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Panel Data Inference under Spatial Dependence

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PANEL DATA INFERENCE
UNDER SPATIAL DEPENDENCE

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Abstract

This paper focuses on inference based on the usual panel data estimators of a one-way error component regression model when the true specification is a spatial error component model. Among the estimators considered, are pooled OLS, random and fixed effects, maximum likelihood under normality, etc. The spatial effects capture the cross-section dependence, and the usual panel data estimators ignore this dependence. Two popular forms of spatial autocorrelation are considered, namely, spatial auto-regressive random effects (SAR-RE) and spatial moving average random effects (SMA-RE). We show that when the spatial coefficients are large, test of hypothesis based on the usual panel data estimators that ignore spatial dependence can lead to misleading inference.

Keywords: Panel data; Hausman test; Random effect; Spatial autocorrelation; Maximum Likelihood.

JEL Classification: C33

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This paper is written in honor of P.A.V.B. Swamy who has made important contributions to the error component panel data model and the random coefficient regression model.

1 Introduction

This paper considers spatial dependence across panels as a simple way of capturing cross-section dependence among countries, regions, or neighbors. The structure of the dependence can be related to location and distance, both in a geographic space as well as a more general economic or social network space, see Anselin (1988) and Anselin, Le Gallo and Jayet (2008). Following Kapoor, Kelejian and Prucha (2007) and Fingleton (2008*a*), we focus on two spatial error processes: the spatial autoregressive (SAR) and the spatial moving average (SMA) random effects model, namely SAR-RE and SMA-RE. Under the assumption that the true model is SAR-RE or SMA-RE, inference based on the usual panel data estimators including pooled OLS, random and fixed effects, Maximum Likelihood under normality, etc., is investigated using Monte Carlo experiments. In panel data analysis, the one-way error component model is popular for capturing heterogeneity among the cross-sectional units, see Wallace and Hussain (1969), Amemiya (1971), Swamy and Arora (1972), to mention only a few. These random effects estimators are programed using standard software like EViews and Stata. Under the assumption of no cross-section dependence, the small sample efficiency of these estimators as well as inference based upon them is studied, for example, by Maddala and Mount (1973), Baltagi (1981) and Moulton (1980), to mention only a few. This paper introduces cross-section dependence through spatial autoregressive and moving average error component models, and re-examine inference based on these estimators. We show that when the spatial coefficients are large, test of hypothesis based on the usual panel data estimators that ignore spatial dependence can lead to misleading inference. In a similar spirit, it is worth noting that Baltagi, Bresson and Pirotte (2007) studied the performance of panel unit root tests under spatial dependence of RE-SAR and RE-SMA types. They find that for the random effects SAR model for example, panel unit root tests that ignore this spatial correlation, will yield over-sized unit root tests of up to 20% at the 5% significance level. The plan of this paper is as follows: Section 2 presents the model, while Section 3 describes the Monte Carlo design. Section 4 contains the Monte Carlo results, and the last section concludes.

2 The model

Consider a linear panel data regression model:

$$y_{it} = Z_{it}\delta + u_{it} \qquad i = 1, \ldots, N; \, t = 1, \ldots, T \tag{1}$$

where $Z_{it} = [1, X_{it}]$ is $1 \times (k+1)$, $\delta' = [\gamma, \beta']$ is $1 \times (k+1)$. Kapoor, Kelejian and Prucha (2007) proposed a SAR random effects model, hereafter SAR-RE. In their specification, the disturbance term u_t itself follows a SAR process

$$u_t = \rho W_N u_t + \varepsilon_t \tag{2}$$

and the remainder term ε_t follows an error component structure

$$\varepsilon_t = \mu + v_t \tag{3}$$

W_N is an $(N \times N)$ known spatial weights matrix which has zero diagonal elements and is usually row-normalized. Also, μ is an $N \times 1$ vector of individual effects which are assumed to be $IID(0, \sigma_\mu^2)$, and v_t is an $N \times 1$ vector of remainder effects which are assumed to be $IID(0, \sigma_v^2)$. μ and v_t are independent of each other and among themselves. Combining (2) and (3), we obtain the SAR-RE specification for the $(N \times 1)$ error vector u_t at time t :

$$u_t = (I_N - \rho W_N)^{-1} \varepsilon_t = B_N^{-1} \varepsilon_t \tag{4}$$

where I_N is an identity matrix of dimension $N \times N$ and $B_N = (I_N - \rho W_N)$. The matrix B_N is assumed to be non-singular, and the row and column sums of the matrix W_N are bounded uniformly in absolute value. For the full $(NT \times 1)$ vector of disturbances, we have:

$$u = \left(\iota_T \otimes B_N^{-1}\right) \mu + \left(I_T \otimes B_N^{-1}\right) v \tag{5}$$

where ι_T is a vector of ones of dimension $T \times 1$. The corresponding $(NT \times NT)$ covariance matrix is given by:

$$\Omega_u = \left(I_T \otimes B_N^{-1}\right) \Omega_\varepsilon \left(I_T \otimes B_N^{-1}\right)' \tag{6}$$

where

$$\Omega_\varepsilon = \sigma_\mu^2 \left(J_T \otimes I_N\right) + \sigma_v^2 I_{NT} = \sigma_v^2 Q_{0,N} + \sigma_1^2 Q_{1,N} \tag{7}$$

and

$$\begin{aligned} Q_{0,N} &= \left(I_T - \overline{J}_T\right) \otimes I_N & (8) \\ Q_{1,N} &= \overline{J}_T \otimes I_N & (9) \end{aligned}$$

with $J_T = \iota_T \iota_T'$, $\overline{J}_T = J_T / T$ and $\sigma_1^2 = \sigma_v^2 + T\sigma_\mu^2$, see Baltagi (2008). Fingleton (2008*a*) extended this model to the spatial moving average random effects specification, hereafter SMA-RE. In that case, the disturbance term u_t in (2) follows a SMA process

$$u_t = \left(I_N + \lambda W_N\right) \varepsilon_t = D_N \varepsilon_t \tag{10}$$

where $D_N = \left(I_N + \lambda W_N\right)$, and ε_t follows an error component structure as in (3). So, the full SMA-RE $(NT \times 1)$ vector of disturbances is given by:

$$u = \left(\iota_T \otimes D_N\right) \mu + \left(I_T \otimes D_N\right) v \tag{11}$$

and the corresponding $(NT \times NT)$ covariance matrix is given by:

$$\Omega_u = \left(I_T \otimes D_N\right) \Omega_\varepsilon \left(I_T \otimes D_N\right)'. \tag{12}$$

Regression models containing spatially correlated disturbance terms based on the SAR or SMA models are typically estimated using Maximum Likelihood (ML), where the likelihood function corresponds to the normal distribution. However, this can be computationally demanding for large N. Kelejian and Prucha (1999) suggested a generalized moments (GM) estimation method for the SAR model in a cross-section setting, and Fingleton (2008*b*) extended this generalized moments estimator to the SMA model. Kapoor, Kelejian and Prucha (2007) generalized this GM procedure from cross-section to panel data and derived its large sample properties when T is fixed and $N \to \infty$. Kapoor, et al. (2007) proposed three generalized moments (GM) estimators of ρ, σ_v^2 and $\sigma_1^2 \left(= \sigma_v^2 + T\sigma_\mu^2\right)$ based on the following six moment conditions:

$$E \begin{bmatrix} \frac{1}{N(T-1)} u_N' Q_{0,N} u_N \\ \frac{1}{N(T-1)} \overline{u}_N' Q_{0,N} \overline{u}_N \\ \frac{1}{N(T-1)} \overline{u}_N' Q_{0,N} u_N \\ \frac{1}{N} u_N' Q_{1,N} u_N \\ \frac{1}{N} \overline{u}_N' Q_{1,N} \overline{u}_N \\ \frac{1}{N} \overline{u}_N' Q_{1,N} u_N \end{bmatrix} = \begin{bmatrix} \sigma_v^2 \\ \sigma_v^2 \frac{1}{N} tr\left(W_N' W_N\right) \\ 0 \\ \sigma_1^2 \\ \sigma_1^2 \frac{1}{N} tr\left(W_N' W_N\right) \\ 0 \end{bmatrix} \tag{13}$$

where

$$
\begin{aligned}
u_N &= \varepsilon_N - \rho\overline{\varepsilon}_N && (14)\\
\overline{u}_N &= \overline{\varepsilon}_N - \rho\overline{\overline{\varepsilon}}_N && (15)\\
\overline{\varepsilon}_N &= (I_T \otimes W_N)\,\varepsilon_N && (16)\\
\overline{\overline{\varepsilon}}_N &= (I_T \otimes W_N)\,\overline{\varepsilon}_N. && (17)
\end{aligned}
$$

Under the random effects specification (5), the OLS estimator of δ is consistent. Using $\widehat{\delta}_{OLS}$ one gets a consistent estimator of the disturbances $\widehat{\varepsilon} = y - Z\widehat{\delta}_{OLS}$. The GM estimators of σ_1^2, σ_ν^2 and ρ are the solution of the sample counterpart of the six equations given above. Kapoor, et al. (2007) suggest three GM estimators. The first involves only the first three moments which do not involve σ_1^2 and yield estimates of ρ and σ_ν^2. The fourth moment condition is then used to solve for σ_1^2 given estimates of ρ and σ_ν^2. The second GM estimator is based upon weighing the moment equations by the inverse of a properly normalized variance-covariance matrix of the sample moments evaluated at the true parameter values. A simple version of this weighting matrix is derived under normality of the disturbances. The third GM estimator is motivated by computational considerations and replaces a component of the weighting matrix for the second GM estimator by an identity matrix. Kapoor, et al. (2007) perform Monte Carlo experiments comparing ML and these three GM estimation methods. They find that on average, the RMSE of ML estimator and their weighted GM estimators are quite similar[1]. Fingleton (2008*a*) extended this GM estimator to the SMA panel data model with random effects (11). The moment conditions for SMA-RE are similar to those derived by Kapoor, et al. (2007).

The spatial feasible GLS estimator of δ, hereafter S-FGLS, is then obtained by replacing ρ, σ_v^2 and σ_1^2 by their GM estimators. More precisely, we have:

$$
\begin{aligned}
\widehat{\delta}_{S-FGLS} &= \left(Z'\widehat{\Omega}_u^{-1}Z\right)^{-1} Z'\widehat{\Omega}_u^{-1}y && (18)\\
&= \left(Z^{**\prime}\widehat{\Omega}_\varepsilon^{-1}Z^{**}\right)^{-1} Z^{**\prime}\widehat{\Omega}_\varepsilon^{-1}y^{**} && (19)
\end{aligned}
$$

---

[1] In our Monte Carlo experiments, we report the results from the GM estimator called weighted GM estimator by Kapoor, et al. (2007) in order to save space. The differences with the other two GM estimators in our Monte Carlo experiments were minor.

$$
\begin{aligned}
\widehat{V}\left[\widehat{\delta}_{S-FGLS}\right] &= \left(Z'\widehat{\Omega}_u^{-1}Z\right)^{-1} && (20)\\
&= \left(Z^{**\prime}\widehat{\Omega}_{\varepsilon}^{-1}Z^{**}\right)^{-1} && (21)
\end{aligned}
$$

with

$$
Z^{**} = \left[I_T \otimes \widehat{H}_N\right] Z, \; y^{**} = \left[I_T \otimes \widehat{H}_N\right] y \tag{22}
$$

where $\widehat{H}_N = \widehat{B}_N = (I_N - \widehat{\rho}W_N)$ for SAR-RE and $\widehat{H}_N = \widehat{D}_N^{-1} = \left(I_N + \widehat{\lambda}W_N\right)^{-1}$ for SMA-RE.

If $\rho = \lambda = 0$, (1) is reduced to the usual one-way error component model. Thus, the vector of disturbances (5) is reduced to:

$$
u = \varepsilon = Z_\mu \mu + v \tag{23}
$$

with $Z_\mu = \iota_T \otimes I_N$. Following (23), the covariance matrix of u is

$$
\Omega = \sigma_v^2 \Sigma \tag{24}
$$

where

$$
\Sigma = \left(Q_{0,N} + \frac{1}{\theta^2} Q_{1,N}\right) \tag{25}
$$

with $\theta^2 = \sigma_v^2/\sigma_1^2$. In order to estimate the parameters of the one-way error component model, we can use several estimators: Ordinary Least Squares (OLS), Within (W), Feasible Generalized Least Squares (FGLS) and Maximum Likelihood (ML) under normality of the disturbances, see Breusch (1987). Here, we are interested in inference using these estimators under the assumption that the true model is SAR-RE or SMA-RE.

The usual estimated covariance matrices of the OLS, Within, FGLS and ML estimators from a typical regression package are:

$$
\widehat{V}\left[\widehat{\delta}_{OLS}\right] = \widehat{\sigma}_u^2 \left(Z'Z\right)^{-1} \tag{26}
$$

$$
\widehat{V}\left[\widehat{\beta}_W\right] = \widehat{\sigma}_v^2 \left(X' Q_{0,N} X\right)^{-1} \tag{27}
$$

$$\widehat{V}\left[\widehat{\delta}_{FGLS}\right]=\widehat{\sigma}^2_{v,FGLS}\left(Z'\widehat{\Sigma}^{-1}_{FGLS}Z\right)^{-1} \tag{28}$$

$$\widehat{V}\left[\widehat{\delta}_{ML}\right]=\widehat{\sigma}^2_{v,ML}\left(Z'\widehat{\Sigma}^{-1}_{ML}Z\right)^{-1} \tag{29}$$

with $\widehat{\Sigma}^{-1}_{FGLS}=Q_{0,N}+\widehat{\theta}^2_{FGLS}Q_{1,N}$ and $\widehat{\Sigma}^{-1}_{ML}=Q_{0,N}+\widehat{\theta}^2_{ML}Q_{1,N}$. For the FGLS estimator, we need estimates of the variance components σ^2_μ and σ^2_v. We can use the following estimators: Amemiya (1971), Wallace and Hussain (1969), Swamy and Arora (1972), Nerlove (1971*a*), Henderson III (1953) and Minque (Rao (1970, 1972)). Moreover, under normality of the disturbances, we can apply the Maximum Likelihood estimator considered by Breusch (1987).

3 Monte Carlo Design

In this section, we consider the small sample performance of usual estimators of an error component model with spatially autocorrelated residuals. The data generating process (DGP) consider two specifications on the remainder errors, namely SAR, given in (2), and SMA, given in (10). More formally:

$$y_{it} = \gamma+\beta x_{it}+u_{it} \qquad i=1,\ldots,N;\ t=1,\ldots,T \tag{30}$$

with

$$u_t=\begin{cases}\rho W_N u_t+\varepsilon_t & \text{for SAR}\\ \lambda W_N\varepsilon_t+\varepsilon_t & \text{for SMA}\end{cases} \quad \text{with } \rho,\lambda=0.2,0.8. \tag{31}$$

Moreover, the remainder term ε_t follows an error component structure:

$$\varepsilon_t=\mu+v_t \tag{32}$$

with $\mu_i\sim iid.N\left(0,\sigma^2_\mu\right)$, $v_{it}\sim iid.N\left(0,\sigma^2_v\right)$. Throughout the experiment the parameters were set at $\gamma=5$ and $\beta=0.5$. The x_{it} explanatory variable is generated as in Nerlove (1971*b*) with:

$$x_{it}=0.1t+0.5x_{it-1}+\omega_{it} \tag{33}$$

where ω_{it} is a random variable uniformly distributed on the interval $[-0.5,0.5]$ and $x_{i0}=5+10\omega_{i0}$. The first 20 period observations were discarded to minimize the effect of initial values. For the error component (32), three cases for the residuals variances are considered:

$$\left(\sigma^2_\mu,\sigma^2_v\right)=(0.8,0.2)\,,\ (0.5,0.5)\ \text{and}\ (0.2,0.8)\,. \tag{34}$$

For the spatial weights matrices, we use regular and irregular lattices structures, as in Anselin and Moreno (2003) and in Kelejian and Prucha (1999). For the regular spatial case, we use two weight matrices which essentially differ in their degree of sparseness. Following Kelejian and Prucha (1999), the weight matrices are labelled as "j ahead and j behind" with the non-zero elements being $1/2j$, $j = 1$ and 5 (see Figures 1 and 2 in the Appendix). Note that, as j increase, the value of non-zero elements $1/2j$ decreases and, this is turn may reduce the amount of spatial correlation. For the irregular spatial case, we take the spatial groupings of the largest French administrative communes (see Baltagi, Bresson and Pirotte (2007)). These spatial weight matrices may represent high-order contiguity relationships. We consider the structures of 1 (see Figure 3 in the Appendix) and 3-nearest neighborhoods.[2] The one or three-order contiguity matrices reflect the fact that neighborhood j may be one of the 1 or 3-nearest neighborhoods to i, but j may have some other 1 or 3-nearest neighborhoods not including i. We consider several individual and time dimensions $N = 50$, $T = (5, 10)$. We evaluate the efficiency of thirteen estimators under spatial autocorrelation of the disturbances. First, we consider nine estimators of the one-way error component model which ignore spatial dependence:

- Ordinary Least Squares (OLS).
- Feasible Generalized Least Squares (FGLS). To estimate the variances components, we use six methods proposed by Amemiya (1971), Wallace and Hussain (1969), Swamy and Arora (1972), Nerlove (1971*a*), Henderson III (1953) and Minque (Rao (1970, 1972)).
- Maximum Likelihood (ML) under normality, see Breusch (1987).
- Within (W).

Second, we consider four estimators which take into account cross-section spatial dependence:

- The spatial True GLS SAR-RE estimator where σ^2_μ, σ^2_ν and ρ are known.

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[2] The figure and the results of this case are not given here to save space, but they will be provided upon request from the authors.

- The spatial True GLS SMA-RE estimator where σ_μ^2, σ_ν^2 and λ are known.
- The spatial FGLS SAR-RE using the GM estimators of σ_1^2, σ_ν^2 and ρ proposed by Kapoor et al. (2007).
- The spatial FGLS SMA-RE using the GM estimators of σ_1^2, σ_ν^2 and λ proposed by Fingleton (2008a).

For all experiments, 1000 replications are performed. To see how inference based on these estimates of β perform, we focus on the simple test for $H_0 : \beta = 0.4$, 0.5, 0.6 and 0.8 (when the true $\beta = 0.5$). We also calculate the variances of the estimators of β using the following formulas:

- Empirical variances from 1000 replications:

$$\widehat{\sigma}^2_{\widehat{\beta}} = \frac{1}{1000} \sum_{j=1}^{1000} \left(\widehat{\beta}_j - \overline{\widehat{\beta}} \right)^2 . \tag{35}$$

- Computed variances. For OLS, this is given by (26); for Within by (27); for FGLS by (28); for ML by (29); and for S-FGLS by (20). For SAR-RE, Ω_u is given by (6), while for SMA-RE, it is given by (12). For each estimator, we average the computed variances of $\widehat{\beta}$ over the 1000 replications.
- True variances:

$$OLS \; : \; \left(Z'Z\right)^{-1} Z'\Omega_u Z \left(Z'Z\right)^{-1} \tag{36}$$

$$Within \; : \; \left(X'Q_{0,N}X\right)^{-1} X'Q_{0,N}\Omega_u Q_{0,N}X \left(X'Q_{0,N}X\right)^{-1} \tag{37}$$

$$FGLS \; : \; \left(Z'\widehat{\Omega}^{-1}_{FGLS}Z\right)^{-1} Z'\widehat{\Omega}^{-1}_{FGLS}\Omega_u\widehat{\Omega}^{-1}_{FGLS}Z \left(Z'\widehat{\Omega}^{-1}_{FGLS}Z\right)^{-1} \tag{38}$$

$$ML \; : \; \left(Z'\widehat{\Omega}^{-1}_{ML}Z\right)^{-1} Z'\widehat{\Omega}^{-1}_{ML}\Omega_u\widehat{\Omega}^{-1}_{ML}Z \left(Z'\widehat{\Omega}^{-1}_{ML}Z\right)^{-1} \tag{39}$$

$$S-TGLS \; : \; \left(Z'\Omega_u^{-1}Z\right)^{-1} \tag{40}$$

$$S-FGLS \; : \; \left(Z'\widehat{\Omega}_u^{-1}Z\right)^{-1} Z'\widehat{\Omega}_u^{-1}\Omega_u\widehat{\Omega}_u^{-1}Z \left(Z'\widehat{\Omega}_u^{-1}Z\right)^{-1} . \tag{41}$$

For equations (40) and (41), Ω_u is given by (6) for a SAR-RE and by (12) for a SMA-RE. Equations (36) to (41) are true formulas. However, if an estimate of Ω is used, we average these variances of $\widehat{\beta}$ over the 1000 replications.

4 Inference Using the Usual One-way Error Component Estimators

4.1 Size and power

Table 1 gives the percentage of rejections of $H_0 : \beta = 0.4$, 0.5, 0.6 and 0.8 (when the true $\beta = 0.5$), for the case of $N = 50$, $T = (5, 10)$, $\left(\sigma_\mu^2, \sigma_v^2\right) = (0.8, 0.2)$, $(\rho = \lambda = 0.2, 0.8)$, and a W(1,1) matrix, one neighbor ahead and one neighbor behind, see Figure 1 in the Appendix. When the true model is SAR-RE, i.e., the first half of Table 1, and for $T = 5$ and $\rho = 0.2$, the OLS estimator rejects the null hypothesis $H_0 : \beta = 0.5$, when it is true, in 12.2% of the cases at the 5% significance level. This means that the test is over-sized. This gets worse when the spatial coefficient ρ increases to 0.8. In this case, the size of the test becomes 15.6%. When the true model is SMA-RE, i.e., the bottom half of Table 1, and for $T = 5$ and $\lambda = 0.2$, the OLS estimator rejects the null hypothesis $H_0 : \beta = 0.5$, when it is true, in 12.3% of the cases at the 5% significance level. Once again, the test is over-sized, and it gets slightly worse when the spatial coefficient λ increases to 0.8. The size of the test becomes 12.8%. The fixed effects (within) and random effects (FGLS) as well as ML estimators have a size of 6.9% to 10.2%, when the true model is SAR-RE, and $T = 5$ and $\rho = 0.2$. However, this gets worse when the spatial coefficient ρ increases to 0.8. In this case, the size of these tests varies between 28.2% and 35.3%. When the true model is SMA-RE, and $T = 5$ and $\lambda = 0.2$, the size of the fixed effects (within) and random effects (FGLS) as well as ML estimators varies between 6.8% to 9.9%. This gets worse when the spatial coefficient λ increases to 0.8. The size of these tests varies between 12.2% to 17.1%. The size of the true GLS whether it is SAR-RE or SMA-RE is never statistically different from 5% for all cases considered in Table 1. This is also true for FGLS SAR-RE and FGLS SMA-RE except for FGLS SAR-RE when $\lambda = 0.8$ (resp. FGLS SMA-RE when $\rho = 0.8$) in Table 1.

Table 2 shows that the true variances of all the $\widehat{\beta}$ considered, are well estimated by their empirical counterparts using 1000 replications. For example, the OLS true variance is 3.738 for $T = 5$ and $\rho = 0.2$, when the true model is SAR-RE. The empirical estimate of this using 1000 replications is 3.805. However, the usual regression package under-estimates this variance and yields 2.385. This under-estimation of the true variance of OLS under spatial dependence leads to the over-sizing of the test of $H_0 : \beta = 0.5$, in Table 1. This gets worse when the spatial coefficient ρ increases to 0.8. The true variance of OLS increases to 17.468 and its empirical estimate is 16.777, while the usual regression package under-estimates this variance and yields 9.7386. This undermines inference based on OLS estimates which ignore heterogeneity and spatial dependence present in this model.[3] The same is true for fixed effects (within) and random effects (FGLS) as well as ML estimators which deal with heterogeneity but ignore spatial dependence. The true variances are of the order of 0.89 to 0.92 for $T = 5$ and $\rho = 0.2$, when the true model is SAR-RE, and their empirical estimates using 1000 replications vary between 0.83 and 0.85. However, the usual regression package estimates will under-estimate these variances and yield magnitudes between 0.56 and 0.71. This gets worse when the spatial coefficient ρ increases to 0.8. The true variances of fixed effects (within) and random effects (FGLS) as well as ML estimators increase in range to 10.60 and 11.09 and their empirical counterparts vary between 9.49 and 10.00, while the usual regression package under-estimates these variances and yield estimates in the range of 2.40 to 3.09. This seriously undermines inference based on these panel estimates which account for heterogeneity but ignore the spatial dependence present in this model. Note that for the FGLS SAR-RE and FGLS SMA-RE, the computed variances tend to over-estimate the true variances in Table 2.

Tables 1 and 2 give also the results when we double T from 5 to 10, holding $N = 50$ constant. Basically, we get the same results for over-sizing of the test for $H_0 : \beta = 0.4$, 0.5, 0.6 and 0.8 (when the true $\beta = 0.5$), and under-estimation of the true variances for all the estimators considered. Of course, the power now increases drastically with T especially for $\beta = 0.4$, and 0.6.

Tables 3 and 5 replicate the results in Table 1 on size and power of H_0, only now the heterogeneity in the individual effects is reduced from 80% of the total variance to 50% in Table 3, and 20% in Table 5. The results for size

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[3] Moulton (1980) warned about this missleading inference for the case of ignoring the individual effects. Here we show that misleading inference also occurs when one ignores cross-section spatial dependence among these individuals.

and power are the same, but the magnitudes are different. Similarly, Tables 4 and 6 replicate the results in Table 2 for the true and estimated variances, only now the heterogeneity in the individual effects is reduced from 80% of the total variance to 50% in Table 4, and 20% in Table 6. The results for the under-estimation of the true-variances are the same, but the magnitudes are different.

4.2 Spatial weighted matrix effect

Tables 7, 9 and 11 replicate the results in Tables 1, 3 and 5, only now for a weight matrix W(5,5) that allows 5 neighbors ahead and 5 neighbors behind, see Figure 2 in the Appendix. Again, the results for size and power are the same, but the magnitudes are different. Similarly, Tables 8, 10 and 12 replicate the results in Table 2, 4 and 6, only now for a weight matrix W(5,5) that allows 5 neighbors ahead and 5 neighbors behind. The results for the under-estimation of the true-variances are the same, but the magnitudes are different.

4.3 Sensitivity to irregular lattices structures

Tables 13, 15 and 17 replicate the results in Tables 1, 3 and 5, only now for an asymmetric one-order contiguity matrix, from french communes, see Figure 3 in the Appendix. Again, the results for size and power are the same, but the magnitudes are different. Similarly, Tables 14, 16 and 18 replicate the results in Table 2, 4 and 6, only now for an asymmetric one-order contiguity matrix, from french communes. The results for the under-estimation of the true-variances are the same, but the magnitudes are different.[4]

4.4 Hausman Test Performance

Table 19 checks the performance of the Hausman (1978) test when the true model is SAR-RE or SMA-RE. This test is based on the contrast between the within estimator and the Swamy and Arora (1972) feasible GLS RE esti-

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[4] We also considered an asymmetric 3-order contiguity matrix, from french communes. These results are not given here to save space, but they will be provided upon request from the authors.

mator.[5] For all experiments performed, the null hypothesis of no correlation between the individual effects and the regressor is satisfied. Table 19 therefore checks the sensitivity of the usual Hausman test that ignores the spatial correlation of the SAR or SMA type. The results indicate that the Hausman test yields size not significantly different from 5% for all experiments considered when $\rho = \lambda = 0.2$, i.e., weak spatial correlation. One exception is the asymmetric one-order contiguity matrix which is over-sized yielding at worst 7.2% rather than 5%. Things get better when T increases from 5 to 10. However, when $\rho = \lambda = 0.8$, i.e., when the spatial correlation effect is larger, the size of the Hausman test is distorted. It varies from an under-sizing of 1.5% for SAR-RE, $T = 10$ and an asymmetric one-order contiguity matrix, to over-sizing by as much as 14% for SAR-RE, $T = 5$ and a W(5,5) matrix. For the SMA-RE model, this distortion in size for the Hausman test varies from 3.1% to 9.4% rather than 5%.

5 Conclusions

We showed that when the spatial coefficients are large, test of hypothesis based on the usual panel data estimators that ignore spatial dependence can lead to misleading inference. This can be explained by the fact that the variances of these misspecified estimators under-estimate their true variances for the spatial panel models considered. These results are robust to doubling T, various spatial weight matrices, with one or five neighbors ahead and behind, as well as for asymmetric spatial weight matrices based on french communes.

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[5] We have also computed this Hausman test based on other FGLS estimators. The results are similar and are not reported here to save space.

References

Amemiya, T., 1971, The estimation of variances in a variance-components model, *International Economic Review* 12, 1-13.

Anselin, L., 1988, *Spatial Econometrics: Methods and Models*, Kluwer Academic Publishers, Dordrecht.

Anselin, L., J. Le Gallo and H. Jayet, 2008, Spatial panel econometrics. Ch. 19 in L. Mátyás and P. Sevestre, eds., The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice, Springer-Verlag, Berlin, 625-660.

Anselin, L. and R. Moreno, 2003, Properties of tests for spatial error components, *Regional Science and Urban Economics* 33, 595-618.

Baltagi, B.H., 1981, Pooling: An experimental study of alternative testing and estimation procedures in a two-way error components model, *Journal of Econometrics* 17, 21–49.

Baltagi, B.H., 2008, *Econometric Analysis of Panel Data* (Wiley, Chichester).

Baltagi B.H., G. Bresson and A. Pirotte, 2007, Panel unit root tests and spatial dependence, *Journal of Applied Econometrics* 22, 339-360.

Breusch, T.S., 1987, Maximum likelihood estimation of random effects models, *Journal of Econometrics* 36, 383-389.

Fingleton, B., 2008*a*, A generalized method of moments estimation for a spatial panel model with an endogenous spatial lag and spatial moving average errors, *Spatial Economic Analysis* 3, 27-44.

Fingleton, B., 2008*b*, A generalized method of moments estimation for a spatial model with moving average errors with application to real estate prices, *Empirical Economics* 34, 35-57.

Hausman, J., 1978, Specification tests in econometrics, *Econometrica* 46, 1251-1271.

Henderson, C.R., 1953, Estimation of variance and covariance components, *Biometrics* 9, 226-252.

Kapoor, M., H.H. Kelejian and I.R. Prucha, 2007, Panel data models with spatially correlated error components, *Journal of Econometrics* 140, 97-130.

Kelejian, H.H. and I.R. Prucha, 1999, A generalized moments estimator for the autoregressive parameter in a spatial model, *International Economic Review* 40, 509-533.

Maddala, G.S. and T.D. Mount, 1973, A comparative study of alternative estimators for variance components models used in econometric applications, *Journal of the American Statistical Association* 68, 324-328.

Moulton, B.R., 1986, Random group effects and the precision of regression estimates, *Journal of Econometrics* 32, 385–397.

Nerlove, M., 1971*a*, A note on error components model, *Econometrica* 39, 383-396.

Nerlove, M., 1971*b*, Further evidence on the estimation of dynamic economic relations from a time-series of cross-sections, *Econometrica* 39, 359-383.

Rao, C.R., 1970, Estimation of heteroscedastic variances in linear models, *Journal of the American Statistical Association* 65, 161-173.

Rao, C.R., 1972, Estimation variance and covariance components in linear models, *Journal of the American Statistical Association* 67, 112-115.

Swamy, P.A.V.B. and S.S. Arora, 1972, The exact finite sample properties of the estimators of coefficients in the error components model, *Econometrica* 40, 261-275.

Wallace, T.D. and A. Hussain, 1969, The use of error components models in combining time-series with cross-section data, *Econometrica* 37, 55-72.

Appendix

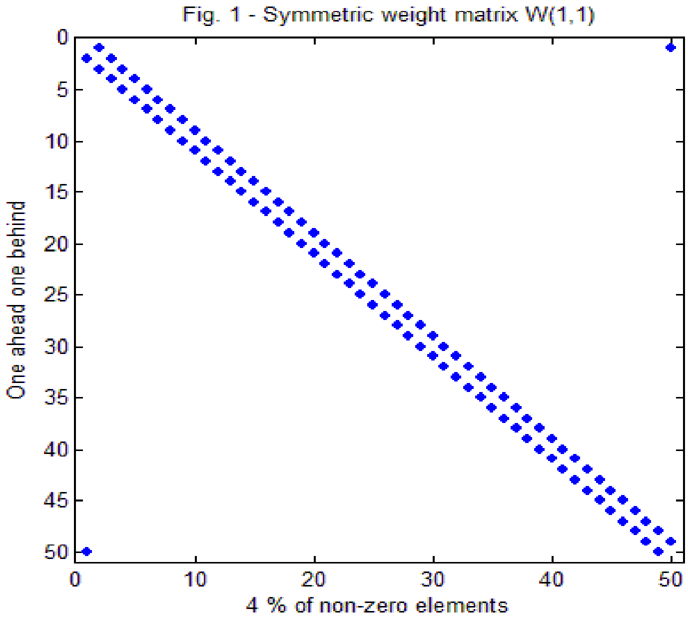
Fig. 1 - Symmetric weight matrix W(1,1)
One ahead one behind
4 % of non-zero elements
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50

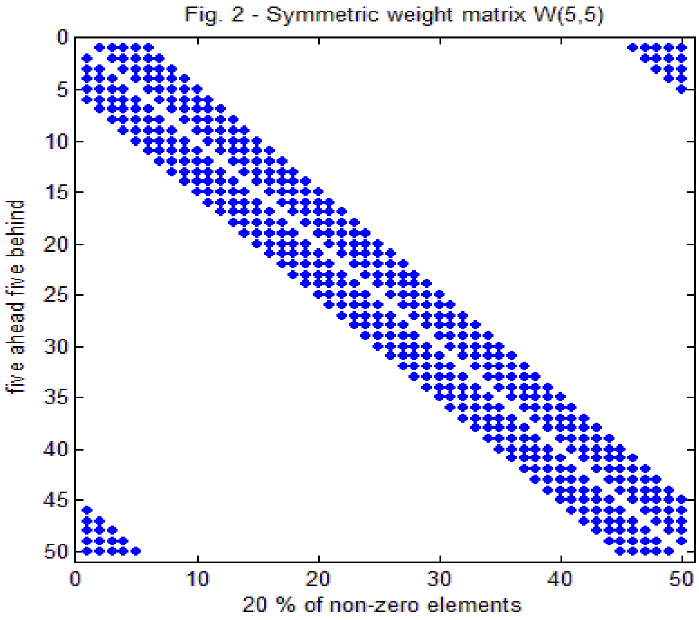
Fig. 2 - Symmetric weight matrix W(5,5)
five ahead five behind
20 % of non-zero elements
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50

Fig. 3 - Asymmetric weight matrix for the 50 largest French communes

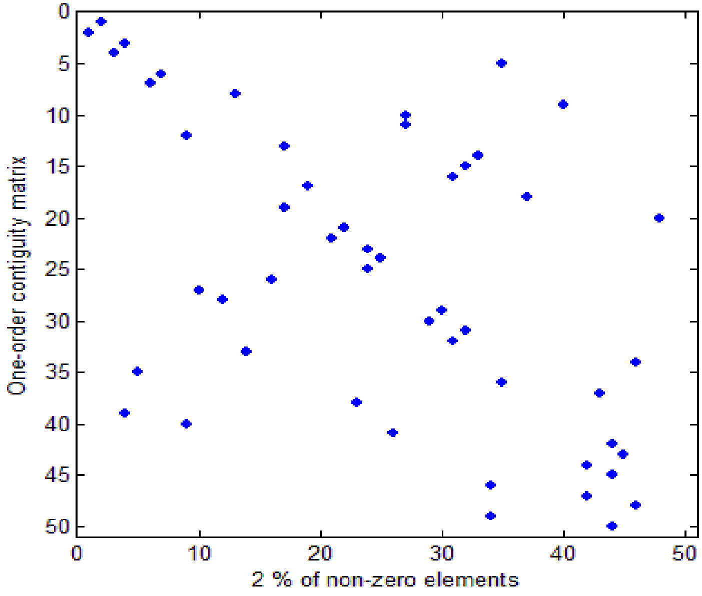

Table 1 – Percentage of rejections of H_0 at 5% significance level for *N*=50, $\sigma_\mu^2 = 0.8$, $\sigma_\nu^2 = 0.2$, W(1,1), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	16.6	12.2	16.9	50.6	33.5	3.4	32.2	99.4	15.9	15.6	15.8	24.8	20.5	15.1	20.7	57.2
FGLS Amemiya (1971)	24.5	7.5	26.1	94.1	83.8	10.3	84.4	100.0	31.5	28.7	30.2	47.8	51.7	37.1	46.5	88.2
Wallace & Hussain (1969)	23.6	7.3	25.2	94.0	83.8	10.1	84.3	100.0	31.2	28.2	29.3	47.0	51.6	36.9	46.3	88.1
Swamy & Arora (1972)	23.9	7.4	25.6	93.9	83.8	10.1	84.3	100.0	31.3	28.5	29.9	47.8	51.4	37.0	46.5	88.2
Nerlove (1971)	31.3	10.2	32.9	96.3	85.3	12.6	86.7	100.0	37.3	35.3	35.4	52.2	54.0	39.9	48.8	89.0
Henderson III (1953)	23.5	7.4	26.1	94.1	83.8	10.3	84.4	100.0	31.4	28.4	29.8	47.8	51.5	37.0	46.5	88.2
Minque (Rao (1970, 1972))	24.2	7.4	25.9	94.0	83.8	10.2	84.3	100.0	31.5	28.7	29.9	47.8	51.5	37.0	46.5	88.2
ML (Breusch (1987))	24.5	7.5	26.1	94.1	83.8	10.3	84.4	100.0	31.5	28.7	30.1	47.8	51.7	37.1	46.5	88.2
Within	23.4	6.9	26.3	93.7	83.6	10.0	84.4	100.0	31.8	30.0	29.7	47.0	51.3	37.0	46.6	87.7
TGLS SAR-RE	18.4	4.5	21.1	91.9	78.1	4.6	77.7	100.0	15.8	4.8	15.7	78.1	43.8	5.5	42.0	99.8
FGLS SAR-RE (Kapoor et al. (2007))	19.1	4.9	20.5	92.2	77.7	4.4	77.7	100.0	14.5	5.3	14.0	76.9	42.8	5.4	42.8	99.8
FGLS SMA-RE (Fingleton (2008*a*))	19.3	5.4	20.9	92.2	78.9	4.4	78.2	100.0	11.6	6.2	14.6	54.2	32.2	6.9	35.6	98.8

	True model SMA-RE															
OLS	16.8	12.3	17.2	51.7	34.3	3.3	33.1	99.4	19.4	12.8	16.0	45.8	30.4	7.0	29.4	97.4
FGLS Amemiya (1971)	24.9	7.3	26.8	95.2	85.1	9.5	86.6	100.0	26.2	13.3	27.8	85.0	72.6	20.3	70.8	100.0
Wallace & Hussain (1969)	24.1	6.8	25.9	95.2	85.1	9.3	86.4	100.0	25.5	13.0	27.1	84.6	72.6	20.3	70.3	100.0
Swamy & Arora (1972)	24.5	7.2	26.4	95.2	85.1	9.4	86.5	100.0	26.0	13.2	27.4	84.5	72.6	20.3	70.6	100.0
Nerlove (1971)	32.5	9.9	33.2	96.7	87.1	11.8	87.7	100.0	33.0	17.1	33.0	88.3	74.4	21.7	74.1	100.0
Henderson III (1953)	24.1	7.2	27.0	95.3	85.1	9.5	86.4	100.0	25.6	13.3	27.5	84.8	72.5	20.1	70.6	100.0
Minque (Rao (1970, 1972))	24.9	7.2	26.6	95.2	85.1	9.5	86.6	100.0	26.1	13.3	27.7	84.7	72.6	20.3	70.6	100.0
ML (Breusch (1987))	24.9	7.3	26.8	95.2	85.1	9.5	86.6	100.0	26.2	13.3	27.8	85.0	72.6	20.3	70.8	100.0
Within	23.8	6.8	27.0	95.1	85.2	9.4	86.1	100.0	26.1	12.2	27.2	83.0	72.3	20.4	70.4	100.0
TGLS SMA-RE	19.4	4.7	22.1	93.6	81.1	4.6	80.3	100.0	39.3	5.0	41.3	99.9	94.6	5.0	93.9	100.0
FGLS SAR-RE (Kapoor et al. (2007))	19.1	4.8	21.1	92.9	80.1	4.1	80.1	100.0	21.2	3.6	23.3	96.4	70.5	2.7	72.3	100.0
FGLS SMA-RE (Fingleton (2008*a*))	19.7	5.2	21.4	93.1	81.3	4.2	80.4	100.0	27.3	5.5	28.1	97.6	85.4	6.3	84.1	100.0

Table 2 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_{\mu}^{2} = 0.8$, $\sigma_{v}^{2} = 0.2$, W(1,1), 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.8057	2.3854	3.7381	0.4189	0.4491	0.4157	16.777	9.7386	17.468	3.1348	1.8247	3.0763
FGLS Amemiya (1971)	0.8333	0.6934	0.8925	0.1480	0.0993	0.1370	9.5321	2.9781	10.610	2.1140	0.4287	1.9499
Wallace & Hussain (1969)	0.8333	0.7095	0.8924	0.1480	0.0999	0.1370	9.4922	3.0401	10.598	2.1138	0.4315	1.9498
Swamy & Arora (1972)	0.8331	0.7009	0.8927	0.1480	0.0998	0.1370	9.5228	3.0105	10.621	2.1140	0.4306	1.9501
Nerlove (1971)	0.8342	0.5579	0.8947	0.1480	0.0894	0.1370	9.6338	2.3995	10.709	2.1147	0.3860	1.9508
Henderson III (1953)	0.8314	0.7001	0.8925	0.1480	0.0997	0.1370	9.5028	3.0076	10.609	2.1133	0.4305	1.9496
Minque (Rao (1970, 1972))	0.8333	0.6972	0.8926	0.1480	0.0995	0.1370	9.5339	2.9942	10.616	2.1141	0.4297	1.9500
ML (Breusch (1987))	0.8333	0.6935	0.8925	0.1480	0.0993	0.1370	9.5291	2.9787	10.610	2.1140	0.4287	1.9499
Within	0.8500	0.7171	0.9158	0.1480	0.1000	0.1373	10.006	3.0938	11.094	2.1206	0.4317	1.9578
FGLS SAR-RE (Kapoor et al. (2007))	0.8040	0.8319	0.8285	0.1411	0.1307	0.1309	1.2302	1.2719	1.1953	0.3356	0.3237	0.3184
FGLS SMA-RE (Fingleton (2008*a*))	0.8044	0.8340	0.8421	0.1413	0.1387	0.1315	2.7800	2.3337	2.2057	0.6001	0.4640	0.5822

	True model SMA-RE											
OLS	3.6616	2.2907	3.5949	0.3974	0.4313	0.3945	4.9165	2.9080	4.9947	0.6447	0.5476	0.6403
FGLS Amemiya (1971)	0.7812	0.6654	0.8349	0.1372	0.0953	0.1271	1.4747	0.8588	1.6027	0.2893	0.1230	0.2668
Wallace & Hussain (1969)	0.7813	0.6809	0.8348	0.1373	0.0959	0.1271	1.4721	0.8790	1.6019	0.2893	0.1239	0.2668
Swamy & Arora (1972)	0.7811	0.6726	0.8350	0.1373	0.0957	0.1271	1.4741	0.8680	1.6034	0.2894	0.1236	0.2669
Nerlove (1971)	0.7817	0.5354	0.8367	0.1372	0.0858	0.1271	1.4815	0.6912	1.6103	0.2893	0.1108	0.2669
Henderson III (1953)	0.7794	0.6718	0.8349	0.1373	0.0957	0.1271	1.4712	0.8671	1.6025	0.2893	0.1235	0.2668
Minque (Rao (1970, 1972))	0.7812	0.6690	0.8350	0.1372	0.0955	0.1271	1.4748	0.8635	1.6031	0.2893	0.1233	0.2669
ML (Breusch (1987))	0.7812	0.6655	0.8349	0.1372	0.0953	0.1271	1.4746	0.8590	1.6027	0.2893	0.1230	0.2668
Within	0.7960	0.6881	0.8561	0.1372	0.0959	0.1274	1.5206	0.8893	1.6550	0.2897	0.1239	0.2676
FGLS SAR-RE (Kapoor et al. (2007))	0.7606	0.7984	0.7920	0.1317	0.1252	0.1227	0.5998	0.6883	0.5813	0.1364	0.1642	0.1311
FGLS SMA-RE (Fingleton (2008*a*))	0.7594	0.7860	0.7815	0.1316	0.1222	0.1221	0.4626	0.5767	0.3470	0.1029	0.1159	0.0799

(1) $\times 10^{-2}$.

Table 3 – Percentage of rejections of H_0 at 5% significance level for N=50, $\sigma_\mu^2 = 0.5$, $\sigma_\nu^2 = 0.5$, W(1,1), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	15.2	11.4	14.7	51.6	35.7	5.7	33.5	99.1	19.8	19.1	18.3	26.3	30.1	25.3	29.3	53.8
FGLS Amemiya (1971)	14.0	7.5	15.4	63.2	52.8	10.3	50.8	99.7	29.5	27.7	27.3	35.6	44.1	36.9	41.1	65.9
Wallace & Hussain (1969)	13.8	7.4	15.2	63.1	52.8	10.3	50.8	99.7	29.4	27.2	27.3	35.5	44.0	36.9	41.0	65.9
Swamy & Arora (1972)	13.7	7.4	14.8	62.7	52.7	10.1	50.6	99.7	29.5	27.1	27.1	35.4	43.8	36.9	41.0	65.6
Nerlove (1971)	19.3	10.3	20.3	70.5	55.6	12.2	54.4	99.7	35.0	33.8	33.7	40.8	46.0	39.7	42.6	67.8
Henderson III (1953)	13.4	7.4	15.2	63.8	52.6	10.3	51.0	99.7	28.7	27.1	26.9	35.3	43.8	36.9	41.1	65.6
Minque (Rao (1970, 1972))	13.8	7.5	15.3	63.0	52.7	10.2	50.8	99.7	29.5	27.4	27.3	35.6	44.0	36.9	41.1	65.8
ML (Breusch (1987))	14.0	7.5	15.4	63.2	52.8	10.3	50.8	99.7	29.6	27.6	27.3	35.6	44.1	36.9	41.1	65.9
Within	13.3	6.9	15.0	60.6	53.0	10.0	50.1	99.7	31.3	30.0	28.6	35.9	43.6	37.0	41.4	65.1
TGLS SAR-RE	10.2	4.6	11.0	58.9	43.0	4.9	41.1	99.7	9.2	4.9	9.4	46.8	21.2	5.2	22.1	91.4
FGLS SAR-RE (Kapoor et al. (2007))	11.3	5.4	11.4	59.1	43.2	5.1	42.6	99.7	10.2	5.4	8.8	46.5	21.0	5.1	21.1	91.6
FGLS SMA-RE (Fingleton (2008*a*))	11.1	5.8	11.3	59.0	44.2	5.1	43.3	99.7	13.3	7.5	14.8	45.4	26.5	7.6	27.2	90.1

	True model SMA-RE															
OLS	15.4	11.4	14.9	52.8	36.4	5.4	34.0	99.1	17.1	12.8	16.6	43.9	34.4	12.3	31.8	94.7
FGLS Amemiya (1971)	13.8	7.2	15.2	64.9	54.2	9.8	52.2	99.7	18.3	13.4	20.2	54.9	49.5	19.7	45.3	98.8
Wallace & Hussain (1969)	13.5	7.3	15.1	64.8	54.1	9.8	52.1	99.7	18.3	13.3	19.8	54.7	49.5	19.7	45.3	98.8
Swamy & Arora (1972)	13.0	7.1	15.1	64.9	53.9	9.7	52.0	99.7	18.0	13.3	19.2	54.5	49.4	19.6	45.4	98.8
Nerlove (1971)	19.2	10.2	20.2	72.7	57.1	11.8	56.1	99.7	23.5	16.8	25.4	59.9	51.2	21.9	47.5	99.0
Henderson III (1953)	12.9	7.2	15.5	65.6	53.9	9.8	52.4	99.7	17.9	13.2	20.2	55.0	49.2	19.2	45.6	98.8
Minque (Rao (1970, 1972))	13.7	7.1	15.1	64.9	54.0	9.8	52.1	99.7	18.3	13.3	19.8	54.8	49.4	19.6	45.3	98.8
ML (Breusch (1987))	13.6	7.3	15.2	65.0	54.2	9.8	52.2	99.7	18.3	13.4	20.0	54.9	49.5	19.7	45.3	98.8
Within	13.2	6.8	14.9	61.8	54.2	9.4	51.4	99.7	18.7	12.2	19.5	51.8	49.6	20.4	44.6	98.8
TGLS SMA-RE	10.4	4.6	11.3	61.2	45.6	4.9	43.3	99.7	21.1	4.8	21.6	92.9	62.3	5.0	61.7	100.0
FGLS SAR-RE (Kapoor et al. (2007))	11.8	5.4	11.7	60.2	45.1	4.7	43.5	99.7	12.5	4.0	11.7	69.9	33.9	2.5	33.9	99.9
FGLS SMA-RE (Fingleton (2008*a*))	11.2	5.3	11.5	60.9	46.2	4.8	44.6	99.7	18.8	6.4	16.7	85.4	62.0	6.4	61.8	100.0

Table 4 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma^2_\mu = 0.5$, $\sigma^2_v = 0.5$, W(1,1), 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.4771	2.4081	3.4339	0.5182	0.4536	0.4942	20.4558	10.0188	21.7309	5.4433	1.8811	5.1325
FGLS Amemiya (1971)	1.9819	1.6389	2.0976	0.3684	0.2470	0.3404	21.5611	6.9772	23.9243	5.2331	1.0645	4.8215
Wallace & Hussain (1969)	1.9811	1.6483	2.0970	0.3684	0.2474	0.3404	21.3989	7.0153	23.8686	5.2323	1.0663	4.8212
Swamy & Arora (1972)	1.9808	1.6589	2.0988	0.3686	0.2481	0.3404	21.4762	7.0620	23.9874	5.2328	1.0695	4.8233
Nerlove (1971)	1.9942	1.3436	2.1215	0.3683	0.2226	0.3406	22.4634	5.7538	24.8963	5.2444	0.9599	4.8336
Henderson III (1953)	1.9675	1.6506	2.0976	0.3682	0.2475	0.3406	21.2013	7.0273	23.8537	5.2203	1.0682	4.8171
Minque (Rao (1970, 1972))	1.9817	1.6488	2.0981	0.3684	0.2476	0.3404	21.5710	7.0201	23.9569	5.2340	1.0670	4.8225
ML (Breusch (1987))	1.9817	1.6392	2.0975	0.3684	0.2470	0.3404	21.5128	6.9779	23.9094	5.2330	1.0645	4.8215
Within	2.1248	1.7927	2.2895	0.3701	0.2501	0.3434	25.0161	7.7346	27.7353	5.3017	1.0794	4.8946
FGLS SAR-RE (Kapoor et al. (2007))	1.9153	1.9065	1.9409	0.3514	0.3222	0.3250	2.7803	2.7320	2.6828	0.8269	0.7868	0.7805
FGLS SMA-RE (Fingleton (2008*a*))	1.9162	1.9540	1.9770	0.3516	0.3356	0.3266	3.7056	2.8242	3.8930	1.5121	0.9077	1.1032

	True model SMA-RE											
OLS	3.3275	2.3119	3.2818	0.4865	0.4355	0.4644	4.8371	2.9536	4.9481	0.8909	0.5562	0.8492
FGLS Amemiya (1971)	1.8613	1.5728	1.9652	0.3417	0.2370	0.3158	3.4440	2.0247	3.7131	0.7185	0.3058	0.6619
Wallace & Hussain (1969)	1.8611	1.5818	1.9648	0.3417	0.2374	0.3158	3.4334	2.0365	3.7093	0.7185	0.3064	0.6619
Swamy & Arora (1972)	1.8605	1.5920	1.9661	0.3419	0.2381	0.3158	3.4387	2.0493	3.7180	0.7188	0.3073	0.6621
Nerlove (1971)	1.8705	1.2893	1.9855	0.3417	0.2136	0.3160	3.5093	1.6626	3.7914	0.7191	0.2757	0.6627
Henderson III (1953)	1.8483	1.5839	1.9655	0.3416	0.2375	0.3161	3.4114	2.0388	3.7086	0.7174	0.3065	0.6619
Minque (Rao (1970, 1972))	1.8611	1.5823	1.9656	0.3417	0.2375	0.3158	3.4446	2.0370	3.7155	0.7186	0.3065	0.6620
ML (Breusch (1987))	1.8613	1.5731	1.9652	0.3417	0.2370	0.3158	3.4416	2.0250	3.7123	0.7185	0.3058	0.6619
Within	1.9899	1.7202	2.1403	0.3432	0.2399	0.3185	3.8015	2.2233	4.1376	0.7244	0.3098	0.6692
FGLS SAR-RE (Kapoor et al. (2007))	1.8151	1.8296	1.8631	0.3280	0.3088	0.3048	1.3886	1.5071	1.3348	0.3380	0.4015	0.3233
FGLS SMA-RE (Fingleton (2008*a*))	1.8117	1.8003	1.8335	0.3278	0.3008	0.3033	0.9179	0.9666	0.7816	0.2370	0.2157	0.1965

(1) × 10^{-2}.

Table 5 – Percentage of rejections of H_0 at 5% significance level for N=50, $\sigma_\mu^2 = 0.2$, $\sigma_\nu^2 = 0.8$, W(1,1), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	13.3	9.5	13.4	50.3	36.3	9.4	33.9	97.9	22.0	20.9	21.4	28.6	37.0	32.3	34.3	53.4
FGLS Amemiya (1971)	13.0	7.7	13.8	50.5	40.4	10.0	37.8	99.1	27.3	25.7	26.4	32.1	41.5	36.8	40.2	56.0
Wallace & Hussain (1969)	13.1	7.7	13.6	50.4	40.4	10.0	37.8	99.1	27.1	25.3	26.1	31.8	41.5	36.7	40.2	56.1
Swamy & Arora (1972)	12.6	7.5	13.2	50.1	40.5	10.0	37.5	99.1	26.2	25.0	26.0	31.2	41.3	36.7	40.2	55.9
Nerlove (1971)	15.2	10.1	16.6	56.1	43.3	12.2	40.9	99.2	32.9	32.1	32.9	37.3	44.0	39.3	41.9	58.5
Henderson III (1953)	12.1	7.8	13.5	51.5	40.2	10.3	39.3	99.0	26.1	25.0	25.3	31.0	41.3	36.6	39.2	56.6
Minque (Rao (1970, 1972))	12.7	7.6	13.5	50.2	40.3	10.0	37.6	99.1	27.1	25.6	26.3	31.7	41.3	36.8	40.2	56.0
ML (Breusch (1987))	13.1	7.6	13.6	50.5	40.4	10.0	37.8	99.1	27.2	25.6	26.1	31.6	41.5	36.7	40.2	56.0
Within	11.0	6.9	12.4	43.3	40.3	10.0	37.2	99.0	30.6	30.0	27.6	32.3	41.6	37.0	40.0	55.9
TGLS SAR-RE	9.4	5.0	9.5	45.8	30.7	5.1	30.2	98.5	7.7	5.3	9.1	38.4	16.1	5.1	15.6	77.4
FGLS SAR-RE (Kapoor et al. (2007))	10.4	6.4	10.4	47.1	30.9	5.4	30.8	98.7	9.2	5.9	8.7	38.3	16.6	5.4	15.6	77.3
FGLS SMA-RE (Fingleton (2008*a*))	10.4	6.7	10.3	47.1	31.9	5.3	31.4	98.8	11.7	8.4	13.2	37.2	20.2	8.1	21.6	75.1

	True model SMA-RE															
OLS	13.2	9.3	13.7	51.5	37.1	8.4	35.0	98.4	15.2	12.5	15.8	43.5	36.5	16.5	32.6	93.6
FGLS Amemiya (1971)	12.9	7.5	13.7	51.3	41.6	9.7	38.6	99.2	16.0	13.4	17.2	45.2	40.9	19.0	36.1	94.0
Wallace & Hussain (1969)	13.0	7.5	13.7	51.5	41.5	9.7	38.5	99.2	16.0	13.1	17.1	45.3	40.9	19.0	36.1	94.0
Swamy & Arora (1972)	12.5	7.3	13.3	51.1	41.4	9.6	38.4	99.2	15.9	12.7	16.9	44.7	40.8	18.9	36.0	93.9
Nerlove (1971)	15.2	9.7	16.3	57.3	44.6	11.6	41.8	99.3	21.0	16.7	21.8	49.3	43.3	21.4	39.4	94.5
Henderson III (1953)	12.3	7.5	13.6	52.8	41.2	10.1	40.8	99.4	15.6	13.0	17.2	46.1	41.0	18.8	37.3	94.6
Minque (Rao (1970, 1972))	12.8	7.4	13.5	51.1	41.4	9.6	38.4	99.2	15.9	13.1	17.1	45.0	40.9	18.9	36.0	93.9
ML (Breusch (1987))	12.9	7.5	13.7	51.3	41.6	9.7	38.5	99.2	16.1	13.3	17.3	45.3	40.9	19.0	36.1	94.0
Within	10.8	6.8	12.0	44.3	40.7	9.4	38.1	99.3	16.3	12.2	17.5	40.3	40.9	20.4	35.9	93.5
TGLS SMA-RE	9.5	4.9	9.8	47.5	32.1	5.2	31.5	99.0	16.6	5.3	17.4	84.6	44.3	5.2	45.5	99.9
FGLS SAR-RE (Kapoor et al. (2007))	10.6	6.2	10.4	48.4	31.3	5.2	31.5	98.9	11.5	5.1	10.4	57.9	23.5	2.8	23.3	98.2
FGLS SMA-RE (Fingleton (2008*a*))	11.2	6.4	10.3	48.3	32.9	5.5	32.4	98.9	18.3	5.9	19.1	85.5	44.4	4.9	45.8	99.9

Table 6 – Mean of empirical, computed and true variances of $\hat{\beta}$ for *N*=50, $\sigma^2_\mu = 0.2$, $\sigma^2_\nu = 0.8$, W(1,1), 1000 replications. $^{(1)}$

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.1387	2.4298	3.1297	0.6128	0.4582	0.5726	24.2668	10.2731	25.9930	7.7379	1.9385	7.1886
FGLS Amemiya (1971)	2.8461	2.3292	2.9432	0.5790	0.3885	0.5345	28.5631	9.8102	31.0907	8.1281	1.6697	7.4739
Wallace & Hussain (1969)	2.8428	2.3292	2.9414	0.5790	0.3887	0.5345	28.1377	9.8067	30.9327	8.1248	1.6705	7.4728
Swamy & Arora (1972)	2.8418	2.3630	2.9468	0.5797	0.3906	0.5346	28.1968	9.9376	31.2136	8.1243	1.6786	7.4818
Nerlove (1971)	2.9280	2.0202	3.0772	0.5806	0.3524	0.5365	32.2521	8.6298	35.3756	8.2275	1.5182	7.5745
Henderson III (1953)	2.8035	2.3171	2.9435	0.5783	0.3835	0.5411	27.0491	9.7631	30.5712	7.9866	1.6663	7.4266
Minque (Rao (1970, 1972))	2.8457	2.3453	2.9446	0.5790	0.3896	0.5346	28.5584	9.8755	31.1567	8.1330	1.6743	7.4784
ML (Breusch (1987))	2.8446	2.3274	2.9424	0.5790	0.3886	0.5345	28.2107	9.7707	30.9514	8.1271	1.6697	7.4734
Within	3.3998	2.8683	3.6632	0.5921	0.4001	0.5494	40.0258	12.3753	44.3765	8.4827	1.7270	7.8314
FGLS SAR-RE (Kapoor et al. (2007))	2.7432	2.6303	2.7079	0.5517	0.5033	0.5094	3.6164	3.4937	3.4660	1.2553	1.1841	1.1812
FGLS SMA-RE (Fingleton (2008*a*))	2.7517	2.6541	2.7698	0.5532	0.5188	0.5126	4.3834	3.6500	4.9940	1.7010	1.1990	1.6001

	True model SMA-RE											
OLS	2.9831	2.3323	2.9688	0.5713	0.4398	0.5342	4.7718	2.9951	4.9014	1.1318	0.5649	1.0579
FGLS Amemiya (1971)	2.6806	2.2355	2.7655	0.5373	0.3728	0.4962	4.8049	2.8663	5.0666	1.1239	0.4807	1.0349
Wallace & Hussain (1969)	2.6786	2.2354	2.7643	0.5373	0.3730	0.4962	4.7755	2.8660	5.0563	1.1238	0.4810	1.0348
Swamy & Arora (1972)	2.6773	2.2679	2.7684	0.5379	0.3748	0.4963	4.7796	2.9066	5.0801	1.1245	0.4833	1.0355
Nerlove (1971)	2.7485	1.9386	2.8822	0.5386	0.3382	0.4978	5.1114	2.4978	5.4547	1.1310	0.4363	1.0419
Henderson III (1953)	2.6438	2.2230	2.7681	0.5382	0.3677	0.5036	4.6845	2.8505	5.0333	1.1116	0.4757	1.0350
Minque (Rao (1970, 1972))	2.6801	2.2509	2.7666	0.5373	0.3738	0.4962	4.8049	2.8859	5.0726	1.1243	0.4820	1.0352
ML (Breusch (1987))	2.6799	2.2337	2.7650	0.5373	0.3728	0.4962	4.7876	2.8629	5.0615	1.1239	0.4807	1.0349
Within	3.1839	2.7524	3.4245	0.5491	0.3839	0.5097	6.0824	3.5573	6.6202	1.1591	0.4956	1.0707
FGLS SAR-RE (Kapoor et al. (2007))	2.6072	2.5250	2.6177	0.5153	0.4824	0.4783	1.8571	1.9413	1.7784	0.5187	0.6092	0.4972
FGLS SMA-RE (Fingleton (2008*a*))	2.6068	2.4923	2.5651	0.5151	0.4701	0.4756	1.0305	0.9757	1.0156	0.3159	0.2956	0.2995

(1) $\times 10^{-2}$.

Table 7 – Percentage of rejections of H_0 at 5% significance level for *N*=50, $\sigma_{\mu}^{2} = 0.8$, $\sigma_{\nu}^{2} = 0.2$, W(5,5), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
H_0 : β=	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	16.3	11.9	16.8	51.7	35.2	3.4	32.5	99.4	19.9	17.5	20.7	39.9	39.0	27.6	35.4	81.0
FGLS Amemiya (1971)	25.4	7.6	27.6	94.9	84.5	11.2	85.3	100.0	47.9	47.2	49.0	65.3	65.4	56.6	61.4	93.2
Wallace & Hussain (1969)	24.6	7.4	26.8	94.6	84.4	10.9	85.3	100.0	47.8	46.8	48.5	65.1	65.3	56.4	61.4	93.2
Swamy & Arora (1972)	24.3	7.2	27.0	94.8	84.3	10.7	85.2	100.0	47.5	46.7	48.4	65.1	65.2	56.1	61.4	93.2
Nerlove (1971)	33.6	10.5	34.3	96.5	86.0	13.2	87.0	100.0	53.3	51.3	54.2	68.1	67.0	58.4	62.7	93.6
Henderson III (1953)	24.2	7.3	26.9	94.8	84.3	10.6	85.2	100.0	47.5	46.6	48.2	64.8	65.0	56.2	61.3	93.2
Minque (Rao (1970, 1972))	24.9	7.6	27.5	94.9	84.5	11.1	85.3	100.0	47.9	47.1	49.0	65.3	65.3	56.6	61.4	93.2
ML (Breusch (1987))	25.4	7.6	27.6	94.9	84.5	11.2	85.3	100.0	47.9	47.2	49.0	65.3	65.4	56.6	61.4	93.2
Within	23.9	7.0	27.0	94.8	84.2	10.6	84.9	100.0	48.3	47.4	49.6	64.1	65.3	55.7	61.1	93.1
TGLS SAR-RE	18.8	4.5	21.2	91.9	78.1	5.1	77.8	100.0	14.5	4.7	14.0	71.8	35.8	5.1	37.7	99.8
FGLS SAR-RE (Kapoor et al. (2007))	19.8	4.7	20.7	90.4	77.2	5.1	77.2	100.0	14.7	5.8	13.8	70.7	37.1	6.2	39.2	99.7
FGLS SMA-RE (Fingleton (2008*a*))	20.2	5.2	21.4	90.8	77.9	5.3	78.4	100.0	18.6	7.4	19.4	65.4	42.2	7.8	43.0	97.2

	True model SMA-RE															
OLS	16.4	11.9	17.0	52.4	34.9	3.2	32.7	99.4	16.8	12.3	17.1	50.8	36.7	5.9	33.3	99.2
FGLS Amemiya (1971)	24.8	7.3	27.4	95.6	85.4	9.6	86.0	100.0	31.4	16.3	31.8	89.2	76.7	24.7	76.5	100.0
Wallace & Hussain (1969)	24.3	6.8	26.8	95.3	85.3	9.6	85.9	100.0	30.6	16.0	31.2	88.9	76.7	24.7	76.4	100.0
Swamy & Arora (1972)	23.9	6.4	26.7	95.0	85.0	9.4	85.8	100.0	30.5	15.5	31.5	88.8	76.5	24.5	76.3	100.0
Nerlove (1971)	33.1	10.0	33.7	96.9	87.3	12.1	88.1	100.0	38.4	21.6	37.5	92.0	78.5	27.2	77.9	100.0
Henderson III (1953)	23.8	6.4	27.0	95.4	85.3	9.3	85.7	100.0	30.3	15.7	31.6	88.9	76.4	24.5	76.3	100.0
Minque (Rao (1970, 1972))	24.6	7.2	27.4	95.4	85.4	9.6	85.9	100.0	31.2	16.2	31.7	89.1	76.6	24.7	76.5	100.0
ML (Breusch (1987))	24.8	7.3	27.4	95.6	85.4	9.6	86.0	100.0	31.4	16.2	31.8	89.2	76.7	24.7	76.5	100.0
Within	23.8	6.4	26.3	95.3	85.4	9.3	86.0	100.0	30.2	14.9	31.1	87.2	76.5	24.0	76.0	100.0
TGLS SMA-RE	19.6	4.5	21.6	93.2	80.3	5.1	80.6	100.0	15.9	5.0	18.3	84.9	60.7	5.5	61.4	100.0
FGLS SAR-RE (Kapoor et al. (2007))	19.9	4.6	20.8	91.5	78.9	4.8	79.0	100.0	16.7	4.9	17.9	82.0	56.6	4.1	57.8	99.9
FGLS SMA-RE (Fingleton (2008*a*))	20.5	5.2	21.6	91.4	80.0	5.0	80.2	100.0	15.8	5.3	20.4	87.5	63.4	5.0	64.2	100.0

Table 8 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_{\mu}^{2} = 0.8$, $\sigma_{\nu}^{2} = 0.2$, W(5,5), 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.5715	2.3139	3.4722	0.3853	0.4319	0.3813	7.8831	3.8965	8.1396	2.1540	0.7181	2.0417
FGLS Amemiya (1971)	0.8145	0.6611	0.8743	0.1469	0.0948	0.1362	8.6398	1.2974	9.5955	2.0611	0.1889	1.9060
Wallace & Hussain (1969)	0.8143	0.6761	0.8740	0.1470	0.0954	0.1362	8.6005	1.3170	9.5815	2.0609	0.1898	1.9060
Swamy & Arora (1972)	0.8143	0.6830	0.8750	0.1470	0.0961	0.1362	8.6618	1.3416	9.6434	2.0620	0.1916	1.9071
Nerlove (1971)	0.8156	0.5319	0.8768	0.1469	0.0854	0.1362	8.7637	1.0468	9.7322	2.0626	0.1701	1.9075
Henderson III (1953)	0.8124	0.6809	0.8741	0.1469	0.0961	0.1362	8.5832	1.3354	9.5711	2.0572	0.1914	1.9047
Minque (Rao (1970, 1972))	0.8145	0.6647	0.8744	0.1469	0.0950	0.1362	8.6428	1.3043	9.6028	2.0613	0.1894	1.9062
ML (Breusch (1987))	0.8145	0.6612	0.8742	0.1469	0.0948	0.1362	8.6359	1.2977	9.5944	2.0611	0.1890	1.9060
Within	0.8312	0.6977	0.8982	0.1470	0.0963	0.1365	9.1563	1.3820	10.1686	2.0718	0.1921	1.9164
FGLS SAR-RE (Kapoor et al. (2007))	0.8451	0.8482	0.8290	0.1434	0.1328	0.1314	1.4771	1.4092	1.4057	0.3946	0.3736	0.3757
FGLS SMA-RE (Fingleton (2008*a*))	0.8476	0.8577	0.8572	0.1457	0.1393	0.1329	2.2699	1.5123	2.7020	1.1450	0.3894	1.1892

	True model SMA-RE											
OLS	3.5399	2.2937	3.4403	0.3742	0.4282	0.3710	3.8265	2.3687	3.7720	0.4985	0.4415	0.4893
FGLS Amemiya (1971)	0.7715	0.6545	0.8263	0.1368	0.0939	0.1268	1.3831	0.6899	1.5142	0.2836	0.0991	0.2623
Wallace & Hussain (1969)	0.7715	0.6694	0.8261	0.1368	0.0944	0.1268	1.3798	0.7049	1.5130	0.2836	0.0997	0.2623
Swamy & Arora (1972)	0.7713	0.6762	0.8268	0.1368	0.0951	0.1268	1.3841	0.7128	1.5177	0.2836	0.1005	0.2624
Nerlove (1971)	0.7722	0.5266	0.8283	0.1368	0.0845	0.1268	1.3910	0.5553	1.5242	0.2836	0.0892	0.2624
Henderson III (1953)	0.7696	0.6741	0.8262	0.1368	0.0951	0.1268	1.3783	0.7105	1.5129	0.2834	0.1004	0.2622
Minque (Rao (1970, 1972))	0.7714	0.6580	0.8263	0.1368	0.0941	0.1268	1.3832	0.6936	1.5148	0.2836	0.0993	0.2623
ML (Breusch (1987))	0.7715	0.6546	0.8263	0.1368	0.0939	0.1268	1.3829	0.6900	1.5142	0.2836	0.0991	0.2623
Within	0.7863	0.6907	0.8480	0.1368	0.0953	0.1271	1.4287	0.7288	1.5698	0.2842	0.1007	0.2632
FGLS SAR-RE (Kapoor et al. (2007))	0.8120	0.8256	0.8319	0.1351	0.1272	0.1252	1.0807	1.0734	1.0672	0.2153	0.2214	0.2014
FGLS SMA-RE (Fingleton (2008*a*))	0.8085	0.8047	0.7958	0.1347	0.1238	0.1237	0.9972	0.9763	1.0181	0.2043	0.1885	0.1964

(1) x 10^{-2}.

Table 9 – Percentage of rejections of H_0 at 5% significance level for N=50, $\sigma_\mu^2 = 0.5$, $\sigma_\nu^2 = 0.5$, W(5,5), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	14.6	11.5	15.0	51.8	36.5	5.9	33.2	99.1	30.1	27.1	30.4	42.3	49.8	43.7	44.8	72.5
FGLS Amemiya (1971)	15.0	8.1	15.8	65.5	53.9	11.0	52.5	99.7	47.0	45.7	44.9	53.0	62.1	56.5	57.8	79.1
Wallace & Hussain (1969)	15.0	8.0	15.6	65.2	53.8	11.0	52.2	99.7	46.6	45.3	44.7	53.0	62.1	56.3	57.8	79.1
Swamy & Arora (1972)	14.1	7.1	15.2	63.8	53.6	10.8	51.7	99.7	46.2	45.0	44.1	52.3	61.8	56.0	57.7	79.0
Nerlove (1971)	19.4	10.6	20.7	72.1	57.3	13.1	56.0	99.7	51.7	50.4	51.3	56.6	64.2	58.4	59.6	80.3
Henderson III (1953)	13.5	7.4	15.5	64.7	53.2	10.6	52.3	99.7	45.5	44.6	43.6	52.4	61.8	55.6	57.7	79.0
Minque (Rao (1970, 1972))	14.8	7.7	15.7	64.9	53.8	11.0	52.4	99.7	47.0	45.7	44.9	52.9	62.1	56.4	57.8	79.0
ML (Breusch (1987))	15.1	8.2	15.8	65.5	53.9	11.0	52.5	99.7	47.0	45.6	44.9	53.1	62.1	56.5	57.8	79.1
Within	13.6	7.0	14.9	61.5	53.7	10.6	51.7	99.7	47.5	47.4	47.3	53.0	62.1	55.7	57.9	78.9
TGLS SAR-RE	9.9	4.5	10.9	58.8	42.8	5.3	41.1	99.7	9.0	4.6	9.8	40.7	18.2	5.2	19.6	87.7
FGLS SAR-RE (Kapoor et al. (2007))	11.9	5.9	12.6	59.0	43.7	5.3	43.8	99.7	11.6	6.9	10.4	40.6	18.9	5.2	20.0	88.1
FGLS SMA-RE (Fingleton (2008*a*))	12.8	6.2	12.4	61.0	45.3	5.6	44.6	99.7	15.8	7.6	16.1	40.5	23.0	8.0	24.9	75.2

	True model SMA-RE															
OLS	14.3	11.5	14.8	52.0	36.2	5.3	33.0	99.1	17.2	13.1	17.7	50.4	39.1	14.1	35.4	96.9
FGLS Amemiya (1971)	14.4	7.3	15.7	66.9	54.5	10.0	52.6	99.7	22.7	16.4	23.2	62.2	53.6	24.9	51.1	99.1
Wallace & Hussain (1969)	14.3	7.3	15.8	66.8	54.4	10.0	52.6	99.7	22.4	16.3	23.1	62.2	53.6	24.9	51.1	99.1
Swamy & Arora (1972)	13.3	6.5	14.6	64.4	54.1	9.4	52.0	99.7	21.3	15.9	22.6	60.4	53.5	24.7	50.4	99.1
Nerlove (1971)	19.2	9.8	20.2	72.7	57.7	11.7	56.5	99.7	28.2	20.6	29.0	66.2	56.7	27.4	53.0	99.2
Henderson III (1953)	13.1	7.0	15.3	66.3	53.8	9.8	52.8	99.7	21.0	15.7	23.0	62.0	53.1	24.7	50.8	99.2
Minque (Rao (1970, 1972))	14.2	7.3	15.7	66.7	54.4	10.0	52.6	99.7	22.4	16.3	23.1	61.8	53.5	24.9	51.0	99.1
ML (Breusch (1987))	14.4	7.3	15.7	66.9	54.5	10.0	52.6	99.7	22.8	16.4	23.2	62.2	53.6	24.9	51.1	99.1
Within	13.1	6.4	14.5	61.6	54.5	9.3	51.8	99.7	22.5	14.9	22.9	58.7	53.7	24.0	50.0	99.1
TGLS SMA-RE	10.0	4.7	11.2	60.3	44.7	5.6	42.9	99.7	9.7	4.8	10.2	51.4	30.5	5.7	31.7	99.1
FGLS SAR-RE (Kapoor et al. (2007))	11.9	5.8	12.6	60.1	45.5	5.2	44.9	99.7	10.2	6.0	10.2	51.1	28.2	5.6	28.5	98.3
FGLS SMA-RE (Fingleton (2008*a*))	12.8	5.9	12.6	62.1	46.7	5.4	45.4	99.7	11.4	6.1	12.0	52.8	32.8	6.2	32.9	99.0

Table 10 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_{\mu}^{2} = 0.5$, $\sigma_{\nu}^{2} = 0.5$, W(5,5), 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.2944	2.3347	3.2293	0.4937	0.4361	0.4697	13.4714	4.1517	14.2953	4.6960	0.7745	4.3736
FGLS Amemiya (1971)	1.9326	1.5628	2.0488	0.3654	0.2357	0.3380	19.0334	3.0109	21.0215	5.0810	0.4686	4.6955
Wallace & Hussain (1969)	1.9312	1.5715	2.0479	0.3654	0.2360	0.3380	18.8225	3.0242	20.9397	5.0800	0.4692	4.6951
Swamy & Arora (1972)	1.9323	1.6225	2.0546	0.3656	0.2391	0.3382	19.1056	3.1300	21.3175	5.0886	0.4755	4.7051
Nerlove (1971)	1.9479	1.2810	2.0768	0.3655	0.2124	0.3383	20.1785	2.4983	22.3255	5.1012	0.4228	4.7153
Henderson III (1953)	1.9172	1.6036	2.0473	0.3649	0.2383	0.3380	18.1778	3.0675	20.6553	5.0026	0.4731	4.6699
Minque (Rao (1970, 1972))	1.9325	1.5722	2.0495	0.3654	0.2362	0.3380	19.0424	3.0296	21.0616	5.0826	0.4697	4.6970
ML (Breusch (1987))	1.9323	1.5630	2.0487	0.3654	0.2357	0.3380	18.9547	3.0109	20.9930	5.0809	0.4686	4.6954
Within	2.0780	1.7443	2.2455	0.3675	0.2408	0.3413	22.8909	3.4549	25.4216	5.1794	0.4802	4.7911
FGLS SAR-RE (Kapoor et al. (2007))	1.9729	1.9078	1.9399	0.3567	0.3230	0.3260	3.3279	2.9761	3.1371	0.9687	0.9090	0.9174
FGLS SMA-RE (Fingleton (2008*a*))	1.9865	1.9541	2.0087	0.3598	0.3340	0.3298	4.2219	3.0973	4.0817	1.2361	1.0477	1.1122

	True model SMA-RE											
OLS	3.2318	2.3133	3.1646	0.4703	0.4321	0.4481	3.9918	2.4060	4.0036	0.7838	0.4491	0.7408
FGLS Amemiya (1971)	1.8350	1.5472	1.9412	0.3404	0.2333	0.3149	3.2067	1.6277	3.4760	0.7030	0.2463	0.6496
Wallace & Hussain (1969)	1.8344	1.5559	1.9406	0.3404	0.2336	0.3149	3.1932	1.6366	3.4709	0.7030	0.2466	0.6496
Swamy & Arora (1972)	1.8342	1.6063	1.9456	0.3405	0.2366	0.3150	3.2137	1.6901	3.5014	0.7037	0.2498	0.6503
Nerlove (1971)	1.8463	1.2682	1.9641	0.3404	0.2103	0.3151	3.2851	1.3358	3.5753	0.7041	0.2220	0.6509
Henderson III (1953)	1.8213	1.5877	1.9405	0.3400	0.2359	0.3150	3.1638	1.6691	3.4605	0.6993	0.2490	0.6483
Minque (Rao (1970, 1972))	1.8349	1.5566	1.9417	0.3404	0.2338	0.3149	3.2076	1.6375	3.4795	0.7031	0.2469	0.6497
ML (Breusch (1987))	1.8349	1.5475	1.9411	0.3404	0.2333	0.3149	3.2033	1.6279	3.4748	0.7030	0.2463	0.6496
Within	1.9659	1.7267	2.1199	0.3421	0.2383	0.3178	3.5718	1.8221	3.9244	0.7106	0.2517	0.6581
FGLS SAR-RE (Kapoor et al. (2007))	1.8967	1.8563	1.9294	0.3359	0.3093	0.3107	2.4944	2.3898	2.4528	0.5460	0.5319	0.4996
FGLS SMA-RE (Fingleton (2008*a*))	1.8880	1.8004	1.8667	0.3350	0.3001	0.3069	2.4453	2.2081	2.3365	0.5404	0.4619	0.4844

(1) x 10^{-2}.

Table 11 – Percentage of rejections of H_0 at 5% significance level for N=50, $\sigma_\mu^2 = 0.2$, $\sigma_v^2 = 0.8$, W(5,5), 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				T=10				T=5				T=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	13.6	9.5	13.3	51.2	37.0	9.1	34.1	97.9	36.5	35.5	36.3	45.2	54.9	51.5	52.3	69.9
FGLS Amemiya (1971)	13.3	7.9	14.4	51.3	41.4	10.7	39.7	99.2	43.2	43.4	42.7	49.0	60.6	55.9	56.2	71.4
Wallace & Hussain (1969)	13.4	7.8	14.5	51.1	41.5	10.7	39.7	99.2	42.5	43.1	42.5	49.0	60.6	55.9	56.2	71.4
Swamy & Arora (1972)	12.2	7.1	13.4	49.6	40.9	10.5	38.8	99.2	42.3	42.3	42.3	48.4	60.3	55.4	56.0	71.2
Nerlove (1971)	16.2	10.4	17.7	56.8	45.3	12.9	42.4	99.2	50.5	49.3	49.9	54.4	62.1	58.2	58.5	72.3
Henderson III (1953)	12.3	7.8	14.1	52.8	41.5	10.5	40.3	99.2	41.6	41.5	41.1	47.6	59.7	54.9	55.7	71.2
Minque (Rao (1970, 1972))	13.1	7.8	14.4	50.9	41.4	10.6	39.6	99.2	43.0	43.2	42.6	49.0	60.5	55.8	56.3	71.4
ML (Breusch (1987))	13.4	7.8	14.5	51.2	41.4	10.7	39.7	99.2	42.5	42.8	42.4	49.0	60.6	55.9	56.2	71.4
Within	11.1	7.0	12.3	43.7	41.5	10.6	38.2	99.1	46.9	47.4	47.1	50.2	60.4	55.7	56.1	70.8
TGLS SAR-RE	9.6	5.1	9.3	46.0	31.2	5.3	29.7	98.6	7.7	5.5	9.2	32.5	14.1	5.4	14.0	73.7
FGLS SAR-RE (Kapoor et al. (2007))	11.1	5.9	10.9	46.4	30.8	6.0	30.3	98.6	8.3	6.5	9.8	33.7	14.3	5.6	15.1	73.6
FGLS SMA-RE (Fingleton (2008*a*))	11.9	6.1	11.2	47.1	31.9	6.3	31.5	98.6	8.0	6.8	9.9	26.9	17.5	6.8	19.9	61.9

	True model SMA-RE															
OLS	13.3	9.0	12.9	51.8	36.7	8.2	34.4	98.6	18.2	12.9	17.7	50.2	40.8	19.4	37.1	94.9
FGLS Amemiya (1971)	12.8	7.5	14.2	51.5	41.4	9.7	39.2	99.2	19.5	15.0	20.1	50.9	45.5	24.2	42.3	95.3
Wallace & Hussain (1969)	13.0	7.3	14.2	51.3	41.3	9.7	39.2	99.2	19.2	15.1	19.9	50.9	45.3	24.2	42.2	95.3
Swamy & Arora (1972)	11.6	6.7	12.6	49.9	41.0	9.7	38.9	99.2	18.7	14.1	19.3	49.7	45.1	23.8	41.5	95.2
Nerlove (1971)	15.6	9.8	17.2	57.2	44.9	11.6	42.6	99.3	24.6	19.8	25.3	55.4	48.3	27.1	44.2	95.9
Henderson III (1953)	11.8	7.3	13.3	53.3	41.0	9.4	40.3	99.4	17.5	13.9	19.7	52.2	45.0	23.6	42.8	95.7
Minque (Rao (1970, 1972))	12.6	7.5	13.9	51.3	41.4	9.7	39.1	99.2	19.3	14.8	20.0	50.7	45.5	24.2	42.1	95.3
ML (Breusch (1987))	12.9	7.4	14.2	51.4	41.3	9.7	39.2	99.2	19.3	15.1	20.0	50.9	45.5	24.2	42.3	95.3
Within	10.3	6.4	12.1	44.3	40.6	9.3	38.7	99.3	20.1	14.9	20.3	44.5	44.7	24.0	41.0	94.9
TGLS SMA-RE	9.5	5.1	9.5	46.8	32.1	5.2	31.6	98.8	8.7	5.5	9.7	40.2	21.4	5.6	22.3	92.3
FGLS SAR-RE (Kapoor et al. (2007))	11.1	5.8	10.9	47.0	32.0	5.7	31.0	98.9	9.7	6.2	10.4	41.1	19.8	4.6	20.4	90.4
FGLS SMA-RE (Fingleton (2008*a*))	11.9	6.0	11.2	48.0	32.9	6.0	32.6	98.9	10.6	6.9	11.6	42.0	23.6	5.8	24.0	92.8

Table 12 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_\mu^2 = 0.2$, $\sigma_\nu^2 = 0.8$, W(5,5), 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.0026	2.3551	2.9864	0.5978	0.4406	0.5582	19.0272	4.3954	20.4510	7.2342	0.8312	6.7055
FGLS Amemiya (1971)	2.7594	2.2214	2.8588	0.5734	0.3708	0.5299	24.1848	4.1860	26.1177	7.8157	0.7328	7.2041
Wallace & Hussain (1969)	2.7550	2.2214	2.8563	0.5734	0.3709	0.5299	23.5410	4.1804	25.8571	7.8110	0.7330	7.2024
Swamy & Arora (1972)	2.7595	2.3272	2.8768	0.5744	0.3768	0.5307	23.9959	4.3697	26.7017	7.8439	0.7453	7.2474
Nerlove (1971)	2.8549	1.9262	3.0080	0.5757	0.3363	0.5326	28.6572	3.7353	31.3774	7.9738	0.6679	7.3627
Henderson III (1953)	2.7137	2.2455	2.8525	0.5711	0.3688	0.5345	21.6411	4.1822	24.7577	7.5441	0.7312	7.0213
Minque (Rao (1970, 1972))	2.7593	2.2368	2.8607	0.5735	0.3718	0.5300	24.1432	4.2121	26.1718	7.8228	0.7348	7.2110
ML (Breusch (1987))	2.7570	2.2197	2.8575	0.5734	0.3708	0.5299	23.5288	4.1504	25.7635	7.8134	0.7328	7.2031
Within	3.3248	2.7909	3.5927	0.5880	0.3852	0.5460	36.6254	5.5278	40.6745	8.2871	0.7683	7.6658
FGLS SAR-RE (Kapoor et al. (2007))	2.7801	2.6123	2.7005	0.5594	0.5019	0.5103	4.2768	3.9508	4.0143	1.4590	1.3556	1.3742
FGLS SMA-RE (Fingleton (2008*a*))	2.7823	2.6478	2.8070	0.5599	0.5380	0.5172	6.1110	5.6117	6.0147	1.9932	1.4618	1.8255

	True model SMA-RE											
OLS	2.9098	2.3326	2.8889	0.5621	0.4364	0.5252	4.1435	2.4414	4.2351	1.0648	0.4571	0.9923
FGLS Amemiya (1971)	2.6318	2.1997	2.7218	0.5345	0.3670	0.4941	4.3981	2.3068	4.6569	1.0958	0.3872	1.0114
Wallace & Hussain (1969)	2.6292	2.1997	2.7200	0.5346	0.3672	0.4941	4.3629	2.3063	4.6427	1.0956	0.3874	1.0113
Swamy & Arora (1972)	2.6306	2.3045	2.7357	0.5354	0.3730	0.4947	4.4127	2.4162	4.7300	1.0989	0.3935	1.0147
Nerlove (1971)	2.7094	1.9070	2.8487	0.5363	0.3329	0.4962	4.7614	2.0071	5.1216	1.1060	0.3514	1.0218
Henderson III (1953)	2.5932	2.2238	2.7202	0.5337	0.3650	0.4997	4.2355	2.3246	4.5770	1.0722	0.3848	0.9992
Minque (Rao (1970, 1972))	2.6315	2.2149	2.7232	0.5346	0.3680	0.4942	4.3989	2.3226	4.6650	1.0963	0.3882	1.0119
ML (Breusch (1987))	2.6305	2.1980	2.7210	0.5346	0.3670	0.4941	4.3759	2.3044	4.6485	1.0957	0.3872	1.0114
Within	3.1454	2.7627	3.3918	0.5474	0.3813	0.5085	5.7148	2.9153	6.2791	1.1369	0.4027	1.0529
FGLS SAR-RE (Kapoor et al. (2007))	2.6833	2.5479	2.7049	0.5275	0.4809	0.4872	3.3678	3.1450	3.2726	0.8230	0.8275	0.9114
FGLS SMA-RE (Fingleton (2008*a*))	2.6821	2.4789	2.6108	0.5259	0.4668	0.4810	3.3204	2.9723	3.1378	0.8141	0.7168	0.7464

(1) × 10^{-2}.

Table 13 – Percentage of rejections of H_0 at 5% significance level for *N*=50, $\sigma_\mu^2 = 0.8$, $\sigma_v^2 = 0.2$, asymmetric one-order contiguity matrix, 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	18.3	13.1	17.3	50.1	34.0	3.2	30.2	99.4	11.2	11.0	12.1	16.7	9.9	7.5	8.8	32.4
FGLS Amemiya (1971)	23.8	6.2	25.4	93.8	83.7	10.2	82.6	100.0	23.5	22.9	24.1	35.9	37.6	29.6	35.5	71.1
Wallace & Hussain (1969)	22.7	5.9	24.8	93.6	83.5	10.2	82.3	100.0	22.4	22.0	23.5	35.1	37.5	29.6	35.5	71.1
Swamy & Arora (1972)	23.8	6.1	25.3	93.6	83.7	10.2	82.5	100.0	23.4	22.7	24.0	35.5	37.5	29.6	35.5	71.1
Nerlove (1971)	30.2	9.6	32.1	96.3	85.2	12.8	84.2	100.0	29.3	27.2	29.5	40.6	40.3	33.1	38.2	72.1
Henderson III (1953)	23.5	6.0	25.5	93.8	83.5	10.3	82.5	100.0	23.4	22.7	24.0	35.8	37.5	29.5	35.5	71.1
Minque (Rao (1970, 1972))	23.6	6.1	25.3	93.6	83.7	10.2	82.5	100.0	23.4	22.7	24.0	35.7	37.5	29.6	35.5	71.1
ML (Breusch (1987))	23.8	6.2	25.4	93.8	83.7	10.2	82.6	100.0	23.5	22.9	24.1	35.9	37.6	29.6	35.5	71.1
Within	23.1	6.5	25.9	93.4	83.7	10.3	82.4	100.0	24.1	22.7	24.4	35.4	37.7	30.1	35.8	70.8
TGLS SAR-RE	19.2	4.2	20.9	92.5	77.6	5.1	78.3	100.0	20.4	6.0	20.8	90.0	48.6	5.4	51.3	100.0
FGLS SAR-RE (Kapoor et al. (2007))	20.0	4.8	20.9	92.4	77.7	4.5	77.7	100.0	16.3	4.7	17.3	85.2	47.1	4.6	50.0	99.9
FGLS SMA-RE (Fingleton (2008*a*))	21.1	5.1	22.8	92.8	78.8	5.1	78.9	100.0	20.8	7.0	23.0	78.0	39.8	7.5	47.5	98.0

	True model SMA-RE															
OLS	18.6	13.8	17.5	51.8	35.5	3.2	31.5	99.4	18.0	15.0	18.3	39.9	26.3	4.9	23.5	95.4
FGLS Amemiya (1971)	24.5	6.0	26.0	95.4	85.2	9.5	84.1	100.0	22.9	10.4	22.8	79.4	67.5	18.9	63.6	99.9
Wallace & Hussain (1969)	23.4	5.7	25.2	95.0	85.2	9.5	84.1	100.0	21.8	10.2	22.4	78.8	67.4	18.8	63.3	99.9
Swamy & Arora (1972)	24.2	6.0	26.1	95.3	85.2	9.5	84.1	100.0	22.6	10.4	22.7	79.4	67.4	18.9	63.6	99.9
Nerlove (1971)	31.4	9.6	32.6	96.8	86.4	11.9	87.0	100.0	28.1	14.8	29.7	82.7	69.9	20.9	66.5	99.9
Henderson III (1953)	23.9	6.0	26.1	95.4	85.2	9.5	84.2	100.0	22.7	10.4	22.9	79.4	67.4	18.9	63.6	99.9
Minque (Rao (1970, 1972))	24.3	6.0	25.9	95.4	85.2	9.5	84.1	100.0	22.6	10.4	22.7	79.3	67.4	18.9	63.6	99.9
ML (Breusch (1987))	24.5	6.0	26.0	95.4	85.2	9.5	84.1	100.0	22.8	10.5	22.8	79.4	67.5	18.9	63.6	99.9
Within	23.6	6.2	27.1	94.9	85.0	9.2	84.0	100.0	22.2	10.7	23.6	77.2	67.5	18.8	63.3	99.9
TGLS SMA-RE	20.1	3.9	21.5	94.3	80.0	5.3	80.7	100.0	79.5	5.1	81.3	100.0	99.9	4.6	99.9	100.0
FGLS SAR-RE (Kapoor et al. (2007))	20.4	4.7	21.6	93.5	79.6	4.4	79.7	100.0	21.7	2.5	20.4	96.6	68.1	2.8	67.0	100.0
FGLS SMA-RE (Fingleton (2008*a*))	22.0	5.0	23.1	94.5	80.8	4.7	81.5	100.0	37.8	4.5	38.9	99.4	87.4	4.7	87.7	100.0

Table 14 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_{\mu}^{2} = 0.8$, $\sigma_{\nu}^{2} = 0.2$, asymmetric one-order contiguity matrix, 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	4.0904	2.4394	3.9652	0.4047	0.4594	0.4098	33.7919	22.8832	33.9755	4.5240	4.2488	4.5331
FGLS Amemiya (1971)	0.8312	0.7084	0.8924	0.1542	0.1019	0.1421	16.6921	6.7291	16.8803	3.5481	0.9791	3.2485
Wallace & Hussain (1969)	0.8315	0.7260	0.8924	0.1542	0.1025	0.1421	16.6442	6.8581	16.8666	3.5478	0.9827	3.2485
Swamy & Arora (1972)	0.8312	0.7122	0.8925	0.1542	0.1021	0.1421	16.6787	6.7657	16.8879	3.5480	0.9813	3.2488
Nerlove (1971)	0.8319	0.5700	0.8940	0.1541	0.0917	0.1421	16.8408	5.4203	17.0101	3.5509	0.8815	3.2502
Henderson III (1953)	0.8291	0.7118	0.8925	0.1542	0.1021	0.1421	16.6551	6.7633	16.8825	3.5473	0.9812	3.2482
Minque (Rao (1970, 1972))	0.8312	0.7123	0.8925	0.1542	0.1021	0.1421	16.6963	6.7645	16.8872	3.5484	0.9813	3.2487
ML (Breusch (1987))	0.8312	0.7086	0.8924	0.1542	0.1019	0.1421	16.6890	6.7302	16.8796	3.5480	0.9791	3.2485
Within	0.8480	0.7289	0.9143	0.1543	0.1023	0.1424	17.4211	6.9487	17.5713	3.5690	0.9837	3.2623
FGLS SAR-RE (Kapoor et al. (2007))	0.7901	0.7993	0.7980	0.1405	0.1310	0.1316	0.9412	1.0439	0.8411	0.2650	0.2735	0.2616
FGLS SMA-RE (Fingleton (2008*a*))	0.7902	0.8001	0.8140	0.1408	0.1377	0.1325	1.4999	1.0940	1.3313	0.3931	0.2951	0.3842

	True model SMA-RE											
OLS	3.9130	2.3257	3.7870	0.3876	0.4382	0.3932	6.7214	3.6134	6.6556	0.6916	0.6790	0.7138
FGLS Amemiya (1971)	0.7743	0.6755	0.8324	0.1409	0.0971	0.1303	1.5973	1.0550	1.7283	0.3313	0.1527	0.3068
Wallace & Hussain (1969)	0.7747	0.6923	0.8324	0.1409	0.0977	0.1303	1.5961	1.0840	1.7278	0.3313	0.1536	0.3068
Swamy & Arora (1972)	0.7743	0.6790	0.8324	0.1409	0.0974	0.1303	1.5969	1.0605	1.7284	0.3313	0.1531	0.3068
Nerlove (1971)	0.7746	0.5435	0.8337	0.1409	0.0874	0.1303	1.6028	0.8491	1.7329	0.3314	0.1375	0.3068
Henderson III (1953)	0.7724	0.6786	0.8325	0.1409	0.0973	0.1303	1.5933	1.0600	1.7284	0.3312	0.1530	0.3067
Minque (Rao (1970, 1972))	0.7742	0.6792	0.8324	0.1409	0.0974	0.1303	1.5973	1.0609	1.7284	0.3313	0.1531	0.3068
ML (Breusch (1987))	0.7743	0.6756	0.8324	0.1409	0.0971	0.1303	1.5972	1.0553	1.7282	0.3313	0.1527	0.3068
Within	0.7890	0.6950	0.8523	0.1410	0.0976	0.1306	1.6446	1.0866	1.7757	0.3322	0.1534	0.3077
FGLS SAR-RE (Kapoor et al. (2007))	0.7482	0.7645	0.7640	0.1300	0.1252	0.1226	0.5682	0.6950	0.5586	0.1394	0.1749	0.1361
FGLS SMA-RE (Fingleton (2008*a*))	0.7395	0.7432	0.7535	0.1298	0.1202	0.1219	0.2381	0.4504	0.1241	0.0824	0.1056	0.0383

(1) × 10^{-2}.

Table 15 – Percentage of rejections of H_0 at 5% significance level for N=50, $\sigma_\mu^2 = 0.5$, $\sigma_\nu^2 = 0.5$, asymmetric one-order contiguity matrix, 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				T=10				T=5				T=10			
$H_0 : \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	15.9	11.7	15.4	49.3	34.8	5.6	32.5	99.1	13.0	12.8	13.9	18.4	19.9	16.6	18.9	35.7
FGLS Amemiya (1971)	13.3	7.1	15.0	63.0	52.1	10.4	49.9	99.7	21.9	21.6	22.8	26.8	33.1	29.7	32.9	49.2
Wallace & Hussain (1969)	13.2	7.2	15.0	62.9	52.0	10.4	49.9	99.7	21.5	21.4	22.6	26.5	33.1	29.7	32.8	49.2
Swamy & Arora (1972)	13.2	7.2	14.9	63.0	52.0	10.3	49.8	99.7	21.7	21.3	22.5	26.6	33.0	29.5	32.8	49.2
Nerlove (1971)	18.3	9.7	19.4	69.5	55.2	13.0	52.7	99.7	27.6	26.3	28.0	33.7	35.5	33.1	34.6	52.7
Henderson III (1953)	12.5	7.2	14.9	63.7	51.7	10.4	50.2	99.7	21.4	21.3	22.6	26.6	33.1	29.5	32.8	49.2
Minque (Rao (1970, 1972))	13.1	7.1	14.9	63.0	52.1	10.3	49.8	99.7	21.9	21.3	22.6	26.7	33.0	29.6	32.8	49.2
ML (Breusch (1987))	13.3	7.1	15.0	63.0	52.0	10.4	49.9	99.7	21.9	21.5	22.7	26.8	33.1	29.7	32.9	49.2
Within	12.9	6.5	14.4	60.5	51.5	10.3	49.0	99.7	24.0	22.7	23.1	27.0	33.0	30.1	32.6	49.0
TGLS SAR-RE	10.6	4.6	11.7	59.9	42.4	5.1	41.5	99.7	11.9	6.1	12.0	59.3	21.5	5.0	25.2	96.4
FGLS SAR-RE (Kapoor et al. (2007))	11.3	4.8	12.4	60.1	43.2	4.9	42.1	99.7	11.5	5.0	10.8	56.0	21.6	4.8	24.6	95.6
FGLS SMA-RE (Fingleton (2008*a*))	11.9	5.0	13.0	61.0	43.6	5.5	43.5	99.7	15.8	8.3	14.5	53.8	27.1	7.7	29.4	94.9

	True model SMA-RE															
OLS	16.1	11.7	15.2	51.1	35.6	5.6	32.7	99.2	16.5	14.0	15.7	38.9	28.4	9.5	25.7	91.4
FGLS Amemiya (1971)	13.5	7.0	15.2	65.4	53.5	9.5	51.2	99.7	15.5	11.4	15.5	48.4	44.6	18.7	39.4	97.2
Wallace & Hussain (1969)	13.4	7.0	15.0	65.4	53.5	9.5	51.2	99.7	15.2	11.4	15.4	48.1	44.6	18.7	39.3	97.2
Swamy & Arora (1972)	13.2	7.0	15.2	65.4	53.4	9.5	51.2	99.7	15.2	11.5	15.3	48.3	44.6	18.7	39.3	97.2
Nerlove (1971)	18.7	9.6	19.5	71.5	57.5	11.8	54.5	99.7	20.3	15.1	20.6	55.3	47.1	21.2	42.3	98.1
Henderson III (1953)	13.0	7.1	15.3	65.9	53.3	9.7	51.0	99.7	15.4	11.2	15.7	48.7	44.6	18.6	39.4	97.3
Minque (Rao (1970, 1972))	13.3	7.0	15.2	65.3	53.4	9.5	51.2	99.7	15.3	11.3	15.4	48.3	44.6	18.7	39.4	97.2
ML (Breusch (1987))	13.5	7.0	15.2	65.4	53.5	9.5	51.2	99.7	15.5	11.4	15.6	48.4	44.6	18.7	39.4	97.2
Within	13.0	6.2	14.4	62.2	53.4	9.2	50.8	99.7	16.2	10.7	15.8	46.8	44.4	18.8	39.9	97.4
TGLS SMA-RE	10.9	4.6	12.3	62.2	44.6	5.2	44.6	99.7	47.6	5.6	47.6	100.0	90.2	5.1	92.2	100.0
FGLS SAR-RE (Kapoor et al. (2007))	11.8	4.9	12.6	61.7	44.7	4.8	43.6	99.7	11.6	2.6	11.6	69.1	32.9	3.1	32.7	99.8
FGLS SMA-RE (Fingleton (2008*a*))	11.8	5.0	13.0	63.0	45.6	5.1	45.3	99.7	15.9	2.4	17.3	92.7	55.3	2.5	55.7	100.0

Table 16 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma_\mu^2 = 0.5$, $\sigma_\nu^2 = 0.5$, asymmetric one-order contiguity matrix, 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.6757	2.4594	3.5949	0.5158	0.4638	0.4982	38.9565	23.2468	38.9238	8.5349	4.3436	8.1544
FGLS Amemiya (1971)	1.9834	1.6748	2.1047	0.3832	0.2532	0.3528	38.0878	15.8013	38.4306	8.7537	2.4309	8.0281
Wallace & Hussain (1969)	1.9840	1.6851	2.1044	0.3833	0.2536	0.3528	37.8708	15.8819	38.3684	8.7524	2.4334	8.0278
Swamy & Arora (1972)	1.9835	1.6846	2.1050	0.3834	0.2538	0.3528	37.9497	15.8976	38.4757	8.7531	2.4369	8.0306
Nerlove (1971)	1.9917	1.3729	2.1235	0.3834	0.2283	0.3531	39.4425	13.0140	39.7273	8.7860	2.1922	8.0507
Henderson III (1953)	1.9679	1.6791	2.1054	0.3829	0.2533	0.3529	37.6573	15.8584	38.3914	8.7402	2.4351	8.0231
Minque (Rao (1970, 1972))	1.9832	1.6850	2.1050	0.3832	0.2538	0.3528	38.1099	15.8979	38.4758	8.7563	2.4367	8.0299
ML (Breusch (1987))	1.9836	1.6752	2.1047	0.3832	0.2532	0.3528	38.0316	15.8027	38.4159	8.7535	2.4310	8.0280
Within	2.1200	1.8222	2.2858	0.3857	0.2558	0.3561	43.5528	17.3717	43.9282	8.9226	2.4593	8.1558
FGLS SAR-RE (Kapoor et al. (2007))	1.8823	1.8392	1.8751	0.3498	0.3233	0.3264	2.1104	2.1114	1.9081	0.6521	0.6599	0.6394
FGLS SMA-RE (Fingleton (2008*a*))	1.8896	1.8494	1.9176	0.3505	0.3351	0.3290	3.3437	3.6396	3.0482	1.2557	0.6701	1.1753

	True model SMA-RE											
OLS	3.4950	2.3446	3.4159	0.4838	0.4423	0.4685	6.1818	3.6533	6.2350	0.9787	0.6886	0.9611
FGLS Amemiya (1971)	1.8508	1.5969	1.9657	0.3505	0.2414	0.3237	3.7592	2.4890	4.0491	0.8213	0.3795	0.7607
Wallace & Hussain (1969)	1.8517	1.6067	1.9656	0.3505	0.2418	0.3237	3.7524	2.5062	4.0469	0.8213	0.3801	0.7607
Swamy & Arora (1972)	1.8511	1.6062	1.9659	0.3506	0.2420	0.3237	3.7542	2.5030	4.0498	0.8215	0.3804	0.7608
Nerlove (1971)	1.8561	1.3090	1.9813	0.3505	0.2176	0.3239	3.8081	2.0431	4.1010	0.8224	0.3421	0.7616
Henderson III (1953)	1.8371	1.6008	1.9666	0.3502	0.2414	0.3239	3.7235	2.4961	4.0490	0.8194	0.3797	0.7603
Minque (Rao (1970, 1972))	1.8505	1.6066	1.9659	0.3505	0.2420	0.3237	3.7594	2.5042	4.0502	0.8214	0.3804	0.7607
ML (Breusch (1987))	1.8510	1.5972	1.9657	0.3505	0.2414	0.3237	3.7577	2.4895	4.0488	0.8213	0.3795	0.7607
Within	1.9724	1.7374	2.1307	0.3524	0.2439	0.3266	4.1115	2.7165	4.4392	0.8304	0.3835	0.7693
FGLS SAR-RE (Kapoor et al. (2007))	1.7838	1.7591	1.8018	0.3240	0.3089	0.3046	1.2952	1.5326	1.2821	0.3476	0.4274	0.3393
FGLS SMA-RE (Fingleton (2008*a*))	1.7658	1.7114	1.7719	0.3237	0.2967	0.3026	0.4719	0.8934	0.2782	0.1696	0.2440	0.0940

(1) × 10^{-2}.

Table 17 – Percentage of rejections of H_0 at 5% significance level for *N*=50, $\sigma_\mu^2 = 0.2$, $\sigma_v^2 = 0.8$, asymmetric one-order contiguity matrix, 1000 replications.

	True model SAR-RE															
	ρ=λ=0.2								ρ=λ=0.8							
	T=5				*T*=10				*T*=5				*T*=10			
$H_0: \beta=$	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8	0.4	0.5	0.6	0.8
OLS	12.8	9.2	13.4	49.8	36.0	8.3	32.3	97.8	16.2	14.1	15.7	20.5	26.8	23.8	25.5	39.3
FGLS Amemiya (1971)	11.9	8.0	12.6	49.9	40.2	10.8	37.2	98.9	19.4	19.2	20.3	23.4	31.8	29.4	32.1	42.2
Wallace & Hussain (1969)	12.0	8.0	12.6	49.9	40.2	10.8	37.1	98.9	19.4	19.0	20.1	23.0	31.8	29.4	32.0	42.2
Swamy & Arora (1972)	11.8	8.0	12.4	49.3	40.1	10.7	37.0	98.9	19.3	18.7	19.9	23.2	31.8	29.4	31.8	42.1
Nerlove (1971)	15.2	9.9	16.3	55.1	43.8	12.7	40.0	99.3	26.2	25.5	26.2	30.2	34.2	32.6	34.4	44.9
Henderson III (1953)	11.2	8.3	12.9	51.9	40.0	10.7	39.0	98.8	19.1	18.0	19.8	22.8	31.7	29.3	32.0	42.2
Minque (Rao (1970, 1972))	11.8	8.0	12.4	49.6	40.0	10.7	37.1	98.9	19.3	19.1	20.0	23.4	31.8	29.4	32.0	42.0
ML (Breusch (1987))	12.0	8.0	12.5	49.7	40.2	10.8	37.2	98.9	19.4	19.3	20.1	23.3	31.8	29.4	32.0	42.2
Within	10.9	6.5	12.1	43.2	39.7	10.3	36.8	98.8	24.2	22.7	22.8	25.6	32.7	30.1	32.4	42.2
TGLS SAR-RE	9.3	4.6	9.7	46.5	30.8	5.6	29.8	98.3	10.7	6.0	10.8	49.1	16.1	5.2	20.0	87.3
FGLS SAR-RE (Kapoor et al. (2007))	10.5	5.6	10.7	47.9	30.5	5.9	31.0	98.4	9.6	5.5	10.0	46.2	16.0	5.2	19.2	86.5
FGLS SMA-RE (Fingleton (2008*a*))	10.4	5.8	11.0	49.5	31.6	6.6	31.2	98.2	15.3	8.0	14.8	43.3	29.5	8.5	28.5	86.0

	True model SMA-RE															
OLS	13.3	9.0	13.6	51.0	36.7	7.5	33.2	98.2	14.7	11.0	14.2	37.9	31.0	13.6	26.4	88.1
FGLS Amemiya (1971)	12.2	8.1	12.6	51.1	41.3	10.0	38.1	99.3	14.5	11.5	14.1	39.7	35.7	18.1	32.3	90.5
Wallace & Hussain (1969)	12.2	8.1	12.6	51.2	41.3	10.0	38.1	99.3	14.5	11.4	13.9	39.6	35.7	18.1	32.3	90.5
Swamy & Arora (1972)	12.1	8.0	12.4	51.1	41.2	9.8	38.0	99.3	14.5	11.3	13.8	39.5	35.7	18.0	32.3	90.5
Nerlove (1971)	15.0	9.9	16.2	56.2	44.3	11.8	40.9	99.4	18.2	14.8	17.5	43.8	39.4	20.7	35.1	91.3
Henderson III (1953)	11.7	8.3	13.0	53.5	41.5	10.1	39.7	99.3	14.0	11.3	14.2	40.6	35.7	17.4	33.1	91.2
Minque (Rao (1970, 1972))	12.0	8.1	12.5	51.0	41.3	10.0	37.9	99.3	14.4	11.5	13.9	39.6	35.6	18.0	32.3	90.4
ML (Breusch (1987))	12.3	8.0	12.6	51.1	41.3	10.0	38.1	99.3	14.6	11.6	14.0	39.7	35.7	18.1	32.3	90.5
Within	10.7	6.2	12.1	44.5	40.6	9.2	37.7	99.3	14.2	10.7	14.1	35.1	35.6	18.8	32.2	89.3
TGLS SMA-RE	9.7	4.6	9.8	48.4	32.5	5.4	31.4	98.9	38.5	5.7	39.4	100.0	73.9	5.3	75.3	100.0
FGLS SAR-RE (Kapoor et al. (2007))	10.4	5.6	10.6	49.8	31.9	5.5	31.5	99.1	9.8	2.8	9.3	57.3	24.3	3.3	21.3	98.0
FGLS SMA-RE (Fingleton (2008*a*))	10.4	5.9	11.0	50.5	33.2	5.8	33.1	99.1	10.5	3.8	11.2	89.5	40.0	4.7	37.6	100.0

Table 18 – Mean of empirical, computed and true variances of $\hat{\beta}$ for N=50, $\sigma^2_\mu = 0.2$, $\sigma^2_\nu = 0.8$, asymmetric one-order contiguity matrix, 1000 replications. (1)

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True	Empirical	Computed	True
OLS	3.2525	2.4785	3.2246	0.6244	0.4687	0.5865	44.3316	23.5632	43.8720	12.6099	4.4467	11.7757
FGLS Amemiya (1971)	2.8707	2.3817	2.9728	0.6012	0.3984	0.5535	50.9360	22.3162	50.8589	13.5122	3.8136	12.4248
Wallace & Hussain (1969)	2.8695	2.3818	2.9717	0.6012	0.3985	0.5535	50.3481	22.3143	50.6755	13.5071	3.8147	12.4235
Swamy & Arora (1972)	2.8679	2.3968	2.9737	0.6017	0.3994	0.5536	50.3644	22.4475	50.9224	13.5063	3.8244	12.4357
Nerlove (1971)	2.9317	2.0647	3.0863	0.6036	0.3613	0.5559	56.7482	19.5482	56.7470	13.7496	3.4673	12.6106
Henderson III (1953)	2.8242	2.3604	2.9777	0.5981	0.3925	0.5586	48.9610	22.2173	50.3975	13.3541	3.8093	12.3727
Minque (Rao (1970, 1972))	2.8700	2.3981	2.9736	0.6013	0.3994	0.5536	50.9490	22.4654	50.9576	13.5235	3.8239	12.4333
ML (Breusch (1987))	2.8705	2.3797	2.9725	0.6012	0.3984	0.5535	50.5122	22.2332	50.7232	13.5135	3.8132	12.4236
Within	3.3921	2.9156	3.6573	0.6171	0.4093	0.5698	69.6845	27.7947	70.2851	14.2762	3.9348	13.0492
FGLS SAR-RE (Kapoor et al. (2007))	2.7010	2.5546	2.6310	0.5496	0.5045	0.5110	2.7785	2.7232	2.5106	0.9936	0.9855	0.9602
FGLS SMA-RE (Fingleton (2008*a*))	2.7022	2.5604	2.7021	0.5508	0.5123	0.5156	4.4004	3.1246	4.0445	1.4442	1.0113	1.3252

	True model SMA-RE											
OLS	3.0687	2.3627	3.0447	0.5773	0.4469	0.5438	5.6865	3.6898	5.8145	1.2680	0.6993	1.2084
FGLS Amemiya (1971)	2.6850	2.2709	2.7835	0.5503	0.3798	0.5082	5.3254	3.5312	5.6428	1.2809	0.5964	1.1885
Wallace & Hussain (1969)	2.6850	2.2710	2.7829	0.5503	0.3800	0.5082	5.3017	3.5314	5.6363	1.2807	0.5967	1.1885
Swamy & Arora (1972)	2.6834	2.2854	2.7843	0.5509	0.3808	0.5083	5.2969	3.5509	5.6443	1.2812	0.5980	1.1889
Nerlove (1971)	2.7335	1.9686	2.8816	0.5521	0.3445	0.5101	5.5645	3.0716	5.9352	1.2915	0.5414	1.1972
Henderson III (1953)	2.6456	2.2496	2.7901	0.5491	0.3738	0.5149	5.1867	3.5046	5.6372	1.2625	0.5907	1.1870
Minque (Rao (1970, 1972))	2.6843	2.2865	2.7841	0.5504	0.3808	0.5083	5.3238	3.5552	5.6462	1.2814	0.5980	1.1889
ML (Breusch (1987))	2.6856	2.2690	2.7835	0.5503	0.3798	0.5082	5.3126	3.5266	5.6413	1.2809	0.5964	1.1885
Within	3.1558	2.7799	3.4092	0.5639	0.3902	0.5225	6.5784	4.3464	7.1027	1.3287	0.6136	1.2308
FGLS SAR-RE (Kapoor et al. (2007))	2.5622	2.4433	2.5437	0.5097	0.4820	0.4772	1.7223	2.0191	1.7173	0.5422	0.6475	0.5362
FGLS SMA-RE (Fingleton (2008*a*))	2.5390	2.3900	2.4901	0.5090	0.4636	0.4741	0.4937	1.0580	0.3583	0.2122	0.3111	0.1428

(1) × 10^{-2}.

Table 19 – Hausman test, percentage of rejections of H_0 at 5% significance level for N=50, 1000 replications.

	True model SAR-RE											
	ρ=λ=0.2						ρ=λ=0.8					
	T=5			*T*=10			*T*=5			*T*=10		
$\left(\sigma_\mu^2,\sigma_v^2\right)$ =	(0.8,0.2)	(0.5,0.5)	(0.2,0.8)	(0.8,0.2)	(0.5,0.5)	(0.2,0.8)	(0.8,0.2)	(0.5,0.5)	(0.2,0.8)	(0.8,0.2)	(0.5,0.5)	(0.2,0.8)
W(1,1)	5.3	6.1	5.6	4.9	5.0	6.1	6.0	7.8	9.9	7.8	7.8	8.8
W(5,5)	4.5	5.0	4.8	3.4	3.0	3.6	4.3	8.2	14.0	2.4	4.2	7.0
Asymmetric one-order contiguity matrix	6.8	7.0	6.8	4.5	4.3	4.7	4.0	5.1	7.3	1.5	1.5	2.5

	True model SMA-RE											
W(1,1)	5.3	6.3	5.6	5.0	5.0	6.1	6.5	6.7	6.1	6.8	7.2	7.7
W(5,5)	4.5	5.0	4.6	3.3	3.0	3.6	4.9	5.8	6.9	3.1	2.9	3.8
Asymmetric one-order contiguity matrix	6.9	7.2	7.1	4.7	4.5	4.9	8.5	9.4	8.6	6.1	6.4	7.2