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Anonymous Markets and Monetary Trading

Anonymous Markets and Monetary Trading[*]

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ABSTRACT: We study infinite-horizon monetary economies characterized by trading frictions that originate from random pairwise meetings, and commitment and enforcement limitations. We prove that introducing occasional trade in "centralized markets" opens the door to an informal enforcement scheme that sustains a non-monetary efficient allocation. All is required is that trading partners' be patient and their actions be observable. We then present a matching environment in which trade may occur in large markets and yet agents' trading paths cross at most once. This allows the construction of models in which infinitely-lived agents trade in competitive markets where money plays an essential role.

Keywords and Phrases: Anonymity, Money, Infinite games, Matching, Social norms

JEL Classification Numbers: C72, C73, D80, E00

1 Introduction

A large segment of monetary literature revolves around the use of models that prominently display various obstacles to the trading process. The motivation behind this modeling choice is that in an ideal monetary framework money should be 'essential,' i.e., eliminating

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it from the economy should result in efficiency loss.[1] This guiding principle has led to the creation of models in which trade is of an intertemporal nature but two intertwined frictions, concerning agents' feasible interactions and access to information, all but rule out credit arrangements. First, a meeting process is imposed that allows only pairwise trade among agents having no lasting relationships and who may be anonymous. Put simply, the 'trading paths' of any two agents are assumed to cross at most once. Second, there are commitment and enforcement limitations, so any allocation must be compatible with individual incentives.

This paper is a theoretical study of the role of meeting and anonymity frictions in modeling money. It is motivated by recent efforts directed at improving the search model of Kiyotaki and Wright [21], which prominently displays an essential role for money. In the typical search model randomly formed pairs of agents use money to overcome exchange problems due to idiosyncratic shocks. Such features, unfortunately, generate analytically intractable distributions of balances when money is divisible (e.g., see [11, 15, 16]). This has inhibited a broader integration of this modeling technique into the "toolbox" of the typical macroeconomist, especially those interested in policy analysis.

This issue has spurred interest in developing frameworks that vary the basic search model with the goal of obtaining degeneracy in equilibrium holdings. This basic remedy is at the core of the work of Shi [27], who cleverly models the population as a continuum of families, each encompassing a continuum of agents. Degeneracy arises from an involuntary redistribution of money holdings within each family, after each round of random matching. Another clever variation is in Lagos and Wright [23]. They achieve degeneracy by introducing a round of Walrasian 'centralized' trading after each round of bilateral random 'decentralized' trading. Though this alters the key meeting friction of the typical search model, the basic premise of the model in [23] is that anonymity and random decentralized pairings are frictions sufficient for money to be essential.[2] Naturally, one wonders whether this intuition is generally accurate and, if it is not, whether we can construct economies in which money is essential to sustain trade in large competitive markets. This would bring us closer to better integrating the literature on the foundations of money with the mainstream macroeconomic literature, as advocated by some observers (e.g., see [20]).

Our study makes two contributions. We start by clarifying that random pairing of anonymous traders is not a friction sufficient to make money essential. To do so we consider environments with alternating decentralized and centralized markets, deterministically or not. We demonstrate that eliminating money need not reduce efficiency, if actions of trading partners are observable and agents are sufficiently patient. The analysis, which complements and develops more formally the arguments advanced in [5], follows the work of [12] and [19], on existence of efficient and individually rational outcomes in

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[1] Methodological observations of this flavor are, for example, in [17, 25, 29]. Studies in [18] and [22] focus on the essentiality of money.

[2] For example, see [23, p. 466], [26, p. 175], or [8, p. 467].

repeated anonymous matching games. The idea is that if agents can quickly inform others of privately observed undesirable behavior, then a credible informal enforcement scheme exists that sustains the efficient allocation. Intuitively, money has a role to play when obstacles to widespread information transmission exist (see [18, 22]) and, indeed, the random matching scheme in [21] is a device to naturally fragment the information exchange process. Introducing centralized trading opens the door to the rapid exchange of information among significant portions of the population, which makes money inessential.[3] However, this finding is not robust to adding a small amount of noise in the observation of individual behavior, since equilibria would arise similar to those in the continuum limit where individual behavior is unobservable (see for instance [6, 13, 14, 24]).

Based on these findings, we then offer a second contribution. We explain how to construct physical environments in which agents interact in markets with (infinitely) many other participants, and yet money is essential. In particular, we present a matching framework, based on the studies in [3, 4], which can be used to model a variety of trade meetings, bilateral and multilateral, deterministic and stochastic. We use this framework to outline an economy in which infinitely-lived agents repeatedly move in and out of markets populated by numerous anonymous agents who, however, are always complete strangers because their trading paths intersect at most once. Such a physical environment gives rise to informational frictions that make money essential. In this manner our study complements recent developments in modeling monetary economies. For instance, the techniques we present allow the design of possible physical environments underlying monetary economies with alternating decentralized and centralized markets in the spirit of [23], or with random matching of continuum of agents as in [27], or with spatially separated competitive markets as in [28]. In addition, these techniques can be used to further the modeling of environments where money is necessary to support trade in large competitive markets (see for instance [9]). This can bring us closer to the desirable stage of better integrating the literature on the foundations of money with the mainstream macroeconomic literature.

2 The physical environment

We describe an environment that captures the salient features of the model in [23]. Time is discrete and infinite, indexed by $t = 0, 1, 2 \ldots$. There is a constant population $J = \mathbb{N}$ of identical infinitely-lived agents and a single perishable good that can be produced by a fraction of the population at each date. Even and odd periods differ in terms of preferences, economic activities, and matchings. We start by formalizing this last element, as it is a central building block.

In each period t, interactions among agents are determined by an exogenous matching process that specifies a partition of the population into trading groups. We define a *match*

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[3]The study in [10], where agents choose to meet pairwise randomly or to costly trade in a centralized market, also suggests that money may not be essential when a centralized market is available.

for agent $j \in J$ in a period t to be a group of people $G_t(j) \subseteq J$, which includes agent j and possibly others. The agents in $G_t(j)$ are called *partners* of j in period t. Let $\beta_t \colon J \to J$ be a stochastic bilateral matching rule, i.e., a function that partitions the population in matches composed of one or two randomly selected agents. (For details see [3].)

Date $t = 0$ is an initial period in which, for convenience, we assume that agents are 'idle,' i.e., $G_0(j) = \{j\}$ for all $j \in J$. In every other date $t \geq 1$ we assume a matching process such that

$$G_t(j) = \begin{cases} \{j, \beta_t(j)\} & \text{if } t \text{ odd} \\ J & \text{if } t \text{ even,} \end{cases} \tag{2.1}$$

where $\beta_t(j) = j$ with probability $1 - \alpha$ for each $j \in J$. That is, in odd periods agents may be paired to someone else, with probability α, while in even periods they all belong to an economy-wide group. We say that trading in odd periods is *decentralized*, while in even periods is *centralized*, as suggested in [23].

Following the matching literature, we identify each match G as a distinct area of economic interaction. To be more precise, it is assumed that agents can exchange objects only with their partners, cannot directly communicate with each other, and can only observe actions and outcomes in their current match—ignoring what has happened in every other match (e.g., [21, 28]).[4] This is referred to as spatial separation and limited communication. There is also anonymity, in that each agent ignores the partition of the population in odd periods and neither observes nor can verify the identity and trading history of others (e.g., [19, 26]). Finally, there is absence of commitment and enforcement, in that agents can always refuse to take an action without being subject to retribution.[5] Thus, actions must be compatible with individual incentives (e.g., [18, 22]).

It is assumed that trade is necessary for consumption to take place. Specifically, in odd periods in each pair of agents a flip of a fair coin determines which one of them is a producer and who is a consumer. In even periods agents can produce and consume. Each producer can supply an amount $a \in [0, \bar{a}]$ of labor to a linear technology that transforms it into a consumption goods. The producer suffers disutility a, and derives no utility from consumption of his own production. In odd periods, every consumer has utility $u_o(c)$ from consuming $c \geq 0$ goods, and in each even period everyone has utility $u_e(c)$. Assume that the functional forms of preferences satisfy the Inada conditions, that zero consumption generates zero utility, and that $\bar{a} \in (c_o^* + c_e^*, \infty)$, where c_o^* and c_e^* satisfy $u_e'(c_e^*) = u_o'(c_o^*) = 1$.

In every period $t > 0$ agents discount next period's payoffs by $\delta \in (0, 1)$ if t is even and $\varepsilon \in (0, 1]$ otherwise (there is no discounting in $t = 0$). To summarize, in even periods

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[4] Working with countable populations simplifies the construction of informationally isolated economies in Section 7 and emphasizes the role played by observability of individual actions.

[5] Lack of formal enforcement institutions suggests that no one can be forced to surrender some of his endowment, produce, or suffer a loss. For example, this means that 'cheating' on a contract cannot trigger a current or future retaliation by the victim or anyone else.

agents are multilaterally matched, can produce *and* consume, while in odd periods only agents who are paired can either produce *or* consume. In essence, in odd periods each agent is randomly assigned to one of three groups, called *producers*, *consumers* and *idle*, corresponding to population proportions $\frac{\alpha}{2}$, $\frac{\alpha}{2}$, and $1 - \alpha$.

3 A trading game

To study non-monetary allocations in this economy, we provide a strategic representation of an infinite horizon trading game. We start by establishing that autarky (zero production) is the unique Nash equilibrium of the static game. Then, we exploit this result in the analysis of the infinite horizon game.

3.1 The static game

Consider a representative one-shot game involving agents in some match $G_t(j)$ generated by the process (2.1), in some period t. Denote by $G_t^P(j)$ and $G_t^C(j)$ the set of producers and consumers, with $G_t(j) = G_t^P(j) \cup G_t^C(j)$. Efficiency of allocations revolves around the amount produced, so we let $\{0\}$ be the action set of consumers and unmatched agents. Producers must choose how much consumption to supply to members of their group, i.e., they choose a non-negative supply of labor input in $[0, \bar{a}]$. Hence, we identify the action set of any agent $k \in G_t(j)$ by

$$A_k = \begin{cases} [0, \bar{a}] & \text{if } k \in G_t^P(j) \\ \{0\} & \text{if } k \in G_t^C(j). \end{cases}$$

We let $a_{t,k} \in A_k$ denote the *action* of agent k in period t, which is precisely the amount produced by agent k due to the linear production technology. Define the action space in the match $G_t(j)$ to be the Cartesian product of the action spaces $\mathbf{A}_{t,j} = \mathsf{X}_{k \in G_t(j)} A_k$ whose elements $\mathbf{a}_{t,j} = (a_{t,k})_{k \in G_t(j)}$ are called action profiles.

Focusing on pure strategies, define the payoff function for agent j by $v_{t,j} \colon \mathbf{A}_{t,j} \to \mathbb{R}$. That is, in period t the payoff to agent j depends only on the actions $\mathbf{a}_{t,j}$ taken in his match $G_t(j)$. It is assumed that payoff functions are common knowledge. If $G_t(j) = \{j\}$, i.e., if the agent is idle in an odd period, then his payoff is zero. Otherwise, since preferences differ in odd and even periods, we define

$$v_{t,j}(\mathbf{a}_{t,j}) = \begin{cases} \frac{1}{2}[u_o(c_{t,j}) - a_{t,j}] & \text{if } t \text{ is odd} \\ u_e(c_{t,j}) - a_{t,j} & \text{if } t \text{ is even}, \end{cases} \tag{3.1}$$

where for $k \in G_t^P(j)$ we have

$$c_{t,j} = \begin{cases} a_{t,k} & \text{if } j \neq k \text{ and } t \text{ is odd} \\ 0 & \text{if } j = k \text{ and } t \text{ is odd} \\ \liminf_{n \to \infty} \left[\frac{1}{n} \sum_{k \in \{1,\ldots,n\} \setminus \{j\}} a_{t,k} \right] & \text{if } t \text{ is even}. \end{cases}$$

An agent's utility depends on how much output the producers in his match deliver to him. His disutility depends on how much output he produces for his partners. If j is a producer in an odd period (with probability $\frac{1}{2}$), then his payoff is $-a_{t,j}$, i.e., the disutility from his labor effort. This effort allows agent j to deliver $a_{t,j}$ consumption to his partner $\beta_t(j)$. Instead, if j is a consumer, then his payoff is $u_o(a_{t,k})$ for $k \neq j$, i.e. it depends on the amount of consumption delivered to him by his partner (a producer). Since in even periods agent j is both a consumer and a producer, his payoff is the utility from consumption minus his disutility from labor. Note that, in calculating the amount $c_{t,j}$ consumed in even periods, we need to consider the function $\liminf$ since the limit of the sequence of averages of individual production $\left\{ \frac{1}{n} \sum_{k \in \{1,\dots,n\} \setminus \{j\}} a_{t,k} \right\}_{n=1}^{\infty}$ does not necessarily exist (see [2, p. 264]). This also has a desirable economic interpretation, since $c_{t,j}$ is simply the *smallest* average quantity that can be produced in the even period.

Let $\mathbf{a}_{t,-j}$ denote the action profile without the action of agent $j \in G_t(j)$, and of course $\mathbf{A}_{t,-j}$ is the set of all these action profiles. Denote the best response correspondence of agent j by $\rho_{t,j} \colon \mathbf{A}_{t,-j} \twoheadrightarrow A_j$, which is defined for each $\mathbf{a}_{t,-j} \in \mathbf{A}_{t,-j}$ by

$$\rho_{t,j}(\mathbf{a}_{t,-j}) = \quad \left\{ a_{t,j} \in A_j : \ v_{t,j}(\mathbf{a}_{t,-j}, a_{t,j}) = \max_{x_{t,j} \in A_j} v_{t,j}(\mathbf{a}_{t,-j}, x_{t,j}) \right\}.$$

A Nash equilibrium for the representative one-shot game is an action profile $\mathbf{a}_{t,j}$ such that $a_{t,k} \in \rho_{t,k}(\mathbf{a}_{t,-k})$ for all $k \in G_t(j)$, i.e., it is a fixed point of the best response correspondence for the match $G_t(j)$. Absence of production, i.e., autarky, is the unique Nash equilibrium of the static game.

Lemma 1. *In the representative static game of period t described above, $a_{t,k} = 0$ for all $k \in G_t(j)$ is the only Nash equilibrium.*

Proof. Consider the representative one-shot game in some period t. Assume that $\widehat{\mathbf{a}}_{t,j}$ is a Nash equilibrium of the game. This implies that $\widehat{a}_{t,k} \in \rho_{t,k}(\widehat{\mathbf{a}}_{t,-k})$ for all $k \in G_t(j)$, i.e., we must have $v_{t,k}(\widehat{\mathbf{a}}_{t,-k}, \widehat{a}_{t,k}) \geq v_{t,k}(\widehat{\mathbf{a}}_{t,-k}, a_{t,k})$ for all $k \in G_t(j)$ and all $a_{t,k} \in A_k$. If t is odd, this implies $u_o(c_{t,k}) - \widehat{a}_{t,k} \geq u_o(c_{t,k}) - a_{t,k}$, hence $\widehat{a}_{t,k} \leq a_{t,k}$ for all k and $a_{t,k}$. It easily follows that in order for $\widehat{\mathbf{a}}_{t,j}$ to be a Nash equilibrium then we must have $\widehat{a}_{t,k} = a_{t,k} = 0$ for all $k \in G_t(j)$, since $v_{t,k}(\widehat{\mathbf{a}}_{t,-k}, a_{t,k})$ is strictly decreasing in $a_{t,k}$ and $0 \in A_k$. An analogous argument applies to even periods. ■

In the static game agents realize the autarkic (or *minmax*) payoff since producers cannot be forced, nor can they commit, to make transfers. Thus, each producer j can always select $a_{t,j} = 0$ and enjoy a payoff $v_{t,j}(\mathbf{a}_{t,j}) \geq 0$. This is a key feature that we exploit later.

3.2 The infinite horizon game

The infinite horizon game is a sequence of even and odd period static games that alternate indefinitely. Specifically, it consists of the population J, the matching process in (2.1), the

action sets described earlier, and payoff functions defined in this section. Notice that this game is one with varying opponents, since no one interacts with a fixed set of partners in every period. It is also a game of imperfect monitoring since during a period t agent $k \in G_t(j)$ observes only the action profile $\mathbf{a}_{t,j}$ in his match, but not in other matches. Thus, before defining payoff functions, we must discuss what information on past play is available to an agent.

At the start of period $t \geq 1$ the information available to agent j can be summarized by the history of actions $h_{t,j}$ he has privately observed in all dates $\tau < t$, with

$$h_{t,j} = (\mathbf{a}_{0,j}, \ldots, \mathbf{a}_{t-1,j}),$$

and we let $h_{0,j} = 0$. The set of histories of j is the Cartesian product $H_{t,j} = \mathsf{X}_{\tau=0}^{t-1} \mathbf{A}_{\tau,j}$, and the history profile of the match at the start of period t is denoted $\mathbf{h}_{t,j} = (h_{t,k})_{k \in G_t(j)}$. Because of random bilateral matches, the elements of $\mathbf{h}_{t,j}$ will generally differ, i.e., partners do not have common histories. It is then conceivable that, due to anonymity and enforcement limitations, agents may be tempted to use these informational disparities to act in a manner that is socially undesirable. To see why, we must study the behavior of the representative agent j.

Define agent j's pure strategy for the infinite horizon game, as the infinite sequence of maps

$$\sigma_j = (s_{0,j}, s_{1,j}, \ldots),$$

where $s_{t,j} : H_{t,j} \to A_j$, the map of the set of histories of agent j into his action set, is defined by $s_{t,j}(h_{t,j}) = a_{t,j}$. Denote the strategy profile in the match $G_t(j)$ by

$$s_t(\mathbf{h}_{t,j}) = (s_{t,k}(h_{t,k}))_{k \in G_t(j)} = \mathbf{a}_{t,j}.$$

Since action sets do not depend on histories, let the sequence of mappings

$$S_{t,j} = \left(A_j^{H_{\tau,j}} \right)_{\tau=t}^{\infty}$$

denote the strategy space of agent j in the subgame starting at period $t \geq 0$, and denote it $S_j = S_{0,j}$ for the infinite horizon game.[6] It follows that every (pure) strategy σ_j gives rise to a strategy $\sigma_{t,j}$ in the subgame starting at t, with $\sigma_{t,j} = (s_{t,j}, s_{t+1,j}, \ldots) \in S_{t,j}$ and $\sigma_{0,j} = \sigma_j$. Finally, let $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots)$ define the collection of strategies of the population J, using $\boldsymbol{\sigma}_t = (\sigma_{t,1}, \sigma_{t,2}, \ldots)$ for the subgame starting in t. Let $\boldsymbol{\sigma}_{-j}$ denote $\boldsymbol{\sigma}$ excluding the strategy σ_j of agent j.

To discuss the payoff to an agent $j \in J$, in the infinite horizon game, let δ_{t+1} denote the discount factor between periods $t > 0$ and $t + 1$. We have

$$\delta_{t+1} = \begin{cases} \varepsilon & \text{if} \quad t \text{ odd} \\ \delta & \text{if} \quad t \text{ even}. \end{cases} \tag{3.2}$$

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[6]For two sets A and B the notation A^B denotes the set of all mappings from B to A.

The factor $\Delta_t(\tau) = \prod_{n=t+1}^{\tau} \delta_n$ will be used to discount back to period t a payoff realized in period $\tau \geq t+1$. The payoff to agent j is the function $V_j \colon \mathsf{X}_{i \in J} S_i \to \mathbb{R}$ defined by

$$V_j(\boldsymbol{\sigma}) = \widehat{v}_0(s_0(\mathbf{h}_{0,j})) + \sum_{\tau=1}^{\infty} \Delta_0(\tau)\widehat{v}_\tau\big(s_\tau(\mathbf{h}_{\tau,j})\big). \tag{3.3}$$

Here,

$$\widehat{v}_t(\mathbf{a}_{t,j}) = \begin{cases} \frac{\alpha}{2}[u_o(c_{t,j}) - a_{t,j}] & \text{if } t \text{ is odd} \\ u_e(c_{t,j}) - a_{t,j} & \text{if } t \text{ is even}, \end{cases} \tag{3.4}$$

is an expected period utility, since in odd periods the agent is consumer or producer with equal probability $\frac{\alpha}{2}$, and earns zero payoff otherwise. Therefore, V_j is the present value of the stream of expected utilities from $t = 0$ on. Since each period $t \geq 1$ defines a proper subgame, we can formalize recursively agent j's payoff in the subgame starting in t by

$$V_{t,j}(\boldsymbol{\sigma}_t) = \widehat{v}_t(s_t(\mathbf{h}_{t,j})) + \delta_{t+1}V_{t+1,j}(\boldsymbol{\sigma}_{t+1}), \tag{3.5}$$

with $V_{0,j} = V_j$. The first term on the right hand side of the functional equation (3.5) represents agent j's current expected utility and the remainder his discounted future payoff.

The best response correspondence of agent j, for the infinite horizon game, is thus

$$\rho_j(\boldsymbol{\sigma}_{-j}) = \left\{ \sigma_j \in S_j \colon \ V_j(\boldsymbol{\sigma}_{-j}, \sigma_j) = \max_{x_j \in S_j} V_j(\boldsymbol{\sigma}_{-j}, x_j) \right\},$$

and the aggregate best response correspondence is $\mathbf{R}(\boldsymbol{\sigma}) = \mathsf{X}_{j \in J}\rho_j(\boldsymbol{\sigma}_{-j})$. The notion of equilibrium for the infinite horizon game is as follows.

Definition 2. *A **subgame perfect Nash equilibrium** for the infinite horizon game is a strategy profile $\boldsymbol{\sigma}$ such that $\boldsymbol{\sigma} \in \mathbf{R}(\boldsymbol{\sigma})$.*

In equilibrium σ_j must be a fixed point of agent j's best response correspondence, i.e., $\sigma_j \in \rho_j(\boldsymbol{\sigma}_{-j})$ for all $j \in J$. Since repeated play of a static game does not decrease the set of equilibrium payoffs, we have that autarky forever is an equilibrium.

Lemma 3. *The strategy $\sigma_j = (0, 0, \ldots)$ for all $j \in J$ is a subgame perfect Nash equilibrium of the infinite horizon game.*

Proof. By Lemma 1, the one-shot Nash equilibrium in any period t is autarky, i.e., $a_{t,j} = 0$ for all $j \in J$. Now fix a period $\tau \geq 1$. Then the strategy "each player j plays $a_{t,j} = 0$ for $t \geq \tau$," is a subgame perfect equilibrium. According to this strategy, the actions taken by player j's future opponents are independent of his current play. In addition, $a_{t,j} = 0$ maximizes period t payoff of agent j. Thus, $\sigma_j = (0, 0, \ldots)$ for all $j \in J$ is a Nash equilibrium of the infinite horizon game. ∎

Permanent autarky is associated with zero payoff to every agent, so it is the worst allocation this economy can achieve. We now demonstrate that there exists a unique socially desirable allocation of labor effort and consumption, called the 'efficient allocation.'

4 The efficient allocation and the optimal trading plan

To find the efficient allocation in this economy, we consider the problem faced by a planner who selects patterns of production and exchange subject to the same physical restrictions faced by agents. In particular, the planner treats agents identically but cannot transfer consumption across matches and over time. The problem is thus to maximize the lifetime utility of the representative agent $j \in J$. Since this agent is in the match $G_t(j)$ in period t, we define relevant action profile and action sets by

$$\mathbf{a}_t = (a_{t,k})_{k \in G_t(j)}, \quad \mathbf{A}_t = \mathsf{X}_{k \in G_t(j)} A_k, \quad \text{and} \quad \mathbf{A} = \mathsf{X}_{t=0}^{\infty} \mathbf{A}_t,$$

omitting the index j, if understood. If we let

$$V(\mathbf{a}) = \widehat{v}_0(\mathbf{a}_0) + \sum_{\tau=1}^{\infty} \Delta_0(\tau) \widehat{v}_\tau(\mathbf{a}_\tau)$$

then the planner's problem is to choose a plan $\mathbf{a} = (\mathbf{a}_t)_{t=0}^{\infty} \in \mathbf{A}$ to solve

$$
\begin{aligned}
&\text{Maximize:} \quad V(\mathbf{a}) \\
&\text{subject to:} \quad a_{t,k} = a_t \text{ for all } k \in G_t^P(j) \text{ and } t \geq 1,
\end{aligned}
\tag{4.1}
$$

where, clearly, $a_{t,k} = 0$ for all $k \in G_t^C(j)$ and $t \geq 1$. The main result is that there exists a unique optimal plan.

Theorem 4. *An optimal plan $\mathbf{a}^* \in \mathbf{A}$ exists and it is unique.*

Proof. We prove the statement by demonstrating that V is a continuous and strictly concave function defined on a compact set.

We start by demonstrating that the function $V \colon \mathbf{A} \to \mathbb{R}$ defined in (4.1) is continuous.

Recall that the functions $\widehat{v}_t \colon \mathbf{A}_t \to \mathbb{R}$ are continuous and uniformly bounded for all t, by assumption. Thus, for all t and all $\mathbf{a}_t \in \mathbf{A}_t$, there exists an $M > 0$ such that $|\widehat{v}_t(\mathbf{a}_t)| \leq M$. Let

$$\mathbf{a}^n = (\mathbf{a}_0^n, \mathbf{a}_1^n, \ldots) \xrightarrow[n \to \infty]{} \mathbf{a} = (\mathbf{a}_0, \mathbf{a}_1, \ldots)$$

hold true in the product topology. That is, $\mathbf{a}_t^n \xrightarrow[n \to \infty]{} \mathbf{a}_t$ for all $t = 0, 1, 2, \ldots$. Since $\widehat{v}_t$ is bounded, it follows from the triangle inequality that

$$|\widehat{v}_t(\mathbf{a}_t^n) - \widehat{v}_t(\mathbf{a}_t)| \leq |\widehat{v}_t(\mathbf{a}_t^n)| + |\widehat{v}_t(\mathbf{a}_t)| \leq 2M.$$

To prove the continuity of V, we must show that $V(\mathbf{a}^n) \xrightarrow[n \to \infty]{} V(\mathbf{a})$. If we fix $\epsilon > 0$, then we need to show that there exists some $n_0 > 0$ such that $|V(\mathbf{a}^n) - V(\mathbf{a})| < \epsilon$ for all $n \geq n_0$. Since $|\widehat{v}_t(\mathbf{a}_t)| \leq M$, we start by picking a natural number $t_1 \in (0, \infty)$ such that

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$\sum_{t=t_1+1}^{\infty} \Delta_0(t) M < \frac{\epsilon}{4}$. Since the functions $\widehat{v}_t$ are continuous, there exists some $n_1 > 0$ such that for all $n \geq n_1$ we have

$$|\widehat{v}_0(\mathbf{a}_0^n) - \widehat{v}_0(\mathbf{a}_0)| + \sum_{t=1}^{t_1} \Delta_0(t) \, |\widehat{v}_t(\mathbf{a}_t^n) - \widehat{v}_t(\mathbf{a}_t)| < \frac{\epsilon}{2} \, .$$

Thus, choosing $n_0 \geq n_1$ we see that for all $n \geq n_0$ we have

$$\begin{aligned}
&\left| V(\mathbf{a}^n) - V(\mathbf{a}) \right| \\
&\leq \left| \widehat{v}_0(\mathbf{a}_0^n) - \widehat{v}_0(\mathbf{a}_0) \right| + \sum_{t=1}^{t_1} \Delta_0(t) \left| \widehat{v}_t(\mathbf{a}_t^n) - \widehat{v}_t(\mathbf{a}_t) \right| + \sum_{t=t_1+1}^{\infty} \Delta_0(t) 2M \\
&< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon \, ,
\end{aligned}$$

which proves the continuity of V.

We now proceed by demonstrating that there is an optimal plan $\mathbf{a}^* \in \mathbf{A}$. Note that $\mathbf{A}$ is a compact space, by the Tychonoff Product Theorem (see [1]). Furthermore, V is continuous. So, by the classical Weierstrass' theorem, V has a maximizer, say $\mathbf{a}^* \in \mathbf{A}$. To prove that $\mathbf{a}^*$ is unique, note that the functions $\widehat{v}_t$ are assumed strictly concave for all t. This implies that V is strictly concave as well, and so the maximizer $\mathbf{a}^*$ is unique. ∎

The optimal plan $\mathbf{a}^*$ assigns a positive amount of consumption to every buyer in every match. To see this, one needs to realize that the planner cannot transfer resources over time and that current choices do not affect future states (aggregate or individual). Consequently, solving the Problem 4.1 is equivalent to solving a sequence of static maximization problems. In every period t, the planner chooses $\mathbf{a}_t$ to maximize $\widehat{v}_t(\mathbf{a}_t)$.

When $t = 0$ agents are idle and thus the maximizer is $a_{0,k} = a_0^* = 0$ for all $k \in J$. Now consider a period $t \geq 1$. Since $a_{t,k} = 0$ for each $k \in G_t^C(j)$ and since each producer is treated identically, i.e., $a_{t,k} = a_t$ for all $k \in G_t^P(j)$, it follows from (3.1) and (3.4) that the objective function in a period t is $\widehat{v}_t(\mathbf{a}_{t,j})$ with the restriction $c_{t,j} = a_{t,j} = a_t$ for all $j \in J$. The maximizers are given by the production quantities

$$a_t^* = \begin{cases} c_o^* & \text{if } t \text{ odd} \\ c_e^* & \text{if } t \text{ even} \, . \end{cases} \tag{4.2}$$

Since $u_o'(c_o^*) = 1$ and $u_e'(c_e^*) = 1$, the optimal plan requires that each producer delivers the surplus-maximizing quantity, in each period. We are now ready to prove that if agents are sufficiently patient, then the optimal plan $\mathbf{a}^*$ can arise as an equilibrium of the infinite horizon game.

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5   A social norm for economic interactions

The key element of analysis in this section is the behavior of producers, since consumers are inactive players. Thus, for expositional ease, we focus exclusively on the choices of a representative producer $j \in J$ in a period t. Recall that the optimal plan (4.2) requires that every producer delivers the surplus-maximizing quantity to his partner(s). In the absence of a medium of exchange, sustaining this plan is difficult due to absence of commitment and lack of formal enforcement.

The work in [12, 19], however, suggests that we can sustain the efficient allocation using a social norm, i.e., a strategy involving an informal enforcement scheme. Specifically, we propose a strategy, called 'altruistic,' that specifies 'desirable' actions (a rule of cooperation) as well as sanctions for 'undesirable' actions (a rule of punishment). We identify desirable behavior with production decisions conforming with the optimal plan, and label every other action as undesirable.

Therefore, let $a_{t,k} = a_t^*$ define a desirable action of producer $k \in G_t^P(j)$ in period t, and let any $a_{t,k} \neq a_t^*$ be undesirable. If we consider agent j in period $t \geq 1$, then we define the desirable history by $h_{t,j}^* = (\mathbf{a}_{0,j}^*, \mathbf{a}_{1,j}^*, \ldots, \mathbf{a}_{t-1,j}^*)$. That is, agent j has observed only desirable behavior up to period t if and only if every producer in his past matches (including agent j himself) followed the optimal plan. Any history $h_{t,j} \neq h_{t,j}^*$, is therefore undesirable because some producer, possibly agent j himself, was seen making a choice that departed from the optimal plan. This leads to the following definition of an altruistic strategy.

Definition 5. *A strategy $\sigma_j^* = (s_{0,j}, s_{1,j}, \ldots)$ for a producer $j \in J$ is called altruistic, if $s_{0,j} = a_0^*$ and in each period $t \geq 1$ he selects*

(i) *$s_{t,j}(h_{t,j}) = a_t^*$, whenever $h_{t,j} = h_{t,j}^*$, and*

(ii) *$s_{\tau,j}(h_{\tau,j}) = 0$ for all $\tau \geq t$, whenever $h_{t,j} \neq h_{t,j}^*$.*

Thus, the altruistic strategy requires that every producer delivers c_t^* consumption to his partners, only if he has observed desirable behavior. However, the producer should play the minmax strategy autarky forever, as soon as he deviates or has knowledge of a deviation. The threat that such a harsh informal punishment spreads to the entire economy, is what can sustain the optimal plan as a subgame perfect equilibrium. We prove it in the following subsections, where for convenience we let $\varepsilon = 1$.

5.1   Individual optimality

Suppose that every agent follows the altruistic strategy $\boldsymbol{\sigma}^*$, and consider the behavior of a representative agent $j \in J$, in some period t. Denote his expected lifetime utility, or continuation payoff, at the beginning of period t by $V_t = V_{t,j}(\boldsymbol{\sigma}_t^*)$, where the subscript j and the argument of the function are omitted since they are fixed. We say that in

period t we are 'in equilibrium,' if the agent has observed only desirable behavior, and 'off equilibrium,' otherwise.

Denote by V_e^* and V_o^* the equilibrium continuation payoffs, restricted to even or odd periods. They are time-invariant since the strategy and the structure of the game are time-invariant in equilibrium, so each subgame is a replica of the infinite horizon game. Using (3.4), (3.5), and (4.2), we have

$$
\begin{aligned}
V_e^* &= \tfrac{1}{1-\delta}\left\{ u_e(c_e^*) - c_e^* + \delta\tfrac{\alpha}{2}[u_o(c_o^*) - c_o^*]\right\}\\
V_o^* &= \tfrac{1}{1-\delta}\left\{ \tfrac{\alpha}{2}[u_o(c_o^*) - c_o^*] + u_e(c_e^*) - c_e^*\right\}.
\end{aligned}
\tag{5.1}
$$

To study the individual optimality of σ_j^* it suffices to consider one-time deviations in a representative subgame starting in some period t, by the unimprovability criterion. Since the altruistic strategy specifies actions to be taken both in- and off-equilibrium, we must examine one-period deviations in both contingencies.

To this end, consider an agent in some period t, off-equilibrium, under the conjecture that everyone else plays the altruistic strategy. We denote by V_e^d and V_o^d the continuation payoffs (in even and odd periods) of this agent if he observed a deviation for the first time in the prior period. We use the notation $\tilde{V}_e$ and $\tilde{V}_o$, if the deviation was observed earlier. Specifying these payoffs as time-invariant is without loss in generality since a deviation becomes part of the history of the entire population in no more than two periods. All this will be clarified next.

Suppose that t is an odd period and that everyone has played the altruistic strategy up to this date, but that the producer in the match $G_t(j)$ deviates from the optimal plan $\mathbf{a}^*$. In this off-equilibrium contingency we have $h_{t+1,k} \neq h_{t+1,k}^*$ for $k \in G_t(j)$, while $h_{t+1,k} = h_{t+1,k}^*$ for all $k \in J\backslash\{G_t(j)\}$. This deviation implies that j and his partner will select the minmax strategy forever after period t, i.e., $a_{\tau,k} = 0$ in all $\tau \geq t+1$ for $k \in G_t(j)$. However, everyone else follows the optimal plan in $t+1$, since they have not observed a deviation. Thus, in period $t+1$, the continuation payoff of agent $k \in G_t(j)$ from following the altruistic strategy $\sigma_{t+1,k}^*$, under the conjecture that everyone also does the same, is

$$
V_e^d = u_e(c_e^*) - 0 + \delta\tilde{V}_o.
\tag{5.2}
$$

Since $t+1$ is an even period—in which every agent is a producer—and the deviation in period t was observed only by the two agents in $G_t(j)$, it follows that only j and his partner will elect to produce nothing in $t+1$. However, everyone will observe their deviations in period $t+1$, and so $\tilde{V}_o = 0$. To see why, notice that $G_{t+1}(j) = J$. Consequently, since $a_{t+1,k} = 0$ for $k \in G_t(j)$, we have $h_{t+2,k} \neq h_{t+2,k}^*$ for all $k \in J$. Under the premise that agents follow the altruistic strategy, then $a_{\tau,k} = 0$ for all $\tau \geq t+2$ and all $k \in J$. Consequently, $V_{\tau,k} = 0$ for all $\tau \geq t+2$ and all $k \in J$, which implies $\tilde{V}_o = 0$.

Clearly, if a deviation occurs for the first time in a centralized market, then the continuation payoff of every agent in the population is

$$
V_o^d = 0 + \tilde{V}_e = 0,
\tag{5.3}
$$

where clearly $\tilde{V}_e = 0$. Indeed, if everyone plays the altruistic strategy, all production shuts down permanently following a deviation in the centralized market.

The lesson is that a deviation from the optimal plan, in any match, eventually shuts down trade in the economy. It takes two periods for this to happen if the deviation occurs in a decentralized market, and one period otherwise. Intuitively, if actions are observable, multilateral matches allow information to flow across a significant fraction of the population. Small group trade—such as pairwise random trade—can slow down the transfer of information, but cannot prevent it simply because agents are anonymous.

5.2 Sustaining the optimal plan

In this subsection we provide a condition that ensures sustainability of the informal enforcement scheme at the core of the altruistic strategy. To derive it, we must verify not only that equilibrium deviations be suboptimal, but sanctioning everyone after observing a deviation must also be in the agent's best interest. After all, off equilibrium an agent might be tempted to keep producing in order to avoid the spread of the sanction and permanent autarky.

Theorem 6. *If we let*

$$\underline{\delta} = \frac{c_o^* + c_e^*}{c_o^* + u_e(c_e^*) + \frac{\alpha}{2}\left[u_o(c_o^*) - c_o^*\right]} \,,$$

then for each $\delta \geq \underline{\delta}$ the altruistic strategy $\boldsymbol{\sigma}^*$ supports the optimal plan as a subgame perfect Nash equilibrium of the infinite horizon game.

Proof. We need to show that, under the conjecture that everyone else plays according to $\boldsymbol{\sigma}^*_{-j}$, the representative agent j cannot profitably deviate from the altruistic strategy either in-equilibrium or off-equilibrium.

To start, consider an off-equilibrium situation in which partners in the match $G_t(j)$ observe a deviation, for the first time. We will derive a condition, in terms of the parameters of the model, guaranteeing that off-equilibrium deviations from σ_j^* are unprofitable. That is, we find a condition under which it is optimal to play the minmax strategy from period $t+1$. We need to consider two cases:

(i) t is odd: every agent $k \in G_t(j)$ is a producer in period $t+1$. Thus, he sanctions others as specified by the altruistic strategy if this maximizes his payoff, i.e., using (5.2) we need

$$u_e(c_e^*) - 0 + \delta\tilde{V}_o \geq u_e(c_e^*) - a_{t+1,k} + \delta\tilde{V}_o \,.$$

The left hand side represents the payoff, off-equilibrium, from selecting the sanction $a_{t+1,k}^* = 0$. The right hand side is the payoff from choosing not to sanction. Clearly, this inequality holds for all $a_{t+1,k} \in A_k$, and especially for $a_{t+1,k} = c_e^*$.

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(ii) t is even: any agent $k \in J$ who is a producer in period $t+1$ follows the altruistic strategy since

$$0 + \tilde{V}_e \geq -a_{t+1,k} + \tilde{V}_e$$

holds for all $a_{t+1,k} \in A_k$, and especially for $a_{t+1,k} = c_o^*$.

Hence, it is individually optimal to play minmax forever, as soon as the agent deviates or detects a deviation from $\boldsymbol{\sigma}^*$. Intuitively, producing for others not only decreases the agent's current payoff, but it does not slow down the spread of sanctioning behavior in the economy. Indeed, even if the agent chooses not to sanction others, his trading partners (who follow the altruistic strategy) will do so.

We now consider in-equilibrium deviations in an arbitrary period t.

(i) t is odd: every producer $k \in G_t(j)$ follows the altruistic strategy whenever

$$-c_o^* + V_e^* \geq 0 + V_e^d.$$

Using (5.1) and (5.2), this inequality becomes

$$\tfrac{\delta}{1-\delta} \left\{ \tfrac{\alpha}{2} \left[u_o(c_o^*) - c_o^* \right] + u_e(c_e^*) - c_e^* \right\} \geq c_o^* + c_e^*. \tag{5.4}$$

That is, the present value of the net loss from deviating must exceed the current net gain from deviating. The latter is $c_o^* + c_e^*$ since the deviator avoids production in both the odd, and even period, before the economy shuts down. Expression (5.4) yields

$$\delta \geq \delta_o \quad = \quad \frac{c_o^* + c_e^*}{c_o^* + u_e(c_e^*) + \tfrac{\alpha}{2}[u_o(c_o^*) - c_o^*]}.$$

(ii) t is even: every agent is a producer and follows the altruistic strategy whenever

$$u_e(c_e^*) - c_e^* + \delta V_o^* \geq u_e(c_e^*) + 0 + \delta V_o^d.$$

Since V_o^* satisfies (5.1) and V_o^d satisfies (5.3), we see that

$$\tfrac{\delta}{1-\delta} \left\{ \tfrac{\alpha}{2} \left[u_o(c_o^*) - c_o^* \right] + u_e(c_e^*) - c_e^* \right\} \geq c_e^*$$

holds whenever

$$\delta \geq \quad \delta_e = \frac{c_e^*}{u_e(c_e^*) + \tfrac{\alpha}{2}[u_o(c_o^*) - c_o^*]}.$$

From $u_e(c_e^*) > c_e^*$ and $u_o(c_o^*) > c_o^*$, we get $0 < \delta_e < \delta_o < 1$. The intuition behind $\delta_o > \delta_e$, is that when a deviation occurs in an odd period, the informal punishment takes place with one period delay, so that agents need to be more patient to willingly follow the optimal plan. Finally, let $\underline{\delta} = \delta_o$. ∎

Theorem 6 establishes that if agents are sufficiently patient, and can observe their partners' actions, then there exists a subgame perfect equilibrium supporting the optimal plan. This holds for any population J, countable or uncountable, since individual actions are assumed observable without noise (see [24, p. 1161]) so that, as soon as the centralized markets opens, everyone can be informed of a privately observed deviation. This encourages desirable behavior in every random bilateral match even if agents are anonymous and there is no formal enforcement institution. If we associate lack of formal enforcement to inability to impose taxation, we have an additional result.

Corollary 7. *The allocation associated to the optimal plan is generally unattainable in a monetary equilibrium, but is attainable in a non-monetary equilibrium if agents are sufficiently patient.*

Proof. Let π denote the gross inflation rate in a stationary monetary equilibrium. It is immediate from equation (19) in [23], that $c_o < c_o^*$ for all $\pi > \delta$, and $\lim_{\pi \to \delta} c_o = c_o^*$. For example, if pricing is competitive in every period, it is simple to demonstrate that

$$c_o = (u_o')^{-1} \left(\tfrac{2(\pi - \delta)}{\alpha \delta} + 1 \right).$$

For details see the proof of Proposition 1 in [8], letting $\gamma = \pi$, $\alpha = 1$, and $q_1 = c_0$. Since no enforcement implies $\pi \geq 1$ then we have $c_0 < c_0^*$. ∎

To sum up, assuming anonymous agents and decentralized trade does not imply that money is essential. What matters is how information about privately observed actions can spread in the economy, which in turn depends on the assumed matching process. However, we emphasize that the equilibrium we derive is not robust to adding a small amount of noise in the observation of individual behavior. Equilibria would arise similar to those in an economy with a continuum population where individual behavior is unobservable (see [6, 13, 24]).

6   Stochastic trading cycles

We have proved that the existence of centralized markets can discourage defections from socially desirable behavior, but it may be argued that this holds because centralized trading has rapid and deterministic periodicity. In this section we generalize our basic result to environments in which knowledge of deviations spreads slowly and randomly.

For example, this may happen if not everyone participates in centralized markets regularly, if there are many spatially separated centralized markets to which agents are randomly assigned, or if the centralized market opens after a random sequence of decentralized trading dates. We choose to follow this last route, for simplicity. Assume that decentralized trade follows a round of centralized trade, but centralized trade occurs after a round

of bilateral trade with time-invariant probability $b \in (0,1)$. Thus, if t is a period of decentralized trade, then centralized trade occurs in period $t + n$, $n \geq 1$, with probability $b(1-b)^{n-1}$. Hence, there is an expected delay of

$$\sum_{n=1}^{\infty} nb(1-b)^{n-1} = \frac{1}{b}$$

periods, before centralized trade takes place. Since we deal with infinite series, we need $\varepsilon \in (0,1)$. That is, agents always discount adjacent periods.

To distinguish between periods with centralized and decentralized trade, let q denote consumption and U the utility function in a round of centralized trading (instead of u_e), and let c denote consumption and u the utility function in a round of decentralized trading (instead of u_o). Given that the altruistic strategy $\boldsymbol{\sigma}^*$ is followed, consider the behavior of a representative agent $j \in J$, in some period t.

Once again, the equilibrium continuation payoffs restricted to centralized and decentralized markets are time-invariant. Hence, denote expected lifetime utility at the start of a round of centralized trading by V_C, and V_D at the start of the first round of decentralized trading that follows it. We use $\widetilde{V}$ to denote the agent's expected lifetime utility at the start of any decentralized trading round. Specifically, we have:

$$
\begin{aligned}
V_C \quad &= U(q^*) - q^* + \delta V_D \\
V_D \quad &= \frac{\alpha}{2}[u(c^*) - c^*] + \widetilde{V} \\
\widetilde{V} \quad &= \varepsilon b \sum_{n=0}^{\infty} (1-b)^n \varepsilon^n [U(q^*) - q^* + \delta V_D] \\
&\quad + b \sum_{n=1}^{\infty} (1-b)^n \sum_{j=1}^{n} \varepsilon^j \frac{\alpha}{2}[u(c^*) - c^*]
\end{aligned}
\tag{6.1}
$$

Using (6.1) we obtain the closed-form solutions

$$
\begin{aligned}
\widetilde{V} \quad &= \frac{1}{1-\delta\Delta_1} \left\{ \Delta_1[U(q^*) - q^*] + \frac{\alpha}{2}[u(c^*) - c^*](\delta\Delta_1 + \Delta_2) \right\} \\
V_C \quad &= \frac{1}{1-\delta\Delta_1} \left\{ [U(q^*) - q^*] + \delta\frac{\alpha}{2}[u(c^*) - c^*](1 + \Delta_2) \right\} \\
V_D \quad &= \frac{1}{1-\delta\Delta_1} \left\{ \Delta_1[U(q^*) - q^*] + \frac{\alpha}{2}[u(c^*) - c^*](1 + \Delta_2) \right\},
\end{aligned}
\tag{6.2}
$$

where simple algebraic manipulation indicates that

$$\Delta_1 = \frac{b\varepsilon}{1-(1-b)\varepsilon} \quad \text{and} \quad \Delta_2 = \frac{\varepsilon}{1-(1-b)\varepsilon} - \Delta_1.$$

Clearly, when $b, \varepsilon \to 1$, we have $\Delta_1 \to 1$ and $\Delta_2 \to 0$, so we get back (5.1).

Now economy-wide punishment can be triggered only stochastically so there is a stronger incentive to defect, relative to the two-period cycle economy. This reduces the set of values that can be assigned to δ, but does not change the nature of the main result.

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Theorem 8. *If we let*

$$\underline{\underline{\delta}} = \frac{c^*\left(1+\frac{\alpha}{2}\Delta_2\right)+q^*\Delta_1}{\Delta_1\left\{\frac{\alpha}{2}[u(c^*)-c^*]+\left[c^*+\frac{\alpha}{2}u(c^*)\Delta_2+U(q^*)\Delta_1\right]\right\}} \quad > \underline{\delta}\,,$$

then for every $\delta \geq \underline{\underline{\delta}}$ the altruistic strategy $\boldsymbol{\sigma}^*$ supports the optimal plan as a subgame perfect Nash equilibrium of the infinite horizon game with random centralized markets.

Proof. We need to show that, under the conjecture that everyone else plays according to $\boldsymbol{\sigma}^*$, the representative agent can profitably deviate from the altruistic strategy neither in- nor off-equilibrium. To start, consider in-equilibrium deviations, distinguishing between periods with centralized and decentralized trading.

(i) Decentralized trading in t: every producer $k \in G_t(j)$ follows the altruistic strategy whenever

$$-c^* + \widetilde{V} \geq 0 + \varepsilon b \sum_{n=0}^{\infty}(1-b)^n \varepsilon^n \left[U(q^*) + \delta \times 0\right] + b\sum_{n=1}^{\infty}(1-b)^n \sum_{j=1}^{n} \varepsilon^j \frac{\alpha}{2}[u(c^*)]\,.$$

Notice that we are assuming the most extreme form of informational isolation in every random matching cycle, i.e., not only agents are never paired more than once, but they also never meet anyone who is indirectly related to any of their previous random partners. The implication is that if one deviates in a bilateral match today, this will be ignored by every agent he will meet before the centralized market opens. This is why $u(c^*)$ appears as the last element of the inequality. Of course, this means that if agents do not have incentives to deviate in this scenario, they certainly do not have incentives to deviate in economies with weaker restrictions on the path of random encounters (see also [4]).

Using (6.1), the inequality above yields

$$\delta\Delta_1\left\{\frac{\alpha}{2}\left[u(c^*)-c^*\right] + \left[c^* + \frac{\alpha}{2}u(c^*)\Delta_2 + U(q^*)\Delta_1\right]\right\} \geq c^*\left(1+\frac{\alpha}{2}\Delta_2\right) + q^*\Delta_1\,,$$

which never holds true as $b \to 0$, i.e., as we converge to the typical random matching model of money. When $0 < b < 1$, instead, it holds true if

$$\delta \geq \quad \delta_D = \frac{c^*\left(1+\frac{\alpha}{2}\Delta_2\right)+q^*\Delta_1}{\Delta_1\left\{\frac{\alpha}{2}[u(c^*)-c^*]+c^*+\frac{\alpha}{2}u(c^*)\Delta_2+U(q^*)\Delta_1\right\}}\,.$$

(ii) Centralized trading in t: every agent is a producer and follows the altruistic strategy whenever

$$U(q^*) - q^* + \delta V_D \geq U(q^*) + \delta V_D^d\,.$$

Since V_D satisfies (6.2) and $V_D^d = 0$, it follows that

$$\delta\left\{U(q^*)\Delta_1 + \frac{\alpha}{2}[u(c^*) - c^*](1+\Delta_2)\right\} \geq q^*\,.$$

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This implies that

$$\delta \geq \quad \delta_C = \frac{q^*}{U(q^*)\Delta_1 + \frac{\alpha}{2}[u(c^*)-c^*](1+\Delta_2)} \ .$$

Since $U(q^*) > q^*$ and $u(c^*) > c^*$, we see that $0 < \delta_C < \delta_D < 1$.[7] That is, if the representative agent prefers to play according to σ_j^* in a decentralized market, then he certainly does so in a centralized market. It is possible to show in a manner analogous to Theorem 6 that if it is not optimal to deviate in equilibrium, then it is certainly not optimal to deviate off-equilibrium. Finally, let $\underline{\delta} = \delta_D$. Notice that $\underline{\delta} > \underline{\delta}$ since if a deviation occurs in a bilateral random match, an economy-wide punishment is expected to take place with $\frac{1}{b}$ periods of delay, so that agents need to be more patient than when centralized trade follows deterministically a round of decentralized trade. ∎

If we interpret the discount rates δ and ε as probabilities of continuation of the game, this result shows that the game must be sufficiently likely to continue once information about a deviation has reached everyone. The size of δ depends on how fast information can be transmitted (the value of b), and on the average number of decentralized trading rounds (depending on ε). For example any $b \in (0,1)$ can sustain the altruistic strategy when $\delta = \varepsilon \to 1$, since a deviation in a bilateral market is on average communicated to the entire population after $\frac{1}{b}$ periods, which is a finite number.

If $b \to 0$ then the infinite horizon game converges to the typical search model of money, i.e., a repeated random matching game among an infinite number of agents. Here, the altruistic strategy cannot be supported, since $\Delta_1 \to 0$ and so there is no possibility of communicating a deviation to a sufficiently large number of agents.[8] It is important to realize, however, that this happens not just because of random pairings. The central friction is obstacles to information transfers across groups of traders. To prove it, we now construct a general matching framework in which agents deterministically enter infinitely large trading groups and yet money is essential.

7 Matching and information

We have seen that it is the presence of obstacles to rapid and widespread information transmission that prevents the sustainability of the optimal plan. The random pairing scheme assumed in models such as [21], naturally justifies such obstacles and—together with other assumptions—generates an explicit role for money. In this section, we demonstrate that there is nothing special about random bilateral matching in achieving this goal. This is done by introducing a matching process that repeatedly partitions the population

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[7]We have $\delta_C < \delta_D$, whenever $\Delta_1 [U(q^*) - q^*] + \frac{\alpha}{2}[u(c^*) - c^*] (1 + \Delta_2) \geq 0$.

[8]See [19, Proposition 3]. See also [7] where it is shown that social norms have no role to play in the random matching model of [21].

into groups with infinitely many partners. We prove that here, too, partners are complete strangers and autarky is the only subgame perfect equilibrium. This allows the construction of models with large (possibly Walrasian) markets in which money is essential, since no two agents (neither their direct and indirect partners) will ever be in the same market more than once.

7.1   A formalization for exogenous matching processes

The analysis in this subsection is based on the formalization developed in [3] and [4], to which we refer the reader for details and proofs of some claims. For the sake of brevity, here we simply sketch the procedure, which consists of the following steps. First, in the initial date, we partition the population into spatially separated sets of agents, called "clusters." Then, we match the agents within each cluster into groups of partners, using a selection procedure called a *matching rule*. Finally, we define a sequence of partitions and matching rules to obtain a *matching process*, i.e., a time-path for the process of group-formation.

To start, consider a representative period. Since matching agents simply means dividing the population J into disjoint sets of people, we start by defining a partitional correspondence $\psi\colon J \twoheadrightarrow J$ called the *clustering rule*. As a result we have $J = \bigsqcup_{s \in S} J_s$, with clusters $J_s = \psi(x)$ for $x \in J_s$, defined over the index set S, and so $\psi(J_s) = \bigcup_{x \in J_s} \psi(x) = J_s$. Given such a partition, we operate on each cluster, dividing its agents into one or more groups. This is what we call a matching rule. A *multilateral* matching rule is a partitional correspondence $\mu\colon J \twoheadrightarrow J$ such that $\mu(x) \subseteq J_s$ for all $s \in S$ and all $x \in J_s$. A special case of this is a *bilateral* matching rule, which is a function $\beta\colon J \to J$ satisfying $\beta^2(x) = x$ for all $x \in J$ that maps every cluster onto itself, i.e., $\beta(J_s) = J_s$.

Since $J = \mathbb{N}$, any matching rule forms groups containing a countable number of agents, finite or infinite. Consider an agent x. Under bilateral matching his trading group is $G(x) = \{x, \beta(x)\}$. Under multilateral matching, his group is $G(x) = \mu(x) \subseteq \psi(x)$, where, for convenience, we assume that $\mu(j) = \psi(j) = \psi(x)$ for all $j \in \psi(x)$ and $x \in J$, i.e., the cluster $\psi(x)$ is exactly the trading group $G(x)$ of agent x.

We call a sequence of matching rules a *matching process* and we construct it as follows. First, we specify an infinite sequence $\Psi = (\psi_0, \psi_1, \ldots)$ of clustering rules on the population J, which we call a *clustering process*, assuming that $\psi_0(x) = \{x\}$ for each $x \in J$. Subsequently, we define a matching process relative to Ψ as an infinite sequence of matching rules (bilateral or multilateral). For practical purposes, we assume that agents know the matching process but do not know Ψ. This means that agents do not know the composition of groups (other than the one in which they currently are), since they do not know the sequence of partitions induced by Ψ.

This formalization allows us to easily keep track of matching histories (and hence action histories). For each $t \geq 0$ we denote by $\mathcal{P}_t(x)$ the set of all partners of any agent x

in periods up to and including t. That is,

$$\mathcal{P}_t(x) = \bigcup_{\tau=0}^{t} G_\tau(x),$$

and observe that $\mathcal{P}_0(x) = \{x\}$ since $\psi_0(x) = \{x\}$. Now, denote by $\Pi_t(x)$ the set of all of x's past and current partners (including x), the past partners of x's current partners, the partners that x's partners in $t-1$ met prior to that date, and so on. This set is given by the recursive formula

$$\begin{aligned} \Pi_0(x) &= \mathcal{P}_0(x) \\ \Pi_t(x) &= \Pi_{t-1}(x) \bigcup \left[\bigcup_{b \in G_t(x)} \Pi_{t-1}(b) \right] \quad \text{for } t = 1, 2, \ldots. \end{aligned}$$

Following [22], we concentrate on matching processes satisfying

$$\Pi_{t-1}(x) \bigcap \Pi_{t-1}(b) = \emptyset \tag{7.1}$$

for every agent $x \neq b \in G_t(x)$ and all $t \geq 1$. If (7.1) holds, then we say that the economy is *informationally isolated*, and the agents in $G_t(x)$ are "complete strangers." To see why, define an 'event' as an action taken by some agent at some date. It can be proved (see [3]) that when (7.1) holds agents never have and never will share any direct or indirect partner over their lifetimes. This is the case even if histories can be freely shared during the course of a match since on every date t the history $h_{t,x}$ of any agent x includes events that are ignored by x's current partners.

It is now obvious that monetary economies based on the matching scheme adopted in [23] are not informationally isolated since the entire population regularly trades in the centralized market. Indeed, from (2.1), we see that for all x we have $G_t(x) = J$ when t is even, which implies $\Pi_{t-1}(x) \cap \Pi_{t-1}(y) = J$ for all $y \in G_t(x)$ in any period $t \geq 3$. Technically, this is at the heart of Theorem 6. The natural question now is: can we construct informationally isolated economies with large (perhaps infinite) recurring trade groups? The answer will be given in the next subsection.

7.2 Modeling informationally isolated economies

A general procedure to construct informationally isolated economies with large trading groups consists of three basic steps. In $t = 0$ we partition the population J into a countable number of sets $P_{0,1}, P_{0,2}, \ldots$ of identical cardinality. We then recursively construct partitions for each subsequent date. Finally, we create clusters out of these partitions, and apply a matching rule within each cluster.

The recursive partition is created as follows (details are in the appendix):

Period   Partition of the set of traders J

0 $J = P_{0,1} \bigsqcup P_{0,2} \bigsqcup P_{0,3} \bigsqcup P_{0,4} \bigsqcup P_{0,5} \bigsqcup P_{0,6} \bigsqcup \cdots$

1 $J = \langle P_{0,1} \bigsqcup P_{0,3} \bigsqcup \cdots \rangle \bigsqcup \langle P_{0,2} \bigsqcup P_{0,6} \bigsqcup \cdots \rangle \bigsqcup \cdots$
        $= P_{1,1} \bigsqcup P_{1,2} \bigsqcup \cdots$

2 $J = \langle P_{1,1} \bigsqcup P_{1,3} \bigsqcup \cdots \rangle \bigsqcup \langle P_{1,2} \bigsqcup P_{1,6} \bigsqcup \cdots \rangle \bigsqcup \cdots$
        $= P_{2,1} \bigsqcup P_{2,2} \bigsqcup \cdots$ (7.2)

$\vdots$ $\vdots$

$t+1$ $J = \langle P_{t,1} \bigsqcup P_{t,3} \bigsqcup \cdots \rangle \bigsqcup \langle P_{t,2} \bigsqcup P_{t,6} \bigsqcup \cdots \rangle \bigsqcup \cdots$
        $= \bigsqcup_{k=0}^{\infty} P_{t+1,k+1} = \bigsqcup_{k=0}^{\infty} \bigsqcup_{n=0}^{\infty} P_{t,(2n+1)2^k}$

$\vdots$ $\vdots$

On each $t \geq 1$, there are countably many sets $P_{t-1,k+1}$, $k = 0, 1, \ldots$, which have the same cardinality, and are pairwise disjoint.[9] We use them to construct infinitely many *matching blocks* in t, designated by the brackets $\langle \cdot \rangle$, each of which is defined by the infinite union

$$P_{t,k+1} = \bigsqcup_{n=0}^{\infty} P_{t-1,(2n+1)2^k} \quad \text{for all} \quad k = 0, 1, \ldots .$$

We use these matching blocks to define a clustering process Ψ on J that delivers informational isolation. Working with the partition (7.2), we select a clustering process $\Psi^* = (\psi_0^*, \psi_1^*, \ldots)$ with the following properties.

First, we let $\psi_0^*(x) = \{x\}$ for all $x \in P_{0,k+1}$ and all k so that $\psi_0^*(P_{0,k+1}) = P_{0,k+1}$ for all k. Then, in each period t we create an infinite sequence of clustering rules $\psi_{t,k+1} \colon P_{t,k+1} \twoheadrightarrow P_{t,k+1}$, for $k = 0, 1, \ldots$. To understand their role, consider a typical matching block $P_{t,k+1}$ on some date $t \geq 1$. We use the clustering rule $\psi_{t,k+1}$ to partition the set $P_{t,k+1}$ into a countable (finite or infinite) number of pairwise disjoint sets, called "clusters." Each cluster is constructed in such a way that it contains countably many agents. Specifically, each agent $x \in P_{t,k+1}$ is placed into a cluster $\psi_{t,k+1}(x) \subseteq P_{t,k+1}$ created by selecting a single agent from each of the sets $P_{t-1,(2n+1)2^k}$ that compose $P_{t,k+1}$ (the proof can be done as in [4, Theorem 4]). Since $P_{t,k+1}$ is composed by countably many sets $P_{t-1,(2n+1)2^k}$, then each cluster $\psi_{t,k+1}(x)$ has countably many agents. Clearly, the union of all the clusters created from the matching block $P_{t,k+1}$ is equal to the matching block itself since the correspondence $\psi_{t,k+1}$ is partitional, i.e., $\psi_{t,k+1}(P_{t,k+1}) = \bigcup_{x \in P_{t,k+1}} \psi_{t,k+1}(x) = P_{t,k+1}$.

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[9]The cardinality is identical since each set $P_{t,k+1}$ is the countable union of sets $P_{0,k+1}$ that have the same cardinality. They are pairwise disjoint by construction.

Next, since each clustering rule operates on a single matching block $k + 1$, define $\psi_t^* \colon J \twoheadrightarrow J$ for each $x \in P_{t,k+1}$ by

$$\psi_t^*(x) = \psi_{t,k+1}(x).$$

Having partitioned each matching block into clusters, we apply a matching rule to each cluster. This allows us to divide the agents forming the cluster into (a countable number of) trading groups or matches. Clearly, there are several matching rules we could use in a period, for example bilateral or multilateral. Considering again a typical matching block $P_{t,k+1}$ of a representative period t, we assume a multilateral matching rule $\mu_t(x) = \psi_t(x)$, so that each cluster simply corresponds to a trading group. Specifically, if in period t agent x belongs to the cluster $\psi_t(x)$, then this set coincides exactly with the agent's trading group $G_t(x)$. Notice that in this case, every agent trades with infinitely many partners. What's crucial is that every agent x will always be in a trading group populated by complete strangers since the clustering process Ψ^* insures total informational isolation. This holds independent of whether agents are anonymous.

Formally, we have the following result.

Theorem 9. *Every matching process based on Ψ^* guarantees informational isolation as defined in (7.1).*

Proof. We want to demonstrate that for any two partners x and $z \neq x$, who belong to the trading group $\psi_{t+1}^*(x) \subseteq P_{t+1,k+1}$ in period $t + 1$, we have $\Pi_t(x) \cap \Pi_t(z) = \emptyset$. The outline of the demonstration is as follows. Start by identifying the matching blocks to which the agents x and z belonged on date t. Suppose $x \in P_{t,h+1}$ and $z \in P_{t,j+1}$, with $j \neq h$ by construction. The proof revolves around demonstrating that $\Pi_t(x) \subseteq P_{t,h+1}$ and $\Pi_t(z) \subseteq P_{t,j+1}$. Indeed, since $P_{t,h+1} \cap P_{t,j+1} = \emptyset$ and $P_{t,h+1}, P_{t,j+1} \subseteq P_{t+1,k+1}$, if $\Pi_t(x) \subseteq P_{t,h+1}$ and $\Pi_t(z) \subseteq P_{t,j+1}$, then we must have $\Pi_t(x) \cap \Pi_t(z) = \emptyset$.

To prove the above, we must rely on the following two properties of the matching process Ψ^*. For each $k = 0, 1, \ldots$, each $t \geq 0$, and each $0 \leq \tau \leq t$ we have:

(i) $\psi_\tau^*(P_{t,k+1}) = P_{t,k+1}$, and

(ii) $\Pi_\tau(x) \subseteq P_{t,k+1}$ for all $x \in P_{t,k+1}$.

The proof of (i) is by induction on t. For $t = 0$ it is obvious that $\psi_0^*(P_{0,k+1}) = P_{0,k+1}$ for all k, since by our definition $\psi_0^*(x) = \{x\}$ for all $x \in J$. Therefore, for the induction step, assume that for some $t \geq 0$ we have $\psi_\tau^*(P_{t,k+1}) = P_{t,k+1}$ for all k and all $0 \leq \tau \leq t$. We want to prove that for any k we have $\psi_\tau^*(P_{t+1,k+1}) = P_{t+1,k+1}$ for each $\tau = 0, 1, \ldots, t + 1$. Start by observing that by the induction hypothesis $\psi_\tau^*(P_{t,k+1}) = P_{t,k+1}$ holds true for all $\tau = 0, 1, \ldots, t$. Now, note that $P_{t+1,k+1} = \bigsqcup_{n=0}^{\infty} P_{t,(2n+1)2^k}$. But then for each $\tau = 0, 1, \ldots, t$ we have

$$
\begin{aligned}
\psi_\tau^*(P_{t+1,k+1}) &= \psi_\tau^*\left(\textstyle\bigsqcup_{n=0}^{\infty} P_{t,(2n+1)2^k}\right) = \textstyle\bigsqcup_{n=0}^{\infty} \psi_\tau^*(P_{t,(2n+1)2^k}) \\
&= \textstyle\bigsqcup_{n=0}^{\infty} P_{t,(2n+1)2^k} = P_{t+1,k+1}.
\end{aligned}
$$

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Also, by definition $\psi_{t+1}^*(P_{t+1,k+1}) = P_{t+1,k+1}$. Therefore, $\psi_\tau^*(P_{t+1,k+1}) = P_{t+1,k+1}$ holds true for each k and all $\tau = 0, 1, \ldots, t+1$ and the validity of (i) has been established.

The proof of (ii) is by induction on τ. For $\tau = 0$ notice that for each $x \in P_{t,k+1}$ we have $\Pi_0(x) = \{x\} \subseteq P_{t,k+1}$. For the inductive step assume that for some $0 \leq \tau \leq t-1$ we have $\Pi_\tau(x) \subseteq P_{t,k+1}$ for all $x \in P_{t,k+1}$. We must show that $\Pi_{\tau+1}(x) \subseteq P_{t,k+1}$ for all $x \in P_{t,k+1}$. Fix $x \in P_{t,k+1}$. From (i) we get $\psi_{\tau+1}^*(P_{t,k+1}) = P_{t,k+1}$, and so $\psi_{\tau+1}^*(x) \subseteq P_{t,k+1}$. Therefore, each element $y \in \psi_{\tau+1}^*(x)$ belongs to $P_{t,k+1}$. But then our induction hypothesis yields $\Pi_\tau(y) \subseteq P_{t,k+1}$ for each $y \in \psi_{\tau+1}^*(x)$, and so

$$\Pi_{\tau+1}(x) = \Pi_\tau(x) \bigcup \left[\bigcup_{y \in \psi_{\tau+1}^*(x)} \Pi_\tau(y) \right] \subseteq P_{t,k+1} \,.$$

We are now ready to show that Ψ^* satisfies (7.1). To this end let us consider period $t+1$ and assume that $x, z \in J$ satisfy $x, z \in \psi_{t+1}^*(x) \subseteq P_{t+1,k+1}$ with $x \neq z$ and $t \geq 1$. That is, x and z are partners in period $t+1$. Since $x \in J = \bigsqcup_{k=0}^\infty P_{t,k+1}$ there exists a unique natural number h such that $x \in P_{t,h+1}$. Clearly, $P_{t,h+1} \subseteq P_{t+1,k+1}$, i.e., the period t matching block $P_{t,h+1}$ must be a component of the matching block $P_{t+1,k+1}$, in the subsequent period. By construction of $P_{t+1,k+1}$, it also follows that there exists some set $P_{t,j+1} \subseteq P_{t+1,k+1}$, with $j \neq h$, such that $z \in P_{t,j+1}$. That is, since x and z are partners in period $t+1$, each of them must belong to one of the (countable) collection of pairwise disjoint sets $\{P_{t,(2n+1)2^k}: \ n = 0, 1, 2, \ldots\}$ that compose $P_{t+1,k+1}$. But then from (ii) it follows that $\Pi_t(z) \subseteq P_{t,j+1}$. Using (ii) once more we get $\Pi_t(x) \subseteq P_{t,h+1}$. Finally, taking into account that $P_{t,h+1} \cap P_{t,j+1} = \emptyset$ we infer that $\Pi_t(x) \cap \Pi_t(z) = \emptyset$. ∎

This theorem demonstrates that, given any infinite population J, a matching process exists that insures complete informational isolation. The necessary ingredient is an initial partition of the population into countably many pairwise disjoint sets of identical cardinality. For example since $J = \mathbb{N}$ we can use the partition $J = \bigsqcup_{k=0}^\infty P_{0,k+1} = \bigsqcup_{k=0}^\infty \{k+1\}$. Then, we follow (7.2) to obtain

$$\begin{array}{llll} P_{1,1} = & P_{0,1} \sqcup P_{0,3} \sqcup P_{0,5} \sqcup \cdots & = \{1, 3, 5, \ldots\} \\ P_{1,2} = & P_{0,2} \sqcup P_{0,6} \sqcup P_{0,10} \sqcup \cdots & = \{2, 6, 10, \ldots\} \\ P_{1,3} = & P_{0,4} \sqcup P_{0,12} \sqcup P_{0,20} \sqcup \cdots & = \{4, 12, 20, \ldots\} \end{array}$$

in $t = 1$ and so on. This means that in every period we can have countably many groups of traders, each of which has countably many agents. These groups could form, for instance, countably many Walrasian markets across which communication is impossible and which are composed by complete strangers.

7.3 Informational isolation and the essentiality of money

We are now ready to demonstrate that if the matching process is based on Ψ^*, then the optimal plan cannot be supported by a social norm.

Theorem 10. *If the matching process is based on Ψ^*, then $\sigma_j = (0, 0, \ldots)$ for all $j \in J$ is the one and only subgame perfect Nash equilibrium of the infinite horizon game.*

Proof. Lemma 1 established that the minmax play $a_{t,k} = 0$ for all $k \in G_t(j)$ is a stationary equilibrium. Now focus on a representative agent j and his group $G_{t+1}(j)$ and let $k \neq j$. By (7.1) we have that $\Pi_t(j) \cap \Pi_t(k) = \emptyset$ for all $k \in G_{t+1}(j)$. It follows that agent k has not observed any action taken in periods $\tau \leq t$ by agent j, his partners, the partners of his partners, and so on. Thus, even if j selects $a_{t,j} \neq a_{t,j}^*$, then $h_{t+1,k} = h_{t+1,k}^*$ for all $k \in G_{t+1}(j)$. In addition, note that $\mathcal{P}_t(j) = \bigcup_{\tau=0}^t G_\tau(j) \subseteq \Pi_t(j)$ so that by (7.1) if j has observed some action of some agent $y \in \mathcal{P}_t(j)$, then k has never observed (and will never observe) any action of y, nor the actions of those who have observed the actions of agent y, and so forth. Considering a deviation, this implies $V_{t+1,j}^d(\boldsymbol{\sigma}_{t+1}) \geq V_{t+1,j}(\boldsymbol{\sigma}_{t+1})$, for any $\boldsymbol{\sigma}_{t+1}$, since every future partner of j will have no history in common with j, and so current actions will not affect j's continuation payoff. In particular, this means that in equilibrium j's future partners will be unaware of any of his prior deviations. It follows that $a_{t,j}^* > 0$ cannot be a best response. To see why, notice that by virtue of being a best response $a_{t,j}^*$ must satisfy

$$-a_{t,j}^* + \delta_{t+1} V_{t+1,j}(\boldsymbol{\sigma}_{t+1}) \geq -a_{t,j} + \delta_{t+1} V_{t+1,j}^d(\boldsymbol{\sigma}_{t+1}),$$

which implies $a_{t,j}^* \leq a_{t,j}$ for all $a_{t,j} \in A_j$. Note that $0 \in A_j$ contradicts the optimality of $a_{t,j}^*$. $\blacksquare$

Matching processes based on Ψ^* destroy all possible links, direct and indirect, among trading partners. Thus, the representative agent $j \in J$ knows that his current actions cannot affect the choices of his future partners, as their histories will have no element in common. Effectively, the matching process ensures that the infinite horizon game is an infinite sequence of one-shot games, since every agent will always trade in markets where no one has knowledge of any of his past deviations.

Specifically, the expected lifetime utility

$$V_{t,j}(\boldsymbol{\sigma}_t) = \widehat{v}_t(s_t(\mathbf{h}_{t,j})) + \delta_{t+1} V_{t+1,j}(\boldsymbol{\sigma}_{t+1}),$$

is maximized when the current payoff $\widehat{v}_t$ is maximized. Indeed, for any given $\boldsymbol{\sigma}_{t+1}$, the continuation payoff $V_{t+1,j}(\boldsymbol{\sigma}_{t+1})$ is unaffected by j's current actions. Hence, in every t agent j should play $s_t(h_{t,j}) = 0$ for all histories $h_{t,j}$, i.e., deviating from the optimal plan in every date is weakly optimal.

The preceding discussion leads to the following.

Corollary 11. *Money is essential if the matching process is based on Ψ^*.*

The informational isolation guaranteed by Ψ^* eliminates the possibility that informal punishment schemes based on social norms can be used to sustain equilibria with positive

consumption. Economies with this trait represent the prototypical case in which money can help in sustaining production and trade. Our discussion of Ψ^* indicates that economies of this type can incorporate many different matching environments, including ones in which infinitely-lived agents interact in trading groups that include infinite many participants.

8 Concluding remarks

We have considered an infinite-horizon economy in which trade is of an intertemporal nature but two frictions rule out credit arrangements. First, a matching process is imposed such that agents' trading paths do not cross more than once. Second, agents must select a course of action that is compatible with individual incentives. We have proved that if we introduce (occasional) centralized trade, then money has hardly a role to play when agents are patient and actions of partners are observable. Trading in a central market allows a quick transfer of the information necessary to sustain informal punishment schemes. Hence, as in [12, 19], desirable allocations can be sustained by means of social norms, even if other assumptions, such as anonymity, rule our credit trades. This suggests that the role of money is weakened when trading institutions exist that foster rapid and inexpensive informational flows.

Based on this intuition, we have developed a matching framework that allows the construction of economies in which infinitely-lived agents trade in infinitely large but informationally isolated markets. Our technique can be used to improve the modeling of pairwise matching economies characterized by spatial separation, such as in [28], or characterized by search with or without rounds of Walrasian trading, such as in [23, 27]. It can also be used to construct monetary models in which agents trade exclusively in large competitive markets. This is a further step toward developing models in which not only money has an explicit role, but which can also be studied using standard general equilibrium tools and are better integrated within the rest of macroeconomic theory.

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