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Anisotropic Inflation and the Origin of Four Large Dimensions

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Abstract

In the context of $(4 + d)$ -dimensional general relativity, we propose an inflationary scenario wherein 3 spatial dimensions grow large, while d extra dimensions remain small. Our model requires that a self-interacting d -form acquire a vacuum expectation value along the extra dimensions. This causes 3 spatial dimensions to inflate, whilst keeping the size of the extra dimensions nearly constant. We do not require an additional stabilization mechanism for the radion, as stable solutions exist for flat, and for negatively curved compact extra dimensions. From a four-dimensional perspective, the radion does not couple to the inflaton; and, the small amplitude of the CMB temperature anisotropies arises from an exponential suppression of fluctuations, due to the higher-dimensional origin of the inflaton. The mechanism triggering the end of inflation is responsible, both, for heating the universe, and for avoiding violations of the equivalence principle due to coupling between the radion and matter.

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I. INTRODUCTION

Most theories which attempt a unification of gravity with matter require the existence of additional spatial dimensions. This gives rise to a pressing question: Why do the extra dimensions remain unobserved? Traditionally, this question has been addressed by arguing that the extra dimensions are “small” compared to experimentally accessible scales, and hence, hitherto undetected. Insofar as the dynamics of these dimensions is gravitational, it is believed that the relevant scale is the Planck length. Thus, it is natural for the extra dimensions to be Planck-sized. However, the spirit of this “naturalness” argument requires that we apply it to dimensions containing the observable universe, too. This leads to the conclusion that we populate a Planck-sized universe, in obvious disagreement with observations.

The latter of the two problems stated above was one, amongst others, which inflation was originally intended to address [1]. In the conventional picture, an initially Planck-sized universe is “blown up”, by a sufficiently long stage of inflation, into a size much larger than that of our present horizon, whilst also producing many other features of the universe we observe today [2]. Hence, from the inflationary point of view, the problem is not the “smallness” of the extra dimensions, but rather the “largeness” of the observable ones. In other words, we must explain how three spatial dimensions inflated while the remaining ones kept their natural, Planckian size.

Conventionally, inflation is driven by a scalar field. Instead of considering theories where a particular subset of dimensions has been singled out, as in brane-world models [3], in this paper we shall deal with theories where all spatial dimensions are equivalent. Then, since a scalar does not single out any direction in space either, it generally yields accelerated expansion in all dimensions. In order to preferentially inflate only four dimensions, a mechanism to keep the extra dimensions at constant size is required. From a four-dimensional perspective, the size of the extra dimensions is characterized by a single scalar field, the radion. Even if we manage to stabilize the radion, say, by the addition of a magnetic flux threading the extra dimensions [4], it is unclear whether the extra dimensions would remain stable during an inflationary phase. In fact, generally the inflaton is expected to couple to the radion, and it is likely that this coupling does violence to the stabilization mechanism during inflation [5, 6]. The stabilization of compactified spaces has been extensively studied in [7].

In this paper we propose an alternative minimal model which achieves inflation in four dimensions whilst keeping the extra dimensions “small”. In order to facilitate this, we require that inflation be driven by a form, rather than a scalar field. The benefit of introducing a form lies in the fact that, unlike a scalar, a form may point in several directions. By wrapping the extra directions with a form, we are able to introduce an anisotropy that is necessary to distinguish four dimension from the additional ones. The details of four-dimensional inflation are then determined by the self-interaction of the form. Furthermore, it is not necessary to provide an additional stabilization mechanism for the radion, and the interactions of the inflaton allow it to decouple naturally from the radion.

Massless forms are ubiquitous in superstring theory and in supergravity. In string theory, forms are a particularly crucial part of the spectrum since a $(p + 1)$ -form couples to a Dp -brane, much like a vector boson (1-form) couples to a charged particle (0-brane) in gauge theories. We, however, will require a massive form. While massive forms are not as familiar as their massless counterparts, they are certainly not unheard of. For instance, in supergravity, a 2-form may acquire mass by eating a vector [8].

Forms also have a history in cosmology. Form fluxes in compact spaces have long been considered as a way to stabilize internal dimensions [4]. Okada studied how the presence of these fluxes might bring about radion induced inflation [9]. The role of 2-forms in Pre-Big Bang scenarios [10] has been discussed by several authors (see for instance [10, 11]). Recently, forms have appeared in attempts to model late-time acceleration in supergravity. [12]. Massive vectors (1-forms) were considered by Ford, who proposed a model wherein a four-dimensional universe undergoes inflation driven by a self-interacting vector [13].

The paper is organized as follows. In Section II we present our model. It is described, both, from the higher-dimensional point of view, and from a dimensionally-reduced perspective. Section III contains details of the inflationary solutions, and in Section IV we compute the spectrum of primordial density fluctuations generated during inflation. In Section V we discuss the end of inflation, and propose a way of stabilizing the radion whilst avoiding the severe constraints on violations of the equivalence principle. Two examples illustrating phenomenologically realistic choices of parameters are presented in Section VI. We conclude in Section VII.

II. THE MODEL

Consider a $(4 + d)$ -dimensional spacetime, i.e., one which contains d additional spatial dimensions. We will propose a mechanism which allows three spatial dimensions to inflate, while keeping the d remaining ones small. Towards this end, we introduce an anisotropy via fields living on the spacetime, rather than through a violation of $(4 + d)$ -dimensional diffeomorphism invariance [3]. This can be achieved by spontaneously giving an expectation value to a field that transforms non-trivially under rotations in $4 + d$ dimensions (see [14] for an alternative possibility). Such fields can be fermionic or bosonic. Conventional wisdom suggests that only bosonic fields may acquire large expectation values. While this need not always be true [15], we will adopt such a viewpoint here. This leaves us with one choice, vector fields and their generalizations, differential forms.

A. Higher-dimensional equations

Consider a (totally antisymmetric) d -form $A_{M_1 \dots M_d} = A_{[M_1 \dots M_d]}$ minimally coupled to gravity,

$$S = \int d^{4+d}x \sqrt{-g} \left[\frac{R}{6} - \frac{1}{2(d+1)!} F_{M_1 \dots M_{d+1}} F^{M_1 \dots M_{d+1}} - W(A^2) \right]. \quad (1)$$

The $(d+1)$ -form F is the field strength of A , $F_{M_1 \dots M_{d+1}} = (d+1)\partial_{[M_1} A_{M_2 \dots M_{d+1}]}$, and the self-interaction term W is an as of yet unspecified arbitrary function of $A^2 \equiv A_{M_1 \dots M_d} A^{M_1 \dots M_d}$. We work in units where the four and higher-dimensional Newton's constant is $8\pi G = 3$, and our metric signature is $(-, +, \dots, +)$.

Although we have not written the additional matter terms in Eq. (1), we assume that the form does not couple to them. Hence, the form $A_{M_1 \dots M_d}$ only interacts with gravity. In a theory of massless forms, invariance under the gauge transformations $\delta A = dB$, where B is any $d - 1$ -form, guarantees that appropriate components of the form decouple from matter [16]. In our theory, the form A is not massless, and it does not couple to additional matter sources either, so we shall not require gauge invariance. In fact, the self-interaction terms W explicitly violates this symmetry.

Varying the action (1) with respect to the metric one obtains Einstein's equations

$G_{MN} = 3 T_{MN}$, where the energy momentum tensor is given by

$$T_{MN} = \frac{1}{d!} F_{MM_2 \dots M_{d+1}} F_N^{M_2 \dots M_{d+1}} + 2 d W' A_{MM_2 \dots M_d} A_N^{M_2 \dots M_d} - g_{MN} \left(\frac{F^2}{2(d+1)!} + W \right). \quad (2)$$

and a prime means a derivative with respect to A^2 . Varying the action with respect to $A_{M_1 \dots M_d}$ one obtains the field equation

$$\nabla_M F^{MM_1 \dots M_d} = 2 d! W' A^{M_1 \dots M_d}, \quad (3)$$

which implies the constraint $\nabla_{M_1} (W' A^{M_1 M_2 \dots M_d}) = 0$.

In this paper we are interested in cosmological solutions of the equations of motion. Hence, we consider a $(4+d)$ -dimensional factorizable spacetime g_{MN} consisting of a four-dimensional Friedmann-Robertson-Walker metric $g_{\mu\nu}$ times a d -dimensional compact internal space with metric G_{mn} of constant curvature $R^{(d)}$,

$$ds_{4+d}^2 \equiv g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + b^2 G_{mn} dx^m dx^n. \quad (4)$$

The coordinates x^μ label our four dimensional world, and the coordinates x^m label what we shall call the “internal” or “compact” space (the dimensions that remain small during inflation). Once the four dimensions start to inflate, the spatial curvature of the four-dimensional metric soon becomes negligible. So, without loss of generality, we consider a spatially flat universe,

$$ds_4^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (5)$$

where t is cosmic time and $a(t)$ is the scale factor. We normalize the internal space metric by the condition

$$\int d^d x \sqrt{G} = 1. \quad (6)$$

Hence, the volume of the internal space is b^d . Then, the four-dimensional scalar b can be interpreted as the radius of the internal space, so we shall call it the radion.

Substituting the metric ansatz (4) into the $(4+d)$ -dimensional Einstein equations and assuming that the radion only depends on time one obtains

$$H^2 + dHI + \frac{d^2 - d}{6} I^2 + \frac{R^{(d)}}{6b^2} = \rho \quad (7a)$$

$$3H^2 + 2\dot{H} + \frac{d^2 + d}{2} I^2 + d\dot{I} + 2dHI + \frac{R^{(d)}}{2b^2} = -3p \quad (7b)$$

$$6H^2 + 3\dot{H} + \frac{d^2 - d}{2} I^2 + (d-1)\dot{I} + 3(d-1)HI + \frac{(d-2)R^{(d)}}{2db^2} = -3P. \quad (7c)$$

In the previous equations we have introduced the Hubble parameter $H \equiv d \log a/dt$ and the expansion rate in the compact space $I \equiv d \log b/dt$. A dot means a derivative with respect to cosmic time t . The energy density ρ , the three-dimensional pressure p , and the pressure along the compact space P are the corresponding components of the energy momentum tensor

$$T^M_N = \text{diag}(-\rho, p, p, p, P, \dots, P). \quad (8)$$

Thus, the energy momentum tensor (2) has to be diagonal in order for solutions of Einsteins equations to exist. An ansatz that satisfies this condition is

$$A_{M_1 \dots M_d} = \sqrt{G} \varepsilon_{0123M_1 \dots M_d} \phi, \quad (9)$$

where $\varepsilon_{M_1 \dots M_{d+4}}$ is totally antisymmetric and $\varepsilon_{0 \dots d+4} = 1$. Consequently, A has non-vanishing components only along the compact dimensions. In fact, A is proportional to the volume form in the compact space, and the proportionality factor is the four-dimensional scalar ϕ , which will turn out to be the inflaton. In a FRW-universe, the field ϕ can only depend on time. From the ansatz (9) and Eq. (4) the square of A given by

$$A^2 = d! b^{-2d} \phi^2, \quad (10)$$

i.e. A^2 is a combination of the inflaton ϕ and the radion b .

Substituting Eq. (9) into Eq. (2) one finds that the energy momentum tensor is indeed of the form (8), where energy density and pressures are given by

$$\rho = \frac{b^{-2d}}{2} \dot{\phi}^2 + W \quad (11a)$$

$$p = \frac{b^{-2d}}{2} \dot{\phi}^2 - W \quad (11b)$$

$$P = -\frac{b^{-2d}}{2} \dot{\phi}^2 + 2W' A^2 - W. \quad (11c)$$

One can derive the equation of motion for ϕ from Eqs. (7) and (11) or directly by considering Eq. (3) for the internal components,

$$\ddot{\phi} + (3H - dI) \dot{\phi} + 2d! W' \phi = 0. \quad (12)$$

Because of the symmetry of the ansatz, the remaining components of the equation of motion are identically satisfied.

By definition, inflation is a stage of accelerated expansion $\ddot{a} > 0$. From a four-dimensional perspective, any inflationary stage can explain the flatness and homogeneity of the universe. However, in an expanding universe [17], only a stage of inflation close to de Sitter yields in general the nearly scale invariant spectrum of primordial density fluctuations that observations seem to favor [18]. Therefore, in order to get a feeling of the constraints our d -form has to satisfy we shall consider a de Sitter stage $H = \text{const}$. In addition, we want to explain why our four-dimensions are large compared to the internal ones, and the simplest way to accomplish that is to assume that the size of the internal dimensions remains constant. Therefore, we shall look for solutions with static internal dimensions, $I = \dot{I} = 0$. With these assumptions Eqs. (7) take the form

$$H^2 + \frac{R^{(d)}}{6b^2} = \rho \quad (13a)$$

$$3H^2 + \frac{R^{(d)}}{2b^2} = -3p \quad (13b)$$

$$6H^2 + \frac{(d-2)R^{(d)}}{2db^2} = -3P. \quad (13c)$$

Solutions of the previous equations exist if and only if

$$\rho + p = 0 \quad (14a)$$

$$2\rho + P = \frac{d+2}{6d} \frac{R^{(d)}}{b^2}. \quad (14b)$$

Eq. (14a) is the familiar inflationary relation satisfied by a cosmological constant or a frozen scalar field. The second condition, Eq. (14b), implies that for flat or negatively curved internal dimensions the null energy condition has to be violated [19, 20].

We restrict now our attention to the energy momentum tensor of the d -form, Eqs. (11). Then, Eq. (14a) implies that the inflaton is frozen, $\dot{\phi} = 0$. Although an exactly frozen field is in general not solution of Eq. (12), we shall later see that a *nearly* frozen, slowly-rolling field actually is. On the other hand, Eq. (14b) constraints the form of the interaction W ,

$$W + 2W'A^2 = \frac{d+2}{6d} \frac{R^{(d)}}{b_0^2}, \quad (15)$$

where b_0 is the constant value of b . The solution to the previous equation is

$$W(A^2) = W_0 + W_1 \cdot (A^2)^{-1/2}, \quad (16)$$

where W_1 is an arbitrary integration constant.¹ If $R^{(d)} = 0$, then Eq. (15) implies $W_0 = 0$. If $R^{(d)} \neq 0$ the $(4 + d)$ -dimensional cosmological term W_0 is related to the constant value of b by

$$b_0 = \sqrt{\frac{d + 2}{6d} \frac{R^{(d)}}{W_0}}. \quad (17)$$

In summary, if a (nearly) frozen field ϕ is a solution of the equations of motion, there exist solutions where four dimensions inflate and the where radius of the internal dimensions is constant. If the compact internal dimensions are flat, $R^{(d)} = 0$, any constant value of b is possible. If the internal dimensions are positively curved, $R^{(d)} > 0$, a positive $(4 + d)$ -dimensional cosmological constant W_0 is required, whereas if the internal dimensions are negatively curved, $R^{(d)} < 0$, a negative cosmological constant W_0 is needed to stabilize the radion during inflation.

Compact spaces of constant negative curvature can be constructed by acting on the d -dimensional hyperbolic plane H^d with a free discrete isometry group of the space [21]. For $d \geq 3$, these compact hyperbolic manifolds are “rigid”, in the sense that the radius is the only massless moduli they admit. In that case, our metric ansatz for the metric in the internal space is the most general one. The application of compact hyperbolic manifolds to cosmology has been pioneered by Starkman and collaborators (see [22] and references therein). Recently, compact hyperbolic spaces have also received attention in the context of cosmological solutions in supergravity [12].

Below we shall see that a nearly frozen field is indeed a solution of the equations of motion, completely analogous to a conventional slow-roll inflationary regime. We shall also show that if the compact space is positively curved, solutions with constant radion are unstable. In order to understand these properties though, it is going to be convenient to work with the dimensionally reduced action.

B. Dimensionally reduced action

In the cosmological setting we have been dealing with, the radion b and the inflaton ϕ only depend on time. In particular, b and ϕ do not depend on the internal coordinates x^m .

¹ One could also allow for functions that are only *approximately* described by Eq. (16), though for simplicity we shall not explore this possibility here.

If the different fields in the action (1) do not depend on the internal space, it is possible to integrate over the internal space and obtain a four dimensional action that describes the dynamics of the four-dimensional scalars b and ϕ . Substituting Eq. (9) into the Lagrangian of Eq. (1), and integrating over the internal space we obtain

$$S = \int d^4x \sqrt{-g} b^d \left[\frac{R^{(4)}}{6} + \frac{R^{(d)}}{6b^2} + \frac{d^2 - d}{6} \frac{\partial_\mu b \partial^\mu b}{b^2} - \frac{1}{2} \frac{\partial_\mu \phi \partial^\mu \phi}{b^{2d}} - W(A^2) \right], \quad (18)$$

where we have used Eq. (6) and A^2 stands for the r.h.s of Eq. (10). $R^{(4)}$ is the four-dimensional scalar curvature and wherever the metric tensor is involved the four-dimensional metric $g_{\mu\nu}$ is to be used. Note that the kinetic terms of the different fields are not in canonical form. For convenience, we would like to work with canonical kinetic terms for the metric and the radion, so we shall rename those fields,

$$g_{\mu\nu} \equiv b^{-d} \tilde{g}_{\mu\nu} \quad \text{and} \quad b \equiv e^\sigma. \quad (19)$$

Plugging the last expressions into Eq. (18) we get

$$S = \int d^4x \sqrt{-g} \left[\frac{R^{(4)}}{6} - \frac{d^2 + 2d}{12} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} \frac{\partial_\mu \phi \partial^\mu \phi}{e^{2d\sigma}} + e^{-(d+2)\sigma} \frac{R^{(d)}}{6} - e^{-d\sigma} W(A^2) \right], \quad (20)$$

where all quantities refer to the tilded metric (we have dropped the tilde). Because for the solutions we are interested in b is (nearly) constant, the behavior of the scale factor in the Jordan frame (18) and the Einstein frame (20) is essentially the same. Let us stress though, that in any case all ‘‘conformal’’ frames are physically equivalent, since physical predictions should not depend on the way fields are named (see for instance [23]).

The action (20) describes two interacting scalar fields ϕ and σ . Substituting Eq. (16) into Eq. (20) and using Eq. (10) one obtains a potential term

$$U(\sigma) + V(\phi) \equiv -e^{-(d+2)\sigma} \frac{R^{(d)}}{6} + e^{-d\sigma} W_0 + \frac{W_1}{\sqrt{d!}} \frac{1}{\phi}, \quad (21)$$

where by ‘‘potential’’ we mean the sum of those terms in the Lagrangian that do not contain derivatives of the fields, with an overall minus sign. Remarkably, this potential does not involve any coupling between the radion σ and the inflaton ϕ , which is part of the origin of the condition (15). Therefore, as far as the potential is concerned, the two fields are decoupled, and we can write down the potential terms for the radion and the inflaton separately. For non-vanishing internal curvature the radion potential is

$$U(\sigma) = -e^{-(d+2)\sigma} \frac{R^{(d)}}{6} + e^{-d\sigma} W_0, \quad (22)$$

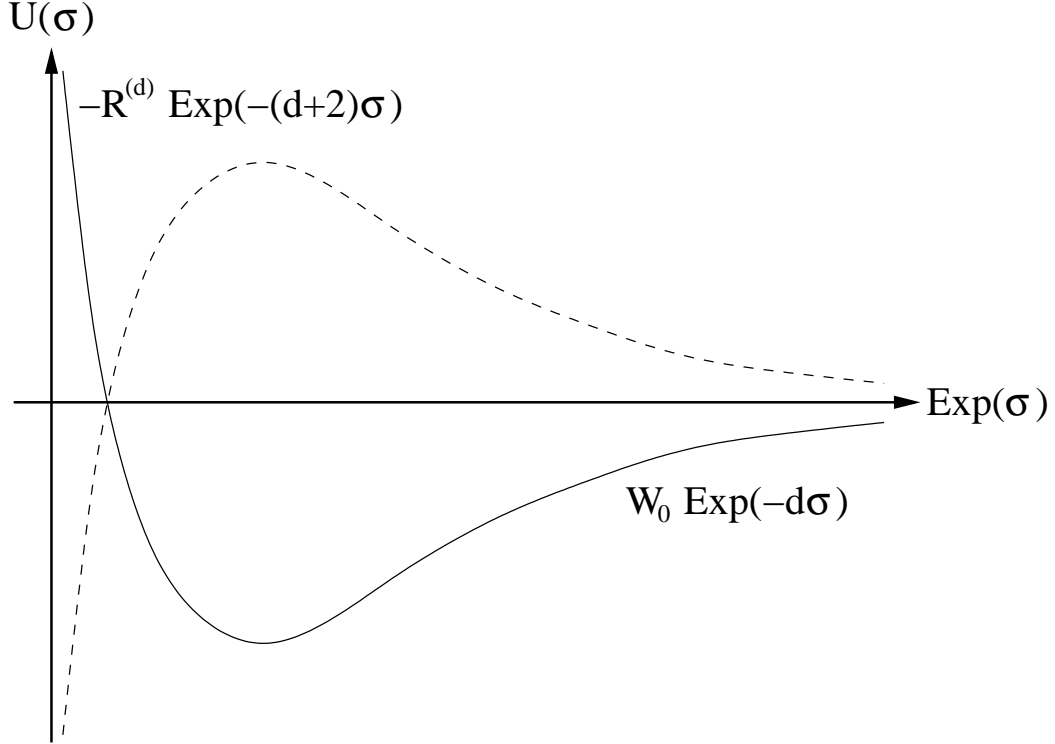


FIG. 1: A plot of the radion potential, Eq. (22), for different signs of the compact space curvature $R^{(d)}$. In order for the potential to have an extremum, $\text{sgn}(R^{(d)}) = \text{sgn}(W_0)$. The extremum is then a minimum if $W_0 < 0$. For $R^{(d)} = 0$, the radion potential identically vanishes.

which has an extremum at $\sigma_0 = \log b_0$, and b_0 is given by Eq. (17). The second derivative of $U(\sigma)$ at σ_0 is

$$m_\sigma^2 \equiv \frac{6}{d^2 + 2d} \frac{d^2 U}{d\sigma^2} \Big|_{\sigma_0} = -\frac{2e^{-(d+2)\sigma_0}}{d} R^{(d)}. \quad (23)$$

Therefore, for $R^{(d)} > 0$ (i.e. $W_0 > 0$) a solution with constant $b = b_0$ is unstable, whereas for $R^{(d)} < 0$ (i.e. $W_0 < 0$) a solution with constant $b = b_0$ is perfectly stable. In the latter case, the minimum of $V(\sigma)$ occurs at a negative value of the four-dimensional cosmological constant

$$U(\sigma_0) = \frac{2}{d+2} \left(\frac{d+2}{6d} \frac{R^{(d)}}{W_0} \right)^{-d/2} W_0. \quad (24)$$

If the internal curvature vanishes (i.e. $W_0 = 0$), so does the radion potential. A constant b is then marginally stable, and any value of $b_0 \equiv e^{\sigma_0} = \text{const}$ is a solution of the equations of motion. A plot of the potential for the different signs of $R^{(d)}$ is shown in Fig. 1.

III. INFLATIONARY SOLUTIONS

Our next goal is to study the inflationary solutions with nearly constant radion that we have hinted in the previous sections and to investigate some of their properties. Let us begin with the inflaton equation of motion,

$$\ddot{\phi} + (3H - 2d\dot{\sigma})\dot{\phi} + e^{2d\sigma}\frac{dV}{d\phi} = 0, \quad (25)$$

where the inflaton potential is given by

$$V(\phi) = \frac{W_1}{\sqrt{d!}} \frac{1}{\phi}. \quad (26)$$

Inverse power-law potentials such as the one in Eq. (26) are known to admit “slow-roll” inflationary solutions, since they satisfy the slow-roll conditions for large values of the field [25]. We will show that during slow-roll, the radion is nearly constant and the field acceleration is negligible. At this point let us just assume

$$\frac{\dot{\sigma}}{H} \ll 1 \quad \text{and} \quad \ddot{\phi} \ll 3H\dot{\phi}, \quad (27)$$

which implies that the speed of the inflaton field is given by the slow-roll expression

$$\dot{\phi} = -\frac{e^{2d\sigma}}{3H} \frac{dV}{d\phi}. \quad (28)$$

Once we have found the solutions of the equations of motion in the slow-roll regime, we will show that the assumptions made in their derivation hold. Note that the first condition in Eq. (27) means that the radion is nearly constant during slow-roll.²

The radion equation of motion is

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{6}{d+2}e^{-2d\sigma}\dot{\phi}^2 + \frac{6}{d^2+2d}\frac{dU}{d\sigma} = 0, \quad (29)$$

where $U(\sigma)$ is given by Eq. (22). If $R^{(d)}$ is negative, $U(\sigma)$ has a minimum at σ_0 . Let us assume that the radion remains in the vicinity of the minimum, $\sigma \approx \sigma_0$, which implies $dU/d\sigma \approx 0$. If $R^{(d)}$ is zero the potential $U(\sigma)$ identically vanishes, and so does $dU/d\sigma$. Then, if the slow roll assumption

$$\ddot{\sigma} \ll 3H\dot{\sigma} \quad (30)$$

² Quantum fluctuations are also responsible for a growth in the expectation value $\langle \sigma^2 - \langle \sigma \rangle^2 \rangle \approx H^3 t$ during inflation [24]. We shall assume that this effect is negligible.

is satisfied, the speed of the radion turns to be

$$\dot{\sigma} = -\frac{2e^{-2d\sigma}}{(d+2)H}\dot{\phi}^2. \quad (31)$$

Note that $\dot{\sigma}$ is negative, i.e. the extra dimensions contract. Because $\dot{\sigma} \equiv \dot{b}/b$, the first condition in (27) means that the extra dimensions evolve much slower than the inflating ones, as desired.

From the higher-dimensional Friedmann equation (13a) or directly from Eq. (20), the four-dimensional Friedmann equation reads

$$H^2 = \frac{e^{-2d\sigma}}{2}\dot{\phi}^2 + \frac{d^2 + 2d}{12}\dot{\sigma}^2 + V(\phi) + U(\sigma), \quad (32)$$

where $V(\phi)$ and $U(\sigma)$ are respectively the inflaton and radion potentials. If $R^{(d)} = 0$, $U(\sigma)$ identically vanishes. Inflation occurs if the energy density in the universe is dominated by potential energy of the scalars. Assuming that

$$\frac{d^2 + 2d}{12}\dot{\sigma}^2 \ll \frac{e^{-2d\sigma}}{2}\dot{\phi}^2 \ll V(\phi) + U(\sigma) \quad (33)$$

the Friedmann equation reads

$$H^2 \approx V(\phi) + U(\sigma). \quad (34)$$

Inserting (34) into Eqs. (28) and (31) one can express H , $\dot{\phi}$ and $\dot{\sigma}$ entirely in terms of the values of the radion and inflaton fields. The conditions (27), (30) and (33) then reduce, up to factors of order one, to the slow-roll conditions

$$\epsilon \equiv e^{2d\sigma} \left(\frac{dV/d\phi}{V(\phi) + U(\sigma)} \right)^2 \ll 1, \quad \eta \equiv e^{2d\sigma} \frac{d^2V/d\phi^2}{V(\phi) + U(\sigma)} \ll 1, \quad (35)$$

which up to the exponential of σ are the conventional slow-roll conditions of single-field slow-roll inflation [2]. For the inflaton potential (26), both conditions are satisfied in the inflaton lies in the range

$$\frac{\exp(d\sigma)}{\sqrt{18}} \lesssim \phi \lesssim -\frac{1}{\sqrt{d!}} \frac{W_1}{U(\sigma)}. \quad (36)$$

Note that for flat internal dimensions ($U(\sigma) = 0$) the upper limit is shifted to infinity. For simplicity, we shall assume in the following that during inflation, even if $R^{(d)} \neq 0$, the field ϕ is much smaller than the upper limit in Eq. (36). Then, $V(\phi) \gg U(\sigma)$ and $U(\sigma)$ can be neglected.

The slow-roll solutions we have discussed are attractors for the evolutions of the field [26]. This means that even if they are initially not satisfied, the equations of motion will

drive the field to a regime where they are [27]. Therefore, in our scenario cosmic evolution is quite insensitive to initial conditions. During slow-roll, the relative change of the Hubble parameter during a Hubble time is small,

$$\frac{\dot{H}}{H^2} = -\frac{\epsilon}{6}, \quad (37)$$

i.e. during slow-roll the universe inflates almost like in a de Sitter stage. At the same time, the radion and inflaton fields are nearly frozen,

$$\frac{\dot{\sigma}}{H} = -\frac{2\epsilon}{9(d+2)}, \quad e^{-d\sigma} \frac{\dot{\phi}}{H} = -\frac{\sqrt{\epsilon}}{3} \quad (38)$$

although the inflaton evolves much faster than the radion. Here, ϵ is the slow-roll parameter defined in Eq. (35). Thus, to leading order in the slow-roll approximation, we can assume that the inflaton slowly changes while the radion remains frozen at $\sigma = \sigma_0$. Here, σ_0 is the minimum of the radion potential if the compact dimensions are negatively curved, or the initial value of the radion if the extra dimensions are flat.

In order for inflation to successfully explain the flatness and homogeneity and the visible universe, it is necessary for inflation to last more than around 60 e-folds (the exact number depends on the unknown details of reheating [28]). We will discuss ways to terminate inflation end in Section V. For our present purposes it will suffice to assume that because the inflaton potential deviates from its functional form (26), inflation ends when the inflaton reaches the value ϕ_0 , i.e. when the form reaches the value $A_0^2 = d! \exp(-2d\sigma_0)\phi_0^2$. Let us then assume that at some moment of time during inflation the value the squared form is A^2 . The number of e-folds of inflation between that time and the end of inflation is

$$N \equiv \log \frac{a_0}{a} \approx \frac{3}{2d!} (A_0^2 - A^2), \quad (39)$$

where we have assumed slow-roll in the derivation. Given $N \approx 60$ and because the inflationary regime is limited by Eq. (36), Eq. (39) constrains the possible values of A_0^2 .

IV. PERTURBATIONS

The most important consequence of a stage of quasi de Sitter inflation is the generation of a nearly scale invariant spectrum of primordial density perturbations [29]. In the simplest inflationary models, inflation is driven by a single scalar field, and its fluctuations are

responsible for the adiabatic primordial spectrum of fluctuation that observations seem to favor. The presence of a second (light) scalar field during inflation is potentially dangerous, since it could lead to entropy perturbations (see for instance [30]).

In our scenario, inflation is driven by the single inflaton field ϕ , but there is an additional field, the radion σ . If the compact dimensions are flat, to leading order in slow-roll, the potential for σ vanishes. Because of that, fluctuations of σ do not contribute to fluctuations in the energy density, so that no entropic component is generated during slow-roll [30]. If the compact dimensions are negatively curved, the field σ is massive. If its mass is bigger than the Hubble factor during inflation, quantum fluctuations of σ are suppressed, so no entropy component is generated either. In both cases, we can therefore regard σ as a constant and study the perturbations solely due to the inflaton ϕ .

The power spectrum $\mathcal{P}(k)$ is a measure of the mean square fluctuations of the generalized Newtonian potential in comoving distances $1/k$. During radiation domination, it is given by [25]

$$\mathcal{P}(k) = \frac{e^{-2d\sigma_0} V^3}{\pi^2 V_{,\phi}^2} \Big|_{k=aH}, \quad (40)$$

where V is the potential (26) and $k = aH$ denotes that the r.h.s. of the equation has to be evaluated at the time the mode crosses the Hubble radius. The factor $e^{-2d\sigma_0}$ shows up because we work with a non-canonically normalized inflaton. Because the radion evolves much slower than the inflaton, we assume that the radion is constant.

The amplitude of the power spectrum is approximately equal to the squared amplitude of the temperature fluctuations in the cosmic microwave background, $\delta T/T \approx 10^{-5}$. Evaluating $\mathcal{P}(k)$ for a mode that crosses the Hubble radius N e-folds before the end of inflation, Eq. (39), we hence obtain the constraint

$$\mathcal{P} = \frac{e^{-d\sigma_0} W_1}{\pi^2 \sqrt{d!}} \sqrt{\frac{A_0^2}{d!} - \frac{2N}{3}} \approx 10^{-10}. \quad (41)$$

Observe the exponential suppression with increasing $d\sigma_0$. The (nearly constant) slope of the power spectrum is parametrized by the spectral index n_s , where

$$n_s - 1 \equiv \frac{d \log \mathcal{P}}{d \log k}. \quad (42)$$

Current observations imply the limits $0.9 < n_s < 1.1$ [31]. Substituting Eq. (40) into the previous definition and evaluating it N e-folds before the end of inflation, Eq. (39), we

obtain

$$n_s - 1 = \frac{1}{3A_0^2/d! - 2N} < 10^{-1}. \quad (43)$$

Note that the quantity in the denominator is always positive. Therefore, the power spectrum is blue ($n_s > 1$). Cosmologically relevant scales typically left the horizon about $N \approx 60$ e-folds before the end of inflation, though this number varies with the details of reheating [28]. We show below that in order to satisfy constraints on the universality of free fall, A_0^2 has to be large, $A_0^2 > d! \cdot 10^7$. Hence, in general one expects $n_s \approx 1$. In Section VI we deal with a concrete example where the different parameters (W_1 , σ_0 , d , etc.) in phenomenologically viable models are specified.

V. REHEATING AND RADION STABILIZATION

In the conventional four-dimensional models of inflation, the universe is reheated when inflation ends and the inflaton starts oscillating around the minimum of its potential [32]. In our model, the inflaton potential is given by Eq. (26). If $R^{(d)} < 0$, inflation ends when ϕ reaches the value $W_1/(d!U(\sigma_0))$. At that value of the field, the potential energy vanishes, and beyond that value the potential becomes negative. The cosmological evolution of a scalar field with such effective potentials was studied in [33]. It was found that in general, once the potential becomes negative, the universe enters a phase of contraction that ends in a singularity. This would prevent the universe from reheating, thus invalidating our model. If the internal space is flat, $R^{(d)} = 0$, once the slow-roll condition is satisfied, it is never violated. In that case inflation never ends. In both cases we have to assume that around some value of the field ϕ_0 the potential deviates from its form in Eq. (26) and is described by a different functional form³. This is the case if at some point the self-interaction $W(A^2)$ develops a minimum at A_0^2 , as shown in Fig. 2. Around the minimum, the function W can be expanded as

$$W(A^2) \approx V_0 + \frac{\lambda}{8}(A^2 - A_0^2)^2, \quad (44)$$

³ For an alternative mechanism to end inflation in a model with $V(\phi) \propto 1/\phi$, see [34].

where V_0 is a cosmological term and λ is a coupling parameter. From the dimensionally reduced point of view, Eq. (20), this yields a radion and inflaton potential

$$V(\sigma, \phi) = -e^{-(d+2)\sigma} \frac{R^{(d)}}{6} + e^{-d\sigma} \left[V_0 + \frac{\lambda}{2} (d! e^{-2d\sigma} \phi^2 - A_0^2)^2 \right]. \quad (45)$$

Again, if $R^{(d)} < 0$, the potential has a stable minimum at $b_0 \equiv e^{\sigma_0}$ given by Eq. (17), with W_0 replaced by V_0 . Even if W has a minimum, a negative cosmological term is needed to stabilize the radion if the internal dimensions are negatively curved [20]. Although the value of the cosmological term during inflation and after the end of inflation might have changed, it can do so only if the value of σ_0 that minimizes the effective potential changes significantly. Because one of our main goals was to study inflationary solutions where σ is constant, we shall not consider this case anymore and proceed with flat internal dimensions.

If $R^{(d)} = 0$, we shall assume $V_0 = 0$, which amounts to tuning the higher-dimensional cosmological constant to zero. During inflation, ϕ evolves while σ stays constant. When A^2 reaches the vicinity of A_0^2 inflation ends, and the fields approach the minimum at A_0^2 . Because we assume that the end of inflation and the minimum of W are not far apart we can assume that σ and ϕ do not change significantly. Then

$$A_0^2 \approx d! e^{-2d\sigma_0} \phi_0^2, \quad (46)$$

where σ_0 is the constant value of σ during inflation and ϕ_0 is the value of ϕ at the end of inflation.

Although there are reheating models where the inflaton potential does not oscillate around a minimum [35], we would like one of our fields to oscillate around the minimum of its potential, i.e. we would like one of them to get a mass. The potential (44) has a minimum at $A^2 = A_0$. Let us denote by σ_0 and ϕ_0 the values of σ and ϕ at that minimum, and let us consider fluctuations around those values,

$$\phi = \phi_0 + e^{d\sigma_0} \delta\phi \quad \text{and} \quad \sigma = \sigma_0 + \sqrt{\frac{6}{d^2 + 2d}} \delta\sigma. \quad (47)$$

Substituting the previous definitions into the Lagrangian of Eq. (20) and expanding to second order in the fluctuations one gets a coupled system of two canonically normalized fields $\delta\sigma$ and $\delta\phi$. Let us introduce the also canonically normalized fields χ_0 and χ_1 , defined by the relations

$$\delta\phi = \cos\theta \chi_0 + \sin\theta \chi_1, \quad \delta\sigma = -\sin\theta \chi_0 + \cos\theta \chi_1, \quad (48)$$

where

$$\sin^2 \theta = \left(1 + \frac{6d}{d+2} \frac{A_0^2}{d!} \right)^{-1}. \quad (49)$$

The fields χ_0 and χ_1 diagonalize the mass matrix of the fluctuations. Because the potential Eq. (44) has a flat direction, the scalar χ_0 turns to be massless. The field χ_1 is massive, and its mass is given by

$$m_{\chi_1}^2 = \frac{\lambda d! e^{-d\sigma_0} A_0^2}{\sin^2 \theta}. \quad (50)$$

We shall show below that in order to satisfy experimental restrictions related to the apparent universality of free fall, we have to consider the small mixing angle limit $\theta \ll 1$. In that case $\delta\sigma \approx \chi_1$ is massive and $\delta\phi \approx \chi_0$ is massless. Therefore, at the end of inflation, the radion starts oscillating around the minimum of its potential, while the inflaton remains essentially constant. Note that for reheating to work it is not crucial that the field that drives reheating is the same as the one that drives inflation. The inhomogeneities seeded during inflation can be transferred to the decay products of the radion because the terms that couple the oscillating radion to matter in general contain metric fluctuations (see [36] for related ideas). There is nevertheless a potential challenge our model has to face. The radion is massless during inflation, but becomes massive at the end of inflation and oscillates around its minimum at a non-vanishing σ_0 . Hence, as pointed out in [37] it is possible that metric fluctuations are parametrically amplified during the reheating process. This will happen if long-wavelength modes lie in a resonance band of the equation that describes the evolution of the radion perturbations. Because in our case the mass (and therefore the oscillation frequency) of the radion can be freely adjusted by changing the parameter λ , one might avoid such resonances.

Once inflation has ended and the universe has been heated, the universe evolves according to the standard Big-Bang scenario. The presence of the massless scalar field χ_0 that generically couples to matter could yield however to violations of several tests on the couplings of matter to gravity and on the validity of the equivalence principle [38, 39]. Presently, the most stringent restrictions arise from experiments on the universality of free fall. Upon the dimensional reduction of the higher-dimensional action (1) (including the matter terms we have not written down), one expects the radion to couple to the four-dimensional fields with different powers of e^σ . For instance, a Maxwell term $F_{MN}F^{MN}$ in Eq. (1) leads to the term $e^{d\sigma} F_{\mu\nu}F^{\mu\nu}$ in Eq. (20) [by F we now mean the electromagnetic field strength, not the

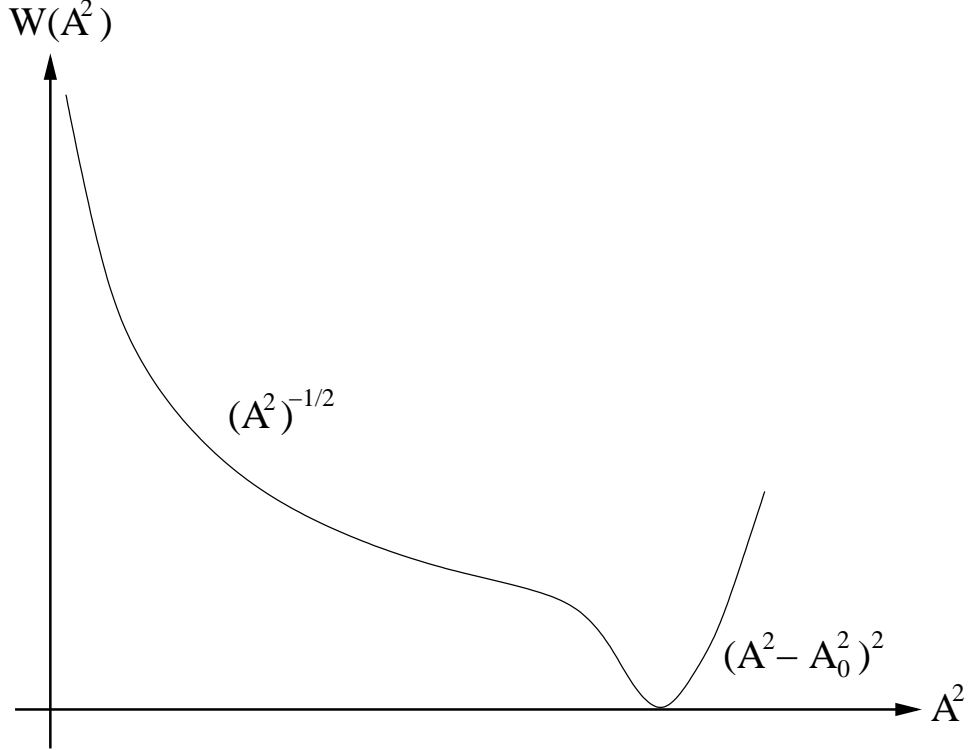


FIG. 2: A plot of the form self-interaction W .

field strength of our form A]. If χ_0 is a massless canonically normalized scalar, the coupling strength

$$\alpha \equiv \frac{\partial \log e^{-d\sigma}}{\partial \chi_0} = -\sqrt{\frac{6d}{d+2}} \frac{\partial \delta\sigma}{\partial \chi_0} \quad (51)$$

is severely restricted by experiments on the universality of free fall, where the undetected differential acceleration of two bodies of different composition puts the limit $\alpha^2 \leq 10^{-7}$ [38].

Using Eqs. (48), (49) and the last limit we find

$$\frac{A_0^2}{d!} \geq 10^7. \quad (52)$$

It then follows that the field $\delta\sigma$ points in the direction of the massive field χ_1 , whereas the field $\delta\phi$ points along the massless direction χ_0 , Eq. (48). Note that since in our model the form $A_{M_1 \dots M_d}$ only couples to gravity, in the dimensionally reduced action the field ϕ only interacts gravitationally. Therefore there are no constraints originating from the massless field $\chi_0 \approx \delta\phi$.

VI. TWO EXAMPLES

In the previous Sections we have formulated the conditions that our scenario has to satisfy in order to provide for a successful inflationary scenario, but we have not verified whether they can be satisfied at all. In the following, we shall fill this gap and comment on the nature of those conditions by providing two explicit examples. Essentially, the conditions our model has to obey have two different origins: Constraints related to inflation and constraints related to radion stabilization after the end of inflaton. Our main goal consisted of formulating an inflationary scenario where a certain number of dimensions remain small while four dimensions become exponentially large. As a bonus, we have also proposed a way to stabilize the radion after the end of inflation. Because inflation and radion stabilization are in principle two different issues, we shall deal with the two sources of constraints separately.

Let us first discuss the constraints associated to inflation. Inflation has to last longer than about 60 e-folds, Eq. (39), it has to account for the correct amplitude of the temperature anisotropies Eq. (41) and must explain the nearly scale invariance of the power spectrum, Eq. (43). In addition, after the end of inflation it is desirable that the universe is reheated when one of our scalars, the radion or the inflaton starts oscillating around the minimum of its potential. Let us assume that the compact dimensions are flat ($R^{(d)} = 0$). String theories require the existence of 6 additional spatial dimensions, hence, we shall set

$$d = 6. \tag{53}$$

Because the compact dimensions are flat, we have to pick $W_0 = 0$ in order to satisfy Eq. (15). This choice corresponds to tuning the cosmological constant to zero. The parameter W_1 is constrained by Eq. (41). Notice that the larger the extra dimensions, the larger W_1 can be. Thus, extra dimensions might “explain” the smallness of the temperature anisotropies. As mentioned in the introduction, one expects all spatial dimensions to be Planck sized initially. In fact, for a smaller size of the universe, our classic description is likely to break down. Therefore, we can safely rely on classical general relativity if for instance $\sigma_0 \approx \log 10$, i.e.

$$b_0 \equiv e^{\sigma_0} \approx 10. \tag{54}$$

With this choice of σ_0 , the slow roll condition (36) is satisfied for $A^2 \gtrsim 40$. Then, Eq. (39)

guarantees that the maximal number of e-folds is more than enough if we choose

$$A_0 \approx 220. \tag{55}$$

which corresponds to $N_{max} \approx 100$. Substituting the former values into Eq. (41) for a scale that left the horizon $N = 60$ e-folds before the end of inflation we obtain

$$W_1 \approx 10^{-2}, \tag{56}$$

a surprisingly “natural” number. Let us point again that its origin is the exponential suppression of the temperature anisotropies due to the internal dimensions. With these numbers the spectral index turns to be

$$n_s - 1 \approx 10^{-2}, \tag{57}$$

quite close to scale invariance and well within the limits of Eq. (43).

If one considers in addition the constraint imposed by the universality of free fall, the numbers are less appealing, though they are in no way more “unnatural” than in other models. Let us keep the same number and size of the internal flat dimensions, Eqs. (53) and (54). Then, the limit in Eq. (52) is satisfied if we set $A_0 \approx 10^5$. Following the same steps as before translates into $W_1 \approx 10^{-5}$ and $n_s = 1$. Therefore, we see that in our model, the source of small parameters is not the smallness of the CMB temperature fluctuations, but the severe limits on the universality of free fall.

VII. SUMMARY AND CONCLUSIONS

We have shown that Einstein gravity in $(4 + d)$ dimensions, when coupled to a self-interacting d -form, possesses solutions which are inflationary attractors. These spacetimes exhibit an inflationary phase in 4 dimensions, whilst being almost static in the d extra dimensions. Such solutions exist both for flat and for negatively curved compact extra dimensions. However, in our minimal scenario, solutions with positively curved extra dimensions and a constant radion are unstable.

An attractive feature of our model is that from the viewpoint of a four-dimensional observer, the inflaton and the radion are very weakly coupled. Therefore, a separate mechanism is not required in order to stabilize the radion. Instead, a bulk cosmological constant will

suffice. More generally, this feature of our model guarantees that if the radion is stabilized by a separate mechanism, inflation will not destabilize it.

Furthermore, we are able to account for the near scale invariant spectrum, and amplitude of perturbations with surprisingly natural input parameters. The compact extra dimensions cause an exponential suppression of fluctuations; this effect is responsible for the smallness of the temperature anisotropies. However, the severe experimental bounds on violations of the equivalence principle drive some of the parameters in the model towards less natural numbers.

In our scenario, the end of inflation is triggered by the departure of the self-interaction W from its functional form during inflation. If W develops a minimum about which the radion may oscillate, then reheating ensues. However, for negatively curved extra dimensions the minimum of the potential occurs at a negative value of the energy density, whence it is problematic, though certainly not impossible to reheat the universe. This mechanism, which ends inflation and causes reheating, also prevents the radion from violating the equivalence principle. Even though the potential of the radion-inflaton system has a flat direction, through an appropriate choice of parameters it is possible to project the radion onto the massive eigenvector of the mass-matrix. Thus, we suppress violations of the equivalence principle which arise from varied coupling of the radion to matter. This idea might be of use, in contexts different from ours, as an alternative method for decoupling the radion from matter in theories with flat directions.

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