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Comments
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CONTAGION EQUILIBRIA IN A MONETARY MODEL

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THE MODEL OF LAGOS AND WRIGHT (2005) alters the meeting friction of the typical search model of money to obtain degeneracy in equilibrium holdings and enhance analytical tractability. It introduces a round of Walrasian "centralized" trading after each round of bilateral random "decentralized" trading. The basic premise is that, although the population meets repeatedly in the centralized market, anonymity and random pairings are frictions sufficient for money to be essential (see Lagos and Wright (2005, p. 466) or Rocheteau and Wright (2005, p. 175); for the essentiality, see Huggett and Krasa (1996) and Kocherlakota (1998)).

This note, based on Aliprantis, Camera, and Puzzello (2005) (where details and proofs can be found), clarifies that anonymity and random pairings are not per se sufficient to generate an essential role for money. Further frictions are generally needed. The starting point is the work of Ellison (1994) and Kandori (1992), who proved that efficient outcomes are supported by "contagion equilibria" in repeated anonymous matching games. We cast the model in Lagos and Wright (2005) as an infinitely repeated game with observable individual actions[2] and we show the existence of contagion equilibria if agents are sufficiently patient. This, however, is not robust to adding a small amount of noise in the observation of individual behavior, because equilibria would arise similar to those in the continuum limit where individual behavior is unobservable (Al-Najjar and Smorodinsky (2001), Fudenberg, Levine, and Pesendorfer (1998), Levine and Pesendorfer (1995)).

The argument goes as follows. There is a unique efficient allocation that is sustained by "desirable" behavior. So, consider a social norm that specifies autarky forever as the sanction rule if a defection from desirable behavior is observed. Centralized trading fosters the rapid spread of sanctions and so discourages defections in all anonymous matches. Intuitively, the random matching friction in the typical search model fragments the process of exchange of goods and of information, but introducing centralized trading allows an informal enforcement scheme to emerge. Consequently, money ceases to be essential for the process of exchange. In fact, eliminating money improves efficiency.

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[2]The model in Lagos and Wright (2005) assumes a continuum of agents. Here we suppose that agents are countable to highlight the fact that individual actions are observable.

1. THE ECONOMIC ENVIRONMENT

Time is discrete and infinite, indexed by $t = 0, 1, \ldots$. There is a constant population $J = \mathbb{N}$ of identical infinitely lived agents and a single perishable good that can be produced by a fraction of the population at each date. In each t a matching process, as specified in Aliprantis, Camera, and Puzzello (2006) partitions J into spatially separated trading groups $G_t(j)$ for $j \in J$. Assume that $G_0(j) = \{j\}$ and, in all $t \geq 1$, let $G_t(j) = \{j, \beta_t(j)\}$ if t is odd and let $G_t(j) = J$ if t is even, where $\beta_t(j) \neq j$ with probability α for each $j \in J$. Thus, trading in odd periods is decentralized (agents are paired with probability α), while in even periods it is centralized (everyone is in an economy-wide group). Agents can only trade and observe actions and outcomes in their match, and are anonymous as in Ellison (1994), i.e., they cannot observe identities and histories. There is no commitment and no enforcement.

Trade is necessary for consumption to take place. In odd periods, in each match a flip of a fair coin determines who is a producer and who is a consumer. In even periods everyone can produce and consume. Each producer can supply $a \in [0, \overline{a}]$ labor to a technology that transforms it into a goods. He suffers disutility a and derives no utility from consumption of own production. In odd (even) periods, every consumer has utility $u_o(c)$ ($u_e(c)$) from consuming $c \geq 0$ goods. Assume that preferences satisfy the Inada conditions and that $\overline{a} \in (c_o^* + c_e^*, \infty)$, where c_o^* and c_e^* satisfy $u_e'(c_e^*) = u_o'(c_o^*) = 1$. Agents discount next period's payoffs by $\delta \in (0, 1)$ only if the current period is even.

Consider a match $G_t(j)$ in period t. Agents have a nontrivial choice of action only as producers, which is when they must choose how much consumption to supply to the members of their group. Hence, we identify the action set of any agent $k \in G_t(j)$ by $A_k = [0, \overline{a}]$ if k is a producer and $A_k = \{0\}$ otherwise. We let $a_{t,k} \in A_k$ be an action, so the action space of profiles $\mathbf{a}_{t,j}$ is $\mathbf{A}_{t,j} = \times_{k \in G_t(j)} A_k$. The payoff function for agent j is $v_{t,j} : \mathbf{A}_{t,j} \to \mathbb{R}$, where

$$v_{t,j}(\mathbf{a}_{t,j}) = \begin{cases} u_o(c_{t,j}) - a_{t,j}, & \text{if } t \text{ is odd,} \\ u_e(c_{t,j}) - a_{t,j}, & \text{if } t \text{ is even,} \end{cases}$$

with

$$c_{t,j} = \begin{cases} a_{t,k}, & \text{if } j \neq k \text{ and } t \text{ is odd,} \\ 0, & \text{if } j = k \text{ and } t \text{ is odd,} \\ \liminf\limits_{n \to \infty} \left[\dfrac{1}{n} \displaystyle\sum_{k \in \{1, \ldots, n\} \setminus \{j\}} a_{t,k} \right], & \text{if } t \text{ is even.} \end{cases}$$

Thus, the agent's utility depends on how much output he receives in his match. His disutility depends on how much he produces for his partners. Clearly, autarky, $a_{t,k} = 0$ for all $k \in G_t(j)$, is the only Nash equilibrium of the static game. Indeed, producers cannot be forced, nor can they commit, to provide consumption. So, producer j can always select $a_{t,j} = 0$ and enjoy payoff $v_{t,j}(\mathbf{a}_{t,j}) \geq 0$.

Now consider the infinite horizon game. For $t \geq 1$ let $h_{t,j} = (\mathbf{a}_{0,j}, \dots, \mathbf{a}_{t-1,j})$ be the history of actions observed by j at $\tau < t$ with $h_{0,j} = 0$. The set of histories of j is $H_{t,j} = \times_{\tau=0}^{t-1} \mathbf{A}_{\tau,j}$. For $G_t(j)$, let $\mathbf{h}_{t,j} = (h_{t,k})_{k \in G_t(j)}$ and note that partners do not have common histories, due to random matching. A pure strategy σ_j for the infinite horizon game is an infinite sequence of maps $\sigma_j = (s_{0,j}, s_{1,j}, \dots)$, where $s_{t,j} : H_{t,j} \to A_j$ is defined by $s_{t,j}(h_{t,j}) = a_{t,j}$. The strategy profile in $G_t(j)$ is $s_t(\mathbf{h}_{t,j})$. Whereas action sets do not depend on histories, let the sequence of mappings $S_{t,j} = (A_j^{H_{\tau,j}})_{\tau=t}^{\infty}$ denote the strategy space of agent j in the subgame starting at $t \geq 0$. Then every σ_j gives rise to a strategy $\sigma_{t,j}$ in the subgame at t, with $\sigma_{t,j} = (s_{t,j}, s_{t+1,j}, \dots) \in S_{t,j}$ and $\sigma_{0,j} = \sigma_j$. Finally, let $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \dots)$ be the set of strategies of the population J, using $\boldsymbol{\sigma}_t$ for a subgame in t.

Let $\delta_{t+1} = \delta$ if t is even and 1 otherwise. Define the expected period utility

$$(1) \qquad \widehat{v}_t(\mathbf{a}_{t,j}) = \begin{cases} \frac{\alpha}{2}[u_o(c_{t,j}) - a_{t,j}], & \text{if } t \text{ is odd,} \\ u_e(c_{t,j}) - a_{t,j}, & \text{if } t \text{ is even.} \end{cases}$$

Whereas each $t \geq 1$ defines a proper subgame, we formalize recursively j's payoff in t using the function $V_{t,j} : \times_{i \in J} S_{t,i} \to \mathbb{R}$ defined by

$$(2) \qquad V_{t,j}(\boldsymbol{\sigma}_t) = \widehat{v}_t(s_t(\mathbf{h}_{t,j})) + \delta_{t+1} V_{t+1,j}(\boldsymbol{\sigma}_{t+1})$$

with $V_j = V_{0,j}$. The best response correspondence of agent j is

$$\rho_j(\boldsymbol{\sigma}) = \left\{ \sigma_j \in S_{0,j} : V_j(\boldsymbol{\sigma}_{-j}, \sigma_j) = \max_{x_j \in S_{0,j}} V_j(\boldsymbol{\sigma}_{-j}, x_j) \right\},$$

so a *subgame perfect Nash equilibrium* for the infinite horizon game is a strategy profile $\boldsymbol{\sigma}$ such that $\sigma_j \in \rho_j(\boldsymbol{\sigma})$ for all $j \in J$. Clearly, autarky forever is an equilibrium because repeated play does not decrease the set of equilibrium payoffs.

What is the efficient allocation in this model? To answer this, consider a planner who treats agents identically and faces their physical restrictions. She will ask each producer to deliver the surplus-maximizing quantity in each match.

THEOREM 1: *An optimal plan exists and it is unique. Specifically, for all producers $k \in J$ we have $a_{0,k} = a_0^* = 0$ and when $t \geq 1$ we have $a_{t,k} = a_t^* = c_o^*$ if t is odd and c_e^* if t is even.*

2. THE MAIN RESULT

The works by Ellison (1994) and Kandori (1992) suggest that we can sustain the efficient allocation using a contagion strategy that specifies desirable actions as well as sanctions for undesirable actions. We identify desirable behavior with production decisions that conform with the optimal plan and we label every other action as undesirable.

DEFINITION 2: A strategy $\sigma_j^* = (s_{0,j}, s_{1,j}, \ldots)$ for a producer $j \in J$ is called a *contagion strategy* if it satisfies $s_{0,j} = a_0^*$ and in each period $t \geq 1$ we have:

(i) $s_{t,j}(h_{t,j}) = a_t^*$, whenever $h_{t,j} = h_{t,j}^*$, and

(ii) $s_{\tau,j}(h_{\tau,j}) = 0$ for all $\tau \geq t$, whenever $h_{t,j} \neq h_{t,j}^*$.

Thus, every producer delivers to his partners c_t^* consumption only if he has observed desirable behavior. The producer selects autarky forever as soon as he deviates or has knowledge of a deviation. Can the threat of this informal punishment sustain the optimal production plan as a subgame perfect equilibrium?

THEOREM 3: *For each*

$$\delta \geq \delta^* \equiv \frac{c_o^* + c_e^*}{c_o^* + u_e(c_e^*) + \frac{\alpha}{2}[u_o(c_o^*) - c_o^*]},$$

the contagion strategy supports the optimal plan as a subgame perfect equilibrium.

Recalling that a monetary authority cannot levy taxes without enforcement, we see that the efficient allocation cannot generally be sustained in a monetary equilibrium, but is attainable in a nonmonetary equilibrium by patient agents. This holds for any J as long as actions are observed without noise (Levine and Pesendorfer (1995, p. 1161)).

To see why, assume that everyone follows the contagion strategy and consider agent j in t. Strategies and the structure of the game are time-invariant in equilibrium, so each subgame is a replica of the infinite horizon game. Thus, the equilibrium continuation payoffs in even or odd periods are time-invariant. If we denote them by V_e^* and V_o^*, using (1) and (2), we have

$$V_e^* = \frac{1}{1-\delta}\left\{u_e(c_e^*) - c_e^* + \delta\frac{\alpha}{2}[u_o(c_o^*) - c_o^*]\right\}, \tag{3}$$

$$V_o^* = \frac{1}{1-\delta}\left\{\frac{\alpha}{2}[u_o(c_o^*) - c_o^*] + u_e(c_e^*) - c_e^*\right\}.$$

For the optimality of σ_j^* consider one-time deviations in a representative subgame, in and off equilibrium (unimprovability criterion). Let V_e^d and V_o^d be the continuation payoffs (in even and odd periods) if a deviation was first seen in the prior period, and use $\tilde{V}_e$ and $\tilde{V}_o$ if the deviation was observed earlier. Therefore, deviating is suboptimal in t odd if $-c_o^* + V_e^* \geq V_e^d$ and in t even if $u(c_e^*) - c_e^* + \delta V_o^* \geq u_e(c_e^*) + \delta V_o^d$, where time-invariance of payoffs is without loss in generality because everyone observes a deviation in at most two periods.

In equilibrium, consider a deviation by a producer in $G_t(j)$. If t is odd, then in $t+1$ all $k \notin G_t(j)$ play a_{t+1}^* because $h_{t+1,k} = h_{t+1,k}^*$. However, all agents $k \in G_t(j)$ have $h_{t+1,k} \neq h_{t+1,k}^*$, so they play $a_{\tau,k} = 0$ in $\tau \geq t+1$. Thus,

$V_e^d = u_e(c_e^*) - 0 + \delta\tilde{V}_o$ for $k \in G_t(j)$. Whereas $G_{t+1}(j) = J$, everyone observes a deviation in $t+1$, so all $k \in J$ have $h_{t+2,k} \neq h^*_{t+2,k}$ and play $a_{\tau,k} = 0$ in $\tau \geq t+2$. Hence, $V_{\tau,k} = 0$ for all $\tau \geq t+2$ and all $k \in J$, and so $\tilde{V}_o = \tilde{V}_e = 0$. Thus, using (3), deviating in t odd is suboptimal if $\delta \geq \delta^*$. For these parameters, defecting is suboptimal also if t is even, which is when a defection is immediately seen by all $k \in J$ and so $V_o^d = \tilde{V}_e = 0$. Finally, off equilibrium, choosing autarky after observing a deviation is clearly in a producer's best interest. Everyone learns of a deviation with at most one period delay, so permanent autarky cannot be avoided by forgiving (producing for) a deviator.

In sum, efficient trades are sustainable without money because any deviation shuts down trade very fast. Pairwise random trade slows the transfer of information, but cannot prevent it simply because agents are anonymous. Observability of actions and centralized trade allow informational flows that encourage desirable behavior in every match. In our working paper (Aliprantis, Camera, and Puzzello (2005)), we develop a matching model that is immune to contagion and present environments where money is essential to support trade in large markets populated by complete strangers. Another approach is Shi (1997).

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