Syracuse University

SURFACE

2007

A new achievable rate region for interference channels with common information

Yi Cao
Syracuse University

Biao Chen
Syracuse University, School of Electrical Engineering and Computer Science, bichen@ecs.syr.edu

Junshan Zhang
Arizona State University, Department of Electrical Engineering, junshan.zhang@asu.edu

A New Achievable Rate Region for Interference Channels with Common Information

Yi Cao and Biao Chen
Department of EECS
Syracuse University
Syracuse, NY 13244
Email: ycao01@syr.edu, bichen@ecs.syr.edu

Junshan Zhang
Department of Electrical Engineering
Arizona State University
Tempe, AZ 85287
Email: junshan.zhang@asu.edu

Abstract— **In this paper, a new achievable rate region for general interference channels with common information is presented. Our result improves upon [1] by applying simultaneous superposition coding over sequential superposition coding. A detailed computation and comparison of the achievable rate region for the Gaussian case is conducted. The proposed achievable rate region is shown to coincide with the capacity region of the strong interference case [2].**

Index terms — interference channels, achievable rate region, capacity region, common information, superposition coding.

I. INTRODUCTION

The capacity region of an interference channel (IC), where the information sources at the two transmitters are statistically independent, has been a long standing problem [3]–[7]. An important milestone in IC is Carleial's work in 1978 [8] where the superposition code idea was used to obtain a much improved inner bound for IC. This inner bound was later improved by Han and Kobayashi [9] who gave an achievable rate region that remains to be the largest reported to this date.

A related problem less well investigated in the past is when the information sources at the two transmitters are correlated, and in particular, they share a common message in addition to their private messages [1]. For example, instead of having only one transmitter obtaining the other user's message a priori, as in the so-called cognitive radio channel [10], [11], the two transmitters can obtain part of the other user's source via conferencing [2]. We can also motivate this problem using sensor network applications, where two nodes need to relay the information of a third node while transmitting their own observations at the same time. This type of correlated information sources has previously been considered in [12] and [13] for a multiple access channel, where the capacity region and optimum conferencing were determined. Interference channel with common message was first considered in [1], where an achievable rate region, an outer bound and a limiting expression for the capacity region were obtained. Later, the capacity region of this channel under strong interference was found in [2], which also showed that the condition of strong interference for this channel is equivalent to that of the classic interference channels. In this work, we propose a new achievable rate region for an interference channel with common information. Our result

improves upon [1] and coincides with the capacity region in [2] under the condition of strong interference.

This paper is organized as follows. In section II, we present the channel model and review the previous achievable rate region [1] and capacity region with strong interference [2]. In section III, we propose a new achievable region and show that it extends the achievable region in [1]. We also establish that the proposed achievable region is consistent with the capacity region for the strong interference case [2]. In section IV, numerical examples are presented for the Gaussian case and the comparison of the two achievable regions are shown. Finally, concluding remarks are given in section V.

II. EXISTING RESULTS

A. Definitions

An interference channel with common information $\mathcal{K}$ is a quintuple $(\mathcal{X}_1, \mathcal{X}_2, p, \mathcal{Y}_1, \mathcal{Y}_2)$, where $\mathcal{X}_1, \mathcal{X}_2$ are two finite input alphabet sets and $\mathcal{Y}_1, \mathcal{Y}_2$ are two finite output alphabet sets, p is the channel transition probability $p(y_1, y_2|x_1, x_2)$, i.e.
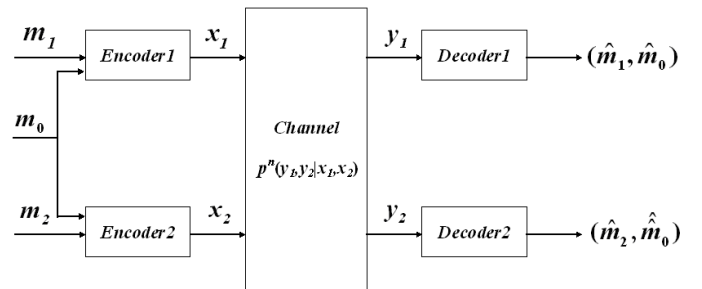

Fig. 1.   Interference channel with common information.

the probability of $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ given $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$. We assume that the channels are memoryless, i.e.

$$p^n(\mathbf{y}_1, \mathbf{y}_2|\mathbf{x}_1, \mathbf{x}_2) \quad = \quad \prod_{t=1}^{n} p(y_1^{(t)}, y_2^{(t)}|x_1^{(t)}, x_2^{(t)}) \quad (1)$$

where for $a = 1, 2$,

$$\mathbf{x}_a = (x_a^{(1)}, \cdots, x_a^{(n)}) \in \mathcal{X}_a^n, \mathbf{y}_a = (y_a^{(1)}, \cdots, y_a^{(n)}) \in \mathcal{Y}_a^n \quad (2)$$

Let $\mathcal{M}_1 = \{1, 2, \cdots, M_1\}$ and $\mathcal{M}_2 = \{1, 2, \cdots, M_2\}$ be sender 1 and sender 2's private message sets, respectively, which are only decoded by intended receivers. Let $\mathcal{M}_0 = \{1, 2, \cdots, M_0\}$

be the common message set, which is to be decoded by both receivers Y_1 and Y_2. Since each sender has knowledge of his own message as well as the common message, there are $M_1 \cdot M_0$ codewords for $\mathbf{x}_1(i,k)$ and $M_2 \cdot M_0$ codewords for $\mathbf{x}_2(j,k)$. Suppose the decoded message indices at receiver 1 are $\hat{m}_1$ and $\hat{m}_0$ while at receiver 2 are $\hat{m}_2$ and $\hat{\hat{m}}_0$. Then, the average probability of decoding error of this channel is defined as

$$P_{e,1}^{(n)} \equiv \frac{1}{M_1 M_2 M_0} \sum_{i,j,k} P(A_{ik}|\mathbf{x}_1(i,k), \mathbf{x}_2(j,k)) \qquad (3)$$

$$P_{e,2}^{(n)} \equiv \frac{1}{M_1 M_2 M_0} \sum_{i,j,k} P(B_{jk}|\mathbf{x}_1(i,k), \mathbf{x}_2(j,k)) \qquad (4)$$

where the events A_{ik} and B_{jk} are defined as

$$A_{ik} \doteq \{\hat{m}_1 \neq i\} \cup \{\hat{m}_0 \neq k\} \qquad (5)$$

$$B_{jk} \doteq \{\hat{m}_2 \neq j\} \cup \{\hat{\hat{m}}_0 \neq k\} \qquad (6)$$

The capacity region of this channel $\mathcal{C}$ is the closure of all the rate triples (R_1, R_2, R_0) such that $P_{e,1}^{(n)} \to 0$, $P_{e,2}^{(n)} \to 0$ as codeword length $n \to \infty$, where $R_1 = \log M_1/n$, $R_2 = \log M_2/n$ and $R_0 = \log M_0/n$.

B. Existing Results

1) Proposition 1: Let $Z = (U_0, U_1, U_2, X_1, X_2, Y_1, Y_2)$ and let $\mathcal{P}$ be the set of distribution on Z that can be decomposed into the form

$$p(u_0)p(u_1|u_0)p(u_2|u_0)p(x_1|u_1)p(x_2|u_2)p(y_1,y_2|x_1,x_2) \quad (7)$$

For any $Z \in \mathcal{P}$, let $R(Z)$ be the set of all triples (R_1, R_2, R_0) satisfying:

$$R_1 \leq I(X_1; Y_1|U_1U_2) + a_i \qquad (8)$$

$$R_2 \leq I(X_2; Y_2|U_1U_2) + b_i \qquad (9)$$

$$R_0 + R_1 + \frac{b_i}{I(X_2;Y_2|U_1U_2) + b_i} R_2 \leq I(U_2X_1;Y_1) \quad (10)$$

$$R_0 + R_2 + \frac{a_i}{I(X_1;Y_1|U_1U_2) + a_i} R_1 \leq I(U_1X_2;Y_2) \quad (11)$$

where

$$a_1 = min[I(U_1;Y_1|U_0), I(U_1;Y_2|U_0)] \qquad (12)$$

$$b_1 = min[I(U_2;Y_1|U_1U_0), I(U_2;Y_2|U_1U_0)] \qquad (13)$$

$$a_2 = min[I(U_1;Y_1|U_2U_0), I(U_1;Y_2|U_2U_0)] \qquad (14)$$

$$b_2 = min[I(U_2;Y_1|U_0), I(U_2;Y_2|U_0)] \qquad (15)$$

$$a_3 = min[I(U_1;Y_1|U_0), I(U_1;Y_2|U_2U_0)] \qquad (16)$$

$$b_3 = min[I(U_2;Y_1|U_1U_0), I(U_2;Y_2|U_0)] \qquad (17)$$

$$a_4 = min[I(U_1;Y_1|U_2U_0), I(U_1;Y_2|U_0)] \qquad (18)$$

$$b_4 = min[I(U_2;Y_1|U_0), I(U_2;Y_2|U_1U_0)] \qquad (19)$$

Then $\mathcal{R}_T = \bigcup_{Z \in \mathcal{P}} R(Z)$ is an achievable region, i.e., $\mathcal{R}_T \subseteq \mathcal{C}$.
Proof: See [1].

2) Proposition 2: For an interference channel $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ with common information satisfying

$$I(X_1; Y_1|X_2, U) \leq I(X_1; Y_2|X_2, U) \qquad (20)$$

$$I(X_2; Y_2|X_1, U) \leq I(X_2; Y_1|X_1, U) \qquad (21)$$

the capacity region $\mathcal{C}_s$ is given by

$$
\begin{aligned}
\mathcal{C}_s = \{ & \bigcup (R_0, R_1, R_2) : \\
& R_1 \leq I(X_1; Y_1|X_2, U) \\
& R_2 \leq I(X_2; Y_2|X_1, U) \\
& R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1|U), I(X_1, X_2; Y_2|U)\} \\
& R_0 + R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\}\}
\end{aligned}
$$
$$(22)$$

where the union is over joint distributions $p(u, x_1, x_2, y_1, y_2)$ that factor as

$$p(u)p(x_1|u)p(x_2|u)p(y_1,y_2|x_1,x_2). \qquad (23)$$

Proof: See [2].

III. A New Achievable Region

A. Modified interference channel with common information $\mathcal{K}_m$

In this modified channel, we allow part of each transmitter's private message to be decoded by both receivers. For each transmitter $a(a = 1, 2)$, the original private message is divided into two parts: private message $i_a \in \mathcal{M}_a = \{1, 2, \cdots, M_a\}$ and common message $j_a \in \mathcal{N}_a = \{1, 2, \cdots, N_a\}$. Besides, each transmitter also has the original common message $k \in \mathcal{M}_0 = \{1, 2, \cdots M_0\}$. So, there are $M_1 \cdot N_1 \cdot M_0$ codewords $\mathbf{x}_1(i_1, j_1, k)$ and $M_2 \cdot N_2 \cdot M_0$ codewords $\mathbf{x}_2(i_2, j_2, k)$. Suppose the decoded message indices at receiver 1 are $\hat{m}_1, \hat{n}_1, \hat{n}_2$ and $\hat{m}_0$, while at receiver 2 are $\hat{m}_2, \hat{\hat{n}}_2, \hat{\hat{n}}_1$ and $\hat{\hat{m}}_0$. Define events

$$E_1 \doteq \{\hat{m}_1 \neq i_1\} \cup \{\hat{n}_1 \neq j_1\} \cup \{\hat{n}_2 \neq j_2\} \cup \{\hat{m}_0 \neq k\} \quad (24)$$

$$E_2 \doteq \{\hat{m}_2 \neq i_2\} \cup \{\hat{\hat{n}}_2 \neq j_2\} \cup \{\hat{\hat{n}}_1 \neq j_1\} \cup \{\hat{\hat{m}}_0 \neq k\} \quad (25)$$

Then, the average probability of decoding error is

$$P_{e,1}^{(n)} \equiv \frac{\sum_{i_1 j_1 i_2 j_2 k} P(E_1|\mathbf{x}_1(i_1, j_1, k), \mathbf{x}_2(i_2, j_2, k))}{M_1 N_1 M_2 N_2 M_0} \qquad (26)$$

$$P_{e,2}^{(n)} \equiv \frac{\sum_{i_1 j_1 i_2 j_2 k} P(E_2|\mathbf{x}_1(i_1, j_1, k), \mathbf{x}_2(i_2, j_2, k))}{M_1 N_1 M_2 N_2 M_0} \qquad (27)$$

The corresponding achievable rate quintuples $(R_{11}, R_{12}, R_{22}, R_{21}, R_0)$ are such that $P_{e,1}^{(n)} \to 0$, $P_{e,2}^{(n)} \to 0$ as codeword length $n \to \infty$, where $R_{11} = \log M_1/n$, $R_{12} = \log N_1/n$, $R_{22} = \log M_2/n$, $R_{21} = \log N_2/n$ and $R_0 = \log M_0/n$. The modified interference channel with common information introduces four auxiliary random variables Q, U_1, U_2, U_0 (Q is the time sharing random variable), defined on arbitrary finite sets $\mathcal{Q}, \mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_0$ respectively. It is easy to see that if a rate quintuple $(R_{11}, R_{12}, R_{22}, R_{21}, R_0)$ is achievable for modified channel $\mathcal{K}_m$, then rate triple $(R_{11} + R_{12}, R_{22} + R_{21}, R_0)$ is also achievable for the original channel $\mathcal{K}$.

B. A new achievable rate region

Theorem 1: Suppose $Z = (Y_1, Y_2, X_1, X_2, U_1, U_2, U_0, Q)$ and let $\mathcal{P}^*$ be the set of distribution on Z that can be decomposed into the form

$$p(q)p(u_0|q)p(u_1|u_0q)p(u_2|u_0q)$$
$$\times p(x_1|u_1q)p(x_2|u_2q)p(y_1y_2|x_1x_2) \qquad (28)$$

For any $Z \in \mathcal{P}^*$, let $S(Z)$ be the set of all quintuples $(R_{11}, R_{12}, R_{21}, R_{22}, R_0)$ of non-negative numbers satisfying:

$$R_{11} \leq I(Y_1; X_1|U_1U_2Q) \qquad (29)$$
$$R_{21} \leq I(Y_1; U_2|X_1U_0Q) \qquad (30)$$
$$R_{11} + R_{12} \leq I(Y_1; X_1|U_2U_0Q) \qquad (31)$$
$$R_{11} + R_{21} \leq I(Y_1; X_1U_2|U_1U_0Q) \qquad (32)$$
$$R_{11} + R_{12} + R_{21} \leq I(Y_1; X_1U_2|U_0Q) \qquad (33)$$
$$R_0 + R_{11} + R_{12} + R_{21} \leq I(Y_1; X_1U_2|Q) \qquad (34)$$
$$R_{22} \leq I(Y_2; X_2|U_1U_2Q) \qquad (35)$$
$$R_{12} \leq I(Y_2; U_1|X_2U_0Q) \qquad (36)$$
$$R_{22} + R_{21} \leq I(Y_2; X_2|U_1U_0Q) \qquad (37)$$
$$R_{22} + R_{12} \leq I(Y_2; X_2U_1|U_2U_0Q) \qquad (38)$$
$$R_{22} + R_{21} + R_{12} \leq I(Y_2; X_2U_1|U_0Q) \qquad (39)$$
$$R_0 + R_{22} + R_{21} + R_{12} \leq I(Y_2; X_2U_1|Q) \qquad (40)$$

Let $\mathcal{R}_m$ be the closure of $\bigcup_{Z \in \mathcal{P}^*} S(Z)$, then $\mathcal{R}_m$ is the achievable rate region of $\mathcal{K}_m$.

Proof of Theorem 1:

Codebook Generation: Let $\mathbf{q} = (q^{(1)}, \cdots, q^{(n)})$ be a random sequence of $\mathcal{Q}^n$ distributed according to the probability $\prod_{t=1}^n p(q^{(t)})$. For the codeword $\mathbf{q}$, generate M_0 i.i.d (independent and identically distributed) codewords $\mathbf{u}_0(k), k = 1, 2, \cdots, M_0$, with each element distributed according to $\prod_{t=1}^n p(u_0^{(t)}|q^{(t)})$. For the codeword $\mathbf{q}$ and each of $\mathbf{u}_0(k)$, generate N_1 i.i.d codewords $\mathbf{u}_1(j_1, k), j_1 = 1, 2, \cdots, N_1$ and N_2 i.i.d codewords $\mathbf{u}_2(j_2, k), j_2 = 1, 2, \cdots, N_2$, with each element distributed according to $\prod_{t=1}^n p(u_1^{(t)}|u_0(k)^{(t)}q^{(t)})$ and $\prod_{t=1}^n p(u_2^{(t)}|u_0(k)^{(t)}q^{(t)})$ respectively. For the codeword $\mathbf{q}$ and each of $\mathbf{u}_1(j_1, k)$, generate M_1 i.i.d codewords $\mathbf{x}_1(i_1, j_1, k), i_1 = 1, 2, \cdots, M_1$, with each element distributed according to $\prod_{t=1}^n p(x_1^{(t)}|u_1(j_1, k)^{(t)}q^{(t)})$. For $\mathbf{q}$ and each of $\mathbf{u}_2(j_2, k)$, generate M_2 i.i.d codewords $\mathbf{x}_2(i_2, j_2, k), i_2 = 1, 2, \cdots, M_2$, with each element distributed according to $\prod_{t=1}^n p(x_2^{(t)}|u_2(j_2, k)^{(t)}q^{(t)})$.

Encoding Rule: For encoder 1, given a message triple (i_1, j_1, k), send the corresponding codeword $\mathbf{x}_1(i_1, j_1, k)$. Similarly, for encoder 2, send $\mathbf{x}_2(i_2, j_2, k)$ for the triple (i_2, j_2, k).

Decoding Rule:

Receiver 1 determines the unique (i_1, j_1, j_2, k) such that

$$\{\mathbf{q}, \mathbf{u}_0(k), \mathbf{u}_1(j_1, k), \mathbf{u}_2(j_2, k), \mathbf{x}_1(i_1, j_1, k), \mathbf{y}_1\}$$
$$\in A_\epsilon^{(n)}(QU_0U_1U_2X_1Y_1) \qquad (41)$$

Receiver 2 determines the unique (i_2, j_2, j_1, k) such that

$$\{\mathbf{q}, \mathbf{u}_0(k), \mathbf{u}_2(j_2, k), \mathbf{u}_1(j_1, k), \mathbf{x}_2(i_2, j_2, k), \mathbf{y}_2\}$$
$$\in A_\epsilon^{(n)}(QU_0U_2U_1X_2Y_2) \qquad (42)$$

where $A_\epsilon^{(n)}(\cdot)$ denotes the jointly typical set.

Analysis of Error Probability: By symmetry of the random code construction, the error probability for a specific message quintuple is independent of that quintuple. Therefore we can assume without loss of generality that $(i_1, i_2, j_1, j_2, k) = (1, 1, 1, 1, 1)$ was sent. We first consider the average error probability $P_{e,1}^{(n)}$ for receiver 1 and suppose $\mathbf{y}_1$ was received. Let $E_1(i_1j_1j_2k)$ denote the event (41). Then we have

$$P_{e,1}^{(n)} \equiv Pr\{E_1^c(1111) \quad or \bigcup_{i_1j_1j_2k \neq 1111} E_1(i_1j_1j_2k)\} \qquad (43)$$

$$\leq Pr\{E_1^c(1111)\} + \sum_{k \neq 1, i_1j_1j_2} Pr\{E_1(i_1j_1j_2k)\} \qquad (44)$$

$$+ \sum_{j_1 \neq 1, i_1} Pr\{E_1(i_1j_111)\} + \sum_{i_1 \neq 1} Pr\{E_1(i_1111)\} \qquad (45)$$

$$+ \sum_{i_1 \neq 1, j_2 \neq 1} Pr\{E_1(i_11j_21)\} + \sum_{j_2 \neq 1} Pr\{E_1(11j_21)\} \qquad (46)$$

$$+ \sum_{j_1 \neq 1, j_2 \neq 1, i_1} Pr\{E_1(i_1j_1j_21)\} \qquad (47)$$

From the way the random sequences $\mathbf{q}, \mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{x}_1$ are generated and by the property of jointly typical set, it follows that

$$Pr\{E_1^c(1111)\} \leq \epsilon \qquad (48)$$

Now, let us evaluate $Pr\{E_1(i_1j_1j_2k)\}$ for $k \neq 1$. From the way the random sequences $\mathbf{q}, \mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{x}_1$ are generated, we know $\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{x}_1$ are all independent from $\mathbf{y}_1$ given $\mathbf{q}$ due to the fact $k \neq 1$. So,

$$Pr\{E_1(i_1j_1j_2k)\}$$
$$= \sum_{(\mathbf{q}\mathbf{u}_0\mathbf{u}_1\mathbf{u}_2\mathbf{x}_1\mathbf{y}_1) \in A_\epsilon^{(n)}} p(\mathbf{q})p(\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{x}_1|\mathbf{q})p(\mathbf{y}_1|\mathbf{q})$$
$$\leq |A_\epsilon^{(n)}|2^{-n(H(Q)-\epsilon)}2^{-n(H(U_0U_1U_2X_1|Q)-\epsilon)}2^{-n(H(Y_1|Q)-\epsilon)}$$
$$\leq 2^{-n(H(Q)+H(U_0U_1U_2X_1|Q)+H(Y_1|Q)-H(QU_0U_1U_2X_1Y_1)-4\epsilon)}$$
$$= 2^{-n(I(Y_1;U_0U_1U_2X_1|Q)-4\epsilon)}$$
$$= 2^{-n(I(Y_1;X_1U_2|Q)-4\epsilon)}$$

Using similar techniques to evaluate the probabilities of other error events, we have

$$P_{e,1}^{(n)} \leq \epsilon + 2^{-n(I(Y_1;X_1U_2|Q)-(R_{11}+R_{12}+R_{21}+R_0)-4\epsilon)} \qquad (49)$$
$$+ 2^{-n(I(Y_1;X_1|U_0U_2Q)-(R_{11}+R_{12})-4\epsilon)} \qquad (50)$$
$$+ 2^{-n(I(Y_1;X_1|U_1U_2Q)-R_{11}-4\epsilon)} \qquad (51)$$
$$+ 2^{-n(I(Y_1;U_2X_1|U_0U_1Q)-(R_{11}+R_{21})-4\epsilon)} \qquad (52)$$
$$+ 2^{-n(I(Y_1;U_2|U_0X_1Q)-R_{21}-4\epsilon)} \qquad (53)$$
$$+ 2^{-n(I(Y_1;U_2X_1|U_0Q)-(R_{11}+R_{12}+R_{21})-4\epsilon)} \qquad (54)$$

ϵ can be arbitrarily small by letting $n \to \infty$. The conditions (29)-(34) will make sure that $P_{e,1}^{(n)} \to 0$ when $n \to \infty$.

For receiver 2, we consider the event $E_2(i_2j_2j_1k)$ specified by (42). With the similar techniques, the decoding error probability $P_{e,2}^{(n)}$ will vanish on the basis of conditions (35)-(40) and letting $n \to \infty$.   Q.E.D.

From the relation of the modified channel $\mathcal{K}_m$ and the original channel $\mathcal{K}$, we give the following theorem without proof.

Theorem 2: Let $\mathcal{R}(Z), Z \in \mathcal{P}^*$ be the set of all (R_1, R_2, R_0) such that $R_1 = R_{11} + R_{12}, R_2 = R_{22} + R_{21}$ for some $(R_{11}, R_{12}, R_{22}, R_{21}, R_0) \in S(Z)$, then $\mathcal{R}_I^* = \bigcup_{Z \in \mathcal{P}^*} R(Z)$ is achievable for the original channel $\mathcal{K}$.

Now, let us define a subset of $\mathcal{R}_I^*$. Denote by $\mathcal{P}$ the set of all distribution $Z = (Y_1, Y_2, X_1, X_2, U_1, U_2, U_0, Q) \in \mathcal{P}^*$ such that $Q = \phi$, where ϕ is a constant. Define $\mathcal{R}_I = \bigcup_{Z \in \mathcal{P}} \mathcal{R}(Z)$. It is easy to see that $\mathcal{R}_I \subseteq \mathcal{R}_I^*$.

Corollary 1: The achievable region proposed by Tan [1] is a subset of $\mathcal{R}_I$, thus a subset of $\mathcal{R}_I^*$, i.e., $\mathcal{R}_T \subseteq \mathcal{R}_I \subseteq \mathcal{R}_I^*$.

Proof: We can always express any $(R_0, R_1, R_2) \in \mathcal{R}_T(Z)$ as $R_1 = R_{11} + R_{12}, R_2 = R_{22} + R_{21}$ for some $(R_{11}, R_{12}, R_{22}, R_{21}, R_0)$ such that:

$$R_{11} \leq I(Y_1; X_1|U_1U_2) \tag{55}$$

$$R_{22} \leq I(Y_2; X_2|U_1U_2) \tag{56}$$

$$R_{12} \leq a_i, R_{21} \leq b_i, i = 1, 2, 3, 4 \tag{57}$$

$$R_0 + R_{11} + R_{12} + R_{21} \leq I(U_2X_1; Y_1) \tag{58}$$

$$R_0 + R_{22} + R_{21} + R_{12} \leq I(U_1X_2; Y_2) \tag{59}$$

It can be easily seen that conditions(55)-(59) imply conditions (29)-(40) with $Q = \phi$. So $\mathcal{R}_T \subseteq \mathcal{R}_I \subseteq \mathcal{R}_I^*$.

Corollary 2: Under the condition of strong interference in (20)-(21), we have $\mathcal{R}_I^* = \mathcal{R}_I = \mathcal{C}_s$.

Proof: When we apply Fourier-Motzkin Elimination on the inequalities (29)-(40) and then remove those redundant inequalities, we get (60)-(72). Under the condition of strong interference (20)-(21), the original channel becomes a compound MAC channel with common information, i.e., the messages of both senders can be all decoded by each receiver [2]. In this situation, there is no "private message" any more, so

$$X_1 = U_1, \qquad X_2 = U_2. \tag{73}$$

Substituting all U_1, U_2 with X_1, X_2 in (60)-(72) and then remove those newly generated redundant inequalities, we get exact (22).   Q.E.D.

IV. NUMERICAL EXAMPLES IN GAUSSIAN CHANNEL

The standard form of a Gaussian interference channel is as follows:

$$Y_1 = X_1 + a_{21}X_2 + Z_1 \tag{74}$$

$$Y_2 = a_{12}X_1 + X_2 + Z_2 \tag{75}$$

where Z_1, Z_2 are arbitrarily correlated zero mean, unit variance Gaussian random variables. Suppose the power constraints of X_1 and X_2 are P_1 and P_2 respectively.

Although we have the mathematical formula for the achievable region, the computation of it seems formidable because we need to exhaust all kinds of distributions. In order to see the shape of the region and compare $\mathcal{R}_I^*$ and $\mathcal{R}_T$, we have to make some constraints to the model. First, we constrain all the input signals to be Gaussian distributed. Second, we set the time sharing variable $Q = \phi$, i.e., the region we compute is actually $\mathcal{R}_I$ instead of $\mathcal{R}_I^*$. Consider, for certain $\lambda, \bar{\lambda}, \gamma, \bar{\gamma}, \mu, \bar{\mu}, \theta, \bar{\theta} \in [0,1]$, with $\lambda + \bar{\lambda} = 1, \gamma + \bar{\gamma} = 1, \mu + \bar{\mu} = 1, \theta + \bar{\theta} = 1$, and additional auxiliary variables W_1, V_1, W_2, V_2, U_0, the following hold:

$$U_0 \sim N(0, P_0) \tag{76}$$

$$W_1 \sim N(0, \lambda\bar{\theta}P_1) \tag{77}$$

$$V_1 \sim N(0, \bar{\lambda}\bar{\theta}P_1) \tag{78}$$

$$X_1 = W_1 + V_1 + \sqrt{\theta P_1/P_0}U_0 \sim N(0, P_1) \tag{79}$$

$$W_2 \sim N(0, \gamma\bar{\mu}P_2) \tag{80}$$

$$V_2 \sim N(0, \bar{\gamma}\bar{\mu}P_2) \tag{81}$$

$$X_2 = W_2 + V_2 + \sqrt{\mu P_2/P_0}U_0 \sim N(0, P_2) \tag{82}$$

where W_1, W_2 are the private messages, $U_1 = V_1 + \sqrt{\theta P_1/P_0}U_0, U_2 = V_2 + \sqrt{\mu P_2/P_0}U_0$ are common messages. After computing those mutual information formula in (29)-(40) and taking the convex hull of all the power allocation, we have the achievable region $\mathcal{R}_I$ in Fig.2. Since this is a
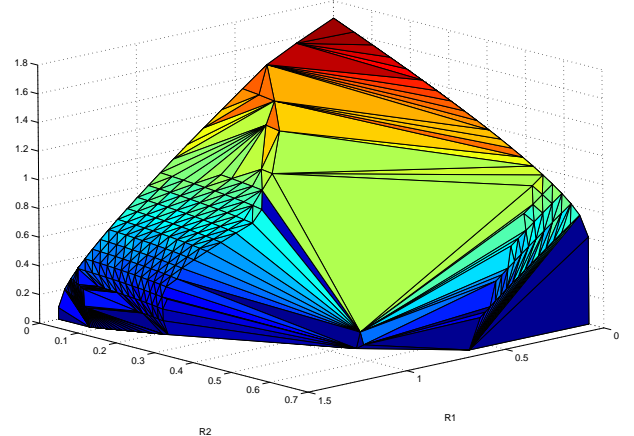

Fig. 2. Achievable rate region for $\mathcal{K}$. $P_1 = 6, P_2 = 1.5, P_0 = 1, a_{12} = a_{21} = 0.74$

3-dimensional region, it is hard to illustrate the comparison of the proposed achievable region $\mathcal{R}_I$ and the previous result $\mathcal{R}_T$. So, we slice the 3D achievable region with planes parallel to (R_1, R_2) plane (i.e., planes with constant R_0) and compare the two regions. The comparison is shown in Fig.3.

Remarks:

1) The 3-dimensional region in Fig.2 agrees with our intuition that it is a convex region.

2) The intersection of the proposed achievable region and the plane with $R_0 = 0$ is actually $\mathcal{G}'$ in Han and Kobayashi's paper [9].

$$R_1 \leq I(Y_1; X_1 | U_2 U_0 Q) \tag{60}$$
$$R_1 \leq I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; U_1 | X_2 U_0 Q) \tag{61}$$
$$R_2 \leq I(Y_2; X_2 | U_1 U_0 Q) \tag{62}$$
$$R_2 \leq I(Y_2; X_2 | U_2 U_1 Q) + I(Y_1; U_2 | X_1 U_0 Q) \tag{63}$$
$$R_1 + R_2 \leq I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_2 | U_0 Q) \tag{64}$$
$$R_1 + R_2 \leq I(Y_1; X_1; U_1 U_2 Q) + I(Y_2; X_2 U_1 | U_0 Q) \tag{65}$$
$$R_1 + R_2 \leq I(Y_1; X_1 U_2 | U_1 U_0 Q) + I(Y_2; X_2 U_1 | U_2 U_0 Q) \tag{66}$$
$$2R_1 + R_2 \leq I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; X_2 U_1 | U_2 U_0 Q) + I(Y_1; X_1 U_2 | U_0 Q) \tag{67}$$
$$R_1 + 2R_2 \leq I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_2 | U_1 U_0 Q) + I(Y_2; X_2 U_1 | U_0 Q) \tag{68}$$
$$R_0 + R_1 + R_2 \leq I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_2 Q) \tag{69}$$
$$R_0 + R_1 + R_2 \leq I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; X_2 U_1 Q) \tag{70}$$
$$R_0 + 2R_1 + R_2 \leq I(Y_1; X_1 | U_1 U_2 Q) + I(Y_2; X_2 U_1 | U_2 U_0 Q) + I(Y_1; X_1 U_2 Q) \tag{71}$$
$$R_0 + R_1 + 2R_2 \leq I(Y_2; X_2 | U_1 U_2 Q) + I(Y_1; X_1 U_2 | U_1 U_0 Q) + I(Y_2; X_2 U_1 | Q) \tag{72}$$
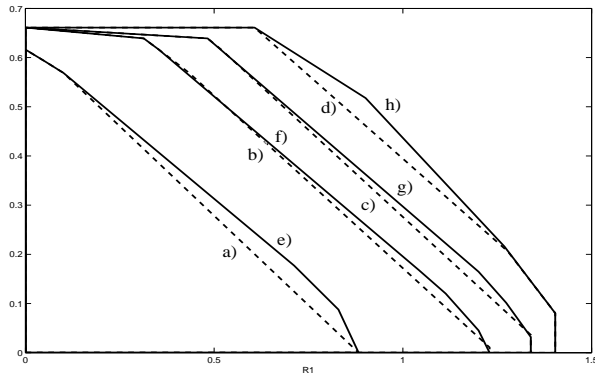

Fig. 3. Comparison of achievable rate region $\mathcal{R}_T$ and $\mathcal{R}_I$. $P_1 = 6, P_2 = 1.5, P_0 = 1, a_{12} = a_{21} = 0.74$. Dashed lines are for $\mathcal{R}_T$ while solid lines are for $\mathcal{R}_I$. a) and e) are for $R_0 = 0.8$; b) and f) are for $R_0 = 0.4$; c) and g) are for $R_0 = 0.2$; d) and h) are for $R_0 = 0$.

3) The comparison in Fig.3 shows that for those parameters $P_1, P_2, P_0, a_{12}, a_{21}$ with the given values, our proposed region $\mathcal{R}_I$ strictly extends $\mathcal{R}_T$. However, our numerical simulation shows that for some other values of the parameters, our achievable region coincides with that of $\mathcal{R}_T$. In fact, the achievable region derived in [1] follows Carleial's idea of sequential superposition coding [8] while our proposed achievable region follows Han and Kobayshi's idea of simultaneous superposition coding [9]. Therefore, our gain in the achievable region is from the superiority of simultaneous superposition coding over sequential superposition coding.

V. CONCLUSION AND DISCUSSION

We proposed a new achievable rate region for the interference channel with common information, which extends the achievable region previously proposed by Tan [1]. Numerical examples in Gaussian case is presented to show for certain values of the parameters, our achievable region can strictly extend $\mathcal{R}_T$. It is also shown that our proposed achievable region coincides with the capacity region under the condition of strong interference.

For future work, one can combine FDMA/TDMA with Han and Kobayashi's simultaneous superposition code, i.e., consider the case when the time-sharing variable $W \neq \phi$. It is conjectured that this combination will yield an even larger achievable region.

REFERENCES

[1] H. H. Tan, "Two-User Interference Channels with Correlated Information Sources," *Information and Control*, vol. 44, pp. 77–104, 1980.

[2] I. Maric, R. D. Yates, and G. Kramer, "The Capacity Region of the Strong Interference Channel With Common Information," in *Asilomar Conference On Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2005.

[3] C. E. Shannon, "Two-way communication channels," in *Proc. 4th Berkeley Symp. on Mathematical Statistics and Probability*, Berkeley, CA, 1961.

[4] R. Ahlswede, "The capacity region of a channel with two senders and two receivers," *Annals Probability*, vol. 2, pp. 805–814, 1974.

[5] A. B. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inform. theory*, vol. 21, pp. 569–570, Sep. 1975.

[6] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inform. theory*, vol. 27, pp. 786–788, Nov. 1981.

[7] M. H. M. Costa and A. A. ElGamal, "The capacity of the Discrete Memoryless interference channel with strong interference," *IEEE Trans. Inform. theory*, vol. 33, pp. 710–711, Sep. 1987.

[8] A.B. Carleial, "Interference Channels," *IEEE Trans. Inform. Theory*, vol. 24, Jan. 1978.

[9] T.Han and K.Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. on Information Theory*, vol. IT-27, pp. 49–60, Jan. 1981.

[10] N.Devroye, P.Mitran, and V.Tarokh, "Achievable Rates in Cognitive Radio Channels," *IEEE Trans. on Information Theory*, vol. 52, pp. 1813–1827, May 2006.

[11] A.Jovicic and P.Vishwanath, "Cognitive Radio: An Information-Theoretic Perspective," *submitted to IEEE Trans. on Information Theory*, 2006.

[12] D. Slepian and J. K. wolf, "A coding theorem for multiple access channels with correlated sources," *Bell Syst. Tech. J.*, vol. 52, pp. 1037–1076, 1973.

[13] F. M. J. Willems, "The discrete memoryless multiple channel with partially cooperating encoders," *IEEE Trans. on Inf. Theory*, vol. 29, pp. 441–445, May 1983.