparently) identical workers. In particular, the model yields a theoretical density function of realized wages that displays positive skewness whenever the mean number of job-seekers per vacancy is large and workers can renegotiate the posted wage. Such a distribution is realistic-looking and yet it does not require additional assumptions on heterogeneity that, in fact, are necessary in the benchmark wage dispersion model in Burdett and Mortensen (1998) (see Mortensen 2003, Chapter 3).[4]

The paper proceeds as follows. Section 2 introduces the basic framework. Section 3 discusses the game played by buyers and sellers. Section 4 studies symmetric equilibrium in the extreme cases of no commitment and full commitment to the list price. Sections 5 and 6 analyze economies with limited commitment. Section 7 concludes.

2 The framework

The economy is composed of a large number of agents of two types. There are $S \in \mathbb{N}$ capacity-constrained sellers, each of whom is endowed with one identical indivisible good from which they derive no utility. The rest are $B \in \mathbb{N}$ identical buyers who have no endowment and derive unit utility from consumption of the sellers' good. Let $\lambda > 0$ denote the ratio of buyers to sellers, which we will call "market tightness." Agents are

[4] Indeed, a referee notes that one way to employ this model—and an interesting problem to pursue—is to use observed distributions of realized wages to back out a possible reduced form relationship between job applications and advertised wages (neither of which are directly observable from realized wage data).

risk neutral and since utility is transferable there are gains from trade.

Agents play a strategic game of complete information composed of three stages. In the first stage, sellers simultaneously, independently and costlessly advertise a single price $r \in [0,1]$, the "list" or "posted" price. In the second stage, buyers costlessly observe prices, and simultaneously and independently choose to meet one seller. In the third stage, matches are realized and a trade process takes place. It is assumed that, after reaching a seller, buyers cannot search again. So, buyers can meet only one seller, but a seller can meet $n = 0, 1, 2, \ldots, B$ buyers. We assume that all players in a match observe n and, if $n \geq 2$, then each of these buyers has an equal chance of being served by the seller.[5] At the end of the trade process players realize their gains. Then, the game ends.

The players' gains are as follows. Those who do not trade earn zero. If a seller and a buyer agree to trade at price $p \in [0,1]$, then the seller realizes utility p and the buyer $1-p$. We call p the sale price, and here is where our model differs from the typical directed search model. The typical model assumes full commitment to the posted price r, so in the third stage $p = r$. Instead, we relax the assumption of full commitment and allow for renegotiation to occur in the third stage. A seller and a buyer may now agree to trade at a price that differs from what had been previously posted.

[5] Buyers' mobility constraints and seller's capacity constraints are common assumptions in the directed search literature, and are crucial to generate equilibrium search frictions (e.g., see Burdett, Shi and Wright 2001, Peters 1984, Shimer 2005).

3 The three stages of the game

We study subgame perfect equilibria, focusing the analysis as follows. First, we are interested in (strongly) symmetric outcomes in which all sellers post the same price and all buyers direct their search randomly and identically across all sellers. Here, buyers are indifferent to where they shop and all sellers face the same distribution of demand. Second, we specialize the analysis to the limiting case of "large markets," which means that we fix $B = \lambda S$ and let $S \to \infty$. This focus is standard in the literature (e.g., see Albrecht, Gautier, and Vroman 2006, Burdett, Shi, and Wright 2001, Shimer 2005). Symmetry is used to capture a notion of anonymity in market interactions and to rule out the implicit coordination associated to asymmetric strategies.[6] The focus on large markets, instead, enhances analytical tractability because the equilibrium distribution of demand is i.i.d. across sellers.

We will move backwards in the analysis, first discussing the sale price in a trade meeting. Then, we study buyers' optimal search behavior, and sellers' optimal advertisements.

3.1 The sale price

Consider a match in the third stage of the game between a seller who advertised the price $r \in [0, 1]$ and $n \in \mathbb{N} = \{1, 2, ...\}$ buyers. We incorporate the possibility of ex-post renegotiation by assuming that the sale price—the price at which trade takes place—might depend on realized demand and posted price, and we denote it $p_n(r)$. This means that

[6] The directed search literature has focused on strongly symmetric outcomes (play is symmetric on and off the equilibrium path) in which sellers play pure strategies. In Galenianos and Kircher (2008) sellers can mix.

$p_n(r)$ is the seller's utility from trade and $1 - p_n(r)$ is the buyer's.

The formulation accommodates various commitment possibilities. In the ordinary case of *full commitment* sale and posted price coincide, so $p_n(r) = r$ for all (r, n) and buyers' search is affected by posted prices. The opposite is *no commitment*, when sale prices are independent of posted prices, though they may depend on realized demand, i.e., $p_n(r) = p_n$ for all (r, n) and r does not affect search. Between these extremes lies *limited commitment*, when it is possible to renegotiate posted prices, at least to some extent. We remain agnostic on the renegotiation mechanism, and for generality we simply define the sale price as a function of the posted price and of the number of buyers.

Assumption 1 *Given limited commitment, sale prices are determined by a function*

$$p : \mathbb{N} \times [0, 1] \to [0, 1],$$

which is continuous and differentiable in r for all $n \in \mathbb{N}$, increasing in each argument, and strictly positive for all $r > 0$.[7]

The central feature of the limited commitment model is that trade can occur at prices that increase in the number of buyers who demand the good. The assumptions on the sale price function have three implications for our analysis of symmetric equilibrium. First, posted prices affect sale prices to some degree and so are meaningful to buyers. A seller can thus encourage buyers' visits by posting a price below the market's. Second, the expected trading price rises in the posted price and so sellers will not list prices at random.[8] Third, unlike the ordinary directed search model, the equilibrium displays sale price dispersion.

[7]Increasing means for $r \in [0, 1)$ we have $p_{n+1}(r) > p_n(r)$ and $\partial p_n(r)/\partial r > 0$ for some n (possibly all).

[8]If sellers mix, then buyers observe the outcome of the mixture (not the mixture itself) and choose

Assuming properties of renegotiated prices without specifying mechanisms, simplifies the analysis of directed search equilibrium with limited commitment. But what trading mechanisms have outcomes compatible with Assumption 1? A possibility is a process of alternating offers, spanning an indefinite number of rounds within a period, between a seller and a buyer selected at random among n. Assume that n is constant across rounds, the seller makes the initial offer, and there is a random stopping rule. Here, there are no agreement delays, the subgame perfect price is unique and rises in n. Indeed, buyers are randomly selected in each round, so their reserve offer rises in the number of competing buyers. Appendix B works out such bargaining solution. The bidding process considered in Albrecht, Gautier, and Vroman (2006), Julien, Kennes, and King (2000) also fits our formulation since auctions lead to $p_1(r) = r$ and $p_n(r) = 1$ if $n \geq 2$. Renegotiation, or its outcome, could also be random, i.e., p could be an expected price (players are risk-neutral). Studying all possible transaction mechanisms, however, is not what we are after. So, we move on to analyze the problem of buyers and sellers.

3.2 The buyer's problem

Consider the second stage of the game, when buyers observe posted prices and then choose to visit one seller. They take the renegotiation outcome function p as given. Since buyers may mix, their payoff corresponds to the expected utility from visiting the various sellers.

Recall that we are interested in symmetric equilibrium. So, buyers must be indifferent to where they shop, and direct their search independently and identically across all sellers.

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where to go. But then, buyers would not direct their search identically across all sellers because p is increasing in r. This is inconsistent with symmetry.

In a finite market this means that every buyer visits any seller with identical probability $1/S$. Hence, the equilibrium distribution of demand faced by every seller is binomial with parameters given by the number of buyers B and the probability $1/S$. In the limiting case of a large market, instead, demand is i.i.d. across sellers as a Poisson with parameter λ. These are well-known results (e.g., see Burdett, Shi, and Wright 2001). So we let

$$m_n(\lambda) = \tfrac{e^{-\lambda}\lambda^n}{n!}, \quad \text{for } n = 0, 1, \ldots. \tag{1}$$

denote the equilibrium probability that a seller meets exactly n buyers, given λ. This is the basic building block of the model.

Two implications immediately follow. First, if a symmetric equilibrium exists, then all sellers face the same distribution of demand; its first two moments coincide with the market tightness parameter λ. In the full commitment case λ is key to pin down the posted price. Without full commitment λ is also key to determine the equilibrium distribution of sale prices.

Second, a buyer's expected utility from meeting any seller in equilibrium is a function of expected demand λ and the posted price r. We denote it $U(\lambda, r)$, with

$$U(\lambda, r) = \textstyle\sum_{n=0}^{\infty} m_n(\lambda) \times \frac{1-p_{n+1}(r)}{n+1}. \tag{2}$$

To understand (2), recall that search decisions are uncoordinated. So, a buyer who reaches a seller finds $n = 0, 1, \ldots$ competing buyers with probability $m_n(\lambda)$. The buyer trades with probability $1/(n+1)$, since it is assumed that all customers have an equal chance of being served by the seller. The payoff for the buyer who trades is $1 - p_{n+1}(r)$.

To study how posted prices affect buyers' search, suppose that a seller deviates by posting r' instead of r. If this affects the deviator's possible sale prices, then the deviation will affects search strategies. To see how, recall that in a (strongly) symmetric outcome buyers out of equilibrium must behave identically and must be indifferent across sellers. So, the out-of-equilibrium distribution of demand at the deviant seller is a Poisson(λ'), where expected demand $\lambda' \in \mathbb{R}_+$ depends on r' via the buyers' indifference condition

$$\sum_{n=0}^{\infty} m_n(\lambda') \times \frac{1-p_{n+1}(r')}{n+1} - U(\lambda, r) = 0. \tag{3}$$

Equation (3) tells us that if the posted price affects sale prices, then the deviator's expected demand λ' must adjust to keep buyers indifferent. A larger λ' shifts the probability mass from low to high demand realizations; see (1). So, as in the ordinary directed search model, the deviation affects *trade risk*. A customer is less likely to be served by sellers who expect greater demand. Trade risk hinges on capacity constraints, a standard friction in the ordinary directed search model. Sellers cannot satisfy excess demand because they can neither increase production, nor can they acquire and resell the idle inventory of others. This means that buyers can be indifferent to visiting a deviator who sets $r' > r$, as long as they expect fewer competitors, $\lambda' < \lambda$.

With limited commitment, however, the deviation also affects *price risk* because the sale price generally depends on demand. A customer who visits sellers with higher expected demand is more likely to pay more than the posted price. The impact of price risk hinges on the renegotiation process, i.e., on the characteristics of the outcome function p.

Notice from (3), that the defection r' does not influence trade or price risk at non-deviators. Indeed, U is simply a function of r and λ. The reason is that in a large market changes in the probability to visit one seller cannot affect the distribution of demand at other sellers because the covariance of demand is zero (e.g., see Burdett, Shi, and Wright 2001, Shimer 2005).[9] Consequently, the buyers' payoff U from visiting any non-deviant seller is unaffected by λ' and r'. This is a key consideration for the seller's optimal choice.

3.3 The seller's problem and the definition of equilibrium

Consider the first stage of the game, when list prices are posted simultaneously and independently by all sellers. Given our focus on symmetric outcomes, suppose that a seller considers a deviation $r' \in [0,1]$ from equilibrium play, when all other sellers play r. The deviator takes as given the sale price function p, the market parameter λ, and the fact that all buyers behave identically.

The deviator's payoff is the expected profit function $W : \mathbb{R}_+ \times [0,1] \to [0,1]$ defined by[10]

$$W(\lambda', r') = \textstyle\sum_{n=1}^{\infty} m_n(\lambda') p_n(r'). \tag{4}$$

If $n \geq 1$ customers arrive, then the deviator earns $p_n(r')$. Otherwise, he earns zero. The probability of meeting n buyers is $m_n(\lambda')$, where the expected demand λ' is pinned down by the indifference constraint (3). So, we define the deviator's problem as follows.

The deviator's objective is to maximize W subject to the constraint (3), i.e., buyers

[9] This is not the case in a finite market, since the distribution of demand in the market is multinominal, and demand covaries negatively across sellers. So, if buyers raise the probability of visiting the deviant seller, then expected demand at all other sellers must fall.

[10] The function W is mapped into $[0,1]$ since $\sum_{n=0}^{\infty} m_n(\lambda') = 1$ for all λ' and $p_n(r') \in [0,1]$ for each r'.

must be indifferent to visit non-deviators. Since

$$\frac{m_n(x)}{n+1} = \frac{1}{x}\frac{e^{-x}x^{n+1}}{n+1!} = \frac{m_{n+1}(x)}{x},$$

we have

$$\textstyle\sum_{n=0}^{\infty} \frac{m_n(\lambda')[1-p_{n+1}(r')]}{n+1} = \frac{1}{\lambda'}\sum_{n=1}^{\infty} m_n(\lambda')[1-p_n(r')]. \tag{5}$$

Fixing r and λ, the deviator's problem is to choose $\left(\lambda', r'\right) \in \mathbb{R}_+ \times [0,1]$ to solve

$$\begin{aligned} &\text{Maximize:} \quad \textstyle\sum_{n=1}^{\infty} m_n(\lambda')p_n(r') \\ &\text{Subject to:} \quad \textstyle\sum_{n=1}^{\infty} m_n(\lambda')\left[1-p_n(r')\right] - \lambda' U(\lambda, r) = 0. \end{aligned} \tag{6}$$

This is akin to a monopolist's problem, who chooses either price or quantity, given that demand is a function of the price. A demand-price relationship arises in our model because in order to get some visits the deviant seller must offer buyers expected utility $U(\lambda, r)$; this is the payoff that buyers expect to get anywhere else. So, if the deviator offers different sale prices, then demand for the deviator's good must adjust. Of course, in symmetric equilibrium $r' = r$ and so $\lambda' = \lambda$.

Definition 1 *Given a sale price function p, a strongly symmetric subgame perfect equilibrium, an **equilibrium** for short, is an identical visiting strategy for all buyers, and an identical posted price r for all sellers, which support the demand distribution (1) at each seller, satisfy buyer's indifference (3) and profit maximization (6).*

To recapitulate, in equilibrium the posted price r might be renegotiated once buyers meet sellers. The outcome of the renegotiation process is given by the sale price function p. In the second stage, buyers must be indifferent to where they go, given the strategies of all others and the possible sale prices. In the first stage, sellers choose r optimally given p and given that everyone acts optimally in each stage.

4 Equilibrium in two extreme cases

For expositional reasons, we first touch upon the full- and no-commitment cases. Full commitment is the standard assumption in the ordinary directed search model and so we consider it for comparison purposes. Studying no-commitment, instead, helps us fix ideas on the connection between equilibrium distribution of demand and of sale prices.

4.1 The ordinary directed search model (full-commitment)

Assume sellers can fully commit to the posted price. The model is equivalent to that in Burdett, Shi, and Wright (2001). We exploit results in that paper to conclude that an equilibrium exists and it is characterized by a unique list price $r = \hat{r}$ defined by

$$\hat{r} = 1 - \tfrac{m_1(\lambda)}{1-m_0(\lambda)}. \tag{7}$$

Given (1), we have $\hat{r} \in (0,1)$, $\lim_{\lambda\downarrow 0} \hat{r} = 0$, $\lim_{\lambda\uparrow\infty} \hat{r} = 1$, and $d\hat{r}/d\lambda \propto 1-e^{\lambda}(1-\lambda) > 0$. Intuitively, $r = 0$ is suboptimal for any seller, as it implies zero profit. If $r = 1$, then a seller can capture the entire market by posting r' slightly below r. So, $\hat{r}$ is interior. The equilibrium posted price monotonically rises in λ since sellers more easily meet buyers. This extensive margin effect lowers the sellers' desire to compete by advertising low prices.

4.2 The distribution of sale prices under no commitment

Assume no commitment. Sale prices are independent of posted prices, though they may depend on the number of customers: $p_n(r) = p_n$ for all (r, n). Here, posted prices cannot impact search behavior, so advertisements are indeterminate, i.e., any $r \in [0,1]$ is an equilibrium. Indeed, W is independent of r' and we have $\lambda' = \lambda$ from (6).

It is immediate that, given some sale price function $p : \mathbb{N} \to [0,1]$, the distribution of demand pins down that of sale prices. If a trade takes place, then the probability of observing the price p_n is simply the probability that, conditional on a meeting, there are $n = 1, 2, \ldots$ buyers.

The equilibrium probability that a seller meets n buyers satisfies (1), so the probability of selling is $1 - m_0(\lambda)$. Therefore in equilibrium we define

$$\Pr[p_n] = \tfrac{m_n(\lambda)}{1-m_0(\lambda)} = \tfrac{\lambda^n}{n!(e^\lambda - 1)}, \quad \text{for } n = 1, 2, \ldots \tag{8}$$

as the probability that a seller trades at price p_n. For any price $x \in [0,1]$ we have

$$\Pr[x] = \begin{cases} \Pr[p_n] & \text{if } x = p_n,\ n = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

We let $\bar{p}$ denote the expected sale price, defined by

$$\bar{p} = \tfrac{\sum_{n=1}^{\infty} m_n(\lambda) p_n}{1-m_0(\lambda)}.$$

To characterize the distribution of sale prices we need more structure on p and so we will assume $\{p_n\}_{n=1}^{\infty}$ is increasing. This means that sellers who meet more buyers end up charging greater prices. We are especially interested in assessing the impact of λ on the mean and the dispersion of sale prices.

Lemma 2 *If $\{p_n\}_{n=1}^{\infty}$ is increasing, then $\bar{p} \in (0,1)$ and $d\bar{p}/d\lambda > 0$.*

(All proofs are in Appendix A.) Average sale prices monotonically rise in λ because sellers are more likely to meet many buyers, hence they are more likely to trade at high prices. If we measure price dispersion by the coefficient of variation, then it monotonically declines in λ for λ sufficiently large, generally. To see why, note that the variance is hump shaped; most trades occur at low prices if λ is small, and at high prices if λ is large.[11] As a consequence, the coefficient of variation can be hump-shaped, depending on $\bar{p}$. Both of these findings will carry through the case of limited commitment, in the next section.

Numerical illustration: price distributions. Suppose sellers are job-seekers and buyers are vacancies, so p_n is the wage to a job seeker who has met n vacancies. We wish to interpret no commitment as wage negotiation that is unavoidable ex-post, perhaps because commitment is very costly (e.g., see Bester 1994). For illustrative purposes we adopt the strategic bargaining game of Appendix B, for $\gamma = 0$ and $\beta = 0.99$. This means that only buyers can make counteroffers and there is almost no risk of breakdowns in bargaining. Hence, $p_n = q_n \in [0.01, 1]$ and $\{p_n\}$ is strictly increasing. This is a world in which workers can bargain a higher wage by playing vacancies against each other.

Figure 1 approximately here

The circles in the four panels of Figure 1 trace the distribution of sale prices under no commitment and $\lambda = 0.5, 2, 4, 8$. The coefficient of variation in each case is identified by c.v.$^{\mathrm{O}}$.[12] Starting with the top left panel, and moving by row, the average number of workers per vacancy increases from half all the way to eight. The key observation

[11] We have $dm_n/d\lambda = (n - \lambda)m_n/\lambda$, which is negative for $n < \lambda$.

[12] The + identifies a benchmark parameterization of the limited commitment model. See next section.

is: unless labor demand outstrips significantly labor supply, then the model predicts a positively skewed density of realized wages, and quite a bit of dispersion. Instead, in a tight labor market (λ is large) realized wages are high and concentrated. This holds true if we reverse the role of buyers and sellers, i.e., if $1-p$ is the wage and the labor market is tight (λ is small).

5 Equilibrium with limited commitment

We now introduce limited commitment, retaining Assumption 1. Our first task is to determine the equilibrium posted price. To do so start by noticing that a version of the law of demand holds in our framework.

Lemma 3 *Given λ, r, and Assumption 1, buyers' indifference (3) defines λ' as an implicit and decreasing function of r'.*

The message is that a seller who deviates faces a trade off between revenue and expected demand, much as in the typical directed search model of price commitments. To see why, suppose a deviator posts r' slightly above the "market" price r. Given Assumption 1, this implies higher expected sale prices (intensive margin). Hence, buyers reduce their visits, and λ' falls below λ (extensive margin). This is as in the ordinary directed search model. With limited commitment, however, demand influences sale prices, also. So, buyers' indifference hinges on the fact that the deviator's customers face less trade risk and less price risk; they are more likely to be served and less likely to pay more than the posted price.

This suggests we can reformulate the seller's problem (6) focusing either on the intensive or the extensive margin choice. Given Lemma 3, we can choose r' and solve

$$\max_{r' \in [0,1]} \sum_{n=1}^{\infty} m_n(\lambda'(r')) p_n(r'),$$

where $\lambda'(r')$ satisfies the indifference constraint in (6).

A simpler alternative is to use the constraint to eliminate r' from W, and then choose λ'; see also Burdett, Shi, and Wright (2001, p.1074) or Montgomery (1991, p.172). This approach is simpler since W becomes an explicit function of λ'. To see this, fix (λ, r) and rewrite the constraint in (6) as

$$\sum_{n=1}^{\infty} m_n(\lambda') p_n(r') = \sum_{n=1}^{\infty} m_n(\lambda') - \lambda' U(\lambda, r).$$

The left hand side is the deviator's payoff. Since $\sum_{n=1}^{\infty} m_n(\lambda') = 1 - m_0(\lambda')$, the deviator's problem becomes

$$\max_{\lambda' \in \mathbb{R}_+} \left[1 - m_0(\lambda') - \lambda' U(\lambda, r)\right]. \tag{10}$$

We can interpret $1 - m_0(\lambda')$ as an expected revenue. It is the value created by a sale (one), multiplied by the probability of trading. Given (λ, r), we interpret $\lambda' U$ as an expected cost. The seller must promise U payoff to each buyer, and since n buyers arrive with probability $m_n(\lambda')$ we have $\sum_{n=1}^{\infty} m_n(\lambda') n U = \lambda' U$. Indeed, $\sum_{n=1}^{\infty} m_n(\lambda') n = \lambda'$ by definition, and U is independent of λ' (due to large markets).

The function in (10) is strictly concave in λ'. Indeed $dm_0(\lambda')/d\lambda' = -m_0(\lambda')$. So,

$$m_0(\lambda') - U(\lambda, r) = 0 \tag{11}$$

is the first order necessary and sufficient condition for a maximum. Given λ and r, there is a unique maximizer $\lambda' > 0$ that equates expected marginal revenue to cost. Higher expected demand λ' lowers the probability of having no customers, so it rises expected revenue by $m_0(\lambda')$. The expected cost rises by U, i.e., by the compensation promised to each additional customer.

The central lesson is that if markets are large, then directed search with renegotiation is analytically similar to the ordinary model with price commitments. Indeed, (11) corresponds to the first order condition in Burdett, Shi, and Wright (2001) for large markets. The reason is that players are risk-neutral. So, given that buyers are indifferent, the profit-maximizing choice λ' must be independent of the type of commitment. Of course, in symmetric equilibrium we must have $\lambda' = \lambda$ so the possibility of renegotiation will affect the equilibrium posted price relative to the ordinary model.

Theorem 4 *Given Assumption 1, there exists an equilibrium with $r = r^* \in [0, 1]$. If*

$$\textstyle\sum_{n=1}^{\infty} m_n(\lambda) p_n(0) < 1 - m_0(\lambda) - m_1(\lambda) < \sum_{n=1}^{\infty} m_n p_n(1), \tag{12}$$

then r^* is unique and interior. If the left inequality is violated, then $r^* = 0$, and if the right inequality is violated, then $r^* = 1$.

Corollary 5 *If $p_n(r) = r$ for all $n \in \mathbb{N}$, then $r^* = \hat{r}$. If $p_1(r) = r$ and $p_n(r) = 1$ for all $n = 2, 3, \ldots$, then $r^* = 0$.*

The posted price is interior—as in the ordinary model of price commitment—when (12) holds. To understand the meaning of this inequality observe that $m_1(\lambda) = \lambda m_0(\lambda)$. Hence, (10)-(11) imply that in an interior equilibrium the seller's payoff is $1 - m_0(\lambda) - m_1(\lambda)$. That is also the payoff from the following pricing plan. If only one buyer arrives, then the good is sold at the seller's reserve value, i.e., $p_1 = 0$ (monopsony price). But if demand exceeds the seller's capacity, then the price jumps up to the buyer's reserve value, i.e., $p_n = 1$ for all $n \geq 2$ (competitive price). This is equivalent to using auctions; see Albrecht, Gautier, and Vroman (2006). So, (12) roughly says that—given all sellers act identically in the model with renegotiation—posting the monopsony price generates a payoff worse than auctions; the converse holds if sellers post the competitive price.

When $r^* \in (0, 1)$ there is revenue equivalence between renegotiation, price commitment, and auctions. To see why, rewrite the first order condition (11) as

$$\textstyle\sum_{n=1}^{\infty} m_n(\lambda) p_n(r) = 1 - m_0(\lambda) - m_1(\lambda). \tag{13}$$

The seller's equilibrium payoff (on the left) is the probability of meeting two or more buyers (on the right). Since utility is linear in prices, this payoff is attainable with different pricing mechanisms. One such instance is the ordinary model with price commitments where $p_n = \hat{r}$ for all n, defined in (7). Another instance is auctions. Of course, there are many other functions p that satisfy (13) for some $r \in [0, 1]$. Indeed, we can think of the outcome p as a lottery parameterized by r; if n customers arrive, then the seller charges one with probability $p_n(r)$ and zero otherwise. For instance, auctions are equivalent to having sellers charge one with probability zero if one buyer arrives, and otherwise charging one with certainty.

Unlike the ordinary model, renegotiation opens the door to corner solutions, $r^* = 0, 1$. To understand why, notice that—depending on the renegotiated price function p and the market parameter λ—there may be no posted price $r \in [0, 1]$ that satisfies the first order condition (13). Recalling that $\sum_{n=1}^{\infty} m_n(\lambda)p_n(r)$ increases in r, we have the corners $r^* = 0, 1$ when the left and right inequalities in (12) are violated, respectively.[13]

To sum up, if (12) holds, then renegotiation does not affect (ex-ante) payoffs relative to the ordinary model. In this case equilibria with and without price commitments are payoff equivalent. Intuitively, players are risk neutral so they only care about average sale prices. Of course, renegotiation does shift surplus between buyer and seller ex-post, which is what gives rise to distributions of sale prices. Instead, if (12) does not hold, then renegotiation does affect payoffs relative to the ordinary model. Whether it is the buyers or the sellers who benefit depends on market parameters and structure of renegotiation.

6 Characterization of limited commitment equilibrium

This section characterizes analytically the limited-commitment equilibrium list price and the distribution of sale prices in terms of the model's parameters. It complements the analysis with numerical simulations of economies that differ in λ and price commitments.

[13] The intuition is: the LHS of (12) is likely to be violated when λ is small, since as $\lambda \to 0$ then $m_0 \to 1$. This is an economy with very few buyers per seller so the (negative) *intensive margin* effect of lowering the posted price (lower expected sale price) is always dominated by a very strong (and positive) *extensive margin* effect (getting more customers), for any $r^* > 0$. Therefore, we have the corner $r^* = 0$. Conversely, the RHS of (12) is likely to be violated when λ is very large, since as $\lambda \to \infty$ then $m_0, m_1 \to 0$. In this case the negative *intensive margin* effect from lowering the posted price always dominates a very weak *extensive margin* effect, for any $r^* < 1$. Therefore, we have the opposite corner $r^* = 1$.

6.1 List prices

A first finding is that equilibrium posted prices tend to behave as in the ordinary model seen in Section 4.1. In particular, if r^* is interior, then it rises monotonically in λ.

Proposition 6 *If $r^* \in (0,1)$, then $dr^*/d\lambda > 0$. Also, $\lim_{\lambda \downarrow 0} r^* = 0$ and $\lim_{\lambda \uparrow \infty} r^* = 1$.*

The intuition hinges on the "law of demand" of Lemma 3. As λ rises the equilibrium posted price rises because the risk of not trading falls. Indeed, indifferent buyers visit sellers at random, so sellers reduce trading risk by attempting to attract buyers with a low price. Of course, in equilibrium every seller behaves identically and so the posted price is positively associated to λ. This is as in the ordinary model with price commitment. With renegotiation, however, the expected sale price increases in the number of visits, also. This provides an added incentive to advertise a low price when demand is scarce, and a high price if demand is considerable.

This last feature emerges clearly if we compare the equilibrium posted price in the models with and without renegotiation. To do so, the sale price function p needs more structure, in addition to Assumption 1. For the sake of simplicity, consider two opposite cases, $p_n(r) \geq r$ and $p_n(r) \leq r$ for $n = 1, 2, \ldots$. Call them *price floors* and *price ceilings*, respectively.[14]

Proposition 7 *Under price floors we have $r^* < \hat{r}$, while $r^* > \hat{r}$ under price ceilings.*

[14] These are of some practical relevance. Automobile or housing markets in the U.S. tend to display price ceilings; buyers either pay the advertised price r or a lower price p. Price floors emerge when firms compete for workers by bidding the wage up above what they originally advertised.

To fix ideas, let's say we compare an economy where it is prohibitively costly to renegotiate the posted price, to one in which buyers can negotiate discounts. The second economy will display a higher posted price. By doing so, sellers offset the expected ex-post shortfall in profit, while keeping risk-neutral buyers indifferent. Of course, one can think of renegotiation schemes that generate trades both below and above the posted price; for example, p can be a random draw. The lesson of Proposition 7 remains the same. The equilibrium posted price will differ from the one in the full commitment model depending on how surplus is expected to shift ex-post. If buyers expect to surrender *additional* surplus, then r^* lies below $\hat{r}$. Conversely, it will lay above.

Analytical example. Consider economies characterized by sale price functions linear in r:

$$p_n(r) = g_n + rh_n, \qquad \text{for } n = 1, 2, \ldots \tag{14}$$

This specification is interesting for two reasons. It encompasses the ordinary model and the model with auctions. It also allows us to easily recover r^* as an explicit function of the model's parameters. If an interior equilibrium exists, then (13)-(14) imply

$$r^* = \frac{1-m_0-m_1-\sum_{n=1}^{\infty} m_n g_n}{\sum_{n=1}^{\infty} m_n h_n}.$$

To fix ideas, assume $\{g_n\}_{n=1}^{\infty}$ increasing, bounded by zero and one, and let $h_n = 1-g_n$. Interior as well as corner solutions r^* may exist depending on $\{g_n\}$. Obviously, the ordinary model has $g_n = 0$ for all n, in which case $r^* = \hat{r}$. We model renegotiation assuming $g_1 = 0 \leq g_n \leq 1$ for all $n > 1$ with strict inequalities for at least some n. Clearly, Assumption 1 holds. Also, (12) holds. To see it, let $G(\lambda) := \sum_{n=1}^{\infty} m_n g_n$. Since

$g_1 = 0 < g_n < 1$ for some n we have

$$G(\lambda) = \textstyle\sum_{n=1}^{\infty} m_n g_n = \sum_{n=2}^{\infty} m_n g_n < 1 - m_0 - m_1.$$

So, $G(\lambda) = \sum_{n=1}^{\infty} m_n p_n(0) < 1 - m_0 - m_1 < \sum_{n=1}^{\infty} m_n p_n(1) = \sum_{n=1}^{\infty} m_n$. We have

$$r^* = 1 - \tfrac{m_1}{1-m_0-G(\lambda)} < \hat{r},$$

which is positive since $G(\lambda) < 1 - m_0 - m_1$, and less than $\hat{r}$ since $G(\lambda) > 0$.

Alternatively, fix $g_1 = 0 < g_n = 1$ for all $n \geq 2$. This corresponds to auctions as in Albrecht, Gautier, and Vroman (2006), Julien, Kennes, and King (2000). Indeed, $G(\lambda) = 1 - m_0 - m_1$ so $r^* = 0$. Of course, $r^* = 0$ for all $g_1 > 0$, since the left hand side of (12) is violated.

Numerical illustration: posted prices Figure 2 reports posted prices for $\lambda \in (0, 12]$ and different commitments. The horizontal axis is the posted price under auctions and the dashed line is $\hat{r}$ (full commitment). The solid line is r^* for a "benchmark parameterization" of the renegotiations model; it assumes p as in (14), with $1 - h_n = g_n = q_n$ with q_n as in Section 4.2. The posted price r^* is always interior and it is positively associated to expected demand λ. The possibility of renegotiation sustains a posted price r^* below the full commitment price $\hat{r}$. The difference is nonlinear since both prices fall to zero as λ vanishes, and approach one as λ grows large.

Figure 2 approximately here

This last observation gives us a chance to speculate on what would happen if sellers could compete in commitment mechanisms, as well as posted prices. One type of commitment could drive out the others, depending on the outcome function p. Indeed, we have seen that corner solutions emerge with renegotiation only if the seller's payoff is bounded by the auctions payoff either below or above. So, perhaps full-commitment would emerge as the equilibrium choice if renegotiated prices cannot exceed a sufficiently low value, and no-commitment would emerge in the opposite scenario.[15]

6.2 The equilibrium distribution of sale prices

The key implication of renegotiation is equilibrium sale price dispersion. To characterize the equilibrium distribution of sale prices we build on Section 4.2. The main difference is that now we must also consider the equilibrium list price, since it affects the support of the price distribution $\{p_n(r^*)\}_{n=1}^{\infty}$.

Given an equilibrium posted price r^*, we define the probability $\Pr[p_n(r^*)]$ as in (8). Let $\Pr[x|r^*]$ denote the equilibrium probability that a seller trades at price $x \in [0,1]$, with

$$\Pr[x|r^*] = \begin{cases} \Pr[p_n(r^*)] & \text{if } x = p_n(r^*),\ n = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

[15] In Coles and Eeckhout (2003) sellers compete in direct mechanisms, search is directed, and there is equilibrium indeterminacy. Other works study if sellers benefit from committing to a price rather than bargaining or using auctions. E.g., see Bester (1994), with search and commitment costs, or Camera and Delacroix (2004), with random matching and heterogeneous buyers. Examples with directed search include Peters (1991), where bargaining is not a stable institution since sellers may benefit from price posting; in McAfee (1993), sellers compete in mechanisms and in equilibrium use identical auctions; Julien, Kennes and King (2001) compares equilibria when prices are determined by posting or by competing auctions; in Acemoglu and Shimer (1999) and Michelacci and Suarez (2006) vacancies choose between posting a wage or bargaining ex-post with heterogeneous workers.

The equilibrium mean sale price is denoted $\bar{p}(r^*)$, defined by

$$\bar{p}(r^*) = \frac{\sum_{n=1}^{\infty} m_n(\lambda) p_n(r^*)}{1 - m_0(\lambda)}.$$

Two elements affect the distribution. The buyer-seller ratio λ influences the price distribution via two channels: the distribution of demand $\{m_n(\lambda)\}$ and, possibly, the posted price r^*. Indeed, if λ affects r^*, then it also affects the support of the distribution. The (underlying) renegotiation scheme also plays a role, because it is presumed to affect the outcome function p. To see how this works out, start by considering the equilibrium average sale price.

Proposition 8 *For all $\lambda > 0$ we have $d\bar{p}(r^*)/d\lambda > 0$. If $r^* \in (0, 1)$, then $\bar{p}(r^*) = \hat{r}$; if $r^* = 0$, then $\bar{p}(r^*) \geq \hat{r}$; and if $r^* = 1$, then $\bar{p}(r^*) \leq \hat{r}$.*

Average sale prices monotonically rise in expected demand because sellers can compete less for buyers, and so are free to raise the posted price (Proposition 6). Assumption 1 thus implies that (some) sale prices will increase. This intensive margin effect is augmented by an extensive margin effect. Since $\{p_n(r^*)\}$ is increasing, then the average sale price $\bar{p}(r^*)$ increases with λ even if the posted price r^* does not increase. That is, average sale prices increase in expected demand even if we have corner solutions. This result has emerged earlier, in Lemma 2, for economies where sale prices are completely independent of the posted price.

A second lesson concerns players' payoffs relative to the ordinary model. If $r^* \in (0, 1)$, then average sale prices are identical in the two models. This implies payoffs are unaffected by price commitments. Instead, if we have corner solutions, then average sale prices differ from the ordinary model. We know from Theorem 4 that corner solutions depend on λ and p. A quick look at (12) suggest that λ small helps sustain $r^* = 0$, while λ large helps sustain $r^* = 1$.[16] Consequently, price commitments can benefit sellers in a seller's market (many buyers per seller) but can hurt them in a buyer's market (many sellers per buyer).

Numerical illustration: average sale prices. The dashed line in Figure 3 is the equilibrium average sale price $\bar{p}(r^*)$ in the benchmark parameterization of the renegotiations model. Since the equilibrium is interior $\bar{p}(r^*)$ coincides with the curve $\hat{r}$ of Figure 2. This confirms the result in Proposition 8. The solid line reports the equilibrium average sale price for a different renegotiations scheme p that supports a different equilibrium price $\tilde{r}^*$. For comparison purposes, this price emerges from a market where we have assumed all renegotiated prices exceed the threshold $a = 0.3$ and $p_n = rq_n(1-a) + a$. Here $\tilde{r}^* = 0$ for λ sufficiently low (approx. 0.65), and $\tilde{r}^* = 1$ for λ sufficiently high (approx. 1.3). As a result $\bar{p}(\tilde{r}^*)$ exceeds $\hat{r}$ for λ small, is equal to $\hat{r}$ for λ moderate, and is lower than $\hat{r}$ for λ large.

Figure 3 approximately here

The density function can be characterized similarly to Section 4.2. In particular, if λ is sufficiently small, then it has a long strung-out right tail. Consequently, the renegotiation model can generate equilibrium realized wage densities that are similar to realistic-looking density functions of earnings (see Mortensen 2003). This holds if buyers are vacancies, sellers are job-seekers, and λ is sufficiently small; it also holds if we reverse the players'

[16] If $\lambda \approx 0$, then $m_0 \approx 1$, hence the left hand side of (12) can be easily violated. The opposite holds when λ is large.

role and λ is sufficiently large. The coefficient of variation of sale prices behaves similarly to the no commitment case, also. It vanishes as λ grows large because sellers increasingly end up trading at similarly high prices, eventually reaching the buyers' reserve value.

Numerical illustration: density functions. The crosses in Figure 1 identify the benchmark parameterization of the renegotiations model. The coefficient of variation is identified by c.v.$^{+}$. Start with the top left panel. Expected demand is less than the seller's capacity ($\lambda = 0.5$) and there is substantial trading risk. So, sellers post a low price but earn more that r^* on average. ($m_0 = 0.60$, $r^* = 0.11$ in Figure 2 and $\bar{p}(r^*) = 0.23$ in Figure 3). The density function of sale prices is downward sloping because low demand realizations are very likely. As expected demand rises (next three panels) the posted price rises, the support of the price distribution shrinks and sale prices become more concentrated. With eight times more buyers than sellers ($\lambda = 8$) there is almost no trading risk. Sellers post almost the buyer's reserve value and trade close to it. ($m_0 = 0.0003$, $r^* = 0.98$ in Figure 2 and $\bar{p}(r^*) = 0.99$ in Figure 3). Indeed, in the benchmark parameterization $dp_n(r)/dr = 1 - g_n$ falls with n. That is, changes in r have little impact on the renegotiation outcome with many customers.

Finally, we can compare the model with renegotiations to the model with no-commitment (circles). In both cases sale price dispersion falls in λ. However, there is more dispersion with no commitment since the support of the distribution is fixed. Under renegotiations, instead, as λ rise sellers not only trade at higher prices because they meet more buyers, but they charge more in every match, also.

6.3 Endogenous price formation and renegotiation

The basic model can be easily enriched. In this section we provide a brief example that deals with endogenizing renegotiation outcomes. In fact, so far we have assumed that renegotiation leads to a unique outcome in the third stage, defined by a function p that satisfies Assumption 1. Here we exploit the strategic renegotiations process that follows the game in Appendix B. We show that the subgame perfect outcome is a function p that satisfies Assumption 1. We also show that the model determines the equilibrium incidence of renegotiation and allows the identification of market conditions under which fixed-price trading emerges in equilibrium, even if list prices are non-binding.

Consider a trading process in the third stage of the game in which, for concreteness, we let sellers be job-seekers, and let buyers be vacancies. So, r is the posted wage and $p_n(r)$ is the realized wage for a worker who has met $n \geq 1$ vacancies. A job-seeker who has met n vacancies selects one at random. At this point, the worker has the option to request renegotiation of the posted wage r. If the option is not exercised, then the vacancy is filled and the worker earns r. Otherwise, a strategic bargaining game starts; see Appendix B. Its outcome is a wage $q : \mathbb{N} \to [0,1]$ with $\{q_n\}_{n=1}^{B}$ positive, monotone, and increasing, which satisfies (B.1).

The subgame perfect outcome of the third-stage game is the wage $p_n(r) = \max\{q_n, r\}$ because workers renegotiate only if they expect to obtain more than r. Notice that p satisfies Assumption 1. The model can be solved in a manner that is similar to what we have done earlier. The basic qualitative results obtained earlier go though. In particular, posted wages and average realized wages rise with λ. An example is in Table 1.

Table 1 approximately here

The main difference with the earlier version of the model with renegotiations is that if $q_1 > 0$, then we may have a continuum of corner solutions $r^* \leq q_1$. That is, we can have indeterminacy of posted wages. Of course, this will depend on market tightness since r^* increases in λ. In particular, an interior solution satisfies $r^* \in (q_1, 1)$, defined by

$$r^* = \frac{1-m_0-m_1-\sum_{n=\bar{n}+1}^{\infty} m_n q_n}{\sum_{n=1}^{n} m_n}.$$

The variable $\bar{n}$ above denotes a unique value greater than or equal to one, which depends on λ. This value $\bar{n}$ defines the endogenous threshold above which the worker chooses to renegotiate the posted wage. If $n \leq \bar{n}$, then trade occurs at the posted wage r^*. Otherwise, the wage is renegotiated and the worker earns q_n. In other words, $q_{\bar{n}} < r^* < q_{\bar{n}+1}$.[17]

The incidence of renegotiation is now endogenous. It depends on market tightness λ. To see why, observe that, given r^*, renegotiation hinges on realized demand n because $\{q_n\}$ is increasing. In addition, r^* increases with λ. So, if λ is sufficiently small, then we have a corner solution $r^* \leq q_1$. In this case $p_n(q_1) = q_n$ for all n. Workers always renegotiate the posted wage, even if they meet only one vacancy ($\bar{n} = 0$). Instead, if the

[17] We have also executed numerical experiments to study how market size affects prices when λ is fixed. We find that prices are higher in smaller markets (all else equal). Intuitively, a seller worries not only about the expected demand, but also about the covariance of demand, which is negative in a finite market. So, if a seller meets customers in a small market, then they represent most of the demand (think of B = 2). Hence, sellers compete aggressively posting a low price. This implies that prices are *less* scattered in a small market, for this specific example.

market is sufficiently tight, then we have $r^* \in (q_{\bar{n}}, q_{\bar{n}+1})$ for some $\bar{n} \geq 1$. Only workers who meet more than $\bar{n}$ vacancies renegotiate. All others accept jobs at the posted wage.

Since the posted wage and the distribution of vacancies depends on λ, it follows that the equilibrium incidence of wage negotiations depends on market tightness. Table 1 provides an example for an economy in which $q_1 = 0.01$; the last two columns report the approximate probability of renegotiation, and the coefficient of variation of realized wages. We see that for λ small posted wages are low and renegotiation is more frequent. For λ large we have high posted wages and infrequent renegotiation. If $\lambda = 4, 8$, then there is renegotiation when matches include more than 14 and 368 buyers, respectively. Such large matches are rare, which is why the probability of renegotiation is approximately zero.

7 Final remarks

We have presented a model of price dispersion with directed search. Renegotiation might take place once matches are realized because there is limited commitment to posted prices, sellers are constrained in their capacity, and buyers cannot re-match to sellers. The sale price depends on the number of buyers who want to purchase the good. An analytically tractable distribution of sale prices emerges in equilibrium.

The proposed theory is consistent with the empirical evidence on cross-employer wage differentials for identical workers. The model yields a density function of realized wages that displays positive skewness if the mean number of job-seekers per vacancy is sufficiently large and workers can renegotiate the posted wage. Such a distribution is empirically plausible and yet it does not require additional assumptions on heterogeneity. So, the theory can account for some of the residual wage heterogeneity observed in the data.

Some natural extensions suggests themselves. It would be interesting to consider a dynamic version with re-matching, or to allow buyers to match to multiple sellers. We think that, as long as buyers cannot simultaneously match to every seller the basic results of the paper should still hold.

Appendix A

Proof of Lemma 2

Clearly, $\bar{p} > 0$. Omitting the argument of $m_n(\lambda)$ we have $\sum_{n=1}^{\infty} m_n p_n < 1 - m_0$ since $p_n < p_{n+1}$ for at least some $n \in \mathbb{N}$, and p is bounded above by one. So $\bar{p} < 1$. We have

$$\frac{d\bar{p}}{d\lambda} = \frac{1}{(1-m_0)^2} \times \left[\sum_{n=1}^{\infty} m_n' p_n (1 - m_0) + m_0' \sum_{n=1}^{\infty} m_n p_n\right]$$

with $m_n' = dm_n/d\lambda = (n - \lambda)m_n/\lambda$. Denoting the term in square brackets by H, we have

$$H = \sum_{n=1}^{\infty} m_n p_n \left[\frac{n}{\lambda} (1 - m_0) - 1\right].$$

We claim $H > 0$. Since $\{n (1 - m_0) / \lambda - 1\}$ and $\{p_n\}$ are increasing sequences, a lower bound for H is obtained by setting $p_n = p_1$. Therefore we have

$$\begin{aligned} H &> \sum_{n=1}^{\infty} m_n p_1 \left[\frac{n}{\lambda} (1 - m_0) - 1\right] = p_1 \sum_{n=1}^{\infty} m_n \left[\frac{n}{\lambda} (1 - m_0) - 1\right] \\ &= p_1 \left[\sum_{n=1}^{\infty} m_{n-1} (1 - m_0) - \sum_{n=1}^{\infty} m_n\right] = p_1[(1 - m_0) - (1 - m_0) = 0 \end{aligned}$$

where $m_n n/\lambda = m_{n-1}$ from (1).■

Proof of Lemma 3

Fix r and λ and omit them as arguments of U. For notational convenience let $x = \lambda'$ and $y = r'$. So we rewrite (3) as $\Delta(x, y) = 0$ by defining

$$\Delta(x, y) := \sum_{n=1}^{\infty} m_n(x) [1 - p_n(y)] - xU.$$

Define

$$m_n'' := d^2 m_n(x)/dx^2 = \frac{(n-x)^2 - n}{x^2} m_n \quad \text{and} \quad p_n'(y) := \frac{\partial p_n(y)}{\partial y}.$$

Omitting the arguments x and y when understood, consider the partial derivatives

$$\begin{aligned}\Delta_x = &\quad \textstyle\sum_{n=1}^{\infty} m_n'(1-p_n) - U = \frac{1}{x}\sum_{n=1}^{\infty} m_n\,(n-x)\,(1-p_n) - U \\ \Delta_y = &\quad -\textstyle\sum_{n=1}^{\infty} m_n p_n'.\end{aligned}$$

Both derivatives are nonzero. Clearly, $\Delta_y < 0$, since $p_n'(y) \geq 0$ with strict inequality for at least some n. To prove $\Delta_x < 0$ notice that when (3) holds $\Delta(x,y) = 0$, i.e., $\sum_{n=1}^{\infty} m_n(1-p_n)/x = U$. Substitute U into Δ_x and get

$$\Delta_x = \tfrac{1}{x}\textstyle\sum_{n=1}^{\infty} m_n\,(n-1-x)\,(1-p_n).$$

We have

$$\begin{aligned}\textstyle\sum_{n=1}^{\infty} m_n\,(n-1-x) &= \textstyle\sum_{n=1}^{\infty} m_n n - (1+x)\sum_{n=1}^{\infty} m_n \;=\; x-(1+x)(1-m_0) \\ &= m_0 + m_0 x - 1 \;=\; m_0 + m_1 - 1 < 0.\end{aligned}$$

Assumption 1 guarantees that $\{1-p_n\}_{n=1}^{\infty}$ is a decreasing sequence. Instead, $\{n-1-x\}_{n=1}^{\infty}$ is increasing, but it is negative for $n \leq k$ where k solves $k-1-x=0$. Define the floor function $\lfloor k \rfloor = \max\{n = 0,1,\ldots : n \leq k\}$, i.e., the largest integer less than or equal to k. So, for $n \leq \lfloor k \rfloor$ we have $(n-1-x) < 0$, and > 0 otherwise. Now we can write

$$\begin{aligned}\Delta_x = \tfrac{1}{x}\textstyle\sum_{n=1}^{\infty} m_n\,(n-1-x)\,(1-p_n) &< \tfrac{1}{x}\textstyle\sum_{n=1}^{\lfloor k \rfloor} m_n\,(n-1-x)\,(1-p_{\lfloor k \rfloor}) \\ &\quad + \tfrac{1}{x}\textstyle\sum_{n=\lfloor k \rfloor+1}^{\infty} m_n\,(n-1-x)\,(1-p_{\lfloor k \rfloor}) \\ &= \tfrac{1}{x}\,(m_0+m_1-1)\,(1-p_{\lfloor k \rfloor}) < 0.\end{aligned}$$

So $\Delta(x,y) = 0$ defines x as an implicit function of y (Implicit Function Theorem) with

$$\frac{dx}{dy} = \frac{\sum_{n=1}^{\infty} m_n p_n'}{\sum_{n=1}^{\infty} m_n'(1-p_n) - U} = \frac{x\sum_{n=1}^{\infty} m_n p_n'}{\sum_{n=1}^{\infty} m_n(n-1-x)(1-p_n)} < 0.$$

$\blacksquare$

Proof of Theorem 4

Fix (λ, r). Use the earlier definition $\Delta(\lambda', r') = U(\lambda', r') - U(\lambda, r)$. The equilibrium value r^* must satisfy individual optimality—i.e., indifference $\Delta(\lambda', r') = 0$ from (3) and profit maximization $m_0(\lambda') = U(\lambda, r)$ from (11)—and symmetry $(\lambda', r') = (\lambda, r)$.

Impose $(\lambda', r') = (\lambda, r)$ and omit the argument of $m_n(\lambda)$. Note that there is always a value λ' that satisfies (11). However, (11) may not hold for $\lambda' = \lambda$. So, we study two cases: (i) interior solutions $r^* \in (0, 1)$, when (11) holds for $\lambda' = \lambda$; (ii) corner solutions $r^* = 0, 1$, when (11) does not hold for $\lambda' = \lambda$. To do so we consider (12), i.e.,

$$\textstyle\sum_{n=1}^{\infty} m_n p_n(0) < 1 - m_0 - m_1 < \sum_{n=1}^{\infty} m_n p_n(1).$$

Interior solutions: If the equilibrium r is interior, then we can substitute $U(\lambda, r) = m_0$ into $\Delta(\lambda, r) = 0$, which implies $r = r^*$ must be a solution to

$$\Delta(\lambda, r) = 1 - m_0 - m_1 - \textstyle\sum_{n=1}^{\infty} m_n p_n(r) = 0. \tag{A.1}$$

If (12) holds, then, $\Delta(\lambda, 0) > 0 > \Delta(\lambda, 1)$. For all λ the function $\Delta(\lambda, r)$ is continuous in r, and $\partial\Delta(\lambda, r)/\partial r < 0$ since $p_n'(r) > 0$ for some n (zero,otherwise). By the Intermediate Value Theorem, there exists a unique $r^* \in (0, 1)$ such that $r = r^*$ satisfies (A.1).

Corner solutions: Suppose first that the left inequality in (12) is violated.

1) If $1 - m_0 - m_1 = \sum_{n=1}^{\infty} m_n p_n(0)$, then $r^* = 0$ is the unique solution. It uniquely satisfies $\Delta(\lambda, r) = 0$ and $m_0(\lambda) = U(\lambda, r)$.

2) If $1 - m_0 - m_1 < \sum_{n=1}^{\infty} m_n p_n(0)$, then $1 - m_0 - m_1 < \sum_{n=1}^{\infty} m_n p_n(r)$ for all $r \geq 0$

because $\sum_{n=1}^{\infty} m_n p_n'(r) > 0$ (Assumption 1). Since $m_1 = \lambda m_0$ we have

$$m_0 > \tfrac{1}{\lambda} \sum_{n=1}^{\infty} m_n [1 - p_n(r)] = U(\lambda, r) \quad \text{for all } r \geq 0. \tag{A.2}$$

The equality on the right hand side follows from (5). To prove $r^* = 0$ is the unique equilibrium we use a contradiction. Suppose $r^* > 0$ is an equilibrium. Then, (A.2) implies the seller's first order condition holds strictly, i.e.,

$$m_0(\lambda) - U(\lambda, r^*) > 0.$$

But $U(\lambda, r)$ falls in r, so $r = r^* > 0$ cannot be optimal for the seller. Posting $r = 0$ gives the seller an even better payoff.

We can prove, as done above, that if $\sum_{n=1}^{\infty} m_n p_n(1) \leq 1 - m_0 - m_1$, then $r^* = 1$ is the unique solution.■

Proof of Proposition 6

Consider an interior solution $r^* \in (0, 1)$. We want to prove $dr^*/d\lambda > 0$. From (13), the value r^* is the unique solution to

$$\Delta(\lambda, r) = \sum_{n=1}^{\infty} m_n p_n(r) - [1 - m_0(\lambda) - m_1(\lambda)] = 0.$$

Fix and omit the arguments λ and r. Consider the partial derivatives

$$\begin{aligned} \Delta_\lambda = &\quad \sum_{n=1}^{\infty} m_n' p_n + (m_0' + m_1'), \\ \Delta_r = &\quad \sum_{n=1}^{\infty} m_n p_n'(r), \end{aligned}$$

where $m_n' := dm_n(\lambda)/d\lambda = (n - \lambda) m_n(\lambda)/\lambda$. Clearly $\Delta_r > 0$ from Assumption 1.

We wish to prove $\Delta_\lambda < 0$. Since $m_1/\lambda = m_0$ we have $m_0' + m_1' = -m_1$ and so

$$\Delta_\lambda = \sum_{n=1}^{\infty} m_n (\tfrac{n}{\lambda} - 1) p_n - m_1 = \sum_{n=1}^{\infty} m_n \tfrac{n}{\lambda} p_n - (1 - m_0).$$

The second equality comes from imposing $\Delta(\lambda, r) = 0$, i.e., $\sum_{n=1}^{\infty} m_n p_n = 1 - m_0 - m_1$. Now notice that many sequences $\{p_n\}$ satisfy $\Delta(\lambda, r) = 0$. A suitable sequence is $p_1 = 0$ and $p_n = 1$ for all $n \geq 2$; another is $p_n = \hat{r}$ for $n = 1, 2, \ldots$, with $\hat{r}$ defined in (7). In particular, zero is the minimum value that p_1 can take, and $\hat{r}$ is the maximum. To see that $p_1 \leq \hat{r}$ notice that $\{p_n\}$ is non-decreasing and so for any $p_1 > \hat{r}$ we have

$$\sum_{n=1}^{\infty} m_n p_n > \sum_{n=1}^{\infty} m_n \hat{r} = 1 - m_0(\lambda) - m_1(\lambda),$$

which is inconsistent with $\Delta(\lambda, r) = 0$. We will use this information below.

Observe that $m_n n/\lambda = m_{n-1}$ since $m_n = e^{-\lambda}\lambda^n/n!$. So we write

$$\Delta_\lambda = \sum_{n=0}^{\infty} m_n p_{n+1} - (1 - m_0) = \sum_{n=1}^{\infty} m_n p_{n+1} + m_0(1 + p_1) - 1.$$

Recall that $\{p_n\}$ is increasing and strictly positive for all $r > 0$ (Assumption 1). We want to find an upper bound for Δ_λ. So, we have two cases.

1) If $\sum_{n=1}^{\infty} m_n > m_0$, then an upper bound for Δ_λ is achieved by an increasing sequence $\{p_n\}$ that satisfies $\Delta(\lambda, r) = 0$, has the smallest possible p_1 and the largest possible p_n for $n \geq 2$. Such a sequence is $p_1 = 0$ and $p_n = 1$ for $n \geq 2$. In this case we have

$$\Delta_\lambda \leq \sum_{n=1}^{\infty} m_n + m_0 - 1 = 0.$$

If $r^* \in (0, 1)$, then $\Delta_\lambda < 0$ since $p_1 > 0$ and $p_n < 1$ for some n by Assumption 1.

2) If $\sum_{n=1}^{\infty} m_n \leq m_0$, then an upper bound is achieved by a constant sequence $\{p_n\}$ that satisfies $\Delta(\lambda, r) = 0$ and has the largest possible p_1. Such a sequence is $p_n = \hat{r}$ for all $n \in \mathbb{N}$. In this case we have

$$\Delta_\lambda \leq \hat{r} + m_0 - 1 \quad = \frac{(1-m_0)m_0 - m_1}{1 - m_0}$$

Now, $\sum_{n=1}^{\infty} m_n \leq m_0$ is $1 - m_0 \leq m_0$. This holds only if $\lambda \in (0, 1)$. In this case

$$(1 - m_0)m_0 - m_1 = (1 - e^{-\lambda})e^{-\lambda} - \lambda e^{-\lambda} < 0.$$

Clearly, $\Delta_\lambda < 0$ because with limited commitment $p_1 < \hat{r}$ and $p_n > \hat{r}$ for some $n \in \mathbb{N}$.

So we have $\Delta_\lambda < 0$ for all λ and r, and $\Delta_r > 0$ for all λ and r. Using the Implicit Function Theorem we have $dr^*/d\lambda = -\Delta_\lambda/\Delta_r > 0$. It is immediate from (13) that $\lim_{\lambda \downarrow 0} r^* = 0$ since $m_0 \to 1$. Also, $\lim_{\lambda \uparrow \infty} r^* = 1$ since $m_0, m_1 \to 0$.■

Proof of Proposition 7

The proof is by contradiction. Fix $\lambda > 0$ and omit it as argument of $m_n(\lambda)$. The renegotiations equilibrium posted price $r^* \in (0, 1)$ is the unique solution to (13).

If we assume full commitment, then the equilibrium posted price is $\hat{r}$ defined in (7). Indeed (Corollary to Theorem 4), $p_n = \hat{r}$ for all $n \in \mathbb{N}$ satisfies (13), i.e.,

$$\textstyle\sum_{n=1}^{\infty} m_n \hat{r} = 1 - m_0 - m_1.$$

Now consider limited commitment. Assumption 1 holds. First, assume price floors, i.e., $p_n(r) \geq r$ for all r and all $n \in \mathbb{N}$, with strict inequality for some n. We wish to prove that $r^* < \hat{r}$. Since r^* is an equilibrium it must satisfy

$$\textstyle\sum_{n=1}^{\infty} m_n p_n(r^*) = 1 - m_0 - m_1.$$

Suppose, by means of contradiction, that in fact $r^* \geq \hat{r}$. Since $p_n(r^*) \geq r^*$ for all $n \in \mathbb{N}$ with strict inequality for some n, we must have

$$\textstyle\sum_{n=1}^{\infty} m_n p_n(r^*) > \sum_{n=1}^{\infty} m_n \hat{r} = 1 - m_0 - m_1,$$

which provides the desired contradiction.

Second, assume price ceilings, i.e., $p_n(r) \leq r$ for all r and all $n \in \mathbb{N}$, with strict inequality for some n. The proof for $r^* > \hat{r}$ is similar to the above. ■

Proof of Proposition 8

Suppose $r^* \in (0,1)$. Then (13) must hold, and we rearrange it as

$$\frac{\sum_{n=1}^{\infty} m_n(\lambda) p_n(r^*)}{1-m_0(\lambda)} = 1 - \frac{m_1(\lambda)}{1-m_0(\lambda)}. \tag{A.3}$$

The left hand side is $\bar{p}(r^*)$. The right hand side is $\hat{r}$, from (7). So, $\bar{p}(r^*) = \hat{r}$. Clearly, $\bar{p}(r^*)$ monotonically increases in λ because $\hat{r}$ monotonically increases in λ.

Now consider the corners $r^* = 0, 1$, when (12) does not hold. We know from Theorem 4 that if $\sum_{n=1}^{\infty} m_n(\lambda) p_n(0) \geq 1 - m_0 - m_1$, then $r^* = 0$. This implies $\bar{p}(0) \geq \hat{r}$, from (A.3). Instead, if $\sum_{n=1}^{\infty} m_n(\lambda) p_n(1) \leq 1 - m_0 - m_1$, then $r^* = 1$. This implies $\bar{p}(1) \leq \hat{r}$.

To prove $d\bar{p}(r^*)/d\lambda > 0$ for $r^* = 0, 1$ note that if $r^* = 1$, then $r^* = 1$ as λ rises. Since $\{p_n(1)\}$ is independent of λ, we can prove $d\bar{p}(1)/d\lambda > 0$ as in Lemma 2. Simply replace p_1 by $p_1(1)$. Now suppose $r^* = 0$. If r^* does not rise with λ, then $d\bar{p}(0)/d\lambda > 0$ also is proved as in Lemma 2. The result holds if $dr^*/d\lambda > 0$, since in this case sale prices rise as well.■

Appendix B

We present a strategic and costly bargaining process in which trade occurs without delay at a unique price q_n increasing in n. Consider a seller who has met $n \in \mathbb{N}$ buyers. A random device selects a buyer, then seller or buyer can request renegotiation of the list price. This initiates a strategic bargaining process that may span an indefinite number of rounds $t = 0, 1, ...$ The number n is assumed constant across rounds (mobility constraints). Assume the seller makes the initial offer q_n. If it is accepted then payoffs are realized and the game ends. If it is rejected, then a new round occurs with probability $\beta \in (0, 1)$. If the game stops without reaching an agreement, then everyone has zero payoff.

If the game continues, a random device selects a new proposer. The seller is selected with probability $\gamma \in (0, 1)$; a recipient is selected with probability $1/n$. With probability $(1 - \gamma)/n$, each buyer is selected to make an offer. If there is agreement the game ends or else a new round occurs with probability β. This process is repeated until it either randomly stops or an agreement is reached, since participation cost and outside options are zero.

This alternating offers bargaining game has a unique subgame perfect equilibrium characterized by symmetric offers and no delays. The (re)negotiated price is

$$q_n = \tfrac{(n-\beta)[1-\beta(1-\gamma)]}{n(1-\beta)+\beta\gamma(n-1)} \quad \text{for } n = 1, 2, ... \tag{B.1}$$

increasing since in n. Intuitively, the seller's inventory constraint exposes buyers to consumption risk, as rejecting the initial offer gives others a chance to buy. So, buyers compete (via random selection) to make and receive offers. With more buyers the seller's bargaining power is stronger and the initial offer rises.

To get (B.1), conjecture an equilibrium in which offers are accepted immediately, are symmetric and time-invariant. Let q_n^s denote the seller's equilibrium offer to any buyer, and q_n^b any buyer's offer, in any round. Since negotiations may randomly stop, the seller makes an initial offer that is weakly preferred by any buyer. Any buyer who makes a counteroffer will act in a similar manner. So, we have two usual indifference conditions

$$\begin{aligned} 1 - q_n^s &= \tfrac{\beta}{n}[\gamma\,(1 - q_n^s) + (1-\gamma)(1 - q_n^b)], \\ q_n^b &= \beta[\gamma q_n^s + (1-\gamma) q_n^b]. \end{aligned}$$

The seller's offer q_n^s (first line) accounts for the number of buyers n, since the buyer who rejects an offer makes a counteroffer only with probability $\beta(1-\gamma)/n$. A buyer's offer q_n^b (second line) makes the seller indifferent given that, if he rejects, he counteroffers with probability $\beta\gamma$. Since the seller moves first, we get $q_n = q_n^s$ defined in (B.1). If buyers can make the initial offer, then we obtain a lower expected price. A detailed study is in Camera and Selcuk (2006).

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	r^*	$\bar{n}$	Prob. renegotiate	c.v.
$\lambda = 0.5$	0.017	1	.229	.642
$\lambda = 2$	0.669	3	.165	.562
$\lambda = 4$	0.925	14	0	0
$\lambda = 8$	0.997	368	0	0

Table 1: Endogenous price formation

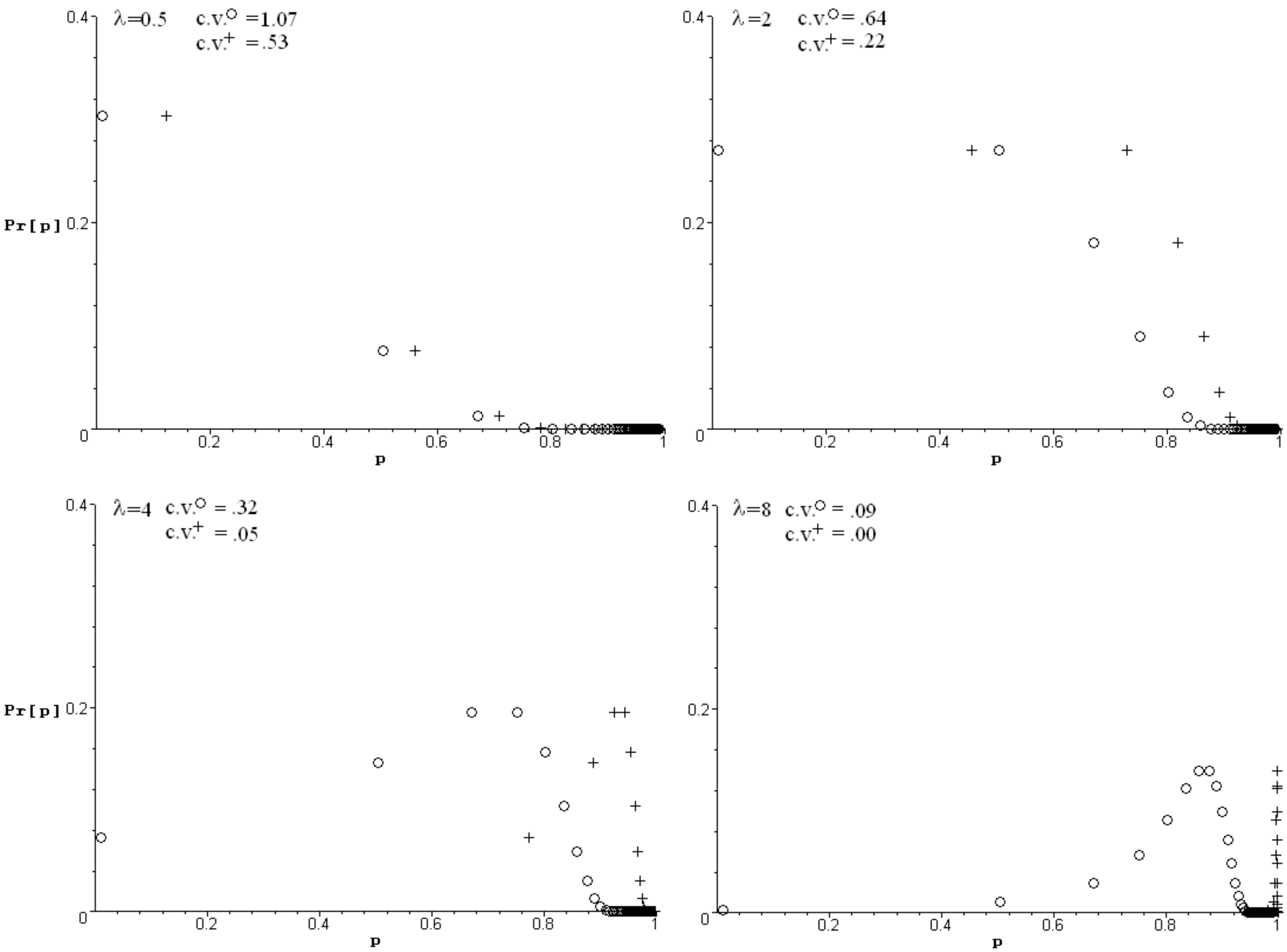

Figure 1

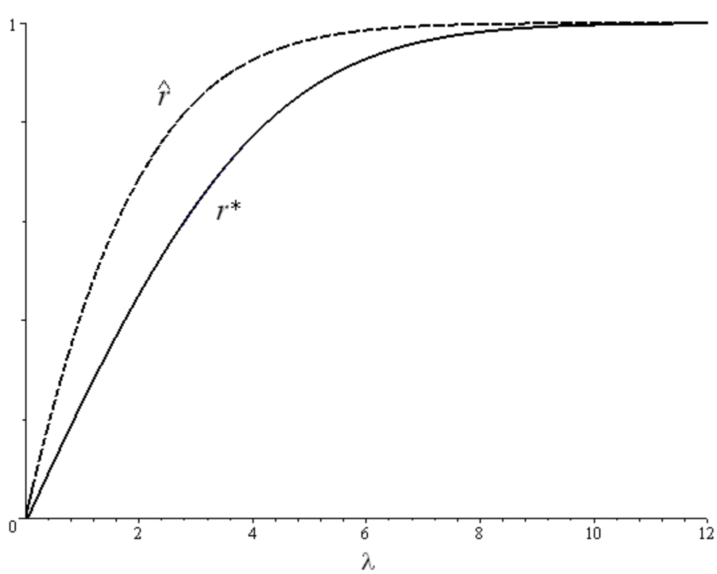

Figure 2

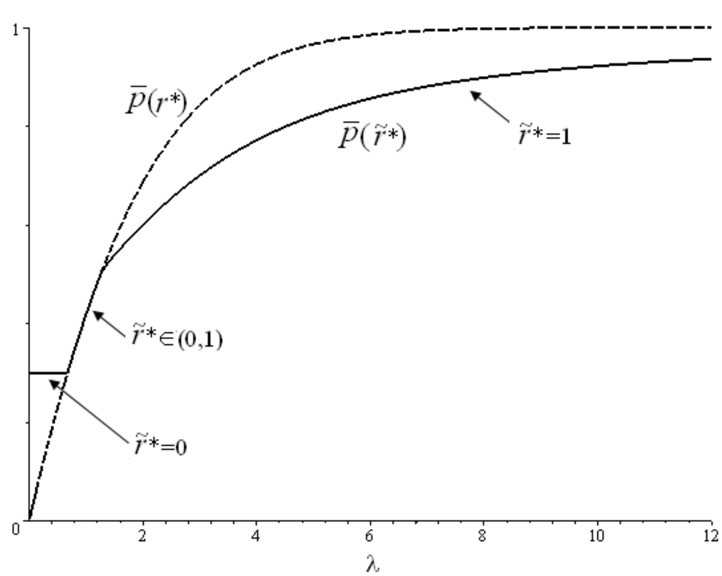

Figure 3