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We use the recently proposed node decimation algorithm for blocking dynamical geometries to investigate a class of models, with central charge greater than unity, coupled to $2D$ gravity. We demonstrate that the blocking preserves the fractal structure of the surfaces.

1. MODEL

The model we examine here is a two dimensional dynamically triangulated surface coupled to Gaussian fields. It has the (fixed area) partition function [1]

$$Z_N \;=\; \sum_{\{\tau,\phi\}} \mathrm{e}^{-S_\tau[\phi]}, \tag{1}$$

where the sum is over all combinatorial triangulations τ and field configurations ϕ. The action,

$$S_\tau[\phi] \;=\; \sum_{\mu=1}^{D} \sum_{\langle i,j\rangle} (\phi_i^\mu - \phi_j^\mu)^2, \tag{2}$$

depends on the configuration of Gaussian fields ϕ. Here i and j label nodes that are nearest neighbors.

This model is characterized by a central charge $c = D$, where D is the number of Gaussian fields. For $1 < c < 25$ there exists no meaningful analytical solution. To test a recently proposed Monte Carlo renormalization group method, *node decimation* [2], we investigated the cases $c = 1$ and $c = 10$. The former was chosen because it is right at the limit of where a useful analytical solution can be found, and the latter because it was our expectation that a $c = 10$ model should closely resemble the $c = \infty$ limit where the surface is known to be a branched polymer.

2. SIMULATIONS

We update the lattice by proposing standard link flip moves, which are either accepted or rejected according to the outcome of a Metropolis test. Similarly, the field configuration is updated by subjecting a proposed change in the field at a node to a Metropolis test. In addition we have employed an overrelaxation update of the Gaussian fields, which we find to significantly reduce autocorrelation times.

An overrelaxation move involves replacing the value of one of the fields as,

$$\phi_i \;\rightarrow\; \phi_i' \;=\; -\phi_i + \frac{2}{q_i} \sum_j \phi_j, \tag{3}$$

where the nodes j are the nearest neighbors of node i, and q_i is its coordination number. We see that since

$$\sum_j (\phi_i - \phi_j)^2 \;=\; \sum_j (\phi_i' - \phi_j)^2, \tag{4}$$

the action is preserved automatically. Hence, these moves are always accepted. We find that using a ratio of only one Metropolis field update to eleven overrelaxation updates decreases the autocorrelation time by an order of magnitude compared to pure Metropolis updates.

3. MEASURING γ_s

We chose to investigate how the value of the string susceptibility exponent γ_s behaves under successive renormalization group transformations. The exponent γ_s defines the behavior of the grand canonical partition function $\mathcal{Z}(\mu)$ near the critical value of the cosmological constant μ:

$$\mathcal{Z}(\mu) \;=\; \sum_{N=0}^{\infty} Z_N e^{-\mu N} \tag{5}$$

$$\;\sim\; (\mu - \mu_c)^{2-\gamma_s} \quad \text{as} \quad \mu \rightarrow \mu_c. \tag{6}$$

It turns out that the value of γ_s is related to the size distribution of *minimal neck baby universes*. A minimal neck on a $2D$ dynamical triangulation is defined as a three link loop on the surface, that is not the boundary of one of the triangles. Such a neck divides the surface into two parts. The baby universe is defined to be the smaller of the two parts. It can be shown that for a surface of size N, the number of baby universes of size B should go like [3]:

$$n_N(B) \sim B^{\gamma_s - 2}(N-B)^{\gamma_s - 2}. \quad (7)$$

Therefore the recipe for finding γ_s is as follows: First, search the surface for minimal necks. Second, upon finding one, count the number of triangles on each side of the surface and call the smaller of these two numbers B. Finally, fit the distribution $n(B)$, averaged over many surfaces, to Eq. (7) and extract γ_s. Since γ_s characterizes the long distance properties of the surface, it is an appropriate observable with which to test how well the node decimation preserves the fractal structure of the surface.

4. NODE DECIMATION

On a regular lattice implementing a real-space renormalization group is straightforward. A familiar example is a field of Ising spins on a square lattice. Here one merely has to block the spins in some regular way, for example by using a majority rule to convert each block of spins into one spin on the reduced lattice. On a random lattice, such neat blocking is not possible. Here the lattice itself is a dynamical variable. We need a way to reduce the size of the surface, while retaining its large scale features. To accomplish this we require a Monte Carlo renormalization group adapted to random surfaces.

We have used the node decimation proposed in [2]. The main idea is to remove nodes, at random, to reduce the size of the surface. It's obvious that removing a node of coordination number greater that three will result in a non-triangular hole. To keep the lattice triangular throughout the process, we first identify the node we want to remove, then flip links around it until its coordination number is three. Only then do we remove the node, leaving a new triangle in its place. In rare cases, it may be impossible to reduce the coordination number of a node to three by flipping links, in which case the change will be abandoned and the node restored to its original condition.

The next step is to block the Gaussian fields themselves. One obvious approach is to let the fields on the blocked nodes be given by

$$\phi'_i = \xi \left[(1-\alpha)\phi_i + \frac{\alpha}{n_i} \sum_j \phi_j \right]. \quad (8)$$

Here n_i is the coordination number of the i-th node. The first term is the direct contribution of the field at that node to the blocked lattice, and the second term is an average of the neighbors of the node on the original lattice. We have introduced a relative weight α between those two terms, which in principle depends on the blocking factor $b = N/N'$, where N and N' are the volumes of the bare and blocked lattices, respectively. Clearly α should go to zero as $b \to 1$. As b becomes larger, α should grow as well. Choosing an α that is too large for a given blocking factor may, on the other hand, result in a cooling effect where the Gaussian fields on the blocked lattice are overly correlated. Preliminary results indicate that this scheme is fairly robust under choice of α.

In general, we must re-scale the blocked fields by an overall factor ξ. Simple arguments suggest that ξ should be given by

$$\xi = b^{\frac{-\beta}{d_H}}, \quad (9)$$

where d_H is the fractal dimension of the surface and $\beta = -\eta/2$ is the length dimension of the scalar field, defined by the scaling of the two-point function;

$$\langle \phi_i \phi_j \rangle \sim r_{ij}^{\eta}. \quad (10)$$

From dimension counting we get the undressed length dimension β_o for a scalar field in two dimensions;

$$H = \int d^2x (\partial\phi)^2 \quad \Longrightarrow \quad \beta_o = 0. \quad (11)$$

In the presence of quantum gravity β_o must be

Table 1
The string susceptibility exponent γ_s measured on surfaces obtained via node decimation (using $b = \sqrt{2}$)

Blocking	$c = 0$	$c = 1/2$	$c = 1$	$c = 10$
0	-0.495(12)	-0.209(8)	$-0.038(58)$	0.570(7)
1	-0.498(7)	-0.365(5)	0.030(57)	0.562(16)
2	-0.500(12)	-0.334(10)	0.124(67)	0.569(9)
3	-0.495(15)	-0.341(6)	0.04(17)	0.542(14)
4	-0.505(19)	-0.315(11)		0.582(11)
5	-0.497(20)	-0.348(3)		0.493(21)
6				0.515(34)
Analytic result	$-1/2$	$-1/3$	0	$1/2$

replaced by β, the dressed field dimension. For $c \leq 1$ we can use the KPZ scaling [4];

$$\beta - \frac{\beta(1-\beta)}{1-\gamma_s} \;=\; \beta_o, \tag{12}$$

where

$$\gamma_s \;=\; \tfrac{1}{12}\left(c - 1 - \sqrt{(25-c)(1-c)}\right). \tag{13}$$

So, for $c = 1$, $\beta = \beta_0 = 0$. For $c > 1$ however, the process is more complicated. The scale factor ξ can be determined numerically by measuring some operator $\mathcal{O}$, that has n powers of ϕ in it. After m blockings, $\mathcal{O}$ will scale like

$$\mathcal{O}_m \;=\; \xi^{nm}\mathcal{O}_0. \tag{14}$$

It is thus possible to determine ξ by constructing a ratio of operators, obtained by different amount of blocking, but compared at the *same* volume. In particular, we have

$$\xi^n \;=\; \frac{\mathcal{O}_{m+1}}{\mathcal{O}_m}. \tag{15}$$

Preliminary investigations of the blocking of the Gaussian fields for $c = 1$ seem to confirm that the overall scale factor $\xi = 1$.

It is worth noting that the corresponding exponent η, has recently been determined numerically for an Ising model coupled to $2D$ gravity, by a direct fit to Eq. (10) [5].

5. RESULTS

We have measured the value of γ_s as a function of blocking level for: $c = 0$ (pure gravity), $c = \frac{1}{2}$ (Ising spins), $c = 1$ and $c = 10$. Bare lattice $N_o = 2000$ was used for $c = 1/2$, $N_o = 1000$ for the other models. These simulations were performed with several million sweeps apiece. The results are summarized in Table 1.

These results clearly show that the node decimation preserves the analytically predicted value of γ_s, even after several iterations. In the case of an Ising model coupled to gravity, the measured value of γ_s agrees more closely with the analytic prediction after one or two blocking iterations than on the bare lattice. This indicates that the blocking procedure may be useful in minimizing finite volume effects. The $c = 1$ case is notoriously difficult to fit, and even after fitting $n(B)$ to a form with logarithmic corrections the results have larger errors.

REFERENCES

1. D.V. Boulatov, V.A. Kazakov, I.K. Kostov, A.A. Migdal, Nucl. Phys. B275 (1986) 641.
2. G. Thorleifsson and S.M. Catterall, Nucl. Phys. B461 (1996) 350.
3. S. Jain and S.D. Mathur, Phys. Lett. B286 (1992) 239; J. Ambjørn, S. Jain, and G. Thorleifsson, Phys. Lett. B307 (1993) 34.
4. F. David, Mod. Phys. Lett. B186 (1987) 379; J. Distler and H. Kawai, Nucl. Phys. B321 (1989) 509; V.G. Knizhnik, A.M. Polyakov and A. Zamolodchikov, Mod. Phys. Lett. A3 (1988) 819.
5. J. Ambjørn, K.N. Anagnostopoulos, U. Magnea and G. Thorleifsson, NBI-HE-96-23, SU-4240-637 (July 1996) (`hep-lat/9606012`).