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An Investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving among Swahili-speaking African High School Bi-/Multilingual Students in the United States

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Abstract

In this study, I examined the practices, resources, and challenges in mathematics word problem solving (MWPS) among the Africa Swahili-speaking refugee high school students in the United States. Specifically, I investigated the language practices and linguistic resources the participants used during MWPS, as well as the linguistic challenges they faced. I also explored the mathematics practices and mathematical resources the participants used as well as the mathematical challenges they faced during MWPS. Lastly, I determined the role of the language practices and/or resources (LPRs) in the participants' mathematical processes.

To accomplish this study, I used a language background survey (LBS) and task-based interviews which were administered to 12 participants who were selected through criterion purposive sampling technique. The tasks were three problems adopted and modified from the National Assessment of Education Progress (NAEP)-released algebra problems. Guided by a Vygotskian perspective of mathematics practices, I allowed the participants a safe translanguaging space as they solved the problems. I then studied how they used their language and mathematics practices, linguistic and mathematical resources, and I noted the linguistic and mathematical challenges they faced in the process.

The analysis revealed that the participants faced various mathematical and linguistic challenges, and they also drew on their LPRs to comprehend the problems, communicate their understanding, develop their mathematics practices, and as a means of identifying with some meaningful social groups. The findings of this study showed that bi-/multilinguals translanguage in mathematics where they use their LPRs in an integrated manner, not in isolation. Since bi-/multilingual students draw on various discursive practices, their mathematics practices are oftentimes informal, making it difficult to demarcate between the students' everyday and

mathematics practices (Barwell, 2013). Also, the findings showed that bi-/multilinguals need support to use the their LPRs in a mathematical sense and to develop more formal mathematical practices.

The findings of this study have implications on the validity of assessments, and how teachers can be prepared to teach bi-/multilinguals, even when they don't share the students' home languages. Drawing on the work of Civil (2012) and Sigley and Wilkinson (2015), I argue that valid assessments would have to valorize bi-/multilingual students' ways of communicating mathematically, even those that may not seem precisely mathematical. Moreover, teachers are to be cognizant of the bi-/multilingual students' ways of mathematical communication and determine ways they could use those ways to enhance the students' learning of mathematics. I also present de Jong et al.'s (2013) conceptual framework that can be used to enhance the preparation of mainstream teachers to support ELLs in content areas. This study suggests the need for further research on translanguaging in mathematics classrooms and how teachers can implement pedagogies that support translanguaging to enhance learning. There is also a recommendation for studies investigating the kinds of professional development mainstream mathematics teachers would need to be effective in the instruction and assessment of students whom they don't share the home language. Also, there is need for further research on how students solve problems and generalize and how they can be supported to develop these processes.

An Investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving among Swahili-speaking African High School Bi-/Multilingual Students in the United States

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List of Abbreviations and Acronyms

ACF	Adventist Christian Fellowship
AMTE	Association of Mathematics Teacher Educators
DIF	Differential Item Functioning
ECR	Extended Constructed Response
ELLs	English Language Learners
ESL	English as A Second Language
ICE	International Conference on Education
KCPE	Kenya Certificate of Secondary Education
LBS	Language Background Survey
LPRs	Language Practices and/or Resources
MWPS	Mathematics Word Problem Solving
MWPs	Mathematics Word Problems
NAEP	National Assessment of Educational Progress
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
PME-NA	North American Chapter of the International Group for the Psychology of Mathematics Education
PrA	Problem A
PrB	Problem B
PrM	Math Eliciting Task
TIMSS	Third International Mathematics and Science Study

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Chapter 1: Introduction

Background of the Study

Current reforms in mathematics education view communicating mathematically as the central indicator of mathematics learning (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Students are to be able to communicate mathematical ideas and relationships in either oral or written form in the English language. The mathematical ideas and relationships that students are to develop and communicate are the mathematics practices that have been highlighted in the standards for mathematics practices. For instance, students are to be able to abstract, generalize, conjecture, test conjectures, construct arguments, and subject claims and arguments to discussion and evaluation by a classroom community (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

While communicating mathematically is central to the learning of mathematics, there has existed tension as to how mathematics practices should be defined or described (Moschkovich, 2013). Mathematics practices have been dichotomized as either everyday/academic, formal/informal, or in-school/out-of-school (Moschkovich, 2013). Traditional mathematics instruction views the relationship between school-based practices and out-of-school practices as "unidirectional", where the out-of-school practices are to be informed by the school practices and not the other way around (Dominguez, 2011). Such dichotomies are not sufficient ways to describe mathematics practices since mathematics practices occur in multiple contexts that may be academic, workplace, playground, street selling, and home among others (Moschkovich, 2013).

In order to address the question of how mathematics practices should be described, there first a need to understand how mathematics practices are appropriated and manifested by all students, especially those for whom English is not their first language. According to Grosjean (1999) and Halai (2007), there are differences in the factors and processes involved in developing communicative competence of English language learners (ELLs) and that of their native counterparts. These findings imply that ELLs may communicate their mathematics practices in ways that are different from those of a native English speaker. Moschkovich (2013) emphasized the need for studies that seek to understand the nature of mathematics practices students use in different contexts and settings; studies that "make visible the ways that learners reason mathematically across settings" (Moschkovich, 2013, p. 264).

Moschkovich (2013) used a Vygotskian perspective and Scribner's (1984) definition of practice as "culturally organized in nature and involving different technologies or symbols systems" (p. 265), to define mathematics practices as social, cultural, cognitive, and semiotic. Mathematics practices are socio-cultural because they originate from social interaction, where the learner is actively involved in a joint activity that "supports their appropriation of goals, focus of attention, and shared meanings" (p. 271). Mathematics practices are cognitive because they involve thinking, and they are semiotic because they involve semiotic systems such as signs, tools, and their meanings. According to a Vygotskian perspective, mathematics practices are embedded in mathematical discourse practices and meaning of utterances. These discourse practices not only involve the use of language, but also the use of other symbolic expressions, objects, and communities. Discourse practices thus entail aspects of both academic and everyday practices. In brief, mathematics practices are expressed in the meanings generated from both everyday and academic discourse practices that students engage in while working on a joint mathematical activity.

From a Vygotskian perspective, everyday discourse practices are not obstacles to participation in academic mathematics practices, but resources for students' participation in formal mathematics practices (Moschkovich, 2013). It is therefore pertinent to study how students, especially those for whom English is not their first language, use language, symbolic expressions, objects, and other tools, as well as the role of the use of these resources in their mathematical processes. Such knowledge can inform how mathematics practices should be described, as well as how ELLs' mathematical communication can be supported.

Statement of the Problem

Research has shown that students whose first language is not English face certain linguistic and cultural biases in standardized examinations such as the National Assessment of Education Progress (NAEP) (Abedi & Lord, 2001; Kieffer, Lesaux, Rivera, & Francis, 2009; Martiniello, 2009). These students, commonly known as ELLs, not only underachieve in NAEP mathematics tests, but they are also underrepresented in the nation's students' report card (Pellegrino et al., 1999). Research on mathematics and language among the ELLs has limited ELLs' identity to the acquisition of English language. The result has been the assertion that the complexity in the natural language of a mathematics text or test, in this case, English, is the cause for ELLs' underperformance in mathematics (Abedi, 2004; Barbu & Beal, 2010; Cuevas, 1984; Martiniello, 2008, 2009; Schleppegrell 2007).

Current research has problematized this model of viewing ELLs, terming it as a deficit model (Garcia, 2009; Moschkovich, 2007). Researchers are now being urged to shift from viewing ELLs in terms of their deficiency in English, into seeing them as learners possessing certain linguistic and cultural resources, which they tap into to make sense of mathematics (Moll, 2010; Moschkovich, 1999; Yosso, 2005). There is a need for studies on what bi-/multilinguals actually do with their language, what language does to them, and what language means to them (e.g., Martinez-Roldan, 2014). Instead of comparing mathematical performance between bi-/multilinguals and monolinguals, researchers have been called to investigate in detail the students' communicative practices and report on differences that favor bi-/multilinguals, and those that are relevant to their mathematical instruction (Moschkovich, 2007). Researchers should seek to know the nature of bi-/multilinguals' language practices and the kinds of resources they draw upon to make sense of mathematics, in order to "make visual" the mathematics practices that result from their use of these language practices and linguistic resources in different contexts (Moschkovich, 2013, p. 264).

A number of studies have looked at how ELLs make sense of mathematics by drawing on various resources such as code-switching (Moschkovich, 2002, 2007), translation (Halai, 2007), gestures (Dominguez, 2005), and everyday/cultural experiences (Dominguez, 2011; Vomvoridi-Ivanovic, 2011), as well as inscriptions (Moschkovich, 2008), and various semiotic features and structures (Solano-Flores, Barnett-Clarke, & Kachchaf , 2013; Solano-Flores & Nelson-Barber, 2001; Solano-Flores, Wang, & Shade, 2016). However, ELLs do not use these practices and resources in isolation, but they employ multiple, complex, and integrated discursive practices to make sense of mathematics. The use of multiple interrelated discursive practices that cannot be easily assigned to one or another traditional definition of language, but that make up the speaker's complete language repertoire, has been termed as translanguaging (Garcial & Wei, 2014, p. 22). According to Garcia and Wei (2014), "translanguaging better captures the sociolinguistic realities of everyday life" (p. 29). Sociolinguists stress the social nature of language and its use in varying contexts under the assumption that language is not only cognitive, but also cultural, social, and situated (Moschkovich, 2007). Most of these studies have been done among the English-Spanish speaking bilinguals at lower grade levels. English-Spanish bilinguals are only a portion of the ELL population in the United States. There is thus a need for studies addressing practices, resources, and challenges in mathematics learning among other ELL populations at higher grade-levels such as high school.

In this study, I considered ELLs' language practices as translanguaging practices and viewed mathematics practices from a Vygotskian perspective. I studied these phenomena among the Swahili-speaking African refugee high school bi-/multilingulal students. According to the United States Department of State Bureau of Population, Refugees and Migration records, as of March 2017 refugees from Africa form the second largest population of refugees in the U.S., with 37.46% (14,277) out of the total refugee population of 38,111 (U.S. Department of State, 2017). The leading refugee population originates from the Near East/South Asia, with 43.56% (16,600). Moreover, many of the refugees from Africa speak Swahili as their native language since Swahili is the lingua franca for many African countries. According to these records, Swahili is the sixth of the top ten native languages spoken by the refugee communities currently living in the U.S. To these students, English is either a second or a third language, making them bi-/multilinguals. No research in mathematics education has yet investigated the practices, resources, and challenges in mathematics word problem solving (MWPS) in the context of students from the Swahili-speaking African refugee community.

By providing these students with an opportunity to translanguage, I identified the language practices and other resources the students drew on to solve three algebra word problems adopted and modified from the NAEP-1990, 1992, and 2009 released test items, as well as the challenges they faced during problem solving. I also investigated the mathematics practices emanating from the meanings of the mathematical discourse practices the students used, together with the mathematical resources and knowledge they drew upon and the mathematical challenges they seemed to face during problem solving. Hopewell (2011) noted that there is a dearth of studies examining when, why, and how students use their linguistic repertoire. Therefore, in this study, I not only investigated the language practices and other resources (LPRs) the participants used in MWPS, but I also sought to know the role of LPRs in the participants' mathematical processes of problem solving, reasoning and proof, connecting, communication, and representation (National Council of Teachers of Mathematics [NCTM], 2000).

Purpose of the Study

The purpose of this study was to investigate the practices, resources, and challenges in MWPS among bi-/multilingual high school students from the African Swahili-speaking refugee community, who recently immigrated to the U.S. Specifically, I investigated the linguistic and mathematical practices and resources the participants used during mathematics word problem solving (MWPS), as well the linguistic and mathematical challenges they faced during this process. I also investigated the role of the LPRs in the participants' mathematical processes of problem solving, reasoning and proof, communication, connection, and representation.

Research Questions

This study was guided by the following research questions:

1. What language practices do the participants use during MWPS? What linguistic challenges do they encounter? What linguistic resources do they use?

2. What mathematics practices do the participants use during MWPS? What mathematical challenges do they encounter? What mathematical resources do they use?
3. What role, if any, do the LPRs play in the participants' participation in mathematical processes?

Significance of the Study

This study strives to complement and advance existing research on mathematical communication by studying the practices, resources, and challenges in MWPS among a population that has not been studied before. The study also advances research on mathematics education among ELLs by employing frameworks that do not view ELLs as deficient, but as possessing certain practices and resources that they draw upon to make sense of mathematics. To the research world, the findings of this study on how bi-/multilinguals manifest their mathematics practices may contribute to the debate of how mathematics practices should be described. To curriculum developers, these findings provide information that might be used in developing a curriculum that prepares teachers to teach bi-/multilingual students. A mathematics teacher might also use the findings of this study to determine ways of supporting bi-/multilingual students develop mathematics practices.

Theoretical Frameworks

This study is premised upon a Vygotskian perspective of mathematics practices, which views mathematics practices as social, cultural, semiotic, and cognitive (Moschkovich, 2013). According to this framework: (a) students develop mathematics practices as they work jointly on a mathematics task, (b) mathematics practices are embedded in the discourse practices students engage in as they solve a mathematics task, (c) the discourse practices that students engage in entail both everyday and academic practices. Everyday practices involve students drawing on their language practices and linguistic resources, while academic practices involve students drawing on various mathematical resources. In this study, I framed language practices and resources of bi-/multilingual students as translanguaging practices and I positioned the language of mathematics as a multi-/semiotic system. In the following sections, I discuss a Vygotskian perspective on mathematics practices, bi-/multilinguals' LPRs as translanguaging practices, and the language of mathematics as a multi-/semiotic system.

Vygotskian Perspective on Mathematics Practices

In this study, I use the term practice or practices in the sense used by Moschkovich (2013). Following Scribner's (1984) usage of the term practices, Moschkovich presented a Vygotskian perspective of a practice or practices as "culturally organized in nature and involving different technologies or symbols systems" (p. 265). Based on a Vygotskian definition of practice or practices, Moschkovich also defined mathematics practices as social-cultural, cognitive, and semiotic. Mathematics practices are socio-cultural because they originate from social interaction where the learner is actively involved in a joint activity. Mathematics practices are cognitive because they involve thinking, and they are semiotic because they involve semiotic systems such as signs, tools, and their meanings (Moschkovich, 2013).

A Vygotskian perspective on mathematics practices has a number of implications including (a) social interaction where learning is predominantly through joint activity, (b) goals are implicit but fundamental aspects of practices, (c) discourse is central to participation in practices, (d) meanings for words are situated and constructed while participating in practices, and (e) appropriation is a central description for learning, but learners do not simply imitate practices, they sometimes transform them (Moschkovich, 2013). According to the Vygotskian perspective, mathematics practices are not as the practices we tell students or model on the board instead, they are the practices that learners develop when they are engaged in discourse during a joint mathematical activity. Mathematics practices are embedded in mathematical discourse practices, where discourse is more than language use since it involves other symbolic expressions, objects, and communities. Within the Vygotskian framing, mathematics practices are not purely cognitive accounts of mathematics but accounts that assume the social, cultural, and discursive nature of mathematical activity (Moschkovich, 2008). Therefore, "students are likely to need time and support as they move from expressing reasoning and arguments in imperfect form towards more academic ways of talking" (Moschkovich, 2013, p. 271).

In this study, students worked in pairs to solve two algebra word problems adopted and modified from the NAEP-1990 and 1992 released test items because NAEP assessment provides data on achievement that is tailored to students' school experiences in the U.S. and these tests contained problems that I determined would be useful for these tasks. I asked the students to talk aloud about the problems and their solution processes. I presented the problems using different semiotic features in an attempt to explore the role different semiotic resources play on MWPS among bi-/multilinguals (Solano-Flores et al., 2013).

Bi-/multilinguals' Language Practices as Translanguaging

From a Vygotskian perspective, mathematics practices are constructed when students participate in discourse while engaged in a joint mathematics activity (Moschkovich, 2013). The mathematical discourse practices that students engage in during problem solving include the use of language(s), symbolic expressions, and other objects and visual devices. In this study, I consider language as a social and cultural practice (Palmer & Martinez, 2016); a form of action that emerges within particular social and cultural contexts (Garcia, 2009). Seeing language as a practice or an action occurring within a social or cultural context means that bi-/multilinguals are not bounded in the manner of use of their language(s). Consequently, bi-/multilingualism is not simply a combination of two or more separate linguistic systems (Palmer & Martinez, 2010), but a dynamic set of language practices developed to "varying degrees in order to interact with increasingly multilingual communities" (Garcia, 2009, p. 42). These dynamic sets of language practices are the translanguaging practices. Garcia and Wei (2014) defined translanguaging as:

> A process by which students … engage in complex discursive practices that include ALL the language practices of ALL students in a class in order to …, communicate and appropriate knowledge ... (p.121, emphasis in original)

In translanguaging, a bi-/multilingual engages in complex and interrelated language practices that cannot be easily assigned to one or another traditional definition of language, but that make up the speaker's complete linguistic repertoire (Garcia, 2009; Garcia & Wei, 2014). Translanguaging allows a bi-/multilingual to flexibly and fluidly 'select' features, which may be visual, textual, symbolic, or in other forms, from their entire semiotic repertoire, and not from an inventory that is limited to the societal definition of what is an appropriate language (Garcia & Wei, 2014). According to Garcia (2009), translanguaging is the normal, observable, and often unmarked way of "languaging" between individuals who belong to the same bi-/multilingual group. Translanguaging better captures the sociolinguistic perspective of language, where language is viewed as social-cultural and situated (Garcia & Wei, 2014). I, therefore, consider my participants' language practices as translanguaging practices. I allowed the students to tap into their entire linguistic repertoire and other semiotic resources within their reach as they solved two algebra word problems adopted and modified from the NAEP-1990 and 1992 released test items.

The Multi-/semiotic Perspective on Mathematics

In this study, I considered the language of mathematics as a multi-/semiotic system consisting of the natural language, technical language, symbols and visuals, and the meanings generated by interactions between these systems (Fang, 2012; O'Halloran, 2005; Schleppegrell, 2007). Within this system, the natural language presents the context of the problem, while symbols give patterns of relationships between entities, and the visuals provide a connection between the physical world and the mathematical process constructed in the problem (Schleppergrell, 2007). Solano-Flores et al. (2013) defined the semiotic perspective of an academic discipline as the view that meanings in the discipline are conveyed through multiple representations, called the meaning-making systems. A meaning-making system is "a set of interpretive resources students use in combination to integrate information represented through multiple semiotic features according to the context of the discipline" (Solano-Flores et al., 2013, p. 148). Students do not draw on the resources for meaning-making in isolation, rather they do so in an integrated manner.

Considering the language of mathematics as a multi-/semiotic system means that mathematics is a complex language and that there are multiple ways learners can make sense of and express or represent mathematical understanding. The multi-/semiotic framework, therefore allows us to study the sources of complexity in MWPS (either mathematical or linguistic challenge) as well as the sources for meaning-making during problem solving (either mathematical or linguistic resource). According to Solano-Flores and Nelson-Barber (2001), meaning-making is shaped by cultures, proficiency in the language of the problem, first-language influences, socioeconomic factors, and opportunity to learn. For instance, a student may draw on a cognate word in their first language to make sense of an English word within a mathematics text or test.

Students draw on various resources including linguistic, everyday/cultural experiences, as well as visual devices such as symbols, diagrams, equations, and graphs (Lemke, 2003) to make sense of a mathematics text. Students also draw on their basic mathematical knowledge/ experiences during problem solving (Schoenfeld, 1985). Schoenfeld (1985) terms these mathematical knowledge/experiences as “resources” that an individual brings to bear on the problem at hand. According to Schoenfeld, resources include intuitions and informal knowledge regarding the domain of mathematics, facts, algorithmic procedures, routine/nonalgorithmic procedures, and understandings or propositional knowledge about the agreed-upon rules for working in the domain.

Contrary to most research that has focused on ELLs’ challenges due to complexity in the natural language (e.g., Abedi & Lord, 2001; Martiniello, 2008, 2009; Messick, 1989), for this study, I paid attention to both the linguistic and mathematical challenges students encounter during problem solving. In this study, I did not consider the challenges in order to expose the participants’ deficits, but to make visual the role of LPRs in the participants’ mathematical processes. Challenges pose opportunities for learning and at the point of challenge, bi-/multilinguals tend to draw upon their LPRs so as to seize certain learning opportunities (Civil, 2012). I also considered the resources (both mathematical and linguistic) students draw upon to make meaning of the mathematical problems. I used two problems that basically involve the same mathematical ideas though represented with different semiotic modalities in an attempt to understand the role of different resources in the participants’ sense-making of the problems and participation in mathematics practices. One problem had a visual component, while the other one did not.

Definition of Key Terms

Bi-/multilingual students were students who spoke at least two languages including Swahili. I termed the ability to speak at least two languages including Swahili bi-/multilingualism.

English language learners (ELL)s are the student for whom English is not their first language.

Language as a practice means seeing language as a practice or an action occurring within a social or cultural context means that bi-/multilinguals are not bounded in the manner of use of their language(s) (Palmer & Martinez, 2016).

Linguistic challenges are the features that can cause students misunderstanding of or confusion about the text, or difficulties that can make them lose focus on the problem (Abedi & Lord, 2001).

Language practices are the features of communication used by bi-/multilinguals such as code-switching, translating, and so on.

Language resources are other features of communication such as gestures and other body expressions that may be relevant to bi-/multilinguals "yet they are less salient to the untrained ear" (Moschkovich, 2007, p. 139).

Mathematical challenges are the aspects that are mathematically challenging during problem solving. Caused by certain linguistic challenges or by lack of the necessary mathematical resources for solving a given problem.

Mathematics practices are ways of reasoning, arguing, symbolizing, revealed in the participants' goals, actions, and perspectives during their problem solving.

Mathematical problem is a task for which one must find a solution but has no ready access to a solution schema or procedure (Charles & Lester, 1982; Schoenfeld, 1985)

Mathematical resources are mathematical concepts and knowledge (formal or informal) that students bring to bear during problem solving (Schoenfeld, 1985).

Mathematics as a multisemiotic system involves seeing the language of mathematics as consisting of the natural and technical language, symbols and visuals, and meanings generated from interactions between these systems (Fang, 2012; O'Halloran, 2005; Schleppergrell, 2007).

Translanguaging is the process by which students engage in complex discursive practices that include all the language practices of all students in a class in order to communicate and appropriate knowledge. (Garcia & Wei, 2014). Translanguaging involves the use of features that are visual, textual, symbolic, drawn from an inventory that is not limited to the societal definition of what is an appropriate language.

Vygotskian perspective of mathematics practices is the view that mathematics practices are social, cognitive, semiotic, and cultural and that they are embedded in a mathematical discourse which consists of both academic and everyday practices.

Chapter Summary

In this chapter, I laid the foundation for my study. I presented the background of my study, stated the problem addressed in this study as well as the purpose and significance of my study. I also discussed the theoretical underpinnings upon which the study was framed.

This study was motivated by the current reforms in mathematics education that place communicating mathematically at the center of mathematical learning for all students. My study thus sought to understand the nature of mathematical communication among high school students in the U.S. for whom English is not their first language. Students who are learning mathematics in a language different from their home language often underachieve in standardized tests such as NAEP, as they face certain linguistic and cultural biases in those examinations.

Specifically, I focused on bi-/multilinguals high school students from the African Swahili-speaking refugee community. These students represented a bi-/multilingual population that has never been studied before. In studying the students' mathematical communication, I sought to understand the language practices and linguistic resources the students used, as well as the linguistic challenges they faced during MWPS. I also sought to know the mathematics practices and mathematical resources they used, as well as the mathematical challenges they faced during MWPS. I also investigated the role the LPRs played in the students' mathematical processes of problem solving, communication, representation, connection, and reasoning.

I framed my study using a Vygotskian perspective where mathematics practices are embedded in mathematical discourse that includes both everyday and academic discourse practices. I viewed everyday practices as consisting translanguaging practices and academic practices as consisting the multi-/semiotic nature of the language of mathematics. I viewed bi-/multilingualism as a resource for sense making and sought to know how bi-/multilinguals draw on their LPRs to communicate mathematically. Understanding how bi-/multilinguals communicate mathematically informs how mathematics practices should be described, how bi-/multilinguals can be supported to learn mathematics, and how teachers can be prepared to support bi-/multilingual students in their mathematics classrooms.

Chapter 2: Review of the Literature

The general aim of this study was to investigate the practices and resources bi-/multilingual high school students from the African refugee community use, as well as the challenges they encounter during MWPS. Specifically, the study investigated the mathematics practices and language practices these students engage in, as well as the linguistic and mathematical challenges they encounter during MWPS. The study also sought to know the resources these students draw on during problem solving, as well as how the students' use of language practices and other resources influence their participation in mathematical processes. The bodies of research discussed in this section include: (a) the language of mathematics, (b) mathematical word problem solving, (c) ELLs' challenges in mathematical word problem solving, (d) bi-/multilinguals' language practices, and (e) bi-/multilinguals' resources for mathematical processes.

The Language of Mathematics

Mathematics is not language free (Ní Ríordáin & O'Donoghue, 2009; Schleppegrell, 2007). Mathematics has its own specialized language, which is conveyed through the natural (everyday) language (Kenney & de Oliveira, in press; Ní Ríordáin & O'Donoghue, 2009). In conveying the language of mathematics, the natural language assumes certain specialized forms, styles, and ways of presenting arguments within the context of mathematics (Cuevas, 1984). For instance, the phrases, *the area under the given curve* and *the sum of the first n terms of the sequence* represent a specialized use of the natural language. The specialized language of mathematics, also known as the technical language consists of terms which are words that are solely mathematical - these are usually of Latin and/or Greek origin such as *hypotenuse*, and those that are uniquely mathematical as well as everyday words such as *irrational* (Fang, 2012).

According to Halliday (1978), the natural language within a mathematics text must always express if it is being used for mathematical purposes or not. In other words, the mathematical use of natural words should be precise, and always give rise to "an almost totally nonredundant and relatively unambiguous language" (Brunner, 1976, p. 209). This kind of nonredundancy and unambiguity is not always preventable and it calls for learners to understand the meanings of these words in a mathematical context.

Other aspects of the language of mathematics are the symbols and visual displays. Visuals and symbols can represent information in ways that language cannot (O'Halloran, 2000). In a mathematics text, symbolic representations play numerous roles including labeling, naming, signifying, communicating, simplifying, representing, revealing structure, and displaying relationships (Pimm, 1995; Stacey & MacGregor, 1999). Within a single statement, symbols can act as generalized numbers, arguments of a function, parameters, unknown numbers, or variables (Usiskin, 1988). According to Schleppergrell (2007), symbols give a pattern of relationships between entities, and the diagrams provide a connection between the material world and the mathematical process constructed in the problem. In mathematics, all these systems work together to construct particular meanings (O'Halloran, 2005) that students should be able to unpack within a given text. The language of mathematics is thus a multi-/semiotic system (O'Halloran, 2005) consisting of the natural and technical language, symbols and visuals, and meanings generated by interactions between these systems (Fang, 2012).

Numerous studies have considered the role of language of mathematics among students for whom English is not their first language (Abedi, 2004; Abedi & Lord, 2001; Abedi, Lord, & Plummer, 1995; Cuevas, 1984; Martiniello, 2008, 2009). However, most of these studies have considered complexity in the natural language as the cause for the achievement gap between

ELLs and non-ELLs (Messick, 1989). Consequently, research on ways to analyze and minimize natural language-related factors of test items has suggested accommodative measures such as language modification (e.g., Abedi, Hofstetter, & Lord, 2004; Kieffer, Lesaux, Rivera, & Francis, 2009).

Few studies have investigated the role of other aspects of the semiotic system of mathematics. Martiniello (2009) considered the role of nonlinguistic schematic representation using differential item functioning (DIF) and found that the impact of linguistic complexity on DIF is attenuated when items provide more nonlinguistic schematic representations that help ELLs make meaning of the text. Solano-Flores et al. (2013) used a semiotic perspective, where meaning is conveyed through multiple ways of representing information, to study the relationship between semiotic features of mathematics items and the performance of ELLs on those items. These researchers compared the semiotic features of test items designed to assess content knowledge and those designed to assess academic language. Their study used semiotic features as the basic unit of analysis and grouped the semiotic features into semiotic modalities such as notation (e.g., +, =, ½, x, 182), mathematical register (e.g., place value, numerator, whole number), natural/mathematical language (e.g., circle, equivalent to, fewer), testing register (e.g., cloze questions, which of the following, best way), and visual representation (e.g., charts, number sentences, proportions). They found that items with higher semiotic load (number of different semiotic features) were more challenging than those with fewer semiotic loads. In another study using PISA-2009 data, Solano-Flores et al. (2016) examined the correlation of the difficulty of science items and the complexity of their illustrations. They found increased illustration complexity to be an affordance in sense-making of science items for students from the high-ranking regions and a challenge to students from lower ranking areas.

Martiniello's (2009) study showed that ELLs depended on nonlinguistic schematic representations to make sense of mathematics text, while Solano-Flores et al.'s (2013) and Solano-Flores et al.'s (2016) studies showed that the number of different semiotic features may afford or hinder students' sense-making in academic subjects. All these three studies are quantitative in nature and do not offer empirical evidence of how students interacted with these semiotic features during problem solving. For Martiniello's (2009) study, it is not clear how and when the nonlinguistic representations helped the students overcome which linguistic complexities of the mathematics text. For Solano-Flores et al.'s (2013) and (2016) study, there are no details of how and when the illustrations afforded or hindered the students sense-making. In an attempt to address this gap, my study examined how students interacted with various semiotic features in two Algebra word problems with different semiotic loads, in order to make sense of the problems. The semiotic features of the two problems were described based on Solano-Flores et al.'s (2013) model for semiotic features and modalities. More broadly, in this study, I reported on how the students used various resources within the semiotic system of mathematics to make sense of the problems and to participate in mathematical processes.

Mathematical Word Problem Solving

Mathematical Problems

As noted by Schoenfeld (1985), defining the term mathematical problem is difficult because problem solving is relative. What is a problem to one student may not be a problem to another student. Schoenfeld (1985) and Charles and Lester (1982) defined a mathematical problem as a task for which one must find a solution but has no ready access to a solution schema or procedure. The opposite of a problem is an exercise. An exercise is mostly a task for which the person who encounters it has ready access to a solution procedure. Kilpatrick (1985) and NCTM (2000) refer to a mathematical problem as a task involving mathematical concepts and principles for which the solution method is unknown in advance by the person(s) engaged in it. In this study, I used the term mathematical problem for a mathematical task which one should solve but has no ready access to the solution procedure.

Researchers have used various words for what they term good mathematical problem. For instance, good mathematical problems are said to be problematic, meaningful, worthwhile, interesting, and beautiful (Crespo & Sinclair, 2008). Other descriptions of good mathematical problems entail simplicity, brevity, clarity, elegance, fruitfulness, mathematical deepness, complexity, cleverness, cognitive demand, novelty, and surprise, among others (Koichu, Katz, & Berman, 2007; Lester, 2007). According to Albayrak, Ipek, and Isik (2006) a good mathematical problem is real (appropriate for students' level and daily life), interesting (motivates students' curiosity), uses suitable language (appropriate for students verbally and in writing), and requires use of basic skills (the ability to use obtained knowledge). The multi-/semiotic nature of the language of mathematics means there can be a wide range of problems with different numbers of semiotic features (Solano-Flores et al., 2013). For instance, there could be problems consisting of a combination of natural/mathematical language and symbols, or problems combining natural language and visuals only across various mathematics strands such algebra, arithmetic, and so on. One common type of mathematical problems is the mathematics word problems (MWPs) which I discuss in the following section.

Mathematical word problems. A mathematical word problem (also known as a mathematical story problem or simply a story problem) is a mathematical problem with words (Jan & Rodrigues, 2012). MWPs are a key educational resource in mathematics curricula and assessments at all levels of education and there is little evidence that they will cease to be fundamental (Jonassen, 2003). MWPs connect mathematical concepts with language or reality (Gasco, Villarroel, & Zuazagoitia, 2014). MWPs are daily life-related problems and are presented orally or in written form and in many cases, they blend relevant data with irrelevant information (Gasco et al., 2014). Sometimes MWPs are presented using contrived problem situations, and in such cases, students should withhold their belief and treat these situations as if they were true (Ambrose & Molina, 2013).

In brief, MWPs are a genre of text (Ambrose & Molina, 2013) and they hold special a place in mathematics curricular at all levels. This the reason their consideration was important in this study. MWPs from the domain of algebra will be used in this study. Algebra is a gate-keeper. In solving algebraic problems, students are transitioning between concrete arithmetic to the symbolic language of algebra, as they develop abstract reasoning skills necessary to excel in math and science. Algebraic thinking has been associated with either the ability to think abstractly or represent relations among variables or to model problems (Driscoll, 1999). According to Driscoll (1999), algebraic thinking entails three habits namely: doing-undoing, building rules to represent functions, abstracting from a computation. The algebra problems used in this study entail the concepts of patterns, functions, and relations. These are some of the topics in algebra that have been termed fruitful to focus on, since they are well researched, well understood, and are seen as key to studying the development of conceptual understanding (Moschkovich, 2008).

In this study, I had three algebra problems, namely, the math eliciting task (PrM), problem A (PrM), and Problem B (PrB). PrM was an algebra problem I adopted from the NAEP-2009 released test items. I used PrM as a math eliciting task to assess the participants' progression with both spoken and written English. I adapted PrB from NAEP-1990 and 1992 released algebra word problems. I then modified PrB into PrA such that the two problems were similar except that they had different combinations of semiotic features (Solano-Flores et al., 2013). All the three problems were used to elicit the participants' mathematical understanding, which helped answer the study's research questions. I selected the original NAEP test items because they were algebra problems testing students' problem-solving ability. In adapting the NAEP-1990 and 1992 test items, I sought to create an extended constructed response type of a question, which would call for a demonstration of conceptual understanding. The three problems that were used in this study, together with a description of their semiotic features are listed in Appendix A1. The three original NAEP-1990, 1992, and 2009 released test items are also listed in Appendix A2.

While it is impossible for a single mathematics problem to satisfy all the features of a good mathematical problem, I considered the three problems planned for use in this study problematic enough as they do not have a ready solution path. The problems employ various semiotic features, for instance, one has a figure while the other does not. Additionally, the problems were appropriate for the daily life of a ninth or tenth-grade student and these students could apply the knowledge they have obtained to solve them (J. Berger, personal communication, April 24, 2017). I discussed the natural language used in the problems with literacy and mathematics experts to ascertain that it matched the expected English proficiency for a ninth or tenth-grade student. I piloted the problems with two pairs of students to ascertain their appropriateness in this study and make the necessary adjustments. The four students whom I used in piloting did not participate in the actual study.

Solving mathematical word problems. Lester (2007) defined mathematical problem solving as the process of interpreting a given situation mathematically, while employing several “iterative cycles of expressing, testing and revising mathematical interpretations - and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics” (p. 782). Moreover, problem solving involves the process of coordinating prior experience, knowledge, and intuition in an attempt to find a method for resolving a situation whose outcome is unknown (Charles & Lester, 1982).

Competence in MWPS is one of the most important competencies that learners should master (Barake, El-Rouadi, & Musharrafieh, 2015). Through problem solving, children can make connections between conceptual and mathematical knowledge (Artut, 2015). According to NCTM (2000), MWPS is “not only a goal of learning mathematics but also a major means of doing so” for “by learning problem solving in mathematics, students should acquire ways of thinking, of perseverance, and curiosity, and confidence in unfamiliar situations” (p. 52). It is therefore pertinent to study how students, and especially bi-/multilinguals problem-solve and to describe the practices and resources they use, as well as the challenges they encounter during this process.

Competence in MWPS is however different from other forms of mathematics competencies because it requires the student to comprehend the text describing the problem situation and derive a number sentence representing the situation (Fuchs et al., 2015; Walkington et al., 2015). Researchers seem to unanimously agree that text comprehension is pivotal to successful MWP solving (Bernardo, 2002; Fuchs et al., 2015; Jan & Rodrigues, 2012; Oviedo, 2005; Ní Ríordáin & O’Donoghue, 2009). The first of Polya’s (1957) four stages of problem solving is understanding the problem. All students are mathematics language learners (Kersaint, Thompson, & Petkova, 2013) and as such, they struggle with comprehending MWPs. However, ELLs face an additional challenge with comprehending the ‘third’ language of mathematics in a ‘second’ language such as English (Barbu & Beal, 2010; Cuevas, 1984; Kenney & de Oliveira, in press; Martiniello, 2008, 2009; Schleppegrell 2007). The aforementioned studies focus on complexity in the natural language as the main source of challenge in MWPS for bi-/multilinguals. However, the urge to shift from a deficit view of ELLs demands that we focus more on what these students bring to bear during MWPS and how what they possess influences their problem resolution and participation in mathematical processes. This was the goal of this study. I investigated the practices, resources, and challenges in MWPS among the bi-/multilingual high school students from the African Swahili-speaking refugee community. I also sought to understand how the LPRs that the students used afforded them participation in mathematical processes.

Schoenfeld (1985) and (2011) presented a framework for studying mathematical problem solving which entails examining resources (knowledge base), heuristics (problem-solving strategies), control (metacognition), and beliefs. According to Schoenfeld (2011), every root cause of success or failure in problem solving would be found within these four categories. In 1992, Schoenfeld added a fifth category of practices to his earlier framework. Schoenfeld acknowledged the role of interactions with others as central in mathematical learning and he posited that students’ mathematical thinking should be understood in terms of the “mathematical communities in which students live and the practices that underlie those communities” (p. 363). According to Moschkovich (2013), the addition of practices to Schoenfeld’s (1985) framework for studying mathematical problem solving made the framework more complete “setting the stage for deeper analyses of mathematics practices” in mathematics education research (p. 269). Schoenfeld (1992) however noted that mathematics practices cannot be studied without considering the resources (knowledge base), heuristics (problem-solving strategies), control (metacognition), and beliefs.

Schoenfeld (1985) defined resources as the mathematical knowledge that the individual brings to bear on a particular problem. For example, intuition and informal knowledge regarding the domain of mathematics, facts, algorithmic procedures, routine nonalgorithmic procedures and understandings regarding the nature of argumentation/rules for working in the domain, and other relevant competencies. Knowledge of the resources the participants bring to bear during problem solving helps with an understanding of what the participants do while working on the problems. There is, therefore, a need to know what the individual believes to be true even if it is not (Schoenfeld,1985). Heuristics are problem solving strategies which may include introducing auxiliary elements in a problem or working auxiliary problems, arguing by contradiction, working forward from the data, decomposing and recombining, exploiting related problems, drawing figures, generalizing and using the inventor's paradox, specializing, using reduction ad absurdum and indirect proof, varying the problem, and working backward. Heuristics provide a means for stretching resources as far as possible. Control decisions determine the efficiency with which facts, techniques, and strategies are exploited. Belief systems are one's mathematical worldviews, the perspective with which one approaches mathematics and mathematics tasks. Belief determines how one chooses to approach the problem, which techniques to use or avoid, how long and how hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control can operate.

In this study, I was concerned with the practices and resources that students use during MWPS and the challenges they face in the process. In order to understand the nature of practices and resources the participants used during MWPS, I analyzed the students' problem solving using Schoenfeld's (1985) stages for problem solving, which include: analysis, design/exploration, implementation, and verification. The analysis stage involves understanding the problem statement, simplifying and reformulating of the problem. Design entails control or balance and it ensures that one is engaged in activities that are most likely profitable to their problem solving. Schoenfeld noted that design is not an isolated box on the flowchart, but something that permeates the entire problem-solving process. Design means keeping a global perspective of what one is doing and proceeding hierarchically. For instance, one should not get involved in detailed calculations or complex operations until they have explored alternatives or have clear justification for using those alternatives. Exploration is the stage where the majority of problem-solving strategies come into play. Exploration entails strategies such as solving equivalent problems or slightly modified or broadly modified problems. If the possible strategies in one stage prove insufficient, one would proceed to the next. If substantial progress is made at any stage of the exploration stage, then one may either return to design to plan again or may decide to reenter analysis in the hope that the insights gained in the exploration stage can help them recast the problem and allow them to approach it differently. Implementation entails the actual problem solution and verification entails checking the solution locally to catch silly mistakes or globally to seek alternative solutions, connections, and for other useful aspects of the problem that may make one a better problem solver.

Studies that have investigated the strategies ELLs use for MWPS have focused on ELLs' text comprehension strategies or practices (Ambrose & Molina, 2013; Artut, 2015; Barake et al., 2015; Gasco, et al., (2014); Jan & Rodrigues, 2012; Mangulabnam, 2013). For instance, bilinguals were found to use direct translation and keywords (Hegarty, Mayer, & Monk (1995). In Barake, et al. (2015) and Ambrose and Molina (2013), some students, driven by "compulsion to calculate", read the text superficially and missed the implicit data, rendering their solution process incorrect. Again, these studies were founded on the deficit model which sees ELLs as deficient in a language and their use of language practices such as direct translation as a sign of deficiency. My study positioned ELLs as possessing certain linguistic and cultural resources which are valuable for sense-making during MWPS. I, therefore, allowed students to draw upon their linguistic resources and any other available resources in order to solve selected algebra word problems. The resources and practices students employed during the various stages of problem solving were investigated, as well as the challenges they encountered at each stage. Schoenfeld (1985) defined resources as the mathematical knowledge students bring to bear on problem solving. However, in this study, I also considered students' knowledge of various linguistic and cultural systems as resources, in an attempt to understand the students' mathematical thinking in terms of the practices that underlie the communities they come from (Schoenfeld, 1985).

ELLs' Challenges in Mathematical Word Problem Solving

In this section I discuss three sources of ELLs' challenges in MWPS: (a) English language related factors, (b) problem context-related factors, and (c) mathematics knowledge related factors.

English-language Related Factors

Most of the existing research on ELLs' linguistic challenges during MWP solving has identified complexity in the natural language as the only challenging aspect of the language of mathematics. Complexity in the natural language is thus termed irrelevant to the constructs a mathematics test intends to measure (Messick, 1989). Technical language is termed "construct relevant" and as such, students are expected to master it for examinations. Complexities in the natural language may be at the lexical (vocabulary), grammatical (morphological and syntactic), or text (cohesion and rhetorical organization) levels (Avenia-Tapper & Llosa, 2015).

At the vocabulary level, challenging features may include unfamiliar words, phrases, and connotations of words with multiple meanings (Moschkovich, 2015). These challenges may also be stated in terms of the number of low-frequency words, abstractions, polysemy of words (multiple usages of words), and idiomatic and culture-specific nonmathematical vocabulary words (Martiniello, 2008). Grammatical complexity may occur when a text contains long dense noun phrases, prepositional phrases, participial modifiers, relative, complement, adverbial and conditional clauses, conjunctions with or without technical meanings, passive voice (missing agent), abstract nouns, interruption construction, *being* and *having* verbs, implicit relationships, and ellipsis (omitted text) (Abedi & Lord, 2001; Avenia-Tapper & Llosa, 2015; Fang, 2006; Martiniello, 2008; Martiniello, 2009; Moschkovich, 2015; Schleppergrell, 2007).

Complexity at the general text level entails relationships between words and visuals (Avenia-Tapper & Llosa, 2015). For instance, the proportion of language to non-language, the number of visuals, and the extent to which the text reader needs language knowledge in order to comprehend the text (Wolf & Leon, 2009). Text level complexity also includes text structure, genre, and rhetorical organization of ideas in the text (Ambrose & Molina, 2013).

In this study, I considered the language of mathematics as multisemiotic consisting of the natural and technical language, symbols and visuals, and meanings generated from interactions between these systems. As such, challenges emanating from all aspects of the language of mathematics were considered. The goal was not to expose these challenges as researchers have done in the past, but to investigate the LPRs bi-/multilingual students may use to negotiate these challenges in order to make sense of MWPs and proceed with problem solving.

Problem-context Related Factors

MWPs take on irrelevant information (situations, events, topic, or actions) and combine them with relevant data (Gasco, et al., 2014). Some of the irrelevant information may present an unfamiliar situation to the students. For instance, Oviedo (2005) found that unfamiliar situations described in her percentage problems (e.g., making concrete, the role of a keypunch operator) were a distraction to her students. She thus concluded that there was a relationship between problem contexts and the language of presentation. Martiniello (2008) found that her students' unfamiliarity with using coupons at the store caused them difficulty with understanding the phrase "coupon for $1.00 off" and hence were unable to solve the problem. According to Ambrose and Molina (2013), "unfamiliarity with a problem context potentially affects all students but especially those that do not belong to the majority cultural group" (p. 1471). The two algebra problems selected for this study were established to be appropriate for the participants' grade-level and they did not present the participants with unfamiliar topics or contexts (J. Berger, personal communication, April 24, 2017).

The irrelevant information of a word problem may be a topic that students have no experience with or they have no interest in. For instance, Celedo-Pattichis (2003) found that her Hispanic middle school students had difficulty with a story problem about astronomy because they lacked experience in that topic. Walkington et al. (2015) studied how readability and topic incidence affected performance on MWPs and found that the topic of the problem impacted students' engagement and performance. These authors noted that the "sheer scale of topics touched by the mathematical story problems can be a formidable barrier to relating language to mathematical reasoning" (p. 1051).

Another challenging aspect of MWPs related to context is the implicit data. Implicit data is information that is not clearly stated in the text which students should find using other information and clues for the text (Barake et al., 2015). According to Barake et al. (2015), every text contains some implicit data, and MWPs are not an exception. After studying how students in grade seven and eight read and understood implicit data when solving MWPs, Barake et al. (2015) found that very few students understood the implicit data and were able to use it in problem solving even when the data was crucial to problem solving. Ambrose and Molina (2013) stated that syntax and vocabulary were not so much a hindrance to students' text comprehension as situations and the implicit nature of the problems.

The aforementioned studies are investigations of challenges ELLs face with MWPS. In this study, I did not simply identify the challenges the participants faced during MWPS. Rather, I allowed the participants to draw on their semiotic resources to make sense of the problems, and I sought to describe the resources the participants drew on, as well as how those resources influenced their sense-making and participation in mathematical processes.

Mathematics-knowledge Related Factors

Apart from being proficient in English and familiar with the context of the problem, successful MWPS requires a knowledge of mathematical concepts and procedures. These mathematical concepts and procedures are what Schoenfeld (1985) termed resources and heuristics respectively. MWPS entails transforming the written words into mathematical operations and symbolization (Jan & Rodrigues, 2012). According to Oviedo (2005), most of the difficulty with MWPs arise from the mismatch between text comprehension, situation comprehension, and problem-solving procedures. Understanding the problem is only the first stage of problem solving (Polya, 1957; Schoenfeld, 1985). After understanding the problem, the student still needs to devise a strategy for solving the situation and had to implement the strategy correctly. Instead of focusing on the sources of difficulties, in this study, I investigated the resources student draw upon to negotiate challenges at various stages of problem solving as well as the mathematics practices they were able to engage in despite the challenges.

The three factors discussed in this section reiterate that successful MWPS entails: (a) understanding of the language conveying the problem (natural language) and the technical language, (b) knowing the mathematical meanings of the symbols and visuals used in the problem, and (c) knowing how language, symbols, and visuals interact, as well as the meaning their interactions generate. In other words, MWPS requires an understanding of the language of mathematics in its entirety. In this study, I aimed at investigating the language practices and any other resources that bi-/multilinguals used to make sense of the language of mathematics in selected algebra word problems, as well as how their use of these resources afforded them participation in mathematical processes.

Bi-/multilinguals' Language Practices

Bi-/multilingualism has been conceptualized in different ways that have led to different models of bi-/multilingualism such as the additive and subtractive models of the 20th century and other more dynamic models of the 21st century (Garcia, 2009). In the subtractive model, a bi-/multilingual initially speaks the first language(s) and a second (or third) one is added while the initial one(s) gets withdrawn. The end result of the subtractive model is a second (or third) language, monolingual speaker. In the additive model, a second (or third) language is added to the first while preserving the first, leading to a two (or three) monolingual individual who is never supposed to mix the two (or three) languages s/he has acquired (Ovando & Combos, 2012).

The need for models that better represent who bi-/multilinguals are and those that are responsive to the 21st-century societies which are increasingly heterogeneous necessitated the birth of the dynamic models of bi-/multilingualism. In the dynamic bi-/multilingualism, several languages are used in varying degrees of competence for different purposes (Garcia, 2009). Dynamic bi-/multilingualism is enacted precisely through of translanguaging (Garcia & Wei, 2014; Lewis, Jones, & Barker, 2012). In other words, dynamic bi-/multilingualism is exhibited through translanguaging.

Translanguaging

In translanguaging, bi-/multilinguals use their two (or more) languages "to make meaning, shape experiences, understandings, and knowledge" (Barker, 2011, p. 288). Translanguaging challenges the monoglossic view of bi-/multilingualism as parallel monolingualism where the bi-/multilinguals' languages are used separately (Garcia, 2009, Garcia & Wei, 2014). Translanguaging is the normal way of languaging among bi-/multilinguals, where they employ hybrid language practices with no bounds (Garcia, 2009; Palmer & Martinez, 2010).

Translanguaging incorporates both code-switching and translating, but it goes beyond both (Garcia, 2009). Whereas code-switching and translating could be used in ways that reflect a monolingual perspective of bilingualism, translanguaging cannot. Translanguaging practices cannot be easily assigned to one or another traditional definition of language, but they make up the speaker's complete language repertoire (Garcial & Wei, 2014, p.). Other translanguaging practices include code-mixing, language switching, and language brokering and interpreting (Daniel & Pacheco, 2015; Moschkovich, 2007; Palmer & Martinez, 2010). Colindres (2015) investigated the use of language practices by Latino teachers in a school district that had reduced the achievement gap between Latino students and the state average for White students by 12 percentage points in middle school mathematics between the years 2004 and 2007 (Tupa & McFadden, 2009). He reported on the teachers use of translanguaging practices such as translating, translanguaging in speaking, student paraphrasing, and stimulus of inner speech, revoicing, previewing, and mnemonic devises, collaborative dialogue, and cognates. Garcia and Wei (2014) also noted that some students translanguage in an inner speech in order to make meaning of situations. Translanguaging is, therefore, an umbrella term for a range of language practices bi-/multilinguals draw on to make sense of situations.

As noted by Moschkovich (2007), most of these language practices overlap in how researchers in mathematics education define or use them. For instance, some sociolinguistics use "code mixing" to refer to the switching for immediate access to an unknown term, such as a single noun or noun phrase, and reserve "code-switching" to refer to changing completely from one language to the other at major boundaries. Others define code mixing as the transferring of linguistic units from one code to another and code-switching as the alternation of one language to the other due to a change in participants, social situation, and so on. Instead of using code-switching, some researchers prefer to use code mixing or borrowing.

Moschkovich (2007) cautioned that making subtle distinctions among different language practices is not relevant to the issues encountered in a mathematics classroom. According to Moschkovich, what is relevant about these practices is an investigation of how their use influences or may influence the teaching and learning of mathematics. This is what part of what I was concerned with in this study. I investigated the practices and resources the participants drew upon during MWPS and the challenges they faced as they solved selected MWPs. Most importantly, I investigated the participants' use various LPRs and how the use of the LPRs influenced their participation in mathematical processes. In the following, I discuss two translanguaging practices that have been well researched.

Code-switching. Code-switching is mixing both language codes in speech, alternating between the two languages; it is like style-switch for monolinguals when they switch from a formal to an informal register (Garcia, 2009). It entails the use of more than one language in a single speech act (Setati, 1998). According to Moschkovich (2007), code-switching is not a single agreed-upon language practice. She noted certain distinctions among types of code-switching and disagreements on when each definition is applicable in the following passage:

Some researchers use code-switching to refer to switches within one speaker turn, others to switches within one conversational episode, and others to within sentence switches. The use of a single word from one language in an utterance that is in another language is a difficult phenomenon to classify; While some researchers consider using single words in another language as cases of code-switching (Sanchez, 1994, p.169), other researchers prefer to call these loans. (p. 15)

Code-switching is common in multilingual classrooms (Akindile & Letsoela, 2001; Martin, 2002; Setati, 2005) especially in classrooms where the teacher and students share a common first language but have to use an additional language for learning (Setati, 2005). According to Chitera (2009), code-switching is one way that multilingual students use to overcome challenges they face in mathematics classes. In this study, I explored the participants' use of language practices such as code-switching and other resources and the role of the use of these resources on their participation in mathematical processes.

Though a potentially useful resource for teaching and learning, code-switching has been stigmatized in a number of ways (Grosjean, 1999). Moschkovich (2007) noted an example in the U.S. where teachers working with Latino students were reported to consider code-switching as unacceptable. Code-switching is still held as a deficiency or a sign of deficiency with admonitions such as "it's not good English" or it's not good Spanish" and in other cases, its use has been outrightly prohibited (Moschkovich, 2007). Code-switching has been regarded as a grammarless mixture of two languages which monolinguals may see as an insult to their own language (Setati, 2005). It is generally believed that people who code-switch do not know either language well enough to be able to converse in either of them alone (Setati, 2005). However, Setati (2005) argued that code-switching does not occur because a child is unable to handle content subjects in English. In other words, code-switching is not a deficit or a sign of other deficiencies (Valdes-Fallis, 1978; Moschkovich 2007). In fact, in some communities, it is the ability to code-switch that marks fluent bi-/multilinguals (Zentella, 1981).

Someone's code-switching cannot be used to "reach conclusions about their language proficiency, ability to recall a word or knowledge of a particular technical term." (Moschkovich, 2007, p.18). Code-switching does not mean someone is not proficient in one language or the other or has not mastered certain technical terms, rather, it is the normal way of languaging among bi-/multilinguals. In this study, I sought to understand the language practice that bi-/multilinguals tap into when solving MWPs. The study assumed that language practices such as code-switching are not a deficit or a sign of deficiency and sought to validate this assumption by examining the role of language practices such as codes-switching play in a bi-/multilingual's mathematical processes.

Translating. Translating is another language practice common among bi-/multilinguals. Translation has been likened to language brokering (Orellana, Reynolds, Dorner, & Meza, 2003). Yosso (2005) noted a case of a bilingual who acted as a language broker at home by translating her mother's emails, phone calls, and coupons. Yosso (2005) used this example to highlight that bi-/multilinguals' have certain language practices that are valuable in their homes and that these practices should also be valued in the classroom. Halai (2007) investigated students' movement between languages during learning of mathematics and noted how students in Pakistan translated English terms to Urdu – their first language. Her assertion was that when translating, students not only need to be able to translate the words, but they also are required to translate mathematical meanings to their preferred language(s).

One challenge that Halai (2007) noted with translation as a language practice was with students' inability to translate English phrases according to the mathematical discourse. In other words, the linguistic structures in the natural language of a mathematics text were in conflict with students' thinking in Urdu. She, therefore, concluded that movement between languages in the course of learning mathematics cannot be regarded as a straightforward resource. She suggested the need for further research into why students move across languages and how this movement may hinder or facilitate learning of mathematics. In this study, I investigated the participants' use of language practices such translating and how the use of such practices may influence their engaging in mathematics practices during MWPS.

Bi-/multilinguals' Resources for Mathematical Processes

Most of the studies that have investigated MWPS among bi-/multilinguals have termed bi-/multilingualism a resource for meaning-making (for example, Adetula, 1989; Barwell, 2009a, 2009b; Chitera, 2009; Clarkson, 2006; Cuevas, 1984; Halai, 2007; Setati, 1998). According to these studies, diverse bilingual students perform better when solving problems in their native language than in their second language, especially when the problems are difficult. A study by Dominguez (2011) showed that the use of language practices aligned with problem difficulty and improved performance, portraying language practices as cognitive resources for learning mathematics (Dominguez, 2011).

Other studies have focused on the influence of specific language practices to bi-/multilingual learners during problem solving. For instance, Adler (2001) found students consistently using code-switching to explore new ideas. Moschkovich (2007) analyzed an excerpt where two ninth-grade Spanish-English students used code-switching to describe patterns, clarify mathematical meanings, state assumptions, support mathematical claims, make generalizations, and make connections to mathematical representations, all of which she termed "valued mathematical discourse practices" (p. 138).

Dominguez (2011) investigated the role of bilingualism and everyday experiences as a cognitive resource for mathematical learning. In his study, Dominguez found that when students used Spanish, they were more able to reinvent and reproduce situations than when they used English. Moreover, there were more reinventions and reproductions of problems with familiar contexts than for the same problems under unfamiliar contexts. Moschkovich (1996) offered an empirical evidence of how everyday experiences with natural phenomena may provide resources for communicating mathematically. For instance, students in her study used experience climbing hills as a resource for describing the steepness of lines.

In an attempt to understand factors that promote language switching among Catalan-Spanish bilingual secondary school students (12-year-olds) in Barcelona, Spain, Planas and Setati (2009) using a sociolinguistic approach, found that shifts from Catalan to Spanish coincided with shifts in the complexity of the students' mathematics practices. For instance, there was a shift in language use as students shifted from the initial familiarization with the task and the new mathematical vocabulary to solving the task and explaining, arguing, and representing symbols. Students thus shifted from use of Catalan to Spanish as it was easier for them to complete and communicate their mathematical processes in their first language.

Upon investigating how seven second-grade Spanish-English bilingual students solving addition and subtraction problems used language to communicate their mathematical reasoning, Dominguez (2005) found that these students used words and gestures simultaneously during their communication. This offers a ground for critiquing studies that focus on students' use of certain recourses in isolation from others and offers justification for why students' resources for mathematical learning should be studied holistically. The use of gestures as a resource for conveying mathematical meaning among bilinguals has also been studied (Moschkovich, 1999, 2002).

Other studies such as Solano-Flores et al. (2013), Solano-Flores et al. (2016), and Martiniello (2009) that investigated the influence of semiotic features such as symbols, diagrams, equations, graphs, and other schematic representations during MWPS. They found that these features tend to attenuate the linguistic complexity of a mathematics text in some cases. Being quantitative studies, these researchers do not provide details of how students, and particularly students for whom the language of the text is not their native language, use these resources during problem solving.

Research on literacy and biliteracy has associated translanguaging with several educational benefits such as deepening bilinguals' metalinguistic awareness, strengthening their comprehension skills, successful development of new languages, and a deeper understanding of content (Daniel & Pacheco, 2014; Garcia & Wei, 2014; Martin-Beltrain, 2014). In translanguaing, bi-/multilinguals' use their LPRs in an integrated manner, not in isolation. Apart from Colindres (2015) who studied the translanguaging strategies used by teachers in Latino mathematics classrooms, no other study has investigated translanguaging in mathematics. Contrary to most studies that have investigated the role of isolated bi-/multilinguals' language practices, my study used a translanguaging lens to explore the LPRs that bi-/multilingual participants from the Swahili speaking African refugee community used during MWPS, together with the role the LPRs played in the participants' mathematical processes which include problem solving, reasoning and proof, communication, connection, and representation. Contrary to other studies that have focused on students in lower grade levels, the participants used in this study were high school students who recently immigrated in the U.S. This population is critical in this study because the academic material at high school is more challenging and these students are more likely to translanguage in order to make sense of it and communicate their understanding (Garcia & Wei, 2014).

Chapter Summary

In this chapter, I reviewed literature related to my study. My study concerned mathematical communications among bi-/multilinguals students from the African Swahili-speaking refugee community. Specifically, I examined how the practices (language and mathematics practices), resources (linguistic and mathematical resources) these students use, as well as the and challenges (linguistic and mathematical challenges) they face during problem solving. The bodied of research that I reviewed to situate my study were therefore those related to the language of mathematics, mathematical word problem solving and the challenges ELLs face during MWPS, as well as those that consider bi-/multilingualism as resources for sense making and not a deficit.

The studies that I reviewed showed that the language of mathematics is a multi-/semiotic system consisting of the natural language, the technical language, symbols and visuals. Students are to not only understand how each of these systems functions within a mathematics problem, but they are to also understand the meanings they generate when they interact with each other. We also noted that each of these aspects of the language of mathematics is a potential source of challenge in MWPS for all students. However, students for whom the natural language is different from their home language face additional challenges with MWPS. Most studies, using a deficit model, have considered complexity in English as the only source of ELLs' challenges in MWPS. My study, however, viewed bi-/multilingualism as a resource for sense making. I therefore sought for the practices and resources these students use during MWPS. I also investigated the challenges they faced during MWPS, whether emanating from complexity in the natural language or from other aspects of the language of mathematics. The aim was not to expose these challenges, but to find out the LPRs the students draw upon at these points of challenge and how drawing on the LPRs enhances their mathematical processes.

Chapter 3: Research Methodology

Introduction

Guided by a constructivist (social-constructivist) worldview, I employed a qualitative approach with case study design to investigate the practices, resources, and challenges in MWPS among the Swahili-speaking bi-/multilingual African refugee high school students in the U.S. Specifically, I investigated: (a) the language practices and linguistic resources the students used and the linguistic challenges they faced, (b) the mathematics practices and mathematical resources the students used and the mathematical challenges they faced, and (c) the role of LPRs in the students' mathematical processes. In this chapter, I discuss the constructivist worldview, qualitative research approach, and the case study research design, and how they fit in this study. I also discuss my role and ethical considerations. I then discuss data collection and analysis procedures for both the piloting and the actual study phases including the selection of participants and the setting for the studies. I also include details of how the pilot study informed the actual study phase. Lastly, I explain how I ensured validity and reliability in the study.

Constructivist Worldview

A worldview is "a basic set of beliefs that guide action" (Guba, 1990). Constructivist beliefs guided the design of this study and data collection and analysis procedures. Citing the work of Crotty (1998), Creswell (2014) noted three assumptions held by constructivists: (a) human beings construct meanings as they engage with the world they are interpreting, (b) humans engage with their world and make sense of it based on their historical and social perspectives – in other words, we are all born into a world of meaning bestowed upon us by our culture, and (c) the basic generation of meaning is always social, arising in and out of interaction with a human community. In the following sections, I discuss how each assumption fits in my study.

Human beings construct meaning as they engage with the world they are interpreting. In order to understand the practices, resources, and challenges in MWPS among the Swahili-speaking bi-/multilingual participants from the African refugee community, I engaged the participants through task-based interviews. The tasks were three algebra word problems. While the participants solved these problems, I sought to understand the practices and resources they used, as well as the challenges they faced.

Humans engage with their world and make sense of it based on their historical and social perspectives. In order to carry out my study, I had to visit the participants in their homes, agree on an appropriate research setting, and gather the information personally. The participants either chose their homes or a nearby public library since, in those settings, they would feel comfortable to draw upon their linguistic repertoires and resources. I interpreted what I found in the field based on the participants' meaning, as well as my own experiences and background. For instance, because I have experience with code-switching between English and Swahili it was easier to notice the participants' use of similar language practices.

The basic generation of meaning is always social, arising in and out of interaction with a human community. The meanings I generated in this study arose from the data collected in the field; LBS, transcripts of video recorded task-based interviews, my brief interview notes, and participants' written work. The Vygotskian perspective of mathematics practices that I used views mathematics practices as social, cognitive, semiotic, and cultural. As such, I considered the meaning of utterances within the mathematical discourse that the participants engaged in as they jointly solved mathematics tasks.

Qualitative Research Approach

According to Creswell (2014), a constructivist worldview "is typically seen as an approach to qualitative research" (p. 204). In other words, the features of qualitative research fit well into the assumptions of constructivism. Qualitative research is concerned with people's experiences and provides richer information into the phenomenon under study. Creswell (2014) identified the main characteristics of qualitative research approach as follows: (a) occurs in natural setting, (b) researcher is a key instrument, (c) multiple forms of data, (d) inductive and deductive data analysis, (e) focus on participants' meaning, (f) emergent design, (g) reflexivity, and (h) holistic account. In the following sections, I discuss each of these features, showing how they relate to my study.

Occurrence in a natural setting. In qualitative studies, the participants are talked to directly and observed as they act within their context. Qualitative research involves face-to-face interaction. In this study, I visited the participants and interviewed them face-to-face and recorded their behaviors and actions, which included their discourse as they solved MWPs. I did not bring the participants to a lab or a contrived setting, but I had them work within their natural environment – an environment where human behavior and events occur, such as their homes or public libraries.

Researcher as a key instrument. The researcher is the primary instrument in data collection. Even though they may use a protocol for collecting data, they the ones who actually gather the information. In this study, I developed instruments such as interview protocols and interviewed the participants by myself.

Multiple forms of data. Qualitative research does not rely on one data source but involves multiple forms of data such as interviews, observations, documents, and audiovisual information. This study generated data from a LBS, audio and video recorded task-based interviews, participants' written work, and my own interview notes. I reviewed all these data to make sense of it and organized it into categories or themes that cut across the interviews, participants' written work, and the survey, and my notes.

Inductive and deductive data analysis. Qualitative researchers develop patterns, categories, and themes as they move from concrete to more abstract units of the information. Analysis of qualitative data begins inductively as the researcher move back and forth between the themes and the database until a more comprehensive set of themes are attained. The analysis then becomes deductive as the researcher works from the established themes trying to see if more evidence and support is needed from the database or if additional information is needed. This is how I worked on my data analysis. I started with looking at the entire set of data in order to make a general sense of it. I then began with specific themes, which were based on my three research questions. I then moved back and forth between the themes and the database looking for categories that fit in each theme. Finally, starting with the themes, I sought the database for information that could further support of my themes.

Focus on participants' meaning. Qualitative research focuses on learning the meaning that the participants hold about the phenomenon under study. In this study, I focused on the participants' meaning at every stage of the research. I did not impose the meaning I hold, or the meaning writers express in literature. My study sought to understand the practices, resources, and challenges for bi-/multilinguals Swahili-speaking African refugee high school participants in the U.S during MWPS. As such, during the task-based interviews, I focused on how the participants understood and solved the problems. Where it was not clear to me what the participants meant, I sought for their explanation. During analysis, I sought for meanings of the utterances in the participants' mathematical discourse.

Emergent design. This feature asserts that a qualitative researcher's initial plan can be altered or shifted even after the researcher has entered the field and began to collect data. The main aim of conducting qualitative research is to learn about the phenomenon from the participants and to address the research to obtain that information. It does not, therefore, make sense to be rigid with an initial plan and miss on important information about the phenomenon one is studying. For instance, my initial plan was to have participants perform the mathematics eliciting task in English. When I piloted the protocol, I did not see the need to alter the plan. However, when I began the actual data collection, I met some participants who said they were only comfortable talking about mathematics in Swahili or with mixing English and some other language(s). In this case, I altered my initial plan and allowed them to discuss the problems in the language they were comfortable with. Without restricting language used by these participants for that task, I would obtain important information for my study.

Reflexivity. Qualitative researchers are required to reflect on their role in the study and how their personal background, culture, and experiences can potentially shape their interpretations of the study's findings. Creswell (2013) noted that discussing reflexivity is more than advancing biases and values in the study, but how the researcher's background may shape the direction of the study. My choice of a qualitative approach for this study is partly influenced by my personal background as a multilingual speaker of five languages and my experiences with learning mathematics in a third language. Although I could relate to most of the participants' experiences, throughout the data collection process I suspended my own perceptions and sought only to understand the participants' experiences. I believed that differences in language backgrounds and past mathematical experiences could render our experiences different. However, my experiences partly influenced my interpretations of the findings. I discuss in detail my role in this study in the role of the researcher and ethical considerations section.

Holistic account. Qualitative researchers try to develop a complex picture of the phenomenon under study. They do this by reporting multiple perspectives, by identifying factors involved in a situation, and by generally sketching the larger picture that emerges. In this study, I was concerned with the practices, resources, and challenges in MWPS among Swahili-speaking African Refugee bi-/multilingual high school participants. I sought to gain an in-depth understanding of all the aspects of this study and to present the multiples perspectives of the participants concerning the aspect. My aim was to provide my readers with a larger picture of the issues I was studying. For instance, while some participants perceived the visual used in PrB as challenging their comprehension of the problem, others saw it as a resource for sense making. As a qualitative researcher, I reported on both perspectives.

Case Study Research Design

Case study design is one of the many qualitative research approaches in existence. Although there are variations in the definition of case studies (Marshall & Rossman, 2016), researchers unanimously agree that case studies provide an in-depth understanding of a case, often a program, event, activity, process, or one or more individuals (Creswell, 2013, 2014; Flyvbjerg, 2011; Marshall & Rossman, 2016). Qualitative researchers need to define their unit of analysis, which might be an individual, a small group, an intervention, as well as set the boundaries around the case (Marshall & Rossman, 2016; Yin, 2014). Cases are bounded by time and activity, and researchers collect detailed information using various data collection procedures over a sustained period of time (Stake, 1995; Yin, 2012). When conducting case studies, researchers need to clarify the selection process of cases (Marshall & Rossman, 2016). A researcher may be interested in an intrinsic case (a case that represents nothing but itself), or an instrumental case (an exemplar of a more general phenomenon) or multiple cases (several instances of a phenomenon) (Carla, 2001; Stake, 1995). Sometimes it is difficult to categorize a case study as intrinsic or instrumental (Stake, 1995). However, the choice of one or the other or both should be based on the learning opportunities the case affords, as well as the challenges it may present (Marshall & Rossman, 2016; Stakes, 1995). The characteristics of case studies fit well into the constructivists' worldview (Stakes, 1995). Hayes (2000) identified four main characteristics of case studies: (a) descriptive study, (b) narrowly focused, (c) combines objective and subjective data, and (d) process-oriented. In the following sections, I briefly describe each of these features and show how my study fits each description.

Descriptive. A case study being descriptive means that all data constitute processes and events, and the contexts in which they occurred. Case study researchers mainly provide details of behaviors or experiences. My study is primarily descriptive; I described the practices and resources that the participants used, as well as the challenges they faced while solving algebra word problems. I also described the role the LPRs played in the participants' mathematical processes.

Narrowly focused: A case study may be a description of a single individual or about groups. My study focused on the case of a group of 12 Swahili-speaking bi-/multilingual African refugee high school students. My study is both intrinsic and instrumental in that the participants used are not only a particular case in and of themselves, but they are also representative of larger bi-/multilingual cases that have been studied in the U.S., for example, by Moschkovich (2007) and in South Africa, for example, by Setati (2005). In other words, my unit of analysis in this study is secondary to understanding (Stake, 1995) the practices, resources, and challenges in MWPS among bi-/multilingual participants.

Combines objective and subjective data: Objective data consists of descriptions of behavior and context, while subjective data details aspects of feelings, beliefs, impressions or interpretations. I collected all these forms of data in my study. My data consisted of the participants' actual discourse and behavior as they solved algebra word problems as well as my interpretations of their feelings and beliefs as expressed in their hand and body gestures.

Process-oriented: Case studies enable the researcher to explore and describe the nature of processes that occur over time. I aimed to describe the practices and resources that Swahili-speaking African refugee bi-/multilinguals high school students in the U.S. use and the challenges they encounter as they solve algebra word problems. A case study made it possible for me to study the participants' mathematical processes for a given period of time, instead of a still snapshot of the process that an experimental study would yield. For example, I spent a minimum of 80 minutes in the field studying the participants' MWPS and interrogating them about the various problem-solving paths they pursued.

Researcher's Role and Ethical Considerations

According to Creswell (2014), a qualitative researcher is an interpretive inquirer who is involved in a sustained and intensive experience with participants. As such, there are some ethical and personal issues a qualitative researcher brings into the research process (Locke, Spirduso, & Silverman, 2013). In this section, I discuss reflexively my biases, values, and personal background that shaped my interpretations of the finding from this study.

As earlier mentioned, I am a multilingual speaker of five languages, including Kamba (my mother tongue), Swahili (like another first language), English (foreign language), and Luo and Kisii (other African local languages). During PreK to grade 3 (grades nursery to standard 3), I learned mathematics primarily in my mother tongue. From grade 4 (standard 4) up to the end of college or university education, I learned mathematics in English. My experience with learning mathematics in a third language and the difficulties that go with solving MWPs written in a language different from home language motivated my interest in this study.

Moreover, my ability to speak Swahili, a language that is shared by many Africans as the lingua franca (Orado, 2014) informed my choice of participants. I view language practices as translanguaging practices, and so my fluency in Swahili helped me understand most of the discourse that went on between the participants. My experience as a volunteer mathematics tutor in a bi-/multilingual community learning center influenced my interest in studying MWPS among bi-/multilinguals and my choice of setting for my study. At the learning center, students had the freedom to communicate in whatever languages they liked. Such a setting would be the best for my study as I desired to create a safe translanguaging space for my participants.

When I visited the participants, I mostly spoke in Swahili as a way of connecting with them. This might have influenced some of the participants' choice of language(s) to use during their discourse as they solved the problems. For example, most participants used Swahili. The reason could have been that they thought Swahili was my preferred language. However, I constantly reminded them to speak naturally, and in the language(s) they were comfortable with, and not to bother with whether I understood what they said or not.

I recruited my participants from two churches, church C and S, in a large city in the northeastern part of the United States where many African refugee communities worship. I visited the churches and discussed my research with the church leaders. Both leaders agreed to support my research by allowing me to recruit participants from their congregation and to even use the church facility to conduct my research, if need be. The leaders wrote and signed letters of cooperation. I prepared the announcement scripts I would use to recruit my participants and submitted my forms for review by the Syracuse University's Institutional Review Board (IRB) office in charge of research and integrity. As soon as I got the authorization to conduct my research, I visited the churches to recruit my participants. The approval letter from the IRB, the letters of cooperation, and scripted announcements are attached as appendices G, H, and I respectively.

In church C, I read the scripted announcement to the high school students in an after-church gathering where a representative from a group that supports students in going to college was addressing high school students. In church S, I also read a scripted announcement to the high school students during a brief after-service meeting. The scripting and reading of the announcement were done in line with the specifications of Syracuse University's IRB.

I used purposive sampling with a criterion that required each participant to be an African refugee high school student who speaks Swahili and/or other native language(s) in addition to English. Moreover, the participant needed to be in grade nine or ten and to have done Algebra 1 or 2 regardless of whether s/he passed the exam or not. I selected a total of sixteen students from those who volunteered after the announcements and met the set criterion to participate in this study. I had four out of the sixteen students be in a pilot for this study, while the remaining 12 participated in the actual study. I deemed a sample of 12 participants adequate for the actual study since, according to Creswell's (2013) review of qualitative research studies, case studies include about four to five cases, where a case can be one or more individuals. This study was a case study of 12 high school students from the African refugee community who had recently immigrated to the U.S. However, my goal for selecting my participants was I would collect data up to the saturation point; the point where data no longer sparks new insights and can adequately address the research questions (Charmaz, 2006; Glaser & Strauss, 2017) and I achieved this goal with the 12 participants.

Data Collection

Case studies help researchers to develop an in-depth understanding of the participants' behaviors and experiences. In order to develop a deep understanding of the practices, resources, and challenges in MWPS for the participants in this study, I used task-based interviews. Task-based interviews have the potential for opening a "window into the subjects' knowledge, problem solving behaviors, and reasoning" (Koichu & Harel, 2007, p. 349). Interviews provide researchers with firsthand experience with the participant. During the interaction with the participant, the researcher can record information as it occurs and can also notice unusual aspects. Through interviews, researchers can control the line of questioning in order to gather an in-depth information about a phenomenon.

On the other hand, interviews have the disadvantage that information gets filtered through the views of the interviewee. Moreover, the responses might be biased because of the interviewer's presence and the fact that not all people are equally articulate and perceptive (Creswell, 2014). I also used LBS before administering the tasks in order to the determine participants' demographic information, experiences with using their language(s) both at home and school, as well as their past language(s) of mathematical instruction (Moschkovich, 2010) and their perception of proficiency in each of their language(s).

I conducted this study in two phases. The first phase was the piloting phase. The second was the actual data collection phase. Each phase entailed the administration of LBS and two task-based interviews, namely, the first and the second task-based interviews. Before I discuss the two phases in detail, I will describe the nature of the instruments and tasks I used in this study.

Data Collection Instruments

In this study, I collected data through a LBS and two task-based interviews. The tasks were three algebra problems, namely, PrM, PrA, and PrB. These problems were either adopted or adapted from the NAEP released algebra test items. Since my study focused on algebra word problem solving among high school bi-/multilinguals in the U.S., I deemed NAEP released test items the best to use because NAEP reflects the performance of ELLs in the content areas covered in class nationally. Moreover, NAEP released mathematics test items have been well researched for linguistic challenges faced by ELLs, e.g., Abedi, 2004, Abedi & Lord, 2001, Abedi, Lord, & Plummer, 1995, and Martiniello, 2009. While NAEP is administered for students in grades four, eight, and twelve, I deemed the eighth-grade algebra test items truly accessible and appropriate for my ninth or tenth-grade participants (J. Berger, personal communication, April 24, 2017). I required tasks from the content of patterns, functions and relations and that called for extended or short constructed response. According to Moschkovich (2008), these content areas are deemed productive to focus on as they are well researched and understood and they are key in studying conceptual understanding. I applied my search criteria at the national center for education statistics (NCES) website and the NAEP released test items that met my search criteria were from years 1990, 1992, and 2009. See Appendix A2 for these original test items. In the following sections, I discuss the LBS and the two task-based interviews that I used in this study.

LBS. The LBS was meant to help provide the participants' demographic and language background information. The LBS I used in this study was adapted from Abedi et al. (1995) (see

Appendix C). I used the survey to determine the participants' demographic information including age, gender, grade level, and length of stay in the U.S. The survey also provided information about the participants' perception of proficiency in each of their language(s) and their experiences with using those language(s) at home, at school, and away from school. For instance, there was a question about how often the participants used their language(s) to speak to their parents, grandparents, brothers, and sisters, as well as friends at school and away from school. The participants were also required to state the language they prefer to use while talking about mathematics and to say their past language(s) of mathematical instruction (Moschkovich, 2010). For instance, they needed to say if they have ever been taught mathematics in their languages. The participants' responses to the LBS formed a large part of the section on Description of Participants.

First task-based interview. I used the first task-based interview to elicit participants' progress with speaking and writing in the English language. The task used in this interview was a problem adapted from the NAEP-2009 released algebra test items. This problem, also referred in this study as the math eliciting task or simply PrM required the participants to continue an algebraic number pattern and to generate a rule for doing so, a concept that is also covered by the tasks used in the second task-based interview. I adapted Kitchen, Burr, and Castellon's (2012) protocol for administering mathematics tasks to bi-/multilinguals. The protocol required each participant to estimate the solution to the task at a glance and explain their choice (speaking), write down the process for their solution (writing and speaking), explain the solution process (speaking), and then write down the solution to an imaginary friend who does not know how to solve the problem (writing and speaking) (Kitchen et al., 2012) (see the English assessment protocol in Appendix B1). Through this protocol, the task-based interview yielded data on the participants' speaking and writing skills, which were the communication modalities I was interested in in this study.

I then used Heritage, Chang, Bailey, Jones, and Peterson's (2015) and Bailey and Heritage's (2018) language progress assessment frameworks to analyze the participants' speaking and writing progress, respectively. In the framework for assessing speaking, Heritage et al. (2015) describe the language features one should target when assessing bi-/multilinguals' progress with speaking in English. These four language features include sophistication of topic vocabulary, the sophistication of sentence structure, the establishment of advanced relationships between ideas, and coherence of explanations. Each of these language features is scored between 0-3, with 0 denoting "not evident", 1 denoting "emergent", 2 denoting "developing", and 3 representing "controlled" (Heritage et al., (2015) (see further description of target language speaking features in Appendix B3). In the writing framework, Bailey and Heritage highlight three features one should target when assessing bi-/multilinguals' writing in English. The three features include opening and closing statements, decontextualization, and graphic representation supporting meaning making. Each of these features is assessed as either "not evident", or "emerging", or "developing", or "controlled" with a score of 0, 1, 2, and 3, respectively (see Appendix B4).

Second task-based interview. The second task-based interview was used mainly to elicit the participants' mathematical processes. The tasks were two algebra word problems with different combinations of semiotic features (Solano-Flores et al., 2013). One problem, Problem B or PrB, was an adaptation of two NAEP released test items; one from the NAEP-1990 and the other one from NAEP-1992 (see Appendix A2 for the original test items). The other problem, Problem A or PrA, was a modification of PrB that had different semiotic features. I selected the original NAEP test items because they both assessed participants' problem-solving skills, and my study was concerned with the behavior of my participants' while solving mathematical problems. The 1990 NAEP-released item was a multiple-choice response type, while the 1992 NAEP-released item required students to provide an extended constructed response.

In adapting the test items to create PrB, I combined the NAEP-1992 test item's main instruction with the triangular figure in the NAEP-1990 test item. This way, I formed a mathematical problem that had at least two semiotic features and that could attract participants' explanation of their solution process. To form PrA, I created a different context for PrB using words and numbers, but no figure. The original NAEP-1990 and 1992 released test items were categorized as "complex" or "hard" and so I had to ascertain that my adaptations were appropriate for my participants (Albayrak, Ipek, & Isik, 2006). I then checked with an experienced high school teacher, who after consulting with several other teachers, confirmed to me that the problems were appropriate since a ninth or tenth-grade student could apply the knowledge they have to solve them (J. Berger, personal communication, April 24, 2017). The two problems that I used in the second task-based interview, together with their semiotic features (Solano-Flores et al., 2013) have been listed in Appendix A1.

In the following section, I discuss the two phases of this study. I first discuss the piloting phase, including the selection of participants and the setting. I then briefly discuss data collection and analysis procedures, and how the pilot study informed the actual data collection phase.

Phase One: Pilot Study

I piloted this study for four reasons. First, I wanted to test if my instruments, which included the tasks, interview protocols, and the LBS, were viable and that they would yield the information I needed to address my research questions. Second, I wanted to know the best way I could pair the participants during the second task-based interviews to increase their chances of translanguaging. Third, I wanted to test the language assessment framework with the kind of responses I got from the first task-based interview. Lastly, I wanted to find out the challenges I might anticipate in this study and how I could possibly address them. In the following, I discuss the selection of participants and setting and how the data collected. Lastly, I discuss the data analysis procedures and how the pilot study helped shape the study.

Participants and Setting

Participants. I selected four out of the sixteen participants who had volunteered to participate in this study and had met the selection criteria to be participants in the pilot study. I chose the four participants so that two had lived in the U.S. for more than three years and the other two had lived for at most three years. The two participants who had lived in the U.S. for at most three years worked together during the second task-based interviews and the other two participants who had been in the U.S for more than three years also worked together. I chose to pair the participants in this way so I could learn information that might assist me in pairing participants during the actual study so as to increase their chances to translanguage as they solve the problems. Table 3.1 below shows the demographics of the four piloting participants. In the sections below the table, I provide details of each participants' language use at home and school, as well as their language of mathematical instruction and language preference and proficiencies in different languages. I also describe the participants' progress in English based their scores on the language assessment task. The participants' information provided on Table 3.1 together with the details of their language use at home and at school, their language of mathematical instruction and language preference and proficiencies was based on the participants' responses to the LBS. All the piloting participants revealed that they had some level of proficiency in Swahili and Kinyarwanda. The participants also stated that they did not receive English as a second language (ESL) services in their school. To protect the identities of the participants, all the names of participants used in this study are pseudonyms.

Table 3.1

Piloting Participants' Demographics

Name	Length of stay in the U.S.	Grade	Gender
Otman	7 years	9	M
Shuman	9 years	10	M
Emman	10 months	9	M
Dinan	3 years	10	F

Otman. Otman was a 15-year-old male ninth-grade student who had lived in the U.S. for seven years. Otman stated that he spoke Kinyarwanda, in addition to English and Swahili.

Language use at home and school. Kinyarwanda is not Otman's first language. He decided not to share what his first language is. He, however, indicated that he started to speak Kinyarwanda when he was 15 years of age. Otman stated that he can speak Kinyarwanda fairly well. He noted that he can also read and write it fairly well. While at home, Otman stated that he speaks Kinyarwanda with his grandparents only. He speaks to everybody else, including his parents, brothers, and sisters, in Swahili. While at school, Otman reported that he speaks neither Swahili nor Kinyarwanda to his friends. Away from school, Otman reported that he speaks to his friends in either Kinyarwanda or Swahili. Otman reported that he understands Swahili very well and he can speak, read, and write it fluently.

The language of mathematical instruction and language preference and proficiencies. Otman stated that he was not taught mathematics in either Kinyarwanda or Swahili. He prefers to talk about mathematics in English. He reported that he understands English very well. He reported that he can speak, read, and write English very well. Otman said he finds it very easy to understand his mathematics teacher's explanations, but he has some difficulties with understanding both his textbook and questions on a mathematics test.

Language assessment score. Otman wrote his responses for both the writing and speaking prompts in English. I scored Otman as a 3 on both modalities, meaning that his speaking and writing were at the controlled stage of English language progression (Heritage et al., 2015). Otman's oral responses contained the appropriate and accurate use of topic and technical vocabulary. He used a variety simple and compound sentences that were accurate and grammatically correct. He maintained clarity in relationships between ideas, and his explanations would require very little or no effort from a listener to understand the steps or process he was explained (Heritage et al., 2015). On his writing, Otman had an opening statement, but not a closing statement. He used no deictic language and he had the appropriate co-references and a naïve reader could understand his explanation based on what he wrote. Moreover, his punctuations, spelling, and other representations were clear and easy for a reader to make meaning of (Bailey & Heritage, 2018)

Shuman. Shuman was a 17-year-old male grade ten student who had lived in the U.S. for nine years. Shuman reported that he speaks Kinyarwanda in addition to English and Swahili.

Language use at home and school. Kinyarwanda is Shuman's first language. Shuman reported that he understands Kinyarwanda fairly well, but he cannot read and write Kinyarwanda well. He reported that he speaks with his grandparents in Kinyarwanda only. Shuman reported that he speaks to his parents, brothers, and sisters in both Swahili and Kinyarwanda. Shuman reported that he speaks to his friends at school or away from school in either Swahili or

Kinyarwanda. Shuman reported that he understands Swahili fairly well and he can speak, write and read it quite well.

The language of mathematical instruction and language preference and proficiencies. Shuman reported that he has not been taught mathematics in either Swahili or Kinyarwanda. He reported that he prefers talking about mathematics in English. He reported that he understands spoken English fairly well, and he can speak, read, and write English quite well. Shuman stated that he finds it fairly easy to understand his mathematics teacher's explanations. He reported that he can read his mathematics textbook with ease, but he has difficulties understanding questions on mathematics tests.

Language assessment score. Shuman responded to both the speaking and writing prompts of PrM in English. I rated him as a 3 on speaking, meaning he was at the controlled stage of English language speaking progression (Heritage et al., 2015). Shuman used precise topic and technical vocabulary appropriately and accurately. He used simple and compound sentences that were grammatically correct. He used more than two discourse connectors, but some of his explanations may require some effort from a listener to understand the step or process he is explaining (Heritage et al., 2015). With regards to writing, Shuman scored a 2, meaning he was at the developing stage of English language writing progression. Shuman's written response had a closing statement, but not an opening statement. He did not use any deictic language, but he ambiguously tied or omitted referents, making it difficult for a naïve reader to follow his explanation (Bailey & Heritage, 2018). He also used some symbols in a nonconventional manner and this could interfere with meaning making for the reader (Bailey & Heritage, 2018).

Emman. Emman is an 18-year-old male ninth-grade student who had lived in the U.S. for 10 months. Emman reported that he speaks Kinyarwanda in addition to English and Swahili.

Language use at home and school. Emman's first language is Kinyarwanda. Emman reported that he understands Kinyarwanda very well, and he can speak, write, and read in Kinyarwanda fluently. Eman reported that he speaks to his parents, grandparents, brothers, and sisters mostly in Kinyarwanda, but he sometimes used Swahili. While at school, Emman reported that he uses English. Emman reported that he speaks to his friends both at school and away from school in English, Swahili, and Kinyarwanda. He reported that he understands Swahili very well. He reported that he can read and write Swahili fluently, but he can only speak it fairly well.

The language of mathematical instruction and language preference and proficiencies. Emman reported that he was not taught mathematics in Kinyarwanda or Swahili. He prefers talking about mathematics in English. Emman reported that he understands spoken English very well and he can speak, read, and write English fluently. Emman stated that he has difficulties understanding both his mathematics teacher's explanations and questions on a mathematics test. He, however, finds it fairly easy to understand his mathematics textbook.

Language assessment score. Emman completed his speaking and writing responses in English. He scored a 2 on speaking, meaning he was at the developing stage of speaking progression in English language (Heritage et al., 2015). Emman's oral responses had sufficient topic vocabulary, but his sentences lacked complex clause structure. He used two different discourse connectors to establish relationships, but his explanations required some effort from the listener to understand the steps or process he was explaining (Heritage et al., 2015). Emman scored 1 on writing, meaning he was at the emerging stage in the English language writing progression. Emman's writing did not have either opening or closing statements. Although he did not use deictic language, he ambiguously omitted referents and used punctuations, spellings, and other representations in a nonconventional manner that could severely interfere with the reader's sense making (Bailey & Heritage, 2018).

Dinan. Dinan was a 17-year-old female tenth-grade student who had lived in the U.S. for three years. Dinan reported that she speaks Kinyarwanda, in addition to English and Swahili.

Language use at home and school. Dinan's first language is French. She reported that she started to speak Kinyarwanda at the age of 5. Dinan reported that she understands Kinyarwanda very well. She stated that she can speak Kinyarwanda fluently, but she cannot read and write it very well. At home, Dinan reported that she speaks to her parents and grandparents mostly in Kinyarwanda, although sometimes she uses Swahili. She reported that she mostly speaks to her brothers and sisters in Swahili, but sometimes she uses Kinyarwanda. At school, Dinan speaks to her friends in English or Swahili. Dinan reported that she only uses Kinyarwanda at school when she is translating information to new students. Dinan reported that she understands Swahili very well and she can speak, read, and write Swahili fluently.

The language of mathematical instruction and language preference and proficiencies. Dinan reported that she was not taught mathematics in Kinyarwanda or Swahili, but she was taught mathematics in French from grades 2-5 (Standard 2-5). She reported that she prefers talking about mathematics in English. Dinan reported that she understands spoken English very well. She reported that she can speak, read, and write English fluently. Diana stated that she has difficulties understanding her mathematics teacher's explanations and she finds it very hard to understand questions on mathematics tests. She, however, finds it fairly easy to understand her mathematics textbook.

Language assessment score. Dinan responded to both the writing and speaking prompts in PrM in English. Her score on speaking was 3, meaning she was at the controlled stage of English language speaking progression (Heritage et al., 2015). Dinan's oral responses included a variety of precise topic and technical vocabulary that was used appropriately and accurately. She also used both simple and compound sentences that were accurate and grammatically correct. Her response had at least three discourse connectors and her explanations would require little or no effort from the listener to understand the steps or process being explained (Heritage et al., 2015). On her writing, Dinan was at the developing stage of English language progression. Although her written response had both opening and closing statements and contained no deictic language, she ambiguously omitted certain referents, and this made it difficult for a naïve reader to follow her explanation. Moreover, some of her punctuation and symbolization was used in a nonconventional manner that could severely interfere with meaning for the naïve reader (Bailey & Heritage, 2018).

Setting. Gee (1996) showed that bilingual teens' translanguaging is responsive to their environment. As such, studies that investigate bi-/multilingual students' use of language practices should carefully consider the setting in which their study occurs (Garcia, 2009; Moschkovich, 2007) because the setting for such research affects the outcomes of the study. Since my study viewed language practices as translanguaging practices, I wanted to choose a setting in which the participants would feel comfortable to use their translanguaging practices (Daniel & Pacheco, 2014). This setting needed to be an out-of-school location because the restrictive language policies that insist on English as the standard language in most U.S. schools (U.S. English, 2006) would make the participants shy away from drawing on their entire linguistic repertoires. The hesitance of bi-/multilingual students to use any other language other than the school language while they are within a school environment was recently noted (Orado, 2014). Before I embarked on my research, I discussed my research goals with the participants and we mutually agreed on the best out-of-school sites for the study. I conducted my pilot study in three different sites including a quiet room in the house of one of the participants, a small room in the back of a coffee shop, and a nearby public library. Decisions on which out-of-school sites to use were based on convenience and the participants' preferences.

Data Collection

Data were collected in two stages. In stage one, I administered the LBS in order to know my participants' demographics, their experiences with using their language(s) both at home and school, as well as their past language(s) of mathematical instruction and their perception of proficiency in each of their language(s). I also administered the first task-based interview that was meant to help assess the participants' progress in English. The participants' responses to the LBS and the language assessment task have been used to describe the participants in the previous sections.

In the second stage, I administered the second task-based interviews to pairs of participants. The tasks were two problems; PrB and PrA, that were described earlier. I paired the four participants such that Otman worked with Shumam, and Emman worked with Dinan. My thinking was to pair two people, Otman and Shuman, who had lived in the U.S. almost the same length of time, and two other people, Emman and Dinan, whose duration of stay in the U.S. varied, work together in order to know how length of stay in the U.S. may influence how the participants used their linguistic repertoires. By putting Emman and Dinan together to work, I was also aiming to determine if gender would influence the participants interaction during problem solving. I allowed participants to solve the tasks in whatever language(s) or solution paths they liked. The protocol that guided this interview can be found in the Appendix A3. The two task-based interview sessions were audio and video recorded. I took short interview notes and collected the participants' written work.

Data Analysis

Data for this study consisted of participants' responses on the LBS data, transcripts of the first and the second task-based interviews and interview notes, as well as the participants' written work. For the LBS, I input participants' responses in an Excel file for better visualization of the information. I have used this information to analyze the participants' language as I described above. I analyzed the first task-based interview using Heritage et al.'s (2015) language assessment framework. According to Heritage et al. (2015), the four basic language features one should focus on when assessing a student's progression with speaking in English are: sophistication of topic vocabulary, the sophistication of sentence structure, the establishment of advanced relationships between ideas, and coherence of explanations. Each of these features is scored between 0-3, with 0 denoting "not evident", 1 denoting "emergent", 2 denoting "developing", and 3 representing "controlled" (Heritage et al., 2015). I scored and averaged each participants' spoken and written responses separately. The average score was the participants' speaking or writing score and it showed the stage they were in their speaking and writing modalities respectively. I have included this information in the description of each participant under the Language Assessment Scores.

The transcripts of the second task-based interview amounted to a data set that I analyzed for: (a) language practices and linguistic resources the participants used and the linguistic challenges they faced, (b) mathematics practices and mathematical resources the participants used and the mathematical challenges they faced, and (c) the role of LPRs in the participants' mathematical processes. There were no changes in the steps I followed in analyzing these data except that I also used data from the first task-based interview to respond to the study's research questions. I discuss details of the data analysis in the second phase of this study.

The main aim of the analysis of the pilot data, as I mentioned earlier, was to: (a) to test the research instruments, (b) to see how best the participants could be paired during the second task-based interviews session, (c) to see if the language assessment framework works, and (d) to find out the challenges I might expect in the study and how I could possibly address them. In the following sections, I discuss how the analysis addressed these goals and how the pilot study shaped my study.

Instruments for data collection. The two main instruments in this study were the LBS and two interview protocols. Piloting showed that the LBS was adequate in establishing the participants' demographics, their experiences with language use mathematical instruction and their perceptions about their proficiency in their language(s). The first task-based interview was meant to assess the participants' English language progression in the speaking and writing modalities, and it seemed to meet this expectation. However, I also noted that the same interview was rich and like the second task-based interview, it could yield information that addressed the three research questions. The researcher is a key instrument in qualitative research (Creswell, 2014). Through the pilot study, I also had a chance to reconsider my role during throughout the research, and especially during the interviews. I saw that I needed to give meaningful prompts, prompts that would help elicit information needed to address my research questions, namely: (a) What language practices and linguistic resources do the participants use during MWPS? What linguistic challenges do they face? (b) What mathematics practices and mathematical resources do the participants use during MWPS? What mathematical challenges do they face? and (c)

What is the role of the LPRs in the participants' mathematical processes? I also needed to consider when to give the prompts and in what language. I saw that I would need to remain flexible (Frisoli, 2010) as I administered the tasks. I would need to pay keen attention to participants' ideas and seek for the participants' clarification, so I would not misinterpret what they say.

Pairing participants. In the second task-based interview, I paired Emman and Dinan. Their difference in the length of stay in the U.S. was 2 years and 2 months. I noted that Emman and Dinan appeared to not feel free to speak with each other. I thought the fact these participants were strangers to each other and that they were not of the same gender might have caused the anxiety. I constantly encouraged them to talk to each other and to feel free to use any language(s) they like. On the other hand, Otman and Shuman seemed to enjoy working together. The two participants were friends from their church and they had lived in the U.S. for almost the same number of years. I, therefore, resolved to do the following during the actual study: (a) pair participants who are friends and are of the same gender, (b) pair participants who have lived in the U.S. the same or almost the same number of years, (c) create an opportunity for the participants to meet and get to know each other before the study begins or use ice breakers before the study begins, (d) encourage participants to speak if they remain silent for two minutes, and (e) remind them to feel free to use any language or solution paths.

Language assessment framework. The participants I piloted with responded to the speaking and writing prompt of the first task-based interview in English. So, I was able to score their responses and to determine the stage they were at in their progression with English (Heritage et al., 2015). However, when I assessed participants' writing I became concerned that the analysis framework I was using could not address certain aspects of their writing. For instance, I did not know how to deal with punctuations and symbolization that showed up in the participants' writing. The framework I was using was only suitable for assessing participants' oral language. Through personal communication, Bailey introduced me to an upcoming framework for assessing students' progress in writing (Bailey & Heritage, 2018). The framework consists of three written language target features namely: opening and closing statements, decontextualization, and graphic representation supporting meaning making. Each of these features is assessed as either "not evident", "emerging", "developing", or "controlled" with a score of 0, 1, 2, and 3, respectively. See Appendix B4 for a full description of the features and details on scoring. I, thus, used this framework to reassess all the participants' writing for both the pilot and actual study.

While I analyzed the participants' responses, I imagined a situation where a student could not respond to the prompts in English. I resolved that, since qualitative studies are emergent (Creswell, 2013), I would allow such participants to use their preferred language. By allowing participants to use the language they are most comfortable with, I would be creating an opportunity to acquire further in-depth information about the participant's use of language practices in mathematics problem solving.

Challenges to anticipate and ways to address them. Piloting this study provided insights into some challenges I might encounter later in the study, including parents' resistance, recruits' hesitance to participate, and silence during problem solving.

Parents' resistance. I met some parents who were not willing to sign the consent forms for a number of reasons. First, they saw signing the form as committing themselves to something and they were not ready to be held accountable for anything in the research. Second, they failed to understand why I chose to study their children, and not others. Third, they were uncertain about how their children and the community would benefit from this study. This form of suspicion or fear could be attributed to their past history and experiences as refugees. In other cases, there was a language barrier in that some parents understood their first language and only a bit of Swahili, the only language I could share with them. Although I had a Swahili version of the consent letters, I recognized that I needed someone to translate the information to the respective first language.

Addressing parents' resistance. To address parents' resistance in future data collection, I resolved to seek for someone trustworthy in the African refugee community to accompany me during the visits. Through his interactions with the refugee community, my husband, who is a pastor, realized that the person the community trusted most was their leader, commonly known as Presidentio. I therefore arranged to meet Presidentio so as to explain my research and the challenges I was anticipating with gaining entry into this community. Presidentio accepted to accompany me from house to house as I sought the parents' consent for their children's participation in my study. At every house we visited, Presidentio briefly introduced me and then he invited me to explain my research agenda. In my explanation for the research, I emphasized how my identity and past experiences drove me into this study and how the recruits and the community might benefit from the study (more details can be found on both the consent and assent forms in Appendix E). While I explained my research, "Presidentio" intervened to translate or clarify ideas in the manner he thought the parents and their children could understand better. During these visits, we agreed with the parents and the recruits on the day and time I would come back to carry out the research. We also agreed on the best location for the study and how the participants would be transported to the venue. This arrangement made the second phase of data collection very smooth.

Recruits' hesitance to participate. The recruits' hesitance to participate in the study would be due to mathematics anxiety, feeling that they were not good enough in mathematics to participate, and uncertainty about the benefits of participation.

Addressing recruits' hesitance. I minimized the recruits' hesitance to participate in the study through a number of ways: (a) by assuring the recruits at every stage of the research that my aim is not to see how good or bad they are at mathematics, but how they approach problem solving and the challenges they face in the process, (b) by emphasizing the free tutoring I would offer anyone who needs assistance with mathematics, (c) by being friendly to the recruits, and (d) by assuring them of my guidance as they solve the problems and that they would learn some mathematics during problem solving.

Silence during problem solving. There were times during the task-based interviews when the participants remained quiet. I interpreted participants' silence during problem solving to mean that the participants were thinking, they were stuck, they were waiting on their peer's contribution (if they are working in pairs), or they are waiting for my feedback. Another important challenge to anticipate during problem solving was instances when participants pursued a wrong solution path and ended up with the wrong answer.

Addressing silence during problem solving. The aim of my study was to understand practices, resources, and challenges in MWPS. As such, I saw the moments when participants were silent as opportunities to know what they were thinking and if they were facing any linguistic or mathematical challenges. I also took the chance to see the practices or resources the participants would draw on in order to address their challenges and move on to problem solving. I used my own intuition to judge what the cause for silence could have been. I would then ask relevant questions in order to keep the participants thinking and speaking. For instance, if I sensed that the silence was because the participants had no idea how to proceed, I could ask them to check their work, or I could ask them to think of a different solution path, especially if I saw that they were pursuing a wrong one. Alternatively, I could have the participants revisit the question and explain to me what the problem was about and what they needed to do to solve it. My main role was to ask meaningful questions, questions that could help uncover the participants' thoughts, the challenges they were facing, and the practices and/resources they drew upon during their mathematical processes. In the next section, I discuss the second phase of this study.

Phase Two: The Actual Study

In this phase, I discuss the participants, setting, and data collection and analysis procedures. In all the discussions, I include the information I gained from the pilot study. I first discuss the participants in this study and the setting where the research took place.

Participants and Setting

Participants. There were twelve participants in this phase of study: four ninth-grade and eight tenth-grade participants who I selected through a criterion-purposive sampling approach. In criterion-purposive sampling, the researcher identifies the population meeting some predetermined criterion of importance. Purposive selection ensures that only participants who will best help one understand the problem and research question are selected (Creswell, 2014). The population of interest in this study was the African refugee high school students who spoke Swahili and/or other native languages in addition to English, and had recently immigrated to the U.S. A recent immigrant in this study was someone who had lived in the U.S. for at most three years. The piloting phase showed participants who had recently immigrated to the U.S. were more likely to rely on their linguistic repertoires to make sense of the problems. Thus, the criterion of selection was that the participant is a recent immigrant, an African refugee who is in grade nine or ten and has completed Algebra 1 or 2 whether s/he passed the New York State Algebra exam or not. My study investigated participants' practices, resources, and challenges in algebra word problem solving. Therefore, the participants needed to have completed grade nine or ten because, in the U.S., participants completing these grade levels are typically done with Algebra 1 or 2. Typically, a participant with knowledge and skills in any of these courses would be able to solve the problems in this study without difficulties.

I chose to study this population of participants for two main reasons. First, participants from the Swahili-speaking African refugee community constitute a different bi-/multilingual case in the U.S., whose mathematics practices have not been addressed in any study. Most studies on bilinguals' mathematical performance have focused on Spanish-English bilinguals. Second, contrary to most studies that focus on bi-/multilingual participants at lower grade-levels, I focused on bi-/multilingual high school students. Studying the practices, resources, and challenges in MWPS for bi-/multilinguals at high school level is significant because this is the level at which most recent U.S. immigrants tend to rely on their linguistic repertoires and other resources in order to make sense of school. The rationale for my choice of the study population is well captured by Garcia and Wei (2014):

> The number of recent immigrants entering U.S. schools after age 15 has increased in the last decade. Thus, scholarly attention is turning from elementary to secondary schools where learning challenges are greater because the content taught is more difficult and there is less time to develop new language practices capable of expressing more sophisticated content. Translanguaging … then serves as an important practice to … these adolescents. (p. 95)

Table 3.2 below shows the demographics of the twelve participants. In the sections that follow Table 3.2, I describe the participants' language use at home and at school, their experiences with language use in mathematical instruction, and their language preference and proficiencies. I also provide details of the participants' progress with English as assessed based on both Heritage et al.'s (2015) and Bailey and Heritage's (2018) language assessment frameworks. All the information on table 3.2, together with details of the participants' language use at home and at school, language of mathematical instruction and language preference and proficiencies in different languages was based on the participants' self-reports on the LBS. The twelve participants revealed that they had some level of proficiency in one or more other languages including Swahili, Kinyarwanda, Bembe, and French. These participants also stated that they did not receive English as a second language (ESL) services in their schools. To protect the identities of the participants, all the names of participants used in this study are pseudonyms.

Table 3.2

Participants' Demographics

**These names are pseudonyms to protect the identity of the participants*

Name	Length of stay in the U.S.	Grade	Gender
Tamani	1 year	10	M
Samba	1 year	9	M
Berina	2 years	10	F
Rehana	2 years	10	F
Neha	1 year 8 months	9	F
Zawiri	1 year 8 months	9	F
Exmon	2 years	10	M
Kwisha	8 months	10	M
Azina	8 months	10	F
Solana	3 years	10	F
Diodo	1 year	9	M
Fareli	8 months	10	M

Tamani. Tamani was a 17-year-old, grade 10 male student who had lived in the U.S. for one year. Tamani speaks Bembe in addition to English to and Swahili.

***Language use at home and school*.** Bembe is his first language. Tamani reported that he understands and speaks Bembe very well, but he reads in the language only fairly well and does not write well in the language. Tamani reported that he uses Bembe mostly to communicate with his parents, grandparents, sisters and brothers, and friends away from school. At other times, he uses Swahili at home and while away from school. Tamani reported that he understands Swahili very well. He reported that he can read, write, and speak Swahili very well. While at school, Tamani uses English and sometimes speaks Bembe with some of his friends.

***The language of mathematical instruction and language preference and proficiencies*.** Tamani reported that he has not been taught mathematics in Bembe or Swahili. He preferred talking about mathematics in English. Although Tamani said he can read and write English very well, he thought he understands and speaks English only fairly well. Tamani finds his mathematics teacher's explanations and questions on mathematics tests fairly easy to understand and finds his mathematics textbook easy to understand.

Language assessment score. Tamani responded to the speaking prompts in English, but he responded to the writing prompts in Swahili. His oral responses were marked by one word or simple sentences that contained the topic vocabulary or sometimes words borrowed from the question. He also used one discourse connector repeatedly and only one conjunction between words or phrases. His explanations would require a lot of effort from the listener to understand the process he was explaining (Heritage et al., 2015). Tamani, therefore, scored 1 on his speaking, meaning he was at the emergent stage of English language speaking progression. Tamani said that it was hard for him to respond to the writing prompt in English. I could not assess his progress in writing because the framework I used only assesses responses written in English (A. Bailey, personal communication, January 16, 2018).

Samba. Samba was a 16-year-old male, ninth-grade student who had lived in the U.S. for a year. Samba speaks Bembe, in addition to English and Swahili.

Language use at home and school. Bembe is Samba's first language. Samba reported that he understands and speaks Bembe very well, but he reads in the language only fairly well and does not write well in the language. Samba reported that he uses Bembe mostly to communicate with his parents, grandparents, sisters, and brothers. At other times, he uses Swahili at home, while with friends at school, and away from school. Samba said that his friends do not like it when he speaks to them in Bembe at school. Samba reported that he understands and speaks Swahili very well. He can read, write, and speak Swahili well. While at school, Samba uses English and sometimes he speaks Swahili with some of his friends.

The language of mathematical instruction and language preference and proficiencies. Samba reported that he has not been taught mathematics in Bembe or Swahili. He prefers talking about mathematics in English because English is the language commonly used to teach the subject. Although Samba said he understands spoken English and can read and write English fairly well, he admitted that he does not speak English well. Samba finds his mathematics teacher's explanations and questions on mathematics tests fairly easy to understand and his mathematics textbook easy to understand.

Language assessment score. Like Tamani, Samba also responded to the speaking prompts in English with only one switch to Swahili. His oral language had many one-word responses and simple sentences. He used some essential topic vocabulary, not from the prompt. He used one discourse connector and only one conjunction to sequence his words or phrases. His explanation would require a lot of effort from the listener to understand the process he was explaining (Heritage et al., 2015). Samba scored 1 on his speaking, meaning he was at the emergent stage of English language speaking progression. Samba responded to the writing prompt in Swahili and so his writing was not assessed because the framework I used only assesses responses written in English.

Berina. Berina was a 16-year-old female, tenth-grade student who had lived in the U.S. for two years. Berina speaks Bembe, in addition to English and Swahili.

***Language use at home and school*.** Bembe is Berina's first language and she reported that she always speaks Bembe at home with parents and grandparents. Berina reported that she uses both Bembe and Swahili when communicating with her brothers and sisters, and with friends away from school. Berina reported that she never speaks Bembe with her friends at school; she only uses English. Berina reported that she understands and speaks Bembe very well, but she cannot read and write in the language at all. Berina reported that she understands and speaks Swahili very well. She can read, write, and speak Swahili very well.

***The language of mathematical instruction and language preference and proficiencies*.** Berina reported that she has not been taught mathematics in Swahili or Bembe, but she studied her primary (nursery to standard 8) mathematics in French. Berina prefers talking about mathematics in English. Berina said that she understands spoken English very well and that she can speak, read and write English very well. Berina confessed that she has had difficulties understanding her mathematics teacher's explanations and in following her mathematics textbook. She, however, said that she finds questions on mathematics tests very easy to understand.

Language assessment score. Berina wrote her response to PrM in English and she scored an average of 1, meaning that her progress in the written modality of English was at the emergent stage (Bailey & Heritage, 2018). Berina's written response had an opening statement, but no closing statement. She did not use any deictic language, but she ambiguously omitted referents and used symbolic representations in a manner that might severely confuse a naïve reader (Bailey & Heritage, 2018). Berina's speaking was also at the emergent stage. Her oral response was characterized by the repetition of vocabulary from the prompt, use of simple sentences, and repetitive use of one discourse connector. Berina's explanations would require a lot of effort from the listener to understand the steps or the process being explained (Heritage et al., 2015).

Rehana. Rehana was a 17-year-old female, grade ten student who had lived in the U.S. for two years. Other than English and Swahili, Rehana speaks Bembe.

Language use at home and school. Bembe is Rehana's first language. Rehana reported that she understands Bembe very well and she speaks it fluently. However, she reported that she cannot read or write in Bembe at all. Rehana reported that she uses Bembe mostly at home when speaking to parents and grandparents, and sometimes when speaking to brothers and sisters and friends away from school. At school, Rehana speaks to her friends mostly in English and sometimes in Swahili, but never in Bembe. Berina also speaks to her brothers and sisters and friends away from school in Swahili. Rehana reported that she understands Swahili very well. She also speaks, reads, and writes Swahili very well.

The language of mathematical instruction and language preference and proficiencies. Rehana reported that she has not been taught mathematics purely in Swahili or Bembe. However, she mentioned that sometimes her teachers in Tanzania, where they camped before coming to the U.S., would switch between English, Swahili, and French to enhance the students' understanding of mathematics. Rehana prefers speaking about mathematics in English since she claimed that she understands English and can speak, read, and write English very well. Rehana said that she finds it very easy to understand her mathematics textbook and questions from mathematics tests, but she has difficulties understanding her mathematics teacher's explanations.

Language assessment score. Rehana wrote her response to PrM in English. She scored a 2 on her written modalities, meaning her writing was at the developing stage. Her writing had an opening statement, but no clear closing statement. She did not use a deictic language and she used co-references within the text appropriately. She, however, used some representations in a nonconventional manner that could partly interfere with meaning making (Bailey & Heritage, 2018). Rehana responded to the speaking prompts of PrM in English. Rehana also scored 2 on her speaking modality, meaning she was at the developing stage of English speaking progression (Heritage et al., 2015). Rehana's oral responses were characterized by use of sufficient topic vocabulary (excluding words from the prompt), attempt to use complex clause structures, use of more than two different discourse connectors, and logical sequencing of most her statements (Heritage et al., 2015).

Neha. Neha was a 16-year-old female, ninth-grade student who had lived in the U.S. for 1 year and 8 months. In addition to English and Swahili, Neha also speaks Kinyarwanda.

Language use at home and school. Kinyarwanda is Neha's first language. Neha reported that she understands and speaks it very well, but she cannot read and write Kinyarwanda at all. Neha reported that she mostly speaks to her parents, grandparents, brothers, and sisters in Kinyarwanda, although sometimes she speaks Swahili. At school, she speaks to her friends in either English or Kinyarwanda. She reported that she hardly speaks Swahili at school. Away from school, Neha speaks to her friends in Kinyarwanda. Neha reported that she understands and speaks Swahili fairly well, but she does not know how to read and write in Swahili.

The language of mathematical instruction and language preference and proficiencies. Neha reported that she learned mathematics through a mix of Kinyarwanda and Swahili from grade one to grade four (standard 1 to 4). She prefers to talk about mathematics in English. Neha said that sometimes her friends at school explain mathematics to her in Kinyarwanda and Swahili. Neha does not understand spoken English well. She also does not speak, read, and write English very well. Neha finds it fairly difficult to understand her mathematics teacher's explanations and questions on mathematics tests. However, she finds it fairly easy to understand her mathematics textbook.

Language assessment score. Neha wrote her PrM response in English and during our discourse she always responded in English. Neha scored a 1 on her speaking modality, meaning she was at the emergent stage. Her spoken responses were marked by the use of essential words that are not topic vocabulary, use of simple sentences with repetitive use of one discourse connector, and explanations that may require a lot of effort from a listener to understand the steps of process being explained (Heritage et al., 2015)

Neha responded to the writing prompt in one simple sentence. There was no opening or closing statements. Neha did not use deictic language, but her omitted referents could make it difficult for naïve readers to follow her explanation. Moreover, some spelling and punctuation was not conventional and could interfere with a naïve reader's sense-making. Neha scored 1 on writing meaning that she was at the emerging stage in the progression (Bailey & Heritage, 2018).

Zawiri. Zawiri was a 15-year-old female, grade-nine student who had lived in the U.S. for one year and eight months. Zawiri spoke Kinyarwanda, in addition to English and Swahili.

Language use at home and school. Kinyarwanda is Zawiri's first language. Zawiri reported that she understands Kinyarwanda very well and she can speak, read, and write Kinyarwanda very well. Zawiri reported that she mostly uses Kinyarwanda at home to communicate with parents, grandparents, brothers, and sisters. However, she sometimes speaks to her parents in Swahili as well. At school, Zawiri speaks to her friends in English, Swahili, or Kinyarwanda. She mostly speaks Kinyarwanda with her friends away from school. Zawiri reported that she understands Swahili very well and can read and write Swahili very well, but she cannot speak the language well.

The language of mathematical instruction and language preference and proficiencies. Zawiri said that she was taught mathematics in Kinyarwanda at grades 1 to 3 (standard 1 to 3), but she had not been taught mathematics in Swahili. Zawiri prefers speaking about mathematics in English. Zawiri said that she understands spoken English very well and can speak, read, and write English fluently. Zawiri claimed that she finds it very easy to understand her mathematics teacher's explanation, but she has difficulties understanding her mathematics textbook and questions on mathematics tests.

Language assessment score. Zawiri wrote her PrM response in English and scored a 2, which means she was at the developing stage of English progression (Bailey & Heritage, 2018). Zawiri's written work had an opening statement, but no closing statement. She did not use any deictic language, used the appropriate co-reference, and a naïve reader could understand her explanations. Her spellings and punctuations were mostly accurate, and other representations were clear and easy for any reader to make meaning (Bailey & Heritage, 2018). Zawiri responded to the speaking prompts of PrM in English and her score for speaking modality was 2 indicating that she was at the developing stage. Zawiri's spoken responses were characterized by the use of sufficient topic vocabulary (including words from prompt) to make the content clear, sentences with complex structures, use of at least two discourse connectors, and a logical sequence of most statements (Heritage et al., 2015).

Exmon. Exmon was a 16-year-old male, tenth-grade student who had lived in the U.S. for two years. Exmon spoke Bembe, in addition to English and Swahili.

Language use at home and school. Exmon reported that he began speaking Bembe when he was four years of age. Exmon mostly speaks to his parents, brothers and sisters, and friends from school in Swahili, although he sometimes uses Bembe. Exmon understands Bembe fairly well. Although Exmon can speak Bembe very well, he reported that he cannot read and write well in Bembe. Exmon reported that he understands Swahili and can speak and write it very well. He, however, has challenges with reading Swahili. At school, Exmon speaks to his friends in English, although he sometimes uses Swahili and Bembe.

The language of mathematical instruction and language preference and proficiencies. Exmon reported that he was not taught mathematics in Bembe or Swahili. He prefers speaking about mathematics in English. Exmon reported that he understands English fairly well, but he cannot speak, read, and write English well. Exmon has difficulties understanding his mathematics teacher's explanations. He also finds it fairly difficult to understand his mathematics textbook and questions on a mathematics test.

Language assessment score. Exmon wrote his response in English and scored a 1, meaning that his writing modality was at an emerging stage. Exmon's written response lacked opening and closing statements. Although he did not use any deictic language, his omission of certain referents would render his work difficult to follow for naïve readers. Moreover, there were errors in some of his spellings and punctuations, and these could severely interfere meaning making (Bailey & Heritage, 2018). Exmon scored a 1 on speaking. Although Exmon's oral responses included some essential topic vocabulary, most of his sentences were simple and did not contain a complex structure I could assess for advanced relationships between ideas and coherence of explanations (Heritage et al., 2015). Most of the time, Exmon seemed to struggle with explaining clearly what he knew and hence his reliance on responses in the form of one-word, short phrases, or simple sentences.

Kwisha. Kwisha was a 17-year-old male, tenth-grade student who had lived in the U.S. for 8 months. In addition to English and Swahili, Kwisha speaks French and Bembe.

Language use at home and school. Bembe is Kwisha's first language, but he began speaking French at 14 years of age. At home, Kwisha reported that he mostly speaks Bembe to his parents, grandparents, brothers, and sisters, but sometimes he also speaks Swahili. At school, Kwisha mostly speaks to his friends in French, and sometimes he uses Swahili and English. With friends away from school, Kwisha speaks Swahili, French, and Bembe. Kwisha reported that he can read French very well. He also understands French and can speak and write it fairly well. He also understands Bembe and speaks it very well, but he said that reading and writing Bembe is hard. Kwisha reported that he understands Swahili and he can read and write Swahili very well, but he does not speak it well.

The language of mathematical instruction and language preference and proficiencies. Kwisha reported that he was taught mathematics in French at grade eight, but he did not receive mathematics instruction in Swahili. He prefers speaking about mathematics in English, but he thinks "both [English and Swahili] may be awesome" to use. Kwisha reported that he understands spoken English fairly well and can speak, read, and write English very well. He easily understands his mathematics teacher's explanations, and he finds it fairly easy to understand his mathematics textbook and questions on mathematics tests.

Language assessment score. Kwisha responded to the writing part of PrM in English and he scored 1, meaning he was at the emerging stage of the English progress. Kwisha's writing lacked opening and closing statements. He did not use any deictic language, but his omission of some referents could make it difficult for a naive reader to understand his explanation. There were some spelling and punctuation errors, and some of his representation could partly interfere with meaning making (Bailey & Heritage, 2018). On speaking, Kwisha responded to the speaking prompts in PrM in English and he scored 1. In most cases, Kwisha accurately used a variety of topic vocabulary, but his responses lacked complex structures, adequate discourse connectors, and conjunctions to logically sequence his statements (Heritage et al., 2015).

Azina. Azina was a 17-year-old female, tenth-grade student who had stayed in the U.S. for 8 months. In addition to English and Swahili. Azina spoke Bembe.

Language use at home and school. Bembe is Azina'a first language. Azina reported that she understands Bembe very well and she can speak and write Bembe fluently. She, however, does not read Bembe very well. At home, Azina mostly speaks to her parents, grandparents, brothers, and sisters in Bembe. She also sometimes uses Swahili at home. At school, Azina speaks to her friends in Bembe, Swahili, or English depending on the language that particular friend understands best. With her friends away from school, Azina speaks Bembe and Swahili. Azina reported that she understands Swahili very well and she can read, write, and speak Swahili fluently.

The language of mathematical instruction and language preference and proficiencies. Azina reported that she was not taught mathematics in Bembe, but she had an experience learning mathematics in French and Swahili in grades K-12, while her family was in a refugee camp in Tanzania before joining the U.S. Her teacher used French, but also switched to Swahili for those who did not understand the material in French. Azina prefers speaking about mathematics in Swahili. Azina reported that she does not understand English well, and she cannot speak, read, and write English well. She finds it very difficult to understand her mathematics teacher's explanations and questions on mathematics tests. She also has difficulties understanding her mathematics textbook.

Language assessment score. Azina wrote her response to PrM in Swahili, but she had one switch to a phrase that consisted words from the prompt. Azina prefers discussing mathematics in Swahili and she even requested that I allow her to respond to the prompts in PrM in Swahili. As such, both her written and spoken responses could not be scored using Heritage et al.'s (2015) framework. According to this framework, Azina's progress in both speaking and writing in English would be rated as "not evident" with a score of 0. However, through personal communication, Bailey advised that it is important to give such students multiple opportunities to work on familiar tasks in English before arriving at the conclusion that English is "not evident" in their oral and written responses.

Solana. Solana was a 15-year-old female, tenth-grade student who had lived in the U.S. for three years. In addition to English and Swahili, Solana speaks Kinyarwanda.

Language use at home and school. Solana's first language is Kinyarwanda. Solana reported that she understands and speaks Kinyarwanda very well. However, Solana does not read Kinyarwanda well and cannot write it at all. Solana speaks to her parents and grandparents mostly in Kinyarwanda. She speaks to her brothers and sisters in Swahili and she sometimes speaks to her parents in Swahili. At school Solana never speaks Kinyarwanda. However, she often speaks Swahili when she is translating information to new students and to those having difficulties with understanding English. While away from school, Solana speaks to her friends in Swahili. Solana reported that she understands Swahili. She can read and write it fairly well and speak it fluently.

The language of mathematical instruction and language preference and proficiencies. Solana reported that she had an experience learning mathematics in both Kinyarwanda and Swahili when she was in grades 3 and 4 (standard 3 and 4). Her teacher taught the subject in French, but she could switch to Swahili or Kinyarwanda to enhance the students' understanding. Solana prefers speaking about mathematics in English. Solana understands spoken English very well. She also can speak, read, and write English fluently. Solana finds it fairly easy to understand her mathematics teacher's explanations. She also finds it fairly easy to understand her mathematics textbook and questions on mathematics tests.

Language assessment score. Solana wrote her response to PrM in English and she also responded to the speaking tasks of PrM in English. She scored 3 on her spoken responses, meaning she was at the controlled stage of English progression (Heritage et al., 2015). Solana's oral responses contained a variety of precise topic and technical vocabulary. She used simple and compound sentences that were accurate and grammatically correct. She mostly used a minimum of 2 discourse connectors and she logically sequenced most of her statements.

Solana's written response lacked a closing statement. She did not use deictic language and tried to unpack the prompt with full noun referents. However, there were errors in some of her spellings, punctuations, and representations that could partly interfere with meaning making (Bailey & Heritage, 2018). Solana thus got 1 in her writing, meaning she was at the merging stage of English language writing progression.

Diodo. Diodo was a 19-year old male, ninth-grade student who had lived in the U.S. for a year. In addition to English and Swahili, Diodo speaks Kinyarwanda.

***Language use at home and school*.** Diodo's first language is Kinyarwanda. Diodo reported that he understands Kinyarwanda very well and can speak, read, and write Kinyarwanda fluently. Diodo reported that he always uses Kinyarwanda at home to communicate to his parents, grandparents, and brothers and sisters. At school, Diodo mostly speaks to his friends in Swahili, though he sometimes uses Kinyarwanda. Away from school, Diodo speaks to his friends in both Swahili and Kinyarwanda. Diodo reported that he reads and writes Swahili fluently, but he does not speak it well.

***The language of mathematical instruction and language preference and proficiencies*.** Before coming to the U.S., Diodo had lived in camps in Uganda for about 10 years where he learned mathematics in English. Diodo reported that he was not taught mathematics in either Kinyarwanda or Swahili. He prefers speaking about mathematics in English since he has always been taught mathematics in English. Diodo understands English fairly well and speaks it quite well. He can read and write Swahili fluently. Diodo finds it very easy to understand his mathematics teacher's explanations, but he does not understand his mathematics textbook and questions on mathematics exams very easily.

Language assessment score. Diodo's spoken and written responses to PrM were in English. He scored 2 on his speaking, meaning he was at the developing stage of English progression (Heritage et al., 2015). Diodo's oral responses were marked by the appropriate and accurate use of a variety of precise topic and technical vocabulary, use of simple and compound sentences that were mostly accurate and grammatically correct, use of at least two discourse connectors and logical sequencing of most of his statements (Heritage et al., 2015).

On his writing, Diodo scored 2, meaning that he was at the developing stage of English language progression in terms of writing. His written work had an opening statement, but no closing statement. He did not use deictic language and he tried to unpack the prompt with full noun referents. His work, however, had some spelling errors and representation that could partly interfere with meaning (Bailey & Heritage, 2018).

Fareli. Fareli was a 17-year-old male, tenth-grade student who had lived in the U.S. for 8 months. Fareli spoke Bembe, in addition to English and Swahili.

Language use at home and school. Fareli understands Bembe very well. Bembe is his first language. He reported that he can speak Bembe very well and can read and write the language fairly well. At home, Fareli always speaks to his parents, grandparents, brothers, and sisters in Bembe. But he sometimes speaks to his brothers and sisters in Swahili. Fareli reported that he understands Swahili very well and can speak, read, and write Swahili fluently. At school, Fareli speaks to his friends in both Swahili and Bembe. While away from school, Fareli mostly speaks to his friends in Swahili, and sometimes uses Bembe.

The language of mathematical instruction and language preference and proficiencies. Fareli reported that he has not been taught mathematics in Bembe or Swahili. He prefers speaking about mathematics in Swahili. He understands spoken English fairly well and he can speak, read, and write English somewhat. Fareli finds it very difficult to understand both his mathematics teacher's explanations and questions on mathematics tests. He, however, finds his mathematics textbook fairly easy to understand.

Language assessment score. Even though Fareli preferred speaking about mathematics in Swahili, he tried responding to both the writing and speaking prompts in English. He scored 1 for both speaking, meaning that he was in the emerging state of English progression (Heritage et al., 2015). I characterized Fareli's oral responses by the use of some essential topic vocabulary not from the prompt, simple sentences, repetitive use of one discourse connector, and explanations that may require a lot of effort from a listener or reader to understand the steps or process being explained (Heritage et al., 2015).

Fareli's written response contained opening and closing statements. He did not use deictic language and he tried to unpack the prompt with full noun referents. Although his work had some spelling and punctuations errors, he made up for these errors by using other symbolic representations (Bailey & Heritage, 2018). Overall, Fareli scored 2 on his writing, meaning he was at a developing stage in the English writing progression. In the next section, I discuss the setting for this study.

Setting. Research shows that bi-/multilingual teens actively use multiple language practices daily for sense making (Daniel & Pacheco, 2015; Martinez, Orellana, Pacheco, & Carbone, 2008). These multiple language practices that bi-/multilinguals use are called translanguaging practices (Garcia, 2009; Garcia & Wei, 2014). Gee (1996) noted that the bi-/multilingual teens' participation in translanguaging practices is responsive to their environments. In other words, the adolescents' use of their translanguaging practices depends on the setting and the people with whom they are interacting. From the description of my research participants, it is evident that bi-/multilinguals use certain languages practices at certain places with certain people for various reasons. For instance, the participants' choice of language to use while at home is different from the language they use while at school; most of the participants chose to use their mother tongue at home, but not at school.

Researchers of the language practices among bilingual teens should carefully consider the setting in which their study occurs (Garcia, 2009; Moschkovich, 2007) because the setting for such research affects the outcomes of the study. In this study, I view language practices as translanguaging practices. As such, I had to choose a setting in which the participants would feel comfortable to use translanguaging practices (Daniel & Pacheco, 2014). The setting had to be an out-of-school location because the restrictive language policies that insist on English as the standard language in most U.S. schools (U.S. English, 2006) would make the participants shy away from drawing on their entire linguistic repertoires. In a recent study conducted by Orado (2014), the bi-/multilingual participants resisted speaking Kimaragoli in a school setting where the study took place because English was the language of instruction and they saw speaking in mother tongue as a violation of the school regulation. During the visits, I made it clear to the participants that they could draw on all their languages practices and other resources while working on the tasks. So, we mutually agreed on the best meeting locations for the study.

While some participants chose a nearby public library, others chose a quiet room within their homes. The participants who chose a nearby library were familiar with the site as they often went there to study or meet other participants for discussions. Those who chose quiet rooms at home considered their homes the places where they are most comfortable to speak in any language they like. In the following sections, I discuss how I secured these two types of settings for this study.

A quiet room within a public library. Since I anticipated conducting my research in local public libraries, I carried with me a brochure that contained a list of the nearby public libraries and their hours of operation. This list helped me plan my research at times when the selected library would be open. I did not book an appointment with the library staff, but simply walked in with the participants and talked to the staff at the reception. I explained that I needed a quiet room where I could conduct my research. I needed a room with enough tables and chairs, and charging outlets as I charged my audio recorder and video camera as I recorded to avoid loss of data due to low battery interruptions.

A quiet room within a participant's house. The pairs of participants who lived in the same house preferred being studied from within their homes. The quiet rooms were either personal study rooms within the house or some other room that the host participant prepared by arranging a table and chairs. The host participant also made sure that there was a nearby charging outlet for my audio recorder and video recorder. The host participant(s) ensured that no other member of the household interrupted while the research was in progress by instructing younger siblings to keep out of the room, and by asking other members of the household to turn down the volume of sound producing devices in the house such as the television, and so on.

Data Collection

Like the piloting phase, data collection in the actual phase involved two stages. There was the first stage where I administered a LBS and the first task-based interview, and the second stage where I administered the second task-based interview. In the following sections, I discuss each stage with details of the instruments used and how they were administered.

Data collection at the first stage. I used two instruments at this stage: a LBS and an English language assessment task-based interview. In the following sections, I briefly discuss these instruments as well as how they were administered.

LBS. The LBS elicited the participants' language use both at home and at school, as well as their past experiences with learning mathematics in all their language(s), and their perception of proficiency in each language.

Administration of LBS. Each participant wrote by hand their responses to a list of questions typed on a paper. I worked with each participant one-on-one so I could clarify the questions they did not understand. As the participants responded to the questions, I sought for more information where need be. For instance, when a student said that Kinyarwanda is not their first language, I sought to know what their first language is. I used polite language, such as "Would you please tell me what language is your first language?" In some cases, some participants were unwilling to say what their first language is and so I chose not to ask them further. I collected the completed survey sheet. It took the participants about 10 minutes to complete the survey. I entered participants' responses to the LBS in a Microsoft Excel file as a way to better visualize them.

The first task-based interview. The first task-based interview was an English assessment interview. The task used was a mathematics-eliciting task on generalization and analysis of patterns in algebra that I adapted from the NAEP-2009 released algebra test items. I deemed the concepts addressed in the problem as fairly basic to ninth and tenth graders. I determined this because in the U.S. students are introduced to the concept of patterns and relationships from fourth grade and they continue to build on it through ninth and tenth grade as they learn how to use functions to model patterns and real-life situations. I anticipated that most of the participants would be in a position to write and explain their problem-solving process. But for those who would face challenges in solving the problem, I was prepared to give them certain prompts to support their understanding of the problem and its solution. At the end of the task, the participants were required to write an explanation of their solution process.

The aim of the task was not to test the participants' content knowledge, but rather to elicit their English speaking and writing skills in the domain of algebra. I adapted Kitchen et al.'s (2012) discursive assessment protocol, which entails four steps, namely: making an estimation, writing a solution, explaining solution process, and a simulation via telephone. In adapting

Kitchen et al.’s protocol, I replaced their last step on “telephone simulation”, with a prompt requiring the participant to write the solution to the task for a friend who is unable to solve the problem (Heritage et al., 2015). Thus, my protocol involved the steps: making an estimation of the solution (speaking), writing a solution (writing and speaking), explaining solution process (speaking), and writing the solution to a friend (writing and speaking). I discuss what each of these steps entailed in the sections below.

Estimating the solution. In making an estimation of the solution, the participants were required to say how each of the next two numbers in the pattern would compare with the number that precedes it. For instance, one could say that the number that comes after 10 should be bigger while the one that comes next would be smaller than the one preceding it. In other cases, I approached this stage by asking the participants to guess the next two numbers and to give a reason for their guesses. The aim was to elicit the participants’ oral communication as well as their mathematical reasoning.

Writing the solution. At this stage, I required the participants to solve the problem and to write a rule for finding the next two numbers in the pattern. I anticipated that some participants would face difficulties with this step. So, I asked supportive questions that could help them unpack the problem and establish a rule used for the pattern. As the participants solved the problem, I asked clarifying questions to make sure that I understood what they were doing.

Explaining their solution. This is where the participants verbally explained their work from the previous step. Again, I asked clarifying questions to get them to speak more and to articulate their mathematical understanding.

Writing the solution as to a friend. Here the participants needed to put together their work from step two and their explanation from step three and write the solution to that problem to a friend who does not know how to solve it.

Administration of the first task-based interview. The participants solved the task individually and in English where possible. Piloting this study revealed that some participants preferred to speak about mathematics in language(s) other than English. I, therefore, allowed participants to complete this task in their preferred language. Part of my aim in this study was to see how participants use their languages when solving mathematics problems. I thus saw participants' flexibility in language use during problem solving as more important in addressing my research questions than restricting their language use in order to assess their progress with English.

Each participant worked on the problem as per the protocol. I read aloud the steps one at a time and allowed the participants time to respond. The first step was to estimate the next two numbers in the pattern. The second was to write the process for solving the problem. The next was to verbally explain their solution process, and finally, to write the solution to a friend who is unable to solve the problem (Kitchen et al., 2012). I asked the participants questions related to their work whenever there was a need. Asking them questions elicited their speaking skills and hence provided rich data for analyzing vocabulary, sentence structures, and relationships between ideas in the context of algebra (Heritage et al., 2015). Moreover, the participants' responses to my clarifying and supportive questions elicited their mathematical thinking, which was also important in answering the study's research questions. The kind of questions I asked varied from one participant to another depending on how they approached solving the problem. On average, each interview lasted 17.23 minutes. The shortest interviews lasted 8.1 minutes while the longest took 27.47 minutes. Table 3.3 below shows the actual time taken for each interview session.

Table 3.3

Length of First Task-based Interviews

Participant	Time (minutes)
Rehana	8.1
Beh	17.41
Solana	17.08
Azina	27.18
Diodo	27.47
Fareli	14.2
Zawiri	15.36
Neha	19.42
Exmon	12.04
Kwisha	13.42
Samba	17.54
Tamani	17.55

I audio and video recorded the participants as they worked through the tasks. The audio recorder was placed on the table where the participants sat while the video camera stood about three meters away from the participants (Orado, 2014). I made interview notes for the instances that I thought were important and needed to be captured in the analysis. I also collected participants' written work. The video recording was the main source of data as it included both the sound (spoken language) and gestures, facial expressions and other body languages, and details of the participants' behaviors.

Transcription. The video recorded data was the main source of data. I created a dissertation data project on Maxqda12, a computer-assisted data analysis program. I then uploaded the video recordings, one at a time, for transcription. Maxqda12 allows one to transcribe video files while watching and listening to them. I first watched the video clip without transcribing in order to get a sense of what happened and see if there were instances where the sound was unclear. For the instances where I did not get understand well what the participants said, I used the respective audio recordings to clarify. The audio recordings had a better sound better quality and were hence clearer. I then transcribed both the participants' verbal language as well as their nonverbal communication.

Data collection at the second stage. I used one task-based interview as the main instrument in this stage. In the following sections, I briefly discuss the task that I used and how I administered it.

The second task-based interview. The tasks were two algebra word problems, PrB and PrA, that I discussed in detail earlier under Data Collection Instruments. PrB was an adaptation of two NAEP-releases test items, one from the NAEP-1990 and the other one from the NAEP-1992 released test items. I made the adaptation so that the two problems would require explanations. My aim in this study was to understand the practices, resources, and challenges the participants face when solving MWPs. Therefore, I deemed problems that called for deep conceptual understanding and required explanations rather than the statement of formulas the best in addressing the aims of this study. Moreover, I required that the problems contain different semiotic features in order to provide a ground for investigating the resources participants use during MWPS (Solano-Flores et al., 2013).

Administration of the second task-based interview. At this stage, the participants worked on the task in pairs. According to the Vygotskian perspective, students develop mathematics practices as they interact with each other while working on a joint activity that "supports their appropriation of goals, focus of attention, and shared meanings" (Moschkovich, 2013, p. 271). The goal was to have participants engaged in a mathematical discourse out of which I could determine their practices, resources, and challenges in MWPS. I also allowed participants to use language(s) of their choice and to draw on any resources they liked and could access in order to solve the problems.

A major semiotic difference between the two problems is that one had a visual while the other one did not. I, therefore, administered the problems one at a time, starting with PrB, which was the one without a visual. After the participants had solved the first problem, I allowed them a five-minute break where necessary, before introducing the second problem. I designed my own protocol for administering this task with five main steps: (a) introducing the task, (b) reading the problem, (c) recalling the problem, (d) follow-up questions, and (e) problem solving. In the following, I discuss what each step was about.

Introduction to the task. In introducing the task, I thanked the participants for agreeing to participate and I mentioned that I would be willing to reward their time by offering them free tutoring in mathematics. In cases where the participants were unfamiliar with each other, I took some time before the start of the first phase to have them introduce themselves to each other and to develop some familiarity. In this step, I also reiterated that my aim was not to grade them and see who scored better than the other; rather I wanted to know how they solved the problems and the challenges they faced. I also mentioned that I would allow them to solve problems in any language they liked. In order to create a translanguaging space, I opened the session by greeting the participants in Kinyarwanda or Swahili. I also switched between English and Swahili words and phrases during the introduction in order to model the use of language practices.

Reading of the task. For this step, I had the participants read the first problem silently for two to three minutes. After that, I asked them to read the problem to each other. As the participants read silently, I observed their behaviors, including reading to self aloud, body gestures showing they were having difficulties, and so on. Also, as they read to each other, I noted any words and phrases they seemed to struggle reading and if there was any interference in how they pronounced certain words or phrases. I also noted any gestures and body language.

Recalling the task. I asked the participants to retell the problem in turns and in their preferred language(s). I used prompts such as: What is the problem about? What is the question in the problem? What are you looking for? As the participants responded to these questions, I sought to know, among other things, if they (a) understood the text in English, (b) rephrased the problem in English or any other language to demonstrate their understanding of its meaning, and (c) drew on other resources within the text or outside the text to make meaning of the problem or part of the problem.

Follow-up questions. At this step, I sought to know what specific aspects of the problem were challenging to the participants. I asked participants, one at a time, to share any word(s), phrases, sentences, or other aspects of the problem they found confusing or hard to understand or those that may have made them lose focus on the problem (Abedi & Lord, 2001). In order to gain a deeper insight into the nature of the challenge participants were facing, I asked that they explain how a certain aspect was challenging them.

Problem solving. At this step, I asked the participants to talk to each other about the problem and to begin solving it. I asked them to discuss the solution process and to write their agreed solution process and ultimate solution in the spaces that were provided on the worksheet. I asked them to feel free to write their work whenever they felt like they had reached an agreement.

There were certain instances during the participants’ problem solving where I needed to interrupt in order to ensure that the task accomplished the aim for which it was designed. Such were the instances where participants seemed to face a difficulty, or remained quiet for more than a minute, or when they seemed to pursue a wrong solution path, or when one or both of the participants reverted to working alone instead of solving the problem together. At the points where participants seemed to face difficulties with problem solving, I sought to know what caused them the challenge and if there were resources they could draw upon to address the challenge. When they got too absorbed with solving the problems independently, I sought to understand what they were thinking. When they remained silent for more than a minute, I encouraged them to keep talking to each other. I also reminded them to use any language(s) and resources they liked. On several occasions, I urged the participants to clarify their thinking. At the points where I saw participants pursuing a wrong solution path, I first sought to know the reasoning behind their work. I then encouraged them to rethink their strategies and to even explore alternative approaches and resources. My role during the tasks was not only to elicit the participants' mathematical thinking, but to also support them in learning during the problem-solving process.

During the interjections, I tried to speak in a language or a manner that was similar or that matched the one used by the previous speaker. For instance, if I cut in after a student had spoken in English, I would then use English. If the student spoke in Swahili, I would also speak in Swahili. If a student spoke in Kinyarwanda or Bembe, I would ask them to translate what they are doing or saying in Swahili. For example, if a participant said, "you plus" and I needed to cut in, I would them ask in a similar manner, "what are you plusing?" Although "plusing" is not an official English word, I used it so as to keep the discourse pattern uninterrupted so that the student could continue in the same line of thought and perhaps reveal more of their mathematical understanding. I tried modeling various language practices by switching codes between English and Swahili. After the participants were done solving the first problem, I presented the second one and administered it in a like manner.

At the end of the entire problem-solving session, I sought the participants' general comments about the problems administered and what they found challenging as they solved them. On average, the second task-based interviews took about 50 minutes. The longest interview session took 71 minutes while the shortest took 34.20 minutes. Table 3.4 below shows the actual time taken by each interview session.

Table 3.4

Length of Second Task-Based Interviews

Participants	Rehana & Beh	Solana & Azina	Diodo & Fareli	Zawiri & Neha	Exmon & Kwisha	Samba & Tamani
Time (minutes)	34.20*	71.63*	51.9*	67.96*	34.19	46.36

* Sessions split into two due to some form of interruption or based on the availability of the participants

I audio and video recorded this session, took brief notes, and collected participants' written work. The video records of the participants' problem solving were the main source of data at this stage. The audio recording supplemented the video recording because the sound in the audio files was clearer and of a better quality. Participants' written work also supplemented the video data in that while the video showed how the participants worked and what they said as they worked, their written work represents what they actually did or wrote. Participants' written work contains information about how they used various semiotic features including symbolization and other mathematical representations.

Transcriptions. I first created a dissertation data project on Maxqda12. I then uploaded the video recordings of the participants' problem solving into the project for transcription. My transcriptions captured both the spoken language as well as gestures, body motions and expressions. I also captured the participants' behaviors and reactions to each other. I used the audio recordings to clarify any instances where video sound was not clear. At the end of each transcription, I included the notes I took during the interviews.

Table 3.5 below summarizes the data collection procedures for the two phases of this study.

Table 3.5

Summary of Data Collection Procedures

Phase	Participants	Stage	Data collection methods	Type of data analyzed
Pilot study	4	First stage	LBS FTBI	LBS responses Video recording, Participant's work
		Second stage	STBI	Video recording, Participant's work
Actual study	12	First stage	LBS FTBI	LBS responses Video recording, Participant's work
		Second stage	STBI	Video recording for FTBI and STBI, participant's work

FTBI and STBI are first task-based interviews and second task-based interviews respectively.

Data Analysis

Data for this study consisted of transcriptions of video-recorded interviews and interview notes, LBS responses, as well as participants' written work for both the first and second interviews. I analyzed these data in two stages. In the following sections, I provide details of analysis at each of the two stages.

Data analysis at the first stage. Data for the first stage of this study consisted of the LBS responses, participants' written work and transcripts of video-records of their problem solving during the first task-based interview. I discuss how the LBS and the first task-based interviews were analyzed in the following sections.

Analysis of LBS responses. I entered participants' responses to the LBS in an Excel file, not to analyze them, but to better visualize them. There was not a need for elaborate descriptive statistics since there were only twelve rows and each row contained details of an individual participant's language use in different settings (at home, school, and away from school), frequency of use of the languages, and their perception of proficiency in either of the languages, and mathematical instruction in either of the languages. This information constituted the description of participants. I used the information on the participants' language background to decide on how the participants should be paired for the second task-based interview.

Analysis of the first task-based interview. I administered this task-based interview for two main reasons: to provide information that could be used to assess participants' progress with English, and to elicit the participants' mathematical problem solving, which could address the study's research questions. In this analysis section, I focus on how the task was used to assess the participants' English language progression in the speaking and writing modalities.

To assess participants' speaking, I analyzed their spoken responses in the transcribed data. I used Heritage et al.'s (2015) framework for analyzing oral language, which consists of four main language features: (a) sophistication of topic vocabulary, (b) sophistication of sentence structure, (b) establishment of advanced relationships between ideas, and (d) coherence of explanations. According to the framework, each of these features is assessed as either "not evident" (with a score of 0), "emergent" (with a score of 1), "developing" (with a score of 2), or "controlled" (with a score of 3) (Heritage et al., 2015). I identified the target language features in the participants' transcripts of spoken responses using Heritage et al.'s (2015) description and I assigned them a score accordingly. I then found the average score by summing the independent scores and dividing them by four. This final score provided evidence of where the student best fitted within the English language speaking progression.

I analyzed the participants' writing using Bailey and Heritage's (2018) framework. This framework presents a description of three language features to focus on when analyzing ELLs' writing. These features include: (a) opening and closing statements, (b) decontextualization, and (c) graphic representation supporting meaning making (see Appendix B4 for further details on the writing framework). Each of these features is assessed as either "not evident" with a score of 0, as "emerging" with a score of 1, as "developing" with a score of 2, or as "controlled" with a score of 3. I identified the said target features in the participants' writing using Bailey and Heritage's description and scored them as per the rubric they have provided. I then found the average score by adding the score assigned to each language feature and dividing the total by three, since there were three features. The average was the final score on the participants' writing. An average score of 1 showed that the participant was at the emerging stage, a 2 meant that the participant was at the developing stage, while a 3 meant that the participant was at the controlled stage of English writing progression. I have provided details of each participants' language assessment in their descriptions.

According to Bailey (personal communication, January 16, 2018), both the speaking and writing frameworks typically provide evidence of where the student is in his or her day-to-day progress with learning to speak or write in English. For this study, participants' scores offered evidence of their best fit within the English learning progression. Knowing the participants' language progression was useful to me while I decided how to pair them during the second task-based interview. For instance, pairing participants who were at the same stage in the progression or pairing participants who were at different levels of English progression, but shared a first language encouraged translanguaging in some cases.

Data analysis at the second stage. Data analyzed at this stage consisted of transcripts of the video recordings from both the first and the second task-based interviews. The first task-based interview also yielded information on the students' mathematical problem solving that could address the study's three research questions. The aim of this study is to understand the practices, resources, and challenges in MWPS among high participants from the Swahili-speaking African refugee community in the U.S. Before I discuss how I analyzed the main data to address my research questions, I will first discuss my data analysis procedures.

Data analysis procedure at the second stage. I followed Creswell's (2013) hierarchical approach for qualitative data analysis. I first organized and prepared the data for analysis by transcribing it on Maxqda12. I then read through the transcripts to get a general sense of the information and to reflect on its overall meaning. As I read the transcripts, I wrote short memos in the margins of the transcripts of the incidences I desired to revisit later during the coding process. In most cases, my memos were comments and reflections on particular aspects of my evaluation of the data. Most of the codes in this study emerged from the comments and reflections that I wrote down as memos. In other cases, my memos yielded information that confirmed certain preexisting existing codes. Table 3.6 provides a codebook that include the codes I used in this study, a description of the codes, and a line or a short excerpt that illustrates the code. Table 3.7 gives three examples of memos that I used in this study. I will use memo 3 to illustrate how I used the memos in my analysis. Memo 3 was a reflection on Fareli's response when I asked him to guess the next two numbers in PrM. Fareli's response involved the use of words and hand and body gestures. Several codes arose from this memo. For instance, the use of hand body gestures as a resource in communication, the word pattern as both a linguistic and a mathematical challenge, and the challenge of an unfamiliar language.

This being a case study, I aimed to gain an in-depth understanding of the practices, resources, and challenges in MWPS among the participants. I coded the data as per the study's research questions: (a) What language practices and linguistic resources do the participants use during MWPS? What linguistic challenges do they face? (b) What mathematics practices and mathematical resources do the participants use during MWPS? What mathematical challenges do they face? and (c) What is the role of the LPRs in the participants' mathematical processes? The main themes of my analysis were: (a) the participants' language practices, linguistic resources, and challenges in MWPS, (b) the participants' mathematics practices, mathematical resources, and challenges, and (c) the role of LPRs on the participants' mathematical processes.

Rossman and Rallis (2012) defined coding as the process of organizing data by bracketing chunks and writing a word representing a category in the margins. My process of coding entailed finding categories that fell under each of the themes I have previously stated. I, therefore, took the transcripts and segmented sentences and paragraphs into categories according to the themes and labeled those categories. The labels I gave to the categories were my codes. As earlier mentioned, most of the coded emerged from the memos that I had written down before the actual analysis. Each theme consisted of codes. For each of these codes, there was evidence from the data that was either generated by a single participant or several participants. Evidence for most of the codes could be obtained from multiple participants. Also, one episode from an individual could serve as evidence for multiple codes and themes.

I based the codes I used in this study on the research literature, common sense, as well as the ideas that emerged from the data and those that readers would expect to find in such a study

(Creswell, 2013). For instance, anyone familiar with the language practices used by ELLs would expect *code-switching* to be one of my codes. Also, just as the word *Spanglish* is used to mean a mixture of Spanish and English, it made sense to use *Swanglish* for a word or noun phrase that was partly Swahili and partly English. Moreover, the *missing addend* code emerged in PrM at the instances where participants had challenges figuring out the missing addends in order to discover the rule in the given pattern. Table 3.6 below is the codebook that shows the main themes in this study, the codes for each theme, together with the descriptions and quotes illustrating the codes. In the sections that follow the codebook, I discuss the details of how my analysis sought to address each research question. I then discuss how I presented my findings and how I interpreted them.

Table 3.6

Codebook

Themes		Codes	Definitions	Exemplars
RQ1	Language practices	Code-switching	Use of more than one language in the same conversation (Adler, 2001; Setati, 1998)	“I don’t know I guess *ni unachukua hizi zote* [is you take all these],
		Translation	Act or process of rendering meaning of what is said or written in one language into another language (Setati, 1998, p. 37)	*Siku ya kwanza*, [first day] first day, uhmm, *aliweka* gallon *tatu*, [he put three gallons]…
		Translanguaging in the inner speech	Using preferred language when reading/talking in the inner speech (Colindres, 2015; Zahner & Moschkovich, 2010). May or may not be audible.	Zawiri talked to herself saying “to get to 12 they took out two, so to get to ….”
		Restating/rereading	Repeating what you said or reading again	“then you get more, add more”
		Invention of terms	Use of inscriptions to communicate certain mathematical processes	“*zinapandana kwa mbili* [they are rising by two]”
	Linguistic challenges	Unfamiliar words/complex syntax	Words that seemed new or sentences complex to some participants	Words like pattern, write, dots, were unfamiliar to some
		Unfamiliar language	Language you cannot communicate in because you do not have the skills to use it.	*chenye kinasumbua hapa ni lugha* [my biggest challenge is the language]
		Abstract/passive representation	Aspects of an item/problem not situated/insufficiently situated (Abedi & Lord, 2001; Abedi et al.,1995)	PrM was non-situated. The shape and size of the container in PrA is unknown
	Linguistic resources	Hand and body gestures	Non-verbal communications through hands and other body parts	Waving hands to show how a pattern repeats
		Familiar objects/words/signs	Objects/words/things students were familiar with	Azina used “Kadum” instead of container

		Cognates	Words with similar sounds and meanings in two languages. Spellings may differ	"Conteneur and "container" in French and English respectively

RQ2	Mathematics practices	Sense making	Unpacking meaning of a problem	Exmon and Kwisha discuss PrA in Bembe
		Looking for/discovering patterns	Finding rules for always getting the next number in a sequence	“so one, two, ooh, are we skipping numbers?”
		Describing/using structure	Discerning pattern and structure and using it in new problems	‘cause *zinapandana kwa mbili* [they are rising by two]”, “pattern *iko* [is] up *kwa* [by] two”
		Looking for/expressing regularity in repeated reasoning	Noticing repeated calculations, finding general methods and shortcuts	‘cause three plus two equals to five and five plus two equals to seven *kwa hivyo tunaenda kwa ile ingine*
		Constructing arguments/conjectures	Making and using assumptions, responding to other’s arguments	‘cause here we skipped like two-two-two and we haven’t gotten to 200 so I think …
	Mathematical challenges	Math symbols and visuals	Inability to represent operations symbolically misunderstanding symbols and visuals	Labeling of figure in PrB
		Technical math lexicons	Technical words like pattern have both everyday and math meaning	Like mathematical meaning of pattern
		Abstraction	Relating quantities, representing relations with symbols, manipulating representations	Solana not able to represent the nature of skipping in PrM, how can n represent any number?
		Generalization	Generalizing too quickly, or about the wrong properties, or trivially	Rehana saying “minus two” is the rule in PrM by considering 12 and 10 only;12-10=2
		Checking work/answers	Verifying accuracy of work and solutions	Kwisha set one proportion and did not try others to check his answer.
		Implicit data/abstract representation	Information hidden from text; require other info and text cues	PrM had a pattern that participants needed to find using the given information.
	Mathematical resources	Prior knowledge of facts/algorithms	Arithmetic skills and basic counting strategies	Kwisha justifying his proportions, Kwisha said, “… I just did it the way I used to do at school.”
		Counting fingers/calculator	Using fingers to find missing addend	Zawiri counted up on one hand from seven 8,9,10,11,12) and said, they added five
		Familiar objects	Objects from everyday life	Rehana and Berina using calendar days in PrA
		Visuals/symbols	Using symbols and visuals to make sense and communicate mathematics	Azi using visual to make sense of PrB: “[*…nikahesabia miviringoso hapa ni tatu* [I counted the dots here three, …]

RQ3	Role of LPRs in mathematical processes	Communication	Explain understanding, state mathematics practices, explain PS process, clarify or substitute verbal language	Saying, "*tunaruka mbili* [we skip two]", which is a conjecture
		Comprehension	Make sense of problems or unpack meaning	Azi using kadum to make sense of the unfamiliar word "gallon"
		Development	Advance from everyday to more mathematical language	"then you get more, add more"
		Identification	Feeling a sense of belonging to a certain meaningful social group (Gee, 1999)	Berina reading "container" and "continues" in PrA as if they were the French words, "conteneur" and "continuer".

Table 3.7

Examples of Memos

Memo 1 Memo 1 was extracted from the following excerpt: **Rehana:** Eh! *Inauliza kwa siku mia*?[It is talking about 100 days?] **Berina:** So, so here, *siku ya kwanza*, first day, *aliweka* [he put] gallons *tatu* [three], *muile* [in the] container right?… *siku ya tatu* [day three], *aliweka saba* [day seven], so, so I think we have to add, right? **Rehana:** *Kwa siku mia atakuawa ameweka gallon gapi*?[How many gallons will he pump in for 100 days] …. **Berina:** Multiplication, *manake ni hii* [is this the meaning?] (she writes the symbol of multiplication to confirm if it is true) **Researcher:** Yea, *hiyo ndiyo* [that is it] multiplication	Berina read the first problem while pointing at the words. Berina and Rehana seemed to like the idea of being allowed to use the language they choose. They both seemed to understand all the words in the question but were confused about the symbol, 100^{th} day. They initially thought you will find the gallons for 100 days. Their translation "*Kwa siku mia ama siku ya mia?"* [for 100 days or 100^{th} day?] illustrates the confusion they had. Rehana suggested that they should add the gallons for each day like, up to day 3 there were 15 gallons altogether. I asked them to think of what that means. Is 15 the number of gallons for 3^{rd} day or for 3 days? They talked about it and realized that the question was about the number of gallons pumped in on the 100^{th} day not for 100 days. They noted that the gallons increased by 2 each day and so they figured that in 7 days, 15 gallons will have been poured, thus, 60 gallons are pumped within a month and so within three months, 180 gallons will have been pumped in and so they added two gallons each day for 10 days and ended up with 200 as they answer. They were thinking about gallons pumped within a certain number of days and not on a certain day. They used the idea of days of a week and months to figure out their answer even though they narrowly missed it. Also, Berina is not sure about the symbol for multiplication. About the challenges they were facing, Berina and Rehana thought that the missing information on how many gallons were pumped in for day 4 and day 5 was necessary. (Memo from interview on PrA with Berina and Rehana)
Memo 2 Memo 2 was extracted from the following excerpt: **Zawiri:** So, let me put them in order first, 1, 2, 3, … (talking to self as she puts them in order) so we have one here, there is no 2 there is no 3 we have 4 yes we have 5, no, uhmm, we have 6, we have 7, 9 we have, 10 we do, 12 yes, so what we are missing is 2, 3, 5, 8 and 11 **Researcher:** So, uhmm, it may not be an issue of what is not missing, maybe we could try other strategies to	Zawiri made a wrong guess, based on what numbers were missing in the pattern of PrM. I asked her to check her answer again since there were other numbers such as 2, 3, 4 and 5 which were missing yet they were not part of her answers. She thought the numbers needed be ordered such as the order of natural numbers in order to generate a pattern. Zawiri did not seem to understand what a pattern means? I pushed her to see the pattern and asked that she writes the explanation. Then she wrote: I subtracted or "took out" 2 and add or "get 5 more". (Memo from interview on PrM with Zawiri)

see how the pattern is because there is already a pattern in the question. … **Researcher:** Careful. From 7 to 12 how many are those? **Zawiri:** (She counts up again, okay 7, counts up from one hand 8, 9, 10, 11, 12, … they added 5 … **Researcher:** … So how will you know the next two numbers? **Zawiri:** (Excited) Take out 2 add 5 (using gestures as she restates the pattern)	
Memo 3 Memo 3 was extracted from the following excerpt: **Researcher:** So, if somebody told you to guess, are you able to guess what the next two numbers will be by just looking at the pattern **Fareli:** (scratching his head) **Researcher:** Just making a guess say they will be this and that **Fareli:** May be 8 and 11 **Researcher:** Why? **Fareli:** Because most of these are like there is kind like they write they didn't make in order, they make like, they put everything like, oh I don't know…(sighs, like fatigued, or facing difficulties, closes his eyes as if pained) (silence…using gestures hands showing continuity)	Fareli used certain hand and body gestures that showed he was facing difficulties with either the mathematical knowledge that was required in this question or with having to explain his answers in a language different from his first language (English). He made a guess, but he was unable to explain why his guesses were 8 and 11. Also, to him, the numbers in the sequence provided needed to be in some order before he could figure out what the pattern was. This indicated that he did not know what the word "pattern" meant. He did not understand that the arrangement already had a pattern and all he needed to do was to find out the rule used in the pattern. (Memo from interview on PrM with Fareli)

Research question one: What language practices do the participants use? What linguistic challenges do they encounter? What linguistic resources do they use? I frame bi-/multilinguals' language practices as translanguaging practices where bi-/multilinguals draw on their entire linguistic repertoire to make meaning of the content. I view the language of mathematics as a multi-/semiotic system that entails multiple meaning making representations (Solano-Flores et al., 2013). I see every element in the multi-/semiotic system of mathematics as a potential source of challenge to students in mathematics (Abedi et al., 1995; Fang, 2012; Schleppegrell, 2007). In this study, linguistic resources are other features of communication that may be relevant to bi-/multilinguals "yet they are less salient to the untrained ear" (Moschkovich, 2007, p. 139).

I used the examples of language practices identified in the literature as the codes for language practices. For instance, I coded incidences where participants used more than one language within the same conversation as code-switching. Other translaguaging practices identified in the literature include student restating, and translanguaging in inner speech, revoicing, and use of mnemonic devices and symbols, and collaborative dialogue (Colindres, 2015). I also looked for instances where such practices were used and coded them appropriately. Moschkovich (2007) identified linguistic resources as features of talk such as gestures and other body expressions. In this study, I looked for instances where these resources showed up and coded them accordingly. In addition to using codes from past literature, I also used common sense to identify other LPRs and to code them and I allowed the codes to emerge from the data (Creswell, 2013).

To identify the linguistic challenges that participants faced with MWPS, I considered challenging linguistic features of the MWPS as defined in the literature. Challenging linguistic features are features that can cause participants misunderstanding of or confusion about the text, or difficulties that can make them lose focus on the problem (Abedi & Lord, 2001). From participants' responses to the question on the aspects of the problems they found challenging during and after each problem-solving session, I was able to sort out the linguistic challenges they faced with the MWPS. I also identified instances within the participants' interaction where they seemed to face certain linguistic challenges. I analyzed the comments participants provided during and at the end of the second task-based interview for linguistically challenging aspects of the questions and the entire problem-solving process.

What mathematics practices do the participants use? What challenges do they encounter? What mathematical resources do they use? From a Vygotskian perspective and Scribner's (1984) view of practice, mathematics practices are socio-cultural, cognitive, and semiotic (Moschkovich, 2013). Cobb, Stephan, McClain, and Gravemeijer (2001) defined mathematics practices as "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (p. 126). In adopting a Vygotskian perspective, we shift from focusing on "purely cognitive accounts of mathematics to accounts that assume the social, cultural, and discursive nature of mathematical activity" (Moschkovich, 2008). For instance, we begin to see mathematics practices, not as practices we "tell" students or model on the board, rather as practices that learners develop when engaged in discourse during a joint activity. Also, a Vygotskian perspective assumes that "students are likely to need time and support as they move from expressing reasoning and arguments in imperfect form towards more academic ways of talking" (Moschkovich, 2013, p. 271).

To determine the mathematics practices the participants engaged in during problem solving, I considered practices as constituted in actions, goals, perspectives, and meanings of utterances. Consequently, I sought mathematics practices within the participants' actions, goals, perspectives, and meanings of utterances (Moschkovich, 2004). The participants' written and verbal solution processes of the tasks consisted of their actions, goals, perspectives, and meanings of utterances. Guided by Schoenfeld's (1985) framework for problem solving, I identified the various stages of problem solving in the participants' solution processes. I then examined the ways of reasoning, arguing, and symbolizing used by the participants as revealed by their actions, goals, perspectives, and meanings of their utterances at various stages of problem solving. These ways of reasoning, arguing, and symbolization constituted the participants' mathematics practices.

Some of the ways of reasoning algebraically, as identified by Driscoll (1999), include building rules to represent functions, doing-undoing, and abstracting from a computation. The three problems I used in this study focused on patterns, functions, and relations. The problems were therefore suitable for eliciting the three ways of reasoning algebraically described by Driscoll. According to Driscoll, some of the indicators that students are building rules to represent functions include: (a) using the repeating information to develop a process for problem solving, and (b) working to develop a rule for relations. An indicator of doing and undoing would be when students work to relate certain numbers to previous ones. Indicators that students are abstracting from computation would be when they: (a) generalize using relationships among addition, multiplication, subtraction, and division, (b) operate readily with letter symbols without the need to know what the symbols stand for, and (c) apply knowledge of properties to develop shortcuts for problem solving.

The standards of mathematics practices highlight the mathematics practices that students are to develop in any domain of mathematics (National Governors Association Center for Best

Practices & Council of Chief State School Officers, 2010). Some of these practices include: abstracting, making conjectures, looking for patterns and using structures, constructing arguments, and sense-making, among others. My analysis of the mathematics practices the participants used was guided by both the standards for mathematics practices' definitions of mathematics practices and Driscoll's (1999) ideas about ways of reasoning algebraically. I identified the sentences or paragraphs where participants revealed any form of reasoning, arguing, or symbolizing and coded those instances with the relevant name from the standards for mathematics practices. For instance, an instance where a participant was using repeating information to develop process for problem solving was coded *looking for/expressing regularity in repeated reasoning* which is the relevant name of what the participant was doing, according to the description given in the standards for mathematical practice.

Schoenfeld (1985) defined mathematical resources as the mathematical concepts and knowledge that students bring to bear during problem solving. These resources include intuition and informal knowledge regarding the domain of mathematics, facts, algorithmic procedures, routine nonalgorithmic procedures and understanding regarding the nature of argumentation/rules for working in the domain, and other relevant competencies. I used these examples of mathematical resources as codes. I coded my data for instances where these mathematical resources showed up. Mathematical challenges result from lack of the necessary mathematical resources for solving a given problem. For instance, knowledge of how things are changing and the amount by which they are changing from one step to another was an important resource for solving the algebra problems in this study. Lack of this knowledge or other related knowledge made the participants' solution processes unsuccessful and was termed a mathematical challenge. Driscoll (1999) identified other challenges in algebra as generalizing too quickly, trivial pattern spotting, generalizing about wrong properties, and difficulty translating from natural language to algebraic expressions. I looked for sentences and paragraphs within the data where the stated mathematical challenges showed up and coded them accordingly. I also used common sense and allowed other codes to emerge (Creswell, 2013).

What role, if any, do the LPRs play in the participants' participation in mathematical processes? The NCTM (2000) presents five mathematics process standards including problem solving, communication, reasoning and proof, connection, and representation. Problem solving entails finding a way towards a new solution or understanding. Reasoning and proof involve seeing and establishing relationships among ideas and facts. Communication entails sharing or recording of one's understanding. Connection deals with relating mathematical ideas to each other and to phenomena outside mathematics. Representation involves understanding the modes of communicating mathematically.

Problem solving is central and the other four processes connect to it powerful ways. I sought to understand the role LPRs played in the instances where participants engaged in these mathematical processes during problem solving. To determine the role LPRs played towards the participants' mathematical processes, I considered the following questions: how did the participant use each language LPR? Did the LPRs serve to reduce the abstractness of the problem, attenuate certain linguistic challenges, or help with symbolization of the problem, or conjecture? Did the LPR help the participant communicate his or her mathematical thinking better? I explored the contribution of each LPR to how a solution is developed. I also explored any relationship, connection, or pattern between the mathematics practices and the language practices. I used a table to represent the LPRs and the role(s) they seemed to play in the participant's mathematical processes. I then categorized these roles into four main themes which were general words for defining the roles of LPRs.

Writing and interpreting the findings. I presented the findings of this study according to the research questions. I drew my main themes for the study from the three research questions. These themes formed the headings of findings section of my thesis. During analysis, I looked for categories that described each theme. In writing my findings, I presented narratives for each theme and its categories. For each description, I supplied evidence in form of diverse quotations from across the participants. In describing the findings for research question three, I included a table of the LPRs and the role they seemed to play towards the participants' mathematical processes.

In presenting the findings, I noted that some quotations served as evidence for more than one categories. For instance, an instance where a participant codeswitched while trying to conjecture could serve as evidence for the participants' use of both a language practice and mathematical practice, as well as evidence for the role of LPRs to the participants' mathematical process. My interpretation of the findings was partly based on the meaning from the participants' side and partly on the understanding I brought to the study due to my personal culture, history, and experiences. I also drew meaning by comparing my findings with information gleaned from the literature in order to see if the findings confirm past information or diverge from it.

Validity and Reliability of the Study

Validating qualitative research entails checking the trustworthiness, authenticity, and credibility of the findings (Creswell, 2013; Moschkovich & Brenner, 2000). According to Creswell and Miller (2000), findings should be validated from the standpoint of the researcher, participant, or the readers of an account. I ensured validity from the participants' side by member checking. In other words, I shared my transcripts and findings with some participants to check if they were true recordings of what took place during the research. To ensure validity from the researcher's standpoint, I piloted the study, triangulated data, and provided a thick description of findings. Piloting the instruments of this study helped me with ideas on how they could be improved, how better to administer them, as well as ideas on the challenges to anticipate while carrying out the study. Details of how piloting this study helped improve the actual study have been discussed earlier under piloting phase.

In the case of triangulation, I used data from multiple sources including LBS, task-based interviews, brief interview notes, and participants' written work. In describing my finding, I made sure to provide a thick description which involved illustrations across the study participants. To ensure validity from readers of the account, I used peer debriefing and external auditors who reviewed the whole project. A colleague in literacy education reviewed my scores for language assessment data to make sure that I scored the participants as per the rubric. My external auditors were the dissertation advisor and the other dissertation committee members who read the final thesis and checked my methods, themes and codes, interpretation of findings, and any bias issues (Creswell, 2008).

Reliability ensures that the researcher's approach is consistent across different researchers and different projects (Gibbs, 2007). To check if approaches are reliable qualitative researchers should document the procedures they used and include as many steps of the procedures as possible (Yin, 2009). In this study, I sought to ensure reliability by documenting as many details of my data collection and analysis procedures as possible. I also used transcripts checking and a codebook to ensure reliability. I detailed every step I undertook in this study so that someone else can be able to follow the same procedures. After each transcription, I read through the transcripts as I played the videos to make sure they did not contain obvious mistakes made during transcription. Also, I created a codebook that guided my analysis, so I could not drift in the definition of codes or shift the meaning of the codes during coding process.

Chapter Summary

In this chapter, I presented the methodology I used in this study. I described the worldview that governed my study, as well as the research approach and design. I employed a constructivist worldview and used a qualitative research approach with a case study design. In discussing each of these aspects of the methodology, I also explained how each one of them fits into my study. I also discussed my role as research and ethical considerations. Although my experience as multilingual learner in a mathematics classroom motivated this study, I tried as much as I could to withhold my experiences, so I could understand other bi-/multilinguals' experiences.

I also discussed data collection instruments as well as the data collection and analysis procedures I employed at both the piloting and the actual study phases. There were three main data collections instruments including the LBS and two task-based interviews. The tasks were three algebra problems adopted and modified from NAEP released test items from three different years. The piloting phase informed the actual study on the feasibility of the instruments in addressing the study's research questions, and the challenges to anticipate and how to address them. My discussion of data collection during the actual study included details of how the three instruments were administered and the role I played as the key instruments in this study.

I also discussed how I analyzed the data I obtained from the three instruments. I analyzed data from the first task-based interview for the participants' speaking using Heritage et al.'s

(2015) framework and their writing using Bailey and Heritage's (2018) analysis framework. I analyzed data from the second task-based interview using Creswell's (2013) hierarchical approach. my analysis sough for codes that would fall under the three main themes derived from my three research questions. I based the codes I used in this study on existing research literature, common sense, and the ideas that emerged from the data and those that I thought readers would expect to find in a study such as this. I discuss the themes and the codes I used in this study in details in the next chapter.

I ended the chapter by discussing how validity and reliability were ensured in this study. I ensured validity from the participants' side through member checking. I ensured validity from the researcher's standpoint by piloting data, triangulation, and by providing a thick description of the findings. From the readers of the account, I ensured validity by peer debriefing and external auditors of the study who include my dissertation advisor and committee members. I ensured reliability of this study by documenting as much details of my data collection and analysis as possible. I also ensured reliability by checking to ensure that there were no obvious mistakes in my transcription and by creating a codebook to guide my coding process.

Chapter 4: Results and Findings

In this chapter, I present the findings from this study that answer my three research questions. In the first section, I describe the language practices and linguistic resources that the participants used, together with the linguistic challenges they encountered. I then discuss the mathematics practices and mathematical resources used by the participants, and the mathematical challenges they encountered. Finally, I discuss the role of the LPRs in the participants' mathematical processes. There are two things you will need to keep in mind as you read through these findings. First, participants did not use language practices in isolation, but they combined them with other LPRs. Therefore, for each language practice, I have included a short paragraph stating the other LPRs that seem to accompany the main one. Second, some data has been used to illustrate multiple themes. For instance, a code-switching episode can illustrate both the use of a language practice and a mathematics practice, as well as the role of language practice in the participant's development of a mathematics process.

Bi-multilinguals' Language Practices, Resources, and Challenges

My first research question was: What language practices do the participants use? What linguistic challenges do they encounter? What linguistic resources do they use? In order to address this research question, I used transcriptions of the problem-solving interviews of the math-eliciting problem (PrM), Problem A (PrA) and Problem B (PrB). I coded all these transcriptions for both the language practices, resources, and challenges known from literature and any others that I observed (see the codebook for details on coding). In the following, I describe the salient language practices, resources, and challenges from this study.

Language Practices

The following were the salient language practices: translation, translanguaging in inner speech, code-switching, restating, and invention of terms. One important observation is that participants used these language practices in combined forms with other practices, not in isolation. In the following sections, I discuss each of these practices and present data to illustrate how the practices were used and how they were combined with other practices or resources.

Translation. Participants translated words, phrases, and sentences in order to unpack the meaning of the problems. For example, when I asked Berina to explain her understanding of Problem A (PrA), she started to translate the problem in Swahili, saying:

> *Siku ya kwanza*, first day, uhmm, *aliweka* [he put] gallon *tatu* [three], right? *muile* [in the]container, *na siku ya pili* [on the second day], he pumped in five gallons, uhmm, so, *na siku ya tatu* [on the third day], uhmm, *aliweka saba* [he pumped in], right? so, uhmm, so, so I think, we have to add, right?

At the beginning of the conversation, Berina said *siku ya kwanza,* which is Swahili for "first day" and she also repeated it in English. Berina's explanation consisted of words from the original problems, some translated from English to Swahili, with the questioning word "right?" and placeholder word "uhmm" separating her thoughts. There are some switches between English and Swahili, which show her use of code-switching during translation.

In another instance, Azina, a tenth-grade student who preferred talking about the problems in Swahili, sought to understand the problems through translation as the excerpt below shows:

> **Researcher:** Okay, so *hiyo swali inakuuliza ufanye nini, umeelewa inakwambia ufanye nini*? [Okay, so what is the question asking you to do? Do you understand what the question requires you to do?]

Azina: *Mimi sijaielewa* [I do not understand the question]

Researcher: So *inakuambia*, [it tells you] "write the next", write *ni kufanya nini*? [what does write mean?]

Azina: *Sijui vizuri* [I do not know quite well]

Researcher: *Andika* [write]

Azina: *Kuandika*? ooh [to write]

Researcher: the next, next *inammaanish nini*? [next means?]

Azina: Next, *inayokuja* [the one that follows]

Researcher: Two numbers

Azina: *Namba mbili* (she translates two numbers on her own)

Researcher: In the pattern below (silence)

Azina: Uhmm, pattern, pattern (silence)

In this conversation, it is clear that we were trying to unpack the language of the problem word by word, phrase by phrase, through translation. Azina was only able to translate to Swahili some of the words used in the problem and I had to help her unpack the rest, then she could continue with solving the problem. There was another case where Azina requested her partner, Solana, to explain Problem B (PrB) to her in Swahili so she could understand it. The following conversation took place.

Azina: *Sijaelewa hii* [I have not understood these] dots

Solana: Dots *ni hizi miviringo* [are these circles], *ndizo* [they are the] dots (pointing at the dots)

Azina: *Halafu hii* count [then this]

Solana: Count *ni kuhesabu* (translates count)

Azina: Ooh, *haafu hii*, [and this] draw

Solana: Draw *ni* [is]

Azina: *Hii ya hapa* [this one here] (pointing at the word word)

Solana: Draw *ni kuchora* [is to draw]

Researcher: *So hiyo swali inasema nini*? [what is the question saying?] Grace *anataka kufanya nini*? [What does Grace intend to do?]

Azina: *Atafute* [she wants to find] number of dots, *yaani anataka atafute namba ya hii michororo* [in other words, she wants to find the number of these dots]

After being taken through the meanings of the words in the question in Swahili, Azina was able to say what the question is about, combining both Swahili and English. In other cases, participants attempted to translate symbols such as 100th day as *siku mia*, which means 100 days and could lead to an incorrect solution path.

Translanguaging in inner speech. Translanguaging in inner speech involved participants talking or reading to self. Sometimes the participants read/talked to themselves in their home language only or they combined multiple languages to make sense of the problems or do arithmetic computations. Zawiri talked to self the most number of times of any participant. For example, she talked to self while performing the arithmetic of missing addends as she solved the PrM in order to establish the pattern and find the next two numbers in the pattern. In all these instances, Zawiri counted on her fingers and used various hand and body gestures. For instance, she once talked to herself saying “to get to twelve they took out two, so to get to ….” With this phrase, Zawiri was trying to remind herself of certain aspects of the pattern involved in the problem in order to find the next two numbers in the pattern. She used the word “they” to fill the gap of a missing agent in the problem and hence relate with the context.

Code-switching. Code-switching is the language practice that I coded the most number of times. There were multiple forms of code-switching; there were switches within one speaker turn, within one conversational episode, and switches within a sentence. Moreover, participants used single words (nouns) or phrases (noun phrases) from one language in an utterance in another language. In the following, I present data that illustrates where some of these variations occurred. I do not distinguish between the variations since within one conversational episode speakers tend to use more than one variation of code-switching.

In another instance, when I asked Exmon in English if there was another way he could have solved PrA, Exmon responded as follows:

"I don't know I guess *ni unachukua hizi zote* [is you take all these], *unachukua hizi* [you take these] numbers *alikuwa mu*-one day [that he put on one day], *na ile ya pili na ya tatu* [and for the second and the third day] *una*add [you-] *kisha una*times [then you-] *kwa* [by]100, I don't know." Exmon started his sentence in English and then he made within sentence switches, inserting Swahili words and phrases. He created Swanglish words and phrases such as "*una*add" and "*una*times". After encouraging Exmon to think further about a quicker way to solve PrA, Exmon made a conjecture and justified it as follows:

Researcher: Is there another way you think you can do this?

Exmon: How about *tuchukue* [we take] two times hundred?

Researcher: Why two times 100?

Exmon: 'Cause three plus two equals to five and five plus two equals to five and five plus two equals to seven *kwa hivyo tunaenda kwa ile ingine* [so we are going to the next one], next day (uses hands to show continuity as he speaks of next day) *atukuwa ana*add [he will be adding] two *itakuja kuwa* [it will be] nine.

Again, Exmon started his responses in English and then he switched to Swahili sentences, words, and phrases. Additionally, Exmon combined code-switching with hand gestures. He used hand gestures to show the continuity of the pattern and, through code-switching, he stated his conjecture that the pattern was growing by two.

Restating/rereading. To restate is to repeat or say something differently in order to make it clear or convincing. I mostly observed this practice when I asked the participants to explain their understanding of the question. In many cases, participants' explanations contained the exact words, phrases or sentences from the original problems, regardless of whether they understood the meaning of these words, phrases, and sentences or not. For instance, after reading PrA, I asked Berina to explain what the problem required her to do and she said, "is to write the next two numbers in the pattern", which is what was stated in the question. In other cases, participants first said a word, phrase, or sentence in one language and then they would say it a second time in a different language or with gestures. For instance,

Researcher: If you had one orange and you want to make them six whole

Zawiri: Cut them

Researcher: Why do you want to cut them?

Zawiri: Then you get more, add more

Researcher: How many more do you add?

Zawiri: You add four (holding fingers) wait to get six you add five!

Zawiri's response "then you get more, add more" appears repetitive, but she repeats herself with a reason. The second part of her response involves a more precise mathematical term, "add". Also, her final response was tied to her hand gesture. In another instance, when I asked Neha what she could do with four to make it nine and she said, "you put together, like you use plus.” Again, here Neha began with a more general response, but restated her response with a mathematical term, “plus”, which represents the process of “putting together”.

Invention of terms. The participants invented terms by coining familiar English words. For example, “galoni/magaloni” was coined from “gallon(s)”. Moreover, participants invented terms in English and other languages that they thought would express the meaning they held for certain situations, regardless of whether the invention was commonly used in mathematics or not. Some of the phrases that the participants used were: “you skipped two”, “*tunaruka mbili* [we skip two]”, “going up by two”, “you minus two and plus two”, “*zinapandana kwa mbili* [they are rising by two]”, “pattern *iko* [is] up *kwa* [by] two”, and so on. Here is an excerpt in which Solana used the phrase “skipping two numbers” to justify her conjecture about multiplying by two.

Researcher: What did you do?

Solana: We times 1 to 200, so it will give us 200

Researcher: What did you do? … Why 100 times two?

Solana: Because we skipped two numbers so that’s why I was trying to see if we times 100 to two gives us 200, and it is ‘cause here we skipped like two-two-two and we haven’t gotten to 200 so I think we will get to 200.

After a brief moment of thought, Solana changed her mind and the conversation went on:

Solana: Oh no! It’s not!

Researcher: Why do you think it’s not?

Solana: So, we skipped two numbers, no, one number, so I think it is one hundred and one, it is two hundred and one

Researcher: Two hundred and one. Why?

Solana: Because skipped two numbers and then…

Researcher: So, like you skipped two numbers, which ones are you referring to?

Solana: I was referring … 'cause last time we said 50, then I was going to keep going and now we stopped here (showing on her worksheet where they got to day 50) and I was like I think it is 201 because if you keep going and we got 101, then for 100 it will be 201 and because we skipped two numbers so I think it is 201.

Solana used the term "skipping by two" to communicate the rule she followed to arrive at the answer. Her argument is that by skipping two numbers for 50 days they got 101 meaning for 100 days they would get 201. She used the phrase, "we kept going" to communicate how the "skipping two numbers" was continued.

Linguistic Challenges

Linguistic challenges emanated from features that caused participants' misunderstanding of or confusion about the text, or difficulties that made them lose focus on the problem (Abedi & Lord, 2001). Participants faced the following linguistic challenges: unfamiliar words and complex syntax, unfamiliar language, abstract representation and passive voice. In the following sections, I describe each of these challenges and provide data to illustrate it.

Unfamiliar words and complex syntax. The word that seemed most unfamiliar to many participants was "pattern". In PrM, participants were supposed to find the next two numbers the pattern: 1, 6, 4, 9, 7, 12, 10. Most participants had a challenge with explaining both the mathematical and linguistic meaning of the word pattern. None of the twelve participants was able to define the word pattern verbally; most of the participants waved their hands to indicate something repeating or continuous. Others said that they did not know what it meant or they knew but could not explain it. For instance, when I asked Rehana what the pattern was in PrM, she said:

Rehana: Pattern, pattern is like a number, uhmm, I don't know how to explain because sometimes English is confuse me, but I understand.

Rehana was trying to say that she understood the meaning of word pattern (either mathematically or linguistically), but she could not explain because of challenges with the English language. I went on to find out what Rehana understood regarding the word pattern:

Researcher: You understand? So, what is going on? What is the pattern?

Rehana: Add and subtract

Researcher: What are you adding?

Rehana: Add five (shows five with a fist) subtract two (shows with two fingers)

This excerpt shows that Rehana knew a pattern involved a certain regularity and, in this case, it entailed "adding five and subtracting two". She reinforced her words with hand gestures. In other words, Rehana understood the mathematical meaning of the word pattern. She just could not explain the linguistic meaning in English.

Unfamiliar language. While most linguistic challenges were at the lexical (word) level, some participants, such as Azina, said that their challenge with comprehension worsens when the unfamiliar words are put together into sentences. For example, when I asked Azina if any words in PrA posed her any challenges, she said, "*mi naona sio ngumu chenye kinanisumbu ati ni kuelewa, sijaelewa lugha vizuri, ni chenye kinasumbua hapa … ingekuwa ni ile lugha najua vizuri ningeelewa na ningefanya* [I think it is not difficult. My biggest challenge is the language. I do not understand the language quite properly. That is my greatest challenge here … Had the problem been presented in a language I understand very well, I could make sense of it and solve it.]" Azina's challenge with unfamiliar language is further confirmed when she requested her partner, Solana, to explain to her the question part of PrA in Swahili: "*tuseme saa hii swali* *wametuuliza, nataka nijue kwa Swahili* [Regarding the question they are asking here, I would like to understand it in Swahili]". Solana did as per Azina's request, and translated the sentence, "how many gallons of water will Tom pump in on the 100th day?" word by word.

Abstract representation and passive voice. An incidence of an abstract representation is seen in PrA, where the nature of the container into which water was being poured was unknown. Zawiri thought details of the size, shape, and type of this container, together with the reason as to why Tom had to pump water into it, should have been provided. Much as these details seem irrelevant to the process of finding the number of gallons for the 100th day, such information could help the problem solver identify with the context of the problem. It is easier to solve a problem when the context makes sense to the problem solver than when the problem situation is implicit, and the problem solver has to invent his or her own assumptions.

PrM was a list of the numbers 1, 6, 4, 9, 7, 12, and 10 with a sentence instructing participants: "Find the next two numbers in the pattern below." Most participants did not recognize that the wording meant that a pattern existed already in the list of numbers. Perhaps the passive nature of the problem and the fact that the problem lacked a context that participants could relate with caused the comprehension difficulties. In an attempt to fill the gap of a missing agent some participants used words like, "they" in their discourse.

Linguistic Resources

I used Moschkovich's (2007) definition of linguistic resources as other features of communication that may be relevant to bi-/multilinguals "yet they are less salient to the untrained ear" (Moschkovich, 2007, p. 139). Linguistic resources include non-verbal forms of communication such as gestures, as well as other aspects of language a bi-/multilingual might use to make sense of content. Linguistic resources do not include language practices in this case.

The participants used the following linguistic resources: hand and body gestures, familiar words/objects/signs, and cognates. In the following sections, I discuss each of these resources and how they were used by the participants.

Hand and body gestures. Hand and body gestures were the most commonly used gestures that I observed. The participants mostly used their hands while they solved the PrM to indicate continuity, repetitiveness, and periodicity, especially when asked to explain the meaning of pattern. For instance, when I asked Rehana the meaning of pattern in the PrM, she said: “of the next is like pattern, I don’t know (waving hands like a sine wave).

In other cases, the participants simply pointed at objects, symbols, words, instead of calling them by names or saying the words. During a conversation with Fareli as he solved PrM, I noted how he preferred to simply write down the answer instead of saying what it is verbally as shown in the following excerpt:

Researcher: So, what did you do here?

Fareli: Plus five, minus two, plus five, minus two, the next number will be like, like this (he writes 15 and looks up at me smiling)

Researcher: And the next one?

Fareli: Will be like, (talks to self as he writes 13 then looks up again with a smile)

Fareli did not appear confident with pronouncing the names of these numbers in English. This could be the reason he chose to simply write them down instead of saying the names incorrectly. In this excerpt, Fareli seems to combine gesturing and symbolization to communicate his understanding. He uses the phrase “will be like” or “like this” to introduce his gesture which entails symbolizing the names of numbers.

Familiar words/objects/signs. Some participants relied on familiar words in the text to make sense of others within the same text. In one instance, I was asking Tamani and Samba if there were words in PrA that posed a challenge to them. The following excerpt shows how Tamani used other words in the text as cues for understanding others.

> **Tamani:** Me *nilieewa hii* [I have understood] fill *hapa kwa sababu niliona hii* [here because I saw this] container (apparently, he had used cognates to understand container)
>
> **Researcher:** How about this 100th day, *hiyo mliielewa namna gani* [how did you understand that phrase]? *Ilileta changamoto kidogo* [was the phrase a little linguistically challenging]?
>
> **Tamani:** *Niliielewa kwa sababu walikuwa wanauliza kwa sababu niliona nanii hii* [I understood it because I saw this sign] *quentel* (pointing to the question mark).

In the first case, Tamani said he understood the meaning of "fill" because he saw the word container (a word he had made sense of using its French cognate "conteneur") and water. He automatically knew that "fill" was related to putting water in the container. For the second part, Tamani claimed that the question mark, which he called "quentel", helped him identify the question part of the problem. He thus read the question and knew that he needed to find the number of gallons of water that Tom would pump on the 100th day.

In other cases, participants related words with names of familiar objects in daily life. For instance, Azina, in trying to explain what gallon meant in PrA, said: "dum, kale kadum [container, that container] (hand gesture for how you hold a container)". Azina coined the word "kadum" from her mother tongue (Bembe) and Swahili as she unpacked the word gallon, which is not a common unit of measuring volume in Africa. She incorporated a hand gesture to show something that can hold some substance. When discussing the meaning of 100th day in PrA, Fareli insisted that the word “on” in the question part “how many gallons of water did Tom pump on the 100th day” implied a specific day, not days: “*ni siku, hii ni siku ya mia, yaani ni zile* gallons *zenye ata*make *hiyo siku ya mia* [it is a day, this is the 100th day, as in the] gallons [he will] make [on that 100th day]. Even as he drew upon a familiar word, “on”, Fareli combined other LPRs like restating, translation, code-switching and some swanglish, “*ata*make” as he affirmed the meaning of the problem.

Cognates. Cognates are words that are semantically and phonologically similar in two languages. Cognates sound similar and have similar meanings, but the spellings may differ. Some participants used cognates to infer the meaning of unfamiliar words. For instance, Tamani drew from the French cognate word “conteneur” to make sense of the English word “container”, which later helped him make sense of the word “fill”. During one of our conversations, Azina related the English word “formula” with the French word “formule”. When she read PrA, Berina pronounced the words container and continues, as if they were French words “conteneur” and “continuer”, respectively.

Bi-/multilinguals’ Mathematics Practices, Resources, and Challenges

My second research question was: What mathematics practices do the participants use? What mathematical challenges do they encounter? What mathematical resources do they use? I found the mathematics practices that the participants engaged in by analyzing their discourse practices and meanings of their utterances. Since I used tasks from algebra, I looked for indicators of algebraic thinking such as doing and undoing, building rules to represent patterns, and abstracting from computation (Driscoll, 1999). The algebraic ways of thinking were in essence the participants’ mathematics practices. Mathematics practices are ways of reasoning, arguing, symbolizing, which I saw in the participants’ goals, actions, and perspectives during their problem solving. Mathematical resources entail the basic knowledge/experiences that participants drew upon during their problem solving. In the following sections, I discuss the salient mathematics practices, resources, and challenges from this study.

Mathematics Practices

Some of the salient practices included: Sense making, looking for/discovering patterns, describing/using structure, looking for/expressing regularity in repeated reasoning, constructing arguments/making conjectures. I discuss each of these practices as well as provide data to illustrate where they occurred.

Sense making. Sense making entails uncovering the meaning of the problem, reflecting on understanding of the task, looking for entry point, considering what is known or unknown about the task and how to arrive at a solution. Making sense of the problem is the first step in problem solving (Polya, 1957). Most participants sought to understand the problems first before solving them. However, some participants, due to the “compulsion to calculate”, did not think carefully about the problems, especially PrM. Participants who sought to make sense of the problems drew on language practices such as translation, code-switching, and resources such as cognates, cues from the texts and familiar objects. For instance, after reading PrA, Exmon and Kwisha discussed the problem using a mixture of Bembe, Swahili, and English, as they tried to make sense of it. At this instance, Exmon first spoke in Bembe then Kwisha joined him in Bembe with a switch to English, “on day one, Tom…gallons”. Kwisha then continued explaining in Bembe. Exmon later joined in Swahili, “siku ya kwanza…”. These students were drawing on their various linguistic repertoires to make sense of the problem.

Looking for/discovering patterns. Most participants misunderstood the mathematical meaning of pattern in PrM, and as an initial step, the participants rearranged the numbers in increasing order, such as the order of natural numbers. This was an attempt to look for patterns they could use to figure out the next two numbers in the sequence. In such cases, I often reminded the participants that the numbers already formed a pattern, and that rearranging them would mess up the pattern and perhaps complicate the problem. So, some participants would immediately revise their strategies.

Zawiri was one of those participants who initially used the wrong basis for finding patterns. When I asked her to look for the pattern within the given numbers, she reread the problem several times and then exclaimed, “so one, two, ooh, are we skipping numbers?” I let her continue examining the numbers while I asked supporting questions to check her work. In the end, she discovered the pattern, which she stated proudly: “Add five, subtract two!” Like Zawiri, most participants needed some form or support in order to discover patterns within the problems.

Describing/using structure. The practice of describing/using structure entails discerning and applying patterns or structures. I mostly observed this practice when participants began solving PrB. PrB was similar to PrA in structure except it was presented within a different context and it contained a visual in the form of figures with dots growing by two. All of the participants were able to discern that these PrA and PrB had the same structure and would thus be solved in the same manner. The following conversation illustrates this.

Researcher: Uhmm, how are they the same?

Kwisha: ‘Cause *zinapandana kwa mbili* [they are rising by two]”, “pattern *iko* [is] up *kwa* [by] two”

Researcher: *Kuna usawa mwingine mnaona* [any other similarity you can see]?

Exmon: *Halafu tena ukipenda* [also if you like] I guess *tutapata ile jibu yetu tena itakuwa* [we will get our same answer, it will be] plus one so *ukipenda ku*check *tuseme,* *ukichukua* [if you'd like to check let's say, if you take]100 times two *itakuwa* [it will be] 200 *sasa itakuwa* [now this will be] wrong *na ukatuambia tuanze kupima* [and you told us to be checking our answers] *ku*check [to] *na* [with] small numbers *uki*check [if you] *hapa* [here] three times two *itakuwa ni* [will be] six *na uki*add [and if you add]one *itakuwa* [it will be] seven *na* [and] seven *ndio tukonayo hapa* [is what we have here] (pointing at figure 3 which had seven dots).

Kwisha not only related the structure of PrA and PrB in terms of the patterns they formed, but he also related their solution strategies and how the answers could be checked and generalized.

Looking for/expressing regularity in repeated reasoning. This practice entails noticing if calculations are repeated and looking both for general methods and for shortcuts. One instance in which the practice was revealed was when Exmon tried to justify his conjecture for finding the number of gallons Tom would pump into the container on the 100th day. Exmon had conjectured multiplying two by 100. He explained his reasoning as follows:

'Cause three plus two equals to five and five plus two equals to seven *kwa hivyo tunaenda kwa ile ingine* [so we are going to the next one], next day (uses hands to show continuity as he speaks of next day) *atukuwa ana*add [he will be adding] two *itakuja kuwa* [it will be] nine.

This was an attempt to express regularity in repeated reasoning that the number of gallons pumped in rose by two each day and so to find the number for 100th day, one would need to consider the amount gained each day by multiplying 100 by two. His conjecture for multiplying 100 by two in order to find the number of gallons pumped in on the 100th day was not totally correct. He needed to add one to it. However, this was an attempt to state a shortcut for solving the problem, an indicator that the participant was looking for/expressing regularity in repeated reasoning.

In another instance, Solana and Azina worked on PrA by listing the number of gallons that Tom pumped into the container daily. After working for the first 50 days, Solana stopped and conjectured that the number of gallons for 100th day would need to be 201. She justified her conjecture using repeated reasoning she had gathered from her previous work as follows:

> **Solana**: I was referring … 'cause last time we said 50, then I was going to keep going and now we stopped here (showing on her worksheet where they got to day 50) and I was like I think it is 201 because if you keep going and we got 101, then for 100 it will be 201 and because we skipped two numbers so I think it's 201.

Solana reasoned from their earlier work that since Tom pumped 101 gallons on the 50th day, then he would pump 201 gallons on the 100th day. By stopping the lengthy calculations at the 50th day in order to figure out a shortcut, Solana was expressing regularity in repeated reasoning.

Constructing arguments/making conjectures. This practice entails understanding and using stated assumptions, definitions, and previously established results to construct arguments. It also involves both making and justifying conjectures, analyzing situations by breaking them into cases, and recognizing and using counterexamples. Furthermore, it includes making conclusions, communicating them to others, and responding to others' arguments. After working on the tasks and discovering patterns, participants made conjectures for finding the number of gallons for the 100th day using words and phrases such as: "It is going up by two. I guess we multiply by two (Zawiri)", "How about *tuchukue* [we take] two times 100? (Exmon)".

Solana had been working with Azina to solve PrA by listing the days and the number of gallons for each day using the fact that two gallons were gained each day. In the middle of their work, Solana stopped to think and conjectured multiplying 100 by two in order to find the number of gallons of water that Tom would pump into the container on the 100th day. Solana started justifying her conjecture as follows:

> **Solana:** Because we skipped two-two numbers so that's why I was trying to see if we times 100 to two give us to 200, and it is 'cause here we skipped like two-two-two and we haven't gotten to 200 so I think we will get to …

Solana's justification is not as precise and loaded with mathematical terminology and coherence as a mathematician would expect, but it has some mathematical sense in it. She communicated her reasoning using her own invented terms. Furthermore, the fact that she saw it necessary to justify her reasoning without being prompted to do so shows she has attained a certain level of growth in mathematical communication. Solana argued that because they gained two gallons each day, then it makes sense to multiply 100 by two in order to find out the number of gallons for 100th day. However, she makes it clear that "we haven't gotten to 200". When I asked her to clarify what she meant by the phrase "we haven't gotten to 200", she said the following:

> **Solana**: I was referring … 'cause last time we said 50, then I was going to keep going and now we stopped here (showing on her worksheet where they got to day 50) and I was like I think it is 201 because if you keep going and we got 101, then for 100 it will be 201 and because we skipped two numbers so I think it's 201.

By saying “we haven’t gotten to 200”, Solana was inferring from their earlier work where on the 50th day they had gotten 101 gallons instead of 100. So, 100th day would not get them 200 gallons, but 201.

Mathematical Challenges

I analyzed the participants’ problem-solving process for aspects that were mathematically challenging. Other mathematical challenges could be attributed to certain linguistic challenges. Some of the salient mathematical challenges included: mathematics symbolization and visuals, technical math lexicons, abstraction, generalization, checking answers/work, implicit data/abstract representation, and missing added.

Mathematics symbolization and visuals. Some participants did not know how to symbolically represent certain mathematical operations and misunderstood the meaning of other symbols. Some challenges of mathematical symbolization have been discussed under abstraction since making representing quantities using symbols is an aspect of abstraction. For instance, Neha was not sure how the subtraction sign is written. In another instance, Berina had to confirm how to write the multiplication sign. The mathematical meaning of 100th day was also confusing to many participants. The following conversation with Rehana and Berina confirms this confusion:

Researcher: Okay, na hiyo 100th day, inamaanisha nini [so that 100th day, what does it mean?]

Berina: Kwa siku mia, ni siku mia [for hundred days, it is hundred days]

Rehana: Ni siku ya mia [it is the hundredth day]

Researcher: Ni siku mia ama ni siku ya mia [is it 100 days or 100th day?]

Berina: Ni siku ya mia [it is the hundredth day]

Rehana: Hii haieleweki [this is not understandable]

Berina and Rehana used translation to make sense of the symbol 100th day. However, they did not translate the symbol according to the mathematical discourse. Their translation did not consider the meaning of the superscript "th". Lack of a proper understanding of 100th day made participants such as Berina and Rehana take incorrect steps in their solution process. For instance, by taking 100th day to mean "*siku mia* [100 days]", Berina and Rehana chose to multiply the number of gallons pumped in on the first day, the second, and the third in order to find the number of gallons pumped in on the 100th day:

Researcher: *Sasa utatafuta aje hiyo?*[so how do you go about solving this problem?]

Berina and **Rehana**: We have to multiply.

Researcher: Multiply *nini* [what]? Multiply what?

Berina: We have to multiply three multiply five,

Researcher: And then?

Berina: I think three multiply five is fifteen, right?

Researcher: Yea that is fifteen.

Berina: Then fifteen multiply seven.

Had I not intervened and encouraged them to think more carefully about the problem and the meaning of their strategy, they would not have made it to the correct solution. My study was concerned with the participants' linguistic and mathematical challenges and so I interacted with them and sometimes influenced their problem solving in ways I would not do if I were I strictly examining what they could do mathematically on their own. For instance, I intervened when the participants remained quiet for more than a minute, when they pursued wrong solution paths, when they stopped working together, or when I needed further clarification on their work.

One of the problems, PrB, had a visual. Although some participants thought the visual was a useful resource, others thought the visual obstructed their comprehension of the problem. The problem required participants to find how many dots would be in the 100th figure, but the figures were not labeled as figure 1, figure 2, and so on. Instead, the figures were labeled 1, 2, 3, and so on. Some participants, like Rehana, confused the meaning of these numbers with the manner in which the dots were to be counted.

Technical math lexicons. The mathematics meaning of the word pattern posed the greatest challenge to the participants. While the participants drew on various resources, such as gestures, to explain their understanding of the term, the greatest challenge was with relating the linguistic meaning to the mathematical meaning. In other words, it was not enough to understand a pattern as something repeating, participants needed to know what was repeating in that context. Most participants reduced the mathematical meaning of pattern to numbers increasing by one, as is the case for counting/natural numbers. This is the reason many participants based their guesses for the next two numbers on the numbers that would be missing after ordering the ones given in the problem. Participants needed to symbolize the rule for finding numbers in the pattern. It was also challenging that two separate rules, +5 and -2, would be required in order to find the next two numbers. Knowing which rule to apply and the number to apply the rule on, was also challenging to most participants. The following conversation with Kwisha illustrates this point:

Kwisha: The pattern is five, five, and five

Researcher: And in between we have what?

Kwisha: In between we have negative two, negative two, negative two

Researcher: So, what should be the next number here?

Kwisha: A five and two?

Researcher: Okay, so what are you going to do to 10 to get the other number?

Kwisha: What I will do to 10 to get this number (pointing at the first blank space)? to add?

Researcher: What do you add?

Kwisha: You have to add five

Researcher: Then the next one will be?

Kwisha: The next one 12? So, I have to add two again to five to ten? Ooh minus two

Researcher: So, it's going to be (he writes 8), so what do you minus? Do you minus from this number (pointing at 10) or the immediate number?

This excerpt shows that Kwisha was confused about the nature of the rules. At first, he omitted the plus sign and the second rule. Later he also confused which rule to use and the number in sequence on which to apply which rule.

Abstraction. Abstraction entails making sense of quantities used in a problem together with their relationships, representing problem situations symbolically, manipulating the representations, and probing the referents for the symbols involved. Eleven out of the twelve participants had difficulties relating the numbers provided in PrM in order to find out the rule involved. Some participants understood the problem and the numbers involved but could not establish a relationship between them. For instance, Fareli said: "I don't know how to solve it". Others like Tamani, did not make sense of the problem at all and simply said: "I don't understand this question". On the same problem, Solana figured out that "two numbers were being skipped" but she did not specify the nature of skipping. In some instance, she used the term, "skipping small to large numbers" and vice versa, but symbolization was still needed to show exactly how this process was happening. Then she could apply it to find the next two numbers in the pattern.

For PrA, I tried to extend the problem beyond finding the gallons that Tom put on the 100^{th} day. As I talked with Fareli and Diodo, I asked them to think of the number of gallons Tom would put on the nth day. The conversation we had showed they had a challenge abstracting:

Fareli: What, n?

Researcher: If you want to give a general formula, instead of 100th you had nth?

Diodo: Like they didn't mention like 100?

Researcher: Yea, they just said n, what would your answer be? n can be any number (they don't seem to understand how you can have n; Fareli looks disturbed, I cross out 100^{th} and replace 100^{th} with nth in the problem)

Fareli: n equal what? Equal 100?

Diodo: Can it be five, 11?

Researcher: n can be any number, it can be 20,100, 5, 11, what will be your answer?

Fareli: Can it be five? I don't get it.

Even though Fareli and Diodo had solved this problem for the number of gallons Tom put on the 100^{th} day, this excerpt showed that they had difficulty with symbolizing the quantities involved in the problem. It seems to ßsurprise them how they can find the number of gallons pumped on the container on the nth day. They are happy to find the number of gallons put on a known day, not on an abstract day.

Generalization. For problems PrA and PrB, I tried to encourage the participants to find a general formula for finding the number of gallons of water that would be poured on any day, n or the number of dots in any figure, n. For PrM, participants were to find the general rule used in the pattern provided and to use it to find the next two numbers in the sequence. However, participants' challenge with abstraction led to further challenges with generalization. Participants either generalized too quickly, or about the wrong properties, or they generalized trivially. For PrM, participants were too quick to suggest some form of operation as required in order to find out the rule involved in the problem. For instance, Rehana quickly suggested that subtracting two from twelve gives ten and so she concluded that the rule entailed subtracting two to get the next number. For PrA, participants suggested either dividing or multiplying 100 by two to get the number of gallons that Tom pumped into the container on the 100th day. Others suggested dividing 100 days by the product of the number of gallons pumped in on the first, second, third, and the fourth day. Participants often had an urge to calculate something, regardless of whether the computation was necessary or not. Most participants guessed that the next two numbers in the pattern of PrM were 5 and 8 or 8 and 11 because these numbers were missing after ordering the numbers in the sequence. Generalizing on the basis of missing numbers, in this case, would be termed trivial because other numbers like 2, 3, and 11 were also missing in the sequence. There was therefore a need for a criterion for choosing one number over another to fill the two spaces.

Checking work/answers. Checking own work is a necessary metacognitive skill for successful problem solving (Schoenfeld, 1985). However, in this study, participants made quick guesses and generalizations for the problems, they seemed not to care to check their answers or conjectures. For instance, after suggesting that the next two numbers in PrM are eight and eleven, or five and eight, none of the participants bothered to check if those were the only numbers that were missing in the sequence. It came as a surprise when I mentioned other numbers that would be missing as well, yet they were not part of their choices.

One participant, Kwisha, assumed the relationship between the number of days and the number of gallons of water pumped in for PrA was proportional. He thus set up proportions using the number of gallons pumped in on the first day as follows.

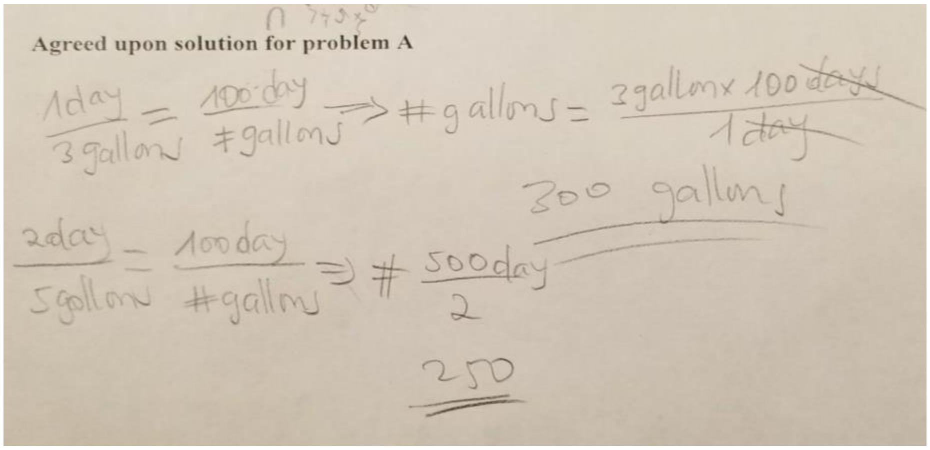


Figure 4.1: Kwisha's Solution to PrA. This figure shows Kwisha’s use of proportions to solve PrA.

Kwisha appeared very confident with his solution path. When I suggested that we check his answer by setting up a proportion using gallons pumped on the second day instead, Kwisha was quick to say “*yea natarajia zitakuwa sawasawa* [I expect both answers will be the same]”. The participants’ failure to check their solution paths may be attributed to their being overconfident or to the lack of knowledge on how to check their work or knowledge of the necessity to check their work.

Implicit data and abstract representation. Implicit data are information that are hidden from the text and participants have to find them using other information and clues from the text. Implicit information may consist of an unclear message in the text or it may be completely unavailable. For instance, in PrM, participants were told there was a pattern formed by the numbers: 1, 6, 4, 9, 7, 12, and 10, but they had to figure it out and use it to find the next two numbers in the sequence. Another aspect of implicit data in this question relates to the missing numbers. Eleven out of the twelve participants misunderstood the meaning of implicit information in this problem. Participants guessed that the next two numbers would have to be chosen from those that were missing from the sequence like, 2, 3, 5, 8, and so on. For instance, Neha suggested 2 and 3 as the next two numbers in the pattern because they were missing. Neha misunderstood the meaning of what was missing. She did not know that the missing information was meant to help her figure out the rule involved, which in turn would help her figure out the next two numbers.

Missing addend. Capability of finding the missing addend was one of the mathematical skills that participants required in order to succeed in solving the problems in this study. Unfortunately, thinking of subtraction as the process of finding the missing addend was a challenge to most participants. In PrM, the participants were to find out what number is added to or subtracted from a given number in order to get certain other numbers. The following excerpt illustrates the struggle some participants, like Neha, had with figuring out the missing addend:

Researcher: So how do you move from four to nine?

Neha: To move from?

Neha: You put together, like you use plus

Researcher: You add

Neha: Yes, you add

Researcher: So, from four to nine you said, plus, what are you plusing?

Neha: Its four plus nine

Researcher: I want to know like if I have four things, to get nine things what do I do with the four?

Neha: (Silence)

Researcher: If I have four oranges and want nine oranges, what am I going to do with the four? How many am I going to add to get nine?

Neha: You have to like, if you have four and you want to get nine.

Researcher: Yes, what do I add to the four?

Neha: You add 1,2,3,4, when you, uhmm, you add (closes eyes as she thinks through)

Generally, when participants needed to figure out the missing addend they counted up or down using their fingers.

Mathematical Resources

Mathematical resources are the mathematical concepts and knowledge that participants bring to bear during problem solving (Schoenfeld, 1985). Participants drew from resources such as prior knowledge of mathematical facts/algorithms, counting fingers/calculator, familiar objects such as calendar days, and visuals/symbols.

Prior knowledge of mathematical facts/algorithms. Participants drew on their past arithmetic skills and basic counting techniques involving the use of the operations of addition, subtraction, multiplication, and division in solving problems. When solving PrM, participants related their knowledge of patterns with the pattern formed by counting numbers, 1, 2, 3, 4, 5, …, claiming that the numbers in the problem had to be ordered likewise. After ordering the numbers in the problem, participants then noted that some of the counting numbers were missing in the new list. They then guessed that the next two numbers would have to be chosen out of the missing ones. This was an incorrect strategy since there were more than two numbers missing. Furthermore, the problem said that the numbers listed formed a pattern. Reordering them would mess up the pattern and perhaps complicate it.

Some participants also drew on the algorithms they have learned in school, though sometimes inappropriately. For example, Kwisha used his knowledge of proportions to solve PrA. In using proportions, Kwisha assumed that the relationship between the number of days and the gallons pumped in daily was proportional. This assumption was not correct. Instead, the relationship was linear. When I asked Kwisha why he chose to use proportions to solve PrA he said, "… I just did it the way I used to do at school." Kwisha did not have a mathematical justification for why he used this strategy. Meaning he had simply memorized the algorithm without knowing what it means and when is the appropriate time to use it in problem solving.

Counting fingers/calculator. Finger counting was the most common resource used by participants, especially when they needed to find the missing addend in PrM. Most participants voluntarily counted on their fingers and ended up getting the correct answers. Zawiri is one participant who, though she had initially pulled up the calculator on her phone, she solely counted on her fingers as she tried to figure out the pattern and the next two numbers in the sequence 1, 6, 4, 9, 7, 12, 10 of PrM:

> **Zawiri:** They added five, … so to get to four they took out two, to get to nine they (silence as she counts up with figures, 4 5,6,7,8,9) they added five again, so to get to seven, (holding her fingers, saying nine), so to get to seven they took out two, for 12, (she counts up, 6,7,8,9,10,11,12) they took out six? No, they added six? They added six?
>
> **Researcher:** Careful; from seven to 12 how many are those?
>
> **Zawiri:** (She counts up again, okay seven, counts up from one hand 8,9,10,11,12) they added five

One participant, Samba, seemed unwilling to either count on his fingers or use a calculator. Despite my suggestion that he count on his fingers in order to figure out the solution, Samba did not bother. He simply smiled. It took him long to mentally figure out what the missing addend was. On another instance, Tamani was also having difficulty with finding what to subtract from 9 to get 7. When I suggested that he count on his fingers, he was indifferent at first. However, as soon as he held his fingers up and began to count, he quickly figured out the answer:

Researcher: Uhmm, nine minus six is three and this is not it. I want to know this, what do you subtract nine to get seven? You can use your fingers, look at your fingers, just see nine and how you get seven from nine

Tamani: Minus

Researcher: Minus what?

Tamani: Six

Researcher: What do I minus nine to get seven? How will I get seven?

Tamani: (He picks on his fingers and starts to count) like 1,2,3, 4,5,6,7 (he stops counting while displaying the seventh and the ninth fingers) it is two!

Some participants used calculators, but they needed to know the kind of equation or expression to input into the calculator. Other participants like, Azina, lacked knowledge on how to use calculators.

Familiar objects. In solving PrA, Berina and Rehana drew on their knowledge of calendar days and months to figure out the problem. The question required that they find the number of gallons Tom would pump in on the 100th day. The participants argued that since Tom pumped 15 gallons on the 7th day, then he would pump 60 gallons (i.e., four times 15) on the 30th day), meaning for by the 90th day, he will have pumped in 180 gallons (i.e., 60 gallons times three). They then added two gallons for each day till the 10th day, which gave 200 gallons for

100 days. They needed to add one gallon to make it 201 gallons, which was the correct answer. This was a much faster strategy than increasing by two till the 100th day, which was the solution path taken by most participants.

Visuals/symbols. After reading PrB, Fareli said that he had not understood the question. He repeatedly read through the question sentence, stating that the problem was confusing. After a short while he said that PrA and PrB were the same and would yield the same answer. When I inquired how he knew they were similar questions, yet he did not understand the problem at first, Fareli said the following:

> **Fareli:** *Zinafanana kwa sababu* [they are similar because] (pointing at the picture) *zote ziko vile vile tu hii hapa ni ku*add two [the two questions are alike here you just add two] (gestures showing to carry on)
>
> **Researcher:** *Hiyo picha imekusaidia kuelewa hiyo swali na unaona hiyo picha ni muhimu hapo?*
>
> [so that picture helped you understand the question and you think the picture is important?]
>
> **Fareli:** Eeh [yes] (pointing the diagram saying, three, five, seven, nine, whispers eleven then he waves his hand to show how the pattern builds up by two while smiling)

In this excerpt, Fareli explained how the picture helped him make sense of the problem, establish a pattern, and draw connections between PrA and PrB. In a similar instance, Azina, a participant who at first did not understand the textual part of the problem, used the visual to make sense of PrB and to relate it with PrA. Explaining how she drew on the visual, Azina said:

Azina: *Niliangalia kwanza na*[I first looked and] oh, *nikahesabia miviringoso hapa ni tatu nikaelewa hapa nitano, hapa saba, hapa tisa nikajua, nikajua nikaweka mbili* [I counted the dots here three, I saw here five, here seven, here nine, then I knew, I put two]

Researcher: *Zilikusaidia kuona nini* [what did the dots help you to see]?

Azina: *Ile* formile [that formula]

Through the visual, Azina was able to connect the pattern of PrA and that of PrB, and to even relate their formulas. By formula here Azina meant the generalization, $2n + 1$, where n is the day number, which we had arrived at in PrA. Azina expressed her preference for word problems that have pictures and numbers over those that are loaded with text and no visuals or numbers.

Some participants preferred using symbols to represent mathematical operations and activities to verbalizing them. For instance, when I worked with Azina to find out the pattern for PrM, Azina symbolized her responses as follows:

Researcher: *Kutoka nne hadi tisa unafanya nini*? [from four to nine what are you doing]?

Azina: (She writes +5),

Researcher: *Kutoka tisa hadi saba unafanya nini* [from nine to seven what are you doing]?

Azina: (She writes -2),

Researcher: *Kutoka saba hadi kumi na mbili unafanya nini* [from seven to twelve what are you doing]?

Azina: (She writes +5)

Researcher: *Kutoka kumi na mbili hadi kumi unafanya nini* [from twelve to ten what are you doing]?

Azina: (She writes -2),

Researcher: *Kutoka kumi na next itakuwa ngapi* [from ten to the next number what are you doing]??

Azina: (She writes 15)

Azina was one of those participants who had no difficulties carrying out arithmetic operations. In all the steps highlighted in this excerpt, Azina did not use a calculator or count on her fingers as did others. Her biggest challenge was with comprehending the problems and responding to questions in English.

The Role of LPRs in Bi-multilinguals' Mathematical Processes

The third research question was: What role, if any, do the LPRs play in the participants' mathematical processes? To answer this question, I considered instances where participants used certain LPRs and sought for the possible reasons for their using those LPRs. I asked questions like, did the participant use the LPR to reduce the abstractness of the problem? Overcome linguistic challenges? Or to develop or communicate certain mathematics practices? I also considered the contribution of the LPRs towards the participants' mathematical processes (i.e., problem solving, communication, reasoning and proof, connection, and representation). I found that the participants used LPRs for comprehension, communication, development, and identification. Table 4.1 below summarizes the LPRs and the role they played in the participants' mathematical processes. Since the LPRs were not used in isolation, I listed the primary LPR and its accompanying LPRs.

Table 4.1

LPRs and their Role in Bi-/multilinguals' Mathematics Processes

LPRs	Role	Description
Translation Accompanied by restating, questioning “right?”, hand gestures, and code-switching	Communication Comprehension	Participants explained their understanding To comprehend unfamiliar words like write, dots, count, draw
Translanguaging in the inner speech Accompanied by counting fingers and other gestures	Identification	Participants talked to self in English or home language (Bembe, Kinyarwanda, Swahili) or in both as they did arithmetic computations or as they counted fingers.
Code-switching Accompanied by hand gestures, Swanglish	Communication	Participants used code-switching to communicate their mathematics practices such as conjecturing, justifying claims. They also used code-switching to explain their solution process
Restating/rereading Accompanied by hand gestures	Development Communication	Participants restated as they developed mathematical precision in language use They also restated to clarify meaning
Invention of terms Accompanied by gestures and code-switching	Communication	Participants invented terms while trying to communicate mathematics practices such as conjecturing, observing patterns, and as they expressed regularity in repeated reasoning
Hand and body gestures	Communication	Participants used gestures to supplement or substitute their verbal language
Familiar words/objects Accompanied by code-switching, translation, restating	Comprehension	Participants used familiar words or cues from the text in order to make sense of the problems.
Cognates Accompanied by hand gestures	Comprehension	Participants inferred meanings of unfamiliar words

	Identification	from cognates in a more familiar language Participants also used cognates as a means of identification

In the following sections, I expound on these uses. I include examples of instances where the role was displayed.

Comprehension

The participants used the LPR of comprehension to unpack the meaning of the problems both linguistically and mathematically. The participants drew on their LPRs when they encountered linguistic difficulties in terms of unfamiliar words, abstract representation. Some of the language practices used included translation, code-switching, and translanguaging in the inner speech. The linguistic resources used for comprehension included familiar words or objects and cognates.

Some participants, like Azina, totally depended on their repertoire of language practices in order to make sense of the problems. Had she not drawn on various interrelated language practices, Azina would not have succeeded in solving any of the problems in this study. She did not initially make sense of the problems because of challenges with English. When I asked her what PrM was about, Azina said that she did not understand the question. She asked for translation of words like write and pattern, which she claimed were unfamiliar to her. The conversation went as follows:

Azina: *Mimi sijaielewa* [I do not understand the question]

Researcher: So *inakuambia*, [it tells you] “write the next”, write *ni kufanya nini*? [what does write mean?]

Azina: *sijui vizuri* [I do not know quite well]

Researcher: *Andika* [write]

Azina: *kuandika*? ooh [to write]

Researcher: The next, next *inammaanish nini*? [next means?]

Azina: Next, *inayokuja* [the one that follows]

In PrA, Azina also asked Solana to translate the question part of the problem. She also referred to familiar objects like "kadum" to make sense of the word gallon in PrA. After reading PrB, Azina again said she did not understand the problem well and so she asked that words like dots, count, and draw be translated to Swahili, the language she preferred discussing mathematics in.

Azina: *Sijaelewa hii* [I have not understood these] dots

Solana: Dots *ni hizi miviringo* [are these circles], *ndizo* [they are the] dots (pointing at the dots)

Azina: *Halafu hii* count [then this]

Solana: Count *ni kuhesabu* (translates count)

Azina: Ooh, *haafu hii*, [and this] draw

Solana: Draw *ni* [is]

Azina: *Hii ya hapa* [this one here] (pointing at the word word)

Solana: *Draw ni kuchora* (translates draw)

The words that Azina grappled with appear basic given that she was a 10th grader: draw, write, count, dots, pattern. As Azina mentioned, had the words been presented in a language she understood, she would not have had problems comprehending the problem. The language of the problem was her main hindrance to comprehending the problem: "… *ingekuwa ni ile lugha najua* *vizuri ningeelewa na ningefanya* [… had the problem been presented in a language I understand very well, I could make sense of it and I could solve it.]"

Communication

The participants drew on various LPRs to communicate their understanding of the problem, their solution processes, as well as their mathematics practices. When I asked Berina to explain her understanding of PrA she fluidly responded:

> *Siku ya kwanza*, first day, uhmm, *aliweka* gallon *tatu*, right? *muile* container, *na siku ya pili*, he pumped in five gallons, uhmm, so, *na siku ya tatu*, uhmm, *aliweka saba*, right? so, uhmm, so, so I think, we have to add, right?

The excerpt shows how Berina used code-switching mixed with translation to communicate her understanding of the problem. Hand and body gestures were also a common resource for communication in this study. In many cases, gestures were used to supplement other LPRs. For instance, participants used hand gestures when explaining their understanding of "pattern". They waved hands or moved them in a manner to show that something was continuing or repeating.

Azina used nonverbal communication when she only wrote down her responses instead of using spoken language:

> **Researcher:** *Kutoka saba hadi kumi na mbili unafanya nini* [from seven to 12 what are you doing]?
>
> **Azina:** (She writes +5)
>
> **Researcher:** *Kutoka kumi na mbili hadi kumi unafanya nini* [from 12 to 10 what are you doing]?
>
> **Azina:** (She writes -2),

Researcher: *Kutoka kumi na next itakuwa ngapi* [from 10 to the next number what are you doing]??

Azina: (She writes 15)

By writing +5, -2 and so on, Azina communicates her arithmetic abilities. By writing 15 as the next number in the pattern, Azina showed her understanding of the rule that was being used in the pattern and her ability to apply the rule to find the next numbers in the pattern. A similar instance occurred in PrA when I asked Fareli to explain how he understood Diodo's conjecture that Tom increased the number of gallons by two each day. Fareli responded by quickly writing: "5-3= 2, 7-5=2". Through this symbolization, Fareli was communicating that the difference in the number of gallons pumped between any two consecutive days was two.

It is not easy to settle on one reason for why these participants used nonverbal responses in the instances highlighted, but difficulties with explaining ideas in English could be one of those reasons. Fareli had a desire to explain in English but he was not able to do so. He thus drew on other modes of communication. For example, earlier in PrM, I asked Fareli to explain why he thought 8 and 11 would be the next two numbers in the pattern. The excerpt below shows the difficulty Fareli had:

Fareli: Because most of these are like there is kind like they write they didn't make in order, they make like. They put everything like, oh I don't know… (he sighs like fatigued, or facing difficulties, closes his eyes as if experiencing pain) (silence…then waves hands to show continuity)

Fareli attempted to explain by incorporating the word "like" as a place holder while he organized his thoughts, but at some point he gave up. He then used hand gestures to show how a pattern behaves. Fareli's body language communicated his frustration and fatigue with explaining in English. In another episode, Fareli supplemented his limitations in English with nonverbal language as follows:

> **Fareli**: Plus five, minus two, plus five, minus two, the next number will be like, like this (he writes 15 and looks up at me smiling)
>
> **Researcher**: And the next one?
>
> **Fareli**: Will be like, (talks to self as he writes 13 then looks up again with a smile)

This time round Fareli decided to symbolize his mathematical ideas. Indeed 15 and 13 were the next two numbers in the pattern that was provided in PrM. Even though Fareli did not say the names of these numerals, his representation was correct. It showed that he understood the rule in the pattern and that he was able to apply it correctly.

Participants also used LPRs to communicate their conjectures, justification of conjectures, and to express regularity in repeated reasoning. The participants commonly drew upon code-switching and invention of terms to communicate their mathematics practices. Participants used terms such "you skipped two", "*tunaruka mbili* [we skip two]", "going up by two", "you minus two and plus two", "*zinapandana kwa mbili* [they are rising by two]", "pattern *iko* [is] up *kwa* [by] two", and so on. When solving PrA, Solana made a claim that "they are going by two-two-two" but "will not get to 200". When I sought for her to clarify what she meant by not getting to 200, Solana made her justification as follows:

> **Solana**:… 'Cause last time we said 50, then I was going to keep going and now we stopped here (showing on her worksheet where they got to day 50) and I was like I think it is 201 because if you keep going and we got 101, then for 100 it will be 201 and because we skipped two numbers so I think it's 201.

Solana invented “mathematical” terms such as “keep going”, “stopped here”, “skipped two numbers” and so on in an attempt to justify her conjecture. These phrases are everyday words, but their usage in this excerpt represents important mathematical processes. For instance, “keep going” represents continued addition of two because “we skipped two numbers”. By saying we “stopped here”, Solana was referring to where they stopped adding by two-two-two. They stopped at the 50th day. They noted that 101 gallons corresponded to the 50th day. Meaning, for 100th day would correspond to 201, not 200. Which is what she meant by saying “we will not get to 200”.

Development

The participants used LPRs to develop certain mathematics practices such as attending to precision, abstracting, conjecturing, and arguing. There are some instances where participants restated in order to clarify meaning and be more precise. For instance, when I asked Zawiri what she would do with four oranges to make them nine without cutting them, she replied: “then you get more, add more”. Her response has two parts. In the first part she says, “get more”, but in the second part she replaced “get” with “add”, which sounds more mathematical. In a similar instance, when I asked Neha what she could do with four to make it nine, she responded: “you put together, like you use plus.” Again here, Neha starts with everyday language “put together”, then she advances to a mathematical symbolization for putting together, “plus”. These were the only instances where participants seemed to exercise caution with the use of language in mathematical communication. All other instances where participants seemed to abstract or develop conjectures and arguments were marked with the use of invented terms, which were mostly non-mathematical.

Identification

Participants draw on their past experiences to comprehend and represent mathematical ideas. However, some past experiences are better accessed through certain LPRs. For instance, by translanguaging in the inner speech, the participants were able to revert to a “space” where they could count in the language and manner in which they first learned to count. Moreover, by talking to self while doing arithmetic computations, some participants were actually able to access their past experiences with arithmetic computations. For instance, Zawiri talked to self while she counted on her fingers in an attempt to figure out the missing addends in PrM. In most cases, I could not hear what Zawiri was saying as her voice was too low and she spoke in a language I could not understand. Cognates were not only useful in comprehending the problems, but they also offered the participants an opportunity to identify with a certain meaningful social group. For instance, Berina read “container” and “continues” in PrA as if they were “conteneur” and “continuer”, which are the French cognates of “container” and “continues”.

Chapter Summary

In this chapter, I provided the main findings of this study as per the research questions. The first research question sought to know the language practices and linguistic resources the participants used during MWPS as well as the challenges they faced. The study showed that the participants used language practices such as translation, translanguaging in inner speech, code-switching, restating/rereading, and invention of terms and linguistic resources such as hand and body gestures, familiar words/objects/signs, and cognates. The findings showed that the participants used their LPRs in an integrated manner, not in isolation. The participants faced linguistic challenges including unfamiliar words and complex syntax, unfamiliar language, abstract representation and passive voice.

The second research question sought for the mathematics practices and mathematical resources the participants used during MWPS and the mathematical challenges they faced. The study showed that the participants engaged in mathematics practices of sense making, looking for/discovering patterns, describing/using structure, looking for/expressing regularity in repeated reasoning, constructing arguments/making conjectures. The mathematics practices that the participants engaged in were mostly informal since they were embedded in their LPRs. The participants drew on mathematical resources such as prior knowledge of mathematical facts/algorithms, counting fingers/calculator, familiar objects such as calendar days, and visuals/symbols. The participants faced challenges with mathematics symbolization and visuals, technical math lexicons, abstraction, generalization, checking answers/work, implicit data/abstract representation, and missing added.

The third research question sought for the role of the LPRs in the participants' mathematical processes. The findings showed that the participants drew on their LPRs to comprehend mathematics and to communicate their mathematical understanding. They also drew on their LPRs to develop certain mathematics practices. The participants also used their LPRs for identification in order to support their problem solving.

One relationship or connection between the LPRs and the mathematics practices is that the participants drew on their LPRs to develop, manifest, and to communicate their mathematics practices. As such, the participants' mathematics practices were mostly informal. The participants described mathematical situations in ways that made sense to them. Ways that were not precisely mathematical.

Chapter 5: Discussion

The aim of this study was to investigate the practices, resources, and challenges in MWPS among Swahili-speaking bi-/multilingual high school students from the African refugee community. Specifically, I sought to know the language practices and linguistic resources the participants used during MWPS, and linguistic challenges they faced. I also sought to know the mathematics practices and mathematical resources they used during MWPS, as well as the mathematical challenges they faced in the process. Lastly, I sought to understand the role of LPRs in the participants' mathematical processes.

To carry out this study, I used a Vygotskian perspective of mathematics practices, which asserts that mathematics practices are embedded in the discourse practices that students engage in as they jointly work on a mathematics task. Moreover, these discourse practices consist of both everyday and academic practices. I framed everyday practices as consisting of bi-/multilingual's translanguaging practices and the academic practices as consisting of the multi-/semiotic language of mathematics. I collected data for this study through surveys and task-based interviews. The tasks were three NAEP-released algebra problems that the participants solved within a space where translanguaging was allowed.

In this chapter, I discuss the findings from this study and link them to existing knowledge. To do this, I first present a summary of the findings of this study and provide some concluding remarks for each set of findings. I then summarize the conclusions into three topics, each of which I discuss and link to existing knowledge on that subject. I then suggest some implications for the study and share the limitations of the study and state some recommendations for future research. I end the chapter by summarizing it.

Summary of the Findings

In this section, I present a summary of the findings of this study and the respective conclusions as per the research questions. I start with the language practices and linguistic resources the participants used, as well as the challenges they faced during MWPS. I then move to the mathematics practices and mathematical resources the participants used, as well as the challenges they faced during MWPS. Lastly, I discuss the role of the LPRs in the participants' mathematical processes.

Bi-multilinguals' Language Practices, Resources, and Challenges

The participants used language practices such as translation, code-switching, translanguaging in the inner speech, restating/revoicing, and invented terms. In translating, the participants tried to unpack the meaning of the problems word by word, phrase by phrase. Code-switching entailed the participants switching codes within one speaker turn or within one conversational episode and using words or phrases from one language in an utterance in another language within sentences. The participants read some words, phrases, sentences more than once in order to comprehend the problems. They also re-voiced and invented words and phrases that made sense to them while explaining ideas. There were instances where some participants engaged in inner speech in a language of their choice. At these instances, the participants either talked to themselves quietly or loudly and in their preferred language.

The participants used linguistic resources such as hand and body gestures, familiar words/objects/signs, and cognates. Hand and body gestures were the most commonly used form of gesture. Oftentimes, the participants used gestures to show their understanding of technical words like "pattern". There were instances where gestures were used together with other signs and symbols. Participants also relied on familiar words to make sense of unfamiliar ones. They also drew on their knowledge of certain objects to make sense of words and names of other objects in the problem. In other cases, the participants used words from their familiar languages that had their cognates in English to make sense of other words in the problems. For instance, the participants who spoke French were easily able to comprehend the English words "container" and "continue" because these words have their cognates "conteneur" and "continuer", respectively, in French. When the participants pronounced the English version of these words they pronounced them as if they were French words. It was evident that the participants appreciated the fact that the English words sounded like words they were familiar with in a familiar language and so they were easily able to infer their meaning from the French cognates.

The linguistic challenges that the participants faced included unfamiliar words and complex syntax, unfamiliar language, abstract representation and passive voice. Some words like draw, write, dots, container, and so on, that might appear rather basic for a ninth- or tenth-grader, were unfamiliar to some of the participants. Most of the participants were at the emerging stage of English speaking and writing according to the language assessment frameworks by Heritage et al. (2015) and Bailey and Heritage (2018). Some participants, like Azina, could not speak or write in English and so they preferred to talk about mathematics in Swahili. There was a challenge with abstract representation in some problems that left participants asking questions they did not have immediate answers for. For instance, some participants wondered what the size and shape of the container in PrA were, as well as why Tom had to pump water into the container. Much as these details appear unrelated to mathematics, they are helpful in establishing the context of the problem, which is helpful in problem solving (Oviedo, 2005). PrM was passive in that it lacked the context that students could use to build an understanding. On the other hand, researchers have shown that context can be a source of difficulty for students especially if it is unfamiliar (Ambrose & Molina, 2013; Martiniello, 2008; Oviedo, 2005). Njagi (2015) found that students' performance improved when MWPs were stripped of their contexts and presented as mathematical expressions.

Conclusion. From the findings on the bi-/multilinguals' language practices, resources, and challenges, I made the following conclusions: (a) the participants drew on various LPRs for sense making and mathematical communication, (b) participants used LPRs in an integrated manner, not in isolation, and (c) the participants needed support to use their LPRs.

Bi-multilinguals' Mathematics Practices, Resources, and Challenges

Mathematics practices were the ways of reasoning, arguing, and symbolizing that the participants engaged in during problem solving. I sought for mathematics practices in both the academic and everyday practices that participants engaged in during MWPS. The participants engaged in mathematics practices such as sense making, looking for/discovering patterns, describing/using structure, looking for/expressing regularity in repeated reasoning, constructing arguments/making conjectures. Sense making was the first step in problem solving for every participant. Sense making was seen in the way the participants sought to unpack the meaning of the problems, and how they sought for entry points and considered what is known or unknown about the task in order to arrive at the solution.

The participants also made attempts to look for/discover patterns although sometimes they looked for the patterns from incorrect sources. For instance, arranging the numbers in PrM in an order such as that of natural numbers so as to find a pattern was unnecessary since the arrangement already formed a pattern. This indicated that the participants had a limited mathematical understanding of the term "pattern". Participants also spent time trying to discover patterns in the other problems. The participants also made attempts to describe/use structure.

This practice was evident when they tried to relate PrA with PrB. The participants were able to discern that these two problems were basically same except that one had a visual and the other one did not. In describing the structure, the participants drew on various LPRs. The participants also noticed if calculations were repeated and they and made attempts to find general methods and shortcuts. The participants were quicker to express their repeated reasoning regarding the number of gallons in PrA that increased by two than for PrM where the pattern was increasing and decreasing interchangeably. Again, the participants drew on a wide range of LPRs as they expressed their repeated reasoning. The participants constructed arguments and made conjectures in the ways that made sense to them. They mostly expressed their conjectures using everyday language practices. For instance, they often used phrases like "skipping by two", "*zinapandana na mbili*" [they are going up by two], and so on, to express the rule of the pattern in PrA.

The participants drew on a number of mathematical resources including prior knowledge of mathematical facts/algorithms, counting fingers/calculator, familiar objects such as the number of days in a week and month, and visuals/symbols. Basic counting techniques and arithmetic skills were important in solving the tasks used in this study. One participant, Kwisha, used his prior knowledge of proportions to solve PrA, but this was incorrect since the relationship between the number of gallons pumped in and the number of days was linear, not proportional. Kwisha's rush to set up proportions and make substitutions without careful thinking confirms the "compulsion to calculate" that many students experience during problem solving (Ambrose & Molina, 2013; Barake, et al., 2015).

Almost all of the participants counted on their fingers at some point although some used the calculators on their phones. The participants also drew on familiar objects to make sense of the problems and to devise solution strategies. While relating problems with familiar objects helped with comprehension and increased the concreteness of the problem and its solution process, the participants needed to apply their knowledge of the objects in the mathematical sense. For instance, while grouping days into weeks and months was a quicker way to arrive at the solution for PrA, Berina and Rehana needed to know that mathematically, they would need to add one to their final answer. This means that some LPRs may not be used as a straightforward resource in problem solving (Halai, 2007).

The participants had mixed reactions regarding the role of the visual in PrB. While some participants saw the visual as enhancing their comprehension of the problem, others saw it as hindering it. Participants like Azina, who were not comfortable using English, found the visual helpful as they did not need to fully understand the textual part of the problem. In another instance, Azina indicated that she preferred symbolizing her responses over verbalizing them. Although Azina had challenges with comprehending the problems and communicating her understanding verbally in English, she accurately symbolized answers to arithmetic problems that posed a challenge to other participants who were higher in English progression. Azina did not need to count on her fingers or to even use a calculator as did other participants. As soon as she made sense of the problems and noted the pattern, like for the case of PrM, she was able to apply the rule correctly in finding the missing numbers.

The mathematical challenges that the participants had included mathematics symbolization and visuals, technical math lexicons, abstraction, generalization, checking answers/work, implicit data/abstract representation, and missing addend. Some participants had a challenge with the meaning of basic symbols and how to represent them. For instance, Neha was not sure how the subtraction sign is written. In another instance, Berina had to confirm how to write the multiplication sign. Some participants did not understand the meaning of the symbol

100th and so their Swahili translation for this symbol was erroneous. For instance, they said "*siku mia*" which means "hundred days" rather than saying "hundredth day". It, therefore, did not matter whether they translated this symbol to Swahili or not. As long as they did not understand its symbolic meaning, their translation would still be erroneous. The mathematical meaning of the word "pattern" in PrM was challenging to most participants. Participants reduced the meaning of "pattern" to numbers increasing by one such as the case for natural numbers. After arriving at the rule +5 and -2 for PrM, some participants still could not apply it to find the next two numbers in the sequence. PrB had a visual, but contrary to many participants who thought the visual enhanced their comprehension of the problem, Rehana found the way the visual was labeled confusing because each figure was labeled with numbers like 1, 2, 3, and so on, while the text said figure 1, figure 2, figure 3, and so on.

Another mathematical challenge was with abstraction. Abstraction entails the participants making sense of the relationships between quantities and representing them symbolically. Eleven out of the twelve participants had a challenge relating the numbers in PrM in order to find a rule for finding the next two numbers in the pattern. Other participants claimed that they did not understand PrM at all. Some participants drew on their LPRs in order to establish relationships between the quantities, but they still needed to symbolize these relationships. For instance, Solana said that two numbers were being skipped, but she still needed to specify the nature of skipping and even symbolize it. When I asked the participants to find the number of gallons Tom would pump on the nth day most of them could not imagine how one can find the number of gallons of water pumped in for an abstract number of days, n. Participants often asked, "Can n be five?" and I often replied, "n can be any number". They had a challenge with thinking abstractedly. Participants' challenge with abstraction led to further challenges with generalization. Participants generalized too quickly, trivially, or about the wrong properties (Driscoll, 1999).

Checking work is an important step in problem solving (Polya, 1957). Unfortunately, the participants were either not accustomed to checking their work or they did not know if it was important to do so. Some participants, like Kwisha, seemed overconfident with their answers to the extent that they did not see the need to check their work even when I suggested that they do so.

The implicit and abstract representation of some aspects of the problems were also challenging to the participants. Most participants did not understand the meaning of the implicit data in the mathematics problems (Barake et al., 2015). Implicit data is the information the problem solver needs to figure out as s/he solves the problem. Most participants expected every detail of the problem to be provided. Figuring out the missing addend was also a challenge to many participants. Although a number of the participants counted on their fingers and others used calculators, they first needed an understanding of what to count on or what to input into the calculator. In other words, they needed to have a mathematical understanding of what they were counting or calculating.

Conclusion. Some of the conclusions I draw from findings on the mathematics practices, resources, and challenges are: (a) some mathematical challenges are caused by certain linguistic or cultural challenges, (b) the participants drew heavily on their LPRs to manifest and communicate their mathematics practices, and (c) while participants drew on their LPRs during MWPS, they still needed support to use those LPRs in the mathematical sense.

The Role of LPRs in Bi-multilinguals' Mathematical Processes

LPRs played a number of roles in the participants' mathematical processes. I grouped these roles into four main categories: comprehension, communication, development, and identification. The participants used LPRs to make both linguistic and mathematical sense of the problems. For some students like Azina, it would have been impossible for them to solve any of the problems in this study if they did not draw on their linguistic repertoire. Participants also drew on their LPRs in order to manifest and communicate their understanding of the problems, their solution process, and their mathematics practices. Some participants, like Azina and Fareli, seemed to prefer gesturing and symbolizing their responses over expressing them verbally. Azina confirmed her difficulties with conversing in English. Fareli also expressed frustration with having to explain ideas in English. Where he faced frustrations, Fareli often got fatigued and gave up explaining in English and either turned to using a familiar language or stopped talking altogether. Gesturing and symbolization were the tools these participants relied on primarily to reveal their mathematical skills and understanding. As Azina noted in her comment, as long as she understood the problem or question, she was sure she could handle the problem as she did not have challenges with basic arithmetic skills and using rules to complete patterns.

Participants also used LPRs to develop mathematics practices such as attending to precision, abstracting, conjecturing, and arguing. There were two instances where participants restated to express more precision with their use of the language of mathematics. In one instance, Zawiri said, "you get more, you add more". In another instance, Neha said, "you get more, you plus". These were the only instances where participants attempted to express precision. There were many other instances where participants used LPRs to develop conjectures and arguments, but they needed support in order to make these conjectures and arguments more mathematically precise. As Bruner (1976) noted, the mathematical use of natural words should be precise, and always give rise to “an almost totally nonredundant and relatively unambiguous language” (p. 209).

Instances when participants translanguaged in inner speech or used cognates seemed to afford them an opportunity to revert to space where they could draw on the practices and experiences that shape who they are, and to identify with certain meaningful social groups (Gee, 1999). This form of identification is important since by it the participants were able to access their arithmetic skills and other mathematical experiences in the manner and the language in which they first learned them.

Conclusion. From the findings on the role of LPRs in the participants’ mathematical processes I conclude the following: (a) it seems impossible to separate the participants’ everyday practices from mathematics practices, (b) without access to their LPRs, most of these participants would have been denied a chance to manifest and communicate their rich mathematical thinking, and (c) the participants needed support to fully develop their mathematics practices.

Summary of the Conclusions

To summarize the sets of conclusions for findings in this study, I note that the participants drew on various LPRs to make sense and to manifest and communicate their mathematics practices. However, they used their LPRs in an integrated manner, not in isolation. Without access to their LPRs, most of the participants would not have been able to manifest and communicate their rich mathematical thinking. Also, the participants’ everyday practices seem inseparable from their mathematics practices. Some mathematical challenges faced by the participants emanated from certain linguistic and cultural practices. Lastly, the participants needed to be supported in order to use these LPRs and to use them in a mathematical sense in order to fully develop precise mathematics practices. From these conclusions, three main areas worth consideration emerge: (a) mathematical communication among bi-/multilingual students, (b) What counts as mathematics practices, and (c) supporting bi-/multilingual students' mathematical problem solving. In the following, I discuss each of these areas as they relate to this study and to other studies in the literature.

Mathematical Communication Among Bi-/multilingual Students

As I noted in Chapter 1, current reforms view mathematical communication as the central indicator of the learning of mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, for communication to occur in a classroom, students need to be given an opportunity to participate in what is going on there. The Vygotskian perspective of mathematics practices upon which this study was framed asserted that students develop mathematics practices when they are jointly engaged in solving a mathematics task. Moreover, the mathematics practices are embedded in the students' discourse practices which consist of both academic and everyday practices. A Vygotskian perspective, therefore, posits participation in mathematical communication as the path towards development of mathematics practices. Using a sociocultural approach, Civil and Planas (2004) also emphasized the role of participation in mathematics saying:

> The acquisition of concepts and skills is not enough in the process of becoming a mathematical learner. There also needs to be an active participation in the reconstruction of a specific kind of discourse. (p. 8)

This emphasis on participation in mathematical discourse by all students brings out the question of which language is appropriate for use in these discussions. In this study, participants were allowed to participate in solving algebra MWPs in pairs. In solving PrM, the participants were to use English only while for PrA and PrB, they were allowed to use their preferred languages. The study showed that most participants had challenges comprehending and solving these problems as well as communicating their mathematical ideas in English. Allowing the participants to use their everyday practices, which includes their everyday language (Orado, 2014), made it easier for them to engage in mathematical discourse with their partners. The participants drew on their LPRs to make sense of the problems and to show and communicate their mathematical thinking. Azina was a participant who barely understood the problems that were presented, yet when the problems were translated she understood them and was able to engage in a discussion where she ended up revealing arithmetic skills that many participants who were at a higher level of English progression did not seem to have. This finding confirms Civil's (2012) assertion that by limiting bi-/multilingual students' communication to English only we miss much of the richness of their mathematical thinking. Indeed, limiting Azina's problem solving to using only English could have portrayed her as not having any mathematical abilities.

The findings of this study agree with many other studies that view bi-/multilingualism as a resource for meaning-making and not a deficit or a sign of deficiency (Dominguez, 2011; Garcia, 2009; Moll, 2010; Moschkovich, 1999, 2007; Vomvoridi-Ivanovic, 2011; Yosso, 2005). Solano-Flores et al. (2013) define a meaning-making system as "a set of interpretive resources students use in combination to integrate information represented through multiple semiotic features according to the context of the discipline" (p. 148). The participants in this study used their LPRs in an integrated manner, not in isolation, confirming the use of translanguaging in mathematics (Garcia & Wei, 2014). The use of multiple interrelated discursive practices that cannot be easily assigned to one or another traditional definition of language, but that make up the speaker's complete language repertoire, has been termed as translanguaging (Garcial & Wei, 2014, p. 22). The participants in this study thus engaged in translanguaging practices (Garcia, 2009; Garcia & Wei, 2014) during MWPS. The knowledge of how translanguaging happens in mathematics and the role it plays during MWPS is new and adds to other studies that have investigated translanguging in literacy and biliteracy, as well as among Latino mathematics teachers. Moreover, the ELL population studied in this case was from the Swahili speaking bi-/multilingual high school students in the U.S, who had not been studied before.

What Counts as Mathematics Practices

In Chapter 1, we saw that there exists a debate on how mathematics practices should be defined. Research shows that mathematics practices have been dichotomized as either academic or everyday, and that only academic mathematics practices should inform everyday mathematics practices and not the other way around. Moschkovich (2013) termed these dichotomies insufficient ways to describe mathematics practices and called for studies that investigate mathematics practices in different contexts using more rigorous frameworks.

The findings of this study showed that the bi-/multilingual students drew on their LPRs in an integrated manner, to develop, manifest, and to communicate their mathematics practices. For instance, participants like Solana made attempts to conjecture and to justify her conjectures although she used her own inventions that were not precisely mathematical (see more details in Chapter 4). This finding aligns with Moschkovich's (2008) notion of mathematics practices as mathematics accounts that assume the social, cultural, and discursive nature of mathematical activity rather than pure cognitive accounts of mathematics. In other studies, Moschkovich (2002, 2003) noted the role of language, gestures, symbols and other resources in doing mathematics. Using Gee's (1999) concept of Discourse, Moschkovich (2013) posited how difficult it is to demarcate clear boundaries between non-mathematics practices and mathematical ones. Thus, she asserted the need for ‘multiple’ view of mathematical discourse and hence mathematics practices because mathematics practices are embedded in mathematical discourse.

The findings of this study show that bi-/multilingual students engage in mathematics practices in the manner that makes sense to them regardless of whether they are precisely mathematical or not. This means someone looking for precise mathematics statements in bi-/multilingual students’ discourse practices may miss on their developing mathematics practices. However, Moschkovich (2008) hinted that more informal, everyday discourse has a role in how students make sense of mathematics. Schoenfeld added that students’ mathematical thinking should be understood in terms of the “mathematical communities in which students live and the practices that underlie those communities” (p. 363). This means that an investigation of the mathematics practices that students engage in should consider the informal aspects of their discourse as well.

After carrying out an analysis on how mathematicians talk, Barwell (2013) noted that everyday language does not disappear in mathematics, rather “it is used in new, more mathematical ways” (p. 221). In brief, mathematics practices and everyday practices are not mutually exclusive, rather, everyday practices find new meaning in mathematics; they complement each other rather than substitute for each other. This finding responds to Moschkovich’s (2013) call for researchers to make visual how bi-/multilingual students communicate mathematically. It reveals the informal and imprecise language used by bi-/multilingual students from the African Swahili-speaking community and the role such language played in the students’ development of more formal mathematical communication.

Supporting Bi-/multilingual Students' Mathematical Problem Solving

The findings of this study showed that the bi-/multilingual students drew on their LPRs to develop, manifest, and communicate their mathematics practices which were in many cases informal. The study found that these students needed support to use the LPRs and to use the LPRs in a mathematical sense, and to fully develop their mathematical processes. Bi-/multilinguals' participation in translanguaging is responsive the environment (Gee,1996). These students thus need a safe translanguaging space; a learning environment that valorizes their LPRs and sees them as resources for sense making. Civil (2012) asserted the need to prioritize bi-/multilingual students' dominant languages in the classroom. Indeed, language should be viewed as a tool for enhancing human communication (Shaltz & Wilkinson, 2010; Vygotsky, 1978).

Current research opposes a deficit model that focuses on bi-/multilinguals deficiency in a specific language, without regard to their rich linguistic and cultural resources. The case of Azina, who barely understood problems written in English, but drew on certain LPRs to comprehend the problems and to communicate her mathematical skills, confirms that a student's deficiency in a language does not mean that s/he is deficient in mathematics. For Azina, the problem was access. As long as she could access the problems, she was able to apply her mathematical skills to solve the problems. Moschkovich (2007) also urgued that bi-/multilinguals' use of LPRs is not a sign of deficiency. After studying the use of code-switching among Latino students, Moschkovich (2007) concluded that people's code-switching cannot be used to "reach conclusions about their language proficiency, ability to recall a word, or knowledge of a particular technical term" (p.18). We, therefore, cannot judge bi-/multilinguals' mathematical problem solving on the basis of their lack of comprehension of the language of the problem; these students have LPRs they can draw upon to be successful in MWPS.

Bi-/multilinguals also need support to use their LPRs in a mathematical sense. While some participants, like Solana, invented terms such as "skipping two numbers" to express a rule for the pattern in PrA, they still needed support to specify the nature of "skipping" and perhaps symbolize it. Students like Berina and Rehana translated the symbol 100^{th} day as "*siku mia*" [100 days], which is not in the mathematical sense of the symbol 100^{th} day. Halai (2007) noted a similar incidence with her Pakistan students where they were unable to translate mathematics problems according to the mathematical discourse. In other words, the linguistic structures in the natural language of a mathematics text were in conflict with students' thinking in Urdu. Berina and Rehana misunderstood the mathematical symbol "100^{th}" and so in this case translation was not going to be a straightforward resource. This finding confirms that movement between languages in the course of learning mathematics cannot be regarded as a straightforward resource (Halai, 2007), meaning that bi-/multilinguals need support in order to tap the full benefit of the LPRs they use.

The findings of this study also showed that bi-/multilinguals need support to fully develop their mathematical processes. Although the participants drew on their LPRs for a number of reasons, a number of instances indicated that they needed a "boost" in order to proceed with problem solving. For instance, Berina and Rehana drew on their knowledge of days and months to figure out a short-cut for solving PrA, but they needed to add one to their final answer. Kwisha drew on his knowledge of proportions, but the relation between quantities in PrA was linear, not proportional. He thus needed support in checking his solution process. Berina and Rehana drew on the visual in PrB, but they found the way it was labeled confusing. They needed support with comprehending the features of the visual. Moreover, while many participants waved their hands periodically to show their understanding of the term "pattern", they still needed to relate this understanding to the context of the problem and to even symbolize it before arriving at the answer. The fact that bi-/multilinguals using their LPRs still needed support to succeed in problem solving does not mean that their use of the LPRs is useless. According to Sigley and Wilkinson (2015), bi-/multilingual students should be encouraged to communicate their understandings by employing all of their semiotic resources - linguistic, symbolic, and gestural" (p. 85). After analyzing Ariel's problem solving, Sigley and Wilkinson (2015) found that Ariel first employed his entire semiotic resources to communicate his mathematical thinking before he could attain a command of more a precise mathematical language. Everyday practices are thus good starting points from which to support students gain proficiency in using more precise content-related language (Herbel-Eisenmann, 2002; Pimm, 1987). This finding contributes to new knowledge by providing an empirical evidence of the connection between students' use of LPRs and the kind of support they need while they use these LPRs. Three forms of support emerge; support to use the LPRs, support to use the LPRs in a mathematical sense, and support to develop more formal mathematical communication.

While solving PrA, Exmon conjectured that 100 days would be multiplied by two because the number of gallons increased by two daily. Unfortunately, this was only a part of the conjecture. Exmon still needed to add one to this product. At this point, Exmon needed support to develop his conjecture fully. So, I asked him to test his conjecture with a fewer number of days whose number of gallons could be easily found. Soon he realized that his conjecture gave gallons that were one less than the actual number. When Kwisha used proportions in solving PrA, I suggested that he check his answer by setting up another proportion. He then got a different answer and realized that proportions would not work in this case. It became clear to me that these students did not know that checking their own work is a valuable step in problem solving.

There were many other instances where the participants did not pay attention to the details of the problems, including the implicit data, and they simply started to calculate. This is the kind of "compulsion to calculate" that Barake et al. (2015) and Ambrose and Molina (2013) noted among the students they studied. Driscoll (1999) also noted how students' perception of problem solving as a series of calculations makes them shift their thought processes from algebraic thinking to arithmetic thinking when solving algebra word problems. These illustrations show that students, especially bi-/multilinguals, need support to develop problem-solving skills since word problems form a special genre of problems in mathematics (Ambrose & Molina, 2013) and they present additional challenges to these students (Barbu & Beal, 2010; Cuevas, 1984; Jan & Rodrigues, 2012; Kenney & de Oliveira, in press; Martiniello, 2008, 2009; Schleppegrell 2007).

Implications for the Study

This study has implications for policy and practice that touch on two major areas: (a) validity of formative and summative assessments, and (b) preparing mathematics teachers to teach bi-/multilingual students.

Validity of Formative and Summative Assessments

Research has shown that students whose first language is not English face certain linguistic and cultural biases in standardized examinations such as the NAEP (Abedi & Lord, 2001; Cuevas, 1984; Kieffer, Lesaux, Rivera, & Francis, 2009; Martiniello, 2009). These students, who are bi-/multilinguals, not only underachieve in the NAEP exams, but they are also underepresented in the nation's students' report card (Pellegrino et al., 1999). Messick (1989) termed linguistic complexity and unfamiliar contexts in exams irrelevant to the construct a mathematics test is meant to measure. Linguistic complexity makes a problem appear harder than it actually is. Barbu and Beal (2010) noted how linguistic complexity hinders students' success with MWPS:

> … if students must devote significant cognitive resources to text comprehension, fewer working memory resources will be available for mathematical problem solving, including identifying the appropriate math operation, forming the problem representation, performing computations and checking progress towards the solution. (p. 4)

The findings of this study indeed confirmed that students solving mathematics problems written in a second or third language face certain linguistics challenges. As Cheuk, Daro, and Daro (2018) argued, test item developers should guard against irrelevant text complexity that can hinder students' reading and comprehension. Although bi-/multilinguals face linguistic challenges in mathematics, my study also showed that these students draw on their LPRs to communicate their mathematical thinking in ways that may not be precisely mathematical. As Sigley and Wilkinson (2015) noted, any valid formative and summative assessment would have to allow students "to demonstrate what they know about mathematics and not solely what they know about the mathematics register" (p. 86). Moreover, teachers are to look beyond such things as spellings or spoken accuracies and focus on the mathematics knowledge a student displays during problem solving (Bailey, Maher & Wilkinson, 2018). This does not mean that content being assessed should be watered down, but that bi-/multilinguals' everyday practices and the resources they bring to bear in problem solving should be valorized. Addressing the issue of validity of assessments, Civil (2012) cautioned that "whatever problems we use, we need to understand that the answers students give reflect their experiences" (p. 92).

Garcia and Wei (2014) presented a possibility for translanguaged-mode assessments where questions are posed in many languages from which students would choose and would be free to reply in whatever semiotic systems they think would display their understanding. According to Garcia and Wei, such assessments would allow students to show what they know using their entire linguistic repertoire. But unfortunately, translanguaging in assessment has neither been "accepted by policy makers who commission the development of tests nor by many teachers who have been taught to assess knowledge in accordance with artificial bounds of social norms and language" (p. 135). When we only consider the practices of the monolingual elite as the norm, we render bi-/multilinguals' languaging illegitimate and deficient (Garcia & Wei, 2014; Moschkovich, 2007). We then use invalid assessment scores to "justify inequalities among class, racial, ethnic and linguistic groups" (p. 134). This study serves as evidence that bi-/multilinguals' languaging is not deficient and as such, there needs to be a change in how we assess mathematical learning and communication, especially among bi-/multilingual students.

Preparing Mathematics Teachers to Teach Bi-/multilingual Students

The role of a teacher in supporting bi-/multilinguals learning mathematics cannot be overemphasized. Throughout this study, we have seen that the participants needed support to use their LPRs in a mathematical sense, and to develop more formal mathematical communication. The participants also needed help to be able to think algebraically and to succeed in their problem solving. As Driscoll (1999) noted, all students "help to develop habits of algebraic thinking related to generalization because they cannot develop this on their own" (Driscoll, 1999, p.65). Moreover, the fact that most students perceive problem solving process as a series of calculations implies that teachers are to help students develop problem-solving skills (Driscoll, 1999). Although teacher preparation needs to equip teachers with knowledge of the mathematics they will teach and the manner in which they will teach it, de Jong et al.'s (2013) noted the need to prepare mainstream teachers to address ELLs' needs. His study of mainstream elementary teachers showed that these teachers were more willing to implement strategies that create a welcoming environment and those that make content instruction comprehensible than strategies that support ELLs language and literacy development.

Garcia (2009) noted that all 21st century teachers would need to be prepared to teach bi-/multilingual students. I see two main reasons as why all teachers will need to be prepared to be able support ELLs' learning. First, ELLs continue to outnumber the non-ELLs population in K-12 school enrollments in the U.S. (de Jong, Harper, & Coady, 2013; Padolsky, 2006; Shin & Kominski, 2010). This compounded with the longstanding shortage of bilingual and ESL teachers and severe budgetary constraints, particularly in rural school districts (American Associations for Employment in Education, 2008) has resulted to placement of ELLs in mainstream classrooms, where they occasionally receive support from an ESL/bilingual teacher through collaborative models (de Jong et al., 2013). Second, the demands of 21st century high-stakes educational environment and the current push to evaluate teachers on the basis of student learning outcomes makes the ELL-enhancement in the preparation of mainstream teachers essential. An important question at this point is how mainstream teachers can be prepared to teach bi-/multilingual students especially when they do not share their home languages (Barwell, 2009).

de Jong et al. (2013) conceptualized ELLs teacher expertize as enhanced expertise. The authors proposed three dimensions that teacher preparation for ELLs must address: (a) contextual understanding of ELLs, (b) understanding the role of language and culture in school, and (c) preparation for situated practice. In the following sections, I discuss what each of these dimensions entail.

Contextual understanding of ELLs. This dimension entails teacher's awareness of the ELLs' personal linguistic histories, cultural experiences, and how the linguistic and cultural experiences can influence their participation, engagement, and learning in the classroom. Garcia and Kleyn (2013) saw the development of the teacher's understanding of the bi-/multilingual students and their families as the most important step towards gathering vital linguistic and cultural information that may help them address the bi-/multilinguals' learning needs. However, de Jong et al.'s (2013) research among elementary mainstream teachers showed that these teachers did not know where and how to get ELLs' linguistic and cultural information. This lack of diagnostic information compromised the teachers' ability to identify and support ELLs' language development. Teachers need to be able to "access and leverage linguistic and cultural resources that may not be readily available in the classroom" (de Jong et al., 2013, p. 94). Moreover, there needs to be ways teachers can share information on ELLs' learning progress and persistent needs with faculty, ELLs' families, and other key stakeholders (Herrera & Murray, 2005)

Understanding the role of language and culture in school. Teachers need pedagogical knowledge and skills informed by an understanding of the role of language and culture in school. Teachers are to be helped build their knowledge about language and culture as processes, means, and goals for instruction and for creating inclusive learning environments and for expanding learning opportunities for bi-/multilingual students (Ainkhead, 2001, de Jong et al., 2013; Sigley &Wilkinson, 2015). In a recent study, Colindres (2015) investigated how high school mathematics teachers in a school district in Texas supported their ELLs learning. He found that the teachers used translanguaging strategies such as cognates, collaborative dialogue for ELLs sharing home languages, paraphrasing and revoicing across different languages, stimulus of inner speech for students to translanguage in inner speech, translanguaging in speaking, translating, and mnemonic devices. Other translanguaging strategies that teachers could use to support ELLs include multilinguals word walls, multilingual sentence starters, peer grouping to home languages to enable collaborative dialogue and cooperative tasks using translanguaging, and research tasks where students translanguage as they find new information, and so on (Garcia& Wei, 2014). According to Garcia and Wei (2014), the potential of translanguaging as a pedagogical practice can be fully realized by acknowledging translanguaging in our teacher education programs and by running professional development to help educators, especially the school leaders, understand the complex practices of bi-/multilingual students and how these practices can be leveraged to enhance learning.

Preparation for situated practice. Other than equipping mainstream teachers with knowledge and skills of ELLs' linguistic and cultural experiences and the role or language and culture in the students' learning, teachers also need to be prepared to navigate and negotiate educational policies and reform activities that may ignore or impede ELLs' learning. Teachers need "to know when and how to adapt proposed and accepted best practices and be able to articulate why alternatives are necessary" for bi-/multilingual learners (de Jong et al., 2013, p. 94). de Jong et al. (2013) also noted how teacher disposition may influence their attitudes towards bi-/multilingual students. As Nieto (2000) asserted:

> Teaching language minority students successfully means above all challenging one's attitudes toward the students, their language and cultures, and their communities. Anything short of this will result in repeating the pattern that currently exists. (p. 196).

Teachers need to see themselves as cultural brokers for their own students (Ainkhead, 2001). In order to effectively mediate between students' linguistic and cultural resources and the classroom content, teachers are to surrender their perceptions about the students, and be willing to learn and use the students' experiences to enhance their learning.

Limitations and Suggestions for Further Research

There were limitations related to my positionality in this study and how it affected the validity of the findings. During my visits to the recruits' homes, I often spoke in Swahili. There is a possibility that the participants took Swahili as my preferred language, causing them to speak more Swahili during problem solving. I am not sure how different the findings would be if I shared the participants' home languages; a further study could investigate that. Also, the problems I gave were prepared in English and this necessitated the participants' use of LPRs to make sense of them and to communicate their understanding. Future studies could investigate the participants' mathematical communication with problems that have been translated to the participants' first language.

My analysis of mathematics practices was based on Driscoll's (1999) indicators of algebraic thinking and the descriptions of mathematics practices provided in the NCTM standards for mathematics practices. In future, a more in-depth study might examine the development of specific mathematics practices in more specific mathematics topics. Generalization as a mathematics practice was a difficult area for all the participants. Most of the participants did not seem to know what it means to generalize and how generalization comes about (Driscoll,1999). Future studies might conduct an in-depth examination of the properties students attend to while generalizing, why they do so, and how they can be supported to attend to the right properties.

My study was a case study and a case study approach often limits the scope and generalizability of findings. Although the findings of this study agree with most findings from other studies among bi-/multilingual students in different contexts, the study only involved 12 African refugee high school bi-/multilinguals who had recently immigrated to the U.S. I am not sure how the findings would be for students at lower grade levels. Moreover, I used problems in the domain algebra from the NAEP released examination, but the findings might differ for problems from other domains of mathematics and standardized examinations.

The study showed that bi-/multilingual students need support to use their LPRs in a mathematical sense and to develop more formal mathematical communication. There is a need for studies that investigate how teachers at various levels of school mathematics engage bi-/multilingual students' in complex mathematical reasoning even when they do not share the students' home languages. Also, there is a need for studies that investigate the kinds of professional development programs mainstream teachers would need to be effective in instruction and assessment of ELLs (Bailey et al., 2018).

Chapter Summary

In this chapter, I highlighted the main findings of this study and how they relate to the existing knowledge. I also discussed the implications of this study on policy and practice in assessments and teacher preparation. The findings touched on three main subjects including mathematical communication among bi-/multilinguals, what counts as mathematics practices and supporting bi-/multilingual students.

Mathematical communication is critical in mathematical learning, but communication cannot occur without participation and language use. Bi-/multilinguals rely on their entire linguistic repertoires and resources to communicate their mathematical understanding. If we limit the language of use during mathematical communication, we then limit our access to the bi-/multilingual students' rich mathematical thinking (Civil, 2012). Since bi-/multilinguals communicate their mathematical thinking in ways that may be informal, the question of what count as mathematics practice arises. We have seen from this study and from existing literature that any analysis of mathematics practices should consider bi-/multilinguals' informal practices because everyday practices do not disappear in mathematics, but they are used in "new, more mathematical ways" (Barwell, 2013, p. 221).

The findings of this study imply the need to valorize bi-/multilingual students' informal ways of thinking mathematically in both formative and summative assessments. The possibility of administering translanguaged-mode assessments needs to be explored (Garcia & Wei, 2014). Bi-/multilinguals need to be supported to use their linguistic repertoires and resources in a mathematical sense. This implies that teachers need to be prepared to harness the potential of translanguaging by implementing the pedagogies suggested by Colindres (2015) and Garcia and Wei (2014).

Students need support to develop both problem-solving skills and habits of thinking related to generalization, for they cannot develop these skills on their own. Before helping bi-/multilingual students advance to more precise mathematical communication, it is important to encourage them to seek to understand the mathematics in the problems by drawing on their entire linguistic and semiotic resources.

While this study corroborates existing literature in a number of ways, it also adds to the existing research in three main ways. First, the study showed that bi-/multilingual students use their LPRs in an integrated manner, not in isolation. In other words, bi-/multilingual students translanguage in mathematics. Translanguaging has been studied in literacy and biliteracy, and among Latino mathematics teachers, but not among bi-/multilingual students during MWPS. Moreover, this study investigated mathematical communication among Swahili-speaking bi-/multilingual high school students in the U.S, an ELL population that had not yet been studied. Lastly, the study pointed to three kinds of support that bi-/multilinguals would require during MWPS; support to draw upon their LPRs, support to use their LPRs in a mathematical sense, and support to develop more formal mathematical communication.

Appendix A1: Adopted and Modified Problems

Math Eliciting Problem (PrM)

Write the next two numbers in the number pattern.

1 6 4 9 7 12 10 ____, ____

Write the rule that you used to find the two numbers you wrote.

Types of semiotic features by semiotic modality and potentially challenging linguistic features

Mathematics register	**Linguistic features**
Types of numbers: *whole numbers*	Implicit information: *rule unknown*
Natural/mathematics language	Polysemy: *pattern, rule*
Technical word: *pattern*	**Visual representation**
Everyday word: *pattern*	Figure: *sequence of numbers with two blanks*
Testing register	
Question phrase: *Write the rule…. Write the next…*	

Problem A

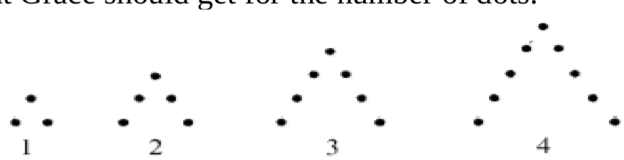
Tom has to fill a certain container with water, but he does not want to fill it in one day. So, each morning he has to pump in some amount of water. On the first day, he pumps in 3 gallons of water to the container, on the second day he pumps in 5 gallons, on the third day he pumps in 7 gallons, and so on. He continues this for many days. How many gallons of water will Tom pump in on the 100^{th} day?

Types of semiotic features by semiotic modality and potentially challenging linguistic features

Mathematics register	**Linguistic features**
Types of numbers: *whole numbers*	Implicit information: *gallons pumped in each day after day three water unknown*
Notation: *100^{th}*	Abstraction: *Nature of the container not known, number of days of pumping in water unknown*
Natural/mathematics language	
Measurement units: *Gallons, days*	
Testing register	
Question phrase: *How many gallons of water…*	

Problem B

Grace has to determine the number of dots in the 100^{th} figure, but she does not want to draw all 100 pictures and then count the dots. Explain or show how she could do this and give the answer that Grace should get for the number of dots.



Types of semiotic features by semiotic modality and potentially challenging linguistic features

Mathematics register	**Linguistic features**
Types of numbers: *whole numbers- 100*	Implicit information: *number of dots in figures after the fifth figure unknown*
Notation: *100^{th} figure*	
Testing register	
Question phrase: *Explain or show how…*	
Visual representation	
Figure: *dot pattern*	

Appendix A2: Original NAEP Test Items

NAEP-2009 released test item's attributes
Content: Algebra; Classifications: Problem solving; Type: Short constructed response; Difficulty level: Easy

Write the next two numbers in the number pattern.

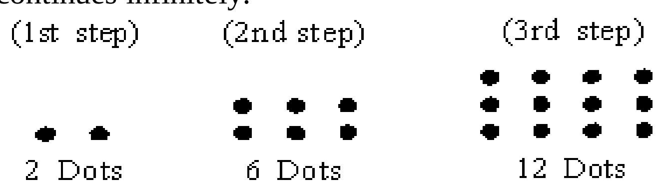
1 6 4 9 7 12 10 ____ ____

Write the rule that you used to find the two numbers you wrote
(NAEP 2009 item 8M5 #11 M066501*)*

NAEP-1992 released test item's attributes
Content: Algebra; Classification: Problem solving; Type: Extended constructed response (ECR); Difficulty level: Hard

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

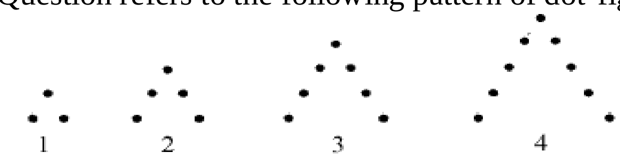
A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.



Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Explain or show how she could do this <u>and</u> give the answer that Marcy should get for the number of dots.
Did you use the calculator on this question?
(NAEP 1992 item 8M12 #9 M054301)

NAEP-1990 released test Item's attributes
Content: Algebra; Classification: Problem solving; Type: Multiple choice; Difficulty level: Hard

Question refers to the following pattern of dot-figures.



If this pattern of dot-figures is continued, how many dots will be in the 100th figure?
100 B. 101 C. 199 D. 200 E. 201
(NAEP 1990 item 8M7 #16 M016901)

Appendix A3: Problem-Solving Protocol

Step 1: Introduction

[Thank the students for accepting to participate in the study and explain the purpose of the study. Explain that they may use whatever languages and resources they like to solve the problems. Ask them to use whatever approaches that may be familiar to them.

[Introduce the first problem. Use code-switching to model use of language practices]

Step 2: Students read problem A

Read the problem individually for 2 minutes, then read it out aloud to your peer.

Step 3: Students recall problem A

Read the problem again out aloud. What's the problem about? What is the question in the problem? What are you looking for?

Step 4: Follow-up questions

Are there some linguistic aspects of this problem you found challenging? Which ones were they? Are there other aspects of this question you found challenging? Which ones were they? Explain the nature of the challenge(s) you faced.

Step 5: Students solve problem A aloud

Read the problem a third time. Talk to your partner about the question. What information do you have? What information are you missing? What information do you need to solve the problem? Now discuss your solution to the problem, come to an agreement, and have one of you write down your final solution on the space provided. [Remind the participants to feel free to use the language(s) they like as well as the problem resolution methods and resources they prefer.]

Repeat steps 1 through 5 for problem B

Appendix A4: Problem-Solving Worksheet

Name:_______________________ Grade: _______________ Gender: _______ Age: ____

Thank you for accepting to participate in the study. The findings from this study will go a long way to help with improving mathematics instruction for students especially those who speak languages other than English. I therefore ask you to feel free to use language(s) of your choice. Also, as you solve the problems, try to use whatever methods and any resources you like. Please follow through the rest of the steps carefully.

Problem A

Tom has to fill a certain container with water, but he does not want to fill it in one day. So, each morning he has to pump in some amount of water. On the first day, he pumps in 3 gallons of water to the container, on the second day he pumps in 5 gallons, on the third day he pumps in 7 gallons, and so on. He continues this for many days. How many gallons of water will Tom pump in on the 100^{th} day?

What do you know?

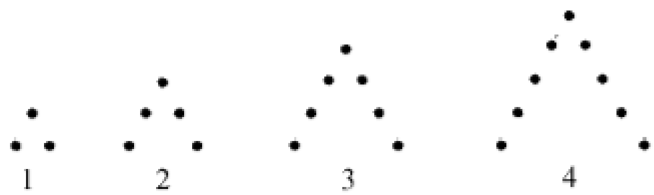
What don't you know?

What do you want to know?

Agreed upon solution for problem A

Problem B

Grace has to determine the number of dots in the 100th figure, but she does not want to draw all 100 pictures and then count the dots. Explain or show how she could do this and give the answer that Grace should get for the number of dots.



What do you know?

What don't you know?

What do you want to know?

Agreed upon solution for problem B

Appendix B1: English Assessment Protocol

Name of Participant: __________________________ Grade: ________________

Mathematics activity elicitation task

Materials: 3 blank sheets of paper, pencil, eraser.

Write the next two numbers in the pattern below.

1, 6, 4, 9, 7, 12, 10, __, __

Stage 1: Making estimations.

[Present student with the problem]

Please read this problem out aloud and make an estimation of the solution without using any tools and without writing down the ideas.

[Ask clarifying questions based on the students' response.]

Stage 2: Writing solution.

Please to write the solution to the task you had estimated previously on a sheet of paper. Please show all of your work. Write a rule that you used to find the two numbers you wrote.

Stage 3: Explaining reasoning.

Please explain the reasoning you used to solve the problem. Explain the rule you use to find the two numbers in the pattern,

[Have them write on a scrap paper to demonstrate mathematical thinking. Ask clarifying questions, make references to aspects of the students' work.]

Stage 4: Writing the explanation of solution

Please write down on the provided sheet of paper the solution procedure to this problem for another student who does not know how to do this task

Appendix B2: English Assessment Worksheet

Name of Participant: __________________________ Grade: ________________

Write the next two numbers in the pattern below.

1, 6, 4, 9, 7, 12, 10, __, __

Stage 2: Writing solution.

Stage 4: Writing the explanation of solution

Appendix B3: Heritage et al.'s (2015) Spoken Language Target Features

Sophistication of topic vocabulary
Small, essential topic vocabulary progressing to a more extensive topic lexicon and use of precise/low frequency topic vocabulary

Not evident - 0	Emergent - 1	Developing - 2	Controlled - 3
No use of topic vocabulary[1] in English or only repeating vocabulary from prompt [i.e. *clean*(v), *teeth*] [1]*Topic vocabulary*: words that would be typically used to explain details about the topic	Use of some essential topic vocabulary[2] not from prompt No use of topic vocabulary beyond the essential words [2]*Essential topic vocabulary*: the relatively small set of topic vocabulary words most speakers or writers are likely to rely on in order for the listener or reader to understand the topic being explained	Mostly accurate use of a variety of topic vocabulary (including essential topic vocab not from prompt and some precise, topic-related words beyond the essential words) Use of sufficient topic vocabulary (including words from prompt) to make the context clear Possible use of imprecise/general terms in place of technical vocabulary[3] or deictic referents (*it*, *that*, *these*, etc.) in place of topic words	Appropriate and accurate use of a variety of precise topic and technical vocabulary (comprised of essential topic vocab not from prompt, as well as many words beyond the essential words, including at least one technical word[3]) Possible use of low-frequency words that enliven the explanation or evoke an image (aka vivid vocab) [3]*Technical topic vocabulary*: words not likely to be encountered outside a discipline; specialized and content-specific words

Sophistication of sentence structure
Simple sentences progressing to complex sentences

Not evident - 0	Emergent - 1	Developing - 2	Controlled - 3
One word responses 2 or more word phrases not in English word order Response in a language other than English	Simple sentences Compound sentences May or may not be accurate No use of embedding (dependent clauses)	Must attempt sentences with complex clause structures (i.e., an independent clause and at least one dependent clause) May have repetitive use of one dependent structure, such as relative, adverbial, or noun clauses	Use of a variety of complex clause structures, including relative, adverbial, or noun clauses Simple and compound sentences are accurate and grammatically correct

Sentence fragments placed in English word order		May or may not be accurate Simple and compound sentences mostly accurate and grammatically correct	

Establishment of advanced relationships between ideas Causal, adversative, conditional, comparative, and contrastive discourse connectors			
Not evident - 0	**Emergent - 1**	**Developing - 2**	**Controlled - 3**
No discourse connectors (causal, conditional, comparative, contrastive) between phrases and clauses to link advanced relationships between ideas No clarity in relationships between ideas	Singular or repetitive use of 1 discourse connector to establish an advanced relationship Possible use of inaccurate or illogical discourse connector within context of establishing distinct relationships between ideas	Minimum of 2 different discourse connectors to establish an advanced relationship Most often displays clarity in relationships between ideas	At least 3 different discourse connectors to establish an advanced relationship **AND** a minimum of 2 different connector words for the same type of relationship (causal, conditional, etc. – see below) Maintains clarity in relationships between ideas

Below is a list of common connectors that establish advanced relationships (this is not an exhaustive list):

- Causal connectors: *because, so, since, therefore, as a result, (in order) to, for,* and**
- Conditional connectors: *when,** if (…then), whenever*
- Comparative connectors: *like, as (though/if/…as), likewise, [adj.] + than*
- Contrastive connectors: *but, or, otherwise, however, instead of, though, although, even though, even so, except, while,** otherwise, on the other hand, whereas, nevertheless, meanwhile***

* Non-conventional use of a discourse connector ** Depending on use, may not be a connector

Coherence of the explanation Temporal connectors			**(*Content Guide*, pp. 68-72)**
Not evident - 0	**Emergent - 1**	**Developing - 2**	**Controlled - 3**
Lack of coherence in sequencing *any* statement No mental schema for explaining in a way that makes sense to the naïve listener Steps or process being explained are largely incomprehensible to the listener	Some coherence by logically sequencing of 1-2 statements using at least 1 conjunction (*and, but, in addition*, etc.), or 1 transitional word (*then, next, first, finally*, etc.) to make the linkage Some evidence of a mental schema but may include several incomplete thoughts/sentences Explanations may require a lot of effort from a listener to understand the steps or process being explained	Logical sequencing of *most* statements Repertoire includes some different discourse connectors (should include both conjunctions and transitional words) Evidence of a mental schema but may include 1-2 incomplete thoughts/sentences Explanations may require some effort from a listener to understand the steps or process being explained	Logical sequencing of *all* statements Repertoire includes many different discourse connectors (should include both conjunctions and transitional words) Evidence of a clear schema from which the explanation is crafted Explanations require very little or no effort from a listener to understand the steps or process being explained

Coherence shows that a child is taking account of the listener's needs as they explain. Coherence is established in the following ways:

1) Successful sequencing of statements using organizing discourse connectors including the following:
   - *Conjunctions* include (but are not limited to): *and, plus, in addition*
   - *Transitional words* include (but are not limited to): *first, next, after, once, finally, (and) then, while*
2) The use of complete sentences, even if the sentences are not necessarily syntactically complex. Use of complete sentences suggests evidence of the child's mental schema for the steps or process being explained. In contrast, sentence fragments or disconnected phrases can lead to an overall lack of coherence.

Appendix B4: Bailey and Heritage's (2018) Written Language Target Features

Opening and Closing Statements

Not evident - 0	Emerging - 1	Developing - 2	Controlled - 3
No use of opening or closing statements • Student launches into explanation without providing **content** (i.e. that the following text will contain an explanation/ instructions about a procedure) or **context** (cleaning teeth or using cubes with the goal of finding the total) • Student does not make closing statement that indicates that explanation is over Note that the use of letter salutations (e.g., "Dear" or "Hi [name]") or closings (e.g., "Sincerely" or "Bye") are not considered opening or closing statements	Use of opening <u>or</u> closing statement, used in a minimal form: • Opening orients the reader by providing **content** OR **context**, but not both • Closing indicates that explanation is over, but does not refer to preceding information (includes statements like, "the end" or "that's it") Starting the explanation with justification ("Why?" portion) may count as employing a minimal opening if one of two conditions are met: • Additional justification is provided elsewhere in the explanation, **or** • Following the justification, the student includes a minimal opening to initiate the procedural ("How?")	Opening used appropriately: • Occurs towards beginning of explanation • Announces both the **content** and the **context** of the explanation that follows (i.e., signals that the note contains an explanation about a specific procedure [teeth/math activity]) …but closing is absent Starting the explanation with justification ("Why?" portion) may count as employing an appropriate opening if one of two conditions are met: • Justification announces context and content of explanation, <u>and</u> additional justification is provided elsewhere in the explanation, **or** • Following the justification, the student includes an appropriate opening to	Opening used appropriately and closing is used appropriately or in a minimal form OR Closing used appropriately and opening is used appropriately or in a minimal form

	component of the explanation Opening or closing may contain extraneous information	initiate the procedural ("How?") component of the explanation OR Closing used appropriately: • Occurs towards end of explanation • Refers back to content of explanation without adding new information • Is not the last step of the procedure …but opening is absent OR Both opening and closing are used in minimal form OR Two minimal openings are used (to introduce procedure and justification sections)	
Examples			
frist you get your brush and you put it on your teeth brush brussh to sayt nice and clean why you should sayt nice and clean because that is going to make you best to eat and rest. (304_B3_T1) *<corrected> First you get your brush and you put it on your teeth*	Ashley, <u>using these cubes to find out how many they are </u>you nead to put them in grups. You nead to put them in grups because it is esyer than caunting one by one. It is a helpful way than counting all of theme because you are going to get confised when you don't put theme in grups. It is a helpful way	Well, you need to count Is the first thing. Second If you want to go fast you need to know to times (? x ?= ?) third, If you want to go slow you need to count on ones. The point Is adding Is put one thing to another. It works like for example A farmer has 5 sheeps But his cousin gave him four more. how much Does the	<u>First I will teach you how to clean your teeth</u>. 1. Wet the tooth brush, 2. Put the tooth paste on the tooth brush, 3. Wet the tooth paste, 4. Brush your teeth 2 minutes, 5 your done spit the water, 6 rinse your mouth and wet your tooth brush. You have

brush brush to stay nice and clean why you should stay nice and clean because that is going to make you best to eat and rest. (304_B3_T1) • Launches into explanation without orienting the reader • No closing *<corrected> You could do it by counting by 1,2,3,4,5 all the numbers. And you should do it this way because you will find your answer faster.* (310_M_T1) • Student does not provide context for explanation, and does not announce content of the explanation	alse becase it is a faster way to count the cubes. (346_M_W_T1) *<corrected> Ashley, using these cubes to find out how many they are, you need to put them in groups. You need to put them in groups because it is easier than counting one by one. It is a helpful way than counting all of them because you are going to get confused when you don't put them in groups. It is a helpful way also because it is a faster way to count the cubes.* (346_M_W_T1) • Opening announces the full context, including the subject matter and end goal, but does not announce the content	famer have all together? So you put 5 sheeps next to four sheeps. And the farmer will have nine sheeps. And time tables I just like adding It just you add the number the times of the question example: I have 5 pencils but My friend gave 2x5 the pencis I have so what you do Is just like adding 5+5=5x2. That Is how you count, add, or times. (521_M_W_T2) *<corrected>Well, you need to count is the first thing. Second, if you want to go fast, you need to know to times. (? x ?= ?) Third, if you want to go slow, you need to count on ones. The point is: adding is put one thing to another. It works, like for example: a farmer has 5 sheeps but his cousin gave him four more. How much does the famer have all together? So you put 5 sheeps next to four sheeps. And the farmer will have nine sheeps. And time tables. I just like adding. It just you add the number the times of the question. Example: I have 5 pencils but my friend gave 2x5 the pencils I have. So what you do is just like adding 5+5=5x2. That is how you count, add, or times.* (521_M_W_T2)	to clean your teeth because you don't want cavity, tooth pain or food in your teeth stuck. So it is important to brush your teeth to be clean. Brush your teeth every day. (525_B3_W1_T2) *<corrected> First I will teach you how to clean your teeth. 1. Wet the tooth brush, 2. Put the tooth paste on the tooth brush, 3. Wet the tooth paste, 4. Brush your teeth 2 minutes, 5 your done spit the water, 6 rinse your mouth and wet your tooth brush. You have to clean your teeth because you don't want cavity, tooth pain or food in your teeth stuck. So it is important to brush your teeth to be clean. Brush your teeth every day.* (525_B3_W1_T2) • Appropriate opening announces content and context • Appropriate closing restates the overall argument of the explanation

		• Although student's explanation does not respond directly to the prompt, his closing summarizes the content of his explanation	

Decontextualization

Not evident - 0	**Emerging - 1**	**Developing - 2**	**Controlled - 3**
Use of **<u>deictic language</u>** (i.e., relies on information about a specific aspect of student's here and now; verbal "pointing": *this, that, these, those*, *here, there*) that interferes with meaning making. Note: Consider everything on the written page (including prompt/title on the top) when determining decontextualized language usage; if child is "pointing" or referring to information in the written prompt or title, this is pragmatically appropriate (they may assume the naïve reader will see this information as well and therefore not fully decontextualize their explanation). OR Explanation does not contain enough content in the text to allow for decontextualization	No use of deictic language Ambiguously tied or omitted referents make it difficult for a naïve reader to follow the explanation For example, in the case of the math task, the student may use <u>pronominal forms</u> to refer back to the spoken prompt rather than paraphrasing the prompt in a lexicalized manner (e.g., *You should group them* instead of *You should group the cubes*). For example, the student may use quantified adjectives (*both*, *each*, *other*, *either*, *neither*) that have not previously been introduced Note: In the case of teeth cleaning explanations, referents using pronouns that co- reference the written material prompt would be acceptable as long as they are not themselves ambiguous.	No use of deictic language Student unpacks the prompt with a full noun referent (e.g., *Group the cubes*) Use of appropriate co-reference within the text or to the written prompt/title on the page (e.g., "do it" [clean teeth] or "them" [teeth] for social task, "this activity" for math) Naïve reader can understand the explanation based on what is on the written page (including prompt/title) Note: Cataphoric uses of "this" are acceptable (e.g., *You should count the cubes using this method [that I will be explaining in this paragraph]*) as are existential uses of "it" (e.g., *It helps to…*) BUT Explanation does not exceed <u>*four*</u> clauses containing an	No use of deictic language Student unpacks the prompt with a full noun referent (e.g., *Group the cubes*) Use of appropriate co-reference within the text or influenced by the written prompt/title on the page (i.e., "do it" [clean teeth] or "them" [teeth for social task, "this activity" for math) Naïve reader can understand the explanation based on what is on the written page (including prompt/title) Note: Cataphoric uses of "this" are acceptable (e.g., *You should count the cubes using this method [that I will be explaining in this paragraph]* as are existential uses of "it" (e.g., *It helps to…*)

		overt or imperative subject (i.e. insufficient evidence of control)	
You could use it by difrent ways as one dy one or by 2. I like doing it this way because it is more easy for me and if I get stuck from the cubes but I see how I already counted **that cube**. (344_M_W_T1) *<corrected> You could use it by different ways as one by one or by 2. I like doing it this way because it is more easy for me and if I get stuck from the cubes, but I see how I already counted **that cube**.* (344_M_W_T1) • Use of deictic referent (*that cube*) that points to the child's physical context • Use of *it* to refer to spoken prompt • Use of *this way* in second sentence is not deictic (it is an anaphoric reference that is poorly tied to language in the explanation)	You should group it by 5 because it is a more ifishent way of counting things. It is also a easier way of counting. When you are done grouping them by fives you should count by fives. Each group is one five. (8941_M_W_T2) *You should group it by 5 because it is a more efficient way of counting things. It is also an easier way of counting. When you are done grouping them by fives you should count by fives. Each group is one five.* (8941_M_W_T2) • Refers back to oral prompt using pronoun (*it*) with unclear referent	You scrub you teeth with the tooth Brush. You shud do it to not get carudeas. and to keep them clean. (9552_B3_W1_T3) *<corrected> You scrub your teeth with the toothbrush. You should do it to not get cavities. And to keep them clean.* (9552_B3_W1_T3) • Student does not refer to prompt using pronominal forms • Use of *do it* is decontextualized anaphoric reference to language in explanation (i.e., *scrub you teeth*) • However, explanation lacks sufficient length to provide evidence of control	In order to find how many cubes there are, you have to sort the cubes out in a certain way (in this case by color). If you find out how many cubes are in each group and multiply that by the amount of groups there is This is the best way to do it because that way, you can just simply multiply instead of having to add a bunch of 1s together. (6412_M_W_T2) *<corrected> In order to find how many cubes there are, you have to sort the cubes out in a certain way (in this case by color). If you find ou how many cubes are in each group and multiply that, by the amount of groups there is. This is the best way to do it because that way, you can just simply multiply instead of havin to add a bunch of 1s together.* (6412_M_W_T2) • Refers to prompt using full noun referents (e.g., *sort the cubes*)

Graphic Representation Supporting Meaning Making

Not evident - 0	Emerging - 1	Developing - 2	Controlled - 3
No use of written language to convey meaning	Nonconventional use of punctuation, spelling, and graphic representation (handwriting, capitalization, etc.) severely interferes with meaning making for the reader	Some nonconventional use of punctuation, spelling, and graphic representation that partly interferes with meaning Student may compensate by making meaning for the reader in some other way (sequencing, formatting, organization of text, etc.)	Use of punctuation, spelling, and graphic representation (handwriting, capitalization, etc.) may or may not be conventional, but it is clear and easy for reader to make meaning Occasional spelling or punctuation errors may occur, but they are minimal and do not interfere with meaning making
Examples			

Examples would include illegible scribbling or drawing in lieu of writing	You need to dith yor teth pecis you will get cadibs. And you shid dish yor teth on yor mlrs on yor noo teth and on yor tin. (9677_B3_W1_T3) *<corrected> You need to brush your teeth because you will get cavities. And you should brush your teeth on your molars on your new teeth and on your tongue.* (9677_B3_W1_T3) Invented spelling severely interferes with meaning making (e.g., *dith*)	My mom doesn't now how to clean her teeth very well First you need to brush your teeth in circles then you need to clean your tung after you clean your top part at last you need to spit. (357_B3_W1_T2) *<corrected> My mom doesn't now how to clean her teeth very well. First you need to brush your teeth in circles. Then you need to clean your tongue after you clean your top part. At last you need to spit.* (357_B3_W1_T2) • Lack of punctuation partly interferes with meaning, but student compensates by capitalizing sentence initial words	First I will teach you how to clean your teeth. 1. Wet the tooth brush, 2. Put the tooth paste on the tooth brush, 3. Wet the tooth paste, 4. Brush your teeth 2 minutes, 5 your done spit the water, 6 rinse your mouth and wet your tooth brush. You have to clean your teeth because you don’t want cavity, tooth pain or food in your teeth stuck. So it is important to brush your teeth to be clean._Brush your teeth every day. (525_B3_W1_T2)

			<corrected> First I will teach you how to clean your teeth. 1. Wet the tooth brush, 2. Put the tooth paste on the tooth brush, 3. Wet the tooth paste, 4. Brush your teeth 2 minutes, 5 your done spit the water, 6 rinse your mouth and wet your tooth brush. You have to clean your teeth because you don't want cavity, tooth pain or food in your teeth stuck. So it is important to brush your teeth to be clean. Brush your teeth every day. (525_B3_W1_T2) • Minor spelling errors (e.g., *your* instead of *you're*) do not interfere with meaning • Missing punctuation does not interfere with meaning

Appendix C: Language Background Questionnaire

Thank you for considering completing this survey questionnaire. The survey instrument will not take more than 10 minutes of your time. I will request that you kindly take some time and fill this survey out honestly and accurately. The results will go a long way towards improving high school mathematics teaching and learning among students for whom English is not their first language in the United States.

Participant's name: ______________________Grade: _______Gender:________ Age:___

1. How long have you been in the United States? ___ years/months
2. Do you speak a language other than English and Swahili? ☐ Yes ☐ No
3. If you speak a language other than English and Swahili:
   a. What is that language? ________________
   b. Was that the first language you learned when you were a child? ☐ Yes☐ No
   c. If not, how old were you when you began speaking that language? ________
   d. How often do you speak that other language: (Check one for each item)

	Always or most of the time	Sometimes	Never or hardly ever
i. with your parents?	☐	☐	☐
ii. with your grandparents?	☐	☐	☐
iii. with your brothers and sisters?	☐	☐	☐
iv. with your friends away from school?	☐	☐	☐
v. with your friends at school?	☐	☐	☐

   e. How well do you: (check one for each item.)

	Very well	Fairly well	Not well	Not at all
i. understand that other language?	☐	☐	☐	☐
ii. speak that language?	☐	☐	☐	☐
iii. read that language?	☐	☐	☐	☐

iv. write that language?	☐	☐	☐	☐

4. Have you ever been taught mathematics in that other language? ☐ Yes ☐ No
   a. If yes, at what grade level(s)? _________
5. How often do you speak Swahili? (Check one for each item.)

	Always or most of the time	Sometimes	Never or hardly ever
i. with your parents?	☐	☐	☐
ii. with your grandparents?	☐	☐	☐
iii. with your brothers and sisters?	☐	☐	☐
iv. with your friends away from school?	☐	☐	☐
v. with your friends at school?	☐	☐	☐

6. How well do you: (check one for each item.)

	Very well	Fairly well	Not well	Not at all
i. understand Swahili?	☐	☐	☐	☐
ii. speak Swahili?	☐	☐	☐	☐
iii. read Swahili?	☐	☐	☐	☐
iv. write Swahili?	☐	☐	☐	☐

7. Have you ever been taught mathematics in Swahili? ☐ Yes ☐ No
   a. If yes, at what grade level(s)? ______________
8. Do you prefer talking about mathematics?
   ☐ in English? ☐ in Swahili? ☐ in your other language?

9. How easy has it been for you in the past to understand: (Check one for each item)

	Very easy	Fairly easy	Fairly difficult	Very difficult
your mathematics teacher's explanations?	☐	☐	☐	☐
your mathematics textbook?	☐	☐	☐	☐
questions on mathematics tests?	☐	☐	☐	☐

10. How well do you: (Check one for each item.)

	Very well	Fairly well	Not well	Not at all
understand spoken English?	☐	☐	☐	☐
speak English?	☐	☐	☐	☐
read English	☐	☐	☐	☐
write English?	☐	☐	☐	☐

Appendix D: Research Approval Letter, IRB - Syracuse University

SYRACUSE UNIVERSITY

INSTITUTIONAL REVIEW BOARD
MEMORANDUM

TO: Joanna Masingila
DATE: April 25, 2017
SUBJECT: **Expedited Protocol Review - Approval of Human Participants**
IRB #: 17-149
TITLE: *An Investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving Among the Swahili-Speaking Congolese High School Bi-/Multilingual Students in the United States*

The above referenced protocol was reviewed by the Syracuse University Institutional Review Board for the Protection of Human Subjects (IRB) and has been given **expedited approval.** The protocol has been determined to be of no more than minimal risk and has been evaluated for the following:

1. the rights and welfare of the individual(s) under investigation;
2. appropriate methods to secure informed consent; and
3. risks and potential benefits of the investigation.

The approval period is **April 24, 2017** through **April 23, 2018**. A continuing review of this protocol must be conducted before the end of this approval period. Although you will receive a request for a continuing renewal approximately 60 days before that date, it is your responsibility to submit the information in sufficient time to allow for review before the approval period ends.

Enclosed are the IRB approved date stamped consent and/or assent document/s related to this study that expire on **April 23, 2018**. **The IRB approved date stamped copy must be duplicated and used when enrolling new participants during the approval period** (may not be applicable for electronic consent or research projects conducted solely for data analysis)**.** Federal regulations require that each participant indicate their willingness to participate through the informed consent process and be provided with a copy of the consent form. Regulations also require that you keep a copy of this document for a minimum of three years after your study is closed.

Any changes to the protocol during the approval period cannot be initiated **prior** to IRB review and approval, except when such changes are essential to eliminate apparent immediate harm to the participants. In this instance, changes must be reported to the IRB within five days. Protocol changes must be submitted on an amendment request form available on the IRB web site. Any unanticipated problems involving risks to subjects or others must be reported to the IRB within 10 working days of occurrence.

Research Integrity & Protections | 214 Lyman Hall | Syracuse, NY 13244-1200 | 315.443.3013 | **orip.syr.edu**

Appendix E: Consent and Assent Forms

INFORMED CONSENT

SYRACUSE UNIVERSITY
College of Arts & Sciences
Mathematics

Mathematics Department
215 Carnegie Building, Syracuse NY 13244, Tel: 315-443-3849

An investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving among the Swahili-Speaking Congolese High School Bi-/Multilingual Students in the United States.

My name is Victoria Mwia Wambua, and I am a graduate student at Syracuse University. I am inviting your child to participate in a research study. Involvement in the study is voluntary, so your child may choose to participate or not. This sheet will explain the study to you and please feel free to ask questions about the research if you have any. I will be happy to explain anything in detail if you wish.

I am interested in learning more about the "Practices, Resources, and Challenges in Mathematical Word Problem Solving among the Swahili-Speaking Congolese High School Bi-/Multilingual Students in the United States". I will conduct the study in two phases. In the first phase, I will ask your child to respond to some survey questions regarding their language background, and to work on one mathematics task individually in order to elicit their proficiency in speaking and writing in English. The language background questions will touch on when your child began using certain languages and how often they use those languages across contexts, as well as their perception about their proficiency in those languages. In the second phase, your child will be asked to solve two other mathematics problems with another student. The whole session will take approximately 70 minutes of your child's time.

All information will be kept confidential. I will assign a number to your child's responses, and only I, Victoria Mwia Wambua, will have the key to indicate which number belongs to which participant. Also, in any articles I write or any presentations that I make, I will use a made-up name for your child, and I will not reveal details about your child and where he or she lives.

Confidentiality cannot be guaranteed in group situations. Since your child will work on some mathematics problems with another child, the other child will know how your child answers questions. While we will discourage anyone from sharing this information outside of the group, we cannot guarantee confidentiality by the other child. We will do our best to keep all of your child's personal information private and confidential but absolute confidentiality cannot be guaranteed.

I plan to audio and video record your child while he or she does or performs the mathematics tasks, but I will first seek for your child's permission before I do so. The audio and video recorded data will be retained in my password-enabled laptop computer for purposes of data analysis only. I will keep the data for five more years after the analysis is done, and thereafter I will discard it by permanently deleting the files. During that time, only me, Victoria Mwia Wambua, the principal investigator, and anyone helping with translation and/or transcription will have access to the data.

By taking part in this research your child will receive free tutoring if she needs it and asks for it, regardless of whether he or she will withdraw along the way or not.

The benefit of this research is that your child will be helping us to understand the practices, resources, and challenges of high school students from the Swahili-speaking Congolese community during mathematical word problem solving. This information should help us to better understand how students from this community, and bi-/multilinguals in general can be better instructed in mathematics. You child might improve her problem solving skills by participating in this research.

The risks to your child participating in this study are getting frustrated with problem solving and feeling tired. These risks will be minimized by letting the child know it is okay if the problem is challenging as the performance will not be graded since all we are looking for is their mathematical reasoning. We will also allow your child some few minutes of rest in case he or she feels tired.
If your child does not want to take part, he or she has the right to refuse to take part, without penalty. If your child decides to take part and later no longer wish to continue, he or she has the right to withdraw from the study at any time, without penalty.

If you have any questions, concerns, complaints about the research, contact me, Victoria Mwia Wambua at 315-450-8205. If you have any questions about your rights as a parent to a research participant, you have questions, concerns, or complaints that you wish to address to someone other than me, Victoria Mwia Wambua, or if you cannot reach Victoria Mwia Wambua, contact the Syracuse University Institutional Review Board at 315-443-3013.

All of my questions have been answered, I understand my child will be audio and video recorded. I am 18 years of age or older, and I wish for my child to participate in this research study. I have received a copy of this consent form.

____
Name of the child

____________________________________ ____________________
Signature of parent Date

____
Printed name of parent

____________________________________ ____________________
Signature of researcher Date

____
Printed name of researcher

INFORMED ASSENT FORM

Syracuse University IRB Approved

SYRACUSE UNIVERSITY

College of Arts & Sciences APR 2 4 2017 APR 2 3 2018

Mathematics

An investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving among the Swahili-Speaking Congolese High School Bi-/Multilingual Students in the United States.

My name is Victoria Mwia Wambua, and I am from the Mathematics Department, at Syracuse University (SU). I am asking you to participate in this research study because you are in ninth or tenth grade and you have done Algebra I and are taking or have already taken Algebra II, and can speak Swahili and you are from the Congolese refugee community in the United States. A research study is a way to learn more about people. In this study, I am trying to learn more about the practices, resources, and challenges high school students from the Congolese community have as they solve mathematics (algebra) word problems. If you decide you want to be part of this study, you will be asked to respond to some survey questions on your language background and to solve three mathematics word problems. The first mathematics task will be a mathematics eliciting task which will be used to determine your proficiency in speaking and writing in English. The language background survey will touch on when you began using certain languages, and how often you use the languages across contexts, as well as your perception of your proficiency in those languages. You will do the first problem individually and then you will work with another student to complete the last two. All of this should take about 70 minutes of your time.

Your willingness to be audio and video recorded while you do the mathematics tasks is a requirement for participation in this study. The audio and video recorded data will be coded using pseudonyms and retained in my password-enabled laptop computer for purposes of data analysis only. I will keep the data for five more years after the analysis is done, and thereafter I will discard it by permanently deleting the files. During that time, only me, Victoria Mwia Wambua, the principal investigator, and anyone helping with translation and/or transcription will have access to the data.

While you solve the problems, you may feel tired or frustrated, but I will allow you some minutes of rest should you feel tired and I will encourage you to solve the problems to your best of ability. Everyone who takes part in this study will benefit by getting free tutoring and by probably improving their problem solving and group work skills. When I am finished with this study, I will write a report about what was learned. This report will not include your name or that you were in the study.

I have already asked your parents if it is ok for me to ask you to take part in this study. Even though your parents said I could ask you, you still get to decide if you want to be in this research study. You can also talk with your parents, grandparents, and teachers before deciding whether or not to take part. No one will be mad at you or upset if you

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decide not to do this study. If you decide to stop after we begin, that's okay too. You can also skip any of the questions you do not want to answer.

By taking part in this research you will receive free tutoring if you need it and if you ask for it, regardless of whether you will withdraw along the way or not.

Confidentiality cannot be guaranteed in group situations. Since you will work on some mathematics problems with another child, the other child will know how you answer the questions. While we will discourage anyone from sharing this information outside of the group, we cannot guarantee confidentiality by the other child. We will do our best to keep all of your personal information private and confidential but absolute confidentiality cannot be guaranteed.

You can ask questions now or whenever you wish. If you want to, you may call me at 315-450-8205. If you are not happy about this study and would like to speak to someone other than me, you or your parents may call the Syracuse University Institutional Review Board (IRB) at 315-443-3013.

Please sign your name below, if you agree to be part of my study and you understand that you will be audio and video recorded. You will get a copy of this form to keep for yourself.

Signature of Participant ___________________________ Date _________________

Name of Participant ___________________________

Signature of Investigator or Designee ______________________Date ____________

Syracuse University IRB Approved

APR 2 4 2017 APR 2 3 2018

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Appendix G: Translated Consent and Assent Forms

IDHINI

SYRACUSE UNIVERSITY
College of Arts & Sciences
Mathematics

Syracuse University IRB Approved

APR 24 2017 APR 23 2018

Idara ya Hisabati
215 Jengo la Carnegie, Syracuse NY 13244, Simu: 315-443-3849

Uchunguzi wa Shughuli, Rasilimali, na Changamoto katika Kutatua Maswali ya Hisabati Miongoni mwa Wanafunzi wa Shule ya Upili wa Jamii ya Wakongo Wananaozungumza Kiswahili katika Marekani.

Jina langu ni Victoria Mwia Wambua, na mimi ni mwanafunzi mhitimu katika Chuo Kikuu cha Syracuse. Nina kukaribisha mtoto wako kushiriki katika utafiti. Kuhusika katika utafiti huu ni hiyari, hivyo mtoto wako anaweza kuchagua kushiriki au la. Barua hii itaelezea utafiti huu kwako. Tafadhali jisikie huru kuuliza maswali kuhusu utafiti kama una yoyote. Mimi nitakuwa na furaha kueleza chochote kwa undani ukipenda.

Nina nia ya kujifunza zaidi kuhusu mazoea, rasilimali na changamoto katika kutatua maswali ya hisabati miongoni mwa wanafunzi wa shule ya upili wanaozungumza Kiswahili na wanaotoka Kongo waishiyo hapa Marekani. Mtoto wako atatakiwa kujibu baadhi ya maswali ya utafiti kuhusu historia ya lugha yake, na kutatua maswahi tatu ya hisabati; swali moja atalitatua yeye peke yake na hayo mengine atatatua na mtoto mwingine. Hii itachukua takribani dakika 70 za muda wa mtoto wako.

Taarifa yote yatawekwa siri. Majibu yote ya mtoto wako yatapewa nambari ya siri, na ni mimi tu, Victoria Mwia Wambua, nitaweka ufunguo unaoonyesha nambari ipi ni ya mtoto mgani. Pia, katika makala yoyote nitakayoandika au mawasilisho yoyote nitakayofanya,nitatumia majina ya bandia kwa ajili ya mtoto wako, na sitawahi kufichua maelezo kuhusu mtoto wako na ambapo yeye anaishi.

Usiri hauwezi kuhakikishwa katika hali ya kikundi. Mtoto wako atayajibu baadhi ya maswali mengine ya hisabati na mtoto mwingine. Hivyo, mtoto mwingine atajua jinsi mtoto wako anajibu baadhi ya maswali. Sisi tutajaribu tuwezavyo kuzuia huyo mtoto asiitoe habari hii nje ya kundi, lakini hatuwezi kuhakikisha usiri unaotokana na huyo mtoto mwingine. Tunaweza kufanya tuwezavyo ili kuweka kazi ya mtoto wako binafsi na siri lakini usiri halisi hauwezekani kwa hakika.

Ninapanga kurekodi sauti na video ya mtoto wako wakati yeye anafanya kazi ya hisabati, lakini, kwanza nitatafuta kibali cha mtoto wako kabla ya kufanya hivyo. Sauti na video iliyorekodiwa yatawekwa katika tarakilishi yangu iliyo na nywila kwa madhumuni ya uchambuzi wa data tu. Nitaiweka hiyo data kwa miaka mitano zaidi baada ya uchambuzi kufanyika, na baada ya hapo mimi nitaitupilia hiyo data kwa kufuta hiyo faili kabisa. Kwa wakati huo wote, mimi pekee, Victoria Mwia Wambua, mchunguzi mkuu, na yeyote atakayesaidia na tafsiri na/au kunukuu atakuwa na ufikivu wa hiyo data.

Faida ya utafiti huu ni kwamba mtoto wako atakuwa akisaidia sisi kuelewa mazoea, rasilimali, na changamoto katika hisabati za wanafunzi wa shule ya sekondari kutoka jumuiya ya Wakongo wanaozungumza Kiswahili. Taarifa hii lazima itatusaidia kuelewa jinsi ya kutoa elimu bora katika hisabati kwa wanafunzi kutoka hii jamii, na jamii zingine za wanaojifunza kiingereza, kwa ujumla. Kwa kuchukua sehemu katika utafiti huu mtoto wako atapokea usaidizi kwa hisabati bila malipo kama atauhitaji, bila kujali kama yeye atajiondoa njiani au la. Hatari kwa mtoto wako kushiriki katika utafiti inaweza kuwa kuchanganyikiwa katika kutatua maswali na kuchoka. Nitapunguza hatari hizi kwa kumueleza asiwe na wasiwasi kwani kazi yake haitapewa daraja na pia nitaruhusu dakika chache iwapo mtoto wako ataonyesha amehisi uchovu ili apumzike. Kama mtoto wako hataki kuchukua sehemu, yeye ana haki ya kukataa kuchukua sehemu, bila adhabu. Kama mtoto wako ataamua kuchukua sehemu na baadaye tena ajihisi hataki kuendelea, yeye ana haki ya kujiondoa kushiriki wakati wowote, bila adhabu.

Kama una maswali yoyote, wasiwasi, malalamiko kuhusu utafiti, wasiliana nami, Victoria Mwia Wambua katika 315-450-8205. Kama una maswali yoyote kuhusu haki zako kama mzazi wa mshiriki wa utafiti, una maswali, wasiwasi au malalamiko ambayo ungependa kushughulikia na mtu mwingine zaidi yangu, Victoria Mwia Wambua, au kama hawawezi kufikia Victoria Mwia Wambua, wasiliana na bodi ya taasisi ya mapitio ya utafiti ya chuo kikuu cha Syracse kwa nambari 315-443-3013.

Maswali yangu yote yametolewa, naelewa kwamba mtoto wangu atachukuliwa rekodi ya sauti na video. Nina miaka 18 ya umri au zaidi, na ninapenda kwamba mtoto wangu ashiriki katika utafiti huu. Nimepokea nakala ya fomu hii ya kibali.

____
Jina la mtoto

____________________________________ ______________________
Sahihi ya mzazi Tarehe

____
Piga chapa jina la mzazi

____________________________________ ______________________
Sahihi ya mtafiti Tarehe

____
Piga chapa jina la mtafiti

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