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STATISTICAL MODELLING OF COMPUTER SYSTEMS: A REVIEW*

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STATISTICAL MODELLING OF COMPUTER SYSTEMS: A REVIEW

ABSTRACT

This paper briefly reviews the data dependent statistical methods useful for computer systems modelling. The techniques are classified according to their applicability toward comparison, tuning and design of computer systems. A review of publications dealing with statistical modelling of computer systems is presented and a comprehensive bibliography is included to provide a useful source of reference toward the present and potential applications of statistical methods for computer system modelling.

1. INTRODUCTION

Modern computer systems with multiprogramming and time sharing capabilities constitute complex processes to study. The complexity stems from the multiplicity and the interdependence of the important system variables, the difficulty of workload characterization, the lack of suitable performance measures and the need to evaluate performance in the presence of changing workload. Two commonly used approaches to solve the problems of comparison of computer systems and system features, performance optimization and cost effective design of computer systems are the analytic approach and the simulation approach. The assumptions inherent in analytic approaches, such as the assumptions of independence and exponential distribution in queue theoretic models, are questionable for real systems [14] and need empirical validation. The simulation approach can be more realistic but may be expensive. This approach also requires a reasonable characterization of workload, a problem which is yet to be satisfactorily solved. The limitations of these two approaches point to the use of statistical modelling involving experimentation under actual operating conditions and to the development of models based upon the available data.

This paper briefly reviews the data dependent statistical methods useful for computer systems modelling. The techniques are classified according to their applicability toward comparison, tuning and design of computer systems. A review of publications dealing with statistical modelling of computer systems is presented and a comprehensive bibliography is included to provide a useful source of reference toward the present and potential applications of statistical methods for computer system modelling.

2. THE STATISTICAL TECHNIQUES

Figure 1 [24] is a black box representation of a computer system. The system is subjected to a workload \underline{W} consisting of demands for the use of system resources such as CPU, I/O devices, memory, etc. Depending upon the values of the controllable and the uncontrolled variables \underline{X}_1 and \underline{X}_2 , the system control program allocates the resources to the jobs being processed. The efficiency with which the system processes the workload is termed as the system performance \underline{Y} (e.g. throughput, response time). The response \underline{Y} depends upon the workload \underline{W} , the adjustable variables \underline{X}_1 , the uncontrolled variables \underline{X}_2 and the parameters $\underline{\theta}$. Thus, the performance of the system can be expressed as the following model:

$$\underline{Y} = f(\underline{W}, \underline{X}_1, \underline{X}_2, \underline{\theta}) + \underline{\varepsilon},$$

where $\underline{\varepsilon}$ is the error vector. With this model in mind, various relevant statistical techniques are discussed in the following section.

2.1 COMPARISON

A comparative study may be undertaken to choose between alternative hardware and software features, to select a computer system suitable for the needs of a potential customer or the like. If experiments can be conducted under a constant representative workload, then the problem is one of comparing the observed performance measures for different computer systems or system features. A complete statistical description of the performance measures is required for this purpose and is usually given by the joint distribution function of \underline{Y} . If the observed values of the performance measures form a correlated sequence, techniques such as the auto correlation and cross correlation functions and time series analysis

[41] can be useful. The data for the analyses should be obtained through planned experiments [43, 45, 46] to minimize the effect of extraneous factors and to increase the efficiency of experimentation.

In the presence of changing workload, either a model $\underline{Y} = f(\underline{W}, \underline{X}_1, \underline{X}_2, \underline{\theta}) + \underline{\epsilon}$ can be built to separate the effects of workload and changes on \underline{Y} or an experiment can be designed to minimize or compensate for the effect of changing workload. Since the workload fluctuations will usually be large, blocking should be introduced to maintain a constant environment via designs such as randomized block designs, Latin square designs, etc.

2.2 TUNING

Tuning concerns system improvements by changing the levels of the controllable variables \underline{X}_1 . Interest centers on answering questions of the following kind. Which of the variables \underline{X}_1 control system performance? Can the system performance be predicted for specified values of \underline{W} and \underline{X}_1 ? How to choose settings for \underline{X}_1 to optimize performance? The answers to these and similar questions are provided by the use of techniques for screening of variables, empirical modelling and empirical optimization.

- (a) Screening of variables: The purpose here is to determine that subset of variables which influences system performance. Due to the presence of interaction between the controllable variables, the usual one variable at a time approach can be misleading and expensive. In such a situation planned experimentation is highly desirable. The use of fractional factorial and factorial designs has been found to be best and quite useful for this purpose. If designed experiments are not possible,

regression analysis [48] based upon passive observations of the system (as against active observations from designed experiments resulting from purposeful interference with the system) may be used. Extreme care must be exercised in interpreting the results of regression analysis. Due to the presence of latent variables, correlation between variables and responses may exist without a causal relationship. Furthermore, important variables may be dubbed as unimportant due to their small range in the observed data [39].

- (b) Empirical Modelling: Once the important variables are known, the model $\underline{Y} = f(\underline{W}, \underline{X}_1, \underline{\theta}) + \underline{\epsilon}$ can be obtained for purposes of prediction and system optimization. The unknown functional relationship, which may be linear or nonlinear, and the unknown parameters $\underline{\theta}$ are determined by using the iterative approach of Section 2.3. If the model is based upon passive observations, it provides useful predictions of \underline{Y} within the experimental zone but its applicability toward system optimization may be limited due to the possibility of a lack of causal relationship. Hence, whenever possible, designed experiments should be used. In any case, the empirical nature of the model limits the capability to extrapolate and implies that the experiments should cover the entire zone of interest.

- (c) Empirical Optimization: If the purpose of the investigation is to obtain optimum settings for X_1 , this can be accomplished by the use of techniques such as response surface methodology (RSM) and evolutionary operation (EVOP). RSM consists of conducting a sequence of factorial or fractional factorial designs with X_1 as design variables and Y (or a suitable function thereof) as the response variable. The designs sequentially indicate better settings for X_1 until the optimum is reached. In EVOP [40] changes in X_1 are kept to a minimum and each design is replicated a large number of times to determine the settings for the next design. This permits optimization without impairing the 'normal' operation of the system.

2.3 SYSTEM DESIGN

System design requires a detailed analysis involving considerable physical understanding of the system. In this context, statistical methods provide experimental design and inference techniques to further scientific understanding and to integrate available physical knowledge regarding the system with empirical data to build useful models. Models for computer systems may be mechanistic, empirical--mechanistic or empirical. The choice of model class depends upon the degree of knowledge regarding the system and the objectives of the analysis. A mechanistic model requires extensive knowledge about the system such that the form of the functional relationship is known. It has the advantage of meaningful extrapolation, physical interpretation and usefulness in system design. An empirical model can be built when very little is known about the system. Most frequently, the situation

is in between the two extremes and an empirical-mechanistic approach is followed.

The iterative modelling procedure [35, 42, 44] is shown in Figure 2. The initial formulation of the model may be based upon theoretical or empirical considerations depending upon the model class. The unknown parameters of the model are estimated for best fit to the data obtained by planned experiments using methods such as linear and nonlinear least squares [49]. The fitted model is diagnostically checked for adequacy of fit. Such checks indicate the nature and possible causes of model inadequacy. A theoretical or experimental investigation of these possible reasons for model inadequacy may reveal the appropriate model corrections to be made. This iterative procedure is stopped when an adequate model has been obtained.

3. LITERATURE REVIEW

Papers on the philosophy of modelling as applied to computer systems have been written by Grenander and Tsao [14], Kimbelton [15, 16], Kobayashi [18] and Schatzoff [25].

A variety of descriptive techniques have been used for modelling purposes by several authors. Bryag [10] and Sutherland [27] employed graphical means of description while Anderson and Sargent [1] used distribution theory to fit empirical service time distributions. Empirically valid stochastic models were used for the page exception process by Lewis and Shedler [20] and for the input traffic by Anderson and Sargent [1].

A simple example of balanced design, to block the effect of changing workload, is due to Margolin, Parmelee and Schatzoff [21] for the comparison

of two different algorithms to manage free storage. System containing the first algorithm was run on Monday and Thursday of week 1 and Tuesday and Wednesday of week 2. System employing the second algorithm was run on Tuesday and Wednesday of week 1 and Monday and Thursday of week 2. The design ensures an analysis which is free of any day to day or week to week variation or of linear or quadratic trend within weeks. Another example is the rapid on line switching approach due to Bard [6, 9]. In [9], Bard compared the effects of two page replacement algorithms by switching back and forth from one to the other every five minutes. Measurements were taken every minute and the first observation was discarded to eliminate the transient effects of switching. A paired comparison was made to block the variation due to workload. In [6], Bard gave expressions for the optimum switching rate and the run length required to obtain significant results by considering a compromise between the loss of data due to transients and the loss of discriminatory power due to load fluctuations.

Friedman and Waldbaum [12] describe the use of regression analysis to separate the effects of workload and system changes. Experiments were performed on System/360 Model 91 under OS/MVT to evaluate the effects of changes in the maximum workspace size and the number of workspaces simultaneously in core. The workload was characterized by the number of conversational inputs per hour, the percent CPU utilization for small and large CPU requests, the number of large CPU requests per hour, the number of commands per hour requiring two workspaces in core simultaneously and the number of log ons per hour. Three percentile points on the cumulative distribution function of the response time were taken as performance measures. The effects of system changes were evaluated by relating the three responses to the two system changes and the six workload variables.

A similar study by Waldbaum is given in [31]. Regression analysis to compare system features has been used by Bard [5] for software modifications to CP-67, by Silverman and Yue [26] for software modifications to an information retrieval system, by Waldbaum [30] for hardware and software modifications to a time sharing system and by Watson [32] to evaluate the effect of an additional 256K bytes of high speed core upon the operating system.

Regression analysis has also been used as a tool for screening variables and to obtain predictive models. It was used by Yeh [33] to express CPU utilization as a function of the number of instructions executed/number of bytes transferred and the CPU/CH overlap. Bard applied regression analysis to a set of data collected by monitoring an IBM System 360 Model 67 computer running under CP-67 time sharing system. The results of the analysis to obtain predictive equations for CP overhead based upon significant system functions such as paging, spooling, virtual I/O, etc. are given in Bard and Margolin [3], Bard [4] and Bard and Suryanarayana [7]. In [7], transformations of input variables were used to improve the predictive equations. Schatzoff and Bryant [24] considered the same problem to indicate the difficulties associated with regression, in particular the effect of sample interval on the regression coefficients. Some application of non-linear regression are given by Racite [22].

A comprehensive application of factorial experiments to computer systems is found in Tsao et al [28, 29]. They used a 3^4 factorial design to study the effects of memory size, problem program, load sequence of system sub-routines and replacement algorithm upon the paging process for IBM 360/40. Anderson and Sargent [1] employed a 3^2 factorial experiment to characterize the degradation of response time in terms of the number of active users and the traffic rate per user. In [2], they used experimental design techniques to improve the performance of the swap scheduling algorithm of an interactive

computer system. Schatzoff, Tsao and Wiig [23] used planned experiments to assess the effects of batch processing versus time sharing on programmer productivity. Goel and Liu [13] employed two level factorial and fractional factorial designs to evaluate the effects of file size, record length, percentage of overflow records, presence or absence of master index, etc. on retrieval and insert times for the indexed sequential access method.

Use of response surface techniques to optimize the settings of five variables of the CP-67 paging priority dispatcher is described by Schatzoff and Bryant [24].

4. CONCLUDING REMARKS

Statistical modelling of computer systems is in its infancy with most of the work done in the past five years. This effort is primarily limited to the use of regression analysis for comparison, screening and predictive modelling and some applications of experimental design techniques to evaluate the effects of changes in the system variables. The techniques of empirical-mechanistic and mechanistic modelling, RSM, EVOP, time series analysis, etc. are yet to be fruitfully exploited. The difficulty of workload characterization, the consequent need for on line experimentation and the non-availability of a dedicated computer for sufficiently long periods of time point to the use of EVOP for empirical optimization. To reduce experimentation, sequential design of experiments [36] may be necessary. The statistical iterative modelling procedure appears to provide a suitable vehicle to integrate the large body of existing theoretical research with experimental data to build useful models.

Statistical methods can be misused and attention should be paid to the selection of proper techniques, the verification of the underlying assumptions and the interpretation of results. For example, empirical statistical models

are applicable only within the experimental region and are not useful in predicting the effects of contemplated changes in the system. For this problem, an empirical-mechanistic or mechanistic model may provide the answer. Similarly, the usual methods of statistical inference assume the errors to be normally, independently distributed with zero mean and constant variance. These assumptions have usually not been verified in the literature. If violated, alternate available methods of inference should be used or new methods may have to be developed.

Finally, it is important to note that every effort should be made to obtain a better understanding of the computer systems. Statistical techniques are not a substitute for physical knowledge. These techniques provide tools for data collection and analysis, to gain and quantify knowledge, and the means of making decisions under uncertainty.

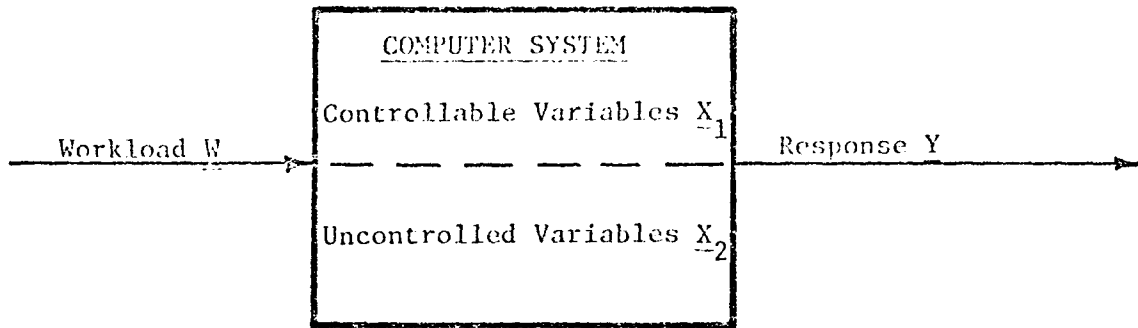
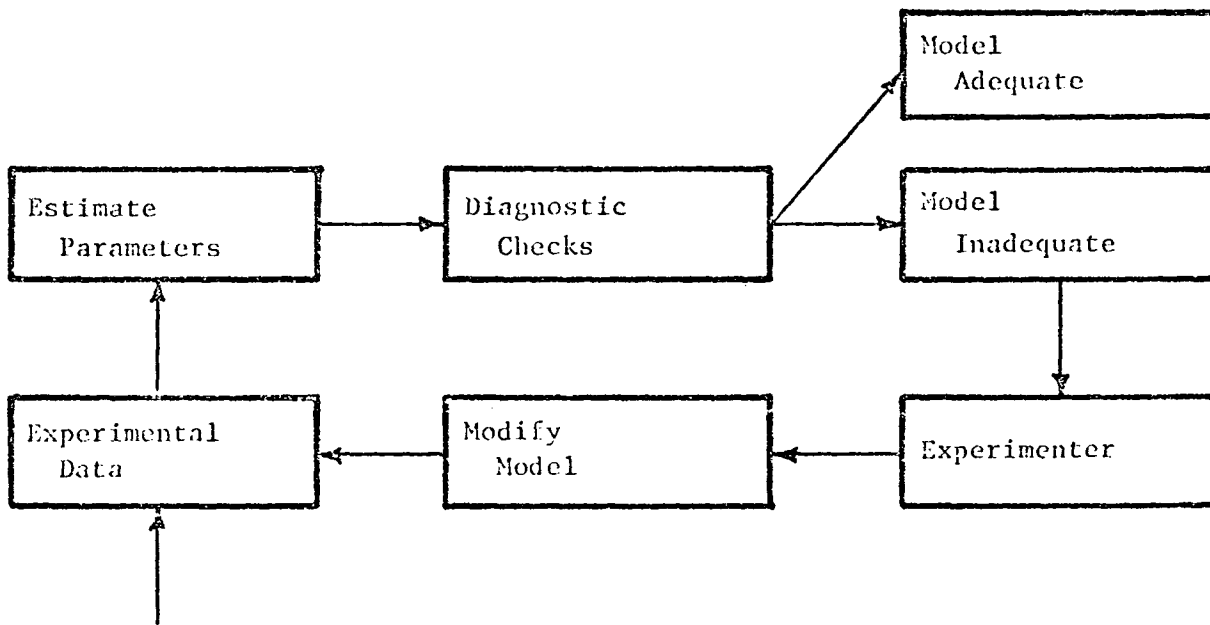


FIG. 1 A COMPUTER SYSTEM



Experimenter Formulates Model.

FIG. 2 MODEL BUILDING CYCLE

REFERENCES

Statistical Modelling of Computer Systems

- [1] H. A. Anderson and R. G. Sargent: A statistical evaluation of the scheduler of an experimental interactive computing system. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 73 - 98.
- [2] H. A. Anderson and R. G. Sargent: Investigation into scheduling for an interactive computing system. IBM Journal of Research and Development 18 (1973), 2, 125 - 137.
- [3] Y. Bard and B. Margolin: CP-67 measurement and analysis I: regression analysis. IBM Cambridge Scientific Center Technical Report G320-2061, 1970.
- [4] Y. Bard: CP-67 measurement and analysis II: overhead and throughput. IBM Cambridge Scientific Center Technical Report G320-2062, 1970. Also in: SIGOPS Workshop on System Performance Evaluation, 1971, 246 - 283.
- [5] Y. Bard: Performance criteria and measurement for a time sharing system. IBM Systems Journal 10 (1971), 3, 193 - 216.
- [6] Y. Bard: A technique for performance comparison of system software features. IBM Cambridge Scientific Center Technical Report 320-2081.
- [7] Y. Bard and K. V. Suryanarayan: On the structure of CP-67 overhead. In: Statistical Computer Performance Evaluation. Academic Press, New York 1972, 329 - 345.
- [8] Y. Bard: An analytic model of CP-67-VM/370. In: ACM SIGARCH-SIGOPS Workshop on Virtual Computer Systems. Association for Computing Machinery, New York 1973, 170 - 176.
- [9] Y. Bard: Experimental evaluation of system performance. IBM Systems Journal 12 (1973), 3, 302 - 314.
- [10] G. E. Bryag: Joss 20,000 hours at console - a statistical summary. In: Proceedings of Fall Joint Computer Conference, New Jersey 1967, 31, 769 - 777.
- [11] P. J. Denning: Book review - Statistical computer performance evaluation. Walter Freiberger, Editor, Academic Press(1972). In: Performance Evaluation Review. Association for Computing Machinery 2 (1973), 1, 16 - 22.
- [12] H. P. Friedman and G. Waldbaum: Evaluating computer system changes under uncontrolled workload conditions. In: Proceedings of Computer Science and Statistics: 7th Annual Symposium on the Interface, Iowa State University, Ames, Iowa 1972, 65 - 71.

- [13] A. L. Goel and Y. Liu: An analysis of ISAM by two level factorial designs. In: Proceedings of Computer Science and Statistics: 7th Annual Symposium on the Interface, Iowa State University, Ames, Iowa 1972. 16 - 20.
- [14] U. Grenander and R. F. Tsao: Quantitative methods for evaluating computer system performance: a review and proposals. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 3 - 24.
- [15] S. R. Kimbleton: Performance evaluation - a structured approach. In: Spring Joint Computer Conference 1972, 411 - 416.
- [16] S. R. Kimbleton: The role of computer system models in performance evaluation. In: Communications of the ACM 15, 1972, 7, 586 - 590.
- [17] D. E. Knuth: An empirical study of fortran programs. IBM Research Report RC-3276, 1971.
- [18] H. Kobayashi: Some recent progress in analytic studies of system performance. In: Proceedings of First USA-Japan Computer Conference, 1972, 130 - 138.
- [19] E. R. Lassetre and A. L. Scherr: Modelling the performance of the OS/360 time sharing option (TSO). In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 57 - 72.
- [20] P. A. W. Lewis and G. S. Shedler: Empirically derived micromodels for sequences of page exceptions. IBM Journal of Research and Development (1973), 86 - 100.
- [21] B. H. Margolin, R. P. Parmelee and M. Schatzoff: Analysis of free storage algorithms. IBM Systems Journal 10 (1971), 4, 283 - 304.
- [22] M. P. Racite: The use of pure and modified regression techniques for developing systems performance algorithms. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 347 - 370.
- [23] H. M. Schatzoff, R. Tsao and R. Wiig: An experimental comparison of time sharing and batch processing. In: Communications of the ACM 10, 1967, 5.
- [24] M. Schatzoff and P. Bryant: Regression methods in performance evaluation some comments on the state of the art. In: Proceedings of Computer Science and Statistics: 7th Annual Symposium on the Interface, Iowa State University, Ames, Iowa 1972. 48 - 57.
- [25] M. Schatzoff: Some approaches to the modelling and evaluation of programs and systems. IBM Cambridge Scientific Center Technical Report 320-2092, 1973.
- [26] H. F. Silverman and P. C. Yue: The response time characterization of an information retrieval system. IBM Research Report RC 3796, 1972.

- [27] G. C. Sutherland: Measurement of Lawrence Livermore Laboratory CDC-7600 system performance. In: SIGME, 1973, 78 - 99.
- [28] R. F. Tsao, L. W. Comeau and B. H. Margolin: A multifactor paging experiment: I. the experiment and the conclusions. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 103 - 134.
- [29] R. F. Tsao and B. H. Margolin: A multifactor paging experiment: II. statistical methodology. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press New York 1972, 135 - 158.
- [30] G. Waldbaum: Large capacity storage and time sharing system/360. IBM Research Report RC 2698, 1969.
- [31] G. Waldbaum: Evaluating computer system changes by means of regression models. In: SIGME, 1973, 127 - 135.
- [32] R. Watson: Computer performance analysis: applications of accounting data. Rand Report R-573-NASA/PR, 1971.
- [33] A. C. Yeh: An application of statistical methodology in the study of computer system performance. In: Statistical Computer Performance Evaluation. Ed. W. Freiberger, Academic Press, New York 1972, 287 - 328.

Statistical Methods

- [34] G. E. P. Box and H. L. Lucas: Design of experiments in nonlinear situations. *Biometrika* (1959), 77 - 90.
- [35] G. E. P. Box and W. G. Hunter: A useful method for model building. *Technometrics* 4 (1962), 301 - 318.
- [36] G. E. P. Box and W. G. Hunter: Sequential design of experiments for nonlinear models. In: Proceedings of IBM Scientific Computing Symposium on Statistics, 1963.
- [37] G. E. P. Box and N. R. Draper: The Bayesian estimation of common parameters from several responses. *Biometrika* 52 (1965), 355 - 364.
- [38] G. E. P. Box and W. G. Hunter: Experimental study of physical mechanisms. *Technometrics* 7 (1965), 23.
- [39] G. E. P. Box: Use and abuse of regression. *Technometrics* 8 (1966), 4, 625 - 629.
- [40] G. E. P. Box and N. R. Draper: Evolutionary operation. Wiley, New York 1969.
- [41] G. E. P. Box and G. M. Jenkins: Time series analysis, forecasting and control. Holden-Day, San Francisco 1970.
- [42] G. E. P. Box: Lecture notes for a course on advanced experimental design and model building. Department of Statistics, University of Wisconsin, Madison, 1972.

- [43] W. G. Cochran and G. M. Cox: Experimental design. Wiley, New York 1957.
- [44] C. Daniel and F. Wood: Fitting equations to data. Wiley, New York 1971.
- [45] O. L. Davies (Editor): Statistical methods in research and production. Oliver and Boyd, London 1967.
- [46] O. L. Davies (Editor): Design and analysis of industrial experiments. Hafner, New York 1971.
- [47] N. R. Draper and W. G. Hunter: Design of experiments for parameter estimation in multiresponse situations. *Biometrika* 53 (1966), 525 - 533.
- [48] N. R. Draper and H. Smith: Applied regression analysis. Wiley, New York 1966.
- [49] D. W. Marquardt: An algorithm for least squares estimation of nonlinear parameters. *Journal of the Society of Industrial and Applied Mathematics* 2 (1963), 431 - 441.