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PRE-SERVICE ELEMENTARY TEACHERS LEARNING TO FACILITATE STUDENTS' ENGAGEMENT OF THE COMMON CORE STATE STANDARDS' MATHEMATICAL PRACTICES: BALANCING ATTENTION TO ENGLISH LANGUAGE LEARNERS, TO ALL LEARNERS, AND TO ONE'S OWN MATHEMATICAL LEARNING

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ABSTRACT

Keywords: English language learners; Mathematical Practices; pre-service teachers; mathematics methods course.

In this dissertation I examine the experiences of eight pre-service elementary teachers (PSTs) in a mathematics methods course as they learned how to teach Mathematical Practice 1 (make sense of problems and persevere to solve them) and Mathematical Practice 3 (construct viable arguments and critique the arguments of others) from the Common Core State Standards to elementary students in general, and to English language learners in particular. While the principal question that motivated this study concerned PSTs' preparation to teach mathematical practices to English language learners, it became apparent that that question could not be answered without considering how PSTs prepare to teach the mathematics practices to all learners, and how they learned mathematics themselves.

This descriptive case study, which uses qualitative methods, involves collection of the following data: open response surveys (pre-and post); homework reflections; lesson plans; university supervisors' and host teachers' reports; and semi-structured interviews of PSTs, university supervisors, and host teachers. I drew initial categories for coding these data from relevant literature, and transcribed, coded, categorized, and generated additional themes from the data (Glaser & Strauss, 1967). The study benefited from, and was limited by, the fact that I was both the researcher and the instructor of the course in which the PSTs were enrolled.

Applying sociocultural theory, I consider both PSTs' personal experiences and their interpretation of their students' personal experiences in their field placements (Forman, 2003). They understood these experiences in terms of six themes: making personal connections with mathematical content, providing access for individual students, holding high expectations for

each student, facilitating productive struggle, facilitating social interactions, and developing students' mathematical language and discourse. All of these themes are important in preparing PSTs to teach mathematics to all elementary students, but each of them has special significance for their preparation to teach English language learners. The PSTs appeared to learn the Mathematical Practices deeply, in part, by reflecting on the significance of these themes for their own mathematical learning.

The PSTs had similarities and differences in their beliefs and practices. As other researchers have suggested, it appeared that PSTs needed positive dispositions toward mathematics and the ability to help students make personal connections with mathematics to engage students in Mathematical Practice 1 (Kilpatrick, Swafford, & Findell, 2001), they needed high expectations and strategies for facilitating access in order to engage students in productive struggle (Moschkovich, 2013), and they needed strategies for facilitating social interactions and developing students' mathematical language and discourse to develop their facility with Mathematical Practice 3.

There seemed to be both a general and specific order for learning these themes. In general, PSTs grasped easily and most thoroughly those themes that were similar to ones infused throughout the teacher education program, and they were more likely to struggle with those that were new to them. PSTs who had no direct experience working with ELLs in their field placements had fewer opportunities to develop an understanding of how to engage these students, but some of these PSTs nevertheless developed understandings and skills that would be valuable for teaching ELLs, while one who did work with ELLs still maintained low expectations for their performance. Writing lesson plans helped PSTs think through how to facilitate students' engagement with mathematical practices; a student whose host teachers insisted they follow scripted lesson plans did not have that opportunity. My summary chapter presents two trajectories for depicting PSTs’ overall learning: one a general trajectory that seems to apply to their learning of any particular theme, and one a trajectory for depicting the order in which different themes are likely to be mastered.

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CHAPTER 1: INTRODUCTION

The Common Core State Standards in Mathematics have established Mathematical Practices that describe how students should be learning to approach mathematics from kindergarten through high school (Koestler, Felton, & Bieda, 2013). The Mathematical Practices place an emphasis on solving real-world word problems and developing critical reasoning skills that require students to communicate their understandings with others. Koestler et al. posit that it is up to teachers to decide how they are going to facilitate these practices among their students. The literature, however, is scant as to how English language learners (ELLs) can be taught to engage in these practices. Lucas (2011) believes that many teacher educators are not preparing pre-service teachers (PSTs) to recognize the importance that language plays in teaching ELLs. Schleppegrell (2010) points out that language plays a vital role in all students' mathematical learning. My study is based on the premise that the PSTs, their elementary students, and their ELL students in particular are all learning to master mathematics as a new language. In this process the PSTs reflect on their own experience as learners and consider how instruction of ELLs can be integrated with (rather than segregated from) instruction of other students. A course based on these same premises provides context for this study.

1.1 Rationale for Study

In Zeichner's (2005) research agenda for teacher education, he calls for more research on preparing PSTs to meet the needs of diverse students. Zeichner argues that teachers are not diverse and that more studies need to address how to meet all students' needs. He specifically states that there is a lack of research on PSTs and ELLs: "This research on the preparation of teachers to teach underserved populations should pay special attention to the preparation of teachers to teach English-Language Learners because almost no research has been conducted on this aspect of diversity in teacher education" (p. 745). Six years later, Zeichner (2011) wrote in the forward of an edited book dedicated to preparing PSTs to work with ELLs that there is a lack of research regarding teachers' preparation to instruct ELLs. He points out that most studies in this area are about ESL specialists. However, mainstream teachers, teachers who teach most of the subjects, spend more time with ELLs (Lucas, 2011) and Zeichner calls for research examining how those teachers can meet ELLs' needs. He praises the book for researching how teacher educators can help PSTs meet ELLs' needs, which he describes as an area of social justice.

In a summary chapter of this book, Lucas (2011) reaffirms the need for more studies on how teacher educators can prepare future teachers. Lucas claims that as a result of the growing ELL population more teachers will have ELLs in their classrooms and it is the responsibility of teacher educators to prepare PSTs to teach ELLs. The current practice of making slight adjustments to teacher education programs is not enough—a major transformation is necessary, states the author. Lucas summarizes that a common theme running through the book is that teaching does not meet ELLs' needs if teachers are not skilled at addressing ELLs' language needs (de Jong & Harper, 2011; Lucas, 2011; Lucas & Villegas, 2011; Valdes & Castellon, 2011; Villegas & Lucas, 2011). She reiterates that this preparation has been lacking in teacher education programs.

Another important theme that Lucas (2011) reports is that presently PSTs believe that it is not their role to develop ELLs' language. She describes de Jong and Harper's (2011) study, which claims that PSTs currently focus on attempting to allow ELLs access to content, and yet fail to see that it is also important for them to develop ELLs' English so that ELLs can be successful in monolingual classrooms. Lucas concludes that teacher educators themselves must be more prepared in regard to developing ELLs' English so that they can help PSTs achieve the same goal (Gort et al., 2011; Sakash & Rodriguez-Brown, 2011; Walqui, 2011).

Lucas (2011) also reports that attention to teaching ELLs should be infused throughout teacher education programs. She summarizes studies (Athanases & de Oliveira, 2011; Gort, Glenn, & Settlage, 2011) that depict that changes to teacher education programs are more effective when many instructors commit to discussing how to meet ELLs' needs. Lucas further says that there may be a variety of ways to change teacher education programs, but that we need research that informs decisions. She calls for research in all areas of teacher education related to preparing PSTs to work with ELLs.

My study addresses these authors' call for research to prepare PSTs and will focus on preparing PSTs to meet ELLs' needs in mathematics, an area for which such research is especially sparse (Turner, Drake, McDuffie, Aguirre, Bartell, & Foote, 2012). Turner et al. maintain that PSTs need to acquire the proper knowledge, dispositions, and practices to instruct meaningful mathematics to ELLs. The authors call for more research to determine how PSTs can acquire such traits. In addition to knowing how to prepare PSTs to teach mathematics to ELLs effectively, with the coming of the Common Core State Standards, it appears important to learn how PSTs can teach ELLs in ways that align with these Standards.

The Common Core State Standards of Mathematics (CCSSM) have eight Mathematical Practices that describe the process of how all students should be learning mathematics from kindergarten through high school (Koestler et al., 2013; Reys, 2013; Russell, 2012). If all students should be guided by the Mathematical Practices, then it will be imperative to research how PSTs can facilitate all students', including ELLs', engagement in these practices. This review will discuss two of the eight Mathematical Practices—the two that seem most relevant to ELLs due to their strong links to language.

Mathematical Practice 1, making sense of problems and persevering to solve them, has a link to language because if students do not understand the problems it will be challenging for them to make sense of and persevere to solve the problems. Teachers can help students make sense of problems by helping them connect their personal backgrounds to mathematical problems (Koestler et al., 2013). However, this is a challenging task for many teachers of ELLs because they typically have different backgrounds than ELLs (Howard, 2006; Nieto, 2003).

Mathematical Practice 3 is linked to language because it requires students to develop viable arguments and critique other students' arguments. The CCSSM report that successful students of Mathematical Practice 3, "justify their conclusions, communicate them to others, and respond to the arguments of others" (CCSSI, 2010, p. 6). Unfortunately, teachers do not have the preparation to facilitate ELLs to develop arguments and critique the arguments of others (Lucas, 2011; Lucas & Villegas, 2011; Zeichner, 2005). Furthermore, the language used in mathematics is more challenging than that of other subjects (Schleppegrell, 2010; Zwiers, 2008), which makes it difficult for ELLs to participate in discussions. Teachers often assume that all students understand words such as "plot," "calculate," and "compare," which Zwiers refers to as "mortar terms." These words lack context and thus are challenging. Additionally there are many words that have different meanings inside and outside of mathematics classes.

It is also important for PSTs to know how to engage all students, including ELLs, in the other six Mathematical Practices. My study examines PSTs who take the first of two mathematics methods course in which Mathematical Practices 1, 3, 4, and 5 were addressed. The remaining four Practices were taught the second methods course. Mathematical Practice 4

(Model with mathematics) appears to be relevant for ELLs because it involves students applying mathematics to their everyday life (CCSSI, 2010). If PSTs and teachers have a different background from the ELLs in their class (Howard, 2006; Lucas, 2011; Sleeter, 2001), PSTs may need to be taught how to engage ELLs in this Practice. Koestler et al. interpret this practice as an opportunity for teachers to tie mathematics into students' everyday lives. Furthermore the authors state that students can engage in this practice as they are learning about equity issues. Mathematical Practice 5 (Use appropriate tools strategically) also appears to be relevant to ELLs because the literature documents that ELLs can learn concepts more deeply when they are provided with tools (Maldonado et al., 2009; Moschkovich, 2013). Maldonado et al. (2009) studied how to encourage English Language Learners to share in the class discourse. They found that manipulatives empowered ELLs by allowing them to explain concepts to their peers that would not be possible to do without these tools.

The Common Core State Standards state that all students, including ELLs, should be engaged in the Mathematical Practices. While the including the other Mathematical Practices in my study could have elicited more information, I have chosen to focus on the first and third Practices because of their strong language components. By studying these Practices I respond to Lucas' (2011) call for more research about preparing PSTs to meet ELLs' language needs. However, I encourage more studies about how PSTs and teachers can engage ELLs in the other Mathematical Practices as well.

This multiple descriptive case study is important because it investigates PSTs learning Mathematical Practices themselves, and documents PSTs facilitating elementary students', including, ELLs', engagement in these Mathematical Practices.

1.2 Research Questions

This research employs case studies of eight PSTs and documents the PSTs learning to facilitate elementary students', including ELLs', engagement in Mathematical Practices 1 and 3 from the Common Core State Standards. The research question that motivates this study is:

1) How do PSTs learn to facilitate ELLs' engagement in the Common Core State Standards' Mathematical Practices?

The first question cannot be answered without addressing three additional questions:

2) How do PSTs make sense of the Mathematical Practices?

3) How do PSTs perceive they should facilitate elementary students' learning of the Mathematical Practices and how do these perceptions change over time**?**

4) How are the experiences of PSTs learning to teach elementary students, including ELLs, similar or different; and what may account for any differences?

Many teacher educator programs do not offer PSTs opportunities to instruct ELLs in their placements (Athanases & Oliveira, 2011). Only two of the eight PSTs in my study had ELL placements, but all eight needed to prepare to teach ELLs (Lucas, 2011). That is why in my study I am interested in examining both PSTs who have ELLs in their placements and those who do not. I consider how all of the PSTs understand and perceive their preparation for teaching ELLs. I examine whether those who do not have this experience in their field placements are completely unprepared to teach ELLs, or whether their preparation to teach elementary students in general—and their own experience with learning mathematics themselves—lay the foundation for working with ELLs in the future.

Therefore, there are parts of my study that focus specifically on how PSTs experience learning mathematics themselves, parts that focus on the process of preparing PSTs to teach ELLs, and also parts that focus on the more general preparation of elementary students. I use case study research to focus on themes from the literature about ELLs, PSTs, and mathematics to compare the experiences of PSTs who taught ELLs and those that did not.

1.3 Definitions of Key Terminology

Deficit model: A focus on ELLs' weaknesses, such as not being proficient in English, rather than valuing and building on ELLs' strengths (Moschkovich, 2013).

English Language Learners (ELLs): For the purpose of this study, ELLs are students who are currently receiving English as a second language service.

English as a second language (ESL): A service that is offered to ELLs to assist them with their English.

Field placement: This is when a PST is assigned to a school to assist a host teacher.

Host teacher: This is the teacher that monitors PSTs in their placement.

Linguistics course: A course that teaches about the structure of English and other languages.

Mainstream teacher: A teacher who teaches all or most subjects to students.

Mathematical Practices: Eight practices from the Common Core State Standards that guide how students should be doing mathematics from kindergarten to twelfth grade.

Mathematical Practice 1: To make sense of problems and persevere in solving them (CCSSI, 2010.) The main intent of this Practice is for students to develop problem-solving skills (Koestler et al., 2013)

Mathematical Practice 3: To construct viable arguments and critique the reasoning of others (CCSSI, 2010). Larson et al. (2012) maintain that this Practice involves students discussing mathematics.

Methods course: A course taught in a teacher education program that teaches PSTs pedagogy.

Pre-service teachers (PSTs): College students enrolled in teacher education programs.

Productive disposition: A belief that mathematics is useful and that students are capable of being successful in the subject (Kilpatrick et al., 2001)

Productive Struggle: When students are given problems that are challenging enough for them to struggle and yet still within their grasp to solve (Dixon, Adams, & Nolan, 2015).

Provide access to content: PSTs make an accommodation for a student or students so they can make sense of the mathematical tasks.

University supervisors: In this study, they are retired teachers who observe, support, and assess PSTs in the field.

1.4 Overview of the Dissertation

The dissertation is divided into six chapters, references, and an appendices section. The first chapter consists of an introduction, a rationale for the study, a discussion of my research questions, and definitions of key terminology. Chapter two offers a review of the literature and explains how the literature relates to my study. The third chapter includes a rationale for the study's methodology, a conceptual framework, an explanation of the context, research design, and selection of participants, data collection, data analysis, ethical concerns, and limitations. Chapter four presents findings and an analysis of data related to Mathematical Practice 1. Chapter five presents findings and an analysis of data related to Mathematical Practice 3. The last chapter includes a summary of findings, implications, limitations and recommendations for further research, and conclusions.

CHAPTER 2: LITERATURE REVIEW

In this chapter, I review general teacher education before discussing how PSTs can be prepared to teach ELLs. I also review themes that may help PSTs learn Mathematical Practices and teach according to them. For reasons outlined in Chapter 1, my emphasis will be on Practices 1 and 3. Lastly, I discuss literature on trajectories in mathematics education, or the sequence(s) by which different content or skills are taught and learned. I end by presenting my conclusions about how my own study can build upon these concepts from the literature.

2.1 Teacher Education

The purpose of this first section is to introduce themes about general teacher education that anticipate more specific themes that arise in my data. For example, in my research I focus on the dispositions of PSTs toward mathematics, but in this section I present a somewhat broader view—how dispositions are defined, measuring PSTs' dispositions, and the importance of PSTs having a positive disposition towards all students, including students with disabilities. I discuss cooperative learning and inclusion. The last theme that I include in this section is facilitating PSTs to be culturally responsive.

2.1.1 Fostering PSTs to Have Positive Dispositions

According to National Council for Accreditation of Teacher Education, dispositions are:

> The values, commitments, and professional ethics that influence behaviors toward students, families, colleagues, and communities and affect student learning, motivation, and development as well as the educator's own professional growth. Dispositions are guided by beliefs and attitudes related to values such as caring, fairness, honesty, responsibility, and social justice. For example, they might include a belief that all students can learn, a vision of high and challenging standards, or a commitment to a safe and supportive learning environment. (NCATE Glossary of Terms at, http://www.ncate.org/search/glossary.htm)

This definition of dispositions has the potential for many applications of the concept since it can encompass a variety of beliefs and attitudes towards a variety of subjects. In a sample of 86 undergraduate and graduate students, Singh and Stoloff (2008) attempted to measure the dispositions of PSTs in a teacher education program in Connecticut by using a self-assessment tool. The 46 Likert questions were mostly about self-perceptions, perceptions of others, and perceptions of content. The researchers caution that the tool ought to be tested more to ensure its validity, but they claim that the study suggests that the PSTs at this university had positive dispositions. If teacher educators want PSTs to have positive dispositions, then it will be useful to have ample research on the subject. My multiple case study adds to the research by examining PSTs' dispositions towards mathematics as they facilitated students to engage in mathematics.

Ambrose (2004) also attempted to use a tool to measure PSTs' dispositions towards mathematics. Ambrose created a survey to detect any changes in beliefs that PSTs' had about mathematics after taking a methods course. The survey was similar to Likert scales but differed in that pre-service teachers were given an opportunity to write their own responses rather than choosing pre-selected responses. Ambrose found this research methodology to be more efficient than the Likert scales, but also time consuming because of the time required to code the responses. The survey and scoring rubrics took two years to develop and were tested on 15 mathematics education classes. One hundred fifty-nine prospective teachers were shown video clips before and after a methods course—they were asked to fill out a survey both times. The responses to the survey were classified into four categories, from no evidence to strong evidence that beliefs had been changed. The study suggested that the PSTs changed their beliefs about mathematics after completing the course and were more critical of the teachers they observed in the videos. This study illustrates the use of a tool that may be used to evaluate the effectiveness of methods courses in improving PSTs' dispositions.

In another study, 29 elementary PSTs from New Zealand participated to determine how field placement influenced their dispositions (Grootenboer, 2006). Data were collected three times during the study: before the mathematics methods course; directly after the methods course; and after their field placement. Quantitative data were collected on the PSTs' disposition with a Likert-scale survey. There was a significant increase in the participants' appreciation for mathematics after taking the methods course. This effect diminished after field placement started, but did not disappear. Upon further analysis, the biggest drop in appreciation was from the PSTs who worked with older students. Qualitative data revealed a possible explanation.

Many of the PSTs of younger students were comfortable with what they observed in the classrooms; there was an alignment between what they had learned in the methods classes (e.g., use of manipulatives) and what was used in the classrooms. In contrast, Grootenboer (2006) believes that many of the PSTs who returned from the older student classrooms scored lower because they observed little congruence between the methodology learned in the university and that used in the classroom. Also, many of the practices observed in the classes of the older children reminded the PSTs of their own negative childhood experiences with mathematics. Grootenboer concluded that field experience plays an important role on PSTs' affective views towards mathematics, so it is vital that this experience be positive.

In summary, the studies discussed suggest that it is important to measure PSTs' dispositions towards their subjects to determine the effectiveness of both methods courses and field placements. My multiple case study contributes to such research by examining PSTs' dispositions as they facilitated students to engage in mathematics.

2.1.2 Teaching Cooperative Learning to PSTs

My study focuses on Mathematical Practice 3 (develop arguments and critique the arguments of others), which according to Larson et al. (2012) involves students discussing mathematics. The PSTs solved mathematics problems and discussed pedagogy in cooperative learning groups in a mathematics methods course and were engaged in Mathematical Practice 3 as they were discussing mathematics. Teaching cooperative learning may also encourage PSTs to develop positive dispositions. According to Johnson and Johnson (2009), who summarized the literature on cooperative learning, there have been more than 1,200 research studies in the past 110 years on the topic. The authors did not define the term, but in an earlier article, Johnson, Johnson, and Holubeck (1994) describe cooperative learning as students working together to ensure that all members of the group master the content. In their summary, Johnson and Johnson state that cooperative learning is widely emphasized in many subjects and practiced in schools all over the world. The authors summarize that when students have positive interdependence it is not only more motivating for students than independent work, but higher-level thinking is promoted as well. Cooperative learning groups are not effective, however, when there are not common goals among the group or when the members of the group work individually. The authors also point out that apart from academic achievement, cooperative learning promotes students to have better social skills, and have positive dispositions.

Despite the benefits, the literature suggests that teacher educators are not placing enough emphasis on cooperative learning (Bouas, 1996; Johnson & Johnson, 1985). Using a Likert scale survey with 14 PSTs and interviews of two PSTs, Bouas measured the attitudes of PSTs towards cooperative learning after it was stressed in three methods classes. The author found that the PSTs enjoyed working in cooperative learning groups in the methods class. They also found that the two PSTs interviewed said that they would likely implement cooperative learning when they were teachers even though they felt that they needed more preparation. Bouas comments that it is important for teacher education programs to stress cooperative learning in the field as well as in methods courses. Despite the recommendation, her study did not actually analyze the PSTs in the field. My study considers the PSTs in their methods courses as well as in their field placements. Also, many of the studies I have discussed use Likert scales to measure PSTs' dispositions; I use qualitative case study analysis. Each approach has its advantages, so it good to have evidence from both.

2.1.3 Teaching PSTs to be Inclusive

The participants in my study were in an inclusion program, preparing to be certified as both elementary teachers and special educators in a regular classroom. While their preparation to teach students with disabilities is not directly applicable to their preparation to teach ELLs, their preparation to teach students with special learning needs in regular classrooms, to avoid deficit assumptions about their students' competence is certainly relevant. Researchers have documented that preparing PSTs to have positive dispositions towards students with disabilities has been challenging; therefore, teacher education programs should assess how effective they are at helping PSTs achieve this goal (Shade & Stewart, 2001). Shade and Stewart contend that teachers need to be prepared to teach students inclusively, which they define as teaching students with disabilities in general education classrooms. In a book written for teachers and PSTs, Reynolds and Birch (1977) state that teacher education programs must ensure that all PSTs have training in inclusive education. However, Reynolds and Birch posit that inclusive practices will not be implemented effectively if teachers do not have positive attitudes about students with disabilities themselves. In the next paragraph, I discuss an empirical study that attempted to measure PSTs' dispositions towards students with disabilities before and after taking a methods course that stressed inclusion.

Cook (2002) finds that after studying 181 PSTs in a Midwestern university, they reported that their preparation for inclusion was not adequate. Cook argues for PSTs to be trained to have positive dispositions for students with disabilities, but the PSTs commented that there were not enough field placements offering PSTs opportunities to work with students with disabilities. Cook maintains that to make an impact on PSTs' dispositions, it is imperative to not only discuss important themes in methods classes, but also to reinforce these beliefs in PSTs' placements. In my study, I examined PSTs dispositions towards all students, including ELLs, when only two of the 22 PSTs in the course had ELLs in their placements.

Wood (1998) suggests that teachers make accommodations for all students by adapting teaching strategies, materials, and assessments. The PSTs in my study were taught in the inclusion program to make adaptations for students and also taught the concept of universal design. In Universal Design teachers accommodate all students' needs at the planning stage rather than trying to add on to or "retrofit" lessons that have already been planned (Rose & Meyer, 2002). Rose and Meyer (2002) summarize by saying that students with disabilities are often stigmatized because they are viewed as being different from other students. They state that if teachers view students with disabilities with deficit views--thus focusing on a student's weaknesses, rather than valuing and building on strengths (Moschkovich, 2013)--then the other students in the class may have the same views towards students with disabilities. In my study, three of the PSTs appeared to have deficit views towards ELLs.

Shade and Stewart (2001) measured dispositions of 122 general education and 72 special education PSTs. The course in the study offered to freshman and sophomores included lectures, audiovisuals, small group discussions, role-play, and empathy building activities. They found that according to a Likert survey, the dispositions toward students of disabilities for both the general education and special education PSTs improved after taking a single course. However, as I discuss in the next two sections, many teacher educators argue that offering a course in isolation is not as effective as immersing PSTs in new practices throughout the program. Another question that I discuss in my research is whether PSTs' background experiences affect their dispositions.

2.1.4 Facilitating PSTs to be Culturally Responsive

One of the leading authors in teacher education, Darling-Hammond (2011), in a summary about teacher education and diversity, writes that a very important goal for teacher educators should be to help PSTs be culturally responsive. In an article about culturally responsive pedagogy, Ladson-Billings (1995) describes the term as instruction that considers students' culture as they develop their knowledge, skills, and attitudes. There is much in the literature discussing what skills teachers and PSTs need in order to meet the needs of the diverse students. Gay (2002) theorizes that teachers should consider all students' cultural backgrounds. She posits that ethnically diverse students' learning improves when teachers are culturally responsive. Kinloch (2012) adds that it is imperative for teachers to motivate students by tying school into students' home lives. In a case study, she describes how a teenager improved in English when his teacher let him write about his community. Gay and Kinloch remind teachers of the importance of connecting with their students. One theme that I examine in my research is how PSTs can connect mathematics with students' backgrounds.

Nieto (2002) believes that many teachers are not relating to their diverse students. She argues that too often teachers engage in the practice of "ethnic tidbits" (p. 8), but this does not meet the diverse students' needs. Nieto asserts that education should be based on a premise that all students can learn and deserve the opportunity to do so. Teachers need to value students' backgrounds and culture on a daily basis. Ladson-Billings (2009) maintains that it is unfair to attempt to teach everyone the same. "The notion of equity as sameness only makes sense when all students are exactly the same" (p.36). She states that teachers would not expect a visually impaired student to read small print or a student in a wheelchair to do push-ups, so instead of treating all students the same, teachers need to treat each student differently in order to be fair. Thus, applying Ladson-Billings' theory, instead of teaching all students the same, teachers should consider each student's individual culture when instructing. Marx (2006) argues that teacher education programs are not successful helping PSTs to accommodate all students' needs unless they encourage them to reflect about their privileges. After reflecting on their white privileges, many of the PSTs became aware of social injustices and vowed to be culturally responsive. Marx comments that becoming aware of white privileges is a continuous path and one that should be reflected on often. Nieto, Ladson-Billings, and Marx offer suggestions for meeting diverse students' needs, such as valuing their culture, treating them as individuals, and reflecting on how to be more culturally responsive.

Grossman, Hammerness, and McDonald (2009) add that it is not enough merely to offer PSTs one multicultural course, or require them to take it; it is more effective to infuse multicultural education throughout the program. Darling-Hammond argues that PSTs have different beliefs about diversity depending on their own experiences as students. Teachers find it challenging to meet all students' needs because, as many researchers reveal, there is a mismatch between predominately white teachers and their diverse students (Howard, 2006; Lucas, 2011; Sleeter, 2001). According to Lucas, most white PSTs have had limited experiences outside of teacher education programs with people that are not also white. Therefore, one challenge for teacher educators is to ensure that the predominately white PSTs' dispositions are positive towards diverse students. In my study I discuss how the PSTs' previous experiences affect how they talk about diverse students.

Similarly, the predominantly white teacher educators are challenged to encourage PSTs to work with diverse students (Ladson-Billings, 2011; Nieto & McDonough, 2011). Zeichner (2009) suggests that teacher educators reflect on how well their programs are preparing PSTs to work with diverse students, and he states, "The white, monolingual, English-speaking, education professors and staff that are responsible for educating teachers for diversity often lack experience themselves in teaching in culturally diverse elementary and secondary schools" (p. 3). If the teacher educators do not have classroom experience themselves, then it may be challenging for them to facilitate PSTs to meet diverse students' needs.

Teacher educators have made progress in the last ten years, according to Nieto and McDonough (2011), by offering more social justice and multicultural courses. The researchers reviewed articles published after Nieto's article in 2000 that called for equity to be a central issue in teacher education, and they attest that professors are teaching PSTs to reflect by having discussions and reading about race, reading stories that are aimed at challenging PSTs' deficit beliefs, and encouraging PSTs to write autobiographies that question their white privileges. Marx (2006) too mentions the importance of reflection. After reflecting on their white privileges, many of the PSTs became aware of social injustices and vowed to teach in a multicultural fashion. Marx comments that becoming aware of white privileges is a continuous path and one that should be reflected on often. Nieto and McDonough (2011) argue that more research should examine what impact programs are having on PSTs and whether they are retaining lessons on social justice that are raised during their teacher education preparation. The authors above note that more teacher educators are discussing diversity, but they say that more research is necessary to offer educators effective strategies.

In questioning the effectiveness of methods courses, it is also important to study what impact host teachers and university supervisors have on PSTs. If PSTs are going to apply what they learn in multicultural courses, host teacher and university supervisors should be sympathetic to that goal. However, there is little research on the role of university supervisors in teacher education (Hartwick & Johnson, 2008). One study published twenty years ago may still be relevant today: it maintains that PSTs do not have freedom to take chances in teaching topics such as multicultural education unless these are valued by their university supervisors and host teachers (Grant & Zozakiewicz, 1995). Host teachers are under pressure to follow a curriculum and do not always find time to teach multicultural education (Hartwick & Johnson, 2008). Grant and Zozakiewicz found that classroom management was the most dominant theme in discussions among PSTs, host teachers, and university supervisors—issues such as multicultural education were rarely mentioned. Valencia, Martin, Place, and Grossman (2009) uphold that PSTs are capable of exercising skills learned on campus, but they are afraid to challenge both their supervisors and their host teachers. In my study, I consider the influence that the host teachers and university supervisors may have had on the PSTs attempting to facilitate student engagement in Mathematical Practices.

In a report of teachers who were working with successful teachers of African American students, Ladson-Billings (2009) compared the teaching skills of a PST she observed with those of an experienced teacher. The white PST struggled to keep all students on task during a mathematics class. By contrast, the experienced teacher taught mathematics for an hour and a half without having to reprimand the same students. As Ladson-Billings reports, excellent teachers possess high expectations for their students and strong knowledge about the content and students, and they connect the material to the students' lives. It would be useful to study how PSTs learn to have high expectations for their students, develop strong content knowledge, and make connections for students the way successful teachers do. In my study I pursue all these themes as they apply to PSTs making sense of how to teach the Common Core Mathematical Practices to all students.

2.2 PSTs Need Special Preparation to Teach ELLs

In spite of the recent attention to multicultural teacher education, most researchers have concluded that teachers are not prepared to instruct ELLs (Lucas, 2011; Zeichner, 2005). De Jong and Harper (2011) assert that taking a general diversity course does not prepare PSTs to work with ELLs—being culturally responsive with ELLs is important, but teachers need to have specific strategies and dispositions for instructing ELLs. They state, "Simply 'tweaking' standard curriculum practices for native English speakers by using techniques to increase comprehensible input will not sufficiently, or necessarily, scaffold ELLs' socio-cultural, academic, language, and literacy development" (p. 86). Untrained teachers, according to de Jong and Harper, judge ELLs with a monolingual lens and fail to take advantage of the opportunities to acknowledge ELLs for being bilingual. The authors emphasize that PSTs need to be taught strategies to meet ELLs' specific needs—I now examine the literature in a discussion of how these specific needs might be met.

Literature on how mainstream teachers can best instruct ELLs is at its early stages (Lucas, 2011; de Jong & Harper, 2011). Lucas states that there are three elements that teacher educators need to consider when instructing PSTs how to work with ELLs—curriculum content, program structures, and program coherence.

2.2.1 Curriculum Content

Lucas (2011) describes curriculum content to be "what teachers need to know and be able to do to teach ELLs well" (p. 6). Most of the research and the literature on teaching ELLs is designed for teachers of English as a second language (Lucas, 2011; de Jong & Harper, 2005). The literature on how regular educators teach ELLs is limited, and the literature on how they prepare to teach these students is sparse (Lucas, 2011). Lucas summarizes, "Whatever the reason, the omission of content related to teaching ELLs in many teacher education programs leaves teachers unprepared to teach students who come to their classes not yet fully proficient in English" (pp. 6 & 7). Lucas argues that the research on English as a second language is not always relevant to mainstream teachers—PSTs and mainstream teachers need research-based advice on how to meet ELLs' needs in classrooms with a mix of ELLs and non-ELLs.

In a widely-cited article (over 214 citations according to Google Scholar), Harper and de Jong (2004) argue that the common misconception that ELLs can be taught the same way as non-ELLs is one reason why mainstream teachers are not learning how to develop language skills for ELLs. The authors propose that providing cooperative learning groups and visuals is not enough for ELLs to become proficient in academic English. In fact, though visuals may allow ELLs access to the content being taught, Harper and de Jong point out that this does not necessarily aid ELLs in communicating their understandings with others. The authors add that ELLs do not acquire proficiency by mere exposure to English; teachers need to address their specific linguistic needs as well. For example, to facilitate their success, teachers should provide ELLs with vocabulary words before reading an article or having a discussion. This article is informative, but is not itself an empirical study and the research that it summarizes is outdated. I offer an empirical study that examines PSTs' perceptions of how to best accommodate ELLs' needs and develop ELLs' language skills.

Typically PSTs do not know enough about the structure of English and other languages to be able to develop ELLs' language skills and therefore should be taught in teacher education programs (Lucas, 2011). For example, Harper and de Jong (2004) explain the value of teachers learning cognates—words that have similar meanings in English and in ELLs' first language—to allow ELLs access to the content and to develop their English proficiency. The researchers mention that one component of teacher education should be to teach PSTs linguistics, such as cognates, so they will be more proficient in developing ELLs' language skills.

Schleppegrell (2010) argues that teachers need to build on students' everyday language and help students change from using everyday language to academic language. Academic language, the language teachers, textbooks, and assessments use to address specific content, is more challenging and complex than everyday language (Shatz & Wilkinson, 2010). Schleppegrell believes that it is not possible to learn a subject without also learning the language specific to that subject. Guerrero (2004) explains how academic language affects ELLs. He argues that it takes more than a year for ELLs to acquire academic language and that ELLs are required to learn the academic language of each subject. Zwiers (2007) conducted a study on secondary school teachers and found that those who had strong knowledge about the language involved in their discipline were more likely to teach academic language to students as they teach content. He also concluded that teachers can be taught to teach their students to use academic language correctly by connecting to students' personal lives and providing students with activities that promote students to talk. Moschkovich (2013) add that students should not be taught academic language by memorizing lists of vocabulary; they should learn vocabulary in context.

Although researchers recommend that PSTs acquire knowledge about the English language so they can teach it better, there is a challenge in persuading PSTs that this is important (de Jong & Harper, 2011). These authors interviewed 180 PSTs in their fifth year of studies. They concluded that PSTs did not find linguistic knowledge to be as important as cultural knowledge when teaching ELLs. When asked what a teacher would need to know in order to help ELLs, the PSTs' most common answers were related to culture and affective responses (having patience) rather than developing language. Their study suggests that PSTs believe that it is not their role to develop ELLs' language skills. De Jong and Harper explain that teacher educators need to convey to future teachers the importance of their having a solid background in linguistics in order to meet ELLs' needs. This quantitative study is valuable to help inform teacher educators that PSTs need to be persuaded that they have a role in developing the language of ELLs. However, more research, including qualitative studies, is needed to test their conclusions. In my research I ask the participants in a pre-survey and a post-survey to describe the role of teachers of ELLs; I also use interviews to explore these themes with my subjects.

Apart from learning how to meet ELLs' language needs, Lucas (2011) asserts that scholars agree that PSTs also need to be taught that "language is closely tied to its sociopolitical context and can be a powerful political force for the inclusion or exclusion of particular groups" (p. 7). It is imperative for PSTs to learn that all languages and speakers of language are of equal status (Lucas & Villegas, 2011). Lucas and Villegas also stress that teachers need to value students' efforts to learn a second language rather than paying too much attention to the grammatical mistakes that they make. Teachers must advocate for ELLs—which according to Lucas and Villegas, can be anything from meeting ELLs' needs to welcoming family members to the classroom. The authors add that teachers ought to advocate for ELLs by fighting against the injustices that occur in our monolingual society.

In a description of ELLs in American schools, Valdes and Castellon (2011) maintain that the climate towards Spanish speakers, the biggest group of ELLs, is unwelcoming and that teacher educators should prepare PSTs to be more tolerant with ELLs. I have discussed how PSTs should be taught both how to meet ELLs' language needs and to advocate for their sociopolitical rights.

2.2.2 Program Structures

Lucas (2011) points out that teacher educators need to both determine what skills the PSTs need to best support ELLs and to acquire the skills themselves. Ideally attention to the instruction of ELLs would be infused throughout the program. If this could be done, PSTs would receive the message that teaching ELLs is valued because it is stressed throughout the curriculum (Lucas, 2011; Walker & Stone, 2011). Lucas argues that if it is not possible to train all the teacher educators about teaching ELLs, then at least one professor should become the expert and a course should be offered to PSTs. However, Lucas notes that adding an extra course would be time consuming for PSTs' busy schedules. Athanases and Oliveira (2011) describe a program in which all the teacher educators incorporated teaching to ELLs in their curriculum. The agricultural educator led discussions about advocating for migrant farmers' children who may not have the same knowledge of farm animals as other children have. In the mathematics methods course, vocabulary was considered for the benefit of the ELLs, and in a science methods class, concept maps were used to facilitate ELLs' access to the content.

In a report of case study research Gort, Glenn, and Settlage (2011) discuss how two white, monolingual teacher educators are influenced as a result of professional development on ELLs. A bilingual educator offered to lead her colleagues in discussions about training ELLs. After the professional development, the science and English educators altered their syllabuses to accommodate ELLs. The educators also changed their teaching styles and looked for help from bilingual learners; one asked a graduate student to teach a lesson in Spanish so the PSTs could appreciate how ELLs feel when they learn content in another language. Both instructors had included course materials in their course about diversity, but after professional development, they added components of their methods courses that involved linguistics for ELLs. Their case study documents how two teacher educators were influenced by changes that their university had made. More research is needed in all areas regarding how mainstream PSTs can meet ELLs' needs (Lucas, 2011). My study offers guidance for how PSTs perceive the challenge of meeting ELLs' needs as well as how this challenge is similar to or different from the challenge of meeting all students' needs in mathematics.

Another option for training PSTs to meet ELLs' needs is to offer or require them to learn a second language, study linguistics, or have a minor in ELL training (Lucas, 2011). This option requires teacher educators to be prepared to offer professional development to PSTs and in-service teachers. Lucas encourages teacher education programs to hire new staff with expertise in ELLs and offer training to all teacher educators on campus. If teacher educators are better prepared, they can better train PSTs and teachers to accommodate ELLs' needs. Although more research is necessary to determine how teacher educators can best prepare PSTs to meet ELLs' needs, it stands to reason that as Lucas theorizes, teacher educators need to be prepared themselves.

2.2.3 Program Coherence

Training PSTs to work with ELLs should be a central goal throughout the teacher education programs (Lucas, 2011). This means, according to Lucas, that teacher educators should consider what PSTs are learning on campus as well as in their placements. Lucas discloses that, unfortunately, there are too many teacher educators who themselves are unprepared in how to teach ELLs. As a result, she says, programs are patchy and inefficient in training PSTs to work with ELLs. Lucas, therefore, calls for teacher education programs to have coherence, which she defines as having a common mission throughout. She adds that there are few examples of successful coherent programs to be found in the literature. Athanases and Oliveira (2011) call for teacher educators to have open discussions on how their programs can be more coherent in regard to training PSTs to teach ELLs.

Athanases and Oliveira (2011) designed a framework, based on Darling-Hammond and Bransford's (2005) work, which guided them in analyzing a case study about the coherence of a program that instructs PSTs in how to work with ELLs. The authors examined the program's coursework and placements, and they observed that teacher educators, PSTs, host teachers, and families all had common beliefs about teaching ELLs. They attributed part of the success to the fact that the teacher educators had strong ties with the community and with the classroom teachers. The authors encourage teacher educators to offer professional development to host teachers so that PSTs can benefit from common messages throughout their program.

Although as I have discussed, teacher educators need to reflect on how to train PSTs to better meet ELLs' needs, Settlage (2011) cautions teacher educators against making deficit assumptions about white, monolingual PSTs. He points out that we should be careful to avoid thinking that all PSTs are incapable of teaching ELLs. Settlage interviewed five PSTs who volunteered to reflect on ELLs and found that even though white and monolingual, they showed signs of being culturally responsive, and they had gone through identity shifts. Settlage encourages teacher educators to be open to the fact that PSTs can meet diverse students' needs.

In the previous section I mentioned that my study considers how university teachers and host teachers influence the PSTs. Santos, Darling-Hammond, and Cheuk (2012) add that university supervisors and host teachers should be selected who have training in the area of ELLs. The researchers also call for schools to support teachers who may have expertise in meeting ELLs' needs. They argue that other teachers should be given time to observe, discuss, and reflect about the practices of teachers who have experience working with ELLs. Santos et al. encourage schools to have networks to prepare elementary teachers to teach content and develop ELLs' language skills. In short, teacher education programs and schools need to value and recruit instructors who have training with ELLs.

2.2.4 Concluding Thoughts

In a summary of empirical studies on how PSTs can learn to teach ELLs, Lucas (2011) explains that there is no consensus as to the best method of preparing PSTs to teach ELLs. Lucas calls for more research on preparing PSTs to teach ELLs in all areas: "Recommendations for research are relatively obvious: we need more research on almost every aspect of the preparation of mainstream teachers for teaching ELLs" (Lucas, 2011, p. 220). She calls for the following studies on ELLs—policy, classroom teachers, curriculum, and teacher educators. Lucas continues to argue that most teacher education programs are making changes to meet ELLs' needs, but these changes need to be documented so that we can discuss best practice. Lucas claims that teacher educators have an especially important role in educating PSTs to meet ELLs' needs because both the PSTs and teachers have minimal preparation in this area. I will address her call for how teacher educators can prepare PSTs to meet ELLs' needs. I examine and document how PSTs perceive the challenge of engaging ELLs in the CCSSM in the context of a mathematics education methods course that aims to address how PSTs can meet all students' needs, including those of ELLs.

None of the literature that I have reviewed on teaching ELLs addresses the problem of teaching PSTs to work with ELLs when they don't have opportunities to work with them in field placements, and nearly all of it frames the challenge of teaching ELLs as a separate issue, unrelated to the challenges of teaching other students. Implicitly that frames the issue of teaching ELLs as a stand-alone topic to be tacked onto other objectives. There needs to be research on how preparation to teach ELLs can be integrated with preparation to teach all students. The questions in my study examine PSTs' perceptions of learning to teach mathematics to ELLs, and how these perceptions are similar to or different form the challenges of teaching mathematics to other elementary students.

2.3 Mathematical Practices

My research involves PSTs learning the Mathematical Practices. The CCSSM have specific content standards for each grade, but they also include eight Mathematical Practices that describe how students should be "doing" mathematics from kindergarten to high school (Koestler et al., 2013). Koestler et al.'s comment about the Mathematical Practices describing how students should be doing mathematics is fundamental—they imply that all states that have adopted the CCSSM Standards should be incorporating these eight practices into their teaching. Russell (2012) refers to the Mathematical Practices as the "backbone of the CCSSM" (p. 56). Reys (2013) also stresses the importance of the Mathematical Practices, stating, "These standards are given front-page attention in the CCSSM (that is, they are introduced early in the document and intended to permeate the entire K-12 curriculum)" (p. v). Considering the importance of the Mathematical Practices, it will be imperative to determine how to engage all students in these Practices.

Koestler et al. (2013) theorize that it is the teacher's responsibility to decide how they are going to facilitate these practices for their students. Although Koestler et al. suggest how teachers may engage students in the Mathematical Practices; theirs is not an empirical study. Another author, Russell (2012), theorizes that teachers should make an effort to focus on a different Mathematical Practice during each lesson taught. She recognizes that there will be overlap, and many lessons will address more than one Practice, but by focusing on at least one Practice, the Mathematical Practices will be given the attention they deserve. There is limited research documenting how teachers may facilitate the engagement of students in the Practices and even less research documenting how PSTs may learn to facilitate the engagement. If the Mathematical Practices describe how students should be doing mathematics, then it will be imperative to know how PSTs can facilitate all students' engagement in these practices. This review will discuss the two Mathematical Practices chosen for this study—Mathematical Practices 1 and 3—because these are the two Practices for which a command of English seems most important. My primary focus will be on six themes in the literature that are especially relevant in addressing how teachers and PSTs may facilitate the engagement of elementary students in Mathematical Practices 1 and 3.

2.3.1 Mathematical Practice 1

There are several references in the literature about the importance of problem solving in relation to Mathematical Practice 1 (Making sense of problems and persevere in solving them). According to Schoenfeld (2013), teachers can align to Mathematical Practice 1 by planning opportunities in their lessons for students to make sense of problems. He also stresses the importance of students making sense of problems themselves. Koestler et al. (2013) add that problem solving is the central focus of the first Practice. In a discussion on the Mathematical Practices for professional development, Larson et al. (2012) hypothesizes that the goal of Mathematical Practice 1 "is for students to become successful problem solvers" (p. 31).

Apart from problem solving there are six themes mentioned in the literature that may help PSTs engage elementary students in Mathematical Practice 1. Four of the six appear frequently in my data, and I will discuss them first. The last two themes ("Productive Disposition" and "Content Knowledge") are well documented in the literature (see citations below) and seem important to the interpretation of my data, but they appear in my data with less frequency. I begin the discussion of each theme by summarizing general research on that theme, and then discuss the research applying that theme to ELLs.

Connections to mathematics. Koestler et al. (2013) explain that for students to make sense of mathematical problems and solve them—the first part of Mathematical Practice 1—students need to connect their background knowledge with the mathematical tasks. Students will be more likely to persevere to solve problems if teachers tie the content to students' lives (Koestler et al.). Clarke, Roche, Sullivan, and Cheeseman (2014) add that teachers can make mathematical connections for students by choosing tasks that are relevant to their personal lives. Their design research study about how teachers perceived they could encourage students to persist in solving challenging tasks used surveys and focus groups to explore how students learn to persist, which is similar to the second part of Mathematical Practice 1—persevere to solve problems. One conclusion from the study was for teachers to connect mathematics to the students' lives.

Koestler et al. (2013) summarize the literature and say that students can make better sense of problems when the teachers select problems that are related to students' lives. Koestler et al. claim that the Mathematical Practices are a big step for teachers, but there is little guidance for teachers about how they can guide students' engagement in these Practices. Although Koestler et al.'s discussion on the theme of connection and Mathematical Practice 1 seems helpful, empirical evidence for their viewpoint is lacking.

Connections for mathematics for ELLs. Teachers should help students connect their own personal backgrounds to mathematical problems, so students can recognize the relevance and make sense of the problems (Koestler et al., 2013). This is a challenging task for many teachers, who tend to have different backgrounds than ELLs (Howard, 2006; Nieto, 2003). Kersaint et al. (2009) contend that teachers need to do more than simply replace names used in textbook word problems—they encourage teachers to connect with ELLs by being culturally relevant. This action they define as: "The creation of the space or environment that enables all students to learn" (Kersaint et al., 2009, p. 131). Moschkovich (2013) urges teachers of ELLs to become acquainted with details of their previous background in mathematics. According to the authors mentioned above, teachers can facilitate ELLs to make sense and persevere to solve problems by helping students connect their learning to their background knowledge.

Making the connection between mathematical problems and symbols may be demanding according to Zwiers (2008), because it is necessary to interpret symbols, an abstract process. Depending on the context, symbols can be difficult for ELLs because they need to "identify the functionality of each symbol and understand the rules that govern its usage" (Kersaint et al., 2009, p. 47). Adding to the challenge, symbols in mathematics may have more than one meaning (Kersaint et al.). Bresser, Melanese, and Sphar (2008) suggest that teachers point to symbols during conversations to help ELLs understand the connection between symbols and words.

Kersaint et al. (2009) suggest that ELLs be allowed to speak in their own language whenever possible because this will help ELLs connect to their background knowledge. Zwiers (2008) contends, “knowledge and cultural capital have influenced math learning” (p. 7). He points out that many ELLs are at a disadvantage because they are not familiar with the vocabulary from word problems that involve terminology—such as baseball, snow, and customary measurement units—that is not part of their culture.

It seems that ELLs, like all students, learn mathematics better if they perceive a connection between the subject matter and their personal background. Special attention is warranted and necessary to make sure that such connections are made for them, and much more research is needed as to how this can best be affected. How to train PSTs in helping ELLs make the connections will be an important facet of my research. The literature focuses on teachers connecting to elementary students in general, or on connecting to ELLs in particular, but more research is needed on how teachers can help ELLs connect to their personal backgrounds as part of helping all students to do so.

Providing access. Another way of helping students make sense of problems is for teachers to provide access to the content. Four years after the Mathematical Practices were published, the National Council of Teachers of Mathematics provided guidelines on how teachers could engage students in these Practices (National Council of Teachers of Mathematics, 2014). It discussed how teachers should provide equitable access to all students by offering each student the accommodation appropriate to his or her needs. The authors of the National Council report argue that being equitable is different from giving all students equal treatment. They believe that all students can achieve, but teachers should make the appropriate accommodations so that each child can be successful. The authors recommend that teachers attempt to adjust their curriculum to ensure that all students can engage in the Mathematical Practices.

Schleppegrell (2007) points out that many students are denied access to content because they have not acquired mathematical language. Since learning mathematical language is so challenging, she argues teachers have important roles in helping students make sense of problems. While not all of the challenges that students face when solving problems are related to mathematical language, it is important for teachers to know how to help students obtain access to content.

Koestler et al. (2013) clarify that teachers should select and adapt problems to provide all students with access to the content. Teachers can select problems that have multiple entry points, according to Koestler et al., to provide more students opportunities to access the content—students can choose to make sense of the problems at different points. In summary, if teachers offer students opportunities access to the content, then their students can make sense of problems and persevere to solve them.

Providing access for ELLs. De Oliveira's (2011) study adds to how PSTs can better teach ELLs in mathematics class; she emphasizes that it is imperative that PSTs are not only taught strategies for meeting ELLs' cultural needs, but also taught how to provide access to ELLs who do not have proficiency in English. To correct the misconception that mathematics only involves numbers, not language, de Oliveira made a demonstration to PSTs. In her first lesson, she taught mathematics in Brazilian Portuguese without scaffolding, with the goal that her PSTs would better empathize with ELLs learning mathematics in another language. She followed this with a second lesson in Portuguese, but this time used gestures, allowed the students wait time, used visual cues, and introduced vocabulary terms. The 152 participants, PSTs and in-service teachers, commented that the second lesson allowed them to have better access to the content. The study concluded that the simulation allowed the PSTs and in-service teachers to empathize with how ELLs might feel when they are taught in English without accommodations.

Fernandes (2011) offers another study in which the PSTs reflected on the importance of language when teaching mathematics to ELLs. Fernandes reports that PSTs in his secondary mathematics methods course “tended to view mathematics as being universal and minimally language intensive, and involving symbols that could be transferred across languages” (p. 11). The exception, Fernandes adds, was that the PSTs accepted that ELLs would be challenged with word problems. The author's position on the importance of training PSTs in linguistics is consistent with my earlier discussion in Section 2.2; PSTs need to understand the demands of the mathematics register. In order for PSTs to understand the language challenges that PSTs face, Fernandes (2011) had PSTs videotape ELLs as they solved measurement problems. The PSTs were instructed to observe the process of the ELLs’ explaining their reasoning, rather than focusing on the correct answer. Next the PSTs tutored ELLs in mathematics. The PSTs were instructed in their methods course to provide the ELLs access to the content by telling them to reread the questions, explain certain words in the question, and encourage the use of context clues. One ELL did not understand the directions until the PST changed the context of the problem to candy. Fernandes reasons that PSTs and teachers who are aware of ELLs’ challenges will be more likely to make linguistic accommodations.

By the end of the study, in a discussion, PSTs suggested that the vocabulary in the instructions be simplified because the ELLs were at a disadvantage if they did not understand the task. Another PST suggested that ELLs would have better access to the content by having materials in front of them, such as string and paper as they were writing about mathematics. The PSTs came to realize that the ELLs had to learn content and English at the same time. Some PSTs suggested that ELLs be allotted more time to solve the problems to compensate for the ELLs' English language challenges. Another lesson learned was that not all ELLs are the same in terms of linguistic development. Some ELLs had acquired a spoken proficiency in English but had not yet acquired proficiency in academic language while others struggled with more basic spoken English. In summary, this research provides examples of how PSTs can be encouraged to reflect on the importance of language when teaching mathematics to ELLs. It would be useful for mathematics educators to have more studies similar to Fernandes' study to offer them ideas on how best to encourage PSTs to provide access to ELLs.

Maldonado, Turner, Dominguez, and Empson (2009) studied how to encourage English language learners to share in the class discourse. They found that using manipulatives empowered ELLs by allowing them to explain concepts to their peers that they would not have been able to do verbally. ELLs are not only able to express themselves better through the use of manipulatives, but also when peers use manipulatives as they justify their answers, ELLs can better comprehend this visualization (Maldonado et al., 2009; Moschkovich, 2013). The use of manipulatives may be another resource to provide ELLs access to mathematical content.

PSTs need to learn that providing access to ELLs requires flexibility. Kersaint et al. (2009) maintain that if the teacher is flexible in trying to meet all students' needs, students will be flexible in their mathematical strategies. "Highlighting differences can also help students realize that mathematics is not a discipline that is done in a single way set down by the ancients" (Kersaint et al., 2009, p. 61). In addition teachers need to be aware that ELLs, according to

Kersaint et al., may have learned different algorithms; ELLs should be encouraged to share these algorithms with their peers.

There are many strategies that will help ELLs access mathematics instruction, in spite of their challenges learning English in monolingual classrooms. All of the studies summarized here emphasize the need to inculcate in PSTs appreciation for the barriers that ELLs face as well as the contributions ELLs can make to their classroom peers. While there is literature on teachers providing access for students in general, and other literature on providing access specifically for ELLs, the literature is sparse as to how teachers can provide access to ELLs and to all other students simultaneously.

High expectations. According to sociocultural theory, teachers should provide students opportunities to solve problems within their zone of proximal development (Vygotsky, 1978). Vygotsky maintains that students learn best when they are trying to solve problems that are too challenging for them to solve on their own but possible for them to solve with appropriate supports. The National Council of Teachers of Mathematics (2014) states that it is imperative for teachers to have high expectations for all students so they provide them problems that are within this zone. They say that too often teachers have low expectations for certain students and offer these students mathematical problems below their zone of proximal development. Teachers who have low expectations for students also tend to wait less time for those students to make sense of problems and persevere to solve them (Kilpatrick et al., 2001). Therefore, it seems apparent that students will be more successful if their teachers have high expectations for their capabilities in mathematics. My study examines the PSTs' beliefs about all students being capable of making sense of problems and persevering at solving them.

High expectations for ELLs. Murrey (2008) encourages teachers to help ELLs have access to content by talking slower and using gestures, but says that teachers should not lower their expectations for the mathematics achievements of ELLs. Teachers should be aware of ELLs' possible challenges with vocabulary, but at the same time recognize that ELLs can be used as a resource when areas such as the metric system are used (Kersaint et al., 2009). Turner et al. (2012) encourage teacher educators to make sure that PSTs have high expectations for ELLs, so that ELLs can live up to their expectations.

According to Celedon-Pattichis and Ramirez (2012), teachers should go beyond setting high expectations. They maintain that most teachers will say that they have high expectations for their students, but it is important for teachers' actions to be congruent with this belief. They observed a third and fourth grade teacher who claimed to have high expectations for all students, but the teacher reflected that her actions were not promoting high expectations for all students. The teacher wrote on a chart whenever a student shared in her mathematics' discussions. After recording the responses, the teacher realized that she was calling on ELLs less than on the other students--an example of a teacher who realized that her actions were not congruent with her belief in high expectations for all students. In my study I examine how some PSTs said that they believed that it was important to have high expectations for all students, but their actions were not congruent—they displayed lower expectations for ELLs than for other students in mathematics.

Marx (2006) argues that teacher educators should encourage PSTs to reflect about how they can meet ELLs' needs. Marx found that at the beginning of her study, nine white PSTs from the southwest had low expectations, which she called deficit views, about ELLs. To address this problem, she gained the PSTs' trust and encouraged them to write and discuss their views about

ELLs. She found that it took a semester, but the PSTs in her study had higher expectations of ELLs after deep reflection. Thus, Marx's study suggests that it would be useful for teacher educators to offer PSTs opportunities to reflect on any low expectations that they may have about ELLs' abilities. In my study I examine how PSTs respond when they are encouraged to reflect on their expectations about ELLs' capabilities in mathematics.

Productive struggle. As teachers provide access to mathematical content, they still need to keep the cognitive load high. In a handbook summarizing research on mathematics education, two established researchers, Hiebert and Grouws (2007), discuss effective strategies of teachers. They stress that one of the strategies is for teachers to increase the cognitive load for students by refraining from giving them too much help when solving problems. In a book designed for teachers to apply the Mathematical Practices, Dixon et al. (2015) add that Mathematical Practice 1 "provides a context for higher-level-cognitive demand mathematical task development during the unit of instruction" (p. 77). Students will engage in Mathematical Practice 1 when they engage in productive struggle—they are given problems that will challenge them and yet will be within their zone of proximal development (Dixon et al., 2015, Vygotsky, 1978). Hiebert and Grouws note that students learn mathematical concepts more deeply when they experience struggle; teachers should structure opportunities for students to engage in productive struggle throughout the curriculum (Clarke et al., 2014; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Human, & Olivier, 1997; Larson et al., 2012; National Council of Teachers of Mathematics, 2014).

According to Warshauer (2014), literature is sparse on how students can engage in productive struggle in mathematics. Her exploratory case study concluded that it was better for teachers to question students when they were struggling instead of telling them answers. She maintains that questioning is a strategy that can help students organize their thoughts and continue making sense of problems. Warshauer's study was an exploratory case study. It would be beneficial to heed her call for more research on how to engender productive struggle.

Hiebert et al. (1997) suggest that in addition to keeping the cognitive load high, teachers should provide a learning environment where students learn from mistakes. In a non-empirical study, Buschman (2007) theorizes that teachers ought to avoid teaching students problem solving strategies; students are more likely to make sense of problems, he argues, if they develop their own strategies. The literature suggests that teachers should facilitate students to engage in productive struggle by providing opportunities to make sense and persevere at solving problems that are challenging.

Productive struggle for ELLs. Murrey (2008) extends that it may be necessary to scaffold the ELLs' understanding, but the mathematics needs to remain rigorous. Students will not have opportunities to persevere with problem solving if teachers scaffold too much (Hiebert & Grouws, 2007; Larson et al., 2012.) In a discussion of the Common Core State Standards (CCSS), Larson et al. add that the goal for teachers should not be to treat all students the same, but for the teacher to look for ways to challenge each student with the ultimate goal of enabling each student to meet the same high standards. Doing so may require them to pose different challenges to different students at any particular point in time. In their words, "The CCSS require all students be held to and supported in meeting the same rigorous standards" (Larson et al., 2012, p. 138).

In a chapter dedicated to ELLs solving problems, Celedón-Pattichis and Musanti (2012) hypothesized that ELLs are capable of solving challenging problems if they are given help to make sense of the problems. They examined two classes to test their hypothesis. The teachers used multiple ways to communicate the mathematical concepts such as gestures, drawings, and manipulatives. The authors concluded that with the aid of multimodal communication, the ELLs found it easier to make sense of the problems and, therefore, they persevered longer at solving challenging problems. In my study, PSTs reflect on their mathematics methods course, in which they attempted to use multimodal communication to teach all students, including ELLs, to engage in the Mathematical Practices.

The authors above discuss the importance of teachers aiding ELLs to gain access to the content while ensuring that the mathematics is still challenging. There is limited literature on how teachers and PSTs can specifically facilitate ELLs to engage in productive struggle in mathematics, so my study has the potential to contribute in this area. Throughout my study I examine PSTs' beliefs about engaging ELLs in productive struggle and ways of developing those beliefs.

Productive disposition. In a book geared for teachers to apply the Mathematical Practices in their teaching, Larson et al. (2012) maintain that having a productive disposition is related to the second part of Mathematical Practice 1, persevering to solve problems. Productive disposition is defined by Kilpatrick et al. (2001) as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 5). Kilpatrick et al. report that students are excited about mathematics when they first come to school but that, unfortunately, our present school system is making most students lose this disposition. They call for teachers to change their practices to make mathematics more engaging. The authors report that kindergartners "have high levels of persistence and eagerness to learn" (p. 171) mathematics, and that it is the responsibility of parents and educators to sustain this disposition in students. It is important for teacher educators to help teachers have productive dispositions—if teachers have productive dispositions, they can positively influence their students to have them (Kilpatrick et al., 2001). Students with productive dispositions can persevere to solve challenging problems, and if they are interested and curious this positive attitude can last for the rest of their lives (National Council of Teachers of Mathematics, 2014). Thus, teachers need to encourage students to view themselves as capable of doing mathematics in order to persevere at solving problems and to have long-term, positive dispositions towards mathematics.

An exploratory case study by Jansen (2012) involved interviewing sixth graders. She concludes that teachers can encourage students to have productive dispositions by providing them opportunities to work in small groups. Akin to the suggestions for productive struggle, Jansen claims that teachers should offer students access to content, but still maintain the high cognitive load. Furthermore, she concludes that when teachers transfer the responsibility to solve problems to the group of students rather than rely on teachers for answers, the students are more likely to report having productive dispositions. Jansen calls for more research on students solving mathematics in small groups and on how this practice may affect their dispositions.

In a non-empirical article, Hiebert, Morris, and Glass (2003) maintain that PSTs themselves need to be proficient in mathematics in order to teach the five strands of mathematical competencies (Kilpatrick et al., 2001), in which the last strand is productive disposition. If so, having a strong understanding in mathematics will help teachers develop productive dispositions in their students. The authors suggest that PSTs consider their lessons as experiments and, after teaching, reflect on how they can improve. They also posit that teacher education courses should examine how students can learn all five competency strands, including having a productive disposition. These ideas about productive disposition make sense, but should be supported with empirical research.

Clarke et al. (2014) maintain that students will not persevere at solving challenging problems unless they believe that they are capable of solving them. The authors define perseverance and explain how teachers can promote it:

> …actions that include students' concentrating; applying themselves, believing that they can succeed, and making an effort to learn, and we term the tasks that are likely to foster such actions cognitively demanding*,* or challenging in that they allow the possibility of sustained thinking, decision making, and some risk taking by the students (p. 67).

Clarke et al. advocate that teachers help students to persist at solving problems by giving them challenging problems, having high expectations for their students, showing enthusiasm for the content, encouraging students to take risks by creating an environment that makes mistakes acceptable, making the problems relevant to students, and allowing ample time for students to solve problems.

The literature discussing how productive disposition is related to Mathematical Practice 1 is important to my study because I observe PSTs facilitating elementary students' engagement in this Practice. The literature is sparse as far as making explicit connections between productive struggle and Mathematical Practice 1, but without mentioning the terms "productive struggle", "productive disposition" or Mathematical Practice 1, Clark et al. (2014) suggest that students who have the belief that they are capable of solving challenging problems will be more likely to persist. In my study I discuss a possible relationship between productive disposition, productive struggle, and Mathematical Practice 1. The literature is also sparse in regards to developing PSTs to have productive dispositions and how that disposition affects specifically ELLs as they learn mathematics.

Content knowledge. Ma (1999) compares elementary teachers from United States and China and found that teachers from the latter country have a stronger understanding of conceptual knowledge in mathematics. She claims that American teachers focus on mathematical procedures rather than teaching why the procedures work. As a result American students have a weaker understanding of mathematical concepts compared to Chinese students (Ma, 1999). Thames and Ball (2010) studied what teachers need to know in order to be effective mathematics teachers. They concluded that effective teachers should have both strong content knowledge of mathematics and strong knowledge of how to teach mathematics.

Hill, Rowan, and Ball (2005) examined whether there was a correlation between teachers' content knowledge (both the teachers' mathematical knowledge and the pedagogy to teach the knowledge) and student achievement. The authors state that in the past, teachers' content knowledge has often been measured by the degrees or courses taken in various disciplines, but that this does not necessarily reflect mastery of the mathematical content knowledge. The study used mixed methods and many variables were considered, such as teachers' experience, mathematical knowledge, and content knowledge for teaching mathematics. The authors found an overall correlation between teachers' content knowledge and student achievement. Even though they might have anticipated a correlation for high school students, they had assumed that all teachers would have a mastery of first grade mathematics. The study suggests that content knowledge for teaching mathematics is important for those who teach students of any age. The correlation was especially strong in the case of teachers in the bottom third of content knowledge, so these teachers should be the focus of teacher training programs.

Zevenbergen (2004) studied whether reducing PSTs' anxiety would increase their ability to learn content. The author observed how study groups at two university mathematics courses were used for improving the PSTs' cognitive, social, and affective learning. The PSTs relied less on the professor as time progressed, and more on fellow students. The PSTs that chose to work in the optional study groups also learned to support each other and use tools, such as manipulatives. PSTs reported that they learned more content as a result of the support groups and improved their self-confidence as well. Many studies have shown that learning content is important for PSTs (Hill, Rowan, & Ball, 2005; Ma, 1999), but Zevenbergen's study suggests how PSTs can be taught the content.

Ball and Forzani (2009) theorize that teacher educators ought to shift their focus from knowledge to learning actual practices that teachers know. They do not suggest that learning content knowledge is not important, but rather that the practice that PSTs need to learn is part of the knowledge that PSTs need to acquire. We need a better system for training PSTs, according to Ball and Forzani that places more emphasis on practice. They claim that there are many skills involved in teaching—such as leading discussions, preparing assignments in and outside of the classroom, grading students' papers, and talking to parents—that could be best learned by practicing. The authors urge teacher education programs to instruct PSTs to learn pedagogical knowledge as well as academic knowledge. Furthermore, the authors call for more research in education to determine best practice for teacher education programs.

Akin to Ball and Forzani (2009) Grossman, Hammerness, and McDonald (2009) theorize that today PSTs are too often confined to playing student roles in methods classes and need more opportunities to engage in activities similar to teaching. Grossman et al. propose that PSTs be taught core practices that are targeted in teacher education to ensure that all PSTs receive this training. The authors suggest that PSTs should have opportunities to learn these practices and receive feedback from teacher educators. While acknowledging that empirical research is needed, Grossman et al. (2009) suggest that teacher educators should teach PSTs core practices, such as asking students effective questions, re-voicing, and scaffolding instructions. There is still not an agreement on all the practices PSTs should learn, but Grossman et al. say that most researchers agree that practices should be taught that have high frequency in the classroom and are research based.

There are many studies that discuss the importance of teachers having strong content knowledge (Hill, Rowan, & Ball, 2005; Ma, 1999; Zevenbergen, 2004). Grossman et al. (2009) and Ball and Forzani (2009) raise the question of exactly what content knowledge PSTs need to be effective teachers. They call for research on how PSTs can learn practices that are in alignment to teaching. What the authors fail to address is what are the best practices for PSTs meeting all students' needs, including ELLs. Furthermore, with the arrival of the Common Core State Standards, more studies need to address whether PSTs need not only knowledge of the Mathematical Practices, but also knowledge of how to teach the Practices to their students. In my study I discuss how PSTs learn the Practices and how their knowledge of mathematics content may affect how they learn the Practices. I also examine what PSTs perceive as best practice to facilitate all students, including ELLs, to engage in the Mathematical Practices.

2.3.2 Mathematical Practice 3

Larson et al. (2012) maintain that the intent of Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others) is for students to justify their answers; teachers should facilitate students to communicate their mathematical thoughts. I discuss how two themes may help PSTs engage elementary students in Mathematical Practice 3. Again, I begin by discussing the general literature on how a theme applies to all learners, and then discuss its particular application to ELLs.

Social interaction. One strategy for encouraging students to participate in class discussions is providing students with the appropriate social environment. In a framework designed to help students learn mathematics with understanding, Hiebert et al. (1997) posit that teachers should provide a community in which students feel comfortable making mistakes or admitting that they don't understand something. Hiebert et al. suggest that teachers encourage students to critique each other, but in order for this to occur, norms need to be established that require students to respect each other.

Wilcox (1991) states that PSTs have pre-conceived ideas about the pedagogy of mathematics, which were influenced by the way they were taught themselves. Wilcox argues that PSTs' beliefs that mathematics has to be learned by memorization can be changed by PSTs learning mathematics together as a community. Wilcox found that when the PSTs experienced learning in a community in their mathematics methods courses, they were more likely to provide their students with the same experiences they had when they themselves were students. Larson et al. (2012) hypothesize, in a non-empirical study, that when teachers emphasize the process of problem solving, rather than stressing the right answer, classroom community is created, which facilitates students being perseverant. In my study, I document PSTs' responses to my attempts to create an atmosphere for the PSTs in which there are strong classroom bonds, and in which there is a focus on the process of mathematics.

The National Council of Teachers of Mathematics (2014) adds that teachers can encourage students to develop arguments and critique each other by listening and valuing one another. I observe how well PSTs facilitate elementary students' engagement in Mathematical Practice 3.

Social interaction for ELLs. Leonard and Guha (2002) advocate that teachers can help ELLs feel welcome by becoming familiar with their students' community and culture. Coggins (2007) extends that teachers should include ELLs in class discourse and prevent some students from dominating the discourse. Allowing students time to discuss in pairs provides opportunities for everyone to participate (Coggins, 2007; Moschkovich, 2013). Bresser et al. (2008) add that when teachers learn a few words of the ELLs' heritage vocabulary, it helps ELLs value their own language.

Turner et al. (2012) offer a study in which they summarize that PSTs typically learn from methods courses on campus and apply what they have learned in the classroom during their placements. They propose that PSTs can be better instructors if they spend time in students' communities, because this experience helps them connect with their students.

Aguirre, Turner, Bartell, Kalinec-Craig, Foote, McDuffie, & Drake (2012) also studied how PSTs could connect ELLs' communities with their schools. They examined 70 projects from 113 PSTs from three universities to analyze how they made connections in the lessons they designed for students of diverse populations. Aguirre et al. assert that teachers often fail to make realistic connections to students' communities when they are designing mathematics problems. One activity that helped the PSTs become more familiar with communities, according to Aguirre et al., was for them to visit two sites near their placement. Knowledge of ELLs' communities helped PSTs develop culturally relevant questions for the students. Similar to Turner et al.'s (2012) study, Aguirre et al. encourage teacher educators to offer PSTs more experiences in diverse students' communities in order to make connections. The literature does not address how teachers can integrate connecting to all students' communities, including ELLs' communities.

Developing mathematical language and discourse. Mathematical Practice 3 involves students listening to others and communicating ideas to others. The CCSSM state that successful students of this practice justify their answers, argue with others, and "reason inductively about data, making plausible arguments that take into account the context from which the data arose" (CCSSI, 2010, p. 6). Koestler et al. (2013) stress the importance of students evaluating each other's ideas. In their words, "Describing ideas accurately can help students correctly interpret the meaning of terms used in another student's argument" (Koestler et al., 2013, p. 33). Thus, students learn content more deeply when they develop arguments and critique each other.

Schleppegrell (2010) also points out that when students are asked to justify their answers, then language necessarily plays an important part in the mathematics curriculum. Furthermore, mathematical language "tends to be conceptually dense, interpersonally alienating, and highly structured textually in unfamiliar ways" (Schleppegrell, 2010, p.74). Teachers need to know how to teach students to understand the language of mathematics. According to Zwiers (2008), teachers typically spend too much time modeling and not enough time developing mathematical language. He posits that in mathematics "there is an increasing need to know why something works, not just how. Knowing why requires students to acquire the proper language and terminology" (p. 92). Thus, the authors imply that students need a special type of vocabulary to justify their mathematical arguments.

Bruun, Diaz, and Dykes (2015) compared learning mathematics to learning another language. Story problems are important in mathematics today, they note, so students need to have a large mathematical vocabulary. Even though Bruun et al. point out that students require an extensive vocabulary to solve word problems, according to Stein and Smith (2011), teachers often dominate the mathematical discourse, and as a result students do not have many opportunities to justify their answers. Teachers should develop students' mathematical language, so they can better argue and critique mathematics (Stein & Smith). Since Mathematical Practice 3 involves students expressing their ideas and critiquing other's ideas, it will be important to understand how teachers can facilitate all students' engagement in this Practice.

One way to develop students' mathematical language is to offer them many opportunities to discuss mathematics. Larson et al. (2012) propose that the intent of Mathematical Practice 3 (constructing viable arguments and critiquing arguments of others) is for students to "make and test conjectures and to communicate their mathematical thinking" (p. 40). The researchers explain that the teacher's role is to guide the students to justify their answers. Larson et al. state that when students discuss each other's ideas they engage in Mathematical Practice 3. Hakuta, Santos, and Fang (2013) explain that the Mathematical Practices require students to be involved in more discourse and hence will benefit them with stronger language skills.

Although the importance of mathematical discourse is broadly accepted, there is much less agreement on the definition. Ryve (2011) chose 108 articles from six peer-reviewed journals about discourse. She found great disparities among the articles and concludes that the authors did not agree on how to define discourse. She states that although there has been a vast amount of literature on discourse in the last two decades, there has been little building-on from one article to another. The author points out that it is difficult to build on an article if one does not know how the concept of discourse is being used. Only 19% of the 108 articles that Ryve reviewed defined what discourse meant to them. Ryve urges the authors who write about discourse to define the word. She explains that different researchers examine discourse from different perspectives, some in terms of how it shapes and what it says about the relationships among the people involved, and some in terms of how it shapes and what it says about how people make sense of the phenomena they're talking about. Her third category, having to do with "cultural and social relations" on a macro scale, is not relevant to my study.

Ryve (2011) states that the two perspectives draw on Wetherell's (2001) principles that people use discourse to create knowledge, not just to convey pre-existing knowledge; that they use it for a purpose that influences how they use it; that the meaning they make is "co-constructed" by the persons involved, rather than conveyed by one person to others; and that this co-construction always occurs in a social context that has particular conventions and implicit or explicit rules that participants may or may not understand or need to learn (e.g., classroom discourse guided by teacher questions has one set of conventions and rules; gossiping among peers has another; and conducting a job interview has another). Applying Wetherell's principles, the PSTs' discourse is important to my study. I examine how my students use discourse to form relationships, how they collaboratively construct meaning of math problems and concepts, and how they learn how to conduct classroom discussions.

The research is abundant that students' mathematical content is promoted when they are in rich discourse classrooms (Forman & Ansell, 2001; Sfard, 2007; Yackel, 2002; Zack & Graves, 2001.) Forman and Ansell urge instructors to change from a traditional teaching style, in which the instructor attempts to transfer her knowledge to the students, to a style that is student-centered. They claim that students learn more if they are encouraged to talk as they make sense of mathematical problems. "Students make sense in mathematics by explaining their invented strategies for solving problems and by listening to and reflecting on the strategies of others" (p. 116). In an empirical study of a third grade classroom, Forman and Ansell conclude that revoicing—repeating what students say—after a student justifies an answer encourages other students to join the conversation.

A shift in discourse may be facilitated by a shift in focus (Dixon, Egendoerfer, & Clements, 2009). The authors found surprising results when students in a second grade were not made to raise their hands when they wanted to speak. The teacher in this study contacted the researchers because she was curious to see if she could promote more discourse among her 16 students. The researchers observed and videotaped the discourse among the students during whole-group discussions for three weeks during one-hour mathematics lessons before the rule about hand raising was established. Next the authors observed the same class for four weeks after they were allowed to speak without raising their hands. The researchers also interviewed each student before and after the study. Students were asked questions about their mathematical abilities and questions about the class discussions. The researchers attempted to determine if the students' oral and written mathematical explanations and justification shifted.

Dixon et al. (2009) conclude that by doing away with the hand-raising rule, students began to engage in discussions among themselves. The second graders would justify their answers while the teacher would talk less and listen more. The teacher's role shifted from that of an expert to that of a facilitator. The researchers found that as the students became more interested in mathematics and more involved, they also made conceptual leaps in their mathematical learning. This study seems convincing because of the rich data collected—teacher journal, observations, video recordings of activities, interviews, and student work. The authors contend that learning mathematics was less procedural and more conceptual as students became more involved in the discourse. In the seven-week study, Dixon et al. observed the students' discourse shift as a result of allowing them not to raise their hands before they spoke.

Sfard (2007) calls for teachers of mathematics to promote learning in students by encouraging them to participate more often in discourse. Zack and Graves (2001) assert that the teacher's role has changed from a "provider of knowledge to a knowledgeable orchestrator" (p. 234). Zack and Graves maintain that teaching mathematics should involve students participating in discourse in inquiry-based classrooms. The literature suggests that students do well in rich student discourse classrooms; students also need to be given opportunities to discuss mathematics to develop arguments and critique others (Mathematical Practice 3). I observe, in my study, how the PSTs learn Mathematics Practices 1 and 3 as they discuss the Practices, as well as how they learn to encourage students to learn mathematics through discussions. Much of the literature on mathematics discourse appears to emphasize the role of the teacher in shaping mathematics discourse, especially in the use of questions that initiate and drive deeper discussion. As an instructor in a mathematics methods course, I did not always find the time to scrutinize this aspect of mathematics discourse. However, I encouraged the PSTs to ask their students probing questions to encourage students to have deep discussions. More research should examine how teacher educators' and teachers' questions facilitate (or not) all students to develop arguments and critique the arguments of others.

Developing ELLs' language and engagement in discourse about mathematics. The Common Core State Standards require students to have a higher command of written and oral language, and Santos et al. (2012) argue that this can be challenging for ELLs. These Standards specify that successful students of Mathematical Practice 3, "Justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose" (CCSSI, 2010, p. 6).

One way for teachers to aid ELLs' language development while learning patterns is by using sentence frames (Bresser et al., 2008). Sentence frames consist of key words that provide parts of the sentence for the students. This visual approach makes it easier for ELLs to make a connection between the mathematical concepts and the language. Another activity that involves pattern recognition is sorting objects. Once again, the beginners could describe their patterns briefly with the aid of a sentence frame, while the more advanced ELLs could speak in longer sentences (Bresser et al.). For the beginner, the teacher could ask questions that require a one-word answer such as, "What color comes next?" ELLs with more advanced English proficiency could be asked, "Explain why you put this here." According to Bresser et al., other activities—ones that promote pattern recognition but include all students and at the same time develop language—are drawing patterns, acting out patterns, and folding.

Borgioli (2008) maintains that teachers should have language objectives as well as mathematics objectives for their lessons. Murrey (2008) complements this, "By differentiating instruction in mathematics for ELLs, teachers can plan and provide access to mathematics curriculum for all students, with the added goal of language instruction for students learning English" (p. 146). Teachers should have mathematical conversations with students, including ELLs, rather than lecturing, recommend Kersaint et al. (2009). In this manner, the students will listen closely because they know that they will have opportunities to talk. Kersaint et al. suggest that the teacher use controlled vocabulary, speak slowly, avoid idioms and pronouns, and increase wait time so that the ELLs can have more access to the mathematics.

The fact that mathematics has its own language, which is more challenging than that of other subjects (Schleppegrell, 2010; Zwiers, 2008), makes it even harder for ELLs to participate in discussions. Mathematical language is problematic for many students, especially ELLs, because "…there is less overlap with concepts, ideas, and terms found in other subjects. Math has many distinct vocabulary terms only used in math" (Zwiers, 2008, p. 92). Furthermore, Zwiers adds that mathematics is more sequential than other subjects, so if an ELL lacks mastery of prior knowledge, it will be difficult to acquire new concepts. One of the guiding principles of Ramirez and Celedón-Pattichis (2012) for teaching mathematics to ELLs is for teachers to support ELLs while they learn mathematics.

Teachers should consider ELLs' language development in order to determine best practice. At the beginning stage, ELLs may point to objects to express their mathematical knowledge. As they become more fluent, teachers can ask more open questions, ones that require ELLs to justify their answers in more depth. Bresser, Melanese, and Sphar (2008) add that if ELLs struggle to justify their answer, then teachers should facilitate their success by asking questions to ELLs that require minimal discourse. Assessing ELLs' English proficiency will aid the teacher in making appropriate decisions to facilitate their participation.

Martiniello (2008) also found that language plays an important part when ELLs solve mathematics problems. She studied Spanish-ELLs and non-ELLs taking a standardized mathematics assessment given to fourth grade students in Massachusetts. She found that as the questions became more complex linguistically, the ELLs performed worse than their peers that were at a similar level in regards to mathematics ability. She also found that the ELLs performed better if Spanish-English cognates were used. Furthermore, the ELLs performed better at word problems that had school-related vocabulary words as opposed to home-based words. Martiniello concluded that while assessments should use controlled vocabulary to measure ELLs' aptitudes in mathematics, teachers should develop ELLs' English so they will perform better on mathematics assessments.

According to Garrison, Ponce, and Amaral (2007), word problems, although challenging, are imperative for ELLs. Teachers may be tempted to give computational problems without language to make it easier for ELLs to participate, but this would not help ELLs develop their language proficiency. Teachers can use the present tense, high frequency words, and short sentences for ELLs with limited English proficiency. As the year progresses, more challenging words and longer sentences can be used. Using word problems, although challenging, is another opportunity to develop ELLs' language. The authors point out that if ELLs get interested, then they can be encouraged to discuss the word problems, further developing their mathematical language.

There is little reported literature on the combination of "mathematics," "discourse," and "ELLs." However, an article by Hufferd-Ackles, Fuson and Sherin (2004)—which deals with discourse in teaching mathematics—is useful to examine. They did not refer to their participants as ELLs, but the majority of the students spoke Spanish at home. Hufferd-Ackles et al. trained four teachers to shift the discourse from the teachers to the students in an urban, Catholic school by teaching inquiry-based lessons. With the advice of the researchers, the teachers asked students to justify their answers to the problems they were given. They suggested that the teachers use visual aids and encourage their students to draw pictures and give explanations when they solved their problems. The researchers observed the four teachers for a year, after which one teacher and her third grade class were studied for an additional year.

Hufferd-Ackles et al. (2004) attempted to determine if a shift was made by quantifying students' discourse. The authors conclude that by the end of the year the students in the class studied were speaking for longer periods of time and asking more questions. Although it may still be challenging for ELLs with limited English proficiency to develop arguments in English and critique their peers, they will benefit from classrooms like the one mentioned above where students have many opportunities to talk. This one-year case study in an urban setting makes a positive contribution to the literature of ELLs, discourse and mathematics.

Moschkovich (2010) believes that it is important to recognize that language is complex and that researchers need to refrain from making generalizations about mathematical discourse—it will vary depending on the context. Teachers should have mathematical conversations with students, including ELLs, rather than lecturing them, recommend Kersaint et al. (2009). In this manner, the students will listen closely because they know that they will have opportunities to talk. Kersaint et al. suggest that the teacher use controlled vocabulary, speak slowly, avoid idioms and pronouns, and increase wait time, so that the ELLs can have more access to the mathematics. ELLs, in particular, can be given drawings to facilitate their comprehension. The teacher should consider ELLs' language development. At the beginning stage, ELLs may point to objects to express their mathematical knowledge. As they become more fluent, teachers can ask more open questions that require ELLs to justify their answers in more depth. Bresser et al. (2008) add that if an ELL struggles to justify their answer, then teachers should facilitate their success by asking questions to ELLs that require minimal discourse.

Koestler et al. (2013) encourage teachers to focus on students' grammar—if students are precise, then it is easier for their peers to understand them. On the other hand, Moschkovich (2007) cautions teachers from placing too much emphasis on grammar with ELLs when they are justifying their ideas. For example, she cites a student who mispronounced the word 'rectangle' although she understood the concept mathematically. Moschkovich feels that this student should be praised because even though she lacked the English vocabulary to define the geometric terms, if the teacher were to pay attention to her gestures, it would be clear that she understood the mathematics. Moschkovich applauds another teacher for not correcting an ELL for mispronouncing the word ‘parallel’ and instead asking the student to explain. The author believes that if assessment in mathematics is limited to defining terms, ELLs will always be at a disadvantage. Moschkovich encourages teachers to assess students’ mathematical content by allowing ELLs to use gestures, their own language, and objects while they are expressing their ideas. It appears that as Koestler et al. state, teachers can encourage students to communicate with precision, but that teachers also need to heed Moschkovich and focus on ELLs’ mathematical communication. If teachers focus too much on ELLs’ grammar, ELLs might be hesitant to participate in class discussions.

The literature implies that Mathematical Practice 3 is facilitated for ELLs when teachers use techniques to engage discourse and remember to scaffold the mathematical tasks while keeping the mathematical concepts rigorous. Teachers can help ELLs obtain access to the mathematical content by using gestures, speaking slowly, and being respectful. However, there is a paradox. Mainstream teachers bear the responsibility of teaching English to their students, as well as that of teaching the content of their discipline/s (Shatz & Wilkinson, 2013; Verplaetse & Migliacci, 2008). Thus teachers must balance the need to facilitate ELLs’ access to the mathematics with the need to develop ELLs' English by offering them opportunities to listen, speak, read, and write about mathematics in English (Moschkovich, 2013). There is much in the literature about the importance of developing students’ mathematics language (Forman & Ansell, 2001; Schleppegrell, 2010; Sfard, 2007; Yackel, 2002; Zack & Graves, 2001.) There is also much in the literature about developing ELLs’ language skills (Bresser et al., 2008; Kersaint et al., 2009; Moschkovich, 201; Shatz & Wilkinson, 2013; Verplaetse & Migliacci, 2008). I examine both how PSTs develop their own mathematical language and how they develop the language skills of ELLs; and I examine whether the one aids the other.

2.4 Relationships and Trajectories Among Themes

Engaging in Mathematical Practices 1 and 3 is important for all students, but especially challenging for ELLs if teachers do not have the ability to facilitate this engagement. There is much overlap between the Mathematical Practices, so if the ELLs manage to engage in one of the Mathematical Practices, it will be easier to engage in another one at the same time (Santos et al. 2012).

The social interaction and developing language themes are related—if teachers create environments where students are comfortable, they are more likely to participate in class discussions, which develop students' language. This is evidenced by Pinnow and Chaval (2014), who theorize that it is vital for teachers to plan strategically where they place ELLs in the room. They write for teacher educators and teachers, and they encourage developing ELLs' social competency so that they will feel comfortable in participating in meaningful mathematical discussions.

I have discussed how PSTs need preparation to effectively teach ELLs to engage in Mathematical Practices 1 and 3. The themes mentioned above may provide such preparation. In my study I examine how the PSTs apply these themes as they learn to teach ELLs to engage in Mathematical Practices 1 and 3. I have discussed how the literature is sparse on how teachers and PSTs can integrate the teaching of ELLs at the same time they teach all students. As I analyze my data, I look for evidence that affirms or refutes the value of such integration.

In my study I present a portion of my data using a trajectory. Trajectories are tools that can be used by mathematics educators to consider the sequences by which PSTs and other students learn mathematics. A research report issued by the Consortium for Policy Research in Education states that trajectories, also called learning progressions, are "empirically supported hypotheses" (Daro et al. p. 12, 2011) that consider both the order and the process of students learning mathematics. The authors claim that since trajectories help teachers understand how students learn, they can likewise help teachers to make decisions on how to support their students. Trajectories are affected by cultural and educational backgrounds, so they lack universal application, but despite limitations, they are useful to examine student thinking within given contexts (Daro et al., 2011).

Being novices, many PSTs will struggle to fully integrate how diverse students think in mathematics, but Turner et al. (2012) consider it important to introduce PSTs to effective teaching practices for diverse students—practices that the PSTs can develop throughout their teaching careers. Turner et al. encourage teacher educators to make sure that PSTs have high expectations for ELLs so that ELLs can live up to their expectations. Their study refers to linking ELLs' community and school experiences, but does not mention the importance of considering ELLs' language experiences. Their empirical study had six researchers who used surveys, interviews, and PST lesson plans to collect data from 200 PSTs. This is useful for my study since I also employ the concept of trajectories in PSTs' development to explore overall patterns and sequences in how PSTs learn to meet all students' needs; I compare my data to the data from the large study of Turner et al.

While Daro et al. (2011) recognize that not all students go through exactly the same order as they learn mathematics; in general there are typical learning trends. Thus, not all students follow the exact trajectory—there are some variations. There is additional variation because students from different backgrounds start at different points on a given trajectory. However, despite the variations, the researchers urge teachers and mathematics educators to consider the trajectories when making curriculum decisions. Daro, the lead author in the report mentioned above, was also one of the authors of the CCSSM. Evidence on trajectories was considered in the sequencing of the content standards from the CCSSM (Daro et al., 2011). The authors state in the report that there are gaps in understanding how students learn topics such as “algebra”, “geometry”, and “measurement”, and they suggest that trajectories may have an important role in helping mathematics educators better understand how students learn these topics. My review has discussed that there are also gaps in understanding how PSTs can learn to teach ELLs, so perhaps looking at data using the concept of trajectories can help fill this gap.

Turner et al. (2012) are helpful in this respect; they use a trajectory to describe PSTs learning to teach mathematics to ELLs. They posit that learning to teach itself follows a trajectory and that the PSTs may follow slightly different paths as they learn about teaching mathematics to ELLs. In Turner et al.’s words, “As PSTs interact with various participants across multiple spaces, we consider how they develop knowledge, dispositions, and practices that position them as effective teachers of mathematics for diverse students” (Turner et al., 2012, p. 69). Turner et al. (2012) found that as the PSTs were learning to make adaptations for ELLs, there were different levels of practices. They referred to the beginning of the trajectory as the “initial practice,” followed by “making connections,” and lastly “incorporating.” They found that each PST they studied had different starting points on the trajectory depending on their previous background. At the beginning of the trajectory, some PSTs had fragmented practices—they believed in creating meaningful experiences, but still held onto previous beliefs in practice. Furthest along the path were the PSTs who not only believed in making meaningful connections for ELLs learning mathematics, but also applied this in their practice.

Daro et al. (2011) discuss trajectories in detail and show their importance in developing a logical sequence of the Common Core State Content Standards. However, there is no discussion of a trajectory for learning the Mathematical Practices. Turner et al. (2012) discuss PSTs learning to teach ELLs meaningful lessons. This raises the question of whether it might be useful to investigate the development of trajectories for PSTs learning the Mathematical Practices, and their possible application to PSTs teaching the Practices to all students, including ELLs. In my study I look for a possible trajectory to examine how PSTs learn the Mathematical Practices themselves as well as how they perceive that the Mathematical Practices should be taught to all students, including ELLs.

2.5 Conclusions

The cited research is related to my study about PSTs facilitating elementary students', including ELLs, engagement in Mathematical Practices 1 and 3.The research lays the foundation for considering how PSTs learn to facilitate all students' learning and ELLs' learning of the Mathematical Practices. I started by reviewing general themes in the literature on teacher preparation and preparation to teach ELLs. I then reviewed how teachers could help students in general, and ELLs in particular, engage in Mathematical Practice 1 and help them be better problem solvers (Larson et al., 2012). Four themes examined for PSTs to engage students, including ELLs, in Mathematical Practice 1 are: connecting with students' backgrounds; providing students with access to the content; having high expectations for all students; and engaging students in productive struggle. In addition, it is important for PSTs to have a productive disposition and possess strong content knowledge. In order for PSTs to engage students in Mathematical Practice 3, PSTs need to create a classroom environment where all students, including ELLs, feel safe to participate in small and whole class discussions. PSTs need not only to have goals for their students in mathematics, but they should also have goals to develop students' language.

The literature about best practices for developing strategies for equipping PSTs to teach ELLs in mathematics is limited and there are not many empirical studies (Moschkovich, 2013; Turner et al., 2012). With the arrival of the Common Core State Standards it is imperative to know how PSTs may facilitate all students, including ELLs, to engage in practices aligned to the Common Core State Standards. Therefore, drawing on the cited literature, this empirical study proposes to study elementary PSTs as they learn Mathematical Practices 1 and 3 themselves, and as they develop strategies to facilitate elementary students', including ELLs, engagement in Mathematical Practices 1 and 3.

CHAPTER 3: RESEARCH METHODOLOGY

Chapter three includes a rationale for my research approach, a conceptual framework, an explanation of the context of my study, a research design, and my selection of participants. I also include sections on data collection, data analysis, ethical concerns, and limitations to the study.

3.1 Rationale for Research Approach

I used a multiple-case study with qualitative methods in this study to serve my purpose of describing how PSTs experienced the process of preparing to engage ELLs and other elementary students in Mathematical Practices 1 and 3. Merriam (1998) defines qualitative case study as: "an intensive, holistic description and analysis of a bounded phenomenon" (p. xiii). Case study research differs from ethnographies and grounded research in that the purpose of these latter qualitative research methods is to describe, not explain and that they may involve less preparation before the collection of data (Yin, 2014). Thus, my research differs from ethnographies and grounded research because I defined the case and research questions before collecting data—my purpose was to describe how PSTs experience learning and teaching Mathematical Practices and to offer possible suggestions for why the PSTs may develop different beliefs.

The "contemporary phenomenon" (Yin, 2014) that I investigated was how the PSTs enrolled in my mathematics methods course prepared to engage students, including ELLs, in Mathematical Practices from the recent Common Core State Standards. The context of the study was "real-world"—the PSTs reflected on how they could teach elementary students, including ELLs. In keeping with Yin's description of answering how and why questions, my research questions examined how PSTs learned the Mathematical Practices themselves and how they taught such practices to other students, including ELLs. I also examined whether PSTs followed similar or different trajectories of learning even though they were all in the same mathematics methods course and the same teacher education program.

I was more interested in the PSTs' developing beliefs and practices about these themes (and what might account for differences among them) than in "mere frequencies" of occurrence. As Yin (2014) suggests for case studies, I analyzed multiple data sources which included: open response surveys (pre-and post), homework reflections, lesson plans, university supervisors' reports, host teachers' reports, and semi-structured interview of PSTs, university supervisors, and host teachers.

The cases in this study consisted of eight PSTs enrolled in the mathematics methods course during the 2014 spring semester. I also interviewed host teachers and university supervisors about the PSTs they had been assigned. There were 22 PSTs in the mathematics methods course; ten were invited to participate in the research and eight of those agreed to be in the study. Thus, the cases were bounded by eight participants and data were collected from the beginning of the course until a final interview after the course's completion (Merriam, 1998). The study was also bounded in the context of the methods course. Although the PSTs reflected about their past experiences, other methods courses, and field placements, their reflections all occurred within the boundaries of the mathematics methods course.

Yin (2014) states that there are three types of case studies: exploratory, descriptive, and explanatory, even though there is the misconception that case studies can only be exploratory. Furthermore, he argues against the beliefs that descriptive studies are limited to surveys and histories and that explanatory research is limited to experiments.

Yin (2014) defined the descriptive case study as "a case study whose purpose is to describe a phenomenon ('the case') in its real-world context" (p. 238). Hancock and Algozzine

(2011) add “descriptive designs attempt to present a complete description of a phenomenon within its context” (p. 37). Hancock and Algozzine give an example of a descriptive case study: “A hospital administrator who attempts to gain a thorough description of her hospital’s emergency room procedures…” (p. 37). I attempted to obtain a detailed description of the PSTs’ experiences and emerging beliefs as they adapted strategies to engage all elementary students, including ELLs, in Mathematical Practices 1 and 3. Merriam (1998) points out that descriptive case studies are typically qualitative and involve thick description. Again, I attempted to use thick, qualitative description while describing PSTs’ experience.

Yin (2014) describes explanatory case studies as “a case study whose purpose is to explain how or why some condition came to be” (p. 238). Hancock and Algozzine (2011) shed light on explanatory case studies: Their primary purpose is to determine how events occur and which ones may influence particular outcomes” (p. 37). They offer an example of an explanatory case study: “A middle school teacher may conduct an explanatory case study to identify factors in his students’ home environments that affect their class performance” (p. 37). Similar to the example provided by Hancock and Algozzine, my study attempted to “identify factors” that might have “affected” how the PSTs both learned and taught the Mathematical Practices. Yin also points out that it is common and acceptable if there is an overlap between the three types of case studies. Thus, my dissertation is a descriptive case study with elements of explanatory case study.

The unit of analysis or case helps the researcher define the case study as single or multiple (Yin, 2014). Because each of the eight PSTs had unique experiences and backgrounds, this is a multiple case study. Multiple case studies tend to be more robust (Yin, 2014) and provide better explanations and generalizations compared to single case studies (Merriam, 1998).

Thus, my dissertation uses a, multiple, descriptive, case study approach with qualitative methods.

3.2 Research Design

The design of the study is important because it connects the research questions to the empirical data and conclusions (Yin, 2009). Yin explains that the purpose of the research design is to guide the researcher in how to answer questions by analyzing data and drawing conclusions. He declares that an effective research design will assure that the data answer the research question. Merriam (1998) claims that there are a variety of data collections and analyses: “Unlike experimental, survey, or historical research, case study does not claim any particular methods for data collection or data analysis” (p. 28).

I heeded Miles and Huberman’s (1994) advice and attempted to avoid the two extremes of tight and loose approaches to study design. Unlike the loose approach, I established research questions before the study and a theoretical proposition that connected them; unlike the tight approach, I attempted to be open to alternative explanations throughout the study. My design was in between the two approaches: I had structure, but was open to any changes that might be required.

In my data analysis I also attempted to balance between an inductive approach and a deductive one as I generated themes. Miles and Huberman (1994) suggest that researchers avoid the two extremes as they develop their analyses. They suggest that researchers with an inductive approach also have research questions beforehand to keep them focused and that researchers with a deductive approach be open to any data that may occur even though it is not in alignment with the research questions. The development of my themes was structured around the review of the literature, the design of my course, and my data analysis. I generated tentative categories based on the literature and themes that I decided to stress in the course and then balanced these against a fairly open-ended analysis of my data, looking for themes that were representative of one or more of them.

Developing my codes was an iterative process. The themes in the literature regarding learning and teaching mathematics to all students, including ELLs, guided me in designing my course and developing codes in my data analysis. Three of my codes, "connection," "social interaction," and "provide access," were key ideas that I learned from the literature that guided my teaching of the mathematics methods course. The PSTs actually used the term "connections," so I used that term for my code as I determined what "connections" meant to them. In interviews, the PSTs also mentioned learning the terms "developing class community" and "peer support" from the mathematics methods course and other methods courses in the inclusion program. These became subsections of the theme "social interaction," which was a term that I found in the literature. The PSTs were in an inclusion program, so they discussed accommodating and differentiating for students. I used the term "providing access," which was prevalent in the literature, to describe this process.

I chose "High expectations" to be a code after analyzing the data and noticing that students were taught in another course to have "assumed competence" for all students. However, I observed that three PSTs still did not apply the concept of assumed competence for ELLs. I changed the code's name from "assumed competence" to "high expectations" to be consistent with the ELL literature.

"Productive struggle" was a theme that I had stressed in my mathematics methods course, but the data did not reflect that this theme received much attention in the other courses in the inclusion program. However, three PSTs said that it was important to ask questions instead of telling answers to make the work challenging for the students, and this process is akin to the descriptions of productive struggle in the literature in which the teacher keeps the cognitive load high for students and yet within their grasp (Hiebert & Grouws, 2007; National Council of Mathematics, 2014; Warshauer, 2014).

I was aware of the theme “developing language” as a result of reading the ELL literature, but I did not see evidence from the data that the theme was stressed in any of the other courses. After interviewing a PST who believed in developing language, I chose that to be a code.

I did not use all the themes raised in the literature, however—only those themes for which I had data to document participants’ experience. I did not have a separate theme on the depth of PSTs’ content knowledge of mathematics, for example, because that was not a theme that I explored explicitly or in any depth in the course. There was one PST who mentioned the importance of having strong content knowledge, which suggests that it ought to be explored more explicitly in the course and in future research. In Chapters 4 and 5 I discuss in more detail how I developed each theme.

The context of this study was a mathematics methods course in which I was the instructor. The course was one semester long, with field placements, and each of its 22 elementary PSTs was part of a teacher education program with a common prescribed curriculum. My design involved the collection of all data that impacted these students, as they became teachers of ELLs in elementary classrooms during the mathematics learning experience. Details will be presented in Section 3.4.

I analyzed the following data: open response surveys (pre-and post), homework reflections, lesson plans, university supervisors’ reports, host teachers’ reports, and semi-structured interview of PSTs, university supervisors, and host teachers. The focus of this study was on documenting and analyzing the experience and learning of the eight PSTs, not on evaluating my effectiveness as an instructor or the course per se; the course provided the context for the PSTs' learning.

Special precautions were taken to protect my subjects and ensure a measure of objectivity in my analysis. Another researcher passed out the consent forms so the PSTs could freely decide if they wanted to participate in the study. The consent forms were not revealed to me until 48 hours after I had handed in the grades; all members of the class gave me permission to use their data, and I selected ten out of the 22. Since one of my research questions was related to ELLs, I selected PSTs who had a connection to ELLs to participate in the study. I decided to invite all five PSTs with ELL connections and five with no apparent connection to balance the number of participants with and without ELL connections. The PSTs were chosen purposefully to answer my research questions about the PSTs facilitating students' engagement in the Mathematical Practices, and also to answer the same question about ELLs.

For the purpose of this paper, it was necessary to define ELLs—I referred to ELLs as students who were receiving ESL services. According to Lucas, (2011) ELLs are students who speak a language other than English at home, but this led to confusion. When they came back from their first placement, three PSTs reported that they thought that they had students who spoke another language besides English at home. Emily said that she had a student who was adopted from China at age three and no longer spoke Chinese; she did not receive ESL services. There were other PSTs who were unsure what language their students spoke at home.

My new definition reduced the confusion because the PSTs only had to ask their host teacher if there were any students in the class that received ESL services. Two PSTs had students receiving ESL services in their placement with this new definition; two may have experienced being ELLs themselves because they were spoke a language other than English at home; and one PST had been in a previous placement with ten ELLs.

3.3 Conceptual Framework

My study is grounded in sociocultural theory. This includes the importance of social interaction--speaking about a topic and participation in shared activities for deep learning of concepts (Kennedy, 2009; Vygotsky, 1978). One of the Mathematical Practices that I studied, the third one, involves students vocalizing ideas as they justify their arguments and critique each other's arguments. As the participants in my study learned mathematical concepts and pedagogies while working in groups, they were vocalizing their ideas in mathematical communities. Furthermore, in the mathematics methods course the PSTs were encouraged to ask questions when they taught their classmates and students in the field. By asking questions, the PSTs motivated students to develop arguments, which is part of Mathematical Practice 3 and involves students participating in the social interaction described by Vygotsky. In my data analysis, I observed how PSTs facilitated students, including ELLs, to engage in Mathematical Practice 3, which also involved social interaction.

Applying Kennedy's (2009) theory, ELLs learn mathematical concepts deeper if they get a chance to develop and share arguments. Moschkovich (2002) concludes that teachers should offer ELLs opportunities to develop arguments by using gestures and manipulatives when they communicate. In my study I collected and analyzed data on how the PSTs perceived what were the best practices for facilitating ELLs to engage in the social interactions required in Mathematical Practice 3.

Vygotsky's (1978) idea of the zone of proximal development also supports facilitating the engagement of students in Mathematical Practice 1. PSTs should use strategies to allow students, including ELLs, to make sense of the problems and find each student's zone of proximal development as they offer assistance. Hiebert et al. (1997) assert that teachers typically offer students too much assistance and this reduces the cognitive load. As PSTs offer students access to the content, it is important that they make the mathematics rigorous enough to be in each student's zone of proximal development. Additionally, PSTs should develop students' language, including ELLs, as they are learning mathematics (Coggins, 2014). Similar to mathematics, students need to be challenged within their zone of proximal development. For example, if ELLs are given too much assistance as they learn English, then their zone of proximal development will be narrowed and their learning limited. Therefore, I analyzed whether and how PSTs considered elementary students, including ELLs,' zone of proximal development in both English and mathematics as they taught.

Sfard (2003) maintains that sociocultural theory involves the 'process of learning.' My study examines the process of PSTs learning the Mathematical Practices and engaging elementary students, including ELLs, in these Practices. Vygotsky (1978) states that it is not possible to study individual learning without studying the social processes and contexts of that learning. Studying the social process is particularly relevant in this study because the PSTs used their social skills as they reflected in groups in their methods course and negotiated with their instructors, host teacher, university supervisor, classmates, and students.

Sociocultural theory suggests that teachers should consider students' background knowledge and the "motives, beliefs, values, norms, and goals" (Forman, p. 337, 2003). My study considered both PSTs' background knowledge and how they considered their students' background knowledge in their placements.

3.4 Research Background and Setting

3.4.1 Pilot Study

De Oliveira (2011) found that her PSTs empathized with ELLs after she taught two mathematics lessons in Brazilian Portuguese to English speaking PSTs. For her first lesson, she did not use gestures and talked at a normal rate. The PSTs in her study commented that they had more access to the content for the second lesson when she spoke slowly and used gestures. Similar to de Oliveira's study, in a pilot study, I taught a mathematics lesson in Spanish to elementary PSTs in a private university in northeastern United States. The PSTs attempted to complete a worksheet but were denied access because they did not understand the Spanish symbols for long division. In addition, the directions were in Spanish, and similar to de Oliveira, I spoke at a normal rate and did not use gestures. The PSTs reflected that this simulation helped them empathize how ELLs might feel when they do mathematics if the teacher did not assist them with the English. Some of the PSTs said that they had thought that mathematics was a universal language, but after this simulation, they realized that they were at a disadvantage for not understanding the language of instruction.

Like de Oliveira (2011), in contrast to the first lesson, I modeled how mathematics might be taught to ELLs more effectively in the second lesson. I attempted to allow the PSTs access to the content by speaking slowly in Spanish, using gestures, translating strategic words into English, and allowing them to speak in their own language (English). At the end of the lesson, similar to the PSTs in de Oliveira's study, my students said that it was imperative to teach ELLs in a way that facilitates their access to the content.

The next semester I taught the same first lesson in Spanish in another mathematics methods course and did not facilitate the PSTs in accessing the long division worksheet. After being taught in Spanish, the PSTs divided into groups of four to discuss the lesson and were audio taped. Similar to the previous course and the PSTs in Oliveira's (2011) study, the PSTs mentioned feelings of frustration, as they were unable to engage in the content. The PSTs shared that they had realized that mathematics was not a universal language, and similar to Schleppegrell's (2010) argument, the PSTs said the language of instruction plays an important part in learning mathematics. In this course, there was a strong emphasis on learning the Mathematical Practices from the Common Core State Standards. The PSTs also commented that they were not able to engage in any of the Mathematical Practices when I had spoken to them in Spanish because they did not understand the directions.

Unlike the previous pilot study, this lesson was not followed up with the model lesson; instead I asked the PSTs to identify strategies that would have allowed them access to the content and enabled them to engage in the Mathematical Practices. Some of the strategies that the PSTs suggested were for me to speak slowly, use visuals, gestures, translate strategic words, and allow them to speak in their own language (English). The PSTs suggested that just as the strategies that they had generated may have helped them understand the content when I spoke Spanish to them, the same strategies may be helpful for ELLs learning mathematics that have limited English proficiency. I incorporated this exercise of having PSTs generate strategies to facilitate students' engagement in targeted Mathematical Practices into the methods course that provided the context for the PSTs in this study.

3.4.2 The Mathematics Methods Course

I taught the mathematics methods course in the spring semester to 22 elementary PSTs—17 sophomores and five juniors. There was one male and the rest were females.

Each mathematics methods class lasted three hours, and we met for fifteen sessions. In addition to these classes, each PST was assigned to an urban or suburban placement from first to third grade and each assisted a host teacher. They had gone into the field previously, but this was the first time the PSTs taught whole class lessons. They were observed by retired teachers, referred to as university supervisors. In the methods course, I gave the PSTs lesson plan guidelines, and they turned in their lesson plans to me for feedback and grading. When they first went to the field, the PSTs planned, implemented, and reflected on a mathematics lesson that lasted approximately an hour. In their second visit, the PSTs extended the process to three connected lessons. I asked the PSTs to choose three target students and ensure that these students' needs were met. The two PSTs who had ELLs in their placement included these students in their list of target students. The rest of the PSTs reflected in their lesson plans on how they would have modified their lessons to accommodate ELLs. After teaching in the field PSTs were asked to write how they had applied in their lessons what they had learned from the methodology class. PSTs wrote about the targeted Mathematical Practices in both the planning and reflecting stages.

The course focused on teaching the PSTs four of the eight Mathematical Practices. In an attempt to teach the Mathematical Practices deeply, I taught the PSTs in a similar format that Moschkovich (2013) suggests for teaching new concepts to students. Moschkovich states that teachers should facilitate understanding of new vocabulary terms by presenting them at the end of the lesson after students have had experience with mathematical tasks. Therefore, rather than presenting a list of the Mathematical Practices to the PSTs, I introduced the Practices gradually. On the first lesson the PSTs played a card game to encourage them to write their own mathematical practices. Unaware, the PSTs secretly played the card game with different rules; when the groups were switched and they were asked not to talk, there was confusion. After the game, the PSTs discussed how there are often different rules for learning mathematics, and they were asked to develop their own rules for guiding mathematics lessons. We discussed that Mathematical Practices have been designed to guide students in learning mathematics from Kindergarten to 12^{th} grade. We gradually refined the rules that we had made together and after the PSTs were familiar with the rules, we compared them to the four Mathematical Practices that were studied in the course.

I asked the PSTs to critique and apply the Mathematical Practices throughout the semester. Mathematical Practice 1: (Make sense of problems and persevere in solving them) and Mathematical Practice 3: (Develop viable arguments and critique the arguments of others) were taught in the course and will be discussed in Chapters 4 and 5 of this study. The PSTs also learned about Mathematical Practice 4: (Model with mathematics), which involves making mathematics relevant to students' lives. All students, including ELLs, are more interested in solving mathematical problems if the problems are relevant to their lives (Clarke et al, 2014). Mathematical Practice 5: (Learn to use tools strategically) was taught in the course; all students, especially ELLs, can learn mathematical concepts more deeply with the aid of manipulatives (Hiebert et al., 1997, Coggins, 2014). The remaining four Mathematical Practices are taught in the PSTs' second mathematics methods course, which is not discussed in this study.

As I mentioned in Chapter 1, although the mathematics methods course addressed four Mathematical Practices, the focus of my dissertation is on Mathematical Practices 1 and 3 because their language implications are more direct. Even though I focus on Mathematical Practices 1 and 3, the data are likely to be relevant to the other Practices as well. For example, if students engage in Mathematical Practice 4 (Model with mathematics) they may be more likely to make sense of problems and persevere to solve them. Likewise, if students engage in

Mathematical Practice 5 (use tools strategically) they may be more likely to develop arguments and critique others.

In course evaluations, the PSTs from the pilot study had reported favorably on their development of strategies to engage ELLs, so I followed similar pedagogies when I taught the PSTs four classes about teaching mathematics to ELLs. After the simulation experience of being taught a mathematics lesson in Spanish, the PSTs divided into groups of five to construct their own strategies for ELLs. The PSTs agreed on 12 strategies that they thought would help ELLs engage in the Mathematical Practices. The 12 strategies were adjusted by the instructor to reflect the strategies most relevant to the literature, put into a Likert scale, and used as an assessment when the PSTs taught their peers. (The assessment is included in Appendix C).

In their groups the PSTs planned a mathematics lesson to engage elementary students, including ELLs. One PST from each group taught the lesson while the other PSTs from the group role-played that they were ELLs. After each lesson was taught, the rest of the PSTs offered oral feedback. All of the PSTs, including the PST who had the role of the teacher, filled out the assessment forms mentioned above. After the feedback, each of the four lessons was modified, retaught, and reassessed on another day.

3.4.3 The Inclusion Program

All of the PSTs from this private university in northeastern United States were enrolled in an inclusion program as both general education and special education majors. The PSTs take more than 70 credits of education courses that teach strategies for meeting all students' needs in core content areas. The PSTs have opportunities to experience classroom community, as they stay with their peers in the same cohort. Each PST is also required to have an Arts and Science concentration of at least 30 credits. Before taking the mathematics methods course, the PSTs were required to take two semesters of college level mathematics consistent with the Common Core State Standards and had to obtain an average grade of B minus or higher between the two courses. In the semester that the PSTs were taking the Mathematics methods course, they were also taking a Social Studies methods course, a course in differentiation, a seminar that stressed teaching techniques, and an information technology course. The PSTs had previously been in the field as part of an extension of a reading and language arts class—they had opportunities to observe teachers and instruct small groups of children.

3.4.4 Role of the Researcher

I taught the PSTs the methods course that is the context of the study and I was also the researcher for this study. Ball (2000) states that it is an advantage for researchers to have the role of an insider because they are better able to reflect on their practice than would be an outside researcher relying on interview or observation data. However, Wong (1995) maintains that first-person research raises methodological challenges because there is tension between the goal of the teacher and the goal of the researcher. According to Wong, the goals of the researcher might influence the teacher's style of instruction. Wong emphasizes that if the two roles are to be combined and there is ever tension between instructing and researching, the teaching should be considered first.

I had authority over the PSTs because I assigned them grades. Even though, as discussed in Section 3.2, I had another researcher pass out the consent forms to address this problem, and did not know until after the course was over which students had given me permission to use their data in my research, it is possible that the students nevertheless were influenced by my dual role.

Ball (2000) asserted that when an instructor is also the researcher of a study, it is important for the researcher to be able to create distance when he/she reflects and is doing research. I attempted to create that distance partly because my unit of analysis is the PSTs, not my teaching. While my teaching undoubtedly affected how PSTs learned strategies to engage ELLs, the focus of the study was on the PSTs' learning. In order to create more distance, I heeded Wong's (1995) advice—whenever there was a conflict between teaching and researching, I gave preference to the teaching. For example, during lesson study the PSTs adapted mathematics lessons to accommodate all students' needs, including ELLs. The PSTs taught a lesson and then retaught it after receiving feedback from their peers. This was a time-consuming process, so only four PSTs had the role of teaching—the rest of the PSTs had other roles. From a researcher's perspective it would have been beneficial to have had the same four PSTs reteach the lesson and observe the progress that they had made. However, from an instructor's perspective, it was more beneficial to give different PSTs the opportunity to teach the second lessons. I allowed and encouraged different PSTs to teach the second lessons; this decision was based on what I thought was the best teaching practice.

One example of how doing research on my own students benefitted them became apparent when I interviewed the PSTs at the end of the course. I have not done interviews in the past because it is time consuming. However, this was an excellent opportunity for the students because I asked each student to justify their answers, which was one of the course objectives. Instead of creating tension, this is an example of students benefiting from the fact that I was researching them.

3.5 Participant Selection

I used purposeful sampling to select participants who might help me answer my research questions (Miles & Huberman, 1994; Tracy, 2013). Since one of my questions was about ELLs, I selected the two PSTs who had ELLs in their placements to collect data on facilitating ELLs' engagement in the Mathematical Practices. In addition, I selected a PST who had ELLs in their previous placement. I also selected the two PSTs who had reported in the survey that they spoke a language other than English at home to collect data on any personal experiences they may have had as they learned English. Below is a brief description of the PSTs chosen with possible ELL connections:

ELL Connection:

Kim:	8 ELLs in an urban placement
Abigail:	6 ELLs in a suburban placement
Judy:	Had 11 ELLs in previous placement
Charles:	Reported speaking both Chinese and English and taught Chinese summer school
Maria:	Parents are from Peru and speak Spanish to her, and she answers in English. In an interview she reported that her grandfather often lives with the family and he does not speak English.

In an attempt to contrast the five participants with possible ELL connections, I also selected five PSTs who had no apparent ELL connection—five whose backgrounds varied widely (Patton, 1990). One PST mentioned in interviews and homework assignments that she valued challenging her students—I selected her to gather data on productive struggle, because it has ties to Mathematical Practice 1 (Clarke et al., 2014; Hiebert et al., 1997; Larson et al., 2012). Another PST discussed the importance of including English in mathematics lessons, so I chose her to collect data on developing language, which ties into Mathematical Practice 3 (Lucas, 2011; Shatz & Wilkinson, 2013; Verplaetse & Migliacci, 2008). I selected Sarah to participate in my study to select a "negative example." Connecting mathematics to the students' personal lives is discussed in the literature as a way to facilitate students to engage in Mathematical Practice 1

(Clark et al., 2014; Koestler et al., 2013) and Sarah said in an interview that she felt disconnected in the mathematics methods course. I wanted to collect data on her experience of feeling disconnected. I invited Fatima and Mary to participate to seek variety in my sample— one expounded at length in answering questions and in her writing; the other answered and wrote tersely. Below is a brief description of the PSTs chosen with no apparent ELL connections:

No ELL Connections:

Amanda:	Showed interest in challenging her students
Emily:	Incorporated writing in her mathematics lesson
Fatima:	Wrote descriptive lesson plans and had long answers in the initial interview
Mary:	Wrote brief lesson plans and answered interview questions briefly
Sarah:	In her first interview she said that the course was disconnected from the field because she did not have ELLs and she did not use the Mathematical Practices in the field.

After interviewing the entire class, and handing in grades, I invited the PSTs above to be participants. Mary and Judy did not respond to two emails inviting them to participate in a second interview, so I chose the other eight PSTs mentioned to be participants in my study.

I wrote emails to the host teachers and university supervisors of all eight of the PSTs who had agreed to be in my study. One university supervisor worked with Kim, Charles, and Amanda. Another university supervisor worked with Emily and Sarah. Both of those supervisors agreed to be interviewed. In addition two more supervisors were interviewed, so all of the supervisors of the participants were interviewed except for Abigail's. Only three of the eight teachers were interviewed: Maria's, Amanda's and Sarah's. Although I wrote the remaining five host teachers two emails, they either wrote me back to say that they were too busy to participate in the interview or did not respond. Table 1 is a chart of the participants that I invited to be in the study.

Table 1: Participants Invited in the Study

Participants Invited to Participate in Study						
Pseudonym	Agreed to Participate?	Host Teacher Interviewed?	University Supervisor Interviewed by me?	Grade Taught in Placement	Number of ELLs in Placement	Urban or Suburban Placement
Abigail	Yes	No	No	1st	6	Suburban
Amanda	Yes	Yes	Yes	3rd	0	Urban
Charles	Yes	No	Yes	1st	0	Urban
Emily	Yes	No	Yes	2nd	0	Suburban
Fatima	Yes	No	Yes	2nd	0	Suburban
Judy	No	No	No	2nd	0	Suburban
Kim	Yes	No	Yes	3rd	8	Urban
Maria	Yes	Yes	Yes	3rd	0	Suburban
Mary	No	No	No	1st	0	Suburban
Sarah	Yes	Yes	Yes	2nd	0	Suburban

3.6 Data Collection

This case study included collecting: open response surveys (pre-and post), group discussions, homework reflections, lesson study, lesson plans, supervisors' reports, and semi-structured interviews with pre-service teachers, their host teachers, and their university supervisors.

Open Response Surveys (Pre-and Post) — on the first day of class open response surveys were administered to the PSTs. Yin (2014) posits that closed surveys are not effective tools for obtaining case study data because they are typically short, and case studies require rich data. In an effort to obtain richer data, I administered open response survey questions at the beginning and end of the course. [Surveys are included in Appendix B.] PSTs described their past experiences with ELLs, whether they spoke another language, and how they might have accommodated ELLs in mathematics. The surveys were an opportunity for students to show

how they would accommodate the ELLs in mathematics. The survey was administered at the beginning and end of the course, and changes were analyzed.

Group Discussions—in each of the 17 class sessions, PSTs were asked to discuss topics in groups of four or five. A compact voice-recording microphone was provided to one member of each group. These were attached to their smartphones, and were used as tape recorders. After class the PSTs sent the recordings to me via email.

Homework Reflections—PSTs wrote private reflections for homework, which were posted online. Over the course of the semester, 15 homework reflections based on class readings and class discussions were reviewed for this case study. In contrast to the group discussion data, the homework reflections were done individually. The individual reflections offered me the chance to examine how each PST developed their own understanding of the Mathematical Practices.

Lesson Plans— PSTs wrote lesson plans about their guided teaching experiences in a classroom from first to third grade. This data source was an opportunity to connect the strategies learned in the course with PSTs' placements. In the second month they planned a one hour-lesson, and in the fourth month, the PSTs extended the process to three connected lessons. The PSTs were asked to choose three target students and to plan, implement, and reflect on how they met their needs. Data were collected on this process and the strategies (if any) were analyzed. One question asked on the lesson plan was for the PSTs to reflect on how they had incorporated class discussions and reading into their lessons.

University Supervisors' Reports—university supervisors observed at least one mathematics lesson and one social studies lesson. Supervisors filled out a form that required them to state how the PSTs addressed issues of diversity, language, and pedagogy for both the social studies and mathematics lesson. In addition, the university supervisors completed a form at the end of the semester describing the PSTs' progress. Supervisors' written comments from the forms about ELLs or accommodating students were collected and analyzed.

Host Teachers' Reports—host teachers completed the same form as the university supervisor at the end of the semester, describing the PSTs' progress. Written comments from the forms were collected and analyzed.

Semi-structured Interviews of PSTs—the participants were given two interviews. The first interview on campus was a class assignment and lasted thirty to forty minutes. The PSTs were asked about their experiences teaching mathematics to all students. After the grades were turned in, the eight PSTs were each asked to participate in another interview. The PSTs were asked indirect questions to attempt to determine what resources they drew on to help ELLs, and all students, in mathematics. (The interview questions are included in Appendix A.)

University Supervisor Interviews—After submitting grades and selecting the eight PST participants, I emailed the corresponding university supervisors and asked to interview them for an hour. I asked the host teachers open questions about the PSTs' lessons. I formulated the interview questions after one round of coding analysis of the PST interviews.

Host Teacher Interviews—I emailed the host teachers of the eight PST participants asking for an interview of an hour length. I asked the host teachers open questions about the PSTs' lessons. I formulated the interview questions after one round of coding analysis of the PST interviews.

3.7 Data Analysis

I transcribed my audio and interview data to form a solid database so another person could also examine the data (Yin, 2014). The purpose of a database, according to Yin (2014) is to have a "systematic archive of all the data (field notes, documents, archival records, etc.) from a case study, assembled to enable the later retrieval of specific pieces of evidence" (Yin, 2014, p.

238). I separated the data into eight sections—each section included all the data from each PST and their corresponding university supervisor and host teacher—to set up my data analysis. First I analyzed each case individually to focus on each PST's data in an attempt to better answer my research question about how each PST's beliefs and practices may have differed. Then I put the eight cases together for a cross case analysis to look for how the PSTs' beliefs and practices were both similar and different (Miles & Huberman, 1994; Tracy, 2013).

I considered Miles and Huberman (1994) as I analyzed my data; they state that researchers, especially novices, benefit from "tightening" their data analysis by focusing on their research questions, but it is also important to be open to any surprises that may arise. For my initial coding I read the data three times to get an overall understanding of the data and made an effort to understand it even if it was not related to my research questions (Saldana, 2009). I noticed that there were many references to the PSTs making accommodations for their students, which became an initial code that I titled "strategies."

Saldaña (2009) suggests that researchers start data analyzing by playing with their data. I searched for words using Atlas in order to look for patterns. Amanda used the word "supports" 20 times out of the 31 times that it was used by the PSTs. However, the word search alone only gave me the frequency of words used, it did not help me explain what she or others meant when they used the term. I took further actions to analyze my data in depth. For example, I ordered the data for each PST to look at patterns over time and coded line for line. After putting the data in chronological order, I noticed that Amanda began to discuss supporting her students more and more as the course developed. While Amanda used the word "supports" to describe how she would promote students to discover mathematics on their own, without telling them the answers, Fatima used the terms "guide" and "questioning" to describe a similar process of allowing students to discover mathematics on their own. I chose the term “Productive Struggle” to describe the PSTs’ engaging students in solving problems that are within their grasp, but still challenging enough to cause tension (Hiebert & Grouws, 2007). When students engage in productive struggle, they engage in behaviors described in Mathematical Practice 1 (Clarke et al., 2014; Hiebert et al., 1997; Larson et al., 2012).

Mere frequency of words used was not as useful as knowing the context of the codes. I changed my coding schemes to distinguish between homework reflections, lesson plans, PST interviews, host teacher interviews, university supervisor interviews, university supervisor observation forms, host teacher observation forms, and whether the code referred to ELLs or not. For example, the code STRAT, ELLs, i, K would be a code from an interview with Kim about how she would use strategies to accommodate for ELLs. This gave me insight to different meanings of the same code. Data from PSTs writing about strategies in a homework assignment (perhaps to get a high mark) is different from data from the PSTs’ university supervisors commenting that they had actually used strategies to accommodate students in their lesson.

One theme that I became aware of during the interview process was “developing language.” I had read in the ELL literature how it was important for teachers to develop ELLs’ language, so I was open to looking for data about this theme. As I was interviewing Charles, it appeared that he made some comments about developing ELLs’ language that would be relevant to my primary dissertation question. “Developing language” became a code in my data analysis.

After each PST was analyzed individually, I used a cross-case analysis, which according to Miles and Huberman (1994) increases the possibility to generalize. As Miles and Huberman recommended, I made a case-level display. In the cross case analysis, I compared and contrasted emerging patterns from the themes from each participant. For example, I included in the display the PSTs' previous experiences of being with students who spoke a language other than English and made this a code.

As I mentioned earlier, Miles and Huberman (1994) advise researchers to be open to any surprises that may occur. As I searched for themes by comparing data within the same PSTs' data set, and as I applied constant comparative analysis, new themes emerged (Glaser & Strauss, 1967). Abigail, Maria, and Sarah made comments suggesting that they had lower expectations for ELLs than the other PSTs, so that became a key code. Two more key codes emerged when I reviewed the data and saw the terms "community" and "peer support" mentioned extensively throughout by all eight of the PSTs. I made "community" and "peer support" codes, and they were key themes in my data presentation.

3.8 Ethical Concerns

My biggest ethical concern was that I had authority over my participants inasmuch as I gave them grades. In order for them to freely decide if they wanted to participate in my study, I did not know which PSTs had granted me permission to use their data until after the course grades had been submitted. Secondly, in an effort to prevent or minimize my manipulating the PSTs in order to generate data that I needed for my study, I made a conscientious effort to put the needs of the students first.

To ensure confidentiality, PSTs', university supervisors', and host teachers' names were changed or removed from all data. My committee and I are the only ones to have access to the data. All data from this study are kept in a locked file and will be destroyed once the study is completed. The data from this study may be published or presented at conferences; however, no identifiable information will be used.

3.9 Limitations

Considering that I was both the course instructor and researcher, it was especially important that I developed checks to limit my bias as I analyzed my data. Yin (2014) describes a high quality analysis to be one that examines all of the evidence, considers rival explanations, and looks for the most significant parts of the case study. Therefore, I looked for multiple explanations that contradicted my developing theories. I also looked for other themes that were not directly derived from my research questions. For example, one PST discussed being culturally relevant throughout the semester. This did not appear to be connected to my research questions, but I examined the data to look for connections. I also found other examples of PSTs who mentioned being culturally relevant to ELLs. I found that as the PSTs attempted to facilitate ELLs' engagement in the Mathematical Practices (Research question #1) they tended to focus more on being culturally relevant and less on developing their language.

I have attempted to describe the process of my data reduction in detail so that if they so desire, other researchers can check the process. My data analysis began after my first class when the PSTs had filled out a pre-survey. I had planned to write participant observation notes after each class, but that turned out to be impractical—I typically spent time before my three-hour class preparing the lesson and talking to the PSTs, and time after class was used for picking up materials and talking to PSTs. Also, it was not possible to focus observations on a small sample inasmuch as I did not select my participants until the course was completed.

While my research questions focus on how PSTs experience the process of learning to teach Mathematical Practices to ELLs and other students, the bulk of the data documents their perceptions of this process through semi-structured interviews and homework assignments. Apart from data involving the PSTs' perceptions, I also examined data from university supervisors' reports and interviews, and data from host teachers' reports and interviews, but I acknowledge that field observations (and other kinds of data) could have provided a more robust analysis. I discuss other limitations and their implications for further research in the final chapter.

CHAPTER 4: FINDINGS—MAKING SENSE OF PSTS' PERCEPTIONS OF MATHEMATICAL PRACTICE 1

In Chapter Four I report and analyze my research findings related to Mathematical Practice 1. The order in which I present data from the individual PSTs varies from one section to another, beginning each time with the PSTs whose data are most extensive and pertinent. Some themes appear to be especially important to particular PSTs, and (while still relevant) much less important to other PSTs. I report on all subjects, if only briefly in each section. To aid in keeping track of the PSTs, I summarize background information in Table 2 for each one. In a summary at the end of each section I again provide a table that shows in thumbnail form how data for each PST were related to the various themes. These ratings are intended only as shorthand aids in following the data and should not be viewed as a substitute for the fuller descriptions presented in the text.

According to Koestler et al. (2013), problem solving is the central focus of Mathematical Practice 1 (making sense of problems and persevering to solve them). Students successful in this practice persist in understanding the meaning of a problem as they plan how to solve it, and they consider previous background knowledge while questioning if their solution is logical (CCSSI, 2012). Teachers can facilitate students' engagement in Mathematical Practice 1 by connecting the content to students' background knowledge, making accommodations, having high expectations, and providing challenging problems (Clarke et al., 2014; Hiebert et al., 1997; Larson et al., 2012). I have divided Chapter Four into four sections: "Connections," "Providing Access," "High Expectations," and "Productive Struggle."

In each section I discuss:

1) The PSTs themselves developing an understanding of Mathematical Practice 1;

2) The PSTs' beliefs and practices of facilitating elementary students' engagement in Mathematical Practice 1;

3) The PSTs' beliefs and practices of facilitating ELLs' engagement in Mathematical Practice 1.

When PSTs, according to Turner et al., were learning new concepts about ELLs that had not yet been internalized, they often had inconsistent beliefs and practices. For example, PSTs at the beginning of the trajectory would say that family support is important and yet later on say that the families' lack of English proficiency was unfavorable to the ELLs' progress. Throughout my analysis, I will be aware of possible inconsistencies from the PSTs as they learn the Mathematical Practice 1 themselves and attempt to apply what they have learned in their teaching.

Data considered include: Open response surveys (pre-and post), homework reflections, lesson plans, university supervisors' reports, host teachers' reports, and semi-structured interview of PSTs, university supervisors, and host teachers. The data are presented in Chapters 4 and 5. In this chapter, I discuss the PSTs who have the most pertinent data at the beginning of each section. Some themes appear to be especially important to particular candidates, and (while still relevant) much less important to other PSTs. I discuss how some terms may have a different meaning for different candidates. For the reader's convenience, I provide background information in Table 2 below on each PST. I provide a table in the summary at the end of each section. I distinguish between the term "moderate" and "strong" by only giving the later ranking if I have sufficient data.

Table 2

Backgrounds of PSTs for Mathematical Practice 1

Name of Participant	Grade Taught in Placement	Number of ELLs in Placement	Urban or Suburban Placement	Mathematics Instruction Taught in Placement	Type of Lessons Taught by the PSTs in the Field
Abigail	1st	6	Suburban	My Math Textbook	LES Model (Inquiry-based)
Amanda	3rd	0	Urban	EngageNY Modules	LES Model (Inquiry-based)
Charles	1st	0	Urban	EngageNY Modules	LES Model (Inquiry-based)
Emily	2nd	0	Suburban	My Math Textbook	LES Model (Inquiry-based)
Fatima	2nd	0	Suburban	My Math Textbook	LES Model (Inquiry-based)
Kim	3rd	8	Urban	EngageNY Modules	LES Model (Inquiry-based)
Maria	3rd	0	Suburban	My Math Textbook	LES Model (Inquiry-based)
Sarah	2nd	0	Suburban	EngageNY Modules	Direct Instruction

4.1 Connections

I chose the term "connections" because the PSTs used this term throughout the data. The literature suggests that teachers should make connections with students in order to facilitate them to make sense of problems (Koestler et al., 2013), and Clarke et al. (2014) note that connections are central to supporting perseverance to solve problems. Being in a dual major program of general and special education, the PSTs had already learned strategies to connect with students. In the mathematics methods course, the PSTs were introduced to an inquiry-based lesson format that was derived from The Connected Mathematics Project called Launch, Explore, Summary (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002). The purpose of the launch was to capture the students' interest at the beginning of the mathematics lesson. Clarke et al. (2014), adds that teachers should connect mathematical tasks with students at the beginning of the lesson to increase their perseverance. Furthermore, as part of the lesson plan, the PSTs were guided to explain how they would make connections with their students. I use the term, "connections" to describe the PSTs' beliefs and practices for connecting students to mathematics. Throughout this section I discuss what connections may mean to the PSTs.

Amanda. Amanda shared in a homework reflection on the first day of the mathematics methods class that she had been challenged and had been a capable mathematics student:

> In elementary, middle, and high school, I was very good at math. I really enjoyed solving problems and learning new strategies. In elementary school, I tested into the gifted and talented program for math. I was definitely challenged by the curriculum (Mathematics autobiography, January 15, 2014).

Amanda took responsibility for her own learning—she said that she enjoyed solving problems suggesting that if she put in the effort, she could make sense of problems by herself (Kilpatrick et al., 2001).

On the second day of class Amanda reflected on how she had been taught as a student. Amanda disagreed with the way in which she had been taught mathematics:

> I do not remember trying to solve problems in different ways. I was always taught with an acronym, like PEMDAS, and felt I had to memorize certain strategies, then demonstrate them in a worksheet. Although this helped me with order of operations, I didn't see the true meaning of what I was doing. Learning math in that way just seems like learning a recipe you must follow with no alterations. Sometimes you need to experiment to get it to taste just right. I also do not remember sharing ideas and critiquing classmates ideas (Mathematics autobiography, January 15, 2014).

In the mathematics methods class I suggested that students be encouraged to solve problems in a variety of ways. Amanda reflected that she had not been taught with that philosophy. Amanda felt like she was just repeating the strategies that her teachers deemed relevant, but this process did not develop for her a deep understanding of concepts. She argued that mathematics would have been more meaningful if she had been allowed to experiment and critique ideas with her peers. Her teachers did not appear to connect the mathematics for her because she did not know why she was doing the mathematics.

Amanda continued to reflect on how she had been taught mathematics:

> I feel that my math education in elementary school provided me with enough ways to see the purpose of what I was learning. However, as time went on, and I took higher-level math courses, I began to ask myself, "When will I ever use this again?" (Mathematics autobiography, January 15, 2014).

She reflected that it is important to have a purpose for learning mathematical tasks. Clarke et al. (2014) state that students will be more likely to persevere to solve challenging problems if teachers explain the purpose of the lesson and connect it to the students' personal lives.

As the semester progressed, Amanda began to focus on the importance of the concept of connecting students to mathematics. A group of Amanda's classmates taught a lesson to the class, and in Amanda's critique, she praised the group for creating rapport with the class before teaching the mathematics:

> One member launched with an anecdotal story about how previous methods of teaching mathematics did not work for her. She helped us relate our negative experiences to her own and made us hopeful that her group would help us see different ways to solving problems (Lesson critique, January 29, 2014).

Amanda reported that her classmate had been effective at aiding the PSTs to persevere and search for a variety of solutions to solve problems by telling a personal story. She commended her peers for connecting the mathematics at the beginning of the lesson—akin to Clarke et al. (2014)—by having a brief discussion about personal preferences for learning mathematics.

In an interview, Amanda reported that she applied what she had learned in the methods course to include students' names and interests to make the content more relevant:

> I'm not opposed to worksheets because I think with addition you need to do multiple problems to practice and I'm not opposed to them, but make them more relevant like you did when you added our names and interests and stuff like that (Second interview, April 28, 2014).

Amanda reasoned that at times students need to practice procedural fluency, but if teachers accommodate their students by adding the students' names and interests, the procedural work becomes more relevant. This is an example of how Amanda extended to her teaching what she had learned in methods class about connections. In the mathematics methods class we had discussed using students' names to make word problems more relevant, but Amanda said that using students' names and interests can make developing fluency more relevant too. In other words, Amanda implied that doing the worksheets could be uninteresting, but by making the mathematics more personal to the students, they may be more likely to persevere at solving them. In this example, Amanda suggested connecting with students by making the tasks relevant and interesting.

Amanda felt that her host teacher did not connect the mathematics tasks to her students' lives. In an interview, Amanda alleged that when her teacher taught, the only motivation to finishing the packet of worksheets was to get it over with:

> A: I saw my students like copying just to get it done because it was their only motivation. They had no motivation to learn it; they had motivation to get the packet done.
> J: What was their motivation for finishing the packet?
> A: Not having to do any more of that packet.
> J: (Laughs.) That's it?
> A: (Laughs.) Yeah (Second interview, April 28, 2014).

When I asked how she motivated her students, Amanda replied:

> They had a riddle and it was written in a letter format. It was a riddle about fractions and there was a person pretending not to understand fractions who asked for their help with learning fractions so basically I called them fraction experts. We need to learn this so we can teach someone else… it was pretty motivational for them (Second interview, April 28, 2014).

It was discussed earlier in this section how Amanda felt that she was not given a purpose for doing mathematics; she believed that she had given the students a purpose for doing mathematics by calling the students "fraction experts" who could help a fictitious person learn. By calling her students experts that needed to help someone else, Amanda connected the mathematics to their lives because they had a purpose for solving the tasks.

Not only did Amanda attempt to make whole group connections, she also planned to make connections for individual students. Contemplating how to improve her lesson, Amanda suggested that she could have had one of her students read the mathematics to the class in order to engage her more at the beginning of the lesson:

> Maybe, since she loves to read, I could have had her read the letters from the mystery student to the rest of the class. I could have also tied in student interests if I had an extra day to work on word problems (Lesson plan reflection, April 4, 2014).

Amanda attempted to motivate her students by allowing them opportunities to do things that they enjoyed and found interesting. This is an example of connecting mathematics by relating it to a personal interest of the student.

Abigail. Abigail wrote in a homework assignment about her own experience with connections to mathematics as a student:

> My favorite learning style is hands on learning. In mathematics, I like to work through the problem step by step after being taught the strategy or technique. I like real word examples because sometimes I feel like some of math we learn is pointless unless it have a connection to something somewhat relevant in my life (Mathematics autobiography, January 15, 2014).

Abigail shared that connecting mathematics to her meant to be relevant and real. She also said that mathematics should not be pointless, which is similar to Amanda's comment about mathematics having a purpose.

Abigail wrote in a homework assignment how I had made a connection with the PSTs in the beginning of methods class. She noted the connection on the first day of class: "I really liked that the professor took the time to tell the students about his life, his experiences and his interests because that established a connection between the students and the professor" (Class reflection, January 17, 2014). Abigail felt that I had connected to the PSTs and her by telling about my personal life.

Throughout the semester, Abigail mentioned the importance of teachers being culturally relevant in mathematics. In a homework reflection, she wrote what this concept meant to her: "Culturally relevant teaching allows the teacher to teach in a way that connects learning to the students' lives and shows how math problems are in everything we do" (Reading reflection on cultural relevance, January 22, 2014). Abigail said that culturally relevant teaching is effective because teachers can accommodate students to connect to mathematics by relating to their personal lives.

Abigail also mentioned language when referring to being culturally relevant with ELLs. When Abigail and a partner analyzed an assessment designed to measure how students engage in the Mathematical Practices, the PSTs wrote: “One suggestion would be to make them more culturally relevant by having language options” (Classroom reflection, March 18, 2014). Thus, Abigail and her partner discussed the importance of considering ELLs’ heritage languages when being culturally relevant.

In the post-survey, Abigail, who had six ELLs in her placement, wrote that she would help ELLs to learn mathematics by “being culturally relevant by connecting math to their lives and communities so they can see the importance of math not just as something they have to learn” (Post-survey, April 22, 2014). Similar to Amanda, Abigail suggested giving students a purpose for doing mathematics.

An examination of her lesson plan indicates what being culturally relevant with ELLs might have meant to her:

> I learned to make learning math more accessible to ELL students which were simplify language, use manipulatives and provide a sufficient amount of time. I incorporated being culturally relevant into my lesson by making real world connections with my students so that they may be able to see the importance of learning the material for their personal benefit. I used examples like roller skates, pizza pie, earrings and shapes they are familiar with in order to have an ease of lesson where it builds upon some prior knowledge to teach new content (Lesson plan reflection, April 15, 2014).

Abigail said that she had made connections with ELLs by using “real world” materials that relate to their “personal benefit.” Her use of common objects that were part of her students' experience appears to be an effective strategy to connect the students to the mathematics.

In order to be culturally relevant, Abigail would need to know her ELLs and their culture well, but in an interview Abigail did not know which country one of her ELLs was from: “I don't know. Somewhere from Africa. An African country. I forgot” (Second interview, May 2, 2014). When discussing another ELL, Abigail described her student: “The mother looks Spanish but maybe she's Arabic. The father’s from Australia but he came to the States” (Second interview, May 2, 2014). In a description of a third ELL, Abigail said: “One girl was from Ghana or something like that but she speak English at home” (Second interview, May 2, 2014). Abigail knew which country the remaining three ELLs were from. Abigail taught first graders, so skating, pizza and earrings may have been relevant to these ELLs’ lives. Another option for PSTs to make personal connections specifically with ELLs is to include problems that relate to the ELLs’ heritage country. It is important for PSTs to know all of their students well, including ELLs, to choose tasks that make personal connections.

Charles. Charles wrote about his learning style and personal experiences learning mathematics as a student:

> I openly admit that I struggle with math, however I also realize that if a teacher teaches to my learning style then this enables me to learn any kind of math. The style that I learn best with is when a teacher takes their time to slowly explain to me in detail, the different formulas, symbols, graphs, and rules. I also appreciate it when teachers constantly ask me questions to see if I understand the math. Furthermore, when doing problems I like it when teachers explain each step and then ask me questions in order to check my understanding. I have this unique learning style because I feel that a lot of my struggles in math came from teachers assuming that I knew how to do the different steps in their math problems, even when I didn’t. This would later hinder my development in math when I learned new problems, because I did not have the math background that my peers did (Mathematics autobiography, January 15, 2014).

Charles preferred his teacher to slowly explain concepts to him step by step. Instead of teachers passing the responsibility to the students to makes sense of problems (Wilcox, 1991), Charles wanted his teacher to explain and connect the mathematics for him.

According to his university supervisor, Charles made more progress in a semester than any of the other four PSTs who she had worked with. At first, according to his supervisor, he was impersonal and did not connect with his students. She suggested that he stop referring to himself in third person: “We talked a lot about opening up and being yourself. It's okay. You need to get rid of the Mr. Lee [pseudonym] thing and use I and me” Charles’ university supervisor told him that he would be more effective if he were more personal with his students (University supervisor interview, July 25 2014). “Once he finds that personal stuff and finds out how to relate to the kids and make the kids interested he will be fabulous. If he sticks with this more formal thing, it is not going to work at all” (University supervisor interview, July 25 2014). Charles’ university supervisor argued that when he was more personal with his students he connected with them and they were more motivated.

By the end of the semester Charles shared personal life experiences with his students, and they were engaged. The university supervisor said that she had heard two students converse about the importance of solving a mathematics problem in order to help Charles: “I heard one kid say, ‘I can’t do this,’ and then another student said, ‘we need to help Mr. Lee find out if he can fit the students on the subway,’ and the student got right back into it” (University supervisor interview, July 25 2014). The supervisor said that the students were more engaged as a result of Charles sharing his personal life with them. She described how she had told Charles that he had taught an effective lesson as a result of being personal: “If you had said, ‘There was a teacher and he took his class on...’ it wouldn’t’ have worked” (University supervisor interview, July 25 2014). Therefore, according to his university supervisor, Charles had motivated his students to solve the mathematics problems by being more personal with his students.

Taken from his lesson plan, Charles described how he was going to connect his lesson on subtraction during the launch:

> I will be telling you a story to help us understand how math can be used in the real world! Last week, a student asked me what he [I] does during the summer. What are some things that you guys do during the summer? (Lesson plans, April 9, 2014).

Charles asked this question to start a brief discussion on summers to get the students interested. He attempted to connect the students to the mathematics task by making the problems real. Next he described his summer experience of taking a train with students. In an interview, he reflected on how he had connected his students in the Launch:

> You can make the math more engaging for them so they will persevere. Like I did with my lesson one kid said "Oh, my gosh we have to solve this because we need to know whether Mr. Lee can get on the subway or not"…They were persevering; it was just something I had never thought about before. I wanted to engage them…I was never thinking before to tell them a story in order to engage them so they would persevere (Second interview, May 5, 2014).

Thus, Charles learned from the methods course to tell a story about his personal experiences to connect with his students, which he said helped them persevere to solve problems.

In an interview, Charles said that when I taught a mathematics lesson in Spanish to the class, it reminded him of when he was a young student and felt disconnected: "When you did that lesson where you spoke in Spanish, that reminded me of when I was younger and I wasn't as culturally connected with my peers around me because I grew up in a very white suburb" (Second interview, May 5, 2014). Charles reflected how not knowing a language made him feel the lack of connection. Charles continued to argue that he could better empathize with ELLs because he had had this experience of learning English himself and feeling disconnected. He had worked with ELLs from Boston during the summer who mostly spoke Mandarin and were learning English. Charles stated in an interview that his knowledge of learning a second language helped him connect with ELLs and then connect with mathematics:

> I think it gives me an understanding of why they struggle with the language sometimes because growing up I struggled a little bit with the language but then again I feel like it gives me a closer connection with ELLs than someone who is not bilingual but it's not as close as it would be with a student who spoke Chinese (Second interview, May 5, 2014).

Charles felt that although he had felt disconnected when he had not understood English when he was younger, this experience could help him connect with students who are also learning English as a second language.

Emily. Emily said that she had had positive experiences with mathematics and enjoyed solving problems:

> I loved math in every experience I had with it. I was always fortunate to have teachers who were passionate and caring. The relationship I built with my teachers allowed me to enjoy impressing them with my math skills. I also enjoy having a problem to solve. It is nice for me to know a couple strategies but to always come up with a correct answer (Mathematics autobiography, January 15, 2014).

She partly attributed her past positive experiences in mathematics to her caring and passionate teachers. In addition, Emily focused on impressing her teachers and obtaining the correct answer. Kilpatrick et al. (2001) posit that while precision is important, students also need practice developing their own strategies for problem solving.

The PSTs learned about the importance of connecting to students as they taught each other in the methods class. Emily praised a lesson taught by her peers because they were able to make the mathematics relevant by connecting fractions and candy:

> Kim had gone to the movies and could not decide which candy to buy. Then, we were asked to vote as a class on which type of candy is the best candy. This tied relevance to the lesson because students were able to connect from a past experience and give an opinion when learning about the importance of fractions (Lesson critique, February 5, 2014).

She wrote that the students had identified with Kim's personal experience of going to the movies. Emily realized that her peers had made a connection to fractions when there was a short discussion first about how mathematics tied into their personal lives.

Emily discussed another lesson that her peers had taught and mentioned connecting:

> Jane [pseudonym] explained how she always had frustration with combinations and that is why she wanted to teach them…it shows relevance, purpose to our lives…This was purposeful because we were able to connect the combinations lesson that was being taught with the food that was offered (Lesson critique, January 29, 2014).

In this example, Emily explained how her peer had connected the mathematical task by sharing personal experiences with the class. She also mentioned that her peer had given the PSTs a purpose for doing the mathematics.

Above, Emily explained how her peers had made connections by having a discussion about the task at the beginning of the lesson. Emily also said that teachers can aid students to make connections by allowing them to talk during the lessons: “Teachers should have students not only working with manipulatives but also in groups so that they can explain what they are doing and the connections they are making” (Reading reflection on number sense, March 17, 2014). In Emily’s lesson plan reflection, she wrote that her students were able to make connections and as a result they discussed mathematics:

> I believe that I gave many opportunities of relevance and the students showed interest in these. I also was very proud of the beginning of the lesson when students were able to connect their prior knowledge of addition and subtraction and successfully complete the launch. Students were also very engaged because they were eager to participate (Lesson plan reflection, February 12, 2014).

Emily pointed out that not only can students make connections by talking, but they also are encouraged to talk if they have previously made a mathematical connection.

More evidence that the PSTs were making content relevant for their students was from other teacher educators. Emily’s host teacher said how she “has a strong connection with the students. Each day she is there to support the students in their learning and has good insight into individual student's needs. Her lessons always keep the students engaged and active in their learning" (Host teacher evaluation, April 11, 2014). Emily's host teacher implied that she made the work relevant for students by getting to know their personal needs.

In a homework reflection about math identity, Emily wrote how she would connect with ELLs:

> I could have them begin by writing about what math means to them. For them to have a math identity I would allow them to develop their own importance with math and for me to be able to adapt my lesson plans so these students can succeed. Having an ELL student be the one who has a math identity would allow me to include relevance in my math lessons so that they are more likely to understand and develop learning rather than just memorize what I had taught for the test. As the article says, this will result in long-term knowledge. Also, the importance for the ELL student and their identity is that it can create a connection between the math content and their primary language and culture (Reading reflection on mathematics identity, February 23, 2014).

Emily suggested that mere memorization of empty facts, without having a purpose and a connection, would not result in retaining the mathematical concepts. She argued to allow ELLs to be involved in the planning process so that they are more connected to the material.

Fatima. As I discuss in section 4.2 Fatima reported having negative experiences in mathematics and she felt that she had been denied access to subject. She wrote that she didn't like mathematics, although she liked solving word problems:

> I am not a fan of math but I love word problems and problem solving type questions. I am a kinesthetic and intrapersonal learner…While I do not necessarily love group projects, I love group work because it allows me to see other people's methods, strategies and ideas, in comparison to my own (Mathematics autobiography, January 15, 2014).

Fatima liked working in groups and listening to her classmates' ideas.

When reflecting on her experiences, Fatima wrote that her statistics teacher used manipulatives: "This was a big practice used in elementary school and in my statistics class by my teacher. He brought in coffee cake and string. It was the highlight of my not-so-great mathematics education" (Mathematics autobiography, January 15, 2014). In an interview, she continued to talk about her statistics teacher after I had asked Fatima if any of her beliefs about mathematics had changed this semester. She said that she used to think that mathematics should be taught in a repetitive fashion because with the exception of her statistics teacher, that is how she was taught:

> I thought it should be repetitive because that's how I was taught. Then my senior year I had an awesome teacher who taught statistics and he made the material relevant and he wasn't too repetitive, but because I had spent so much time all through middle school and all through high school learning that way, so then I thought that is how you had to do math—by being repetitive (Second interview, April 28, 2014).

Fatima's comments imply that she enjoyed how her statistics teacher had connected with her by using manipulatives and making the material relevant rather than repetitive. According to Fatima, she had already learned techniques for making students interested in the content in other methods courses. All of the PSTs at this teacher education program were enrolled in dual majors of general and special education; thus they had already learned many strategies to connect with all elementary students, including special education students. Fatima said: "It was teachers [instructors] that targeted our interests… those are lessons that I remember and the information that I remember" (First interview, April 24, 2014). Fatima's comments suggest that she learned the most from the instructors who made the content interesting to her.

On the last day of class each of the PSTs was asked to give a presentation to the class about the most important aspects of the mathematics methods course. Fatima included the following:

> In order for mathematics to be a positive learning experience for the maximum number of students, teachers should target instruction that engages students in a variety of the mathematical practices. These practices create a learning environment where math is relevant, fun, accessible, and engaging (Emerging theories presentation, April 22, 2014).

Just as Fatima said that she had learned the most as a student from professors who had "targeted" her interests, she also said that teachers should target students' interests. Fatima suggested that when students are engaged in the Mathematical Practices a learning environment is created in which mathematics is relevant, fun, accessible, and engaging.

Her next comment adds more evidence for how Fatima did not approve of mathematics being too repetitive. She criticized the textbook used in her placement because there were too many problems: “We teach you and then give you 23 problems to use it [apply what was taught]. Even though the last three are word problems, it doesn't change the fact that they are doing a problem over and over again” (First interview, April 24, 2014). Fatima may have meant that the three word problems offered variety, but the 23 previous problems by contrast did not; her statistics teacher had made the work relevant and more varied too.

In a critique of one of her peers’ lesson, Fatima also discussed repetitive problems:

> The word problems eventually became slightly repetitive and monotonous but had they made the problems more varied in type/difficulty and the nature of the word problems exciting and interactive, the[y] would have retained everyone’s attention better and provide a more compelling lesson (Lesson critique, January 24, 2014).

Although in this comment Fatima did not mention connecting with students, she said that varying the problems and making them more exciting would better capture students’ attention. This is in alignment with the literature about encouraging students to engage in Mathematical Practice 1. Hiebert et al. (1997) maintain that students need to spend more time solving tasks that require problem solving and less time “practicing paper-and-pencil skills on sets of worksheet exercises” (p. 17). Larson et al. (2012) claim that the purpose of this Practice is to improve students at problem solving. In the last four paragraphs, I have discussed how Fatima learned from her own experiences about the importance of connecting mathematics by making it relevant, varied, and engaging.

One challenge that Fatima had was to connect the mathematics for her students and keep them quiet at the same time when her university supervisor observed her: “Not that the kids weren't interested; they were. The noise level was on task noise but still was getting a little bit loud so was up to her to redirect it" (University supervisor interview, July 9, 2014). On the student evaluation the supervisor had a similar comment: "Room got noisy frequently (on task chatter but the volume needed to be lower)" (University supervisor evaluation, April 16, 2014). Therefore, Fatima had connected with her students and they were excited to persevere to solve problems, but she had to negotiate with the goal of the Mathematical Practice and her university supervisor, who wanted the students to be quieter. I asked Fatima to comment on that experience and she said:

> It was so exciting being able to explore different 3-D shapes and since they were still exploring, it was hard for them to articulate. And with my supervisor being there, I was trying so hard to keep the noise level down (Second interview, April 28).

Despite Fatima's connecting the mathematics with her students, she had the restraint of her university supervisor to expect her to connect the mathematics with her students and keep them quiet at the same time.

Another strategy for connecting to all students is for teachers to be culturally relevant. Fatima commented how she had learned about being culturally relevant in many of her courses:

> I think it was a process and I understood at the beginning, but the fact that we talked about cultural relevance in every single one of our courses... It's really important that we do because it makes us see how important it is to every aspect of instruction. We started in the beginning [of the mathematics methods course] by reading Ladson-Billings book and talking about cultural relevance. Then we talked about it in the classroom and how it helped.… if you start talking about it at the beginning before you talk about math or social studies then you have a different mindset (Reading reflection on cultural relevance, January 22, 2014).

She had discussed being culturally relevant in other classes, but when it was mentioned in the first lesson of the mathematics methods course it helped Fatima understand being culturally relevant in the context of mathematics.

Fatima also attempted to make her lessons relevant for her students through student interests. Fatima wrote in her lesson plan that she had helped one particular student "make sense and persevere through the problems presented" (Lesson plan, February 13, 2014) by connecting the problems to basketball. According to Fatima's university supervisor, Fatima made the mathematics tasks interesting and connected for her students. Her supervisor said: "Lessons were motivating and engaging. Launches [hooks at the beginning of the lesson] were focused and goal oriented, activities were age appropriate" (University supervisor evaluation, April 16, 2014). Although doing mathematics problems about basketball may not have been relevant for all students, Fatima attempted to connect mathematics with the students' interests.

Her host teacher agreed that Fatima had attempted to make personal connections with her students. She wrote: "Fatima's lessons included creative introductions and activities that the students could relate to and would be interested in for example, basketball, a 'mystery box,' airplane travel, and passports" (Host teacher evaluation, April 16, 2014). After Fatima had taught the lesson she wrote:

> Many students also benefitted from the relevance all of the problems provided to them... The use of basketball, splitting candy/food, and Frozen, all had students modeling mathematics as they solved the real life situations. They were clearly intrigued and engaged with the lesson as I received questions and comments including, "Wait so when is this party? I think you should invite more people" (Lesson plan, February 13, 2014).

Above Fatima noted that her students were intrigued; her students' happiness was also a factor that Fatima looked for when evaluating her lessons. Earlier I discussed how Fatima had reflected that her teachers had not made mathematics engaging for her; however, when she herself was a teacher, she noticed whether her students were engaged. She engaged her students by appealing to her students' different learning preferences. When teaching, Fatima not only focused on whether the students were getting the correct answer, but also whether they were having fun. In other words, Fatima was developing students to have positive dispositions (Kilpatrick et al., 2001), which encourages students to engage in Mathematical Practice 1 (Larson et al., 2012).

Kim. Kim described her experiences with mathematics and her learning preferences:

> The first B I ever got in a math class happened senior year. In calculus, I had a teacher who did problems out of the book then gave us worksheets, and I was always confused. I did not like math then and it clearly showed in my struggles. In college, I took statistics and did really well again, but felt that I needed to put more effort into things because the professor did not do much teaching. I took the elementary program math class next and did a great job. I have always been a "math and science person," and I think it will be hard for me to have the patience with students who struggle, so I am hoping I will learn how to! I always get the "logical/mathematic" intelligence when I take a learning style test. I do not think that is necessarily how I learn best though. I like visuals. I picture things in my brain when I think. Like if I take a test, I remember where the answer is in my notes in a picture in my head. So I see words and numbers and things in my head. (Mathematics autobiography, January 15, 2014).

Kim reported that she was a visual learner and had always been successful in mathematics. She did not make connections with her teacher that gave her worksheets, but did well in the mathematics content course offered for teacher education majors. Kim's description suggests that she is responsible for her own learning—she wrote "I" throughout, and also realized that she could get a good grade regardless of how her teacher taught if she put in the effort (Kilpatrick et al., 2001).

The PSTs were asked to write rules to guide how teachers should teach mathematics. One of Kim's rules was: "Connect math to real life. Math is everywhere is [in] real life so by seeing how it affects life in real world problems or in areas that interest students will motivate them to try hard" (Homework reflection on rules for mathematics, January 15, 2014). Since Kim wrote this rule on the first homework assignment, presumably she had already obtained the belief about mathematics connecting to students by doing real world problems. She also indicated that when the students are interested in the problems, they would be more motivated to solve them.

When Kim analyzed some assessment questions she and her partner discussed connections: “Many of the questions were connected to real life situations so that the students could relate to the content. However, some of the questions were basic math problems that did not relate to students’ lives” (Classroom reflection, March 18, 2014). Kim and her partner described connection as relating to students’ lives by using problems that are real.

Kim continued to describe connection and real problems in her discussion of being culturally relevant:

> Culturally relevant teaching practices emphasize the importance of making connections to real-world experiences of children. This directly relates to MP4 [Mathematical Practice 4: Model with Mathematics], which states that math should be realistic and about everyday life. If teachers use relevant scenarios for different students in the class, they are being culturally relevant (Reading reflection on cultural relevance, January 22, 2014).

Parallel to Kim, Koestler et al. (2013) also state that mathematical modeling allows teachers to offer all students opportunities to connect to real world problems, especially diverse students, who are often ignored in classrooms.

In regard to her teaching, Kim noticed that the inquiry-based model offered her opportunities to connect with her students: “I also see now how the LES [Launch, Explore, Summary model] can be successful: especially the launch. I engaged the students completely like never before and connected it to real life and things they were interested in” (Lesson plan, February 24, 2014). Kim felt that the model was effective at connecting with students and as a result, they were more interested in the mathematical tasks.

Kim reflected in her lesson plan how she would help two of her ELLs engage in the Mathematical Practices:

> To help him engage in the MPs, I will start with a story about how I baked brownies for the kids and gain his interest in the delicious dessert and to be relevant to real life. I will also incorporate a rap to engage interest and connect to real life… I will connect the lesson to her interests, again, because she likes snacks, like the brownies, and she has a

> musical and talkative personality that would benefit from the rapping part of the lesson (Lesson plan, February 24, 2014).

Kim argued that telling a story, eating a brownie, and singing a rap song at the beginning of the lesson would connect the task on fractions to the ELLs' personal lives.

Lastly, Kim felt that the curriculum that her host teacher was using with the students, the EngageNY modules, were not connecting with a particular ELL, nor with other students:

> He gets frustrated easily and gives up almost immediately if he does not understand and make sense of the problem at hand (MP1). MP4, again, is not very observable because of how the modules are and do not connect to real life situations that he has been in, as well as other students. He also does not get to talk to other students in groups much, which is one of his interests (Lesson plan, April 16, 2014).

Students are not connected to the mathematics with the EngageNY modules, according to Kim, because they do not connect to the students' personal lives. Furthermore, this ELL would have benefited from making a connection to mathematics through talking to his peers, and Kim stated that this was not happening with the use of the EngageNY modules.

Maria. She described her preference for learning mathematics: "When I'm learning math I need to practice the same problem many times so I can understand it and come up with a solution" (Mathematics autobiography, January 15, 2014). In this comment, Maria takes responsibility for making sense of mathematics on her own; unlike Charles who preferred for his teachers to make sense of the problems for him.

Maria reflected in a homework assignment that she had acquired a belief about connecting with students because she experienced how I had connected with the PSTs: "An example of this [connecting] can be seen what you (Jim) [researcher/instructor] did on the first few days of classes. Getting to know your students' personal identity, interests and hobbies, so you can connect it to our learning styles" (Class reflection, January 17, 2014). Maria felt connected because I, as an instructor, showed an interest in her life.

In an interview, Maria, whose parents were from Peru, shared that she had learned about being culturally relevant because she experienced this in the mathematics methods class: “Bringing culture into your teaching style… going to school growing up; that wasn't done for me so much. Like, I can hear Peru very often. I like how you did that. It was interesting to see it all put together” (First interview, April 24, 2014). Maria may have felt a personal connection with me as I talked about her culture in the mathematics methods course.

On the last day of class, Maria mentioned how I had connected with her when I taught a mathematics lesson to the PSTs in Spanish. She felt valued because she was the only one in the class who understood. I also wrote emails in Spanish to Maria and gave her written feedback in Spanish. For one methods class, I invited a woman in from Peru so the PSTs could practice teaching lessons to an English language learner. After the class, Maria danced with the Peruvian woman. I attempted to connect with Maria through dance because in a homework reflection she wrote that she enjoyed dancing: “I absolutely love to dance and it’s something that I've enjoyed since I was a young girl” (Mathematics autobiography, January 15, 2014). These are examples of an instructor connecting with Maria’s personal life through language and culture.

When asked about being culturally relevant, Maria shared that she had learned about the importance of cultural relevance this semester: “I understand it more. To be proud and everything. I didn't know culturally relevant in high school. I just learned it this semester. Being culturally relevant I can incorporate my own culture” Maria implied that being culturally relevant involves being proud of one’s culture (Second interview, April 24, 2014).

Maria discussed making mathematics relevant to students in a reflection about a lesson plan based on the Launch, Explore, Summary model. She wrote that she had used what she had learned in methods class about connecting with students in her placement. Maria stated that she used the first part of the lesson, the Launch, to make mathematics exciting for her students:

> It is important that math relates to our lives so that we have students engaged and actively participating at all time, or else they won't feel the motivation to learn any new concepts. To make math fun and exciting I've used our launch practices from class and using it as a base for the launch I actually used (Lesson plan, February 13, 2014).

Maria discussed how she would motivate her students to solve problems. She reasoned that it was important to connect mathematics to the students' lives so they are more motivated to solve the mathematics problems. By motivating students through relevance, Maria was supporting students to make sense of problems and persevere in solving them (Mathematical Practice 1).

Maria included language in her explanation of being culturally relevant. When writing about ELLs, she also mentioned language: "The importance for the ELL student and their identity is that it [being culturally relevant] can create a connection between the math content and their primary language and culture" (Reading reflection on ELLs, February 19, 2014). After reading about identities in mathematics for elementary students, Maria specifically applied the concept to ELLs saying that getting to know ELLs' identities will help them make a connection between mathematics and their personal lives.

Sarah. Sarah reported on her learning experiences as a mathematics student:

> I have had many amazing math teachers who gave me positive experiences about math. In elementary school, I had a goofy and funny male teacher who made learning math fun. As I got older, the teachers became more serious and the math got harder. Depending on the topic of math changed my perspective on math…Overall, I have had good experiences in math throughout high school and college. If a new topic is being introduced, I like to take it slow and start with simple math before moving on to the challenging stuff. I also like working with groups but I don't like when my learning is mostly based off of the help of other students, which was my experience in Math 117 and 118 (Mathematics autobiography, January 15, 2014).

Sarah reported having positive experiences with mathematics and teachers until she got to high school because the high school teachers were too serious. Sarah did not like making sense of mathematics with her peers. In Section 4.4 I discuss how Sarah preferred her teacher to make sense of mathematical tasks for her.

In a homework reflection, Sarah claimed that if a teacher were culturally relevant, then students would be more interested:

> It is important that the teacher knows information about the students to relate math to their culture. Not only does including culture help students relate to learning, but it also can keep their interest and engagement when they know they are included. Students should not be taught based off of societies dominant culture (Reading reflection on culturally relevance, January 22, 2014).

Sarah's description of cultural relevance includes getting to know all students personally as well as their culture. She extends that being culturally relevant can encourage students to be interested and engaged as they feel included.

Sarah provides another example—a negative case—of how doing relevant work provides more motivation to learn. She commented in an interview that she did not feel the connection between the methods class and her field placement. She said that she did not have any ELLs in her placement and therefore she could not relate to the lessons:

> I felt that I couldn't relate to what everyone else was doing. Like for ELLs, we were doing those lessons for two weeks. I felt like I didn't benefit from that. I wasn't the one teaching and I was the one that didn't have any ELLs either. I kind of felt like I didn't need to bother learning any of this when I probably should have (First interview, April 24).

For Sarah, motivation to learn was lacking because she did not feel a connection; she implied this was because she did not have ELLs and thus the work was not relevant for her, so she was not motivated to do it.

Sarah's host teacher commented how she improved at connecting with her students as the semester progressed:

> It took some time for her to start really engaging with the kids, but once she did she started forming connections with each of the kids and had the desire to want the best for

> them and to know them as more than just what you see at that desk (Host teacher interview, July 30, 2014).

According to her host teacher, when Sarah learned to connect with her students, they were more engaged with the content.

I did not find data suggesting that Sarah had connected her students with the mathematics.

Summary of Connections

Table 3

Summary of PSTs Connecting Mathematics to Students[1]

Name of Participant	Connected Mathematics to Students?
Abigail	Strong
Amanda	Strong
Charles	Strong
Emily	Strong
Fatima	Strong
Kim	Strong
Maria	Strong
Sarah	Felt Restricted to Make Connections

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

I have discussed examples of the PSTs' beliefs and practices about how to make connections with elementary students. PSTs learned about connections by reflecting on their past experiences as learners, as well as the experiences in the mathematic methods course and from critiquing their peers' teaching as part of this course. When reporting their personal experiences about mathematics, Charles and Sarah suggested that they relied on others to make connections for them, while Maria, Amanda, and Kim were more autonomous.

One common theme mentioned by the PSTs was the importance of teachers making personal connections with students. These connections could be made through the teacher talking about his/her personal experiences or the teacher asking the students about their personal experiences. Another common theme was that when teachers connect with students, they give them a purpose for doing mathematics. Charles said that it was easier for him to connect with ELLs because he had personally experienced learning another language himself. Kim reported connecting with ELLS by using manipulatives to which they could relate as they performed mathematics tasks. The PSTs implied that if teachers connect with students by being personal, the students were more interested and engaged in mathematics. Clarke et al. (2014) claim that making the mathematics content relevant to students will help them be more persistent in solving problems. Thus, by connecting with students, the PSTs could be helping them to persevere to solve problems.

The PSTs shared how teachers could connect with all students by including their culture in the classroom. Teaching a mathematics lesson in Spanish helped both Maria and Charles reflect on their past cultural experiences. Maria mentioned that my speaking Spanish helped her feel proud of her culture, whereas Charles was reminded of an experience when not knowing a language had isolated him from his culture. Charles and Maria, who were both bilingual, were the only PSTs to include language in their definition of cultural relevance.

Turner et al. (2012) explain how PSTs have different trajectories for learning about ELLs and that the practices towards the end of the trajectory involve making mathematics meaningful for ELLs. Turner et al. claim that at the beginning stages of learning new concepts, PSTs often have inconsistencies in their learning—they say one idea but fail to apply it later on. The PSTs in this study did not appear to have inconsistencies in learning the concept of connections because generally they both discussed it and attempted to apply it to their lessons.

The PSTs' description of connections in this study appears to depart from the Connections Standard from Principles and Standards, which involves students connecting mathematical ideas (Koestler et al., 2013). The discussion in this study focused on the importance of teachers making personal connections with students so they are more motivated to engage in mathematics. Koestler et al. (2013) explain that the Connections Standard involves students using mathematics outside of class, and Mathematical Practice 1 involves students making sense of problems by relating content to their personal experiences. Hence, the PSTs' definition of making personal connections to content may encourage students to engage in Mathematical Practice 1, but it differs from the Connections Standard.

4.2 Providing Access

The PSTs commented in homework assignments and interviews how they made accommodations for students in order for them to better access the content. I chose the theme of providing access to content to add to the literature. Koestler et al. (2013) claim that it is the teacher's role to select and adapt problems to facilitate students' understanding so that they can make sense of problems. Murrey (2008) adds that teachers should plan and differentiate instructions so that all students, including ELLs, can have access to the content. My contribution will be to discuss how PSTs mentioned providing access to mathematical content and how this may facilitate all students', including ELLs', engagement in Mathematical Practice 1.

Emily. In an interview, Emily reflected how she had experienced in the mathematics methods course how it felt as a student when an instructor attempted to adapt the course to meet students' needs. She appreciated how I had allowed the PSTs to have different options for learning in the mathematics methods course:

> I also like how you were understanding of how people thought because you actually did. A lot of people don't practice what they preach, but you do. If that's not working for you, you changed it. You are very accommodating… I think it's important for teachers to set that up in the classroom, because before I didn't understand how could you have students do different things and be fair; but we say it's important to accommodate, so we have to be honest about it with the rest of the students. We're probably like, 'I don't need that accommodations so it's okay,' but I think that was really nice of you (First interview, April 24, 2014).

She suggested that accommodating is when the teacher does not treat all students the same, but attempts to make changes to meet students' individual needs. Emily said that the students that did not need accommodations would still accept that their peers might need such accommodation if the teacher was honest about it. Koestler et al. (2013) point out that teachers can help students make sense of problems by adapting tasks to meet each student's needs.

In an interview, I asked Emily whether teachers should make different accommodations for ELLs than for special education students:

> Every student is different and every student needs different things. ELL students need more support with language and translations and things like that and they need things like gestures and manipulatives and then there are so many students with different disabilities and they just need a certain thing that will help them. To answer your question every student needs different things. An ELL student maybe would need the same thing as a student with a disability but maybe not (Second interview, April 29, 2014).

Emily recognized that it was difficult to generalize, but ELLs would possibly benefit from having accommodations such as translations, gestures, and manipulatives. She explained how each student is different and therefore it is important for teacher to accommodate each student.

Emily also wrote how she had planned to accommodate her students:

> I used different accommodations to meet my students' needs. One thing that I did for the students was providing a scaffolded worksheet so that students were able to go step-by-step without adult support. Another accommodation I used was pictures and manipulatives in the Launch so that students who were not strong readers could participate as well (Lesson plan, February 12, 2014).

Emily pointed out above that by providing her students accommodations she helped them make sense of the problems on their own without assistance from an adult.

Emily described what she had learned from teaching a lesson to her peers who pretended to be ELLs:

> When we planned it, it was interesting to see how much a teacher needs to go into students' backgrounds and getting to know them and making it relevant for them so all those things will make them engaged. Just by doing that sole activity, I will never forget that in Yemen they eat goat. And by doing that you use visuals, you have the manipulatives and translations when you are teaching (Second interview, April 29, 2014).

Emily reported that she had learned to accommodate for ELLs by planning. She claimed that she had retained information about one of the mock ELLs' diets from the process of planning. She planned to accommodate the ELLs in the methods class by using visuals, manipulatives, and translations.

Although Emily did not teach any student who received ESL services, there was one student who was born in China and who, according to Emily, had not yet acquired proficiency in English. She described her accommodations for this student: "Another adaptation I used for this student was to provide pictures when doing my lesson and to check in on her" (Second interview, April 29, 2014). Providing pictures, especially for this student, is an example of how Emily tried to help this student by adapting to her learning style. She pointed out that her accommodation would help all of the students, especially the ELLs. Therefore, Emily's accommodation did not hinder the other students' progress. In an interview, Emily reflected on how she had an ELL in her placement previously and she did not even know what language the

ELL spoke. Emily said, “I wasn't being a good teacher and being involved in trying to accommodate her needs” (Second interview, April 29, 2014). Emily suggested that being a good teacher involves accommodating ELLs’ language needs.

Amanda. Amanda discussed her learning preferences when she critiqued her classmates’ lesson. Amanda explained that she was not supported sufficiently in the lesson because she was not given the option to use manipulatives:

> In the explore part of the lesson, we worked in groups and developed our own methods of solving the different word problems by drawing them out. However, this strategy does not help all learners. I like physical manipulatives better than drawing, so maybe the group could have had the learners select an option that would better suit their needs. This strategy would also have helped them achieve MP5 [Mathematical Practice 5: Use tools strategically] more fully if we used tools more strategically (Lesson critique, January 29, 2014).

Amanda argued that the lesson would have reached more learners if they had been given a choice of tools to problem solve (Hiebert et al., 1997). She referred to Mathematical Practice 5 and said that if she had been given the option of using manipulatives she would have been able to use that tool more strategically—perhaps she would use the manipulatives better since she liked them more than drawing. Allowing her the option of which tool to use, might have provided Amanda better access to the content.

In an interview, Amanda expressed frustration at how her host teacher and other teachers put down the students as they were planning their lessons. According to Amanda, instead of focusing on their students’ weaknesses, Kim (her classmate) and she looked for ways to support their students to succeed. Contrary to the teachers at the school, who would complain about the students’ inability to perform challenging tasks, Kim and Amanda would support them so that they would have success:

> “The kids aren’t going to be able to do this,” [said the teachers as they were planning.] And they said, “We'll put it down but the kids can't do this.” I was like, “What's the

> point? You know…" and when Kim and I would say, "We don't think the kids could do that," we would talk about what we could change so they could do that. That's a difference, that's a huge difference I saw with the collaboration between the teachers and the collaboration between Kim and me. We'd be like, "Okay well, how can we get them to do this?" (Second interview, April 28, 2014).

Amanda used the word "support" to describe how Kim and she would provide her students to have access to the content. Amanda said that her teachers would give up on the students, but Amanda and Kim planned to provide the students access to the content by making accommodations for the students.

Amanda allowed drawing in the middle of the lesson to motivate her students: "Also, since he loves to draw, I thought that drawing to represent different types of fractions would benefit him. He made many different representations using the plates and sentence strips" (Lesson plan, April 4, 2014). Drawing, using plates, and sentence strips were strategies for allowing students to have better access to the mathematical tasks.

Amanda discussed how she had learned the importance of providing access to ELLs in mathematics. After I taught a mathematics lesson in Spanish Amanda wrote:

> In our class on Tuesday, we were given a division worksheet in Spanish. Some students did not even try to figure out what we were supposed to be doing and just gave up. In the end, we realized that math is NOT universal. We decided that if a few action words were translated into English, like "divide," then we could have actually attempted the problems. We also suggested that the teacher could model a similar problem on the board to help. In the end, we discovered the struggle that ELLs face when teachers do not have the training to modify their lessons (Homework reflection, February 19, 2014).

In this simulation, according to Amanda, she understood how ELLs might have felt if their teacher did not modify the lesson. Some of her peers did not have access to the content because they did not understand Spanish and therefore did not persevere to make sense of the problems. Amanda noted that by changing the lesson slightly, I might have provided them access to the lesson. Lastly, Amanda said that her peers and she had learned that mathematics is not universal—not knowing Spanish affected the PSTs' access to the content, and accommodations such as translating target words would have helped the PSTs to have better access to the content.

In an evaluation, Amanda's university supervisor wrote: "Amanda has consistently demonstrated her understanding of diverse learning needs and the need to differentiate instruction" (University supervisor evaluation, June 15). Amanda's host teacher wrote similar comments: "Amanda worked hard to include all learning styles and all kids" (Host teacher evaluation, April 18, 2014). Therefore, according to the university supervisor and her host teacher Amanda allowed her students access to the mathematics by differentiating her instruction to make the content more relevant to each student.

Amanda explained in an interview that she did not have ELLs in her class, but she learned about teaching ELLs by talking with Kim, who had eight. Amanda stated that she compared strategies that Kim used for accommodating ELLs with strategies that she had used for special education students:

> Talking to Kim was good because she did have ELLs in the class; so talking to her to see what she did in comparison to what I did was interesting because I had kids with IEPs in my class. And I think either way you need to have some sort of accommodation there. And I kind of compared mine with hers and we both used a lot of visuals. That's really helpful for anyone even for everyone, I think…I still used a lot although I didn't use translations, but I did use some other manipulatives for some students in my class and visuals, so I think any way you accommodate, it is going to be helpful no matter who's in your class… I use a lot of gestures too (Second interview, April 28, 2014).

Even though Amanda did not have the opportunity to teach ELLs, she said that it is important to make accommodations for many students. Amanda used visuals and gestures for her students, which she thought would help ELLs have access to content as well. She added that ELLs tend to need more accommodations involved with translations.

Abigail. Abigail discussed the importance of teachers being culturally relevant and differentiating instruction. She wrote in a homework assignment: "A culturally relevant teacher differentiates his or her teaching in a way so that all students can learn, regardless of what it may take" (Reading reflection on cultural relevance, January 22, 2014). She emphasized that teachers should consider students' culture and differentiate their instruction accordingly to attempt to provide all learners access to the content.

In a homework reflection, Abigail wrote that students are not interested in mathematics because of the way it is taught: "Student interest in mathematics is very low, not because they are not interested in mathematics but because they are being taught mathematics in a way that causing them to sit, listen and get the right answer on the worksheet" (Reading reflection on discourse, April 15, 2014). In an interview, I asked Abigail about worksheets and she said:

> I didn't really care for school when I was a kid so I didn't care for worksheets but just being a teacher in the inclusive program…you're not going to give your students worksheets…I feel like I want to have fun when I become a teacher and be creative and think outside the box because I never had teachers think outside the box. I would just do the worksheet and they would give me books to work on my own and then take a test (Second interview, May 2, 2014).

Abigail made another reference to worksheets being boring: "My teacher uses My Math [class textbook] on the interactive board so it will not be a boring worksheet" (Class reflection, March 18, 2014).

In her reflection, Abigail shared that she had been taught mathematics with worksheets and that they were boring and yet when she taught her lessons, she also used worksheets in her lesson. She attempted to structure her lesson plans to make the worksheet more engaging for her students: "I used worksheets, but interactive like, 'Talk to your neighbor and have stuff to touch,' so that helped them out a lot because they were able to touch things instead of looking at the board and working independently" (First interview, April 23, 2014). Allowing her students to use manipulatives and talking were strategies for Abigail to provide access that the worksheet alone would not have provided.

Abigail said that the students used manipulatives in her lesson:

> Yes, for this lesson they did use manipulatives. I drew circles and squares and triangles and they cut out different parts of it and the next day I gave them circles, triangles, and squares. They made composite shapes and the last day we used paper plates so we definitely used manipulatives (Second interview, May 2, 2014).

Abigail stated that she hadn't made accommodations for her ELLs, yet using manipulatives is an example of an accommodation for ELLs. According to Moschkovich (2007), the use of manipulatives is an accommodation that is useful for ELLs because it allows them better access to the content. Perhaps Abigail said that she did not make accommodations for ELLs when she used manipulatives because all of the students, including the ELLs, used manipulatives.

Abigail wrote in a homework reflection that teachers need to consider ELLs' language when they are doing mathematics:

> ELL's have a math identity because not only are they learning mathematical skills, they are learning the language that coincides with it and English language as well. Their math identity is defined by them as a math student and as an English language learner. For them to be successful in the math world, a teacher must consider how they are as a math student and their level of English proficiency and comprehension in order to successfully accommodate to their needs and actually learn the necessary material (Reading reflection on ELLs, February 19, 2014).

She stated that teachers need to make special language accommodations for ELLs in order to provide them access to the mathematics content. Abigail maintained that teachers need to know ELLs' level of English well in order to make the appropriate accommodations.

In class discussions Abigail suggested that it was important for teachers to make accommodations, such as translations for ELLs. When asked if she had made special accommodations for the ELLs, she said that she hadn't. Furthermore, Abigail said that the ELLs struggled in mathematics and needed to "catch up" If the ELLs were struggling with the mathematics enough that they had to "catch up," then it would appear that Abigail and her teacher should have developed accommodations for their ELLs.

Charles. In Section 4.1 I discussed how Charles had had negative experiences in mathematics as a student because he felt that his teacher assumed that he understood concepts when he didn't. He felt that his teacher had denied his access to the content. In a reflection at the beginning of the semester, Charles reflected on his opinion of good teacher qualities:

> In my opinion, a teacher who gives good instruction is one that carefully explains directions, makes sure to constantly check on their students, and to scaffold their teachings when necessary. On the other hand, a teacher who is poor at giving instruction is one who gives vague directions, does not check on their students, and does not teach at an age appropriate level (Second interview, May 5, 2014).

One of the qualities that Charles suggested was for teachers to scaffold for students.

As Charles planned his lessons, he also mentioned scaffolding for his students: "I will scaffold my questions if students have a hard time understanding them" (Lesson plan, February 13, 2014). Charles did not explain exactly how he was going to scaffold the questions. He wrote how he had planned to provide a student access to the content: "When given manipulatives Anna will use them to help her solve math problems" (Lesson plan, April 9, 2014). Charles was the only one to mention making accommodations for "high achievers": "To accommodate for my students' needs, I made sure to make my material challenging for high achieving students" (Lesson plan, April 9, 2014).

Charles also reflected in his lesson plan after he had taught about providing access: "Some students were also struggling with the language of the word problems that I was giving them, which led me to reread the problem in different ways in an attempt to clarify" (Lesson plan, April 9, 2014). Charles provided his students' access to the subtraction by rereading the problems for his students.

Charles wrote in a lesson plan reflection that he had attempted to accommodate his students in his lesson:

> I made sure to add pictures to my agenda and presentations to help my English Language Learners understand what I was teaching. There are many things that I have learned in class which I have applied to this lesson. The first thing that I learned was the use of visuals, which is something that I would not have thought of before this course (Lesson plan, April 9, 2014).

He commented that he had learned in the mathematics methods course to use visuals to accommodate his ELLs. (While Charles taught two students who were bilingual, they did not receive ESL services, so I am not classifying them as ELLs in this study.) Using visuals and pictures are two ways Charles accommodated his students.

Charles explained that in the mathematics methods course he had learned strategies to accommodate ELLs: "I liked the role-playing. It gave us opportunities to develop our teaching; for example, the visuals. Once I used it on you guys, I was more comfortable using it in the classroom" (Second interview, May 5, 20140). Even though Charles did not have any ELLs in his placement, he found it effective to learn about using visual strategies to connect to other learners in his placement.

Fatima. At the beginning of the semester Fatima wrote in a homework assignment how she felt that she did not have access to the content when she learned mathematics as a student. She was denied the opportunity to take an assessment in third grade and as a result she had doubts about her capabilities in the subject. Fatima felt that this negative experience of mathematics stayed with her during middle school and high school:

> My math experiences in elementary. Middle and high school were very similar. I feel that I was tracked in math since 3rd grade, with my teacher only giving certain students the privilege of taking a test to score that math ability while the rest of us she assumed didn't know enough to take the test. This created not only a lack of motivation for me in school but also an element of self-doubt regarding my mathematics skills. That single third grade test created a domino effect on all the other skill/placement tests, affecting my sicth [sixth] grade placement which therefore determined my high school placement (Mathematics autobiography, January 15, 2014).

Other PSTs have discussed providing access by making an accommodation, but Fatima felt that she was actually denied access and not allowed to take an assessment; this negative experience stayed with her through high school.

Fatima wrote in her lesson plan that she would allow her students various options to solve tasks: “Students who are more visual can write them down” (Lesson plan, April 14, 2014). Fatima also explained why she encouraged students to use manipulatives: “Students who are more hands on will be able to solve the problem realistically using manipulatives. This will also benefit students who have trouble verbalizing their answer/explanation because they can use the bags to help them” (Lesson plan, April 14, 2014). Fatima allowed her students better access to the content by considering their preferred learning preferences.

An example of Fatima applying what she had learned in her methods courses to the field was in her same lesson plan:

> I accommodated my lesson to meet student’s needs by making my lesson appealing to multiple types of learners. The use of blocks and group work addressed the interpersonal and bodily kinesthetic needs of the learners while the recording sheet and anchor chart addressed logical and visual learners (Lesson plan, February 13, 2014).

Fatima planned her lessons to meet the needs of various learners. The other examples of providing students access to content were the differentiation of instruction for an individual or group, but Fatima refers to accommodating by teaching the whole group the same. She referred to accommodation because she had designed certain activities to target various learners’ needs.

Like Emily, Fatima did not have students that were classified as ELLs, but she had a student who was bilingual. Fatima’s university supervisor told me in an interview that she had motivated a student by writing some Arabic words on a worksheet: “It just made that child's day!” (University supervisor interview, July 9). Fatima’s accommodation was unique because this student had already acquired proficiency in English, but by including words in his language, he was more motivated to solve the problems.

Kim. As Kim critiqued her peers' lesson, she mentioned how the use of manipulatives was helpful to her:

> To figure out these problems, we were offered cubes to figure out the answers. They did not tell us to use them, but they really helped and were right in front of us to use and see the actually meaning behind the word problems. This is MP5, using appropriate manipulatives, or appropriate tools strategically in order to solve the problem. I really liked being able to use the cubes; they helped me visualize the addition and subtraction problems very concretely (Lesson critique, January 24).

Although Kim might have been able to solve the problems without the use of manipulatives, she wrote that the manipulatives helped her visualize while solving mathematics problems.

After reading an article about engaging ELLs in mathematics, Kim compared it to what she had learned in class when I had taught the mathematics lesson in Spanish. She said that after not being able to complete the division worksheet because she did not understand the language, she realized that it was necessary to develop strategies to facilitate ELLs' access to the content:

> Another strategy we talked about in class and aligned with the article was the use of concrete materials being very helpful for ELL students to understand and explain their thinking…As a teacher, I will need to incorporate all of these strategies and mainly continue to notice the struggles and successes of the ELL students in my current class to figure out my own ways of meeting my ELL students needs (Class reflection, February 19, 2014).

Kim recognized that allowing ELLs to use manipulatives could provide them access to the content. Moschkovich (2013) maintains that successful teachers of ELLs adapt their instruction to allow ELLs' access content to the content. Kim pointed out that the use of manipulatives would not only help ELLs understand the content, but as well could provide them with access to justify their answers.

Kim, who had eight ELLs, commented that the mock lesson working with ELLs in the methods course gave her practice connecting with all students in the field: “I think actually doing mock ELL lessons in class helped me prepare and be much more aware of my lesson this time around. I made the lesson accessible for all students, culturally relevant, and included all the MPs” (Second interview, April 30, 2014). Kim reported that she had made her lesson accessible to all of her students.

Kim used multiple learning differences to provide ELLs in her placement better access to the content. In her lesson plans, she planned, implemented, and reflected on strategies. She wrote how she would help an ELL to engage in Mathematical Practices with the use of strategies:

> I will make sure this student is able to use the manipulatives, like cubes, sticky notes, and actual chairs, tables, etc. (MP5) to solve the problem; I feel like this will help this student because he is a visual/ spatial learner and never gets much of an opportunity to actually touch 3D manipulatives instead of a drawn representation…I will also assist him and the other ELL students by writing the key [strategic] words of the tasks on the board and maybe even model with the cubes (Lesson plan, April 13, 2014).

Kim gave another example of making an accommodation that was not linguistic for her ELLs: “I let certain students, [ELLs] who are more bodily kinesthetic, collect the graphs at the end of the day” (Lesson plan, April 13, 2014). By Kim making non-linguistic accommodations for ELLs, she demonstrated understanding that these students were not only ELLs, who needed to have English accommodations, but they needed other accommodations as well.

In Kim’s lesson plan reflection, she wrote that she had used strategies with her ELLs to allow them access to the content during the lesson she had taught:

> I also made many gestures for the ELL whenever I asked them about “bigger or smaller” or when I was pointing to the parts of a graph I was talking about. I also gave them the cubes as a manipulative that accommodated the ELL in expressing their ideas (Lesson plan, April 13, 2014).

Kim not only made accommodations for the ELLs when they were solving the mathematics problems, but also when she was giving directions and when she asked them questions. Perhaps ELLs were better able to make sense of problems because they understood the directions and questions as a result of Kim's accommodations.

Kim's university supervisor said that she used strategies to make the directions clear to the ELLs:

> She asked great questions and used all sorts of visuals. From not only looking at your clothes but even saying that; then she demonstrated it on one of the kids to see where your shirt came from. So even if I did not speak the language, it was pretty clear what was expected. She would do things like write it down (University supervisor evaluation, June 15, 2014).

Kim used smiling, gestures, and visuals to allow ELLs access to mathematics. Murrey (2008) encourages teachers to help ELLs with language by talking slower and using gestures.

Maria. Maria revealed in an interview how one of her professors outside of the teacher education program had been accommodating with her:

> My writing professor was easy to talk to and easy to approach and she cracked jokes and was friendly so it was easy for me to ask for an extension. She was accommodating and I told her that I wanted to do my best work and I needed an extension in order to do that (Second interview, April 29, 2014).

Maria said that her professor was accommodating to her by allowing her an extension. Maria explained that she had approached her professor, which implies that not all of the students were given the extension, but by being granted that extension, Maria felt that she was able to complete her work to a better standard. The accommodation for Maria was not to change the task, but to allow her more time to complete it.

Maria explained in lesson plan reflection how she had made learning easier for a student in her class: "This chart is helpful because it allows for him to remember multiplication facts and visually see how two factors make the product" (Lesson plan, February 13, 2014). Maria also made pictures of the agenda to accommodate her visual learners and made the print bigger for a student in order for him to have access to the content:

> The agenda will be printed for each group with pictures and sentences, so that those visual learners can understand how the lesson will be organized. For the group that has one of my focus learners, I will have that agenda in a larger font than the other ones to accommodate to her needs (Lesson plan, February 13, 2014).

Maria described how she made different adaptations for three different learners so that they would have better access to the content.

Maria wrote in a homework reflection that teachers could provide ELLs access to mathematics content by using manipulatives and other accommodations:

> Materials are much needed for effective mathematical performances. In the reading they stated that the use of concrete materials would be beneficial in work with ELL students, but with some assistance of course in relation to the mathematical language. Our group, suggested for manipulatives to be used because it gives ELLs a way to understand and communicate their thinking, in a way words cant. For any visual or hands-on learners this method would be even more beneficial because it appropriately accommodates to their needs and motivates their behavior to learn (Reading reflection on ELLs, February 19, 2014).

Maria, like Kim, pointed out that use of manipulatives by ELLs not only helped them understand mathematics better, but using manipulatives also provided ELLs access to communicating their ideas. Mathematical Practice 3 states that students should develop arguments; by using manipulatives, teachers can provide ELLs better access to this Practice.

Sarah. In a homework reflection, Sarah shared one of her preferences for learning mathematics:

> I did not use manipulative[s] to learn math until I got to college. In the previous math courses I took in college, we always used manipulatives and it helped me so much. Sometimes I cannot visualize in my head so it is helpful to actually see math being worked out. I know if this could help me as a college student, then it could definitely help an elementary school student who is just learning math (Mathematics autobiography, January 15, 2014).

She reflected that using manipulatives while solving mathematics helped her visualize the process. As Sarah reflected on her own learning preferences, she also speculated that providing elementary students with manipulatives would provide them access to the content.

I have discussed the PSTs using accommodations such as making content interesting, being culturally relevant, and tapping into students' multiple learning differences. However, Sarah noted that due to her placement she did not have opportunities to connect with her students by using accommodations. She was in a placement where EngageNY modules were used, and she said that she had to follow the curriculum; her teacher said that she should follow the scripted lessons rather than write her own lesson plan. The other PSTs were instructed to write in their lesson plans adaptations that they would make for their students. However, since Sarah said that she could not write a lesson plan, I attempted to accommodate her needs and allowed her to use the scripted lessons. I asked Sarah to spend more time reflecting on her lesson. She said the following about accommodations: "I didn't use them [accommodations] so much because I couldn't come up with my own lessons so I couldn't include students' interests or put in manipulatives" (First interview, April 24, 2014). Therefore, even though Sarah discussed in the methods course how to connect with her students, she did not have the opportunity to write lesson plans to accommodate her students to have better access to the content. Sarah also reported that the curriculum she used prevented her from using accommodations to connect with her students.

Sarah offered suggestions in a homework reflection for how teachers can modify their lessons to provide ELLs access to mathematics content:

> Teachers can give additional support to English language learners. It is important for the teacher to give every student the opportunity to succeed, even if that teacher needs to make accommodations to their lesson. One thing teachers should do is a lot of modeling and demonstrating. This takes away the linguistic challenge and focuses more on the

> math. The student can visually see the work and understand that better than being explained to. Another thing is zone of proximal development. Teachers should scaffold information that is in their range of learning that is slightly challenging for the student. This way the student is prepared and recognizes what they are learning (Reading reflection on ELLs, February 19, 2014).

Like the other PSTs, Sarah mentioned strategies for teachers to accommodate ELLs to provide them access. She felt that it was necessary for teachers to meet all students' needs and suggested that ELLs have a linguistic challenge when doing mathematics and therefore would benefit from accommodations, such as teacher demonstrations.

In Sarah's lesson plan she wrote that if she had an ELL she would make the following accommodations to her lesson:

> If I had an English language learner, I would accommodate the lesson by adding manipulatives. I would have blocks that represent the hundreds, tens, and ones place and demonstrate how to decompose. That way he is visually seeing it on paper and with blocks (Lesson plan, February 27, 2014).

Verplaetse and Migliacci (2008) contend that adaptations to lessons, such as visual representations, are one way for teachers to facilitate ELLs' access to content. Sarah said that using manipulatives is visually appealing for ELLs.

Summary of Providing Access

Table 4

Summaries of PSTs Connecting Mathematics to Students and Providing Access to Content[1]

Name of Participant	Connected Mathematics to Students?	Provided Students Access to Content?
Abigail	Strong	Strong
Amanda	Strong	Strong
Charles	Strong	Strong
Emily	Strong	Strong
Fatima	Strong	Strong
Kim	Strong	Strong
Maria	Strong	Strong
Sarah	Felt Restricted to Make Connections	Felt Restricted to Provide Access

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

I have discussed the PSTs' beliefs and practices about providing access to mathematics content. All of the PSTs said that they need to make accommodations for different students rather than teaching everyone the same way. Sarah, Amanda, and Kim all mentioned the importance of being allowed to solve mathematics the way they wanted to. They agreed that having the choice of how to solve problems provided them with better access to mathematics content. The PSTs reflected that they had their own learning preferences, so it was logical that each student may have different preferences, and by making accommodations the students would have better access to the content. Some of the PSTs modified their lessons for individual students, while others modified their lessons for the entire class to target certain students' needs.

Another theme mentioned was that teachers and PSTs should attempt to provide all students with access to content. They suggested that providing all students with access does not mean teaching all students the same. The National Council of Teachers of Mathematics (2014) clearly state that equity does not mean that all students should be treated the same. They maintain: "Equity in school mathematics outcomes is often conflated with equality of inputs" (p. 60, 2014). This idea of equity is in alignment with the PSTs' beliefs that in order for all students to succeed, different accommodations need to be made.

According to Fernandes (2011) there is a misbelief that mathematics is universal and that teachers do not have to make modifications to lessons for ELLs because there is little language involved in the subject. However, after the PSTs had experienced being taught a mathematics lesson in Spanish, they seemed to appreciate the value of making linguistic accommodations for ELLs. Some of the accommodations for ELLs mentioned were using translations, gestures, visuals, manipulatives, and teacher demonstrations.

Four of the PSTs were at placements that had adopted the use of EngageNY modules. Three of these PSTs planned and implemented inquiry-based lessons instead of the modules, but Sarah was told that she should follow the school curriculum. The lessons were already prepared and this PST did not have the experience of writing her own lesson plan to include accommodations. Experienced teachers have already had opportunities to develop lesson plans and make accommodations for students, but PSTs may benefit from the experience of planning lessons to accommodate all learners to have access to content. Furthermore, not only did Sarah lack the opportunity to experience planning lessons, she also felt that she had no freedom to adapt the lessons to accommodate the students. As school districts adopt scripted curriculums, it is important to examine how this may affect the training of PSTs. Turner et al.'s (2012) trajectory of learning calls for all PSTs to learn how to write meaningful lessons for ELLs, but Sarah was not given the opportunity to write a lesson plan that applied what she had learned in methods courses about providing access to content for all of her students.

4.3 High Expectations

In contrast to the two previous themes, the PSTs did not use the term "high expectations"—I chose the term after discovering in the data that two PSTs appeared to have low expectations for ELLs. The literature describes how teachers and PSTs should have high expectations for students. Moschkovich (2013) maintains that teachers should focus on ELLs' strengths and aid them to engage in rigorous mathematics rather than focus on their lack of English proficiency. Clarke et al. (2014) discuss that teachers can promote being persistent at solving challenging problems by having high expectations for students. Thus, in order for the PSTs to facilitate all of their students', including the ELLs', engagement in Mathematical Practice 1, it is imperative for them to have high expectations for all students. Throughout this section I discuss PSTs' expectations of elementary students' capabilities to learn mathematics. Furthermore, I will discuss why some PSTs may have different expectations even though they all attended the same mathematics methods course and teacher education program.

Amanda. As I discuss in Sections 4.1 and 4.4 Amanda had high expectations for herself in mathematics—she enjoyed solving challenging problems and had the belief that if she put in the effort, she would get the results (Kilpatrick et al., 2001). In a reflection Amanda wrote for homework after reading an article about math identity, she demonstrated that she has high expectations for students:

> Every student, no matter what language he or she speaks or where he or she is from, has had some sort of experience with math. Teachers who take the time to support the students and allow them to express their ideas in different ways besides traditional worksheets and traditional test scores that show achievement gaps give their students a chance to develop a math identity. In addition, a math identity for all students is necessary to facilitate participation. I like the idea of calling all students mathematicians so that they feel empowered as math learners. Another strategy to promote the math identity of all students is to give students more time on a problem. If you call on a student to answer a problem, give them some time to gather their thoughts. Students work at different paces, and the one who finishes a problem in ten seconds should have just as positive of a math identity as the one who takes a minute to get the same answer (Reading reflection on mathematics identity, February 23, 2014).

It would be possible for a PST to say that they have high expectations for all students, but not apply it (Turner et al., 2012); however, Amanda wrote the comment above without being cued to express an opinion on high expectations—a clue that this may be an accurate reflection of her outlook. She argued that all students should participate, be given enough wait time, and have positive math identities. An example of Amanda having high expectations for her students is her belief that they will be empowered if teachers call them mathematicians.

Amanda applied her belief about calling students mathematicians in the field:

> I also said that it was important for students to be told that they can succeed and that they are good at math. Everyone has had some sort of math experience and should be given the opportunity to discover new content for themselves to build on previous knowledge. This is why I decided to call my students mathematicians at the beginning of my lesson. It's something so small, but can make a huge difference when you presume competence in your learners (Lesson plan, February 26, 2014).

The PSTs had learned about the importance of having presumed competence with all students in another course in the education program. Although we did not mention the term in the mathematics methods course, Amanda applied the concept in her mathematics lesson. Furthermore, she said that students would do well if you tell them that they are capable. She also suggested that since all students are capable mathematicians, they should be give opportunities to discover mathematics for themselves.

Amanda also had high expectations for diverse students, which is conveyed in her homework reflection:

> When students enter the classroom with different approaches and different points-of-view, it is crucial to entertain them all and put them into discussion with each other. Students of different cultures have different levels of understanding of mathematics. It is important not to racially profile or assume on[e] race knows more than another (Homework reflection on ELLs, February 19, 2014).

The comments show that Amanda distinguished between being different and having a different level of knowledge. She argued that students may have different mathematics understandings, but one was not better than the other. I did not find more data about Amanda's expectations for ELLs.

Abigail. In Sections 4.1 and 4.4 I discuss Abigail's experiences about mathematics as a student. In a discussion about race, Abigail hints at her expectations for herself:

> My other black friends who is in VPA [Visual Performing Arts]; that's where all the black people's at…Everybody with the lesser grades, they got to go to VPA or something like that rather than coming to something like this [teacher education program] (Second interview, May 2, 2014).

The comments above indicate that Abigail may have high expectations for herself as she is in the more challenging program.

The only data I found in support of Abigail's positive expectations for students was from her university supervisor: "Abigail is respectful and maintains high expectations for students" (University supervisor evaluation, April 21, 2014). However, in an interview Abigail's comments suggested that she had low expectation for ELLs. When I asked Abigail how to aid ELLs to persevere in mathematics, she suggested reducing the cognitive load for ELLs: "Plan good and have different worksheets. You can have the same worksheets but you can say that they can do two or one or something like that so they won't feel pressure to do the same thing as everyone else" (Second interview, May 5, 2014). Abigail's comments about having the ELLs do less content than their peers is not in alignment with the literature that maintains that teachers should have high expectations for ELLs in mathematics. For example, Murrey (2008) points out that teachers can help ELLs understand the content, but they should expect ELLs to engage in rigorous mathematics.

Turner et al. (2012) reported that when PSTs had deficit beliefs, this slowed down their trajectory for making mathematics meaningful for ELLs. Although Abigail had learned the concept of presumed competence in the teacher education program, she did not apply this concept to ELLs. In an interview, Abigail referred to ELLs as “hopeless,” “lesser people” and “language struggling:”

> J: Were the ELLs in your classroom separated or were they together?
> A: They were separated.
> J: What do you think of that?
> A: It was good because for a lack of a better word if you have hopeless people sitting at one table, how are they going to help each other? If you have stronger people mixed with the lesser people then they can help each other at the table.
> J: Okay and why do you call them hopeless?
> A: I didn't. I said for the lack of better word.
> J: What would a better word be then?
> A: Two language struggling or language something. If you have a bunch of them at one table that speak different languages, how will they be able to help each other? If you have a strong English and one language difficult whatever word person and then some people that sit in the middle you can have them collaborate and embrace some ideas rather than just having a bunch of people who possibly if they spoke in their own language they couldn't even understand each other in their own language (Second interview, May 5, 2014).

Abigail’s comments appear to be examples of the deficit model (Moschkovich, 2013) as she considered the ELLs to have a problem. Abigail corrected herself for calling the ELLs “hopeless,” but she referred to the ELLs in her class as “language struggling” when in fact the ELLs could have been “language experts” (Aguirre, Mayfield-Ingram, & Martin, 2013), as they already knew one language and were learning another. Abigail’s comment that the ELLs may not have understood each other even in their own language was another example of her low expectations for ELLs. She implied that since the ELLs did not speak English, they could not even communicate. If Abigail has low expectations for her ELLs, then it may be difficult for her to engage ELLs in the practices called for in Mathematical Practice 1.

Charles. Charles reflected in an interview how he was not given the accommodations that he felt were necessary due to his race. In a course discussion he shared that his teachers had high expectations for him because he was Asian, so I asked him to elaborate on this in the interview: "I think that was the case. I remember once in elementary school I would ask for help but the teacher would not circulate to my side of the room because she automatically assumed that I could do it" (First interview, April 23, 2014). In the previous discussion with Amanda, I discussed how the students benefited from their teachers' high expectations, but Charles felt that he was at a disadvantage—his teacher assumed that he was capable and hence he was not given support.

Despite his negative experience, Charles still argued for teachers to have high expectations for ELLs. Similar to Kim, Charles argued for high expectations for ELLs in mathematics when he was asked to write a rule to guide teachers to help ELLs reach their mathematical potential: "Presume competence & don't view the native language as a hindrance. Just because an ELL student cannot do the mathematics, doesn't mean he/she can't do it but it may just mean they are still learning the language" (Homework reflection on rules for mathematics, January 15, 2014). The term "presumed competence," an expectation that every student can achieve, was taught in another methods course in special education. Charles applied these words to ELLs and argued that teachers need to have high expectations for ELLs. Charles, who was bilingual, warned teachers not to have low expectations for ELLs in mathematics just because they speak another language.

Emily. I did not find data about Emily's personal experience as a student regarding her expectations. In an interview, Emily told me how she had made accommodations for one of her students:

> It is funny because you learn about all this stuff here and you assume that everyone else knows it. I had a student that doesn't do anything, but he's one of the top students. He always got all A's in the pre-assessments, but you have to beg him to do the work. He hates any kind of worksheet and writing is just so boring for him…That's why he's not doing it and no one else saw that, so I was just like, "okay if this is too easy for you then you are going to write five sentences instead of one (Second interview, April 29, 2014).

As a result of Emily's high expectations, she demanded more work from her student and he responded. Emily continued to explain how having high expectations for students means that teachers should adapt tasks so all students are challenged: "We always talk about differentiating and challenging all students and having high expectations but when you make a universal thing how are you going to challenge the students that need to be challenged?"

Kim. I did not find data regarding Kim's experiences as a student about expectations. Like Charles, Kim made comments that reflect that she had high expectations for ELLs' ability in mathematics. After being taught a mathematics lesson in Spanish, Kim wrote:

> I think the biggest thing that related to our class was that it is not okay to assume that the ELL student is behind in math because of lack of proper schooling or because they are not at correct "grade-level ability;" it is important to realize that ELL students have double the work: understanding English in order to understand the math that everyone else is learning for the first time too (Class reflection, February 19, 2014).

Kim stated that even though ELLs might not have acquired English proficiency, it is important to have high expectations for their ability in mathematics.

When Kim was in the field, she attempted to have high expectations for all students. She reflected in her lesson plan that she told her students that she wanted all of the students to be involved: make it a class expectation that everyone must handle the paper at some point during the group portion to ensure they try to gain understanding, which is MP1.

Maria. Although I did not find data regarding Maria's own experiences with expectations, her host teacher wrote in an evaluation form that Maria had high expectations for students: "She used wait time and she set high expectations for her students" (Host teacher evaluation, April 14, 2014).

In an interview, I asked Maria about how to motivate students and she said:

> M: Just like not give up and don't think the first thought is "You can't do it. You can do it." Even those with a severe disability. I didn't have any with a severe disability, but I'm just saying. Take different methodologies to try and reach their highest work potential.
> J: Where did you learn that?
> M: In differentiation. [Another methods course]
> J: Presumed competence?
> M: Yeah. (First interview, April 24, 2014).

Maria associated teachers having high expectations with motivating students. She said that she had learned to have presumed competence in another methods course and yet, like Charles and Amanda, she discussed this concept in the mathematics methods course. Maria argued that regardless of whether students have a disability, teachers should have high expectations for students. Maria implied that if teachers have presumed confidence or high expectations for students, then they would look for strategies to help each student reach their potential. Maria's comment about presuming that students are competent and telling them not to give up is in alignment with PSTs having high expectations for students so that they will persevere to solve problems.

Sarah. The data are mixed in regard to Sarah's expectations for students learning mathematics. Her university supervisor wrote in an evaluation: "She has high expectations" (University supervisor evaluation, April 12, 2014). In an interview, I asked Sarah's host teacher what Sarah's core beliefs were:

> I think she has the core belief that all kids can learn but I don't know that she necessarily truly understands. I think every preservice teacher comes out and says that, but don't

> know that until you have the experiences… I think the belief became more solidified…She had some challenges adapting lessons for the special ed kids. I think she also learned from what we were doing as to how she taught. We have in our classroom some kids who have unique needs and I think that intimidated her. I think that she also learned from what we were doing. As time wore on she saw that they were treated like everyone else and they were expected to learn with accommodations (Host teacher interview, July 30, 2014).

Sarah's host teacher appeared to have high expectations for his students so he may have been a good role model for her. He said that she was intimidated by some students, but she improved over time.

Although Sarah did not write a lesson plan in the same way that her peers did because in her placement they were using EngageNY modules, I still asked her to take the module lessons and attempt to adapt them to the lesson template. In the section that asked to predict students' responses, she wrote: "I do not expect every student to answer. There are students who outshine others in math and tend to compete and finish first. The shy students or the struggling ones may not answer what's expected" (Lesson plan, February 27, 2014). Thus, it could be argued that Sarah was demonstrating low expectations, as she did not consider it her responsibility to challenge all of her students to answer the questions. However, another explanation for Sarah's response is that the EngageNY modules restricted her. She shared in Section 4.2 that she did not feel that she was able to adapt her lessons to meet each student's individual needs. According to the National Council of Teachers of Mathematics (2014), equity does not mean treating everyone equal. Sarah felt that she had to teach everyone equally so under those circumstances it is understandable how she might have not expected everyone to answer the questions the same way.

The data are also mixed about Sarah's expectations for ELLs. She argued in a homework reflection: "It is extremely important for teachers to realize that [the] ELL is being set back because teachers haven't been meeting their academic needs" (Reading reflection on ELLs, February 19, 2014). Sarah contended that it is the teacher's responsibility to ensure that ELLs are successful as opposed to accepting that ELLs are weak at academics. Sarah implied that if teachers have high expectations for ELLs and change the way that they teach them, then the ELLs could be successful. In the same reflection, her comments also showed high expectations for ELLs in mathematics:

> They may not be able to complete work because they don't understand the directions or how to explain a concept, but if that student understands the math they need to do, then they can do it (Reading reflection on ELLs, February 19, 2014).

In contrast to Sarah's earlier comments, in an interview at the end of the semester, Sarah said the following:

> S: I just can't imagine a student who doesn't understand any English being able to learn in a classroom. I just can't visualize it. I can't understand how they can learn math and understand it the same as everyone else.
> J: Kim had eight.
> S: That is crazy to me (Second interview by Skype, May 8, 2014).

Sarah's comments above contradict her earlier comments about having high expectations for ELLs in mathematics. Sarah knew what she was expected to say, but not how to enact it. Sarah implied above that ELLs are not capable of doing mathematics if they do not speak English. Rather than focusing on what these students could offer, she thought that it would be "crazy" to have eight ELLs in the class. In the same interview, Sarah said that her sister had ten ELLs in the class. I asked what strategies she would use if she had ten ELLs in her class:

> That is ten students that you know are at a disadvantage in your classroom that you need to keep up with and work hard to include them. At the same time teaching everything to everyone else. It's a huge challenge (Second interview by Skype, May 8, 2014).

Having ten hypothetical ELLs in her classroom appears to be a burden to Sarah. Instead of focusing on the positive aspects that the ELLs could contribute to the class, Sarah focused on the negative. Furthermore, Sarah was not referring to any ELL in particular; without knowing the ELL, she assumed that just because the 10 students were ELLs, they would be at a "disadvantage" This is not in alignment with what she had learned in her other methods course about presumed competence—presuming that all students, including ELLs, are competent. Turner et al. (2012) found that the PSTs often had "inconsistent awareness" when they first learned about meeting ELLs' needs. Sarah appears to have inconsistency with her expectations for ELLs. In homework reflections Sarah wrote how teachers should have high expectations for ELLs, and yet by the end of the semester, she made comments that were inconsistent and did not reflect high expectations for ELLs' mathematics' abilities.

When I interviewed Sarah's host teacher, I shared her comments and he said: "I can see her saying that. The comments she made about her sister in California. I can see her saying that. Having ten ELLs in the classroom. That I can see scaring her" (Host teacher interview, July 30, 2014). Sarah's host teacher stated that she might have been scared to have ELLs in her class. I mentioned that Sarah might have low expectations for ELLs, but her host teacher suggested that she might have low expectations for herself; perhaps she was insecure and questioned her capabilities of teaching ELLs.

Summary of High Expectations

Table 5

Summaries of PSTs Connecting Mathematics to Students, Providing Access to Content, and High Expectations for Students[1]

Name of Participant	Connected Mathematics to Students?	Provided Students Access to Content?	High Expectations for Students?
Abigail	Strong	Strong	Weak
Amanda	Strong	Strong	Strong
Charles	Strong	Strong	Strong
Emily	Strong	Strong	Strong
Fatima	Strong	Strong	Strong
Kim	Strong	Strong	Strong
Maria	Strong	Strong	Strong
Sarah	Felt Restricted to Make Connections	Felt Restricted to Provide Access	Weak

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

The PSTs' comments suggest that they had different expectations for ELLs' abilities to solve mathematics. Turner et al. (2012) maintain that deficit views slow down PSTs' progress in developing meaningful lessons for ELLs. Clarke et al. (2014) add that having high expectations for students will help students persevere at solving problems. All of the PSTs learned the concept of "presumed competence" in another course in the teacher education program.

Amanda associated presumed competence in her students with students discovering content for themselves. She stated the importance of improving the students' dispositions for mathematics by telling them that they can succeed. Larson et al. (2012) maintain that the second part of Mathematical Practice 1 is linked to students having a productive disposition—a belief that the mathematics is worth doing and they are capable of doing it. Therefore, when Amanda praised her students for being competent at mathematics, she may have been improving their dispositions. Applying Turner et al.'s (2012) trajectory to Amanda, she was far along the trajectory for teaching presumed competence; she discussed it in another methods course, and also applied this concept in a meaningful way for her students.

Sarah discussed the importance of having presumed competence for students, but she made comments that reflected having low expectations for ELLs' mathematics ability. As I have discussed throughout these studies, Sarah had a unique placement in that she was the only one to teach the EngageNY modules and therefore did not have the opportunity to write her own lesson plans.

The PSTs had different expectations for students in mathematics. What may have accounted for these differences if all of the PSTs took the same mathematics education course and were in the same teacher education program? Rather than lecture, I attempted to have the PSTs construct their own knowledge of the Mathematical Practices in the methods course. According to Marchionada et al. (2014) the difference between traditional and constructivist mathematics courses is that the latter involves students being active in the learning process. The authors posit that students in constructivist courses focus on the content that is most relevant to them. Similar to the study by Marchionada et al. the PSTs in my research may have each focused on the content from the methods courses in the teacher education program that they considered to be most relevant. The PSTs had different placements and host teachers, which may be one explanation for the PSTs' different focus on expectations for their students.

Another explanation for the difference of PSTs' beliefs about having high expectations for all students may be related to their experiences outside of the teacher education program. Fatima said in an interview that they had not learned much about ELLs in the teacher education program. As the PSTs had limited opportunities to discuss their biases about ELLs in the teacher education program, they developed their beliefs from their outside experiences, which could be another explanation for the PSTs having different expectations for ELLs.

Turner et al. (2012) noted that PSTs have different paths as they learn to accommodate ELLs. Some PSTs in their study had inconsistencies in their learning—they said one thing but did not apply it. Similar to Turner et al.'s study, Abigail had some inconsistencies in her path to learning to have high expectations for students. She reflected that it was important for teachers to presume competence for all students, but she also made some deficit remarks about ELLs. Turner et al. explain that deficit ideas slow down PSTs' progress of teaching ELLs in a meaningful way. Kim, by contrast, had high expectations for her students. She reflected in the mathematics methods class that it was important for teachers to have high expectations for ELLs, and she also looked for strategies to engage her ELLs in rigorous mathematics lessons when she was in the field. I did not find much data on Charles, Emily, Kim, and Maria--these PSTs were focused on other aspects of education (Marchionada et al., 2014).

4.4 Productive Struggle

I have chosen the theme "productive struggle" after observing how two PSTs showed signs of guiding their students while two others reduced the cognitive load by offering the students a great deal of help. Hiebert and Grouws (2007) maintain that teachers in United States need to raise the cognitive load for students and give them problems that are within their grasp, but still challenging enough to cause tension. I chose the theme "productive struggle" because it is mentioned in the literature. Teachers should empower their students by helping them engage in productive struggle, which promotes Mathematical Practice 1 (Clarke et al., 2014; Hiebert et al., 1997; Larson et al., 2012). In Section 4.2 I discussed the PSTs making accommodations for their students. However, as they make such accommodations, PSTs should also make the content challenging (Murrey, 2008).

Amanda. One PST who was focused on productive struggle throughout the semester was Amanda. In homework and class discussions, Amanda commented many times how the students in her placement were not challenged. Amanda reflected that she had been challenged as a student in mathematics:

> In elementary, middle, and high school, I was very good at math. I really enjoyed solving problems and learning new strategies. In elementary school, I tested into the gifted and talented program for math. I was definitely challenged by the curriculum (Mathematics autobiography, January 15, 2014).

Amanda was proud of how she was successful as a student, despite the challenge: “During senior year, I took calculus 2 and earned a 5 on the AP test for that class. I am very proud of this accomplishment” Mathematics autobiography, January 15, 2014). Amanda experienced productive struggle as a student because she was challenged in mathematics and yet still successful. The fact that Amanda felt good about achieving a good grade after working hard reflects that she has a productive disposition about mathematics (Kilpatrick et al., 2001)

Amanda used the word “support” throughout the course. In an interview Amanda reflected on her educational experiences as a child and how her father had supported her in mathematics:

> I also had a lot of support at home and other kids don't. My dad’s an engineer so I got through Calc 2 and he couldn't help me with that because he learned that so long ago and even he doesn't really use Calc.

Amanda still felt supported by her father, even though he himself could no longer do the calculus. She did not say how her father supported her, but she was not looking for her father to do the calculus for her.

More evidence that she believed in keeping the cognitive load high was in Amanda's homework reflection. She wrote that by asking questions instead of telling students the answers, they could learn more: "I think that the main idea of this article is to not simply do the work for the kids" (Reading reflection on discourse, March 30, 2014).

I met Amanda before a methods class after she had observed her host teacher and students for two days—she was visibly distraught. She shared that she was disturbed by the way her students were not being challenged. In methods class I allowed the PSTs time to discuss their experiences in their placements and discuss which Mathematical Practices, if any, their host teachers were facilitating. Amanda described to her classmates what she had experienced observing her teacher in placement:

> My teacher just stands in front of the room and does the problems on the board and that is the part where there is no accountability. And the discourse is just certain students raising their hands answering and her writing their answer on the board. And not making sure that everyone has the answer written down and actually making sure that everyone understands the answer (Classroom reflection, February 4, 2014).

Amanda had been challenged as a student in mathematics so she disagreed with her host teacher's writing the answers on the board before students had had opportunities to make sense of problems on their own.

Amanda noticed that not everyone was being challenged in the problems solving process in her classroom and hence the students may not have had a deep conceptual understanding of area: "We are working on area but I don't think that the kids know what it means" (Classroom reflection, February 4, 2014). Amanda described how the students were not engaging in

Mathematical Practices because they were forced to complete the worksheets so quickly that the teacher had to give the students the answers: “The fact that we stick [with] and we are so strict with keeping up with the worksheets, we don’t have the MPs because there is no perseverance with the kids to do the worksheets” (Classroom reflection, February 4, 2014). Amanda felt that the teacher gave the students the answers too soon in order to finish all of the worksheets. As a result, the students were not making sense of the problems themselves; they were not being challenged if the teacher wrote the answers on the board and they merely copied them down.

In an interview at the end of the course, Amanda told me that her experience of observing her teacher do the work for her students was frustrating to watch:

> The first question was 9+6 and the next question was 19+6 and 29+6 and she wanted them to see the pattern that all of them would end in five and then just change the tens. And some of the kids definitely got that; some were on a roll and others were still counting on their fingers and she said, “No look at the pattern,” and she would tell students the answers. She was like, “25, 35.” I think her motive was to get them to see the patterns but the kids were just waiting for her to say the answers and they weren't even trying so I was like, “Noo!” (Second interview, April 28, 2014).

Amanda was frustrated watching her teacher tell her students the answers because she wanted the students to explore on their own, which she deemed more challenging.

Amanda said that she did not know how to motivate one of her students to finish the mathematical tasks that were being used in her classroom. As Amanda explained to her classmates what was going on in her classroom, she commented that she felt lost: “I saw this one kid in my class and I don’t know how to convince him to actually do it. I can’t think of a good way with the modules to get him to do it. I am so lost” (Classroom reflection, February 4, 2014). As a result of their frustration, Amanda and Kim talked to their host teachers and asked if they could teach their own lessons rather than follow the EngageNY modules. This was more work for the two PSTs because the modules had already been written—if they did not follow them, Amanda and Kim would have to write the lessons themselves. Thus by talking to her host teacher and writing her own lesson plans to motivate her students, Amanda also increased her own cognitive load.

I explained that it was important for the PSTs not to show the students how to solve the problems—that should be done by the students. As I had suggested, Amanda explained in her lesson plan how she would get the students interested in the mathematics during the launch part of the lesson, but would not tell them how to solve the problems:

> MP1: Students will be given a task to fold a paper into equal parts. I will be asking students questions to guide their thinking through this problem solving activity. I will encourage them to think of different ways to persevere through the problem. Since I do not model [demonstrate] the correct way in the launch, this will give students a chance to make sense, then persevere in the explore (Lesson plan, February 26, 2014).

While I instructed the PSTs not to demonstrate how to do the problems because it lowers the cognitive load, Amanda extended my instructions by making the connection herself that by guiding them with thought provoking questions she was promoting Mathematical Practice 1. She said that she would ask questions rather than telling answers, which would promote the students to make sense and persevere to solve the problems themselves without her help.

In a lesson plan, Amanda discussed how she would challenge her students:

> I also added a challenge for my students to compare fractions with different numerators and denominators to test whether or not they really could reason about fractions and what they represent. I realized that it was difficult for some students to grasp… That is why I wanted students to tell me what the numerator and denominator actually mean (Lesson plan, February 26, 2014).

By encouraging her students to justify their answers, Amanda believed that she was making the content more challenging. Amanda had stated in methods class that many of her students did not have deep understanding of measurement. Amanda echoed this earlier concern when she planned her lesson and wrote that some of her students had a surface understanding of fractions. Instead of making the work easy enough for them to complete, Amanda made the work challenging for the students.

After the lesson was taught, Amanda discussed how she had allowed her students to work on their own, even though some of them had errors in their reasoning:

> I had some difficulty with the summary part because students were sharing shapes that were not partitioned equally. I tried to rely on other students to critique their models, but some had done the same exact thing. I had to work hard to hold back and let students figure it out for themselves. Finally one student who did have a pretty firm grasp on partitioning critiqued the models by saying they weren't the same shape parts, so they were not the same size (Lesson plan, February 26, 2014).

She refrained from correcting her students because Amanda wanted to promote discovery learning and keep the tasks challenging. According to Hiebert et al. (2003), although the teacher should not dominate the discourse, it is a useful strategy for teachers occasionally to share their own ideas. Therefore, instead of waiting for a student to come up with the correct answer, another option could have been for Amanda to suggest a possible answer.

Amanda's university supervisor reported in her evaluation that Amanda had engaged her students in challenging tasks. In the university supervisor's words: "This lesson was very well done, based on a concept that is very hard for third graders" (University supervisor interview, June 15, 2014). Amanda thought that she had been taught as a student to do challenging mathematics and she attempted to challenge her students as well. Amanda's host teacher commented on one of her lesson plans that the students were working on a task independently. Amanda reflected earlier how she disagreed with the way her teacher would tell her students answers; it follows that Amanda would want her students to persevere at solving the problems independently.

Rather than helping them right away, Amanda encouraged students to read the instructions: "When I first handed out the pre-assessment, many students asked for help. I asked them to try it on their own and use what they know about fractions to answer the questions" (Lesson plan, April 4, 2014).

Amanda wanted her students to make sense of the problems on their own rather than helping them right away. In an interview, Amanda described how she had guided a student to explore instead of telling them the answers:

> A: I asked her to draw the number line. I guided her rather than modeling [demonstrating] and she was doing the work.
> J: Why did you guide her? What's the point of guiding?
> A: Guiding helps them to develop the right idea but having them figure out on the own. (Second interview, April 28, 2014).

Again, Amanda wanted to guide her students to explore because it was more cognitively challenging than merely telling them the answer.

Amanda was upset with herself because she had delegated the responsibility to a teaching assistant to guide some students. Amanda felt that the assistant was telling the students what to do rather than guiding them: "Next time, I would spend more time observing these students and assisting them [rather] than relying mostly on the TAs for support" (Second interview, April 28, 2014). Amanda was also upset when the substitute teacher rushed to give a student an answer, rather than allowing her to solve it:

> There was also a sub for the special educator in the room and I was guiding her with questions; I wasn't telling her the answer and I could see the sub in the back of the room holding up numbers and I was like, "Just let her figure it out. She's got it; she just needs time to do it." And the sub was like, "Two it's two." "Stop it! (Amanda laughed) We got this and get out of here." So I wasn't telling her the answers. I was having her make a number line…(Second interview, April 28, 2014).

Although Amanda did not believe this was a prime goal of her host teacher, Amanda wanted to challenge all of her students. In her lesson plan, Amanda reflected how she had decided to give a student the same work as everyone else, even though it was challenging. By placing the student with a peer, the student responded and persevered at solving the task:

> First, I accommodated R.K. by putting her in a small group with E.S. because E.S. helps keep her on task and focused. I did not want to give her separate work to complete because I wanted her to be included in the lesson. I know she likes working with her peers, so it seemed counter productive to have her doing something different than everybody else (Lesson plan, April 4, 2014).

Amanda had experienced success at solving challenging problems when she was a student and she attempted to engage all students to engage in Mathematical Practice 1 by allowing them to struggle productively.

Fatima. In an interview Fatima said that her teachers had not taught in alignment with the Mathematical Practices, and as a result Fatima had had a negative mathematical disposition as a young student:

> I like them because I've tried to include them in my lessons. It was beneficial to learn about the Mathematical Practices and then see how effective they were in improving my students' learning. And the fact that when I was in school, I don't remember my teachers using any of them and I didn't have a positive math identity or positive experiences with math (Mathematics autobiography, January 15, 2014).

Fatima appears not to have had a productive disposition about mathematics growing up as she said that mathematics was not a positive experience for her; students who have a productive disposition are excited about mathematics (National Council of Teachers of Mathematics, 2014).

Similar to Amanda, Fatima attempted to guide students to make sense of problems without reducing the cognitive load. In an interview, Fatima said that she had experienced as a student how some of her classmates would tell her the answers:

> F: In high school I had students take over for me on group projects. I knew someone was going to take over for me so I didn't bother understanding.
> J: And that was a negative experience?
> F: I guess so. At the time I thought it was cool that I didn't have to do much work but I didn't learn as much. (Second interview, April 28, 2014).

Fatima reasoned that although the work was easier when her peers told her the answers, she did not learn deeply because she did not discover these concepts herself.

At the beginning of the semester, when Fatima taught her lesson with five other classmates, she had doubts about how much to assist her classmates:

> We attempted to help our students make sense of a problem by giving them time and hints to solve our problems. The hints helped jumpstart their thinking but I believe if we really want to help them make sense of the problem, we would have gone back and reviewed the terms and strategies for the Cartesian problems (Lesson critique, January 29, 2014).

In the first sentence Fatima said that she wanted to give her peers hints and time to solve the problems because she did not want to tell the answers. However, when there was some confusion, Fatima wrote that it would have been easier for the students if she had reviewed the terms with the class in advance even though this practice would have decreased the cognitive load. Fatima struggled herself trying to decide how much to help her peers.

In class Fatima argued that instead of telling students how to solve problems, it was more effective to ask students probing questions that guide them to solve for themselves, and during an interview she again argued to keep the cognitive load high:

> I think questioning is an important part of exploring. I think if some of my classmates used it more then it might have made the LES model more successful. Questioning itself isn't giving away the problem. You can question in a way that helps them understand and think of the problem without giving away the answer. For some people it's hard and [they] think that asking the question is giving the answer when it isn't (Second interview, April 28, 2014).

As already noted, similar to Amanda, Fatima wanted to guide her students to discover on their own rather than telling them the answers.

Fatima told me again that teachers should keep the cognitive demand high by not telling the students the answers: “There's no benefit telling students the answers. They are not learning from the sheet or activity” (Second interview, April 28, 2014). Fatima used an analogy of her getting lost to students solving problems. She stated that she needed to find her own way with directions and students need to find their own way when solving problems:

> F: I do recognize the importance of exploring. I don't know how to get somewhere unless I get lost. It's funny because when I get lost then I can understand how to get there. I was telling Kim that if I go with another person then I don't remember how to get there on my own so I have to get lost first and then find my way back. That's how we understand.
> J: Do you apply that to your students when you teach?
> F: Yes I think they need to get lost. They need to get lost in the problem and explore the problem. (Second interview, April 28, 2014).

Just as Fatima liked to explore and find her own way with directions, she believed that students should be allowed to explore in mathematics.

At the end of the semester Fatima told me that it was frustrating for her because her university supervisor did not support her with the ideas that we were learning in our methods class, and that created tension for her. I asked her if tension was a bad thing and if she would plan tension in her lessons:

> If a lot comes out of it, tension is okay but I don't think I would plan my lessons to have a lot of tension. I would target my lesson to have a little bit of tension or a little bit of arguing or critiquing (Second interview, April 28, 2014).

Students engage in Mathematical Practice 1 when they are given problems that require them to have tension or struggle (Dixon, et al., 2015; Hiebert, 2003). Similar to Hiebert, Fatima stated that it is the teacher's goal to find each student's zone and provide some tension, but not too much. Fatima described how students could be guided to solve problems in such a way that the cognitive load still remains high. One technique for guiding was to ask students questions so that they would solve the problems themselves rather than her telling them answers which would reduce the cognitive load.

When discussing Mathematical Practice 1, Fatima stated she had understood the concept of persevering intellectually, but when she went to facilitate students' engagement in this practice she learned more deeply:

> MP1 is making sense of the problem and persevering to solve it. It is important because students need to look at all parts the problem and understand the problem first and then

> take their time to solve the entire problem. They need to make sense of it first and truly understand the content being taught in order to solve the problem so they gain the learning from the problem. I learned when I try to incorporate it is that it's a lot harder to incorporate then you think it is because if you have 25 students, they may appear to understand the problem. I had one student who would rush through it. He solved the problem but missed parts of it. Then he would say that he did not get it and push the paper away. That's another example of persevering. Before I just thought of the one way of not persevering, by not starting; so I've learned you have to go both ways. Take your time to go through it (Second interview, April 28, 2014).

Before her placement, Fatima thought that the main strategy for facilitating students' engagement in Mathematical Practice 1 would be to ask the students to try the problem. However, when a student rushed his work and did not take enough time to make sense of the problems or persevere to solve the problems, she realized that merely asking a student to try a problem was not always enough to facilitate the engagement of Mathematical Practice 1; it was also imperative that this student worked more slowly. Clarke et al. (2014) also states that students need ample time to persevere at solving problems.

In an interview Fatima described how she aided students to make sense of problems on their own: "I also think you need to give it time. You can't wait five minutes and walk around and say, 'Whoops, nobody is getting it.' You have to give them time" (Second interview, April 28, 2014). Fatima argued that teachers need to give students time to solve the problems and not give in to the temptation of helping them right away.

Fatima reflected in a lesson plan how questioning was effective: "By questioning my student's answers and responses, it had them delve further into their thinking of the math concept and engage with Mathematical practice #1 as they made sense of odd and even's role in division" (Lesson plan, April 14, 2014). She commented that it was easy for students to decide if a number was odd or even, but it was more cognitively challenging for them to explain why numbers were odd or even. Fatima maintained that by questioning her students to explain why, they learned more deeply.

One of the objectives for the lesson plan in the mathematics methods course was to have the PSTs write down questions that they would ask their students during the lesson. Fatima planned to ask students probing questions: “What is different? What is the same? Which of the shapes roll? Which of shapes do not roll? How do [are] the shapes that roll are similar? How would they differ? How do the shapes that don’t roll similar? What is different?” (Lesson plan, February 13, 2014). These questions were designed to guide students to make sense of problems on their own.

Fatima reasoned that having a deep conceptual understanding of the content of her lesson would benefit her students in the future when they divide numbers with remainders:

> Odds and Evens, is a seemingly easy topic considering students prior knowledge but it is important that you emphasize and direct student’s thinking to the why’s of the problem so that they build upon their knowledge of odds and evens within a different context (Lesson plan, April 14, 2014).

Thus, Fatima maintained that by guiding her students into deep learning, they would not only understand the present topic, but would be more successful in future topics.

As she planned her lessons, Fatima looked for ways to guide her students to explore without her providing the students the answers. Fatima planned to encourage students to make sense of the problems by discussing in pairs how to solve the questions rather than her telling them: “Explore with your partner all of the different ways we can help Anna set up the meeting for everyone to see” (Lesson plan, April 14, 2014). She also planned to allow students to use blocks as tools to make sense of the problems. Fatima engaged students to engage in productive struggle by questioning them and allowing them to work in groups to make sense of problems with minimal teacher assistance.

Maria. On the first homework assignment Maria reflected on her experience learning mathematics as a student:

> Elementary math was good and pretty straight to the point. I didn't have much trouble with it…High school was a roller coaster. Geometry was decent, Trigonometry was the absolute worst, and Pre-Calculus and Statistics were really good. The only math I've taken in college was Math 118 for elementary teachers so that wasn't hard (Mathematics autobiography, January 15, 2014).

At first glance, it appears that Maria had a productive disposition as she said that mathematics was "good." However, students with productive dispositions not only enjoy mathematics but also have the belief that if they struggle, then they will be able to solve the problem. Maria said that mathematics was good because it was straight to the point, indicating that it was good precisely because there was no struggle.

At first glance, it also appeared that Maria understood Mathematical Practice 1. In both her lesson plan and her interviews she described how she had encouraged her students to persevere. However, upon careful examination, Maria often had a shallow interpretation of this practice. In Maria's lesson plan reflection she implied that a student completed his work without struggle so he was engaged in the first Mathematical Practice:

> He is on task all the time and participates when he can. There doesn't seem to be any struggle in his math capabilities because he is actively solving out problems, and aware of the content being taught. I believe he used MP1 while completing each problem (Lesson plan, February 13, 2014).

Maria wrote that the student was engaged in Mathematical Practice 1 even though he did not struggle. Dixon et al. (2015), in contrast, explains that Mathematical Practice 1 centers on students solving challenging problems, which encourages students to struggle and persevere.

Maria stated she was proud that after she had helped a student for 20 minutes, the student persevered to do 50 math facts on an assessment:

> With the student I had who doubted herself she would pout until someone noticed her, but if you build her confidence, that's like the perseverance part and tell her she can do it. At the end she was able to solve all 50 problems on math facts. That feels good on my part. I didn't take out too much of the day; only 20 minutes and she felt good (Second interview, April 29, 2014).

Hiebert and Grouws (2007) discuss struggle in mathematics to be more than just persevering with tasks that have already been shown to them. In their words, “By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated” (Hiebert & Grouws, 2007, pp. 387-388)? A drill activity followed by an assessment is what the authors refer to as memorizing information; not struggling to learn mathematical concepts. Furthermore, according to Kilpatrick et al. (2001), students need to develop a disposition that they are capable problem solvers and that their criteria for success is not solely measured by assessments, but rather in their ability to solve problems. Maria was not encouraging the student to be a successful problem solver who enjoyed the challenge of persevering and finally solving a problem; instead the student and Maria were proud that the student endured the assessment and got a good grade. In this example, Maria did not help her student to develop a productive disposition (Kilpatrick et al., 2001).

In Maria’s lesson plan reflection, she said that there was one pattern that she was “hoping they learned”:

> Another pattern that was found was that the sum of the two numbers in the product equals 9. The one pattern that I was hoping they learned was just adding nine after each fact, similar to all the other multiplications facts, and the easiest (Lesson plan, February 13, 2014).

When Maria said that she was hoping that students would get an answer it suggests that Maria may have been steering the students to solve the problem the way that she wanted them to solve it, rather than guiding them to develop their own strategies. Larson et al. (2012) believe that successful problem solving does not necessarily mean that students figure out the correct answer;

instead it means that students persevere and attempt various strategies. The authors argue that teachers can guide students to engage in challenging mathematical tasks. Maria appeared to be too direct in her approach to teaching students to solve mathematical tasks.

Maria explained how she had helped her students in a lesson that she had taught:

> One will say it should be a scale of four and the others say it should be a scale of two. And I would say, "Look it's 4, 7, 11." I modeled to do a scale of two but that one group did a scale of four. That got everyone else thinking and confused a little bit but then they were able to see (Second interview, April 29, 2014).

The fact that Maria allowed the students above to choose their own scale encourages students to be invested in their learning. However, by modeling or demonstrating Maria reduced the cognitive load. Furthermore, when the students were confused, instead of allowing the students time (Clarke et al., 2014) to solve the problems, she stepped in and explained so everyone understood. Hiebert (2003) encourages teachers to move away from the temptation to explain to students how to solve problems and to welcome student confusion in order for students to develop their own problem solving techniques. Maria appeared to direct her students to get the right answer rather than support them as described in Mathematical Practice 1.

Sarah. Like Maria, Sarah did not focus on students engaging in productive struggle. On her first homework assignment Sarah wrote rules to guide how mathematics should be taught, and four out of five of the rules that Sarah wrote showed alignment to the literature on facilitating the engagement of Mathematical Practice 1. She said that students should use manipulatives; Hiebert et al. (1997) also emphasize the importance of manipulatives for learning content deeply. Sarah wrote that mathematics should be relevant, and Clarke et al. (2014) maintain that making mathematics relevant is a strategy for students to persevere at solving problems. In methods class we discussed the advantage of students having multiple strategies to solve problems (Larson et al., 2012), which was Sarah's rule number three. Sarah believed in students working together in small groups. When students are in small groups they have more opportunities to discuss and make sense of how to solve problems (National Council of Teachers of Mathematics, 2014). Even though these rules were in alignment with the literature for best practice, the concept of productive struggle was missing.

In one of Sarah's rules she suggested that students be taught easy problems first in order to avoid confusion; yet Hiebert (2003) maintain that some confusion is effective for students to develop problem solving dispositions. Sarah wrote:

> When I understood math, I loved it but when I was confused and lost in a class, it was my absolute worst subject. I achieve in math when I am in a class where I am confident and confortable [sic] to ask questions (Homework reflection on rules for mathematics, January 15, 2014).

Sarah did not like confusion as a student, and in one of her rules she wrote that confusion should be avoided.

In yet another rule, she stated that teachers should "have a back up explanation to offer," (homework reflection on rules for mathematics, January 15, 2014) to their students. This rule depicts how Sarah may view the teacher's role. Rather than being a facilitator for students to solve problems themselves, she implied that the teacher's role was to explain to the students how to solve problems. Hiebert et al. (1997) believe that teachers are giving students too many explanations and that this reduces the cognitive load; instead, teachers should encourage students to make sense of problems themselves.

More evidence of Sarah's initial beliefs on the role of teachers was Sarah saying: "As a learner, I need concepts explained simply yet thoroughly. If a new topic is being introduced, I like to take it slow and start with simple math before moving on to the challenging stuff" (Mathematics autobiography, January 15, 2014). As mentioned earlier, Sarah viewed the teacher's role to be the one who is responsible for explaining concepts to students, and this should be done without confusing the students. Thus, it would appear that Sarah did not have a productive disposition—she did not have a solid belief that she could solve problems herself by putting in the effort; she preferred for her teacher to explain concepts to her.

Later in the semester, in a critique of her classmates' lesson, Sarah wrote that she liked the idea that there was confusion because it helped them learn:

> I thought it was interesting when the class got into a discussion about a specific problem. There was confusion about if order counts or if it did not count. We have different opinions and one student said that combinations means that order matters. .. I also liked how there was discussion and confusion because it helped us continue to learn and solve the problem the correct way (Lesson critique, January 29, 2014).

In class, we had discussed how confusion could promote students to learn mathematical concepts more deeply, so this may be why Sarah wrote that it was positive even though her comments earlier in the semester had discussed avoiding confusion.

However, Sarah did not apply the concept of allowing students to be confused when she taught her next lesson. Sarah critiqued her own lesson that she taught with three other peers. In this reflection about her teaching, Sarah stated that she wanted to explain the confusion: "I also used real life situations to help explain confusion. I used a pizza pie as the whole (denominator) because people were confused with the total number" (Lesson critique, January 31, 2014). Sarah mentioned that the confusion was positive in one occasion, but she did not implement that strategy when she taught her peers.

Sarah said that students were making sense of problems at the application center, but it seems that she encouraged the students to solve the problems her way:

> MP1: This mathematical practice is used at the application center of the module lesson. The students are given one problem and before solving it, they must read it as a group and everyone make sense of the problem. They must understand and determine if the problem is addition or subtraction and what exactly is being asked of them. This is when they have time to talk as a group about the problem and then persevere in solving it. At my center for concept development, they have the chance to make sense of the first

> problem. They understand that they can't subtract because there are no tens in the whole number. They talk about why that can't work and how to solve it. They do it for the other problems too. The second problem they have to figure out how to use the same strategy but now the number has a zero in the tens and ones. As a group, we understand the problem before solving it (Lesson plan, February 27, 2014).

Larson et al. (2012) argue that the intent of Mathematical Practice 1 is for students to develop their problem solving skills. However, Sarah claimed to have promoted the Practice even though she seemed to deny her students opportunities to solve the problems on their own. She said that after the students understood the problems, they would solve more. However, if the students already understood the problems then they would have limited opportunities to struggle to make sense of them. Sarah did not select the tasks that were taught; her host teacher gave her the lesson plans from Engage NY.

Although the lesson plans were already written for Sarah, I asked her to attempt to fill out the lesson plan template the best she could. Sarah was asked to predict how her students would respond. She wrote:

> I cannot predict if the students will answer in the way the module expects. I think that if I go at a good pace, the students will keep up and understand the lesson...I expect the students to follow along at my pace (Lesson plan, April 11, 2014).

The comments above depict that Sarah went at a fast pace and attempted to have her students work fast to complete the work. Clarke et al. (2014) argue that students need sufficient time to persevere and solve challenging problems. Again, Sarah mentioned the lack of time in a reflection after she taught:

> If I had the time and flexibility, I would do my lesson based on this but I had to teach the module in centers. There was no time for a launch, and the students did no exploring. The lessons were less about figuring out the math and learning from peers and more about the teaching giving the information. The students had never worked with graphs so I had to show them how to label it and how to gather the information to put in the graph. There was no way the students could explore how to do this on their own because there is no problem to solve (Lesson plan, April 11, 2014).

Sarah's comments depict that under the restrictions of the scripted curriculum, her students were not given opportunities to solve problems; in fact, she said that there were no problems to solve—she merely told the students what to do.

I asked Sarah's host teacher about her questioning techniques and he said:

> There are times when she jumped in too soon… I think looking at those Modules was intimidating for her. I know I keep using that word intimidating, but I think that describes her and her initial experiences. You are looking at an hour lesson, and there were 15 pages of stuff that I have to get through (Host teacher interview, July 30, 2014).

Her host teacher sheds more light on how Sarah might have taught. Her main goal was to get her students to complete the up to 15 pages that the students were required to complete. Thus, it was faster for Sarah to tell her students answers rather than wait for them to discover the answers on their own.

Sarah's classmates taught at least four whole group lessons, but Sarah taught in centers instead because that was her teacher's routine. Sarah described how she taught at her centers: "At my center, there are no word problems. I write a problem vertically and visually and they do the same. There is no relating to life because they are just working with numbers and equations" (Lesson plan, February 27, 2014). If the students mostly did arithmetic with her, she had limited opportunities to help students to makes sense of word problems and persevere to solve them.

In a lesson plan reflection, Sarah wrote that she was not able to engage her students in the inquiry-based lessons that were taught in the methods class:

> Unfortunately, there was not much that I was able to use in my math lessons that I learned in class. This is because I taught the module, which is a structured and set lesson for math that the class has to learn by. The module had set lessons and they told me what I should say and expect to hear from my students. It also gave me all of the post assessments and activities. A big part of our class in using the LES model: Launch, Explore, and Summary. If I had the time and flexibility, I would do my lesson based on this but I had to teach the module in centers. There was no time for a launch, and the students did no exploring. The lessons were less about figuring out the math and learning from peers and more about the teaching giving the information. The students had never

> worked with graphs so I had to show them how to label it and how to gather the information to put in the graph. There was no way the students could explore how to do this on their own because there is no problem to solve (Lesson plan, April 11, 2014).

Therefore, she did not get much practice facilitating students' engagement in productive struggle because, according to Sarah, there were no problems to solve. Sarah recognized that it takes time for students to do discovery learning—time which she did not have. When she said that the students did not figure out the problems so she had to teach them, she implied that she used a direct approach.

Sarah's university supervisor said that another one of her PSTs was effective at guiding her students. I asked her if she had noticed Sarah guiding her students and she said that she did not:

> I didn't have that many opportunities to see that. She was using the module and I really didn't see whole lesson. She was working with students in groups. She was doing a practice and the teacher's aide was doing work with the lower level of students. They did it together and they talked it through and she had them making decisions. There were certain questions she would ask them why they did things. They had to defend but they weren't challenging enough. There wasn't much perseverance (University supervisor interview, June 15, 2014).

The university supervisor has an understanding of productive struggle because she gave an example of the other PST guiding her students and keeping the cognitive load high. I asked her university supervisor what Sarah's core beliefs were and she replied: "I don't know. Probably that they can learn from each other, but probably more that they can learn from me [learn from Sarah]" (University supervisor interview, June 15, 2014). Not only did Sarah's university supervisor's comments add support that Sarah believed that her role was to explain to students how to solve problems, she also stated that the students did not persevere because the problems were not challenging.

Upon completion of the course, in an interview, Sarah explained that she liked the way her teacher also explained to her students how to solve problems because they learned from him. Sarah stated that this method was better than students trying to solve the problems themselves because if they try to solve problems in groups, the high achieving student would dominate. Below is Sarah's reasoning:

> J: Who does most of the talking?
> S: The teacher.
> J: How do you feel about that?
> S: Sometimes I don't see any other way working. What he does is good because they learn from him; he is good at explaining. It's better than trying to get them to figure it out. There are students in the classroom that are high achieving and low achieving. If you put them together without supervision and let them figure it out, the high achieving student is going to do all the work. The other student isn't going to learn anything (Second interview, May 8, 2014).

Telling and explaining to students mathematical concepts may be faster than allowing them to solve for themselves, but it does not promote students to make sense of problems themselves. Sarah reported that her teacher had dominated the discourse and explained mathematics concepts to the students, and yet in the mathematics methods course, we had discussed how teachers could share the discourse with the students and encourage them to solve challenging problems on their own. It might have been frustrating for Sarah to balance between her host teacher's expectations and my own.

I asked Sarah what she would do if a student could not figure out a problem: "I would give them help. Giving individual help is a faster route than just letting them keep trying to figure it out by themselves" (Second interview, May 8, 2014). Clarke et al. (2014) state that in order for students to solve challenging problems, they will need time, but Sarah preferred the faster telling approach to the students struggling to solve problems. I wanted to know how Sarah helped her students, so I asked her:

> J: What did that look like? What was your help like?
> S: Mostly during the problem sets or if you see them doing something wrong. During the lessons I would call on them. Making sure they participate. Making sure they are writing and they're looking at you. All the signs that you can see.
> J: Great, but for some reason if they give up what would you do?
> S: You just tell them that they can do it and then you explain how they can do it (Second interview, May 8, 2014).

Just as Sarah described her teachers' role was to explain mathematics to her when she was a student, Sarah said that she would also explain to students how to solve problems.

By helping the students too soon, Sarah was not facilitating that they make sense of problems and persevere to solve them. This method of telling instead of allowing the students to solve problems is not in alignment with Mathematical Practice 1. Koestler et al. (2013) maintain that the essence of Mathematical Practice 1 is to engage students in problem solving. If the students were not involved in productive struggle because Sarah gave too much guidance (perhaps due to the use of the scripted curriculum and the host teacher's example) and the tasks were not challenging, then she had limited experience facilitating students' engagement in Mathematical Practice 1.

Kim. Kim had a productive disposition as she reported excelling at mathematics:

> I was ahead of the class in math; I always knew my math facts because I had a lot of practice at home with my mom. Math always came easy to me...I met all the prerequisites to be in algebra in 8th grade and never got below an A in any math class. In high school, the success in math continued (Mathematics autobiography, January 15, 2014).

Similar to Amanda, Kim was disappointed with the way her host teacher instructed her students. Kim's goal was to make the students more excited in mathematics because she observed that they were bored and not persevering. In an interview Kim described how she motivated students to persevere at solving problems:

> "They got learned helplessness…they wanted me to do it for them. They got used to the worksheets. It is not okay. They loved my lessons because they didn't have to do

> worksheets. One kid said, "Are we doing math again in groups?" "Yes." I'm glad that people were getting up instead of sitting (First interview, April 23, 2014).

Kim believed that if she made the content more exciting, then her students would persevere to solve them.

In an interview, Amanda described how she had worked with Kim to plan a meaningful social studies lesson plan for their students. Amanda mentioned that they guided their students, rather than telling them the answers:

> So that's when Kim and I said, "Okay how do you teach global economy to third-graders?" And it [lessons online] was all for seventh and eighth graders. We found some terms and did some activities with them and in the end it was in their zone of proximal development. We guided them to be there. They really enjoyed it (Second interview with Amanda, April 28, 2014).

As Amanda pointed out above, the two PSTs guided their students to engage in content that was challenging. Apart from making content interesting, Kim also used questioning to facilitate her students' engagement of productive struggle. In an interview, she emphasized how important questioning was:

> A lot of us didn't talk about questioning in our final presentations, but I think that was one of the biggest things from the course at the beginning of the semester. I would say, "Give me your pencil and do this" and do it for them. After we talked about questioning, [in the mathematics methods course] I started to do that. They weren't learning if I did it for them. They had learned helplessness. So after a while I would ask questions (First interview, April 23, 2014).

Kim's techniques of questioning her students allowed them to become interested in the problems and make sense of the problems on their own, so Kim did not have to tell them how to solve. In an interview Kim disagreed with her host teacher's giving a group of students too much guidance: "They had the best graph because she gave them the answers" (Second interview, April 30, 2014). Even though the group had achieved the goal by making a good graph, Kim would have preferred for them to complete the assignment with less assistance from the host teacher. By helping students believe that they could solve the problems themselves, Kim was facilitating her students to develop productive dispositions (Kilpatrick et al., 2001). Kim believed that teachers should make mathematics content interesting and should ask questions, which I interpret as encouraging productive struggle.

Kim, who had eight ELLs in the field, made the content challenging for her ELLs by asking questions. The university supervisor said that Kim's social studies lesson was engaging and she asked a lot of questions:

> They loved it and even the kids that didn't speak English could find it and write it down. Then they plotted it all out on the maps. How do you think they got here from Nicaragua? How do you think they got here from China? What are some of the choices of all the different places that the clothes came from? It was fascinating (University supervisor interview, June 15, 2014).

The fact that Kim's university supervisor noticed that she was asking probing questions to all students, including the ELLs, suggests that Kim not only believed in asking probing questions, but also was also capable of implementing them. By asking questions rather than telling ELLs the answers, Kim allowed her students to make sense of the problems themselves.

Murrey (2008) summarizes that it is usually best to scaffold the ELLs' understanding, but the mathematics needs to remain rigorous. Kim's university supervisor said that Kim helped her ELLs understand the language by using strategies, but the work was still challenging for them. In one of Kim's lesson plan reflections; she discussed how she would only help the ELLs if they needed it:

> I will encourage him positively so that he feels the need to really challenge himself to talk to the whole class and persevere in that aspect because he usually perseveres on the daily (MP1). I will also assist him and the other ELL students by writing the key [target] words of the tasks on the board and maybe even model with the cubes, sticky notes, and numbers on the side of the graphs, if they are struggling, but only if they are struggling, which I do not really expect from B.L (Lesson plan, April 16, 2014).

Kim was willing to accommodate her ELLs, but she would try to keep the cognitive load high by allowing them to make sense of the problems on their own if possible.

Abigail. Abigail offers a detailed description of her mathematics experiences and preferences:

> My overall math experience has been okay. In elementary school, I was the star student in mathematics, always working ahead in the workbook or given the next grade up workbook. I also did well on tests. I liked math as a young student because there is always a definite answer, I just have to find my way to it unlike in English where you have to think about stuff and analyze. In middle school, I continued to stay good in math but I went through a rough patch in school so I really wasn't doing any academics. In high school, I started to not care about mathematics just as long as I passed the regents exam. The teachers in high school took teach to the test literally in every way, shape and form. My college experience with mathematics has been a positive experience so far. I have only taken three math courses and got good grades in all my courses. Personally, I did not like the way things were taught to me but I understood the reasons behind it… As long as the teacher is actually teaching what is suppose to be taught and will be tested on, I am pretty much fine with taking any math course. When I get confused or stumbled, I will ask for help (Mathematics autobiography, January 15, 2014).

We could assume that Abigail has a productive disposition because she generally speaks positively about the subject. However, she has a strong emphasis on assessment throughout her description. For example, she did not like it when her teachers taught mathematics that was not followed up by tests. Kilpatrick et al. (2001) state that students that have productive dispositions typically enjoy solving problems, not merely because they do well on assessments, but because they enjoy the process and that mathematics is useful for their personal lives.

The data on Abigail's beliefs and practices regarding productive struggle are mixed. In a homework reflection she commented on the importance of teachers asking questions to students rather than telling them answers:

> Good questioning has a huge impact on mathematical lessons especially in the LES model. The teacher must be have the right questioning skills to guide their students in the write direction but asking student's questions that makes them wonder and connects to real life. This connects to MP1 because the teacher is not giving them the answer; they are asking questions that lead the students to the path towards the correct answer. That

> right there is so hard to do and take times to perfect. I know when I first got introduced to the LES model, the most difficult part for me was the questioning part (Reading reflection on discourse, March 30, 2014).

In her essay, she made the connection that asking students questions, is related to Mathematical Practice 1—when teachers ask students questions, they can make sense and persevere on their own. However, she also said that asking questions was challenging for her, especially at the beginning.

I did not find evidence of Abigail teaching students to engage in productive struggle in her lesson plans, and I will discuss two situations where she had opportunities to do so. She wrote:

> I notice that my students struggles [struggled] with making equal parts on the triangle. They were so focused on making four equal parts that they were unable to realize that what they drew where not equal parts. I also had to stop at the second center to do a mini lesson on how to correctly make equal parts but making the lines that touch opposite sides and/or vertices (Lesson plan, April 15, 2014).

Instead of allowing the students to make sense and persevere to solve the problems on their own, Abigail showed the students how to make equal fraction parts. I do not have the data to determine whether the students were struggling outside of their zone of proximal development Vygotsky (1978). However, it is important for teachers to allow students opportunities to make mistakes on their own without the teacher stepping in to aid them too soon (Hiebert et al, 1997).

On the lesson plans that the PSTs designed they are asked to select three target students that they observe throughout the lesson. At the beginning they write a short description of each student and how they plan to help these three students. The PSTs are also asked to reflect on the progress of all the students, including these three. Abigail described one of her target students:

> MC is one of the advanced learners in the class. She is often completing her work before the other students in the class and will quietly read a book or do another non-distracting activity. She also plays the role of the teacher assistant for my teacher, especially before I

> got there by helping run errands and marking off morning work (Lesson plan, April 11, 2014).

According to Abigail's observations, this student completed her mathematics early and would read and help her teacher with non-mathematical tasks. Abigail did not make it one of her goals to challenge this student so that she would be doing mathematics the entire lesson. More evidence for not challenging her students was also written in one of Abigail's lesson plans: "The more advanced mathematical thinkers in the class will finish first which is why I gave them to option to color the different equal parts if they finish before the time is called" (Lesson plan, April 11, 2014). Engaging students in productive struggle was not evident in Abigail's lesson plans.

Emily. Emily shared on the first homework reflection that she had had positive experiences with mathematics:

> I loved math in every experience I had with it. I was always fortunate to have teachers who were passionate and caring. The relationship I built with my teachers allowed me to enjoy impressing them with my math skills. I also enjoy having a problem to solve. It is nice for me to know a couple strategies but to always come up with a correct answer (Mathematics autobiography, January 15, 2014).

From Emily's comments it could be that she had a positive disposition towards mathematics.

Emily expressed interest in guiding students so they could develop their own strategies. Emily reflected: "As teachers, we are not supposed to look at the cookie-cutter formulas for math problems but allow the students to explore their own ways to solve problems and figure out which strategy works best" (Reading reflection on cultural relevance, January 22, 2014). Emily said that teachers should guide students to discover for themselves.

Emily struggled when she was making her first lesson plan. Instead of answering her questions, I guided her to develop her lesson plan on her own. At the end of the semester, she recognized that although she was frustrated at first, she learned more when she was encouraged to struggle and work out the lessons on her own. In an interview, Emily said that just as she experienced developing her lesson plan on her own with little guidance, she would also aid her students to solve mathematics problems on their own with little guidance from her.

Emily shared her thoughts on the importance of teachers guiding students:

> When a student says an answer and it's not right there is still thinking going on that is going to lead them to the right answer and you can guide them with proper peer support. You can't just say yes or no the students need to explain (Second interview, April 29, 2014).

Akin to Hiebert et al. (1997) Emily said that teachers should allow students to make mistakes because they can learn from this process.

Emily guided her students instead of telling them the answers, according to Emily's supervisor (who also supervised Sarah). Contrasting Emily to Sarah, she said: “I think Emily, when they did the strategies and some of them floundered, she didn't directly say, ‘You're going about it wrong.’ She was more of a, ‘Keep at it and use your partners. Use each other's strengths’” (University supervisor interview, June 15, 2014).

In Emily’s lesson she planned to engage one of her students in productive struggle: “I will incorporate Mathematical practice 1 for this student because I will give her the opportunity to think on her own. I will give her time to persevere” (Lesson plan, April 2, 2014). Emily’s goal was for the student to persevere and solve the problem on her own, and this would be possible if she allotted this student ample time.

Charles. Charles reflected that he did not have opportunities to make sense of problems and solve them when he was a student. He said that he was supposed to solve mathematical problems using his teachers’ strategies, but he often got confused because his parents signed him up for mathematics support outside of school. The program outside of school also insisted that Charles solve problems using their strategy, but since Charles had a shallow understanding of the mathematical concepts, he could not apply the strategies when the problems were slightly changed:

> You just learn one way. You're taught, "Here is the way we solve 16×15." And there is one specific way. "Solve it." Because I was not able to use different ways to solve the math, I did not retain it. So I learned and I would forget. I learned it and I would forget. So when I was growing up my parents would constantly give me worksheets to do over and over again. That's how they learned to do math. Their way of doing math was doing it until it gets ingrained into your brain…I feel like for me, it didn't work. When I got to tenth grade or even eighth grade, I forgot all that math that I had been taught. I feel like if I had learned my way I would've remembered more (Second interview, May 5, 2014).

Charles argued that he had not had opportunities to make sense of mathematics himself—he merely did what his instructors told him to do. As a result, Charles said that he did not retain what he had previously studied.

Charles was in a classroom where the EngageNY modules were used. Unlike Sarah, Charles was able to teach a unit following the inquiry-based model introduced in class. The problem he chose in one of his lessons was challenging for first graders:

> Students are expected to do the problem by adding 19+13 using any strategies that they have learned in class. The answer should be 32 which is greater than 30. This means that Mr. Lee's class cannot get on the train (Lesson plan, April 9, 2014).

Not only was the problem that Charles chose challenging, his university supervisor indicated that he had encouraged productive struggle:

> He went from group to group. Most kids didn't need support because they were working so well. It was only five minutes. If you let it go on for 10 or 15 minutes, he would have lost them. It was a challenging problem. For kids that had problems he asked them, "What do we need to do first?" without giving them answers and saying it is 36-15… He sort of talked them through it. It was a brilliant lesson and I thought the kids were so much farther ahead then they might have been if he hadn't done it that way. He could've tried it by saying, "Take this number and then you subtract" (University supervisor interview, June 15, 2014).

According to Hiebert et al., (1997), teachers often reduce the cognitive load on challenging tasks by helping students too much. However, his university supervisor reported that Charles allowed the students to struggle and only helped them when they needed it. Furthermore, he asked them questions so that they could solve the problem on their own.

Charles said that he had not always been able to make sense of problems himself, but in the field he allowed his students to struggle and make sense of problems. Charles is an example of a PST who may have had a somewhat negative disposition towards mathematics at the beginning of the semester, but as he connected with his students throughout the semester his disposition improved, and he engaged his students in productive struggle.

Summary of Productive Struggle

Table 6

Summaries of PSTs Connecting Mathematics to Students, Providing Access to Content, High Expectations for Students, and Providing Students to Struggle Productively[1]

Name of Participant	Connected Mathematics to Students?	Provided Students Access to Content?	High Expectations for Students?	Provided Students to Struggle Productively?
Abigail	Strong	Strong	Weak	Weak
Amanda	Strong	Strong	Strong	Strongest
Charles	Strong	Strong	Strong	Moderate
Emily	Strong	Strong	Strong	Moderate
Fatima	Strong	Strong	Strong	Strong
Kim	Strong	Strong	Strong	Strong
Maria	Strong	Strong	Weak	Weak
Sarah	Felt Restricted to Make Connections	Felt Restricted to Provide Access	Weak	Weak

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

I have discussed the PSTs' beliefs and practices related to the concept of productive struggle. According to National Council of Teachers of Mathematics (2014), if students have a productive disposition—a belief that mathematics is worthwhile combined with the belief of self-efficacy—they will have a positive attitude about mathematics for the rest of their lives. Larson et al. (2012) state that the "fortitude to participate or persevere in the problem solving process toward a successful end" (p. 27) is an "important component of a productive disposition" (p. 27).

Perseverance is also a component of productive struggle (Warshauer, 2014)—implying a possible linkage between productive disposition and productive struggle.

It would be interesting to see whether PSTs who have a productive disposition are more likely to encourage their students to engage in productive struggle. I do not have sufficient data to test such a relationship (especially not for Emily and Charles who focused on other topics, such as developing students' language). However, I will discuss possible patterns that I noted in my data and discuss how they align with the literature.

The data suggested that Amanda had a productive disposition—she said that she loved mathematics and was confident of her abilities in the subject. She had experienced being engaged in productive struggle as a student and was frustrated when she saw her students not being challenged. Amanda discussed how she had guided her students to keep the cognitive load high, rather than telling them answers, suggesting that she had encouraged her students to engage in productive struggle.

Kim—who reported that she had excelled in mathematics throughout elementary, high school, and college—showed clear signs of a productive disposition. Kim taught eight ELLs in her placement and focused on ways to encourage her ELLs, and all her students, to engage in productive struggle.

It would appear from Maria's comments that she was at the beginning stages of learning the concept of productive struggle because she did not pay attention to it. Turner et al. (2012) argue that PSTs have a trajectory for learning new concepts about ELLs; they explain that the first step in the trajectory is for PSTs to pay attention to the new concept. Maria said that she had had some positive experiences and some negative experiences with mathematics as a student, but she did not like mathematics when it was too challenging. Thus, Maria did not appear to have experienced much productive struggle as a student, or to focus on her students experiencing this concept.

Sarah liked mathematics but avoided learning and teaching mathematics that was too confusing. Kilpatrick et al. (2001) state that if you have a productive disposition, you have the belief that you can solve challenging problems by working hard. Thus, Sarah may not have had a productive disposition, and she avoided struggle as well. Sarah implied that the teacher's role was to explain mathematics concepts to students. She had liked it when her teachers did this and she said that she approved of her host teacher explaining concepts to her students because it was faster than if they discovered for themselves. Sarah had to balance between the pedagogies taught in the mathematics methods course and those of the host teacher. Furthermore, Sarah did not have the opportunity to write a lesson plan for her students to engage in productive struggle because the lesson plans were already planned for her.

The data on Abigail suggest mixed results. She reported being successful in mathematics, but also had a limited view of mathematics. Abigail's homework reflection demonstrated an understanding of productive struggle, but this concept was missing in her lesson plans. I do not have the data to discuss possible relationships between having a productive disposition and teaching productive struggle for Charles and Emily.

Fatima may not have had a productive disposition—she commented in two different occasions that she had had negative experiences with mathematics as she grew up. In an interview, Fatima revealed that she did not feel confident about mathematics content, but she was confident about her effectiveness as a mathematics educator. However, Thames and Ball (2010) argue that in order to be effective at teaching mathematics, teachers should not only have strong pedagogical skills, but also have strong content knowledge. Fatima told how she believed in guiding students by asking them questions. She learned in the field that it is important for students to take their time when they are persevering to solve problems (Clarke et al., 2014).

One of my research questions is to examine how the experiences of PSTs learning to teach are similar or different; and what may account for any differences. The PSTs who I have discussed had different beliefs and practices about facilitating students' engagement in productive struggle. One explanation could be that having a productive disposition (or not) may account for PSTs engaging (or not) students in productive struggle. Fatima was the exception because her comments suggested her not having a productive disposition or strong content knowledge, but nevertheless showing signs of engaging students in productive struggle. In chapter 6 I will continue the discussion about productive struggle and productive dispositions.

CHAPTER 5: FINDINGS—MAKING SENSE OF PSTS' PERCEPTIONS OF MATHEMATICAL PRACTICE 3

In Chapter Five I report and analyze my research findings related to Mathematical Practice 3 (construct viable arguments and critique the reasoning of others). As in Chapter 4, the order in which I present data from the individual PSTs varies in descending order according to the pertinence of the data. Also as in Chapter 4, I provide tables to aid in keeping track of the PSTs. Table 7 summarizes background information pertinent to this chapter; Tables 8 and 9 provide "thumbnail summaries" of the results.

Larson et al. (2012) maintain that the intent of Mathematical Practice 3 is for students to “make and test conjectures and to communicate their mathematical thinking” (p. 40). The researchers explain that the teacher’s role is to guide the students to justify their answers. Teachers can facilitate students’ engagement in Mathematical Practice 3 by creating an atmosphere where students feel comfortable making mistakes (Hiebert et al., 1997) and there is ample student discourse (Sfard, 2007). I have divided Chapter Five into two sections: “Social Interaction” and “Developing Language.”

In each section I discuss:

1) The PSTs themselves developing an understanding of Mathematical Practice 3;

2) The PSTs’ beliefs and practices of facilitating elementary students’ engagement in Mathematical Practices 3;

3) The PSTs’ beliefs and practices of facilitating ELLs’ engagement in Mathematical Practices 3.

Data considered include: Open response surveys (pre-and post), homework reflections, lesson plans, university supervisors’ reports, host teachers’ reports, and semi-structured interview of PSTs, university supervisors, and host teachers.

Table 7

Backgrounds of PSTs for Mathematical Practice 3

Name of Participant	Language/s Spoken	Number of ELLs in Placement	Mathematics Instruction Taught in Placement	Type of Lessons Taught by the PSTs in the Field
Abigail	English	6	My Math Textbook	LES Model (Inquiry-based)
Amanda	English	0	EngageNY Modules	LES Model (Inquiry-based)
Charles	English, Mandarin	0	EngageNY Modules	LES Model (Inquiry-based)
Emily	English	0	My Math Textbook	LES Model (Inquiry-based)
Fatima	English	0	My Math Textbook	LES Model (Inquiry-based)
Kim	English	8	EngageNY Modules	LES Model (Inquiry-based)
Maria	English, some Spanish	0	My Math Textbook	LES Model (Inquiry-based)
Sarah	English	0	EngageNY Modules	Direct Instruction

5.1 Social Interaction

I chose the terms "Community" and "Peer Support" because the PSTs used the two terms throughout the semester in homework reflections and lesson plans. While the two terms are similar, I refer to creating a classroom community when the whole class is involved, while the second term only requires a small group. Hiebert et al. (1997) claim that teachers should develop social interaction in the classroom to aid students to communicate in mathematics as they build concepts together.

Throughout the data when the PSTs referred to creating community, they meant the class community. While it would be possible to have smaller communities within a class or a school community, the PSTs limited their use of developing community to the class. I will discuss in more detail what the terms community and peer support meant to the PSTs and how they tie into helping elementary students to construct viable arguments and critique the arguments of others.

Kim. In Kim's first reflection, she wrote that I had created community in the mathematics methods course on the first class: "The most impressive part about the first class was that the teacher was very intimate and friendly. He made the community happen right away, which is nice to see since we learn about community all throughout this program" (Class reflection, January 17, 2014). She implied that I had created community by being intimate and friendly, so it can be assumed that these are some traits that are important to Kim for creating classroom community.

Kim said in an interview: "We did a lot of community last year. We are respectful. I can see that that works. You can do it on campus. It is harder with younger kids because they have quick tempers. I see why build a community" (First interview, April 23, 2014). From these comments we can infer that Kim believed that it is useful to build classroom community; she was aware that teacher educators have developed the community with the PSTs in the past; and it was more challenging for younger students because they do not control their tempers. She also associated community with being respectful.

In an interview, Kim said that as a result of her teacher educators establishing a classroom community, there was more student discourse: “And it worked. By the end of the Block One [last year] we were friends and everyone participated more and felt more comfortable” (First interview, April 23, 2014). This comment not only confirms that the PSTs have learned about creating a classroom community in the teacher education program and that they felt comfortable in the community, but it also makes the link to student discourse. Kim said that as a result of having a classroom community, the PSTs talked more.

Kim described her host teacher in an interview and said that shouting does not create classroom community:

> She got frustrated and would yell at them. The kids are used to that at home. I wouldn't start that way. I asked if she tried to build a community but she said there's no time with the state testing (First interview, April 23, 2014).

The fact that Kim asked her host teacher about creating community after mentioning that her teacher yelled at her students suggests that Kim did not agree with the yelling and that it was detrimental for creating community in the classroom. I asked her if she thought there was time to create classroom community and she replied:

> I think she could've done more. She talked bad about the kids to me. I didn't talk back to her, but it was strange because she is supposed to be the professional and she was badmouthing the kids. I don't agree with that. I think that is even more disrespectful. One of the kids was on time out and I asked why he was disrespectful, and he said because she is disrespectful to us. They listened more to me because I was a good guy. I don't know how it would play out if I were the only teacher, but I think if you respect them, they respect you. If you want them to do well... I didn't like that. I would never have that in my classroom (First interview, April 23, 2014).

Kim said that her teacher should have created more community and then she mentioned how the teacher did not respect her students. This is more evidence for Kim’s belief that teachers should respect their students to create a classroom community. Kim disagreed with how her host teacher treated her students; she argued that if teachers treated their students with respect and were caring, the students would reciprocate. Kim also argued that it was not respectful for teachers to talk badly about their students.

Not only did Kim discuss the importance of creating community, she also applied the concept in the field. Kim said that creating community involved students feeling comfortable and criticized her host teacher for shouting at her students. In contrast, she planned to have her students be nice to each other:

> I will put the students into heterogeneous groups of three so that they can talk through their problem in a group setting and be able to explain their ideas in their groups. I will also be sure to tell them to talk about others' ideas especially if they do not agree with them, in a NICE way. I will definitely need to stress the nice part in my behavioral expectations (Lesson plan, February 24).

Kim capitalized the word "nice" to emphasize that the students could better express their ideas if they did so in a caring manner. Perhaps Kim had observed that her teacher was ineffective at engaging her students to discuss mathematics, so she was trying to engage them by developing community. In fact, one of Kim's goals for a student was to be nicer when he critiqued his peers: "So his critiquing needs to be more positively aimed" (Lesson plan, April 16, 2014). She reasoned that merely critiquing arguments is not enough; it should be done nicely. Kim had a strong focus of encouraging the students to be caring with each other as they discussed mathematical concepts.

Even though Kim had planned for students to be nice to each other, which would develop the classroom community, she reflected after her lesson that she had not achieved the goal:

> I would make the groups a little different, maybe try to have less in a group because some groups as I said, had difficulty being respectful to each other and I feel that if I more strategically put the groups together or made them with fewer people, less issues would occur in that area (Lesson plan, February 24, 2014).

These comments are more evidence that Kim valued having the students to be nice to each other. Even though she felt that she had made a mistake by putting too many students in a group, she reflected on how she could structure her class so that the students would be more respectful to each other in the future.

Not only did Kim expect her students to be caring, but also she was also caring with one of her students to encourage her to participate:

> If I make the expectation that everyone collects data, helps construct the graph, and tries to explain and help the others in their group by critiquing their ideas, she will be engaged in MP3. I will prompt her with a positive attitude and tell her personally to try her hardest and talk to her group if she does not get it, and then to ask me or the other teacher for help (Lesson plan, February 24, 2014).

It was important for Kim that one of her students participated in the group discussions. Kim hoped that she could get her student to participate by asking her kindly. By being kind to one student, Kim was developing the whole class's community as the other members could witness this kindness. Furthermore, Kim's expectation for the whole class to participate and work together also developed the classroom community. Expecting students to help each other by critiquing each other's ideas encouraged student mathematical discourse.

Apart from creating a classroom community, Kim used a similar social interaction strategy, "peer support," to encourage one of her students to participate in discussions: "To include MP3 beneficially, I will have C.M. in a group with math students who are leaders that will keep him on track and give him good examples to talk about" (Lesson plan, April 16, 2014). If Kim's host teacher had better developed classroom community, then it would not have been so important to strategically place students to ensure participation—all of the students should have had the social interaction skills to invite this student to participate in the discussions. However, Kim reported that her host teacher had said that the class community had not been developed because there was not enough time. Therefore, Kim strategically placed her student with a group of peers that she thought would support her and encourage her to participate in mathematical discussions. This social interaction strategy will be referred to as peer support.

In a homework assignment that asked Kim to reflect on math identity, she linked the concepts of mathematical identity and building classroom community. Kim wrote that all students, especially ethnic students, should be included in the community in order to express their mathematical arguments:

> It makes sense; if a student feels like they are part of their math class through a sense of belonging and community where they can celebrate who they are, where they are from, and how they perceive math, then they will be able to embody both an identity through math in the classroom and the skills necessary to understand math. This combination of math identity and learned math skills causes the student to have the motivation and understandings to be effective in learning math (or any subject for that matter). This applies to all students, like I said, but is most important with students in ethnic groups that are not the 'norm' so that they can feel like there is no 'norm' in the classroom (Reading reflection on mathematics identity, February 23, 2014).

Kim argued that teachers need to build ELLs' confidence in their ability to be successful in mathematics so that they will feel that they are part of the mathematical community. The teacher can improve students' self-efficacy by accepting students for who they are.

After reading an article about increasing elementary students' discourse in mathematics, Kim extended the concept in the article to ELLs. She continued to argue for welcoming ELLs into the mathematical community:

> I think that it makes so much sense [developing community] and goes along with everything we have been talking about: cultural relevance, being able to explain your own strategies and critique others respectfully, and getting motivation from engaging and more personal math lessons (Reading reflection on discourse, March 30, 2014).

Kim expressed that teachers need to give ELLs a positive mathematical identity and that this can be done by connecting with students and creating community; once ELLs have a sense of belonging, they will be more likely to justify their answers. She wrote how teachers can facilitate ELLs' engagement in Mathematical Practice 3 by including them in the community.

Kim planned to develop classroom community in order to increase student discourse. She explained in her lesson plan how she had strategically placed her ELLs with other ELLs so that they felt more supported: "I put each ELL with another ELL so they had someone who supported them positively as well" (Lesson plan, April 16, 2014). Kim prearranged putting the ELLs together to support each other because she felt that they were not part of the community. Kim realized the importance of the ELLs feeling supported as they were learning mathematical concepts and so she was concerned about how the other students treated them. Kim described how she took measures to improve a problem situation, and personally showed interest for an ELL:

> One of the kids who is from Japan loves to draw so I always make a note to ask him what he has drawn today. I feel like he doesn't feel included at all, apart from those other kids. There is one kid in the class that will play with them but he is still kind of mean about it (Second interview, April 16, 2014).

Earlier it was discussed how Kim felt that her host teacher was unkind to her students and that kindness was one of the ways that teachers could create community. She pointed out that the students were unkind to the ELLs. Kim realized that it is important for all students to feel a sense of belonging, and she paid special attention to one of the ELLs to make him feel welcome.

In an interview, Kim was upset as she told me how one of the students had made fun of one of the ELLs during her lesson. Kim said the ELL struggled with both the language and mathematics, but she was pleased that the student was attempting to persevere with her lesson. Unfortunately, after a student made fun of her, she stopped participating and cried:

> She wanted to do this. She had a marker in her hand; she was looking; she was ready and I put her in one of the worst groups by accident because one of the kids is an instigator and he said, "No, you don't know English. What are you going to write!" In front of her

> face and there was another girl in her group that backed him up and I heard the whole thing and came over. The girl started crying and I said, "I can't believe you would treat someone in your group that way!" And I brought her in the hallway to comfort her and I said, "Don't listen to them. Don't worry about what they say. They are going to let you do things. That's not fair; don't worry about what they say." I had to be really nice to her (Second interview, April 16, 2014).

Kim understood that she had to make the student feel part of the community before she could continue to engage in Mathematical Practices 1 and 3. This example depicts how Kim believed that to facilitate her ELLs', and all students' construct viable arguments and critique others, they should cooperate in groups.

In a homework reflection about teachers sharing discourse with students Kim said:

> I also believe that think-pair-share is important for ELL and all students, because it supports peer interaction which can benefit social interactions and also help students who are not as confident in answering questions by themselves, like ELLs (Reading reflection on discourse, March 30, 2014).

Kim argued that providing ELLs opportunities to work in pairs would help ELLs be part of the community and increase ELLs' mathematical discourse. This is an example of a PST suggesting peer support to facilitate ELLs' engagement in Mathematical Practice 3.

In her lessons, Kim tried to create a classroom atmosphere that valued all students' ideas, including ELLs', by having an ELL become part of the group and engage in Mathematical Practice 3. Instead of the ELL asking Kim or her teacher for help, Kim urged her to ask her classmates: "I also will have her in a group with some students who are high achieving in math so she gets the opportunity to listen to their ideas and get her talking about her own strategies" (Lesson plan, February 24, 2014). Earlier it was discussed how Kim took an interest in an ELL's drawing in order to make him feel better. However, in this example she attempted to encourage the ELL to justify her ideas in the group. Kim strategically placed her student to facilitate her engagement in mathematics.

Amanda. Amanda reported having learned about the importance of creating classroom community in previous methods courses. She explained what creating community meant to her: “Throughout the past couple of years, I have learned the value of classroom community, and that each student should be included as a valued member of that community” (Homework reflection, January 14, 2014). For Amanda, creating classroom community included valuing each student. Hiebert et al. (1997) call for classroom communities that value each student’s opinion, whereas Amanda described community as valuing each member.

Amanda’s early homework reflection adds insight to the connection she made between creating class community and student discourse: “Fostering positive relationships between all students in the classroom has visible benefits in terms of active participation during instruction” (Homework reflection, January 14, 2014). Thus, Amanda posited that students would be more willing to engage in Mathematical Practice 3 if the classroom community is developed.

Amanda tried to make the students feel safe when she told them that she would not get angry if they made a mistake. In an interview Amanda stated:

> So I was just guiding them with the questions and if they didn't answer them correctly I didn't say, ‘Okay you failed.’ I said, ‘Look at them again. Look at your yarn and the things you have available to reason about sizes (Second interview, April 28, 2014).

By reassuring the students that she would not get angry, Amanda endeavored to create an atmosphere where the students felt comfortable to make mistakes. According to Hiebert et al. (1997), teachers need to create an atmosphere where the students are comfortable to make mistakes so that the whole class can learn from the mistakes.

Amanda’s university supervisor said in an interview that she had instructed her students how to behave at the beginning of her lesson:

> She said when we work in groups, “How should we treat each other? Everything was stated positively; everything was stated positively and the kids came up with specifics

> about how to work in the groups and had to be positive (University supervisor interview, January 15, 2014).

This is another example of Amanda creating a safe atmosphere for her students.

Knowing how important it was for all of her students to participate, Amanda facilitated two students' collaboration with the rest of the group: "I made S.C. and R.K. social stories to prepare them for working in groups. I am proud that I really thought through different strategies that would help my focus learners be included in the group activity" (Lesson plan February 26, 2014). Just as Amanda had discussed her behavior expectations to the whole class, she also practiced with two students before the lesson. As a result of Amanda's intervention, the students were able to work with their peers. Amanda said that she was proud of how students had collaborated and discussed mathematical concepts together, which indicates that she valued this behavior.

Amanda explained how her goal was to have one of her students participate in a group discussion. She planned to do this by placing her strategically with other students that would support her: "I want to place B.S. in a heterogeneous group so that he can be supported by his peers and given a chance to participate since he often does his modified work separately from the rest of the class" (Lesson plan, February 26, 2014). Just as Amanda attempted to create strong community ties to make the students feel safe to engage in discussions about mathematics, she also used peer support to encourage a student's participation.

Another example of peer support to foster student discourse was written by Amanda in her lesson plan: "M.S. is a third grade girl who loves to be the center of attention. She is usually very helpful and friendly to R.K. I am thinking of putting them in a group to reason together" (Lesson plan, April 4, 2014). This is another example of Amanda placing the students together strategically so they collaborate with one another to discuss tasks.

Not only did Amanda plan for her students to collaborate, she also noticed if they had collaborated and reflected about this collaboration in her lesson plan after teaching: "They worked together to achieve a common goal. I was proud of how well they were encouraging each other and sharing the materials" (Lesson plan, April 4, 2014). Her university supervisor also commented in her evaluation that Amanda discussed collaboration with her: "Amanda reflected that the launch went well, that several of the groups worked really well together" (University supervisor interview, June 15, 2014). The university supervisor's comments supports that Amanda focused on students collaborating with one another when she taught.

Amanda said that she had received information about ELLs from her friend, Kim, who had eight in her placement:

> I think I was lucky because I had Kim and she would tell me her stories about her English-language learners like segregated from the rest of the class and people were so mean to them. How could this possibly happen? So, stories definitely helped me. I could see what it might be like but you really don't know until you have ELLs (Second interview, April 28, 2014).

Amanda reflected that the ELLs in Kim's class were treated unkindly. Although she did not experience working with ELLs, she had an opportunity to reflect on how to treat ELLs by listening to her classmate's accounts.

Lastly, Amanda experienced peer support with Kim in the field. In an interview Amanda said:

> Yes we collaborated so much. We had lunch together every day, we shared materials, and we talked about our lessons. It was definitely a good collaborative effort. I was lucky I was placed with her. I was literally right across the hall. I would go in there and she would talk to me. It was awesome (Second interview, April 28, 2014).

Not only did Amanda use peer support to encourage her students to feel comfortable, but she also got support from one of her peers.

Abigail. Abigail wrote in a homework reflection about the concept of creating a classroom community:

> I have read the Common Core Standards and use it a basis of student learning and accountability while designing instruction that keeps in mind differentiation, interactivity and community building. I am learning now about different disabilities (learning, speaking, physical) that may appear in my future classrooms so that I may create a classroom community that respects all differences where everyone is incorporated into the classroom productively (Homework reflection, January 14, 2014).

Abigail reported that she used the Standards as she planned her lessons. She attempted to have all students meet the Standards by developing class community, which meant to respect differences.

In a homework assignment Abigail wrote her views on discourse: “Students should work together to support; peer support and interaction” (Reading reflection on discourse, March 30, 2014). Abigail said that when students supported each other, they were more willing to interact with each other. In her Power Point summary at the end of the semester, Abigail asked her peers to discuss the importance of Mathematical Practice 3. She maintained that the discussions in the mathematical methods course: “allowed us to share ideas, brainstorm solutions, and notice the differences between districts” (Emerging theory presentation, April 22, 2014). Abigail made the association between the discussions in the mathematics methods course and their engagement in Mathematical Practice 3.

Despite advocating for students to support each other in a classroom community, Abigail described her placement differently:

> The discourse in my math community is very independent and worksheet focused. I haven't gotten a chance to observe a lesson where they are working on a concept; they mostly just do worksheets independently and can work with a partner. My teacher does flex groups with certain students that I guess changes daily, but mostly, independent quiet, no explaining. It's kind of just busy work (classroom refection, February 4, 2014).

This description contrasts with Abigail's earlier description of class community. She reported that the students tended to work quietly and did not critique each other's arguments.

This was Abigail's second placement and she compared the two schools in an interview; she explained that the two schools were different in regards to race: "Two totally different environments; two different sets of kids. Teachers. Everything. There were black teachers. There were black students [at her first school]" (Second interview, May 2, 2014). Abigail continued to maintain that the first school had stronger ties than the school where she had just been placed:

> Maybe it's a cultural thing because black people, we are more like a family. It's not like you go fend for yourself; it's more like a family environment. Everything is more helpful. In Smith School, [pseudonym] it's like you go and do what you have to do and go home. People don't see each other outside. You just don't see me outside of school. It's a different kind of environment. I'm thinking it's probably a cultural thing because black people are family-oriented or together and more close-knit rather than... (Second interview, May 2, 2014).

Abigail said that the second school was individualistic and she did not see the students after school. Instead of "community," Abigail used the term "environment" to describe the two schools. Abigail's description of the first school being helpful and like a family is similar to Amanda's description of a community being safe and comfortable to make mistakes.

Abigail continued to explain the difference between the two environments: "I'm not white, so I don't know, but I think white people is more, 'I'm going to do what I have to for myself'" (Second interview, May 2, 2014). Abigail argued that schools with high white populations might be more individualistic and less supportive than schools with high black populations. Abigail reflected on the two schools where she had been placed and concluded that one was more friendly and family-oriented.

In a written evaluation Abigail's university supervisor added more insight:

> For most of this placement Abigail appeared to be uncomfortable in this setting, and she did not readily engage in conversations with her host teacher or other colleagues.

> However, as time progressed she did become more conversant with her host teacher. For further improvement it would be helpful for Abigail to readily engage in professional discussions…The cooperating teacher has expressed that Abigail should be approaching her more frequently for an exchange of ideas (University supervisor evaluation, April 21, 2014).

The university supervisor's comments about Abigail being uncomfortable and not talking to her host teachers suggest that Abigail herself may not have felt like she was part of the community. Abigail reflected in a homework assignment that creating classroom community was a strategy for helping all students meet the Standards; yet in her lesson plans she did not mention how she would develop community as she was teaching her students fractions. Perhaps Abigail did not include developing community in the field, even though she had discussed it in the methods course, because she did not feel the community in her placement—exemplifying the importance of the placement and methods courses being in alignment. Furthermore, Abigail was in a placement with six ELLs. If a strong classroom community had been established, it might have been an opportunity for Abigail to experience ELLs developing mathematical arguments within their classroom community.

Charles. Charles briefly described his neighborhood community as he was growing up:

> I went to elementary school in the states in a white suburb. I started out in Chinatown and then I went to a white suburb but I maintained a connection with my community…It's pretty much a typical Asian American culture; academics is valued (First interview, April 23, 2014).

Just as Charles said that he had been part of a community growing up, he also believed in creating positive classroom communities. Charles described his opinion in a homework assignment about classroom communities:

> Building community is another strategy that I am learning in order to become an effective teacher. Through many of my classes, I have learned that an inclusive classroom community is key in order for effective learning to take place. This is because students will feel more welcome and comfortable when their classroom feels like a community, and they will become responsible for each others' learning by making sure to help each

> other when they need to. Therefore, one action that I am taking as a preservice teacher, is to learn many different types of community builders to help facilitate my future classroom community's growth (Homework reflection, January 14, 2014).

Similar to Amanda, Charles described the importance of students feeling comfortable in their classroom community. He mentioned students helping one another and being responsible for each other's learning. Lastly, Charles said that he believed in creating strong classroom community.

Parallel to Abigail, Charles also compared the classroom community at his two placements. He described the classroom teacher he had at the time of the interview: "I think there is community. It is much better than last semester where my teacher yelled all the time. My teacher was able to create community, especially since some of the kids come from rough backgrounds, so I'm very impressed" (Second interview, May 5, 2014). For Charles, class community was created because his present teacher did not yell at her students. Charles valued creating classroom community because he was impressed by how his teacher had managed to achieve this goal. He also mentioned when discussing community: "You have to know your students" (Second interview, May 5, 2014). Thus, not yelling at students and getting to know them were ways for a teacher to create a classroom community according to Charles.

Charles did not have an opportunity to teach ELLs in his placement, but there was a student whom he felt would benefit from ESL services. He said that she still had not acquired academic English. He explained how this student could carry on a conversation in the playground, but did not talk much in the classroom: "At first she was super quiet and didn't say anything. So you knew that she was quiet or there was a language issue and even now she is still very were quiet in the classroom" (Second interview, May 5, 2014). I asked how he could meet her needs and he suggested peer support: "Just having someone to talk with her. I could have probably paired her with one of the high achieving student" (Second interview, May 5, 2014). Charles argued that strategically placing this student next to a supportive peer would have helped her engage in conversation. Although Charles did not actually have practice working with ELLs, this comment suggests that he had knowledge of how to facilitate ELLs' engagement in Mathematical Practice 3 by creating peer support.

Emily. When I asked Emily about implementing Mathematical Practice 3, she mentioned creating community: "That's why it's so important at the beginning of the year to establish that community that we learned about in Block One [last year]. You need to make sure that the students feel comfortable saying that they don't know" (Second interview, April, 29, 2014). Like Kim, Emily related establishing community with being comfortable and added that if students were comfortable they would be more likely to speak up when they did not know an answer. She expanded on this idea in the same interview:

> I think that it's exciting because if you build a good community then the students are going to be able to defend themselves and explain why they put what they do and you can also incorporate having other students argue or critique and even though they might not agree it's going to make the confirmation of your answer even stronger (Second interview, April, 29, 2014).

Analogous to Hiebert et al. (1997), Emily maintained that if there were a strong classroom community, then the students would be comfortable developing arguments and critiquing each other. She also stated that discussing mathematics would allow students to learn concepts more deeply.

When in an interview I asked Emily what was challenging about Mathematical Practice 3, she again showed her belief that students will be more likely to share if strong community ties are established. Her response was:

> One of the difficult parts of it is that when you have a student who either isn't confident in themselves and their answer or they don't really know what's going on, and then you

> ask them to explain it to other friends, that's where you might lose students because they might say, “I don't know; you just tell me what you did, and I'll copy it down.” But that's why it’s so important at the beginning of the year to establish that community that we learned about in Block One. You need to make sure that the students feel comfortable saying that they don't know, so “teach me,” instead of just copying down what you put (Second interview, April, 29, 2014).

Again, Emily explained her belief that if there is a classroom community developed then the students will feel more comfortable to engage in Mathematical Practice 3.

Emily mentioned establishing a classroom community when she described mathematical pedagogy. She argued that it is preferable to give students one rather than many mathematics questions:

> You don’t have to practice it over and over and over again…what good does that do them? It makes students stressed because they only have one minute to do the problems and it will make students feel bad about themselves… I don't see the point of that when you can build community and build your support and language skills by just having one question (Second interview, April, 29, 2014).

The comments above depict how Emily valued creating a classroom community and developing students’ language. Emily was making pedagogical decisions based on how well the curriculum developed class community and language—she was not in favor of giving students multiple questions because she did not want the students to feel bad. For Emily, having fewer questions for students allowed the students to feel good about themselves, consequently creating a community, which facilitated student discussions of mathematics.

In Emily’s lesson plans she depicted her behavior expectations for her students:

-Be respectful

-Be ready to learn

-Be responsible

-Listen to your partners

-Share your ideas (Lesson plan, February 12, 2014).

Emily's five rules described how she wanted the students to behave. She planned for the students to be respectful and share their ideas. It stands to reason that Emily planned for her students to be respectful in order for them to better develop and critique each other's arguments.

One example of peer support promoting students to discuss mathematics was written by Emily:

> They also were allowed to converse with each other about how to approach the problem, how to solve the problem, and then what to do after when I gave them an extension. By using peer support, they were able to come up with multiple strategies of how to find halves and fractions. This helped students understand the content because they were able to see how to solve in different ways in case they were confused on one way (Lesson plan, April 2, 2014).

Emily implied that as the students supported each other, they were able to discuss multiple strategies and understand the concepts deeper.

Emily had a student who was learning English although she did not receive ESL services. She wrote about her accommodations for this student: "She had the opportunity of drawing instead of writing her responses, getting peer support, and also having support from the adults in the classroom…her peers were able to describe different problems to her and how she could do them" (Lesson plan, April 2, 2014). Emily took advantage of peer support to help her student discuss mathematical concepts.

Fatima. As I discussed in Section 4.1 Fatima considered herself an intrapersonal learner and enjoyed working in groups. Fatima mentioned learning about community in the field; she said in an interview:

> F: Last year we talked a lot about the importance of building community. In IDE [Instructional Design in Education] we discussed that. Last semester I was in a placement that did not have a community and this semester I was in a placement where they did have community.
> J: So you talked about it and you experienced it?
> F: Yes. I also think it was important to see how not having a community does not help (First interview, April 24, 2014).

In addition to learning about creating classroom community from previous methods courses, Fatima pointed out that she had become aware of the importance of building classroom community when she was in a classroom where she deemed that it was missing.

When I asked Fatima about the Mathematical Practices she discussed creating a community:

> I think it's [creating a community] really important to help them to get each other to understand the problem. Also you use social skills and create relationships when you have to talk about a problem. That's a huge part of building community. It's weird because you need a community in order to do it well. It's important to have a community before you can critique each other without it feeling like an insult. Then at the same time, when you're critiquing each other's reasoning, you are creating relationships and you're helping to build the community because you're helping each other learn (Second interview, April 28, 2014).

Fatima connected Mathematical Practice 1, Mathematical Practice 3, and creating a community. She argued that students could make sense of problems when working together and that not only was it important to have a strong community in order for students to engage in Mathematical Practice 3, but as they engaged in Mathematical Practice 3, they were also developing community skills.

Fatima noticed that a student was relying on her or her host teacher to help her through the work. Fatima's goal was to have students learn from each other instead of from the teacher:

> I had one student who would look at the problem and stop. I would come over to her and asked her and helped her at first, but realized she could do the problems on her own or with peer support. She wouldn't do it unless I was there. It was hard for me because I wanted her to persevere through the problem, but she wouldn't unless there was some teacher support (Second interview, April 28, 2014).

Thus, Fatima used the term peer support when the students helped another student solve a problem rather than relying on the teacher to solve it. She explained how peer support helped another one of her students: "She also benefits from peer support and explanation, which helps her understand a way to solve the problem" (Lesson plan, April 14, 2014). Fatima implied that students learned concepts more deeply when they support each other to solve them rather than the teacher explaining how to solve the problems.

Fatima did not have ELLs, but one of her students was bilingual. Her university supervisor said: "She did a better job organizing the groups so that this little guy who was new to the country; he was with a group of kids that were going to support him" (University supervisor, July 9, 2014). The university supervisor pointed out that it is important for PSTs to know their students well and that Fatima had used her knowledge about her students to make this student feel comfortable and supported in his group.

Maria. Maria described in a homework assignment how she was part of a community on campus: "I enjoy being a part of the Latin@ [Latino] community and being surrounded with things that associate with my culture" (Mathematics autobiography, January 15, 2014). Growing up, Maria also felt part of her family community as she shared in an interview:

> Family and culture are two big things I think when you incorporate as much culture as my family did in my household that is why I mentioned it. Maybe someone who is from... I don't know with white people it is different. They don't incorporate as much of their culture into their lives. I am surrounded by different foods, my family members, like the physical artifacts; my house has everything; different games that we play and everything (Second interview, April 29, 2014).

In regard to classroom community, Maria shared in a homework reflection how she thought it was important: "I strongly believe that a teacher's role in the classroom is to create a safe environment that nurtures students' growth by helping them think critically, independently and compassionately" (Homework reflection, January 14, 2014).

Maria described in a homework reflection how she had developed her ideas about creating strong classroom community by observing her host teacher:

> Building community is an essential part in student learning I feel that I can create quite well. I'm a very flexible person so when I see something being done I can easily adapt to it. For instance, our cooperating teacher has set up a distinct feature during her morning routine. Every morning there is a small community builder, but little do the students know that these are used to build their social skills and be comfortable with one another as a community. I feed off this already established routine only letting it continually grow and keep growing. My actions to build community include partner work, group work, asking your classmates for help then asking the teacher, and simple good mornings when the students enter the room. This helps unify the classroom and have things work efficiently to reach the desired goal (Homework reflection, January 14, 2014).

Similar to the other PSTs, Maria also explained that students felt more comfortable when strong classroom community ties were established.

In her homework reflection Maria applied the concept of developing classroom community to being culturally relevant:

> To unite culture in the context of mathematics, it is imperative for teachers themselves to learn about the culture of their community or their students. It doing so, they can support their ideas and give purpose and meaning to any context area. Culturally relevance can also be tied with MP1 by trying to solve problems in different ways. For instance, as teachers we should be using various teaching strategies that can be successful with certain ethnic groups, such as small-group and cooperative-group learning (Reading reflection on cultural relevance, January 22, 2014).

Maria pointed out that teachers should learn about students' cultures to include all students in the classroom community. She also stated that ELLs tend to learn better in small groups (Coggins, 2014).

Maria continued to describe in a homework assignment students feeling comfortable in classroom communities:

> A teacher must begin [to] include how math was like from their own country. With this information ELLs can share how math is the same and different in separate countries. This makes ELLs feel included and comfortable in the classroom community. I think the focus for all learners to have a math identity would be to begin with the teacher explaining their own math and even life experiences (Reading reflection on mathematics identity, February 23, 2014).

In Section 4.1 I discussed how the PSTs described connecting with students by teachers making personal connections. Maria suggested that teachers build the classroom community by sharing their own personal experiences of learning mathematics and afterwards inviting ELLs to do the same.

Sarah. In one of Sarah's homework reflections she outlined her views on classroom communities and mathematical discussions:

> The inclusive program has taught me the importance of building community in the classroom. The students should be in a classroom where everyone feels safe, supported, and accepted. This helps students to learn in a positive environment where they can participate and be included. A way to build community is to have the students learn about social skills and being friendly. I think it is also important for the students to talk about each other's interests and facts to discover commonalities and differences. The learning environment should respect and value everyone's culture so that the students could feel confident and safe to be who they are (Homework reflection, January 14, 2014).

Sarah shared that she had learned about building communities from the teacher education program. Sarah uses words such as "comfortable," "friendly," and "safe" to describe community.

In the same reflection Sarah broadened her description of community to include diverse classrooms:

> Diverse classrooms should involve community builders as a strategy to learn about each other's cultures to make everyone feel accepted and welcomed. From my own experiences, I am more open to participate when I feel comfortable amongst my teacher and peers. I have learned in school and witnessed in my field placements that many students feel the same (Homework reflection, January 14, 2014).

Sarah's goal was to build community by having all of her students feel comfortable, accepted, and welcome.

Sarah and four more peers taught a lesson on fractions to the rest of the class. After she taught she wrote:

> In small groups, the students discussed different ways to create the same fraction. People had agreements and disagreements and they explained why it worked and how they did

> the solution. As a whole group, we discussed the different strategies and forms of fractions…I found that this lesson was productive and the students were constantly engaged in small and whole group (Lesson critique, January 31, 2014).

In Sarah's essay she explained how the PSTs did not all agree with one another's views, but the peers were able to explain their ideas and they were still engaged. Hiebert et al. (1997) maintain that teachers should have the expectation for students to value each other's ideas and that they have opportunities to learn from each other. While Sarah did not specifically say that the social interaction was positive while the PSTs were discussing mathematical ideas, she said that the lesson was productive.

An example of peer support is from Sarah's lesson plan. She wrote how she planned for students to work with their peers supporting one another:

> I will be making homogeneous [heterogeneous] groups so that the students are mixed based on their abilities. This way, when I have the students work together, there is a lot of peer support, which is especially beneficial for the students who are struggling…I also encouraged peer support so that students can work together to help each other learn (Lesson plan, April 11, 2014).

For Sarah, peer support involved students working together cooperatively, which was especially valuable for the "struggling students." Sarah said that she would encourage peer support, which indicates that it is something that she valued.

Kim and Charles spoke of pairing ELLs with non-ELLs, but Sarah offered a different suggestion. Sarah believed in the importance of having students, including ELLs, support each other. She wrote in a homework reflection: "Peer support is very important in math. Students can help model and teach each other. It is especially great if there is a bilingual student to sit next to the ELL that can help" (Reading reflection on ELLs, February 19, 2014). Sarah pointed out that peer support is useful for all students, especially ELLs. She stated that ELLs could support each other by allowing access to the content through translation.

Sarah also argued for ELLs to be in small groups as opposed to large: "This style is beneficial to ELLs because these students do not benefit from lecture, whole group, and teacher-talking lessons. These students need peer support, hands on learning… Every student is responsible for explaining an answer" (Reading reflection on discourse, March 30, 2014). Sarah maintained that all students, especially ELLs, benefit from supporting each other in small groups and justifying their answers.

Summary of Social Interactions

Table 8

Summary of PSTs for Social Interaction[1]

Name of Participant	Provided Social Interaction?
Abigail	Strong
Amanda	Strong
Charles	Strong
Emily	Strong
Fatima	Strong
Kim	Strong
Maria	Strong
Sarah	Strong

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

The PSTs discussed two types of social interaction. They had learned about the first, establishing community, in the mathematics methods course, in other methods courses in the education program, and in the field. There was general agreement that creating classroom community was important and teachers could create community by being respectful, intimate, friendly, not shouting, valuing students, and making students feel safe. Emily noted a connection between creating class community and students engaging in Mathematical Practice 3. She stated that when there is a strong classroom community, students are more likely to participate in mathematical discussions.

In homework reflections in the mathematics methods course, Abigail wrote about the importance of creating classroom community. She also said that she had experienced being part of a classroom community in her previous placement. However, she was not part of the classroom community in her placement in this study. Mewborn (2003) believes that teachers are more likely to apply pedagogies to their students when they have previously experienced these themselves. Abigail had experienced being part of a community in the teacher education program and in her previous placement, but perhaps she would have learned the importance of creating classroom more deeply if she had been part of the classroom community. The implication for teacher educators is that it is important to ensure that field experiences reinforce practices that are valued in the teacher education program.

The second social interaction, peer support, also involved the students feeling comfortable, respected, and feeling supported. In addition, the PSTs used peer support to help students learning English to construct viable arguments and critique others. Kim felt that the ELLs in her class were not part of the community, so she used peer support in order to encourage the ELLs to discuss mathematics. Pinnow and Chaval (2014) maintain that it is vital for ELLs to be positioned strategically to help them develop arguments. They advocate developing ELLs' social competency so that they will feel comfortable in participating in meaningful mathematical discussions. Kim planned to position the ELLs in her classroom with students that she thought would help them develop arguments. As Kim suggested, there may not be sufficient time for PSTs to establish classroom community, but using peer support to encourage students, such as ELLs, to engage in Mathematical Practice 3 may be a viable alternative.

One of my research questions is to examine how the experiences of PSTs learning to teach elementary students, including ELLs, are similar or different. In regard to beliefs and practices of social interaction, I found the PSTs to have similar views. They mentioned that social interactions such as creating strong classroom communities and peer supports would encourage students to participate in mathematical discussions. One possible explanation for the similarities among the PSTs is that they had discussed and experienced being part of a community throughout the teacher education program. I assume that the PSTs also discussed peer support in the teacher education program— I did not discuss it in the mathematics methods course and yet all eight of the PSTs mentioned this term. Therefore, as the concept of social interaction was infused throughout the program, the PSTs had similar beliefs and practices.

5.2 Language Development

Mathematical Practice 3 states that students should construct viable arguments and critique arguments of others. According to Stein and Smith (2011), many students are not accustomed to developing mathematical arguments because traditionally the teachers have dominated the discourse. Students benefit from having a better command of mathematical language to argue and critique mathematics (Stein & Smith, 2011). With the growing population of ELLs (Lucas, 2011), it is also important for PSTs and teachers to develop ELLs' English and native language so that they can engage in Mathematical Practice 3. According to Shatz and Wilkinson (2013), teachers need to teach students English as well as content in all subjects.

I observed in the surveys that all of the PSTs except Charles described how they would provide access for ELLs to solve mathematics, but did not describe developing ELLs' language. This omission is similar to de Jong and Harper's study (2011) that concluded that PSTs focus on providing access to content for ELLs more than developing their language. De Jong and Harper assert that providing access is not enough—teachers should develop ELLs' language as well. After observing how the PSTs developed non-ELLs' language by using strategies such as sentence frames, I included the sub-themes "Developing Mathematical Language" and "Developing English for ELLs." The first concentrates on developing mathematical language to all learners and the second is developing English specifically for ELLs. According to Moschkovich (2007), teachers should also develop ELLs' native language, but that will not be included in this study as it was not present in my data.

Developing Mathematical Language

Larson et al. (2012) explain that the intent of Mathematical Practice 3 (constructing viable arguments and critiquing arguments of others) is for students to "make and test conjectures and to communicate their mathematical thinking" (p. 40). According to Zwiers (2008), since today in mathematics we are requiring students to justify their thinking, we also need to help them develop their mathematical language—language used when communicating in mathematics. Schleppegrell (2010) adds that mathematical language is dense but that acquiring this language is necessary for students to be successful in the subject. Although the PSTs did not mention the importance of developing students' mathematical language, they discussed strategies that may develop language. Because of the connection to Mathematical Practice 3 I have chosen to include this theme and will analyze PSTs' beliefs and practices that are related to it.

Developing ELLs' English

In order for ELLs to be successful at developing arguments and critiquing others (Mathematical Practice 3) in predominately English classrooms, teachers need to help ELLs learn English (Lucas, 2011). Therefore, this section will include the PSTs' beliefs and practices about developing ELLs' English. Of course, there is an overlap between this section and the previous because ELLs, like all students, also need to develop their mathematical language. However, this section is about PSTs' developing ELLs' English because it is especially challenging for ELLs, who lack English proficiency, to engage in Mathematical Practice 3. Because of this challenge, Murrey (2008) reasons that teachers should not only develop mathematical concepts in ELLs, but also develop their English language skills. As was true for developing mathematical language, the PSTs rarely focused on the importance of developing ELLs' English skills, but I have chosen this theme because again there is a connection to Mathematical Practice 3. I will analyze PSTs' beliefs and practices that relate to developing ELLs' English.

Charles. Charles was the only PST to write in a pre-survey (See Appendix D) that ELLs should not be allowed to speak in their native language during mathematics class: "No, because this promotes dependency. It is important to respect students' culture and promote their use of language while outside of the class, but students should continually practice the language that they are learning while in the classroom" (Pre-survey, January 14, 2014). Charles argued that teachers should encourage ELLs to speak in English throughout the day in the classroom. In his post-survey, in response to the same questions, he said that ELLs could "only speak their language when they are truly struggling with English. However, if their English is simply poor but still good enough to be understood they should use it so that they become more fluent" (Post-survey, April 22, 2014).

Charles argued for ELLs to speak in English to improve their ability to do so, even if it was challenging for them, whereas the other PSTs had a stronger focus on providing ELLs access to the content. De Jong and Harper (2011) argue that general education PSTs typically undervalue the importance of developing ELLs' English. In a survey they found that most general education PSTs thought it was their job to provide students access to content, but did not feel that it was their job to develop ELLs' language—that was the job of the ESL teachers. Charles is the only PST who focused on developing ELL's English more than providing them access to the content.

I will examine Charles' past experiences to determine why Charles might have had a different approach about teaching ELLs to the other PSTs. In an interview, Charles reported that for three summers he had worked in his church school voluntarily with ELLs living in Boston who mostly spoke Mandarin and were learning English:

> A lot of the students there only speak Mandarin Chinese so I've had a lot of experience going back and forth with the language…That's why I emphasize speaking English in class because if they don't then they won't learn English. …That is one of the mistakes that I made because in the beginning I would speak Chinese to them and then they would never speak to me in English. Later I learned to tell them that I would not listen to them unless they use English, even if it was broken English…I told them that I wanted them to learn English. I think that is a better approach. Again because I don't want them to become dependent on Chinese because they could get away with it for their entire lives if they stay within the Boston Chinatown area. But once they get out [of Boston] they are not going to be able to communicate with people because they have been using Mandarin Chinese the whole time (First interview, April 23, 2014).

As a result of Charles' growing up with Mandarin Chinese students and teaching them, he believed that it is important for teachers to focus on teaching ELLs English. Charles learned English and he wanted ELLs to learn English as well.

This experience of working with a population of predominantly Chinese-American students and teachers is different from that obtained in schools that often have teachers who do not understand ELLs' language or culture. It is not the purpose of this dissertation to discuss whether it is best practice to encourage students to speak English in schools similar to where Charles had been teaching, but Moschkovich (2007) maintains that teachers should allow ELLs in public schools to speak their own language in order to have access to the content and to develop their own language. Therefore, while Charles may have found it useful to speak English exclusively at his summer program, if Charles were to be assigned to teach ELLs in public schools, it could be useful to allow ELLs opportunities to speak in their own language.

However that may be, Charles had a stronger focus on developing ELLs' language than did his peers. In an interview I asked Charles why he was the only PST to require ELLs to speak English in school. He reasoned that he had a different opinion because of his background:

> It's based on experience. Because I have grown up in the Asian-American community in Boston, I've heard stories of people who come from China and because they are so dependent on Chinese, they don't learn English. For example, there are students who come from China and go to the two public schools and because most students speak Mandarin or Cantonese, now most students coming from China speak Cantonese, they can use their Cantonese without learning English. Everyone in Chinatown speaks Cantonese or Mandarin; they could survive in that environment and never learn English (First interview, April 23, 2014).

Charles stated that it was important for Chinese ELLs to learn English at schools, so he advocated teaching English in all subjects, including mathematics. This is an example of Charles making a decision about ELLs based on his experience growing up as a student.

As the conversation continued, Charles brought out the advantages of teachers' learning another language. I asked Charles how his experience of being bilingual helped him in the mathematics methods course:

> I think it gives me an idea of what the students don't have…some students they go, "Oh, because I'm teaching ELLs." Because they are not bilingual, they only see one side of it. "Oh I can make some translations, I can use some visuals, I can use peer support, and I can differentiate different material for these students." But because they don't have that skill they don't realize, "Hey, maybe I could do some of the explaining in another

> language to help guide my students or have my students say something to me in another language and help them to say that again in English." That's an aspect that if you are not bilingual, it's not possible for you to do unless you know the language that the student is speaking (Second interview, May 5, 2014).

There were opportunities that Charles had with his Mandarin ELLs that monolingual PSTs could not have had. When he mentioned that he could explain concepts to his students in Mandarin, Charles was focusing on the students' access to content rather than developing their language. However, Charles also said that the students could speak to him in Mandarin and then translate into English—the purpose of having the students speak in English was to develop their proficiency in this language.

Charles continued to discuss the positive aspects of being bilingual. Charles recognized that learning content in another language is challenging because he experienced it himself. However, he argued that it was a challenge that the ELLs should have and that he was better prepared to help Mandarin ELLs with grammar because he is bilingual:

> C: Another perspective is understanding the differences and the hardships that the students are going through when they are learning English. For example, when I taught in the organization where I am at; last summer I realized that one of the biggest issues with Chinese American students who just came from mainland China is the tenses. They struggle a lot with the tenses. They don't know past tense, present tense, future tense because in Chinese there are no tenses. So that is something that I realized when I am teaching them. "Oh you guys need to know your tenses." That's something I hammer into them. I'm trying to think of other examples.
> J: Tell me more. That is interesting. In Chinese you don't have articles, right? "The" and "a?"
> C: Yeah, no articles but that's an example... because I'm bilingual, I recognize these struggles that the students go through, so if they are writing something and they don't have the correct tense, maybe for someone who doesn't speak Chinese they will think that it's because they don't know how to spell the word but with the background that I have, I know that they never grew up speaking a language that has tenses. Therefore, that's why they don't identify that that's a problem that they have.
> J: So you spend some time addressing that?
> C: Yeah. It gives you more perspective of why the students are struggling or what aspects the students are struggling with.
> J: That's interesting. Can you think of more examples?

> C: Pronunciation. So if I say “fifth.” I even make that mistake. I say “fith” instead of “fifth.” That's a pronunciation error but that is a small detail. For example, if we were talking and I said that I came in fifth place you would never realize that I didn't pronounce “fth” I said “ith” instead (Second interview, May 5, 2014).

Charles realized that knowing Mandarin helped him with Mandarin ELLs. As Charles had experienced making mistakes with past tense verbs and pronunciation of certain phonemes himself, he was able to focus on these areas with his students. This is an example of how Charles’ experience of being bilingual allowed him to better meet ELLs’ needs. De Jong and Harper (2011) explain that parallel to Charles’ comments; mainstream teachers need to have knowledge about linguistics so they can develop ELLs’ English in the same manner as ESL teachers.

Not only did Charles have an advantage working with Mandarin ELLs because he understood the difficulties that Mandarins typically have when learning English, but he also felt that being bilingual helped him be more empathetic with all ELLs learning English, especially the Mandarin ELLs:

> I think it gives me an understanding of why they struggle with the language sometimes because growing up I struggled a little bit with the language. But then again I feel like it gives me a closer connection with ELLs than someone who is not bilingual, but it's not as close as it would be with a student who spoke Chinese (Second interview, May 5, 2014).

Because of being bilingual, Charles asserted that he could develop ELLs’ language in ways that his monolingual peers could not. It was easier for him to identify with Mandarin ELLs, but he also thought that he could connect with other ELLs because he had a common experience of learning English himself. Harper and de Jong (2004) maintain that ELLs do not acquire proficiency by mere exposure to English; teachers need to address their specific linguistic needs as well. Charles appeared to be aware of the importance of meeting such needs because of his experiences being bilingual and teaching ELLs.

Charles's experience of teaching ELLs may have helped him analyze practices in the field. Charles learned that one of his students was no longer receiving ESL services, but he said that she had not acquired academic English. He questioned the practice of treating this student as if she had a disability instead of assisting her with the language:

> She clearly needs ESL services…The Special Ed teacher would pull out students that struggled with reading and she was included in that group…but for her, she never had a disability. Does that count? If you are an ELL that's not a disability, is it? She didn't have a disability but she was still being pulled out and with kids that did have disabilities…for all we know she may be a high achiever. Language is the issue, and sadly she's being treated like student with a disability. What is that doing for her self-esteem? (Second interview, May 5, 2014).

This is another example of Charles focusing on the importance of developing ELLs' language. In this case, Charles argued that the student's lack of access was a result of not being proficient in English. Charles argued to develop the student's English in order to have more access to the content. Furthermore, Charles considered the importance of the student's self-esteem or disposition.

Charles described how he helped this student:

> I spent a lot of time sitting next to her and reading to her to see how she would respond. I spent a lot of time with her and I used a lot of visuals. I would ask her to explain what she was doing and she would try using her limited English words (Second interview, May 5, 2014).

Charles used scaffolding techniques such as visuals as well as proximity to develop the student's English.

Harper and de Jong (2004) reveal that the common misconception that ELLs can be taught the same way as non-ELLs is one reason why mainstream teachers are not learning how to develop language skills for ELLs. Teachers need to have specific teacher development and dispositions for instructing ELLs (de Jong and Harper, 2011). Unlike the other seven PSTs, Charles had a strong belief that teachers need to develop ELLs' English. Charles might have acquired this disposition partly from his background, as he himself experienced learning English as a second language.

Maria. Maria's parents were from Peru; she wrote that she understood Spanish, but felt uncomfortable when I spoke to her in Spanish:

> I was taken off guard when you started speaking Spanish, so I kind of felt in the hot seat. Sometimes I feel that my Spanish isn't very good… My mom tells me *'saluda a tu abuelo'* [say hi to your grandfather] and I'm walking down the steps to go see him because he lives downstairs and I think what to say in English and translate it. That's where I struggle. It's hard for me to have a conversation. I think in English when I should think in Spanish (Class reflection, January 15, 2014).

I asked her how language ties into culture and identity and she said:

> It has a lot to do with who you are. At times I feel like I'm embracing all about me when I speak my language. When I'm not I still feel like I am because I have something different to offer to other people who have different cultures (First interview, April 24, 2014).

In this example, Maria was proud of her language and culture, but later on in the same interview she used the word "deficiency" to describe her culture. She wanted to make sure that other people knew that she was not born in Peru: "I don't know, just to differentiate the two because if someone asks where you're from and I say I'm from Peru they will think that I have this deficiency. That I am illiterate or something. I don't know" (Second interview, May 5, 2014). I asked Maria to explain and she said, "I don't know. It's different like I can't speak or write in English or something" (Second interview, May 5, 2014). Lucas (2011) proposes for PSTs to learn another language to develop a positive disposition toward teaching ELLs. Maria understands a second language and at times feels proud of her language, but she also made a comment about not wanting to be classified as being born in Peru so that people won't think that she is illiterate.

In the pre-survey Maria wrote the most important thing for meeting ELLs' needs is "making them feel comfortable in the classroom." In the post-survey she wrote: "Incorporating their culture into the math context will make ELL's feel more included into the classroom and feel less isolated" (Post-survey April 22, 2014). Thus, unlike Charles, Maria had a strong focus on providing ELLs' access to the content, but did not mention developing language.

I did not find data in regard to Maria developing mathematical language or ELLs' English.

Abigail. One PST who appeared to have a negative disposition towards ELLs was Abigail. Although Abigail said in an interview that she had had experience listening to Spanish while working, she wrote that she did not speak any languages besides English. I discussed in Section 4.3 how Abigail had low expectations for ELLs and referred to them as "language struggling." There are more examples of Abigail having low expectations for ELLs, specifically in regards to language. Abigail shared in an interview how language affected ELLs when they did mathematics: "You have to know the language in order to write a language. If you don't know how to write or spell then it is kind of hard, especially if you have to understand the question" (Second interview, May 2, 2014). This comment about not being able to spell or write may not have been accurate because the ELLs may have been able to spell and write in their own language. De Jong and Harper (2011) maintain that many teachers judge ELLs with a monolingual lens and focus on what is missing from ELLs instead of acknowledging ELLs for what they do well. Abigail appeared to be focused on the ELLs' lack of English proficiency rather than acknowledging them for speaking another language (Moschkovich, 2013).

Abigail continued referring to ELLs' problem of not being fluent in English and suggested "fixing" the problem by teaching English to the parents:

> The root cause of English language learners is not the students; it is the parents because they don't have the background language at home and so if they speak some other language at home they are definitely not going to be practicing English at home. I'm not saying it is wrong for the parents not to speak their language. It is great that they still

> know their home base language because that's part of their culture, and you can't take that away from them but that's the missing piece. So when students are able to practice their language at home and bring it back to school, that will help them succeed at school when they go home and have a totally different language and forget about English and only pickup English from 8 to 3 PM, they are not really practicing their language [refers to English as their language]. That's the hard part and that's what keeps them behind rather than moving forward. Personally that's what I think…maybe some English classes for the parents…teaching them [parents] how to incorporate into different languages so that way the students are practicing language at home. That's a whole another problem (Second interview, May 5, 2014).

Abigail acknowledged parents speaking their own language with ELLs, but her comments about the students not practicing language at home unless they were speaking English is another example of her monolingual disposition—she suggested that she could “fix” ELLs’ language problem by teaching English to the parents. According to Lee and Suarez (2009), it is important for ELLs to learn their own language because it promotes more adjusted students as well as developing their language. Rather than over-emphasizing ELLs’ lack of English proficiency, teachers can make ELLs feel more welcome, according to Lucas and Villegas (2011), by showing interest in their language. By recommending that parents speak English to ELLs instead of their heritage language, Abigail failed to appreciate the importance of offering ELLs opportunities to develop their first language.

In the comments below, Abigail spoke of how it was easy to accommodate one ELL and include their language in the lesson, but it was not possible with six ELLs in the classroom:

> Often times I've felt that a lot of English language learners in the classroom are like ignored. Especially when you have more than one because it's easy to differentiate and incorporate their language in the classroom, but when you have like six different languages in your classroom and you don't know the background of all of them, it's kind of hard to incorporate all the languages. That's going to be a lesson in itself rather than you teaching actual content. Trying to incorporate everyone's language. More times I felt that they were ignored and you have to try and play catch-up constantly rather than... And having to decide if they're going to get pulled out to have their ESL services or are they going to stay in the classroom and challenge themselves constantly so... (Second interview, May 5, 2014).

Once again, Abigail's comments depict that she viewed the ELLs as a problem and the more ELLs she had the harder it would be to teach them. She did not consider developing ELLs' language so that they could better participate in mathematics. Furthermore, by saying that she would not know the background of the ELLs she implied that it is harder to get to know ELLs than other students.

There were more examples of comments made by Abigail that suggested that she had a negative disposition for ELLs. I asked Abigail to say how it was possible to facilitate ELLs' engagement in Mathematical Practice 3:

> It's not, because if they don't understand the language then how are you going to feed off another person? Or the other person is somewhat stronger than them in the class and they say the right answer rather than explaining. [This happens] especially with younger kids (Second interview, May 5, 2014).

Even though Abigail had discussed strategies throughout the mathematics methods course for how to facilitate ELLs' engagement in the Mathematical Practices, Abigail said that without understanding the language [English] then ELLs cannot engage in Mathematical Practice 3. Abigail did not consider that ELLs could develop arguments by speaking in their own language, using gestures, and/or using manipulatives; nor did she consider that she could develop ELLs' English so that they would be more successful at engaging in Mathematical Practice 3 in the future.

In a homework reflection, Abigail wrote that teachers need to consider ELLs' language proficiency:

> ELL's have a math identity because not only are they learning mathematical skills, they are learning the language that coincides with it and English language as well. Their math identity is defined by them as a math student and as an English language learner. For them to be successful in the math world, a teacher must consider how they are as a math student and their level of English proficiency and comprehension in order to successfully

> accommodate to their needs and actually learn the necessary material (Reading reflection on mathematics identity, February 23, 2014).

Similar to all PSTs except for Charles, Abigail focused on providing ELLs with access to the content more than developing their language. Even though Abigail mentioned ELLs' English proficiency, her focus was on making appropriate accommodations so that the ELLs could have access to the content.

The only data that I found related to Abigail developing ELLs' English in the field were from an interview. Abigail said that she used manipulatives with ELLs: "That was very helpful to them; they were able to make the composite shapes with manipulatives" (Second interview, May 5, 2014). The comment suggests that Abigail used a scaffolding strategy, manipulatives, to help her ELLs develop their English.

Amanda. I reported in Section 4.4 that Amanda had learned the importance of asking questions to facilitate students' engagement in productive struggle. Amanda was guided on her lesson plan template to ask her students questions. She wrote: "Michael has 6/8 and Brandon has 4/8. Which one is greater based on your yarn pieces? How do you know?" (Lesson plan, February 26, 2014). Amanda not only asked for the answer, but also wanted the students to develop an argument to justify their answers. By asking students to justify their answers, Amanda may have developed their mathematical language. Mathematical Practice 3 states that students should ask their peers questions to understand and strengthen each other's arguments (CCSSI, 2010); teachers can set an example for students by asking questions themselves (Stein & Smith, 2011). Lampert (1998) adds that teachers can develop students' mathematical language by asking them questions.

Apart from questioning her students, Amanda also helped develop her students' mathematical language by using a scaffold. In order to help all of her students, including her special education students, Amanda used a sentence starter in one of her lessons: "I will encourage them to say: "____ will get _____ out of ______ pieces of the snowflake/ pieces of cake/ sections of the treasure chest" (Lesson plan, April 4, 2014). Without this aid, some of the students would not have been able to participate in the mathematical discussions. Coggins (2014) explains that the goal of the class should be to hear all students' arguments. She says that sentence starters not only support certain students to participate in whole class discussions who ordinarily would not be able to do so because they lack the mathematical and English proficiency, but sentence starters can also be used in small group discussions to help all students feel that their ideas are valued. According to Ramirez and Celedon-Pattichis (2012) sentence starters develop students' mathematical concepts and language. Therefore, Amanda's use of the sentence starter was consistent with the literature about developing students' mathematical language.

Amanda wrote in her pre-survey that she only spoke English and she had had no training, but she had had some experience with an ELL in her first placement. In an interview I asked Amanda to describe her initial experience with this ELL:

> At first I was confused because I really had no training. I never talked about them [ELLs], and I was talking to one kid one day and another kid came up in the class, and said, "He doesn't speak English," and so I was gesturing and actually realized that whenever they talk to him, they called over the other student, and I wasn't sure how I felt about that. I said, "How can I reach this kid without always calling over this translator who was luckily willing to help, but at the same time I needed to reach him some other way so I really didn't know I do that (Second interview, April 28, 2014).

Amanda shared that she only spoke English and she had not learned about ELLs in the teacher education program, and yet she tried to experiment with pedagogy that was different from the class norm. The students were used to communicating with this student by solely using a translator, but Amanda tried to use gestures.

Amanda also discussed scaffolding with ELLs. Below is an example of how in the mathematics methods class Amanda learned pedagogy for supporting ELLs even though, like all but two of the PSTs, she had no ELLs in her placements. Abigail had shared how she was struggling to get one of her ELLs in the field to participate in class discussions, and Amanda commented on that discussion in reflecting on an article assigned for homework as she wrote about helping an ELL:

> I feel that this student could benefit from the counting board described in the chapter. If she cannot effectively engage in the activity, then it is possible that she missed the foundational skill of learning numbers. How can she move on to more complex activities with word problems when she does not understand the math language that the teacher and other students are using? Although this student may seem too old for number lines or counting activities, if the student began to learn more about the base-10 system and actually experiment with it, wouldn't she be able to apply her new knowledge to the word problems? Wouldn't the exploration show her how relevant the material is and encourage her to persevere and achieve the MPs? I believe that it will take some hard work and dedication, but the individualized instruction will benefit this student in the long run and she will learn the vocabulary in the context of counting and number relations (Reading reflection on number sense, March 17, 2014).

This is an example of how discussing ELLs in class impacted Amanda—she wrote about it even though the article did not mention ELLs. Amanda reflected that scaffolds such as the counting board could help the ELL develop her mathematical language. Amanda was the only PST to use the term "math language"—I did not discuss it with the PSTs, nor was it in the readings. She argued that this ELL would benefit from developing her language through exploring in the context of number sense. Amanda explained how the ELL could not only engage in the Mathematical Practices if Abigail offered her support, she could both learn the mathematical content and develop her mathematics language at the same time (Moschkovich, 2013).

Another example of a scaffold to develop ELLs' English was Amanda writing in her post-survey that she would meet ELLs' needs in mathematics by using a word wall. The fact that

Amanda proposed using this aid in mathematics class indicates that she was open to developing ELLs' language while they learned mathematics.

Sarah. Sarah had a negative disposition towards ELLs. In the pre-survey she said that she did not speak any language besides English, and she later wrote: "My best friend teaches me Russian. I took Spanish classes for 4 years" (Pre-survey, January 14, 2014).

Sarah spent six weeks with her host teacher, but she explained in an interview that he did not help her learn the concept of teaching ELLs even though they had talked about it:

> We talked about ELLs. He was also saying that he had experience with ELLs and he said there wasn't much that he did to focus on them. He said that he was putting most of his focus into the rest of the class. He knew that it wasn't good, and it wasn't right and he needed to incorporate them, but he didn't know how or what to do. I said that I was taking a class learning about it, but I don't know what to do either. When I am sitting in the classroom and seeing what is actually happening, I wouldn't know what to do either (Second interview, May 8, 2014).

Sarah said that even though she had taken a course that emphasized teaching ELLs, she would not know how to meet their needs if she were to teach ELLs. Therefore, Sarah did not feel that she had yet received preparation to teach ELLs from the field or from the teacher education program.

I asked Sarah how I could have better prepared her to meet ELLs' needs and she replied:

> We did everything we could in your class. Practicing lessons and things like that which is really good? But then how do you teach?… It is really hard to… you need to know their language in order to help them and it is really hard with the language; you need language to communicate. Like math can't be your communication all the time. As much demonstrating and modeling, they need explanations. How can they explain themselves if they don't know how to explain? (Second interview, May 8, 2014).

Sarah did not consider developing ELLs' English as a strategy of meeting ELLs' needs. Moschkovich (2013) maintain that ELLs can learn mathematics content and English at the same time, but Sarah disagreed. She said that ELLs couldn't learn mathematics unless teachers know their native language. After taking the mathematics methods course, she realized the importance of language and mathematics, but she did not know how to develop ELLs' language. Furthermore, she did not know how to encourage ELLs to develop viable arguments if ELLs did not know English.

Sarah felt overwhelmed with the idea of teaching students who are not proficient in English and she explained in an interview how she could not imagine teaching ELLs mathematics: "I just can't imagine a student who doesn't understand any English being able to learn in a classroom" (Second interview, May 8, 2014). If Sarah can't imagine teaching ELLs mathematics, it would also be difficult for her to imagine both developing ELLs' language in mathematics class and teaching content at the same time. Turner et al. (2012) point out that PSTs need to have a positive disposition about ELLs in order to write meaningful lesson plans.

Sarah wrote on her post-survey that the most important thing to meet ELLs' needs is to "make sure they have the supports so they understand" (Post-survey, April 22, 2014). Sarah described how she would provide ELLs with access to the mathematical content, which would help them engage in Mathematical Practice 1. However, an additional need of ELLs is to improve their English proficiency.

Sarah reflected that she had limited experience developing arguments in mathematics as a student because her teachers usually dominated the discourse:

> I learned math in more of a lecture style where the teacher does most of the talking and explaining. Every now and then the teacher calls on a student to explain but many students did not engage and participate. If a student does not need to talk, they usually did not. Students were never motivated to learn math, therefore, we did not interact with each other to try to learn and solve problems (Mathematics autobiography, January 15, 2014).

Sarah recognized that when the teacher dominated the discourse, she and her classmates had fewer opportunities to discuss problems. Although she did not mention it, encouraging students to argue and questioning them, instead of dominating the discourse, are strategies for teachers to develop students' mathematical language (Lampert, 1998; Stein & Smith, 2011).

Emily. Emily had beliefs and practices regarding developing students' mathematical language. She told me in an interview that one of the most important ideas that she had learned in the mathematics methods course was that mathematics is linked to English Language Arts (ELA):

> My biggest take away is promoting English and all languages into math because I want to create strong students in all areas…I realize that it's good because they get more practice with language during math time without thinking that it's reading time or language time and so they learn language subliminally… A big part of math, especially now that it's becoming more English integrated; I mean you have to be able to read a question. And that's huge in high school and college and in all of our life (First interview, April 24, 2014).

Emily recognized that it is important to develop students' English during mathematics—not just in ELA. According to Emily, there are language components such as reading, which have to be acquired in order to be successful in mathematics. Apart from having students learn ELA in mathematics class, she also was in favor of teachers including other languages. Moschkovich (2007) argues that ELLs should be encouraged to develop their own heritage language as well English as they make mathematical arguments.

When asked how she formed this belief about mathematics being linked to ELA, Emily said:

> I remember the first thing you asked us was about English language in math class and this made me think more. Like, "Why would he ask me that? How is this important?" Obviously it is important if it's being asked. And then the whole thing in the classroom and witnessing it and seeing the students actually be successful with the strategies and learn more than the other days. I watch them in math when they're not doing these. It's just very... It's just proof that this whole thing works and students are getting more out of it than just, "Do you know how we have to write the standards that we are using?" Well, you can meet so many standards from other subjects like social studies, math, reading, writing standards in every lesson (Second interview, 2014).

Emily referred to meeting the ELA writing standards and she argued that these could be met by providing students opportunities to write in mathematics and social studies classes. She said that she had learned about helping students discuss mathematics by talking about it in the mathematics methods class and observing students be successful in the field as they discussed mathematics.

Even though Emily said that she had learned to include ELA in mathematics from the mathematics methods course, she already had a focus on developing language from the beginning of the course. After the first methods class she wrote in a homework reflection that she liked how I had included English in the class: "I also enjoyed how we can read in order to learn math because we can apply it to something rather than just numerical problems" (Mathematics autobiography, January 15, 2014). This interest continued throughout the semester and the questions that she asked her peers at the end of the mathematics methods course were also about speaking English. Emily asked them the following questions in her final presentation: "How often do you use the English language? Talk about different times during the day that you are communicating with others. What do you use language for?" (Emerging theory presentation, April 22, 2014). After asking the questions, Emily facilitated a discussion about how English was important in mathematics. Emily argued that we need to teach ELA in all classes, including mathematics.

By the end of the semester, Emily talked about her new belief that all subjects are interrelated. I asked Emily if the theme about language and mathematics was a new idea for her:

> I never thought of these things before and math is not just... Well, nothing is just that one thing. Math is not just numbers. Writing is not just written language and reading is not just reading because there are so many other interchangeable subjects like content in everything. You can't just say, "Okay, it's math time, here's just numbers to do. Don't look at any words, just do the numbers." It's a whole different philosophy. You have to understand symbols; you have to understand context; you have to be able to comprehend.

> It's neat. I really like…I'm just going to repeat what I said before, but I really like having math, reading, and ELA in one thing (Second interview, April 29, 2014).

This is another instance of when Emily believed that teachers should teach ELA in mathematics class. She used to think that learning mathematics was simply working with numbers, but now she claimed that mathematics is linked to language. Emily's description of teaching students to read and write during mathematics class would also develop their mathematical language—the ELA teaching is done in the context of solving mathematics problems.

At the beginning of the semester Emily reflected on a homework assignment and wrote about students working together in groups and talking: "Students should explain their work so they don't just get the problem wrong but the teacher can see where they went wrong and where they need to be fixed" (Mathematics autobiography, January 15, 2014). However, her focus in this comment was on students talking so the teacher could catch and correct mistakes. As I have discussed, by the end of the semester, Emily had acquired a new belief. Emily understood the value of students talking in mathematics not only so that the teacher could correct the students' mistakes, but also because students learned concepts more deeply. Although Emily did not use these words, as the students were discussing mathematics, they had opportunities to develop their mathematical language.

Emily not only discussed in the methods course the importance of teaching ELA in mathematics, she also applied this idea in the field:

> Well, in my post assessment I was assessing their writing so I am assessing a writing standard and if they don't tell me what they think and they are just showing me numbers then how do I know if they really understood it? (Second interview, April 29, 2014).

The advantage of writing in mathematics, according to Emily, is that her students could practice the mathematics and writing at the same time. In the mathematics methods class we had discussed that students learn concepts more deeply when they discuss them, but Emily extended this idea by developing students' mathematics language through writing assignments in mathematics class. Emily also implied that from their written work, she could assess the process of how they solved problems better than if she merely looked at their numerical answers.

Emily wrote in her pre-survey that she had limited experience teaching ELLs—"not much at all." She also wrote that her father was Norwegian and had taught her some words. After Charles, Emily was one of the PSTs who was most open to developing ELLs' language. She planned to develop students' English in mathematics and said that it was important for ELLs to learn both their own language and English. Thus Emily appears to have a positive disposition towards developing ELLs' language, but she also discussed providing access to ELLs. For example, Emily stated in her post-survey: "I would be giving them translations, modifications, images, and extra support. ELLs easily get lost in subjects and I would make sure they had access to all information and support so they would not get lost" (Post-survey, April 22, 2014). This quote portrays how Emily would help ELLs in mathematics class by providing them access to the content. De Jong and Harper (2011) argue that it is important for PSTs to provide access to content, but they also need to focus on developing ELLs' English—Emily did both.

Emily said that using scaffolds, such as manipulatives, could help ELLs develop their English:

> These students need to not only be progressing in their language but also using math as a time to practice their English skills. Using manipulatives and working together are becoming very important in a class so that students get the tactile feeling of numbers and also can communicate about them (Reading reflection on number sense, March 17, 2014).

Emily referred to one of her students as an ELL, even though she did not receive ESL services. Not only did the student learn the content more deeply from speaking, according to Emily, but she also learned from writing:

> Another aspect from the reading that I focused on in my unit was the integration of English in math. I had the student's practice writing and speaking fluent sentences about their mathematical thinking. This engaged all of my students, especially my English Language Learner. I believe it is effective because they [the] way that I had the students explaining how they got their answer, what they did, and writing it out strengthened understanding of the content and also had them practicing those important skills (Lesson plan, April 2, 2014).

Emily wrote in her lesson plan reflection that her ELL student benefited from speaking and writing their mathematical ideas. Emily's comment is comparable to Moschkovich (2013), who contends that ELLs need practice with listening, speaking, reading, and writing in English.

Fatima. Fatima used arguments, questioning, and scaffolding to develop students' mathematical language. I discussed in Section 4.4 how Fatima, Amanda, and Kim used questioning techniques to facilitate students' engagement in productive struggle. In the mathematics methods class Fatima argued that instead of telling students how to solve problems it was more effective to ask students probing questions that guide them to discover for themselves: "Questioning is something I learned this semester that really helps. If you sit there and tell them they are not going to pay attention about how to do the problem" (Second interview, April 28, 2014). Fatima learned that questioning was important, but she said that it was important so that students could better engage in productive struggle. Not only does asking students questions encourage students to make sense of problems, it also gives them opportunities to develop their mathematical language (Lampert, 1998).

One of Fatima's lessons contains questions that she was planning to use for her topic on shapes:

> These questions I can ask throughout the lesson: What is different? What is the same? Which of the shapes roll? Which of shapes do not roll? How do the shapes that roll are similar? How would they differ? How do the shapes that don't roll similar? What is different? What can you tell me about this shape? What else do you notice? Do you see any other shapes within this shape? Talk with your group about each shape and choose 3 or 4 important parts about that shape that we see (Lesson plan, February 13, 2014).

As the students answered the questions, Fatima would be developing their mathematical language on the area of geometrical shapes—studying a theme in depth is an effective strategy for developing mathematical language (Lampert, 1998). Furthermore, she asked them to discuss their ideas in groups so the students had opportunities to develop arguments and critique others as they answered her questions.

Fatima is one PST that reflected on the advantages of encouraging arguments in mathematics. When Fatima and her group were teaching a lesson, they were initially going to stop a class argument and tell their peers the answers, but I asked them to allow the argument to continue so the PSTs could experience developing arguments and critiquing each other. Hiebert et al. (1997) maintain that students learn concepts more deeply if they are encouraged to justify their answers. Fatima stated that their peers were more engaged as a result of the argument and not only did they push their learning further as they attempted to make sense of the problem, but they were also engaged in Mathematical Practice 3:

> The truly varying strategies and answers allowed members of our classroom to engage in the arguments and peer critique that, up until that point, we had been pretending to experience. The legitimacy of this MP allowed me to see the practice in motion; watching as the class continued to question themselves and others in order to help them make sense of the problem. The conflict over the problem made it so that the entire class stayed engaged and listened to other classmates' ideas, actually thinking about them and sometimes even applying them to their problem (Lesson critique, January 31, 2014).

Fatima reflected that she had learned how students could benefit from arguing inasmuch as she had experienced it unintentionally in her own lesson.

Fatima described the lesson as a disaster as she and her group had not planned the argument, but she realized that it was a rewarding experience for her because she learned that students can be more engaged when they argue about mathematics: "After reflecting on the lesson, I have come to view this situation as a beautiful disaster, due to the way the exploring and investigating of the various problems, experiences, and observations of the lesson contributed to my developing pedagogical knowledge" (Lesson critique, January 31, 2014).Fatima focused on the fact that the argument allowed students to become more engaged in the mathematical content. Not only did the PSTs delve deeper into the content as they were arguing during Fatima's lesson, they were also developing their mathematical language as they attempted to express their ideas orally to peers.

In one of her lessons in the field she also planned for her students to argue about the properties of shapes. Fatima gave certain students leadership roles to promote discussions about the characteristics of shapes:

> Target [student] D would verbalize his answer like "Oh yeah it slides" when the object was actually rolling and student B would help him by saying, "Wait does it slide? I thought it rolled." Or "I think you mean rolled" all while exemplifying this to my target student. This was a great way for my student to learn and exampled to him how to critique and question, with him asking similar questions to other students in the group (Lesson plan, February 13, 2014).

In her placement, Fatima made use of what she had learned about encouraging mathematical arguments. Hiebert et al. (1997) state that it is important for teachers to create an atmosphere where arguing about mathematics is valued.

In a scaffolding strategy discussed in the mathematics methods course for developing her students' mathematical vocabulary, Fatima purposely used her students' language at the beginning of the lesson. Instead of using informal vocabulary such as "spheres," "cones," and "edges" with her second graders, Fatima used the students' vocabulary such as "shapes," "rolls," and "balls." After the students had acquired experience with the terms, she introduced the more technical terms. In an interview she explained this idea:

> The idea is you need to use their language first and then you use the language that you want them to use and slowly it becomes interchanged. Rather than making them learner at the beginning. We talked about rote memorization ruining learning sometimes. If you

> let them learn what they need to and slowly scaffold the language it works better (First interview, April 24, 2014).

Scaffolding students' language offered students opportunities to develop their own language. By introducing the technical words at the end of the lesson after students had acquired experience with the shapes, it would be easier for the students to retain the new words (Moschkovich, 2013).

In Fatima's lesson plan, she discussed how she would help one student in particular learn mathematical language: "She understands language when it is presented in a more kid friendly version. Once she understands the concept fully, she will begin to use the mathematical language" (Lesson plan, April 14, 2014). Fatima felt that this student would benefit if she started using the student's informal language and gradually introduced more formal language. By scaffolding the language, she planned to develop the student's mathematical language.

Fatima also used a visual scaffold, a word web, to aid her students develop their language. Her university supervisor said that she developed her students' language in a social studies lesson:

> She basically created a vocabulary web in her social studies lesson…She asked what they saw and then wrote down what they said. For example, food. As the kids were observing then she would put down the words. It was coming from the kids (University supervisor interview, July 9, 2014).

The fact that the students generated the words themselves may have been motivating for the students because it was more personable than if Fatima had given them a list of vocabulary words to memorize (Moschkovich, 2010). Using a web could expand her students' vocabulary, another example of how a PST developed language for her students. According to Coggins (2014), word webs and other visual scaffolds are more effective at developing language than merely listening to a teacher talk. Fatima developed students' vocabulary in social studies, but if she had used the scaffold in mathematics it might have developed their mathematical language.

Another example of Fatima using a scaffold to develop her students' language, this time in mathematics, was a discussion sheet used to facilitate her students' mathematical discourse:

> I had a discussion sheet. They were question starters like, "Why do you think that?" or "I think this way because…" or "I think this; does anyone agree? Why do you think that way?" These are little things they can use when they talk to each other (Lesson plan, April 14, 2014).

Similar to her word web and Amanda's sentence starter, Fatima attempted, through the use of the discussion sheet, to include all students in mathematics discussions. She could have asked the students the questions verbally, but by writing them down, Fatima helped more learners justify their answers with a visual scaffold.

A slightly different scaffold used by Fatima that may have developed students' mathematical language was using manipulatives with her students. Fatima explained why she encouraged students to use manipulatives: "Students who are more hands on will be able to solve the problem realistically using manipulatives. This will also benefit students who have trouble verbalizing their answer/explanation because they can use the bags to help them" (Lesson plan, April 14, 2014). Fatima planned these provisions to help students make sense of problems on their own, but using manipulatives was also an example of aiding student to develop their language. Coggins (2014) explains that the use of manipulatives is easier for students to discuss because there is a physical representation, whereas merely discussing mathematical concepts without manipulatives is more challenging for students. Fatima made a similar argument about manipulatives: "The availability of concrete materials to aid in the understanding and communicating are vital for these ELL students and should be used extensively in the classroom" (Reading reflection on ELLs, February 19, 2014). Fatima also said that ELLs could be supported to develop their English with the use of manipulatives.

Fatima said in an interview that although she did not have ELLs in her placement, she learned strategies that can be applied to many learners:

> The idea that mathematics has nothing to do with language is a common idea that a lot of us had before we took the class and then after the class we learned that it is always there. Languages impacts mathematics a lot! The course helped us learn how to teach ELLs in our future and the different strategies which also help to teach. You see I didn't have any ELLs in the classroom, but knowing those strategies can help your classroom that don't have any ELLs in them (Second interview, April 28, 2014).

In her pre-survey Fatima said that she had no experience teaching ELLs, nor did she speak another language, although she had taken six years of Latin. However, perhaps Fatima's experience developing students' mathematical language would help her develop ELLs' English in mathematics classes.

Kim. Kim used questioning and vocabulary exercises to develop students' mathematical language. She criticized her teacher for not encouraging her students to justify their answers and talk about mathematics:

> Then she will call on the kids that she knows are going to get it right. She doesn't really have confidence in her students. She just gives them the answers. She doesn't ask them how they got the answers to anything. When the kids talk to each other, they are not talking about math; they are probably talking about how dumb the other kid is. It's pretty messed up (Second interview, April 30, 2014).

Kim guessed that the student discourse was not even about mathematics; if it was not; perhaps this was because the teacher had failed to ask questions that required the students to justify their answers. By asking students questions, teachers can develop students' mathematical language (Lampert, 1998).

According to Kim's university supervisor, Kim expanded her students' mathematical vocabulary: "She certainly talked about math vocabulary and how it's different from other and you need to know 'parts' and 'equal'" (University supervisor interview, June 15, 2014). With a richer mathematical vocabulary, students can better develop viable mathematical arguments and critique others (Stein & Smith, 2011).

In the pre-survey, Kim wrote that she did not have training with ELLs and said: "I know random Spanish words. I took Spanish for five years in high school." Kim changed her view by the end of the semester about developing ELLs' English. In the pre-survey Kim wrote that she would make accommodations for ELLs to speak their own language, but in her post-survey she said that students should be able to speak their own language and English: "Yes, but they should be encouraged to learn English." In other words, Kim could be arguing that the ELLs could speak in their own language in order to have access to the content, but they also should speak in English in order to develop their language. This is an example of developing ELLs' English in the context of mathematics.

As well, Kim advocated supporting ELLs by using manipulatives: "Another strategy we talked about in class and aligned with the article was the use of concrete materials being very helpful for ELL students to understand and explain their thinking" (Reading reflection on ELLs, February 19, 2014). Kim added that she had learned the idea of using manipulatives with ELLs from class discussions and from an assigned article.

Below are three examples of Kim developing ELLs' English through questioning. She planned to modify her questioning format for her ELLs:

> Encourage all students to participate by asking specific questions to specific students: if I ask an ELL a question, I will make sure I point to aspects directly on the graph and use gestures to be clear in my questions (Lesson plan, April 16, 2014).

Not only was Kim aware that she could engage her students to participate in the mathematical discussions by questioning them, but she also planned how she was going to ask the ELLs the questions. As Kim realized that asking questions was important, she facilitated the ELLs' access to the questions by using visual strategies. In her lesson plan reflection, Kim wrote more about her belief on questioning ELLs:

> I am very proud of the accommodations I made for the ELLs. I think they all understood bar graphs, and in some cases, more than the other students in the whole class. I maybe would have liked to question them more and get them expressing their ideas, MP3, because that was hard to get them to do on their own. I should have questioned them directly, but it is very hard to do since we only had 2 teachers and 5 groups (Lesson plan, April 16, 2014).

While Kim admitted that she should have asked even more questions in order for the ELLs to verbally justify their answers, she realized that questioning facilitated ELLs' engagement in Mathematical Practice 3.

Another instance of Kim recognizing the importance of questioning her ELLs was when she noticed that an ELL student was answering his classmates' questions: "I was very impressed with him, especially in his group; he led his group and respected them all by answering their questions" (Lesson plan, April 16, 2014). Again, one of Kim's goals was for her ELLs to answer questions so they could develop their language, so she was proud of the ELL when he achieved this. Kim not only asked the ELLs questions herself, but also observed how ELLs responded to peers' questioning.

Kim used questioning techniques to develop ELLs' language, and she also used scaffolding strategies. She facilitated her ELLs' engagement in Mathematical Practice 3 by allowing them to develop their arguments with tools: "He used the graphs his group made and his cubes to back up his reasoning for things, which is very impressive and essential for an ELL. I am very pleased" (Lesson plan, April 16, 2014). It was argued by Kim that the ELLs were more likely to express their ideas through the use of tools, so she provided these in the lesson. Another tool used by Kim was manipulatives. She wrote in her lesson plan how she would use them: "Use the manipulatives when questioning them so ELL[s] understand that I am questioning about cubes and connecting it to their data they collected. Point back and forth to their data and the cubes" (Lesson plan, April 16, 2014). In addition to allowing her ELLs to use manipulatives, she planned to use the manipulatives herself as she asked the ELLs questions. In a lesson reflection she mentioned manipulatives with ELLs again:

> I also used manipulatives throughout the lessons so that the ELL could express their reasoning using the strategic tools at hand instead of words. This helped them out a lot and also let them see how the math actually worked. I also used several visuals for every part of my lesson, either pictures by themselves for all learners or pictures and words for those students who need to be challenged; the ELLs still benefit and get a better sense of the general concept of the lesson (Lesson plan, April 16, 2014).

Kim used manipulatives and visual scaffolding strategies to help her ELLs develop their language.

Table 9

Summaries of PSTs for Social Interaction and Developing Language[1]

Name of Participant	Provided Social Interaction?	Developed Students' Language?
Abigail	Strong	Weak
Amanda	Strong	Strong
Charles	Strong	Strongest
Emily	Strong	Strong
Fatima	Strong	Strong
Kim	Strong	Strong
Maria	Strong	Weak
Sarah	Strong	Weak

[1]Ratings are intended only as shorthand aids to follow the data, not as substitutes for fuller descriptions presented in the text.

The PSTs discussed beliefs related to developing students' mathematical language. PSTs said that they had learned the importance of questioning students and allowing them to argue or justify their ideas. Emily also pointed out that it is beneficial to teach ELA in mathematics class. Fatima and Kim used scaffolds that may have encouraged all students to participate in discussions. I did not find evidence of the PSTs saying that their goal was to purposely develop students' mathematical language. However, the PSTs' beliefs and practices of questioning, allowing students to argue, including ELA in mathematics class, and scaffolding may have helped students develop their mathematical language, which may in turn help students' engagement in Mathematical Practice 3.

The ELL literature suggests that PSTs be taught a second language so that they can better empathize with ELLs and are better prepared to develop their language linguistics (Harper & de Jong, 2004; Lucas, 2011; Valdes et al., 2005). Like Charles, Maria was exposed to a second language, but she had not had the experience that Charles had with teaching ELLs. In addition, Charles reported that he had experienced growing up with students who spoke in their native language, but Maria had not had this experience; she had not spoken Spanish at school. Despite having knowledge of a second language, Maria still did not focus on developing students' language. Therefore, although I do not dispute the importance of PSTs learning a second language, I call for more studies to determine if there are other factors, such as disposition, that may affect how PSTs assist ELLs develop language.

Charles and Emily had had previous experiences with learning a second language and they also had a positive disposition about developing ELLs' language. In contrast, Sarah had limited experience with other languages and felt overwhelmed by teaching ELLs. Abigail did not speak other languages and had low expectations for ELLs. I have discussed how feeling overwhelmed and having low expectations may limit PSTs from developing ELLs' language. However, not all the monolingual PSTs who had limited training with ELLs had negative dispositions. Kim said that she had no training at all with ELLs and despite taking five years of Spanish, only knew some words; and yet she employed strategies to support ELLs as they were learning English in mathematics class.

The PSTs had different experiences and dispositions about ELLs and this may have accounted for differences in their beliefs and practices about developing students' language. Although only two of the PSTs had experience developing ELLs' English, the PSTs learned about this concept in the mathematics methods course. Furthermore, PSTs had experience developing students' mathematical language, which would help students engage in Mathematical Practice 3. The PSTs used questioning techniques, arguing, and scaffolding to develop mathematical language. Perhaps this experience of developing mathematical language would be useful for PSTs if they have opportunities to develop ELLs' English.

CHAPTER 6: CONCLUSIONS AND IMPLICATIONS

The following chapter includes findings, implications, limitations and recommendations for further research, and conclusions. In this multiple descriptive case study, I have examined how PSTs experienced the process of learning Mathematical Practice 1 (make sense of problems and persevere in solving them) and Mathematical Practice 3 (construct viable arguments and critique the reasoning of others) from the Common Core State Standards. In addition, I have studied PSTs facilitating students', including ELLs', engagement in Mathematical Practices 1 and 3. I have examined the experiences of the PSTs and offered suggestions to account for similarities and differences. I have analyzed open response surveys (pre-and post), group discussions, homework reflections, peer teaching, lesson plans, supervisors' reports, and semi-structured interviews. The participants are eight PSTs enrolled in their first mathematics methods course, for which I was the instructor. Additional participants include their corresponding host teachers and university supervisors, if they agreed to be included.

6.1 Summary of Findings

The processes by which the PSTs learned the Mathematical Practices themselves, and the processes by which they learned to facilitate students' learning (and ELLs' learning) of the Practices were invariably linked. As demonstrated in previous chapters, all of the PSTs learned the Mathematical Practices more deeply by teaching in the field. To determine how deeply the concepts were learned ~~I~~ analyzed whether or not they had applied the concepts in their teaching. I will also discuss the link between research questions two and three. Lastly, the question regarding similarities and differences between PSTs will be woven throughout discussions of the other three questions. Although I will discuss each research question separately, the links will be present throughout.

6.1.1 How do PSTs Make Sense of the Mathematical Practices?

Mathematical Practice 1. There are four themes for PSTs facilitating elementary students' engagement in Mathematical Practice 1: connecting mathematics with students' personal experiences, providing students access to the content, holding them to high expectations, and providing opportunities for productive struggle.

Connections. Having been in a dual major program of general and special education, the PSTs learned strategies throughout the program to connect content with students' lives and backgrounds. In the mathematics methods course the PSTs reflected on whether before college their teachers had connected with them in mathematics, and they practiced connecting with their peers as they planned, implemented, and reflected on lessons in the methods course. The PSTs were taught to use an inquiry-based lesson format designed to capture the students' interest at the beginning of the mathematics lesson (Clarke et al., 2014).

Although the PSTs used a variety of words to describe connecting students to mathematics, the most common theme was making personal connections with students, which could be made by the teacher talking about her personal life or asking questions about the students' personal life and then connecting those personal details to the mathematics they were learning. The PSTs said that teachers could make personal connections with students to create more interest and engagement in mathematics, which in turn encourages mathematical discussions. Another common theme was that when teachers connect with students, they give them a purpose for doing mathematics

Another form of connection for the PSTs was being culturally relevant, which also involved making a personal connection with students. All eight PSTs were given a reading reflection for homework in the mathematics methods course on being culturally relevant, and their reflections were similar. The PSTs suggested that teachers make an attempt to celebrate all students' cultures. One PST maintained that students that are not part of the dominant society's culture might need to have the mathematical tasks specifically connected to their culture. The PSTs said that teachers could connect with all students by including their culture in the classroom. One PST had a unique experience of cultural relevance—she was the only PST to understand Spanish, and she said that when I spoke Spanish to her, she learned the importance of teachers being culturally relevant.

My data on connections support the findings of Koestler et al. (2013) that PSTs should connect students' background knowledge with mathematical tasks to assist them in making sense of problems and also those of Clark et al. (2014), who argued that connecting students' mathematics aids them in persevering to solve problems.

Providing access. The PSTs had learned about providing access to all students throughout the teacher education program. In the mathematics methods course the PSTs practiced teaching lessons to their peers; in a homework reflection, three PSTs commented about the importance of their peers making accommodations for them while they were teaching—if their peers had considered the PSTs' learning preferences, then the PSTs reported it had been easier to access the content.

After I taught a mathematics lesson to the PSTs in Spanish, all eight reflected that not knowing the language of instruction impeded gaining access to the content. After being taught in Spanish, the PSTs discussed strategies that would have allowed them access to the content when I had been speaking in Spanish. The most common strategies for allowing ELLs to have access to the content were for the teacher to use visuals, gestures, and speak slower. As the National Council of Teachers of Mathematics (2014) asserts, the eight PSTs contended that providing all students with access does not mean teaching all students the same. Apart from making connections, teachers can facilitate engagement in Mathematical Practice 1 for students by selecting and adapting problems to provide them access to the content (Koestler et al., 2013). In summary, the PSTs had learned from the methods courses, including the mathematics course, that it is important to provide students access to solve content.

High Expectations. The PSTs had different expectations for their students' mathematical abilities. Two PSTs manifested high expectations for their students; two made comments that revealed low expectations, and four did not comment about their expectations for students. The PSTs had been introduced to the concept of "presumed competence"—the belief that all students can learn—in another course in the teacher education program. One PST mentioned that teachers should presume that students are competent, and yet her comments reflected that she had low expectations for ELLs' mathematics ability. Similar to the study by Turner et al. (2012), this PST had inconsistencies in her learning—she said one thing but did not apply it. In contrast a second PST had a strong understanding of presumed competence as she applied this concept to her students in the field. She argued that it was important to presume that all students are competent; for this reason she called them mathematicians and allowed them to discover on their own.

Another PST had low expectations for students as evidenced by deficit remarks about ELLs (Moschkovich, 2013). A fourth PST appeared to have learned in depth the concept of having high expectations for students because she not only discussed this concept in homework reflections, but she applied it to her teaching of all students, including ELLs. I did not find significant data about the other four PSTs' beliefs regarding expectations. This is not surprising, since I attempted to use a constructivist approach, and Marchionada et al. (2014) point out that PSTs in constructivists classes do not all focus on the same topics.

One explanation for why PSTs' have different expectations for ELLs is due to their varied outside experiences. Although the PSTs discussed ELLs in four modules of the mathematics education class, the topic was not infused throughout the program. Since the PSTs had limited opportunities to discuss best practice for ELLs throughout the teacher education program, they relied heavily on their outside experiences, which appear to have varied widely. Marx (2006) argues that PSTs need opportunities to reflect on their deficit beliefs, so perhaps the PSTs would have had higher expectations if they had had more opportunities to reflect on high expectations.

Productive struggle. The PSTs were in an inclusion program, so they had been encouraged to make accommodations for their students, but while making these accommodations they were to keep the cognitive load high (Hiebert, 2003). There were times when the PSTs appear to have made the content too easy for their students, whereas I did not find instances when their content was too challenging for the students. According to the sociocultural theory, the PSTs should have had students solve problems within their zone of proximal development (Vygotsky, 1978).

It appears that the PSTs' own dispositions towards mathematics varied and that these variations may have affected their beliefs around productive struggle. Two PSTs had positive dispositions and experienced solving challenging mathematics problems as they grew up; they also focused on engaging their students in productive struggle. Another PST, who also shared that she was good at mathematics, did not show the passion that the previous two expressed for the subject. This PST did not apply her beliefs about productive struggle in her lesson plans. Two more PSTs avoided productive struggle growing up and teaching. For the previous five PSTs, their dispositions towards mathematics appeared to influence their beliefs and practice on productive struggle.

A sixth PST did not enjoy mathematics, and still does not, yet she provided some evidence of engaging her students in productive struggle. She was convinced that she was effective at mathematics pedagogy even though she had had negative experiences with mathematics and thus was not strong in understanding the content of mathematics; Thames and Ball (2010) argue that teachers need both strong content and pedagogy to be effective. I did not observe whether this PST was effective in engaging her second grade students to struggle in mathematics, but her lesson plan appeared to be challenging for second graders. For the remaining two PSTs, I found nothing in my data regarding their experience with or beliefs on productive struggle.

The PSTs in this study had taken at least two mathematics content courses with an emphasis on problem solving prior to the mathematics methods course studied here. However, when asked to write about their learning experiences in mathematics all of the PSTs discussed in detail their experiences in elementary and high school. Four did not mention the two content courses, and the other four wrote no more than a sentence—two saying they did well, and the other two saying that they did not like the way the courses were taught.

Teaching mathematics effectively requires a combination of content knowledge and pedagogical knowledge (Thames & Ball, 2010). According to Masingila, Olanoff, and Kwaka (2012), research is sparse concerning the instructors of the mathematics content courses for elementary PSTs. They also suggest that most instructors do not have experience teaching at the elementary level. Furthermore, the authors of the study report that there is limited training and support for the instructors who teach content courses to elementary PSTs. Greenberg et al.

(2013) maintain that most mathematics content courses in teacher education do not provide PSTs with enough pedagogical content knowledge. Despite taking two content courses in mathematics, the PSTs in my study only briefly mentioned, if at all, these courses. I call for research for teacher educators to study how these content courses may help PSTs learn the concept of productive struggle.

Mathematical Practice 3. (Construct viable arguments and critique the reasoning of others). There are two themes for PSTs facilitating elementary students' engagement in Mathematical Practice 3: social interaction and developing language.

Social interaction. All eight PSTs used the terms "community" and "peer support" to describe the social interaction that would be supportive for all students. Five of the PSTs reported learning these terms throughout the teacher education program. The PSTs referred to creating a classroom community when the whole class was involved, while peer support only required a small group or one classmate. Three PSTs commented that creating a classroom community and using peer support encouraged students to participate in Mathematical Practice 3.

One PST shared that she had talked to her host teacher about the importance of creating strong classroom community ties, but her host teacher replied that she did not have time for that. This PST noticed that the eight ELLs in her placement were not accepted by the other students, so she used peer support to encourage her ELLs to talk. While all of the PSTs discussed the use of peer support, this PST was unique in using peer support to encourage ELLs to talk in a classroom where she deemed that there was not an established sense of community. This PST's practice was in alignment with Pinnow and Chaval's suggestion (2014) that teachers place ELLs strategically so that they sit next to students who will encourage and support them to talk.

Developing language. Mathematical Practice 3 calls for students to construct viable arguments and critique arguments of others, which requires that teachers develop students' mathematical language so that students can justify their arguments (Stein & Smith, 2011). Teachers should also develop ELLs' English at the same time they are teaching content (Shatz and Wilkinson, 2013). I discussed the PSTs' beliefs and practices related to "Developing mathematical language" and "Developing Language for ELLs."

The PSTs in my study focused on accommodating ELLs' language so they could access the mathematics content. One bilingual PST had the strongest belief that teachers need to develop ELLs' English. He was the only one to say that ELLs should speak in English at all times to improve. The other PSTs said they would permit ELLs speak in their own language. When asked what the most important role was for teachers of ELLs, these other seven focused on providing ELLs access to content, such as translating content, using gestures, and making ELLs feel comfortable. One of the two bilingual PSTs said that he focused on teaching ELLs' English in all subjects, including mathematics, because he had learned English as a second language. However, the seven other PSTs, including one who was herself bilingual, did not consider the importance of developing ELLs' English as well. This finding in my study is consistent with the finding of a study by de Jong and Harper (2011) who found that PSTs tend to focus on providing ELLs' access to content and ignore the importance of improving their English language skills.

In order for PSTs to value developing ELLs' language they can be taught a second language so that they can better empathize with ELLs (Harper & de Jong, 2004; Lucas, 2011; Valdes et al., 2005). Consistent with the suggestion in the literature that PSTs take more language courses, the PST who wrote that it is the teacher's role to develop language was bilingual.

There were four other PSTs who reported not knowing a second language and not having field experience with ELLs, but who still paid attention to developing their students' mathematical language. These PSTs' beliefs and practices of questioning, allowing students to argue, incorporating reading and writing in mathematics class, and scaffolding their students' learning, may have helped their students develop their mathematical language (Stein & Smith, 2011). These same practices are strategies for developing ELLs' English (Coggins, 2014) and may have been useful for preparing these PSTs to facilitate ELLs' engagement in Mathematical Practice 3, despite their lack of direct experience with ELLs.

The two remaining PSTs did not focus on developing mathematical language or ELLs' English. These two PSTs also had low expectations for ELLs. Perhaps teacher educators should consider other factors, such as dispositions, in addition to learning a second language, to encourage PSTs to develop students' mathematical language.

6.1.2 How do PSTs perceive they should facilitate elementary students' learning of the Mathematical Practices?

I gave the PSTs a lesson plan template to guide them as they planned, implemented, and reflected on skills such as connecting with students, providing access to all students, and creating social interaction with all students. As discussed in the previous section, all of the PSTs seemed to understand these three themes, and all but two also applied these themes to their teaching. For example, they modified their lessons for individual students, which provided their students access to the content.

One of the two PSTs mentioned above did not write lesson plans and said that she did not have time to make accommodations for students because she had to follow the scripted EngageNY module lessons that were used in her classroom. Without planning her own lessons to provide connections, access to content, productive struggle, social interaction, and language development, she did not experience developing learning goals for her students. Hence, although she discussed these themes in the mathematics methods course, she did not have the opportunity to plan these themes for her specific students in the field. There were three other PSTs who also were in placements with the same scripted programs, but they negotiated with their host teachers to teach the inquiry-based approaches taught in the mathematics methods course. Hiebert et al. (2007) claim that it is important for teachers to have learning goals for students as they plan. They also state that when PSTs plan the goals themselves then they are more likely to facilitate students in achieving these goals. The seven PSTs who planned lessons had opportunities to make learning goals to meet students' needs.

A second PST had opportunities to write her own lesson plans and discussed how she had connected mathematics for students and provided them with access. However, even though she wrote in a homework assignment that students should support each other, she did not apply this strategy in her lesson plans in the field. This lack of application to the field may have been a result of not feeling strong personal ties to a school that was predominately white, in which she was placed. (She herself was African American.) This PST reported that the school where she was placed did not have the same family feeling as the school in her previous placement.

In contrast, there was more variety in teaching the themes of productive struggle and developing language. There were three PSTs who did not show signs of engaging students in productive struggle, and they also showed signs of having low expectations for students. Of these three, one said that she did not like mathematics if it is too challenging, and her direct approach did not seem to help students to struggle productively. She commented that it was important for teachers to assume that all students are capable, yet she also commented that she would accommodate ELLs by giving them less work. The second PST referred to some of her elementary students as “low” and implied that these students would benefit from more explanations instead of keeping the cognitive load high and engaging them in struggle. The third had deficit views for ELLs and did not discuss or facilitate elementary students’ engagement in productive struggle. The data from these three PSTs is consistent with the findings of researchers who observe that teachers who have low expectations for students do not encourage students to engage in productive struggle (Hiebert et al., 2007; Clarke et al., 2014).

The PSTs who showed signs of learning the Mathematical Practices themselves, based on the six themes presented, usually made use of these in the field. For example, one PST had a strong focus on productive struggle in the mathematics methods class, and this interest continued in the field. In contrast, another PST did not discuss productive struggle in the course, nor did she apply the concept of productive struggle in the field. Mewborn (2003) believes that teachers learn practices best when they have experienced these practices themselves. Extending Mewborn’s argument to the PSTs, the ones who experienced practices in the methods class by writing about them in assignments were more likely to implement the same strategies in the field with their students.

6.1.3 How do PSTs learn to facilitate ELLs’ engagement in the Common Core State Standards’ Mathematical Practices?

Only two PSTs had opportunities to teach ELLs in the field. One appeared to be successful at facilitating ELLs’ engagement in Mathematical Practices 1 and 3. According to her university supervisor, she provided the eight ELLs in her class the opportunity to engage in meaningful mathematics. In interviews and lesson plans, this PST explained how she had connected with the ELLs by taking a personal interest in their lives, providing them access to the content by using gestures, and keeping the tasks challenging by asking them questions instead of telling them answers. She strategically placed her ELLs with peers who she thought would encourage the ELLs to develop viable arguments, and she also developed her ELLs' mathematical and English vocabulary. This PST displayed a positive disposition towards mathematics and high expectations for her ELLs. Not only did this PST connect the mathematics, provide access to content, and facilitate ELLs to engage in productive struggle, as mentioned in 6.1.2, she did the same for all of her students.

However, although it is advantageous for PSTs to have experience teaching ELLs in their placements (Lucas, 2011; Walker & Stone, 2011), such experience may not be sufficient. A second PST had the opportunity to teach ELLs in her placement, and yet her outcome was different from the first. She made comments about ELLs that showed signs of the deficit model (Moschkovich, 2013). Whether for this reason or not, she did not show evidence of connecting well with her ELLs. Since she did not become well acquainted with them, a strong personal connection was absent. As mentioned above, she reported that she missed the strong family feeling that had been present in her previous placement. Perhaps for this reason I did not see evidence that she attempted to promote a family feeling among her ELLs or among her other students.

Lucas (2011) claims that learning a second language would help PSTs to acquire a positive disposition to teach ELLs. One PST was bilingual, and his comments about developing language appear to be in alignment with what de Jong and Harper (2011) suggest about having an appropriate disposition towards ELLs. However, another bilingual PST did not focus on developing ELLs' English language skills. Thus, mastery of another language may not be enough for PSTs to acquire a positive disposition for ELLs that would encourage them to develop ELLs' English in all subjects, including mathematics. The first PST also had prior experience teaching ELLs, although not in the field experience associated with my mathematics methods course, so maybe the combination of knowing a second language and teaching ELLs in the field would foster such a disposition in PSTs.

In the mathematics methods course, the PSTs taught mini-lessons to their peers who pretended to be ELLs, but one PST said that she would not know how to teach ELLs unless she had the actual experience of doing so. Even though only two of the PSTs had opportunities to teach ELLs in the field, several had experiences, beliefs, and teaching skills that they had acquired within or outside of the teacher education program that will be useful when they do teach ELLs. As discussed, one PST appeared to be successful at facilitating ELLs in her placement to engage in Mathematical Practices 1 and 3 despite reporting a limited knowledge of a second language and little training to teach ELLs. This PST did have a positive disposition, so if the three other PSTs who also had positive dispositions for ELLs had been given opportunities to teach ELLs in their placements, they may have met ELLs' needs as well. Although these PSTs did not have experience with a second language or teaching ELLs, they had beliefs and teaching experience about connecting mathematics with students' personal experiences, providing students access to the content, holding high expectations, providing productive struggle, using social interaction, and developing mathematical language which might serve them in facilitating the engagement of ELLs in Mathematical Practices 1 and 3.

The PST who did not appear to develop any of those understandings was in a placement with a scripted program. She had similar experiences in the teacher education program and in my mathematics course as the other PSTs, but she did not have opportunities to plan for all of her students to engage in the six themes discussed in this study. Furthermore, this PST made comments that revealed that she had low expectations for ELLs. Therefore, before teaching ELLs she would likely benefit from reflecting on her biases toward ELLs (Marx, 2006).

6.1.4 How are the Experiences of PSTs Learning to Teach Elementary Students, Including ELLs, Similar or Different; and what may Account for any Differences?

The PSTs reported learning beliefs and practices from reflecting on past experiences, from other methods courses, from the field, and from the mathematics methods course. The PSTs in this study shared their beliefs and practices about connecting mathematics with students; they majored in special education and general education, so they had many opportunities to develop an understanding of the importance of connecting, as in social connections or personal connections with students; and they experienced connection to mathematical problems in the mathematics methods course. The teacher education program also stressed the importance of providing access for all students. The PSTs in this study were adept at creating social interactions for their students. Three of the PSTs said that they had learned about creating strong communities in other methods courses and recognized that this was reinforced in the mathematics methods course. By creating safe classroom atmospheres, elementary students (both ELLs and non-ELLs) are more likely to develop viable arguments and the ability to critique the arguments of others (Clarke et al., 2014; Hiebert et al., 1997). Connecting with students, providing access, and providing social interactions were areas where the PSTs had similar beliefs and practices.

In contrast, the PSTs showed a wider range of beliefs and practices in the areas of holding high expectations, providing productive struggle, and developing ELLs' facility with English. Because the emphasis (perhaps too much emphasis) of the mathematics methods course was on pedagogy, the PSTs did not experience as much productive struggle in solving challenging problems as I had hoped. The PSTs had previously taken two mathematics content courses that focused on mathematics as a problems solving process, but there were still PSTs who reported not liking mathematics and who thus did not have a productive disposition (National Council of Teachers of Mathematics, 2014). The PSTs who reported enjoying mathematics mentioned productive struggle and applied this concept in their teaching. Apparently, one reason for the difference in the PSTs' beliefs and practices towards using productive struggle was their own range of productive dispositions and attitudes toward struggle. Those who felt confident about solving challenging problems themselves were better prepared to present challenging problems to their students.

It would appear that the PSTs made the most progress on those themes that were emphasized throughout the teacher education program. One university supervisor pointed out in an interview that the PSTs were well prepared to connect with students because of their dual major in special and general education. The teacher education program also offered the PSTs experiences in being members of a strong class community and in doing activities with their peers. However, according to one PST, the preparation of PSTs to teach ELLs was not infused throughout this program. This may explain why some of the PSTs had low expectations for ELLs despite their general understanding of the theme of "presumed competence," and why the PSTs did not focus on developing ELLs' language skills. One PST had a strong focus on developing language, but he reported acquiring this belief from experiences outside the program.

In summary, in areas such as connection, providing access, and social interaction the PSTs had similar experiences and developed skills and understandings that are likely to serve them well when they teach ELLs and all students. However, in the areas of holding high expectations, providing opportunities for productive struggle, and developing ELLs' English, the data were more dispersed. Hiebert et al. (2007) caution teachers to be skeptical of causal explanations of how students learn, but it is plausible that the PSTs were primarily influenced by their outside experiences, which were varied, in the areas that were not infused throughout the teacher education program. Some PSTs will have had outside experiences, but in an effort to help PSTs facilitate all students' engagement in Mathematical Practices 1 and 3 mathematics educators might consider introducing the six themes discussed in this study into teacher education programs.

6.2 Conclusions and Implications for Practice

6.2.1 Conclusions

The literature in mathematics has established that students and teachers benefit from having productive dispositions—the belief that mathematics is useful and that they are capable of solving mathematical problems (Kilpatrick, 2001). Studies suggest that methods courses can improve PSTs' attitudes towards mathematics and students with disabilities (Ambrose, 2004; Shade & Stewart, 2001). There is ample literature measuring PSTs' dispositions with Likert scales before and after taking methods courses. There is also ample literature about the challenges that the predominately white teacher educators face to prepare predominately white PSTs to be culturally responsive to meet diverse students' needs (Howard, 2006; Ladson-Billings, 2011; Nieto & McDonough, 2011). According to the ELL literature, being culturally responsive is important for meeting ELLs' needs, but it is not enough— teacher educators need to prepare PSTs to develop ELLs' English skills (Harper & de Jong, 2004, Lucas, 2011). My own findings are consistent with all of these conclusions.

The findings of this study shed new light on these arguments, however, and have important implications for how teacher educators can provide the experience that PSTs need to learn Mathematical Practices themselves, and hence to teach the Practices to all students, including ELLs. Unlike the studies mentioned above, I did not evaluate the PSTs' dispositions before and after the mathematics methods course; rather I observed the PSTs' perceptions over time using qualitative data. My findings suggest that there are many factors that may affect the PSTs' dispositions, such as their placements, other methods courses, the mathematics methods course, and their previous experiences. Furthermore, the PSTs' dispositions towards mathematics appear to affect how they learn the Mathematical Practices themselves and how they facilitate (or fail to facilitate) all students, including ELLs, to engage in these Practices. Although my data are necessarily limited, they suggest that the PSTs who have productive dispositions are more ready to facilitate students to engage in productive struggle than those who do not have productive dispositions.

My data suggest that when themes are infused throughout PSTs' preparation programs, they are more likely to apply these themes in their field placements. However, when themes are not infused, but only introduced in a single course, PSTs' general dispositions are more likely to play a more important role. For example, creating classroom community was widely taught throughout the program, so all the participants wrote about this theme and appeared to have applied it in their field placements. In contrast, productive struggle had not been stressed as a theme in prior courses, and the only PSTs who reflected it in their lesson plans and field teaching were ones who had experienced it in their own mathematics learning and had overall positive dispositions towards mathematics.

My study is unique in that most of the ELL literature discusses in isolation how to prepare PSTs to meet ELLs' needs (Lucas, 2011; Lucas & Villegas, 2011; Walker & Stone, 2011). I have examined how PSTs can meet ELLs' needs and non-ELLs' needs at the same time. Whether PSTs have placements with ELLs or not, all of them need to be prepared to meet ELLs' needs. Clearly, preparation progress ought to strive to give all PSTs opportunities to work with ELLs whenever possible, but not being able to provide such preparation does not mean such programs cannot prepare them at all. My findings indicate that while teaching non-ELLs PSTs can develop skills that could be helpful in teaching ELLs. For example, one PST offered her non-ELLs sentence frames to facilitate their engagement—a technique that is valuable for teaching ELLs but valuable for teaching other students as well (Coggins, 2014). Many PSTs are not placed in schools with large ELL populations (Walker, 2011); my study examining how to engage both ELLs and non-ELLs in mathematics is relevant to their situation.

The Common Core State Standards expect teachers to engage all students in the Mathematical Practices. However, they do not specifically state how teachers and PSTs can achieve that goal. After examining both the Mathematic and ELL literature, I offer themes that may aid in learning the Mathematical Practices. I focus on Mathematical Practices 1 and 3 and examine how the PSTs learn the Practices themselves, and how they perceive over time to engage all students, including ELLs. I found evidence that the better the PSTs learned the Mathematical Practices themselves, the better they could teach them. If this is the case, it follows that mathematics teacher educators should teach PSTs the Mathematical Practices in methods courses.

More research is desirable, but perhaps teacher educators can teach the Mathematical Practices to PSTs by connecting them with mathematical problems, providing them access, and developing social interaction in the classroom. Furthermore, PSTs need to engage in productive struggle in methods courses; if teacher educators have high expectations for PSTs and select challenging mathematics problems, they are more likely to do the same for students in their field placements and in their eventual classrooms.

The literature regarding ELLs argues that PSTs also need to develop ELLs' language skills similar to the way that an ESL instructor might do. My study looks at PSTs developing ELLs' language while teaching them mathematics; a subject that many PSTs mistakenly believe is universal and does not involve language (Fernandes, 2011; de Jong & Harper, 2011). PSTs need to learn how to challenge ELLs within their zone of proximal development so the ELLs can better develop their English skills (Lucas, 2011). The mathematics literature suggests that ELLs should be provided scaffolds to assist them to make sense of problems (Fernandes). I agree, and I propose that the PSTs be taught how to help all students, including ELLs, to make sense of problems (first part of Mathematical Practice 1) by providing them access to the content, while keeping the cognitive load high so they will persevere to solve problems (second part of Mathematical Practice 1). With the burgeoning ELL population, it is imperative that PSTs be prepared to facilitate all students to engage in the Mathematical Practices from the Common Core State Standards.

6.2.2 Trajectories

In my discussion of implications I introduce two trajectories. The first trajectory addresses the question of whether there was a predictable sequence to how the PSTs learned, or did not learn, each of the six themes discussed in the study. As I suggested in my data analysis, these six themes provide PTS opportunities to learn Mathematical Practice 1 and 3 themselves. The second trajectory suggests that there may be an order for PSTs learning the themes that facilitate the engagement of students in Mathematical Practice 1. I do not provide a combined trajectory for Mathematical Practice 3 because I did not find a link between its two themes.

For the first trajectory, I turn to Turner et al. (2012), who designed a trajectory for PSTs to engage ELLs in multiple mathematics knowledge bases. Turner et al. developed this trajectory after examining data from 200 PSTs from six teacher education programs. The goal of the teacher educators in their study is to take the PSTs from where they started in regard to meeting ELLs' needs, and to advance them along the trajectory. The researchers recognized that although there were common tendencies, each PST had different paths. I have used their trajectory (please refer to the literature review from Section 2.6) as a basis of my first trajectory, simplifying and adapting it to PSTs learning each of the six themes from my study: connections, providing access, high expectations, productive struggle, social interactions, and developing language.

In the first trajectory under Phase 1, I include the PSTs learning the themes prior to and during methods courses; under Phase 2, I include the PSTs applying the themes as they write their lesson plans and anticipate how to meet students' needs; and under Phase 3, I have included the PSTs applying the themes in their teaching. I include the term "experience" in the first phase because throughout my study, the PSTs discussed how they had experienced the themes in elementary or high school, during another methods course in the teacher education program, and/or in the mathematics methods course. The PSTs also reflected on their experiences, so I used "reflect" to represent how the PSTs reflected on their experiences in the methods courses from the teacher education program (see Figure 1). I posit that the PSTs ideally should experience each theme and reflect on it before advancing to the next phase. The first phase ties directly into my research question about how PSTs learn the Mathematical Practices themselves both prior to and during the methods courses, but they continued to learn the themes more deeply as they made lesson plans and taught in the field. Thus, as I discussed in Section 6.1, there is an overlap between the first two questions, because the PSTs learned the six themes themselves as they grappled with the ways to incorporate them in their teaching.

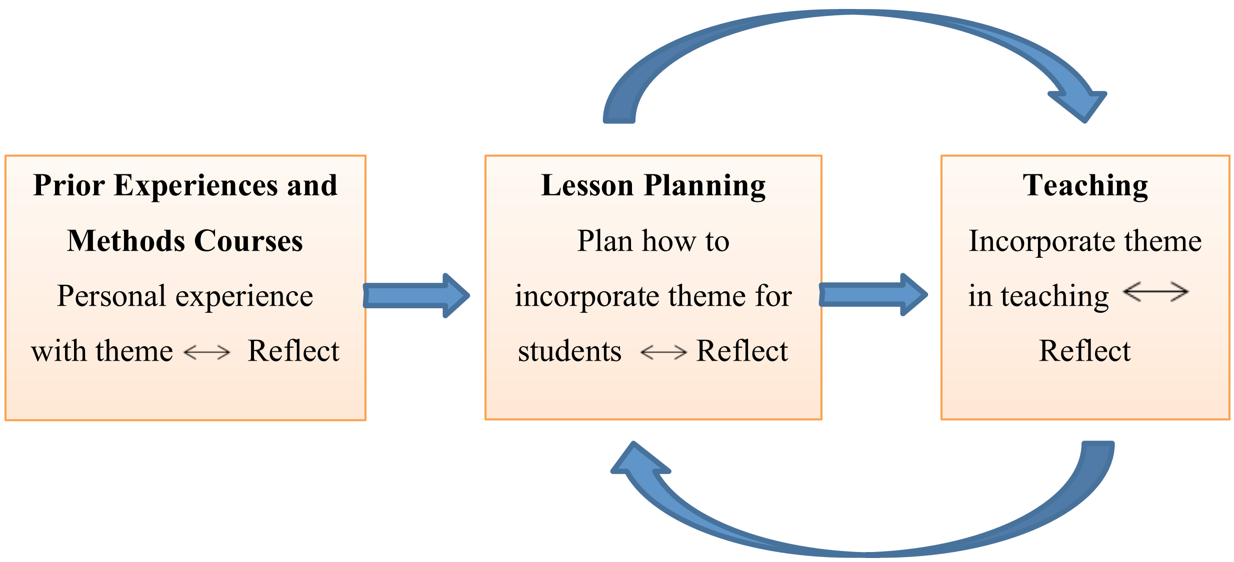

Figure 1: **A Possible Trajectory for Learning Themes**

This trajectory represented in Figure 1 can be applied to PSTs learning the themes of this study—connections, providing access, high expectations, productive struggle, social interactions, and developing language and discourse. The PSTs learned the themes more deeply as they reflected on their experiences in methods courses, planned lessons, and taught in the field. Figure 1 depicts that advancement to the second phase of lesson planning for PSTs is promoted by repeated cycles of experiencing each theme and reflecting on it. Experience in the first phase can be prior experiences from the PSTs or experience from the teacher education program. I found that all the PSTs, except for the one whose placement provided no opportunity to develop lesson plans, moved along the second phase with the themes of connections and providing access. For example, after experiencing and reflecting on the concept of connections in the methods course, seven PSTs used the launch (the first part of an inquiry-based lesson) to consider how they would connect their students to the mathematics in their lesson plans. Each of these seven PSTs selected three target students and described in their lesson plans how they would provide access for these students. These same two themes had been infused throughout the teacher education program.

On the other hand, the PST whose lesson plans were written for her did not have this opportunity (nor did she do anything to create it), and she did not advance. This suggests that experienced teachers who have had multiple opportunities to make connections with their students may be able to stay in the third phase of teaching, but before advancing along the trajectory, PSTs need opportunities to write their own lesson plans and reflect on them (Hiebert et al., 2007). One PST noted as she was writing lesson plans that she experienced tension or productive struggle as she attempted to make connections for her students. Before advancing to the third phase, the PSTs benefit from planning and reflecting—talking to their peers, host teacher, university supervisor, and professors—many times.

Another theme that was infused throughout the teacher education program in this study was social interaction. All of the PSTs appeared to have completed the second phase in this theme except the PST who did not feel part of the class community and did not include social interaction in her lesson plans. Although this PST did not enter the second phase with the theme of social interaction, she experienced and reflected on this theme in the methods course, which indicates that perhaps she would be ready to enter the second phase in her next placement. Making connections, providing access, and social interactions were all infused throughout the teacher education program. These examples seem to reflect the effectiveness of infusion and the importance of engage in repeated cycles of exposure to a theme.

In contrast, three of the eight PSTs did not experience or reflect on the theme of productive struggle in the mathematics methods course, nor include it in their lesson plans or in teaching. All of the PSTs presumably experienced some productive struggle in two mathematics content courses, which required them to solve challenging problems, before taking this methodology course. One explanation for these PSTs' lack of assimilation in this theme may have been that although the PSTs experienced engaging in productive struggle, they did not have enough opportunities to reflect. I suggest that to help PSTs advance along the trajectory, mathematics methods courses need to offer PSTs opportunities to solve challenging mathematics problems and more time to reflect on solving such problems. It is interesting to note that the same three PSTs who did not advance along the trajectory for productive struggle also did not advance in expectations (having high expectations for students). More research is needed on theI connections between the PST's content courses and their methods courses.

Clearly such learning experiences are not limited to the teacher education program. The PST who had experience outside the program of developing ELLs' English was able to reflect on that experience through homework reflections and interviews in the mathematics methods course. This could help explain why he was further along the trajectory than were the other PSTs, who did not have such experience. Although he did not have the opportunity to teach ELLs in his field placement, he had a positive disposition towards ELLs and was ready to move along the trajectory in advancing ELLs' English. The other seven PSTs had limited experience and did not reflect on developing ELLs' English. Since the seven PSTs did not have experience developing ELLs' English, teacher education programs should infuse this theme throughout (Athanases & de Oliveira, 2011; Harper & de Jong, 2004; Lucas, 2011).

The third phase, teaching, refers to how the PSTs applied each theme while teaching in their placements. There are often constraints as PSTs attempt to apply their lesson plans to their teaching. For example, one PST struggled to apply her lesson plan in the field because she wanted to connect the mathematics to her students, and to encourage them to persevere to solve her tasks, but her university supervisor wanted her to keep the students quiet. By reflecting on these restraints—she discussed her experience with me in an interview—she could advance along the trajectory. Two PSTs felt that scripted lesson plans would not meet their students' needs, so they negotiated with their host teachers and taught the inquiry-based lessons that they had learned from the mathematics methods class. All of the PSTs reflected how they had to balance the expectations of their host teachers, university supervisors, and university instructors and negotiate space for exercising their own judgment.

After teaching a lesson, all of the PSTs reflected on their lessons. They reflected orally with their host teachers and university supervisors and in written form in their lesson plan reflections. Then the PSTs were asked in a lesson plan reflection how they could have improved their lessons to meet all students' needs and how they would have planned to meet ELLs' needs if they had had the opportunity. This reflection returned the PSTs to the second phase as they pondered how to teach the next lesson before returning to the third phase of teaching. Although I do not have evidence that the PSTs carried this cycle on between phases two and three, Hiebert et al. (2007) encourages teachers to constantly reflect back and forth between teaching and planning to meet all students' needs. The first trajectory describes a possible path for developing each of the separate six themes discussed in my study, whereas the second trajectory attempts to shed light on possible connections among those themes.

In my discussion of Figure 1, I suggested that one explanation for why all of the PSTs seem to have understood and mastered the themes of “Social Interaction”, “Connections”, and “Provide Access” was that these themes were infused throughout the program, whereas fewer PSTs showed evidence of understanding and mastering other themes, which were newer to them. But another (not necessarily mutually exclusive) explanation for this difference could be that there is a necessary trajectory or sequence for learning these themes (See Figure 2).

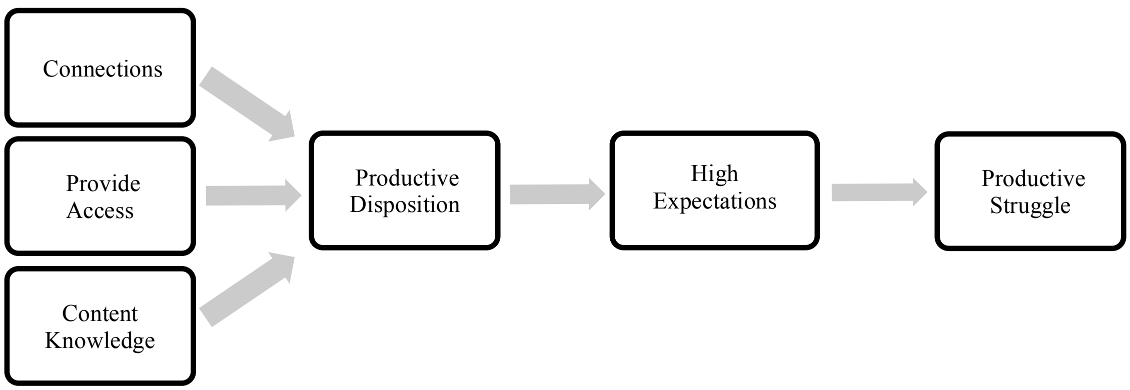

Figure 2: **A Possible Trajectory for Engagement in Mathematical Practice 1**

The trajectory presented in Figure 2 represents a possible order for learning Mathematical Practice 1. PSTs may need to learn how to connect students to mathematics, provide them access to content, and have strong content knowledge themselves before developing a productive disposition, having high expectations for students, and teaching productive struggle.

All eight of the PSTs experienced making connections and providing access for students, but three of them never mastered the concept of productive struggle. Progress toward that end, I suggest, required a productive disposition—a belief that mathematics is useful and that they themselves are capable of being successful in the subject (Kilpatrick et al., 2001). Four PSTs did have positive dispositions—they believed that mathematics was useful and that they were capable of mastering it—and as a consequence, I believe, they could hold high expectations for their own students and thereby teach their students to engage in productive struggle. I posit that it is necessary for PSTs themselves to have a positive disposition toward their subject before they can hold high expectations and teach students to engage in productive struggle. Teacher educators can better foster PSTs' disposition in mathematics methods courses by giving the message to PSTs that they have control of their own learning (Kilpatrick et al., 2001; Marchionada, et al., 2014).

In my study I have discussed how connecting students with mathematics, providing them access to the content, and engaging students in productive struggle are pedagogies that may help PSTs engage students in Mathematical Practice 1. Thames and Ball (2010) argue that teachers should have a strong knowledge of pedagogies, but they also argue for teachers having a strong content knowledge of mathematics. As an instructor I did not provide the PSTs with many opportunities to explore and reflect on their own content knowledge (a point I address below), but there is evidence that their confidence (or lack of confidence) in their own content knowledge affected their disposition and receptivity to productive struggle. (I did not directly measure their content knowledge: in retrospect, I should have, as both instructor and researcher.) Thus, I have included content knowledge on the trajectory. Teachers who have strong content knowledge are more likely to have productive dispositions (Kilpatrick et al., 2001). The implication is that teacher educators should take responsibility for developing PSTs' content knowledge in order to move them along the trajectory.

In contrast to the literature (Thames & Ball, 2010), one PST presented plausible evidence that she was effective at pedagogy, even though she said that she did not have a strong content knowledge of mathematics. She also reported having negative experiences learning mathematics herself, and therefore may have had a negative disposition. Nevertheless, this PST wrote lesson plans that included productive struggle, and her university supervisor said that she asked students probing questions to engage them in productive struggle. One explanation for this apparent contradiction to the literature (and my proposed trajectory) could be that this PST advanced along the trajectory because she enjoyed teaching shapes to second graders and had strong knowledge in that particular topic. Another explanation for her reporting that she did not like mathematics and yet appeared to engage her students in productive struggle was because she was modest in reporting her content knowledge. (I did not collect data to assess the PSTs' content knowledge.)

Turner et al. (2012) contend that those PSTs who reach the end of their trajectory not only acquire the skills at the end of the path, but also acquire all the skills along the way. I suggest that PSTs who reach the end of this trajectory and create opportunities for productive struggle will also be able to teach students to make connections, provide access to students, develop strong content knowledge, and have a positive disposition towards mathematics. Hence, such PSTs would be prepared to facilitate student engagement in Mathematical Practice 1. The trajectory offers teacher educators a visual representation of the importance of developing PSTs' own content knowledge and dispositions towards mathematics so that PSTs can engage their students in productive struggle.

There were two PSTs who mentioned the importance of having high expectations for all students, but who had low expectations for ELLs. These PSTs showed evidence of connecting with students and providing access for students, including ELLs. This suggests that it may be possible to connect mathematics with students and provide them access without having high expectations. However, these two PSTs did not show signs of engaging students in productive struggle, so it may be necessary for PSTs to have high expectations for their students before they can engage their students in productive struggle. These two PSTs showed little indications of productive dispositions to mathematics. Two other PSTs had productive dispositions and also engaged their students in productive struggle. It will be important to investigate to what extent it is necessary for PSTs to have high expectations of their students before they can engage students in productive struggle.

I did not find a parallel trajectory for PSTs facilitating the engagement of students in Mathematical Practice 3—I did not note a connection between the two themes of social interaction and developing language.

Two PSTs experienced social interaction—one of the themes for engaging Mathematical Practice 3— in the field as they collaborated with one another. The implications are for teacher educators not only to provide PSTs with opportunities to collaborate with each other in the methods class, but also to encourage PSTs to work together in the field whenever possible. For example, teacher educators can allow and even encourage PSTs to plan some of their lesson plans together.

Merely giving PSTs opportunities to work with ELLs does not guarantee that they will facilitate ELLs' engagement in Mathematical Practices 3. The PST who had six ELLs in her placement had deficit views about ELLs. Marx (2006) offers a study where PSTs worked on their deficit views through reflection, which is time consuming but perhaps necessary. Therefore, another implication for teacher educators is that PSTs need opportunities to reflect on their deficit views about ELLs.

A bilingual PST, who had previous experience working with ELLs, had a strong belief in developing ELLs' English. This suggests another reason why placements should provide opportunities to teach ELLs, thus helping PSTs to acquire such a belief (Lucas, 2011). The PST who had eight ELLs in her placement did not focus on developing the ELLs' language as much as she did on the other themes, but after being in the field, she also began to realize the importance of this concept. This implies that teacher education programs should discuss how to meet ELLs' needs in methods courses, while also providing them with opportunities to practice teaching ELLs in their placements (Lucas, 2011).

The PSTs in this study had similarities and differences in their beliefs and practices. Like Turner et al. (2012), I do not assume that all PSTs begin their professional training to teach mathematics at the same point. As is evidenced in both trajectories, the PSTs' own experiences and attitudes toward learning mathematics appear to influence how they prepare to teach mathematics. It would appear that the PSTs made the most progress on particular themes of this study—and consequently advanced further along the trajectories—when there also had been a strong focus on the themes throughout their teacher education program. This implies that the themes such as productive struggle, high expectations, and developing language need to be infused throughout teacher education programs (Athanases & de Oliveira, 2011; National Council of Teachers of Mathematics, 2014).

It is possible that as useful as the initial distinction between Mathematical Practice 1 and Mathematical Practice 3 was for collecting and analyzing data, there may be an overall trajectory or sequence that encompasses both. For example, there may be connections between the themes of "productive struggle" and "developing language." According to sociocultural theory, students should be challenged to solve problems within their zone of proximal development as they learn, so there may be similar connections between the two themes. There may also be connections between "providing access" and "developing language" and overlaps between "making connections” and “social interactions”, or perhaps there is a logical sequence to all six themes. This would open up an exciting new set of possibilities about sequencing the instruction of the Mathematical Practices. The Common Core authors pay much attention to sequencing for the Content Standards, my study suggests that we need to consider ways of sequencing the Mathematical Practices as well.

6.3 Limitations and Recommendations for Further Research

This study centered on PSTs’ experiences during one semester. They were engaged in my mathematical education course, and the many factors involved in teaching make it virtually impossible for an instructor to draw causal connections between one's teaching and what was learned in the course rather than elsewhere (Hiebert et al., 2007). This limits my ability to arrive at firm conclusions and underscores the need for further testing of the tentative conclusions and recommendations that I offer.

Ball (2000) maintains that researchers who study their own teaching have an advantage over other researchers because they are insiders and know their participants well. However, having the dual role of researcher and instructor also had potential for bias. I had authority over the PSTs, as I had the responsibility of assigning them a grade at the end of the semester. Despite my attempts to reduce the bias by not knowing who had given me permission to use their data until after the grades had been turned in, the PSTs may still have been trying to tell me what they thought I wanted to hear, rather than expressing their true opinion, to get an optimal grade in their homework reflections. Furthermore, even though the last interviews were administered after the grades had been turned in, I had taught them for a semester, so again they may have been inclined to say what I wanted to hear. To this research base I suggest adding research from outside researchers about PSTs teaching mathematics to ELLs.

There are other limitations to having a dual role. One is that I may have had bias as an instructor because it was to my advantage to look at the data favorably—thus enhancing my reputation or self-image as an effective instructor. Another is that the broader, more impartial view of an outsider might have stimulated questions that did not occur to me as an instructor. Although as already mentioned, I may have acquired richer data by being an insider (Ball, 2000); an outside researcher may have had insights that I did not have because I was too close to my data.

I should also note that because I only looked at one course, it was impossible to compare this course with other mathematics methods courses. Whether carried out by inside or outside researchers, additions to this research base by making course comparisons may prove to be fruitful.

Some scholars insist that to learn how to teach ELLs, PSTs must have opportunities to instruct ELLs (Athanases & de Oliveira, 2011). Only two of the 22 PSTs in my class, both of whom were included in this study, actually had ELLs in their placements, and only one of these two appeared to have benefitted from that experience. On the other hand, most of the PSTs in my study developed skills that would logically be important in instructing ELLs, if and when they have them in their classes. Therefore, the data about ELLs was based on PSTs' perceptions of how to teach, but only two PSTs had actual practice teaching ELLs. Future researchers should look for more opportunities to study PSTs who have placements with ELLs. Furthermore, future research could investigate whether it is common practice to have placements with fewer ELLs than is representative of the population and, if so, why more PSTs are not being placed with ELLs in their placements.

Although I had many data sources—open response surveys (pre-and post); homework reflections; lesson plans; university supervisors' and host teachers' reports; and semi-structured interviews of PSTs, university supervisors, and host teachers—the bulk of my data came from interviews of PSTs. My findings and conclusions would be more robust if I had had more data sources. For example, if I had gone to the schools where the PSTs were placed and explained my study in person, more of the host teachers may have chosen to participate in direct observations of field lessons. Therefore, I encourage teacher educators to have similar studies with a broader base of data.

Of the two PSTs who had ELLs in their placements, I had limited data on one's teaching experiences because neither her university supervisor nor her host teacher agreed to interview with me. Only three of the eight host teachers agreed to interviews. In contrast, as an instructor in the teacher education program, I knew the university supervisors—four out of five agreed to interviews. If I had been an observer in the field, the lack of interviews with host teachers would not have been as much of a limitation.

Another limitation is that I studied early PSTs who were predominately sophomores. These students spend most of their time in their coursework and have limited opportunities to teach mathematics in their field placements. If I had studied student teachers, who spend more time in their placements, more data on these later stages of their trajectories would have been produced. Follow-up studies with student teachers as participants would be useful.

The PSTs in this study were students in an inclusion program with dual majors in general education and special education. Lucas (2011) argues that PSTs need training in how to specifically meet the needs of ELLs. The PSTs who chose to be in an inclusion program may have unique characteristics that may limit generalizations to other types of programs. The PSTs in this study were in a teacher education program where there was a strong emphasis on connecting all students to learning content, providing access to content, and creating social interactions for all students. Furthermore, it is difficult to determine what, if any, pedagogies in the inclusion program helped the PSTs learn to teach using the Mathematical Practices. This study may be most relevant to teacher programs that offer dual majors of general education and special education. Further research should study general education PSTs who are not special education majors.

Although content knowledge was addressed in this study because one of the PSTs said that she was effective at pedagogy even though she did not have strong content knowledge, I had minimum data on this concept. The PSTs self-reported their content knowledge, but their content knowledge was not actually assessed. In retrospect, in the interviews, I could have asked more questions that required the PSTs to discuss the content of mathematics. More importantly, I could have explored the significance of content knowledge more deeply I had given it if it more explicit attention in my course. Future research might consider some of the themes discussed in this study as well as study possible effects of PSTs' content knowledge.

This study was not designed to evaluate the mathematics methods course in which I was the instructor, nor did it consider alternative ways of designing this course. Furthermore, there was limited attention to alternative ways to design field experiences for PSTs. One PST had the constraint of teaching at a school where there was a scripted curriculum for mathematics. Future studies could examine mathematics methods courses to determine how to best meet the PSTs' needs to learn and teach Mathematical Practices to all students. In addition, studies can examine how host teachers' expectations affect PSTs' learning to teach all students.

Clarke et al. (2014) maintain that we can help students to persevere to solve problems by allowing them more time. The literature reports that ELLs especially benefit from having more time to make sense of problems (Verplaetse & Migliacci, 2008). In this study, the PSTs were observed by their university supervisors, who observed them teach twice, once for a mathematics class and once for a social studies lesson. The PSTs gave their students pre-assessments and post-assessments and then were evaluated by their university supervisors. The fact that the PSTs had only one lesson to demonstrate to their university supervisors that they could make an impact on their students' learning of mathematics may have encouraged the PSTs to tell their students' answers so that they would do well on the post-assessments. One PST reported that she was observed for 40 minutes instead of the suggested hour. If teachers should facilitate students' engagement in Mathematical Practice 1 by allowing them to have more time (Clarke et al., 2014), then PSTs should also be given ample time to teach to their students. Research on placements that allow PSTs more time to teach might reveal different results.

Not only could the suggested framework be examined in a variety of mathematics methods education programs, it could also be applied to various subjects. For example, it would be valuable to examine how useful (or not) the trajectory concept might be to explain PSTs' learning the Next Generation Science Standards or the Common Core Literacy Standards, each of which has dimensions similar to the Mathematical Practice Standards.

6.4 Concluding Observations

The purpose of this descriptive case study was to document the experience of eight PSTs learning to facilitate elementary students', including ELLs', engagement in Mathematical Practices 1 and 3 from the Common Core State Standards. This dissertation contributes to Zeichner's (2005) call for research on PSTs meeting ELLs' needs. My study and the course that served as its context were grounded in sociocultural theory, including Vygotsky's (1978) conviction that providing students with opportunities to speak about a topic promotes deep understanding of concepts. The PSTs discussed ideas related to the Mathematical Practices with their peers, host teachers, university supervisors, students, and with me as both their instructor and researcher. In alignment with sociocultural theory, my study considered both PSTs' personal experiences and how they viewed their students' personal experiences during their placements (Forman, 2003).

With our country's burgeoning ELL population and deepening commitment to the Common Core State Standards, it is imperative that teacher education programs give close attention to how they prepare PSTs to facilitate ELLs' engagement in the Mathematical Practices. It is of the utmost importance that we advance PSTs along their trajectory of meeting ELLs' needs—and all students' needs—in mathematics. One way of furthering this goal might be for researchers to use strategies, similar to the ones I have presented here, to study their own courses as well as by studying PSTs' experiences in classes taught by others.

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APPENDICES

Appendix A:

Semi-Structured Questions for First Interview of 22 Preservice Teachers

1. Describe the Mathematical Practices from the Common Core State Standards.
2. Name five strategies for teaching mathematics to ELLs and why they are important.
3. What did you learn about the connection between language and mathematics from this course?
4. What did you learn about teaching mathematics to ELLs from this course?
5. How is your preparation to teach mathematics to ELLs after taking the course compared to before the course? How could you be more prepared to teach mathematics to ELLs? What experienced besides the course help you teach mathematics to ELLs?
6. Explain how this course will help you teach mathematics when you are a teacher and give suggestions for how to improve the course.

Semi-Structured Questions for Second Interview of 8 Preservice Teachers

1. Describe how you would teach an addition lesson to accommodate all learners, including ELLs. (I would analyze how much they promote Mathematical Practices1 (‘productive struggle’) and Mathematical Practices3 (discourse) for the ELLs and all students.)
2. How could probing questions facilitate ELLs’ engagement of the Mathematical Practices?
3. How did your teacher promote Mathematical Practices to engage all learners?
4. What changes would you make to engage all learners more deeply in the Mathematical Practices?
5. Please comment on the supervisor’s comments about your lesson.
6. Tell me more about your field placement.

Semi-Structured Questions for Host Teachers

1. Create rapport. How is the year going?
2. Why did you choose to work with a pre-service teacher?
3. Describe the class environment.
4. What are your pre-service teacher's best qualities?
5. In what areas does she need to improve?
6. How are the University's ideas different from the school's ideas?
7. What parts of the math course, if any, did the pre-service teacher discuss with you?
8. What is your opinion of the LES model? How effective was the pre-service teacher at implementing the LES model?
9. What is your opinion of the Common Core State Standards? In what ways, if any, did the pre-service teacher engage her students in the Common Core State Standards?
10. Discuss Mathematical Practice 1. In what ways, if any, did the pre-service teacher promote students to engage in Mathematical Practice 1?
11. Discuss Mathematical Practice 3. In what ways, if any, did the pre-service teacher promote students to engage in Mathematical Practice 3?
12. One theme in the class was whether math is like learning another language. Comment on that. In what ways, if any, did the pre-service teacher implement this idea in the class?
13. Discuss modeling. What are the pros and cons? How much did the pre-service teacher model in the class?
14. What else would you like to tell me about anything discussed?

Semi-Structured Questions for University Supervisors

1. Why did you choose to supervise a pre-service teacher/s?
2. Describe the pre-service teacher.
3. Describe the class environment.
4. How are the University's ideas different from the school's ideas?
5. What parts of the math course, if any, did the pre-service teacher discuss with you?
6. What is your opinion of the LES model? How effective was the pre-service teacher at implementing the LES model?
7. What is your opinion of the Common Core State Standards? In what ways, if any, did the pre-service teacher engage their students in the Common Core State Standards?
8. Discuss Mathematical Practice 1 (Make sense and persevere in problems). In what ways, if any, did the pre-service teacher promote students to engage in Mathematical Practice 1?
9. Discuss Mathematical Practice 3 (Develop an argument and critique the argument of others). In what ways, if any, did the pre-service teacher promote students to engage in Mathematical Practice 3?
10. One theme in the class was whether math is like learning another language. Comment on that. In what ways, if any, did the pre-service teacher implement this idea in the class?
11. Discuss modeling. What are the pros and cons? How much did the pre-service teacher model in the class?
12. What else would you like to tell me about anything discussed?

APPENDIX B

Pre-Survey (January, 2014)-EED 323

Please answer the questions fully. You may write on the back of the paper if you need more space.

Name ______________________

1. Do you speak any other languages besides English? Explain which languages and to what degree.

2. Describe your race and ethnicity.

3. What training have you had with English language learners (ELLs) on and off campus?

4. The most important thing for meeting ELLs' needs is…

5. Do you believe that English language learners should be able to speak in their own language in the classroom? Explain.

6. Explain in detail how you would help English language learners in math.

Post-Survey (April, 2014)-EED 323

Please answer the questions fully. You may write on the back of the paper if you need more space.

1. The most important thing for meeting ELLs' needs is…

2. Do you believe that English language learners should be able to speak in their own language in the classroom? Explain.

3. Explain in detail how you would help English language learners in math.

APPENDIX C

Mathematical Assessment for PSTs Working with ELLs

Circle one of the answers below and justify why you chose your answer in the space provided.

(**SD** Strongly disagree **D** Disagree **N** Neutral **A** Agree **SA** Strongly agree)

Task

The mathematical task was within ELLs' zone of proximal development SD D N A SA

Justification ______________________________

The task was culturally relevant for ELLs. SD D N A SA

Justification ______________________________

The task developed ELLs' language skills. SD D N A SA

Justification ______________________________

Launch

The PST spoke at a pace that was accessible for ELLs. SD D N A SA

Justification ______________________________

The teacher (PST) made the content accessible for ELLs.
(Examples: gestures, clear directions, visuals, translations) SD D N A SA

Justification ______________________________

Explore

The teacher facilitated ELLs to develop mathematical arguments. SD D N A SA

Justification ____

The teacher facilitated ELLs to critique the argument of others. SD D N A SA

Justification ____

The teacher facilitated ELLs to persevere with task/s. SD D N A SA

Justification ____

The teacher asked ELLs probing questions. SD D N A SA

Justification ____

Summary

The teacher facilitated ELLs to use multiple representations.
(Examples: Manipulatives, pictures, gestures) SD D N A SA
Justification ____

The teacher used strategies to include ELLs in conversations.

(Revoiced, smiled, translated, praised, wait time) SD D N A SA

Justification ____

*Assessed by*______________________________

APPENDIX D

Summary of PSTs' Responses from Pre- and Post-surveys

Question 1. The most important thing for meeting ELLs' needs is…		
PST	January (pre-survey)	April (post-survey)
Abigail	Making every language "normal" so that everyone feels included.	Is to simplify their language and make modifications that will meet their needs individually. No matter what content, subject, language being spoken, the needs of the student should be put first and foremost.
Amanda	To provide services, translations, and resources to help them feel like part of the class. It will help them keep a part of their culture while learning English.	-Differentiation: make sure you have peer supports, translations, and gestures -Actually recognize that there are ELLs and provide supports tailored to their needs
Charles	To understand their culture, and where they came from, as well as their background, and how they became your students. Also, to learn about their experiences with English.	Visuals, language translations, peers support, gestures, body language.
Emily	Making the student comfortable and accessible to both languages.	-providing language enforcement in both English and their own language. -giving translations, images, props, extra support, culturally relevant materials.
Fatima	To incorporate their culture and language into their learning. Teach an additional language, not as a replacement language	<u>Top 5</u> Manipulatives Gestures Cultural relevance Peer support Visuals
Kim	Patience and being able to communicate in the classroom somehow.	Smiling, using body language and gestures to grab attention and focus. Visuals probably come in a near second place to body language.
Maria	Making them feel comfortable in	Incorporating their culture into the math

	the classroom.	context will make ELL's feel more included into the classroom and feel less isolated. Not only does it make Ell's feel like they have a role as a student, but incorporating culture allows for others to learn about other content areas.
Sarah	To be able to have a form of communication with them.	Make sure they have the supports so they understand. Just because math is a lot about numbers, its more about language and strategies.

Question 2. Do you believe that English language learners should be able to speak in their own language in the classroom? Explain.		
PST	January (pre-survey)	April (post-survey)
Abigail	Yes to the extent that learning continues to happen at all levels.	If we lived in an open world, I would say yes. If the teacher can speak another language and incorporate it meaningfully into the classroom.
Amanda	Yes, because they should be able to feel comfortable and learn in a way that most benefits their own needs.	Yes, they may need to in order to persevere and make sense of problems (MP1) They also need to feel included and important to the community.
Charles	No, because this promotes dependency. It is important to respect students' culture and promote their use of language while outside of the class, but students should continually practice the language that they are learning while in the classroom.	Yes, but to an extent. I believe they should only speak their language when they are truly struggling with English. However, if their English is simply poor but still good enough to be understood they should use it so that they become more fluent.
Emily	Yes, ELL student should be progressing in both their own language and English.	Yes, students need to be practicing and growing in their own language as well as English to keep their identity.
Fatima	Yes, like I said above. It is part of who they are.	Of course! You want [to] embrace their language as enrichment to the classroom community's culture. Your overall instruction should make English an additional language rather than a replacement language.
Kim	Yes, it is part of them; I would make sure I made the right	Yes, but they should be encouraged to learn English. Make their language

	accommodations to make sure they can speak their own language, but still communicate throughout the class.	something they can share with and teach the class.
Maria	Yes, because that way other people can learn words in other languages.	Yes! I believe it so important to know a second language because it will benefit people in all areas. With young learners, they absorb things like a sponge and they will be engage and motivated to learn.
Sarah	Yes, but I also believe they should try to speak English so that other students understand them.	Yes, it should be a culturally relevant class, and their language should be used and respected. Then other students can learn their language which creates a supportive environment.

Question 3. Explain in detail how you would help English language learners in math.		
PST	January	April
Abigail	Have #s available in different languages. Look for commonalities/differences in math across the different cultures.	I would help English language learners learn math by simplifying or by providing translations to make the language an easy thing for them to understand and to jumpstart their comfortableness of math as a subject area. Another thing I would do to help ELL's learn math is to be cultural relevant by connecting math to their lives and communities of math not just something they have to learn.
Amanda	I would provide translations of numbers and use a variety of visuals to help represent computations. I could also provide hands-on activities to represent the math.	-I will keep the MPs in mind -I will use LES and question -Incorporate culturally relevant manipulatives -translations -Word wall -Peer support

		-gestures
Charles	I think that math is universal, so I would explain to the students using English terms to try and get them acquainted with the language. However, I would mostly try to explain the math through examples, letters used and symbols.	I would study their culture and understand it better. I would also make sure to use a lot of gestures and body language as well as visuals. If possible, I will make translations however this may not be possible if the language is too challenging. At all times, I will have other students provide peer support.
Emily	I would use models and also use their native language.	I would be giving them translations, modifications, images, and extra support. ELLs easily get lost in subjects and I would make sure they had access to all information and support so they would not get lost.
Fatima	I would make sure that their language/ability to speak English doesn't affect their learning of math.	Give them manipulatives, have them work in groups with individual jobs that give them role and accountability in the group. I would make sure to use a lot of manipulatives. They would use it to solve the problem (input) as well as (output) showing of their answer.
Kim	I would use both metric and standard systems of measurement, teach numbers and math terms in English and ELL language. It would be very hard to explain things; I feel like I would have to make things very visual for students to see and understand no matter what language is spoken. Math seems like it could be its own language.	I would be there to support them, honestly, before some other students. I would use visuals, written explanations, gestures, and some applicable translations for everything. Obviously, this is very idealistic but may not be realistic. But any gesture or positivity helps ELLs from my observations. As much clarity, simplicity, support by visuals, directions are crucial to ELLs understanding.
Maria	I believe math to be the same in every country so it is only a matter of knowing how to say numbers in a specific language.	To help English language learners in math I would incorporate vocabulary words into the lesson prior to the lesson. I believe that the culture of the ELL should be taught, so I would focus on this and that way other students are aware of who is in the class. When I teach my math lessons I plan to use culture and reference to my students when I teach new content. I would also incorporate hands on learning that will benefit ELL's and use visuals to explain the content.

Sarah	I would learn vocabulary and numbers in their language.	-I would model and demonstrate -peer support, small group -translations, dictionary -use assistive technology: ex IPad -USE MANIPULATIVES -labels -talk slow, soft tone, clear, smile

VITA

James Ewing was born in Ithaca, New York. After graduating from Ithaca High School in 1979, James received a Bachelor's of Arts in 1983 from the University of Connecticut where he majored in sociology. He was an elementary teacher in Dryden, New York in 1985 for a year, and then at American School of Bilbao, Spain for three years. From 1989 to 2010, James taught grades three to six at Sunny View School, Torremolinos, Spain. For the last eight years at this school, he was both a teacher and head of primary school. James received his Master's degree in Elementary Education from the State University of New York at Cortland in 1993. James received his Doctors of Philosophy in Teaching and Curriculum from Syracuse University in 2016. At Syracuse University James received the Outstanding Teaching Assistant Award and the Berj A. Harootunian Research Award in 2014.