Syracuse University

SURFACE

Physics

College of Arts and Sciences

2-1-1995

Exploring \pp Scattering in the \1N Picture

Joseph Schechter
Department of Physics, Syracuse University, Syracuse, NY

Francesco Sannino
Syracuse University, Dipartimento di Scienze Fisiche, Mostra d'Oltremare Pad.19, and Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Mostra d'Oltremare

Follow this and additional works at: https://surface.syr.edu/phy

Part of the Physics Commons

Exploring $\pi\pi$ scattering in the $\dfrac{1}{N_c}$ picture

Francesco Sannino [a,b,c] [1]   *and*   *Joseph Schechter* [a] [2]

[a] *Department of Physics, Syracuse University, Syracuse, New York, 13244-1130.*

[b] *Dipartimento di Scienze Fisiche, Mostra d'Oltremare Pad.19, 80125 Napoli, Italia.*

[c] *Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Mostra d'Oltremare, Pad.19 I-80125 Napoli, Italia.*

Abstract

In the large N_c approximation to QCD, the leading $\pi\pi$ scattering amplitude is expressed as the sum of an infinite number of tree diagrams. We investigate the possibility that an adequate approximation at energies up to somewhat more than one GeV can be made by keeping diagrams which involve the exchange of resonances in this energy range in addition to the simplest chiral contact terms. In this approach crossing symmetry is automatic but individual terms tend to drastically violate partial wave unitarity. We first note that the introduction of the ρ meson in a chirally invariant manner substantially delays the onset of drastic unitarity violation which would be present for the *current algebra* term alone. This suggests a possibility of local (in energy) cancellation which we then explore in a phenomenological way. We include exchanges of leading resonances up to the 1.3 GeV region. However, unitarity requires more structure which we model by a four derivative contact term or by a low lying scalar resonance which is presumably subleading in the $\dfrac{1}{N_c}$ expansion, but may nevertheless be important. The latter two flavor model gives a reasonable description of the phase shift δ_0^0 up until around 860 MeV, before the effects associated which the $K\overline{K}$ threshold come into play.

---

[1] *e-mail:* sannino@nova.npac.syr.edu, sannino@axpna1.na.infn.it

[2] *e-mail:* schechter@suhep.phy.syr.edu

1 Introduction

The study of $\pi\pi$ scattering has been one of the classical methods for investigating the nature of the strong interaction. Many elegant ideas have been proposed [1]. At the present time the standard approach at low energies is based on chiral perturbation theory [2]. This enables one to nicely understand the scattering amplitudes near threshold($< 400~MeV$). However it is very difficult to extend this treatment to higher energies since the pole structure of the resonances in this region cannot be easily reproduced by a truncated power series expansion in energy. Our interest in this paper will be to investigate in a schematic sense how the description of $\pi\pi$ scattering might be extended up to slightly past the 1 GeV region in a new chiral picture.

It has been clear for many years that in the region just beyond the threshold region the effects of ρ exchange dominate. However in the chiral perturbation program the effects of the ρ arise in the second order of the energy expansion [3]. This is of course due to the fact that the usual chiral program is mainly devoted to improving the description of the dynamics in the threshold region in which the ρ does not explicitly appear. For going beyond the threshold region we would like an approach which can treat the ρ and other resonances at the first stage of an iterative procedure.

Such an approach is suggested by the large N_c approximation to QCD. As reviewed in [4] for example the leading order $\dfrac{1}{N_c}$ approximation to $\pi\pi$ scattering is obtained by summing all possible tree diagrams corresponding to some effective Lagrangian which includes an infinite number [5] of bosonic resonances of each possible spin. In addition it is allowed to include all possible contact terms. This clearly has the right structure but initially seems to be so general as to be practically useless. Here we will argue that this may not be the case.

1

An amplitude constructed according to the above prescription will automatically satisfy crossing symmetry. On the other hand just calculating the tree approximation to an effective Lagrangian will not guarantee that unitarity is satisfied. This is the handle we will use to try to investigate additional structure. Unitarity has of course the consequence that the amplitude must have some suitable imaginary term which in the usual field theory is provided by loop diagrams. However the leading $\frac{1}{N_c}$ approximation will give a purely real amplitude away from the singularities at the direct $s - channel$ poles. We may consider the imaginary part of the leading $\frac{1}{N_c}$ amplitude to consist just of the sum of delta functions at each such singularity. Clearly, the real part has a much more interesting structure and we will mainly confine out attention to it. Furthermore we will assume that the singularities in the real part are *regularized* in a conventional way.

Unitarity has the further consequence that the real parts of the partial wave amplitudes must satisfy certain well known bounds. The crucial question is how these bounds get satisfied since, as we will see, individual contributions tend to violate them badly. At first one might expect that all of the infinite number of resonances are really needed to obtain cancellations. However the success of chiral dynamics at very low energies where none of the resonances have been taken into account suggests that this might not be the case. At the very lowest energy the theory is described by a chiral invariant contact interaction which, however, quite soon badly violates the unitarity bound. It will be observed that the ρ exchange tames this bad behavior dramatically so that the unitarity bound is not badly broken until around an energy beyond 2 GeV. This suggests that there is a *local* cancellation between resonances which enforces the bound. The local cancellation is not easily predicted but if true in general it would greatly simplify the task of extending the phenomenological description of scattering processes to higher energies. Including just the effects of the ρ and the π particles corresponds to including

2

just the $s - wave$ quark antiquark states in our model. A natural next step , which we shall also explore here, would be to include the exchange of the allowed $p - wave$ quark antiquark states. At the same time the quark model suggests that we include the first radial excitations of the s-wave states which in fact lie near the $p - wave$ states. Theoretically glueball and exotic states are suppressed in $\pi\pi$ scattering according to the large N_c approximation [4].

In our analysis the pions will be treated as approximate Goldstone bosons corresponding to the assumption that the theory has a spontaneously broken chiral symmetry. Actually the need for spontaneous breakdown of this symmetry can be argued from the $\frac{1}{N_c}$ approach itself [6]. The effective Lagrangian we use shall be constructed to respect this symmetry. Furthermore for the purpose of the initial exploration being performed here we shall consider resonance interaction terms with the minimum number of derivatives and shall also neglect chiral symmetry breaking terms involving the resonances. When going beyond the initial stage we will be forced to proceed in a more phenomenological way.

In section 2, after the presentation of the partial wave amplitudes of interest, we show how the introduction of the $\rho-$meson in a chirally invariant manner substantially delays the onset of the severe unitarity violation which would be present in the simplest chiral lagrangian of pions. The program suggested by this *local cancellation* is sketched.

Section 3 is concerned with the contribution to the pion scattering amplitude from the *next group* of resonances - those in the range of the $p - wave$ $\overline{q}q$ bound states in the quark model. It is observed that a four derivative contact term can be used to restore unitarity up to about $1\ GeV$.

In section 4, a possibly more physical way to restore unitarity is presented which makes use of a $\frac{1}{N_c}$ subleading contribution due to a very low mass scalar (presumably $\overline{q}q\overline{q}q$) state.

Finally, section 5 contains a brief summary and discussion of some directions for future

work.

2 Current algebra plus ρ exchange

In this section we will study the partial waves for $\pi\pi$ scattering computed in a chiral La-
grangian model which contains both the pseudoscalar and vector mesons, (i.e., the lowest
lying s-wave quark antiquark bound states).

The kinematics are discussed in Appendix A, where the partial wave amplitudes T_l^I are
defined. They have the convenient decomposition:

$$T_l^I(s) = \frac{(\eta_l^I(s) \ e^{2i\delta_l^I(s)} - 1)}{2i} \tag{2.1}$$

where $\delta_l^I(s)$ are the phase shifts and $\eta_l^I(s)$ (satisfying $0 < \eta_l^I(s) \leq 1$) are the elasticity
parameters. Extracting the real and imaginary parts via

$$R_l^I = \frac{\eta_l^I \ sin(2\delta_l^I)}{2}, \tag{2.2}$$

$$I_l^I = \frac{1 - \eta_l^I \ cos(2\delta_l^I)}{2}, \tag{2.3}$$

leads to the very important bounds

$$|R_l^I| \leq \frac{1}{2}, \qquad 0 \leq I_l^I \leq 1. \tag{2.4}$$

For fixed η_l^I the real and imaginary parts lie on the well known circle in the Argand-plane
$R_l^{I\,2} + (I_l^I - \frac{1}{2})^2 = (\frac{\eta_l^I}{2})^2$. This formula also enables us to solve for I_l^I as :

$$I_l^I = \frac{1}{2}\left[1 \pm \sqrt{\eta_l^{I\,2} - 4R_l^{I\,2}}\right]. \tag{2.5}$$

Let us use (2.5) for an initial orientation. Near threshold $\eta_l^I = 1$, R_l^I is small and we should
choose the minus sign in (2.5) so that

$$I_l^I(s) \approx [R_l^I]^2. \tag{2.6}$$

4

In the large N_c limit the amplitude near threshold is purely real and of the order $\frac{1}{N_c}$. This is consistent with (2.6) which shows that $I_l^I(s)$ is of order $\frac{1}{N_c^2}$ and hence comes in at the second order. This agrees with the chiral perturbation theory approach [2] in which $R_l^I(s)$ comes from the lowest order tree diagram while I_l^I arises from the next order loop diagram. On the other hand, when we depart from the threshold region the $\frac{1}{N_c}$ approach treats the contribution of the ρ-meson at first order while the chiral perturbation theory approach treats it at second and higher orders.

Now, pion physics at very low energies is described by the effective chiral Lagrangian,

$$L_1 = -\frac{F_\pi^2}{8} Tr(\partial_\mu U \partial_\mu U^\dagger) + Tr(B(U + U^\dagger)) \tag{2.7}$$

wherein $U = e^{2i\frac{\phi}{F_\pi}}$ and ϕ is the 3×3 matrix of pseudoscalar fields. $F_\pi = 132\ MeV$ is the pion decay constant. Furthermore $B = diag(B_1, B_1, B_3)$, where $B_1 = \frac{F_\pi^2 m_\pi^2}{8}$ and $B_3 = \frac{F_\pi^2(m_K^2 - \frac{m_\pi^2}{2})}{4}$, describes the minimal symmetry breaking. We shall choose $m_\pi = 137\ MeV$.

A straightforward computation using (2.7) yields the $\pi\pi$ scattering amplitude [7] defined in (A.1):

$$A(s,t,u) = 2\frac{(s - m_\pi^2)}{F_\pi^2}. \tag{2.8}$$

This equation will be called the *current algebra result*. With (A.2)-(A.4) we obtain $R_0^0(s) = T_0^0(s)$ as illustrated in Fig. 1. The experimental Roy curves [8] are also shown. Up till about 0.5 GeV the agreement is quite reasonable (and can be fine tuned with second order chiral perturbation terms) but beyond this point R_0^0 keeps increasing monotonically and badly violates the unitarity bound (2.4). We will see that the introduction of the ρ-meson greatly improves the situation.

5

There are several different but essentially equivalent ways to introduce vector mesons into the chiral invariant Lagrangian. A simple way [9] is to treat the vectors as gauge particles of the chiral group and then break the local symmetry by introducing mass-type terms. The 3×3 matrix of the vector fields, ρ_μ is related to auxiliary linearly transforming gauge fields A_μ^L and A_μ^R by

$$A_\mu^L = \xi \rho_\mu \xi^\dagger + \frac{i}{\tilde{g}} \xi \partial_\mu \xi^\dagger \tag{2.9}$$

$$A_\mu^R = \xi^\dagger \rho_\mu \xi + \frac{i}{\tilde{g}} \xi^\dagger \partial_\mu \xi, \tag{2.10}$$

where $\xi \equiv U^{\frac{1}{2}}$ and $\tilde{g}$ is a dimensionless coupling constant. Under a chiral transformation $U \to U_L U U_R^\dagger$ [10],

$$\xi \to U_L \xi K^\dagger \equiv K \xi U_R^\dagger \tag{2.11}$$

(which also defines the matrix $K(\phi, U_L, U_R)$ and ρ_μ behaves as

$$\rho_\mu \to K \rho_\mu K^\dagger + \frac{i}{\tilde{g}} K \partial_\mu K^\dagger. \tag{2.12}$$

It is convenient to define

$$v_\mu = \frac{i}{2} \left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) \tag{2.13}$$

$$p_\mu = \frac{i}{2} \left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) \tag{2.14}$$

which transform as

$$p_\mu \to K p_\mu K^\dagger \tag{2.15}$$

$$v_\mu \to K v_\mu K^\dagger + i K \partial_\mu K^\dagger. \tag{2.16}$$

These quantities enable us to easily construct chiral invariants and will also be useful later [11]. The chiral Lagrangian including both pseudoscalars and vectors that one gets can be rewritten as the sum of L_1, in (2.7) and the following:

6

$$L_2 = -\frac{1}{4}Tr(F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)) - \frac{m_\rho^2}{2\tilde{g}^2}Tr\left[(\tilde{g}\rho_\mu - v_\mu)^2\right], \tag{2.17}$$

where $F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu]$. The coupling constant $\tilde{g}$ is related to the ρ-meson width by

$$\Gamma(\rho \to 2\pi) = \frac{g_{\rho\pi\pi}^2 p_\pi^3}{12\pi m_\rho^2} \qquad g_{\rho\pi\pi} = \frac{m_\rho^2}{\tilde{g}F_\pi^2}. \tag{2.18}$$

We choose $m_\rho = 0.769 \ GeV$ and $g_{\rho\pi\pi} = 8.56$. Symmetry breaking contributions involving the ρ are given elsewhere [12] but are small and will be neglected here. The Lagrangian piece in (2.17) yields both a pole-type contribution (from the $\rho_\mu v_\mu$ cross term) and a contact term contribution (from the $v_\mu v_\mu$ term) to the amplitude at tree level [9]:

$$A(s,t,u) = (2.8) - \frac{g_{\rho\pi\pi}^2}{2}\left(\frac{u-s}{m_\rho^2 - t} + \frac{t-s}{m_\rho^2 - u}\right) + \frac{g_{\rho\pi\pi}^2}{2m_\rho^2}[(t-s) + (u-s)] \tag{2.19}$$

We notice that the entire second-term in (2.17) is chiral invariant since v_μ and $\tilde{g}\rho_\mu$ transform identically. However the $Tr(\rho_\mu v_\mu)$ and $Tr(v_\mu v_\mu)$ pieces are not separately chiral invariant. This shows that the addition of the ρ meson in a chiral invariant manner necessarily introduces a contact term in addition to the minimal pole term. Adding up the terms in (2.19) yields finally

$$A(s,t,u) = 2\frac{(s-m_\pi^2)}{F_\pi^2} - \frac{g_{\rho\pi\pi}^2}{2m_\rho^2}\left[\frac{t(u-s)}{m_\rho^2 - t} + \frac{u(t-s)}{m_\rho^2 - u}\right] \tag{2.20}$$

In this form we see that the threshold (current algebra) results are unaffected since the second term drops out at $t = u = 0$. An alternative approach [13] to obtaining (2.20) involves introducing a chiral invariant $\rho\pi\pi$ interaction with two more derivatives.

$A(s,t,u)$ has no singularities in the physical region. Reference to (A.2) shows that the isospin amplitudes T^0 and T^2 also have no singularities. However the T^1 amplitude has the expected singularity at $s = m_\rho^2$. This may be cured in a conventional way, while still

maintaining crossing symmetry, by the replacements

$$\frac{1}{m_\rho^2 - t, u} \rightarrow \frac{1}{m_\rho^2 - t, u - im_\rho\Gamma_\rho} \tag{2.21}$$

in (2.20) [3]. A modification of this sort would enter automatically if we were to carry the computation to order $\frac{1}{N_c^2}$. However we shall regard (2.21) as a phenomenological regularization of the leading amplitude.

Now let us look at the actual behavior of the real parts of the partial wave amplitudes. R_0^0, as obtained from (2.20) with (2.21), is graphed in Fig. 2 for an extensive range of $\sqrt{s}$, together with the *pions only* result from (2.8). We immediately see that there is a remarkable improvement; the effect of adding ρ is to bend back the rising $R_0^0(s)$ so there is no longer a drastic violation of the unitarity bound until after $\sqrt{s} = 2$ GeV. There is still a relatively small violation which we will discuss later. Note that the modification (2.21) plays no role in the improvement since it is only the non singular t and u channel exchange diagrams which contribute.

It is easy to see that the *delayed* drastic violation of the unitarity bound $|R_l^I| \leq \frac{1}{2}$ is a property of all partial waves. We have already learned from (2.20) that the amplitude $A(s, t, u)$ starts out rising linearly with s. Now (2.19) and (A.3) show that for large s the ρ exchange terms behave as s^0. The leading large s behavior will therefore come from the sum of the original *current-algebra* term and the new *contact-term*:

$$A(s, t, u) \sim \frac{2s}{F_\pi^2}\left(1 - \frac{3k}{4}\right), \qquad k \equiv \frac{m_\rho^2}{\tilde{g}^2 F_\pi^2}. \tag{2.22}$$

But k is numerically around 2 [14], so $A(s, t, u)$ eventually *decreases* linearly with s. This turn-around, which is due to the contact term that enforces chiral symmetry, delays the onset of drastic unitarity violation until well after the ρ mass. It thus seems natural to speculate

---

[3]One gets a slightly different results if the the regularization is applied to (2.19).

that, as we go up in energy, the leading tree contributions from the resonances we encounter (including both crossed channel as well as s-channel exchange) conspire to keep the $R_l^I(s)$ within the unitarity bound. We will call this possibility, which would require that additional resonances beyond the ρ come into play when $R_l^I(s)$ from (2.20) start getting out of bound, *local* cancellation.

In Fig. 3 we show the partial waves R_1^1 and I_1^1 computed using (2.20) and (2.21). Not surprisingly, these display the standard resonant forms. For completeness we present the R_0^2 and R_2^0 amplitudes in Fig. 4. We may summarize by saying that the results of this section suggest investigating the following recipe for a reasonable approximation to the $\pi\pi$ scattering amplitude up to a certain scale E_{max}.

1. Include all resonances whose masses are less than $E_{max} + \Delta$, where $\Delta \approx several\text{-}hundred$ MeV. This express the hoped for local cancellation property.

2. Construct all possible chiral invariants which can contribute, presumably using the minimal number of derivatives. Compute all $\pi\pi \to \pi\pi$ tree diagrams, including contact terms. *Regulate* the resonance denominators in a manner similar to (2.21), but restrict attention to the real part. Interpret the manifestly crossing symmetric result as the leading order in $\dfrac{1}{N_c}$ real $\pi\pi$ scattering amplitude.

3. Obtain the imaginary parts of the partial wave amplitudes, using (2.5). The $\eta_l^I(s)$ might be computed by including channels other than $\pi\pi$.

We will start to explore this program by checking whether the inclusion of the *next group* of resonances does enable us to satisfy the unitary bound for R_0^0. For simplicity we restrict ourselves to a two-flavor framework. It is straightforward to generalize the scheme to three flavors.

3 The next group of resonances

To keep our investigation manageable we shall mainly restrict attention to the partial wave amplitude $R_0^0(s)$. As we saw in the last section, this is the one most likely to violate the unitarity bound. The first task is to find the effective lagrangian which should be added to (2.7) plus (2.17). There is no *a priori* reason not to add chiral invariant $\pi\pi$ contact interactions with more than two derivatives. But the most characteristic feature, of course, is the pionic interactions of the new resonances in the energy range of interest, here up to somewhat more than 1.0 GeV. Which resonances should be included ? In the leading $\dfrac{1}{N_c}$ approximation, $\overline{q}q$ mesons are [4] ideally mixed nonets, assuming three light quark flavors. Furthermore, the exchange of glueballs and exotic mesons are suppressed in interactions with the $\overline{q}q$ mesons. The $\dfrac{1}{N_c}$ approximation thus directs our attention to the $p-wave$ $\overline{q}q$ resonances as well as the radial excitations of the $s-wave$ $\overline{q}q$ resonances.

The neutral members of the $p-wave$ $\overline{q}q$ nonets have the quantum numbers $J^{PC} = 0^{++}$, 1^{++}, 1^{+-} and 2^{++}. Of course, the neutral members of the radially excited $s-wave$ $\overline{q}q$ nonets have $J^{PC} = 0^{-+}$ and 1^{--}. Only members of the 0^{++}, 1^{--} and 2^{++} nonets can couple to two pseudoscalars [4]. By G-parity conservation we finally note that it is the $I = 0$ member of the 0^{++} and 2^{++} nonets and the $I = 1$ member of the 1^{--} nonet which can couple to two pions. Are there good experimental candidates for these three particles ?

The cleanest case is the lighter $I = 0$ member of the 2^{++} nonet; the $f_2(1270)$ has, according to the August 1994 Review of Particle Properties (RPP) [15], the right quantum numbers, a mass of 1275 ± 5 MeV, a width of 185 ± 20 MeV, a branching ratio of 85% into two pions, and a branching ratio of only 5% into $K\overline{K}$. On the other hand the $f_2'(1525)$

---

[4]It is possible to write down a two point mixing interaction between 0^{-+} and radially excited 0^{-+} particles etc.., but we shall neglect such effects here.

has a 1% branching ratio into $\pi\pi$ and a 71% branching ratio into $K\overline{K}$. It seems reasonable to approximate the 2^{++} nonet as an ideally mixed one and to regard the $f_2(1270)$ as its non-strange member.

The $\rho(1450)$ is the lightest listed [15] particle which is a candidate for a radial excitation of the usual $\rho(770)$. It has a less than 1% branching ratio into $K\overline{K}$ but the $\pi\pi$ branching ratio, while presumably dominant, is not yet known. With this caution, we shall use the $\rho(1450)$. The $\rho(1700)$ is a little too high for our region of interest.

An understanding of the $I = 0$, 0^{++} channel has been elusive despite much work. The RPP [15] gives two low lying candidates: the $f_0(980)$ which has a 22% branching ratio into $K\overline{K}$ even though its central mass is below the $K\overline{K}$ threshold and the $f_0(1300)$ which has about a 93% branching ratio into $\pi\pi$ and a 7% branching ratio into $K\overline{K}$. We shall use the $f_0(1300)$ here. It is hard to understand why, if the $f_0(980)$ is the $\overline{s}s$ member of a conventional 0^{++} nonet, it is lighter than the $f_0(1300)$. Most likely, the $f_0(980)$ is an exotic or a $K\overline{K}$ *molecule* [16]. If that is the case, its coupling to two pions ought to be suppressed in the $\dfrac{1}{N_c}$ picture. This is experimentally not necessarily true but we will postpone a discussion of the $f_0(980)$ as well as other possible light 0^{++} resonances to the next section.

Now we will give, in turn, the $\pi\pi$ scattering amplitudes due to the exchange of the $f_0(1300)$, the $f_2(1270)$ and the $\rho(1450)$.

3.1   The $f_0(1300)$

Denoting a 3×3 matrix of scalar fields by S we require that it transform as $S \to KSK^\dagger$, (see (2.11)) under, the chiral group. A suitable chiral invariant interaction, using (2.14), is

$$L_{f_0} = -\gamma_0 F_\pi^2 Tr(Sp_\mu p_\mu) = -\gamma_0 Tr(S\partial_\mu\phi\partial_\mu\phi) + \cdots \tag{3.1}$$

11

where the expansion of ξ was used in the second step. It is interesting to note that, in the present formalism, chiral symmetry demands that the minimal $S\phi\phi$ interaction must have two derivatives. Specializing to the particles of interest and taking f_0 to be ideally mixed leads to

$$L_{f_0} = -\frac{\gamma_0 f_0}{\sqrt{2}}(\partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}) + \cdots .\tag{3.2}$$

The partial width for $f_0(1300) \to \pi\pi$ is then

$$\Gamma(f_0(1300) \to \pi\pi) = \frac{3\gamma_0^2}{64\pi M_{f_0}}\left(1 - \frac{4m_\pi^2}{M_{f_0}}\right)^{\frac{1}{2}}\left(M_{f_0}^2 - 2m_\pi^2\right)^2 \tag{3.3}$$

The RPP[15] lists $\Gamma_{tot}(f_0(1300)) = 0.15 - 0.40 \ GeV$ and $M_{f_0} = 1.0 - 1.5 \ GeV$. For definiteness we shall choose $\Gamma_{tot}(f_0(1300)) = 0.275 \ GeV$ and $M_{f_0} = 1.3 \ GeV$. These yield $|\gamma_0| = 2.88 \ GeV^{-1}$. Using (3.2) we find the contribution of f_0 exchange to the $\pi\pi$ scattering amplitude, defined in (A.1), to be:

$$A_{f_0}(s, t, u) = \frac{\gamma_0^2}{2}\frac{(s - 2m_\pi^2)^2}{M_{f_0}^2 - s}.\tag{3.4}$$

Actually, as discussed around (2.21), the singularity in the real part of (3.4) will be regulated by the replacement

$$\frac{1}{M_{f_0}^2 - s} \to \frac{M_{f_0}^2 - s}{(M_{f_0}^2 - s)^2 + M_{f_0}^2\Gamma^2}.\tag{3.5}$$

3.2 The $f_2(1270)$

We represent the 3×3 matrix of tensor fields by $T_{\mu\nu}$ (satisfying $T_{\mu\nu} = T_{\nu\mu}$, and $T_{\mu\mu} = 0$) which is taken to behave as $T_{\mu\nu} \to KT_{\mu\nu}K^\dagger$ under chiral transformation. A suitable chiral invariant interaction is

$$L_T = -\gamma_2 F_\pi^2 Tr(T_{\mu\nu}p_\mu p_\nu).\tag{3.6}$$

12

Specializing to the particles of interest, this becomes

$$L_{f_2} = -\frac{\gamma_2}{\sqrt{2}}(f_2)_{\mu\nu}(\partial_\mu\vec{\pi} \cdot \partial_\nu\vec{\pi}) + \cdots. \tag{3.7}$$

In this case we note that the chiral invariant interaction is just the same as the minimal one we would have written down without using chiral symmetry. The partial width is then

$$\Gamma(f_2(1270) \to \pi\pi) = \frac{\gamma_2^2}{20\pi}\frac{p_\pi^5}{M_{f_2}^2} \tag{3.8}$$

where p_π is the pion momentum in the f_2 rest frame. This leads to $|\gamma_2| = 13.1 \ GeV^{-1}$.

To calculate the f_2 exchange diagram we need the spin 2 propagator [17]

$$\frac{-i}{M_{f_2}^2 + q^2}\left[\frac{1}{2}\left(\theta_{\mu_1\nu_1}\theta_{\mu_2\nu_2} + \theta_{\mu_1\nu_2}\theta_{\mu_2\nu_1}\right) - \frac{1}{3}\theta_{\mu_1\mu_2}\theta_{\nu_1\nu_2}\right], \tag{3.9}$$

where

$$\theta_{\mu\nu} = \delta_{\mu\nu} + \frac{q_\mu q_\nu}{M_{f_2}^2}. \tag{3.10}$$

A straightforward computation then yields the f_2 contribution to the $\pi\pi$ scattering amplitude:

$$\begin{aligned}
A_{f_2}(s,t,u) &= \frac{\gamma_2^2}{2(M_{f_2}^2 - s)}\left(-\frac{16}{3}m_\pi^4 + \frac{10}{3}m_\pi^2 s - \frac{1}{3}s^2 + \frac{1}{2}(t^2 + u^2)\right. \\
&\quad \left. -\frac{2}{3}\frac{m_\pi^2 s^2}{M_{f_2}^2} - \frac{s^3}{6M_{f_2}^2} + \frac{s^4}{6M_{f_2}^4}.\right)
\end{aligned} \tag{3.11}$$

Again the singularity will be regulated as in (3.5).

3.3   The $\rho(1450)$

We may read off the proper chiral invariant contribution[5] of the $\rho(1450)$ to the $\pi\pi$ scattering amplitude from the second term of (2.20)

$$A_{\rho'}(s,t,u) = -\frac{g_{\rho'\pi\pi}^2}{2m_{\rho'}^2}\left[\frac{t(u-s)}{m_{\rho'}^2 - t} + \frac{u(t-s)}{m_{\rho'}^2 - u}\right] \tag{3.12}$$

---

[5]It is not necessary to introduce the $\rho(1450)$ as a massive gauge field as we did for the $\rho(770)$, but the answer is the same. See [13] for further discussions.

where $g_{\rho'\pi\pi}$ is related to the $\rho' \to \pi\pi$ partial width by

$$\Gamma(\rho' \to 2\pi) = \frac{g_{\rho'\pi\pi}^2 p_\pi^3}{12\pi m_{\rho'}^2}. \tag{3.13}$$

We shall use $|g_{\rho'\pi\pi}| = 7.9$ corresponding to $\Gamma(\rho \to 2\pi) = 288 \ MeV$.

3.4 $f_0(1300) + f_2(1270) + \rho(1450)$

Now we are in a position to appraise the contribution to R_0^0 of the next group of resonances. This is obtained by adding up (3.4), (3.11) and (3.12) and using (A.2)$-$(A.4). The individual pieces are shown in Fig. 5.

Note that the $f_0(1300)$ piece is not the largest, as one might at first expect. That honor goes to the f_2 contribution which is shown divided into the s-channel pole piece and the $(t+u)$ pole piece. We observe that the s-channel pole piece, associated with the f_2, vanishes at $\sqrt{s} = M_{f_2}$. This happens because the numerator of the propagator in (3.9) is precisely a spin 2 projection operator at that point. The $\rho(1450)$ contribution is solely due to the t and u channel poles. It tends to cancel the t and u channel pole contributions of the $f_2(1270)$ but does not quite succeed. The t and u channel pole contributions of the $f_0(1300)$ turn out to be negligible. Notice the difference in characteristic shapes of the s and $(t+u)$ exchange curves. Fig. 6 shows the sum of all these individual contributions. There does seem to be cancellation. At the high end, R_0^0 starts to run negative well past the unitarity bound (2.4) around 1.5 GeV. But it is reasonable to expect resonances in the $1.5 - 2.0 \ GeV$ region to modify this. The maximum positive value of R_0^0 is about 1 at $\sqrt{s} = 1.2 \ GeV$. This would be acceptable if the $\pi + \rho$ contribution displayed in Fig. 2, and which must be added to the curve of Fig. 6, were somewhat negative at this point. However this is seen not to be the case, so some extra ingredient is required. The $\dfrac{1}{N_c}$ approach still allows us the freedom of

14

adding four, and higher derivative contact terms. More physically, there is known to be a rather non trivial structure below 1 GeV in the $I = J = 0$ channel.

3.5 4−Derivative Contact Terms.

First let us experiment with four-derivative contact terms. So far we have not introduced any arbitrary parameters but now we will be forced to do so. There are two four-derivative chiral invariant contact interactions which are single traces in flavor space:

$$L_4 = a\; Tr(\partial_\mu U \partial_\nu U^\dagger \partial_\mu U \partial_\nu U^\dagger) + b\; Tr(\partial_\mu U \partial_\mu U^\dagger \partial_\nu U \partial_\nu U^\dagger) \tag{3.14}$$

where a and b are real constants. The single traces should be leading in the $\dfrac{1}{N_c}$ expansion. Notice that the magnitudes of a and b will differ from those in the chiral perturbation theory approach [2] since the latter essentially also include the effects of expanding the ρ exchange amplitude up to order s^2. The four pion terms which result from (3.14) are:

$$L_4 = \frac{8}{F_\pi^4}\left[2a\,(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi})^2 + (b-a)\,(\partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi})^2\right] + \cdots. \tag{3.15}$$

This leads to the contribution to the $\pi\pi$ amplitude:

$$A_4(s,t,u) = \frac{16}{F_\pi^4}\left[a\left((t - 2m_\pi^2)^2 + (u - 2m_\pi^2)^2\right) + (b-a)(s - 2m_\pi^2)^2\right]. \tag{3.16}$$

Plausibly, but somewhat arbitrarily, we will require that (3.16) yields no correction at threshold, i.e. at $s = 4m_\pi^2$, $t = u = 0$. This gives the condition $b = -a$ and leaves the single parameter a to play with. In Fig. 7 we show R_0^0, as gotten by adding the piece obtained from (3.16) for several values of a to the contribution of $\pi + \rho$, plus that of the *next group* of resonances. For $a = +1.0 \times 10^{-3}$ the four-derivative contact term can pull the curve for R_0^0 down to avoid violation of the unitarity bound until around $\sqrt{s} = 1.0\ GeV$. The price to be paid is that R_0^0 decreases very rapidly beyond this point. We consider this to be an undesirable feature

15

since it would make a possible local cancellation scheme very unstable. Another drawback of the four-derivative contact term scheme is that it lowers $R_0^0(s)$ just above threshold, taking it further away from the Roy curves. Let us therefore set aside the possibility of four and higher derivative contact terms and try to find a solution to the problem of keeping R_0^0 within the unitarity bounds in a different and more phenomenological way.

4   Low energy structure

Let us investigate the addition of low energy [18] *exotic* states whose contributions to $\pi\pi$ scattering should be formally suppressed in the large N_c limit. Experimentally we know that there is at least one candidate - the $f_0(980)$ mentioned in the previous section. According to the RPP [15] its width is $40 - 400 MeV$ and its branching ratio into two pions is $78.1 \pm 2.4\%$. To get an idea of its effect we will choose $\Gamma_{tot} = 40 MeV$ and use the formulas (3.2)-(3.5) with the appropriate parameters. In Fig. 8 we show the sum of the contributions of the *next group* added to $\pi + \rho$ with the effect of including the $f_0(980)$. It is seen that the $f_0(980)$ does not help unitarity - below $\sqrt{s} = 980 MeV$ it makes the situation a little worse while above it improves the picture slightly.

What is needed to restore unitarity over the full range of interest and to give better agreement with the experimental data for $\sqrt{s} \lesssim 900\ MeV$?

i. Below $450\ MeV$, $R_0^0(s)$ actually lies a little below the Roy curves. Hence it would be nice to find a tree level mechanism which yields a small positive addition in this region.

ii. In the $600 - 1300\ MeV$ range, an increasingly negative contribution is clearly required to keep R_0^0 within the unitarity bound.

It is possible to satisfy both of these criteria by introducing a broad scalar resonance (like

the old σ) with a mass around 530 MeV. Its contribution to $A(s,t,u)$ would be of the form shown in (3.4) and (3.5) which we may rewrite as:

$$\frac{32\pi}{3H}\frac{G}{M_\sigma^3}\frac{(s-2m_\pi^2)^2(M_\sigma^2-s)}{(s-M_\sigma^2)^2+M_\sigma^2 G'^2}, \tag{4.1}$$

where we have set $\dfrac{\gamma_0^2}{2}\to\dfrac{32\pi}{3H}\dfrac{G}{M_\sigma^3}$, $\Gamma\to G'$ and $H=\left(1-4\dfrac{m_\pi^2}{M_\sigma^2}\right)^{\frac{1}{2}}\left(1-2\dfrac{m_\pi^2}{M_\sigma^2}\right)^2$ is approximately one. If this were a typical resonance which was narrow compared to its mass and which completely dominated the amplitude, we would set $G\approx G'\approx\Gamma$. However for a very broad resonance it may be reasonable to regard G' as a phenomenological parameter which could be considered as a regulator in the sense we have been using. Choosing $M_\sigma=0.53\ GeV$, $\dfrac{G}{G'}=0.31$ and $G'=380\ MeV$, the contribution to R_0^0 of (4.1) is shown in Fig. 9. The curve goes through zero near $0.53\ GeV$ (there is a small shift due to the crossed terms). Below this value of $\sqrt{s}$ it adds slightly in accordance with point i while above $0.53\ GeV$ it subtracts substantially in the manner required by point ii. This is the motivation behind our choice of $M_\sigma=0.53\ GeV$. Adding *everything* - namely the $\pi+\rho$ piece, the *next group* piece together with the contributions from (4.1) and the $f_0(980)$ - results in the curves shown in Fig. 10 for three values of G'. These curves for $R_0^0(s)$ satisfy the unitarity bound $|R_0^0|\le\frac{1}{2}$ until $\sqrt{s}\approx 1.3\ GeV$. After $1.3\ GeV$, the curves drop less precipitously than those for the four-derivative contact term in Fig. 7.

Fig. 10 demonstrates that the proposed *local cancellation* of the various resonance exchange terms is in fact possible as a means of maintaining the unitarity bound for the (by construction) crossing symmetric real part of the tree amplitude. Essentially, just the three parameters M_σ, G and G' have been varied to obtain this. The other parameters were all taken from experiment; when there were large experimental uncertainties, we just selected typical values and made no attempt to fine-tune. Procedurally, G', G and M_σ were adjusted

to obtain a best fit to the Roy Curves below 700 MeV; this turned out to be what was needed for unitarity beyond 700 MeV. It was found that M_σ had to lie in the 530 ± 30 MeV range and that $\dfrac{G'}{G}$ had to be in the 0.31 ± 0.06 range in order to achieve a fit. On the other hand G' could be varied in the larger range 500 ± 315 MeV. It is also interesting to notice that the main effect of the sigma particle comes from its tail in Fig. 9. Near the pole region, its effect is hidden by the dominant $\pi + \rho$ contribution. This provides a possible explanation of why such a state may have escaped definitive identification. For the purpose of comparison we show in Fig. 11, the total R_0^0 together with the $\pi + \rho$ and *current-algebra* curves in the low energy region.

It is interesting to remark that particles with masses and widths very similar to those above for the σ and the $f_0(980)$ were predicted [19] as part of a multiquark $qq\bar{q}\bar{q}$ nonet on the basis of the MIT bag model. Hence, even though they do not give rise to formally leading $\pi\pi$ amplitudes in the $\dfrac{1}{N_c}$ scheme, the picture has a good deal of plausibility from a polology point of view. It is not hard to imagine that some $\dfrac{1}{N_c}$ subleading effects might be important at low energies where the QCD coupling constant is strongest.

Other than requiring $|R_0^0| \leq \dfrac{1}{2}$ we have not attempted to fit the puzzling experimental results in the the $f_0(980)$ region. Recent interesting discussions are given in refs [18]. It appears that the opening of the $K\overline{K}$ channel plays an important role and furthermore, additional resonances may be needed. In this paper we have restricted attention to the $\pi\pi$ channel (although the effective Lagrangian was written down for the case of three light quarks). Clearly, it would be interesting to study the $f_0(980)$ region in the future, according to the present scheme.

4.1 Imaginary part and Phase Shift

Finally, let us discuss the imaginary piece I_0^0. In the leading $\dfrac{1}{N_c}$ limit the imaginary part vanishes away from the singularities at the poles, whereas R_0^0 has support all over. This suggests that we determine an approximation to I_0^0 from the $\dfrac{1}{N_c}$ leading R_0^0 using dispersion theory, rather than getting it directly from the tree amplitude with the regularization of the form (2.21). The latter procedure picks up pion loop contribution to the $\rho - propagator$, for example, but misses very important direct pion loop contributions. A dispersion approach will include both. In the low energy region, we can proceed more simply by just using the unitarity formula (2.5) directly. Up until the $K\overline{K}$ threshold it seems to be reasonable to approximate the elasticity function $\eta_0^0(s)$ by unity [18]. Strictly speaking $\eta_0^0(s)$ may depart from unity at the $4\pi^0$ threshold[6] of $540\ MeV$. In Fig. 12 we show I_0^0 obtained from (2.5) on the assumption $\eta_0^0 = 1$ for several values of G'. Both signs in front of the square root are displayed. Of course, the correct curve should start from zero at threshold ($-$ sign in front of the square root). Continuity of $I_0^0(s)$ would at first appear to suggest that we follow along the lower curve. In order to go continuously to the upper curve it is necessary that the argument of the square root vanish at some value of $\sqrt{s}$. With the approximation $\eta_0^0 = 1$, this vanishing occurs if $|R_0^0|$ is exactly $\dfrac{1}{2}$. In Fig. 12, the discontinuity in the $\sqrt{s} = 540\ MeV$ region is extremely sensitive to tiny departures of $|R_0^0|$ from $\dfrac{1}{2}$. However, both experiment and the expectation that $\dfrac{d\delta_l^I}{d\sqrt{s}} \geq 0$ [7] suggest that beyond $\sqrt{s} \approx 540\ MeV$ we should actually go to the upper curve ($+$ sign in front of the square root). This can be accomplished without

---

[6]It is amusing to note that each of the low energy resonances, i.e. $\sigma(530)$ and $f_0(980)$, are located just below threshold; for the σ it is the 4π threshold while for the $f_0(980)$ it is the $K\overline{K}$ threshold.

[7]In potential theory Wigner [20] has shown that $\dfrac{d\delta_l^I}{d\sqrt{s}} \geq -\dfrac{a}{\beta}$ where a is the approximate interaction radius and β is the pion velocity in the center of mass. Strictly speaking, for $a \lesssim 1.7\ fm$ the lower curve is also allowed.

violating continuity by assuming that η_0^0 is not precisely one. For the curves shown all that is required is a decrease in η_0^0 of not more than 0.04. Alternatively, we could choose parameters so that R_0^0 reaches 0.5 precisely. Then the fit at higher energies is slightly worse. The corresponding three curves for the phase shifts are shown in Fig. 13. The discontinuity should be smoothed over in accordance with our discussion above. The agreement with experiment is quite reasonable up to about 860 MeV. We did not go beyond this point for the purpose of obtaining the phase shift because we are neglecting the $K\overline{K}$ channel which becomes relevant in the computation of the imaginary part.

5 Summary and discussion

In the leading large N_c approximation to QCD, $\pi\pi$ scattering corresponds to the sum of an infinite number of tree diagrams which can be of the contact type or can involve resonance exchange. This can only be a practically useful approximation if it is possible to retain just a reasonably small number of terms. The most natural way to do so is, of course, to consider contact terms with as few derivatives as possible and exchange terms with resonances having masses less than the extent of the energy region we wish to describe. In this paper we have carried out an initial exploration of this program in a step by step way. The first step is to include only the well known chiral contact term which reasonably describes the scattering lengths. However this amplitude badly violates partial wave unitarity bounds (seen most readily in the $I = L = 0$ channel, see Fig. 1) at energies beyond 500 MeV. We observed that the introduction of the ρ meson dramatically improved the situation, delaying drastic violation of the unitarity bound till around 2 GeV (see Fig. 2). We noted that this effect could be nicely understood as the result of an extra contact term which must be present when the ρ is introduced in a chirally invariant manner. Furthermore, this feature holds in

the strict large N_c limit, i.e., without including the phenomenological regularization (2.21). The observed cancellation encouraged us to investigate the possibility of a more general *local cancellation*, due to inclusion of all (large N_c leading) resonances in the energy range of interest and, perhaps, higher derivative contact terms. The program is sketched at the end of section 2.

Taking the large N_c approach as well as the standard $\overline{q}q$ spectrum literally, we argued that the *next group* of resonances whose exchange contributes to the leading amplitude should comprise the $f_0(1300)$, the $f_2(1270)$ and the $\rho(1450)$. We observed (section 3) that there was a tendency for these to cancel among themselves; for example the crossed-channel exchanges of the $\rho(1450)$ tended to cancel against those of the $f_2(1270)$. In our analysis, the complications due to enforcing chiral symmetry and using the full spin 2 propagator were taken into account. However, the cancellation with both the $\pi + \rho$ and *next group* was not sufficient to satisfy the unitarity bound $|R_0^0| \leq \dfrac{1}{2}$ in the energy range till 1.3 GeV. An allowed leading N_c way out - by adding four derivative contact terms - was thus investigated. This enabled us to restore unitarity till about 1.0 GeV (see Fig. 7). The drawback was that $R_0^0(s)$ dropped off rather precipitously afterwards, which would make a local cancellation scheme very unstable.

As a more physically motivated alternative we investigated, in section 4, the possibility of including scalar resonances having masses less than 1 GeV. These are presumably not of the simple $\overline{q}q$ type and hence their exchange should be of sub-leading order in the large N_c limit. An interesting interpretation gives these particles a $\overline{q}q\overline{q}q$ quark structure [19]. Then a somewhat narrow state like the $f_0(980)$ is expected together with a very low mass and very broad state like the old σ meson. (Both should belong to a 3−flavor nonet). It was found that the $f_0(980)$ particle did not help much in restoring unitarity. In the experimentally puzzling region close to 980 MeV it is, however, expected to play an extremely important

role. On the othe hand, the further introduction of the other scalar, which we denoted as the $\sigma(530)$, treated with a phenomenological regularization parameter (see (4.1)) enabled us to satisfy the unitarity bound all the way up to 1.3 GeV (see Fig. 10). Thus, if low energy scalars are included, the proposed *local cancellation* may be a viable possibility. The imaginary part of the partial wave amplitude, $I_0^0(s)$ was also computed from the unitarity relation (2.5) and found to lead to a phase shift δ_0^0 in reasonable agreement with experiment until about 860 MeV. Beyond this point, the effect of the opening of the $K\overline{K}$ channel must be specifically included.

There are many directions for further work.

 i. The most straightforward is the investigation of different channels. For example, considering $\pi\pi \to K\overline{K}$ and $K\overline{K} \to K\overline{K}$ should enable us to study the interesting $K\overline{K}$ threshold region in a more detailed way. Looking at channels which don't communicate with $\pi\pi$ would enable one to focus on particular resonance exchanges.

 ii. The greatly increasing density of levels as one goes up in energy clearly indicates that there is a limit to how far one can go with the kind of *microscopic* approach presented here. It is expected that at energies not too much higher than the 1.3 GeV region this analysis should merge with some kind of string-like picture [5]. In that region the question of the validity of the $\dfrac{1}{N_c}$ expansion and a possible *local cancellation* can presumably be approached in a more analytical manner and interesting models can be studied [21]. Here we have tried to follow a phenomenologically oriented path, assuming only chiral dynamics in addition to the $\dfrac{1}{N_c}$ framework.

 iii. One can also imagine a kind of *Wilsonian effective action* [22] with which the present approach can be further discussed. This should allow the systematic calculation of

loops but would be extremely complicated in practice.

Acknowledgments

Appendix A

Here we list the kinematic conventions for $\pi\pi$ scattering. The invariant amplitude for $\pi_i + \pi_j \rightarrow \pi_k + \pi_l$ is decomposed as:

$$\delta_{ij}\delta_{kl}A(s,t,u) + \delta_{ik}\delta_{jl}A(t,s,u) + \delta_{il}\delta_{jk}A(u,t,s),\tag{A.1}$$

where s,t and u are the usual Mandelstam variables obeying $s+t+u = 4m_\pi^2$. Physical values lie in the region $s{\geq}4m_\pi^2,\ t{\leq}0, u{\leq}0$. (Note that the phase of (A.1) corresponds to simply taking the matrix element of the Lagrangian density of a four point contact interaction).

Projecting out amplitudes of definite isospin yields:

$$
\begin{aligned}
T^0(s,t,u) &= 3A(s,t,u) + A(t,s,u) + A(u,t,s),\\
T^1(s,t,u) &= A(t,s,u) - A(u,t,s),\\
T^2(s,t,u) &= A(t,s,u) + A(u,t,s).
\end{aligned}\tag{A.2}
$$

In the center of mass frame:

$$
\begin{aligned}
s &= 4(p_\pi^2 + m_\pi{}^2),\\
t &= -2p_\pi^2(1 - cos\theta),\\
u &= -2p_\pi^2(1 + cos\theta),
\end{aligned}\tag{A.3}
$$

where p_π is the spatial momentum and θ is the scattering angle. We then define the partial wave isospin amplitudes according to the following formula:

$$T_l^I(s) \equiv \frac{1}{64\pi}\sqrt{\left(1 - 4\frac{m_\pi^2}{s}\right)}\int_{-1}^{1} dcos\theta P_l(cos\theta)T^I(s,t,u).\tag{A.4}$$

24

References

[1] A good discussion of early work is given in S. Gasiorowicz, *Elementary Particle Physics* John Wiley and Sons, Inc., (1966).

[2] S. Weinberg, Physica, **96A**, 327 (1979). J. Gasser and H. Leutwyler Ann. of Phys. **158** (1984) 142; J. Gasser and H. Leutwyler **B250** (1985) 465. A recent review is given by Ulf-G. Meißner, Rept. Prog. Phys. 56, 903 (1993).

[3] J. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D39** 1947 (1989). G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B221** (1989) 311. N. Kaiser in *Effective Field Theories of the Standard Model*, Ulf-G. Meißner editor; p41.

[4] E. Witten, Nucl. Phys. **B160** (1979) 57. See also S. Coleman, *Aspects of Symmetry*, Cambridge University Press (1985). The original suggestion is given in G. 't Hooft, Nucl. Phys. **B72** (1974) 461.

[5] Such a situation exists in the Veneziano model; G. Veneziano, Nuovo Cim. **57A** (1968) 190. A modern perspective is given in M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory Vol 1* Cambridge University Press (1987).

[6] S. Coleman and E. Witten, Phys. Rev. Lett. **45** 100 (1980). G. Veneziano, Phys. Lett. **95B**, 90 (1980). A. Salomone, J. Schechter and T. Tudron, Phys. Rev. **D24**, 492 (1981).

[7] S. Weinberg, Phys. Rev. Lett. **17** 616 (1966).

[8] S.M. Roy, Phys. Lett. **36** 353 (1971). J.-L. Basdevant, C.D. Froggatt and J.L. Petersen, Nucl. Phys. **B72** (1974) 413. J.-L. Basdevant, P. Chapelle, C. Lopez and M.

Sigelle, Nucl. Phys. **B98** (1975) 285. C.D. Froggatt and J.L. Petersen, Nucl. Phys. **B129** (1977) 89.

[9] Ö. Kaymakcalan and J. Schechter, Phys. Rev. **D31** 1109 (1985).

[10] C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** 22409 (1969).

[11] A different approach leading to the same Lagrangian is reviewed in M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164** 217 (1988).

[12] J. Schechter, A. Subbaraman and H. Wiegel, Phys. Rev. **D48** 339 (1993).

[13] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. De Rafael, Phys. Lett. **223** 425 (1989).

[14] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** 255 (1966). Riazuddin and Fayyazuddin, Phys. Rev. **147** 107 (1966).

[15] Review of Particle Properties, K. Hikasa *et al.*, Phys. Rev. D **50** 1173 (1994).

[16] J. Weinstein and N. Isgur, Phys. Rev. **D41** 2236 (1990).

[17] G. Wentzel, *Quantum Theory of Fields*, Interscience (1949).

[18] A recent comprehensive discussion of $\pi\pi$ scattering with emphasis on the $f_0(980)$ region is given in D. Morgan and M.R. Pennington, Phys. Rev. **D48** 1185 (1993). Other recent theoretical work on low energy $\pi\pi$ scattering includes: N.N. Achasov and G.N. Shestakov, Phys. Rev. **D49**, 5779 (1994); R. Kaminski, L. Lesniak and J.P. Maillet Phys. Rev. **D50** 3145 (1994); C.D. Roberts, R.T. Cahill, M. E. Sevior and N. Iannella, Phys. Rev. **D49** 125 (1994); G. Janssen, B.C. Pearce, K. Holinde and J. Speth, preprint n. KFA-IKP(TH)-1994-40, ADP-94-23/T163.

[19] R.L. Jaffe, Phys. Rev. **D15** 281 (1977).

[20] E.P. Wigner, Phys. Rev. **98** 145 (1955).

[21] A somewhat different approach is discussed by A.A. Bolokhov, A.N. Manashov, M.V. Polyakov and V.V. Vereshagin, Phys. Rev. **D48** 3090 (1993).

[22] See J. Polchinski, *Effective Field Theory and the Fermi Surface*, in Proceedings Theoretical Advanced Study Institute on Elementary Particle Physics, Boulder, USA, (1993).
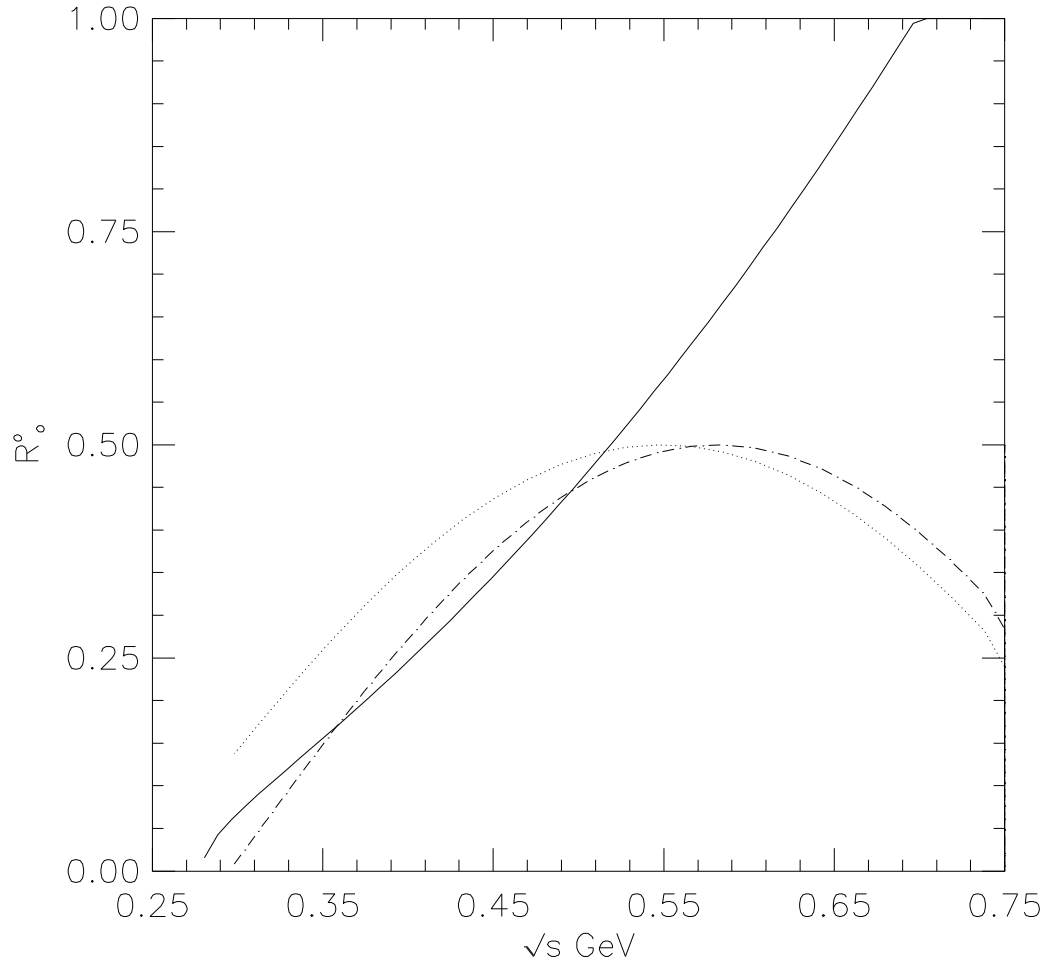
Fig. 1 The solid line is the current algebra result for R_0^0. The dotted and dot-dashed lines are the Roy curves for R_0^0.
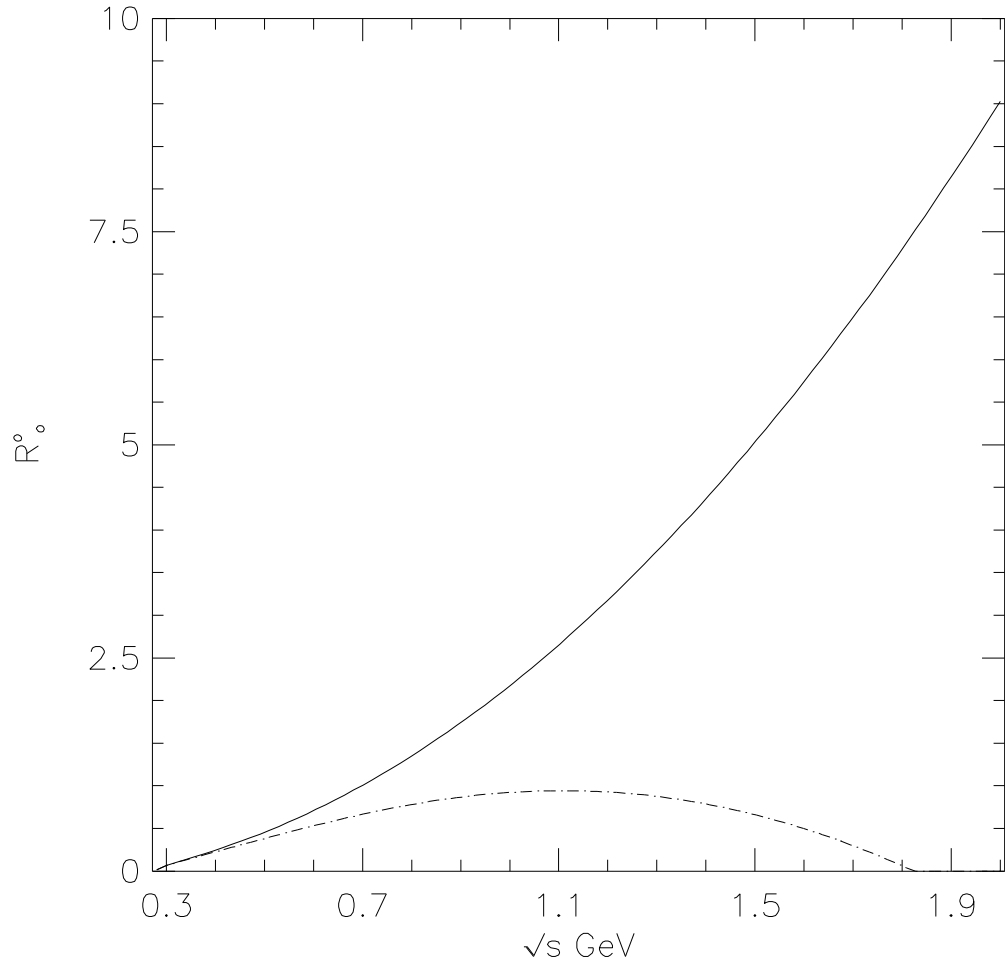
Fig. 2 The solid line is the current algebra result for R_0^0. The dot-dashed line is the $\rho + \pi$ result for R_0^0.
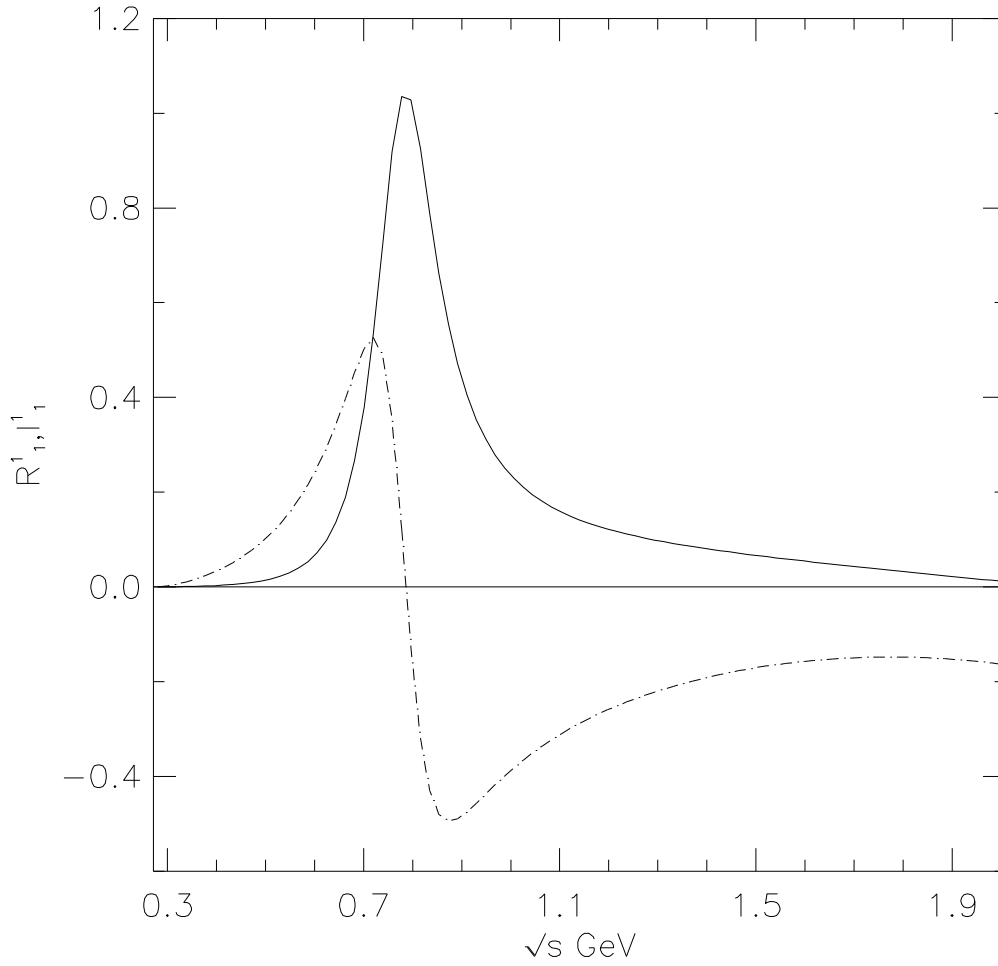
Fig. 3 The solid line is the imaginary part I_1^1. The dot-dashed line is R_1^1.
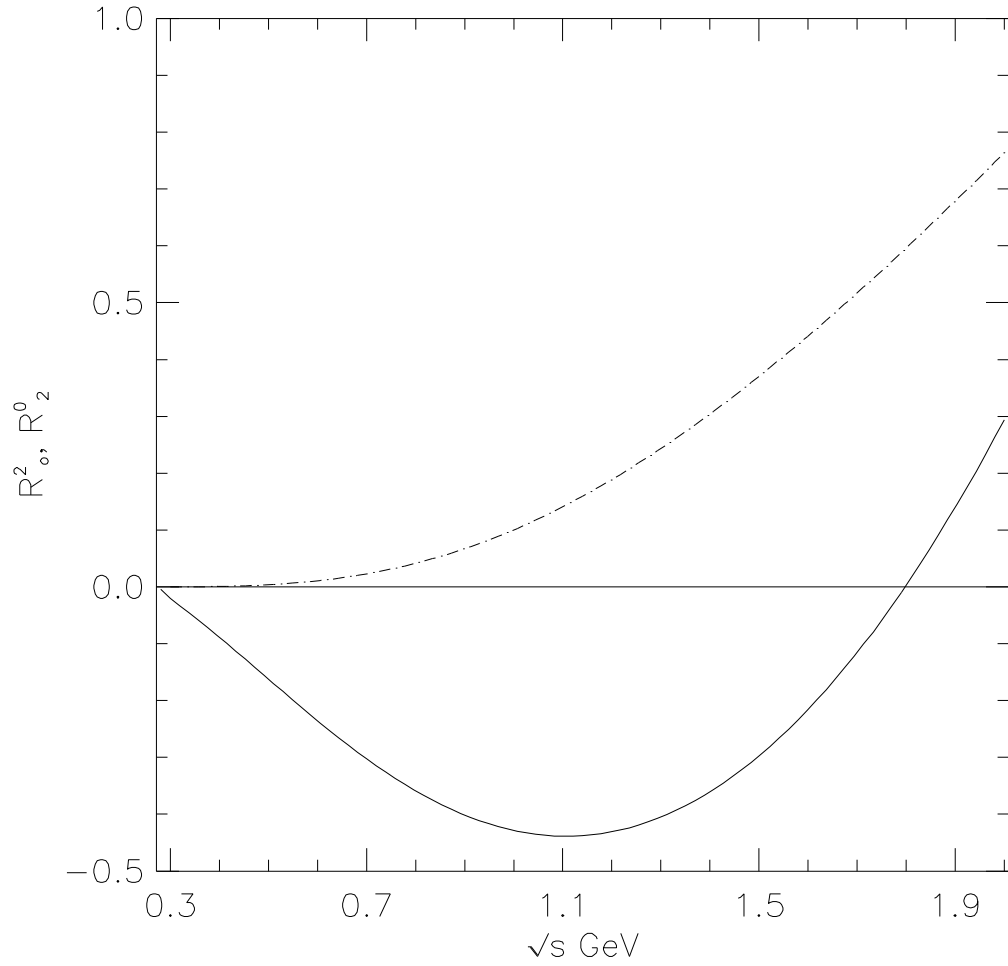
Fig. 4 The solid line is the $\pi + \rho$ contribution to the $I = 2$, $L = 0$ real part. The dot-dashed line is the $\pi + \rho$ contribution to the $I = 0, L = 2$ real part.
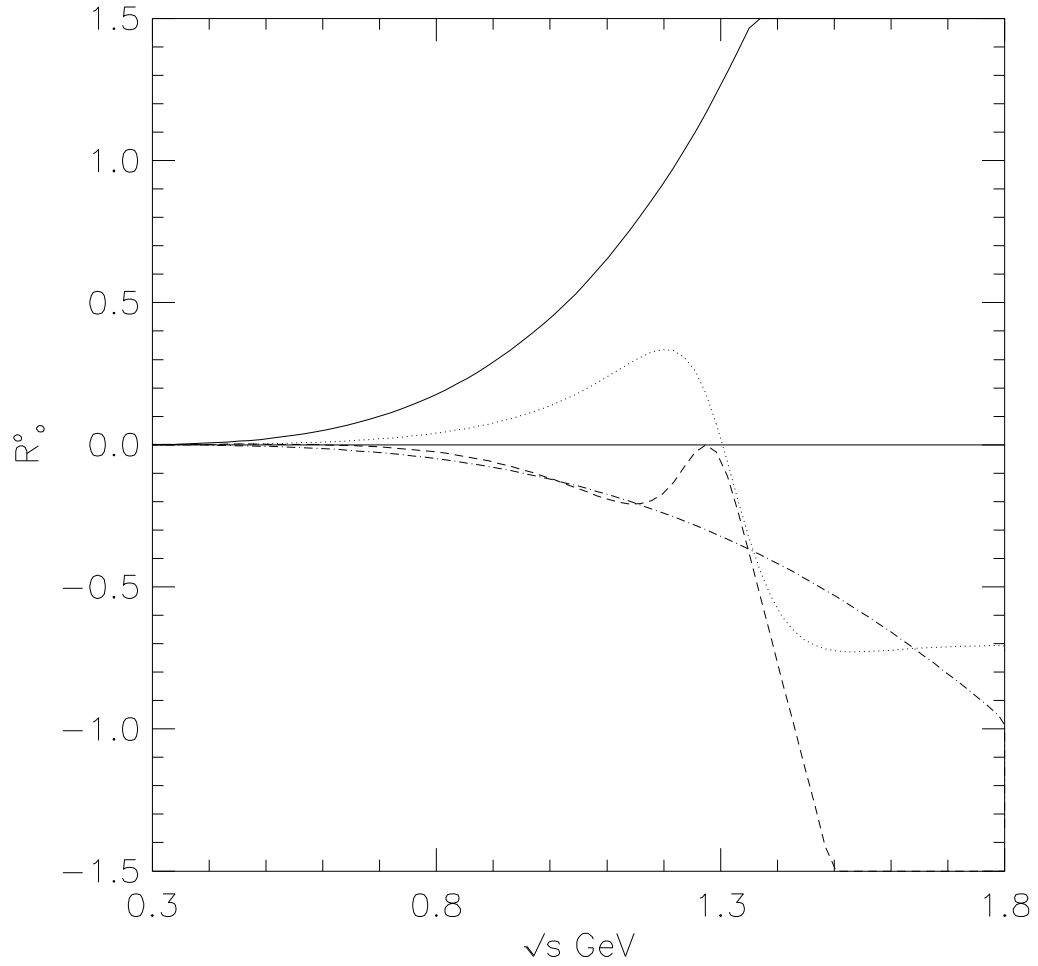
Fig. 5 Contributions to R_0^0. Solid line: $f_2(t+u)$. Dashed line: $f_2(s)$. Dotted line: $f_0(1300)$. Dot-dashed line: $\rho(1450)$.

32
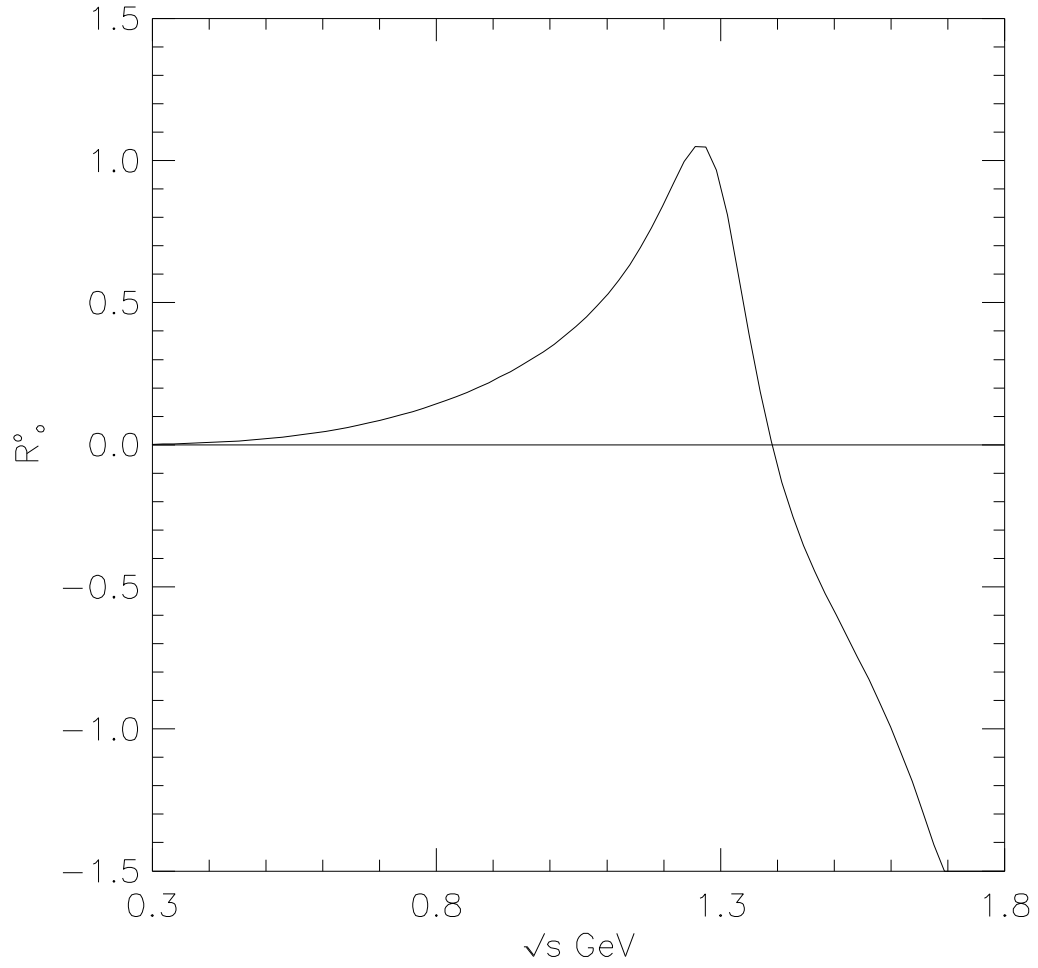
Fig. 6 Sum of all contributions in Fig. 5.

33
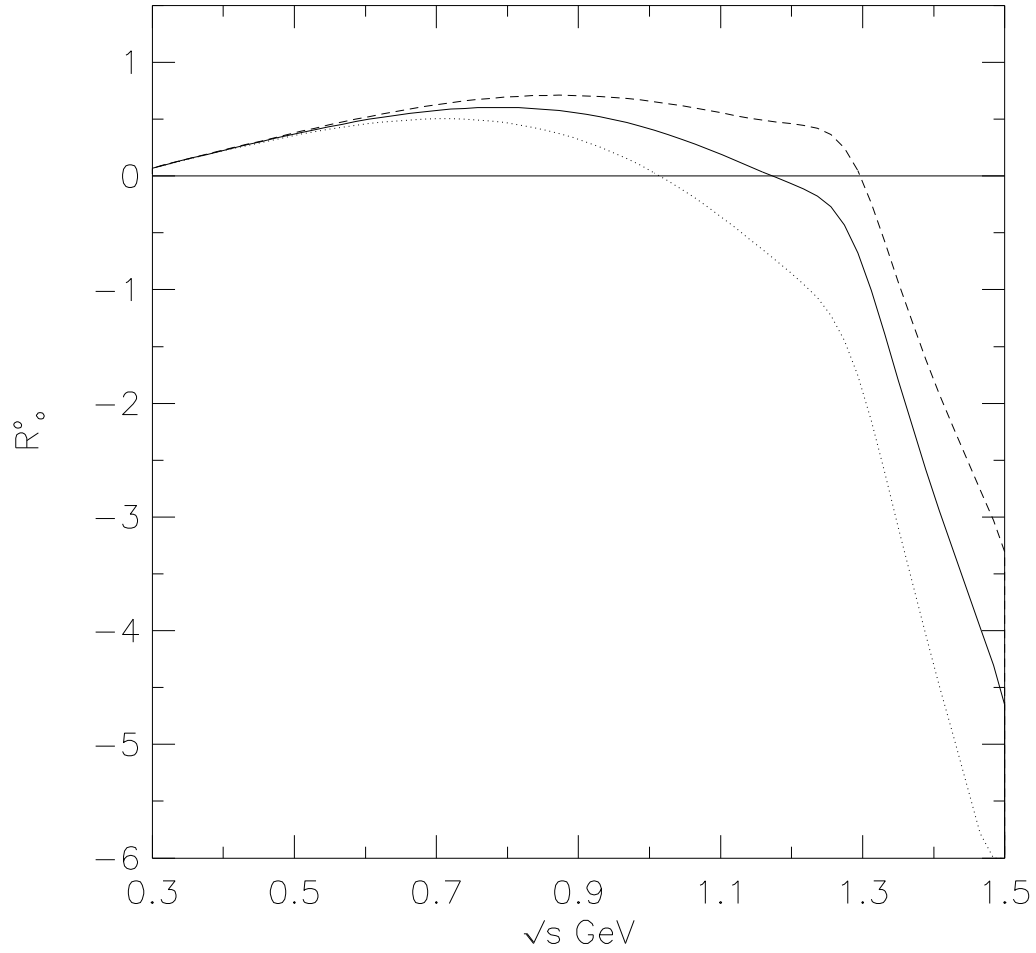
Fig. 7 Effect of four derivative contact term. Solid line: $a = +0.7$. Dashed line: $a = +1.0$. Dotted line: $a = +0.5$, in units of 10^{-3}
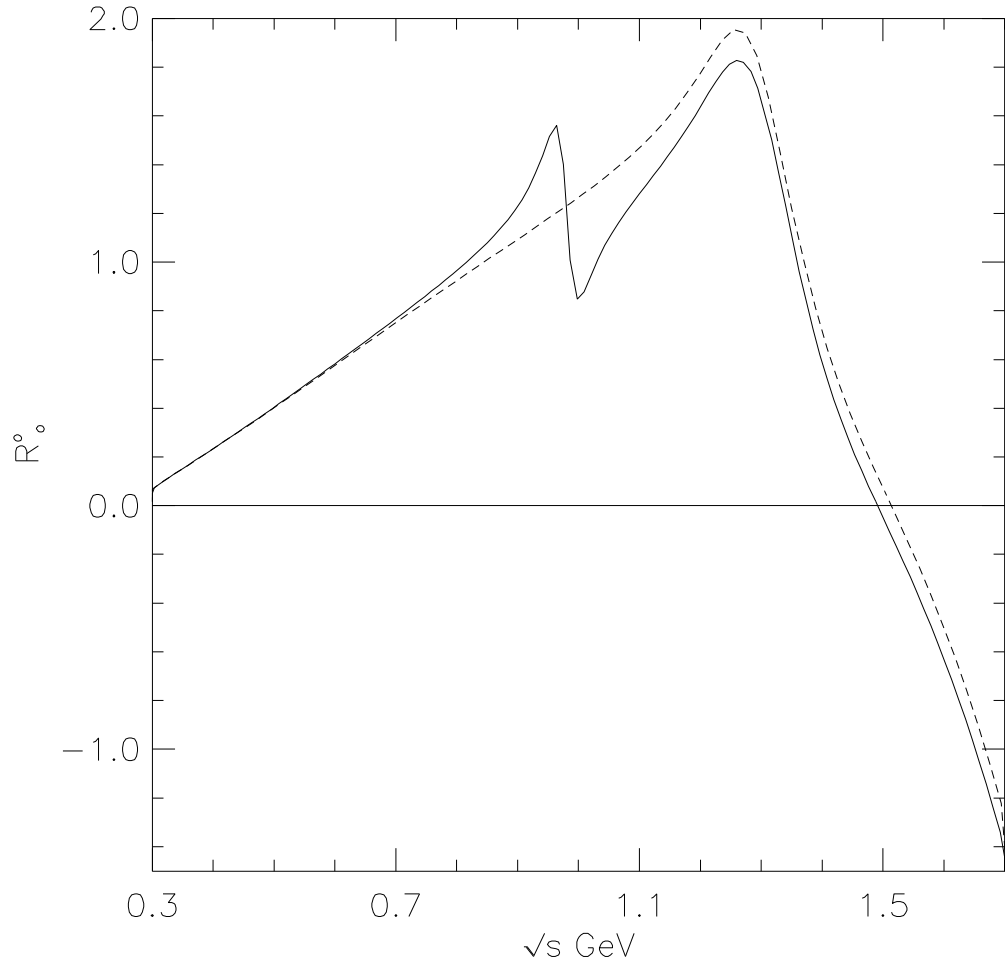
Fig. 8 Solid line: $\pi + \rho(770) + f_0(980) + f_2(1275) + f_0(1300) + \rho(1450)$. Dashed line: without $f_0(980)$.
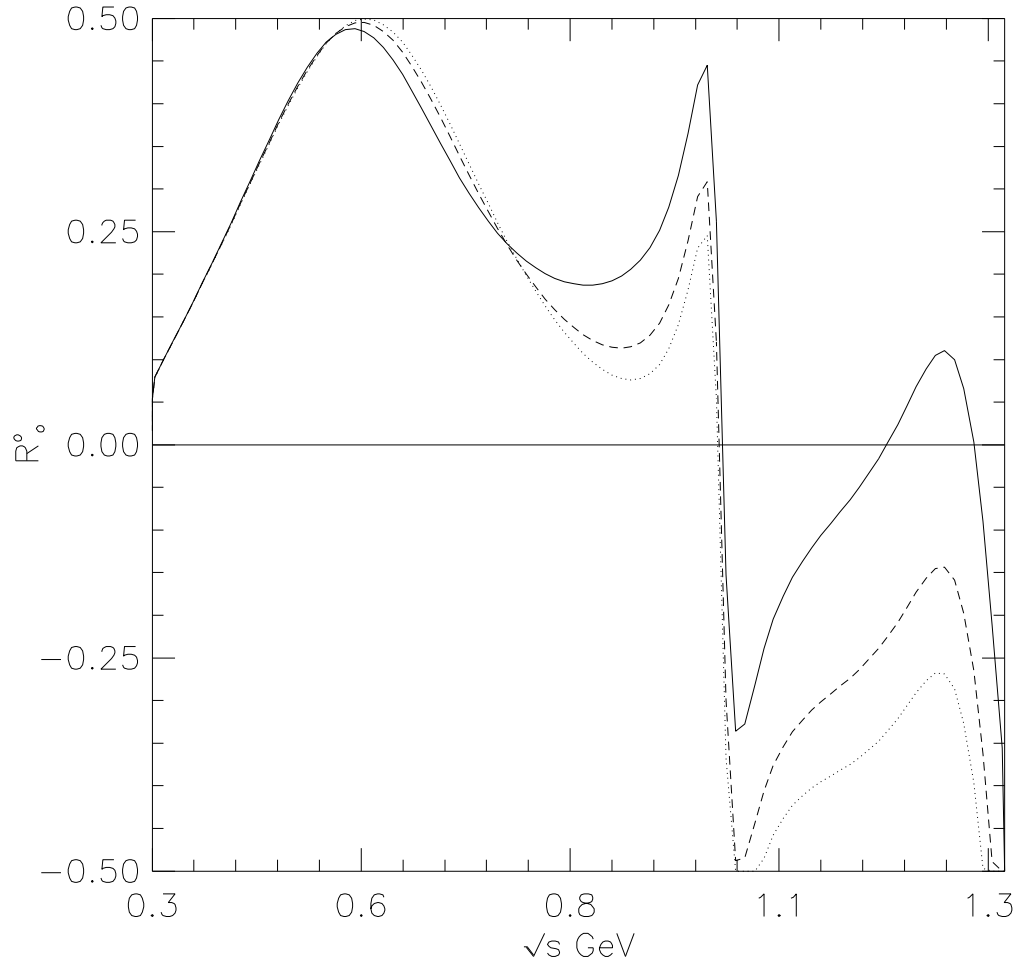
35
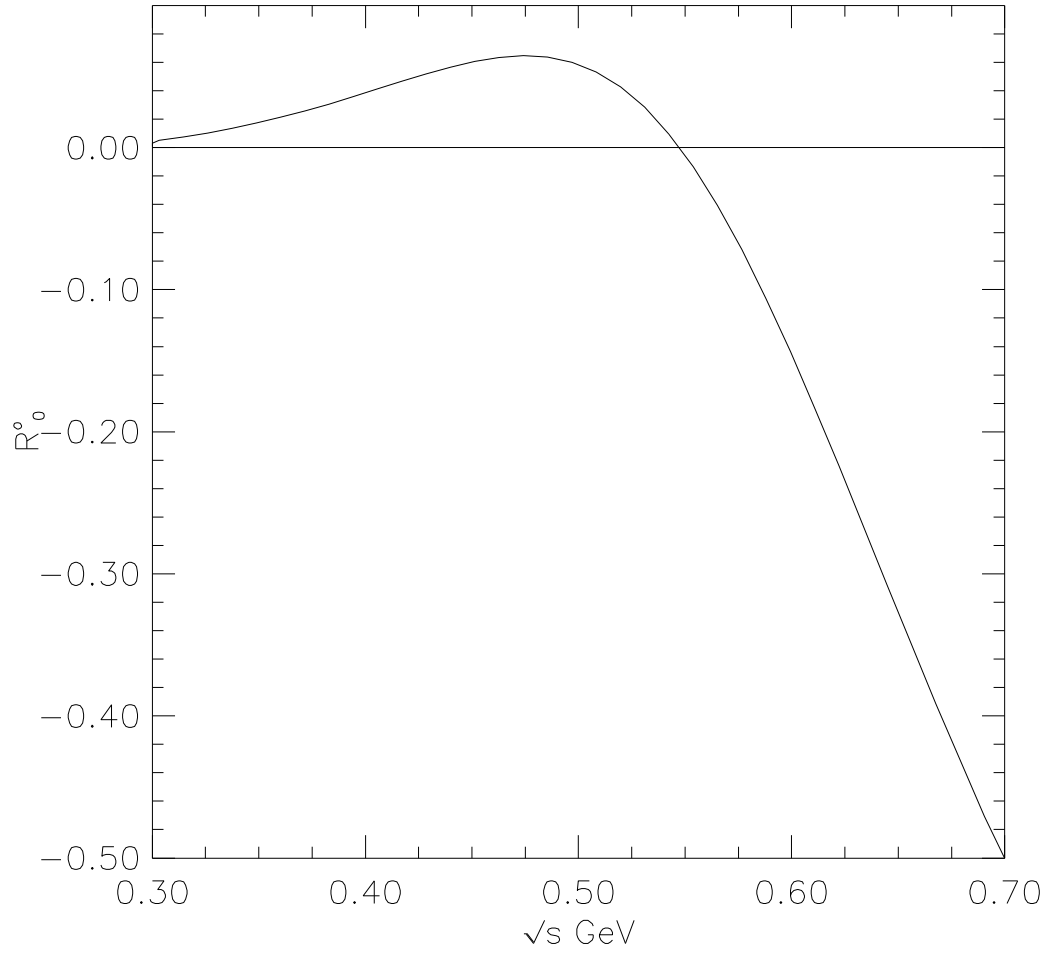
Fig. 9 Contribution of $\sigma(530)$ to R_0^0.

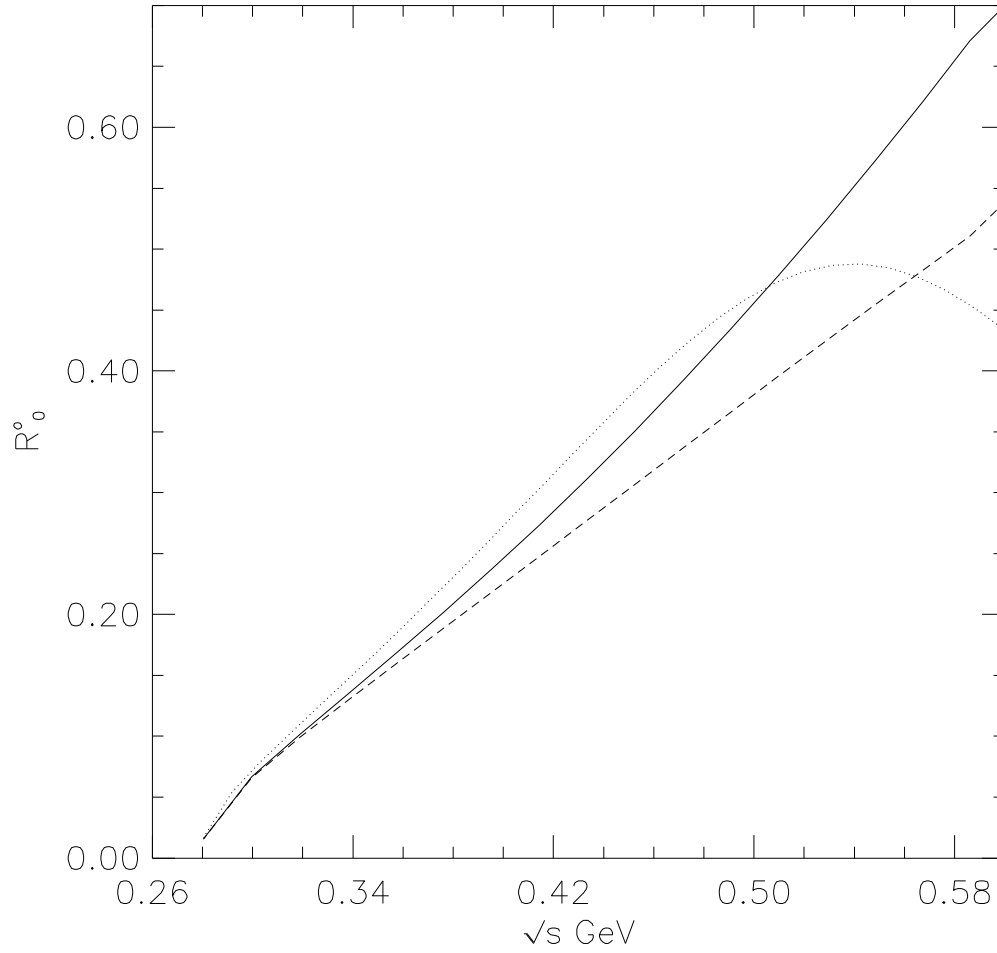Fig. 10 Pattern for R_0^0. Solid line: $G' = 380\ MeV$. Dashed line: $G' = 440\ MeV$. Dotted line: $G' = 470\ MeV$.

Fig. 11 The low energy structure for *current-algebra*, solid-line; $\pi + \rho$, dashed-line; *everything*, dotted-line.
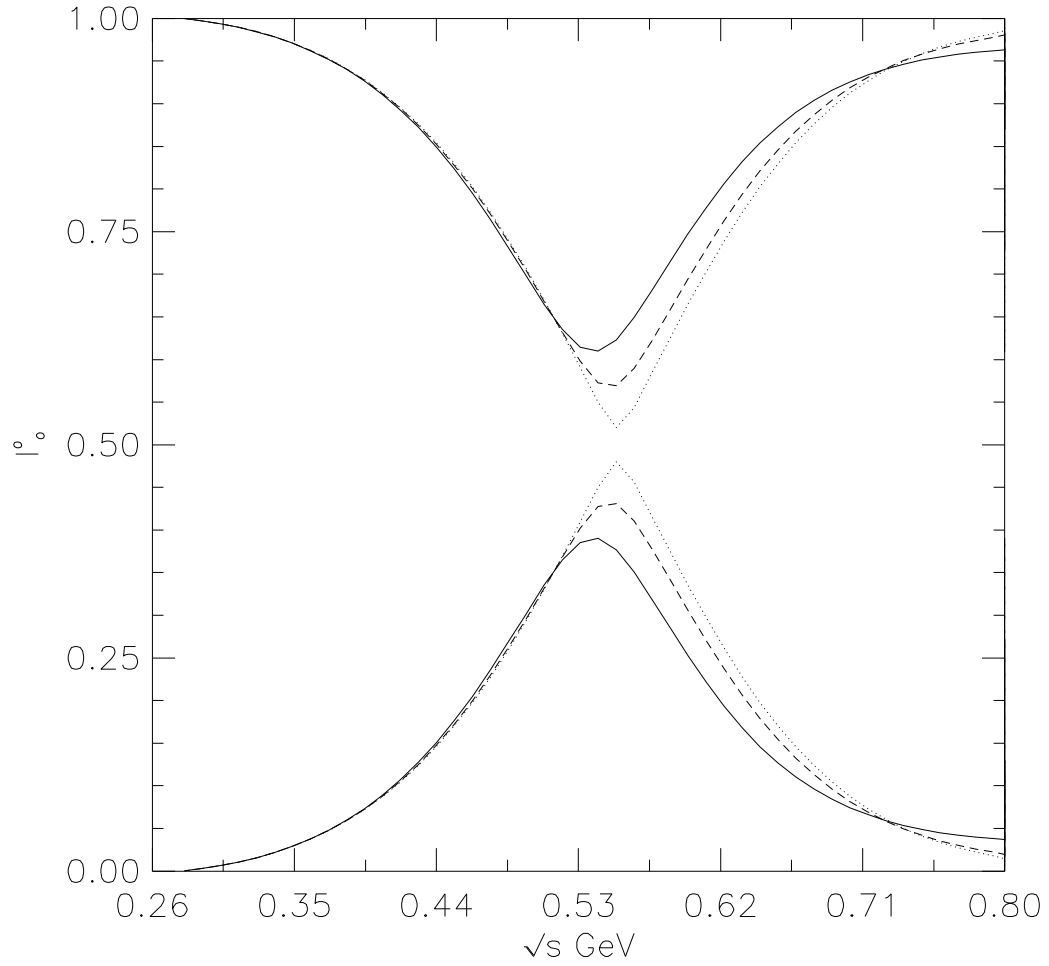
Fig. 12 Estimated imaginary part I_0^0. Upper curves + sign in front of the square root in (2.5), lower curves, − sign. Solid line: $G' = 380\ MeV$. Dashed line: $G' = 440\ MeV$. Dotted line: $G' = 470\ MeV$.

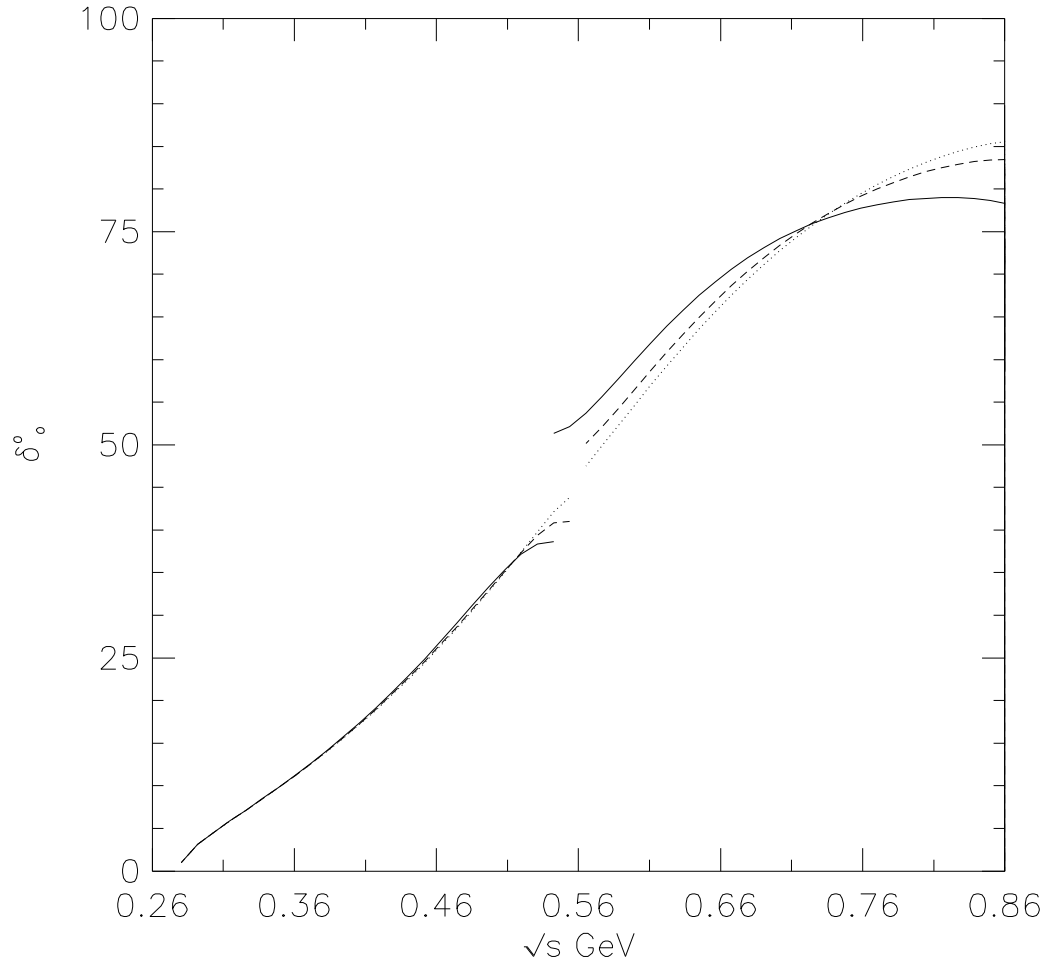Fig. 13 Estimated phase shift δ_0^0. Solid line: $G' = 380\ MeV$. Dashed line: $G' = 440\ MeV$. Dotted line: $G' = 470\ MeV$.