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Radial Excited States for Heavy Quark Systems in NRQCD

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Abstract

Following the Non-Relativistic QCD approach we use a gauge invariant smearing method with factorization to measure the excitation energies for a heavy $Q\bar{Q}$ system on a $24^3 \times 48$ lattice at $\beta = 6.2$. The results come from averaging over an ensemble of 60 QCD configurations. In order to enhance the signal from each configuration we use wall sources for quark propagators. The quark Hamiltonian contains only the simplest non-relativistic kinetic energy term.

The results are listed for a range of bare quark masses. The mass splittings are insensitive to this variable though there are a slight trends with increasing quark mass. For an appropriate choice of UV cut-off ($a^{-1} = 3.2$Gev) the mass spectrum compares reasonably well with the experimental values for the spin-averaged energy gaps of the Υ system.

We also present results for the DE and DT waves for the lowest bare quark mass. The results are consistent with degeneracy between the two types of D wave. This encourages the idea that even with our simple quark Hamiltonian the departure from rotational invariance is not great.

1 Introduction

In a previous paper [1] we studied heavy quark bound states appropriate to a description of the J/ψ andΥ systems using the non-relativistic approach of Lepage (NRQCD) [2, 3, 4, 5]. We investigated the lowest bound states for S, P and D waves ignoring spin effects for the quarks using gauge configurations from the UKQCD collaboration on a $16^3 \times 48$-lattice with a β-value of 6.2. In the present paper we report results on the first excitations in the S and P-channels for this system (again without spin). We obtained these results using 60 quenched QCD configurations from the UKQCD collaboration on a $24^3 \times 48$ lattice at $\beta = 6.2$. The new results are reasonably consistent with our previous ones but considerably more precise.

Our results are based on the construction of a number of smeared and unsmeared operators that couple to the appropriate channels and the measurement of their cross correlators. The smeared operators are constructed in a gauge invariant manner. Using a simple subtraction procedure we show that the correlation functions do indeed have a multi-exponential structure. Our best estimates of the lowest states and the first excited states in both the S and P-channels of the $Q\bar{Q}$ system are established by performing consistent correlated fits to the measured operator correlators using appropriately factorizing two-exponential forms. Some three-exponential fits were attempted to test the range of applicability of the fits but did not lead to different conclusions.

2 Quark Propagators

The quark propagator in a given gauge field background is

$$G(x,y) = \langle \psi(x)\psi^\dagger(y)\rangle \quad , \tag{1}$$

where the angle brackets indicate averaging over the quark degrees of freedom $\{\psi(x)\}$ and $x = (\mathbf{x}, t)$, $y = (\mathbf{y}, 0)$. When $t = 0$, $G(x,y) = \delta_{\mathbf{xy}}$.

The evolution for the (non-relativistic) quark propagator $G(x,y)$ is

$$G(x+\hat{t}, y) = U_{\hat{t}}^\dagger(x)\left(1 - \frac{H_0}{n}\right)^n G(x,y) + \delta_{\mathbf{xy}}\delta_{t0} \tag{2}$$

where $\hat{t}$ denotes a unit step in the time direction and n is the order of the time-step update as discussed by Thacker and Davies [6]. We set $G(x,y) = 0$ for $t \leq 0$. The modified update is necessary for stability at certain values of the bare quark mass. In this paper we use $n = 3$. The hamiltonian H_0 is that appropriate to non-relativistic propagation ignoring spin of a quark of mass M, namely

$$H_0 = \frac{-1}{2Ma}\sum_{\hat{\mu}=1}^{3} \Delta_{\hat{\mu}}^{+}\Delta_{\hat{\mu}}^{-} \tag{3}$$

The covariant finite differences Δ^+, Δ^- are given by their usual expressions (we have suppressed all colour and spin indices)

$$\Delta^+_{\hat{\mu}} G(x,y) = U_{\hat{\mu}}(x) G(x+\hat{\mu},y) - G(x,y) \quad , \tag{4}$$

$$\Delta^-_{\hat{\mu}} G(x,y) = G(x,y) - U^\dagger_{\hat{\mu}}(x-\hat{\mu}) G(x-\hat{\mu},y) \quad . \tag{5}$$

3 Smeared Operators

The ideal method for detecting excitated states in a given channel is to construct operators each of which couples only to one of the states. An alternative approach is to accept as a starting point a basis set of operators with quantum numbers appropriate to the channel of interest and to recognize that an intermediate state will couple to each of these operators in a unique way so that the exponential contribution associated with that state to the cross correlators of the basis operators will have a factorizing form. This is the approach we have adopted in dealing with the gauge invariant smeared operators we construct and use in our simulation.

The operators we investigated for the S-channel were in addition to the standard point operator (we use $\chi(x)$ to denote the anti-quark degrees of freedom)

$$O^{(0)}(x) = \chi^\dagger(x)\psi(x) + \text{h.c.} \quad , \tag{6}$$

a set of operators of the form

$$O^{(m)}(x) = \sum_{\hat{\mu}} \chi^\dagger(x) \left(M^m_{\hat{\mu}}(x)\psi(x+m\hat{\mu}) + \hat{M}^m_{\hat{\mu}}(x)\psi(x-m\hat{\mu}) \right) + \text{h.c.} \quad , \tag{7}$$

where the $\hat{\mu}$-sum is over space like directions and the matrices $M^m_\mu(x)$ and $\hat{M}^m_\mu(x)$ have the (appropriately ordered) product forms

$$M^m_{\hat{\mu}}(x) = \prod_{\nu=0}^{m-1} U_{\hat{\mu}}(x+\nu\hat{\mu}) \;\; \text{and} \;\; \hat{M}^m_{\hat{\mu}}(x) = \prod_{\nu=1}^{m} U^\dagger_{\hat{\mu}}(x-\nu\hat{\mu}) \quad . \tag{8}$$

For the P-channel we use a family of operators of the form

$$O^{(m)}_{\hat{\mu}}(x) = \chi^\dagger(x) \left(M^m_{\hat{\mu}}(x)\psi(x+m\hat{\mu}) - \hat{M}^m_{\hat{\mu}}(x)\psi(x-m\hat{\mu}) \right) + \text{h.c.} \quad , \tag{9}$$

The DE wave operators are

$$\begin{aligned} O^{DE(m)}_{\hat{\mu}\hat{\nu}}(x) = \chi^\dagger(x) \Big(& M^m_{\hat{\mu}}(x)\psi(x+m\hat{\mu}) + \hat{M}^m_{\hat{\mu}}(x)\psi(x-m\hat{\mu}) \\ & - M^m_{\hat{\nu}}(x)\psi(x+m\hat{\nu}) - \hat{M}^m_{\hat{\nu}}(x)\psi(x-m\hat{\nu}) \Big) + \text{h.c.} \quad , \end{aligned} \tag{10}$$

and the DT wave operators are

$$O_{\hat{\mu}\hat{\nu}}^{DT(m)}(x) = \chi^\dagger(x)\left(\Delta_{\hat{\mu}}^{(m)}\Delta_{\hat{\nu}}^{(m)} + \Delta_{\hat{\nu}}^{(m)}\Delta_{\hat{\mu}}^{(m)}\right)\psi(x) + \text{ h.c. } , \tag{11}$$

where

$$\Delta_{\hat{\mu}}^{(m)}\psi(x) = M_{\hat{\mu}}^m(x)\psi(x+m\hat{\mu}) - \hat{M}_{\hat{\mu}}^m(x)\psi(x-m\hat{\mu}) \quad . \tag{12}$$

The correlation functions we measure are

$$F_{nm}^S(t) = \frac{1}{V}\sum_{\mathbf{x},\mathbf{y}}\langle O^{(n)}(x)O^{(m)}(y)\rangle \quad , \tag{13}$$

for S-wave analysis, and

$$F_{nm}^P(t) = \frac{1}{3V}\sum_{\hat{\mu},\mathbf{x},\mathbf{y}}\langle O_{\hat{\mu}}^n(x)O_{\hat{\mu}}^m(y)\rangle \quad , \tag{14}$$

for the P-wave analysis. Here $V = 24^3$, the spatial lattice volume.

All of the above operators are of the form

$$O(x) = \chi^\dagger(x)\Psi(x) \quad , \tag{15}$$

where for an appropriate set of ($SU(3)$-matrix) coefficients $\{C_{\mathbf{x}\mathbf{x}'}\}$

$$\Psi(x) = \sum_{\mathbf{x}'} C_{\mathbf{x}\mathbf{x}'}\psi(x') \quad , \tag{16}$$

and $x' = (\mathbf{x}', t)$. A typical correlation function can be expressed as

$$F_{12}(t) = \frac{1}{V}\sum_{\mathbf{x}\mathbf{y}}\langle \mathrm{Tr}\bar{G}_{12}(x,y)G^\dagger(x,y)\rangle + \text{c.c.} \quad , \tag{17}$$

where

$$\bar{G}_{12}(x,y) = \langle \Psi^{(1)}(x)\Psi^{(2)\dagger}(y)\rangle \quad , \tag{18}$$

where 1 and 2 indicate the two (possibly the same) smeared operators. Of course the Green's function $\bar{G}_{12}(x,y) = \sum_{\mathbf{x}'} C_{\mathbf{x}\mathbf{x}'}^{(1)}\mathcal{G}^{(2)}(x',y)$ where $\mathcal{G}^{(2)}(x',y)$ can be calculated with an appropriate change of initial condition by the same method as the original quark Green's function.

4 Wall Source Method

Because the computing overhead is considerable it is desirable to extract as much signal as possible from each pass through a gauge field configuration. To this end we modify the measured correlators $F_{nm}^S(t)$, $F_{nm}^P(t)$ and $F_{nm}^D(t)$as follows. Eq(17) leads to a method of computation for our typical correlation function $F_{12}(t)$ that requires the evaluation of the Green's functions for at least a representative sample of y-values on the initial time slice if we wish to maximize the information to be extracted from each configuration. This is computationally onerous. An alternative procedure is the following. We replace the single $\mathbf{y}$-summation in eq(17) with a double sum thus

$$F_{12}(t) = \frac{1}{V} \sum_{\mathbf{x},\mathbf{y},\mathbf{y}'} \langle \mathrm{Tr}\bar{G}_{12}(x,y) G^{\dagger}(x,y') \rangle + \mathrm{c.c.} \quad , \tag{19}$$

where $y' = (0, \mathbf{y}')$. We now rely on the gauge field averaging to eliminate the contributions from the gauge non-invariant off-diagonal terms in the above double $(\mathbf{y}, \mathbf{y}')$-sum leaving only the contribution from the gauge invariant diagonal terms for which $\mathbf{y} = \mathbf{y}'$. The disadvantage of the method is that the off-diagonal contributions provide noise even if they do average to zero. The advantage of the method is that we pick up all the diagonal terms in one pass since it is only necessary to compute the objects of the form

$$g(x) = \sum_{\mathbf{y}} G(x,y) \quad , \tag{20}$$

which satisfies the same equation as $G(x,y)$ and the initial condition

$$g(0, \mathbf{x}) = 1 \quad . \tag{21}$$

Similar remarks apply to $\bar{g}_{12}(x) = \sum_{\mathbf{y}} \bar{G}_{12}(x,y)$. We have then

$$F_{12}(t) = \frac{1}{V} \sum_{\mathbf{x}} \langle \mathrm{Tr}\bar{g_{12}}(x) g^{\dagger}(x) \rangle \quad , \tag{22}$$

In practice we do find that the method works well and does provide a good signal relatively economically. It is implicit in the discussion that no gauge fixing has been imposed on the ensemble of gauge fields. However because of the limitations of the data set the averaging procedure may not work perfectly. The different treatment of the two operators in the correlation function may mean that the symmetry $F^{S,P,D}_{mn}(t) = F^{S,P,D}_{nm}(t)$ no longer holds. This does not destroy factorization and we allow for the asymmetry in fitting the data. This procedure is similar to the multiple origin approach first utilised by Kenway[7] and Billoire et al.[8], except we seed all sites on the initial timeslice. A quark wall source was also used by Gupta et al.[9] although they fix to Coulomb gauge.

5 Data Fitting

As indicated above we try to fit the correlation function $F^S_{mn}(t)$ with the multi-exponential form

$$F^S_{mn}(t) = \sum_a \gamma_{(a)m} \gamma'_{(a)n} e^{-M_{(a)}t} \quad . \tag{23}$$

Our main results are obtained using a two exponential form requiring eight parameters. This allows us to obtain estimates for the lowest S and P states together with the first excitations. Our statistical method involved a correlated least squares fit based on an estimate of the complete set of variances and cross correlators of all the fitted quantities. Since our results are based on 60 statistically independent gauge field configurations we felt that eight parameters was a reasonable number for the fitted form. We did perform three exponential fits in certain cases with twelve parameters. Where these seemed reliable they were consistent with the two exponential fits but with considerably less tight errorbars. The results we quote are from separate S and P-channels fits. We also carried out a combined S and P-channel fit but obtained results that were little different.

The limitations of the data led us to confine ourselves to two operators per channel in any one fit. The precise form of the 2×2 matrix of correlators was

$$\begin{pmatrix} F_{mm}(t) & F_{mn}(t) \\ F_{nm}(t) & F_{nn}(t) \end{pmatrix}$$
$$= \begin{pmatrix} (\gamma_m^{(1)})^2 & \gamma_m^{(1)}\gamma_n^{(1)}\rho^{(1)} \\ \gamma_n^{(1)}\gamma_m^{(1)} & (\gamma_n^{(1)})^2\rho^{(1)} \end{pmatrix} e^{-Mt} + \begin{pmatrix} (\gamma_m^{(2)})^2 & \gamma_m^{(2)}\gamma_n^{(2)}\rho^{(2)} \\ \gamma_n^{(2)}\gamma_m^{(2)} & (\gamma_n^{(2)})^2\rho^{(2)} \end{pmatrix} e^{-(M+\Delta M)t} \quad . \tag{24}$$

This is equivalent to the form in eq(23). The asymmetry in the factorized forms is represented by the departure of the parameters $\rho^{(1)}$ and $\rho^{(2)}$ from unity. Note that we have parametrized the splitting ΔM between the two levels explicitly since this is the quantity of direct interest.

The basis of the fitting procedure is the estimate of the correlation matrix of results. At any one time these comprised the two direct and two cross correlators for two operators evaluated on 48 time slices. The correlation matrix was therefore of dimension 192×192 . Our data is extracted from 60 independent gauge configurations. The correlation matrix is therefore of rank $r \leq 60$ and therefore necessarily singular. In practice the effective rank of the correlation matrix is even less than this since beyond a certain point the eigenvalues become so small their estimation from the data is not reliable. The least squares fitting procedure and the associated error estimates require the use of the inverse of the correlation matrix. It is necessary and indeed correct to restrict the inversion of the matrix to an appropriate subspace that is spanned by eigenvectors with eigenvalues large enough for reliable estimation from the data. The dimension of the subspace is referred to as the Singular Value Decomposition (SVD) cut.

In assessing the results of the fitting procedure we examined cases with a range of values of initial off-set and SVD cuts for different combinations of smeared operators. Our criterion for a choice of result was that the χ^2-value be acceptably near unity per degree of freedom and that the error was the best (usually the first) of a range of reasonably good and statistically consistent fits.

6 Explicit Diagonalization Scheme

Before exhibiting the results of the correlated fits we show directly the existence of a second exponential by means of a diagonalization method. Lüscher and Wolff [10] have shown that the eigenvalues of the correlation matrix are of the form

$$e^{-M_{(a)}t}\left(1 + O(e^{-\Delta M_{(a)}t})\right) \tag{25}$$

where $\Delta M_{(a)}$ is the distance of state $M_{(a)}$ from other states. Thus we evaluate the eigenvalues of the correlation matrix using the appropriate Numerical Recipes routines [11]. Fig. 1 shows the result for $Ma = 1.5$ for the S-wave combination $O_0^S(x)$ and $O_4^S(x)$ - the ground state S-wave is suppressed revealing the existence of the exponential associated with the first excited state. Fig. 2 shows effective mass plots for the $1S$ and $2S$ states obtained from these graphs. Figs. 3 & 4 show similar results for the $1P$ and $2P$ states. The results are reasonably consistent with those of the correlated fits discussed below which were used to produce the quoted numbers.

In order to obtain reasonably smooth plots the effective mass was defined as

$$M(t) = 0.25 * \log\left(\frac{A(t)}{A(t+4)}\right) \quad , \tag{26}$$

where

$$A(t) = \left(F(t) + wF(t+1) + w^2F(t+4) + w^3F(t+3)\right)/4 \quad , \tag{27}$$

and w is chosen to render the terms in the sum of comparable size.

7 Correlated Fits for S and P Waves

The results of the correlated fits are shown in Table 1 . Also shown are the operators used to obtain the results, the χ^2 per degree of freedom, the SVD-cuts and the off-sets at which a reasonable statistical stability set in.

M_0a	Chan.	Ma	ΔMa	χ^2/dof	SVD-cut	Off-set	Operators m
1.5	S	1.1152(8)	0.198(24)	25.6/18	26	17	0 and 4
	P	1.243(12)	0.116(12)	19.2/20	28	11	2 and 6
2.0	S	0.9788(7)	0.194(12)	11.53/12	20	12	1 and 4
	P	1.112(13)	0.105(20)	14.48/20	28	11	2 and 6
3.0	S	0.8287(7)	0.179(17)	21.68/18	26	15	0 and 4
	P	0.969(17)	0.090(17)	11.6/14	22	11	2 and 6

Table 1: The results for the ground states and first excited splits in the S and the P channels.

It is also interesting to compare the various mass splits with the spin-averaged values of the Υ-system. We use a conversion factor from lattice units to physical units of $a^{-1} = 3.2$. The results are listed in Table 2 .

It is clear that the pattern of mass splitting for the lower two bare masses ($M_0a = 1.5$ & 2.0) is reasonably close to the actual splitting for the Υ except for the $2S$ level which appears to be too high but exhibits a downward trend as M_0a increases. The other splits are less sensitive to changes in the bare mass.

M_0a	$1S-2S$	$1S-1P$	$1S-2P$	$1P-2P$
1.5	0.198(24) 0.634(77) Gev	0.128(12) 0.410(38) Gev	0.244(17) 0.781(54) Gev	0.116(12) 0.371(38) Gev
2.0	0.194(8) 0.621(26) Gev	0.133(13) 0.426(42) Gev	0.238(24) 0.762(77) Gev	0.105(20) 0.336(64) Gev
3.0	0.179(17) 0.573(54) Gev	0.140(17) 0.448(54) Gev	0.230(24) 0.736(77) Gev	0.090(17) 0.288(54) Gev
Expt(Υ)	0.563 Gev	0.430 Gev	0.795 Gev	0.365 Gev

Table 2: The results for various mass splits in lattice and physical units compared to the spin-averaged results for the Υ-system. The conversion factor is $a^{-1} = 3.2$ Gev.

The ratios of mass splits is independent of the choice for a^{-1} . For each bare quark mass these ratios are listed in Table 4 taking the central value of the $1S-1P$ split as the base.

M_0a	$1S-2S$	$1S-1P$	$1S-2P$	$1P-2P$
1.5	1.55(19)	1.00(12)	1.91(13)	0.91(9)
2.0	1.46(6)	1.00(13)	1.79(18)	.79(15)
3.0	1.28(12)	1.00(17)	1.64(17)	0.64(12)
Expt(Υ)	1.31	1.00	1.85	0.85

Table 3: The results for various mass splits expressed as ratios to the central value of the $1S-1P$ split and compared to the spin-averaged results for the Υ-system.

These results are not dissimilar to those of the phenomenological non-relativistic quark models [12]. The spectrum is relatively independent of the bare quark mass though the $2S$ state is rather high. Apart from the anomalously high $2S$ state the ratios that fit best correspond to a bare quark mass somewhere between $M_0a = 1.5$ and $M_0a = 2.0$. Using the same conversion factor as above we find these correspond to $M_0 = 4.8$ Gev and $M_0 = 6.4$ Gev. This is to be compared with the B-quark mass of $\sim$ 5 Gev suggested by the mass of the Υ itself. If we take the $1S-1P$ mass split for $M_0a = 1.5$ as the correct basis on which to calculate then we find $a^{-1} = 3.4(3)$ Gev which encompasses the above value.

Our results for a^{-1} are higher than suggested by a measurement of the string tension σ . At $\beta = 6.2$, $\sigma a^2 = 0.026(1)$ [14, 13] that is $\sqrt{\sigma}a = 0.161(3)$. If we use the phenomenological value $\sqrt{\sigma} = .42$ Gev we obtain $a^{-1} = 2.6(1)$ Gev. This is in line with other estimates of a^{-1}. Another way of expressing this discrepancy is to note that our lattice calculation at $\beta = 6.2$ yields a ratio $\sqrt{\sigma}/\Delta M(1S{-}1P) \simeq 1.25$ whereas the phenomenological result is $\simeq 1.0$. The question then is whether or not there is a reasonable explanation of this discrepancy for our model. One answer is to recognize that the string tension is associated with the long range part of the quark potential while the $1S-1P$ split comes about as a result of a balance between long and short range effects in the potential. The short range force is controlled by the strong coupling evaluated at a higher momentum, q^*, than that, $\bar{q}$, associated with the string tension. The main difference between the quenched and unquenched theories is the differential renormalization of the strong coupling $\alpha(q)$ at a given q due to the vacuum polarization effects of light quarks. If we fix the string tension to be the same in both theories then we have $\alpha_U(\bar{q}) = \alpha_Q(\bar{q})$ However because the quenched coupling runs faster than the unquenched one we have $\alpha_U(q^*) > \alpha_Q(q^*)$. The short range force in the quenched theory will therefore be weaker than in the unquenched case. Because the S-wave states are particularly sensitive to the short range part of the $Q{-}\bar{Q}$ force they will be more deeply bound in the unquenched theory than in the quenched one. The P-waves will be controlled more by the longer range part of the force associated with the string tension. This will tend to leave the P-waves unchanged between the two theories with the result that for a given string tension $\Delta M(1S-1P)$ will be greater in the unquenched relative to the quenched theory. In turn this will yield a lower value for the ratio $\sqrt{\sigma}/\Delta M(1S-1P)$ for the unquenched relative to the quenched theory in line with our results. Similar results will hold for for any two quantities associated with different momentum scales. In the quenched theory they will yield different estimates for a^{-1} while unquenched theory (by definition) will produce consistent estimates.

8 Correlated Fits for D Waves

In Table 4 we show the results for a two exponential fit to the two versions of the D wave operators for the bare quark mass $M_0 a = 1.5$.

$M_0 a$	Chan.	Ma	ΔMa	χ^2/dof	SVD-cut	Off-set	Operators m
1.5	DE	1.373(11)	0.315(13)	23.7/18	26	6	2 and 6
	DT	1.388(19)	0.389(41)	22.5/14	22	6	1 and 4

Table 4: The results for the ground states and first excited splits in the DE and the DT channels.

The encouraging feature of these results is the degeneracy within errors not only of the basic states in the two channels but also of the first excited states. The conclusion is that to a good approximation the cubical symmetry of the (spatial) lattice is replaced by rotatational symmetry. Expressed in physical units

$\Delta M(1S - 1D) = 0.83(4)$ Gev if we again use $a^{-1} = 3.2$ Gev. There is so far no observed D wave for the Υ system but the corresponding state for charmonium (J/ψ) has $\Delta M(1S - 1D) = 0.702$ Gev. Given the simple and approximate nature of our heavy quark Hamilton this is an encouraging result. The quality of the results from the simulation restricted the application of the fitting procedure to the range $t < 20$ so the outcome for the mass gap may be expected to be on the high side. This circumstance may also explain why the measured gap $\Delta M(1D - 2D) \simeq 1$ Gev is implausibly high. It is interesting that it showed in both versions of the D wave spectrum.

9 Conclusions

We have measured the $Q\bar{Q}$ mass splittings for the radial excitations of the S and P waves using the non-relativistic heavy quark propagators calculated from quenched gluon configurations from the UKQCD collaboration. Our measurements were based on 60 QCD configurations on a $24^3 \times 48$ lattice. The Hamiltonian we used to calculate the quark propagators was of the simplest kind containing only the kinetic energy contribution and omitting all higher corrections. The results emerged from two-exponential correlated fits to pairs of smeared operator correlators. Our best results yielded statistical errors $\sim 10\%$ for the mass splittings.

On a broad picture our results are not inconsistent with the pattern of spin-averaged splittings of the Υ-system. In particular the ratios $\Delta M(1S - 1P)$: $\Delta M(1S - 2P)$: $\Delta M(1S - 1D)$ seem roughly correct. The $2P$ wave shows a slight dependence on the bare quark mass. We have not yet determined the dependence of the $1D$ wave on this mass. The absence of an experimental $1D$ state for Υ restricts us to a comparison with the corresponding charmonium state or theoretical quark model predictions but that comparison is encouraging. If we base our evaluation of the cut-off strictly on the $1S - 1P$ mass split then we find the value $a^{-1} = 3.4$ Gev. This very much in line with scaling predictions from results of the corresponding calculations performed at values of β =5.7 and 6.0 [15] . This suggested a bare quark mass for the kinetic energy Hamiltonian in the range 4.8 to 6.4 Gev.

However there are two obvious problems that present themselves. The first is that the ratio involving the string tension $\sqrt{\sigma}/\Delta M(1S - 1P)$ is measured as 1.25 compared to a phenomenological value of 1.0 . It is plausible that this discrepancy may be due to the overly strong running of the coupling in the quenched theory. The second is the anomalously high value of $\Delta M(1S - 2S)/\Delta M(1S - 1P)$ and its sensitivity to the bare quark mass. It may be that the deficiencies of the quenched approximation can resolve this problem also. An alternative explanation is that there is a measurement problem with the S wave channel. Future measurements using different smeared operators constructed in the Coulomb gauge from quark model wave functions should help to resolve the issue.

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Figure Captions

Fig. 1 The results for the S-wave propagator $F_{00}^{(S)}(t)$ (circles) together with the the results of the diagonalization procedure applied to the correlation matrix for the operators $O_0^{(S)}$ and $O_4^{(S)}$ (diamonds) revealing the contribution of the $2S$-state.

Fig. 2 The effective mass plot for the $1S$ (upper graph) and $2S$ states (lower graph). The estimates obtained from the correlated fit are indicated by a full line.

Fig. 3 The results for the P-wave propagator $F_{22}^{(P)}(t)$ (circles) together with the the results of the diagonalization procedure applied to the correlation matrix for the operators $O_2^{(P)}$ and $O_6^{(P)}$ (diamonds) revealing the contribution of the $2P$-state.

Fig. 4 The effective mass plot for the $1P$ (upper graph) and $2P$ states (lower graph). The estimates obtained from the correlated fit are indicated by a full line.