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Panel Unit Root Tests and Spatial Dependence

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Center for Policy Research
Working Paper No. 88

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December 2006

$5.00

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Abstract:

This paper studies the performance of panel unit root tests when spatial effects are present that account for cross-section correlation. Monte Carlo simulations show that there can be considerable size distortions in panel unit root tests when the true specification exhibits spatial error correlation. These tests are applied to a panel data set on net real income from the 1000 largest French communes observed over the period 1985-1998.

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SUMMARY

This paper studies the performance of panel unit root tests when spatial effects are present that account for cross-section correlation. Monte Carlo simulations show that there can be considerable size distortions in panel unit root tests when the true specification exhibits spatial error correlation. These tests are applied to a panel data set on net real income from the 1000 largest french communes observed over the period 1985-1998.

1. INTRODUCTION

Panel data unit root tests have been proposed as alternative more powerful tests than those based on individual time series unit roots tests, see Levin, Lin and Chu (2002), Im, Pesaran and Shin (2003), and Breitung (2000) to mention a few of the popular tests used in economics to test purchasing power parity (PPP) and growth convergence in macro panels using country

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data over time. Banerjee (1999), Phillips and Moon (2000), Baltagi and Kao (2000), Choi (2005) and Breitung and Pesaran (2006) provide some reviews of this literature. One of the advantages of panel unit root tests is that their asymptotic distribution is standard normal. This is in contrast to individual time series unit roots which have non-standard asymptotic distributions.

But these tests are not without their critics. The test proposed by Levin, Lin and Chu (2002), hereafter denoted by LLC, is applicable for homogeneous panels where the AR coefficients for unit roots are in particular assumed to be the same across cross-sections. The null hypothesis is that each individual time series contains a unit root against the alternative that each time series is stationary[1]. As Maddala (1999) pointed out, the null may be fine for testing convergence in growth among countries, but the alternative restricts every country to converge at the same rate. Im, Pesaran and Shin (2003), hereafter denoted by IPS, allow for heterogeneous panels and propose panel unit root tests which are based on the average of the individual ADF unit root tests computed from each time series. The null hypothesis is that each individual time series contains a unit root while the alternative allows for some but not all of the individual series to have unit roots. One major criticism of both the LLC and IPS tests is that they require cross-sectional independence. This is a restrictive assumption given the cross-section correlation and spillovers across countries, states and regions.

Maddala and Wu (1999) and Choi (2001) proposed combining the p-values from the individual unit root ADF tests applied to each time series. Once again, these tests follow a standard normal limiting distribution. They have the advantage that N, the number of cross-sections, can be finite or infinite; the time series can be of different length; and the alternative allows some groups to have unit roots while others may not.

Recent studies that try to account for cross-sectional dependence in panel unit root testing include the following: Chang (2002) who explored the non-linear IV methodology to solve the inferential difficulties in the panel unit root testing which arise from the intrinsic heterogeneities and dependencies of panel models. Chang (2002) suggests an average of individual nonlinear IV t-ratio statistics of the autoregressive coefficient obtained from using an integrable transformation of the lagged level as instrument. These methods

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[1]This test is also consistent against a heterogeneous alternative as long as the fraction of cross-sections with a stationary AR coefficient converges to a positive constant (see Moon and Perron (2004a)).

assume cross-sectional correlation in the innovation terms driving the autoregressive processes. In another paper, Chang (2004) applies the bootstrap methodology to unit root tests for dependent panels. She proposes various tests which explicitly allow for the cross-correlation across cross-sectional units as well as heterogeneous serial dependence. Choi (2002), on the other hand, generalizes the three unit root tests (inverse chi-square, inverse normal and logit) to the case where the cross-sectional correlation is modelled by error component models. The tests are formulated by combining p-values from the ADF test applied to each individual time series whose stochastic trend components and cross-sectional correlations are eliminated using GLS-demeaning and GLS-detrending (see Elliott, Rothenberg and Stock (1996)). Choi (2002) shows that the combination tests have a standard normal limiting distributions under the sequential asymptotics $T \to \infty$ and $N \to \infty$.

To avoid the restrictive nature of cross-section demeaning procedure, Bai and Ng (2004), Moon and Perron (2004b) and Phillips and Sul (2003) propose dynamic factor models by allowing the common factors to have differential effects on cross-section units. Phillips and Sul's model is a one-factor model where the factor is independently distributed across time. They propose a moment-based method to eliminate the common factor which is different from principal components. More specifically, in the context of a residual one-factor model, Phillips and Sul (2003) provide an orthogonalization procedure which in effect asymptotically eliminates the common factors before preceding to the application of standard unit root tests. Moon and Perron (2004b) propose a pooled panel unit root test based on "de-factored" observations and suggest estimating factor loadings that enter their proposed statistic by the principal component method. Bai and Ng (2004) consider the possibility of unit roots in the common factors. They apply the principal component procedure to the first-difference version of the model, and estimate the factor loadings and the first differences of the common factors. Standard unit root tests are then applied to the factors and the individual "de-factored" series. Pesaran (2005) suggests a simple way of getting rid of cross-sectional dependence that does not require the estimation of factor loading. His method is based on augmenting the usual ADF regression with the lagged cross-sectional mean and its first-difference to capture the cross-sectional dependence that arises through a single factor model[2].

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[2]See Breitung and Pesaran (2006) and Choi (2005) for excellent surveys of this literature and a more formal treatment of the underlying assumptions behind each test.

This paper considers spatial dependence across the panels as an alternative means of capturing cross-section dependence among the countries. Spatial dependence models — popular in regional science and urban economics — deal with spatial interaction and spatial heterogeneity (see Anselin (1988)). The structure of the dependence can be related to location and distance, both in a geographic space as well as a more general economic or social network space. Section 2 presents some commonly used spatial error processes: the spatial autoregressive (SAR) and the spatial moving average (SMA) error process and the spatial error components model (SEC). Section 3 runs Monte Carlo simulations to compare the empirical size of panel unit root tests with and without spatial error dependence. We find that ignoring spatial dependence when present can seriously bias the size of panel unit root tests.[3] Section 4 provides an empirical illustration based on net real income from the 1000 largest french communes observed over the period 1985-1998. Section 5 summarizes the results and concludes.

2. SPATIAL ERROR MODELS

The spatial autoregressive (SAR) specification for the $(N \times 1)$ error vector u_t in period $t = 1, ..., T$ can be expressed as:

$$u_t = \theta_1 W_N u_t + \varepsilon_t = (I_N - \theta_1 W_N)^{-1} \varepsilon_t \tag{1}$$

where W_N is an $(N \times N)$ known spatial weights matrix, θ_1 is the spatial autoregressive parameter and ε_t is an $(N \times 1)$ error vector assumed to be distributed independently across cross-sectional dimension with constant variance σ_ε^2. The error covariance matrix for the cross-section at time t becomes:

$$
\begin{aligned}
\Omega_{t,N,SAR} &= E\left[u_t u_t'\right] = \sigma_\varepsilon^2 \left(I_N - \theta_1 W_N\right)^{-1} \left(I_N - \theta_1 W_N'\right)^{-1} \\
&= \sigma_\varepsilon^2 \left(B_N' B_N\right)^{-1} \text{ with } B_N = I_N - \theta_1 W_N
\end{aligned}
\tag{2}
$$

So the full $(NT \times NT)$ covariance matrix is:

$$\Omega_{SAR} = \sigma_\varepsilon^2 \left[I_T \otimes (B_N' B_N)^{-1}\right] \tag{3}$$

---

[3]Banerjee, Marcellino and Osbat (2004, 2005) consider the effect of cross-cointegration on the critical values of first generation panel unit root tests that do not account for cross-sectional dependence including LLC, IPS, Breitung and Maddala and Wu. They find these tests to be seriously biased. However, this is different from allowing for spatial error dependence among the cross-section units which may not be necessarily cointegrated.

Even when W_N is sparse, $(B'_N B_N)^{-1}$ will not be sparse. Hence, Anselin (2003) classifies the spatial covariance structure induced by this SAR model as *global*.

In contrast, a spatial moving average (SMA) specification for the $(N \times 1)$ error vector u_t in period $t = 1, ..., T$ can be expressed as:

$$u_t = \theta_2 W_N \varepsilon_t + \varepsilon_t = (I_N + \theta_2 W_N) \varepsilon_t \tag{4}$$

where θ_2 is the spatial moving average parameter. The error covariance matrix for the cross-section at time t becomes:

$$\Omega_{t,N,SMA} = E\left[u_t u'_t\right] = \sigma_\varepsilon^2 \left[I_N + \theta_2 \left(W_N + W'_N\right) + \theta_2^2 W_N W'_N\right] \tag{5}$$

So the full $(NT \times NT)$ covariance matrix is:

$$\Omega_{SMA} = \sigma_\varepsilon^2 \left[I_T \otimes \left[I_N + \theta_2 \left(W_N + W'_N\right) + \theta_2^2 W_N W'_N\right]\right] \tag{6}$$

The covariance matrix in (6) depends only on W_N and $W_N W'_N$ and there is no inverse matrix as for SAR. In fact, if W_N is defined as first order contiguity, such elements consist of location pairs that are first and second order neighbors but there is no higher order contiguity. Hence, Anselin (2003) classifies the spatial covariance structure induced by this SMA model as *local*.

An alternative to SAR and SMA models is the spatial error components (SEC) specification, suggested by Kelejian and Robinson (1995). Anselin, Le Gallo and Jayet (2006) argue that the range of the covariance induced by the SEC model is a subset of that of the SMA model, and hence it is also a case of *local* spatial spillovers.

In the SEC model, the error term is decomposed into a local and a spillover effect. The $(N \times 1)$ error vector u_t in period $t = 1, ..., T$ is expressed as:

$$u_t = \theta_3 W_N \psi_t + \varepsilon_t \tag{7}$$

where ε_t is an $(N \times 1)$ vector of local error components and ψ_t is an $(N \times 1)$ vector of spillover error components. The two component vectors are assumed to consist of *iid* terms with respective variances σ_ε^2 and σ_ψ^2 and are uncorrelated. The resulting $(N \times 1)$ cross-sectional error covariance matrix is then:

$$\Omega_{t,N,SEC} = E\left[u_t u'_t\right] = \sigma_\varepsilon^2 I_N + \sigma_\psi^2 \theta_3^2 W_N W'_N \tag{8}$$

and the overall $(NT \times NT)$ covariance matrix is[4]:

$$\Omega_{SEC} = \sigma_\varepsilon^2 I_{NT} + \sigma_\psi^2 \theta_3^2 \left(I_T \otimes W_N W_N' \right) \qquad (9)$$

In the simplest case, the weights matrix is binary, with $w_{ij} = 1$ when i and j are neighbors and $w_{ij} = 0$ when they are not. By convention, diagonal elements are null: $w_{ii} = 0$ and the weights are almost always standardized such that the elements of each row sum to 1.

3. A MONTE CARLO STUDY

3.1. The design of the Monte Carlo

In this section, we consider the small sample performance of several unit roots tests allowing for spatial error dependence in the true model. Following Pesaran (2005), we consider dynamic panels with individual effects. The data generating process (DGP) in this case is given by:

$$y_{it} = (1 - \rho_i) \mu_i + \rho_i y_{it-1} + u_{it} \ , \ i = 1, ..., N \ , \ t = 1, ..., T \qquad (10)$$

where

$$\mu_i \sim iid.U\,(0, 0.02) \ , \ u_{it} \sim iid.N\left(0, \sigma_i^2\right)$$

with $\sigma_i^2 \sim iid.U\,(0.5, 1.5)$. In another set of experiments, we allow for individual deterministic trends in the DGP. For this case, y_{it} is generated as follows:

$$y_{it} = \mu_i + (1 - \rho_i) \delta_i t + \rho_i y_{it-1} + u_{it} \ , \ i = 1, ..., N \ , \ t = 1, ..., T \qquad (11)$$

with $\delta_i \sim iid.U\,(0, 0.02)$. This ensures that y_{it} have the same average trend properties under the null and the alternative hypotheses.

In each case, i.e., the individual effects without deterministic trends and individual effects with deterministic trends, we consider three alternative specifications for the spatial error dependence. In particular, we let u_{it} follow the SAR, SMA or SEC specifications described in section 2 with $\sigma_\varepsilon^2 = 1$ and

$$\begin{cases} \theta_1 = 0.4, 0.8 & \text{for SAR given in (1)} \\ \theta_2 = 0.4, 0.8 & \text{for SMA given in (4)} \\ \theta_3 = 0.4, 0.8 \quad \sigma_\psi^2 = \begin{cases} 0.1\sigma_\varepsilon^2 & \text{(SEC1)} \\ \sigma_\varepsilon^2 & \text{(SEC2)} \\ 10\sigma_\varepsilon^2 & \text{(SEC3)} \end{cases} \text{ for SEC given in (7)} \end{cases} \qquad (12)$$

---

[4]If $\sigma_\psi^2 = \sigma_\varepsilon^2$ and $\theta_2 = \theta_3$, then the SEC specification is similar to the SMA error process.

For the SEC specification, the three values of σ_ψ^2 allow one to attach more or less importance to the spillover error components (ψ_t) as compared to the local error components (ε_t). If $\sigma_\psi^2 = \sigma_\varepsilon^2$, then the SEC specification is similar to SMA error process because we have the same values for θ_2 and θ_3 (0.8 or 0.4). We vary the panel size so that $N = 50, 100$ and $T = 25, 50$. Following Kelejian and Prucha (1999, p. 520), we use several spatial weight matrices which differ in their degree of sparseness. The first matrix, for example, is a "1 ahead and 1 behind" matrix with the i-th row $(1 < i < N)$ of this $N \times N$ matrix having non-zero elements in positions $i + 1$ and $i - 1$. This describes an i-th cross-sectional unit whose disturbances are related to those of the cross-sectional units immediately before it and immediately after it[5]. This matrix is row normalized so that all its non-zero elements are equal to 1/2. The other spatial weight matrices considered are labelled as "j ahead and j behind" with the non-zero elements being $1/2j$, for $j = 2, ..., 10$.

Following Pesaran (2005), we report size[6] of the panel unit root tests under the null $\rho_i = 1$ for all $i = 1, 2, .., N$, and heterogeneous alternatives $\rho_i \sim iid.U\,(0.85, 0.95)$, using 1000 replications for each experiment. For each y_{it}, we generate $T + 40$ observations and drop the first 40 observations in order to reduce the dependency on initial values. For each experiment, we perform nine panel unit root test statistics: the Levin, Lin and Chu test (2002) (hereafter LLC), the Breitung test (2000) (hereafter B), the Im, Pesaran and Shin test (2003) (hereafter IPS), the Maddala and Wu test (1999) (hereafter MW), the Choi tests (2001, 2002) with and without cross-sectional correlation (hereafter Choi_c and Choi)), the Chang IV test (2002), the Phillips and Sul test (2003) (hereafter PS) and the Pesaran test (2005). Our experiments include a case of no spatial correlation as well as four types of spatial correlation (SAR, SMA, SEC1 and SEC3), with two values of the parameters indicating weak versus strong spatial dependence. We also consider ten weight matrices, differing in their degree of sparseness, four pairs of (N, T) and two models including individual effects and individual deterministic trends. Even with this modest design, the total number of experiments considered is 1600.

3.2. Monte Carlo results

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[5]The matrix is defined in a circular world so that the non-zero elements in rows 1 and N are, respectively, in positions $(1, N)$ and $(N, 1)$.

[6]The size ajusted power tables are also available upon request from the authors.

3.2.1. Cross-section correlation statistic, global and local range of dependence

Pesaran (2004) proposed a general test of error cross-section dependence as an alternative to the Breusch-Pagan LM test. Pesaran's test statistic is based on the pair-wise correlation coefficients themselves rather than their squares:

$$CD = \sqrt{\frac{2T}{N\left(N-1\right)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \widehat{\rho}_{ij} \right)$$

where $\widehat{\rho}_{ij}$ is the sample estimate of the pair-wise correlation of OLS residuals $\widehat{u}_{it}$:

$$\widehat{\rho}_{ij} = \frac{\sum\limits_{t=1}^{T} \widehat{u}_{it}\widehat{u}_{jt}}{\sqrt{\sum\limits_{t=1}^{T} \widehat{u}_{it}^2}\sqrt{\sum\limits_{t=1}^{T} \widehat{u}_{jt}^2}}$$

Under the null (*i.e.*, no cross-section dependence), $CD \Rightarrow N\left(0,1\right)$. Pesaran (2004) shows that this test has good small sample properties for small N and T. The simple average $\widehat{\overline{\rho}}$ of these correlation coefficients across all the $\left(N \times \left(N-1\right)\right)/2$ pairs is given by:

$$\widehat{\overline{\rho}} = \sqrt{\frac{2}{TN\left(N-1\right)}}CD$$

Table 1 gives the average CD statistic and the average cross-section error correlation coefficient $\widehat{\overline{\rho}}$ for $N = 50$ and 100, $T = 25$ and $\theta = 0.8$, using 1000 replications. This is done for the two models given in (10) and (11) and for the three different specifications of spatial error dependence (SAR, SMA and SEC) with ten different weight matrices $W\left(j,j\right)$ for $j = 1, ..., 10$. The average cross-section correlation coefficients increase with $j = 1, ..., 10$. For $N = 50$, this correlation measure is high for SAR (between 8.56% and 28.66%) and also for SEC3 (between 1.74% and 18.30%) when the spillover error components $\left(\psi_t\right)$ is important as compared to the local error components $\left(\varepsilon_t\right)$ $\left(\sigma_\psi^2 = 10\sigma_\varepsilon^2\right)$. Note also that the average correlation coefficients tend to be small for SEC1 with a small spillover error components $\left(\sigma_\psi^2 = 0.1\sigma_\varepsilon^2\right)$. For SMA and SEC2 $\left(\sigma_\psi^2 = \sigma_\varepsilon^2\right)$, these coefficients lie between 2.81% and 4.18% and 0.93% and 3.42%, respectively. As N doubles from 50 to 100, $\widehat{\overline{\rho}}$ becomes half its size.

Except for SEC1 ($\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$), the CD statistics reject the null of no cross-section dependence for all spatial errors processes considered. The highest values for the CD statistic are those of the SAR specification, followed by the SEC3 specification ($\sigma_\psi^2 = 10\sigma_\varepsilon^2$). When N doubles, we get similar results. Introducing a deterministic trend in the dynamic individual effects model does not change the results. In summary, the cross-section dependence measure $\widehat{\overline{\rho}}$ and the corresponding test statistic CD are useful diagnostics and Table 1 shows how they vary with various models of spatial dependence and degree of sparseness of the spatial weights matrix[7].

3.2.2. The Spatial Dependence Specification Effect

Table 2 gives the empirical size of all panel unit root tests considered for the dynamic panel model with individual effects given by (10). For each (N, T) pair ($N = 50, 100, T = 25, 50$), we give the rejection rates of the null hypothesis when it is true at the 5% significance level. This is done for the standard case (*i.e.*, the benchmark model, where there is no spatial dependence) and in cases where spatial effects are included (12). Table 2 reports the results for only two standardized weight matrices labelled $W(1, 1)$ and $W(5, 5)$ for "1 ahead and 1 behind" and "5 ahead and 5 behind". Figures 1 to 5 summarize the effects of varying these weight matrices on the size of the various panel unit root tests.

In Table 2, for $N = 50$, $T = 25$ with *no* spatial correlation, all the tests yield empirical size which is not statistically different from the 5% nominal size. This varies from 3.7% for the Breitung test to 5.6% for the Maddala-Wu and Choi tests. For the SAR specification, with the sparse weight matrix $W(1, 1)$, we observe the following: if we increase the degree of spatial dependence from 0.4 to 0.8, all panel unit root tests considered become oversized yielding rejection rates as high as 17.8% for the B test and as low as 10.4% for Phillips and Sul's test and 10.6% for the Pesaran test. These rejection rates are smaller for tests allowing for cross-section dependence (16% for Choi_c, 13.7% for Chang, 10.6% for Pesaran and 10.4% for Phillips and Sul) than for

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[7]Another useful and well-known test is Moran's I statistic where the null hypothesis is the absence of spatial dependence *versus* no precise expression for spatial dependence, see Anselin (1988). Since this is a cross-section statistic, we ran this test as an extra check using the last time period $T = 25$ for the case where $N = 50$. Like the CD test of Pesaran, this test rejected the null of no spatial dependence no matter what the degree of sparseness of the weight matrix. The only exception is the SEC1 specification where the test cannot detect the small spillover effect with $\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$.

tests assuming cross-section independence (17.8% for Breitung's test, 17.4% for LLC, 16.9% for MW, 16.7% for Choi and 16.3% for IPS). When T increases from 25 to 50 and/or when the number of individuals increases from $N = 50$ to $N = 100$, we get similar results[8]. Including spatial effects with a SAR specification leads to considerable size distortions in panel unit roots tests. These size distortion are not as bad when we consider the SMA specification of cross-section dependence. For $N = 50$, $T = 25$ and when the degree of spatial dependence is 0.8, the size of the tests vary between 7% for the Pesaran test and the Phillips and Sul test and 10.4% for the LLC test. Once again, tests that allow for cross-section dependence like the Chang test, the Pesaran test and the Phillips and Sul test yield a lower frequency of type I error than tests that assume cross-section independence. When T increases from 25 to 50, with $N = 50$ and $\theta = 0.8$, Pesaran's test yields the lowest frequency of type I error (6.4%) followed by Phillips and Sul's test (6.9%) and Chang's (8.8%) test with the highest frequency reported for LLC (12.9%). For the SEC specification of cross-section dependence, size distortions are very small and negligible when the spillover error components (ψ_t) have a smaller variance than the local error components (ε_t) (*i.e.*, SEC1 with $\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$). However, when the spillover error variance is ten times that of the local error components (*i.e.*, SEC3 with $\sigma_\psi^2 = 10\sigma_\varepsilon^2$), the MW and Choi tests become oversized yielding a frequency of type I error as high as 9.5%.

As shown by the CD statistic (Table 1), with the sparse weight matrix $W(1, 1)$, we reject the null hypothesis of no cross-section correlation or no spatial dependence for SAR, SMA and the SEC3 $(\sigma_\psi^2 = 10\sigma_\varepsilon^2)$. In these cases, all panel unit roots tests considered are oversized. When the null hypothesis of no spatial dependence is not rejected, i.e., for SEC1 $(\sigma_\psi^2 = 0.1\sigma_\varepsilon^2)$, all the tests yield empirical size which is not statistically different from the 5% nominal size.

3.2.3. The Spatial Weight Matrix Effect

The second panel of Table 2 reports the size of the panel unit root tests as we increase the number of neighbors in the weight matrix. In this case, we go from $W(1, 1)$ to $W(5, 5)$. For the SAR process, the size distortions for the panel unit root tests, especially those that do not account for cross-sectional dependence, increase. In fact, for $N = 50$, $T = 25$ and $\theta = 0.8$,

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[8]In case N is large (say over 500) and T is small ($T = 25$), we get similar results in terms of empirical size for all the tests except for Phillips and Sul and Pesaran tests for which size distortions strongly decline whatever the spatial error process.

the B test's frequency of type I error increases from 17.8% for $W(1,1)$ to 21.5% for $W(5,5)$. The Phillips and Sul and Pesaran's test seem to be the least affected. These tests yield a frequency of type I error of 10.4% and 10.6% for $W(1,1)$ and 8.9% and 9.9% for $W(5,5)$, respectively. This is not the case for Chang's test where the corresponding frequencies are 13.7% for $W(1,1)$ and 17.6% for $W(5,5)$. Choi_c's comparable frequencies are 16.0% for $W(1,1)$ and 15.3% for $W(5,5)$. Figure 1 plots the empirical size of the panel unit roots tests for various weight matrices, $W(j,j)$, $j = 1,...,10$, for the spatial autoregressive (SAR) specification. Panel unit roots tests that allow for cross-section dependence (Choi_c, Pesaran and Phillips and Sul) have the lowest size distortions and have a decreasing profile as j increases. This is in contrast to the other panel unit root tests that assume cross-section independence whose profiles show an empirical size around 20% for various degrees of sparseness of the weight matrix.

Figure 2 plots the empirical size of the unit roots tests of the spatial moving average error process for various $W(j,j)$, $j = 1,...,10$. For this SMA process, the size distortions are worse for $W(1,1)$ than for $W(5,5)$. Instead of a monotonic evolution as in the case of the SAR, the size distortion reveals a peak around 10% for $W(1,1)$ and a convergence to the 5% nominal size for $W(j,j)$ as j increase from 1 to 10. Again, the Pesaran and Phillips and Sul tests are the closest to the 5% nominal size no matter what weight matrix is considered. Figure 2 suggests that, for the SMA specification, the size distortions of the tests decrease as j increases. This may be due to the specific structure of the error covariance matrix Ω_{SMA} given in (6), which depends only on W_N and $W_N W_N'$. Whatever the order contiguity of W_N, the covariance with the first order neighbor is higher than with all other covariances. So, the range of spatial dependence is much smaller than that for a corresponding SAR model.

Figures 3 to 5 plot the empirical size of the unit roots tests with spatial error components (SEC) with different variances for the spillover effects ($\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$, $\sigma_\psi^2 = \sigma_\varepsilon^2$ and $\sigma_\psi^2 = 10\sigma_\varepsilon^2$). When the spillover error components (ψ_t) do not have a higher variance than the local error components (ε_t), all the tests have empirical size that is close to the 5% nominal size (see SEC1 and SEC2 in Figures 3 and 4). However, when $\sigma_\psi^2 = 10\sigma_\varepsilon^2$ (*i.e.*, SEC3 in Figure 5), the size distortions increase with j. In fact, for $W(10,10)$, we get the following empirical size for the various tests considered: Choi (18.4%), MW (18.2%), Chang (12.5%), Choi_c (12%), Pesaran (8.5%) and Phillips and Sul (7.2%). What is surprising is that the tests that assume cross-sectional

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independence have size close to the 5% nominal size whatever the sparseness of the weight matrix is for the SEC specification. Figure 5 suggests that tests assuming cross-section independence perform overall better than those allowing for cross-section dependence when the spillover component in the SEC process has a higher variance than the local error component. In the group of cross-sectional dependence tests, only Pesaran and Phillips-Sul tests exhibit empirical sizes which are close to 5%. These two tests allow the common factors to have differential effects on cross-section units whereas the Choi and Chang tests do not allow for common factors and suppose that the cross-sectional correlation is modelled by error component models. In fact, Choi uses an error component model rather than a factor structure to handle the cross-dependence. Chang (2002) allows for general cross-sectional dependence of the error terms u_{it}. The key aspect of Chang's approach is to use a different estimator whose t-ratios are independent for all cross-sections even, if they are correlated. However, her non linear IV estimator converges slowly (at a rate of $T^{1/4}$) rather than the usual faster T rate[9]. These characteristics explain the differences in the specification of the cross dependence and may be at the origin of the contrasted results for the SEC specification. In other words, Choi and Chang tests seem to be sensitive to misspecification of the SEC error correlation while tests assuming cross-section independence — and Pesaran and Phillips-Sul tests allowing for common factors — perform better[10].

Figure 6 plots the empirical size at the 5% nominal level of the four tests which explicitly allow for cross-sectional dependence for $N = 100$ and $T = 50$. This is done for the dynamic panel model with individual effects given by (10). We plot the case when $\theta = 0.8$ and $\sigma_\psi^2 = 10\sigma_\varepsilon^2$. We compare the behavior of these tests when the underlying specification includes SAR,

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[9]Chang (2002) proposes a nonlinear instrumental variable approach where the IGF (Instrument Generating Function) $F(y_{i,t-1}) = y_{i,t-1}e^{-c_i|y_{i,t-1}|}$ is used an an instrument for estimating $(1 - \rho_i)$. The choice of the parameter c_i is crucial for the properties of the test. This parameter should be proportional to $(s^{-1}(\Delta y_{i,t}))$, the inverse of the standard deviations of $\Delta y_{i,t}$: $c_i = KT^{-1/2}s^{-1}(\Delta y_{i,t})$. Chang notes that when the time dimension is small $(T = 25)$, her test slightly over-rejects the null and she proposes to use a larger value of K to correct the upward size distortion. For a critique of this test, see Im and Pesaran (2003).

[10]In this context, Breitung and Das (2005) found that the GLS-t statistic may have severe bias if T is only slightly larger than N and the robust OLS t-statistic performs slightly worse but outperforms the nonlinear IV test of Chang (2002).

SMA or the SEC spatial error dependence specifications. For large N and T, the Phillips and Sul test and the Pesaran test have the lowest size distortions for each case (SAR, SMA or SEC). The Choi_c and Chang tests give second best results. Figure 6 also confirms that the SAR specification leads to the largest size distortions, followed by the SEC specification (when $\sigma_\psi^2 = 10\sigma_\varepsilon^2$). However, for the SMA specification, and except for $W(1,1)$, all four tests have empirical size close to the 5% nominal size.

Table 3 gives the empirical size at the 5% nominal level of all tests considered for the dynamic panel model with individual effects *and* deterministic trends given by (11) (see also the Figures 7 to 9). Results are very similar to those obtained in Table 2, but the magnitudes are slightly different. For example, for $N = 50$, $T = 25$ and $\theta = 0.8$, the empirical size in Table 2 for the SAR process with the W(1,1) weight matrix, varied between 10.4% and 17.8%. With a deterministic trend in Table 3, the same empirical size varied between 12.8% and 18%. For W(5,5), the empirical size varied between 8.9% and 21.5% in Table 2 and 11.7% and 23.2% in Table 3. Spatial error processes generate different profiles of size distortions depending on the spatial error specification: SAR, SMA or SEC.

3.2.4. Sensitivity to Irregular Lattice Structures and Row Standardization

The spatial weights matrices considered in the paper are regular lattice structures. Using real irregular lattices structures, as in Anselin and Moreno (2003) and in Kelejian and Prucha (1999), does not change the conclusions of the Monte Carlo study. We used real world matrices by taking spatial groupings of French administrative communes for dimension $N = 50$ and 100 (see section 4 and Figures 10 and 11) and, when the underlying true specification exhibits SAR or SEC error correlations, we show that the magnitude of size distortions are similar to the previous cases of the simulation study when regular lattice structures are used (see the appendix and Tables A1 and A2 which are available upon request from the authors).

We also check if the row-standardization of the spatial weights matrix has an impact on the degree of spatial correlation and on the size of the tests. Using the same real world matrices, we checked the impact of the row-standardization on the size of the test. The non row-standardization increases the empirical size for SAR for the W(1,1) weight matrix and decreases it for the W(5,5) weight matrix. The reverse is true for the SEC3 (*i.e.*, $\sigma_\psi^2 = 10\sigma_\varepsilon^2$) specification, where for W(5,5) the empirical size increases

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for non-row standardization as compared to row-standardization. Note that for the row-standardized weights matrix, as j increases, the value of non-zero elements $(1/2j)$ decreases and, this in turn may reduce the amount of spatial correlation. In contrast, for the non row-standardized weights matrix, the weights are always the same ($w_{ij} = 1$ when i and j are neighbors and $w_{ij} = 0$ when they are not) and the amount of spatial correlation increases with the order of contiguity. So, the empirical size of these tests will be affected by whether we row standardize or not. Even for SEC1 where $\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$, the presence of spatial dependence leads to higher size distortions when the weight matrix is not row-standardized.

4. EMPIRICAL ILLUSTRATION: FRENCH INCOMES PER COMMUNE

Following Anselin and Moreno (2003), we consider irregular lattice structures by taking spatial groupings of $N = 50$, 100 and 1000 French administrative communes covering the period 1985-98 ($T = 14$)[11]. This data was obtained from INSEE census on "fiscal household" from fiscal time series at the municipality level (see the appendix).

Spatial weight matrices may represent high-order contiguity relationships. To illustrate these ideas, we use a k-order contiguity matrix for different data samples containing $N - 1$ potential neighborhoods in French municipalities (for $N = 50$, 100 and 1000). Figure 10 shows the pattern of 0 and 1 values in a $(N - 1 =)$ 49 by 49 grid for the 5-nearest neighborhoods. Note that a non-zero entry in row i, column j denotes that neighborhoods i and j have borders that touch and are therefore considered "neighbors". Of the 2401 possible elements in the 49 by 49 matrix, there are only 250 non-zero elements, designated on the axis of Figure 10 by the sparseness value of 10% ($= 250/2500$). These non-zero entries reflect the contiguity relations between the 5-nearest neighborhoods. The five-order contiguity matrix shown in Figure 10 is asymmetric. This reflects the fact that neighborhood j may be one of the 5-nearest neighborhoods to i, but j may have some other 5-nearest neighborhoods not including i.

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[11]$N = 50$ was obtained by restricting our sample to communes with more than 36,500 fiscal households in 1985. $N = 100$ was obtained by restricting our sample to communes with more than 23,500 fiscal households in 1985, and $N = 1000$ was obtained by restricting our sample to communes with more than 3495 fiscal households in 1985.

When we increase the number of municipalities (from $N = 50$ to $N = 100$, see figure 11), the sparseness (i.e., the percent of non-zero links between the 5-nearest neighbors) decrease to 5%. There are only 500 non-zero elements of the 9801 possible elements in the 99 by 99 matrix. Then, for a k-order spatial contiguity matrix for $(N-1)$ neighbors, the sparseness is given by $k/N\%$.

Using the net real income per fiscal household per commune (in logs) as the single $y_{i,t}$ series, we first checked the degree of spatial dependence with CD statistics. As N increases from 50 to 100 to 1000, the average cross-section error correlation coefficients $\widehat{\overline{\rho}}$ decrease from 65.33% to 64.46% to 42.16%, (see Table 4 which is available upon request from the authors). These results were obtained for the model given by (10) with individual effects only. Similar values were obtained for the model given by (11) with individual effects and deterministic trends. The CD statistics are highly significant, signalling strong cross-section dependence among the net real incomes in French communes. As N increases, the underlying spatial matrix becomes more sparse.

Turning to the unit root tests, we emphasize that $T = 14$ is smaller than the T used in the Monte Carlo experiments. In fact, we know that the size of the unit root tests is sensitive to the time length. For small N ($N = 50$) which is associated with strong cross-section dependence, panel unit root tests which do not account for cross-sectional dependence do not reject the null of panel unit roots except for LLC test. Levin et al. (2002, p. 18) argue that good performance for their test "depends crucially upon the independence assumption across individuals, and hence not applicable if cross sectional correlation is present".

When we increase N from 50 to 1000, with the spatial dependence becoming more sparse, panel unit root tests that do not account for cross-sectional dependence do not reject the null of panel unit roots. The exceptions are the LLC and IPS tests for the model given by (10) with individual effects only. Tests that account for cross-sectional dependence, do not reject the null of panel unit roots except for Pesaran's test and the Phillips and Sul test. These conflicting results may be attributed to the short time series length ($T = 14$). Pesaran and Choi's frameworks are restricted factor models, contrary to the unrestricted factor model of Phillips and Sul and the Chang test which allow for general cross-sectional dependence of the error terms. Breitung and Pesaran (2006) argue that the application of factor models in the case of weak correlation does not yield valid test procedures. They also emphasize that

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testing for unit roots in the common components are likely to require large panels, with the power of the test being good only when T is large.

5. CONCLUSION

Using Monte Carlo simulations, this paper studied the performance of panel unit root tests when spatial effects are present that account for cross-section correlation. Our results show that there can be considerable size distortions in panel unit root tests. The tests of Choi (2002), Chang (2002), Pesaran (2005) and Phillips and Sul (2003) which explicitly allow for the cross-sectional dependence have better performance than other classic panel unit root tests that assume cross-sectional independence. For the SAR specification of cross-sectional dependence, we get the largest size distortions of the panel unit root tests. In contrast, the SMA specification of cross-sectional dependence leads to the lowest size distortions of empirical size except for a "1 ahead and 1 behind" weight matrix. The Kelejian and Robinson (1995) SEC specification also generates small size distortions except when the spillover error components have a higher variance than the local error components. For the applied econometrician, the message from these experiments is that size distortions of panel unit root tests is highly sensitive to the underlying spatial dependence specification (especially to the SAR model and SEC error specifications) and to the sparseness of the spatial weight matrix.

APPENDIX

Following Anselin and Moreno (2003) and Kelejian and Prucha (1999), we use real irregular lattice structures. These were obtained by taking spatial groupings of the largest French administrative communes[12]. Using the 1985 net real income per fiscal household per commune (in logs) as the initial value for y_{i0}, we simulate our model with individual effects. We use $\mu_i \sim iid.U(0,0.02)$, $\delta_i \sim iid.U(0,0.02)$ and u_{it} is either $iid.N(0,\sigma_i^2)$ with $\sigma_i^2 \sim iid.U(0.5,1.5)$ or follows a spatial error specification (SAR, SMA or SEC) with $\sigma_\varepsilon^2 = 1$ and $\theta_1 = \theta_2 = \theta_3 = 0.8$ and $\sigma_\psi^2 = 0.1\sigma_\varepsilon^2$, σ_ε^2 or $10\sigma_\varepsilon^2$. We consider only one time period length ($T = 25$). Figures 10 and 11 show the sparseness of the weight matrix for the 50 and 100 largest French communes. Table A1, which is available upon request from the authors, gives the empirical size of the panel unit root tests as the sparseness of the weight matrix increases for $N = 50$ and 100. There are considerable size distortions when the underlying true specification exhibits SAR or SEC error correlations.

With the same set of simulated data, we checked the impact of the row-standardization on the size of the test (see Table A2, available upon request from the authors). The non row-standardization strongly increases the empirical sizes for SAR for the W(1,1) weight matrix and decreases it for the W(5,5) weight matrix. The reverse is true for the SEC3 (*i.e.*, $\sigma_\psi^2 = 10\sigma_\varepsilon^2$) specification, where for W(5,5) the empirical size increases for non-row standardization as compared to row-standardization.

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[12]This data was obtained from INSEE (the French National Institute of Statistics and Economic Studies) census on "fiscal household" from fiscal time series at the municipality level. This data set covers more than 34,000 municipalities (or communes) over the period 1985-98. As a statistical unit, the "fiscal household" (*i.e.* set of individuals filling the same form for income tax) is generaly smaller than the usual household (*i.e.* set of individuals living in the same dwelling).

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Table 1 - Cross section correlations for dynamic panels (1000 replications)
T=25 and θ=0.8

W(j,j)	Individual effects																			
	Average CD statistic										Average correlation coefficient (%)									
	SAR		SMA		SEC1		SEC2		SEC3		SAR		SMA		SEC1		SEC2		SEC3	
j	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100
1	14.98	14.73	4.91	4.85	0.29	0.27	1.63	1.65	3.05	3.06	8.56	4.19	2.81	1.38	0.16	0.08	0.93	0.47	1.74	0.87
2	22.7	22.28	6.06	5.98	0.47	0.46	3.34	3.42	8.33	8.42	12.97	6.33	3.46	1.7	0.27	0.13	1.91	0.97	4.76	2.39
3	28.81	28.28	6.52	6.44	0.54	0.53	4.26	4.37	12.84	13.05	16.46	8.04	3.73	1.83	0.3	0.15	2.43	1.24	7.33	3.71
4	33.79	33.18	6.79	6.69	0.57	0.56	4.81	4.95	16.77	17.13	19.31	9.43	3.88	1.9	0.32	0.16	2.74	1.41	9.58	4.87
5	37.93	37.27	6.98	6.85	0.59	0.59	5.17	5.34	20.15	20.67	21.67	10.59	3.98	1.95	0.33	0.17	2.95	1.52	11.51	5.87
6	41.36	40.71	7.1	6.97	0.6	0.6	5.43	5.61	23.08	23.78	23.63	11.57	4.05	1.98	0.34	0.17	3.1	1.6	13.19	6.76
7	44.24	43.68	7.18	7.05	0.61	0.61	5.63	5.82	25.74	26.54	25.28	12.42	4.1	2.00	0.35	0.17	3.21	1.66	14.71	7.54
8	46.63	46.24	7.25	7.11	0.62	0.62	5.78	5.98	28.06	28.99	26.64	13.14	4.14	2.02	0.35	0.18	3.3	1.7	16.03	8.24
9	48.58	48.47	7.29	7.15	0.63	0.62	5.9	6.13	30.14	31.2	27.76	13.78	4.16	2.03	0.36	0.18	3.37	1.74	17.22	8.87
10	50.16	50.46	7.33	7.19	0.63	0.63	5.99	6.23	32.02	33.21	28.66	14.34	4.18	2.04	0.36	0.18	3.42	1.77	18.3	9.44
W(j,j)	Individual effects and deterministic trends																			
	Average CD statistic										Average correlation coefficient (%)									
	SAR		SMA		SEC1		SEC2		SEC3		SAR		SMA		SEC1		SEC2		SEC3	
j	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100	N=50	N=100
1	14.99	14.7	4.88	4.83	0.23	0.25	1.64	1.67	3.02	3.07	8.56	4.18	2.79	1.37	0.15	0.07	0.93	0.47	1.72	0.87
2	22.72	22.25	6.03	5.96	0.46	0.4	3.36	3.43	8.31	8.4	12.98	6.33	3.44	1.69	0.26	0.13	1.92	0.97	4.74	2.39
3	28.82	28.24	6.5	6.41	0.52	0.52	4.24	4.38	12.84	13.06	16.47	8.03	3.71	1.82	0.3	0.15	2.42	1.24	7.33	3.71
4	33.79	33.12	6.76	6.67	0.56	0.56	4.78	4.95	16.74	17.11	19.3	9.42	3.86	1.89	0.32	0.16	2.73	1.41	9.56	4.86
5	37.91	37.19	6.94	6.83	0.58	0.58	5.14	5.35	20.13	20.65	21.66	10.57	3.96	1.94	0.33	0.16	2.94	1.52	11.5	5.87
6	41.32	40.61	7.06	6.93	0.59	0.59	5.39	5.62	23.09	23.78	23.61	11.55	4.03	1.97	0.33	0.17	3.08	1.6	13.19	6.76
7	44.19	43.55	7.14	7.00	0.6	0.6	5.58	5.82	25.69	26.54	25.25	12.38	4.08	1.99	0.34	0.17	3.19	1.65	14.68	7.54
8	46.57	46.1	7.21	7.07	0.61	0.61	5.73	5.98	28.01	29.04	26.61	13.1	4.12	2.01	0.35	0.17	3.27	1.7	16.00	8.25
9	48.52	48.31	7.25	7.11	0.62	0.62	5.84	6.12	30.08	31.25	27.72	13.73	4.14	2.02	0.35	0.18	3.33	1.74	17.19	8.88
10	50.1	50.28	7.29	7.15	0.62	0.63	5.92	6.23	32.00	33.23	28.63	14.29	4.16	2.03	0.35	0.18	3.39	1.77	18.28	9.45

SEC1 : $\sigma^2_{\psi} = 0.1\,\sigma^2_{\varepsilon}$, SEC2 : $\sigma^2_{\psi} = \sigma^2_{\varepsilon}$ and SEC3 : $\sigma^2_{\psi} = 10\,\sigma^2_{\varepsilon}$.

W(j,j): "j ahead and j behind" weights matrix

Table 2 - Empirical size (%) of the panel unit root tests at 5% level for dynamic panels with individual effects (1000 replications)

	T	θ	LLC		B		IPS		MW		Choi		Choi_c		Chang IV		Pesaran		PS	
N			50	100	50	100	50	100	50	100	50	100	50	100	50	100	50	100	50	100
No Spatial	25		4.2	5.3	3.7	3.1	3.8	3.7	5.6	5.3	5.6	5.0	5.6	4.7	4.9	4.6	4.4	5.8	3.9	3.1
	50		4.7	8.1	5.3	4.7	4.1	4.2	5.0	4.5	5.1	4.2	5.0	5.2	4.0	4.1	5.3	4.8	3.1	2.7
W(1,1)																				
SAR , θ_1=0.4 ; 0.8	25	0.8	17.4	19.1	17.8	14.5	16.3	14.8	16.9	17.3	16.7	17.7	16.0	17.5	13.7	13.0	10.6	8.8	10.4	10.3
		0.4	7.1	7.5	5.4	5.2	5.6	5.0	7.0	7.7	6.7	6.9	6.7	7.0	5.5	5.9	6.4	6.5	4.7	4.1
	50	0.8	18.9	22.3	18.6	16.7	17.3	12.0	16.1	15.0	15.7	14.9	15.3	13.9	14.5	13.4	9.7	8.7	9.4	9.4
		0.4	9.9	12.1	7.4	6.1	7.8	5.3	6.7	6.0	6.8	5.4	6.3	6.1	6.5	6.2	5.4	5.5	4.0	3.6
SMA , θ_2=0.4 ; 0.8	25	0.8	10.4	11.2	8.9	8.1	9.2	8.1	10.2	11.1	10.0	10.4	9.8	10.3	8.3	8.3	7.0	6.9	7.0	5.3
		0.4	6.3	6.9	4.8	4.7	5.4	4.9	6.3	6.6	6.0	6.6	6.3	6.6	5.2	5.9	6.3	6.0	4.8	3.6
	50	0.8	12.9	14.4	10.6	9.7	10.7	6.9	9.1	9.0	8.9	8.8	10.0	8.5	8.8	8.6	6.4	5.1	6.9	5.5
		0.4	8.8	11.0	6.9	5.9	7.3	4.9	6.5	5.7	6.5	5.5	5.7	5.9	6.0	6.2	5.1	4.7	3.6	4.4
SEC1 , θ_3=0.4 ; 0.8	25	0.8	4.8	6.2	4.3	4.0	4.3	4.3	5.8	6.1	5.4	6.1	5.5	6.4	4.7	3.4	5.2	5.7	4.4	4.0
		0.4	4.3	5.9	4.4	3.8	5.0	4.3	5.7	5.9	5.8	6.0	5.6	6.2	5.0	4.0	4.5	5.6	4.0	3.3
	50	0.8	4.9	7.1	4.5	5.1	4.3	3.7	5.3	5.0	5.2	4.8	5.3	4.6	5.0	4.0	5.4	5.6	3.1	2.6
		0.4	4.9	7.5	5.1	4.7	4.5	3.9	5.0	5.2	4.7	5.2	4.7	4.8	4.5	4.7	5.4	6.0	3.4	2.8
SEC3 , θ_3=0.4 ; 0.8	25	0.8	5.5	7.1	4.1	5.1	4.3	5.3	9.5	8.0	9.4	7.8	8.8	8.5	6.4	7.0	6.6	5.8	6.8	5.9
		0.4	4.8	5.4	4.3	3.1	3.8	3.9	7.9	6.9	7.7	6.8	7.1	7.6	5.9	5.5	5.7	5.8	5.5	4.6
	50	0.8	7.5	9.9	6.3	6.8	4.7	4.1	7.6	7.1	7.9	7.1	8.1	6.9	7.8	6.3	7.1	7.1	5.5	4.2
		0.4	6.4	7.9	6.0	5.8	4.6	3.6	7.6	6.2	7.9	5.9	8.6	6.0	7.1	5.4	6.8	7.4	4.5	3.6
W(5,5)																				
SAR , θ_1=0.4 ; 0.8	25	0.8	18.9	23.7	21.5	20.9	18.6	19.7	19.3	22.4	19.9	21.9	15.3	19.1	17.6	15.7	9.9	10.4	8.9	11.4
		0.4	6.0	6.7	4.1	4.0	3.7	4.9	5.3	6.0	5.2	6.1	5.3	5.6	4.5	4.7	3.8	5.6	4.1	3.7
	50	0.8	21.2	23.3	23.5	24.3	21.2	16.5	19.8	19.8	20.4	19.8	16.4	19.0	17.1	17.6	9.8	11.2	7.9	8.4
		0.4	6.9	8.5	6.1	4.8	5.3	3.9	5.2	5.0	4.9	5.5	5.1	5.6	4.5	4.6	5.5	6.0	3.0	3.8
SMA , θ_2=0.4 ; 0.8	25	0.8	6.9	8.0	5.6	4.6	5.5	5.0	6.0	7.9	5.9	7.9	6.2	6.4	5.5	5.1	4.6	5.5	3.5	6.1
		0.4	5.3	5.8	3.5	3.4	3.4	4.2	4.7	5.9	4.9	5.7	5.1	5.3	4.5	4.8	3.7	5.6	4.5	3.5
	50	0.8	8.7	9.8	6.9	5.7	5.8	4.4	7.0	5.7	6.6	5.7	5.7	6.1	5.8	5.6	6.3	6.0	5.0	3.9
		0.4	6.4	8.1	5.8	4.8	5.3	4.0	5.1	5.0	5.2	5.1	5.4	5.0	4.4	4.6	5.7	5.8	3.8	2.8
SEC1 , θ_3=0.4 ; 0.8	25	0.8	3.8	5.3	3.6	3.1	3.9	4.2	5.6	5.9	5.6	6.0	6.0	5.9	5.1	4.3	5.2	6.2	3.9	3.6
		0.4	4.3	5.2	3.7	2.7	4.1	3.8	5.8	5.9	6.0	5.9	5.8	5.3	4.9	4.5	4.3	5.6	4.6	3.3
	50	0.8	4.1	6.8	5.0	4.6	4.5	3.5	5.7	4.6	5.6	5.0	5.3	4.4	5.1	4.7	5.7	5.7	3.5	1.8
		0.4	4.2	7.3	5.1	4.7	4.2	3.7	5.6	4.6	5.3	4.6	4.9	5.0	4.2	4.8	5.5	5.5	2.6	2.2
SEC3 , θ_3=0.4 ; 0.8	25	0.8	5.2	6.4	5.2	5.4	4.3	4.2	15.1	15.3	15.5	15.9	12.3	15.5	11.3	11.6	9.0	8.0	7.4	7.8
		0.4	4.6	5.2	4.4	3.7	3.2	7.8	7.6	8.5	7.6	8.2	6.5	8.2	6.0	6.1	5.8	5.6	5.9	6.7
	50	0.8	7.4	10.9	6.6	6.1	5.9	3.4	15.2	15.9	16.0	15.7	13.6	15.3	13.0	11.6	10.2	9.6	6.0	6.6
		0.4	5.9	8.6	5.4	4.9	5.3	3.5	7.3	6.1	7.2	6.1	6.7	6.2	6.7	6.1	6.6	7.1	3.9	3.9

No Spatial: standard case without spatial correlation --- W(j,j): "j ahead and j behind" standardized weights matrix --- SEC1: $\sigma^2_\psi = 0.1\ \sigma^2_\varepsilon$ --- SEC3: $\sigma^2_\psi = 10\ \sigma^2_\varepsilon$

Table 3 - Empirical size (%) of the panel unit root tests at 5% level for dynamic panels with individual effects and deterministic trends (1000 replications)

	T	θ	LLC		B		IPS		MW		Choi		Choi_c		Chang IV		Pesaran		PS	
N			50	100	50	100	50	100	50	100	50	100	50	100	50	100	50	100	50	100
No Spatial	25		4.0	4.9	3.8	3.7	4.9	5.8	6.1	6.5	5.9	5.7	6.3	6.0	4.9	4.7	4.1	5.4	5.1	4.2
	50		6.5	8.4	5.6	4.5	4.0	4.6	4.6	4.2	4.4	4.2	4.5	3.5	4.2	3.8	4.7	5.2	1.9	1.9
W(1,1)																				
SAR , θ_1=0.4 ; 0.8	25	0.8	15.7	15.5	18.0	15.7	17.4	15.6	17.4	16.6	17.3	16.6	17.5	15.3	13.5	13.2	13.2	11.7	12.8	12.0
		0.4	7.2	6.9	6.4	6.1	7.3	7.2	7.5	8.0	7.5	7.6	7.2	7.6	5.5	5.9	5.4	6.0	5.5	5.2
	50	0.8	18.6	20.1	19.3	17.7	12.8	9.8	15.5	16.3	16.4	16.0	15.7	14.8	14.2	13.0	11.0	9.9	8.8	7.7
		0.4	8.7	12.7	7.8	6.8	6.2	5.3	6.3	6.2	6.6	6.7	6.5	6.2	5.8	5.8	5.5	5.9	3.3	3.5
SMA , θ_2=0.4 ; 0.8	25	0.8	9.7	10.1	9.1	8.9	10.0	10.1	10.5	12.3	10.9	12.0	10.7	11.1	8.2	8.7	8.2	7.3	7.9	7.4
		0.4	6.5	6.7	5.3	5.6	7.1	7.0	7.3	7.8	7.2	7.2	7.0	7.4	5.1	6.3	5.7	5.7	6.6	4.9
	50	0.8	10.9	15.2	11.1	10.7	7.5	3.8	7.4	8.4	7.4	8.6	7.6	8.4	8.8	8.4	6.8	6.5	5.4	3.9
		0.4	8.5	11.3	7.0	6.1	5.6	5.9	6.1	5.8	5.8	5.8	6.2	6.1	5.8	5.3	5.4	5.9	3.9	2.8
SEC1 , θ_3=0.4 ; 0.8	25	0.8	4.4	6.2	4.8	4.5	4.3	4.9	5.4	5.8	5.3	5.5	5.9	6.3	4.9	3.7	3.8	5.8	3.8	5.2
		0.4	3.8	5.7	4.6	4.5	5.5	4.5	6.0	5.9	5.6	5.6	6.1	5.9	5.1	3.9	4.1	6.2	5.0	4.0
	50	0.8	6.4	8.2	5.0	5.2	3.4	3.5	4.6	4.1	4.7	4.0	4.8	4.0	4.5	3.9	4.6	6.1	2.9	2.0
		0.4	6.7	8.3	5.2	4.8	4.0	4.0	4.0	3.5	4.3	3.7	4.0	4.1	4.6	4.2	4.8	5.0	2.9	2.3
SEC3 , θ_3=0.4 ; 0.8	25	0.8	5.7	5.9	4.5	6.9	5.4	5.6	7.7	10.3	8.2	9.5	8.0	10.2	6.2	7.1	6.2	7.6	5.8	5.8
		0.4	4.8	4.3	4.2	4.2	4.6	5.2	7.6	9.3	7.5	8.7	7.4	8.7	6.1	5.5	5.6	6.7	5.5	4.8
	50	0.8	6.1	9.0	6.8	7.8	4.7	5.6	8.7	9.1	8.6	8.7	8.3	8.4	7.5	6.4	5.8	7.5	3.7	2.9
		0.4	5.0	7.7	6.3	6.0	4.0	4.8	7.0	7.6	7.2	7.0	6.8	7.9	7.2	5.3	5.6	6.9	3.0	2.9
W(5,5)																				
SAR , θ_1=0.4 ; 0.8	25	0.8	19.0	22.5	22.3	21.1	20.8	22.5	23.1	23.3	23.2	22.7	18.4	19.0	18.1	15.2	12.8	13.7	11.7	12.3
		0.4	6.3	6.1	4.4	4.3	6.1	5.5	7.0	7.3	7.2	6.5	6.9	6.9	4.6	4.8	4.5	5.9	4.0	4.5
	50	0.8	23.8	26.0	23.8	25.4	19.7	14.0	20.7	20.3	20.9	20.8	16.8	18.8	16.6	17.6	12.3	12.1	6.3	8.8
		0.4	7.5	10.8	6.6	4.9	4.0	4.4	5.0	4.8	5.1	4.9	4.5	4.5	4.4	4.7	4.5	5.3	2.7	2.6
SMA , θ_2=0.4 ; 0.8	25	0.8	7.0	7.2	5.7	5.7	7.6	6.4	7.7	8.0	8.0	7.7	7.5	7.5	5.3	5.1	5.0	6.2	5.7	5.4
		0.4	5.4	5.9	3.8	4.0	5.4	4.8	7.0	6.5	6.9	6.1	6.4	6.2	4.1	4.9	4.1	5.5	4.9	4.3
	50	0.8	8.7	12.1	7.2	6.7	4.9	4.7	6.5	6.4	5.8	6.7	5.5	5.8	5.1	5.8	5.1	5.8	2.9	2.5
		0.4	7.1	9.4	5.7	5.0	3.9	4.0	4.9	4.2	4.3	4.6	4.3	4.3	4.6	4.3	4.7	4.8	2.5	1.6
SEC1 , θ_3=0.4 ; 0.8	25	0.8	3.7	5.4	4.2	3.9	5.1	5.1	5.7	6.5	5.8	6.3	5.9	6.4	5.1	4.3	3.5	5.5	4.5	4.2
		0.4	3.8	5.6	4.1	3.5	5.3	5.4	5.8	6.2	6.0	6.0	5.7	6.2	4.7	4.8	4.0	6.0	5.3	4.6
	50	0.8	6.1	8.2	5.0	4.9	3.9	4.5	4.5	4.2	4.5	4.1	4.0	4.4	4.8	4.3	5.3	5.5	3.2	2.1
		0.4	6.4	8.0	5.2	4.9	4.0	4.5	4.5	4.5	4.1	4.3	4.2	4.0	4.8	4.1	4.9	5.1	3.7	1.3
SEC3 , θ_3=0.4 ; 0.8	25	0.8	5.3	6.3	5.6	6.4	6.4	6.6	14.5	16.8	15.1	16.3	14.3	15.8	11.3	11.3	10.5	11.4	8.4	9.1
		0.4	3.7	4.7	4.5	3.8	5.0	5.6	9.2	8.6	9.0	8.0	8.2	9.1	6.8	6.2	6.6	7.6	6.0	6.4
	50	0.8	6.4	9.3	6.9	6.3	4.9	5.3	16.5	16.7	16.9	16.5	14.9	18.0	13.0	11.2	11.5	12.7	6.5	6.7
		0.4	6.6	9.4	5.6	5.4	4.6	4.8	6.5	8.5	6.3	8.2	6.4	8.7	6.1	5.8	5.8	8.0	4.1	2.1

No Spatial: standard case without spatial correlation --- W(j,j): "j ahead and j behind" standardized weights matrix --- SEC1: $\sigma^2_\psi = 0.1\ \sigma^2_\varepsilon$ --- SEC3: $\sigma^2_\psi = 10\ \sigma^2_\varepsilon$

Fig. 1 - Spatial AutoRegressive (SAR) error process
Individual effects
N=50, T=25, θ=0.8

Legend:
- LLC
- B
- IPS
- MW
- CHOI
- CHOI_C
- CHANG_IV
- PESARAN
- PS

Y-axis: Empirical size (%)

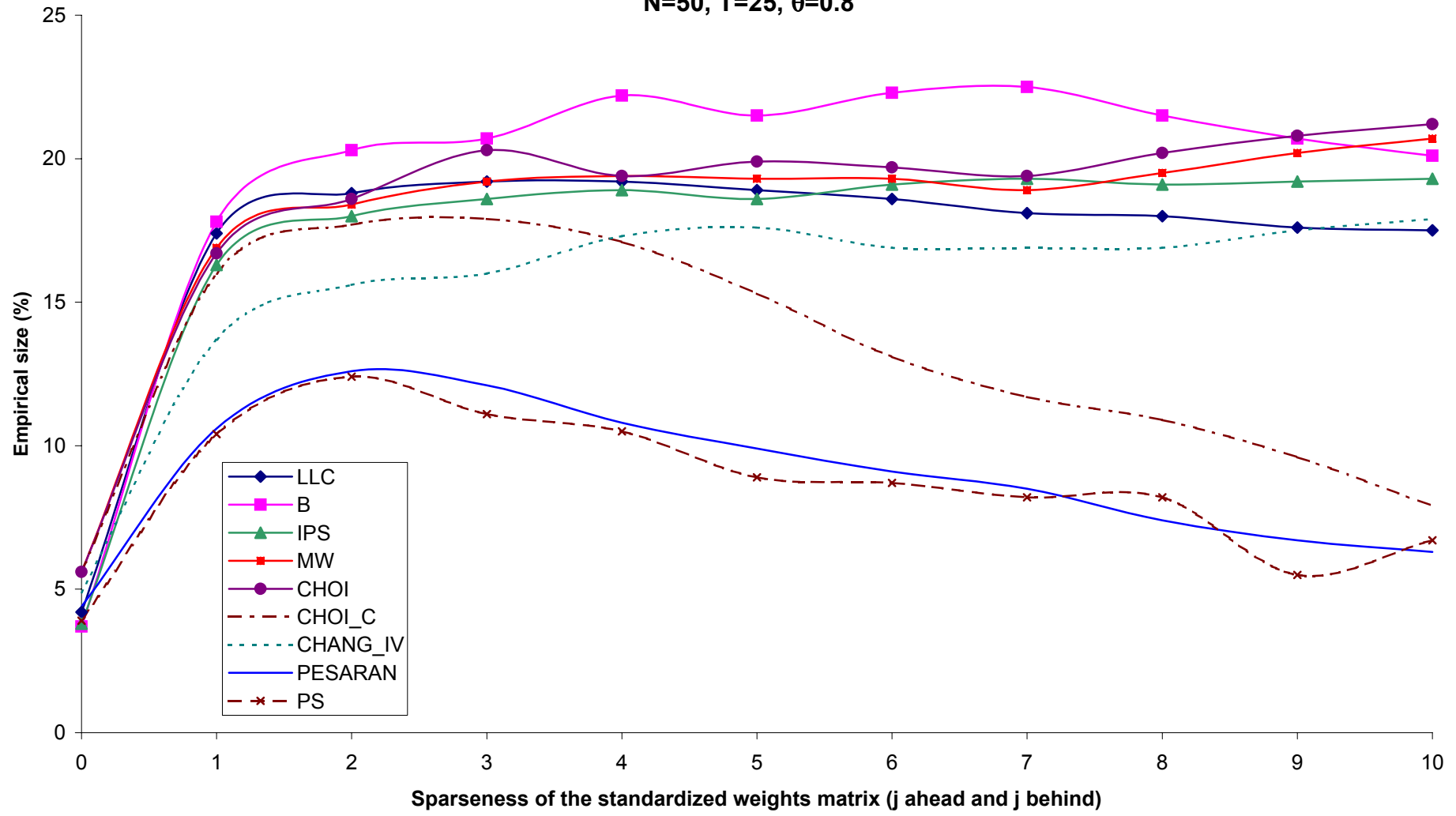X-axis: Sparseness of the standardized weights matrix (j ahead and j behind)

Fig. 2 - Spatial Moving Average error process (SMA)
Individual effects
N=50, T=25, θ=0.8
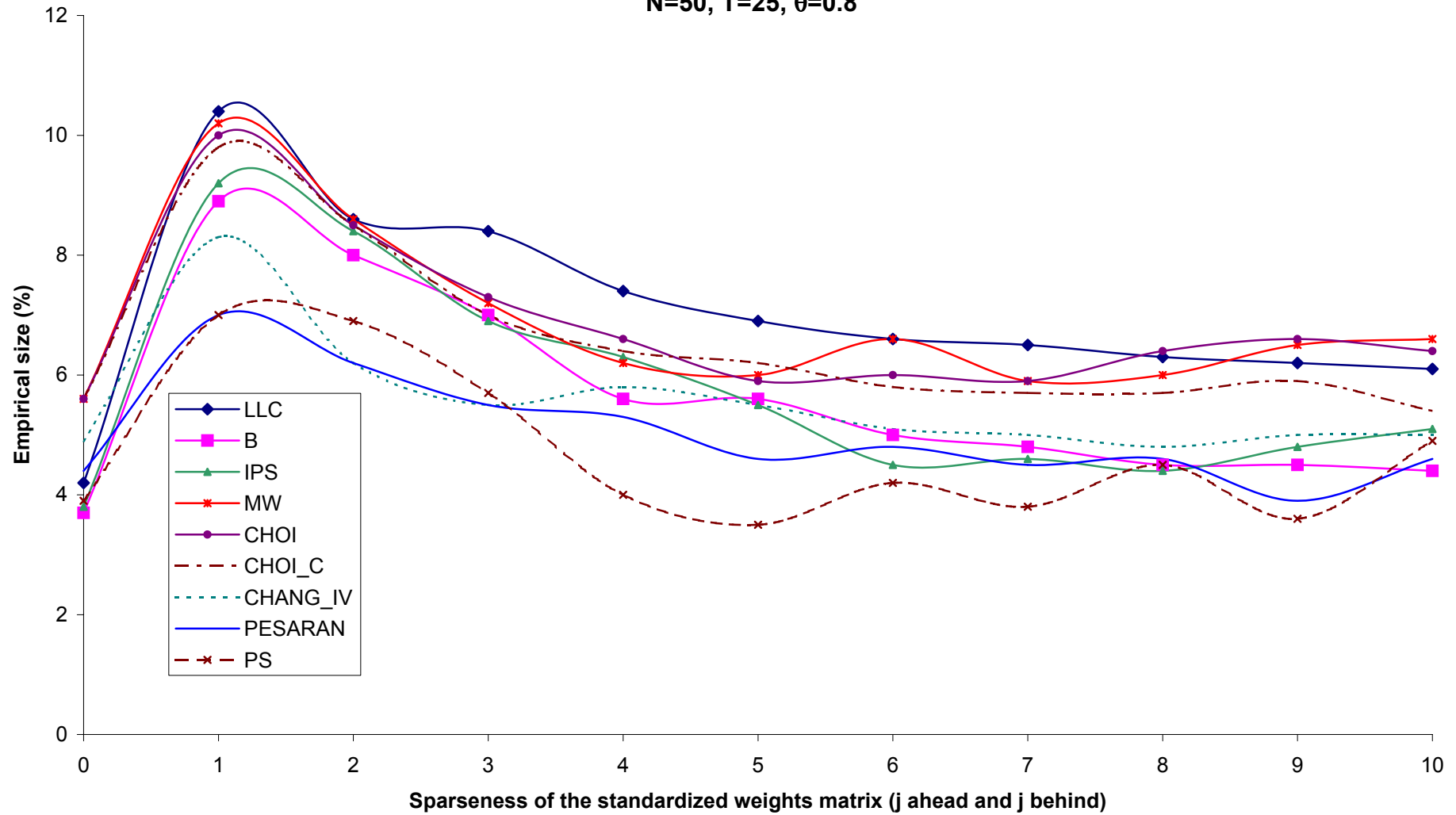
Fig. 3 - Spatial Error Component process (SEC1)
Individual effects
**N=50, T=25, θ=0.8, $\sigma^2_\psi = 0.1\sigma^2_\varepsilon$**
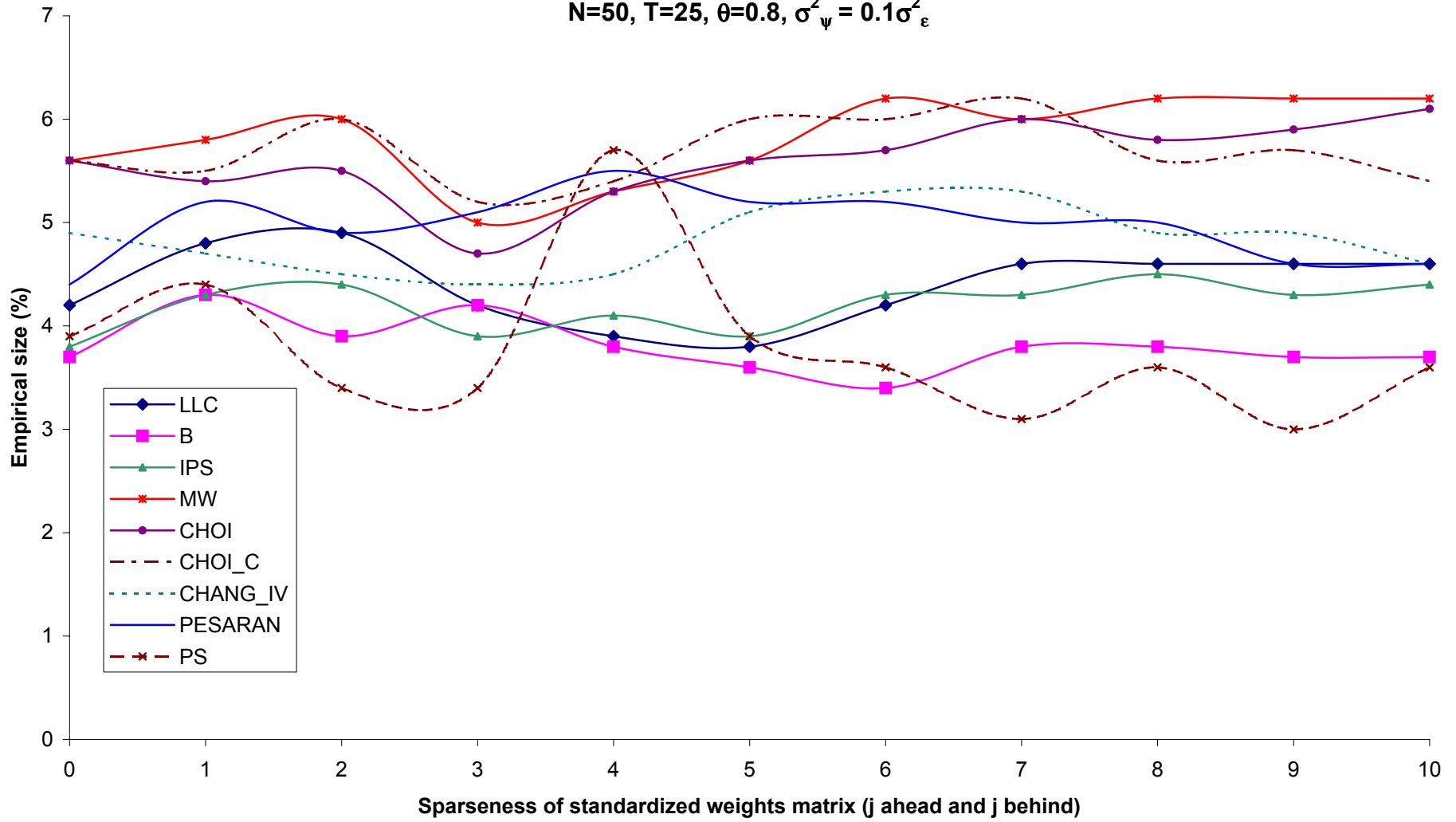
Fig. 4 - Spatial Error Component process (SEC2)
Individual effects
N=50, T=25, θ=8, $\sigma^2_\psi = \sigma^2_\varepsilon$
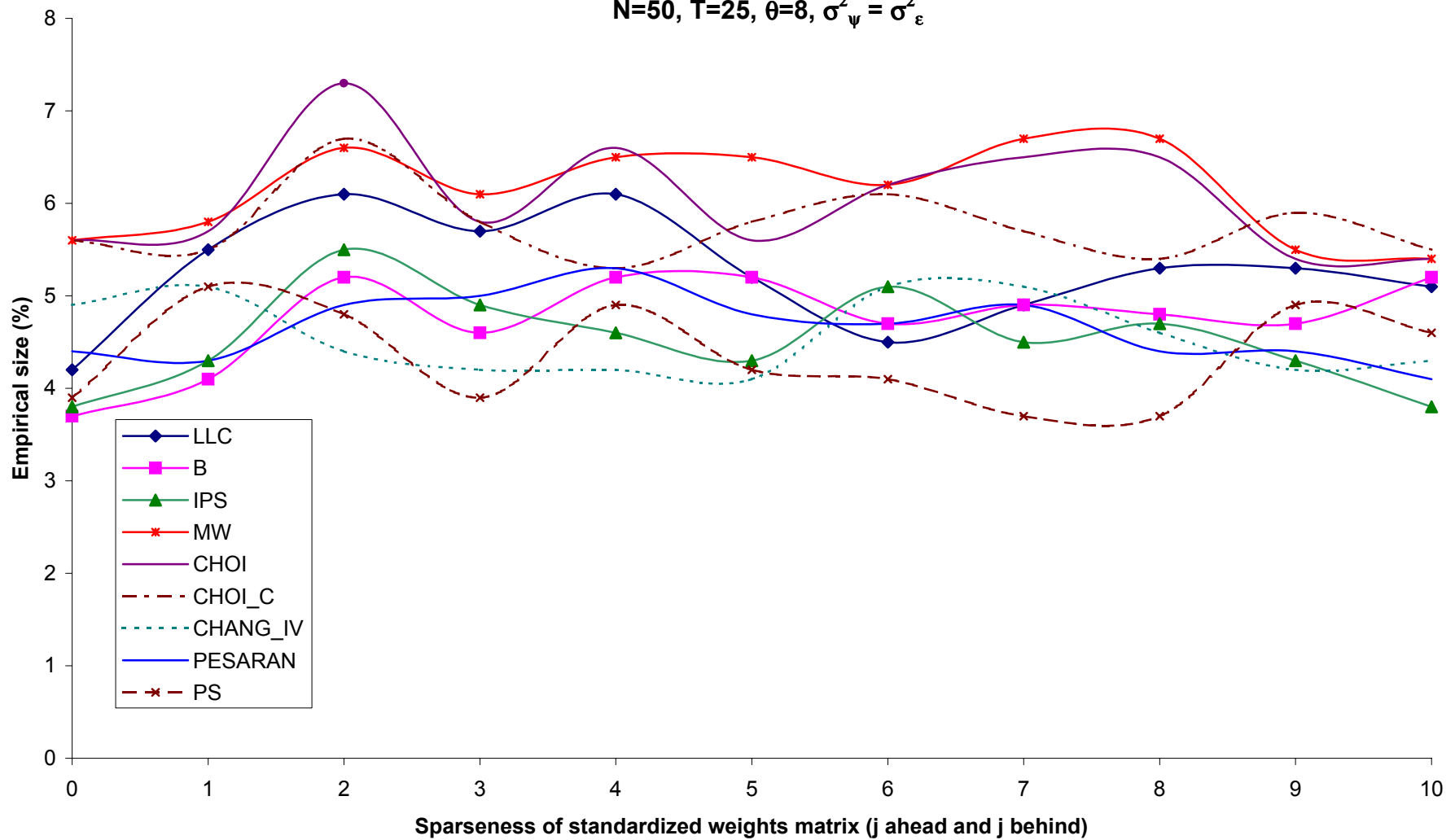
Fig. 5 - Spatial Error Component process (SEC3)
Individual effects
N=50, T=25, θ=0.8, $\sigma^2_\psi = 10\ \sigma^2_\varepsilon$
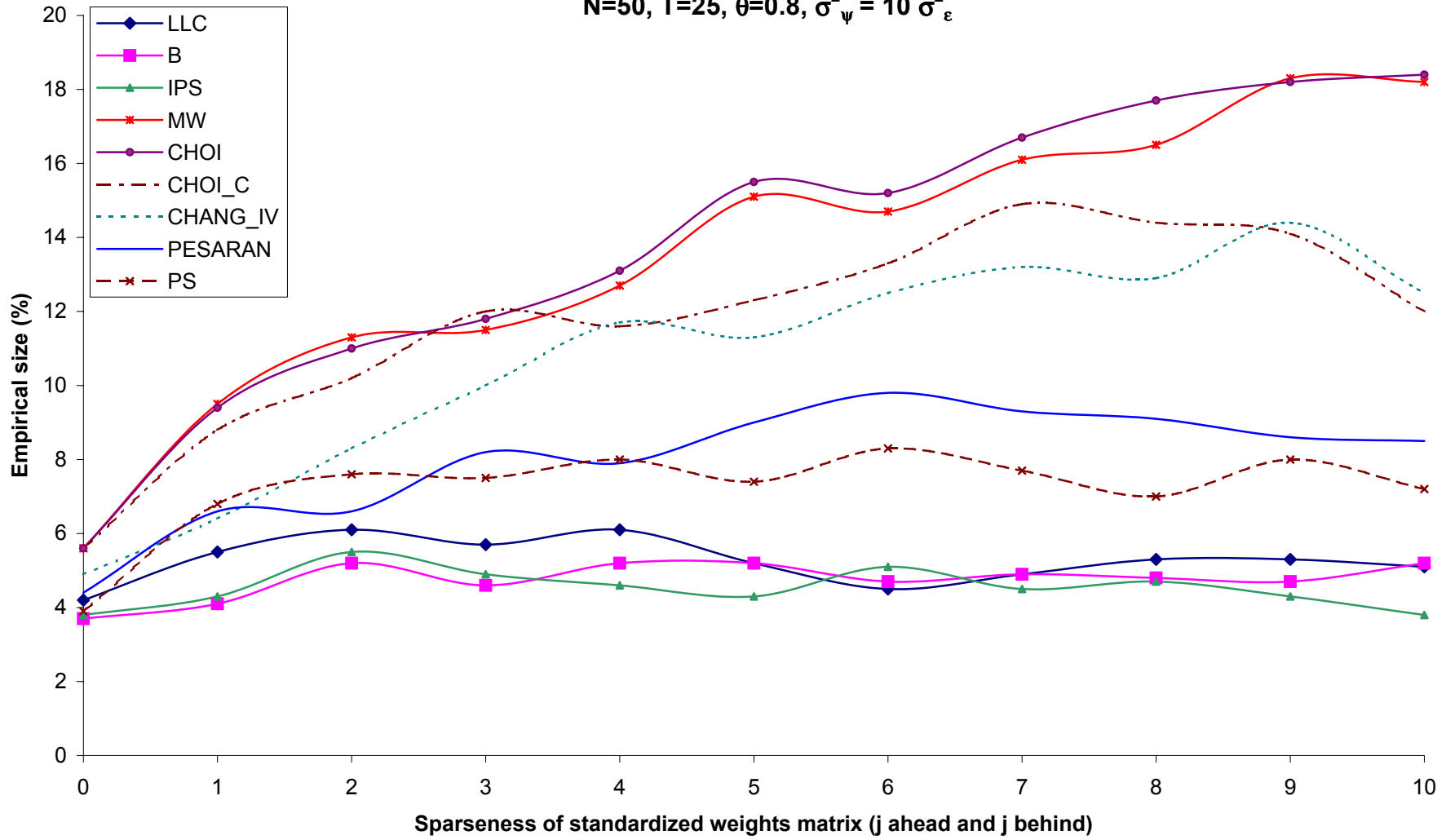
Fig. 6 - Panel unit root tests that allow for cross-section dependence
Individual effects, N=100, T=50, θ=0.8, $\sigma^2_\psi = 10\, \sigma^2_\varepsilon$
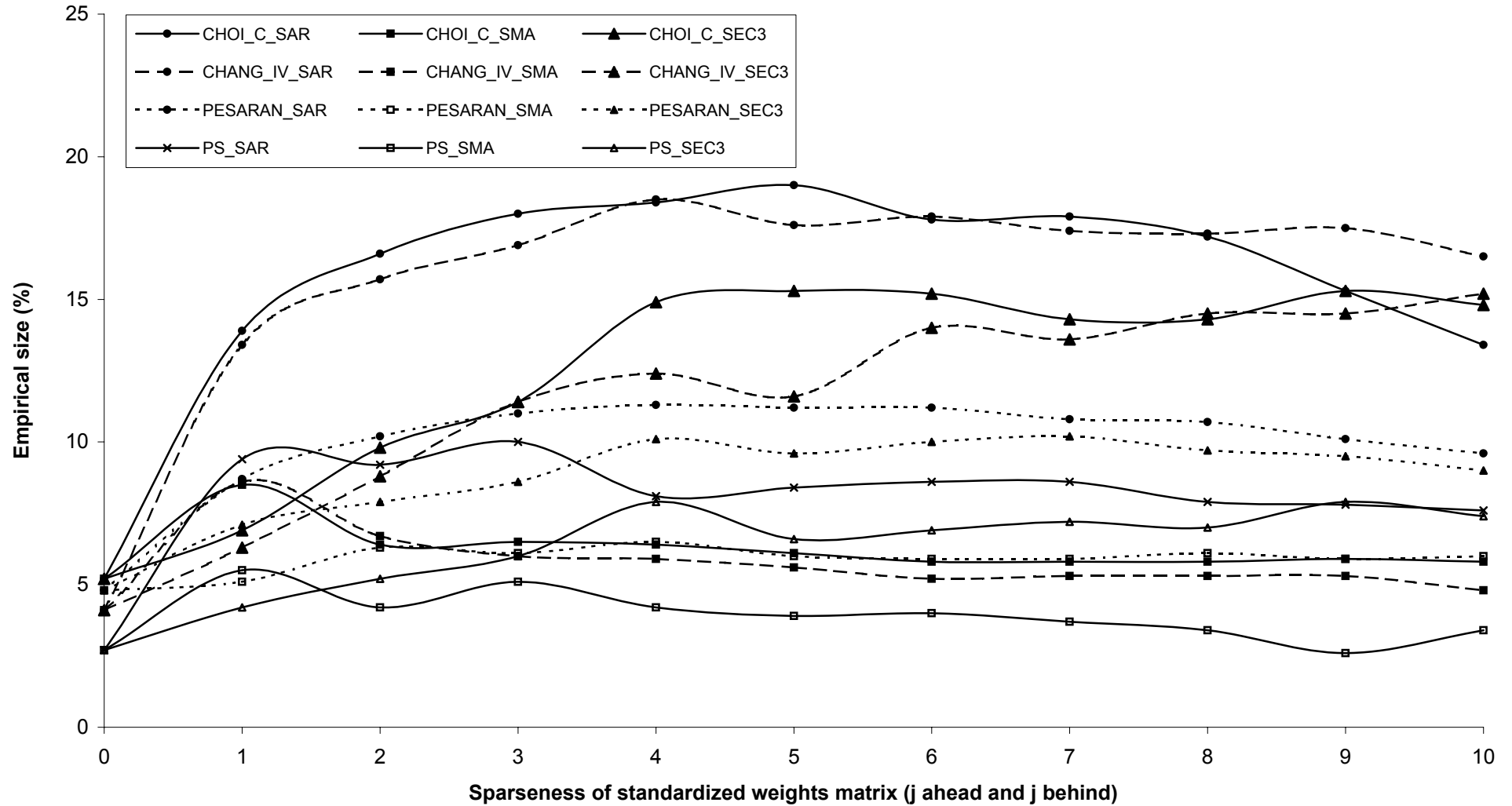
Fig. 7 - Spatial AutoRegressive (SAR) error process
Individual effects and deterministic trends
N=50, T=25, θ=0.8
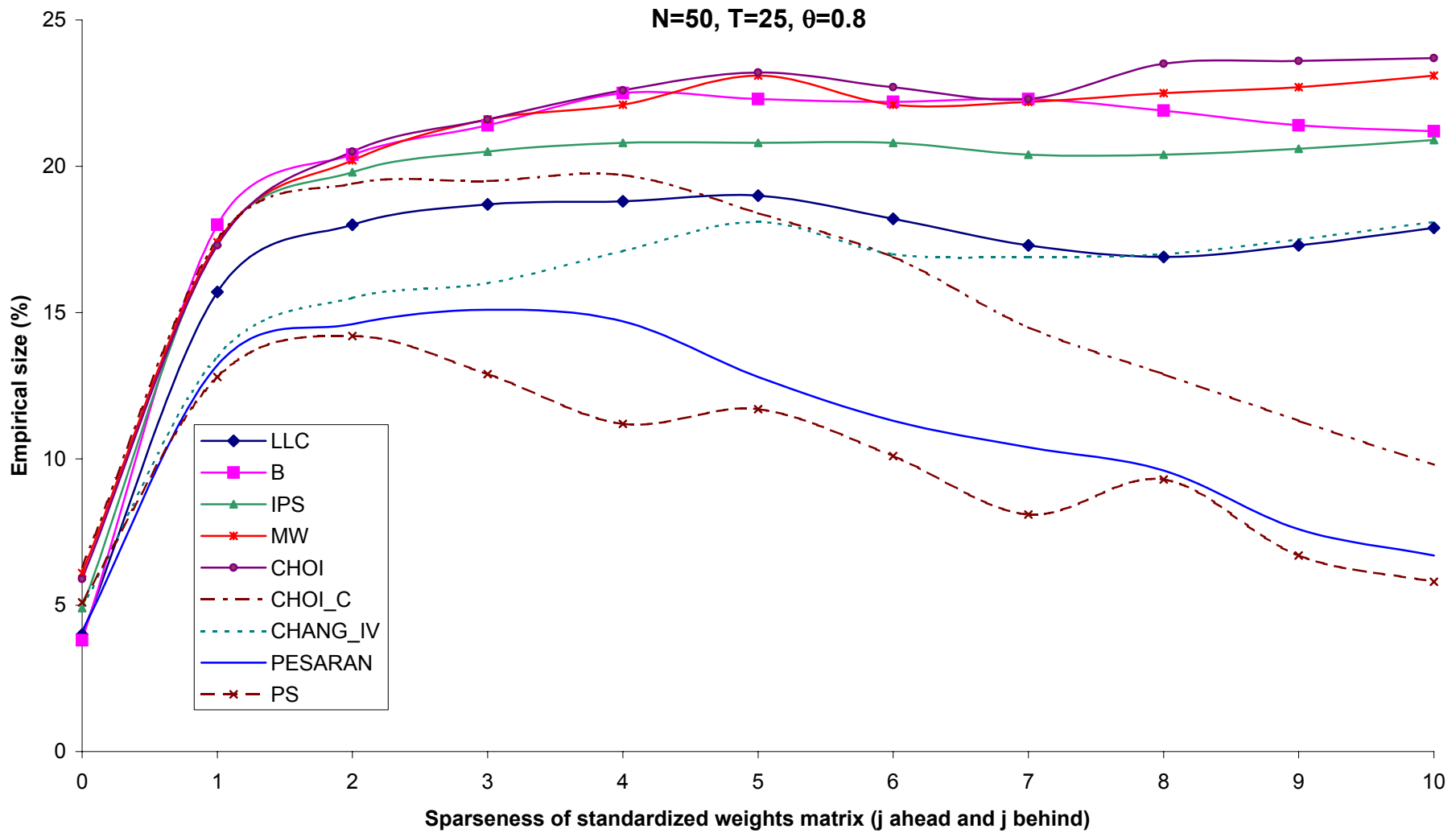
Fig. 8 - Spatial Moving Average (SMA) error process
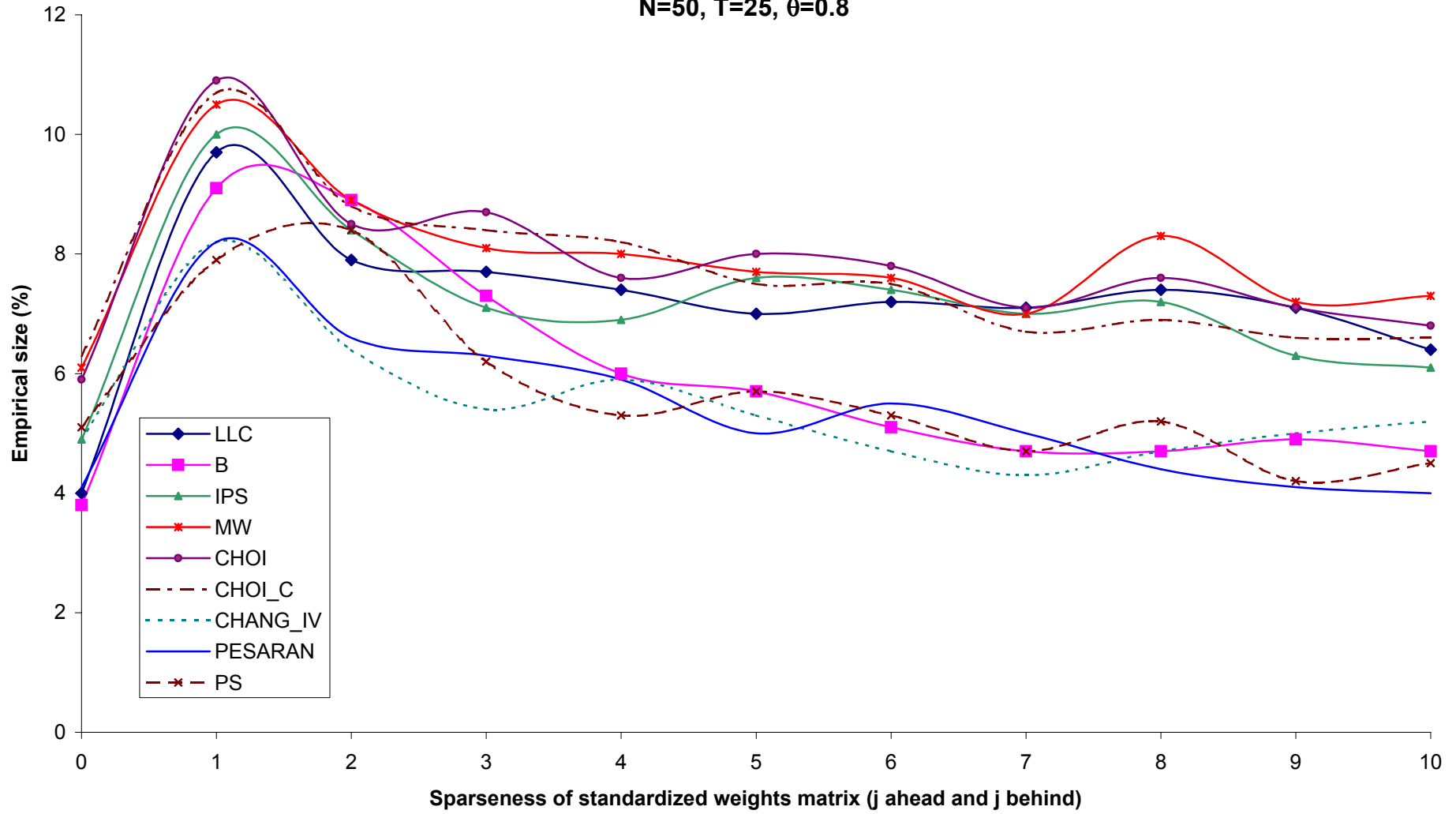**Individual effects and deterministic trends**
N=50, T=25, θ=0.8

Fig. 9 - Spatial Error Component process (SEC3)
Individual effects and deterministic trends
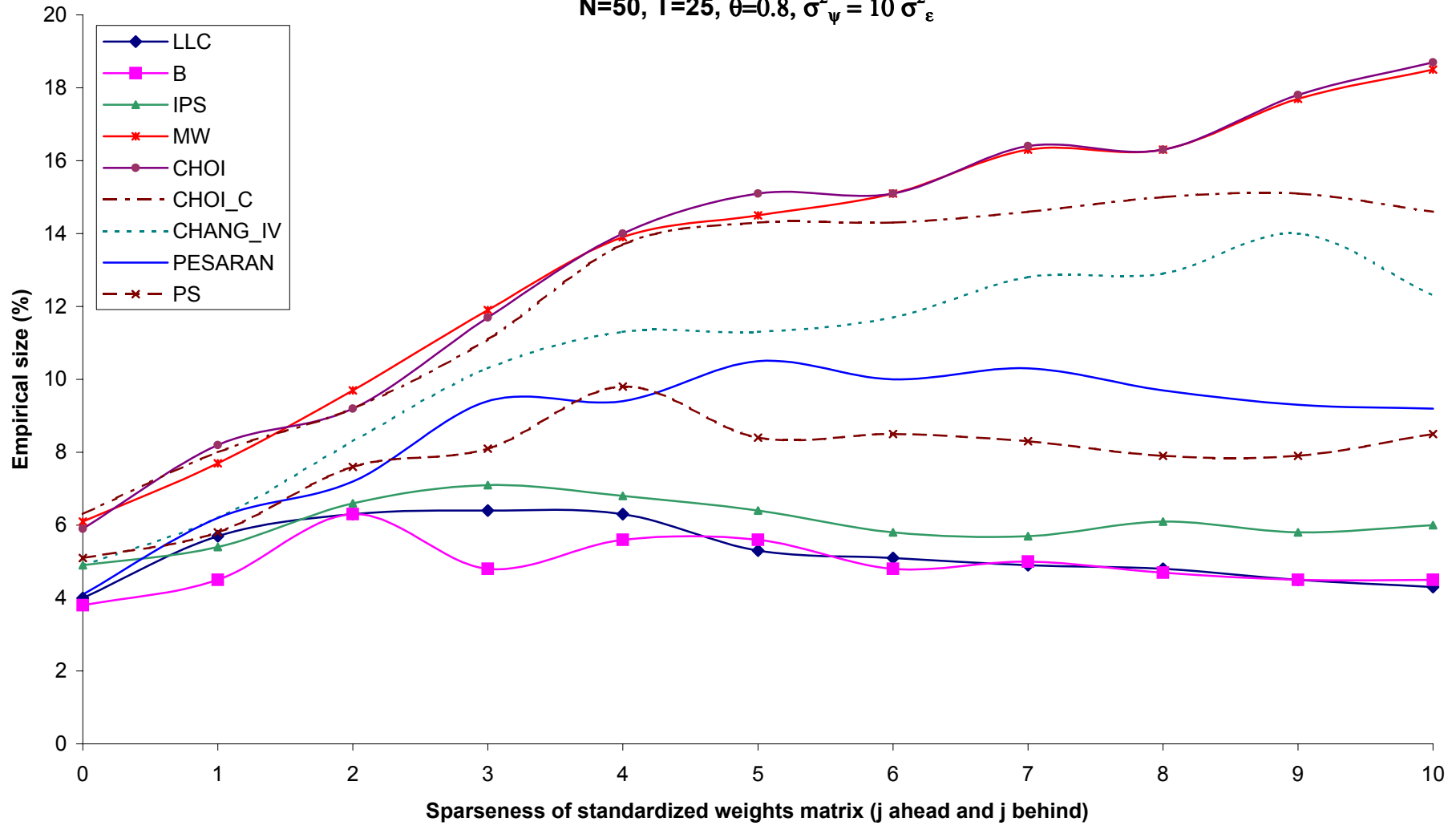**N=50, T=25, θ=0.8, $\sigma^2_{\psi} = 10\,\sigma^2_{\varepsilon}$**

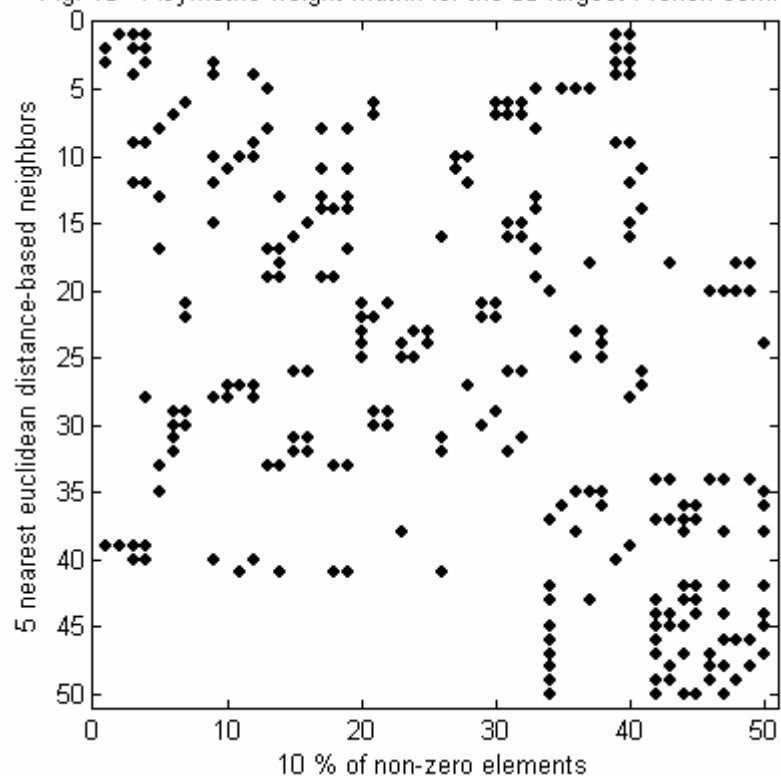Fig. 10 - Asymetric weight matrix for the 50 largest French communes

10 % of non-zero elements

Fig. 11 - Asymetric weight matrix for the 100 largest French communes

5 % of non-zero elements