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Essays on Risk Mitigation Methods in Global Supply Chain Management

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ESSAYS ON RISK MITIGATION METHODS IN GLOBAL SUPPLY CHAIN MANAGEMENT

ABSTRACT

In this dissertation, we study risk mitigation methods in the area of global supply chain management. Recent economic uncertainties such as the U.S. credit and the Greek debt crises, and natural disasters such as the Japanese tsunami create a greater amount of uncertainty and risk for global supply chain operations. The goal of the dissertation is to develop new prescriptive policies for multinational corporations and global relief organizations in order to assist them in mitigating the risks present in their operating environment in an effective manner.

The first essay of the dissertation investigates how a multinational corporation can effectively hedge against the exchange-rate risk by structuring and managing a supply chain in a global setting. Economic uncertainties such as the U.S. credit crises and the debt concerns in southern European countries (e.g., Greece, Italy, Portugal and Spain) call for new policies in order to operate under an increased level of uncertainty. For example, even though the Japanese Yen and the US Dollar are considered to be more stable currencies, they have exhibit significant fluctuations in recent times, putting the US Dollar at its record low against the Yen on October 24, 2011, since the Great Depression. Naturally, such exchange rate fluctuations increase the need to develop new risk mitigation policies that would assist global corporations in managing the fluctuations in their global revenues and profits. This essay provides multinational corporations with prescriptions for ways to cope with the increased amount of global economic uncertainty. It demonstrates that production hedging, defined as producing and supplying less

than the firm's total global demand, can be an effective policy in minimizing the negative implications of the exchange-rate risk while maximizing global profits.

The second essay of the dissertation examines how a global relief organization can effectively hedge against the risk of demand uncertainty in its distribution of essential products to the areas of urgent needs. It is commonly observed that deficiencies in information infrastructure or economic instability contribute to the adversity of forecasting the need for essential supplies. This study stems from recognizing the ineffectiveness in the distribution of humanitarian aid supply in countries of need. Such distribution inefficiencies can lead to harmful consequences for many people. United Nations Children's Fund (UNICEF) reports that in 2007 alone, 9.2 million children worldwide under the age of five died from largely preventable causes. It is important to recognize that many of the well-developed theories are hard to apply in these countries because of inferior infrastructure and the unstable nature of the economy and political environment. Thus, it is critical to develop new policies and insights for the managers of humanitarian organizations in order to increase the efficiency of distribution operations under uncertainty. Specifically, the second essay of this dissertation demonstrates that a relief organization can minimize the risk of shortages by strategically utilizing its limited budget on procurement and the transportation of essential supplies.

ESSAYS ON RISK MITIGATION METHODS IN GLOBAL SUPPLY CHAIN
MANAGEMENT

BY

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DISSERTATION

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TABLE OF CONTENTS

List of Tables

List of Figures

List of Propositions

CHAPTER 1: INTRODUCTION

1.1 Introduction

In this dissertation, we study risk mitigation methods in the area of global supply chain management. Recent economic uncertainties such as the U.S. credit and the Greek debt crises, and natural disasters such as the Japanese tsunami create a greater amount of uncertainty and risk for global supply chain operations. The goal of the dissertation is to develop new prescriptive policies for multinational corporations and global relief organizations in order to assist them in mitigating the risks present in their operating environment in an effective manner.

The first essay of the dissertation investigates how a multinational corporation can effectively hedge against the exchange-rate risk by structuring and managing a supply chain in a global setting. Economic uncertainties such as the U.S. credit crises and the debt concerns in southern European countries (e.g. Greece, Italy, Portugal and Spain) call for new policies in order to operate under an increased level of uncertainty. Moreover, the fluctuations in exchange rates have increased as a result of these global economic crises. While the Yen and the US Dollar are widely considered to be more stable currencies, they can exhibit significant fluctuations in short periods of time, e.g., 6.53% in the third quarter of 2011 alone. The US Dollar reached its record low against the Japanese Yen since the Great Recession on October 24, 2011, trading at 75.71 Yen/USD. More importantly, such fluctuations in exchange rates can have a profound effect on the bottom-line profits of global manufacturers. Thus, it is necessary to provide multinational corporations with prescriptions for ways to cope with the increased amount of global economic uncertainty.

The second essay of the dissertation examines how a global relief organization can effectively hedge against the risk of demand uncertainty in its distribution of essential products

to the areas of urgent needs. It is commonly observed that deficiencies in information infrastructure or economic instability contribute to the adversity of forecasting the need for essential supplies. This study stems from recognizing the ineffectiveness in the distribution of humanitarian aid supply in countries of need. Such distribution inefficiencies can lead to harmful consequences for many people. United Nations Children's Fund (UNICEF) reports that in 2007 alone, 9.2 million children worldwide under the age of five died from largely preventable causes (e.g., illnesses such as pneumonia and malaria, malnutrition, lack of access to safe water). Natural and man-made disasters continue to threaten numerous lives around the world. It is important to recognize that many of the well-developed theories are hard to apply in these countries because of inferior infrastructure and the unstable nature of the economy and political environment. Thus, it is critical to develop new policies and insights for the managers of humanitarian organizations in order to increase the efficiency of distribution operations under uncertainty. Specifically, the second essay of this dissertation demonstrates that a relief organization can minimize the risk of shortages by strategically utilizing its limited budget on procurement and the transportation of essential supplies.

1.2 Overview of Essay 1

The first essay analyzes the influence of exchange-rate risk on a global manufacturer's pricing and production planning decisions. The firm determines its price and production amount in the presence of exchange rate uncertainty with the objective of maximizing its expected profit while complying with a risk mitigation constraint. Upon the completion of production, the firm observes the realization of the random exchange rate, and based on this value, it allocates these products to various markets. An interesting element of this essay is that the firm can produce and supply less than its total global demand. This conservative action of committing to less than the

maximum amount that can be sold is defined as production hedging. The firm also complies with a Value at Risk constraint where its losses are limited in amount and probability.

The main findings of this study are: (1) production hedging is not only a legitimate scheme to maximize expected profits, but also an effective policy to reduce risk. Even under extreme risk aversion, for example, production hedging can eliminate the possibility of risk completely; (2) the more risk-averse the firm is, the more likely it prefers production hedging; (3) the firm uses both price and production quantity as levers to reduce risk; (4) contrary to the common wisdom that the firm would increase its selling price under production hedging, it benefits by reducing the optimal selling price below the alternative of satisfying the total global demand; (5) the optimal selling price can be even lower than the unit manufacturing cost despite the fact that the firm is not trying to remove competition, and thus, this result cannot be classified as predatory pricing; (6) the firm can manufacture more products under production hedging than it does under a policy of producing total global demand; (7) production hedging is still a viable and potentially optimal policy even in the presence of financial hedging.

1.3 Overview of Essay 2

The second essay investigates how relief organizations can minimize the shortage of supplying essential products to regions of desperate need in the presence of demand uncertainty with budget constraints. The difficulty of forecasting the need for essential supplies is a big obstacle for relief organizations in order to effectively supply medical and nutritional products within a given budget and timeframe. This study considers a not-for-profit humanitarian organization that has a limited budget. Given the amount of its budget, the organization has to procure medical and nutritional supplies prior to realizing demand. The organization also has to make a decision regarding whether to supply them using surface transportation in advance in

3

order to provide a faster response at times of emergency. Moreover, the firm can hold back some of its supplies in order to respond to breakout cases using air transportation. Air transportation is more expensive, but the shipment of the supplies can be postponed until after random demand is realized. Thus, given a limited budget, the firm has to make trade-offs between cheaper surface transportation with higher amounts of supply versus expensive, yet responsive, air transportation in its distribution decisions.

This study makes four main conclusions. We use service level, percentage of the population in need of humanitarian supplies that do not experience stockouts, as the primary performance metrics in developing our conclusions. These four major findings can be listed as follows: (1) If the organization has no supply reserved for air shipment or the demand variances are equal between markets, it is optimal to provide equal amounts of essential goods, resulting in equal service levels through surface transportation. However, when some supply is reserved for air shipment and the variances in market demand are not equal, it is optimal to send more essential goods in terms of service levels to the regions that exhibit greater variances in demand; (2) given a fixed amount of total variance, a relief organization is better off having a greater amount of distortion in demand than having balanced variation between market demands; (3) an organization can improve its performance if its budget increases proportionally with the number of regions to serve; (4) a relief organization is better off being assigned non-correlated regions as opposed to positively correlated regions.

CHAPTER 2: ESSAY 1 – RISK MITIGATION OF PRODUCTION HEDGING

2.1 Introduction

On October 24, 2011, the US Dollar reached its lowest ever against the Japanese Yen since World War II, trading at 75.71 Yen/USD. A consequence of this is that it has become virtually impossible for Japanese firms that operate with small margins to remain profitable. Toyota is an example of a company that operates with small margins while manufacturing more than 50% of its cars in Japan and selling a significant proportion of them in the US. When their revenues from the US sales are repatriated, Toyota struggles to remain profitable. In recent years, the fluctuations in exchange rates have increased due to various global economic crises (e.g., the US credit crisis, Japanese tsunami disaster, the Greek debt crisis, etc.), resulting in a greater amount of uncertainty for global businesses. Such fluctuations in exchange rates alone can have a profound effect on the bottom-line profits of global manufacturers. The recent global economic crises has caused a greater need for research in global supply chains, and makes it necessary to provide multinational corporations with prescriptions to cope with the increased amount of global economic uncertainty. This essay responds to this need by investigating the influence of exchange-rate risk on a global manufacturer's pricing and manufacturing decisions.

We consider a global manufacturer that produces a product in one country, and sells it in two countries: one domestic and one international where the revenues, in terms of domestic currency, fluctuate with exchange rates. We model the problem as a two-stage stochastic program, where the firm initially determines a production amount, and after observing the realization of the random exchange rate variable, decides how to allocate its products to two markets. Thus, the initial production decision is made in the presence of exchange-rate uncertainty, and the allocation to markets is made after uncertainty is resolved.

In the problem setting motivating our study, we consider a firm that determines the selling price at the beginning of the planning period, and prior to observing the realized value of the random exchange rate. There are various examples of this practice from different industries. A case study of Harvard Business School, Zara: Fast Fashion (HBS 9-703-497), describes the scenario where the firm applies the price tags on the garment before it gets shipped to its retail stores around the globe. The case makes it clear that Zara determines the selling price of its garment to be sold at various markets before realizing exchange rates, and prior to its production and distribution.

Price-setting in the presence of uncertainty is common among companies that utilize catalog sales and marketing techniques. McMaster-Carr, for example, produces over 480,000 products (SKUs) and sells them all around the world. They typically set their marketing plans for the quarter and send catalogs out to dealers prior to the quarter. In this practice, the selling price for each product (SKU) is determined prior to the quarter. The firm cannot adjust the selling price of individual products with respect to the exchange rate fluctuations as this would be: (1) difficult to implement, and (2) would hurt sales. For firms that set the selling price in advance (such as those firms that sell through catalogs), determining the production quantity and the selling price for each market are not only crucial in order to maximize profits, but also to mitigate risks.

Compared to firms that have the pricing flexibility through the postponement of the pricing decision until after the realization of exchange rates, firms such as Zara and McMaster-Carr are exposed to higher risks since they cannot adjust the selling price in a market in response to the fluctuating exchange rate, and have greater needs for policies that hedge against exchange rate risks in an early pricing scheme. Our study responds to this need by providing an analysis with

6

the pricing and production decisions made in the presence of exchange-rate risk. We next describe the unique features of our study.

Kazaz et al. (2005) is the first to introduce production hedging. They describe production hedging as a policy where the firm deliberately produces less than the total global demand under exchange rate uncertainty. While their study defines production hedging as a potentially optimal policy for a risk-neutral firm, they do not characterize the optimal solution or provide any information about the pricing (and production) behavior. Our study differs from Kazaz et al. (2005) in three ways: (1) we consider a risk-averse firm, (2) characterize how optimal price and production decisions vary at various levels of risk aversion, and (3) while Kazaz et al. (2005) does not account for the international dumping laws (by offering the same product at different prices in each market), our study develops pricing and production policies that comply with this requirement.[1] Examples of the behavior of price and quantity that are not presented in Kazaz et al. (2005) include the following:

1. One may intuit that the firm might increase its selling price under a production hedging policy since the production amount is less than the total demand. However, we show that the firm might actually decrease its choice of the optimal selling price below the equivalent when it manufactures the total demand. The firm reduces its optimal selling price in order to maximize the cherry-picking flexibility that postponed allocation decisions provide.

2. The firm might produce more units under a production hedging policy than it does when it aims to satisfy the total global demand.

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[1] The anti-dumping laws in the General Agreement on Tariffs and Trade (GATT) were originally developed in 1947, and were left unchanged in the adjustments in 1994 and are the underlying set of principles for the World Trade Organization legislation in 1995. Article VI. 1. (a) describes anti-dumping when the following applies: "If the price of the product exported from one country to another is less than the comparable price for the like product when destined for consumption in the exporting country." In business and economics, because multinational firms have landed operations that generate taxable revenues in multiple countries, the anti-dumping law prohibits the firm from declaring a different price in each market at the time of the pricing decision.

3. The optimal selling price can be even lower than the unit manufacturing cost. This

   unexpected result might be incorrectly described as predatory pricing[2] (see Ordover and

   Willig 1981 for its legal description) where the firm determines a selling price below its cost

   and commits to losses for a period of time in order to drive the competitor out of the market.

   Our model, however, does not feature competition, and the optimal selling price is not

   determined in such a way to eliminate the competitor from the market or gain market

   power/share, but rather to maximize expected profits while limiting potential losses. Thus,

   the result cannot be subject to the laws prohibiting predatory pricing.[3]

One common theme in the above counter-intuitive results is that they occur when the firm's unit

cost of manufacturing is high.

    Our analysis introduces a new constraint ensuring that the firm does not lose money more

than a certain amount with a limited probability. This corresponds to a value at risk (VaR)

measure that finds wide acceptance among practicing risk managers. VaR is the preferred

approach in the Basel II Accord, which specifies the banking laws and regulations issued by the

Basel Committee on Banking Supervision (2005). We show that, under risk aversion, a

production hedging policy leads to less risky global production planning decision than the

traditional policy that commits to the production of the total global demand. Moreover, the

interval of unit manufacturing costs that make the policy of manufacturing the total demand

optimal decreases under risk aversion and the interval of unit manufacturing costs that make the

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[2] The international laws established by the General Agreement on Tariffs and Trade clearly states that selling a product below cost can only be classified as dumping when there is no identifiable price in other markets. In our problem, the firm determines a selling price in both markets. The availability of a price in another market eliminates the possibility of classifying the action of selling below cost as dumping.

[3] The game theory literature has identified other reasons why a supplier may set a selling price below cost. Rosenthal (1981) describes the scenario when the firm aims to become a monopolist in the market. In the presence of learning curves, Cabral and Riordan (1984) rationalize predatory pricing from the perspective of increasing market share and power. Our result is characteristically different than these earlier findings, because in our setting, the firm does not increase price once it establishes a higher market share and market power.

production hedging policy optimal increases. Thus, we conclude that production hedging is more frequently preferred by a risk-averse firm than a risk-neutral firm. The size of the international market also plays a role in deciding whether to implement production hedging. Higher foreign market size implies higher risk, and increases the importance of the risk mitigation aspect of the optimal policies. Finally, the variation in exchange rate uncertainty influences the firm's policy decision; higher variation makes production hedging a more attractive policy.

We argue that our production hedging policy helps justify similar practices from various businesses. Zara is an example of companies that value *scarcity* as a marketing strategy to drive sales at the posted listing price rather than at discounted prices. Zara's practice of deliberately under-producing garments resembles production hedging and provides support for its existence and benefits in practice.

The rest of the essay is organized as follows: Section 2.2 provides the review of the literature. Section 2.3 introduces the model and develops the structural properties of the potentially optimal policies. Section 2.4 provides the impact of the unit manufacturing cost on the optimal price and quantity decisions, and the corresponding profit. Section 2.5 provides the impact of financial hedging. Section 2.6 presents a discussion on the influence of potential extensions to the model. Section 2.7 presents conclusions and managerial insights. Section 2.8 provides future research directions for this essay. Section 2.9 provides the appendix where Section 2.9.1 has all proofs and derivations; Section 2.9.2 presents a comprehensive analysis of the special case of extreme risk aversion; Section 2.9.3 demonstrates that our results continue to hold if the firm can set two different prices in each market—a practice that violates anti-dumping laws.

2.2 Related Literature

Research founded on stochastic exchange rates has been active since the break-up of the fixed Bretton Woods system and the alteration to the floating exchange rates in the early 1970s. The mainstream of research in the starting phases was focused on reducing the associated risks by exercising financial instruments, such as currency options or forward contracts (see Cornell and Reinganum 1981, Biger and Hull 1983, Shastri and Tandon 1986, Bodurtha and Courtadon 1986).

While financial schemes dominated the early stages of the literature on hedging against exchange rate uncertainty, Kogut and Kulatilaka (1994) and Huchzermeier and Cohen (1996) introduced operational hedging schemes which stimulated research in the global supply chain literature. Huchzermeier and Cohen (1996) not only demonstrate the validity of hedging via the configuration of the supply chain network, but also claim that there are two major advantages of operational hedging schemes over financial hedging schemes. They list these advantages as follows: (1) contrary to financial hedging instruments where the costs ascend as the planning horizon increases, the cost for operational hedging decreases because the switching costs can be spread over the time horizon, and (2) operational hedging has the advantage of not being restricted to major currencies.

The majority of the global supply chain literature has been investigated under flexibility accrued through excess capacity in multi-national production facilities. Kogut and Kulatilaka (1994) demonstrate that having a supply network located in different countries is equivalent to owning an option, where the value is dependent upon the volatility of the exchange rate. Huchzermeier and Cohen (1996) numerically exhibit that hedging against exchange rate risks can be accomplished by having excess capacity across internationally dispersed sites. Through

the Harvard Business School case of Applichem (Flaherty, 1985), Lowe et al. (2002) evaluate the value of having excess capacity. Rosenfield (1996) proves analytically, under stylized models, that excess capacity can reduce costs compared to a single plant. Kouvelis et al. (2001) focus on the choice of ownership strategies among exporting, joint venture, and wholly owned production facilities for the foreign production facility where they support their findings with empirical results. Chod and Rudi (2006) study the equilibrium investment level of two independent firms located in different countries with pricing flexibility in a game theoretic model.

Based on the assumption of having excess capacity, a significant part of the literature focuses on either finding the optimal supply chain configuration or the optimal operating polices for a multinational company. Dasu and Li (1997) concentrate on the optimal production allocation policy to minimize the production cost assuming a stochastic exchange rate and switching costs. Li et al. (2001) complement Dasu and Li (1997) by having the same objective of pursuing the optimal operating policies, but with different assumptions in the model which are: (1) stochastic demand and processing times, (2) make-to-stock environment. Aytekin and Birge (2004) and Dong et al. (2010) both consider the optimal supply chain configuration and the optimal operating policy for the associated configuration under stochastic demand and exchange rates. The key difference between the two articles is that Dong et al. (2010) include responsive pricing in their analysis. Kouvelis and Gutierrez (1997) focus on the production quantity and the coordination between decentralized production facilities under demand and exchange rate uncertainty. The key difference is that a multinational firm sells its "style goods" in two non-overlapping selling seasons.

Contrary to the majority of the operational hedging related literature, Kazaz et al. (2005) introduce a different operational hedging scheme called production hedging, gaining flexibility by deliberately producing less than the global demand to cope with the uncertain exchange rate. It can be viewed as an opposing operational hedging policy to having excess capacity in multiple countries. By deliberately producing less than the global demand, the firm gains flexibility to select between markets based on the realized exchange rate. The major advantage for corporations that choose the production hedging policy is that the initial investment would be significantly lower compared to policies that gain flexibility through excess capacity. Moreover, Kazaz et al. (2005) prove that under production hedging, the expected profit can be greater compared to fulfilling the global demand which is implicitly assumed in the literature.

To the best of our knowledge, there has not been any additional work on operational hedging achieved by under-producing. Kazaz et al. (2005) allows for independent setting of the selling price in each market, and thus, the firm is likely to violate the anti-dumping laws. Our study, on the other hand, reflects a more realistic setting from a planning perspective, where the ex-ante selling price eliminates potential consideration of dumping. Our work complements Kazaz et al. (2005) in three ways: (1) Whereas Kazaz et al. (2005) solely view the production hedging policy as a profit maximizing scheme in a risk-neutral perspective, our work investigates how production hedging reduces the exchange rate risk for a risk-averse firm and introduces modified production hedging policies that limits the risk of loss; (2) whereas Kazaz et al. (2005) focus on the merits of production hedging as an operational policy under various models, our work characterizes the optimal price and production quantity decisions, (3) our optimal policy structure differs from the set of potentially optimal policies of Kazaz et al. (2005) due to the additional risk and price constraints. For example, even in the risk-neutral analysis, Kazaz et al.

12

(2005) eliminates one of the potentially optimal policies, corresponding to the production of the minimum demand, from the potentially optimal set of policies, due to the fact that selling prices are determined independently. Not only is our pricing scheme more desirable and reflects the reality from an economic perspective, but the new set of optimal policies warrant additional investigation and richer insights.

Few papers incorporate a risk-averse perspective in the global supply chain literature. Myall and Thiele (2007) compare three well-known risk measures: standard deviation, shortfall, and VaR and provide an analysis of how the decision-makers' risk preference influences the operational hedging policy. Ding et al. (2007) and Zhu and Kapuscinski (2011) incorporate a risk-averse perspective by using a mean-variance objective function and the present certainty equivalent value (PCEV) measure in a multi-period setting, respectively, to decide the optimal operational and financial hedging decisions in the presence of demand and exchange rate uncertainty. Other studies that utilize both operational and financial instruments to hedge against exchange rate uncertainty include Mello et al. (1995) and Chowdhry and Howe (1999). Van Mieghem (2007) also utilizes a mean-variance objective function in the presence of demand uncertainty alone, and finds that a risk-averse firm will invest in greater capacities compared to a risk-neutral firm. Under demand uncertainty that is correlated with the price of financial asset, Gaur and Seshadri (2005) incorporate two risk-averse frameworks, the mean-variance and the utility-maximization frameworks, to prove that the optimal inventory level increases when a firm utilizes financial hedging. Under stochastic spot and future prices of commodities, Devalkar et al. (2007) consider an integrated optimization problem for a firm involved in procurement, processing and trading of commodities. Using the VaR measure for risk aversion, they prove that the optimal procurement quantity for a risk-averse firm is never greater than that for a risk-

neutral firm. Our study shows both behaviors as possible actions: similar to the finding of

Devalkar et al. (2007), risk aversion leads to smaller initial manufacturing investment with lower

unit manufacturing costs, and similar to the findings of Van Mieghem (2007) and Gaur and

Seshadri (2005), the firm can produce a larger number of products under higher unit

manufacturing costs.

2.3 The Model

This section presents the modeling approach used for the pricing and production planning

problem of a multinational corporation that sells in both the domestic and the foreign market and

experiences exchange rate uncertainty. The model is a two-stage stochastic program, where the

first stage corresponds to the pricing and manufacturing decisions, and after the exchange rate is

realized, the firm allocates its production to each market in the second stage.

Before the realization of the exchange rate, the firm determines the total production quantity,

denoted X, at a unit manufacturing cost, denoted c, and the selling price in each market p_i, where

$i = 1$ corresponds to the home market and $i = 2$ corresponds to the foreign market. At the time

that prices are determined, the firm is assumed to avoid dumping, and therefore sets a selling

price in the foreign market such that when divided by the mean exchange rate, it equals the same

return from the home market, i.e., $p_2 = p_1 / \bar{e}$. As a result, at the time of determining prices, the

(expected) return from each market is equal, i.e., $p_1 = p_2 \bar{e}$, and the firm complies with the anti-

dumping laws.[4] Randomness in the exchange rate is represented by $\tilde{e}$, where e is the realization,

$f(e)$ is the pdf defined on a support $[e_l, e_h]$ with a mean $\bar{e} = E[\tilde{e}]$ where $e_h > e_l > 0$. We make no

assumptions regarding the distribution of $f(e)$, except that we scale it such that $\bar{e} = 1$ without loss

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[4] As the random exchange rate fluctuates, the actual return from a sale in the foreign market differs from that of the home market. Because the selling price is determined in the presence of exchange-rate uncertainty, the firm's pricing decision is not subject to dumping, even if the firm does not adjust its selling price in the foreign market instantaneously with exchange rate fluctuations.

of generality which results in a single selling price p that applies in both markets, i.e., $p_1 = p_2 \bar{e} = p$. Therefore the conclusions of the study are robust as the results hold under any arbitrary probability distribution function with a positive support.

Demand $d_i(p)$ is decreasing in price, and its unique inverse is denoted $p(d_i)$. We assume that revenue $pd_i(p)$ is concave, i.e., $2d_{ip}(p) + pd_{ipp}(p) \leq 0$ where $d_{ip}(p)$ and $d_{ipp}(p)$ represent the first and second derivatives of the demand function d_i with respect to price.

We utilize the VaR measure to limit the risk under a chosen pair of decision variables in the first stage. In VaR, two parameters describe the firm's risk preference: β represents the loss (value at risk) that the firm is willing to tolerate at probability α. For the VaR measure, there exists a corresponding exchange rate e_α that satisfies $\int_{e_l}^{e_\alpha} f(e) = \alpha$ where $0 \leq \alpha \leq 1$ and the value at risk is the loss at the exchange rate realization $e = e_\alpha$. Note that $\alpha = 0$ and $\beta = 0$ is the case of extreme risk aversion where the firm prefers to eliminate the possibility of loss completely.

The model is a two-stage stochastic program with recourse, where the firm simultaneously chooses the optimal production quantity X and the optimal selling price p in Stage 1. Given the realized exchange rate, at the beginning of Stage 2, the firm determines the allocation quantity to the home and foreign markets, defined as x_1 and x_2, respectively. The sum of the allocation to markets cannot exceed the first-stage production quantity, i.e., $x_1 + x_2 \leq X$.

Our model introduces a constraint that limits the firm's tolerable loss according to the VaR risk measure in Stage 1. If VaR for a given α, X, and p, is more than the tolerable loss β, the pair of decision variables is an infeasible solution, since the risk exceeds the tolerable amount for the firm. For production allocation x_1 and x_2, the VaR constraint is

$$-cX + \max_{\substack{(x_1, x_2) \geq 0 \\ x_1 + x_2 \leq X}} \pi\left(x_1, x_2 \,|\, X, p, e_\alpha\right) \geq -\beta \tag{2.1}$$

where $\pi\left(x_1, x_2 \mid X, p, e_\alpha\right)$ is the second-stage profit function. Constraint (2.1) ensures that the pair

of decision variables (*X* and *p*) results in a feasible operational policy. As a result, β can be

perceived as a measure of risk aversion because the more risk-averse a firm is the lower the

value of β is for a given α.

The model can be expressed as follows:

Stage 1:
$$\max_{(X,p)\geq 0} E\left[\Pi(X, p)\right] = -cX + \int_{e_l}^{e_h} P(X, p, e) f(e) de \tag{2.2}$$

subject to (2.1).

Stage 2: Given *X*, *p*, and *e*, $P(X, p, e) = \max_{\substack{(x_1, x_2)\geq 0 \\ x_1+x_2\leq X \\ x_i\leq d_i}} \pi\left(x_1, x_2 \mid X, p, e\right) = px_1 + pex_2$.

From $\partial\pi / \partial x_i > 0$, it follows that $x_1 + x_2 = X$ in an optimal solution to the second-stage problem.

Thus, the second-stage problem can be restated as

$$P(X, p, e) = \max_{\substack{(x_1, x_2)\geq 0 \\ x_1+x_2=X \\ x_i\leq d_i}} \left\{\pi\left(x_1, x_2 \mid X, p, e\right) = px_1 + pex_2\right\}. \tag{2.3}$$

Thus, the combinations of (*X, p*) that violate (2.1) are infeasible solutions. The combinations of

(*X, p*) that satisfy (2.1) ensures that the chosen operational policy will result in a profit of at least

$-\beta$ with probability 1– α. For example, if $\alpha = 0.05$, $\beta = 500$, and (2.1) is satisfied, the firm's

profit will be more than –500 with a confidence level of 95%. When $\alpha = 1$, $\beta = \infty$, our model is

equivalent to the risk-neutral firm as (2.1) becomes $-cX + \pi(x_1, x_2 \mid X, p, e_h) \geq -\infty$ and is

guaranteed to be non-binding with probability 1. Risk-averse firms are generally concerned with

protecting their losses, rather than taking advantage of the upside potential in exchange rates. To

reflect this nature, the remaining analysis assumes that $e_\alpha \leq \bar{e}$ and $\beta \geq 0$.

2.3.1 Allocation Decisions and the Impact of Exchange-Rate Uncertainty

We proceed with the analysis of the optimal allocation decisions in the second stage for a given production quantity and price. If $e < \bar{e}$, then the revenue from the foreign market is valued less than the domestic revenue; therefore, the firm prioritizes the home market and allocates the products in order to satisfy the home market demand before the foreign market demand. If the firm has any leftovers after satisfying the home market demand, then they are sold in the foreign market. If $e \geq \bar{e}$, however, the revenue from the domestic market is valued less than the foreign revenue. In this scenario, the firm prioritizes the foreign market in its allocation decisions. If the firm has any leftovers after satisfying the foreign market demand, then they are sold in the domestic market. Before proceeding with the analysis, let us define θ as the difference between the cdf and the partial expectation until the switching point for the allocation preference in the second stage (i.e., $e = \bar{e}$):

$$\theta = \int_{e_l}^{\bar{e}} \bar{e} f(e) \, de - \int_{e_l}^{\bar{e}} e f(e) \, de = \int_{\bar{e}}^{e_h} e f(e) \, de - \int_{\bar{e}}^{e_h} \bar{e} f(e) \, de \tag{2.4}$$

As shown in the middle expression of (2.4), θ can be regarded as the "expected loss" for each dollar revenue from selling in the less-desirable foreign market as opposed to the revenue that can be generated from selling in the more-desirable domestic market. When the realized foreign currency is below $\bar{e}$ (it is less desirable to sell in the foreign market), selling in the domestic currency gives the firm a multiplier of 1 (i.e., no risk scalar), while selling in the foreign market gives a multiplier of e scaling the revenue due to the exchange-rate risk. For several common distributions (e.g., normal, uniform, exponential), Section 2.9.1 shows that θ is proportional to the standard deviation. Thus, the value of θ increases with higher variance in exchange rates. As shown in the far-right expression of (2.4), θ can also be regarded as the upside potential corresponding to the expected gain from selling in the foreign market. Thus, we see that θ is a

17

rather rich and illuminating measure: (1) it is an indicator of exchange rate volatility, (2) it is an indicator of downside risk protection associated with producing less than global demand (e.g., the larger the value of θ, the larger the value of the flexibility to not serve the foreign market when the exchange rate is low), and (3) it is an indicator of the upside profit potential of the foreign market over the home market when the exchange rate is high.

2.3.2 Potentially Optimal Policies

The combined pricing and production planning problem presented in (2.2) $-$ (2.3) is not jointly concave in its decision variables. Therefore, it is necessary to explore the structural properties in order to provide insight into the potentially optimal decisions. First, it should be observed that the firm would never manufacture more than its total demand in both markets. Using these structural properties, there are seven potentially optimal policies, which we define as follows:

1. $X = d_1 + d_2$ as the Total Demand (TD) policy,

2. $\max\{d_1, d_2\} < X < d_1 + d_2$ as the Production Hedging Interior (PHI) policy,

3. $X = \max\{d_1, d_2\}$ as the Production Hedging Maximum (PHX) policy,

4. $\min\{d_1, d_2\} < X < \max\{d_1, d_2\}$ as the Production Hedging Interior less than Maximum (PHIX) policy,

5. $X = \min\{d_1, d_2\}$ as the Production Hedging Minimum (PHN) policy,

6. $X < \min\{d_1, d_2\}$ as the Production Hedging Interior less than Minimum (PHIN) policy, and

7. $X = 0$ as the No Production (NP) policy.

The TD policy is where the firm satisfies the demand in both markets. In traditional aggregate planning methods, it is assumed that the firm has to satisfy the demand in both

18

markets. The TD policy is aligned with this traditional perspective. Policies PHI, PHX, PHIX, PHN, and PHIN are production hedging policies where the firm deliberately produces a smaller amount than the global demand. The firm does not produce under the NP policy. The optimal selling price can be determined separately for PHX and PHN policies, where the firm produces exactly one of the two demand values. Policies PHI, PHIX, and PHIN feature interior points in manufacturing quantities which are not equal to any of the two demand values. These interior point policies would not exist in the risk-neutral setting corresponding to the special case with α = 1 and β = ∞. Specifically, they appear in the set of potentially optimal solutions only when the risk constraint (2.1) is binding. Figure 2.1 depicts the optimal production quantity for a given selling price for two different cost parameters, c = 30 in part (a) and c = 60 in part (b), under linear demand functions, d_1 = 100 – p, d_2 = 130 – 1.5p, and when the exchange rate has a uniform distribution with a support of [0, 2], and under risk parameters α = 1 and β = ∞. Figure 2.2 shows the influence of the VaR perspective, and demonstrates how these production decisions get revised for the same set of parameters under a binding risk constraint with α = 0 and β = 0.
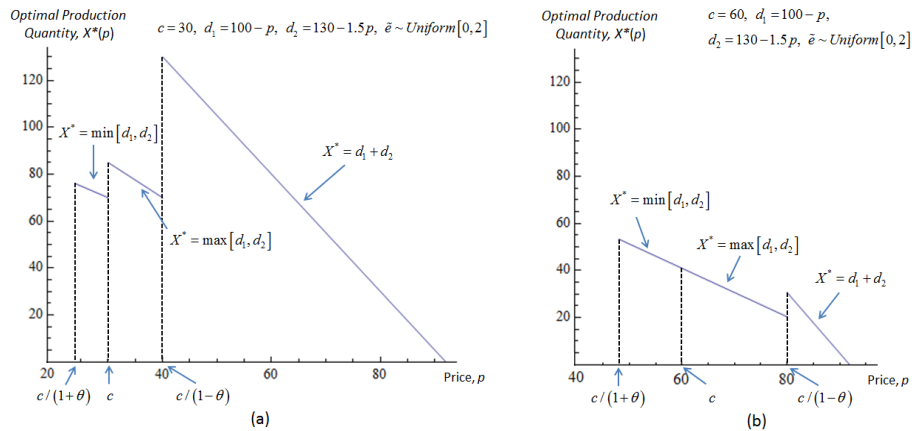
Figure 2.1 Optimal production choices at each price when VaR constraint is not binding.

19
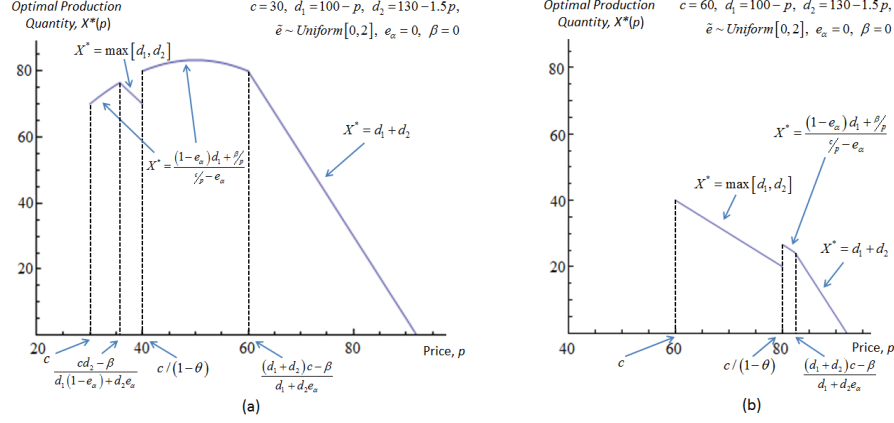
Figure 2.2 Optimal production choices at each price when the VaR constraint is binding.

The following proposition provides the optimal production decisions for a given price level under the VaR constraint. Moreover, it shows that only six of the above seven policies are candidates for the optimal policy.

Proposition 2.1. *a) The objective function* (2.2) *is concave in X for a given p; b) For a given price level, there are six potentially optimal production policies:*

i) $c/(1+\theta) \leq p < \min(\max(c/(1+\theta), c - (\beta/\min(d_1,d_2))), c)$*, then* $X^* = \beta/(c-p) < \min(d_1,d_2) \Rightarrow$ PHIN;

ii) $\min(\max(c/(1+\theta), c - (\beta/\min(d_1,d_2))), c) \leq p < c$*, then* $X^* = \min(d_1,d_2) \Rightarrow$ PHN;

iii) $c \leq p < \min(\max(c, (cd_2 - \beta)/(d_1(1 - e_\alpha)+d_2 e_\alpha)), c/(1 - \theta))$*, then* $d_1 < X^* = ((1 - e_\alpha)d_1+(\beta/p))/((c/p) - e_\alpha) < d_2 \Rightarrow$ PHIX;

iv) $\min(\max(c, (cd_2 - \beta)/(d_1(1 - e_\alpha)+d_2 e_\alpha)), c/(1 - \theta)) \leq p \leq c/(1 - \theta)$*, then* $X^* = \max(d_1,d_2) \Rightarrow$ PHX;

v) $c/(1 - \theta) < p < \max(((d_1+d_2)c - \beta)/(d_1 + d_2 e_\alpha), c/(1 - \theta))$*, then* $\max(d_1,d_2) < X^* = ((1 - e_\alpha)d_1+(\beta/p))/((c/p) - e_\alpha) < d_1+d_2 \Rightarrow$ PHI;

vi) $\max(((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha), c/(1 - \theta)) \leq p$*, then* $X^* = d_1+d_2 \Rightarrow$ TD;

20

c) policy PHX *satisfies the risk constraint* (2.1) *and dominates policy* PHIX *when* $d_1 \geq d_2$ *for* $c \leq p < c/(1 - \theta)$; *d) policy* NP *cannot be the optimal solution.*

The above proposition provides several insights about the problem. First, the firm would not engage in production if the optimal price is less than $c/(1 + \theta)$; however, the proposition shows that at least one policy will have a price higher than $c/(1 + \theta)$ and have positive (expected) profit, eliminating the NP policy. Proposition 2.1 also shows that policy PHIX cannot be optimal when $d_1 > d_2$, because policy PHX is risk free and dominates the expected profit of PHIX at every price in the interval of $c \leq p \leq c/(1 - \theta)$.

Proposition 2.1 proves that the firm is capable of producing when the price is less than the unit manufacturing cost, and yet, generate positive expected profit while satisfying constraint (2.1) according to PHIN and PHN policies. This result implies that the firm is willing to take a loss in the domestic market and in the foreign market at the expected exchange-rate, but the probability of upside potential of the exchange rate can create profits so much that these policies can yield a positive expected profit. Thus, the firm can be viewed to be betting on the potential appreciation of the random exchange rate in these policies.

Even when limited to $\alpha = 1$ and $\beta = \infty$ (i.e., the risk neutral case), Proposition 2.1 differs from a similar result presented in Kazaz et al. (2005), which allows the firm to set a different price in each market at the expense of violating the anti-dumping laws. The consequence of the pricing flexibility is that the firm can eliminate one of the potentially optimal production hedging policies. In Theorem 7 of Kazaz et al. (2005), the set of potentially optimal policies does not include the minimum of the two demand values because the firm can adjust its selling price in a less dominant market, resulting in the elimination of the minimum demand policy. However, in more realistic settings that reflect pricing with the consideration of anti-dumping laws, the policy

where production equals the minimum demand cannot be eliminated from the set of potentially optimal policies. Kazaz et al. (2005) also does not feature any interior point solutions as in policies PHI, PHIX and PHIN either; these policies are consequences of the VaR constraint.

We next establish the structural properties of the potentially optimal policies with respect to price.

Proposition 2.2 *a) The objective function* (2.2) *is concave in p for a given X, however, is not jointly concave in p and X; b) Policies* TD, PHX, PHN *are all concave in p; c) Policies* PHI, PHIX, *and* PHIN *are not concave in p. Thus, a line search is necessary to determine the optimal price and production quantity; d) Policy* PHIN *cannot be the optimal solution; e) the optimal price is never less than c/(1+ θ).*

The consequence of the above proposition is the firm always sets a price greater than or equal to $c/(1+\theta)$. Proposition 2.1 has already shown that the firm always manufactures a positive quantity when $p > c/(1+\theta)$; thus, $c/(1+\theta)$ can be considered as the minimum price level. Considering that there can be a price that drives at least one of the demand curves to zero, the firm is able to establish a minimum and a maximum price level.

Proposition 2.2 shows that policies PHX and PHN where the firm produces exactly one of the two demand values have a well-behaving objective function. However, the risk constraint creates additional complexity for interior point solutions in PHI, PHIX and PHIN policies. Proposition 2.2 proves that policy PHIN cannot be optimal because when the risk-neutral solutions do not satisfy constraint (2.1), the firm is forced to increase price in a way that production meets the minimum of the two demands. As a result, the firm modifies its PHN policy. This is a clear example of when the firm uses its pricing lever to adjust its policy to comply with (2.1).

As a result of propositions 2.1 and 2.2, we conclude that there are only five potentially optimal policies for the problem presented in (2.1) – (2.3). We next investigate the structural properties of each of these five policies. The optimal selling price and production quantities for each policy under binding and non-binding VaR constraints are presented in Table 2.1.

Policy		VaR Constraint Binding?	Optimal Price, p^*	Optimal Production Quantity, X^*
PHN	$d_1 \geq d_2$	nb	$p^{PHN} = \dfrac{c}{1+\theta} - \dfrac{d_2}{d_{2p}}$	$X^{PHN} = d_2^{PHN} = \left[p - \dfrac{c}{1+\theta}\right]\left[-d_{2p}\right]$
		b	$p^{PHN} = c - \dfrac{\beta}{d_2}$	$X^{PHN} = d_2^{PHN} = \dfrac{\beta}{c-p}$
	$d_1 < d_2$	nb	$p^{PHN} = \dfrac{c}{1+\theta} - \dfrac{d_1}{d_{1p}}$	$X^{PHN} = d_1^{PHN} = \left[p - \dfrac{c}{1+\theta}\right]\left[-d_{1p}\right]$
		b	$p^{PHN} = c - \dfrac{\beta}{d_1}$	$X^{PHN} = d_1^{PHN} = \dfrac{\beta}{c-p}$
PHIX	-	b	$p^{PHIX} = \dfrac{cX - \beta}{(1-e_\alpha)d_1 + e_\alpha X}$	$X^{PHIX} = \dfrac{(1-e_\alpha)d_1 + \beta/p}{\beta/p - e_\alpha}$
PHX	$d_1 \geq d_2$	nb	$p^{PHX} = -\dfrac{d_1 - cd_{1p} + d_2\theta}{d_{1p} + d_{2p}\theta}$	$X^{PHX} = d_1^{PHX} = -p\left[d_{1p} + d_{2p}\theta\right] + cd_{1p} - d_2\theta$
	$d_1 < d_2$	nb	$p^{PHX} = -\dfrac{d_1\theta + d_2 - cd_{2p}}{d_{1p}\theta + d_{2p}}$	$X^{PHX} = d_2^{PHX} = -p\left[d_{1p}\theta + d_{2p}\right] - d_1\theta + cd_{2p}$
		b	$p^{PHX} = \dfrac{cd_2 - \beta}{(1-e_\alpha)d_1 + e_\alpha d_1}$	$X^{PHX} = d_2^{PHX} = \dfrac{p\left[(1-e_\alpha)d_1 + e_\alpha d_1\right] + \beta}{c}$
PHI	-	b	$p^{PHI} = \dfrac{cX - \beta}{(1-e_\alpha)d_1 + e_\alpha X}$	$X^{PHI} = \dfrac{(1-e_\alpha)d_1 + \beta/p}{\beta/p - e_\alpha}$
TD	-	nb	$p^{TD} = c + \dfrac{d_1 + d_2}{-(d_{1p} + d_{2p})}$	$X^{TD} = d_1^{TD} + d_2^{TD} = -(p-c)(d_{1p} + d_{2p})$
		b	$p^{TD} = \dfrac{(d_1 + d_2)c - \beta}{d_1 + e_\alpha d_2}$	$X^{TD} = d_1^{TD} + d_2^{TD} = \dfrac{\beta + (1-e_\alpha)pd_1}{c - pe_\alpha}$

Table 2.1 Optimal value expressions for alternative policies.[5]

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[5] In Table 2.1, "nb" abbreviates the scenario when the VaR constraint is "non-binding" and "b" abbreviates the scenario when it is "binding."

2.3.2.1 Total Demand (TD) Policy

The TD policy is the conventional production policy where the firm manufactures the exact

quantity that satisfies the demand in both markets. The expected profit for the TD policy is

$$E\left[\Pi^{TD}\left(X^*(p)=d_1+d_2,p\right)\right]=(p-c)d_1+(p\bar{e}-c)d_2.\qquad(2.5)$$

The optimal production quantity, price, and the corresponding demands in each market under the

TD policy are presented in Table 2.1 (the superscripts specify the production policy). Because

the demand function is not described by a specific function, a closed-form expression is not

provided.

Unlike the production hedging policies, because the firm produces to satisfy the total demand

in the TD policy, it does not have the flexibility to adjust the allocation of products based on the

exchange-rate fluctuations. From this perspective, the TD policy is rigid and less responsive to

exchange-rate uncertainty. The following remark shows that firm charges an optimal selling

price that is always greater than the unit manufacturing cost, but not necessarily always greater

than or equal to $c/(1-\theta)$.

Remark 2.1 *The optimal selling price p^{TD} in Table* 2.1 *is always greater than c, but it can also*

take values that are less than or equal to $c/(1-\theta)$, which causes the TD policy to be dominated

by production hedging.

Higher values of the unit manufacturing cost imply that the TD policy is exposed to a higher

risk of negative profits at low realizations of the exchange rate. However, when the unit

manufacturing cost is considerably low, the secure profit from the domestic market can cover the

possible losses from the foreign market, so the global firm is not exposed to the risk of negative

profits. The following proposition shows that when the risk constraint (2.1) is not satisfied, the

firm can take two alternative paths: (1) revise the TD policy to comply with the VaR constraint, or (2) switch to the PHI policy and produce below the total demand while satisfying the risk constraint (2.1).

Proposition 2.3 *The* TD *policy satisfies the risk constraint* (2.1) *if the unit manufacturing cost is*

$$c \leq \left\lfloor \frac{p^{TD}\left(d_1^{TD}+d_2^{TD}e_\alpha\right)+\beta}{d_1^{TD}+d_2^{TD}} \right\rfloor \tag{2.6}$$

where p^{TD} is the optimal non-binding expression in Table 2.1; *b) if the unit manufacturing cost does not satisfy* (2.6)*, then the firm can either follow the* TD *policy by charging a higher selling price equal to $p^{TD} = ((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha))$ in order to comply with* (2.1), *or follow the* PHI *policy.*

2.3.2.2 Production Hedging Interior (PHI) Policy

When the optimal selling price in the TD policy is greater than $c/(1+\theta)$, but the unit manufacturing cost does not satisfy (2.6), the firm can reduce its manufacturing level below the total demand, and follow the PHI policy that satisfies:

$$X^{PHI} = \frac{(1-e_\alpha)d_1^{PHI} + \beta\!\big/\!{p^{PHI}}}{c\big/\!{p^{PHI}} - e_\alpha}, \quad p^{PHI} = \frac{cX^{PHI} - \beta}{(1-e_\alpha)d_1^{PHI} + e_\alpha X^{PHI}}.$$

One important property of the PHI policy is that the selling price can show both an increasing and a decreasing behavior when compared with the optimal non-binding selling price of the TD policy (as presented in Table 2.1). Example 1 in Section 2.9.2 provides an example of both situations for an extremely risk-averse firm. In general, for lower values of the unit manufacturing cost c, the firm prefers to raise the price and decrease the production amount under the PHI policy compared with the non-binding optimal TD policy. As the unit

25

manufacturing cost c increases, however, the firm can alter its behavior and reduce its selling price below its non-binding TD equivalent. Because the reduction in selling price increases the demand in both markets, the firm can increase the amount of production above the level of the non-binding TD policy. Indeed, the firm might still be producing less than its total demand, but that quantity can be higher than the non-binding TD amount. The motivation for the reduction in the selling price stems from the fact that the firm can increase the revenue from the domestic market, which can pay for the losses incurred at the low realizations of the exchange rate. As a result of this increased home demand, the firm relies more on the domestic market, and can generate higher revenues in the home market under the PHI policy. In conclusion, the firm can exhibit a behavior where it reduces the selling price with or without an increase in its manufacturing output. Example 2 in Section 2.9.2 demonstrates the instance when the firm decreases its selling price and produces a higher quantity than its non-binding TD equivalent. The potential increase in the optimal manufacturing amount can also be seen in Figure 2.2 when the firm switches from a TD policy to the PHI policy.

The following proposition provides several insights into the traditional practice of producing the total demand. First, it shows that with increasing exchange-rate uncertainty, corresponding to a higher value of θ, TD becomes less likely to be optimal, and production hedging becomes more likely to be optimal. Second, it proves that with increasing risk aversion, corresponding to lower values of α and/or β, the TD policy becomes less likely to be optimal. This result implies that with higher risk aversion, the firm is more likely to follow production hedging. Thus, as the level of risk aversion increases with the inclusion of a VaR constraint, the likelihood of having the TD policy as the optimal solution decreases. As a result, it can be concluded that risk aversion makes production hedging a more attractive policy.

Proposition 2.4 *a) With increasing exchange-rate uncertainty θ, the firm is less likely to follow the* TD *policy, and more likely to follow production hedging; b) With increasing risk aversion* (*i.e., with decreasing α and/or β*), *the* TD *policy becomes less likely to be optimal and production hedging becomes more likely to be optimal.*

2.3.2.3 Production Hedging Maximum (PHX) Policy

Under the PHX policy, the firm determines the optimal price and production quantity so that it equals the maximum of the two demand values. Depending on the relative values of the two market demands, the expected profit for the PHX policy is:

$$E\left[\prod^{PHX}\left(X^*\left(p\right)=d_1\geq d_2,p\right)\right]=-cd_1+\int_{e_l}^{\bar{e}}pd_1f(e)de+\int_{\bar{e}}^{e_h}\left[pd_2e+p(d_1-d_2)\right]f(e)de \quad (2.7)$$

$$E\left[\prod^{PHX}\left(X^*\left(p\right)=d_2> d_1,p\right)\right]=-cd_2+\int_{e_l}^{\bar{e}}\left[pd_1+pe\left(d_2-d_1\right)\right]f(e)de+\int_{\bar{e}}^{e_h}pd_2ef(e)de \quad (2.8)$$

Under the PHX policy, the firm produces to satisfy one of its market demand, and has the flexibility to sell all (or most) of its products in the home market if the exchange rate turns out to be unfavorable. PHX is a less risky policy compared to the TD policy for two reasons: First, it always satisfies the risk constraint (2.1) when the domestic market is greater than or equal to the foreign market. Second, when the realized exchange rate is unfavorable to sell in the foreign market, PHX sells only a limited amount of leftovers from the domestic market, whereas the TD policy sells all of its production in the foreign market at a loss, leading to higher expected losses.

When the domestic market is larger than the foreign market, PHX always satisfies risk constraint (2.1). In the presence of a larger foreign market, the following proposition determines the cost threshold that PHX satisfies constraint (2.1). It also shows that when (2.1) is violated,

27

the firm can take two alternative actions: either revise the PHX policy to comply with (2.1), or switch to the PHIX policy and satisfy (2.1).

Proposition 2.5 *The* PHX *policy satisfies the risk constraint* (2.1) *if the unit manufacturing cost is*

$$c \le p^{PHX}\left[\frac{d_1^{PHX}}{d_2^{PHX}}\left(1-e_\alpha\right)+e_\alpha\right]+\frac{\beta}{d_2^{PHX}} \tag{2.9}$$

where p^{PHX} satisfies the optimal non-binding expression in Table 2.1; *b) if the unit manufacturing does not satisfy* (2.9), *the firm can either follow the* PHX *policy by charging a higher selling price equal to $p^{PHX} = (cd_2 - \beta)/(d_1(1 - e_\alpha) + d_2 e_\alpha)$ in order to comply with* (2.1), *or follow the* PHIX *policy.*

When policy PHX violates the VaR constraint, the firm can increase its selling price to comply with (2.1). As the firm becomes more risk averse, corresponding to smaller values of α and/or β, the modified price of PHX monotonically increases. Alternatively, the firm can follow PHIX.

2.3.2.4 Production Hedging Interior less than Maximum (PHIX) Policy

This policy arises only when the foreign market is greater than the domestic market for the selling price $c \le p \le c/(1-\theta)$. When the unit manufacturing cost does not satisfy (2.9), the firm prefers to reduce its manufacturing level below the foreign market demand, and follow the PHIX policy that satisfies:

$$X^{PHIX} = \frac{\left(1-e_\alpha\right)d_1^{PHIX} + \beta/p^{PHIX}}{c/p^{PHIX} - e_\alpha}, \quad p^{PHIX} = \frac{cX^{PHIX} - \beta}{\left(1-e_\alpha\right)d_1^{PHIX} + e_\alpha X^{PHIX}}.$$

28

Similar to PHI, the selling price under the PHIX policy can exhibit both an increasing and a decreasing behavior when compared with the non-binding optimal price under the PHX policy presented in Table 2.1. For relatively small values of the unit manufacturing cost, the firm increases the selling price under PHIX above the non-binding optimal PHX price; and, as the unit manufacturing cost increases, the firm can reduce the selling price below the non-binding optimal PHX price. In the latter case, the firm can also increase its manufacturing output above the non-binding PHX production amount. Therefore, all the price and quantity conclusions made between the TD and PHI policy continue to hold for the comparison of PHX and PHIX policies. We next turn our attention to the policy that manufactures the minimum of the two market demands, and show that the price and quantity conclusions under the VaR constraint have some differences.

2.3.2.5 Production Hedging Minimum (PHN) Policy

Under the PHN policy, the firm determines the optimal price and production quantity so that it equals to the minimum of the two market demand values. The expected profit for the PHN policy is:

$$
\begin{aligned}
&E\left[\Pi^{PHN}\left(X^{*}(p)=\min\{d_{1},d_{2}\},p\right)\right] \\
&=-c\min\{d_{1},d_{2}\}+\int_{e_{l}}^{\bar{e}}p\min\{d_{1},d_{2}\}f(e)de+\int_{\bar{e}}^{e_{h}}p\min\{d_{1},d_{2}\}ef(e)de
\end{aligned}
\tag{2.10}
$$

The firm can always sell all of its entire production in the home market if the exchange rate turns out to be unfavorable. However, because the selling price is less than the unit manufacturing cost in this policy, it cannot eliminate the possibility of losses, and thus, can violate the risk constraint (2.1). Even with its selling price that is below the unit manufacturing

cost, policy PHN presents less risk than TD, and can be less risky than PHX when the foreign market is larger.

Remark 2.2 *The optimal selling price p^{PHN} in Table 2.1 is greater than $c/(1+\theta)$, but it can also take values greater than or equal to c.*

When the non-binding selling price p^{PHN} in Table 2.1 is less than the unit manufacturing cost c and the risk constraint (2.1) is violated, the firm takes only one action: Increase the price so that the production amount equals the foreign market demand (i.e., the minimum demand). Unlike the reaction when non-binding solution of the TD policy violated risk constraint (2.1), the firm does not reduce its manufacturing quantity below the minimum of the two market demands. Thus, as proven in Proposition 2.2, policy PHIN is not among the list of candidate optimal solutions. This result is formalized in the following proposition.

Proposition 2.6 *The PHN policy satisfies the risk constraint (2.1) if the unit manufacturing cost is*

$$p^{PHN} \le c \le p^{PHN} + \frac{\beta}{\min\left\{d_1^{PHN}, d_2^{PHN}\right\}} \tag{2.11}$$

where p^{PHN} satisfies the non-binding expression in Table 2.1; otherwise, the firm continues to follow the PHN policy, but charges a higher selling price equal to $p^{PHN} = c - \beta/\min\{d_1, d_2\}$ and complies with (2.1).

Propositions 2.3, 2.4, and 2.6 have collectively shown the impact of including of the VaR constraint on policies that produce one or sum of the two market demand values. We have concluded that the risk constraint (2.1) increases the selling price in policies TD, PHX and PHN. The price increase results in a smaller cherry picking benefit due to reduced demand values, but

also decreases the downside risk of the random exchange rate. The following proposition shows the influence of the VaR constraint on the optimal manufacturing quantity in policies TD, PHX and PHN. As an outcome of the increased selling price, it shows that the production quantity is lower, reducing the risk as the firm needs to sell less to the foreign market when the foreign currency depreciates. Thus, the firm relies more on the home market revenues in the presence of binding risk constraint (2.1).

Proposition 2.7 *The optimal production quantity for policies* TD, PHX, *and* PHN *with constraint* (2.1) *is smaller than or equal to the non-binding optimal production quantities.*

The risk constraint (2.1) is the reason behind establishing policies PHI and PHIX. These two policies are candidate optimal solutions that reduce the manufacturing quantity below the level of the TD and PHX policies, respectively. It is useful to investigate the likelihood that the non-binding optimal solutions for TD, PHX, PHN, violate (2.1). The following proposition shows that, for a given selling price, the TD policy has the most risk due to not having the flexibility to cope against the stochastic exchange rate and has the highest probability to violate the constraint in (2.1).

Proposition 2.8 *At a given selling price p*, *the profit generated by the* TD *policy has the highest probability to violate the constraint in* (2.1), *and thus presents a higher risk than the* PHX *and* PHN *policies*.

The consequence of the above proposition is that production hedging not only has the ability to increase the expected profit but also can reduce the downside risk of the exchange rate for a global firm. The following proposition elaborates more on the impact of exchange-rate uncertainty and risk on the profits. It makes two important observations: First, it shows that when

the risk constraint is not binding, exchange-rate uncertainty θ has no effect on the expected profit of the TD policy, but the expected profits of all the production hedging policies increase with θ. The impact of θ resembles an option value for production hedging policies. From (2.4), it can be seen that the value of $\int_{e_l}^{\bar{e}} (\bar{e} - e) f(e) de$ is the expected payoff of the option to sell a unit of foreign currency at price $\bar{e}$, and the value of $\int_{\bar{e}}^{e_h} (e - \bar{e}) f(e) de$ is the expected payoff of the option to buy a unit of foreign currency at price $\bar{e}$. The value of θ generally increases with the variance of the exchange rate random variable (see Section 2.9.1). Part a) of the following proposition exposes the positive relationship between the value of the allocation option under production hedging and the value of foreign exchange options. In contrast with the production hedging policies, the TD policy is rigid, and is unaffected by θ or the variance in exchange rates. Second, when the VaR constraint is binding, the proposition shows that the expected profit of the TD policy is not affected by smaller values of θ, but starts to decrease with large values of θ. This is because increasing values of θ implies that the pdf is changing, and the corresponding value of the point e_α of the VaR constraint moves. It is not possible to characterize the direction of the change in the value of e_α for arbitrary distributions. However, for symmetric distributions, e_α either stays the same or decreases with increasing values of θ. The implication of decreasing value of e_α is that the intersection point of the VaR-constrained PHI policy and the TD policy moves to the right in Figure 2.2; and, PHI becomes the dominant policy for a larger range of price values, and TD becomes valid for a smaller range of price values. Thus, it can be concluded that under risk aversion, the increase in exchange-rate uncertainty makes the TD policy less attractive, and as a consequence, production hedging becomes more attractive.

Proposition 2.9 *a) The expected profits of production hedging policies* PHN, PHIX, PHX, *and* PHI *are all increasing in* θ, *whereas the expected profit of the* TD *policy does not change with* θ; *b) When the risk constraint is binding, the expected profit of the* TD *policy is non-increasing in* θ *under symmetric distributions.*

We have investigated the effect of a risk constraint on the selling price, manufacturing quantity and profits. We have shown that when the VaR constraint is binding, while policies TD, PHX and PHN increase the selling price (Propositions 2.3, 2.5, 2.6) and reduce the manufacturing quantity (Proposition 2.7), there are no consistent directional inequalities for policies PHI and PHIX, i.e., the selling price and manufacturing quantity can show both an increasing and decreasing behavior. In settings where the VaR constraint is not binding, the price and quantity relationships between these three policies (TD, PHX and PHN) are primarily influenced by the relative value of the unit manufacturing cost. We next investigate the impact of the unit manufacturing cost on the behavior of the optimal selling price, manufacturing quantity, and expected profit.

2.4 The Impact of the Unit Manufacturing Cost

In this section, we examine the choice of the optimal policy and analyze the price and production decisions with varying unit manufacturing costs under non-binding VaR constraint. Propositions 2.1 and 2.2 establish the optimal quantity and price relationships. For example, when the selling price is above $c/(1-\theta)$, the firm's optimal production decision is the sum of the two demand values. A similar technical result to Proposition 2.4 can be developed for the unit manufacturing: As the unit manufacturing cost c increases, it becomes less likely that the firm's optimal policy choice would be the TD policy. We next analyze the influence of the unit manufacturing cost on the optimal selling price, manufacturing quantity, and expected profit.

2.4.1 Optimal Price Analysis

Propositions 2.3, 2.5, and 2.6 under the binding VaR constraint have established that the relationship between prices has the following order:

$$p^{PHN} < p^{PHIX} < p^{PHX} < p^{PHI} < p^{TD}.$$

Thus, the price of the TD policy is always higher than the price of production hedging policies. However, when the VaR constraint is not binding, the above pricing order can be altered based on the value of the unit manufacturing cost. Following the basic price-demand principles of economics, one would intuit that the optimal price under production hedging policies PHX and PHN would be greater compared to that of the TD policy when the VaR constraint is not binding. This is because by producing less than the total demand in production hedging, a firm can gain flexibility to cope against the fluctuating exchange rate, and increase the selling price for a smaller number of items that will be distributed to the markets. Thus, it appears to be natural to expect the optimal price under production hedging to be greater because of the smaller production mechanism. We next establish the general conditions where the optimal price under production hedging policies PHX and PHN is lower than that of the TD policy under non-binding VaR constraint.

There is a threshold point for the unit manufacturing cost where the order of the optimal prices under each production policy changes under the non-binding VaR constraint. This threshold point, c_p, serves as a sufficient condition for the firm to set a lower price under production hedging, regardless of the effect of the VaR constraint. Specifically, when the unit manufacturing cost is higher than c_p, the optimal price under production hedging is smaller than that of the TD policy. We denote the maximum of the two demand function with $d^{\max} = \max(d_1, d_2)$ and the minimum of the two demand functions with $d^{\min} = \min(d_1, d_2)$.

Proposition 2.10 $p^{PHX}\left(p^{PHN}\right) \le p^{TD}$ *holds when* $c \ge c_p^{PHX}\left(c_p^{PHN}\right)$ *where*

$$c_p^{PHX} = \frac{d^{\max} + d^{\min}\theta}{1 + d_p^{\max}} - \frac{\left[d_1^{TD} + d_2^{TD}\right]\left[d_p^{\max} + d_p^{\min}\theta\right]}{\left[d_{1p}^{TD} + d_{2p}^{TD}\right]\left[1 + d_p^{\max}\right]} \quad and \quad c_p^{PHN} = \left(-\frac{d^{\min}}{d_p^{\min}} + \frac{d_1^{TD} + d_2^{TD}}{d_{1p}^{TD} + d_{2p}^{TD}}\right)\left(\frac{1 + \theta}{\theta}\right).$$

Under production hedging, the firm has the flexibility to not only hedge against the possibility of a low exchange rate but also take advantage of its upside potential. By lowering the price, a firm has greater demands in both markets which results in a greater opportunity to alter the allocation of the products to more desirable markets based on the realization of the exchange rate. It is necessary to point out that in a production hedging policy one might assume that consumers are worse off as they get less number of products. However, some consumers benefit from reduced selling price under a production hedging policy over the TD policy. However, some consumers still face the risk of having limited supply under production hedging.

Figure 2.3 shows how the optimal price under both the TD policy and the production hedging policy change with the unit manufacturing cost under identical linear demand functions, $d_1 = 100 - p$, $d_2 = 100 - p$, and when the exchange rate has a uniform distribution with a support of $[0, 2]$ and the firm's risk preferences are $\alpha = 0.05$ and $\beta = 700$. Because $d_1 = d_2$, PHX and PHN policies merge into one production hedging policy, denoted as PH, in this example. Even though the VaR constraint is not binding in this problem setting, Figure 2.3 demonstrates that the optimal price under production hedging is always lower if both markets have identical linear demand functions. Thus, VaR constraint is not the only reason for a reduced price under production hedging. As explained above, the option value generated from the postponement of allocation can be so useful that the firm might prefer to reduce its selling price in order to increase demand, and thus, to maximize the cherry-picking benefits associated with this allocation postponement. Figure 2.3 also demonstrates that the optimal price under production

35

hedging can be lower than the unit manufacturing cost (e.g., when $c > 83.33$). The conditions for pricing below cost are formalized in the following proposition.
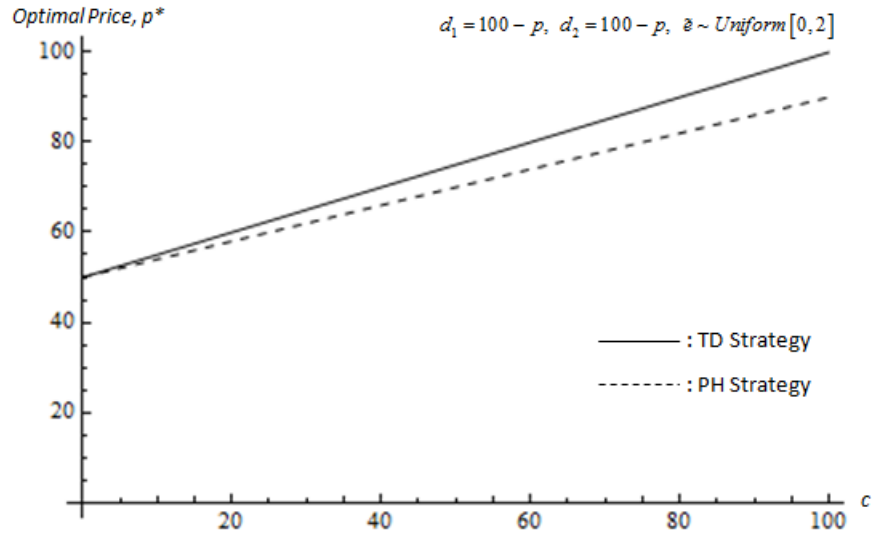
Figure 2.3 Optimal prices for alternative policies.

Proposition 2.11 *When the risk constraint (2.1) is satisfied under the PHN policy, the optimal selling price can be less than the unit manufacturing cost if and only if $c \geq (p/\varepsilon_{min})((1+\theta)/\theta)$ where ε_{min} denotes the price-elasticity of the minimum demand function. When the risk constraint (2.1) is violated under the PHN policy, the optimal selling price can be less than the unit manufacturing cost if and only if $\beta \geq 0$. Moreover, PHN can have a positive expected profit, and can be the optimal policy for the problem.*

The reason why a firm would price their product lower than the unit manufacturing cost under production hedging is to maximize the cherry picking benefit of distributing the product to the more desirable markets. This flexibility dominates the losses that are stemming from sales under an undesirable realization of the exchange rate. In a global business setting, selling below the production cost is generally considered as predatory pricing, and the practice is restricted by

36

laws. However, our model does not incorporate any kind of competition, and the reason for pricing under the unit manufacturing cost is to maximize profits, and certainly, not because of an attempt to eliminate competition, gain market share and power in the future, while enduring losses in the meanwhile. Thus, traditional arguments for predatory pricing do not apply to our finding. It is important to highlight that pricing below the unit manufacturing cost does not occur when there is a single foreign market or a single domestic market. This only appears when there are multiple markets and the firm follows production hedging and prices in the presence of exchange rate uncertainty.

2.4.2 Optimal Production Quantity Analysis

One would expect that the production quantity would always be greater under the TD policy compared to the production hedging policy. However, it is established in Section 2.3.2.2 that the optimal manufacturing quantity under PHI, with a binding VaR constraint, can be greater than that of the TD policy. Thus, the firm can manufacture a larger amount of products under production hedging than it does under the TD policy.

This section develops new threshold expressions for the unit manufacturing cost that enables the firm to manufacture a greater amount under production hedging than it does under TD. Proposition 2.7 states that the optimal manufacturing quantity when the VaR constraint is binding is smaller than its equivalent when the risk constraint is not binding. Because strongly binding VaR constraints make it easier to observe this phenomenon, we restrict the analysis to the comparison of the manufacturing quantities under PHX and PHN with that of TD under non-binding risk constraints. Let c_x represent the value of the unit manufacturing cost that equates the production amount of the TD policy to that of the production hedging policies. The threshold point c_x serves as a sufficient condition for the firm to manufacture a higher quantity under

production hedging regardless of the influence of the VaR constraint. The following proposition identifies the condition where the optimal production quantity is greater under production hedging compared to the TD policy.

Proposition 2.12 $X^{PHX}\left(X^{PHN}\right)\ge X^{TD}$ *holds when* $c\ge c_X^{PHX}\left(c_X^{PHN}\right)$ *where*

$$c_X^{PHX}=\frac{-2p^{TD}\left(d_{1p}^{TD}+d_{2p}^{TD}\right)-\left(d_1^{TD}+d_2^{TD}\right)d^{\max}+p^{PHX}\left[d_p^{\max}+d_p^{\min}\theta\right]+d^{\min}\theta}{-2\left(d_{1p}^{TD}+d_{2p}^{TD}\right)+d_p^{\max}}\ \text{ and }$$

$$c_X^{PHN}=\frac{2p^{TD}\left(d_{1p}^{TD}+d_{2p}^{TD}\right)+\left(d_1^{TD}+d_2^{TD}\right)-p^{PHN}d_p^{\min}}{-2\left(d_{1p}^{TD}+d_{2p}^{TD}\right)+\left(\dfrac{d_p^{\min}}{1+\theta}\right)}.$$

The above condition (i.e., for manufacturing a larger number of products under production hedging), only occurs for products that have considerably large manufacturing costs. We conclude that higher unit manufacturing costs causes the firm to manufacture a larger quantity under production hedging than it does under TD. This counter-intuitive phenomenon arises due to the optimal pricing scheme under production hedging policy which is lower in value than that of the TD policy. Thus, the production quantity is not always greater for firms that employ the TD policy, and greater demands can be generated by firms utilizing the production hedging policy. Consequently, global consumers may not be always worse off under a production hedging policy as the firm can flood the markets with a larger number of units.

Figure 2.4 provides the optimal production quantity under each production policy using the same parameter values as in Figure 2.3. As can be seen from this figure, when the unit manufacturing cost c exceeds the threshold cost value $c_X^{PH}=c_X^{PHX}=c_X^{PHN}$, the optimal production amount under a production hedging policy is greater than that of the TD policy.
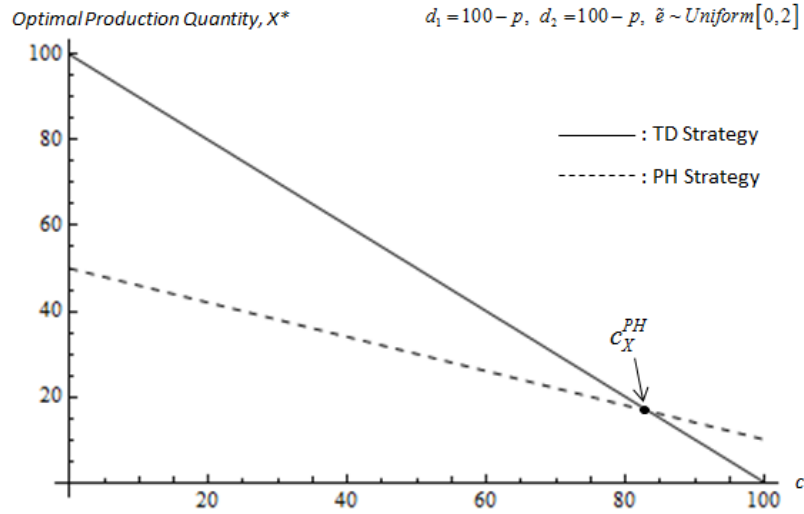
Figure 2.4 Optimal production quantity for alternative policies.

2.4.3 Optimal Profit Analysis

Now that we have examined the impact of the unit manufacturing cost on the optimal price and manufacturing quantity, we next turn our attention to its influence on optimal profit. From Proposition 2.1, we know that with the threshold point $c/(1-\theta)$ increases with higher values of the unit manufacturing cost, making the TD policy less likely to be optimal. Moreover, under binding VaR constraints, the threshold price point (for switching from TD to PHI) is higher than $c/(1-\theta)$, making the TD policy even less desirable.

This section develops new threshold expressions for the unit manufacturing cost that enables the firm to gain higher profit under production hedging than it does under TD. Because the VaR constraint makes it easier for the TD policy to be dominated by production hedging, we restrict the analysis to the comparison of the optimal profits under PHX and PHN with that of TD under non-binding risk constraints. Using the same parameters from figures 2.3 and 2.4, Figure 2.5 provides an example of this scenario by displaying the optimal profits under the TD and PH policies. If a firm operates on a tight profit margin, then it is better to employ production hedging

rather than the TD policy. On the other hand, it is better to satisfy the total demand when there is a considerable amount of profit margin for a firm. The presence of a relatively large profit margin can dominate the value of flexibility gained by producing less than the total demand. Let c_Π denote the threshold cost where production hedging becomes optimal as a replacement for the TD policy. Once again, c_Π serves as a sufficient condition to switch to production hedging regardless of the influence of the VaR constraint. Using these threshold expressions, the following proposition identifies the condition where the optimal expected profit is greater under a production hedging policy compared to the TD policy.

Proposition 2.13 $E\left[\Pi^{PHX}\right]\left(E\left[\Pi^{PHN}\right]\right) \geq E\left[\Pi^{TD}\right]$ *holds when* $c \geq c_\Pi^{PHX}\left(c_\Pi^{PHN}\right)$ *where*

$$
c_\Pi^{PHX} = \frac{-\left(3d_p^{\max}+d_p^{\min}\theta\right)p^{PHX}-d_p^{\min}\theta - \sqrt{\begin{array}{c}\left[-\left(3d_p^{\max}+d_p^{\min}\theta\right)p^{PHX}-d_p^{\min}\theta\right]^2 - \left(\dfrac{\left(d_1^{TD}+d_2^{TD}\right)^2}{d_{1p}^{TD}+d_{2p}^{TD}}\right) \\ +4d_p^{\max}\left[-2\left(d_p^{\max}+d_p^{\min}\theta\right)\left(p^{PHX}\right)^2 - \left(d^{\max}+d^{\min}\theta\right)p^{PHX}\right]\end{array}}}{-2d_p^{\max}} \ and
$$

$$
c_\Pi^{PHN} = \frac{-2\left(1+\theta\right)p^{PHN}d_p^{\min} - \sqrt{\begin{array}{c}\left[2\left(1+\theta\right)p^{PHN}d_p^{\min}\right]^2 \\ -4d_p^{\min}\left[\left(1+\theta\right)^2\left(p^{PHN}\right)^2 d_p^{\min}\right] + \left(1+\theta\right)\left(\dfrac{\left(d_1^{TD}+d_1^{TD}\right)^2}{d_{1p}^{TD}+d_{1p}^{TD}}\right)\end{array}}}{-2d_p^{\min}}.
$$
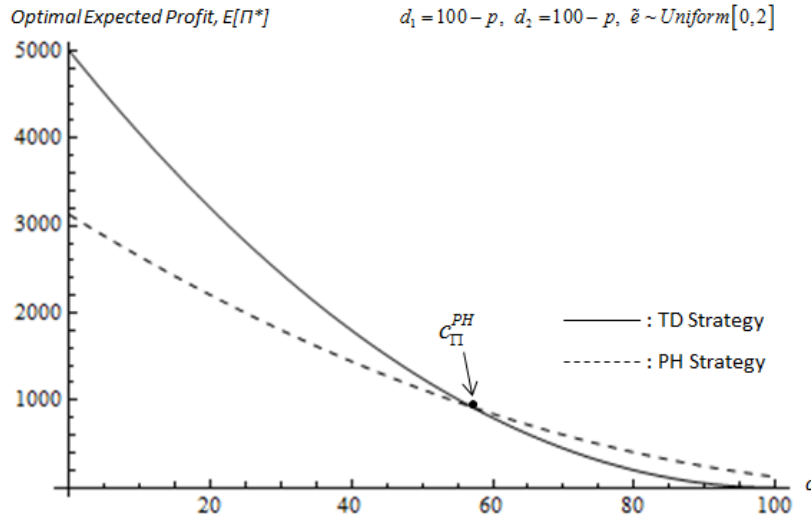
Figure 2.5 Optimal expected profit for alternative policies.

As a consequence of the above proposition, we conclude that production hedging becomes more desirable, and satisfying the global demand becomes less desirable, with higher unit manufacturing costs. Combining the results from propositions 2.10, 2.11, and 2.12, it can be concluded that increasing values of the unit manufacturing costs makes the firm follow production hedging, charge a lower price and manufacture a higher quantity than its TD equivalent.

2.4.4 Extreme Risk Aversion

Our analysis in sections 2.4.1 through 2.4.3 has identified threshold cost expressions and explained the behavior of optimal price, quantity and profits using non-binding VaR constraints. These thresholds can be perceived as sufficient conditions regardless the effect of the VaR constraints. We now turn our attention to the influence of extreme risk aversion with $\alpha = 0$ and $\beta = 0$, which is a special case of the analysis presented in Section 2.3. Under extreme risk aversion, the firm prefers to eliminate the risk of losing money at every realization of the exchange rate.

Specifically, even at the lowest realization of the exchange rate, the firm is required to not incur a loss.

We provide a detailed analysis of the characterization of the policy structure in Section 2.9.2 for the extremely risk-averse firm. The main results from this analysis can be summarized as follows: (1) production hedging has the capability to eliminate risk completely from the life a global manufacturer; (2) the firm charges a price that is strictly greater than or equal to the unit manufacturing cost; and, (3) when the demand of the home market is larger than that of the foreign demand, the firm manufactures a quantity that is greater than or equal to home market demand; in this scenario, the firm eliminates PHN and PHIX policies and gravitates towards PHX and PHI policies. It should be noted here that, when the demand of the home market is smaller than that of foreign market, policy PHN normally cannot be eliminated from consideration; it only gets eliminated under the special case of extreme risk aversion.

In sum, the analysis proves that the TD policy is optimal in a smaller range of cost parameters under increasing risk aversion. Alternatively, it can be concluded that production hedging is more attractive under an increasing level of risk aversion. Using the same parameter values as in figure 2.3, 2.4, and 2.5, Figure 2.6 shows how production hedging becomes the more desirable policy under a wider range of cost parameters and the TD policy is preferred for a smaller range of cost parameters under extreme risk aversion ($\alpha = \beta = 0$) over the risk-neutral model with $\alpha = 1$ and $\beta = \infty$.

$d_1 = 100 - p, \; d_2 = 100 - p, \; \tilde{e} \sim Uniform[0,2]$

	c = 0		57		c = 100
Maximum Expected Profit		TD		PH = PHX = PHN	
Maximum Expected Profit with no risk	TD	PHI		PH = PHX = PHN	
	c = 0	33.3	55		c = 100
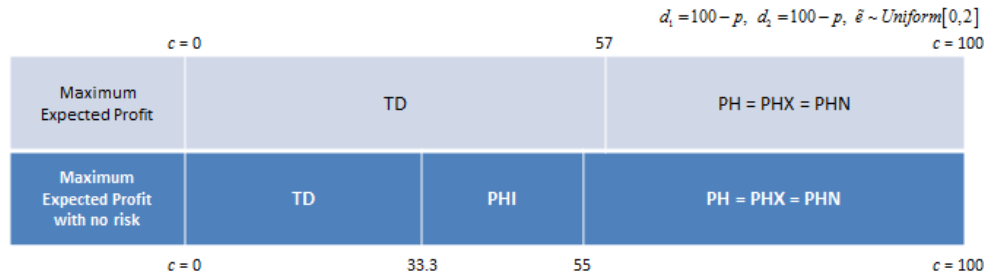
Figure 2.6 Optimal policies for various unit manufacturing cost parameters for the risk-neutral and extreme risk-averse models.

2.5 The Impact of Financial Hedging

This section presents the impact of incorporating financial hedging into the model. The analysis in Section 2.3 presents that the list of potentially optimal policies includes TD, PHX and PHN when the value-at-risk constraint is not binding, and PHI and PHIX when the risk constraint is binding. How would these results change in the presence of the financial hedging capability? Can financial hedging eliminate production hedging? We provide insights into these two questions using both futures and currency options. The derivations assume that the financial institution providing the hedging instrument uses the same exchange rate distribution with the manufacturing firm, and does not profit from the instrument, i.e., it makes zero expected profits from the financial hedging contracts. Therefore, the derivations provide insight into the maximum advantage the manufacturing firm can attain through financial hedging.

Let us consider the event that the optimal policy is TD when constraint (2.1) is ignored. In the absence of financial hedging, this suggests that the firm can either revise the TD policy to one that satisfies the risk constraint but returns a lower expected profit, or alternatively, switch to PHI. Proposition 2.14 proves that the firm can purchase d_2 units of futures, or currency options, and obtain the same expected profit while satisfying constraint (2.1). This implies that policy PHI is dominated by TD in the presence of financial hedging. Similarly, consider next the event

that the optimal policy is PHX when constraint (2.1) is ignored. Note that the risk constraint is only violated when $d_2 > d_1$ under the PHX policy. When financial hedging is not available, the firm either follows PHX at a lower return or switches to PHIX. However, when financial hedging is available, the firm can purchase $d_2 - d_1$ units of futures, or currency options, and retain its original PHX policy while satisfying the risk constraint in (2.1). Thus, financial hedging eliminates PHIX policies.

Proposition 2.14. *Financial hedging eliminates* PHI *and* PHIX *from the list of potentially optimal policies*.

The above proposition proves that financial hedging eliminates intermediate point solutions with PHI and PHIX policies, and reduces the list of potentially optimal policies to TD, PHX and PHN. Thus, it can be concluded that financial hedging can eliminate the negative effects of a Value-at-Risk constraint in the firm's pricing and production decisions. However, financial hedging cannot eliminate PHX and PHN from the list of potentially optimal solutions, and therefore, we conclude that production hedging is still a viable policy under financial hedging.

2.6 Discussion of Potential Extensions

Our study provides a simple model that emphasizes the influence of exchange-rate risk in pricing and production decisions of a global manufacturer. In this section, we discuss how the results alter under various scenarios: excluding the price constraint that avoids dumping, the inclusion of transportation and localization costs, the consideration of multiple periods, and the consideration of demand functions not only influenced by the selling price, but also by the realized exchange rate.

2.6.1 The case when p_2 is not equal to $p_1/\bar{e}$

Our model incorporates the firm's operating environment by determining selling prices in both home and foreign markets at the same time in the presence of exchange-rate uncertainty. While doing so, it locks the relative *ex-ante* prices as $p_2 = p_1/ \bar{e}$. One might intuit that some of the counter-intuitive results are due to this pricing scheme. Therefore, it is necessary to point out that the pricing scheme that complies with the anti-dumping laws is not the reason behind our surprising results.

Production hedging continues to be a viable optimal policy when the firm can set differing prices in each market. When the firm can set a different price in each market, it can adjust the demand accordingly, and policy PHN can be eliminated from the set of potentially optimal solutions, and the list of potentially optimal policies reduces to TD, PHI, PHX, and PHIX.

It is important to note that our surprising price results continue to hold when the firm can determine a different price in each market. Specifically, (1) the firm can charge a lower selling price under production hedging than when it follows the TD policy, (2) the firm can end up producing more units under a production hedging policy than what it manufactures under the TD policy, and (3) the firm might prefer to charge a selling price that is lower than the unit manufacturing cost in both markets, and yet have a profitable expected profit. Consistent with earlier findings, all three results become viable scenarios with increasing values of the unit manufacturing cost. Section 2.9.3 provides examples of these three results occurring under the flexibility to determine a different price in each market.

2.6.2 Transportation/localization costs

Our model ignores the presence of transportation, or possibly localization costs such as final assembly to comply with the requirements and/or tastes of local markets. Let us define t_i as the combination of transportation and localization costs in order to sell the product in market $i = 1, 2,$

45

representing home and foreign markets, respectively. The firm continues to determine the selling price p and production quantity X in the first stage, in the presence of exchange rate risk. However, after exchange rate randomness is revealed in the second stage of our problem, the firm has the ability to not send the product to the foreign market. When the realized exchange rate is extremely low, i.e., $e < t_2/p$, the firm is better off by discarding the product rather than selling it in the foreign market. This is coined as "allocation hedging" in Kazaz et al. (2005), and when the firm follows the TD policy (i.e., must satisfy all demand), it cannot avoid the loss when the exchange rate is such that foreign revenue is less than transportation/localization cost. However, when the firm follows a production hedging policy, the negative effects of allocation hedging are diminished because the firm already prefers to sell in the home market when the exchange rate takes a lousy realization. Therefore, it can be easily seen that when transportation and/or localization costs are incorporated into the model, production hedging policies become even more attractive. Alternatively, the current model provides a more conservative characterization of when a production hedging policy is more desirable than the traditional practice of following the TD policy.

2.6.3 Multi-Period Extension

Our earlier analysis has considered a single-period setting. We argue that even if the optimal policy is the TD policy in a single-period setting, it can switch to a production hedging policy in a multi-period setting. Thus, the analysis of a multi-period setting can increase the use of production hedging beyond a single-period setting.

Let us consider the problem setting with two periods using the exchange rate random variables $\tilde{e}_t$ that are identically and independently distributed (i.i.d.) for periods $t = 1, 2$. Let us assume that the single-period version of the problem (for period 1) results in the TD policy, i.e.,

46

$X_t^* = d_{1t} + d_{2t}$. In the multi-period setting, for the same exchange-rate distribution in period 1, consider the realization of e_1 such that $pe_1 \leq c$. Note that for realizations $e_1 \leq c/p$, the firm is better off by not serving the foreign market, and saving the manufacturing cost of the foreign market demand in period 2. Thus, even if the firm originally produces the total demand in period 1, it does not allocate the product to the foreign market in order to save the manufacturing cost of period 2. This behavior is identical to that of "allocation hedging," which was described earlier as a reaction based on transportation/localization costs. In period 2, because e_2 follows i.i.d., the optimal solution is again the TD policy with the same X^* and p^*; but this time, the firm produces only the amount corresponding to the home market demand due to the leftovers from previous period. In this example, while the firm follows a TD policy from a myopic perspective, it can be seen that it follows a production hedging policy in the aggregate plan that considers the sum of periods 1 and 2. This is because the firm ends up producing a total amount of units that is strictly less than the sum of the demands in two periods.

The above example demonstrates that the firm exercises allocation hedging more frequently as a result of the presence of manufacturing cost savings from future periods. As pointed out earlier, allocation hedging increases the likelihood of production hedging in a single-period, and the above example demonstrates how it can increase the use of production hedging in a multi-period setting. Thus, the multi-period setting can increase the likelihood of following production hedging in at least one of the periods even beyond the levels observed in a single-period model.

2.6.4 Demand influenced by exchange rates

From a Purchasing Power Parity (PPP) Theory perspective, one can argue that the demand for a product in a foreign market is influenced by economic conditions. Specifically, when the foreign-to-domestic exchange rate is low (high), consumers in that market have less (more)

47

purchasing power, resulting in lower (higher) demand. Recall that e is the foreign-to-domestic exchange rate in our model. Thus, the inclusion of a PPP perspective is equivalent to defining the demand function as $d(p,e)$, where foreign demand is positively correlated with e and domestic demand is negatively correlated with e. When the exchange rate realization e is low, the firm prefers to sell in the home market, which due to PPP, is also when home market demand is high. Alternatively, when the exchange rate is high, the firm prefers to sell in the foreign market, which is also when the foreign market demand is high. Thus, the allocation decision on production hedging naturally targets the market with high purchasing power, and as consequence, the profit opportunity under production hedging is enhanced relative to TD. Production hedging is even more desirable when demand is affected by the exchange rate.

From the above discussion, we see that incorporating extensions such as transportation and localization costs, behavior of the firm in a multi-period setting, and the effects of exchange rate on the demand function all make production hedging policies more pronounced and desirable in the presence of exchange rate risk.

2.6.5 Minimum allocation requirements

Our earlier findings show that a global firm can choose to completely abandon a market based on the realized exchange rates under production hedging. However, a firm can face regulations or restrictions that force allocating at least a certain amount of products to a market regardless of the realized exchange rate. Incorporating a constraint that ensures a minimum allocation amount to markets does not change the structural properties of our model. In a model with minimum allocation constraints, our demand functions can be thought as the demand that exceeds the minimum requirement, and similarly, the firm's production quantity decisions can be viewed as the additional amount of manufacturing that exceeds the minimum allocation

48

requirements. As a result, the results and insights in this paper continue to hold under minimum allocation requirements, but the benefits from production hedging reduce because of the reduced amount of flexibility in allocation quantities relative to the total demand.

2.6.6 Postponed pricing

We next analyze a variation of our model where the firm can postpone its pricing decisions until after the exchange rate is realized. We refer to this model as the "postponed pricing model," and alternatively, it can be considered as the "recourse pricing model." In this postponed pricing model, we retain the restriction that the firm complies with the anti-dumping law. Specifically, once the realized value of exchange rate is revealed, the firm sets its price in both markets in such a way that the product returns the same value in home country currency from the sales in both markets, i.e., $p = p_1 = p_2 e$. In the first stage of the model, the firm determines only a manufacturing quantity X in the presence of exchange-rate uncertainty subject to the same risk constraint. In the second stage of the model, the firm maximizes its revenues by determining the two allocation quantities, x_1 and x_2, as well as the selling prices p_1 and p_2. Because of the anti-dumping law restriction, the second stage model becomes an optimization over three decision variables: x_1, x_2, and p where $p_1 = p$ and $p_2 = p/e$. The analysis of the postponed pricing model is presented in the Appendix 2.9.4.

We show that the flexibility of postponing selling prices until after exchange rates are observed does not alter the characteristics of our primary results. First, we show that the firm can follow a production hedging policy where it manufactures a smaller amount than its total demand. Because pricing decisions are postponed until the exchange rate is realized, the demand is not precisely known at the time of the manufacturing decision. Therefore, to create a benchmark, we consider the optimal manufacturing quantity at the expected exchange rate as the

quantity corresponding to the TD policy. We show that the firm can benefit by manufacturing a smaller amount than its TD policy. We demonstrate this result using linear demand function in the Appendix 2.9.4.

Second, the firm can set a price below the optimal price choice of the TD policy. When the realized exchange rate is low, the firm sets a lower selling price in the domestic market and a higher selling price in the foreign market. When the realized exchange rate is high, however, the firm sets a higher selling price in the domestic market and a lower selling price in the foreign market. In either scenario, the optimal selling price can be below the optimal prices of the TD policy. Under lower values of exchange rate realizations, the selling price in the domestic market is lower than the optimal price of the TD policy, and under higher values of exchange rate realizations, the selling price in the foreign market is lower than the optimal price of the TD policy.

Third, and most importantly, the firm can set a selling price below its manufacturing cost. This result can be observed under low and high realizations of exchange rate. When the realized exchange rate is low approaching the lower support of its pdf, the firm can set a price below cost in the domestic market. Similarly, when the realized exchange rate is high approaching the upper support of its pdf, the firm can set a selling price below cost in the foreign market. Thus, the result regarding the firm's action to price a product below cost continues to hold under the postponed pricing flexibility.

In sum, we conclude that our manufacturing quantity and selling price findings are robust as they cannot be eliminated from the optimal solution in the postponed pricing model. Specifically, the firm can manufacture a smaller quantity and charge a lower selling price under a production

hedging policy. Moreover, the firm continues to charge a selling price below cost in the postponed pricing model when the exchange rate is either too low or too high.

2.7 Conclusions

Production hedging policies are conservative actions where the firm deliberately does not manufacture sufficient products to satisfy the global demand. This paper shows how global firms can mitigate exchange rate risk by employing production hedging policies.

The paper makes seven main contributions. First, we show that production hedging is not just capable of maximizing expected profit, but can also minimize the firm's exchange-rate risk. While a firm that commits to satisfying its global demand cannot lower the possibility of losing money, production hedging reduces the probability of losing money significantly. In the event that the firm is extremely risk averse, it cannot avoid the possibility of losing money when it manufactures the total demand; however, production hedging can eliminate the possibility of a loss even at the lowest realization of the exchange rate random variable. Thus, production hedging is a less risky policy when compared to the traditional practice of manufacturing the total demand.

Second, we show that as the firm's level of risk aversion increases (e.g. smaller values of α and β), it becomes more likely to choose production hedging over a policy that satisfies the global demand. Thus, under risk aversion, production hedging is even a more appropriate policy.

Third, we demonstrate that the firm uses both the selling price and production quantity decisions as levers to reduce the firm's exchange-rate risk exposure. Unlike the single directional results reported in earlier publications under other forms of uncertainty, we prove that under the VaR constraint, the selling price and the manufacturing quantities can show both an increasing and decreasing behavior under exchange-rate uncertainty.

51

One might argue that, under production hedging, the firm produces less, and therefore, has the ability to charge a higher price than the price when it satisfies the global demand. Our fourth result proves that the firm charges a lower price under production hedging than the optimal price when it satisfies the global demand. By reducing the selling price, the firm increases demand in markets and benefits more from the postponement of the allocation decisions. This result is consistent with our example of Zara. This apparel company's marketing strategy is based on scarcity, which resembles our production hedging result. Moreover, one can find that the retail prices of Zara are considerably lower than their major competitors which resembles with our finding associated with reduced pricing under a production hedging policy over the total demand policy.

Fifth, the firm can actually set a selling price that is lower than the unit manufacturing cost under production hedging, and yet make a positive expected profit. This unexpected pricing result should not be confused with predatory pricing because our model does not feature competition, and the intention is not to drive the competitor out of business, or increase market share and power. In the event that the firm is extremely risk averse, the firm deviates from this behavior, and does not charge a price below cost.

Under production hedging, some markets are not fully served, and thus one might argue that consumers suffer from such practices. However, our sixth result shows that the firm can produce more units and set a lower price under production hedging than when it satisfies the global demand. As a consequence, consumers are not always worse off under a production hedging policy. Indeed, some consumers can benefit from lower prices when multinational firms utilize production hedging policies. Risk aversion generally causes the firm to increase the selling price and reduce the manufacturing quantity. However, the firm may continue to reduce the selling

price under the binding risk constraints below what it would charge in the absence of a VaR constraint. As a result of reduced selling price, the demand is higher in each market, and the firm might manufacture a higher quantity.

Finally, our seventh result shows that production hedging is still a viable and potentially optimal policy even in the presence of financial hedging. While financial hedging can mitigate the negative effects of a Value-at-Risk constraint, it eliminates the intermediate policies that stems from the risk constraint. Under financial hedging, the list of potentially optimal policies is reduced to the total demand, the maximum of the two market demands, and the minimum of the two market demands.

We identify three parameters that are influential in these rather counter-intuitive results: Higher values of unit manufacturing costs, volatility in exchange rates, and the size of foreign market size can make the firm commit to production hedging. Higher unit manufacturing cost implies higher potential for loss and a bigger risk, and causes the firm to prefer production hedging over the commitment of satisfying the global demand. Greater volatility in the random exchange rate makes production hedging more attractive and reduces the likelihood of manufacturing the total demand in the optimal solution. Similar to the unit manufacturing cost, the size of the international market can also influence the firm's preference towards production hedging. Higher foreign market size implies higher risk, and increases the importance of risk mitigation aspect of production hedging.

2.8 Future Research Directions

The initial modeling approach presented in Section 2.3 has isolated the influence of exchange-rate risk on pricing and production decisions of a firm. This model can be extended to include financial hedging decisions (as demonstrated in Section 2.5) along with pricing and

manufacturing quantity decisions that take place in the presence of exchange-rate uncertainty in Stage 1. While the revised model can provide a richer presentation, it would not change the structural results presented in this section. This is because production hedging cannot be eliminated from the list of potentially optimal decisions, and the firm would never commit to financial hedging exceeding the amount $X - d_1$ units.

Our model has largely ignored the influence of demand uncertainty, and it can be extended by incorporating demand uncertainty. It is noteworthy to mention that, when exchange-rate uncertainty is ignored, the problem becomes a Price-Setting Newsvendor Problem (PSNP). According to PSNP, it is well-known that the firm never sets an optimal selling price that is below its manufacturing cost; price greater than unit manufacturing cost enables the firm to determine a Newsvendor ratio. When uncertainty in exchange rates and demand are simultaneously incorporated into the model, one would expect to find examples where the firm continues to set its selling price lower than the unit manufacturing cost for profit maximization. Such a finding can further enrich the significance of our pricing result, because it never occurs with analytical models that consider demand uncertainty in isolation, ignoring the impact of exchange-rate uncertainty. A rich and complete model can be developed by considering the pricing, manufacturing, and financial hedging decisions simultaneously under the presence of exchange-rate and demand risks; such a model can provide richer insights for practicing managers.

2.9 Appendix

2.9.1 Proofs and Derivations

The derivations demonstrating how θ is related with variance

Because $\theta = \int_{e_l}^{\bar{e}} (\bar{e} - e) f(e) de = \int_{\bar{e}}^{e_h} (e - \bar{e}) f(e) de$, larger values of θ are associated with higher

volatility. The value of $\int_{e_l}^{\bar{e}} (\bar{e} - e) f(e) de$, for example, is the expected payoff of the option to sell

a unit of foreign currency at price $\bar{e}$, and the value of $\int_{\bar{e}}^{e_h} (e - \bar{e}) f(e) de$ is the expected payoff of

the option to buy a unit of foreign currency at price $\bar{e}$.

We show that for several common distributions that θ is proportional to the standard

deviation. We use X to denote the random variable of interest (e.g., exchange rate) with pdf $f(x)$,

and μ and σ denote the mean and standard deviation of X. We define the corresponding

"standardized" random variable $Z = (X - \mu)/\sigma$, which has mean 0 and standard deviation 1. For

realization x of X, the corresponding realization z of Z is $z = (x - \mu)/\sigma$, e.g.,

- at $x = \mu$ we have $z = 0$,

- at $x = \infty$ we have $z = \infty$,

- $x - \mu = z\sigma$,

- $dz/dx = 1/\sigma$, or in alternative form, $dx = \sigma dz$.

Thus, by change of variable,

$$\theta = \int_{\mu}^{\infty} (x - \mu) f(x) dx = \sigma \int_{0}^{\infty} zf(\mu + z\sigma) \sigma dz.$$

For several common distributions, the function $f(\mu + z\sigma)\sigma$ is solely a function of z, and for

distributions with this property, we let $\phi(z) = f(\mu + z\sigma)\sigma$. Substituting into the above, we get

$$\theta = \sigma \int_{0}^{\infty} z\phi(z) dz$$

55

and we see that θ is proportional to standard deviation σ. Below, we provide a few examples:

Normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)}$$

$$\phi(z) = f(\mu + z\sigma)\sigma = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z}$$

$$\theta = \sigma\int_0^\infty z\phi(z)\,dz = \sigma\int_0^\infty \frac{z}{\sqrt{2\pi}}e^{-\frac{1}{2}z}\,dz = \frac{1}{\sqrt{2\pi}}\sigma = 0.399\sigma$$

Uniform distribution:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & x \in [a,b] \\ 0, & x \notin [a,b] \end{cases}$$

$$\mu = (a+b)/2$$

$$\sigma = (b-a)12^{-1/2}$$

$$x = a \Rightarrow z = \frac{a-(b-a)/2}{(b-a)/\sqrt{12}} = -\frac{\sqrt{12}}{2} = -\sqrt{3}$$

$$x = b \Rightarrow z = \frac{b-(b-a)/2}{(b-a)/\sqrt{12}} = \frac{\sqrt{12}}{2} = \sqrt{3}$$

$$\phi(z) = f(\mu + z\sigma)\sigma = \begin{cases} \dfrac{1}{2\sqrt{3}}, & z \in \left[-\sqrt{3},\sqrt{3}\right] \\ 0, & z \notin \left[-\sqrt{3},\sqrt{3}\right] \end{cases}$$

$$\theta = \sigma\int_0^\infty z\phi(z)\,dz = \sigma\int_0^{\sqrt{3}} \frac{z}{2\sqrt{3}}\,dz = \frac{\sqrt{3}}{4}\sigma = 0.433\sigma$$

Exponential distribution:

$$f(x) = \frac{1}{\sigma} e^{-\frac{x}{\sigma}}, x > 0$$

$$\mu = \sigma$$

$$\phi(z) = f(\mu + z\sigma)\sigma = e^{-(z+1)}, z > -1$$

$$\theta = \sigma\int_0^\infty z\phi(z)\,dz = \sigma\int_0^\infty ze^{-(z+1)}\,dz = \frac{\sigma}{e}\int_0^\infty ze^{-z}\,dz = \frac{\sigma}{e} = 0.368\sigma$$

Proof of Proposition 2.1

a) For any price level, the second-order derivative of the objective function with respect to X is zero, implying that the objective function is linearly increasing in the production amount. For this reason, it is sufficient to consider the first-order derivatives to complete the proof.

b-i) and b-ii) When $0 \le X \le \min(d_1, d_2)$,

$$E\left[\Pi(X,p)\right] = -cX + \int_{e_l}^{\bar{e}} pXf(e)\,de + \int_{\bar{e}}^{e_h} peXf(e)\,de.$$

$$\frac{\partial E\left[\Pi(X,p)\right]}{\partial X} = -c + p[1+\theta]$$

The above implies that if $p < c/(1+\theta)$, the optimal production amount is $X^* = 0$; otherwise, if $p \ge c/(1+\theta)$, then the optimal production amount is at least $\min(d_1, d_2)$ if (2.1) is satisfied. When $\min(d_1, d_2) \le X \le \max(d_1, d_2)$, if the home demand is larger than the foreign demand at a price level p, then the objective function can be written as follows:

$$E\left[\Pi(d_2(p) < X < d_1(p), p)\right] = -cX + \int_{e_l}^{\bar{e}} pXf(e)\,de + \int_{\bar{e}}^{e_h} \left[ped_2(p) + p(X - d_2(p))\right]f(e)\,de,$$

$$\frac{\partial E\left[\Pi(X,p)\right]}{\partial X} = -c + p$$

or, alternatively if the foreign demand is higher than the domestic demand at p, then,

$$E\left[\Pi\left(d_1(p) < X < d_2(p), p\right)\right] = -cX + \int_{e_l}^{\bar{e}}\left[pd_1(p) + pe\left(X - d_1(p)\right)\right]f(e)de + \int_{\bar{e}}^{e_h} peXf(e)de,$$

$$\frac{\partial E\left[\Pi(X,p)\right]}{\partial X} = -c + p$$

Both cases imply that if $p < c$, the optimal production amount is $X^* = \min(d_1,d_2)$ if (2.1) is satisfied; otherwise, if $p \geq c$, then the optimal production amount is at least $\max(d_1,d_2)$ if (2.1) is satisfied. However, if $p < c - (\beta/\min(d_1,d_2))$ then (2.1) is violated. Thus, if $c/(1+\theta) \leq p < c - (\beta/\min(d_1,d_2))$, then it is optimal to produce the maximum amount while satisfying (2.1). Because (2.1) is binding, we have $X^* = \beta/(c - p)$ which we label as the PHIN policy. If $\min(\max(c/(1+\theta), c - (\beta/\min(d_1,d_2)))), c) \leq p < c$, then $X^* = \min(d_1,d_2)$, then $X^* = \min(d_1,d_2)$ since (2.1) is satisfied which we label as the PHN policy.

b-iii) and b-iv) Consider the case when $\max(d_1,d_2) \leq X \leq d_1+d_2$.

$$E\left[\Pi\left(\max\left(d_1(p),d_2(p)\right) \leq X < d_1(p)+d_2(p), p\right)\right] = -cX + \int_{e_l}^{\bar{e}}\left[pd_1(p) + pe\left(X - d_1(p)\right)\right]f(e)de$$

$$+ \int_{\bar{e}}^{e_h}\left[ped_2(p) + p\left(X - d_2(p)\right)\right]f(e)de$$

$$\frac{\partial E\left[\Pi\left(\max\left(d_1(p),d_2(p)\right) \leq X < d_1(p)+d_2(p), p\right)\right]}{\partial X} = -c + p[1-\theta]$$

This implies that if $p < c/(1 - \theta)$, then the optimal production amount is $X^* = \max(d_1,d_2)$ if (2.1) is satisfied; otherwise, if $p \geq c/(1 - \theta)$, then the optimal production amount is d_1+d_2 if (2.1) is satisfied. If $d_1 \geq d_2$, $X^* = \max(d_1,d_2)$ always satisfies (2.1) for $c \leq p < c/(1 - \theta)$ which we label as the PHX policy. However for the same price region, if $d_1 < d_2$ and $p < (cd_2 - \beta)/(d_1(1 - e_\alpha)+d_2e_\alpha))$, then $X^* = \max(d_1,d_2)$ violates (2.1). Thus, if $d_1 < d_2$ for $c \leq p < \min(\max(c, (cd_2 - \beta)/(d_1(1 - e_\alpha)+ d_2e_\alpha))), c/(1 - \theta))$, then it is optimal to produce the maximum amount while satisfying (2.1). Because (2.1) is binding, we have $X^* = ((1 - e_\alpha)d_1 + (\beta/p))/((c/p) - e_\alpha)$ which we

58

label as the PHIX policy. If $d_1 < d_2$ for $\min(\max(c, (cd_2 - \beta)/(d_1(1 - e_\alpha) + d_2e_\alpha))), c/(1 - \theta)) \leq p \leq$

$c/(1 - \theta)$, then $X^* = \max(d_1, d_2)$ since (2.1) is satisfied, which we label as the PHX policy.

b-v) and b-vi) If $p \geq c/(1 - \theta)$, then the optimal production amount is $X^* = d_1 + d_2$ if (2.1) is

satisfied. However, if $p < ((d_1 + d_2)c - \beta)/(d_1 + d_2e_\alpha)$, then (2.1) is violated. Thus, if $c/(1 - \theta) < p$

$< ((d_1 + d_2)c - \beta)/(d_1 + d_2e_\alpha)$ then it is optimal to produce the maximum amount while satisfying

(2.1). Because (2.1) is binding, we have $X^* = ((1 - e_\alpha)d_1 + (\beta/p))/((c/p) - e_\alpha)$ which we label as

the PHI policy. If $\max(c/(1 - \theta), ((d_1 + d_2)c - \beta)/(d_1 + d_2e_\alpha)) \leq p$, then $X^* = d_1 + d_2$ since (2.1) is

satisfied which we label as the TD policy. Thus for a given price level, there are six potentially

optimal production policies.

c) When $d_1 > d_2$ for $c \leq p < c/(1 - \theta)$, the PHX policy always satisfies (2.1) because the firm can

always sell all the products in the home market for low exchange rate realizations without a

chance of loss. We already proved that it is optimal to produce $X^* = \max(d_1, d_2)$ for $c \leq p < c/(1 -$

$\theta)$.

d) The six potentially optimal production candidates yield an expected profit that is greater than

or equal to zero under the corresponding optimal price regions. The NP policy always yields an

expected profit that strictly equals to zero. Thus, the NP policy is never an optimal solution. ❑

Proof of Proposition 2.2

a) For a given X, there are four regions of price values that can alter the objective function

expression, and we analyze concavity in each region specifically. For simplicity, we assume that

$d_2(p) > d_1(p)$ for all p in these derivations.

R1) $0 \leq X \leq \min(d_1(p), d_2(p))$: For small p, the demand will be very high, both markets exceeding

the amount of initial production X. The objective function in this region (R1) is the following:

$$E\left[\Pi\left(p,X\,|\,R1\right)\right] - cX + \int_{e_l}^{\bar{e}}[pX]f(e)de + \int_{\bar{e}}^{e_h}[peX]f(e)de\,. \tag{2.12}$$

From the first- and second-order condition of (2.12) with respect to p, we derive the following, respectively:

$$E\left[\Pi_p\left(p,X\,|\,R1\right)\right] = \int_{e_l}^{\bar{e}}[X]f(e)de + \int_{\bar{e}}^{e_h}[eX]f(e)de = (1+\theta)X > 0,\ E\left[\Pi_{pp}\left(p,X\,|\,R1\right)\right] = 0\,.$$

First-order derivative shows that the objective function is always increasing in p for a given X in this interval, and the second-order derivative equals to zero. Thus, the expected profit is linearly increasing in p for a given X in R1. From the first- and second-order condition of (2.12) with respect to X, we derive the following, respectively:

$$E\left[\Pi_X\left(p,X\,|\,R1\right)\right] = \int_{e_l}^{\bar{e}}[p]f(e)de + \int_{\bar{e}}^{e_h}[pe]f(e)de = (1+\theta)p > 0,\ E\left[\Pi_{XX}\left(p,X\,|\,R1\right)\right] = 0\,.$$

In R1, the objective function is linearly increasing in X for a given p. However, because the cross derivative $E\left[\Pi_{pX}\left(p,X\,|\,R1\right)\right] = 1+\theta > 0$, the determinant of the Hessian becomes negative. Thus, the objective function is not jointly concave in p and X in R1.

R2) $d_1(p) < X \le d_2(p)$: Note that in R2, the prices are such that $d_1(p) < X \le d_2(p)$. The objective function for R2 is the following:

$$E\left[\Pi\left(p,X\,|\,R2\right)\right]$$
$$= -cX + \int_{e_l}^{\bar{e}}\left[pd_1(p) + pe\left(X - d_1(p)\right)\right]f(e)de + \int_{\bar{e}}^{e_h}[peX]f(e)de = \left(p\bar{e} - c\right)X + pd_1(p)\theta. \tag{2.13}$$

From the first- and second-order condition of (2.13) with respect to p, we derive the following, respectively:

$$E\left[\Pi_{p}\left(p, X \mid R2\right)\right] = \overline{e}X + \theta\left(d_{1}\left(p\right) + pd_{1p}\left(p\right)\right),$$

$$E\left[\Pi_{pp}\left(p, X \mid R2\right)\right] = \theta\left(2d_{1p}\left(p\right) + pd_{1pp}\left(p\right)\right) \leq 0.$$

First-order derivative can take both positive and negative values, but the second-order derivative is non-positive, so the objective function is concave in p for a given X in R2. From the first- and second-order condition of (2.13) with respect to X, we derive the following, respectively:

$$E\left[\Pi_{X}\left(p, X \mid R2\right)\right] = \left(p\overline{e} - c\right), \; E\left[\Pi_{XX}\left(p, X \mid R2\right)\right] = 0.$$ In this region, the objective function is

concave in X for a given p. However, because the cross derivative $E\left[\Pi_{pX}\left(p, X \mid R2\right)\right] = \overline{e} > 0$,

the determinant of the Hessian becomes negative. Thus, the objective function is not jointly

concave in p and X in R2.

R3) $d_2(p) < X \leq d_1(p) + d_2(p)$: Note that in R3, the prices are such that $d_2(p) < X \leq d_1(p) + d_2(p)$.

The objective function for R3 is the following:

$$E\left[\Pi\left(p, X \mid R3\right)\right]$$

$$= -cX + \int_{e_l}^{\overline{e}}\left[pd_1(p) + pe\left(X - d_1\left(p\right)\right)\right]f\left(e\right)de + \int_{\overline{e}}^{e_h}\left[p\left(X - d_2\left(p\right)\right) + ped_2\left(p\right)\right]f\left(e\right)de$$

$$= \left(p\overline{e} - c\right)X + pd_1\left(p\right)\theta + pd_2\left(p\right)\theta - pX\theta. \tag{2.14}$$

From the first- and second-order condition of (2.14) with respect to p, we derive the following, respectively:

$$E\left[\Pi_{p}\left(p, X \mid R3\right)\right] = \overline{e}X - \theta X + \theta\left[d_1\left(p\right) + pd_{1p}\left(p\right) + d_2\left(p\right) + pd_{2p}\left(p\right)\right],$$

$$E\left[\Pi_{pp}\left(p, X \mid R3\right)\right] = \theta\left[2d_{1p}\left(p\right) + pd_{1pp}\left(p\right) + 2d_{2p}\left(p\right) + pd_{2pp}\left(p\right)\right] \leq 0.$$ First-order derivative

can take both positive and negative values, but the second-order derivative is negative, so the

objective function is concave in p for a given X in R3. From the first- and second-order condition

61

of (2.14) with respect to X, we derive the following, respectively:

$E\left[\Pi_X\left(p,X\,|R3\right)\right]=\left(p\bar{e}-c\right)-p\theta=p(1-\theta)-c,\ E\left[\Pi_{XX}\left(p,X\,|R3\right)\right]=0$. In this region, the

objective function is concave in X for a given p. However, because the cross derivative

$E\left[\Pi_{pX}\left(p,X\,|R3\right)\right]=\left(\bar{e}-\theta\right)>0$, the determinant of the Hessian becomes negative. Thus, the

objective function is not jointly concave in p and X in R3.

R4) $d_1(p)+d_2(p)<X$: Note that in R4, the prices are so high such that $d_1(p)+d_2(p)<X$. The

objective function for R4 is the following: $E\left[\Pi\left(p,X\,|R4\right)\right]=-cX+pd_1(p)+pd_2(p)$. We know

that this expected profit function is concave in p from the concave revenue assumption. Thus, we

conclude that the objective function is concave in p for a given X, but is not jointly concave in p

and X.

b) From the first- and second-order condition of (2.5), we derive the following, respectively:

$E\left[\Pi_p^{TD}\left(X^*(p)=d_1+d_2,p\right)\right]=d_1+(p-c)d_{1p}+d_2+(p-c)d_{2p}$

$E\left[\Pi_{pp}^{TD}\left(X^*(p)=d_1+d_2,p\right)\right]=\left[2d_{1p}+(p-c)d_{1pp}\right]+\left[2d_{2p}+(p-c)d_{2pp}\right]\leq0;$

From the first- and second-order condition of (2.7), we derive the following, respectively:

$E\left[\Pi_p^{PHX}\left(X^*(p)=d_1\geq d_2,p\right)\right]=d_1+(p-c)d_{1p}+\theta\left(d_2+pd_{2p}\right)$

$E\left[\Pi_{pp}^{PHX}\left(X^*(p)=d_1\geq d_2,p\right)\right]=2d_{1p}+(p-c)d_{1pp}+\theta\left(2d_{2p}+pd_{2pp}\right)\leq0;$

From the first- and second-order condition of (2.8), we derive the following, respectively:

$E\left[\Pi_p^{PHX}\left(X^*(p)=d_2>d_1,p\right)\right]=d_2+(p-c)d_{2p}+\theta\left(d_1+pd_{1p}\right)$

$E\left[\Pi_{pp}^{PHX}\left(X^*(p)=d_2>d_1,p\right)\right]=2d_{2p}+(p-c)d_{2pp}+\theta\left(2d_{1p}+pd_{1pp}\right)\leq0;$

From the first- and second-order condition of (2.10), we derive the following, respectively:

$$E\left[\Pi_p^{PHN}\left(X^*(p)=\min\{d_1,d_2\},p\right)\right]=\min\{d_1,d_2\}+(p-c)\min\{d_1,d_2\}_p$$
$$+\left(\min\{d_1,d_2\}+p\min\{d_1,d_2\}_p\right)\theta$$

$$E\left[\Pi_{pp}^{PHN}\left(X^*(p)=\min\{d_1,d_2\},p\right)\right]=2\min\{d_1,d_2\}_p+(p-c)\min\{d_1,d_2\}_{pp}$$
$$+\theta\left(2\min\{d_1,d_2\}_p+p\min\{d_1,d_2\}_{pp}\right)\le 0.$$

Thus, policies TD, PHX, and PHN are concave in p and have a unique optimal solution.

c) The region that we encounter policies PHI, PHIX and PHIN under the VaR constraint are regions R3, R2 and R1, respectively. The VaR constraint is simply cutting the plane and is reducing the feasible set of (p,X) values in the objective function. Because the objective function is not jointly concave in X and p in these regions, a line search is necessary to determine the optimal price and production quantity.

d) Consider PHIN, where the selling price is below the unit manufacturing cost (Proposition 2.1) violates constraint (2.1). Under PHIN, the firm produces a quantity that is less than the minimum of the two demands. In order to satisfy (2.1), the firm has to produce $X^* = \beta/(c-p)$ and charge a price $p^* = c - \beta/X$. Substituting $X^* = \beta/(c-p)$ into the expected profit expression provides:

$$E\left[\Pi^{PHIN}\right]=-c\left(\frac{\beta}{c-p}\right)+\int_{e_l}^{\bar{e}}p\left(\frac{\beta}{c-p}\right)f(e)de+\int_{\bar{e}}^{e_h}pe\left(\frac{\beta}{c-p}\right)f(e)de=-\beta+\frac{p\beta\theta}{c-p}.$$ The first- and

second-order derivatives with respect to price show that the expected profit is convex increasing

in price: $E\left[\Pi_p^{PHIN}\right]=\dfrac{c\beta\theta}{(c-p)^2}>0$, $E\left[\Pi_{pp}^{PHIN}\right]=\dfrac{2c\beta\theta}{(c-p)^3}>0.$

This implies that the selling price under PHIN will increase as much as it can until the production amount becomes equal to the minimum demand, which is decreasing in price. Because at the intersection point, the firm is producing the minimum demand value, the solution

is described as the PHN policy, and its expected profit is higher than that of PHIN. As a result, PHIN is dominated by the PHN policy with revised price levels.

e) The proof follows from the first-order condition of (2.12) with respect to p, where p is small such that $0 \leq X \leq \min(d_1(p), d_2(p))$, corresponding to region R1:

$$E\left[\Pi_p\left(p, X \mid R1\right)\right] = \int_{e_l}^{\bar{e}} [X] f(e) de + \int_{\bar{e}}^{e_h} [eX] f(e) de = (1+\theta) X > 0, \ E\left[\Pi_{pp}\left(p, X \mid R1\right)\right] = 0.$$

First-order derivative shows that the objective function is always increasing in p for a given X in this interval, and because the expected profit is linearly increasing in p for a given X in R1 in this interval, the optimal price is always at least equal to the amount that would satisfy the minimum demand. From Proposition 2.1, we know that the minimum demand is produced when price is greater than equal to $c/(1+\theta)$. Thus, $p \geq c/(1+\theta)$. ❑

Proof of Proposition 2.3

Let $\Pi_{e_\alpha}^{TD}$ denote the optimal profit under the TD policy when the exchange rate is realized at e_α. Thus, $\Pi_{e_\alpha}^{TD} = \left(p^{TD} - c\right) d_1^{TD} + \left(p^{TD} e_\alpha - c\right) d_2^{TD}$. After rearranging the terms for the condition of

$\Pi_{e_\alpha}^{TD} \geq -\beta$, we derive the following condition: $0 \leq c \leq \left[\dfrac{p^{TD}\left(d_1^{TD} + d_2^{TD} e_\alpha\right) + \beta}{d_1^{TD} + d_2^{TD}}\right]$. Hence, if

$c > \left[\dfrac{p^{TD}\left(d_1^{TD} + d_2^{TD} e_\alpha\right) + \beta}{d_1^{TD} + d_2^{TD}}\right]$, then (2.1) becomes binding and the optimal price under the TD

policy no longer follows the non-binding expression in Table 2.1. Thus, to derive the optimal price under the TD policy while satisfying (2.1), we follow the Karush-Kuhn-Tucker condition. Let Z^{TD} denote the Lagrangian function of the TD policy, i.e.,

$Z^{TD} = (p-c) d_1 + (p_2 - c) d_2 - \lambda\left[(p-c) d_1 + (p e_\alpha - c) d_2 + \beta\right]$, where λ denotes the Lagrange

multiplier. If $c > \dfrac{p^{TD}\left(d_1^{TD} + d_2^{TD} e_\alpha\right)}{d_1^{TD} + d_2^{TD}}$, then $\lambda > 0$. Thus, following the Karush-Kuhn-Tucker

condition, $(p-c)d_1 + (pe_\alpha - c)d_2 + \beta = 0$. After rearranging the terms, we find the binding

optimal price under the TD policy which equals to $p^{TD} = \dfrac{\left(d_1^{TD} + d_2^{TD}\right)c - \beta}{d_1^{TD} + d_2^{TD} e_\alpha}$. Instead of serving

the total global demand with a higher selling price, the firm can reduce the manufacturing

commitment in the first stage. Thus, the PHI policy is where $\max(d_1, d_2) < X^*(p) < d_1 + d_2$ while

maximizing the expected profit and satisfying (2.1). $\square$

Proof of Proposition 2.4

a) Proposition 2.1 shows that the TD policy is optimal when the selling price is greater than or

equal to $\max(c/(1 - \theta), ((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha))$. Notice that as θ increases the value of $c/(1 -$

$\theta)$ also increases. Thus, the TD policy is less likely to be optimal for greater values of θ.

b) Proposition 2.1 shows that the TD policy is optimal when the selling price is greater than or

equal to $\max(c/(1 - \theta), ((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha))$. Notice that as the firm becomes more risk-

averse the value of α and β decreases. As α decreases, the value of e_α also decreases

correspondingly, which results to an increase for $((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha)$. Also as β decreases

the value of $((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha)$ increases. Thus, for decreasing values of α and β, the

interval of selling price values that satisfy $\max(c/(1 - \theta), ((d_1 + d_2)c - \beta)/(d_1 + d_2 e_\alpha))$ becomes

smaller. As a result, with higher risk aversion, the TD policy is less likely to be optimal, and one

of the production hedging policies is more likely to be optimal. $\square$

Proof of Proposition 2.5

We only consider the case of $d_2 > d_1$. Let $\Pi_{e_\alpha}^{PHX}$ denote the optimal profit under the PHX policy when the exchange rate is realized at e_α. Thus,

$\Pi_{e_\alpha}^{PHX} = -cd_2^{PHX} + p^{PHX}d_1^{PHX} + p^{PHX}e_\alpha\left(d_2^{PHX} - d_1^{PHX}\right)$. After rearranging the terms for the

condition of $\Pi_{e_\alpha}^{PHX} \geq -\beta$, we derive the following condition:

$c \leq p^{PHX}\left[\dfrac{d_1^{PHX}}{d_2^{PHX}}\left(1-e_\alpha\right)+e_\alpha\right]+\dfrac{\beta}{d_2^{PHX}}$. Hence, if $c > p^{PHX}\left[\dfrac{d_1^{PHX}}{d_2^{PHX}}\left(1-e_\alpha\right)+e_\alpha\right]+\dfrac{\beta}{d_2^{PHX}}$, then (2.1)

becomes binding and the optimal price under the PHX policy no longer follows the non-binding

expression in Table 2.1. Thus, to derive the optimal price under the PHX policy while satisfying

(2.1), we follow the Karush-Kuhn-Tucker condition. Let Z^{PHX} denote the Lagrangian function of

the PHX policy, i.e., $Z^{PHX} = \left(p-c\right)d_2 + pd_1\theta - \lambda\left[-cd_2 + pd_1 + pe_\alpha\left(d_2 - d_1\right)+\beta\right]$.

If $c > p^{PHX}\left[\dfrac{d_1^{PHX}}{d_2^{PHX}}\left(1-e_\alpha\right)+e_\alpha\right]+\dfrac{\beta}{d_2^{PHX}}$, then $\lambda > 0$. Thus, following the Karush-Kuhn-Tucker

condition, $-cd_2 + pd_1 + pe_\alpha\left(d_2 - d_1\right)+\beta=0$. After rearranging the terms, we find the binding

optimal price under the PHX policy which equals to $p^{PHX} = \dfrac{cd_2^{PHX} - \beta}{d_1^{PHX}\left(1-e_\alpha\right)+d_2^{PHX}e_\alpha}$. Instead of

producing to satisfy the foreign market demand with a higher selling price, the firm can reduce

the manufacturing commitment in the first stage. Thus, the PHIX policy is where $d_1 < X^*(p) < d_2$

while maximizing the expected profit and satisfying (2.1). ❑

Proof of Proposition 2.6

a) Let $\Pi_{e_\alpha}^{PHN}$ denote the optimal profit under the PHN policy when the exchange rate is realized

at e_α. Thus, $\Pi_{e_\alpha}^{PHN} = \left(p^{PHN} - c\right)\min\left\{d_1^{PHN}, d_2^{PHN}\right\}$. After rearranging the terms for the condition of

66

$\Pi_{e_\alpha}^{PHN} \geq -\beta$, we derive the following condition: $c \leq p^{PHN} + \dfrac{\beta}{\min\left\{d_1^{PHN}, d_2^{PHN}\right\}}$. Hence, if

$c > p^{PHN} + \dfrac{\beta}{\min\left\{d_1^{PHN}, d_2^{PHN}\right\}}$, then (2.1) becomes binding and the optimal price under the PHN

policy no longer follows the non-binding expression in Table 2.1. Thus, to derive the optimal

price under the PHN policy while satisfying (2.1), we follow the Karush-Kuhn-Tucker condition.

Let Z^{PHN} denote the Lagrangian function of the PHN policy, i.e.,

$$Z^{PHN} = (p-c)\min\left\{d_1^{PHN}, d_2^{PHN}\right\} + p\min\left\{d_1^{PHN}, d_2^{PHN}\right\}\theta - \lambda\left[(p-c)\min\left\{d_1^{PHN}, d_2^{PHN}\right\} + \beta\right].$$ If

$c > p^{PHN} + \dfrac{\beta}{\min\left\{d_1^{PHN}, d_2^{PHN}\right\}}$, then $\lambda > 0$. Thus, following the Karush-Kuhn-Tucker condition,

$(p-c)\min\left\{d_1^{PHN}, d_2^{PHN}\right\} + \beta = 0$. After rearranging the terms, we find the binding optimal price

under the PHN policy which equals to $p^{PHN} = c - \dfrac{\beta}{\min\left\{d_1^{PHN}, d_2^{PHN}\right\}}$. $\square$

Proof of Proposition 2.7

Recall from propositions 2.3, 2.5 and 2.6, that the binding prices under polices TD, PHX and

PHN, respectively, are all greater than their non-binding counterparts. Thus, the binding optimal

production quantities under policies TD, PHX and PHN are less than their non-binding

counterparts. $\square$

Proof of Proposition 2.8

We complete the proof in two parts to show that the TD policy has the highest probability to

violate constraint (2.1): when $p \geq c$, and when $p < c$.

i) First, consider the case when $p \geq c$.

When $p \geq c$, the TD policy violates the risk constraint (2.1) when the following condition holds:

$$e < \left[\frac{c}{p} - \frac{(p-c)d_1 + \beta}{pd_2} \right].$$ Thus, the probability of the TD policy violating (2.1) equals to

$$\int_{e_l}^{\left[\frac{c}{p} - \frac{(p-c)d_1+\beta}{pd_2} \right]} f(e) de.$$

a) If $p \geq c$, then the PHN policy never violates the constraint (2.1). Thus, if the selling price is greater than the unit manufacturing cost, the probability of the PHN policy violating (2.1) strictly equals to zero.

b) If $d_1 \geq d_2$ and $X^* = \max(d_1, d_2) = d_1$, then the probability of the PHX policy violating (2.1) strictly equals to zero.

c) However, if $d_2 > d_1$ and $X^* = \max(d_1, d_2) = d_2$, then the PHX policy violates (2.1) when

$$e < \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)} \right].$$ Thus, the probability of the PHX policy ($d_2 > d_1$) violating (2.1) equals to

$$\int_{e_l}^{\left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)} \right]} f(e) de.$$ To verify that the TD policy has a greater probability violating (2.1) than the

PHX policy, we next prove that $\left[\frac{c}{p} - \frac{(p-c)d_1 + \beta}{pd_2} \right] - \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)} \right] \geq 0$.

$$\left[\frac{c}{p} - \frac{(p-c)d_1 + \beta}{pd_2} \right] - \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)} \right] = \left[\frac{(cd_2 - (p-c)d_1 - \beta)(d_2 - d_1)}{pd_2(d_2 - d_1)} \right] - \left[\frac{(cd_2 - pd_1 - \beta)d_2}{pd_2(d_2 - d_1)} \right]$$

Because the denominators are equal and positive, we only compare the numerators:

$(cd_2 - (p-c)d_1 - \beta)(d_2 - d_1) - (cd_2 - pd_1 - \beta)d_2 = (p-c)d_1^2 + \beta d_1 \geq 0$. Thus,

$$\left[\frac{c}{p} - \frac{(p-c)d_1 + \beta}{pd_2} \right] \geq \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)} \right].$$ This proves that the probability of the TD policy

violating (2.1) is greater than that of the PHX policy.

68

Thus, when $p \geq c$, the TD policy has the highest probability to violate (2.1).

ii) Next, consider the case when $p < c$.

When $p < c$, the TD policy violates the risk constraint (2.1) when the following condition holds:

$e < \left[\dfrac{c}{p} + \dfrac{(c-p)d_1 - \beta}{pd_2} \right]$. Thus, the probability of the TD policy violating (2.1) equals to

$\int_{e_l}^{\left[\frac{c}{p} - \frac{(c-p)d_1 - \beta}{pd_2} \right]} f(e) de$.

Since the selling price is less than the unit manufacturing cost, the PHX and the PHN policy does

not always satisfy (2.1).

a) For the PHN policy where $d_1 \geq d_2$, if $\beta > (c-p)d_2$, then (2.1) is never violated; however, if

$0 \leq \beta \leq (c-p)d_2$ then (2.1) is violated when $e < \left[\dfrac{c}{p} - \dfrac{\beta}{pd_2} \right]$. Thus, for the comparison of the

TD policy and the PHN policy $(d_1 \geq d_2)$, we only need to consider the case for

$0 \leq \beta \leq (c-p)d_2$. Because $\left[\dfrac{c}{p} - \dfrac{\beta}{pd_2} + \dfrac{(c-p)d_1}{pd_2} \right] > \left[\dfrac{c}{p} - \dfrac{\beta}{pd_2} \right]$ for $0 \leq \beta \leq (c-p)d_2$, the TD

policy has a greater probability to violate (2.1) than the PHN policy $(d_1 \geq d_2)$.

b) For the PHN policy where $d_2 > d_1$, if $\beta > (c-p)d_1$, then (2.1) is never violated; however, if

$0 \leq \beta \leq (c-p)d_1$ then (2.1) is violated when $e < \left[\dfrac{c}{p} - \dfrac{\beta}{pd_1} \right]$. Thus, for the comparison of the TD

policy and the PHN policy $(d_2 > d_1)$, we only need to consider the case where $0 \leq \beta \leq (c-p)d_1$.

To verify that the TD policy has a greater probability violating (2.1) than the PHN policy $(d_2 >$

$d_1)$, we next prove that $\left[\dfrac{c}{p} + \dfrac{(c-p)d_1 - \beta}{pd_2} \right] - \left[\dfrac{c}{p} - \dfrac{\beta}{pd_1} \right] > 0$ when $0 \leq \beta \leq (c-p)d_1$.

69

$$\left[\frac{c}{p}+\frac{(c-p)d_1-\beta}{pd_2}\right]-\left[\frac{c}{p}-\frac{\beta}{pd_1}\right]=\left[\frac{\left(cd_2+(c-p)d_1-\beta\right)d_1}{pd_1d_2}\right]-\left[\frac{(cd_1-\beta)d_2}{pd_1d_2}\right].$$ Because the

denominators are equal and positive we only compare the numerators:

$$\left[\left(cd_2+(c-p)d_1-\beta\right)d_1\right]-\left[(cd_1-\beta)d_2\right]=(c-p)d_1^2+\beta(d_2-d_1)>0 \text{ where } 0\leq\beta\leq(c-p)d_1$$

and $d_2>d_1$. Thus, $\left[\frac{c}{p}+\frac{(c-p)d_1-\beta}{pd_2}\right]>\left[\frac{c}{p}-\frac{\beta}{pd_1}\right].$ This proves that the TD policy has a

greater probability to violate (2.1) then the PHN policy ($d_2>d_1$).

c) For the PHX policy where $d_1\geq d_2$, if $\beta>(c-p)d_1$, then (2.1) is never violated; however, if

$0\leq\beta\leq(c-p)d_1$, then (2.1) is violated when $e<\left[\frac{c}{p}-\frac{\beta}{pd_1}\right].$ Thus, for the comparison of the

TD policy and the PHX policy ($d_1\geq d_2$), we only need to consider the case when

$0\leq\beta\leq(c-p)d_1.$

Following the proof of ii-b), we know that $\left[\frac{c}{p}+\frac{(c-p)d_1-\beta}{pd_2}\right]>\left[\frac{c}{p}-\frac{\beta}{pd_1}\right].$ Thus, the TD

policy has a greater probability to violate (2.1) than the PHX policy ($d_1\geq d_2$).

d) For the PHX policy where $d_2>d_1$, if $0\leq\beta\leq(c-p)d_2$, then (2.1) is violated when

$e<\left[\frac{c}{p}-\frac{\beta}{pd_2}\right].$ If $\beta>(c-p)d_2$, then (2.1) is violated when $e<\left[\frac{cd_2-pd_1-\beta}{p(d_2-d_1)}\right].$

First consider the case where $0\leq\beta\leq(c-p)d_2$. Because $\left[\frac{c}{p}-\frac{\beta}{pd_2}+\frac{(c-p)d_1}{pd_2}\right]>\left[\frac{c}{p}-\frac{\beta}{pd_2}\right],$

the TD policy has a greater probability to violate (2.1) than the PHX policy for $0\leq\beta\leq(c-p)d_2$

.

Next consider the case where $\beta > (c-p)d_2$. To verify that the TD policy has a greater

probability violating (2.1) than the PHX policy, we next prove that

$$\left[\frac{c}{p} + \frac{(c-p)d_1 - \beta}{pd_2}\right] > \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)}\right].$$

$$\left[\frac{c}{p} + \frac{(c-p)d_1 - \beta}{pd_2}\right] - \left[\frac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)}\right] = \left[\frac{(cd_2 + (c-p)d_1 - \beta)(d_2 - d_1)}{pd_2(d_2 - d_1)}\right] - \left[\frac{(cd_2 - pd_1 - \beta)d_2}{pd_2(d_2 - d_1)}\right]$$

Because the denominators are equal and positive we only compare the numerators:

$$\left[(cd_2 + (c-p)d_1 - \beta)(d_2 - d_1)\right] - \left[(cd_2 - pd_1 - \beta)d_2\right] = -(c-p)d_1^2 + \beta d_1 > 0 \text{ where}$$

$\beta > (c-p)d_2$ and $d_2 > d_1$. Thus, $\left[\dfrac{c}{p} + \dfrac{(c-p)d_1 - \beta}{pd_2}\right] > \left[\dfrac{cd_2 - pd_1 - \beta}{p(d_2 - d_1)}\right]$. This proves that the

TD policy has a greater probability to violate (2.1) than the PHX policy when $\beta > (c-p)d_2$.

Thus, the TD policy has a greater probability to violate (2.1) than the PHX policy ($d_2 > d_1$).

In sum, the TD policy has the highest probability to violate constraint (2.1). ❑

Proof of Proposition 2.9

a) The objective function for each policy can be rewritten in terms θ as follows:

$$E\left[\Pi^{PHN}\left(X^*(p) = \min\{d_1, d_2\}, p\right)\right] = -c\min\{d_1, d_2\} + p\min\{d_1, d_2\}(1+\theta) \text{ and its derivative}$$

with respect to θ is $E\left[\Pi_\theta^{PHN}\left(X^*(p) = \min\{d_1, d_2\}, p\right)\right] = p\min\{d_1, d_2\} > 0$;

$$E\left[\Pi^{PHIX}\left(X^*(p) = \frac{(1-e_\alpha)d_1 + \frac{\beta}{p}}{\frac{c}{p} - e_\alpha}, p\right)\right] = (p-c)\frac{(1-e_\alpha)d_1 + \frac{\beta}{p}}{\frac{c}{p} - e_\alpha} + pd_1\theta \text{ and its derivative with}$$

respect to θ is $E\left[\Pi_\theta^{PHIX}\left(X^*(p) = \frac{(1-e_\alpha)d_1 + \frac{\beta}{p}}{\frac{c}{p} - e_\alpha}, p\right)\right] = pd_1 > 0$;

$$E\left[\Pi_\theta^{PHX}\left(X^*(p) = \max\{d_1, d_2\}, p\right)\right] = (p-c)\max\{d_1, d_2\} + p\min\{d_1, d_2\}\theta \text{ and its derivative}$$

71

with respect to θ is $E\left[\Pi_\theta^{PHX}\left(X^*(p) = \max\{d_1, d_2\}, p\right)\right] = p\min\{d_1, d_2\} > 0$;

$$E\left[\Pi^{PHI}\left(X^*(p) = \frac{(1-e_\alpha)d_1 + \beta/p}{c/p - e_\alpha}, p\right)\right] = (p-c)\frac{(1-e_\alpha)d_1 + \beta/p}{c/p - e_\alpha} + p\theta\left[(d_1 + d_2) - \frac{(1-e_\alpha)d_1 + \beta/p}{c/p - e_\alpha}\right] \text{ and its}$$

derivative with respect to θ is

$$E\left[\Pi_\theta^{PHI}\left(X^*(p) = \frac{(1-e_\alpha)d_1 + \beta/p}{c/p - e_\alpha}, p\right)\right] = p\left[(d_1 + d_2) - \frac{(1-e_\alpha)d_1 + \beta/p}{c/p - e_\alpha}\right] > 0;$$

$E\left[\Pi^{TD}\left(X^*(p) = d_1 + d_2, p\right)\right] = (p-c)(d_1 + d_2)$ and its derivative with respect to θ is

$$E\left[\Pi_\theta^{TD}\left(X^*(p) = d_1 + d_2, p\right)\right] = 0.$$

b) For symmetric distributions, from the inverse of the cumulative distribution function, if $\theta_1 \geq$ θ_2, then $e_\alpha(\theta_2) \geq e_\alpha(\theta_1)$. Thus, we can conclude that $\dfrac{\partial e_\alpha(\theta)}{\partial \theta} \leq 0$. Notice that the derivative of

$((d_1+d_2)c - \beta)/(d_1+d_2 e_\alpha))$ with respect to e_α is positive, i.e. .

$$\frac{\partial\left\{\left[(d_1 + d_2)c - \beta\right]/\left[d_1 + d_2 e_\alpha(\theta)\right]\right\}}{\partial e_\alpha} \geq 0. \text{ Thus, } \frac{\partial\left\{\left[(d_1 + d_2)c - \beta\right]/\left[d_1 + d_2 e_\alpha(\theta)\right]\right\}}{\partial \theta} \geq 0. \text{ Since,}$$

the risk constraint is binding $((d_1+d_2)c - \beta)/(d_1+d_2 e_\alpha)) \geq c/(1 - \theta)$. Recall from Proposition 2.2

that the objective function is concave in p for a given X. Thus, we can conclude that as θ

increases, the binding optimal TD price, i.e., $p^{TD} = ((d_1+d_2)c - \beta)/(d_1+d_2 e_\alpha))$, further deviates

from the non-binding optimal price which results to non-increasing expected profits. $\square$

Proof of Proposition 2.10

i) From Table 2.1, we know that $p^{TD} = -\dfrac{d_1^{TD} + d_2^{TD}}{d_{1p}^{TD} + d_{2p}^{TD}} + c$ and $p^{PHX} = -\dfrac{d^{max} - cd_p^{max} + d^{min}\theta}{d_p^{max} + d_p^{min}\theta}$. By

rearranging the terms for the condition of $p^{PHX} \leq p^{TD}$, we derive the following condition:

$$p^{PHX} \leq p^{TD} \text{ holds when } c \geq c_p^{PHX} \text{ where } c_p^{PHX} = \frac{d^{max} + d^{min}\theta}{1 + d_p^{max}} - \frac{\left[d_1^{TD} + d_2^{TD} \right]\left[d_p^{max} + d_p^{min}\theta \right]}{\left[d_{1p}^{TD} + d_{2p}^{TD} \right]\left[1 + d_p^{max} \right]}.$$

ii) From Table 2.1, we know that $p^{PHN} = -\dfrac{d^{min}}{d_p^{min}} + \dfrac{c}{1+\theta}$. By rearranging the terms for the

condition of $p^{PHN} \leq p^{TD}$, we derive the following condition: $p^{PHN} \leq p^{TD}$ holds when $c \geq c_p^{PHN}$

where $c_p^{PHN} = \left(-\dfrac{d^{min}}{d_p^{min}} + \dfrac{d_1^{TD} + d_2^{TD}}{d_{1p}^{TD} + d_{2p}^{TD}} \right)\left(\dfrac{1+\theta}{\theta} \right).$ ❑

Proof of Proposition 2.11

Recall from Proposition 2.1 that the optimal price region under the PHN is $c/(1 + \theta) < p \leq c$.

In other words, if the optimal price is less than the unit manufacturing cost (but greater than or

equal to $c/(1 + \theta)$), then it is optimal to produce the quantity that equals to $\min(d_1,d_2)$. Under the

PHN policy, (2.10) can be rearranged as:

$$E\left[\Pi^{PHN}\left(X = \min\{d_1,d_2\}, \frac{c}{1+\theta} \leq p < c \right) \right] = \left[p(1+\theta) - c \right]\min\{d_1,d_2\} \geq 0$$

where we can see that the expected profit is greater than or equal to zero. If the risk constraint

(2.1) is satisfied, the PHN policy follows the non-binding optimal price from Table 2.1 where

$p^{PHN} = \dfrac{c}{1+\theta} - \dfrac{d^{min}}{d_p^{min}}$. By rearranging the terms for the condition of $p^{PHN} \leq c$, we derive the

following condition: $c \geq \left(\dfrac{p}{\varepsilon_{min}} \right)\left(\dfrac{1+\theta}{\theta} \right)$ where ε_{min} denotes the elasticity *of* $\min(d_1,d_2)$. If the risk

constraint (2.1) is violated the PHN policy follows the binding optimal price from Table 2.1

where $p^{PHN} = c - \dfrac{\beta}{d^{\min}}$. By rearranging the terms for the condition of $p^{PHN} \leq c$, we derive the

following condition: $\beta \geq 0.$ $\square$

Proof of Proposition 2.12

i) From Table 2.1, we know that $X^{TD} = -\left(p^{TD} - c\right)\left(d_{1p}^{TD} + d_{2p}^{TD}\right)$ and

$X^{PHX} = -p^{PHX}\left[d_p^{\max} + d_p^{\min}\theta\right] + cd_p^{\max} - d^{\min}\theta$. By rearranging the terms for the condition of

$X^{PHX} \geq X^{TD}$, we derive the following condition: $X^{PHX} \geq X^{TD}$ holds when $c \geq c_X^{PHX}$ where

$$c_X^{PHX} = \frac{-2p^{TD}\left(d_{1p}^{TD} + d_{2p}^{TD}\right) - \left(d_1^{TD} + d_2^{TD}\right)d^{\max} + p^{PHX}\left[d_p^{\max} + d_p^{\min}\theta\right] + d^{\min}\theta}{-2\left(d_{1p}^{TD} + d_{2p}^{TD}\right) + d_p^{\max}}.$$

ii) From Table 2.1, we know that $X^{PHN} = -p^{PHN}d_p^{\min} + \dfrac{cd_p^{\min}}{1+\theta}$. By rearranging the terms for the

condition of $X^{PHN} \geq X^{TD}$, we derive the following condition: $X^{PHN} \geq X^{TD}$ holds when $c \geq c_X^{PHN}$

where $c_X^{PHN} = \dfrac{2p^{TD}\left(d_{1p}^{TD} + d_{2p}^{TD}\right) + \left(d_1^{TD} + d_2^{TD}\right) - p^{PHN}d_p^{\min}}{-2\left(d_{1p}^{TD} + d_{2p}^{TD}\right) + \left(\dfrac{d_p^{\min}}{1+\theta}\right)}.$ $\square$

Proof of Proposition 2.13

i) From Table 2.1, we know that $E\left[\Pi^{TD}\right] = -\dfrac{\left(d_1^{TD} + d_2^{TD}\right)^2}{d_{1p}^{TD} + d_{2p}^{TD}}$ and

$$E\left[\Pi^{PHX}\right] = -d_p^{\max}c^2 + \left[\left(3d_p^{\max} + d_p^{\min}\theta\right)p^{PHX} + d_p^{\min}\theta\right]c - \begin{bmatrix} 2\left(d_p^{\max} + d_p^{\min}\theta\right)\left(p^{PHX}\right)^2 \\ + \left(d^{\max} + d^{\min}\theta\right)p^{PHX} \end{bmatrix}.$$

By rearranging the terms for the condition of $E\left[\Pi^{PHX}\right] \geq E\left[\Pi^{TD}\right]$, we derive the following

condition: $E\left[\Pi^{PHX}\right] \geq E\left[\Pi^{TD}\right]$ holds when $c \geq c_\Pi^{PHX}$ where c_Π^{PHX} solves

$$-d_p^{\max} c^2 + \left[\left(3d_p^{\max} + d_p^{\min}\theta\right)p^{PHX} + d_p^{\min}\theta\right]c - \left[\begin{array}{c} 2\left(d_p^{\max} + d_p^{\min}\theta\right)\left(p^{PHX}\right)^2 \\ +\left(d^{\max} + d^{\min}\theta\right)p^{PHX} + \dfrac{\left(d_1^{TD} + d_2^{TD}\right)^2}{d_{1p}^{TD} + d_{2p}^{TD}} \end{array}\right] = 0 .$$

ii) From Table 2.1, we know that

$E\left[\Pi^{PHN}\right] = -\dfrac{d_p^{\min}}{1+\theta}c^2 + 2p^{PHN}d_p^{\min}c - \left(1+\theta\right)\left(p^{PHN}\right)^2 d_p^{\min}$. By rearranging the terms for the

condition of $E\left[\Pi^{PHN}\right] \geq E\left[\Pi^{TD}\right]$, we derive the following condition: $E\left[\Pi^{PHN}\right] \geq E\left[\Pi^{TD}\right]$ holds

when $c \geq c_\Pi^{PHN}$ where c_Π^{PHN} solves

$$-\dfrac{d_p^{\min}}{1+\theta}c^2 + 2p^{PHN}d_p^{\min}c - \left[\left(1+\theta\right)\left(p^{PHN}\right)^2 d_p^{\min} + \dfrac{\left(d_1^{TD} + d_2^{TD}\right)^2}{d_{1p}^{TD} + d_{2p}^{TD}}\right] = 0 . \ \square$$

2.9.2 The Study of Extreme Risk Aversion

In this section, we analyze the case of extreme risk aversion where the firm prefers not to

lose any money even at the catastrophic event and at the worst realization of the exchange rate

random variable. This goal can be viewed as an extreme risk-averse measure as the firm

eliminates the risk of having negative profits (e.g., value-at-risk is zero with an associated

probability of 100%). Thus, the extreme risk-averse analysis can be considered as a special case

where $\alpha = \beta = 0$. Moreover, the VaR constraint (2.1) becomes

$$-cX + \pi(x_1, x_2 | X, p, e_l) \geq 0. \tag{2.15}$$

The following proposition provides a modified version of Proposition 2.1 where $\alpha = \beta = 0$.

Proposition 2.14 *a) For a given price level, there are five potentially optimal production*

policies:

i) $p = c$, *then* $X^* = \min(d_1, d_2) = d_1 \Rightarrow$ PHN;

ii) $c < p < \min(\max(c, (cd_2 - \beta)/(d_1(1 - e_a) + d_2 e_a)), c/(1 - \theta))$, *then* $d_1 < X^* = ((1 - e_l)/(c/p -$

$e_l))d_1 < d_2 \Rightarrow$ PHIX;

iii) $\min(\max(c, cd_2/(d_1(1 - e_l) + d_2 e_l))), c/(1 - \theta)) \leq p \leq c/(1 - \theta)$, *then* $X^* = \max(d_1, d_2) \Rightarrow$ PHX;

iv) $c/(1 - \theta) < p < \max(c/(1 - \theta), ((d_1 + d_2)/(d_1 + d_2 e_l))c)$, *then* $\max(d_1, d_2) < X^* = ((1 - e_l)/(c/p -$

$e_l))d_1 < d_1 + d_2 \Rightarrow$ PHI;

v) $\max(c/(1 - \theta), ((d_1 + d_2)/(d_1 + d_2 e_l))c) \leq p$, *then* $X^* = d_1 + d_2 \Rightarrow$ TD;

b) policy PHX *satisfies the risk constraint (2.15) and dominates policy* PHIX *when* $d_1 \geq d_2$ *for* $c \leq$

$p < c/(1 - \theta)$;

c) policy NP *cannot be the optimal solution.*

Proof of Proposition 2.14

For a-i), from Proposition 2.1, we know that if $0 \leq p < c/(1 + \theta)$ then $X^* = 0$ and if $c/(1 + \theta) \leq p < c$

then $X^* = \min(d_1, d_2)$ if (2.1) is satisfied. However, under extreme risk aversion, if $p < c$ then

(2.15) is always violated. Thus, $0 \leq p < c$ then $X^* = 0$. Also, if $p < (d_2/(d_1(1 - e_l) + d_2 e_l))c$, then

(2.15) is violated. If $d_2 > d_1$, because $c < (d_2/(d_1(1 - e_l) + d_2 e_l))c$, when $p = c$ it is optimal to

produce under the PHN policy because it is the maximum quantity that can be produced while

satisfying (2.15). If $d_1 \geq d_2$, then (2.15) is always satisfied as long as the selling price is greater

than or equal to the unit manufacturing cost. The rest of the proof follows Proposition 2.1. ❑

The above proposition shows that under extreme risk aversion the PHIN policy is not an

optimal policy because it always violates (2.15). Also, Proposition 2.14 proves that the minimum

price that the firm can charge under extreme risk aversion is equal to the unit manufacturing cost.

Under extreme risk-aversion, the firm can no longer charge a lower price than the unit manufacturing cost because there will always be a chance of yielding negative profits. In sum, under extreme risk aversion, the firm manufactures a quantity that is at least the home market demand, and at most the sum of the two market demand values.

Example 1 – The optimal price can show both an increasing and decreasing behavior when the firm switches from the TD policy to PHI policy:

We provide an example where the optimal policy choice switches from the TD policy (optimal non-binding policy) to the PHI policy (optimal binding policy) and the corresponding optimal price decreases below the level of the non-binding optimal price. We use the same problem parameters with those reported for Figure 2.6, i.e., $d_1(p) = 100 - p$, $d_2(p) = 100 - p$, and the exchange rate random variable is distributed with a Uniform distribution on a support of $[0,2]$. For the non-binding case, the risk parameters are equal to: $\alpha = 1$ and $\beta = \infty$; whereas, for the binding case, the risk parameters are equal to: $\alpha = 0$ and $\beta = 0$. As can be seen from Table 2.2, the optimal price for the non-binding TD and binding PHI policies both increase with unit manufacturing cost. When the unit manufacturing costs is less than 45, the risk-averse firm increases the selling price and therefore, we have $p^{PHI} > p^{TD}$. However, for unit manufacturing costs that exceed 50, the risk-averse firm decreases the optimal selling price below its non-binding optimal level, and we have $p^{PHI} < p^{TD}$.

Example 2 – The optimal price and the optimal production quantity can show both an increasing and decreasing behavior when the firm switches from the TD policy to PHI policy:

We provide an example where the optimal policy choice switches from the TD policy (optimal non-binding policy) to the PHI policy (optimal binding policy). We use the same

77

problem parameters with those reported for Figure 2.1 i.e., $d_1(p) = 100 - p$, $d_2(p) = 130 - 1.5\,p$,

and the exchange rate random variable is distributed with a Uniform distribution on a support of

[0,2]. For the non-binding case, the risk parameters are equal to: $\alpha = 1$ and $\beta = \infty$; whereas, for

the binding case, the risk parameters are equal to: $\alpha = 0$ and $\beta = 0$. As can be seen from Table

2.3, the optimal price for the non-binding TD and binding PHI policies both increase with unit

manufacturing cost; on the other hand, the optimal production quantity for the non-binding TD

and binding PHI policies both decrease with unit manufacturing cost. When the unit

manufacturing costs exceeds 35, the risk-averse firm increases the selling price and decreases the

production quantity. Thus, we have $p^{PHI} > p^{TD}$ and $X^{TD} > X^{PHI}$. However, for unit manufacturing

costs that are less than 31, the risk-averse firm decreases the optimal selling price below and

increases the optimal production quantity above its non-binding optimal level, respectively.

Thus, we have $p^{TD} > p^{PHI}$ and $X^{PHI} > X^{TD}$.

	Non-Binding Optimal Price	Binding Optimal Price
$c = 35$	$p^{TD} = 67.5$	$p^{PHI} = 70.0$
$c = 40$	$p^{TD} = 70.0$	$p^{PHI} = 72.1$
$c = 45$	$p^{TD} = 72.5$	$p^{PHI} = 72.9$
$c = 50$	$p^{TD} = 75.0$	$p^{PHI} = 73.8$
$c = 55$	$p^{TD} = 77.5$	$p^{PHI} = 74.7$

**Table 2.2** Optimal price expressions for the non-binding and binding models.

Unit Mfg. Cost	Non-Binding		Binding	
	Optimal Price	Optimal Production Quantity	Optimal Price	Optimal Production Quantity
$c = 30.7$	$p^{TD} = 61.35$	$X^{TD} = 76.62$	$p^{PHI} = 61.00$	$X^{PHI} = 77.49$
$c = 31.0$	$p^{TD} = 61.50$	$X^{TD} = 76.25$	$p^{PHI} = 61.42$	$X^{PHI} = 76.43$
$c = 35.0$	$p^{TD} = 63.50$	$X^{TD} = 71.25$	$p^{PHI} = 66.56$	$X^{PHI} = 63.58$
$c = 40.0$	$p^{TD} = 66.00$	$X^{TD} = 65.00$	$p^{PHI} = 69.05$	$X^{PHI} = 53.42$
$c = 45.0$	$p^{TD} = 68.50$	$X^{TD} = 58.75$	$p^{PHI} = 69.42$	$X^{PHI} = 47.17$

**Table 2.3** Optimal price and production queantity expressions for the non-binding and binding models.

2.9.3 Analysis with Different Prices

Figure 2.7 provides the optimal prices for alternative policies without the restriction of equal prices using the same parameter values as in Figure 2.3. Let PH1X denote the PHX policy when $d_1 \geq d_2$ and PH2X denote the PHX policy when $d_2 > d_1$. It is clear that charging a lower selling price under production hedging policies is not because of the restriction of equal prices in both markets as p_1^{PH1X} and p_2^{PH2X} is always lower than the optimal price of the TD policy. In addition, Figure 2.7 shows that when $c > 83.33$ the optimal price under production hedging is less than the unit manufacturing cost. Thus, we can conclude that charging a selling price that is lower than the unit manufacturing cost is not because of the restriction of equal prices.

Figure 2.8 provides the optimal production quantity for alternative policies without the restriction of equal prices using the same parameter values as in Figure 2.7. Without the restriction of equal prices, the optimal production quantity under production hedging is greater than that of the TD policy when $c > 83.33$. Thus, we can conclude that manufacturing a larger quantity under production hedging is not because of the restriction of equal prices.
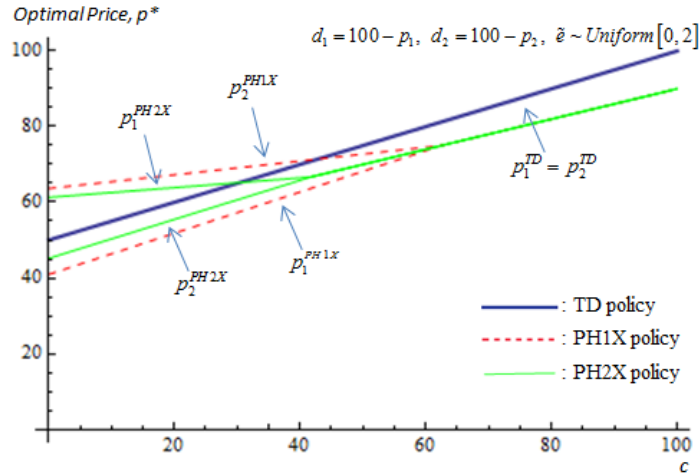
Figure 2.7 Optimal Prices for Alternative Policies (without restriction of equal prices).
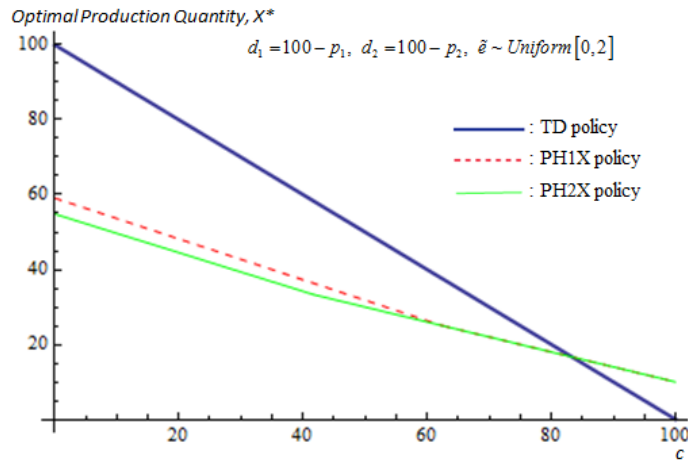
Figure 2.8 Optimal production quantity for alternative policies (without restriction of equal prices).

2.9.4 The Analysis of the Postponed Pricing Model

In this section, we investigate the postponed pricing model where a firm decides the selling price in each market after the realization of the exchange rate as opposed to the early pricing model. We continue to impose the anti-dumping law where the firm sets a single selling price where $p = p_1 = p_2 e$.

In the first stage, the firm determines the production quantity X in the presence of exchange-rate uncertainty. The objective function is

$$\max_{X \geq 0} E\left[\Pi(X)\right] = -cX + \int_{e_l}^{e_h} E\left[\pi\left(p_1, p_2, x_1, x_2 \mid e\right)\right] f(e) de$$

$$\text{s.t. } -cX + \pi\left(p_1, p_2, x_1, x_2 \mid e_\alpha\right) \geq -\beta.$$

In the second stage, based on the production quantity X and realization of exchange rate, the firm determines a selling price in each market as follows: $p = p_1 = p_2\, e$. This condition enables the firm to avoid violating the anti-dumping law. As a result, demand in the foreign market is equal to $d_2(p/e)$. Low realization of e implies that p_2 will be high and demand $d_2(p/e)$ will be small. High realization of e implies that p_2 will be low and demand $d_2(p/e)$ will be high. The second-stage profit function is

$$\pi\left(p, x_1, x_2 \mid e\right) = \max_{\substack{p, x_1, x_2 \geq 0 \\ x_1 \leq d_1(p) \\ x_2 \leq d_2(p/e) \\ x_1 + x_2 \leq X}} \left\{px_1 + px_2\right\}$$

The structural properties of the second-stage problem are as follows: For a given e,

1. if $X > d_1(p) + d_2(p/e)$, then $x_1^* = d_1(p)$ and $x_2^* = d_2(p/e)$; $\pi(p, x_1, x_2 \mid e) = p(d_1(p) + d_2(p/e))$.

2. otherwise if $X \leq d_1(p) + d_2(p/e)$, then $x_1^* + x_2^* = X$; $\pi(p, x_1, x_2 \mid e) = pX$,

Let us define the profit-maximizing price for a given e as $p^*(e)$, and its value is provided by maximizing $\max_{p \geq 0} \pi(p, x_1, x_2 \mid e) = p(d_1(p) + d_2(p/e))$. The following equations prove that the second-stage problem is concave with respect to p.

$$\partial \pi(\bullet)/\partial p = \left(d_1(p) + d_2(p/e)\right) + p\left(d_{1p}(p) + d_{2p}(p/e)\right)$$

$$\partial^2 \pi(\bullet)/\partial p^2 = 2\left(d_{1p}(p) + d_{2p}(p/e)\right) + p\left(d_{1pp}(p) + d_{2pp}(p/e)\right) \leq 0$$

81

because of our earlier assumption regarding concave revenue functions, i.e., $2\,d_p(p) + p\,d_{pp}(p) \leq$ 0 in each market. Therefore, $p^*(e)$ is the price that satisfies $\partial\pi(\bullet)/\partial p = 0$.

We next investigate the above result under the special case of linear demand functions defined as $d_1(p) = a_1 - b_1 p$ and $d_2(p/e) = a_2 - b_2(p/e)$. In this case, the profit-maximizing price becomes $p^*(e) = (a_1 + a_2)/2(b_1 + (b_2/e))$, and the total demand can be expressed as:

$$d_1(p^*) + d_2(p^*/e) = a_1 - b_1((a_1 + a_2)/2(b_1 + (b_2/e))) + a_2 - b_2(((a_1 + a_2)/2(b_1 + (b_2/e)))/e)$$

$$= (a_1 + a_2) - (b_1 + (b_2/e))((a_1 + a_2)/2(b_1 + (b_2/e)))$$

$$= (a_1 + a_2)/2.$$

The second-stage profit function can be written as:

$$\pi(p^*(e)) = p(d_1(p) + d_2(p/e))$$

$$= (a_1 + a_2)/2(b_1 + (b_2/e))((a_1 + a_2)/2) = (a_1 + a_2)^2/4(b_1 + (b_2/e)).$$

Therefore, the optimal properties of the second-stage problem are as follows:

1. If $X \geq (a_1 + a_2)/2$, then $p^* = p^*(e) = (a_1 + a_2)/2(b_1 + (b_2/e))$, and $\pi(p^*(e)) = (a_1 + a_2)^2/4(b_1 + (b_2/e))$.

2. Otherwise if $X < (a_1 + a_2)/2$, then $d_1(p) + d_2(p/e) = a_1 - b_1 p + a_2 - (b_2/e)p = X$, which implies that the optimal price is $p^*(X) = ((a_1 + a_2) - X)/(b_1 + (b_2/e))$, and $\pi(p^*(X)) = ((a_1 + a_2) - X)/(b_1 + (b_2/e))X$

Then, the first-stage objective function becomes:

$$E\big[\Pi(X)\big] = -cX + \int_{e_l}^{e_h} E\big[\pi\big(p_1, p_2, x_1, x_2 | e\big)\big] f(e)\,de$$

$$= \begin{cases} -cX + \int_{e_l}^{e_h} (a_1 + a_2)^2/4\big(b_1 + (b_2/e)\big) f(e)\,de & \text{if } X \geq \big((a_1 + a_2)/2\big) \\[2em] -cX + \int_{e_l}^{e_h} \big((a_1 + a_2) - X\big)/\big(b_1 + (b_2/e)\big) Xf(e)\,de & \text{if } X < \big((a_1 + a_2)/2\big). \end{cases} \qquad (2.16)$$

The first and second-order condition of (2.16) with the respect to X are the following:

$$\partial E\big[\Pi(X)\big]/\partial X = \begin{cases} -c < 0 \text{ if } X \ge \big((a_1+a_2)/2\big) \\ -c + \int\limits_{e_l}^{e_h}\big((a_1+a_2)-2X\big)/\big(b_1+(b_2/e)\big)f(e)de \text{ if } X < \big((a_1+a_2)/2\big) \end{cases}$$

$$\partial^2 E\big[\Pi(X)\big]/\partial X^2 = \begin{cases} 0 \text{ if } X \ge \big((a_1+a_2)/2\big) \\ \int\limits_{e_l}^{e_h}(-2)/\big(b_1+(b_2/e)\big)f(e)de < 0 \text{ if } X < \big((a_1+a_2)/2\big) \end{cases}$$

Thus, the optimal production amount X^* is strictly less than $(a_1+a_2)/2$, and can be obtained from:

$$X^* = \frac{(a_1+a_2)}{2} - \frac{c}{2\left(\int\limits_{e_l}^{e_h}1/\big(b_1+(b_2/e)\big)f(e)de\right)} \le \frac{(a_1+a_2)}{2} \tag{2.17}$$

TD policy: For our TD policy, we consider the case where $e = \bar{e}$, and uncertainty in exchange rate is removed from the problem. The profit for the TD policy is

$$\Pi\big(X = d_1(p)+d_2(p)\big) = (p-c)\big(d_1(p)+d_2(p)\big). \tag{2.18}$$

The first- and second-order conditions of (2.18) with the respect to p are the following:

$$\partial\Pi(\bullet)/\partial p = \big(d_1(p)+d_2(p)\big)+(p-c)\big(d_{1p}(p)+d_{2p}(p)\big)$$

$$\partial^2\Pi(\bullet)/\partial p^2 = 2\big(d_{1p}(p)+d_{2p}(p)\big)+(p-c)\big(d_{1pp}(p)+d_{2pp}(p)\big) < 0$$

because of our earlier assumption regarding concave revenue functions. Therefore, p^{TD} is the

price that satisfies $\partial\Pi(\bullet)/\partial p = 0$. For linear demand functions such as $d_1(p) = a_1 - b_1 p$ and $d_2(p)$

$= a_2 - b_2 p$, the profit-maximizing price becomes $p^{TD} = (c/2) + (a_1+a_2)/2(b_1+b_2)$, and the total

demand can be expressed as:

$d_1(p^{TD}) + d_2(p^{TD}) = a_1 - b_1((c/2) + (a_1+a_2)/2(b_1+b_2)) + a_2 - b_2((c/2) + (a_1+a_2)/2(b_1+b_2))$

$$= ((a_1+a_2) - ((b_1+b_2)c))/2.$$

Therefore,

$$X^{TD} = \frac{(a_1 + a_2) + (b_1 + b_2)c}{2}.$$ (2.19)

Whenever the optimal production amount X^* is below X^{TD} in our original postponed-pricing problem, then the firm is following production hedging. Figure 2.9 demonstrates that X^* is likely to be less than X^{TD} which shows that production hedging continues to exist under postponed pricing.
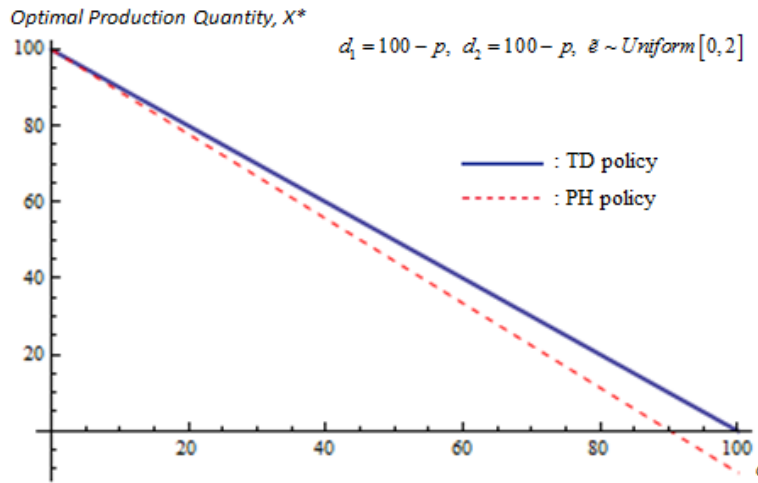

Figure 2.9 Optimal Production Quantity for Postponed Pricing Model.

Figure 2.10 shows that the firm can set a price below the optimal price choice of the TD policy. When the realized exchange rate is low, where $e = 0.5$ for Figure 2.10, the firm sets a lower selling price in the domestic market and a higher selling price in the foreign market. For example, when $c = 30$, the optimal prices for the home market and the foreign market under production hedging are the following, respectively: $p_1^* = 44.42$ and $p_2^* = 88.84$; while the optimal price under the TD policy is $p^{TD} = 65$.
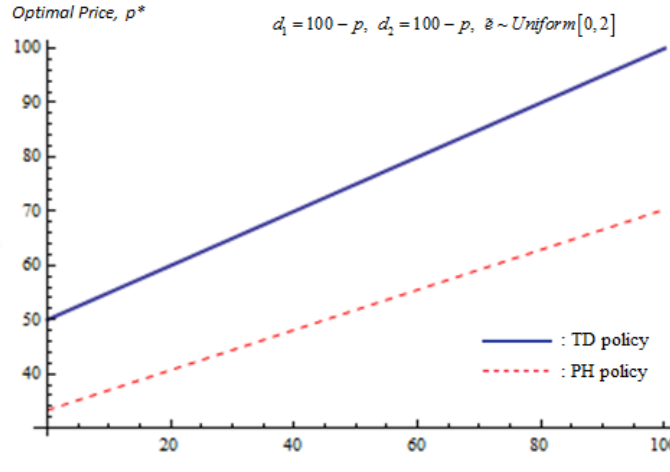
Figure 2.10 Optimal Price for Postponed Pricing Model where $e = 0.5$.

On the other hand, under the scenario where the realized exchange rate is high (i.e., $e = 1.5$), Figure 2.11 illustrates the opposite pricing strategy when compared to Figure 2.10. In this case, the firm sets a higher selling price in the domestic market and a lower selling price in the foreign market. For example, when $c = 30$, the optimal prices for the home market and the foreign market under production hedging are the following, respectively: $p_1{}^* = 79.96$ and $p_2{}^* = 53.30$; while the optimal price under the TD policy is $p^{TD} = 65$. Notice that in either scenario, the optimal selling price can be below the optimal prices of the TD policy.
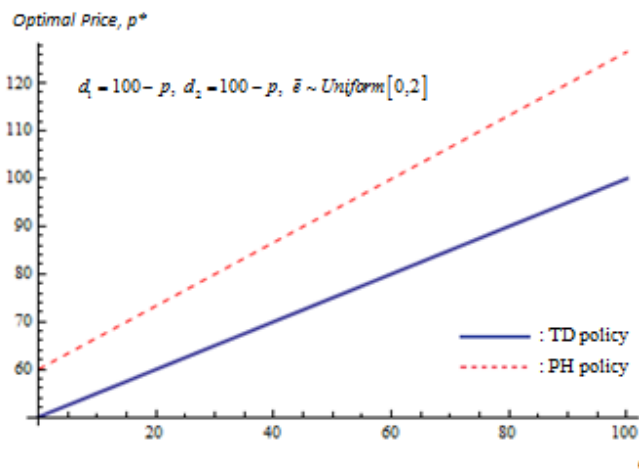
Figure 2.11 Optimal Price for Postponed Pricing Model where $e = 1.5$.

85

From Figure 2.10 and Figure 2.11, we find that the firm can set a selling price below its manufacturing cost. For example, from Figure 2.10, when $c = 80$ the domestic market selling price under production hedging, $p_1^* = 62.91$, where $e = 0.5$. From Figure 2.11, when $c = 80$ the foreign market selling price under production hedging, $p_2^* = 75.50$, where $e = 1.5$. Thus, the result regarding the firm's action to price a product below cost continues to hold under the postponed pricing flexibility.

CHAPTER 3: ESSAY 2 – MINIMIZATION OF EXPECTED SHORTAGE UNDER DEMAND UNCERTAINTY WITH BUDGET CONSTRAINTS

3.1 Introduction

Traditionally, research in operations management is mainly focused on optimization of numerous aspects from a corporation's perspective. Accordingly, a great proportion of research is closely related with the maximization of a firm's profit, or the minimization of various costs. It is important to recognize that research in operations management can benefit other fields excluding the commercial sphere. This essay stems from recognizing the importance of efficiently providing essential supplies (e.g., vaccines, pharmaceuticals, nutritional products) to regions of desperate needs in a timely manner.

Mismanagement of the supply of essential goods can result in many human casualties. United Nations Children's Fund (UNICEF) reports that in 2007 alone, 9.2 million children worldwide under the age of five died from largely preventable causes (e.g., illnesses such as pneumonia and malaria, malnutrition, lack of access to safe water). Natural and manmade disasters continue to threaten numerous lives around the world. It is a great challenge for governments and non-governmental organizations (NGOs) to establish an effective and efficient supply chain for essential supplies in regions of crisis.

Deficiencies of information infrastructure in these regions make it extremely difficult to estimate the need for essential supplies. Moreover, instability in economic and political problems in these regions can further contribute to the adversity in developing a reliable forecast. Therefore, NGOs need to incorporate demand uncertainty into their decision-making process when they are planning the acquisition of essential supplies.

In addition to demand uncertainty, deficiencies in transportation infrastructure result in longer lead-times for the distribution of essential goods, especially for surface transportation (ground and sea transportation). A 2009 UNICEF report shows that, even after target arrival dates were adjusted, orders of ready-to-use therapeutic foods still arrived late 75% of the time to the horn of African region. Thus, it is necessary for an NGO to build flexibility in its supply chain design in order to provide a quicker response at the time of emergencies. This is typically accomplished through stocking essential supplies in a distribution center and by utilizing air transportation for a faster emergency response.

This essay responds to the needs of NGOs for an effective distribution of humanitarian supplies. It investigates how organizations with budget constraints can maximize the supply and distribution of aid in the presence of demand uncertainty. For many of the relief organizations, the orders for essential goods are placed only once upon the receipt of funding. The managers of such organizations need to determine how to utilize the budget between procurement for stock to be deployed using cheaper surface transportation and the stock that is reserved for responsive, but expensive, air transportation. Thus, relief organizations are challenged by limited timeframe as well as limited budget in their preparation for uncertain demand.

Transportation cost of the essential goods contributes to a great proportion of the limited budget, especially for air transportation. The 2009 UNICEF report demonstrates that sea freight contributes to 4% of the total landed cost, whereas, air freight contributes to 39% of the total landed cost.[6] It is reported that the air transportation cost is 14 fold expensive compared to sea transportation from France to eastern Africa, where air freight averages $2.40/kg and sea freight averages $0.17/kg. Thus, air transportation imposes a trade-off between allocation flexibility and minimizing transportation costs to free up more funding to purchase essential goods. However,

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[6] The total landed cost does not account for the transportation cost after arrival to the country of needs.

88

there have been situations where air transportation was the main transportation alternative, albeit the high transportation cost, due to extremely short timeframes given to relief organizations because of unexpected tragedies or due to the need of special treatment of transporting a commodity such as vaccines. Thus, the results of this study are restricted to the commodity groups that are transportable through both the air and surface transportation options.

In the problem setting motivating our study, we consider a relief organization that procures essential supplies for distribution purposes to regions of urgent needs. The organization can distribute the essential supplies through two types of transportation: (1) surface (ground and sea) transportation and (2) air transportation. Surface transportation is a less costly option but the actual delivery lead-time is longer. On the other hand, air transportation is more expensive and has a shorter lead-time. Thus, within a given budget, the organization needs to determine how much to allocate for the procurement of essential goods and for each transportation option. Decisions of procurement and the allocation to each transportation option are made in the presence of demand uncertainty. In addition, the relief organization distributes the essential goods via surface transportation to the nations of need before the realization of demand because of the long lead-times. After the realization of demand in each region, the relief organization can ship the remaining goods to the regions with deficiencies by using the stock reserved for air transportation.

In this essay, we address the following three research questions:

1. How should a relief organization allocate its limited budget in order to minimize the expected shortage of relief aid in regions of need?

2. What is the impact of each market's demand, budget, and number of regions to serve on the relief organization's allocation decision and the expected shortage?

3. How does the correlation of demand between regions affect the relief organization's allocation decision and the expected shortage?

The rest of the essay is organized as follows: Section 3.2 provides the review of the literature. Section 3.3 introduces the model. Section 3.4 provides the characteristics of the optimal policy and how each factor affects the relief organization's decision. Section 3.5 provides conclusions and managerial insights. Section 3.6 discusses future research directions for this essay. The proofs and technical derivations are presented in an appendix in Section 3.7.

3.2 Literature Review

This essay relates and contributes to two areas of literature: (1) the humanitarian supply chain literature, and (2) the literature that incorporates demand uncertainty. In this section, we review the literature in both streams and present how our work differs from these publications.

There has been a growing interest in the activities associated with humanitarian supply chains as evidenced from the ever-growing budgets allocated to their activities. Thomas and Kopczak (2005) report that the budgets of the top ten largest relief organizations exceeded $14 billion in 2004. Recently, Van Wassenhove (2006) addresses the need for humanitarian supply chains to be agile, adaptable and aligned, and stresses the importance of collaboration between humanitarian organizations, businesses and academics in order to achieve this goal. Atley and Green (2006) survey the operations management and management science literature to identify potential research directions in disaster operations. Oloruntoba and Gray (2006) examine the applicability of commercial supply chain concepts to humanitarian supply chains by identifying the difference between the two supply chains. Beamon and Balcik (2008) highlight the importance of developing performance measurement metrics for humanitarian supply chains that

differ from that of commercial supply chains due to the differences in revenue sources, goals and stakeholders.

The main methodology in the humanitarian supply chain literature is based on case studies. Case studies from the Humanitarian Research Group that is a part of INSEAD's Social Innovation Center such as Jahre and Jensen (2009) and Gatignon et al. (2010), for example, describe various emergency situations. Our study differentiates itself from the literature by employing an analytical model that can provide policy prescriptions and insights for the managers of relief organizations in their efforts to plan and respond to emergencies. Moreover, a large proportion of the literature focuses on the improvements in the lower level of the humanitarian supply chain. Van Wassenhove and Martinez (2012) emphasize the importance of allocating scarce resources in an efficient way for humanitarian supply chains. They demonstrate that greater efficiency in humanitarian operations can be achieved by adapting well-developed principles to vehicle fleet management for international humanitarian organizations. Stapleton et al. (2009) also focus on the improvement of vehicle fleet management in terms of agility and cost effectiveness based on a case study of the International Federation of the Red Cross and Red Crescent Societies (IFRC). This essay differs from these studies by focusing on the insights stemming from the allocation of limited budgets to the relief organization's headquarters.

This essay, if stripped of its context, belongs to the literature associated with demand uncertainty. A remarkable amount of work has been done analytically for organizations that operate under random demand. Literature in this area can be categorized in numerous ways, depending on the scope and the featured assumptions. The length of horizon is a common way of classification. Single-period models are developed in Porteus (1990), Petruzzi and Dada (1999), Dana and Petruzzi (2001), Bernstein and Federgruen (2003). Publications that feature the

analysis of a multi-period setting include Zabel (1972), Li (1988), Federgruen and Heching (1999), Bitran et al. (1998). Another form of classification includes the form of the demand function utilized in the analysis. Whitin (1955), Lai (1990), Abad (1996) feature linear demand, Li (1988), Feng and Gallego (1995), and Bitran and Mondschein (1997) feature poisson demand. Sogomanian and Tang (1993), Neslin et al. (1995), Chan et al. (2006) utilize general descriptions of the demand function. There is one common feature in the majority of these articles within the literature that examines the impact of demand uncertainty – the objective is either directly or indirectly related with profit maximization. This essay differs from the aforementioned articles in the sense that our objective is to minimize the expected shortage of demand, and is not associated with profit maximization.

3.3 The Model

This section presents the problem definition and the modeling approach used by the relief organization with a limited budget that purchases and transport essential goods to regions of need in the presence of demand uncertainty. The objective of the relief organization is to minimize the total expected shortage in the regions of need.

The model is formulated as a two-stage stochastic program, where the first stage corresponds to the acquisition of essential goods and supplying the goods to regions of need via surface transportation. After the demand is realized in each region, the organization recognizes and transports the remaining goods to the region(s) with deficiencies via air transportation. The model is initially presented for the case of independent demands. Later, correlation is incorporated introduced to the model and its effects on the technical results are studied. Figure 3.1 describes the timeline of events for the relief organization.

92
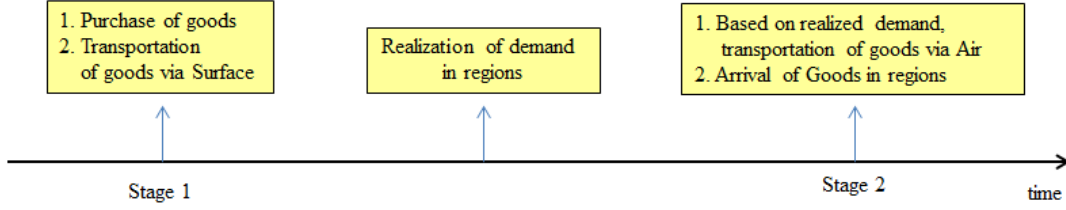
Figure 3.1 Timeline of events for relief organization with demand uncertainty

Before the realization of demands in n regions, the relief organization determines the total amount of essential goods to be purchased, denoted Q, at a given cost per unit, denoted c, within a given budget B. The organization also needs to reserve a proportion of the budget for the transportation of goods. There are two options of transportation: (1) surface transportation and (2) air transportation. In the first stage, the organization needs to determine the amount of goods to be sent to each region via surface transportation, denoted q_i where $i = 1, 2, \ldots, n$. In addition, the organization needs to determine the quantity allocated to air transportation which is denoted as q_a. Let $\mathbf{q} = (q_1, q_2, \ldots, q_n, q_a) \geq 0$. The cost for surface and air transportation is denoted as t_s and t_a, respectively. We assume that $t_a > t_s$, and the transportation cost to each region is equal for each transportation option. The total cost for a unit to be transported via surface and air is denoted as c_s and c_a where $c_s = c + t_s$ and $c_a = c + t_a$. Without loss of generality, we normalize c_s to 1. Thus, $B \geq \sum_{i=1}^{n} q_i + c_a q_a$. Because we have a single period model, there is no incentive for the relief organization not to spend the entire budget. Therefore, the budget consumption for the relief organization always strictly equals B, i.e.,

$$B = \sum_{i=1}^{n} q_i + c_a q_a. \tag{3.1}$$

In our model, we standardize the surface transportation quantity q_i and the random demand in each region D_i (d_i represents the realized demand), so that the comparison of service levels in

93

each region is clearly observed. Let z_i denote the standardized stocking factor for each region via surface transportation where $z_i = (q_i - \mu_i)/\sigma_i$; μ_i represents the expected demand, i.e., $\mu_i = \mathrm{E}[D_i]$, and σ_i denotes the standard deviation of the random demand in each region. Note that z_i can have negative values as opposed to q_i. Let $\mathbf{z} = (z_1, z_2, \ldots, z_n)$. Randomness of demand in each region is modeled as $D_i = \mu_i + Z_i\sigma_i$, where Z_i represents the standardized random error term for each region. We assume that the pdf for the random demand in each region has the same functional form, but not necessarily with identical parameter values. Thus, the pdf of the standardized random error term $\phi(y_i)$ is equal for both regions where y_i denotes the realization of Z_i. We make no other assumptions regarding $\phi(y_i)$.

The general form of the objective function that aims to minimize the expected shortage can be expressed as follows:

$$\min_{\mathbf{q} \geq 0} E\left[S(\mathbf{q})\right] \text{ where } S(\mathbf{q}) = \left(\sum_{i=1}^{n}(D_i - q_i)^+ - q_a\right)^+$$

subject to (3.1).

The standardized version of the general objective function is the following:

$$\min_{\mathbf{z}} E\left[S(\mathbf{z})\right] \text{ where}$$

$$S(\mathbf{z}) = \left(\sum_{i=1}^{n}\left((\mu_i + Z_i\sigma_i) - (\mu_i + z_i\sigma_i)\right)^+ - q_a\right)^+ = \left(\sum_{i=1}^{n}\sigma_i(Z_i - z_i)^+ - q_a\right)^+ \tag{3.2}$$

subject to (3.1).

Due to the fact that the optimization of the objective function is complex even with two regions (which corresponds to an optimization problem with three decision variables), we investigate the problem both analytically and numerically in the following sections.

3.4 Minimization of Expected Shortage

In this section, we examine the optimal budget allocation decisions for a relief organization that aims to minimize the total expected shortage. Furthermore, we present the analysis of a sensitivity parameter where we demonstrate how changing parameter values affect the optimal allocation of budget and the expected shortage.

3.4.1 Characterization of the Optimal Stocking Factors for a given q_a

We proceed with the analysis of the optimal stocking factor for each region via surface transportation for a given amount of goods reserved for air transportation q_a. For a given q_a, the budget parameter B corresponds to a unique k which is the sum of the product of the stocking factor z_i and the standard deviation in each region σ_i, i.e.,

$$k = \sum_{i=1}^{n} z_i \sigma_i .\tag{3.3}$$

Specifically, the relationship between B and k is the following:

$$k = B - \left(\mu + c_a q_a\right)$$

where $\mu = \Sigma \mu_i$. Thus, k can be interpreted as the remainder of budget subtracting the mean demands and the cost of supplying goods via air transportation. Since z_i can have negative values, this implies that k can take negative values, corresponding to the cases when the sum of mean demands exceeds the value of B.

We begin the analysis with the problem setting that features only two regions, i.e., $n = 2$. Notice that z_2 can be substituted with $(k - z_1\sigma_1)/\sigma_2$, reducing (3.2) to a single decision variable optimization problem. Thus, for a given q_a, we can decompose the demand space into the following sets (see Figure 3.2).

$\Omega_0 = \{Z_1 \leq z_1 + q_a/\sigma_1$ and $Z_2 \leq (k - z_1\sigma_1)/\sigma_2 + q_a/\sigma_2$ and $Z_2 \leq (k + q_a - Z_1\sigma_1)/\sigma_2\}$,

95

$\Omega_1 = \{Z_1 \leq z_1 \text{ and } Z_2 > (k - z_1\sigma_1)/\sigma_2 + q_a/\sigma_2\},$

$\Omega_2 = \{Z_1 > z_1 + q_a/\sigma_1 \text{ and } Z_2 \leq (k - z_1\sigma_1)/\sigma_2\},$

$\Omega_3 = \{Z_1 > z_1 \text{ and } Z_2 > (k - z_1\sigma_1)/\sigma_2 \text{ and } Z_2 > (k + q_a - Z_1\sigma_1)/\sigma_2\}.$
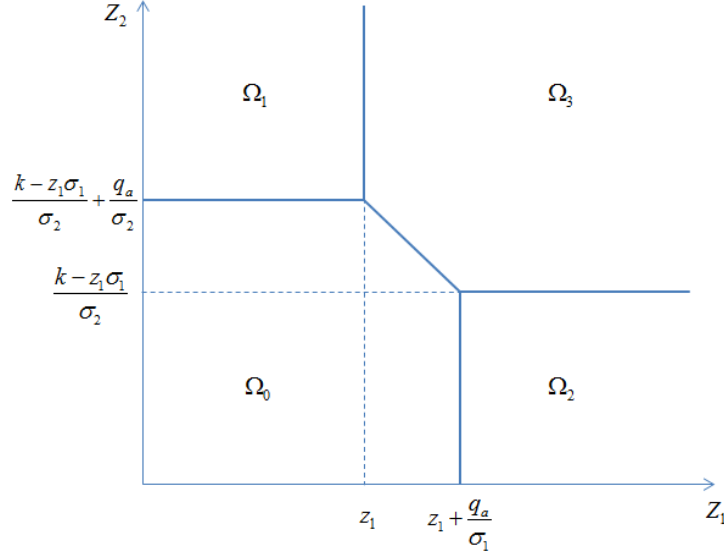
Figure 3.2 Decomposition of the demand space when $n = 2$

The shortage in Ω_0 is equal to zero as the demands in both regions are completely covered by the supply through surface and air transportation. Ω_1 is the set where the realization of the demand in region 2 is greater than the sum of the goods transported via surface and air (q_a) while the demand in region 1 is completely satisfied by the goods transported via surface. Ω_2 is the set where the realization of the demand in region 1 is greater than the sum of the goods transported via surface and air (q_a) while the demand in region 2 is completely satisfied by the goods transported via surface. Ω_3 is the set where the realizations of the demand in both regions exceed the total procurement of essential goods Q. Thus, for a given q_a with two regions, the objective function (3.2) can be expressed as following:

$$\min_{z_1} E\left[S\left(z_1|q_a\right)\right] = \int_{-\infty}^{z_1} \int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \left[\sigma_2\left(y_2 - \frac{k-z_1\sigma_1}{\sigma_2}\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1$$

$$+ \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty} \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}} \left[\sigma_1\left(y_1 - z_1\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1$$

$$+ \int_{z_1}^{z_1+\frac{q_a}{\sigma_1}} \int_{\frac{k-y_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \left[\sigma_1\left(y_1 - z_1\right) + \sigma_2\left(y_2 - \frac{k-z_1\sigma_1}{\sigma_2}\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1 \qquad (3.4)$$

$$+ \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty} \int_{\frac{k-z_1\sigma_1}{\sigma_2}}^{\infty} \left[\sigma_1\left(y_1 - z_1\right) + \sigma_2\left(y_2 - \frac{k-z_1\sigma_1}{\sigma_2}\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1.$$

The following lemma describes the optimal stocking factors for the case of two regions.

Lemma 3.1 a) If $q_a = 0$, then $z_1^* = z_2^*$. b) If $q_a > 0$ and $\sigma_1 = \sigma_2$, then $z_1^* = z_2^*$. c) If $q_a > 0$ and $\sigma_1 < \sigma_2$, then $z_1^* \leq z_2^*$, and the inequality is strict if $\Phi(z_2^*) < 1$.

The above lemma demonstrates that when there are no essential goods allocated for air transportation, it is optimal for the relief organization to allocate the goods for surface transportation so that the service levels in both regions are equal in order to minimize the expected shortage. This proves that if the cost for air transportation is extremely high, it is optimal for the relief organization to abandon the air transportation option and transport the entire essential goods via surface transportation to both regions so that $z_1^* = z_2^*$. Note that the fluctuations in each region do not affect the optimal stocking factor when $q_a = 0$. However when $q_a > 0$, it is optimal for the relief organization to allocate the goods for surface transportation to both regions equally in terms of service levels if and only if the standard deviations in both regions are equal. This implies that if two regions have different level of fluctuations in demand, the optimal stocking factors for each region differ. Then, when $q_a > 0$ and both regions have different levels of demand fluctuations should the relief organization transport more or less via surface to the more volatile region? The above lemma proves that it is optimal to allocate more

essential goods for surface transportation to the region with greater volatility in order to minimize the expected loss. Thus, just by observing the optimal stocking level, one can figure out which region has greater volatility in demand. If $z_1^* > z_2^*$, then the demand in region 1 has greater volatility compared to region 2; otherwise, the demand in region 2 has greater volatility than region 1.

Proposition 3.1 *a*) *If $q_a = 0$, then $z_l^* = z_m^*$ for all l and m. b) If $q_a > 0$ and $\sigma_1 = \sigma_2 = \ldots = \sigma_n$, then $z_l^* = z_m^*$ for all l and m. c) If $q_a > 0$ and $\sigma_l < \sigma_m$ for some l and m, then $z_l^* \leq z_m^*$, and the inequality is strict if $\Phi(z_m^*) < 1$.*

The above proposition shows that the results established in Lemma 3.1 are not restricted to only two regions, but continue to hold with n regions. Thus, if there are no essential goods allocated for air transportation, it is optimal for the relief organization to allocate the goods equally (in terms of service levels) via surface transportation to n regions. If the standard deviations are equal in all n regions, it is optimal to split evenly (in terms of service levels) of the essential goods that are reserved to be transported via surface. However if $q_a > 0$, it is optimal for the relief organization to transport more (in terms of service levels) via surface to regions with greater volatility at the first stage. Note that the above proposition is robust as it is not restricted to a specific form of demand uncertainty or a probability density function.

3.4.2 The Impact of Each Parameter

In this section, we examine through numerical examples how each parameter (e.g., unit cost of air transportation, budget, and variance) affects the relief organization's decision on the allocation of budget and on the expected shortage.

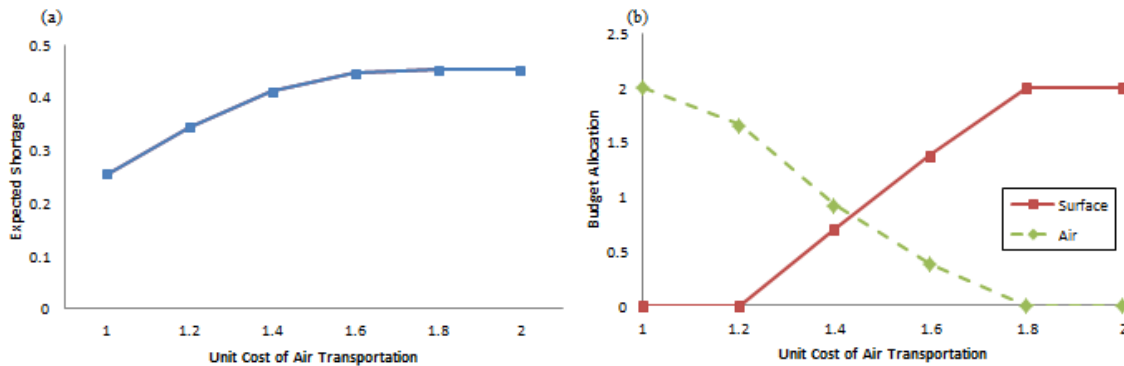### 3.4.2.1 The Impact of the Unit Cost of Air Transportation

The unit cost of air transportation plays an important role on how heavily the relief organization relies on either the surface or the air transportation option. The following numerical example illustrates how the unit cost of air transportation affects the optimal expected shortage ($E[S^*]$), optimal budget allocated to surface and air transportation (k^* and q_a^*), optimal stocking factor in each region (z_1^* and z_2^*), and the difference between the optimal stocking factors ($z_2^* - z_1^*$). For the following numerical example, let $n = 2$, $\mu_1 = \mu_2 = 0$, $\sigma_1 = 1$, $\sigma_2 = 2$, $B = 2$, and pdfs in both regions use the standardized normal distribution.
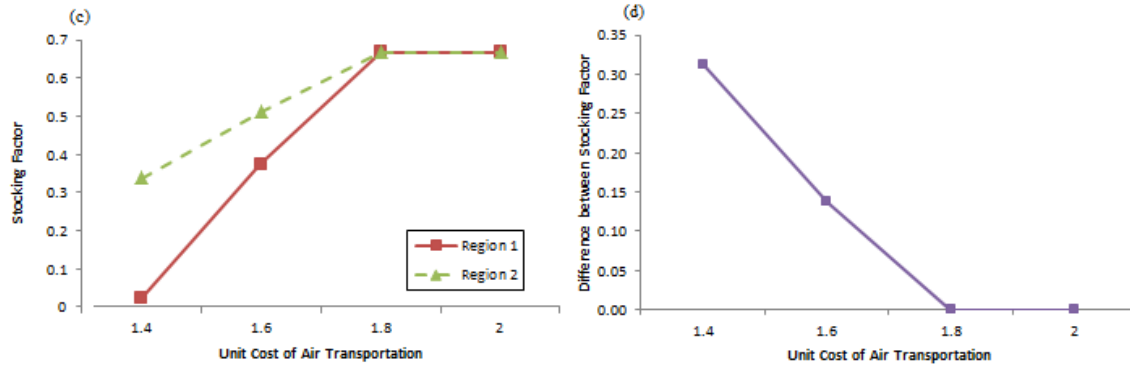
Figure 3.3(a) shows that the optimal expected shortage increases with the unit cost of air transportation. It is obvious that the increase of the unit cost of air transportation has a negative effect on minimizing the expected shortage as the cost of utilizing the flexible transportation option becomes more costly to the relief organization. As the unit cost of air transportation becomes more expensive, the relief organization utilizes less of the air transportation option and more of the surface transportation option which is illustrated in Figure 3.3(b).

When the cost difference between air and surface transportation is minimal, the relief organization allocates its entire budget to air transportation. In contrast, there is a certain threshold unit cost where air transportation becomes excessively expensive that the relief organization abandons the option. In the above numerical example, we can observe that this threshold is strictly less than 2. We know for sure that when $n = 2$, the relief organization abandons the air transportation option if $c_a > 2$ as it is better to serve to each region a unit via surface transportation rather than having the flexibility to serve only one region. Following this logic, the threshold cost where the relief organization abandons the air transportation option increases with the number of regions to supply.

99

Figure 3.3(c) illustrates that $z_1{}^*$ and $z_2{}^*$ increases with higher values of c_a. This is because if the cost of air transportation increases, then the air transportation option becomes less attractive and the relief organization allocates more of the budget to surface transportation. Note that the optimal stocking factors where c_a equals 1 and 1.2 are omitted because there are no goods transported through surface transportation. In this case, the optimal stocking factors goes to negative infinity.

Notice that $z_2{}^* > z_1{}^*$ when $q_a > 0$ because $\sigma_2 > \sigma_1$, and $z_1{}^* = z_2{}^*$ when $q_a = 0$, which is consistent with the results in presented in Proposition 3.1. From Figure 3.3(d), we observe that $z_1{}^*$ increases in a faster rate (compared to $z_2{}^*$) with respect to c_a, and as a result, the difference between the optimal stocking factors decreases monotonously. This shows that the difference between the stocking factors and $q_a{}^*$ has a positive correlation, and as a special case if $q_a = 0$, then $z_2{}^* - z_1{}^* = 0$. In conclusion, with higher incentives to reserve goods for air transportation, the organization increases its stocking factor for the more volatile region and decreases its stocking factor for the less volatile region in its allocation of surface transportation.
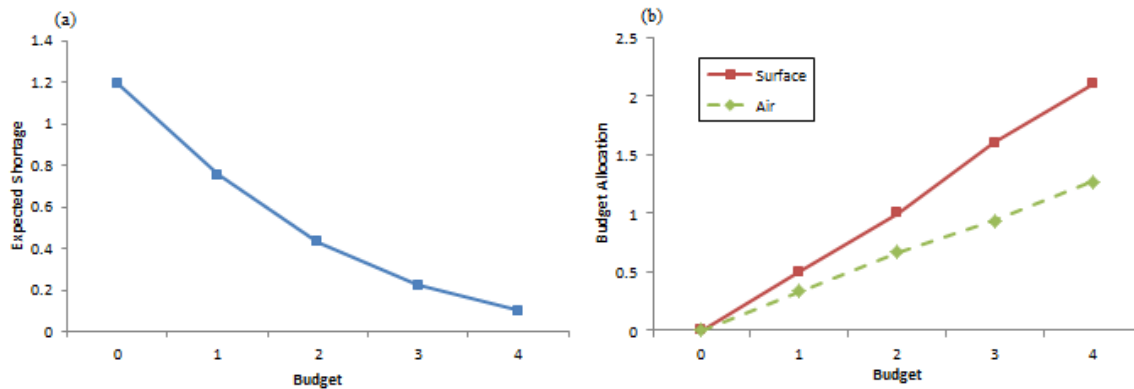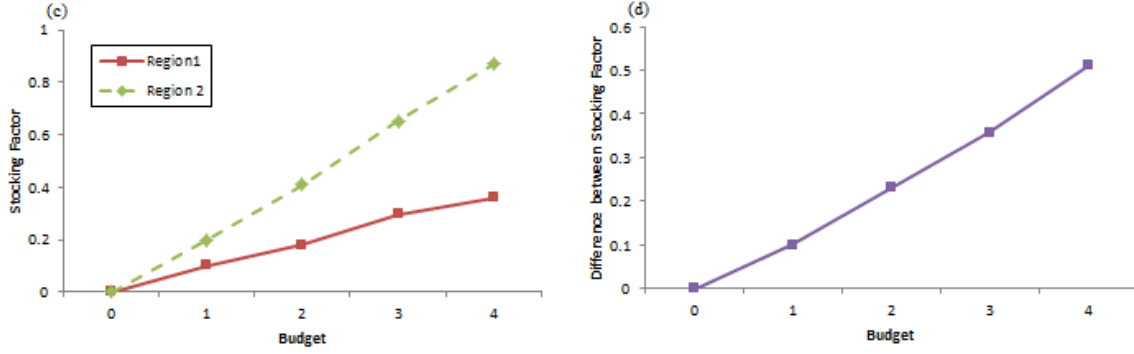
Figure 3.3 Effect of the unit cost of air transportation on (a) optimal expected shortage, (b) optimal budget allocated to surface and air transportation, (c) optimal stocking factor in each region, and (d) difference between the optimal stocking factors.

3.4.2.2 The Impact of the Budget

The impact of the budget on the optimal expected shortage, the optimal allocation of surface versus air transportation, optimal stocking factor in each region, and the difference between the optimal stocking factors is rather straightforward. The magnitude of the budget itself does not affect the characteristic of the relief organization's decisions, but functions more as a scaling factor. For the following numerical example, let $n = 2$, $\mu_1 = \mu_2 = 0$, $\sigma_1 = 1$, $\sigma_2 = 2$, $c_a = 1.5$, and pdfs in both regions use the standardized normal distribution.

Figure 3.4 Effect of budget on (a) optimal expected shortage, (b) optimal budget allocated to surface and air transportation, (c) optimal stocking factor in each region, and (d) difference between the optimal stocking factors.

Figure 3.4(a) shows that the optimal expected shortage decreases as the budget increases. It is obvious that the increase of budget has a positive effect on minimizing the expected shortage. Thus, one of the common concerns for relief organizations is securing sufficient amount of budget. However, notice that there is a diminishing return as the budget increases. Therefore, after obtaining a certain amount of funding, it might enhance the performance of the relief organization to alter its focus and effort to other areas.

Figure 3.4(b) illustrates that the relief organization transports more units of essential goods through both air and surface transportation as the budget increases. Note that the cost of air transportation in the numerical example is $c_a = 1.5$. If the cost of air transportation is extremely high (low), the relief organization abandons the air (surface) transportation option and only increases the quantity of goods to be transported via surface (air).

Figure 3.4(c) and (d) demonstrate that z_1^* and z_2^* as well as the difference between the two stocking factors increase with higher values of the budget. Given that $\sigma_2 > \sigma_1$, we can observe through Figure 3.4(c) that $z_2^* \geq z_1^*$. The increasing difference between the stocking factors can be explained by the increasing amount of q_a^*. From Section 3.4.2.1, we find that the more the relief

organization utilizes air transportation, the greater the difference is between the optimal stocking

factors given that $\sigma_1 \neq \sigma_2$. However, if the cost of air transportation is too high (resulting in $q_a = 0$), the difference between the optimal stocking factors does not change with the budget due to

the fact that $z_1^* = z_2^*$.

3.4.2.3 The Impact of the Degree of Distortion in Demand

It is straightforward to see that the increase of demand fluctuations in regions has a negative

impact on minimizing the expected shortage. We also know that the relief organization serves

more to the region of greater degree of fluctuations via surface transportation. However, it is not

clear how the degree of distortion (δ) affects the relief organization when the sum of the demand

variances is kept constant, i.e., $0.5(\sigma_1^2 + \sigma_2^2) = \bar{\sigma}^2$. Let $\delta \geq 0$ where $\delta = \sigma_2^2 - \bar{\sigma}^2 = \bar{\sigma}^2 - \sigma_1^2$

given that $\sigma_2^2 \geq \sigma_1^2$. In this section, we investigate how the degree of distortion impacts the

optimal expected shortage, optimal budget allocation between surface and air transportation,

optimal stocking factor in each region, and the difference between the optimal stocking factors.

For the following numerical example, let $n = 2$, $\mu_1 = \mu_2 = 0$, $B = 1$, $c_a = 1.5$, $\bar{\sigma}^2 = 0.5$, and pdfs

in both regions use the standardized normal distribution.

When $\delta = 0$, $\sigma_1^2 = \sigma_2^2 = 0.5$, whereas when $\delta = 1$, $\sigma_1^2 = 0$ and $\sigma_2^2 = 1$ given $\bar{\sigma}^2 = 0.5$. Thus, δ

$= 0$ corresponds to the case where the degree of fluctuations are equal in both regions, whereas δ

$= 0.5$ corresponds to the case where the degree of fluctuations is high in region 2 and region 1

has deterministic demand. Figure 3.5(a) shows that for a given $\bar{\sigma}^2$, the optimal expected

shortage decreases as the degree of distortion increases. This is because when the randomness

stems from a single region, there is no need to use the costly air transportation option as a relief

organization can send the exact amount of goods to the region where the demand is precisely

known and use the rest of the budget to send a greater quantity of goods to the region with demand uncertainty. In conclusion, when demand uncertainty is present in a single region (i.e., the other region has deterministic demand) there is no value from the flexibility that can be gained through air transportation. Thus, if a relief organization could choose among scenarios where the total variance is constant but the degree of distortion differs, it is optimal for the organization to choose the scenario with the greatest degree of distortion. It would be ideal for the relief organization if it had the ability to force all the randomness to one region and keep the other region deterministic in terms of demand. Thus, humanitarian organizations can explore the possibility to improve its performance by focusing on one region to obtain a more accurate forecast of demand, at the expense of spending less resource in the other region for forecasting.

Figure 3.5(b) illustrates that as the degree of distortion increases, the relief organization utilizes more of the surface transportation and less of air transportation. When $\delta = 0.5$ (which corresponds to the case where the demand in region 1 is deterministic), notice that the relief organization abandons air transportation. It is because there is no value of having flexibility as the demand is completely revealed in one region, causing the relief organization to use surface transportation alone by procuring the maximum amount of essential goods possible.

Figure 3.5(c) demonstrates that the optimal stocking factor of the region with greater variance (region 2) increases with the degree of distortion continuously. However, the optimal stocking factor of the region with smaller variance (region 1) initially decreases with smaller values of distortion, but exhibits an increasing behavior with respect to the greater degrees of distortion. Notice that the optimal stocking factors are equal when $\delta = 0$ and $\delta = 0.5$. When $\delta = 0$, the standard deviations are equal in both regions. Recall from Proposition 3.1 that if the standard deviations are equal, then $z_1^* = z_2^*$. When $\delta = 0.5$, recall that there is no value gained from air

transportation, therefore $q_a{}^* = 0$. From Proposition 3.1, we know that if $q_a{}^* = 0$, then $z_1{}^* = z_2{}^*$.
This result can also be observed from Figure 3.5(d) which is consistent with the result presented
in Proposition 3.1.
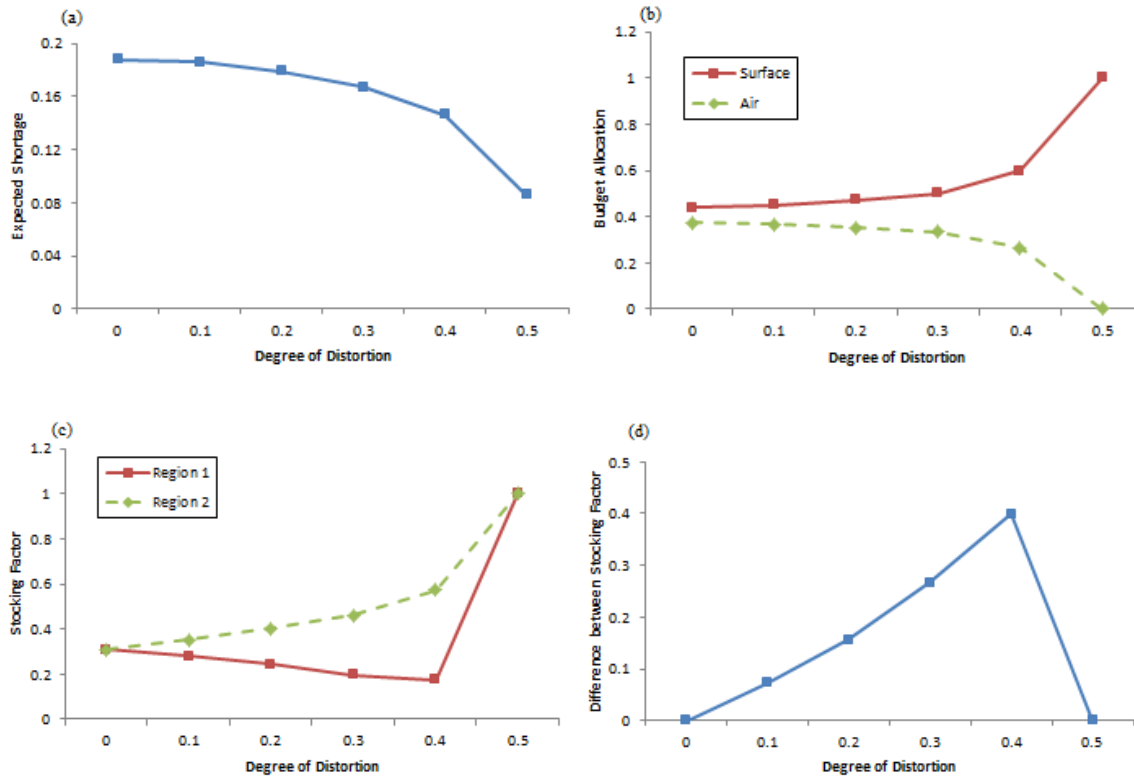
Figure 3.5 Effect of the degree of distortion on (a) optimal expected shortage, (b) optimal budget
allocated to surface and air transportation, (c) optimal stocking factor in each region, and (d)
difference between the optimal stocking factors.

3.4.2.4 The Impact of Number of Regions to Serve

In this section, we investigate how the number of regions impacts the relief organization's
allocation decision and the expected shortage by comparing a relief organization that serves two
regions with an organization that serves three regions. For the following numerical example, the
parameter values for the relief organization where $n = 2$ ($n = 3$) are the following: $\mu_1 = \mu_2 = (\mu_3)$

$= 0$, $\sigma_1 = \sigma_2 = (\sigma_3) = 1$, $c_a = 1.5$, and pdfs in all regions use the standardized uniform distribution. For a given amount of budget, it is straightforward that the expected shortage increases with respect to the number of regions a relief organization needs to serve. For instance, if $B = 2$ then $E[S^*] = 0.135$ when $n = 2$; whereas the optimal expected shortage $E[S^*] = 0.329$ when $n = 3$ using the same amount of budget.

The impact of the number of regions on the optimal budget allocation is not as straightforward as the impact on the expected shortage. For a relief organization that serves two regions ($n = 2$), it is optimal to split the budget approximately into half between air and surface transportation. However, for a relief organization that serves an extra region ($n = 3$) with the same amount of budget, the optimal split of the budget changes dramatically where it is optimal to use all the budget on air transportation while abandoning the surface transportation option. This is because as the number of regions increases the value of flexibility also increases. Previously, we have found that the relief organization abandons the air transportation option if c_a > 2 when $n = 2$. However, when $n = 3$, the threshold cost increases to 2.17. This also illustrates that the value of the air transportation option increases with respect to the number of regions to serve. The air transportation cost threshold also increases with respect to the budget. For example, if $B = 3$ the threshold increases to 2.45 (from 2.17 when $B = 2$).

Now that we have investigated the impact of the numbers of regions, we turn our attention to the following question: Is a relief organization better off by serving more regions with an increased amount of budget? For comparison, we assign one unit of budget for each region where for two regions $B = 2$ and for three regions $B = 3$. As aforementioned, for two regions the minimum expected shortage $E[S^*] = 0.135$ where the optimal split of the budget between air and surface transportation is approximately half. For three regions with a greater budget, $B = 3$, the

minimum expected shortage $E[S^*] = 0.089$ where the optimal split of the budget between air and surface transportation is approximately equal to 3:1 ratio. Thus, the above example demonstrates that relief organizations can investigate on the possibility to improve its performance by integrating their budgets and distribution operations.

3.4.2.5 The Impact of Correlation

We now focus on the effect of correlation between the demand in the two markets described as D_1 and D_2. We only examine the effect of positive correlation as it is typically the case where regions are close enough that they both are affected by natural disasters or other factors. It is unlikely for a humanitarian organization to have negatively correlated demands where high demand of essential goods in one region would indicate low demand in the other region. Let ρ represent the correlation parameter where $0 \leq \rho < 1$. We follow the structure of Van Mieghem (1995) to analyze the effect of correlation where the joint probability density of $f(d_1, d_2)$ is given by

$$f\left(d_1, d_2 \mid \rho\right)$$
$$= \begin{cases} \dfrac{1}{\pi\sqrt{1-\rho^2}} & \text{if} \quad d_1^2 - 2\rho d_1 d_2 + d_2^2 < 1 - \rho^2, \\ 0 & \text{otherwise} \end{cases}. \tag{3.5}$$

Note that the joint density is uniform on an ellipse for (3.5). This attractive feature of (3.5) enables to clearly analyze the effect of correlation. For the following numerical example, let $n = 2$, $\mu_1 = \mu_2 = 0$, $\sigma_1 = \sigma_2 = 1$, $c_a = 1.5$, and $B = 0.5$.

107
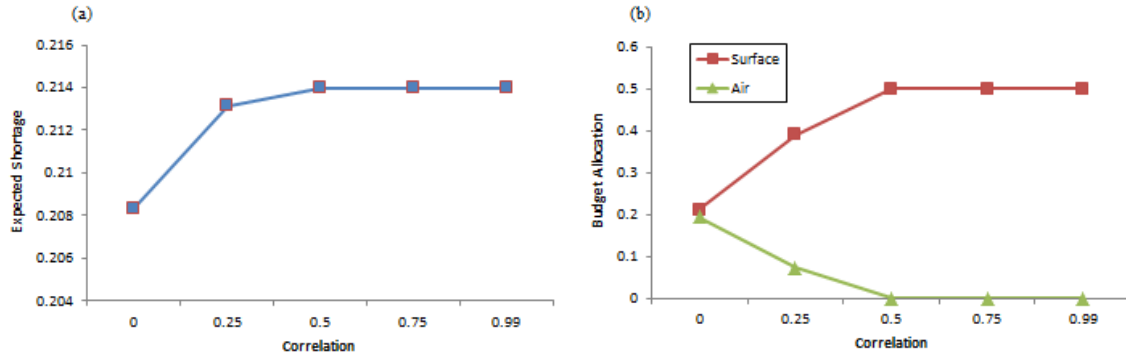
Figure 3.6 Effect of correlation on (a) optimal expected shortage, (b) optimal budget allocated to surface and air transportation.

Figure 3.6(a) demonstrates that the optimal expected shortage is non-decreasing with respect to correlation. As opposed to other factors (e.g., unit cost of air transportation, budget, variance), the impact of correlation is not as quite straightforward. It turns out that a relief organization is better off serving regions that are not correlated, although comparing to other factors, the impact is minimal. Nevertheless, efforts to minimize the correlation between regions can lead to enhanced performance for humanitarian organizations.

Figure 3.6(b) illustrates that the relief organization allocates more budget to surface transportation as the correlation increases. It is because the value of air transportation flexibility decreases with respect to correlation. For instance, if the demands in two regions are perfectly positively correlated (which is a special case where $\rho = 1$), there is no value be gained from air-transportation flexibility as the relief organization knows the exact demand for one region given the demand of the other. Since the variances are equal for both regions, z_1^* and z_2^* are always equal for any given ρ, which is proved in Proposition 3.1.

3.5 Conclusions

It is extremely critical to procure and distribute humanitarian supplies to the regions of need in a timely manner in order to save lives of children and adults. This essay shows how a relief organization can minimize the expected shortage in the presence of uncertain demand. While doing so, it shows the influence of various factors on the relief organization's budget allocation decision.

This essay makes three main contributions. First, we determine the optimal stocking factor corresponding to the surface transportation option in order to minimize expected shortage. We find that when there are no supplies reserved for air shipments, or the demand variances are equal, it is optimal to provide equal amounts of essential goods in terms of stocking factors via surface transportation. However, when there are supplies reserved for air shipment and the variances are not equal, it is optimal to send more essential goods in terms of stocking factors to the regions that exhibit greater demand variances.

Second, we show the influence of each factor on the relief organization's objective function – minimizing the expected shortage, and on the decisions regarding the utilization of the limited budget between procurement and the two transportation options. An increase in the unit cost of air transportation has a negative effect on the expected shortage while giving an incentive for the organization to allocate more of the budget to surface transportation. Clearly, an increase in budget has a positive effect on the expected shortage. However, the budget itself hardly affects the characteristic on how an organization manages its budget, and can be considered as more of a scaling factor. Increases in demand variation also negatively affect the organization. As the demand variance in one region increases, the organization increases its surface transportation allocation to that region.

Third, we provide insights regarding how relief organizations can improve their performance in terms of reducing shortages in regions of need. Given a total amount of demand variance, we discover that an organization is better off having greater amount of distortion between demand variances in order to minimize the expected shortage. We also find that the organization allocates a greater proportion of budget to surface transportation as the degree of distortion increases, reducing the budget reserved for air transportation. Thus, humanitarian organizations can explore the possibility to improve its performance by focusing on one region to obtain a more accurate forecast of demand, at the expense of spending less resource in the other region for forecasting.

For a given amount of budget, an increase in the number of regions that need to be served negatively influences the expected shortage. As the number of regions to serve increases for a relief organization, we find that the proportion of budget towards air transportation increases; this result implies that the value from the air transportation flexibility increases with a higher number of regions. If the budget increases proportionally with the number of regions to serve, we find that the relief organization can provide a better service overall, and further reduce the expected shortage. Thus, this result shows that improvements can be made by coordinating organizations and their distribution operations by integrating their budgets.

We find that (positive) correlation has a negative impact on a humanitarian organization as it increases the expected shortage. Moreover, we find that it is optimal for a relief organization to allocate more budget to surface transportation if correlation increases. Thus, efforts to minimize the correlation between regions can lead to enhanced performance for humanitarian organizations.

3.6 Future Research Direction

In this essay, the model incorporates the risk that stems from the difficulty of forecasting the needs of essential supplies, resulting in demand uncertainty. In addition to the challenges associated with demand uncertainty, relief organizations distribute their supplies in countries with inadequate transportation infrastructure. Thus, the model developed in this essay can be extended to incorporate lead-time uncertainty in responding to the emergency needs. Lead-time uncertainty is much more prevalent in surface transportation (ground and sea transportation) operations. Compared to air transportation, the lead-time for surface transportation is affected by an increased number of factors. For instance, the lead-time for sea transportation can be greatly influenced by the inefficient management of the ports for transshipment (e.g., congested transshipment ports, issues of regulatory paper work in each port, port strikes). For ground transportation, seasonal weather problems, necessity of driving partially empty trucks due to road weight restrictions, and border-crossing delays can greatly affect the arrival of essential goods. Thus, the analysis of a model that considers both demand uncertainty and lead-time uncertainty would provide richer insights for relief organizations.

Another potential extension includes the analysis of the same problem for a relief organization that procures and distributes multiple products. We conjecture that the majority of our surface transportation allocation methods and the ensuing insights continue to hold for the multi-product problem with a single market. Thus, there might be opportunities to extend our results in a problem setting that features multiple products being served in various regions.

3.7 Appendix

Proof of Lemma 3.1

First, we show that (3.4) is a convex function. Recall that (3.4) is an objective function with a single variable z_1. Thus, convexity is proven by showing that the second order derivative with respect to z_1 is positive. The first order derivative of (3.4) with respect to z_1 is the following:

$$
\begin{aligned}
\frac{\partial E\left[S\left(z_1 | q_a\right)\right]}{\partial z_1} =& \int_{-\infty}^{z_1} \int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \sigma_1 \phi(y_2)\phi(y_1)dy_2 dy_1 - \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty} \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}} \sigma_1 \phi(y_2)\phi(y_1)dy_2 dy_1 \\
& + \int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \left[z_1\sigma_1 + \sigma_2(y_2-k)-q_a\right]\phi(y_2)\phi(z_1)dy_2 \\
& + \int_{\frac{k-z_1\sigma_1}{\sigma_2}}^{\infty} \left[z_1\sigma_1 + \sigma_2(y_2-k)-q_a\right]\phi(y_2)\phi\left(z_1+\frac{q_a}{\sigma_1}\right)dy_2 \\
& - \int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \left[z_1\sigma_1 + \sigma_2(y_2-k)-q_a\right]\phi(y_2)\phi(z_1)dy_2 \\
& - \int_{\frac{k-z_1\sigma_1}{\sigma_2}}^{\infty} \left[z_1\sigma_1 + \sigma_2(y_2-k)-q_a\right]\phi(y_2)\phi\left(z_1+\frac{q_a}{\sigma_1}\right)dy_2 \\
=& \sigma_1 \left(\int_{-\infty}^{z_1} \int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \phi(y_2)\phi(y_1)dy_2 dy_1 - \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty} \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}} \phi(y_2)\phi(y_1)dy_2 dy_1 \right)
\end{aligned}
$$

(3.6)

The second-order derivative of (3.4) with respect to z_1 is the following:

$$
\frac{\partial^2 E\left[S\left(z_1 | q_a\right)\right]}{\partial z_1^2} = \sigma_1 \left(\int_{\frac{k-z_1\sigma_1}{\sigma_2}+\frac{q_a}{\sigma_2}}^{\infty} \phi(y_2)\phi(z_1)dy_2 + \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}} \phi(y_2)\phi\left(z_1+\frac{q_a}{\sigma_1}\right)dy_2 \right) > 0.
$$

Because the second-order derivative of (3.4) with respect to z_1 is strictly positive, the objective function is strictly convex. Thus, z_1^* can be derived from (3.6) where the first-order condition equals zero, and z_2^* can be derived from (3.3).

a) By substituting $q_a = 0$ into (3.6) and reorganizing the equation, we derive the following:

$$
\int_{-\infty}^{z_1} \int_{\frac{k-z_1\sigma_1}{\sigma_2}}^{\infty} \phi(y_2)\phi(y_1)dy_2 dy_1 = \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}} \int_{z_1}^{\infty} \phi(y_2)\phi(y_1)dy_2 dy_1 .
$$

Recall that due to standardization, the pdfs are equal for Z_1 and Z_2. Therefore, the intervals are interchangeable within an expression. Also, recall that $z_2 = (k - z_1\sigma_1)/\sigma_2$. Thus, if $z_1 = (k - z_1\sigma_1)/\sigma_2$ which is equal to z_2, the first-order condition is satisfied. In conclusion, if $q_a = 0$ then setting the stocking factors equal, i.e., $z_1{}^* = z_2{}^*$, minimizes the expected shortage.

b) First, we substitute a common standard deviation σ for σ_1 and σ_2 in the first-order condition (3.6) which is the following:

$$\int_{-\infty}^{z_1} \int_{\frac{k-z_1\sigma}{\sigma}+\frac{q_a}{\sigma}}^{\infty} \phi(y_2)\phi(y_1)dy_2dy_1 = \int_{-\infty}^{\frac{k-z_1\sigma}{\sigma}} \int_{z_1+\frac{q_a}{\sigma}}^{\infty} \phi(y_2)\phi(y_1)dy_2dy_1 .$$

If $\sigma_1 = \sigma_2 = \sigma$, then $z_2 = (k - z_1\sigma)/\sigma$. Thus, if $z_1 = (k - z_1\sigma)/\sigma$ which is equal to z_2, the first-order condition is satisfied. In conclusion, if $\sigma_1 = \sigma_2$ then setting the stocking factors equal, i.e., $z_1{}^* = z_2{}^*$, minimizes the expected shortage.

c) The first-order derivative of (3.6) with respect to σ_2 is the following:

$$\frac{\partial(3.6)}{\partial\sigma_2} = \frac{\sigma_1}{\sigma_2} \left(\begin{array}{c} \int_{-\infty}^{z_1} \left(\frac{k - z_1\sigma_1}{\sigma_2} + \frac{q_a}{\sigma_2} \right) \phi\left(\frac{k-z_1\sigma_1}{\sigma_2} + \frac{q_a}{\sigma_2} \right) \phi(y_1)dy_1 \\ + \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty} \left(\frac{k - z_1\sigma_1}{\sigma_2} \right) \phi\left(\frac{k-z_1\sigma_1}{\sigma_2} \right) \phi(y_1)dy_1 \end{array} \right) > 0 \qquad (3.6)$$

Since our objective is to minimize the expected shortage, if the first-order derivative of

is negative with respect to σ_2, it indicates that $z_1{}^*$ should be decreased with respect to σ_2. Recall that $z_2 = (k - z_1\sigma_1)/\sigma_2$. Thus, if $z_1{}^*$ decreases, then $z_2{}^*$ increases. Lemma 3.1b has proven that if the standard deviations are equal in both regions then the optimal stocking factors are also equal. However, if σ_2 increases (while keeping σ_1 constant), then $z_1{}^*$ decreases and $z_2{}^*$ increases, leading to the conclusion that $z_2{}^* > z_1{}^*$. Note that (3.4) can be rewritten in terms of z_2, and the same result can be proven in terms of σ_1 following the identical path. Thus, it is optimal for the relief

113

organization to allocate more essential goods in terms of service level for surface transportation to the region with greater level of fluctuations in demand, i.e., if $\sigma_1 > \sigma_2$, then $z_1{}^* > z_2{}^*$, otherwise $z_1{}^* \leq z_2{}^*$. $\square$

Proof of Proposition 3.1

We may decompose $S(\mathbf{z})$ into two terms as follows:

$$S(\mathbf{z}) = E\left[\sum_{i \in N \setminus \{l,m\}} \left(\sigma_i \left(Z_i - z_i \right)^+ - q_a \right)^+ \right] + E\left[\sum_{i \in \{l,m\}} \left(\sigma_i \left(Z_i - z_i \right)^+ - \left(q_a - \sum_{i \in N \setminus \{l,m\}} \sigma_i \left(Z_i - z_i \right)^+ \right)^+ \right)^+ \right]$$

$$= S_{N \setminus \{l,m\}} (\mathbf{z}) + S_{\{l,m\}} (\mathbf{z}).$$

The second term in the expression gives the expected units short for two arbitrarily selected regions denoted l and m. These two regions are served with air shipments, if needed, after the initial shortages at all other regions have been covered via air to the extent of available supply q_a. Finally, let $z_i' = z_i{}^*$ for all $i \in N \setminus \{l, m\}$, $z_l' = z_m' = 0.5(z_l{}^* + z_m{}^*)$, and $\mathbf{z}' = (z_1', \ldots, z_n')$.

a) Suppose that Proposition 3.1a is not true. Then there exists a problem instance such that

$$S^* = E\left[\sum_{i \in N \setminus \{l,m\}} \sigma_i \left(Z_i - z_i{}^* \right)^+ \right] + E\left[\sum_{i \in \{l,m\}} \sigma_i \left(Z_i - z_i{}^* \right)^+ \right] = S^*_{N \setminus \{l,m\}} + S^*_{\{l,m\}} < S_{N \setminus \{l,m\}} (\mathbf{z}') + S_{\{l,m\}} (\mathbf{z}')$$

But $S^*_{N \setminus \{l,m\}} = S_{N \setminus \{l,m\}} (\mathbf{z}')$, and by Lemma 3.1, $S_{\{l,m\}} (\mathbf{z}') \leq S^*_{\{l,m\}}$, which is a contradiction.

b) Suppose that Proposition 3.1b is not true. Then there exists a problem instance such that $S^* = S^*_{N \setminus \{l,m\}} + S^*_{\{l,m\}} < S_{N \setminus \{l,m\}} (\mathbf{z}') + S_{\{l,m\}} (\mathbf{z}')$.

Let $\tilde{q}_a = \left(q_a - \sum_{i \in N \setminus \{l,m\}} \sigma_i \left(Z_i - z_i{}^* \right)^+ \right)^+ = \left(q_a - \sum_{i \in N \setminus \{l,m\}} \sigma_i \left(Z_i - z_i ' \right)^+ \right)^+$ denote the random leftover air

supply for regions l and m, and let $f_a(x)$ denote the pdf of $\tilde{q}_a$. Then

114

$$S_{\{l,m\}}(\mathbf{z}') = \int_0^\infty E\left[\sum_{i\in\{l,m\}} \left(\sigma_i(Z_i - z_i')^+ - x\right)^+ \bigg| \tilde{q}_a = x\right] f_a(x)\,dx.$$

But $S^*_{N\setminus\{l,m\}} = S_{N\setminus\{l,m\}}(\mathbf{z}')$, and by Lemma 3.1,

$$E\left[\sum_{i\in\{l,m\}} \left(\sigma_i(Z_i - z_i')^+ - x\right)^+\right] \le E\left[\sum_{i\in\{l,m\}} \left(\sigma_i(Z_i - z_i^*)^+ - x\right)^+\right]$$

for any x, which implies $S_{\{l,m\}}(\mathbf{z}') \le S^*_{\{l,m\}}$, and we have a contradiction.

c) We assume that $\sigma_l < \sigma_m$. Let $z_i{}^o = z_i{}^*$ for all $i \in N\setminus\{l, m\}$, $z_l{}^o \le z_m{}^o$ and $\mathbf{z}^o = (z_1{}^o, \ldots, z_n{}^o)$.

Suppose that Proposition 3.1c is not true. Then there exists a problem instance such that

$$S^* = E\left[\sum_{i\in N\setminus\{l,m\}} \sigma_i(Z_i - z_i^*)^+\right] + E\left[\sum_{i\in\{l,m\}} \sigma_i(Z_i - z_i^*)^+\right] = S^*_{N\setminus\{l,m\}} + S^*_{\{l,m\}} < S_{N\setminus\{l,m\}}(\mathbf{z}^o) + S_{\{l,m\}}(\mathbf{z}^o)$$

But $S^*_{N\setminus\{l,m\}} = S_{N\setminus\{l,m\}}(\mathbf{z}^o)$, and by Lemma 3.1, $S_{\{l,m\}}(\mathbf{z}^o) \le S^*_{\{l,m\}}$, which is a contradiction. $\square$

CHAPTER 4: CONCLUSIONS

In this dissertation, we examine risk mitigation methods in the area of global supply chain management. We develop policies and provide insights both for multinational corporations with the objective of maximizing expected profits and minimizing exchange-rate risk, and for global relief organizations with the objective of minimizing expected shortage due to demand uncertainty.

The first essay builds on the idea of production hedging, which is defined as deliberately producing less than the total global demand, in order to mitigate the exchange-rate risk. In this essay, we examine a global firm's selling price and manufacturing quantity decisions in the presence of exchange-rate risk. It is important to note that the selling price is set before the random exchange rate is realized, and it is determined in a way that complies with the international anti-dumping laws (i.e., the selling price is the same amount when using the expected exchange rate). The choice of the selling price influences the demand in markets in multiple countries. Moreover, the selling price and manufacturing quantity decisions are made under a Value-at-Risk constraint that dictates the firm's choice of tolerable loss with a corresponding probability.

After production takes place, the firm observes the realization of the random exchange rate, and determines the amount of products to be allocated to each market for sale. In a two-country setting with one domestic and one foreign market, we show that there are five potentially optimal policies, only one of which satisfies the total demand. The other four polices recommend manufacturing a smaller quantity than the total demand, and are forms of production hedging policies. The reason for a smaller manufacturing quantity can be explained by two factors: (1) the flexibility created by postponing the allocation of the stock to appreciating markets with a

116

higher return in value (resembling the option value), and (2) limiting potential losses due to the VaR constraint (i.e., risk mitigation). We find that as the firm becomes increasingly risk-averse, it is more likely to prefer production hedging. Thus, production hedging is not only a profit-maximizing scheme, but also a risk mitigation technique to battle exchange-rate risk.

The second essay considers a relief organization providing essential products to multiple regions in the presence of demand uncertainty with a limited budget. In this essay, the relief organization is seeking to minimize the shortages of providing essential goods with two options of transportation: (1) surface transportation and (2) air transportation. For the surface transportation option, the organization can deliver greater amounts of essential products because of the cheaper unit transportation cost; however due to the extended lead-time, the products need to be shipped to each region before the demand is known. For the air transportation option, in contrast, the organization can transport products after the random demand is realized in each region which provides the organization with the flexibility to serve the market with excess demand. However, because of its expensive unit transportation cost, the delivered quantity of essential goods decreases due to the limited budget. The relief organization can strategically allocate the budget to each transportation option to benefit from a combination of having flexibility (via air transportation) and conveying greater amount of essential goods (via surface transportation).

We find that the relief organization increases the proportion of the budget towards air transportation, gaining more flexibility, (while subtracting from the budget for surface transportation) when the following occurs: (1) the unit cost of air transportation decreases, (2) the distortion of variance between demands in each region decreases, (3) the number of regions that need to be served increases, (4) correlation between regions decrease. Otherwise, the

117

organization allocates more of the budget to surface transportation. Note that just because the relief organization prioritizes one option over another does not necessarily indicate whether the optimal expected shortage increases or decreases. There are several obvious scenarios that benefit a relief organization in order to minimize expected shortage, such as having increased amount of budget, a decrease in transportation costs, and a decrease in demand variances. There are several surprising scenarios that benefit a relief organization. For instance, for a given total amount of demand variation, as the distortion of variance increases between regions, the expected shortage decreases. Moreover, minimizing correlation between regions can lead to enhanced performance for humanitarian organizations. Besides, we find scenarios where integrating budgets of relief organizations for supplying essential goods lead to reduced amount of expected shortage.

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EDUCATION

Ph.D.   in Supply Chain Management, Whitman School of Management, Syracuse University, August 2012
Dissertation Title: Essays on risk mitigation methods in global supply chain management

M.A.   in Economics, Maxwell School, Syracuse University, December 2010

B.A.   Dual degree in e-business and business administration, College of Business Administration, Ajou University, Suwon, Korea, February 2008

ACADEMIC APPOINTMENTS

Assistant Professor, Malaysia Institute of Supply Chain Innovation, Shah Alam, Malaysia, 2012 − Current

RESEARCH INTERESTS

Supply Chain Risk Management, Global Supply Chain, Currency Risk, Pricing and Production, Humanitarian Supply Chain

RESEARCH IN PROGRESS

Park, J., B. Kazaz, S. Webster. 2012. Risk mitigation of production hedging.

Park, J., B. Kazaz, S. Webster. 2012. Minimization of expected shortage under demand uncertainty with budget constraints.

Lee, C., J. Park. 2012. Efficient consumer returns and resell policies under valuation uncertainty.

HONORS

Robert H. Brethen Operations Management Institute summer research grant, Syracuse University, 2009, 2010, 2011

CONFERENCE PRESENTATIONS

Production and Operations Management Society (POMS) Conference, Chicago, Il, April 2012
Title: Risk Mitigation of Production Hedging
Title: Minimization of Expected Shortage under Demand Uncertainty with Budget Constraints

Institute for Operations Research and the Management Sciences (INFORMS) Conference, Charlotte, NC, November 2011
Title: Risk Mitigation of Production Hedging

Seventh Conference on Integrated Risk Management in Operations and Global Supply Chains, Montreal, Canada, July 2011
Title: Risk Mitigation of Production Hedging

Manufacturing & Service Operations Management (MSOM) Conference, Ann Arbor, MI, June 2011
Title: Risk Mitigation of Production Hedging

Production and Operations Management Society (POMS) Conference, Reno, NV, May 2011
Title: Risk Mitigation of Production Hedging

TEACHING EXPERIENCE

Undergraduate, "Introduction to Supply Chain Management" (SCM 265), Syracuse University
– spring 2011: teaching evaluation 3.9 out of 5.0
– fall 2011: teaching evaluation 4.6 out of 5.0
– this is a course taught in an integrated style with the introductory finance and marketing courses
– prepared own homework assignments, quizzes, exams
– used Capsim simulation in order to teach the integrated core

TEACHING ASSISTANT

MBA, "Global Supply Chain Management" (SCM 600), Syracuse University, fall 2010

Undergraduate, "Introduction to Supply Chain Management" (SCM 265), Syracuse University, fall 2009, spring 2010

Undergraduate, "Marketing Research" (MAR 356), Syracuse University, fall 2008, spring 2009

Undergraduate, "Operations Management," Ajou University, fall 2005, spring 2006, fall 2006, spring 2007, fall 2007

PROFESSIONAL SERVICE

Referee for *Decision Sciences*

Session Chair, Pricing and Risk Management, Production and Operations Management Society Conference, Reno, NV, May 2011

Assisted conference organization, Sixth Conference on Integrated Risk Management in Operations and Global Supply Chains, Whitman School of Management, Syracuse University, August 2010

Assisted conference organization, Fourth Annual Behavioral Operations Conference, Whitman School of Management, Syracuse University, June 2009

Member of *Institute for Operations Research and the Management Sciences, Manufacturing & Service Operations Management, Production and Operations Management Society, Decision Sciences Institute*

INDUSTRY & EXTRACURRICULAR EXPERIENCE

Soldier, Republic of Korean Army, Gapyong, Korea, March 2003−March 2005
  − Served at the Headquarters of the Tiger Division (operations department)

Intern, HaanSoft, Seoul, Korea, Summer 2006
  − Assisted developing marketing strategies for new services and products

Intern, Yahoo Korea, Seoul, Korea, Summer 2007
  − Assisted developing marketing strategies, beta tested and found bugs for new services provided
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