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# Fusing Dependent Decisions for Hypothesis Testing with Heterogeneous Sensors

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## Abstract

In this paper, we consider a binary decentralized detection problem where the local sensor observations are quantized before their transmission to the fusion center. Sensor observations, and hence their quantized versions, may be heterogeneous as well as statistically dependent. A composite binary hypothesis testing problem is formulated, and a copula-based generalized likelihood ratio test (GLRT) based fusion rule is derived given that the local sensors are uniform multi-level quantizers. An alternative computationally efficient fusion rule is also designed which involves injecting a deliberate random disturbance to the local sensor decisions before fusion. Although the introduction of external noise causes a reduction in the received signal to noise ratio, it is shown that the proposed approach can result in a detection performance comparable to the GLRT detector without external noise, especially when the number of quantization levels is large.

## Index Terms

Multimodal signals, Statistical dependence, Copula theory, Hypothesis testing, Multisensor fusion, Quantization, Stochastic Resonance

## I. INTRODUCTION

Decentralized detection has long been an active and important research area [1]. One of the earliest applications to have motivated research in decentralized detection was distributed radar where it was

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essential to compress data at each (local) radar before relaying it to a fusion center (see [2] and references therein). More recently, decentralized detection has found applications in sensor networks. Recent technological advances have enabled the deployment of multiple low cost sensors to monitor a region of interest (ROI) for reliable detection, estimation and/or tracking of events. Each sensor is usually programmed to send only quantized versions of its measurements to a remotely located fusion center due to communication bandwidth and power constraints. The fusion center then consolidates receptions from all the sensors to make a global inference.

The design of a distributed detection system involves designing the local and fusion center decision rules under different criteria and constraints [1], [3], [4]. Under the assumption that local observations are conditionally independent given the hypothesis, and the fusion center receives the local sensor outputs without any loss, the optimality of the LRT for local sensor decision rules under the Bayesian criterion and the Neyman-Pearson criterion have been proved in [5] and [6]. However, the problem becomes highly complex when conditional independence assumption does not hold. The LR based decision rules at the local sensors may no longer result in an optimal system design [7], [8]. It has also been shown that distributed detection with dependent observations is an NP-complete problem; it cannot be solved using a polynomial time algorithm [9], [10]. The problem is usually simplified by constraining the local sensors to be binary quantizers. In [11], Drakopoulos and Lee have derived a rule for fusing correlated decisions under the assumption that the correlation coefficients between the sensor decisions are known and local sensor thresholds generating the correlated decisions are given. Kam et. al [12], employed another approach, namely, the Bahadur-Lazarsfeld series expansion of probability density functions (PDFs) to derive the optimum fusion rule for correlated local decisions. It was, however, assumed that the joint distribution of sensor observations was completely known. In this paper, we consider the scenario in which the dependence structure and hence the joint distribution between the sensor observations may be unknown. Such problems are typical of sensor networks that consist of heterogeneous sensors, i.e., sensors with disparate sensing modalities. For example, it is not immediately clear how one could model the complex relationship between observations of an audio and a video sensor monitoring a common ROI.

The problem of binary hypothesis testing with heterogeneous sensors has been considered in our earlier work [13], where we developed a parametric framework using the statistical theory of copulas. While designing the copula based fusion rule in [13], it was, however, assumed that the fusion center has access to the exact real-valued (analog) version of the locally processed data. In many cases such as in WSNs discussed above, there could be limitations on both the transmission power and the bandwidth available

for sensor-to-fusion center communication. It may, thus, be necessary to quantize the information at each sensor before its transmission to the fusion center. Our goal, in this paper, is to design a decision fusion rule based on copula theory, for combining quantized heterogeneous information, thereby significantly extending the formulation and results in [13]. We note here that the application of copula theory for fusing correlated decisions has been recently considered in [14]. The local sensors were binary quantizers, and it was assumed that the *true* copula function generating the data under each hypothesis is known *a priori* but for some parameters. In this work, we relax this assumption and consider the case when the copula function used to model the dependence structure between the variables may be “misspecified”, i.e., the chosen copula density may not accurately characterize inter-sensor dependence. The formulation is also extended to include multi-bit quantizers at the local sensors.

As will be evident later, one of the main limitations of the copula-based generalized likelihood ratio test (GLRT) for fusing discrete decisions is the considerable increase in computational complexity as the number of sensors and/or quantization levels increases. For example, a system with  $N$  sensors each with an  $M$ -level quantizer requires the computation of  $N$ -dimensional integrals, and optimization over an  $\frac{N(N-1)}{2}$ -dimensional space for maximum likelihood (ML) estimation of parameters associated with elliptical copulas such as the Gaussian and  $t$ -copula functions [15]. This issue of computational complexity is also addressed in this paper, and an alternative computationally efficient fusion rule is proposed that involves deliberately adding external noise to the quantized observations before fusion. We call this noise, the low pass filter (LPF) noise for reasons that will become clear later. The approach completely eliminates the necessity to compute the multidimensional integrals and greatly simplifies the fusion rule. However, the reduced complexity comes at the cost of the decreased signal to noise ratio (SNR) at the fusion center. Thus, the key to the success of this approach is a “good” design of the LPF noise, i.e., we need to derive the form of the LPF-noise PDF that would introduce minimal distortion. We present an approach based on Widrow’s additive quantization noise model. Our approach is similar to Gustafsson and Karlsson [16] who have considered the problem of estimating a deterministic parameter in noise using quantized observations. However, unlike [16], where the authors propose to inject the artificial dither noise before quantization, we add the deliberate disturbance post quantization, and at the fusion center. As we show later, the addition of noise after quantization is equivalent to low pass filtering in the characteristic function (CF) domain, unlike dithering which amounts to anti-alias filtering [16], [17].

The paper is organized as follows. The problem is formulated in Section II, and a copula based rule for fusing dependent local sensor decisions is derived in Sections III and IV. Section V addresses the issue of computational complexity associated with the fusion rule derived in the previous sections. An

alternative computationally efficient fusion rule based on Widrow's statistical theory of quantization is proposed here. An illustrative example is presented in Section VI to elucidate the theory presented in the previous sections. In Section VII, we identify a class of problems for which the detector threshold can be determined to achieve a desired false alarm rate. We summarize our paper and provide some concluding remarks in Section VIII.

## II. PROBLEM FORMULATION

The problem of signal detection is formulated as a binary hypothesis test where the hypothesis  $H_1$  indicates the presence of a signal, while  $H_0$  indicates its absence. A total of  $N$  sensors are used to collect observations,  $Z_n$ , for  $n = 1, \dots, N$ . Observations at each sensor  $n$  are independent and identically distributed (i.i.d.) over time with PDFs  $f_n(z_n; \psi_n)$  and  $g_n(z_n; \lambda_n)$  under  $H_1$  and  $H_0$  respectively, where  $\psi_n$  and  $\lambda_n$  are distributional parameters. We assume that these marginal PDFs are well-specified under both hypotheses (see Definition 1 below).

**Definition 1.** *Well-specified model (White 1994 [18])*

*A parametric model  $\{f(x; \Theta)\}$  is well-specified for a random variable  $X$  if there exists a unique  $\theta' \in \Theta$  such that  $f(x; \theta') \in \{f(x; \Theta)\}$  corresponds to the true density of  $X$ . Otherwise,  $\{f(x; \Theta)\}$  is said to be misspecified for  $X$ .*

However, no knowledge is assumed regarding the dependence structure between the heterogeneous data streams. We approximate this dependence using copula functions (see Section II-A below). Sensor observations are further passed through uniform multi-level quantizers (see Fig. 1) before their transmission to a remotely located fusion center. The input-output transfer function of the quantizer at each sensor is shown in Fig. 2. Thus, the quantizer output, during any time interval  $1 \leq l \leq L$ , can be given as

$$u_{nl} = \mathcal{Q}_m(z_{nl}) = \begin{cases} -m_n q_n - \frac{q_n}{2}, & z_{nl} < -m_n q_n, \\ q_n \lfloor \frac{z_{nl}}{q_n} \rfloor + \frac{q_n}{2}, & -m_n q_n < z_{nl} \leq m_n q_n, \\ m_n q_n + \frac{q_n}{2}, & z_{nl} \geq m_n q_n, \end{cases} \quad (1)$$

where,  $q_n$  and  $2(m_n + 1)$  correspond to the quantizer step size and the number of quantization levels respectively, at sensor  $n$ . Further,  $\lfloor x \rfloor$  stands for the floor operation that denotes an integer smaller than or equal to  $x$ . The quantized value at sensor  $n$  can be represented with an integer  $i_n = -m_n - 1, -m_n, \dots, m_n + 1$ . In this paper, we do not consider quantizer saturation errors. That is, we assume

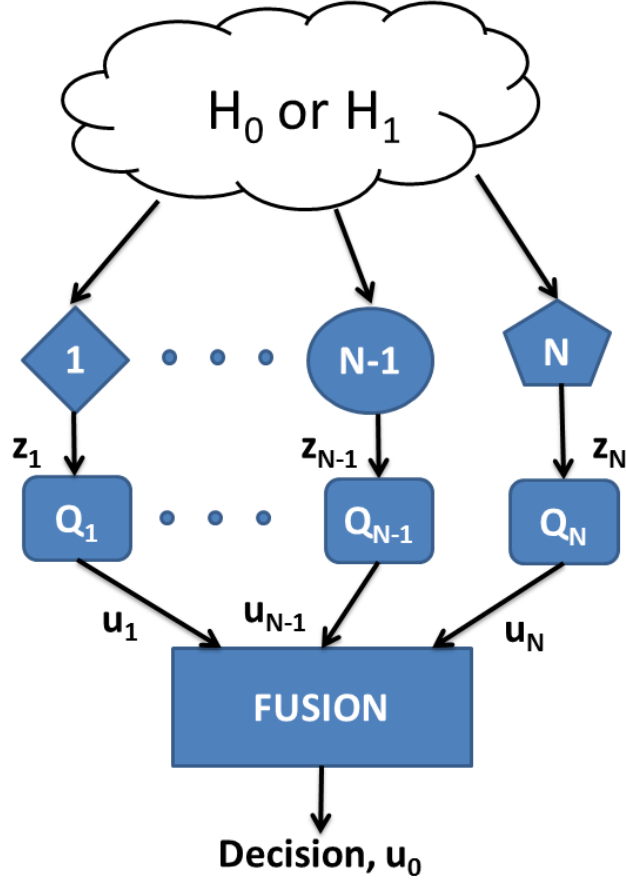


Fig. 1. Distributed heterogeneous sensor network: A parallel architecture.

that the dynamic range of the (analog) signal input to the quantizer is well within the lower and upper limits of the quantizer.

Observations thus received at the fusion center are used to estimate the unknown model parameters, and a GLRT based fusion rule is employed for global decision making. In addition to estimating the model parameters, the selection of copula densities is also embedded in the GLRT formulation and is thus performed in real-time. Sensor observations and hence their quantized versions are assumed to be i.i.d. in time, and, our focus, in this paper, is on designing a fusion rule that could exploit the spatial dependence between sensor decisions for improved detection performance.

Next, we briefly discuss the use of copula theory to approximate joint density functions.

#### A. Joint PDF approximation using copula theory

We begin with the definition of a copula function.

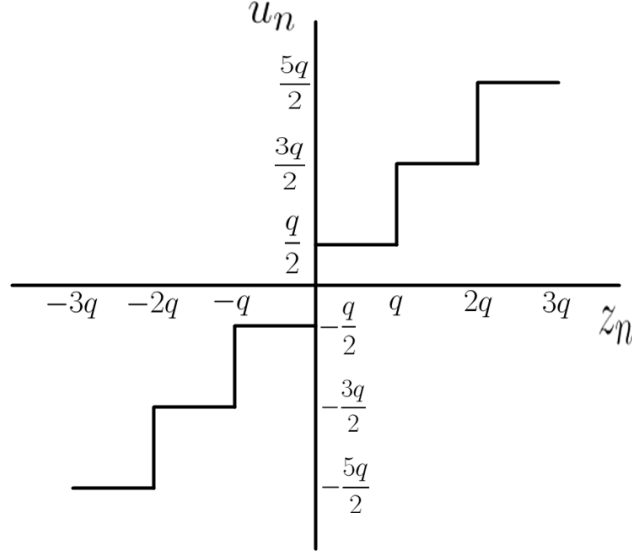


Fig. 2. Input-output transfer function of a uniform scalar quantizer

**Definition 2.** A function  $C : [0, 1]^N \rightarrow [0, 1]$  is an  $N$ -dimensional copula if  $C$  is a joint cumulative distribution function (CDF) of an  $N$ -dimensional random vector on the unit cube  $[0, 1]^N$  with uniform marginals [15], [19], [20].

The following theorem by Sklar is central to the statistical theory of copulas.

**Theorem 1.** (Sklar's Theorem)

Let  $F$  be an  $N$ -dimensional CDF with continuous marginal CDFs  $F_1, F_2, \dots, F_N$ . Then there exists a unique copula  $C$  such that for all  $z_1, z_2, \dots, z_N$  in  $[-\infty, \infty]$ ,

$$F(z_1, z_2, \dots, z_N) = C(F_1(z_1), F_2(z_2), \dots, F_N(z_N)). \quad (2)$$

Note that the copula function  $C(u_1, u_2, \dots, u_N)$  is itself a CDF with uniform marginals as  $U_n = F_n(Z_n) \sim \mathcal{U}(0, 1)$  (by probability integral transform). The joint density can now be obtained by taking the  $N^{\text{th}}$  order derivative of (2),

$$\begin{aligned} f(\mathbf{z}) &= \frac{\partial^N}{\partial z_1 \dots \partial z_N} C(F_1(z_1), \dots, F_N(z_N)) \\ &= \underbrace{\left( \prod_{n=1}^N f_n(z_n) \right)}_{f_p(\mathbf{z})} c(F_1(z_1), \dots, F_N(z_N)) \end{aligned} \quad (3)$$

Thus, in Eq. (3), the copula density,  $c(\cdot)$ , *weights* the product density,  $f_p(\mathbf{z})$ , *appropriately* to incorporate dependence between the random variables  $\{Z_n\}_{n=1}^N$ .

Theorem 1 also admits the following converse, especially useful in practice when the true distribution  $F$  (and hence the true copula  $C$ ) is unknown. It allows one to construct a statistical model by considering the univariate behavior of the underlying marginals and subsequently fitting the desired dependence structure specified by some copula, say,  $K$ .

**Theorem 2.** *If  $F_1, F_2, \dots, F_N$  are univariate marginal CDFs and if  $K$  is an  $N$  dimensional copula, then the function  $\Xi : \mathbb{R}^N \rightarrow [0, 1]$ ,*

$$\Xi(z_1, \dots, z_N) = K(F_1(z_1), \dots, F_N(z_N)), \quad (4)$$

*is a valid  $N$ -variate CDF with marginals  $F_1, F_2, \dots, F_N$ .*

A copula based parametric model can be derived by taking the  $N^{th}$  order derivative of (4) to obtain

$$\hat{f}(\mathbf{z}) = f_p(\mathbf{z})k(F_1(z_1), \dots, F_N(z_N)) \quad (5)$$

$$= f_k(\mathbf{z}). \quad (6)$$

Some of the commonly used copula functions include the Gaussian, Student's t and those belonging to the Archimedean family [13].

It is evident that model mismatch errors are introduced when  $k(\cdot) \neq c(\cdot)$ ; *i.e.*, the selected copula does not represent the true dependence structure. This leads to suboptimal performance. An important question then is, *how does one choose  $k(\cdot)$  from a finite set (say  $\mathcal{A}_k$ ) of copula densities?* As discussed earlier, the selection of copula densities is embedded in the GLRT formulation in this paper (see Eq. (14)). It may also be required to estimate the parameters,  $\psi_d$ , of the chosen copula function from the acquired data. These parameters control the shape of the copula function and can be estimated by exploiting their relations to other nonparametric measures of association such as Kendall's  $\tau$  or Spearman's  $\rho$  [15]. In this paper, we use a maximum likelihood (ML) based approach known as the method of inference functions for margins (IFM) [21] to estimate the copula dependence parameters.

### III. (MISSPECIFIED) GLRT BASED FUSION OF SOFT DECISIONS

In the following, we consider a two-sensor network for simplicity.

Under hypothesis  $H_1$ , the probability that the data  $R_l = (u_{1l}, u_{2l})$  received at the fusion center at the



time instant  $l$  takes a specific value  $(i_1 q_1 + \frac{q_1}{2}, i_2 q_2 + \frac{q_2}{2})$  is

$$P_{i_1, i_2} = \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} f(z_1, z_2) dz_2 dz_1, \quad (7)$$

where  $f(z_1, z_2)$  is the true but unknown joint PDF of unquantized sensor observations under  $H_1$ . Now, approximating the dependence structure using a copula density  $k_1 \{F_1(z_1; \psi_1), F_2(z_2; \psi_2); \psi_d\}$  contained in some set  $\mathcal{A}_k$  of valid copula densities, we have

$$\begin{aligned} \hat{P}_{i_1, i_2}(\psi) &= \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} \hat{f}(z_1, z_2; \psi) dz_2 dz_1 \\ &= \int_{i_1 q_1}^{(i_1+1)q_1} \int_{i_2 q_2}^{(i_2+1)q_2} f_1(z_1; \psi_1) f_2(z_2; \psi_2) k_1(F_1(z_1; \psi_1), F_2(z_2; \psi_2); \psi_d) dz_2 dz_1 \\ &= K_1 \{F_1((i_1+1)q_1; \psi_1), F_2((i_2+1)q_2; \psi_2); \psi_d\} - \\ &\quad K_1 \{F_1(i_1 q_1; \psi_1), F_2(i_2 q_2; \psi_2); \psi_d\}, \end{aligned} \quad (8)$$

where  $\psi = (\psi_1, \psi_2, \psi_d)^T \in \Psi \subset \mathbb{R}^a$  is the  $a$ -dimensional unknown parameter vector that will be estimated from the received data,  $K_1\{\cdot\}$  is the copula CDF and  $F_n(\cdot)$  is the CDF of  $Z_n$  under hypothesis  $H_1$ . The dependence of  $\hat{P}_{i_1, i_2}(\psi)$  on  $K_1\{\cdot\}$  is not made explicit for notational convenience.

The likelihood function of the data  $R_l$  under hypothesis  $H_1$  can now be written as

$$\hat{P}(R_l; \psi, H_1) = \prod_{i_1} \prod_{i_2} [\hat{P}_{i_1, i_2}(\psi)]^{\delta(u_{1l} - i_1 q_1 - \frac{q_1}{2}, u_{2l} - i_2 q_2 - \frac{q_2}{2})}, \quad (9)$$

where  $\delta(\cdot)$  is the two-dimensional Kronecker-delta function defined as

$$\delta(x, y) = \begin{cases} 1, & x = y = 0 \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

The log-likelihood function of  $R_l$  is, therefore,

$$\log \hat{P}(R_l; \psi, H_1) = \sum_{i_1} \sum_{i_2} \delta\left(u_{1l} - i_1 q_1 - \frac{q_1}{2}, u_{2l} - i_2 q_2 - \frac{q_2}{2}\right) \log \hat{P}_{i_1, i_2}(\psi). \quad (11)$$

Similarly, the likelihood function of  $R_l$  under  $H_0$ , when a copula density  $k_0(G_1(z_1; \lambda_1), G_2(z_2; \lambda_2); \lambda_d) \in \mathcal{A}_k$  is used to approximate the joint distribution under  $H_0$ , can be derived as,

$$\log \hat{P}(R_l; \lambda, H_0) = \sum_{i_1} \sum_{i_2} \delta\left(u_{1l} - i_1 q_1 - \frac{q_1}{2}, u_{2l} - i_2 q_2 - \frac{q_2}{2}\right) \log \hat{Q}_{i_1, i_2}(\lambda), \quad (12)$$

where

$$\begin{aligned} \hat{Q}_{i_1, i_2}(\lambda) &= K_0 \{G_1((i_1 + 1)q_1; \lambda_1), G_2((i_2 + 1)q_2; \lambda_2); \lambda_d\} - \\ &K_0 \{G_1(i_1 q_1; \lambda_1), G_2(i_2 q_2; \lambda_2); \lambda_d\}. \end{aligned} \quad (13)$$

$\lambda = (\lambda_1, \lambda_2, \lambda_d)^T \in \Lambda \subset \mathbb{R}^b$ , is the  $b$ -dimensional unknown parameter vector,  $K_0\{\cdot\}$  is the copula CDF and  $G_n(\cdot)$  is the CDF of  $Z_n$  under hypothesis  $H_0$ .

With Eqs. (11) and (12), it is straightforward to derive the test to be employed at the fusion center,

$$\mathcal{T}_k(\mathbf{u}_1, \mathbf{u}_2) \triangleq \log \frac{\max_{k_1(\cdot) \in \mathcal{A}_k, \Psi} \prod_l \hat{P}(R_l; \psi, H_1)}{\max_{k_0(\cdot) \in \mathcal{A}_k, \Lambda} \prod_l \hat{P}(R_l; \lambda, H_0)} \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (14)$$

which results in

$$\sum_l \sum_{i_1} \sum_{i_2} \delta \left( u_{1l} - i_1 q_1 - \frac{q_1}{2}, u_{2l} - i_2 q_2 - \frac{q_2}{2} \right) \log \frac{\hat{P}_{i_1, i_2}^*(\hat{\psi})}{\hat{Q}_{i_1, i_2}^*(\hat{\lambda})} \underset{H_0}{\overset{H_1}{\geq}} \eta, \quad (15)$$

where  $\hat{P}_{i_1, i_2}^*(\hat{\psi})$  and  $\hat{Q}_{i_1, i_2}^*(\hat{\lambda})$  correspond to the copula functions  $K_1^*(\cdot)$  and  $K_0^*(\cdot)$  respectively, which maximize the terms on the left hand side of Eq. (14), and  $l$  is the time index. Thus, the maximization in Eq. (14) is over the copula densities belonging to a set  $\mathcal{A}_k$  of valid copula densities as well as the unknown marginal and copula dependence parameters. Unlike the classical composite hypothesis testing formulation which would have required the knowledge of the true copula densities with possibly unknown parameters, we allow for the case when the set  $\mathcal{A}_k$  may not be inclusive of the true models  $c_1(\cdot)$  and/or  $c_0(\cdot)$ . Thus, the copula functions,  $K_1^*(\cdot)$  and  $K_0^*(\cdot)$ , chosen after maximization may still be misspecified. We, therefore, call the test a misspecified GLRT (mGLRT).

#### IV. EXTENSION TO $N (> 2)$ SENSORS

The copula based fusion rule designed for a two sensor network in the previous section can be easily extended to larger sensor networks. Similar to Eq. (15), the fusion rule for  $N$  sensors can be derived as

$$\sum_l \sum_{i_1} \cdots \sum_{i_N} \delta \left( u_{1l} - i_1 q_1 - \frac{q_1}{2}, \dots, u_{Nl} - i_N q_N - \frac{q_N}{2} \right) \log \frac{\hat{P}_{i_1, \dots, i_N}^*(\hat{\psi})}{\hat{Q}_{i_1, \dots, i_N}^*(\hat{\lambda})} \underset{H_0}{\overset{H_1}{\geq}} \eta, \quad (16)$$

where,

$$\hat{P}_{i_1, \dots, i_N}^*(\psi) = \int_{i_1 q_1}^{(i_1+1)q_1} \cdots \int_{i_N q_N}^{(i_N+1)q_N} \underbrace{\hat{f}(z_1, \dots, z_N; \psi)}_{\text{copula-based estimate}} dz_N \cdots dz_1 \quad (17)$$

$$= K_1^* \{F_1((i_1+1)q_1; \psi_1), \dots, F_N((i_N+1)q_N; \psi_N); \psi_d\} - \\ K_1^* \{F_1(i_1 q_1; \psi_1), \dots, F_N(i_N q_N; \psi_N); \psi_d\}, \quad (18)$$

and,

$$\hat{Q}_{i_1, \dots, i_N}^*(\lambda) = \int_{i_1 q_1}^{(i_1+1)q_1} \cdots \int_{i_N q_N}^{(i_N+1)q_N} \underbrace{\hat{g}(z_1, \dots, z_N; \lambda)}_{\text{copula-based estimate}} dz_N \cdots dz_1 \quad (19)$$

$$= K_0^* \{G_1((i_1+1)q_1; \lambda_1), \dots, G_N((i_N+1)q_N; \lambda_N); \lambda_d\} - \\ K_0^* \{G_1(i_1 q_1; \lambda_1), \dots, G_N(i_N q_N; \lambda_N); \lambda_d\}. \quad (20)$$

Thus, the fusion rule involves evaluating  $N$ -dimensional integrals in real-time where  $N$  is the number of sensors, i.e., the computational complexity is exponential in the number of sensors. This is in addition to the optimization over multiple dimensions to obtain ML estimates of the unknown parameters. Application of mGLRT is, therefore, highly prohibitive as the number of sensors increases due to the increased computational complexity. We derive an alternative computationally efficient test in the next section.

## V. A COMPUTATIONALLY EFFICIENT FUSION RULE

In this section, we propose a computationally efficient approach that involves deliberately injecting noise to the quantized observations before fusion (see Fig. 3). While noise is generally perceived as an unwanted signal, interestingly, several studies have shown that the addition of *controlled noise* could in fact be beneficial in some cases. For example, dithering, the process of adding noise to the signal before quantization has been shown to improve signal quality and mitigate the artifacts introduced due to quantization [22]–[24]. Also, it has been observed by many researchers that some types of signals get amplified by a nonlinear system when noise is added to the input signal (see [25] and references therein). This phenomenon is popularly known as stochastic resonance (SR). Here, we use this approach of adding external noise to reduce computational complexity rather than to enhance the signal to noise ratio (SNR). Our approach is based on Widrow's quantization theory which we review next.

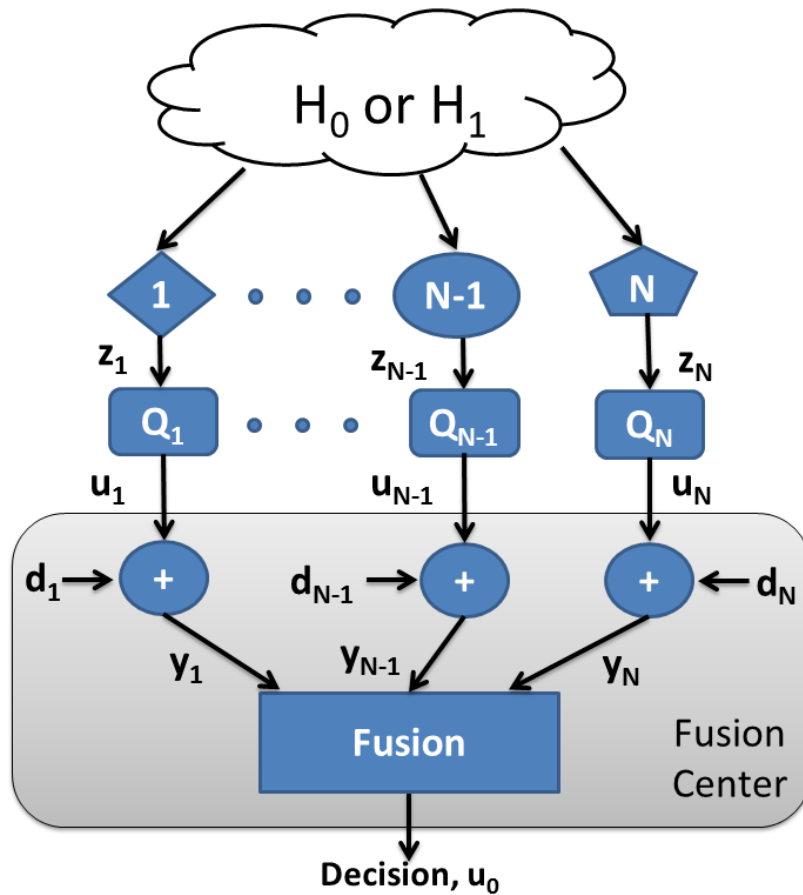


Fig. 3. A *controlled* noise  $d_n$  is added at the output of each sensor  $n$ . The approach greatly simplifies the fusion rule by avoiding the need to compute multidimensional integrals.

#### A. Widrow's Statistical Theory of Quantization: A Review

The statistical theory of quantization was developed by Widrow and co-workers [17], [26], [27]. They interpreted quantization of a random variable as sampling of its PDF, and showed that the PDF of the quantized signal is the convolution of the input signal PDF with a rectangular pulse function followed by conventional sampling. Thus, the PDF of the quantizer output,  $u_{nl}$ , at sensor  $n$  and at any time instant,  $l$ , can be given as

$$p_{U_n}(z) = \left( p_{W_n}(z) \star p_{Z_n}(z) \right) \cdot c_{\delta'_n}(z), \quad (21)$$

where  $p_{Z_n}(z)$  is the PDF of the random variable at the input  $Z_n$ ,  $p_{W_n}(z)$  denotes the rectangular pulse function,

$$p_{W_n}(z) = \begin{cases} \frac{1}{q_n}, & -q_n/2 < z < q_n/2 \\ 0, & \text{elsewhere,} \end{cases} \quad (22)$$

whose width depends on the quantizer step-size ( $q_n$ ) defined in Section II, and  $c_{\delta'_n}(z)$  denotes the impulse train,

$$c_{\delta'_n}(z) = \sum_{i_n \in \mathbb{Z}} q_n \delta' \left( z - i_n q_n - \frac{q_n}{2} \right). \quad (23)$$

The ‘ $\star$ ’ in Eq. (21) denotes the convolution operation, and  $\delta'(\cdot)$  in Eq. (23) is the Dirac-delta function. This process of convolution followed by conventional sampling is popularly known as ‘‘area sampling’’ [27]. Also, note that  $p_{W_n}(\cdot)$  is also the PDF of a uniform random variable,  $W_n \sim \mathcal{U}(-\frac{q_n}{2}, \frac{q_n}{2})$ . Thus, quantization introduces two ‘types’ of distortions or errors: (a) the additive uniform noise (AUN) error, and (b) the aliasing error due to sampling.

The two errors introduced due to quantization can be better visualized in the characteristic function (CF) domain. The CF of a random variable  $X$  is obtained by taking the Fourier transform of its PDF  $p_X(x)$ ,

$$\phi_X(v) = \int_{-\infty}^{\infty} p_X(x) e^{jvx} dx = E [e^{jvx}]. \quad (24)$$

Taking the Fourier transform of Eq. (21), one obtains the CF of output variable  $U_n$ ,

$$\begin{aligned} \phi_{U_n}(v) &= \sum_{i_n=-\infty}^{\infty} \phi_{Z_n} \left( v + i_n \frac{2\pi}{q_n} \right) \text{sinc} \left( \frac{q_n(v + i_n \frac{2\pi}{q_n})}{2} \right) e^{-ji_n \frac{2\pi}{q_n} \frac{q_n}{2}} \\ &= \sum_{i_n=-\infty}^{\infty} (-1)^{i_n} \phi_{Z_n} \left( v + i_n \frac{2\pi}{q_n} \right) \text{sinc} \left( \frac{q_n(v + i_n \frac{2\pi}{q_n})}{2} \right) \end{aligned} \quad (25)$$

where  $\phi_{Z_n}(v)$  is the CF of the input  $Z_n$  and  $\text{sinc}(v) = \frac{\sin(v)}{v}$ . Note that Eq. (25) is different from the one in [17, p. 65, Eq. 4.11] as we have considered a mid-rise quantizer here instead of a mid-tread quantizer used in [17]. Fig. 4 shows the operations in the ‘frequency’ domain. Note that the central lobe ( $i_n = 0$  in Eq. (25)),

$$\phi_{Z_n+W_n}(v) = \phi_{Z_n}(v) \cdot \text{sinc} \left( \frac{q_n v}{2} \right), \quad (26)$$

corresponds to the CF one would obtain by adding an independent and uniformly distributed random variable  $W_n$  to the input  $Z_n$ . It is clear from Fig. 4 that, in addition to the error introduced due to the addition of uniform noise, quantization also causes an aliasing error due to overlapping (and phase

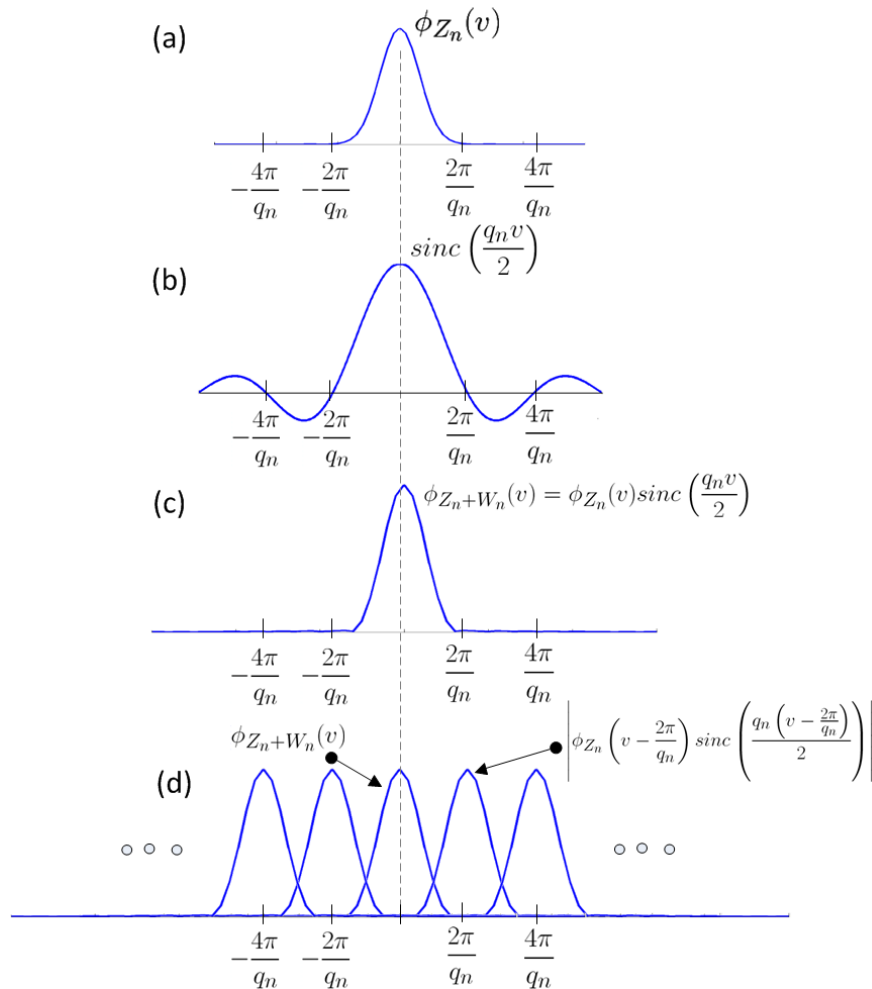


Fig. 4. Illustration of the quantization process in the CF domain: (a) CF of  $Z_n$ ; (b) CF of  $W_n$ , the sinc function; (c) CF of  $Z_n + W_n$ ; (d) Repetition of CF of  $Z_n + W_n$ ; the CF of the quantized variable is given by the summation of these repetitions after weighting each appropriately (see Eq. (25)).

shifted) lobes of  $\phi_{Z_n+W_n}(v)$ . However, if the input PDF is band-limited so that  $\phi_{Z_n}(v) = 0$  for  $|v| > \frac{\pi}{q_n}$ , then the ‘frequency’-shifted versions of  $\phi_{Z_n+W_n}(v)$  do not overlap and, in principle, the original PDF can be reconstructed from the knowledge of  $p_{U_n}(\cdot)$ . This is Widrow’s first quantization theorem:

**Theorem 3.** (*Widrow’s Quantization Theorem I*)

If the CF of the input variable  $Z_n$  is bandlimited so that

$$\phi_{Z_n}(v) = 0, \quad |v| > \frac{\pi}{q_n}, \quad (27)$$

then the different lobes in  $\phi_{U_n}(v)$  do not overlap, and in principle, the original PDF  $p_n(z_n)$  (before quantization) can be recovered from the PDF of  $U_n$ .

When  $\phi_{Z_n}(v) = 0$  for  $|v| > \frac{2\pi}{q_n}$  so that the derivatives of  $\phi_{U_n}(v)$  computed at  $v = 0$  are not affected due to the overlap of CF lobes, then the moments of  $Z_n$  can be recovered from those of  $U_n$ . This is Widrow's second theorem:

**Theorem 4.** (*Widrow's Quantization Theorem II*)

If the CF of  $Z_n$  is bandlimited so that

$$\phi_{Z_n}(v) = 0, \quad |v| > \frac{2\pi}{q_n}, \quad (28)$$

then the moments of  $Z_n$  can be derived from the moments of  $U_n$ .

In the following, we assume that Theorem 3 (and hence Theorem 4) holds, and derive a rule to fuse multi-level decisions at the fusion center. We also note here that Widrow's additive model for quantization noise, and, hence the fusion rule derived in the next section, is better suited for high resolution quantization (See [28] and references therein).

*B. Derivation of a Computationally Efficient Fusion Rule*

As discussed previously, the high complexity in computing the mGLRT statistic for quantized observations stems from the need for computing multi-dimensional integrals. We propose to simplify the fusion process by adding *controlled noise* to the observations received at the fusion center. The system is shown in Fig. 3. An externally generated noise,  $d_n$ , with PDF  $p_{D_n}(d_n)$  is added to the quantized observations from each sensor  $n$  before fusing them for making a global decision. Denote the new observations by  $y_n = u_n + d_n$  whose CF is given by

$$\phi_{Y_n}(v) = \phi_{U_n}(v) \cdot \phi_{D_n}(v). \quad (29)$$

One can choose the noise source with a bandlimited CF to filter out the repeated and phase-shifted CF lobes in  $\phi_{U_n}(v)$ . This is analogous to low pass filtering in signal processing. We, therefore, call the noise  $D_n$ , the LPF-noise. As shown in Fig. 4 (d), an ideal noise source would be one with a rectangular CF in the pass-band,  $-\frac{\pi}{q_n} \leq v \leq \frac{\pi}{q_n}$ , (also see Fig. 5). However, a rectangular function in the CF domain corresponds to a PDF whose shape corresponds to a sinc function, an invalid PDF. Note that this is similar to the non-realizability of an ideal low pass filter in signal processing. One, therefore, needs to carefully design  $D_n$  so that it causes minimal distortion while transforming the discrete-valued random variable,  $U_n$ , into a continuous variable,  $Y_n$ . As long as the input variable  $Z_n$  satisfies Widrow's first

quantization theorem (Theorem 3) under both  $H_1$  and  $H_0$ , we have,

$$Y_n = Z_n + W_n + D_n. \quad (30)$$

Thus, under hypothesis  $H_1$ , the PDF of data,  $y_{nl}$ , at time instant  $l$  is

$$p_{Y_n}(y_{nl}; \psi_n, H_1) = p_{Z_n}(y_{nl}; H_1) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}) \quad (31)$$

$$= f_n(y_{nl}; \psi_n) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}). \quad (32)$$

Using a copula density (say)  $k_1(\cdot; \psi_d) \in \mathcal{A}_k$  to estimate the dependence structure between sensor observations, the joint PDF of the data  $\mathbf{y}_l = (y_{1l}, y_{2l}, \dots, y_{Nl})$  can now be approximated as

$$\hat{p}_{\mathbf{Y}}(y_{1l}, \dots, y_{Nl}; \psi, H_1) = \left\{ \prod_{n=1}^N p_{Y_n}(y_{nl}; \psi_n, H_1) \right\} k_1 \{F_{Y_1}(y_{1l}; \psi_1), \dots, F_{Y_N}(y_{Nl}; \psi_N); \psi_d\} \quad (33)$$

where

$$F_{Y_n}(y) = \int_{-\infty}^y p_{Y_n}(t; \psi_n, H_1) dt \quad (34)$$

denotes the CDF of  $Y_n$  under  $H_1$ .

Similarly, the joint PDF of the data under  $H_0$  can be approximated as

$$\hat{p}_{\mathbf{Y}}(y_{1l}, \dots, y_{Nl}; \lambda, H_0) = \left\{ \prod_{n=1}^N p_{Y_n}(y_{nl}; \lambda_n, H_0) \right\} k_0 \{G_{Y_1}(y_{1l}; \lambda_1), \dots, G_{Y_N}(y_{Nl}; \lambda_N); \lambda_d\}, \quad (35)$$

where

$$p_{Y_n}(y_{nl}; \lambda_n, H_0) = p_{Z_n}(y_{nl}; H_0) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}) \quad (36)$$

$$= g_n(y_{nl}; \lambda_n) \star p_{W_n}(y_{nl}) \star p_{D_n}(y_{nl}), \quad (37)$$

$k_0(\cdot; \lambda_d) \in \mathcal{A}_k$  is the copula density used to estimate the dependence structure of sensor observations under  $H_0$ , and

$$G_{Y_n}(y) = \int_{-\infty}^y p_{Y_n}(t; \lambda_n, H_0) dt \quad (38)$$

denotes the CDF of  $Y_n$  when the underlying hypothesis is  $H_0$ .



With Eqs. (33) and (35), it is now straightforward to derive the (misspecified) GLRT

$$\mathcal{T}'_k(\mathbf{y}) = \log \frac{\max_{k_1(\cdot) \in \mathcal{A}_k, \Psi} \prod_{l=1}^L p_{\mathbf{Y}}(y_{1l}, \dots, y_{Nl}; \psi, H_1)}{\max_{k_0(\cdot) \in \mathcal{A}_k, \Lambda} \prod_{l=1}^L p_{\mathbf{Y}}(y_{1l}, \dots, y_{Nl}; \lambda, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (39)$$

The test derived above involves continuous-valued variables and thus does not involve computation of multidimensional integrals. This greatly simplifies the test. The reduced complexity is, however, at the expense of decreased signal-to-noise ratio due to the injection of noise  $d_n$  at the fusion center. The addition of external noise facilitates *filtering* of the baseband CF,  $\phi_{Z_n+W_n}(v)$ , from the received quantized observations  $\phi_{U_n}(v)$ . This noise should be designed so that it destroys as little information as possible while filtering the required signal.

Next, we present a numerical illustration.

## VI. AN ILLUSTRATIVE EXAMPLE

In this section, we consider the problem of detecting a random phenomenon using a network of two sensors. It is known that the observations received at the local quantizers each follow a Gaussian distribution. That is,

$$\begin{aligned} H_0 &: Z_1 \sim \mathcal{N}(0, \sqrt{10}), \quad Z_2 \sim \mathcal{N}(0, \sqrt{10}) \\ H_1 &: Z_1 \sim \mathcal{N}(\mu_1, \sqrt{10}), \quad Z_2 \sim \mathcal{N}(\mu_2, \sqrt{10}) \end{aligned} \quad (40)$$

where,

$$\mathcal{N}(\mu, \sigma) := \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}(z - \mu)^2\right)$$

is the usual univariate Gaussian density function. The means,  $\mu_1$  and  $\mu_2$ , under the hypothesis,  $H_1$ , are unknown, although a priori we know that they are greater than zero. Further, the observations may be statistically dependent; however, no knowledge about the dependence structure (and hence the joint distribution) is provided.

The observations,  $\{z_{1l}, z_{2l}\}_{l=1}^L$ , at the two local sensors are passed through uniform scalar quantizers before their transmission to the fusion center. Thus, the fusion center has access only to the quantized measurements,  $\mathbf{u} = \{u_{1l}, u_{2l}\}_{l=1}^L$ , to make a global decision in favor of one of the two hypotheses. Then, the GLRT based fusion rule for this problem is the same as the one derived in Eq.(15), with the joint

probabilities,

$$\begin{aligned} \hat{P}_{i_1, i_2}^* \left( \hat{\mu}_1, \hat{\mu}_2, \hat{\psi}_d \right) &= K_1^* \left\{ \Phi_{\hat{\mu}_1, \sqrt{10}} \left( (i_1 + 1)q_1 \right), \Phi_{\hat{\mu}_2, \sqrt{10}} \left( (i_2 + 1)q_2 \right); \hat{\psi}_d \right\} - \\ &K_1^* \left\{ \Phi_{\hat{\mu}_1, \sqrt{10}} \left( i_1 q_1 \right), \Phi_{\hat{\mu}_2, \sqrt{10}} \left( i_2 q_2 \right); \hat{\psi}_d \right\} \end{aligned} \quad (41)$$

and

$$\hat{Q}_{i_1, i_2}^* = \Phi_{0, \sqrt{10}} \left( (i_1 + 1)q_1 \right) \cdot \Phi_{0, \sqrt{10}} \left( (i_2 + 1)q_2 \right) - \Phi_{0, \sqrt{10}} \left( i_1 q_1 \right) \cdot \Phi_{0, \sqrt{10}} \left( i_2 q_2 \right), \quad (42)$$

where  $\Phi_{\mu, \sigma}(\cdot)$  denotes the Gaussian CDF with mean,  $\mu$ , and standard deviation,  $\sigma$ . The unknown marginal parameters,  $\mu_1$  and  $\mu_2$ , and the copula function  $K_1^*(\cdot; \hat{\psi}_d)$  in Eq. (41) are obtained by maximizing the generalized likelihood ratio as shown in Eq.(14). An alternative computationally efficient test was derived in Section V-B which involves injection of LPF-noise before fusion. We evaluate its performance using the example presented here.

Although Gaussian CFs are not perfectly bandlimited, a property necessary for using the LPF-noise based fusion rule, they are very close to being bandlimited for all practical purposes. Fig. 5 shows quantization and the effect of LPF-noise in the CF domain. The quantization step size,  $q_n$  is set to 0.3 of the input standard deviation ( $q_n = 0.3\sigma_n$ ). The CF of the input variable  $z_n$  is shown in Fig. 5(a). Addition of the quantization noise,  $w_1$ , is equivalent to multiplication of  $\phi_{Z_n}(v)$  (shown in Fig. 5(a)) with a sinc function,  $\text{sinc}\left(\frac{q_n v}{2}\right)$ . The resultant CF,  $\phi_{Z_n + W_n}(v)$ , is shown in Fig. 5(b). This CF is repeated and summed in Fig. 5(c) which represents the CF of the quantized signal,  $u_n$  (see Eq. (25)). The CF of the LPF-noise,  $D_n$ , a standard Gaussian distributed variable in this example, is shown in Fig. 5(d).<sup>1</sup> It is clear that multiplication of  $\phi_{U_n}(v)$  with  $\phi_{D_n}(v)$  which is equivalent to addition of  $d_n$  to  $z_n + w_n$  in the random variable domain, ‘filters’ the signal so that only the main lobe ( $v = 0$ ) of  $\phi_{U_n}(v)$  is retained (Fig. 5(e)). Since the LPF-noise is different from the ideal one with rectangular CF, the signal,  $z_n + w_n$ , undergoes some distortion while being ‘filtered’. However, this distortion is almost imperceptible as evident from Fig. 5(e).

<sup>1</sup>It is important to note that a standard Gaussian noise may not be the ‘best’ LPF-noise. It is used here to provide a simple illustrative example.

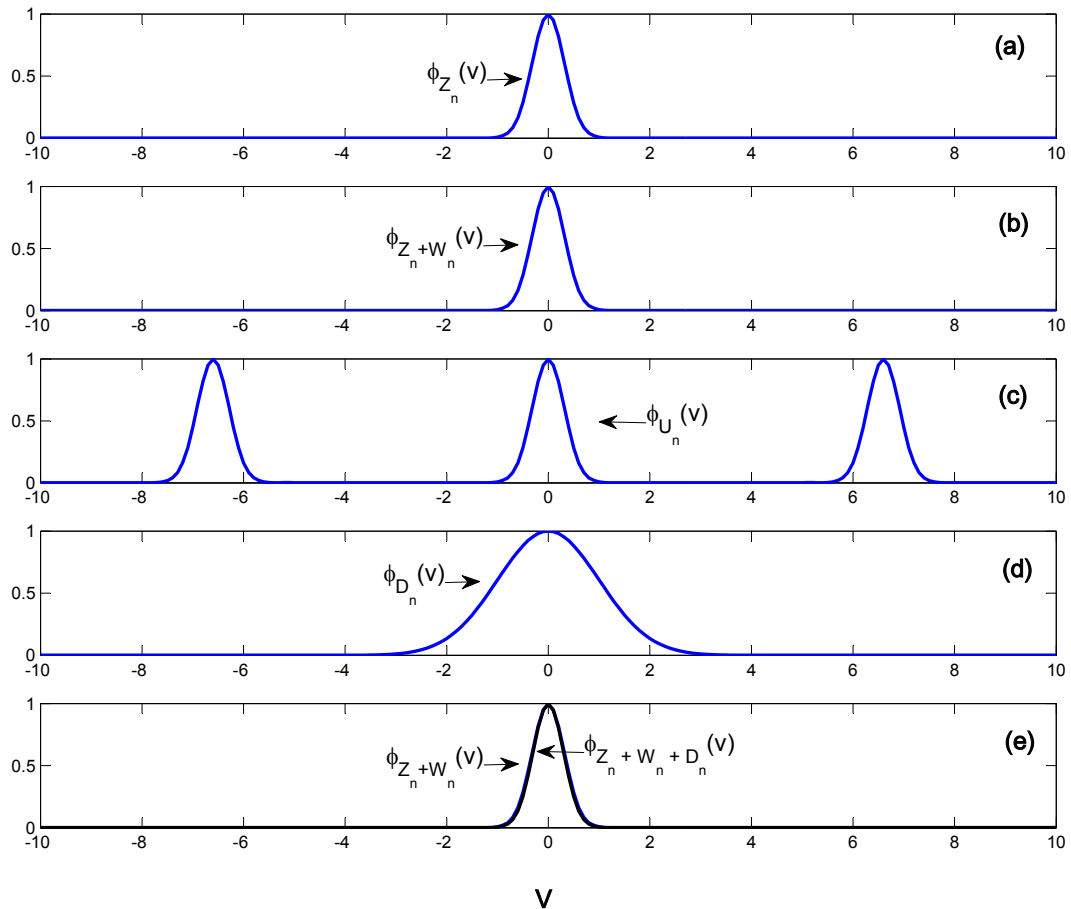


Fig. 5. ‘Filtering’ the quantized signal with LPF-noise. The quantization step size,  $q_n$ , is set to  $0.3\sigma_n$ : (a) CF of  $Z_n$ ; (b) CF of  $Z_n + W_n$ ; (c) CF of  $U_n$ ; (d) CF of the external LPF-noise,  $D_n$ ; (e) CF of  $Y_n = Z_n + W_n + D_n$ .

The PDF of the transformed variable,  $Y_n = Z_n + W_n + D_n$ , under the hypothesis  $H_1$  is given by

$$\begin{aligned}
 p_{Y_n}(y_n; \mu_n, H_1) &= p_{Z_n+D_n}(y_n) \star p_{W_n}(y_n) \\
 &= \mathcal{N}\left(\mu_n, \sqrt{\sigma_n^2 + \sigma_{d_n}^2}\right) \star \mathcal{U}\left(-\frac{q_n}{2}, \frac{q_n}{2}\right) \\
 &= \frac{1}{q_n} \left[ \Phi_{\mu_n, \sqrt{11}}\left(y_n + \frac{q_n}{2}\right) - \Phi_{\mu_n, \sqrt{11}}\left(y_n - \frac{q_n}{2}\right) \right]
 \end{aligned} \tag{43}$$

Similarly, under  $H_0$ , we have

$$\begin{aligned}
 p_{Y_n}(y_n, H_0) &= p_{Z_n+D_n}(y_n) \star p_{W_n}(y_n) \\
 &= \mathcal{N}\left(0, \sqrt{\sigma_n^2 + \sigma_{d_n}^2}\right) \star \mathcal{U}\left(-\frac{q_n}{2}, \frac{q_n}{2}\right) \\
 &= \frac{1}{q_n} \left[ \Phi_{0, \sqrt{11}}\left(y_n + \frac{q_n}{2}\right) - \Phi_{0, \sqrt{11}}\left(y_n - \frac{q_n}{2}\right) \right]
 \end{aligned} \tag{44}$$

Having derived the marginal PDFs (Eqs. (43) and (44)), the LPF-noise based fusion rule (Eq. (39)) is now applied for testing between the two hypotheses. We include the Frank and the Gaussian copula functions in the set,  $\mathcal{A}_k$ , of potential copula models for characterizing dependence between observations under  $H_1$ .

In order to inject dependence between observations under  $H_1$ , we first generate dependent uniformly distributed bivariate samples,  $\mathbf{V} = \{(v_{1l}, v_{2l})\}_{l=1,2,\dots,L}$ , using Clayton copula with Kendall's  $\tau$  set to 0.31. The inverse CDF corresponding to each sensor's observation (specified in Eq. (40)) is then used to transform the bivariate samples,  $\mathbf{V}$ , to give a bivariate vector of dependent sensor observations with the required marginals:

$$z_{nl} = \Phi_{\mu_n, \sigma_n}^{-1}(v_{nl}), \quad \forall n, l. \tag{45}$$

The marginal parameters,  $\mu_1$  and  $\mu_2$ , are set to 0.5. Detection performance of the LPF-noise based GLRT is evaluated using this synthetic dataset. As discussed earlier, the set,  $\mathcal{A}_k$ , of potential copula functions consists of the Frank and Gaussian copula functions. Note that we have deliberately excluded the Clayton copula from this set so that we can evaluate the detection performance when the true underlying copula is unavailable. Hence, we also call the test the misspecified GLRT.

In Fig. 6, we plot the ROC curves using 50,000 Monte Carlo trials. The decision window,  $L$ , is set to 50 samples. That is, we assume that the sensors observe the phenomenon over  $L = 50$  time intervals before making a decision in favor of either hypothesis. It is evident from the figure that the performance of the LPF-noise based fusion rule is very close to the upper bound given by the analog/unquantized transmission case albeit with reduced computational complexity. This is true for both the quantizers,  $q_n = 0.3 \sigma_n$  and  $q_n = 0.6 \sigma_n$ , considered here. The quantization step sizes of  $0.3 \sigma_n$  and  $0.6 \sigma_n$  correspond to 22 and 12 quantization levels respectively in the  $[-3\sigma, 3\sigma]$  region of a Gaussian density function.

Another approach that is often adopted to address the issue of computational complexity is to deliberately neglect statistical dependence between sensor observations while designing the test. The test,

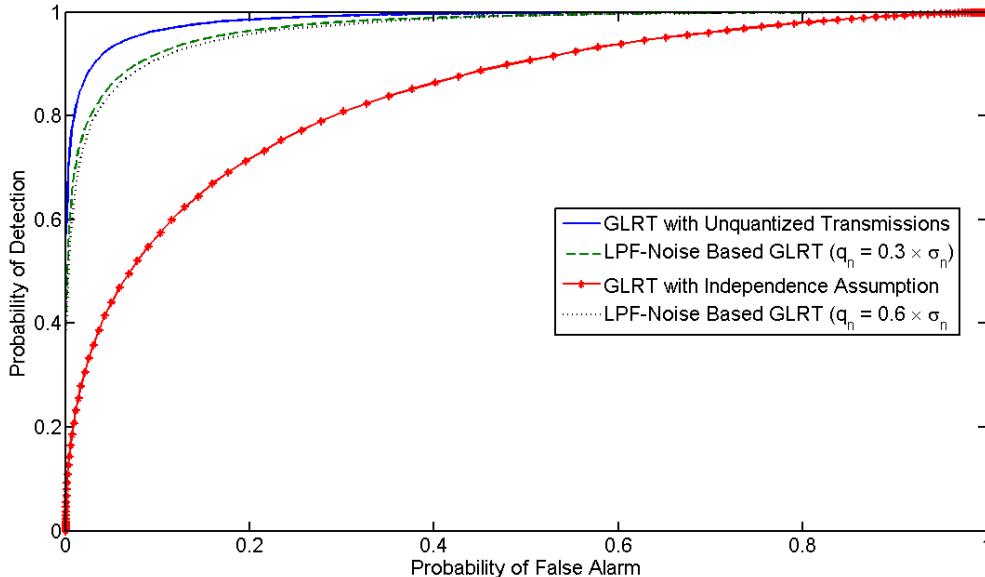


Fig. 6. Monte Carlo based Receiver Operating Characteristics: Performance of the fusion rule based on LPF noise is very close to the upper bound given by the analog transmission case. Also, the LPF-noise based GLRT outperforms the one designed assuming statistical independence between the observations.

so designed, would require computation of  $N$  one-dimensional integration operations as opposed to  $N$ -dimensional integrations where  $N$  is the number of sensors. However, such an approach severely degrades the detection performance as evident from Fig. 6. The LPF noise based GLRT significantly outperforms the one designed with the statistical independence assumption.

## VII. DETERMINATION OF THE DETECTOR THRESHOLD

Following the Neyman-Pearson formulation, we now look for a method to set the detector threshold  $\eta$  in (15) so that the false alarm probability,  $P_F$ , is constrained to  $\alpha \in (0, 1)$ . This, however, requires the knowledge of  $p_{\mathcal{T}_k}(t_k; H_0)$ , the PDF of the test statistic under the null hypothesis. Since the postulated statistical models,  $\{\hat{f}(\mathbf{z}; \Psi \subset \mathbb{R}^a)\}$  and  $\{\hat{g}(\mathbf{z}; \Lambda \subset \mathbb{R}^b)\}$ , under  $H_1$  and  $H_0$  respectively, are only approximations of the true underlying distributions, it is difficult to derive the exact distribution of the test statistic under either hypothesis. However, some advancement is possible for a certain class of problems especially when  $L$  is large. The following theorem, due to Wilks [29], identifies this class of problems.

**Theorem 5.** *Suppose the following conditions hold, in addition to the usual regularity conditions [30] that ensure the validity of asymptotic ML theory:*

**C 1.**  $\{\hat{f}(\mathbf{z}; \Psi \subset \mathbb{R}^a)\}$  and  $\{\hat{g}(\mathbf{z}; \Lambda \subset \mathbb{R}^b)\}$  are well-specified under  $H_0$ ,

**C 2.**  $\{\hat{g}(\mathbf{z}; \Lambda)\}$  is nested in  $\{\hat{f}(\mathbf{z}; \Psi)\}$ , i.e.,  $\hat{g}(\mathbf{z}; \lambda) \in \{\hat{f}(\mathbf{z}; \Psi)\}$ ,  $\forall \lambda \in \Lambda$ .

Then the modified test statistic,  $2\mathcal{T}_k(\cdot)$ , converges in distribution ( $\xrightarrow{d}$ ) to a chi-squared distribution with  $\nu (= a - b)$  degrees of freedom under the null hypothesis.

$$2\mathcal{T}_k(\mathbf{z}) \xrightarrow{d} \chi_{a-b}^2, \quad \text{under } H_0 \quad (46)$$

From (46), the probability of false alarm,  $P_F$ , is

$$\begin{aligned} P_F &= \Pr(2\mathcal{T}_k(\cdot) > 2\eta; H_0) \\ &\stackrel{L \rightarrow \infty}{=} Q_{\chi_{a-b}^2}(2\eta), \end{aligned} \quad (47)$$

where  $Q_{\chi_{\nu}^2}(\cdot)$  denotes the right-tail probability of a chi-squared random variable with  $\nu$  degrees of freedom. One can thus obtain the threshold  $\eta$  so that  $P_F$  is constrained to a desired level  $\alpha \in (0, 1)$  as below:

$$\eta = \frac{Q_{\chi_{a-b}^2}^{-1}(\alpha)}{2} \quad (48)$$

The assumption of a well-specified  $g(\cdot; \lambda)$  is reasonable for many applications. For example, it is always possible to collect enough training data under  $H_0$  (when there is no signal present), so that  $g(\cdot)$  can be consistently estimated.

Note that the illustrative example in Section VI satisfies the conditions of Theorem 5, and thus belongs to the class of problems for which the detector threshold can be determined. Here, the true distribution under  $H_0$ , given by the product of two univariate Gaussian PDFs, was assumed to be completely known (and hence well-specified). Now, given that the data belongs to the null hypothesis,

$$\hat{f}(\mathbf{z}; \hat{\psi}) = \left( \prod_{n=1}^2 f(z_n; \hat{\psi}_n) \right) k(\cdot; \hat{\psi}_d) \xrightarrow{H_0} g(\mathbf{z}; \lambda) = \prod_{n=1}^2 g(z_n; \lambda_n) \quad (49)$$

since both the Frank and the Gaussian copulas converge to the independence copula. Thus, condition **C1** of Theorem 5 holds. Also, it is easy to see that  $g(\mathbf{z}; \Lambda)$  is nested in the family defined by  $\{\hat{f}(\mathbf{z}; \Psi)\}$  since the marginal PDFs under both hypotheses are univariate Gaussian PDFs. Thus, the condition **C2** is satisfied, and we have the asymptotic convergence of the test statistic to a chi-squared distribution with  $a - b = 3$  degrees of freedom, where the number of unknown parameters under  $H_1$ , denoted by ‘a’ is three ( $\mu_1$ ,  $\mu_2$  and  $\psi_d$ ), and that under  $H_0$ , denoted by ‘b’, is zero. Thus,  $P_F = Q_{\chi_3^2}(2\eta)$ . Fig. 7

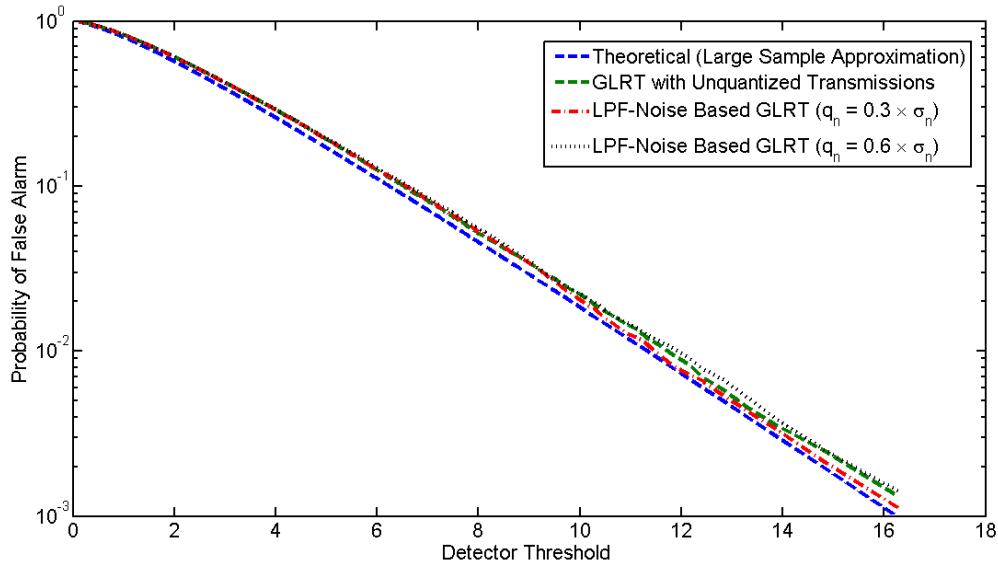


Fig. 7. System probability of false alarm vs. Detector threshold. A good match between the theoretical and simulated  $P_F$  values is evident from the figure.

shows a plot of this theoretical  $P_F$  along with simulated false alarm values obtained using 50,000 Monte Carlo trials with the decision window,  $L$ , set to 50 samples. A good match between the theoretical and simulated  $P_F$  values across the two different quantization step sizes is evident from the figure.

Note that the distribution of  $2\mathcal{T}_k(\mathbf{z}; \hat{\lambda}, \hat{\psi})$  under the null hypothesis depends only on the model complexities of  $\{\hat{f}(\cdot; \Psi)\}$  and  $\{g(\cdot; \Lambda)\}$ , i.e., the number of uncertain parameters  $a$  and  $b$ . If the set  $\mathcal{A}_k$  consists of copula densities with parameters of different dimensions, the threshold  $\eta$  must be adjusted accordingly (see Eq. (48)) to maintain a desired false alarm probability. Alternatively, one could restrict the set  $\mathcal{A}_k$  to include copula models with equal complexity to avoid the extra step of varying  $\eta$  in real-time.

## VIII. CONCLUSION

In this paper, the problem of fusing statistically dependent sensor decisions for the detection of a random event was considered. Sensor observations (or features extracted thereof) are first quantized using uniform multilevel quantizers before their transmission to the fusion center. Inter-modal dependence was assumed to be unknown and was approximated using copula functions. A GLRT based decision fusion algorithm that can fuse both hard and soft local decisions was derived. The important problem of selecting the best copula was embedded in the GLRT formulation. It was noted that the derived copula-based fusion algorithm becomes computationally expensive as the number of sensors and/or number of quantization levels increase. A novel approach based on Widrow's additive quantization noise model was developed

which requires deliberate injection of an external noise at the receiver before fusion. The addition of external noise at the fusion center effectively ‘filters’ the baseband CFs by rejecting the repetitive CF lobes that arise due to quantization. Since this process is analogous to low pass filtering (LPF) in signal processing, we term this noise, the LPF-noise.

As an illustrative example, using different copula functions such as the Clayton, Frank and Gaussian copulas was presented. Gaussian noise sources were used to generate LPF-noise at the fusion center, and results for two different quantization step sizes were obtained. Our results show that the approach based on LPF-noise can be considerably accurate provided the CF of the input signals are bandlimited and Widrow’s first quantization theorem is satisfied. The key to the success of this computationally efficient approach is the choice of the external noise source used for filtering the baseband CF. Design of a noise source that introduces minimal distortion while filtering is a topic of future research.

## IX. ACKNOWLEDGMENTS

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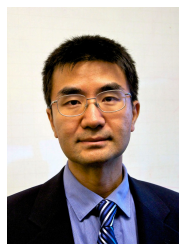
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