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A Statistical Approach to MPEG Video Stream Characterization *†

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Abstract

As video consumes a significant percentage of the available network bandwidth, understanding video bandwidth requirements will translate into better network control schemes. In this study, several commercially available video streams are statistically analyzed and several modeling approaches are developed. The segmentation techniques are found to be rewarding. However, the improvements due to complexity of the polynomial models are insignificant.

Introduction

Within applications that require network resources, video consumes the most bandwidth [2], [4], [5]; constrained MPEG-1 standard bounds the bitrate by 1.86Mbits/sec whereas MPEG-2 requires significantly higher bit rates [3].

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†A previous report about this study where single video stream was investigated in greater detail, was filed with the CASE Center, Statistical Characterization of MPEG Video Streams, CASE Center Technical Report, 9507

As the bitrate usage of the network is dominated by video, understanding bitrate transmission requirements and constructing prediction functions will enable a better utilization of the bandwidth, facilitate more deterministic quality of service guarantees, and allow better resource allocation techniques.

Although MPEG-1 has a well established protocol that can be used to extract the bitrate information, this approach results in much wider variations in bandwidth requirements than observed in typical video streams. Therefore in our approach, we have used actual video streams.

In this report, first the data are presented. The standard deviation of the whole stream is compared to that of different partitioning techniques to establish the motivation for the models presented.

In the next section, the performance measures used to compare different partitioning techniques, are summarized.

In the following sections, time invariant, linear and quadratic models for three videos and their corresponding performance measures are presented.

In the last section, the models are compared.

Throughout this report, the subscripts, 0, 1, 2, are used to identify constant, linear, and quadratic models whereas the superscripts 0, 1, 2, are used to indicate partitionings: Superscript 0 denotes a single whole partition where data are treated as a single segment. Superscript 1, denotes the partitioning technique where data are segmented into 3 groups: I, P, and B segments. Superscript 2 denotes the technique where data are segmented into 15 segments. The segmentation technique is covered in greater detail in Data section. Independent variable, time, is expressed in multiples of $\frac{1}{30}^{th}$ of a second.

Background

MPEG is a coding standard for video and the associated audio. Currently there are three phases of the MPEG standard: MPEG-1, MPEG-2 and MPEG-4. MPEG-1 defines a bit stream for compressed video and the associated video to fit into a bandwidth of 1.5 Mbits/sec. The aim of MPEG-2 is to have a higher quality video coded at 3 to 10 Mbits/sec; while MPEG-4 targets very low bit rates for video-conferencing applications.

The MPEG standards define 3 layers, a system coding layer, a video coding layer and an audio coding layer. The system coding layer specifies a multiplexed structure for combining multiple streams of video and audio data and means for specifying synchronization information for these streams. The video layer specifies a coded representation of video data and the decoding process to recover the original video data; while the audio layer specifies a coded representation of audio data.

Compression of the video data is obtained by eliminating both temporal and spatial redundancies in the video stream. A block-based motion compensation technique is used to reduce the temporal redundancy between frames or fields; and a transform-based technique is used to reduce the spatial redundancy within a frame or a field.

In order to balance the requirement for random access to the video stream and the requirement for low bit rates, three types of pictures are defined. A picture can be either a frame or a single field of a frame. The three picture types are Intra-coded Pictures (I-frames), Predictively Coded Pictures (P-frames) and Bidirectional Predictively Coded Pictures (B-frames).

I-frames use only a transform-based spatial redundancy reduction scheme. They can be decoded independently of any other picture, hence provide access points for random access of the video streams. Since no attempt is made to reduce the temporal redundancy in an I-frame, the compression ratio is only moderate. P-frame are coded in reference with a previous I- or P-frame and can be used as a reference for other frames. The compression of a P-frame takes into account the temporal redundancy between that P-frame and its reference frame. Thus, the compression ratio is higher for a P-frame than an I-frame. The highest compression ratio is obtained for B-frames, which require past and future reference frames. The reference frames can be either I-frames or P-frames. In order to decode a P-frame the past reference frame must be available, while to decode a B-frame both the past and future reference frames must be available. Compression ratios of 26:1 are possible for MPEG-1.

Since the decoding of a B-frame requires both a past reference and a future reference, the order in which the compressed video stream is stored in a digital storage system or transmitted over a network is different from the order in which the frames are to be displayed. The frames are stored and transmitted in the order needed by the decoder. The decoder has the task to display the frames in the proper order once they are decoded.

Data

In this study, the behavior of three commercially available MPEG-1 video streams are investigated. The motivation for the models presented is covered in detail in [1].

Performance Measures

To evaluate the prediction functions and segmentation techniques, three metrics are used:

$$MAPE, \text{ Mean Absolute Percent Error} = \frac{\sum_{t=1}^{n} \frac{|y(t)-\hat{y}(t)|}{y(t)}}{n} * 100$$

$$MAD, \text{ Mean Absolute Deviation} = \frac{\sum_{t=1}^{n} |y(t)-\hat{y}(t)|}{n}$$

$$MSD, \text{ Mean Squared Deviation} = \frac{\sum_{t=1}^{n} (y(t)-\hat{y}(t))^2}{n}$$

Video 1

Time Invariant Models

For time invariant models, the averages and 95% confidence intervals for means that correspond to three segmentation techniques are calculated.

$$f_0^0(t) = \{8,305.15 \pm 83.40$$

$$f_0^1(t) = \begin{cases} 20,671.58 \pm 198.17 & \text{if } t \bmod 15 = 0 \\ 11,866.87 \pm 46.38 & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 5,643.57 \pm 20.01 & \text{else} \end{cases}$$

$$f_0^2(t) = \begin{cases} 20,671.58 \pm 198.17 & \text{if } t \bmod 15 = 0 \\ 4,997.27 \pm 54.18 & \text{if } t \bmod 15 = 1 \\ 6,345.87 \pm 48.56 & \text{if } t \bmod 15 = 2 \\ 11,651.84 \pm 68.36 & \text{if } t \bmod 15 = 3 \\ 5,518.25 \pm 46.07 & \text{if } t \bmod 15 = 4 \\ 4,638.99 \pm 73.84 & \text{if } t \bmod 15 = 5 \\ 11,700.29 \pm 89.91 & \text{if } t \bmod 15 = 6 \\ 6,431.71 \pm 52.44 & \text{if } t \bmod 15 = 7 \\ 5,905.89 \pm 41.51 & \text{if } t \bmod 15 = 8 \\ 12,183.59 \pm 92.41 & \text{if } t \bmod 15 = 9 \\ 5,498.64 \pm 48.34 & \text{if } t \bmod 15 = 10 \\ 5,712.28 \pm 47.42 & \text{if } t \bmod 15 = 11 \\ 11,931.84 \pm 101.11 & \text{if } t \bmod 15 = 12 \\ 5,676.68 \pm 46.61 & \text{if } t \bmod 15 = 13 \\ 5,710.27 \pm 35.37 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the model probability of H_0 to hold is calculated, where H_0 stands for the Null Hypothesis, constant term (interception) being 0. As can be seen from the following table, probability of H_0 being true, p value, is negligible.

Model & Frame Type	Upper Bound for p Value of H_0
$f_1^0(t)$	0.0001
$f_1^1(t)$, **I Frames**	0.0001
$f_1^1(t)$, **P Frames**	0.0001
$f_1^1(t)$, **B Frames**	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001
$f_1^2(t), 2^{nd}$ **Frames**	0.0001
$f_1^2(t), 3^{rd}$ **Frames**	0.0001
$f_1^2(t), 4^{th}$ **Frames**	0.0001
$f_1^2(t), 5^{th}$ **Frames**	0.0001
$f_1^2(t), 6^{th}$ **Frames**	0.0001
$f_1^2(t), 7^{th}$ **Frames**	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001
$f_1^2(t), 9^{th}$ **Frames**	0.0001
$f_1^2(t), 10^{th}$ **Frames**	0.0001
$f_1^2(t), 11^{th}$ **Frames**	0.0001
$f_1^2(t), 12^{th}$ **Frames**	0.0001
$f_1^2(t), 13^{th}$ **Frames**	0.0001
$f_1^2(t), 14^{th}$ **Frames**	0.0001

The higher degrees of segmentation shows significant decreases in error measures:

Prediction Function	MAPE	MAD	MSD
$f_0^0(t)$	46.79	3551.78	19663831
$f_0^1(t)$	10.88	752.73	1360328
$f_0^2(t)$	9.09	662.98	1171230

Linear Models

In this section, a linear model is fitted to each segment. This way, the trend information within each video is utilized. Below are the linear models constructed at 95% confidence level.

$$f_1^0(t) = 8307.8 \pm 170.77 - (0.00051 \pm 0.02461)t$$

$$f_1^1(t) = \begin{cases} 21947 \pm 379.89 - 0.246434t & \text{if } t \bmod 15 = 0 \\ 11638 \pm 92.20 + 0.044247t & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 5606.82 \pm 40.00 + 0.007094t & \text{else} \end{cases}$$

$$f_1^3(t) = \begin{cases} 21947 \pm 379.89 - 0.246434t & \text{if } t \bmod 15 = 0 \\ 5075.95 \pm 107.92 - 0.015199t & \text{if } t \bmod 15 = 1 \\ 6216.35 \pm 96.30 + 0.025017t & \text{if } t \bmod 15 = 2 \\ 11432 \pm 162.36 + 0.042429t & \text{if } t \bmod 15 = 3 \\ 5481.49 \pm 91.92 + 0.007099t & \text{if } t \bmod 15 = 4 \\ 4762.97 \pm 147.09 - 0.023934t & \text{if } t \bmod 15 = 5 \\ 11577 \pm 179.27 + 0.023852t & \text{if } t \bmod 15 = 6 \\ 6207.60 \pm 102.96 + 0.043249t & \text{if } t \bmod 15 = 7 \\ 5837.38 \pm 82.74 + 0.013215t & \text{if } t \bmod 15 = 8 \\ 11943 \pm 183.49 + 0.046385t & \text{if } t \bmod 15 = 9 \\ 5545.67 \pm 96.33 - 0.009072t & \text{if } t \bmod 15 = 10 \\ 5621.35 \pm 94.46 + 0.017535t & \text{if } t \bmod 15 = 11 \\ 11579 \pm 201.76 + 0.06424t & \text{if } t \bmod 15 = 12 \\ 5601.08 \pm 92.96 + 0.014594t & \text{if } t \bmod 15 = 13 \\ 5720.03 \pm 70.71 - 0.001882t & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the linear models, we carry out the statistical hypothesis testing on Null hypothesizes, H_0, H_0' where:

H_0 : The constant term (intercept) is zero.

H_0' : The linear term (slope) is zero.

Below are the probabilities of H_0 and H_0' to be true, p values:

Model & Frame Type	**Upper Bound for p Value of H_0**	**Upper Bound for p Value of H_0'**
$f_1^0(t)$	0.0001	0.9721
$f_1^1(t)$, **I Frames**	0.0001	0.0001
$f_1^1(t)$, **P Frames**	0.0001	0.0001
$f_1^1(t)$, **B Frames**	0.0001	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001	0.0993
$f_1^2(t), 2^{nd}$ **Frames**	0.0001	0.0024
$f_1^2(t), 3^{rd}$ **Frames**	0.0001	0.0023
$f_1^2(t), 4^{th}$ **Frames**	0.0001	0.3656
$f_1^2(t), 5^{th}$ **Frames**	0.0001	0.0568
$f_1^2(t), 6^{th}$ **Frames**	0.0001	0.1192
$f_1^2(t), 7^{th}$ **Frames**	0.0001	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001	0.0614
$f_1^2(t), 9^{th}$ **Frames**	0.0001	0.0031
$f_1^2(t), 10^{th}$ **Frames**	0.0001	0.2705
$f_1^2(t), 11^{th}$ **Frames**	0.0001	0.0297
$f_1^2(t), 12^{th}$ **Frames**	0.0001	0.0002
$f_1^2(t), 13^{th}$ **Frames**	0.0001	0.0662
$f_1^2(t), 14^{th}$ **Frames**	0.0001	0.7551

The performance measures that correspond to linear models:

Prediction Function	**MAPE**	**MAD**	**MSD**
$f_1^0(t)$	46.79	3551.78	19663828
$f_1^1(t)$	10.83	743.47	1319094
$f_1^2(t)$	9.00	651.59	1127355

Quadratic Models

In this section, a quadratic model is fitted to the individual segments. The equations and confidence intervals constructed at 95% confidence intervals are listed below:

$$f_2^0(t) = (8303.77 \pm 256.13) - 0.001823t - 0.000000225t^2$$

$$f_2^1(t) = \begin{cases} 22939.00 \pm 560.44 - 0.822430t + 0.000055652t^2 \\ \quad \text{if } t \bmod 15 = 0 \\ 11635.00 \pm 138.33 + 0.045992t - 0.000000169t^2 \\ \quad \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 5500.26 \pm 59.91 + 0.068795t - 0.000005955t^2 \\ \quad \text{else} \end{cases}$$

$$f_2^2(t) = \begin{cases} 22939.00 \pm 560.44 - 0.822430t + 0.000055652t^2 & \text{if } t \bmod 15 = 0 \\ 5042.48 \pm 161.71 + 0.004220t - 0.000001876t^2 & \text{if } t \bmod 15 = 1 \\ 6059.15 \pm 143.49 + 0.116208t - 0.000008807t^2 & \text{if } t \bmod 15 = 2 \\ 11514.00 \pm 243.32 - 0.004796t + 0.000004560t^2 & \text{if } t \bmod 15 = 3 \\ 5439.33 \pm 137.82 + 0.031533t - 0.000002359t^2 & \text{if } t \bmod 15 = 4 \\ 4754.61 \pm 220.67 - 0.019090t - 0.000000468t^2 & \text{if } t \bmod 15 = 5 \\ 11559.00 \pm 269.00 + 0.034161t - 0.000000995t^2 & \text{if } t \bmod 15 = 6 \\ 5955.53 \pm 152.45 + 0.189189t - 0.000014081t^2 & \text{if } t \bmod 15 = 7 \\ 5736.61 \pm 123.80 + 0.071538t - 0.000005626t^2 & \text{if } t \bmod 15 = 8 \\ 11884.00 \pm 275.48 + 0.080342t - 0.000003275t^2 & \text{if } t \bmod 15 = 9 \\ 5391.38 \pm 144.16 + 0.080158t - 0.000008605t^2 & \text{if } t \bmod 15 = 10 \\ 5464.85 \pm 141.05 + 0.108003t - 0.000008722t^2 & \text{if } t \bmod 15 = 11 \\ 11582.00 \pm 300.70 + 0.073961t - 0.000000938t^2 & \text{if } t \bmod 15 = 12 \\ 5555.52 \pm 139.66 + 0.040948t - 0.000002544t^2 & \text{if } t \bmod 15 = 13 \\ 5604.91 \pm 105.69 + 0.064687t - 0.000006424t^2 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the quadratic models, statistical hypothesis testing is separately carried out on H_0, H_0', and H_0'' where:

H_0 : The constant term (intercept) is zero.
H_0' : The linear term (slope) is zero.
H_0'' : The quadratic term is zero.

The upper bounds on the probabilities, p values, that H_0, H_0', and H_0'' are true, under the observed data are given below:

Model & Frame Type	Upper Bound for p Value of H_0	Upper Bound for p Value of H_0'	Upper Bound for p Value of H_0''
$f_1^0(t)$	0.0001	0.9750	0.9670
$f_1^1(t)$, **I Frames**	0.0001	0.0001	0.0001
$f_1^1(t)$, **P Frames**	0.0001	0.0001	0.0001
$f_1^1(t)$, **B Frames**	0.0001	0.1439	0.9543
$f_1^2(t), 0^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001	0.9088	0.5860
$f_1^2(t), 2^{nd}$ **Frames**	0.0001	0.0004	0.0041
$f_1^2(t), 3^{rd}$ **Frames**	0.0001	0.9310	0.3787
$f_1^2(t), 4^{th}$ **Frames**	0.0001	0.3150	0.4212
$f_1^2(t), 5^{th}$ **Frames**	0.0001	0.7038	0.9206
$f_1^2(t), 6^{th}$ **Frames**	0.0001	0.5768	0.8619
$f_1^2(t), 7^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001	0.0112	0.0327
$f_1^2(t), 9^{th}$ **Frames**	0.0001	0.1999	0.5756
$f_1^2(t), 10^{th}$ **Frames**	0.0001	0.0147	0.0050
$f_1^2(t), 11^{th}$ **Frames**	0.0001	0.0008	0.0037
$f_1^2(t), 12^{th}$ **Frames**	0.0001	0.2800	0.8834
$f_1^2(t), 13^{th}$ **Frames**	0.0001	0.1978	0.3918
$f_1^2(t), 14^{th}$ **Frames**	0.0001	0.0073	0.0044

The performance measures for quadratic models:

Prediction Function	MAPE	MAD	MSD
$f_2^0(t)$	46.79	3551.79	19663825
$f_2^1(t)$	10.80	740.46	1304337
$f_2^2(t)$	8.97	647.57	1111770

Video 2

Time Invariant Models

For time invariant models, the averages and 95% confidence intervals for means that correspond to three segmentation techniques are:

$$f_0^0(t) = \{4, 89783.36 \pm 55.01$$

$$f_0^1(t) = \begin{cases} 18095 \pm 198.21 & \text{if } t \bmod 15 = 0 \\ 6445.776460 \pm 21.72 & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 2958.011862 \pm 5.70\text{else} & \end{cases}$$

$$f_0^2(t) = \begin{cases} 18095 \pm 198.21 & \text{if } t \bmod 15 = 0 \\ 2854.42 \pm 21.14 & \text{if } t \bmod 15 = 1 \\ 2996.04 \pm 16.48 & \text{if } t \bmod 15 = 2 \\ 6425.22 \pm 44.64 & \text{if } t \bmod 15 = 3 \\ 2964.71 \pm 20.52 & \text{if } t \bmod 15 = 4 \\ 2992.41 \pm 17.38 & \text{if } t \bmod 15 = 5 \\ 6446.57 \pm 42.80 & \text{if } t \bmod 15 = 6 \\ 2952.22 \pm 17.97 & \text{if } t \bmod 15 = 7 \\ 2980.81 \pm 17.04 & \text{if } t \bmod 15 = 8 \\ 6449.31 \pm 42.68 & \text{if } t \bmod 15 = 9 \\ 2933.14 \pm 17.71 & \text{if } t \bmod 15 = 10 \\ 2989.00 \pm 16.83 & \text{if } t \bmod 15 = 11 \\ 6462.02 \pm 43.60 & \text{if } t \bmod 15 = 12 \\ 2938.23 \pm 18.46 & \text{if } t \bmod 15 = 13 \\ 2979.15 \pm 17.78 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the model probability of H_0 to hold is calculated, where H_0 stands for the Null Hypothesis, constant term (interception) being 0. As can be seen from the following table, probability of H_0 being true, p value, is negligible.

Model & Frame Type	Upper Bound for p Value of H_0
$f_1^0(t)$	0.0001
$f_1^1(t)$, **I Frames**	0.0001
$f_1^1(t)$, **P Frames**	0.0001
$f_1^1(t)$, **B Frames**	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001
$f_1^2(t), 2^{nd}$ **Frames**	0.0001
$f_1^2(t), 3^{rd}$ **Frames**	0.0001
$f_1^2(t), 4^{th}$ **Frames**	0.0001
$f_1^2(t), 5^{th}$ **Frames**	0.0001
$f_1^2(t), 6^{th}$ **Frames**	0.0001
$f_1^2(t), 7^{th}$ **Frames**	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001
$f_1^2(t), 9^{th}$ **Frames**	0.0001
$f_1^2(t), 10^{th}$ **Frames**	0.0001
$f_1^2(t), 11^{th}$ **Frames**	0.0001
$f_1^2(t), 12^{th}$ **Frames**	0.0001
$f_1^2(t), 13^{th}$ **Frames**	0.0001
$f_1^2(t), 14^{th}$ **Frames**	0.0001

The higher degrees of segmentation shows significant decreases in error measures:

Prediction Function	MAPE	MAD	MSD
$f_0^0(t)$	56.27	2591.77	15935228
$f_0^1(t)$	9.93	546.56	1174230
$f_0^2(t)$	9.89	545.53	1173092

Linear Models

In this section, a linear model is fitted to each segment. Below are the linear models constructed at 95% confidence level.

$$f_1^0(t) = 4896.131512 \pm 110.01 + 0.000121t$$

$$f_1^1(t) = \begin{cases} 18811 \pm 393.88 - 0.070777t & \text{if } t \bmod 15 = 0 \\ 6385.921840 \pm 43.40 + 0.005917t & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 2906.066408 \pm 11.54 + 0.005135t & \text{else} \end{cases}$$

$$f_1^3(t) = \begin{cases} 18811 \pm 393.88 - 0.070777t & \text{if } t \bmod 15 = 0 \\ 2792.160428 \pm 42.11 + 0.006158t & \text{if } t \bmod 15 = 1 \\ 2933.144487 \pm 32.73 + 0.006220t & \text{if } t \bmod 15 = 2 \\ 6366.851232 \pm 89.22 + 0.005772t & \text{if } t \bmod 15 = 3 \\ 2930.250981 \pm 41.00 + 0.003407t & \text{if } t \bmod 15 = 4 \\ 2939.000987 \pm 34.62 + 0.005280t & \text{if } t \bmod 15 = 5 \\ 6342.830558 \pm 85.38 + 0.010255t & \text{if } t \bmod 15 = 6 \\ 2888.884348 \pm 35.75 + 0.006260t & \text{if } t \bmod 15 = 7 \\ 2925.184907 \pm 33.91 + 0.005498t & \text{if } t \bmod 15 = 8 \\ 6396.565802 \pm 85.34 + 0.005212t & \text{if } t \bmod 15 = 9 \\ 2870.275074 \pm 35.23 + 0.006212t & \text{if } t \bmod 15 = 10 \\ 2948.631272 \pm 33.60 + 0.003988t & \text{if } t \bmod 15 = 11 \\ 6437.541034 \pm 87.26 + 0.002420t & \text{if } t \bmod 15 = 12 \\ 2891.815707 \pm 36.84 + 0.004589t & \text{if } t \bmod 15 = 13 \\ 2941.495550 \pm 35.51 + 0.003722t & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the linear models, we carry out the statistical hypothesis testing on Null hypothesizes, H_0, H_0' where:

H_0 : The constant term (intercept) is zero.

H_0' : The linear term (slope) is zero.

Below are the probabilities of H_0 and H_0' to be true, p values:

Model & Frame Type	Upper Bound for p Value of H_0	Upper Bound for p Value of H_0'
$f_1^0(t)$	0.0001	0.9799
$f_1^1(t)$, **I Frames**	0.0001	0.0001
$f_1^1(t)$, **P Frames**	0.0001	0.0018
$f_1^1(t)$, **B Frames**	0.0001	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001	0.0008
$f_1^2(t), 2^{nd}$ **Frames**	0.0001	0.0001
$f_1^2(t), 3^{rd}$ **Frames**	0.0001	0.1389
$f_1^2(t), 4^{th}$ **Frames**	0.0001	0.0574
$f_1^2(t), 5^{th}$ **Frames**	0.0001	0.0005
$f_1^2(t), 6^{th}$ **Frames**	0.0001	0.0060
$f_1^2(t), 7^{th}$ **Frames**	0.0001	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001	0.0002
$f_1^2(t), 9^{th}$ **Frames**	0.0001	0.1621
$f_1^2(t), 10^{th}$ **Frames**	0.0001	0.0001
$f_1^2(t), 11^{th}$ **Frames**	0.0001	0.0066
$f_1^2(t), 12^{th}$ **Frames**	0.0001	0.5257
$f_1^2(t), 13^{th}$ **Frames**	0.0001	0.0044
$f_1^2(t), 14^{th}$ **Frames**	0.0001	0.0165

The performance measures that correspond to linear models:

Prediction Function	MAPE	MAD	MSD
$f_1^0(t)$	56.22	2590.9	15935235
$f_1^1(t)$	9.86	542.80	1161915
$f_1^2(t)$	9.81	541.59	1160679

Quadratic Models

In this section, a quadratic model is fitted to the individual segments. The equations and confidence intervals constructed at 95% confidence intervals are listed below:

$$f_2^0(t) = 4903.102010 \pm 165.02 - 0.001946t + 0.000000102t^2$$

$$f_2^1(t) = \begin{cases} 17321 \pm 580.73 + 0.371769t - 0.000021887t^2 \\ \quad \text{if } t \bmod 15 = 0 \\ 6570.149814 \pm 64.77 - 0.048720t + 0.000002701t^2 \\ \quad \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 2991.191952 \pm 17.20 - 0.020110t + 0.000001248t^2 \\ \quad \text{else} \end{cases}$$

$$f_2^2(t) = \begin{cases} 17321 \pm 580.73 + 0.371769t - 0.000021887t^2 & \text{if } t \bmod 15 = 0 \\ 2885.96 \pm 62.73 - 0.021690t + 0.000001377t^2 & \text{if } t \bmod 15 = 1 \\ 3004.81 \pm 48.80 - 0.015054t + 0.000001052t^2 & \text{if } t \bmod 15 = 2 \\ 6508.63 \pm 133.41 - 0.036305t + 0.000002080t^2 & \text{if } t \bmod 15 = 3 \\ 3005.54 \pm 61.26 - 0.018934t + 0.000001104t^2 & \text{if } t \bmod 15 = 4 \\ 3021.45 \pm 51.59 - 0.019180t + 0.000001209t^2 & \text{if } t \bmod 15 = 5 \\ 6579.87 \pm 126.96 - 0.060051t + 0.000003475t^2 & \text{if } t \bmod 15 = 6 \\ 2989.99 \pm 53.14 - 0.023722t + 0.000001482t^2 & \text{if } t \bmod 15 = 7 \\ 3013.38 \pm 50.50 - 0.020652t + 0.000001292t^2 & \text{if } t \bmod 15 = 8 \\ 6593.37 \pm 127.30 - 0.053126t + 0.000002883t^2 & \text{if } t \bmod 15 = 9 \\ 2975.74 \pm 52.35 - 0.025045t + 0.000001544t^2 & \text{if } t \bmod 15 = 10 \\ 3026.78 \pm 50.13 - 0.019168t + 0.000001144t^2 & \text{if } t \bmod 15 = 11 \\ 6598.83 \pm 130.50 - 0.045398t + 0.000002364t^2 & \text{if } t \bmod 15 = 12 \\ 2981.87 \pm 54.94 - 0.022105t + 0.000001319t^2 & \text{if } t \bmod 15 = 13 \\ 3006.57 \pm 53.12 - 0.015563t + 0.000000953t^2 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the quadratic models, statistical hypothesis testing is separately carried out on H_0, H_0', and H_0'' where:

H_0 : The constant term (intercept) is zero.
H_0' : The linear term (slope) is zero.
H_0'' : The quadratic term is zero.

The upper bounds on the probabilities, p values, that H_0, H_0', and H_0'' are true, under the observed data are given below:

Model & Frame Type	Upper Bound for p Value of H_0	Upper Bound for p Value of H_0'	Upper Bound for p Value of H_0''
$f_1^0(t)$	0.0001	0.9194	0.9116
$f_1^1(t)$, **I Frames**	0.0001	0.0001	0.0001
$f_1^1(t)$, **P Frames**	0.0001	0.0001	0.0001
$f_1^1(t)$, **B Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001	0.0031	0.0001
$f_1^2(t), 2^{nd}$ **Frames**	0.0001	0.0082	0.0041
$f_1^2(t), 3^{rd}$ **Frames**	0.0001	0.0197	0.0053
$f_1^2(t), 4^{th}$ **Frames**	0.0001	0.0081	0.0013
$f_1^2(t), 5^{th}$ **Frames**	0.0001	0.0014	0.0001
$f_1^2(t), 6^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 7^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001	0.0005	0.0001
$f_1^2(t), 9^{th}$ **Frames**	0.0001	0.0004	0.0001
$f_1^2(t), 10^{th}$ **Frames**	0.0001	0.0001	0.0001
$f_1^2(t), 11^{th}$ **Frames**	0.0001	0.0010	0.0001
$f_1^2(t), 12^{th}$ **Frames**	0.0001	0.0029	0.0012
$f_1^2(t), 13^{th}$ **Frames**	0.0001	0.0006	0.0001
$f_1^2(t), 14^{th}$ **Frames**	0.0001	0.0120	0.0013

The performance measures for quadratic models:

Prediction Function	MAPE	MAD	MSD
$f_2^0(t)$	55.76	2582.41	15936614
$f_2^1(t)$	9.73	535.12	1129390
$f_2^2(t)$	9.68	533.95	1128064

Video 3

Time Invariant Models

For time invariant models, the averages and 95% confidence intervals for means that correspond to three segmentation techniques are:

$$f_0^0(t) = \{4774.97 \pm 43.65$$

$$f_0^1(t) = \begin{cases} 16499 \pm 298.57 & \text{if } t \bmod 15 = 0 \\ 6218.27 \pm 28.91 & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 3024.81 \pm 6.76 & \text{else} \end{cases}$$

$$f_0^2(t) = \begin{cases} 16499 \pm 298.57 & \text{if } t \bmod 15 = 0 \\ 3003.74 \pm 391.85 & \text{if } t \bmod 15 = 1 \\ 3077.30 \pm 18.94 & \text{if } t \bmod 15 = 2 \\ 6237.91 \pm 60.43 & \text{if } t \bmod 15 = 3 \\ 3015.06 \pm 38.39 & \text{if } t \bmod 15 = 4 \\ 3065.55 \pm 19.60 & \text{if } t \bmod 15 = 5 \\ 6200.93 \pm 58.40 & \text{if } t \bmod 15 = 6 \\ 2982.31 \pm 21.89 & \text{if } t \bmod 15 = 7 \\ 3040.91 \pm 17.70 & \text{if } t \bmod 15 = 8 \\ 6207.58 \pm 55.91 & \text{if } t \bmod 15 = 9 \\ 2992.03 \pm 21.99 & \text{if } t \bmod 15 = 10 \\ 3034.57 \pm 16.72 & \text{if } t \bmod 15 = 11 \\ 6226.65 \pm 56.43 & \text{if } t \bmod 15 = 12 \\ 2995.28 \pm 22.79 & \text{if } t \bmod 15 = 13 \\ 3041.32 \pm 17.41 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the model probability of H_0 to hold is calculated, where H_0 stands for the Null Hypothesis, constant term (interception) being 0. As can be seen from the following table, probability of H_0 being true, p value, is negligible.

Model & Frame Type	**Upper Bound for p Value of H_0**
$f_1^0(t)$	0.0001
$f_1^1(t)$, **I Frames**	0.0001
$f_1^1(t)$, **P Frames**	0.0001
$f_1^1(t)$, **B Frames**	0.0001
$f_1^2(t), 0^{th}$ **Frames**	0.0001
$f_1^2(t), 1^{st}$ **Frames**	0.0001
$f_1^2(t), 2^{nd}$ **Frames**	0.0001
$f_1^2(t), 3^{rd}$ **Frames**	0.0001
$f_1^2(t), 4^{th}$ **Frames**	0.0001
$f_1^2(t), 5^{th}$ **Frames**	0.0001
$f_1^2(t), 6^{th}$ **Frames**	0.0001
$f_1^2(t), 7^{th}$ **Frames**	0.0001
$f_1^2(t), 8^{th}$ **Frames**	0.0001
$f_1^2(t), 9^{th}$ **Frames**	0.0001
$f_1^2(t), 10^{th}$ **Frames**	0.0001
$f_1^2(t), 11^{th}$ **Frames**	0.0001
$f_1^2(t), 12^{th}$ **Frames**	0.0001
$f_1^2(t), 13^{th}$ **Frames**	0.0001
$f_1^2(t), 14^{th}$ **Frames**	0.0001

The higher degrees of segmentation shows significant decreases in error measures:

Prediction Function	**MAPE**	**MAD**	**MSD**
$f_0^0(t)$	54.3	2410.24	13792612
$f_0^1(t)$	16.73	689.74	2027704
$f_0^2(t)$	16.71	689.59	2027024

Linear Models

In this section, a linear model is fitted to each segment. This way, the trend information within each video is utilized. Below are the linear models constructed at 95% confidence level.

$$f_1^0(t) = 4796.96 \pm 220.91 - 0.001581t$$

$$f_1^1(t) = \begin{cases} 16834 \pm 424.15 - 0.024115t & \text{if } t \bmod 15 = 0 \\ 6240.22 \pm 57.81 - 0.001579t & \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 3014.86 \pm 13.52 + 0.000715t & \text{else} \end{cases}$$

$$f_1^3(t) = \begin{cases} 16834 \pm 424.15 - 0.024115t & \text{if } t \bmod 15 = 0 \\ 2974.95 \pm 58.87 + 0.002071t & \text{if } t \bmod 15 = 1 \\ 3046.71 \pm 37.85 + 0.002200t & \text{if } t \bmod 15 = 2 \\ 6211.93 \pm 120.85 + 0.001869t & \text{if } t \bmod 15 = 3 \\ 3017.40 \pm 47.64 - 0.000168t & \text{if } t \bmod 15 = 4 \\ 3047.61 \pm 39.19 + 0.001291t & \text{if } t \bmod 15 = 5 \\ 6249.88 \pm 116.79 - 0.003521t & \text{if } t \bmod 15 = 6 \\ 2980.05 \pm 43.79 + 0.000162t & \text{if } t \bmod 15 = 7 \\ 3025.89 \pm 35.40 + 0.001080t & \text{if } t \bmod 15 = 8 \\ 6239.12 \pm 111.85 - 0.002268t & \text{if } t \bmod 15 = 9 \\ 3010.12 \pm 43.99 - 0.001301t & \text{if } t \bmod 15 = 10 \\ 3019.50 \pm 33.44 + 0.001083t & \text{if } t \bmod 15 = 11 \\ 6259.96 \pm 112.90 - 0.002395t & \text{if } t \bmod 15 = 12 \\ 3001.57 \pm 45.61 - 0.000452t & \text{if } t \bmod 15 = 13 \\ 3024.89 \pm 34.82 + 0.001181t & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the linear models, we carry out the statistical hypothesis testing on Null hypothesizes, H_0, H_0' where:

H_0 : The constant term (intercept) is zero.

H_0' : The linear term (slope) is zero.

Below are the probabilities of H_0 and H_0' to be true, p values:

Model & Frame Type	Upper Bound for p Value of H_0	Upper Bound for p Value of H_0'
$f_1^0(t)$	0.0001	0.5686
$f_1^1(t)$, **I Frames**	0.0001	0.0737
$f_1^1(t)$, **P Frames**	0.0001	0.3901
$f_1^1(t)$, **B Frames**	0.0001	0.0961
$f_1^2(t), 0^{th}$ **Frames**	0.0001	0.0737
$f_1^2(t), 1^{st}$ **Frames**	0.0001	0.2683
$f_1^2(t), 2^{nd}$ **Frames**	0.0001	0.0675
$f_1^2(t), 3^{rd}$ **Frames**	0.0001	0.6267
$f_1^2(t), 4^{th}$ **Frames**	0.0001	0.9114
$f_1^2(t), 5^{th}$ **Frames**	0.0001	0.3002
$f_1^2(t), 6^{th}$ **Frames**	0.0001	0.3429
$f_1^2(t), 7^{th}$ **Frames**	0.0001	0.9072
$f_1^2(t), 8^{th}$ **Frames**	0.0001	0.3370
$f_1^2(t), 9^{th}$ **Frames**	0.0001	0.5234
$f_1^2(t), 10^{th}$ **Frames**	0.0001	0.3519
$f_1^2(t), 11^{th}$ **Frames**	0.0001	0.3079
$f_1^2(t), 12^{th}$ **Frames**	0.0001	0.5044
$f_1^2(t), 13^{th}$ **Frames**	0.0001	0.7550
$f_1^2(t), 14^{th}$ **Frames**	0.0001	0.2857

The performance measures that correspond to linear models:

Prediction Function	MAPE	MAD	MSD
$f_1^0(t)$	54.33	2410.26	13792451
$f_1^1(t)$	16.73	690.17	2025137
$f_1^2(t)$	16.71	690.02	2024335

Quadratic Models

In this section, a quadratic model is fitted to the individual segments. The equations and confidence intervals constructed at 95% confidence intervals are listed below:

$$f_2^0(t) = 4818.07 \pm 130.94 - 0.006136t + 0.000000164t^2$$

$$f_2^1(t) = \begin{cases} 16815 \pm 635.97 - 0.020006t - 0.000000148t^2 \\ \quad \text{if } t \bmod 15 = 0 \\ 6241.39 \pm 86.72 - 0.001832t + 9.092946510^{-9}t^2 \\ \quad \text{if } (t \bmod 3 = 0) \wedge (t \bmod 15 \neq 0) \\ 3045.75 \pm 20.28 - 0.005947t + 0.000000240t^2 \\ \quad \text{else} \end{cases}$$

$$f_2^2(t) = \begin{cases} 16815 \pm 635.97 - 0.020006t - 0.000000148t^2 & \text{if } t \bmod 15 = 0 \\ 2974.10 \pm 88.28 + 0.002254t - 6.59911710^{-9}t^2 & \text{if } t \bmod 15 = 1 \\ 3044.12 \pm 56.76 + 0.002758t - 2.00611310^{-8}t^2 & \text{if } t \bmod 15 = 2 \\ 6143.29 \pm 181.20 + 0.016688t - 0.000000533t^2 & \text{if } t \bmod 15 = 3 \\ 3073.60 \pm 71.37 - 0.012317t + 0.000000437t^2 & \text{if } t \bmod 15 = 4 \\ 3097.85 \pm 58.70 - 0.009553t + 0.000000390t^2 & \text{if } t \bmod 15 = 5 \\ 6300.53 \pm 175.16 - 0.014450t + 0.000000393t^2 & \text{if } t \bmod 15 = 6 \\ 3045.05 \pm 65.59 - 0.013862t + 0.000000504t^2 & \text{if } t \bmod 15 = 7 \\ 3052.37 \pm 53.09 - 0.004634t + 0.000000205t^2 & \text{if } t \bmod 15 = 8 \\ 6258.12 \pm 167.83 - 0.006365t + 0.000000147t^2 & \text{if } t \bmod 15 = 9 \\ 3048.70 \pm 65.97 - 0.009620t + 0.000000299t^2 & \text{if } t \bmod 15 = 10 \\ 3040.81 \pm 50.17 - 0.003511t + 0.000000165t^2 & \text{if } t \bmod 15 = 11 \\ 6263.80 \pm 169.52 - 0.003222t + 2.974696210^{-8}t^2 & \text{if } t \bmod 15 = 12 \\ 3043.26 \pm 68.41 - 0.009439t + 0.000000323t^2 & \text{if } t \bmod 15 = 13 \\ 3037.61 \pm 52.27 - 0.001561t + 9.858208610^{-8}t^2 & \text{if } t \bmod 15 = 14 \end{cases}$$

To verify the quadratic models, statistical hypothesis testing is separately carried out on H_0, H_0', and H_0'' where:

H_0 : The constant term (intercept) is zero.
H_0' : The linear term (slope) is zero.
H_0'' : The quadratic term is zero.

The upper bounds on the probabilities, p values, that H_0, H_0', and H_0'' are true, under the observed data are given below:

Model & Frame Type	Upper Bound for p Value of H_0	Upper Bound for p Value of H_0'	Upper Bound for p Value of H_0''
$f_1^0(t)$	0.0001	0.5802	0.6715
$f_1^1(t)$, I Frames	0.0001	0.7105	0.9372
$f_1^1(t)$, P Frames	0.0001	0.8031	0.9717
$f_1^1(t)$, B Frames	0.0001	0.0005	0.0001
$f_1^2(t), 0^{th}$ Frames	0.0001	0.7105	0.9372
$f_1^2(t), 1^{st}$ Frames	0.0001	0.7631	0.9798
$f_1^2(t), 2^{nd}$ Frames	0.0001	0.5665	0.9046
$f_1^2(t), 3^{rd}$ Frames	0.0001	0.2774	0.3191
$f_1^2(t), 4^{th}$ Frames	0.0001	0.0419	0.0382
$f_1^2(t), 5^{th}$ Frames	0.0001	0.0550	0.0245
$f_1^2(t), 6^{th}$ Frames	0.0001	0.3305	0.4472
$f_1^2(t), 7^{th}$ Frames	0.0001	0.0127	0.0092
$f_1^2(t), 8^{th}$ Frames	0.0001	0.3031	0.1897
$f_1^2(t), 9^{th}$ Frames	0.0001	0.6545	0.7660
$f_1^2(t), 10^{th}$ Frames	0.0001	0.0854	0.1244
$f_1^2(t), 11^{th}$ Frames	0.0001	0.4089	0.2644
$f_1^2(t), 12^{th}$ Frames	0.0001	0.8224	0.9525
$f_1^2(t), 13^{th}$ Frames	0.0001	0.1035	0.1094
$f_1^2(t), 14^{th}$ Frames	0.0001	0.7243	0.5224

The performance measures for quadratic models:

Prediction Function	MAPE	MAD	MSD
$f_2^0(t)$	54.33	2410.29	13792362
$f_2^1(t)$	16.72	690.01	2025005
$f_2^2(t)$	16.70	689.91	2024036

Conclusion

As can be seen from Figure 2, segmentation techniques result in considerable improvements over single segment case. Segmentation to 15 partitions represent about 17% improvement over the segmentation technique where the data were partitioned to 3 segments.

For the second and third video streams, however, the difference between 3 and 15 segment cases are negligible.

The segmentation techniques, without exception, provide dramatic improvements over single segment model fitting.

The higher degrees of the complexity of the models, however, result in an insignificant improvements.

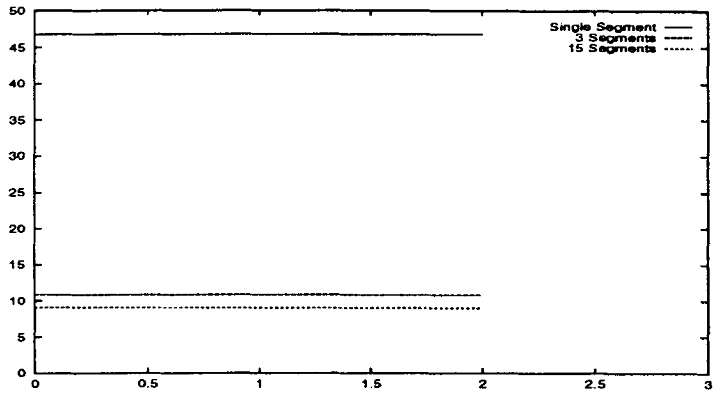

Figure 1: MAPE of different segmentation techniques and models for Video 1. Horizontal axis is the complexity of the polynomial model fitted, where as the vertical axis is MAPE, Mean Absolute Percentage Error.

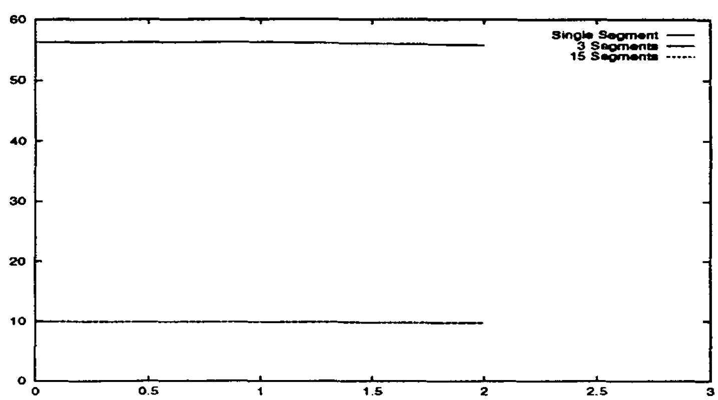

Figure 2: MAPE of different segmentation techniques and models for Video 2. Horizontal axis is the complexity of the polynomial model fitted, where as the vertical axis is MAPE, Mean Absolute Percentage Error.

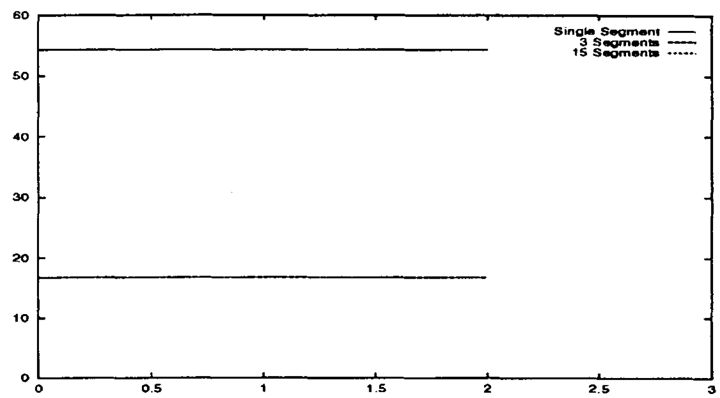

Figure 3: MAPE of different segmentation techniques and models for Video 3. Horizontal axis is the complexity of the polynomial model fitted, where as the vertical axis is MAPE, Mean Absolute Percentage Error.

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