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Effective Field Theory Approach to String Gas Cosmology

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Abstract

We derive the $4D$ low energy effective field theory for a closed string gas on a time dependent FRW background. We examine the solutions and find that although the Brandenberger-Vafa mechanism at late times no longer leads to radion stabilization, the radion rolls slowly enough that the scenario is still of interest. In particular, we find a simple example of the string inspired dark matter recently proposed by Gubser and Peebles.

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I. INTRODUCTION

String theory continues to be our leading candidate for a quantum theory of gravity. However, it continues to be a challenge to find phenomenological predictions that can be verified by experiment. One challenge that stymies the effort towards string phenomenology is our lack of understanding of the ground state of string theory. This has come to be known as the cosmological moduli problem (see e.g. [1] and references within). One approach to resolving this problem is to take the point of view that cosmological evolution should be responsible for determining the value of the moduli. One often finds that, by adopting this view, the moduli relax to special locations in the stringy landscape, which are points of enhanced symmetry. A recent example of such a scenario was presented in [2]. There it was found that the moduli are trapped in orbits around points of enhanced symmetry due to the production of light string modes. Stability then sets in due to the Hubble damping resulting from cosmological evolution. Alternatively, one can also have so-called racetrack models where the moduli continue to roll and do not remain fixed, but could give interesting cosmological consequences [3]. In this latter approach an important issue is fine tuning or the cosmic coincidence problem, i.e. why did the moduli start rolling at such a particular time?

The Brandenberger-Vafa scenario (BV scenario) [4], or Brane Gas Cosmology (BGC) [5, 6, 7, 8, 9, 10, 11, 13] as it has come to be known, is an example of a cosmological model that incorporates both of these approaches to the moduli problem. In these scenarios one generally works in the background of ten dimensional dilaton gravity[1], with sources given by the string winding and momentum modes. It is found that the scale of the extra dimensions (radion) is then stabilized at the self-dual radius, where many of the string modes become massless and the symmetry is enhanced [8][2]. A crucial aspect of

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[1] Although this has been extended to M-theoretical considerations in [7].

[2] We note that the analysis of [8] was performed with the case of bosonic strings in mind. Stabilization at the self-dual radius leading to enhanced gauge symmetry is expected for heterotic strings, but this

these findings was the running of the dilaton to weak coupling, which was driven by the winding and momentum modes of the string. In addition, it has been shown that this model is stable to both anisotropies [9] and inhomogeneities at the linear level [10].

In this paper we would like to extend the BV scenario to better understand its predictions for late-time cosmology. In particular, we would like to see if the stabilization mechanism is still plausible in the $4D$ effective field theory resulting from dimensional reduction. Here, one usually assumes that the dilaton is fixed, since otherwise this would lead to unacceptable observational consequences[3]. Given that at late times we are no longer in the regime of dilaton gravity, perhaps one would naively expect that such stabilization would no longer work. That is, in General Relativity one must generally introduce exotic matter and/or violate the weak energy condition to stabilize extra dimensions [12]. The string modes that we will consider here do not have such properties. To spare the reader suspense, we do in-fact find that stabilization fails, except in the special case of one extra dimension[4]. However, this is not as disastrous as one might first imagine. In fact, although the radion is no longer stable in the effective theory, its evolution is slow enough compared to that of the $4D$ background to be observationally acceptable. In addition, we find that this evolution can lead to interesting phenomenology. As an example, we find an example of a cold dark matter candidate like that recently purposed by Gubser and Peebles [15].

In Section II, we will briefly review the radion stabilization mechanism as presented in [8]. We present the stress energy tensor for the string modes and the corresponding action for the string modes. In Section III, we consider the evolution in the Einstein frame. This is not the correct frame at early times when one is interested in the geodesics followed by the strings, but will be important for the late-time cosmology. In Section IV, we

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does not occur for Type II strings. We thank Steve Gubser for reminding us of this point.

[3] However there have been attempts to keep the dilaton dynamical, see e.g. [14]

[4] We mention that the case of $d = 1$, from the perspective of the $5D$ Einstein frame, has already been considered in [13].

dimensionally reduce the theory and find a form for the string modes in the $4D$ effective theory, which is given by their effective potential. From the effective potential we are able to discuss the stability of the radion, which is shown to exhibit slow rolling behavior. This leads us to the possibility of closed strings in the extra dimensions behaving as dark matter.

II. STRING GASES IN THE STRING FRAME

Our starting point is the action

$$S = \frac{1}{2\kappa_{10}^2} \int d^D x \sqrt{-G} e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{12} H^2 \right), \tag{1}$$

which is the low energy effective action for the bosonic degrees of freedom of string theory [16]. For simplicity, we will ignore flux $(H = 0)$ in the remainder of this paper[5].

We wish to consider space-times of the form

$$ds^2 = G_{MN} dX^M dX^N = g_{\mu\nu}(x) dx^\mu dx^\nu + b^2(x)\gamma_{mn}(y) dy^m dy^n, \tag{2}$$

where x^μ are coordinates in the four dimensional space-time and y^m are the coordinates of the d extra compact dimensions. The scale factor $b(x)$ is the volume modulus of the extra dimensions, which we will take to be strictly a function of time. For realistic string theory compactifications it is necessary that the Ricci curvature of $\gamma_{mn}(y)$ vanishes, i.e. $R[\gamma_{ij}] = 0$ (See e.g. [18]). For simplicity, in this paper we will consider the case of a toroidal background $\gamma_{mn}(y) = \delta_{mn}$. We will take $g_{\mu\nu}$ to be that of a standard, flat FRW universe and (2) can be parameterized as

$$ds^2 = -dt^2 + a^2(t)\, d^2\vec{x} + b^2(t)\, d^2\vec{y}. \tag{3}$$

We now want to consider the effect of a gas of string winding and momentum modes on the cosmological evolution. We can include the string modes by adding a matter term

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[5] See [17] for a treatment where flux was considered.

to the action (1)

$$S_m = \int d^D x \sqrt{-G} \rho \,, \tag{4}$$

$$T_{MN} = -\frac{2}{\sqrt{-G}} \frac{\delta S_m}{\delta G^{MN}} \,, \tag{5}$$

where ρ is the total energy density of the winding and momentum modes. The stress energy tensor of the string gas can be written as

$$T^{MN} = (T_3^w)^{MN} + (T_d^w)^{MN} + (T_3^m)^{MN} + (T_d^m)^{MN} \,, \tag{6}$$

where these terms represent the separate gases of winding and momentum modes in both the three large and d small dimensions. Note that, since we are ignoring interactions between winding and momentum modes and between strings associated with the separate subspaces, all species are separately conserved. We should clarify that by *interactions* we are referring to the long-range forces that result, for example, from the gravitational exchange. String intersections are of course not possible, since strings do not generically intersect in the full $10D$ space-time.

Keeping this in mind, we now consider the components of the *total* stress tensor

$$
\begin{aligned}
-T^0_0 = \rho &= \frac{d\mu N_d}{\sqrt{G_s}} \, b(t) + \frac{d\mu M_d}{\sqrt{G_s}} \, b^{-1}(t) + \frac{3\mu N_3}{\sqrt{G_s}} \, a(t) + \frac{3\mu M_3}{\sqrt{G_s}} \, a^{-1}(t) \,, \\
T^i_i = p_i &= \frac{-\mu N_3}{\sqrt{G_s}} \, a(t) + \frac{\mu M_3}{\sqrt{G_s}} \, a^{-1}(t) \,, \\
T^m_m = p_m &= \frac{-\mu N_d}{\sqrt{G_s}} \, b(t) + \frac{\mu M_d}{\sqrt{G_s}} \, b^{-1}(t) \,,
\end{aligned}
\tag{7}
$$

where $\sqrt{G_s} = a^3(t) b^d(t)$ is the determinant of the spatial part of the metric. The equations that follow from the action (1) with the above sources were studied in detail in [8]. There it was shown that if the size of the extra dimensions is taken initially to be on the order of the string scale, the string modes and dilaton will drive the radion to the self-dual radius ($b = 1$ in string units), while the other three large spatial dimensions continue to expand. This can be understood by considering that the winding modes become more massive as

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the extra dimensions expand and the momentum modes become more massive as they contract. The self-dual radius is chosen as the dynamically favored scale, since the mass is minimized there. Furthermore, many of these massive string modes actually become massless at the self-dual radius, which is a location of enhanced gauge symmetry. This is an example of the Higgs mechanism in string theory, where the role of the Higgs is played by the radion or scale of the extra dimensions [16].

We now wish to see if this argument survives in the lower dimensional effective field theory. This is crucial for the BV scenario, since once the three dimensions have grown large enough, the small dimensions can be integrated out and the theory should be described by a $4D$ low energy effective field theory. But before dimensionally reducing, we would like to discuss the evolution in the Einstein frame.

III. STRING GASES IN THE EINSTEIN FRAME

We can move to the Einstein frame by performing a conformal rescaling of the metric and a field redefinition of the dilaton,

$$\tilde{G}_{MN} = e^{-\frac{4\phi}{(2+d)}} G_{MN} \quad , \quad \tilde{\phi} = \frac{4M_D}{\sqrt{2+d}}\phi \,, \tag{8}$$

where quantities with a *tilde* refer to the Einstein frame metric. In terms of the new fields the action (1) becomes

$$S_E = \int d^D x \sqrt{-\tilde{G}} \left(\frac{1}{2} M_D^2 \tilde{R} - \frac{1}{2} \tilde{G}^{MN} \tilde{\nabla}_M \tilde{\phi} \tilde{\nabla}_N \tilde{\phi} \right) + \tilde{S}_m \,, \tag{9}$$

where we take $M_D^{-2} = 8\pi G_D$ as the D dimensional Planck mass. The matter action is now given by

$$\tilde{S}_m = \int d^D x \sqrt{-\tilde{G}} \tilde{\rho} \,, \tag{10}$$

where the energy density $\tilde{\rho}$ is not simply the transformed density ρ from the string frame, since one has to take the transformation of the determinant of the metric into account,

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that is

$$\sqrt{-G} \;=\; \sqrt{-\tilde{G}}e^{\frac{4+d}{2\sqrt{2+d}}\left(\frac{\tilde{\phi}}{M_D}\right)}\,. \tag{11}$$

Our energy and pressure terms are then given in terms of the transformed stress energy tensor

$$-\tilde{T}^0_0 = \tilde{\rho} \;=\; \frac{d\mu N_d}{\sqrt{\tilde{G}_s}}\,\tilde{b}e^{\frac{1}{\sqrt{2+d}}\left(\frac{\tilde{\phi}}{M_D}\right)} + \frac{d\mu M_d}{\sqrt{\tilde{G}_s}}\,\tilde{b}^{-1} + \frac{3\mu N_3}{\sqrt{\tilde{G}_s}}\,\tilde{a}e^{\frac{1}{\sqrt{2+d}}\left(\frac{\tilde{\phi}}{M_D}\right)} + \frac{3\mu M_3}{\sqrt{\tilde{G}_s}}\,\tilde{a}^{-1}\,,$$

$$\tilde{T}^i_i = \tilde{p}_i \;=\; -\frac{\mu N_3}{\sqrt{\tilde{G}_s}}\,\tilde{a}e^{\frac{1}{\sqrt{2+d}}\left(\frac{\tilde{\phi}}{M_D}\right)} + \frac{\mu M_3}{\sqrt{\tilde{G}_s}}\,\tilde{a}^{-1}\,,$$

$$\tilde{T}^m_m = \tilde{p}_m \;=\; -\frac{\mu N_d}{\sqrt{\tilde{G}_s}}\,\tilde{b}e^{\frac{1}{\sqrt{2+d}}\left(\frac{\tilde{\phi}}{M_D}\right)} + \frac{\mu M_d}{\sqrt{\tilde{G}_s}}\,\tilde{b}^{-1}\,. \tag{12}$$

Finally, the equations of motion are given by

$$\tilde{R}^0_0 - \frac{1}{2}\delta^0_0\tilde{R} \;=\; \frac{1}{M_D^2}\left(\tilde{\rho} + \frac{1}{2}\dot{\tilde{\phi}}^2\right)\,, \tag{13}$$

$$\tilde{R}^\mu_\mu - \frac{1}{2}\delta^\mu_\mu\tilde{R} \;=\; -\frac{1}{M_D^2}\left(\tilde{p}_i + \frac{1}{2}\dot{\tilde{\phi}}^2\right)\,, \tag{14}$$

$$\tilde{R}^m_m - \frac{1}{2}\delta^m_m\tilde{R} \;=\; -\frac{1}{M_D^2}\left(\tilde{p}_m + \frac{1}{2}\dot{\tilde{\phi}}^2\right)\,, \tag{15}$$

$$\tilde{\Box}\,\tilde{\phi} \;=\; -\frac{d\tilde{\rho}}{d\tilde{\phi}}\,, \tag{16}$$

where it is understood that repeated indices are not summed over. We will not be interested in exact solutions to the above equations at this point. However, we would like to pause and make some important comments regarding their interpretation.

A.   Switching Frames and Stability

It is often argued that by moving to the Einstein frame, dilaton gravity is simply general relativity with the addition of scalar matter. That is indeed true for the vacuum case, however when string sources are present the dilaton couples differently to winding and momentum modes. This is manifested by our stress energy terms in (12). For example,

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if we substitute the energy density in (12) back into the matter action (10), we see that momentum modes are conformally invariant (as expected since they behave like radiation, which has a traceless stress energy tensor), but the winding modes are affected by the conformal transformation. This complicated coupling to the dilaton is just an example of why the Einstein frame is the unnatural frame for interpreting the physics at early times[6]. However, here we are interested in making contact with late-time cosmology, when the Einstein frame becomes relevant.

One can understand the difference in the frames quantitatively by examining the Einstein frame line element in terms of the string frame scale factors

$$d\tilde{s}_E^2 = -e^{-\frac{4\phi}{(2+d)}}dt^2 + e^{-\frac{4\phi}{(2+d)}}a^2(t)d^2\vec{x} + e^{-\frac{4\phi}{(2+d)}}b^2(t)d^2\vec{y}. \tag{17}$$

Recalling that the evolution of the dilaton was crucial for stabilizing the scale factor $b(t)$ in the string frame, we can immediately see the difficulty in stabilizing the Einstein frame scale factor $\tilde{b}(t) = e^{-\frac{2\phi(t)}{(2+d)}}b(t)$. That is, we see that an evolving dilaton is in contradiction with the stabilization of $\tilde{b}(t)$. One way to resolve this is to fix the dilaton at late-times, such that the scale factors are fixed at the self-dual radius $b(t) = \tilde{b}(t) = 1$ in both frames. This argument is intuitively correct, since once the dilaton is fixed the string and Einstein frames are equivalent. However, we will see in the next section that fixing the dilaton will not be enough to maintain stabilization for the case of more than one extra dimension.

The reader may argue that one needs a specific mechanism to provide a potential for the dilaton. It is generally believed that this mechanism should come from a better understanding of the nonperturbative theory and is perhaps related to the breaking of supersymmetry. We do not wish to address this issue here, except to comment that our conclusions seem to be robust in this regard, as long as we remain in the weak coupling regime. For example, one could imagine trying to incorporate the scenario we will describe below in a Type IIB construction with fluxes, where all moduli (including the dilaton)

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[6] This has been discussed much in the literature, see for example [19].

have been fixed accept for the radion [20]. Another possibility is to follow the point of view of [14] and keep the dilaton dynamic.

In what follows, we will consider the stabilization in the ten dimensional string frame to be accurate at high energy/small distances. Therefore, we accept the results of [8] as being accurate in this early regime. Then, once we reach the stage of late-time cosmology we assume that the issue of SUSY symmetry breaking and the evolution or fixed dilaton are given, *a priori*. Following this cosmological transition we expect the physics to be described by the $4D$ effective field theory. We will now discuss what influence the closed string modes have in this regime.

IV. STRING GASES IN FOUR DIMENSIONAL EINSTEIN GRAVITY

We now want to consider the four dimensional effective field theory resulting from the compactification of the d internal dimensions [12]. We can dimensionally reduce the action (9) by decomposing the determinant of the metric (3) and the Ricci scalar as

$$\sqrt{-\tilde{G}} = \tilde{b}^d \sqrt{-\tilde{g}}, \tag{18}$$

$$R[\tilde{G}_{MN}] = R[\tilde{g}_{\mu\nu}] - 2d\tilde{b}^{-1}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\tilde{b} - d(d-1)\tilde{b}^{-2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{b}\tilde{\nabla}_\nu\tilde{b}, \tag{19}$$

where we have again used $R[\gamma_{ij}] = 0$. The compactification scale is usually taken to be around the Planck scale. This means that one can integrate out the dependence of the fields on the coordinates of the extra dimensions. We obtain the low energy effective action for the four dimensional theory

$$S_{eff} = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} M_p^2 \left(\tilde{b}^d \tilde{R}[\tilde{g}_{\mu\nu}] - 2d\tilde{b}^{d-1}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\tilde{b} - d(d-1)\tilde{b}^{d-2}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{b}\tilde{\nabla}_\nu\tilde{b} \right) \right.$$
$$\left. -V_d \, \tilde{b}^d \tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\phi}\tilde{\nabla}_\nu\tilde{\phi} + V_d \, \tilde{b}^d \, \tilde{\rho} \right], \tag{20}$$

where we have defined $V_d = \int d^dy\sqrt{\gamma}$ as the spatial volume of the extra dimensions for unit scaling ($\tilde{b} = 1$) and the four dimensional Planck mass is given by $M_p^2 = V_d M_D^2$.

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We now perform another conformal rescaling and field redefinition to put the action (20) in canonical form,

$$\psi = \sqrt{\frac{d(2+d)}{2}} M_p \ln(\tilde{b}), \qquad \bar{g}_{\mu\nu} = e^{\sqrt{\frac{2d}{2+d}}\left(\frac{\psi}{M_p}\right)} \tilde{g}_{\mu\nu}, \qquad \varphi = V_p^{1/2}\tilde{\phi}, \qquad (21)$$

where we have introduced the four dimensional dilaton φ. We parametrize our metric by

$$ds^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + e^{2\lambda(t)}d\vec{x}^2, \qquad \lambda(t) = \ln \bar{a}(t). \qquad (22)$$

Then the four dimensional effective action with canonically normalized fields is given by

$$S_{eff} = \int d^4x \sqrt{-\bar{g}}\left(\frac{1}{2}M_p^2 R[\bar{g}_{\mu\nu}] - \frac{1}{2}\bar{g}^{\mu\nu}\bar{\nabla}_\mu\psi\bar{\nabla}_\nu\psi - \frac{1}{2}\bar{g}^{\mu\nu}\bar{\nabla}_\mu\varphi\bar{\nabla}_\nu\varphi + V(\lambda,\varphi,\psi)\right), (23)$$

where the effective potential is obtained from dimensionally reducing the matter action (10), applying the transformations (21) and plugging in the metric (22). One finds

$$V(\lambda,\varphi,\psi) = \mu V_d e^{\frac{1}{\sqrt{2+d}}\frac{\varphi}{M_p}}\left(3N_3 e^{-2\lambda}e^{-\sqrt{\frac{2d}{2+d}}\left(\frac{\psi}{M_p}\right)} + dN_d e^{-3\lambda}e^{\left(1-\frac{d}{2}\right)\sqrt{\frac{2}{d(2+d)}}\frac{\psi}{M_p}}\right)$$

$$+\mu V_d\left(3M_3 e^{-4\lambda} + dM_d e^{-3\lambda}e^{-\left(1+\frac{d}{2}\right)\sqrt{\frac{2}{d(2+d)}}\left(\frac{\psi}{M_p}\right)}\right). \qquad (24)$$

With this potential, the equations of motion are

$$3\dot{\lambda}^2 = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\psi}^2 + V(\lambda,\varphi,\psi), \qquad (25)$$

$$2\ddot{\lambda} + 3\dot{\lambda}^2 = -\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\dot{\psi}^2 + \frac{1}{3\sqrt{\bar{g}_s}}\frac{\partial(\sqrt{\bar{g}_s}\,V)}{\partial\lambda}, \qquad (26)$$

$$\ddot{\varphi} + 3\dot{\lambda}\dot{\varphi} = -\frac{\partial V}{\partial\varphi}, \qquad (27)$$

$$\ddot{\psi} + 3\dot{\lambda}\dot{\psi} = -\frac{\partial V}{\partial\psi}, \qquad (28)$$

where we set $M_p \equiv 1$ and $\bar{g}_s$ is the determinant of the spatial part of the metric (22). For example, if the dilaton and radion are slowly rolling and we take $N_3 = N_d = M_d = 0$ we are left with the equations for a radiation dominated universe,

$$3\dot{\lambda}^2 \sim e^{-4\lambda} \sim \bar{a}^{-4}(t), \qquad (29)$$

$$2\ddot{\lambda} + 3\dot{\lambda}^2 \sim -\frac{1}{3}\bar{a}^{-4}(t), \qquad (30)$$

as we expect from a gas of massless bosonic strings in the uncompactified directions.

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A. Late time solutions with $d = 1$ and dark matter

Let us first consider the case of one extra dimension $(d = 1)$ and focus on the winding and momentum modes in the extra dimensions only, i.e. $N_3 = M_3 = 0$. We are justified in doing this, since at late times the winding modes have all annihilated in the large dimensions [9] and as we have seen the momentum modes behave as a gas of radiation, which is subdominant in the matter dominated epoch.

To understand the late time evolution, we must decide how to evolve the dilaton. Here we choose to give the dilaton a VEV, which we argued in Section IIIA should be determined by fixing the minimum of the potential to be at $\psi = 0$, i.e. the self-dual radius. Thus, we fix $\phi_0 = 0$ and at the self-dual radius one has $N_d \approx M_d$. The potential is given by

$$V(\psi, \lambda) = \frac{\mu V_d N_d e^{\frac{\sqrt{6}\psi}{6M_p}}}{e^{3\lambda}} + \frac{\mu V_d M_d e^{-\frac{\sqrt{6}\psi}{2M_p}}}{e^{3\lambda}}, \tag{31}$$

so that

$$V(\psi = 0, \lambda) \sim \frac{1}{e^{3\lambda}}, \tag{32}$$

which one recognizes as the energy density for matter. We can take it to be in the dark sector, because of its stringy origin. This leads to a simple example of the string inspired dark matter discussed recently by Gubser and Peebles in [15]. In that paper it was shown that by considering long-range scalar and gauge interactions, one is led to slightly modified ΛCDM models that may still be within observational bounds. We refer readers to that paper for the details, but here we wish to demonstrate that our naive string gas model reproduces one such dark matter candidate. The crux of the modification to the ΛCDM model is a *fifth force* that is provided by the long-range interactions of the string modes. Recall that winding strings do not intersect in greater than three dimensions, that is why we were led to three dimensions decompactifing and the other d remaining compact as motivated by the BV scenario. However, the strings do interact through their effects on

the gravitational background and this leads to a modification of the local gravitational force. For two particles with masses m_i and m_j $(i = 1, 2)$ the force is given by [15],

$$F_{ij} = \beta_{ij} \frac{Gm_i m_j}{r^2}, \quad \beta_{ij} = 1 + 2M_p^2 \frac{Q_i Q_j}{m_i m_j}, \tag{33}$$

where the m_i are obtained from the potential (31) by

$$V(\psi, \lambda) = \sum_q n_i m_i, \tag{34}$$

so that we identify

$$n_1 = \frac{M_d}{e^{3\lambda}}, \quad n_2 = \frac{N_d}{e^{3\lambda}} \tag{35}$$

as the number densities and

$$m_1 = \mu V_p e^{-\frac{\sqrt{6}}{2} \frac{\psi}{M_p}}, \quad m_2 = \mu V_p e^{\frac{\sqrt{6}}{6} \frac{\psi}{M_p}} \tag{36}$$

as the masses. Finally, the *charges* Q_i are given by

$$Q_i = \frac{dm_i}{d\psi}. \tag{37}$$

We also need to introduce the mass fraction

$$f_i = \frac{n_i m_i}{\sum_k n_k m_k},$$

which for our model is $f_1 = f_2 = \frac{1}{2}$.

The statement that $\psi = 0$ is a stable minimum becomes that of charge neutrality for the charges Q_i, which reads

$$\sum_i n_i Q_i = 0, \tag{38}$$

with $3M_d = N_d$.

If we calculate β_{ij} for this model we find

$$\beta_{ij} = 4 \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}.$$

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First, we note that the off-diagonal entries are zero, so that winding and momentum modes are not interacting. This was our starting assumption, so this offers a nice consistency check.

As discussed in [15], the growth of instability is given in terms of the mass density contrast $\delta_i = \delta\rho_i/\rho$ where ρ is the total energy density and

$$\ddot{\delta}_i + 2\dot{\lambda}\dot{\delta}_i = \frac{\rho}{2M_p^2}\sum_j \beta_{ij} f_j \delta_j, \tag{39}$$

where the second term is a damping factor which expresses the decay of peculiar velocity due to cosmological expansion. The problem of finding the modes of instability is made more tractable by introducing

$$\Delta = \sum_i c_i \sqrt{f_i}\delta_i, \tag{40}$$

where the c_i are constants. If one chooses these constants appropriately and introduces the matrix

$$\Xi_{ij} = \sqrt{f_i}\beta_{ij}\sqrt{f_j}, \tag{41}$$

then Δ satisfies

$$\ddot{\Delta}_a + 2\dot{\lambda}\dot{\Delta}_a - \frac{\rho}{2M_p}\xi_a \Delta_a = 0, \tag{42}$$

where the ξ_a are the eigenvalues of Ξ. To find the eigenvalues we must multiply (41) by an extra factor of $\frac{1}{2}$, so that our normalization of ψ agrees with that of [15]. We then find the eigenvalues $\xi_1 = 1$ and $\xi_2 = \frac{1}{3}$. It then follows that

$$\Delta_1 \sim t^{2/3}, \qquad \Delta_2 \sim t^{1/3}, \tag{43}$$

are the fastest growing modes. The first is an adiabatic mode, which corresponds to the motion of the string modes moving together with the expansion in the matter dominated epoch, while the second mode is subdominant. Since models of this type were discussed in detail in [15], we will simply conclude this section by commenting that our naive string gas model appears to lead to the possibility of a cold dark matter candidate for $d = 1$.
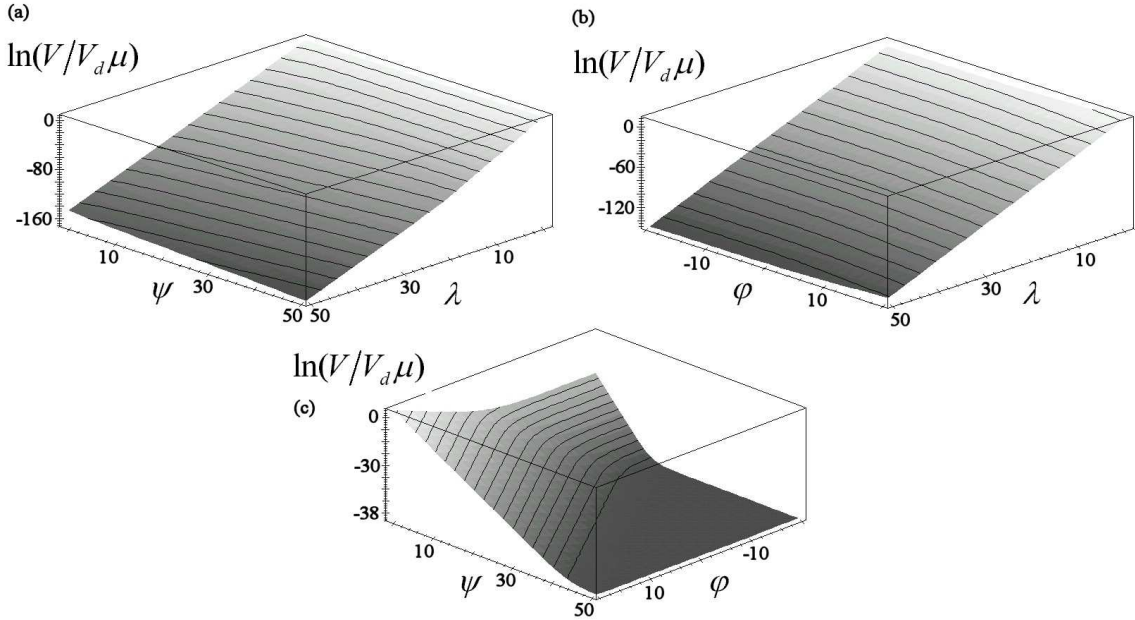
FIG. 1: The potential $\ln(V(\lambda, \varphi, \psi)/(V_d\mu))$ from (24) is plotted with $d = 6$, $N_d = M_d = M_3 = 1$, $N_3 = 0$, $M_p \equiv 1$ and (a) the dilaton held constant at $\varphi = -3/M_p$, (b) the extra dimensions held constant at $\psi = 0$ and (c) the large dimensions held constant at $\lambda = 10$.

However, one finds that in the case of more than one extra dimension no local minimum exists for $V(\psi)$, so that the question of stability becomes more of a sensitive issue, which we now discuss.

B. Late time solutions for $d > 1$

We have presented the $d = 1$ case as an explicit illustration of the possibility of obtaining CDM models from the string gas approach. In the case $d > 1$, one can see that the potential (31) has no local minimum. This means that once the effective field theory becomes relevant, one expects the radion to begin rolling and the extra dimensions to expand. This seems to be a negative finding for the stabilization of extra dimensions

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at late times, within this naive extension of the BV scenario. Moreover, the CDM model relies crucially on the vanishing of the pressure around the minimum $\psi = 0$. Since the minimum vanishes for $d > 1$, one must worry that if the pressure becomes appreciable we will not be in a matter dominated epoch.

To spite these seemingly negative results, a resolution is immediately apparent. If one examines the potential (24), one finds that radion rolls slowly down its potential compared to the expansion. In fact, one finds

$$\frac{\partial (\ln V)}{\partial \psi} < 1, \qquad \frac{\partial^2 V}{\partial \psi^2} < 1, \tag{44}$$

quite generally. As a specific example, let us consider the case discussed in [8]. That is, $d = 6$ and the winding modes have all annihilated in the large dimensions, so $N_3 = 0$. The resulting potential is plotted in Fig. 1(a-c). We see that the potential is steepest in the λ direction as can be seen in Fig. 1 or computed directly using (24). Thus, the three dimensions will grow much faster than the radion. At the same time the dilaton will roll to more and more negative values, which is a result of the string modes driving the model to weak coupling. The dilaton will grow faster in the early phase but slowing down after ψ has grown to an appreciable size. This can be seen in Fig. 1(b,c) by the shallowness in the direction of the dilaton after ψ has grown. The same generic behavior is found for the case of $d = 2, 3, 4$, or 5, since the potential has a similar shape. Thus, once the transition takes place the radion can roll slowly enough that the pressure remains negligible and a dark matter candidate arises as in the $d = 1$ case. In Fig. 2 we present a sample numeric solution to (25)-(28), which exemplifies this slow roll behavior. We note that this behavior is generic for initial conditions that respect the condition $g_{\text{string}} \sim e^{2\phi} \ll 1$.

Although this behavior is promising, a more detailed investigation is needed. In particular, one needs to better understand the dark matter in the case of appreciable pressure. That is, if the pressure becomes significant the analysis reviewed in Section IV-A needs to be revised. One might also entertain the possibility of obtaining dark energy from such a model, however this seems to fail since the pressure is positive (leading to contraction)
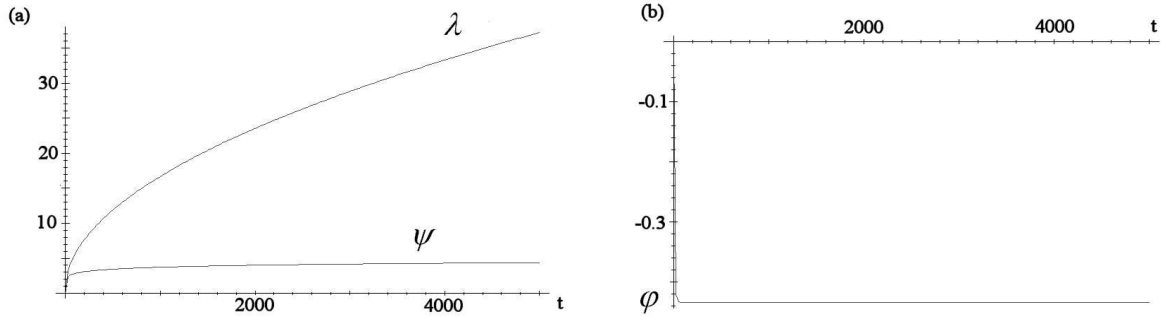
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FIG. 2: The solution to equations (25)-(28) for the potential $V(\lambda, \varphi, \psi)$ from (24) as plotted in Fig. 1 with the same settings and the initial conditions $\varphi = \dot{\varphi} = \psi = \dot{\psi} = \lambda = 0$: (a) evolution of $\lambda(t)$ (large dimensions, upper curve) and $\psi(t)$ (extra dimensions, lower curve), (b) evolution of the dilaton $\varphi(t)$.

as can be seen from (26). Regardless of what properties this new matter has, it seems worthy of future study.

V. CONCLUSION

In this article we have shown that stabilization of the radion in the effective theory at late-times is problematic. This was not the case with one extra dimension, where after passing to the effective theory the potential retains a minimum at the self-dual radius. In that case, we were able to construct a simple realization of scalar dark matter as discussed in [15]. However, in the case of more than one extra dimension the potential does not possess a local minimum. This led to extra dimensions that are slowly growing, but that remain small compared to the large dimensions. This case is interesting in many respects, e.g. it could lead to *large* extra dimensions without the need to invoke brane world scenarios. We also note that these results remain valid for both a fixed and evolving dilaton, where in the later case the string modes drive the dilaton towards the region of weak coupling. We find the interpretation of winding and momentum modes as CDM

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much more involved, but possible. We leave such considerations for future work.

Acknowledgments

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