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EFFECTIVE HADRON DYNAMICS: FROM MESON MASSES TO THE PROTON SPIN PUZZLE

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Abstract

We construct a three flavor chiral Lagrangian of pseudoscalars and vectors with special emphasis on the symmetry breaking terms. Comparing tree level two and three point functions with experiment allows us to first, fix the parameters of the model (including the light quark mass ratios) and second, to predict $m(K^{*+}) - m(K^{*\circ})$, $\Gamma(K^* \rightarrow K\pi)$ and $\Gamma(\phi \rightarrow K\overline{K})$. The last mentioned quantities come out reasonably well, in contrast to an "ordinary" $SU(3)$ treatment. For this purpose we need "second order" symmetry breakers involving the vector fields analogous to those needed for the chiral perturbation theory program with only pseudoscalars. An improved description of the $\eta - \eta'$ system is also given. We then use the soliton sector of this improved chiral Lagrangian to investigate some aspects of baryon physics which are especially sensitive to symmetry breaking. For this purpose a fairly elaborate "cranking" techinque is employed in connection with the collective Hamiltonian. In addition to the "strong" baryon mass spectrum a careful investigation is made of the non-electromagnetic part of the neutron-proton mass difference. This work is needed to improve our previous estimates concerning the two component approach to the "proton spin" puzzle. We find that both the "matter" and "glue" contributions are small but they do tend to cancel each other.

1. *Introduction*

In this paper we first discuss a three flavor chiral Lagrangian of pseudoscalars and vectors with special emphasis on symmetry breaking. The soliton sector is then used to calculate the non-electromagnetic part of the neutron-proton mass difference as well as the nucleon matrix element of the axial singlet current. Our motivation is to update some earlier papers [1,2,3] (which should be consulted for adequate references to background work). However, this updating has turned out to be non-trivial and to yield some results which may be of general interest.

The proton matrix element of the singlet axial vector current has recently attracted a lot of attention because the EMC experiment [4] indicates that it is close to zero at $q^2 = 0$. This is called the "proton spin puzzle" [5,6] since the matrix element should equal unity (twice the proton spin) in the naive non-relativistic quark model. Amusingly, the simplest Skyrme model of pseudoscalars only does predict zero [7]. One might therefore take the point of view that the simple Skyrme model be considered a kind of first approximation to low energy dynamics. We have argued [1] that more is needed because additional "short distance" information is required to adequately explain the neutron-proton mass difference. In fact, the calculations of the $n-p$ mass difference and of the axial proton matrix element are closely linked. Both require (at the dominant two flavor level) the "excitation" of the η meson field; the vanishing of one quantity implies the vanishing of the other. With vector mesons supplying the short distance component it is actually possible to explain [1] the neutron-proton mass difference and obtain [2,8] a suitably small value for the axial singlet matrix element. Another aspect of the proton spin puzzle which this kind of model might illuminate is the so-called "two component decomposition". This is an attempt to make the small value of the axial singlet matrix element plausible from the QCD parton model point of view. The original idea [9] of decomposing the axial singlet current into "matter" and "glue" parts has been criticized [5,10] as not being gauge invariant. However, a way to overcome this objection - by looking instead at a two component Goldberger-Trieman type of relation - has been suggested by Shore and Veneziano [11]. When this mechanism is implemented [3] in the meson Lagrangian it turns out to result, at the soliton level, in an extra contribution to the $n-p$ mass difference proportional to the "glue" component of the axial singlet matrix element. A numerical estimate suggested [3] that the glue component, as well as the matter component, was small.

Now, most of the above work consisted of *calculations* for the two flavor part but merely *estimates* for the effects of including the third flavor. In this paper we discuss the calculations using the full three flavor model throughout. It should be remembered that the $n-p$ mass difference is driven by the symmetry breaking terms in the meson Lagrangian so it is especially important to treat these carefully. The same care is warranted for treatment of symmetry breaking in the collective Hamiltonian which describes the soliton sector of the theory.

The first step of studying symmetry breaking in the meson Lagrangian is discussed in sections 2, 3 and 4. We want to have sufficient symmetry breaking terms to be able to explain at least the pseudoscalar decay constants and masses as well as the mass spectrum of the vectors and the $1^- \rightarrow 0^- 0^-$ decay amplitudes. How to do this for the pseudoscalars is well known. In the chiral perturbation theory (CPT) program [12] a list of symmetry breakers depending on the mass matrix in the underlying theory of QCD gives sufficient flexibility to fit the 0^- decay constants and masses (apart from the $\eta - \eta'$ system for which a special treatment based on the $U(1)_A$ anomaly is required). There are also some loop diagrams resulting in "chiral logs" but these are numerically small for the indicated choice of scale . On the other hand, it has been generally felt that no such elaboration is required for understanding symmetry breaking in the vector multiplet. The vectors are typically considered [13] the most "normal" multiplet. Here we emphasize that this is not really the case. Both the isospin splittings and $1^- \rightarrow 0^- 0^-$ decay amplitudes show large deviations from the simple $SU(3)$ predictions. We find that these *can* be understood if suitable vector symmetry breaking terms which are the analogs of the dominant ones for the pseudoscalars are included. What it boils down to is the use of the symmetry breakers which satisfy Okubo's [14] original form of the quark line rule [14,15], except for the 0^- isosinglet channel. An improved fit for that channel is also discussed here in section 4. In section 7 we make some further remarks related to developing a larger analog of the CPT program which would also include vectors and presumably describe low energy hadron physics up to around 1 GeV.

Returning to the main track of this paper, we describe, in section 5, the calculation of the $n - p$ mass difference using the improved meson Lagrangian. A number of improvements, which have been described in detail elsewhere, are included for treating the $SU(3)$ collective Hamiltonian of the soliton sector. In the first place, the collective Hamiltonian is diagonalized exactly using the Yabu-Ando technique [16]. In addition, "cranking" corrections are included to allow for "centrifugal" effects with the rotating Skyrmions. This means that while the classical soliton involves the π^a, $\omega_\circ$ and $\underline{\rho}^a$ fields, once it starts rotating, fields for $\eta, \eta', K, \overline{K}, \underline{\omega}, \rho_\circ^a, K_\mu^*$ and $\overline{K}_\mu^*$ get "excited" [17-20] with amplitudes proportional to the rotational "angular velocities". This yields improved results for "strong" baryon mass differences, static properties, etc. We find that the model which has been thus improved over that of ref. 1 continues to give a reasonable description of the non-electromagnetic part of the $n - p$ mass difference. What changes is the percentage of this difference due to the excitation of the η and/or η' fields. Previously [1] it was around 70% but is now reduced to about 20% for reasons discussed in section 5. The application of this calculation to determining the "glue" part of the proton's axial singlet matrix element is given in section 6. We continue to find that this contribution is small. However, because there is a smaller η contribution to the $n - p$ mass difference, the accuracy of this statement is actually decreased somewhat.

2. Terms of the Lagrangian

Here we collect together, for convenience, the **a.** chiral invariant, **b.** flavor symmetry breaking but quark line rule conserving and **c.** quark line rule violating terms of the pseudoscalar−vector effective Lagrangian. Some discussion will be given of why we have chosen to include the terms listed, but not others.

The dynamical degrees of freedom are the elements of the 3×3 matrix of the pseudoscalar nonet, ϕ and the 3×3 matrix of the vector nonet ρ_μ. We need the unitary matrices

$$U = e^{2i\phi/F_\pi}, \quad \xi = U^{1/2} = e^{i\phi/F_\pi}, \tag{2.1}$$

where F_π is a *bare* pion decay constant. The vector mesons, which also transform non-linearly under chiral $U(3) \times U(3)$, are related to auxiliary linearly transforming "gauge fields" A^L_μ and A^R_μ by [21]

$$A^L_\mu = \xi\rho_\mu\xi^\dagger + \frac{i}{\tilde{g}}\xi\partial_\mu\xi^\dagger, \quad A^R_\mu = \xi^\dagger\rho_\mu\xi + \frac{i}{\tilde{g}}\xi^\dagger\partial_\mu\xi, \tag{2.2}$$

where $\tilde{g}$ is a *bare* $\rho\phi\phi$ coupling constant.

Note that we have included only the lowest lying s-wave $q\overline{q}$ bound states of QCD as our dynamical fields.

a. *Chiral invariant terms*:[21-23]

These include the kinetic piece for the vectors:

$$-\frac{1}{4}Tr[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)], \quad F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu], \tag{2.3}$$

and pseudoscalar kinetic plus vector interaction pieces:

$$\begin{aligned} & -\frac{m_v^2(1+k)}{8k}Tr(A^L_\mu A^L_\mu + A^R_\mu A^R_\mu) + \frac{m_v^2(1-k)}{4k}Tr(A^L_\mu U A^R_\mu U^\dagger) \\ = & -\frac{1}{2}m_v^2 Tr(\rho_\mu\rho_\mu) - \frac{im_v^2}{2\tilde{g}}Tr[\rho_\mu(\partial_\mu\xi\xi^\dagger + \partial_\mu\xi^\dagger\xi)] - \frac{F_\pi^2}{4}(1+k)Tr(\partial_\mu\xi\partial_\mu\xi^\dagger) \\ & - \frac{F_\pi^2}{4}(1-k)Tr(\xi^\dagger\partial_\mu\xi^\dagger\xi\partial_\mu\xi), \end{aligned} \tag{2.4}$$

where m_v is a *bare* vector meson mass and k is a dimensionless constant chosen as $k = (m_v/F_\pi\tilde{g})^2$ to insure correct normalization of the pseudoscalar kinetic term. Please notice that we are using the "$x_4 = it$" Euclidean-type metric convention.

There are also important interaction terms proportional to the Levi-Civita symbol, $\epsilon_{\mu\nu\alpha\beta}$. To write these compactly it is convenient to use a form notation with one-forms $\alpha = dUU^{-1}$ and A^L and to write the action terms (rather than the Lagrangian density terms):

$$\begin{aligned} \Gamma_{WZ}(U) + \int Tr \quad & [ic_1(A^L\alpha^3) + c_2(dA^L\alpha A^L - A^L\alpha dA^L + A^L\alpha A^L\alpha) \\ + \quad & c_3(-2i(A^L)^3\alpha + \tfrac{1}{\tilde{g}}A^L\alpha A^L\alpha)] \end{aligned} \quad , \tag{2.5}$$

where c_1, c_2, c_3 are constants discussed in [20-23] while $\Gamma_{WZ}(U)$ is the Wess-Zumino-Witten term [24]:

$$\Gamma_{WZ}(U) = \frac{-i}{80\pi^2}\int_{\mathcal{M}^5} Tr(\alpha^5).$$

Eqns (2.3), (2.4) and (2.5) comprise a minimal (though nevertheless complicated in detail) model for pseudoscalars and vectors [25]. One might imagine extending it by systematically adding terms with two more derivatives. This would be analogous to the usual chiral perturbation theory (CPT) program [12] with only pseudoscalars included. It should be remarked, however, that a good deal of the four derivative structure of the pseudoscalar-only effective Lagrangian can be obtained by "integrating out" the vectors of a *minimal*-type pseudoscalar-vector effective Lagrangian.

b. *Symmetry breaking terms*:

In this "updating" of section II of ref. [1] we will include only quark-line rule conserving terms, *i.e.* those which can be written as a *single* trace in flavor space.

The current quark mass terms in the fundamental QCD Lagrangian may be written as

$$\mathcal{L}_{mass} = -\hat{m}\overline{q}\mathcal{M}q, \tag{2.6}$$

where q is the column vector of up, down and strange quark fields, $\hat{m} \equiv (m_u + m_d)/2$ and $\mathcal{M}$ is a dimensionless, diagonal matrix which can be expanded as follows:

$$\mathcal{M} = y\lambda_3 + T + xS, \tag{2.7}$$

with $\lambda_3 = diag(1, -1, 0)$, $T = diag(1, 1, 0)$ and $S = diag(0, 0, 1)$. x and y are the quark mass ratios

$$x = \frac{m_s}{\hat{m}}, \quad y = -\frac{1}{2}\left(\frac{m_d - m_u}{\hat{m}}\right). \tag{2.8}$$

It is also convenient to define [13]

$$R = \frac{m_s - \hat{m}}{m_d - m_u} = \frac{1 - x}{2y}. \tag{2.9}$$

The significance of R is that it can, in principle, also be determined from the rather well-known masses of the $\frac{1}{2}^+$ baryon octet. We have introduced the redundant notations above because of the importance of $\mathcal{M}$ in (2.7), to which all the symmetry breakers are taken to be proportional. We then write for the quark-line-rule conserving but flavor symmetry breaking terms in the effective Lagrangian:

$$\begin{aligned} Tr(\mathcal{M}[&\alpha'(A^L_\mu U A^R_\mu + A^R_\mu U^\dagger A^L_\mu) + \beta'(\partial_\mu U \partial_\mu U^\dagger U + U^\dagger \partial_\mu U \partial_\mu U^\dagger) \\ &+\gamma'(F^L_{\mu\nu} U F^R_{\mu\nu} + F^R_{\mu\nu} U^\dagger F^L_{\mu\nu}) + \delta'(U + U^\dagger - 2)] \\ &+\lambda'^2[\mathcal{M}U^\dagger\mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U - 2\mathcal{M}^2]), \end{aligned} \tag{2.10}$$

where $F^{L,R}_{\mu\nu} = \partial_\mu A^{L,R}_\nu - i\tilde{g}[A^{L,R}_\mu, A^{L,R}_\nu]$ and $\alpha', \beta', \gamma', \delta', \lambda'$ are the symmetry breaking coefficients. The parameters α'', β'' etc. introduced in ref. [1] may be related easily to α', β' etc. via:

$$x = \alpha''/\alpha' = \beta''/\beta' = etc., \quad y = \alpha/\alpha' = \beta/\beta' = etc. \tag{2.11}$$

Our motivations for including the terms shown in (2.10) are as follows. First, the δ' term is the standard one which splits the pseudoscalar masses. Next, the α' term is the one with the correct chiral properties which splits the vector masses. However, as pointed out in ref. [1], the α' term also gives objectionably large derivative-type symmetry breaking for the pseudoscalars. This is partially canceled by the pure pseudoscalar derivative symmetry breaker given by the β' term. The γ' term yields a derivative-type symmetry breaker for the vectors too. Note that the β' term is the #5 type in the CPT classification [12]. The λ'^2 term, which involves two powers of $\mathcal{M}$, is the #8 type. It is argued to be the same order as the β' term. Furthermore, it is the only remaining manifestly quark-line-rule conserving symmetry breaker to "second order". From a practical point of view the

λ'^2 term also enables us, as we shall see, to vary x in (2.8) while keeping the pseudoscalar masses and decay constants fixed at their measured values. In the CPT program, the determination of λ'^2 is subject to the Kaplan-Manohar ambiguity [26]; we shall discuss this point later.

c. *Quark-line-rule violating and symmetry breaking terms*:

One of the remarkable features of low energy dynamics is the success of the OZI or quark-line rule [14,15]. There is however a conspicuous exception associated with the interactions of the pseudoscalar singlet particles $(\eta, \eta'...)$; the physics of this is denoted as the "$U(1)$ problem". In a rough way, this picture is reflected in the CPT fit of ref. [12]. There the OZI rule violating terms of #4 type ($\sim Tr(\partial_\mu U \partial_\mu U^\dagger)\, Tr(\mathcal{M}(U+U^\dagger))$) and of #6 type ($\sim [Tr(\mathcal{M}(U+U^\dagger))]^2$) are both claimed to be negligible. On the other hand the #7 type OZI rule violating symmetry breaker ($\sim [Tr(\mathcal{M}(U-U^\dagger))]^2$) is claimed to be non-negligible. We shall see that the latter is naturally associated with pseudoscalar singlet particle interactions. Incidentally note that we have now accounted for all the symmetry breaking terms involving only pseudoscalars which appear in the second order CPT list. To these we have added the α' and γ' terms in (2.10), which involve the vectors.

Here, we shall write down essentially just those OZI rule violating terms needed to fit the η and η' masses and two photon decay widths in an effective chiral Lagrangian framework. In order that the $U(1)$ anomaly equation for the axial singlet current, $\partial_\mu J^5_\mu = G$, with $G = \partial_\mu K_\mu$, $K_\mu = \frac{-3ig^2_{QCD}}{4\pi^2}\epsilon_{\mu\beta\gamma\delta} Tr(A_\beta \partial_\gamma A_\delta - \frac{2ig_{QCD}}{3} A_\beta A_\gamma A_\delta)$ in terms of the QCD gauge fields, be obeyed in the massless limit it is convenient to introduce G as an "auxiliary" field in the effective Lagrangian. No kinetic term is to be written for G so it gets eliminated by its equation of motion. This procedure might be considered as an effective way of integrating over instanton field configurations. We thus add the following terms to our effective Lagrangian:

$$\begin{aligned}\frac{1}{\kappa}G^2 \;\; &+ \;\; \frac{i}{12}G\,\ln(\det U/\det U^\dagger) + n\,Tr(\alpha_\mu)Tr(\alpha_\mu) + iG\,\epsilon'\,Tr[\mathcal{M}(U-U^\dagger)] \\ &+ \;\; iG\,\zeta'\,Tr[\mathcal{M}(A^L_\mu U A^R_\mu - A^R_\mu U^\dagger A^L_\mu)],\end{aligned} \tag{2.12}$$

where κ, n, ϵ' and ζ' are parameters. The first two terms in (2.12) are the standard ones for mocking up the $U(1)$ anomaly and giving mass to the η' meson [27]. The third term, which gives us the freedom to adjust the η' "decay constant", and the fourth term which is a symmetry breaker of "instanton-induced" type, have been mentioned before [28]. The fifth term is an "instanton-induced" symmetry breaker involving vectors; since it leads only to four and higher point interactions we shall neglect it for our present purposes by setting $\zeta' = 0$. Now G appears only in (2.12). It therefore satisfies the equation of motion

$$G = \frac{\kappa}{4}\chi - \frac{i\kappa\,\epsilon'}{2}Tr[\mathcal{M}(U-U^\dagger)], \tag{2.13}$$

where the mathematical ($SU(3)$ singlet) η' field is defined by $\eta' = \frac{\sqrt{3}}{2}F_\pi \chi$, χ being gotten from the decomposition $U = e^{i\chi}\tilde{U}$ with $\det\tilde{U} = 1$. G in (2.13) should be substituted into (2.12). Among other things it gives a term of #7 type in the CPT list. We have no special reason, on the other hand, to include a term of #6 type.

An additional term involving G will be considered later in connection with an application to the "proton spin" puzzle.

d. *Remarks*:

The total effective action of pseudoscalars and vectors is taken to be

$$\Gamma_{eff} = \int [\,(2.3) + (2.4) + (2.10) + (2.12)]d^4x + (2.5). \tag{2.14}$$

Symmetry breaking terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ are being neglected here for simplicity (they do not contribute to most of the processes which will be discussed in the present paper). Similarly, OZI rule violation for particles other than the pseudoscalars will mostly be neglected.

3. Physical quantities at the OZI rule conserving level

We will find the parameters of the effective Lagrangian by comparing the tree level 2 and 3 point functions with experiment. There is a natural separation into the properties of the pseudoscalar singlets (which we discuss in the next section) and the other particles whose properties follow just from the OZI rule conserving terms.

Compared to the discussion in Sec. II of ref. [1], there is the difference that the λ' term is now included and also the γ' term is not set to zero. The latter results in a very substantial improvement in the predictions for the properties of the $K^\star$ mesons and gives a different fit for the fundamental mass ratios x and y of the light quarks.

First, expanding out the kinetic terms shows we should renormalize the fields as (taking typical examples):

$$\begin{aligned} \pi^+ &= Z_\pi \phi_{12}, \qquad & K^+ &= Z_K \phi_{13}, \qquad & \rho^+_\mu &= Z_\rho\, \rho_{12\mu}, \\ K^{\star +}_\mu &= Z_{K^\star}\, \rho_{13\mu}, \quad & \omega_\mu &= Z_\omega (\rho_{11\mu} + \rho_{22\mu})/\sqrt{2}, \quad & \phi_\mu &= Z_\phi\, \rho_{33\mu}, \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} Z_\pi &= [1 + \frac{4}{F_\pi^2}(\frac{\alpha'}{\tilde{g}^2} - 4\beta')]^{1/2}, \quad Z_K = [1 + \tfrac{2}{F_\pi^2}(1+x)(\tfrac{\alpha'}{\tilde{g}^2} - 4\beta')]^{1/2}, \\ Z_\rho &= Z_\omega = (1 - 8\gamma')^{1/2} \qquad , \qquad Z_K^\star = [1 - 4\gamma'(1+x)]^{1/2}, \\ Z_\phi &= (1 - 8x\gamma')^{1/2}. \end{aligned} \tag{3.2}$$

Next, the computation of the "Noether" currents of this model leads to the identification of the physical pion and kaon decay constants $F_{\pi p}$ and F_{Kp} as:

$$F_{\pi p} = Z_\pi F_\pi, \quad F_{Kp} = Z_K F_\pi\,. \tag{3.3}$$

Expanding out the pseudoscalar mass terms yields, in the isospin limit,

$$m_\pi^2 = \frac{8}{F_{\pi p}^2}(\delta' + 4\lambda'^2), \quad m_K^2 = \frac{4}{F_{Kp}^2}[\delta'(1+x) + 2\lambda'^2(1+x)^2]. \tag{3.4}$$

Note that, as pointed out in Sec.2, if λ'^2 were not present the two equations in (3.4) would lead to a fixed value for $x = m_s/\hat{m}$. However it turns out to be very useful to explore the physical situation for various values of x. The vector meson masses in the isospin limit are:

$$\begin{aligned} m_\rho^2 &= m_\omega^2 = (m_v^2 - 2\alpha')/Z_\rho^2, \quad m_{K^\star}^2 = [m_v^2 - 2\alpha'(1+x)]/Z_{K^\star}^2, \\ m_\phi^2 &= (m_v^2 - 4x\alpha')/Z_\phi^2 \qquad . \end{aligned} \tag{3.5}$$

Next, let us turn to the mass splittings between members of the same iso-multiplet. We are not computing the photon-exchange contributions here so the prediction will be for the *non-electromagnetic* part of the iso-splittings. We then find [1]

$$[m(K^\circ) - m(K^+)]_{non-EM} = \frac{y}{F_{Kp}^2 m_K}[2m_K^2(\frac{\alpha'}{\tilde{g}^2} - 4\beta') - 4\delta' - 16\lambda'^2(1+x)], \quad (3.6)$$

$$[m(K^{\circ *}) - m(K^{+*})]_{non-EM} = y(-4\gamma' m_{K^*}^2 + 2\alpha')/m_{K^*} Z_{K^*}^2, \quad (3.7)$$

$$[M_{\rho\omega}]_{non-EM} = -y(2\alpha' - 4\gamma' m_\rho^2)/m_\rho Z_\rho^2. \quad (3.8)$$

(The $\rho^\circ\!-\!\omega$ transition mass, $M_{\rho\omega}$ is defined in terms of the effective term in the Lagrangian: $-2m_\rho M_{\rho\omega}\rho_\mu^\circ\omega_\mu$).

Finally, consider the $\rho\phi\phi$ coupling constants. The $\rho\pi\pi$ coupling constant, $g_{\rho\pi\pi}$ may be defined in terms of the $\rho \to 2\pi$ width, $\Gamma(\rho \to 2\pi)$ as

$$\Gamma(\rho \to 2\pi) = \frac{g_{\rho\pi\pi}^2\, |\underline{q}(\rho)|^3}{12\pi m_\rho^2}, \quad (3.9)$$

where $\underline{q}(\rho)$ is the momentum of the final pion in the ρ rest frame. In our model,

$$g_{\rho\pi\pi} = \frac{m_v^2 + 4\alpha'}{\tilde{g} F_{\pi p}^2 Z_\rho}. \quad (3.10)$$

For the other vector decays into two pseudoscalars we find

$$\begin{aligned}
\frac{\Gamma(K^\star \to K\pi)}{\Gamma(\rho \to 2\pi)} &= \frac{3}{4}\left(\frac{m_\rho}{m_{K^\star}}\frac{F_{\pi p}}{F_{Kp}}\frac{Z_\rho}{Z_{K^\star}}\right)^2 \frac{|\underline{q}(K^\star)|^3}{|\underline{q}(\rho)|^3}, \\
\frac{\Gamma(\phi \to K\overline{K})}{\Gamma(\rho \to 2\pi)} &= \left(\frac{m_\rho}{m_\phi}\left(\frac{F_{\pi p}}{F_{Kp}}\right)^2 \frac{Z_\rho}{Z_\phi}\right)^2 \frac{|\underline{q}(\phi)|^3}{|\underline{q}(\rho)|^3},
\end{aligned} \quad (3.11)$$

where $\underline{q}(K^\star)$, for example, denotes the *pseudoscalar* momentum in the $K^\star$ rest frame. Also, the small OZI rule violation for ϕ decays was neglected in the second part of (3.11), as well as in the mass formula (3.5).

Now let us consider the experimental determination of the various quantities introduced. This is actually the heart of the matter and contains several very interesting qualitative features. In the first place this determination yields estimates of the fundamental quark mass ratios $x = m_s/\hat{m}$ and $y = (m_u - m_d)/(m_u + m_d)$ in our framework. The dependent quantity $R = (1-x)/2y$ is also relevant since it can be separately determined from consideration of other particle multiplets (like the baryon octet). For comparison we mention the results obtained in earlier work in this model [1] in which $\gamma' = \lambda'^2 = 0$ and wherein $M_{\rho\omega}$ was the only experimental isospin violating quantity used for the fit:

$$x = 37, \quad y = -0.36, \quad R = 50\,. \quad (3.12)$$

[1]Only 3.8 was used for fitting in ref. [1]. The extra factor Z_ϕ^{-2} makes negligible difference. Note that the $\lambda'^2 \to 0$ limit of (3.6) quoted, but not used for fitting, should be replaced by the present formula.

Let us also compare with the determination of Gasser and Leutwyler[13]:

$$x = 25.0 \pm 2.5, \quad y = -0.28 \pm 0.03, \quad R = 43.5 \pm 2.2\,. \tag{3.13}$$

A recent alternate approach to the CPT program by Donoghue and Wyler [29] predicts instead

$$y = -0.54 \pm 0.09\,. \tag{3.14}$$

Those authors do not have a similarly precise estimate of x but consider $R = 32 \pm 5$ and hence $x \approx 36$ to be more acceptable. In the present paper we find a "best fit" for

$$x = 31.5, \quad y = -0.42, \quad R = 36\,. \tag{3.15}$$

This is closer to (3.14) than to (3.13).

The main qualitative difference between our result (3.15) and (3.13) can be roughly understood in the following way. Gasser and Leutwyler consider the vector meson nonet to be the most "normal" one in the sense that old fashioned $SU(3)$ relations can be best trusted. Their value of R, for example, is essentially obtained from the vectors. However we find very important $SU(3)$ violations for the vector nonet system. They did note that the non-electromagnetic part of $[m(K^{\circ *}) - m(K^{+*})]$, which is related to $M_{\rho\omega}$ (their basic isospin violation input) by $SU(3)$, could not be adequately explained but attributed this to uncertainties in interpreting the experimental results. Here, we find that the experimental value for this mass splitting can be fairly reasonably explained if one allows non-trivial wave function renormalization for the K^*_μ and ϕ_μ fields by keeping $\gamma' \neq 0$. This also allows us to dramatically improve the predictions for the widths $\Gamma(K^*)/\Gamma(\rho)$ and $\Gamma(\phi)/\Gamma(\rho)$ in (3.11). It is amusing to note that when $SU(3)$ was first proposed, the symmetry prediction for $\Gamma(K^*)/\Gamma(\rho)$ worked very well. But since then, the measured width of the ρ has increased from about 100 MeV to about 150 MeV. The relatively large wave function renormalization for the K^* tends to restore the agreement between theory and experiment.

It is very easy to fit the symmetry breaking parameters $x, y, \alpha', \beta', \gamma', \delta', \lambda'^2$ as well as the "symmetric" parameters $\tilde{g}$ and m_v^2 to experiment [2] if one temporarily holds x fixed and computes everything else. First, feeding the known values for $F_{\pi p}, F_{Kp}, m_\pi$ and m_K into (3.4) gives δ' and λ'^2. From $F_{Kp}/F_{\pi p} = Z_k/Z_\pi$ we next find the quantity $(\frac{\alpha'}{\tilde{g}^2} - 4\beta')$ from (3.2). Equation (3.6) then yields y while the formulas for m_ρ and m_{k^*} in (3.5) together with (3.8) give us α', γ' and m_v. Next the ρ width via (3.9) and (3.10) gives $\tilde{g}$ and finally β' is found from $(\frac{\alpha'}{\tilde{g}^2} - 4\beta')$. The fitted parameters as functions of x are displayed in Table 3.1.

We note that the strength of the standard non-derivative pseudoscalar symmetry breaker, δ', does not change much with x. The most dramatic effect is the increasing importance of γ' which, from (3.2), is seen to lead to an important wave function renormalization for the K^* and ϕ vector masses. When $x = 25$, the values of x and y agree with the Gasser-Leutwyler values in (3.13). As x increases it is seen that the strengths $|\beta'|$ and $|\lambda'|$ decrease substantially. For our fit in (3.15), α' has also decreased in magnitude

[2]Input parameters: $F_{\pi p} = 0.132\,\mathrm{GeV}, F_{Kp} = 0.161\,\mathrm{GeV}, m_\pi = 0.137\,\mathrm{GeV}, m_{K^*} = 0.892\,\mathrm{GeV}, m_\rho = 0.768\,\mathrm{GeV}, m_K = 0.497\,\mathrm{GeV}, (m_{K^\circ} - m_{K^+})_{non-EM} = 5.28\,\mathrm{MeV}, M_{\rho\omega} = -2.65\,\mathrm{MeV}, \Gamma(\rho \to 2\pi) = 0.1491\,\mathrm{GeV}, \Gamma(K^* \to K\pi) = 0.0498\,\mathrm{GeV}, \Gamma(\phi \to K\overline{K}) = 0.0037\,\mathrm{GeV}$.

x	y	R	$\alpha'(\mathrm{GeV}^2)$	$\beta'(\mathrm{GeV}^2)$	γ'	$\delta'(\mathrm{GeV}^4)$	$\|\lambda'\|(\mathrm{GeV}^2)$	$\tilde{g}$
15	-0.13	53.5	-9.08×10^{-3}	-2.59×10^{-4}	-1.05×10^{-3}	3.24×10^{-5}	1.45×10^{-3}	3.53
20	-0.19	48.9	-4.71×10^{-3}	-1.42×10^{-4}	4.32×10^{-4}	3.72×10^{-5}	9.64×10^{-4}	3.72
25	-0.27	43.8	-2.03×10^{-3}	-7.91×10^{-5}	1.39×10^{-3}	3.92×10^{-5}	6.56×10^{-4}	3.83
28	-0.33	40.5	-8.72×10^{-4}	-5.36×10^{-5}	1.81×10^{-3}	3.98×10^{-5}	5.15×10^{-4}	3.88
30	-0.38	38.3	-2.25×10^{-4}	-4.05×10^{-5}	2.05×10^{-3}	4.01×10^{-5}	4.31×10^{-4}	3.91
32	-0.43	36.1	3.43×10^{-4}	-2.89×10^{-5}	2.26×10^{-3}	4.04×10^{-5}	3.51×10^{-4}	3.93
34	-0.49	33.9	8.46×10^{-4}	-1.88×10^{-5}	2.45×10^{-3}	4.06×10^{-5}	2.71×10^{-4}	3.95
36	-0.55	31.6	1.29×10^{-3}	-9.98×10^{-6}	2.62×10^{-3}	4.07×10^{-5}	1.84×10^{-4}	3.97
38	-0.63	29.3	1.69×10^{-3}	-2.19×10^{-6}	2.77×10^{-3}	4.09×10^{-5}	4.71×10^{-4}	3.99

Table 3.1: Parameters as functions of x

x	$(K^{\circ *} - K^{+*})_{non-EM}$	$\Gamma(\rho)/\Gamma(K^*)$	$\Gamma(\rho)/\Gamma(\phi)$	$m(\phi)$
15	2.04 MeV	5.44	124	1.01 GeV
20	2.44 MeV	4.97	103	1.02 GeV
25	3.05 MeV	4.45	81	1.04 GeV
28	3.55 MeV	4.12	67	1.07 GeV
30	3.97 MeV	3.90	57	1.09 GeV
32	4.47 MeV	3.67	47	1.13 GeV
34	5.09 MeV	3.44	38	1.18 GeV
36	5.84 MeV	3.21	28	1.26 GeV
38	6.78 MeV	2.98	18	1.43 GeV

Table 3.2: Predictions

and has reversed sign. Clearly δ' and γ' are the dominant symmetry breaking parameters for our fit. The value of the "gauge" coupling constant $\tilde{g}$ does not change much with x.

We now have four predictions which are given in Table 3.2: $[m(K^{\circ *})-m(K^{+*})]_{non-EM}$, $\Gamma(K^* \to K\pi)/\Gamma(\rho \to 2\pi)$, m_ϕ and $\Gamma(\phi \to K\overline{K})/\Gamma(\rho \to 2\pi)$. These quantities are sensitive to the wave function renormalizations of the vector particles containing strange quarks (K^* and ϕ). First consider the $K^{\circ *} - K^{+*}$ mass difference. The photon exchange contribution has been estimated to be -0.7 MeV [30]. The experimental value is alternately given as 4.5$\pm$0.4 MeV (if one simply subtracts the two numbers in the Review of Particle Properties [31]) or as 6.7$\pm$1.2 MeV (if one considers just the "dedicated" experiments). So we should have,

$$[m(K^{\circ *}) - m(K^{+*})]_{non-EM} \approx \begin{cases} 5.2\,\mathrm{MeV} \\ 7.4\,\mathrm{MeV} \end{cases}. \tag{3.16}$$

This should be equal (see the limiting forms of (3.7) and (3.8)) in the $SU(3)$ limit to the negative of the non-electromagnetic piece of the $\rho - \omega$ transition mass $M_{\rho\omega}(non - EM)$, which in turn has been found from analysis of experiment by Gasser and Leutwyler [13] to be $-2.65$$\pm$0.20 MeV. Clearly the $SU(3)$ prediction is very bad. However, agreement can be obtained in the present model if $Z_{K^*} \neq 1$ in (3.7). We see from table 3.2 that the range of x between 34 and 38 is suitable for this purpose. This is due to the increase of γ' with increasing x. Next, consider the ratio of widths $\Gamma(\rho \to 2\pi)/\Gamma(K^* \to K\pi)$ which experimentally is 3.0. Eq (3.11) shows that this ratio is also sensitive to Z_{K^*}. We see from table 3.2 that, once again, exact agreement is obtained for x around 38. Thus considering just the properties of the K^* particle one would be tempted to choose x =38. This represents (see table 3.1) a value of y significantly larger than (3.13) in magnitude but in agreement with (3.14). Larger values of x and y (compared to (3.13)) are also favored by consideration of the ratio $\Gamma(\rho \to 2\pi)/\Gamma(\phi \to K\overline{K})$ which is experimentally 40.3. We see from table 3.2 that best agreement for this ratio is obtained for x around 34. On the other hand m_ϕ is fit best with a smaller value of x (about 20). While it is true that the simplification of neglecting $\omega - \phi$ mixing has been made above, this effect is small enough so the m_ϕ prediction should be reasonably accurate. A compromise "best fit" with $x \approx$32 (see (3.15)) improves three predictions remarkably, does not distort the m_ϕ prediction too badly and will be seen in the next section to be needed to lead to a good description of the $\eta - \eta'$ system.

It is of interest to note that the vector meson wave function renormalizations for the best fit x=32 case are $Z_\omega = Z_\rho = 0.99$, $Z_{K^*} = 0.84$ and $Z_\phi = 0.65$. Since Z_ϕ, in particular [3], represents a rather non-trivial correction one might in the future want to investigate other possible higher order terms involving vectors and loop diagrams. In any event, our analysis shows that such large symmetry breaking is required to understand the vector mesons which contain strange quarks.

We remark here that the additional symmetry breaking term:

$$\mu' Tr(A^L_\nu \mathcal{M} A^R_\nu \mathcal{M}) \tag{3.17}$$

can help to fine tune our results. Because it is quadratic in $\mathcal{M}$ it will provide a factor of x^2 for $m^2(\phi)$ and negligible contributions to other vector masses. Similarly, it will mainly affect the $\phi_\mu \to K\overline{K}$ decay amplitude. There will be an extra contribution of $-2x^2\mu'/Z_\phi^2$ to $m^2(\phi)$ and an extra factor of $1 + \frac{4\mu'(x+x^2/2)}{m_v^2+4\alpha'}$ for the $\phi \to K\overline{K}$ amplitude. The choice $\mu' = 2.4 \times 10^{-5}$ GeV^2 then enables us to fit both $m^2(\phi)$ and $\Gamma(\rho)/\Gamma(\phi)$ in Table 3.2 while still keeping $x = 31.5$. Notice that (3.17) is the analog for the vectors of the λ'^2 term in (2.10) for the pseudoscalars; there are then three analogous quark-line-rule conserving but symmetry breaking terms for each multiplet.

4. Physical quantities for the $\eta - \eta'$ system.

In the simplest Lagrangian which can mock up the $U(1)$ anomaly (just the first two terms of (2.12) included) the η' mass and the $\eta - \eta'$ mixing can be satisfactorily fit. However the η mass comes out too low [4]. Here we will show that this problem can be remedied if the second two terms of (2.12) as well as the OZI rule conserving symmetry breakers in (2.10) are all taken into account.

Defining the 2-dimensional vector $\eta = \begin{pmatrix} \eta_T \\ \eta_S \end{pmatrix}$ where $\eta_T = (\phi_{11} + \phi_{22})/\sqrt{2}$, $\eta_S = \phi_{33}$, we collect the quadratic terms in the effective Lagrangian as

$$-\frac{1}{2}\partial_\mu \eta^\top K \partial_\mu \eta - \frac{1}{2}\eta^\top P \eta. \tag{4.1}$$

[3]In the extraction of the parameters c_1 and c_2 of (2.5) from experiment in [22] some ϕ decays are involved. We note that in (4.5) of this reference we should now write $\left|\epsilon/Z_\phi\right|$ instead of $|\epsilon|$. This replacement should also be made in (4.8) so that the old formulas for c_1 and c_2 remain unaltered.

[4]This can be seen by referring to Fig 2 of [32].

Here the "kinetic" matrix is

$$\begin{aligned} K_{TT} &= 1+\frac{4}{F_\pi^2}(\frac{\alpha'}{\tilde{g}^2}-4\beta')+\frac{16\,n}{F_\pi^2}, \\ K_{SS} &= 1+\frac{4x}{F_\pi^2}(\frac{\alpha'}{\tilde{g}^2}-4\beta')+\frac{8\,n}{F_\pi^2}, \\ K_{TS} &= K_{ST}=\frac{8\sqrt{2}\,n}{F_\pi^2}, \end{aligned} \tag{4.2}$$

while the "potential" matrix is

$$\begin{aligned} P_{TT} &= \frac{1}{F_\pi^2}[8\delta'+\frac{8}{3}\kappa\epsilon'+\frac{\kappa}{9}+16\kappa\epsilon'^2+32\lambda'^2(1+y^2)], \\ P_{SS} &= \frac{1}{F_\pi^2}[8x\delta'+\frac{4}{3}x\kappa\epsilon'+\frac{\kappa}{18}+8\kappa x^2\epsilon'^2+32x^2\lambda'^2], \\ P_{TS} &= P_{ST}=\frac{\sqrt{2}}{F_\pi^2}\,[\frac{2}{3}\kappa\epsilon'(1+x)+\frac{\kappa}{18}+8\kappa x\epsilon'^2]. \end{aligned} \tag{4.3}$$

The "kinetic" matrix is diagonalized by

$$R^{-1}(\theta_1)K\,R(\theta_1)=\hat{K}=\begin{pmatrix}\hat{K}_1 & 0\\ 0 & \hat{K}_2\end{pmatrix}, \tag{4.4}$$

where

$$R(\theta_i)=\begin{pmatrix}\cos\theta_i & \sin\theta_i\\ -\sin\theta_i & \cos\theta_i\end{pmatrix}, \tag{4.5}$$

and $\hat{K}_{1,2}=\frac{1}{2}[(K_{TT}+K_{SS})\mp\sqrt{(K_{TT}-K_{SS})^2+4K_{TS}^2}\,]$. The angle θ_1 is obtained from

$$\tan 2\theta_1=\frac{2K_{TS}}{K_{SS}-K_{TT}}. \tag{4.6}$$

The *effective* potential matrix P' is then

$$P'=\hat{K}^{-1/2}R^{-1}(\theta_1)P\,R(\theta_1)\hat{K}^{-1/2}. \tag{4.7}$$

This yields the *physical* η and η' squared masses as the eigenvalues of (4.7):

$$m^2(\eta,\eta')=\frac{1}{2}[(P'_{TT}+P'_{SS})\mp\sqrt{(P'_{TT}-P'_{SS})^2+4P'^2_{TS}}]. \tag{4.8}$$

The "bare" column vector is finally expanded in terms of the *physical* fields $\eta_p=\begin{pmatrix}\eta\\ \eta'\end{pmatrix}$ as

$$\eta=R(\theta_1)\hat{K}^{-1/2}R(\theta_2)\eta_p, \tag{4.9}$$

where θ_1 is obtained from (4.6) while θ_2 is determined from

$$\tan\,2\theta_2=\frac{2P'_{TS}}{P'_{SS}-P'_{TT}}. \tag{4.10}$$

Note that the transformation (4.9) is, in general, non-orthogonal. For comparison, the mixing convention in the octet-singlet basis is

$$\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = R(\theta) \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \tag{4.11}$$

and, to the extent that $\hat{K}$ may be approximated by the unit matrix,

$$\theta \approx \theta_1 + \theta_2 - 54.7^\circ. \tag{4.12}$$

The standard source of information, which we shall also employ here, about mixing in the $\eta - \eta'$ system is comparison of the $\pi^\circ \to 2\gamma$, $\eta' \to 2\gamma$ and $\eta \to 2\gamma$ decays. It is well known that $\pi^\circ \to 2\gamma$ is well-described by "gauging" the first term of (2.5). Thus it is sufficient to consider ratios of the other rates to the π° rate. We find

$$\frac{\Gamma(\pi^\circ \to 2\gamma)}{m_\pi^3} : \frac{\Gamma(\eta \to 2\gamma)}{m_\eta^3} : \frac{\Gamma(\eta' \to 2\gamma)}{m_{\eta'}^3} = \frac{9}{2} : (a_1)^2 : (a_2)^2, \tag{4.13}$$

wherein

$$\begin{aligned} a_1 &= \frac{5}{\sqrt{2}} \left(\frac{\cos\theta_1 \, \cos\theta_2}{\hat{K}_1^{1/2}} - \frac{\sin\theta_1 \, \sin\theta_2}{\hat{K}_2^{1/2}} \right) - \left(\frac{\sin\theta_1 \, \cos\theta_2}{\hat{K}_1^{1/2}} + \frac{\sin\theta_2 \, \cos\theta_1}{\hat{K}_2^{1/2}} \right) \\ a_2 &= \frac{5}{\sqrt{2}} \left(\frac{\cos\theta_1 \, \sin\theta_2}{\hat{K}_1^{1/2}} + \frac{\cos\theta_2 \, \sin\theta_1}{\hat{K}_2^{1/2}} \right) + \left(\frac{\cos\theta_1 \, \cos\theta_2}{\hat{K}_2^{1/2}} - \frac{\sin\theta_1 \, \sin\theta_2}{\hat{K}_1^{1/2}} \right). \end{aligned} \tag{4.14}$$

Experimentally, $(a_1)^2 = 3.98 \pm 0.60$ and $(a_2)^2 = 7.20 \pm 1.40$.

We may now try fit our parameters to these experimental data. Adopting the compromise fit discussed in Sec.3 with $x = 31.5$, we have three new unknown parameters: κ, ϵ' and n (see (2.12)). On the other hand there are four experimental quantities to be fit: $m_\eta, m_{\eta'}$, and the $\eta - \eta'$ mixing information contained in $(a_1)^2$ and $(a_2)^2$ of (4.13). Conducting a numerical scan of the κ, ϵ' and n parameter space we find a best fit for

$$\kappa = 0.105 \, \mathrm{GeV}^4, \quad n = 1.40 \times 10^{-4} \, \mathrm{GeV}^2, \quad \epsilon' = 1.59 \times 10^{-4}. \tag{4.15}$$

With these parameters the physical quantities are

$$m(\eta) = 549 \, \mathrm{MeV}, \; m(\eta') = 959 \, \mathrm{MeV}, \; (a_1)^2 = 3.98, \; (a_2)^2 = 7.19, \tag{4.16}$$

in agreement with experiment. Furthermore, the quantities describing the non-orthogonal $\eta - \eta'$ matrix in (4.9) are

$$\theta_1 = 7.44^\circ, \; \theta_2 = 34.7^\circ, \; \hat{K}_1^{1/2} = 1.07, \; \hat{K}_2^{1/2} = 1.36. \tag{4.17}$$

This would correspond, via (4.12), to a usual $\eta - \eta'$ mixing angle of about -12.6°. However, because $\hat{K} \neq 1$ this comparison is just suggestive.

We have found that it is difficult to achieve a fit for the experimental quantities in the $\eta - \eta'$ system for x significantly different from the value used above. It is also amusing to note that the λ'^2 symmetry breaker in (2.10) plays an important role in enabling us to get a good fit in the first place. Without the λ'^2 term present the only reasonable fit was found with very large and negative ϵ' which would give the wrong sign for the neutron-proton mass difference, as computed in this model.

5. The neutron proton mass difference

In the preceding sections we discussed the effective meson action with emphasis on the symmetry breaking parts, especially those which are isospin non-invariant. In this section we will use this effective mesonic model to obtain information about properties of baryons. In mesonic theories like the one under consideration baryons emerge as soliton solutions.

In order to explicitly test the isospin non-invariant terms we will evaluate the non-electromagnetic contribution to the neutron proton mass difference:

$$\Delta = (M_{\rm neutron} - M_{\rm proton})_{\rm non-EM}. \tag{5.1}$$

Experimentally the neutron proton mass difference is [31] $1.29 MeV$. However the electromagnetic interaction (photon exchange) also contributes to the neutron proton mass difference. Using experimental electromagnetic form factors this contribution is estimated as [13]:

$$\Delta_{EM} = (-0.76 \pm 0.30) MeV.$$

The negative sign agrees with Δ_{EM} being dominated by the Coulomb repulsion. Thus we may extract:

$$\Delta = (2.05 \pm 0.30) MeV. \tag{5.2}$$

a. Description of the approach

It has been demonstrated previously[1] that vector mesons are a necessary ingredient in an effective meson theory in order for it to explain the neutron proton mass difference reasonably well. In ref.[1] however, only the $SU(2) \times U(1)$ version of the model was properly calculated while the $SU(3)$ part was just roughly estimated. It is therefore highly desirable to extend the treatment to $SU(3)$.

In the first step the static soliton with unit baryon number is constructed[5]. The appropriate classical ansätze are:

$$\xi_\pi(\mathbf{r}) = \begin{pmatrix} exp(i\hat{\mathbf{r}} \cdot \boldsymbol{\tau} F(r)/2) & 0 \\ 0 & 1 \end{pmatrix}, \quad \omega_0 = \frac{\omega(r)}{2\sqrt{2}\tilde{g}}, \quad \rho_{i,a} = \frac{G(r)}{\sqrt{2}\tilde{g}r}\epsilon_{ija}\hat{r}_j. \tag{5.3}$$

Substituting (5.3) into the Lagrangian $\mathcal{L}$ yields the classical mass $M_{\rm cl} = -\int d^3r\mathcal{L}$. The isospin breaking terms do not actually contribute to $M_{\rm cl}$. An analytic expression for $M_{\rm cl}$ may be found in refs.[17]. Extremizing $M_{\rm cl}$ yields second order non-linear differential equations for $F(r), G(r)$ and $\omega(r)$. The solutions to these equations are characterized by their topological charge which is identified with the baryon number. The solution of the baryon number one sector has been extensively discussed in the literature[17-20,22]. The static soliton (5.3) is invariant under "grand spin" $\mathbf{G} = \mathbf{J}+\mathbf{I}$ transformations but not under spin ($\mathbf{J}$) and isospin ($\mathbf{I}$) transformations separately. Therefore the static soliton carries neither good spin nor good isospin quantum numbers. In $SU(2)$ the projection onto good quantum numbers is carried out by introducing time dependent collective coordinates for

[5] More details on the notation of this section may be found in refs [17].

the zero modes of the theory. In $SU(2)$ these are the spatial and isospatial rotations. These two transformations are actually equivalent due to the above mentioned grand spin symmetry. Thus in $SU(2)$ the spectrum contains only baryons with equal spin and isospin, *e.g.* N,Δ.

Unfortunately the extension of soliton models to include strange fields is non-trivial due to the presence of $SU(3)$ breaking, *i.e.* flavor transformations are not real zero modes. However, we will consider $SU(3)$ to still be an approximate symmetry and therefore shall introduce collective coordinates for the whole flavor group. This approach allows us to easily make contact with ordinary baryon phenomenology. Furthermore it has been demonstrated in ref.[17] that the collective approach describes static properties of the low-lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons reasonably well. An alternative point of view that $SU(3)$ symmetry is strongly broken leads to the bound state approach[33]. In this approach a kaonic bound state is constructed in the background field of the static soliton. The corresponding bound state energy eigenvalue determines the mass splittings of baryons with different hypercharge since they are characterized by different occupation numbers of this bound state. Numerically the two (somewhat different) approaches yield comparable results for the mass differences of baryons with different hypercharge and spin.

The collective coordinates $A(t) \in SU(3)$ are introduced by flavor-rotating (5.3):

$$\xi(\mathbf{r},t) = A(t)\xi_k\xi_\pi(\mathbf{r})\xi_k A^\dagger(t) \tag{5.4}$$

and

$$\frac{1}{\sqrt{2}}\rho_\mu(\mathbf{r},t) = A(t)\begin{pmatrix} \rho_\mu^\pi + \omega_\mu & K_\mu^* \\ K_\mu^{*\dagger} & 0 \end{pmatrix} A^\dagger(t). \tag{5.5}$$

Here we have also allowed for a more general field configuration parametrized by ξ_k and K_μ^*. We will see shortly how these additional fields get excited. The time dependence of the collective coordinates is made most transparent by the introduction of "angular velocities", Ω_a (a=1,...,8):

$$A^\dagger\dot{A} = \frac{i}{2}\sum_{a=1}^{8}\lambda_a\Omega_a = i\begin{pmatrix} \Omega_\pi + \Omega_\eta & \Omega_K \\ \Omega_K^\dagger & -2\Omega_\eta \end{pmatrix} \tag{5.6}$$

wherein the $\lambda_a, \quad (a = 1,..,8)$ denote the $SU(3)$ Gell-Mann matrices.

Obviously the vector meson terms in (2.4) as well as the "anomalous" part of the action (2.5) contain expressions which are linear in the time derivative. These expressions provide source terms linear in the angular velocity for the fields which vanish classically. Thus additional fields are excited by the collective rotation. Clearly they are linear in the angular velocities. The most general ansatz for the non-strange vector mesons excited by the isospin rotation is[17,19-20]

$$\rho_0^\pi = \frac{1}{2\sqrt{2}\tilde{g}}[\xi_1(r)\boldsymbol{\Omega} + \xi_2(r)(\hat{\mathbf{r}}\cdot\boldsymbol{\Omega})\hat{\mathbf{r}}]\cdot\boldsymbol{\tau}, \quad \omega_i = \frac{\Phi(r)}{2\sqrt{2}\tilde{g}}\epsilon_{ijk}\Omega_j\hat{r}_k. \tag{5.7}$$

The pseudoscalar nonet contains components which are excited by the isospin rotation[20] as well as by rotation into the strange directions[19]. Parametrizing:

$$\xi_k = e^{iz}, \quad z = \begin{pmatrix} \eta_T & K \\ K^\dagger & \eta_S \end{pmatrix} \tag{5.8}$$

suitable ansätze are $\eta_T = \frac{1}{4}(\chi(r)+\chi_8(r))\hat{\mathbf{r}}\cdot\mathbf{\Omega}$, $\eta_S = \frac{1}{4}(\chi(r)-2\chi_8(r))\hat{\mathbf{r}}\cdot\mathbf{\Omega}$ and $K = W(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}\Omega_K$. For later calculations it is convenient to introduce also an ansatz for the glueball field $G = g(r)\hat{\mathbf{r}}\cdot\mathbf{\Omega}$ instead of eliminating G from the beginning via (2.13). From parity and isospin covariance we also get the following ansätze for the K^* isospinor fields:

$$K_0^* = \frac{S(r)}{\sqrt{2}\tilde{g}}\Omega_K, \quad K_i^* = \frac{1}{2\sqrt{2}\tilde{g}}[iE(r)\hat{r}_i + \frac{D(r)}{r}\epsilon_{ijk}\hat{r}_j\tau_k]\Omega_K. \tag{5.9}$$

Substituting (5.3-5,7-9) into the action and expanding up to second order in the angular velocities as well as linear order in isospin breaking yields the collective Lagrangian:

$$\begin{aligned} L = & -M_{\rm cl} + \tfrac{1}{2}\alpha^2\textstyle\sum_{i=1}^3\Omega_i^2 + \tfrac{1}{2}\beta^2\sum_{\alpha=4}^7\Omega_\alpha^2 - \tfrac{\sqrt{3}}{2}\Omega_8 + \alpha_1\sum_{i=1}^3 D_{8i}\Omega_i + \beta_1\sum_{\alpha=4}^7 D_{8\alpha}\Omega_\alpha \\ & -\tfrac{1}{2}\gamma(1-D_{88}) - \tfrac{1}{2}\gamma_S(1-D_{88}^2) - \tfrac{1}{2}\gamma_T\textstyle\sum_{i=1}^3 D_{8i}D_{8i} - \tfrac{1}{2}\gamma_{TS}\sum_{\alpha=4}^7 D_{8\alpha}D_{8\alpha} \\ & -\Gamma_3 D_{38} - \Delta_3\textstyle\sum_{i=1}^3(D_{3i}D_{8i} + D_{38}D_{88}) + \alpha_3\sum_{i=1}^3 D_{3i}\Omega_i + \beta_3\sum_{\alpha=4}^7 D_{3\alpha}\Omega_\alpha, \end{aligned} \tag{5.10}$$

where the D_{ij} denote the matrices $\frac{1}{2}Tr(\lambda_i A\lambda_j A^\dagger)$ of the $SU(3)$ adjoint representation. The moments of inertia α^2 and β^2 are functionals of the radial functions $(\xi_1, \xi_2, \Phi, \chi, \chi_8)$ and (W, S, E, D), respectively. Varying the moments of inertia with respect to these fields yields linear inhomogeneous second order differential equations with the classical fields F, G and ω as source terms. Analytic expressions for the moments of inertia as well as the symmetry breaking parameters γ, α_1 and β_1 may be found in ref.[17]. Here we only wish to explain the mechanism which excites χ, χ_8 and g. The non-strange combination $\chi + \chi_8$ is excited by the terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ in (2.14) exactly as explained in refs.[1,20]. The glueball field and the strange part of the η fields are subsequently induced via the equation of motion (2.13).

The additional symmetry breaking parameters γ_S, γ_T and γ_{TS} are solely due to the $(\lambda')^2$ term in (2.10):

$$\begin{aligned} \gamma_S &= -\gamma_T = -\tfrac{32\pi}{9}(\lambda')^2(1-x)^2\textstyle\int drr^2(1-\cos 2F), \\ \gamma_{TS} &= \tfrac{32\pi}{3}(\lambda')^2(1-x)^2\textstyle\int drr^2(1-\cos F). \end{aligned} \tag{5.11}$$

The isospin symmetry breaking parameters $\Gamma_3, \Delta_3, \alpha_3$ and β_3 are needed for the evaluation of the neutron proton mass difference and we present the somewhat lengthy analytic expressions in an appendix. Here we just wish to mention that Γ_3 and Δ_3 contain only classical fields while α_3 and β_3 contain expressions linear in the excitations (5.7-8) as well.

The Lagrangian (5.10) is quantized canonically by introducing $SU(3)$ right generators, R_a $(a = 1,..,8)$ via:

$$R_a = -\frac{\partial L}{\partial\Omega_a} = \begin{cases} -(\alpha^2\Omega_a + \alpha_1 D_{8a} + \alpha_3 D_{3a}), & \text{a=1,2,3} \\ -(\beta^2\Omega_a + \beta_1 D_{8a} + \beta_3 D_{3a}), & \text{a=4,..,7} \\ \frac{1}{2}\sqrt{3}, & \text{a=8.} \end{cases} \tag{5.12}$$

We separate the isospin breaking part of the Hamiltonian:

$$H = -\sum_{a=1}^{8} R_a\Omega_a - L = H_{I=0} + H_{I=1} \tag{5.13}$$

The isospin symmetric part $H_{I=0}$ may be diagonalized exactly by generalizing the approach of Yabu and Ando[16]. This yields the energy formula:

$$E_{I=0} = M_{cl} + \frac{1}{2}(\frac{1}{\alpha^2} - \frac{1}{\beta^2})J(J+1) - \frac{3}{8\beta^2} + \frac{1}{2\beta^2}\epsilon_{SB}, \tag{5.14}$$

where ϵ_{SB} is the eigenvalue of

$$\begin{aligned} C_2 \quad & +\beta^2\gamma(1-D_{88}) + \beta^2\tfrac{\alpha_1}{\alpha^2}\textstyle\sum_{i=1}^{3} D_{8i}(2R_i+\alpha_1 D_{8i}) + \beta_1 \sum_{\alpha=4}^{7} D_{8\alpha}(2R_\alpha+\beta_1 D_{8\alpha}) \\ & \qquad +\beta^2\gamma_S(1-D_{88}^2) + \beta^2\gamma_T \textstyle\sum_{i=1}^{3} D_{8i}D_{8i} + \beta^2\gamma_{TS}\sum_{\alpha=4}^{7} D_{8\alpha}D_{8\alpha}, \end{aligned} \tag{5.15}$$

and $C_2 = \sum_{a=1}^{8} R_a^2$ denotes the quadratic Casimir operator of $SU(3)$. ϵ_{SB} is obtained, of course, numerically, using differential operator realizations of the R_a [34]. The isospin breaking part is obtained to be:

$$\begin{aligned} H_{I=1} &= \Gamma_3 D_{38} + \Delta_3 \sum_{i=1}^{3}(D_{3i}D_{8i} + D_{38}D_{88}) \\ &\quad + \frac{\alpha_3}{\alpha^2}\sum_{i=1}^{3} D_{3i}(R_i+\alpha_1 D_{8i}) + \frac{\beta_3}{\beta^2}\sum_{\alpha=4}^{7} D_{3\alpha}(R_\alpha+\beta_1 D_{8\alpha}). \end{aligned} \tag{5.16}$$

In the evaluation of the Hamiltonian only terms linear in the isospin breaking have been retained. Finally we have the non-electromagnetic contribution to the neutron proton mass difference:

$$\begin{aligned} \Delta &= \langle n|H_{I=1}|n\rangle - \langle p|H_{I=1}|p\rangle \\ &= -2\Gamma_3\langle p|D_{38}|p\rangle - 2\Delta_3\langle p|\sum_{i=1}^{3}(D_{3i}D_{8i}+D_{38}D_{88})|p\rangle \\ &\quad -2\frac{\alpha_3}{\alpha^2}\sum_{i=1}^{3}\langle p|D_{3i}(R_i+\alpha_1 D_{8i})|p\rangle - 2\frac{\beta_3}{\beta^2}\sum_{\alpha=4}^{7}\langle p|D_{3\alpha}(R_\alpha+\beta_1 D_{8\alpha})|p\rangle, \end{aligned} \tag{5.17}$$

where $|p\rangle$ denotes the exact proton eigenstate of $H_{I=0}$. In writing (5.17) we have made use of the flavor transformation properties of the D-functions.

Although it is obvious, we would like to stress that in this treatment the neutron proton mass splitting is not obtained as the difference of two large numbers. On the contrary, the leading operator in the isospin breaking($H_{I=1}$) has been extracted and its expectation value is identified with the mass difference.

b. Numerical results

We now present our numerical results for the neutron proton mass difference. To proceed we have to fix the parameters of the anomalous part of the action, (2.5). Previously [22] it has been shown that

$$\begin{aligned} \tilde{h} &= 4(2c_1 - c_2/\tilde{g} - c_3/4\tilde{g}^2) \approx 0.4 \\ \tilde{g}_{VV\phi} &= 4c_2 \approx 1.9 \end{aligned} \tag{5.18}$$

fit reasonably well the decay processes $\omega \to 3\pi$ and $\omega \to \rho\pi$. $\tilde{h}$ and $\tilde{g}_{VV\phi}$ are allowed to vary [6] in the range $\tilde{h} = -0.15, .., 0.7$ and $\tilde{g}_{VV\phi} = 1.3, .., 2.2$ subject to the constraint $|\tilde{g}_{VV\phi} - \tilde{h}| \approx 1.5$ due to uncertainties in the determination of the $\omega - \phi$ mixing angle. The third parameter could not be fixed in the meson sector; however, it was argued that

[6] Hans Walliser has pointed out to us that the experimental measurements for the relevant ω and ϕ partial decay widths have changed from the 1986 to the 1992 "Review of Particle Properties". Continuing to use (4.4) [see footnote 3 above], (4.7) and (4.9) of [22] now gives us $|\epsilon|/Z_\phi = 0.059 \pm 0.005$ and central values $\tilde{g}_{VV\phi} = 1.81$, $\tilde{h} = 0.38$. These are substantially similar to the older values.

x	$\Gamma_3(MeV)$	$\Delta_3(MeV)$	α_3	β_3
25.0	-15.32	0.08	-0.0046	-0.0171
28.0	-15.60	0.05	-0.0049	-0.0178
30.0	-16.08	0.04	-0.0052	-0.0185
31.5	-16.45	0.03	-0.0055	-0.0189
34.0	-16.89	0.02	-0.0060	-0.0195
38.0	-17.89	0.00	-0.0070	-0.0205

Table 5.1: The isospin breaking parameters $\Gamma_3, \Delta_3, \alpha_3$ and β_3 as functions of x. The input parameters are according to table 3.1. For parameters of the anomalous sector we take $\tilde{h} = 0.4, \tilde{g}_{VV\phi} = 1.9, \kappa = 1.0$.

x	$-2\langle D_{38}\rangle_p$	$M_1(MeV)$	$M_2(MeV)$	$M_n - M_p(MeV)$
25.0	-0.017	-174.1	-31.0	1.60
28.0	-0.020	-180.0	-31.2	1.75
30.0	-0.021	-183.5	-31.0	1.88
31.5	-0.023	-185.1	-30.6	1.97
34.0	-0.025	-188.6	-29.7	2.12
38.0	-0.028	-193.0	-27.3	2.41

Table 5.2: The relevant matrix elements $M_1 = \frac{-2}{\alpha^2}\sum_{i=1}^{3}\langle D_{3i}(R_i + \alpha_1 D_{8i})\rangle_p$ and $M_2 = \frac{-2}{\beta^2}\sum_{\alpha=4}^{7}\langle D_{3\alpha}(R_\alpha + \beta_1 D_{8\alpha})\rangle_p$ to evaluate the neutron proton mass difference as functions of x. Input parameters as in table 5.1

$\kappa = c_3/2\tilde{g}c_2 \approx 1$ from studies of baryon properties. We will adopt this value together with (5.18).

In table 5.1 we present the results for the isospin breaking parameters $\Gamma_3, \Delta_3, \alpha_3$ and β_3 as functions of x. Obviously Δ_3 is negligible and we will omit it from now on.

In table 5.2 we display the relevant matrix elements and the prediction for the non-electromagnetic contribution to the neutron proton mass difference, Δ in (5.3).

Note that the α_3 term dominates and contributes more than 50% of the neutron proton mass difference. This is expected since the α_3 term represents essentially the pure two flavor contribution. As previously discussed [1] the use of the exact wave functions of the $SU(3)$ Hamiltonian rather than the "unperturbed" wave function drastically suppresses the matrix elements of the Γ_3 piece. Clearly, reasonable agreement with the result extracted from experimental data (5.2) is obtained for $x \geq 30$. Especially the prediction for the neutron mass difference for the favored value in the meson sector $x = 31.5$ turns out to almost coincide with the central value of the experimental data.

Based on the discussion in ref.[17] one might however object that for $\tilde{h} = 0.4$ and $\tilde{g}_{VV\Phi} = 1.9$, the $SU(2)$ moment of inertia $\alpha^2 = 4.52GeV^{-1}$ is somewhat too small, predicting too large mass splittings between the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. A small variation to $\tilde{h} = 0.36$, $\tilde{g}_{VV\Phi} = 1.88$ and $x = 28$ yields $\alpha^2 = 5.00GeV^{-1}$. The corresponding mass differences are displayed in table 5.3 and are found to agree reasonably well with the experimental data. Actually the $SU(3)$ mass differences come out slightly worse than previously since the effect of α' is lowered. This is due to the fact that large, positive α_1 and β_1 terms in (5.10) mitigate the effects of a large collective symmetry breaking parameter γ. In the previous fit (without the γ' type term) α' was negative and

	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
Fit	168.	263.	404.	327.	470.	617.	766.
(5.18)	144.	237.	350.	360.	480.	602.	723.
Expt.	177.	254.	379.	293.	446.	591.	733.

Table 5.3: Best fit to the mass differences of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons with respect to the nucleon for the parameter set $\tilde{h} = 0.36$, $\tilde{g}_{VV\Phi} = 1.88$, $\kappa = 1.0$ and $x = 28$ (see also table 3.1). Also listed are the predicted mass differences for the central values for $\tilde{h}$ and $\tilde{g}_{VV\Phi}$ with $\kappa = 1.0$ and $x = 31.5$ (3.15).

significantly larger in magnitude than in the present case (cf. table 3.1). Noting[7] that α_1 is essentially proportional to $(1-x)\alpha'$ we may understand why the $SU(3)$ mass differences are increased. For this parameter set we obtain:

$$\Delta = 1.77MeV \ , \tag{5.19}$$

which is slightly lower than the central value $2.05MeV$ of the experimental data for Δ but still within the error bars. The dominant contribution to (5.19) stems from the α_3 term: $0.93MeV$. Previously[1] we had found that this contribution was almost entirely due to the η excitation. This is now somewhat modified since firstly in ref. [1] only the unphysical η_T was present and its mass of $550MeV$ was assumed. Employing the treatment of section 4 in the soliton sector as well decreases the effect of the η fields somewhat because they are suppressed by a larger mass. Secondly the γ' term which was not present in the approach of ref.[1] contributes about 30% to α_3. The strange excitations contribute about $0.54MeV$ to the mass difference. The contribution of the Γ_3 term ($0.29MeV$) is strongly suppressed since the matrix element of D_{38} is much smaller than its $SU(3)$ symmetric value when the exact eigenstates of $H_{I=0}$ are used. Comparing (5.19) with the results obtained for the central values (5.18) we recognize a small change for the worse with the baryon best fit parameters. However, the better agreement of Δ for the set (5.18) is merely due to the smaller non-strange moment of inertia so we would expect the baryon best fit set to be more reliable for predicting baryon observables.

In the next section we will discuss a small extension of the model Lagrangian which enables us to fine tune the predicted neutron proton mass difference by adding a term which may be interpreted as providing the gluonic contribution to the proton spin.

6. Two component description of the proton spin puzzle

In this section we will describe the implications of the preceding considerations on the so called proton spin puzzle which actually refers to the surprisingly small matrix element of the axial singlet current between proton states. In a naïve quark model this matrix element measures twice the proton spin. As indicated previously[3] the neutron proton mass difference may be used to determine the gluonic contribution to this matrix element. In ref.[3] only an estimate of this quantity was made. Here we will present the complete calculation since all the necessary tools have been provided in the preceding sections and we may therefore disentangle the two components of the axial singlet matrix element: matter and gluon contribution [11].

[7] Correct charge normalization requires that we only include the contributions of the classical fields (5.3) to α_1 and β_1.

The axial singlet current, J^5_μ may most easily be obtained from

$$J^5_\mu = -2\frac{\partial \mathcal{L}}{\partial_\mu \chi} = \sqrt{3}F_\pi \partial_\mu \eta' + s\tilde{J}^5_\mu \tag{6.1}$$

with χ being defined after eqn. (2.13). It is important to note that the second part,

$$\tilde{J}^5_\mu \;=\; \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\Big\{\frac{i}{2}(\frac{\gamma_1}{3}+\frac{\gamma_2}{2})\tilde{p}_\nu\tilde{p}_\rho V_\sigma - \sqrt{2}\gamma_2\tilde{g}F_{\nu\rho}V_\sigma - 4i\tilde{g}^2(\gamma_2+2\gamma_3)V_\nu V_\rho V_\sigma\Big\} \tag{6.2}$$

is not a pure gradient and has therefore a non-vanishing matrix element at zero momentum transfer in contrast to the first part. In eqn. (6.2) we have used $\frac{1}{\sqrt{2}}V_\mu = \rho_\mu - \frac{i}{2g}v_\mu$ as well as $\tilde{p}_\mu(v_\mu) = \tilde{U}^{1/2\dagger}\partial_\mu\tilde{U}^{1/2} \pm \partial_\mu\tilde{U}^{1/2}\tilde{U}^{1/2\dagger}$. $\tilde{J}^5_\mu$ may be considered as the short distance part of the axial singlet current since it is dominated by the vector mesons and is not present in models with pseudoscalars only. The dimensionless constant s has been introduced by the replacement in the Lagrangian:

$$\partial_\mu\chi\tilde{J}^5_\mu \longrightarrow s\partial_\mu\chi\tilde{J}^5_\mu \tag{6.3}$$

to allow for a deviation from the nonet form for the pseudoscalar fields in the Lagrangian. The nonet form was originally introduced to satisfy the OZI rule; however such an approach seems somewhat debatable for the η'.

Defining axial form factors of the nucleon for the flavor $l = u, d, s$ via

$$\begin{aligned} \frac{\sqrt{p_0p'_0}}{M_P} \qquad & < P(\vec{p}\,')|\overline{q}_l\gamma_\mu\gamma_5 q_l|P(\vec{p}) >= \\ & \overline{u}(\vec{p}\,')[\gamma_\mu\gamma_5 H_l(q^2) + \frac{iq_\mu}{2M_P}\gamma_5\tilde{H}_l(q^2)]u(\vec{p}), \quad \text{with } q_\mu = p_\mu - p'_\mu. \end{aligned} \tag{6.4}$$

the relevant quantity for the axial singlet current is

$$H(q^2) = \sum_{l=1}^{3} H_l(q^2). \tag{6.5}$$

Obviously the first term in (6.1) only contributes to the induced form factor $\tilde{H}(q^2) = \sum_{l=1}^3 \tilde{H}_l(q^2)$. In the original Skyrme model (without vector mesons) $\tilde{J}^5_\mu = 0$ and therefore also $H(0) = 0$, which has been considered as a success[7] of the Skyrme model since it nicely describes the results of the EMC experiment[4].

In order to introduce the two component mechanism [11] it is necessary [3] to allow the pseudoscalar gluon field, G to couple to $\tilde{J}^5_\mu$ via the chirally invariant expression:

$$\frac{2t}{\kappa}\partial_\mu G\tilde{J}^5_\mu \tag{6.6}$$

wherein t is a new dimensionless parameter. This additional term changes the equation of motion for the glueball field (2.13) to

$$G = \frac{\kappa}{2\sqrt{3}F_\pi}\eta' + t\partial_\mu\tilde{J}^5_\mu. \tag{6.7}$$

$s \setminus t$	0.0	-0.5	-1.0	-1.5	-2.0
0.8	1.70	1.84	1.98	2.13	2.25
0.9	1.74	1.87	2.02	2.16	2.27
1.0	1.77	1.91	2.05	2.20	2.29
1.1	1.80	1.95	2.09	2.24	2.32
1.2	1.85	1.98	2.13	2.27	2.35

Table 6.1: The neutron proton mass difference Δ as a function of the parameters s and t.

We furthermore have the $U_A(1)$ anomaly equation which is equivalent to the equation of motion for η' after eliminating the glueball field:

$$(-\partial^2 + \frac{\kappa}{6F_\pi^2})\eta' = \frac{s-t}{\sqrt{3}F_\pi}\partial_\mu \tilde{J}_\mu^5 \tag{6.8}$$

In both equations (6.7) and (6.8) we have neglected small symmetry breaking effects originating from the ϵ' term in (2.12). Taking matrix elements of these equations between nucleon states we may read off the corresponding coupling constants at zero momentum transfer:

$$\begin{aligned} g_{\eta' NN} &= \frac{s-t}{s}\frac{2m_N}{\sqrt{3}F_\pi}H(0) \\ g_{GNN} &= \frac{t}{t-s}\frac{2\sqrt{3}F_\pi g_{\eta' NN}}{\kappa}. \end{aligned} \tag{6.9}$$

Next we may use the coupling constants to reexpress the axial singlet current form factor:

$$H(0) = \frac{\sqrt{3}F_\pi}{2m_N}(g_{\eta' NN} - \frac{\kappa}{2\sqrt{3}F_\pi}g_{GNN}) = \text{“matter”} + \text{“glue”} \tag{6.10}$$

The modifications (6.3) and (6.6) of course enter the evaluation of the non-strange moment of inertia α^2, however we find, even for a large deviation of $s-t$ from unity, no significant change of α^2. This is due to the fact that these modifications only affect the η and glueball fields but leave the vector meson excitations ξ_1, ξ_2 and Φ essentially unaltered. Since the η fields do not explicitly appear in the definition of $\tilde{J}_\mu^5$ the corresponding matrix element numerically turns out to be independent of t; we find for the best fit parameters of section 5 (see table 5.3):

$$H(0) = 0.29s. \tag{6.11}$$

However, we do find a significant dependence of the predicted value for the neutron proton mass difference, Δ on s and t since there the η fields enter crucially. In table 6.1 we display our numerical results for Δ using the “baryon” best fit parameters. We note that for $|t| \leq 1.5$ the mass difference depends linearly on t while for larger values of $|t|$, non-linear effects which enter via the equations of motion are significant.

To proceed, we consider the precise value of $H(0)$ as the parameter which determines the “fudge factor” s via eqn (6.11). Then fixing t from the neutron proton mass difference allows us to separate the matter and gluon contribution to $H(0)$ in eqn (6.10). We present our results for various experimentally allowed values of $H(0)$ in table 6.2 for the parameter

$H(0)$	t	"matter"	"glue"
	Baryon Best fit		
0.0	-1.86 ± 0.90	0.54 ± 0.26	-0.54 ∓ 0.26
0.1	-1.64 ± 1.02	0.57 ± 0.30	-0.47 ∓ 0.30
0.2	-1.31 ± 1.04	0.58 ± 0.30	-0.38 ∓ 0.30
0.3	-0.96 ± 1.00	0.58 ± 0.29	-0.28 ∓ 0.29
	Set (3.15,5.18)		
0.0	-1.09 ± 0.94	0.32 ± 0.26	-0.32 ∓ 0.27
0.1	-0.74 ± 0.93	0.31 ± 0.30	-0.21 ∓ 0.27
0.2	-0.38 ± 0.96	0.31 ± 0.30	-0.11 ∓ 0.28
0.3	-0.00 ± 0.97	0.30 ± 0.29	0.00 ∓ 0.28

Table 6.2: The "matter" and "glue" contribution to the axial singlet matrix element of the proton for two sets of parameters. For the best fit parameters see table 5.3.

set used to fit the baryon mass difference as well as for the central values of $\tilde{h}$ and $\tilde{g}_{VV\phi}$, (5.18) together with (3.15). We think that the results of the "baryon" best fit should be more reliable than the results for the set which gives a best fit to the meson properties. This is because for both the spin and isospin mass splittings, a fine-tuning of the non-strange moment of inertia α^2 turns out to be crucial (see section 5). In any event, the difference between the two sets provides a measure of the "systematic" uncertainties in our calculation. Taking into account that for the set (3.15,5.18) the absolute values for the gluon contribution to the axial singlet matrix element are lower than for the baryon best fit we might consider the results for the latter set as an upper bound. We should furthermore remark that it would be surprising if s were to deviate too much from unity. Thus, in each case the range for $H(0)$ between 0.2 and 0.3 would seem most reasonable.

To sum up, we may conclude from table 6.2 and the discussion above that while there is a tendency for the "glue" part to cancel the "matter" part, both are most likely to be on the small side compared to unity. This agrees with the results of [3], though it must be remarked that the present, more detailed, calculation allows larger uncertainties in the "glue" and "matter" pieces. The reason for this is that, as pointed out in the previous section, the dependence of Δ on the η fields is lessened due to the inclusion of the γ' term in (2.10). This translates into somewhat larger values of $(s-t)$ than were previously estimated [3] as well as large uncertainties in t. Since the present approach "lives off" the deviation of the calculated Δ from its "experimental" value, its accuracy could be improved if a more accurate value of the photon exchange contribution, Δ_{EM} were available.

7. *Remarks on the mesonic Lagrangian*

The effective Lagrangian discussed in sections 2−4 has a number of connections with physical quantities outside the Skyrme model approach. We would like to make some, partly speculative, remarks on this here.

a. *Light Quark Masses*

Their values are of interest, for example, in constructing models of the quark mixing (Kobayashi-Maskawa) matrix of weak interactions. Conventionally, [13] the absolute val-

m_u	m_d	m_s
2.3	5.6	125
3.2	7.9	175
4.2	10.2	225

Table 7.1: Light quark masses.

ues of the (current) quark masses are considered to be "running" masses evaluated at the scale of 1 GeV. Estimates of the kind we have made here yield just their ratios. Evaluation of "QCD sum rules" yields [13] an absolute value for $m_s(1\,\mathrm{GeV})$ in the 125-225 MeV range. Combining this with the "best fit" in (3.15) yields the sets (all in MeV) given in table 7.1.

Note that ratios like $\sqrt{m_d/m_s} = 0.21$ are independent of the absolute value of $m_s(1\,\mathrm{GeV})$.

b. *Confirmation of R*

It is interesting to try to confirm the value R=36 in (3.15) from consideration of other particle multiplets. The most precisely known masses are, of course, those of the ordinary octet baryons. First order perturbation theory (which yields the fairly well satisfied Gell-Mann Okubo mass formula) gives the prediction:

$$R = \frac{\Xi - \Sigma}{n - p} = \frac{\Sigma - N}{\Xi^- - \Xi^\circ}\,, \tag{7.1}$$

wherein each particle symbol stands for its mass, which in turn is assumed to have been "corrected" by subtraction of the photon-exchange contribution. Numerically this reads $R = 60.7 \pm 9.1 = 46.2 \pm 5.9$. The large deviation between the two central values suggests a need to go beyond first order perturbation theory. If one expands around $m_u = m_d = m_s = 0$ the procedure [13] is not straightforward since one encounters non-analytic terms like $m^{3/2}$ and $m \ln m$. A possible way out is to expand around the point $m_u = m_d = m_s = \frac{1}{3}(m_u + m_d + m_s)$ rather than zero. It was recently shown [35] that this method leads to a second order prediction for R:

$$R = \frac{3\Lambda + \Sigma - 2N - 2\Xi}{2\sqrt{3}m_T + (n-p) + (\Xi^\circ - \Xi^-)}\,, \tag{7.2}$$

where m_T is the "corrected" $\Lambda - \Sigma^\circ$ transition mass. In principle m_T could be determined by a precision measurement of the difference between the $pK^- \to \Lambda\eta$ and $n\overline{K}^\circ \to \Lambda\eta$ scattering amplitudes, for example. In the absence of such information we can use (7.2) to construct a lower bound for R if we assume that quantities at second order should not deviate too drastically from their first order values. This yields [35] roughly

$$R > 38 \pm 10\,, \tag{7.3}$$

which is evidently consistent with R=36.

c. *Kaplan-Manohar ambiguity*

If $\mathcal{M}(x)$ is a 3×3 matrix field transforming as $(3, 3^*)$ under $SU(3)_L \times SU(3)_R$, it is easy to see that $[\mathcal{M}(x)]^{\dagger -1} \det \mathcal{M}^\dagger(x)$ transforms in the same way. This suggests investigating

[26] the substitution

$$\mathcal{M} = \mathcal{M}' + b\mathcal{M}'^{-1} \det \mathcal{M}' \,, \tag{7.4}$$

where b is a real parameter and $\mathcal{M} = \mathcal{M}^\dagger$ is the diagonal matrix proportional to quark masses, given in (2.7). Using the characteristic equation for a 3×3 matrix it can be seen that the effect of the substitution (7.4) on the non-derivative symmetry breakers of the second order CPT program [12] is simply

$$\begin{aligned}
&\delta' Tr\left[\mathcal{M}(\tilde{U} + \tilde{U}^\dagger)\right] + \lambda'^2 Tr(\mathcal{M}\tilde{U}\mathcal{M}\tilde{U} + \mathcal{M}\tilde{U}^\dagger\mathcal{M}\tilde{U}^\dagger) \\
&+ c_6[Tr(\mathcal{M}(\tilde{U} + \tilde{U}^\dagger))]^2 + c_7[Tr(\mathcal{M}(\tilde{U} - \tilde{U}^\dagger))]^2 \\
&= \delta' Tr\left[\mathcal{M}'(\tilde{U} + \tilde{U}^\dagger)\right] + (\lambda'^2 - \frac{b\delta'}{2}) Tr(\mathcal{M}'\tilde{U}\mathcal{M}'\tilde{U} + \mathcal{M}'\tilde{U}^\dagger\mathcal{M}'\tilde{U}^\dagger) \\
&+ (c_6 + \frac{b\delta'}{4})[Tr(\mathcal{M}'(\tilde{U} + \tilde{U}^\dagger))]^2 + (c_7 + \frac{b\delta'}{4})[Tr(\mathcal{M}'(\tilde{U} - \tilde{U}^\dagger))]^2 + \text{higher order}
\end{aligned} \tag{7.5}$$

Here we have chosen to use the octet chiral field $\tilde{U}$, with $\det \tilde{U} = 1$, for simplicity. By "higher order" we mean the terms proportional to b which are generated from the λ'^2, c_6 and c_7 terms. These are third order in the CPT program and are to be thrown away when one is working at second order. The significance of (7.5) is that, assuming the higher order terms to be negligible, the identical physics is obtained using either the parameter set $\{m_u, m_d, m_s, \lambda'^2, c_6, c_7\}$ or the set $\{m_u', m_d', m_s', (\lambda'^2)', c_6', c_7'\}$; these are related by

$$\begin{aligned}
m_u &= m_u' + b\, m_d' m_s' \,, \\
m_d &= m_d' + b\, m_u' m_s' \,, \\
m_s &= m_s' + b\, m_u' m_d' \,, \\
\lambda'^2 &= (\lambda'^2)' - \frac{b\delta'}{2} \,, \\
c_6 &= c_6' + \frac{b\delta'}{4} \,, \\
c_7 &= c_7' + \frac{b\delta'}{4} \,,
\end{aligned} \tag{7.6}$$

for any b.

Evidently there is a one parameter family of ambiguities. A lot of attention has been focussed on the possibility of obtaining $m_u = 0$ in some "frame" since that might provide a non-axionic solution of the "θ-problem". Assuming that a solution for the parameters is specified for the primed frame in (7.6), we can transform to $m_u = 0$ with the choice $b = -m_u'/(m_d' m_s')$. The corresponding transformation for R in (2.9) is then

$$R \approx R'/(1 + m_u'/m_d'). \tag{7.7}$$

With the best fit choice (3.15) for the primed frame we find that $R \approx 26$ is needed for $m_u = 0$; according to (7.3) this would perhaps be unlikely.

The present model is, of course, different from the usual CPT program since, among other things, we are including the vectors. In any event, as discussed in section 2c, the assumption that OZI rule violation is dominant for the pseudoscalar singlet channel leads to a type 7 but not type 6 symmetry breaker. This would uniquely fix the "frame" for the present purpose.

d. *Extension of the meson Lagrangian*

It is naturally of interest to consider how the present mesonic Lagrangian can be modified to yield a systematic continuation of the CPT program to higer energies. The CPT program [12] itself is most reliable for energies up to several hundred MeV, which restricts us to using just the pion fields. Each successive order of approximation adds terms with two more derivatives or one more power of the mass matrix (formally introduced as counterterms to loops computed at the previous order). In practice, going beyond second order may be difficult. Perhaps surprisingly, the three flavor continuation, including the K's and the η, seems to work reasonably well. However, it obviously cannot be extended in energy past the vector mesons without including them. Since we would like to use the meson Lagrangian to study baryons-as-solitons in the 1 GeV region, it would seem reasonable to add just the vectors (together, perhaps, with pseudoscalar and scalar singlet glue fields in order to develop a direct connection with QCD) and stop there. This provides a "clean break" in the sense of including just the lowest lying s wave states of the quark model. To model QCD in this way to *all* energies would likely require us to include *all* the higher resonances, as suggested either by their need to produce high energy Regge behavior [36] or by their presence in the large N_c approximation [37].

Once we have agreed to limit the "quarkonium" states to the pseudoscalars and vectors it is necessary to list the chiral invariant interaction terms. One question is whether this will introduce double counting since it is known [38] that "integrating out" the vectors reproduces the coefficients of some of the pure pseudoscalar terms. We think that this should not be a problem; one should include *a priori* both the vector and pseudoscalar terms and suitably readjust the coefficients of the pseudoscalar terms. For the "second order" treatment, loops computed with the "first order" terms ((2.4) and the α' and δ' terms in (2.10)) should be included. For a start, however, it seems reasonable to neglect them (while loop diagrams are of great conceptual interest they are often numerically negligible for the indicated scale choice [8]).

Here we have included those terms which we believe to be dominant. Especially, we have neglected OZI rule violating terms, except as discussed in section 2c. The fit to experiment is reasonable for the 2 and 3 point functions considered, Trying to fit more quantities will probably require fine tuning by adding additional terms and including chiral logs. Considering the tremendous amount of data in the region below 1 GeV, it is clear that the best way to proceed is along an incremental "evolutionary" path.

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Appendix

In this appendix we list the analytic expressions for the isospin breaking parameters in the collective Lagrangian (5.9).

Substituting the classical fields into the symmetry breaking part of the Lagrangian

[8] For example, the "chiral logs" are not qualitatively important in (10.11) of the first of refs. [12].

(2.10) yields:

$$\begin{aligned}
\Gamma_3 &= \frac{8\pi y}{\sqrt{3}}\int dr\tilde{\Gamma}_3 \\
\tilde{\Gamma}_3 &= \frac{\alpha'}{2\tilde{g}^2}[r^2\cos F(F'^2+\omega^2) - 2G(G+2)\cos F + 4(1+G-\cos F)] \\
&-2\beta'\cos F(F'^2r^2 + 2\sin^2F) + 2\delta' r^2(1-\cos F) \\
&-\frac{\gamma'}{\tilde{g}^2}\cos F(2G'^2 + \frac{G^2}{r^2}(G+2)^2 - \omega'^2r^2) \\
&+\frac{4(2+x)\lambda'^2}{3}r^2(1-\cos 2F)
\end{aligned} \tag{A.1}$$

and

$$\Delta_3 = \frac{32\pi}{3\sqrt{3}}(1-x)y\lambda'^2\int dr r^2(2\sin^2F + 3\cos F - 3). \tag{A.2}$$

For the isospin breaking parameters which appear in those terms which are linear in the angular velocities we find:

$$\begin{aligned}
\alpha_3 &= \frac{8\pi y}{3}\int dr\tilde{\alpha_3} \qquad \beta_3 = 4\pi y\int dr\tilde{\beta_3} \\
\tilde{\alpha_3} &= \frac{\alpha'}{\tilde{g}^2}\{\omega r^2[2-2\xi_1-\cos F(2+\xi_1+\xi_2)] - 4\Phi(1+G-\cos F) \\
&+(\chi+\chi_8)\sin F[\frac{1}{2}\omega^2r^2 + \frac{1}{2}F'^2r^2 - G(G+2)] \\
&-[r^2\cos FF'(\chi'+\chi_8') + 2(\chi+\chi_8)\sin F]\} \\
&-2\beta'[(\chi+\chi_8)\sin F(F'^2r^2+2\sin^2F) - 2(r^2\cos FF'(\chi'+\chi_8') + 2\sin F(\chi+\chi_8))] \\
&-\frac{\gamma'}{\tilde{g}^2}[(\chi+\chi_8)\sin F(2G'^2 + \frac{G^2}{r^2}(G+2)^2 - \omega'^2r^2) \\
&+4(\omega'\xi_1'r^2 + 2G'\Phi') + 2\cos F(\omega'(\xi_1'+\xi_2')r^2 + \frac{4}{r^2}\Phi G(G+2))] \\
&-2\delta' r^2(\chi+\chi_8)\sin F - 2\epsilon' r^2 g\sin F - 8\frac{2+x}{3}\lambda'^2r^2(\chi+\chi_8)\sin 2F
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
\tilde{\beta_3} &= \frac{\alpha'}{2\tilde{g}^2}\{W(\sin\frac{F}{2} + \sin F)[\omega^2r^2 - 2(1+G-\cos F)^2] \\
&+2\cos\frac{F}{2}[(1+G-\cos F)(D + 2W\sin\frac{F}{2}(1+\cos\frac{F}{2})) - \omega r^2(S-1+\cos\frac{F}{2})]\} \\
&+\frac{\alpha'}{\tilde{g}^2}\{r^2\omega\sin^2\frac{F}{2} \\
&-W(1+G-\cos F)[\sin\frac{F}{2}(1+2\cos\frac{F}{2}+\cos F) + 2\sin F(\cos\frac{F}{2}+\cos F)] \\
&+\sin\frac{F}{2}[\frac{r^2}{2}F'(E + 2\sin\frac{F}{2}W' - F'W) - \sin F((D + 2W\sin\frac{F}{2}(1+\cos\frac{F}{2}))]\} \\
&+2(\frac{\alpha'}{4\tilde{g}^2} - \beta')\{W(\sin\frac{F}{2} + \sin F)(F'^2r^2 + 2\sin^2F) \\
&-2\cos\frac{F}{2}(1+\cos\frac{F}{2})[r^2F'W' + 2\cos\frac{F}{2}\sin FW]\}
\end{aligned}$$

$$-4\delta' r^2 \sin\frac{F}{2}(1+\cos\frac{F}{2})W$$
$$+\frac{\gamma'}{\tilde{g}^2}\{\cos\frac{F}{2}[G'(2D'+EG)+\frac{G}{r^2}(G+2)^2 D-2\omega' r^2(S'+\frac{\omega}{4}E)]$$
$$-W(\sin\frac{F}{2}+\sin F)[2G'^2+\frac{G^2}{r^2}(G+2)^2-\omega'^2 r^2]\}$$
$$-4\lambda'^2 r^2 W[\sin 2F+2\sin\frac{3F}{2}+\sin F+2x\sin\frac{F}{2}(1+\cos\frac{F}{2})] \tag{A.4}$$

Of course, for the parts of the Lagrangian already investigated in ref.[1] the expressions for Γ_3 and α_3 coincide.

Footnotes

1. Only 3.8 was used for fitting in ref. [1]. The extra factor Z_ϕ^{-2} makes negligible difference. Note that the $\lambda'^2 \to 0$ limit of (3.6) quoted, but not used for fitting, should be replaced by the present formula.

2. Input parameters: $F_{\pi p} = 0.132\,\mathrm{GeV}, F_{Kp} = 0.161\,\mathrm{GeV}, m_\pi = 0.137\,\mathrm{GeV}, m_{K^*} = 0.892\,\mathrm{GeV}, m_\rho = 0.768\,\mathrm{GeV}, m_K = 0.497\,\mathrm{GeV}, (m_{K^\circ} - m_{K^+})_{non-EM} = 5.28\,\mathrm{MeV}, M_{\rho\omega} = -2.65\,\mathrm{MeV}, \Gamma(\rho \to 2\pi) = 0.1491\,\mathrm{GeV}, \Gamma(K^* \to K\pi) = 0.0498\,\mathrm{GeV}, \Gamma(\phi \to K\overline{K}) = 0.0037\,\mathrm{GeV}$.

3. In the extraction of the parameters c_1 and c_2 of (2.5) from experiment in [22] some ϕ decays are involved. We note that in (4.5) of this reference we should now write $\left|\epsilon/Z_\phi\right|$ instead of $|\epsilon|$. This replacement should also be made in (4.8) so that the old formulas for c_1 and c_2 remain unaltered.

4. This can be seen by referring to Fig 2 of [32].

5. More details on the notation of this section may be found in refs [17].

6. Hans Walliser has pointed out to us that the experimental measurements for the relevant ω and ϕ partial decay widths have changed from the 1986 to the 1992 "Review of Particle Properties". Continuing to use (4.4) [see footnote 3 above], (4.7) and (4.9) of [22] now gives us $|\epsilon|/Z_\phi = 0.059 \pm 0.005$ and central values $\tilde{g}_{VV\phi} = 1.81$, $\tilde{h} = 0.38$. These are substantially similar to the older values.

7. Correct charge normalization requires that we only include the contributions of the classical fields (5.3) to α_1 and β_1.

8. For example, the "chiral logs" are not qualitatively important in the first of refs. [12].).

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