1966

A Procedure for Planning Production and Determination of Inventory

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A PROCEDURE FOR PLANNING PRODUCTION

AND

DETERMINATION OF INVENTORY

BY

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A thesis submitted
in partial fulfillment of the requirements for the
degree Master of Science, Major in Mechanical
Engineering, South Dakota
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1966

A PROCEDURE FOR PLANNING PRODUCTION

AND

DETERMINATION OF INVENTORY

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

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ACKNOWLEDGMENTS

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CHAPTER I

INTRODUCTION

The 17th Century concepts of inventory are expressed in extremes. Writing in 1677, A. Pappilon[1] observed that, "The stock or riches of the kingdom doth not only consist in our money, but also our ships for war, and magazines furnished with all necessary materials."

In the past, an individual's wealth was measured by his ownership of tangible, observable commodities such as the size of his flock or his herd, those things considered important by his neighbor.

"Even inventories greatly in excess of the amount needed to carry on the processes of production and distribution were considered beneficial."[2]

The "excess concept" of wealth was carried into the late 20th Century, sometimes directly and sometimes in modified form. Some firms still maintain excessive stock under the assumption that large inventories are beneficial. At one time excess inventories were considered advantageous, but today they are regarded as the major cause of business failures. Most often, inventories deal the severest blow to the new entrepreneur because he cannot afford to have capital tied up in

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[1] A. Pappilon, A Treatise Concerning the East India Trade, quoted in Jacob Vinor, Studies in the Theory of International Trade, Harper & Bros., New York, 1937, p. 20.

[2] T. M. Whitin, Theory of Inventory Management, Princeton University Press, N. J., 1953, p. 3.

inventory during a seasonal slump. One of the factors that has aggravated the problem of inventories has been the trend towards product diversification in the past 25 years.

It appears that many companies still have not accepted the philosophy of planning production and inventory control from the over-all company standpoint which is quite clear from the following comments by a few experts in this field.

Benjamin Melnitsky[3] in "Management of Industrial Inventory" states that:

> The aborigine knew nothing of inventory control, and quite possibly his 20th Century corporate counterpart is equally unenlightened. The changeover from inventory to inventory control bears no date. Some concerns plunged into the healthful waters of scientific management of inventories well before the first World War. Others are still on the shore contemplating on the advisability of wetting their toes.

W. E. Welch[4] observed that:

> The management of inventories is frequently treated as an intuitive process in which management must rely on experienced requisitioners with a 'feel' for the problem in order to interpret broad directives. Lacking a more suitable tool, these directives take the form of 'Use your best judgment in the determination of order quantities, but watch your total inventory,' and 'Arrange the timing of your purchases and your manufacture to avoid interruptions in the line, but do not take excessive risks of obsolescence or unneeded inventory.

This opinion was corroborated by Nyles V. Reinfeild[5] in 1960:

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[3]Benjamin Melnitsky, Management of Industrial Inventory, Conover-Mast Publications, New York, 1951, p. 3.

[4]W. E. Welch, Tested Scientific Inventory Control, Management Publishing Corp., Greenwich, Conn., 1956, p. 121.

[5]Nyles V. Reinfeild, Production Control, Prentice-Hall, Inc., New Jersey, 1960, p. 222.

Modern inventories represent investments far in excess of the average plant expansion program, and yet, as a rule, the men selected to control these inventories have had little or no formal training in the job to be done. They rely on their own experienced judgment, learning by making mistakes. Their only guidelines of operation are those set forth by top management.

The subject of inventory has found little interest from the theoretical viewpoint.

Economists interested in the theory of the firm have devoted very little time to the study of inventory control or of its influence on the theory, although businessmen themselves are keenly aware of the importance of the topic.[6]

Management of inventories is recognized today as one of the key responsibilities in achieving continuous and economical plant operation. Inventory dollars are no longer regarded as a drain on working capital; they are a factor to be used and administered with skill and intelligence.

In a manufacturing concern a heavy factor in determining the amount of profit is its operating cost; and in order to reap maximum profits from operations, companies are attempting to minimize the various components of their over-all operating cost. One such component is the cost of carrying inventory; and in many business concerns this inventory storage cost contributes a sizable portion of the operating cost.

Two situations give rise to an inventory storage cost. First, a certain cost is incurred by having money tied up in unsold inventory; along with this, the physical storage of this inventory contributes to

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[6] T. M. Whitin, op. cit., p. 7.

the cost. If, on the other hand, a factory warehouse stands empty space accrues a certain cost of maintenance. Lack of inventory also may cause a loss of profit through loss of sales. Moreover, most warehouses do indeed have limited capacity. Because such assets as warehouse capacity and capital are not limitless, it is necessary that the firm should use them efficiently as these restrictions do not permit the total average (dollar) inventory that the individual item's optimal policies would require. Regulating the size and composition of an inventory in order to minimize its cost forms an interesting field of study.

The basic problem of an inventory policy is, therefore, to strike a balance between savings and the costs and capital requirements associated with larger stocks. In the past, businessmen have been able to achieve a reasonably balanced inventory policy largely through an intuitive understanding of the needs of their businesses. However, as business grows, it becomes more complex, and as business executives become more and more specialized in their jobs or farther removed from direct operations, achieving an economical balance intuitively becomes increasingly difficult. That is why, more and more businessmen are finding the concepts and mathematics of the growing body of inventory theory to give practical help.

Business management now has a wide range of techniques for attacking production planning and inventory control problems. These are more than new developments in clerical methods for keeping track of orders and inventory balances. They are methods for analyzing the

place of inventories in an individual business organization and for designing production and inventory control systems which will be truly responsive to management policies on investment, customer service, employment, and cost reduction. These techniques have been developed over a period of many decades.

It is along these lines of production planning and inventory control with which this thesis is concerned, the purpose being to outline methods for establishing a production planning and inventory control program that would be beneficial to industry.

CHAPTER II

REVIEW OF LITERATURE

Although inventory problems are as old as history itself, it has only been since the turn of the century that any attempts have been made to employ analytical techniques in studying these problems. The initial impetus for the use of mathematical methods in inventory analysis seems to have been supplied by the simultaneous growth of the manufacturing industries and the various branches of engineering, especially industrial engineering. The real need for analysis was first recognized in industries that had a combination of production scheduling and inventory problems.

The mathematical determination of the quantity of an item to be ordered at any one time was one of the first subjects of investigations by the early pioneers of scientific management. By the early 1900's many formulas had been developed, but until World War II the application of these formulas was limited.

As far back as 1915 an economic-lot-size equation was developed by F. W. Harris[1] which minimized the sum of inventory-carrying and set-up costs where demand was known and constant. This formula was almost identical to the present accepted economic-lot-size formula:

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[1]Ford W. Harris, Operations and Cost, New York, 1955, pp. 48-52, referred to in F. E. Raymond, Quantity and Economy in Manufacture, McGraw-Hill, New York, 1931, pp. 121-122.

$$Q = \sqrt{\frac{PS}{C}} \; K$$

where

Q is economic production quantity,

P is cost of preparing for the manufacture of a lot,

S is daily rate of sales,

C is unit production cost, and

K is a constant which includes not only the interest rate
but also other factors such as storage cost, insurance, and
taxes.

Inventory accumulation and depletion have long been recognized
as a major contributing factor to fluctuations in business activity.
But, the inventory control literature was developed in 1920's, partly
under the impetus of the very considerable losses suffered by American
businessmen during the depression of 1920-1921.

In 1922, W. M. Roming[2] showed how well-managed inventory methods
help to stabilize profits.

In 1923, Kenneth W. Stillman[3] used graphical method for finding
the most economical lot quantity.

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[2] W. M. Roming, How to Cut Cost Corners Through Inventory, Industrial Management, Vol. 63, no. 2, February 1922, pp. 86-87.

[3] Kenneth W. Stillman, Quantities for Lot Manufacture, Industrial Management, Vol. 65, no. 2, February 1923, pp. 84-86.

Two years later, H. S. Owen[4] described a simple method for keeping inventory investment at practical minimum.

Then, in the same year, we find that Ralph C. Davis[5] derived formulas for determining the proper quantity to manufacture and to carry in stock to give least unit cost.

Further, George F. Mellen[6] found a similar solution to the problem.

The general explanation as developed by a number of writers including Davis, Mellen, and Owen, is the economy of placing larger orders. Specifically, they assumed that in addition to the price paid for the goods ordered, there is a procurement cost to each order which is independent of the magnitude of the order. In that case, there is an incentive not to order continuously but to order larger amounts less often.

More complicated analysis involving the use of lot size formulas has been carried out, allowing the inclusion of several additional factors. There is need for further work in adapting these formulas to concrete situations.

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[4] H. S. Owen, How to Maintain Proper Inventory Control, Industrial Management, Vol. 69, no. 2, February 1925, pp. 83-85.

[5] Ralph C. Davis, Methods of Finding Minimum-Cost Quantity in Manufacturing, Manufacturing Industries, Vol. 9, no. 4, September 1925, pp. 353-356.

[6] George F. Mellen, Practical Lot Quantity Formula, Management and Administration, October 1925, p. 155.

Another basic aspect of inventory control that has received much attention involves the effect of uncertainty on inventory levels. The existence of uncertainty brings about a need for safety allowances to provide against running out of stock because of random variations in demand. T. C. Fry,[7] in 1928, was the first to carry out research on this problem.

In 1931, F. E. Raymond[8] wrote a book "Quantity and Economy in Manufacture" while he was at M.I.T. This was the first full length book to deal with inventory problems. It attempts to explain how various extensions of the simple lot size model can be used in practice.

An excellent article on inventories that has received little attention because of its being written in a foreign language was published in 1938 by Erich Schneider and translated by T. M. Whitin[9] in 1954. Schneider addressed himself to the following problem: given a manufacturer's sales forecast as a function of time, his initial inventory, carrying charges, production costs and other conditions, how should he schedule his production in order to minimize costs involved in production and storage adequate to fulfill sales requirements? He used

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[7] T. C. Fry, Probability and Its Engineering Uses, Van Nostrand Company, New York, 1928, pp. 229-232.

[8] F. E. Raymond, Quantity and Economy in Manufacture, McGraw-Hill Book Company, New York, 1931.

[9] T. M. Whitin, Erich Schneider's Inventory Control Analysis, Journal of Operations Society of America, Vol. 2, no. 3, August 1954, p. 331.

simple graphical techniques in conjunction with mathematical analysis
to solve this problem.

Between 1938 and World War II, there is not much to be mentioned
as far as research in this field is concerned, except that of Wilson.
Wilson[10] made an attempt to combine the lot size and safety allowance
aspects of inventory control and described the interaction between them.

It was not until after World War II, when management sciences
and operations research emerged, that detailed attention was focussed
on various inventory problems. Some of the important developments are
given below in chronological order.

M. B. Phillips[11] described the advantages of establishing pur-
chasing office as a division of finance department, centralized
purchasing and storage, stock and inventory control, and carefully
planned and organized purchasing as means for promoting good budgetary
procedure.

R. L. Bowles[12] thought of the maintenance of raw material inven-
tories at a level enough to keep warehouse and storage costs at a mini-
mum and yet high enough to prevent interruptions of manufacturing

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[10] R. H. Wilson, _A Universal System of Stock Control_, Purchasing,
Vol. 11, no. 3, 1941, pp. 80-86.

[11] M. B. Phillips, _Integrating Purchase Practices With Fiscal
Policies_, Purchasing, Vol. 21, no. 4, October 1946, pp. 109-111.

[12] R. L. Bowles, _Controlling Raw Materials to Meet Varying
Production_, Factory Management and Maintenance, Vol. 105, no. 1,
January 1947, pp. 94-96.

operations, as a challenging problem which must be faced constantly by production control managers. He developed a system to control raw materials inventories when the demand is varying. The advantages of his system were better inventory turnover figures and a considerable saving in time.

J. C. Borden[13] observed that when a manufacturing business falls substantially short of meeting the rate of output for which it originally planned, inventory becomes a major problem for management. Based on his experience in Cutler-Hammer company he thought the following five steps as necessary in order to reduce inventory.

1. Take another look at formulas for setting the quantities ordered.

2. Hammer repeatedly on the rule that material must not be ordered for arrival before inventory will reach the planned minimum quantity.

3. Undertake a more detailed reconsideration than normal of all shop orders shortly before they are scheduled to run.

4. Re-examine quantities required for minimum inventories and reduce all that can stand it.

5. Look for points where inventories are now carried but can be dispensed with.

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[13]J. C. Borden, What Close Control of Inventory Means to Management, Factory Management and Maintenance, Vol. 105, no. 2, February 1947, pp. 105-108.

B. D. Henderson[14] explained a system whereby Westinghouse saved money by balancing cost of material and purchasing against cost of inventory investment based on rate of use. Another advantage of this policy of determining order points and quantities on inventory items supported on mathematical basis and record of actual experience was the elimination of material shortages.

C. R. Schubert[15] discussed the inventory control system used by Monarch Machine Tool Company, Sidney, Ohio, and showed how careful planning and control of supply on hand enabled company to maintain high inventory turnover.

L. A. Hradesky[16] outlined three steps for setting up an inventory control system. These are:

1. Creating sufficient quantities of materials to keep division operating over predetermined span of time;

2. Controlling quantities that have been created, for proper disbursement;

3. Purifying materials of excesses that have accumulated.

Coordination of related functions of purchasing, stores, stock

[14]B. D. Henderson, Purchasing Profits Through Inventory Control, Purchasing, Vol. 23, no. 2, August 1947, pp. 92-93.

[15]C. R. Schubert, Low-Cost Inventories Through Close Controls, Factory Management and Maintenance, Vol. 105, no. 11, November 1947, pp. 102-104.

[16]L. A. Hradesky, Good Inventory vs Bad Inventory, Mill & Factory, Vol. 43, no. 4, October 1948, pp. 107-110.

control and surplus disposal, in order to reduce dollar inventory without jeopardizing service requirements, was discussed by J. Albin.[17] His program was adopted by American Airlines for reduction of dollar inventory.

W. Collier[18] and R. Blair established an inventory control program at Tapco plant, Thompson Products, Inc., Cleveland, which resulted in 60 per cent reduction of maintenance stock.

William M. Vermilye[19] of the National Bank of New York explained the effect of the cost of carrying seasonal goods over to another season on company's profit. He gave an example of a leading manufacturing company which had never used any system of inventory control to know how much of its inventory was carried over from one season to the next. The management had the idea that when some of its products were left over from a season and could not be sold except at a sacrifice, it was better to carry it over than to make the sacrifice necessary to sell it, on the theory that the most, it could cost them, would be six per cent on the amount involved. They were very much surprised when Vermilye called their attention to the fact that the six per cent which

[17]J. Albin, Fewer Dollars on Shelf, Purchasing, Vol. 25, no. 4, October 1948, pp. 90-94.

[18]W. Collier, and R. Blair, Inventory Control Reduces Maintenance Stocks 60 Per Cent, Factory Management and Maintenance, Vol. 107, no. 3, April 1949, pp. 68-70.

[19]William M. Vermilye, Economic Trends in Manufacturing and Sales, p. 18, private printing, quoted in Charles A. Koepke, Plant Production Control, Wiley & Sons, New York, 1949, pp. 414-415.

they had figured as the cost of carrying was but a small fraction of what was spent. He showed that the cost of carrying in this particular case was much more than six per cent. He also set up a simple equation which shows the relationship between profit earned and inventory turnover:

Let W = Working Capital

T = Turnover

Then $WT = V$ (volume of business done in dollars)

If P = Rate of profit per dollar per turnover,

Then VP = Total gross profit for the business

If O = The overhead of the business and

P_n = net profit, then

$VP - O = P_n$

Substituting $WT = V$

$$WTP - O = P_n$$

An increase in net profit can be realized if the working capital is increased, if turnover is increased, if the rate of gross profit is increased, or if the amount of overhead is decreased.

D. S. Lisberger[20] showed how Apparatus Department of General Electric Co. carried out program for inventory reduction and permanent and continuing improvement in inventory performance by way of employee's meetings, inventories committees, newsletter, information letter and Inventory Control Manual.

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[20] D. S. Lisberger, It Pays to Sell Inventory Control, American Machinist, Vol. 94, no. 4, February 1950, pp. 106-109.

Another inventory control system established by H. J. Holtz,[21] allowed the company to consolidate several warehouses into one, make efficient use of a four-story building, reduced its manpower requirements and speeded receiving and shipping operations.

In 1950, W. V. Stoughton[22] used a continuous inventory procedure for the parts depots of Caterpillar Tractor Company. This procedure paid close attention to quantity differences disclosed by serial counts made and avoided annual physical inventory in these units of organization.

At the Accounting Conference, Rutgers University, September 1950, M. E. Peloubet[23] described some of the most widely used methods of inventory valuation and the situations where they are applicable.

The problem of controlling the rate of production can be stated in terms of servomechanism theory, and the well developed methods of that theory employed to study the behavior of control system. Richard M. Goodwin[24] has arrived independently at this idea as a means for studying market behavior and business cycles. The applicability of

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[21]H. J. Holtz, *Inventory Control System Saves Floor Space, Cuts Over-head Costs*, Steel, Vol. 127, no. 8, gust 1950, pp. 96-98.

[22]W. V. Stoughton, *Physical Inventory Day-by-Day*, National Association of Cost Accountants, Vol. 32, no. 3, November 1950, pp. 280-285.

[23]M. E. Peloubet, *Choice of Inventory Methods Depends on Specific Needs of Each Business*, Journal of Accountancy, Vol. 91, no. 1, January 1951, pp. 70-77.

[24]Richard M. Goodwin, *Econometrics in Business-Cycle Analysis*, Chapter 22 in Avin H. Hansen, Business Cycles and National Income, W. W. Norton & Co., New York, 1951.

servomechanism models to the theory of the firm has been discussed by
W. W. Cooper.[25] Later Herbert A. Simon[26] showed that the basic ap-
proach and fundamental techniques of servomechanism theory can indeed
be applied fruitfully to the analysis and design of decisional proce-
dures for controlling the rate of manufacturing activity. The problem
of controlling the rate of production of a single item was considered
and a cost criterion was constructed for evaluating alternative deci-
sion rules or constructing an optimum rule so as to minimize the cost
of manufacture over a period of time.

F. B. Newman[27] described technique for adjusting sales predic-
tions to actual sales trends. His technique is based on the concept of
control chart to indicate unexpected deviations from sales trend. He
presented tables and charts showing the derivation and use of control
limits.

The problem of uncertainty of demand was again considered by
Arrow, Harris, and Marschak[28] in 1951. Their analysis constituted a
considerable extension of previous results of T. C. Fry. They derived

[25] W. W. Cooper, A Proposal for Extending the Theory of the Firm,
Quarterly Journal of Economics, Vol. 65, February 1951, pp. 87-109.

[26] Herbert A. Simon, On the Application of Servomechanism Theory
in the Study of Production Control, Econometrica, Vol. 20, no. 2,
April 1952, pp. 247-268.

[27] F. B. Newman, Method of Inventory Control, Industrial Quality
Control, Vol. 7, no. 6, May 1951, pp. 29-31.

[28] K. J. Arrow, T. Harris, and J. Marschak, Optimal Inventory
Policy, Econometrica, Vol. 19, no. 3, July 1951, pp. 250-272.

mathematical models for optimal inventory policy considering the demand flow as a random variable with a known probability distribution. They determined the best maximum stock and the best reordering point as a function of the demand distribution, the cost of making an order, and the penalty of stock depletion.

A year later Dvoretzky, Kiefer, and Wolfowitz[29] showed the conditions required for optimal policy. They pointed out that systems based on lot sizes and safety allowances are not necessarily optimal. They considered the problem of what quantities of goods to stock in anticipation of future demand. Loss is caused by inability to supply demand or by stocking goods for which there is no demand. They tried to strike a balance between over-stocking and under-stocking. In the first part of the paper they treated the case when the demand was given by completely specified probability distribution functions. In the second part,[30] they dealt with the case of unknown distribution of demand. In 1953, they presented another paper[31] which describes the necessary and sufficient conditions for the validity of (s,S) policy.

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[29]A. Dvoretzky, J. Kiefer, and J. Wolfowitz, The Inventory Problem, Econometrica, Vol. 20, no. 2, April 1952, pp. 187-222.

[30]Ibid., Vol. 20, no. 3, July 1952, pp. 450-466.

[31]A. Dvoretzky, J. Kiefer, and J. Wolfowitz, On the Optimal Character of the (s,S) Policy in Inventory Theory, Econometrica, October 1953, pp. 586-596.

A recent book by Arrow, Karlin, and Scarf[32] is mostly concerned with additional mathematical concepts and implications of the (s,S) policy. The (s,S) policy has received more analytical treatment of a general nature than any of the other policies. The (s,S) policy with $0 < s < S$ is implemented as follows.

Whenever the stock level falls below s, the ordering rule calls for replenishing stock to the level S. When the quantity of goods in supply exceeds s, no ordering is done. Delivery of goods when ordered is assumed to be immediate. Decisions whether to order or not are to be made at the start of successive periods. The state of the system at the start of each period is described by current stock level.

Several other techniques for analyzing inventory control problems have been formulated. Most important of them is linear programming. The linear programming models are designed primarily for situations with important seasonal fluctuations in demand from period to period. If fluctuations in production are reduced, costs involved in overtime production are lowered, but only at the expense of increased carrying charges. The linear programming model finds the level of production for each period that minimizes combined overtime and carrying

[32]K. J. Arrow, S. Karlin, and H. Scarf, Studies in the Mathematical Theory of Inventory and Production, Stanford University Press, Stanford, California, 1958.

charges. Charnes, Cooper, and Farr[33] have applied linear programming
to setting over-all production levels where there are significant
seasonal fluctuations in demand and where demand is assumed to be
known.

H. F. Dickie[34] recommended six steps as necessary for better in-
ventory management. These are:

1. recognizing need for control

2. determining optimum turnover

3. analyzing problem

4. economical ordering

5. minimizing work-in-process

6. educating personnel

G. N. Hackett[35] while working at Thompson Products, Inc.,
Cleveland, Ohio, applied certain standards to control of investment in
inventory. He established standards for raw materials, supplies, work-
in-process, and finished goods.

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[33]A. Charnes, W. W. Cooper, and D. Farr, Linear Programming
and Profit Preference Scheduling for a Manufacturing Firm, Journal of
Operations Research Society of America, Vol. 1, no. 3, May 1953, pp.
114-129.

[34]H. F. Dickie, Six Steps to Better Inventory Management,
Factory Management and Maintenance, Vol. 111, no. 8, August 1953, pp.
96-100.

[35]G. N. Hackett, Standards for Inventory Control, Purchasing,
Vol. 36, no. 1, January 1954, pp. 95-96.

B. Grad and R. C. Hartigan[36] described a method for determining the cost of carrying inventory in a plant. This analysis is based on certain basic factors such as:

1. possession costs

2. value losses

3. return on investment

4. general business influences

George L. Sukes[37] came up with a solution for the problem of purchasing and using of bulk expendable materials at Cadillac-Cleveland Tank Plant, Cleveland, which resulted in low inventory and less storage space.

Most inventory control systems are based on predetermined order points. They signal when to place orders for materials and parts normally carried in stock. When the quantity of any material on hand drops below its order point, it is time to order more. But the problem with most systems is that the quantities are only correct when production requirements and parts-delivery schedules remain fairly constant--which they seldom do.

This problem was solved at Reynolds Metals Company's aluminum foil plant in Louisville using a system developed by E. Ken Hedrick[38]

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[36] B. Grad, and R. C. Hartigan, Keep Your Inventory Carrying Cost Down, Mill & Factory, Vol. 54, no. 4, April 1954, pp. 79-81.

[37] George L. Sukes, Better Bulk Buying, Factory Management and Maintenance, Vol. 113, no. 7, July 1955, pp. 138-141.

[38] E. Ken Hedrick, How Much to Re-order When, Factory Management and Maintenance, Vol. 113, no. 8, August 1955, pp. 110-111.

the plant industrial engineer. It keeps order points tied closely to the ups and downs in usage and delivery schedules. This system which is based on the flexible order points has resulted in a more balanced inventory and reduced investment. Under this flexible order-point system, Hedrick used three control factors.

1. Estimate of actual usage as compared with normal usage.

2. Normal and maximum delivery times.

3. Degree of protection against shortages required at the time. The last--number 3--is perhaps the key to the system. Degrees of protection were established for the purpose of guarding against certain conditions, such as excess usage, excess usage and a slight delivery delay, excess usage and maximum delivery delay and protection against maximum delivery delay only.

C. G. McCabe[39] developed a new machine-inventory system for Solar Aircraft Company, San Diego. It identifies equipment, keeps records updated, helps to plan the work load and tracks down maintenance needs and costs. The system requires a lot of paperwork, but results in a far better job.

E. D. Lucas[40] discussed the possibilities of using an electronic system for handling inventory so as to keep it at a minimum safe level,

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[39]C. G. McCabe, Better Machine Inventory, Factory Management and Maintenance, Vol. 113, no. 9, September 1955, pp. 134-136.

[40]E. D. Lucas, Automatic Production Inventory Control, Control Engineering, Vol. 2, no. 9, September 1955, pp. 68-73.

to maintain accessible record of all inputs to and withdrawals from inventory to provide necessary printed reports, to reduce load on clerical staffs, and to secure other benefits.

G. J. Bevans[41] suggested a method of using 3-stub tag system with serially numbered receiving ticket in triplicate for control of 30 different grades of incoming scrap at Robert Gair Company, manufacturers of paper board and paper products.

M. D. Triouleyre[42] developed an improved system for better physical and accounting control when he was working for Shawinigan Resins Corporation, Springfield, Mass. About 4,000 different items were carried in the storeroom at an inventory value of $230,000 and 3,500 withdrawals were made on 1,400 stores requisitions per month. Under new plan, supplies at $22,000 were issued monthly using 800 requisitions.

W. F. Hoehing[43] explained the functions, objectives and characteristics of stores inventory account. He employed probability theory and statistical methods to maximize return-on-assets ratio at Westinghouse Electric Corp., Sharon, Pa. He explained the application of this

[41]G. J. Bevans, Simple Inventory System Increases Productivity, Paper Industry, Vol. 38, no. 3, June 1956, pp. 222-223.

[42]M. D. Triouleyre, Better Control of Supplies--With Economy, National Association of Cost Accountants, Vol. 38, no. 5, January 1957, pp. 647-656.

[43]W. F. Hoehing, Statistical Inventory Control, Industrial Quality Control, Vol. 13, no. 7, January 1957, pp. 7-13.

method to one of the factors that can cause stockouts as well as general operation of procedures introducing two other factors, namely size of demand and delivery time.

W. Karush,[44] in 1957, considered the problem of allocation of inventory dollars among various competing commodities, so as to minimize over-all lost sales dollars. He found out an explicit mathematical solution showing the relationship between lost sales and inventory levels.

C. C. Holt[45] designed an analysis to facilitate decision making for the allocation of inventory to lots. He made an attempt to remove some of the limitations of the traditional lot-size analysis.

L. B. Kahn,[46] in his paper, introduced concept of controlling turnover of multistock inventory, using index of activity as measure of inventory turnover for each of stock items comprising total inventory. A simple formula was used which made possible ready and immediate knowledge of turnover through use of electronic computer.

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[44] W. Karush, Queuing Model for Inventory Problem, Operations Research, Vol. 5, no. 5, October 1957, pp. 693-703.

[45] C. C. Holt, Decision Rules for Allocating Inventory to Lots and Cost Functions for Making Aggregate Inventory Decisions, Journal of Industrial Engineering, Vol. 9, no. 1, January-February 1958, pp. 4-10.

[46] L. B. Kahn, Quality Control of Inventory Turnover, Industrial Quality Control, Vol. 14, no. 10, April 1958, pp. 4-7.

E. Naddor,[47] and S. Saltzman suggested a method for the deter-
mination of how frequently orders should be initiated and how many dif-
ferent items should be listed on orders, so as to minimize the sum of
costs of carrying inventories and costs of ordering.

M. B. Schupack[48] made an attempt to apply operation research
techniques to solve inventory problems. He extended simplest economic
lot-size formula, containing only carrying costs and ordering costs, to
include the case of seasonal demand. But, no allowance was made for
shortages and uncertainty. The key step in this attempt was fitting of
analytical function to seasonal demand pattern by means of harmonic
analysis.

Robert G. Brown[49] discussed the problems of minimizing and
measuring the uncertainty of demand facing a company. For the first
time, he showed that it is possible and practical to measure the cur-
rent distribution of error in the forecast, by item.

K. F. Simpson[50] developed mathematical models and formulas for

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[47] E. Naddor, and S. Saltzman, Optimal Reorder Periods for Inven-
tory System With Variable Costs of Ordering, Operations Research, Vol.
6, no. 5, September-October 1958, pp. 676-692.

[48] M. B. Schupack, Economic Lot Sizes With Seasonal Demand,
Operations Research, Vol. 7, no. 1, January-February 1959, pp. 45-57.

[49] Robert G. Brown, Less Risk in Inventory Estimates, Harvard
Business Review, Vol. 37, no. 4, July-August 1959, pp. 104-116.

[50] K. F. Simpson, Theory of Allocation of Stocks to Warehouses,
Operations Research, Vol. 7, no. 6, November-December 1959, pp. 797-
805.

distribution to number of warehouses of certain quantity of material which has just been received, or is about to be received by the central agency, while another quantity is expected to be received at known time in future.

A. Bhatia, and A. Garg[51] showed that the dynamic programming technique can be employed for efficient handling of inventory control problems with known but variable requirements at discrete points of time and having objective function with fixed and variable portion. The method presented offers substantial savings in computation.

L. G. Spencer[52] discussed the following aspects, giving examples of practice in the Plastics Department of General Electric Company's plant at Decatur, Illinois.

1. Direct and indirect inventory responsibilities.

2. Control procedures.

3. Tabulating routines, and forecasting techniques.

4. Control through standardization.

5. Role of reports in inventory control.

6. Measuring performance.

7. Checking up through audit.

8. Education in inventory control.

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[51] A. Bhatia, and A. Garg, Application of Dynamic Programming to Class of Problems in Inventory Control, Journal of Industrial Engineering, Vol. 11, no. 6, November-December 1960, pp. 509-512.

[52] L. G. Spencer, Many Facts of Sound Inventory Control, National Association of Accountants, Vol. 41, no. 12, August 1960, pp. 5-14.

P. R. Winters[53] described the way of handling inventory deci-
sions as to when to make more (trigger point) and how much to make
(lot size) and developed models for triggering production run of joint
lot, depending on inventory situation at different warehouses of the
factory.

S. Eilon[54] discussed models both for continuous and instanta-
neous demands and showed that minimizing cost due to uncertainty of
demand is a special case of criterion of profit maximization.

William J. Frink[55] applied dynamic quantity control to produc-
tion planning and inventory control in a chemical plant. He showed how
accurately gathered and properly employed inventory information acts as
a throttle on production to optimize total operation. Any failure or
upset in any manipulated variable like reorder point, production and
shipments, scheduling and sales forecasting reflects somewhere as a
change in inventory level which is a controlled variable. Thus when
inventory remains within control limits the manufacturing cycle is cor-
rect and stable. He described a production planning and inventory in-
formation system which resulted in considerable savings at Union Carbide

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[53] P. R. Winters, Multiple Triggers and Lot Sizes, Operations
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[55] William J. Frink, D Q C in Chemical Plants, Control Engi-
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Chemicals Company. This system connected eight bulk plants, five bulk terminals and seventy field warehouses throughout the country and production control was executed through independent but highly interrelated inventory data processing networks.

H. Chestnut, T. F. Kavanagh, and J. E. Mulligan[56] showed that inventory control has much in common with the control of a physical process. They applied tools like forecasting, simulation, and optimization to design automatic ordering systems that minimize costs while keeping up customer service. To test forecasting techniques, the authors developed a simulation program, making use of a GE 225 computer.

Ruddell Reed, and Walter E. Stanley,[57] working on a graduate project in the Department of Industrial Engineering, University of Florida, designed a procedure for improved economic control of hospital general inventories. According to the authors, the design is practical, efficient and readily adaptable to existing hospital situations. Emphasis was placed on determining the order point and order quantity for storage items.

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[56]H. Chestnut, T. F. Kavanagh, and J. E. Mulligan, Applying Control to Inventory Management, Control Engineering, Vol. 9, no. 9, September 1962, pp. 96-102.

[57]Ruddell Reed, and Walter E. Stanley, Optimizing Control of Hospital Inventories, Journal of Industrial Engineering, Vol. 16, no. 1, January-February 1965, pp. 48-51.

Summary

From this review of literature, it is quite clear that much more attention is needed on production planning and inventory control problems than what has been given in the past. The next few years should bring about much additional research which will help in deciding to what extent industrial application of the various inventory control systems will be profitable.

An attempt will be made in this thesis to consider the effect of certain limitations or constraints, such as warehouse capacity, available capital, and available machine time, on economic production quantities for establishing an improved inventory control system.

CHAPTER III

PRODUCTION PLANNING AND CONTROL

Production is generally thought of as the output of a plant or the flow of product through the plant. Our concept of inventories is that they are reservoirs attached to the flow of production for the purpose of keeping the product-supply pipeline full and the flow through it continuous at a level consistent with sales demand. The essence of management is planned and controlled activity. Therefore, we can say that the primary operational function of production planning and control is to manage production and inventory skillfully.

This requires participating in the sales forecast; coordinating inventory and production levels with the sales forecast; planning the mix of product; controlling production, which includes scheduling and follow-up of work-in-process to meet scheduled delivery promises; controlling production materials and supplies inventories and requisitioning those materials and supplies for purchase and delivery on specified dates; maintaining physical control of all production materials and supplies, work-in-process, and finished-goods inventories; providing customer service including promises to customers and information regarding customer's orders; and last, controlling shipments and internal and external transportation. These activities must all be coordinated in any plant, large or small, to achieve good plant performance. The most effective coordination is achieved when it is under one management, production planning and control.

The task of managing production and inventories can be broken into three major functional areas: production planning, production control, and material management.

Production Planning

The necessity for careful planning of production operations arises from four important factors:[1]

1. The increased complexity of production and distribution systems.

2. The need for careful timing of interrelated activities.

3. The necessity for anticipation of changes and orderly reaction to them.

4. The desire to achieve the most economical combination of resources.

A production plan must provide the required quantities of product at the proper time and at a minimum total cost consistent with quality requirements. The plan should be the basis for the establishment of many of the operating budgets. It should establish manpower requirements and hours to be worked, both regular time and overtime. Further, the production plan establishes the equipment requirements and the level of the anticipated inventories.

In setting up the production plan, we must keep in mind that if

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[1]Gordon B. Carson, _Production Handbook_, The Ronald Press Company, New York, 1958, p. 2.9.

demand must be met when it occurs, there are three sources that can be used:

1. Current production.

2. Inventory on hand.

3. Current production and inventory.

If back orders are permissible, the current demand may be deferred to some time in the near future. When materials can be back-ordered, we have a situation which can be compared to something between a continuous manufacturing operation and an intermittent manufacturing operation. It does provide flexibility, but it should not be relied upon to avoid the problem of meeting demand when it occurs.

Another factor that should be considered in production planning is the stability of the work force. The more highly skilled the employees, the more important a stable work force becomes. When demand is nearly constant throughout the year, the necessity for a stable work force creates no serious problem. If demand is cyclic, one must either vary the size of the work force or use inventories to meet demand. The use of inventories and a level work force to meet a cyclic demand results in a lower investment in plant and equipment. If demand is increasing, an expansion in the size of work force, increasing efficiency, or some other means of reducing the number of hours per unit is called for. A decreasing demand usually requires a reduction in the size of the work force if efficiency is to be maintained. Thus, planning under these conditions must be consistent with demand, company policies, and economic production.

Production planning can be subdivided into two sections:

1. Long-range planning.

2. Current planning.

1. Long-range planning:

Long-range planning is the development of a program of more than a year's duration which estimates specific market potentials for a given product or group of products using various estimating techniques. Once the market potentials have been determined, the organization plans are evolved for attaining the established potentials. The long-range planning process could be compared to a blueprint. However, one of the basic differences is that where the engineer has to follow the blueprint dimensions with the utmost exactness in order to accomplish the desired objective, management, once it has established a basic approach, should be flexible in order to meet the changing environment due to anticipated actions of competition. Long-range planning is usually divided into three different areas of planning: long-term plans, short-term plans, and new-product introduction. Long-term plans usually cover a period of three to five years ahead of the current date. Some companies extend their future planning to ten or even twenty years from the current date. The period covered is usually determined by the time requirements of the project to be accomplished. For example, a manufacturer of large power generators for electric companies might plan its sales ten years ahead, based on population growth and future power consumption. It must determine if it has sufficient capacity in its present plant to produce enough generators to

meet the future demands caused by population growth. If it finds that
it does not have enough plant capacity to meet the anticipated demand,
it will then make plans for plant expansion to meet the demands cor-
rectly as they will occur in future.

It is very hard to predict sales and production plans for the
next ten years on the basis of population growth. But many companies
are extending their plans for ten years ahead, with the help of econo-
mists and market researchers. These plans are then altered annually to
include the latest changes in corporate thinking and projected
conditions.

More common practice is to plan from three to five years ahead.
Factors considered in making plans for this period include the general
economic condition, an estimate of the areas in which the consumer is
likely to spend his income during those years, the trend of product
styles, volume and prices within the industry, consumer reactions to
the company's present or planned product styles and prices, company
sales trends, and current demands. When the sales forecast has been
established in units and dollars for each product line, general inven-
tory and production plans can be evolved which will indicate whether
there will be a need for additional machinery and plant facilities.
Plans can then be made for the timing of the acquisition of land,
erection of the plant, and the purchase and installation of equipment.
The firmness of these plans will depend entirely upon the estimated
reliability of the sales and demand forecast.

For production planning purposes it is particularly important

to distinguish between forecasts of demand and forecasts of sales. Whereas forecasts of sales may be important for estimating revenue, cash requirements, and expenses, a production planning system is designed primarily to react to the customer demand. Demand may differ from sales for a variety of reasons. For example, sales may understate demand to the extent that the manufacturing and distribution system would be unable to cope with the volume of customer orders placed. In other words, sales represent an output from, rather than an input to, the production and inventory control system.

Having distinguished between sales and demand forecasts, we will now discuss some of the important characteristics of production planning forecasts. Forecasts of customer demand are fundamental to the operation of a business. Any company is in business primarily to serve its customer's needs in some way. Its survival depends on its ability to adapt its operations to customer's needs and to serve its customers adequately and efficiently when the need arises. The demand forecast is the link between the evaluation of external factors in the economy which influence the business and the management of the company's internal affairs. A forecast of sales or demand of some type exists whenever the company management makes a decision in anticipation of future sales or demand. This decision may be either to build a new plant or to manufacture another run of a particular item to restore inventory balances. Thus, we see that the sales and demand forecast should be reliable in order to reduce planning costs. Many times management fails to recognize that forecasts are made at various levels in the

production and inventory control system. Frequently, one finds that forecasting decisions which have an important influence on production planning operations are made by store keepers or stock-room clerks with no definite procedure or policy being followed. Determination of the types of forecasts required and establishment of procedures to make these forecasts are fundamental steps in the organization of a production and inventory control system. In short, the characteristics of forecasts related to production and inventory control can be summarized as the timing, detail and reliability of forecasts, and the assignment of responsibility for making forecasts and controlling or improving their quality.

Setting up procedures for making required forecasts is only the first step. Another part of the forecasting job, and an important one, is to establish procedures for reviewing forecasts made. This reviewing of forecasts can be divided into two parts:

1. Determination of whether the forecasts are being made according to the procedures established.

2. Measurement of the accuracy of forecasts made and determination of causes for major errors, as a basis for improving the quality or the effectiveness of forecasting procedures.

Short-term plans cover the next one to three years. Usually, these are moving plans which are adjusted quarterly for the first future year and semiannually or annually for the second and third future years. Again, the sales forecast is based on the general economic picture, industry and company market studies, company sales

trends, and current demands. Specific plans are made which project a continuation of the current year's plan. Using the sales forecast as a base, levels of inventory, production, and manpower are planned for specific products. The production plans are then used as the basis for detailed estimates of machine loads, and for specific determinations of the productive capacities of machinery and plant facilities. Firm decisions are made about additional machinery and plant facilities. The orders are then placed so there is coordination between delivery time and the plan made. Particularly important is the balancing of sales and production plans with the financial requirements for inventory, machinery, and plant facilities. As a matter of fact, finance is the dominating factor in making plans and decisions.

New-Product Introduction:

In a competitive economy, few products can maintain their places in the market without change or improvement. The products purchased and used by individuals, usually termed consumer products, must be changed according to the latest technological developments so as to satisfy the public taste. New models of highly fabricated products, such as automobiles, television sets, and refrigerators are introduced yearly to win consumer favor. Often, the most successful industrial firms are those that have been able to use research to produce new products.

The idea of new product introduction starts from research and development. This idea is then discussed between the sales department and research and development. The estimates of manufacturing cost,

selling price, profit, and investment which will be required for new machinery and plant facilities are made. Market research is done to find out sales potential and possible competition. The production planning and control department is aware of all these developments. When it is agreed that the new product can be made and sold for profit, closer estimates are made of the sales potential, manufacturing costs, selling price, profit, and required plant investment. At this point, production planning and control estimates the required investment in inventories of raw materials, work-in-process, and finished product that are necessary to back up the estimated sales. Research and development checks its final design with manufacturing to make sure that the product can be made and specified tolerances maintained. At the same time production planning and control checks with purchasing to be sure that all the materials required for the new product are available. All these estimates are combined into a recommendation to top management to manufacture the new product.

2. Current planning:

Current plans are made for a period of one year or less. They are of two types: fixed plans and moving plans.

The fixed plan is set up for a calendar or fiscal year by quarters. The plan remains constant throughout the entire year unless actual performance deviates from the planned performance to an extent that the plan becomes useless. The fixed plan is realistic, but at the same time it has a disadvantage of not being adjustable to changing conditions.

The moving plan is made for a fixed period of time, such as three or twelve months, and is revised every three months to extend the plan for the full period of time. The first three months of a twelve-month period is considered firm, with the other nine months tentative and subject to change. Naturally, this type of plan reflects changes in conditions and the thinking of the planners every time it is revised. Therefore, the moving plan has the advantage of being more current, but it has the disadvantage of a shorter firm period.

Current plans are much more detailed than short-term or long-term plans and take into account each product and the sales forecast, inventory, and production requirements for that item. The current plan takes into consideration the total sales forecast, current inventory of products, production requirements, capacity of the existing plant and equipment, and the level of manpower needed for the required production.

The sales forecast that is used for current plans is based on a composite estimate of sales from the field. The total dollars of this forecast become the basis for a general plan of production and inventories. The total units of each product are used to plan the production and inventory for that item, if the forecasts for individual items are considered to be sufficiently reliable. A number of companies have found from experience that forecasts can be made more reliable if production planning and control personnel review them with the sales manager before they are finalized.

It is also the responsibility of production planning and control to measure the plant capacity against the sales forecast periodically

so that they can recommend the purchase of needed additional equipment or plant facilities in time to prevent shortages of capacity. A current plan for the management of production and inventories should be an integrated statement which coordinates with the sales forecast the planned levels for finished-goods inventory, work-in-process inventory, manpower, and raw-materials inventory.

Let us suppose that we are planning the operations of a plant making a line of stock and special-order products. A forecast of demand, separately for stock items and special-order items, has been made and converted into production requirements and is shown in Table 1.

Let us assume that it has been decided to plan for uniform production throughout the year. This means that we must plan the operating level at $\frac{121,000 \text{ units}}{242 \text{ production days}} = 500$ units per day. This gives a production plan as shown in Table 2. The cumulative production plan is shown in the second column. From this the allowance for the forecast production requirements for special orders is subtracted, since these cannot be produced to inventory. The remainder, thus obtained, is the cumulative production plan for stock items. The cumulative production requirements for stock items subtracted from the cumulative production plan for these items gives the planned seasonal inventory for stock items, shown in the last column.

Production Control

Production control is a term applied to a group of interrelated management techniques. These techniques were developed with the growth

Table 1

Demand Forecast Converted to Production Requirements

Month	Cumulative Production Days	Stock Items Cumulative	Special Items		Total Cumulative Requirements
			Monthly	Cumulative	
January	22	5,500	1,100	1,100	6,600
February	41	9,000	700	1,800	10,800
March	62	11,600	400	2,200	13,800
April	83	16,000	1,000	3,200	19,200
May	105	22,500	1,800	5,000	27,500
June	125	32,000	1,900	6,900	38,900
July	137	43,500	2,000	8,900	52,400
August	159	55,700	2,300	11,200	66,900
September	179	68,900	3,500	14,700	83,600
October	202	80,700	2,300	17,000	97,700
November	221	91,500	2,000	19,000	110,500
December	242	100,000	2,000	21,000	121,000

Table 2

Production Plan

Month	Cumulative production days	Cumulative total production plan	Cumulative allowance special-order items	Cumulative production plan stock items	Cumulative production requirements stock items	Planned seasonal inventory stock items
January	22	11,000	1,100	9,900	5,500	4,400
February	41	20,500	1,800	18,700	9,000	9,700
March	62	31,000	2,200	28,800	11,600	17,200
April	83	41,500	3,200	38,300	16,000	22,300
May	105	52,500	5,000	47,500	22,500	25,000
June	125	62,500	6,900	55,600	32,000	23,600
July	137	68,500	8,900	59,600	43,500	16,100
August	159	79,500	11,200	68,300	55,700	12,600
September	179	89,500	14,700	74,800	68,900	5,900
October	202	101,000	17,000	84,000	80,700	3,300
November	221	110,500	19,000	91,500	91,500	0
December	242	121,000	21,000	100,000	100,000	0

of the Industrial Revolution.

Although men like Frederick W. Taylor and Henry L. Gantt have made great contributions to this field of activity, it would not be possible to list the names of all those individuals who deserve credit for the development of production control. Each day someone in industry produces a better means of recording data, a better duplicating process, an improved filing system, a different organizational structure, a better means of communication, or in some other way adds to our knowledge and skill in production control techniques. This is the manner in which production control develops, constantly striving for adjustment to each problem as industry expands.

The control of the flow and processes of manufacturing by means of an effective production control program is one of the most important factors in the successful operation of any industrial enterprise. The production control department itself is the lifeline of the industrial organization. It is due to this department and to an effective production control program that the organization is able to:

1. Meet production schedules

2. Maintain control of inventories

3. Utilize production facilities to their fullest capacity

4. Coordinate the introduction of engineering and manufacturing changes into the production program.

The primary objective of a modern factory is the production of economic goods of the proper quality and quantity, utilizing the least expensive methods, and meeting a necessary time schedule for completion.

In order to achieve these ends, the factory must have a medium for co-ordinating its activities into a single organized effort.

Production control is a group of physical activities, based upon modern management principles and concepts, and designed to guide production employees, machines, and materials to the fullest realization of their primary objectives. It is the technique of determining what items are to be produced and in what sequence the production operations must be applied to them. It also determines the quantities, the location, the time, and the order in which the components of a product must be processed. Production control furnishes the factory with the forms, specifications, and detailed instructions that must be followed in order to carry out the predetermined plan. Following this, production control performs a check on the factory's progress by means of current records and reports that reveal troubled areas and also furnish valuable data for future production planning.

It is not sufficient for the production control system simply to detect and repair such troubles as material shortages and machine breakdown. A properly organized and managed production control program will include means for anticipating production bottlenecks so that action may be taken to avoid or minimize the adverse effects of emergency situations.

Production control, then, consists of a group of staff or service functions which is intended to furnish management with the necessary systems, procedures, and forms required for the planning and control of production operations. It is no exaggeration to state that

the success or failure of an industrial enterprise may depend upon the design and operation of its production control program. Inadequate control means production shut-downs for lack of materials, large inventories that are not necessary, failure to incorporate engineering and manufacturing changes at the required time and constant pressure of rush orders. On the other hand, a production control program that has been carefully organized and well-planned and is subject to frequent scrutiny for the purpose of further improvement, serves both the industry and the consumer by effecting full utilization of all production facilities.

In general, successful production control depends on the satisfactory performance of several types of functions. These are:

1. Forecasting and planning

2. Inventory control

3. Control of production operations

4. Process engineering

The size and character of a production control organization will depend on:

1. The duties specifically delegated to the production control group.

2. The degree of control required.

3. The size of the company involved.

A production control rule given by Magee[2] for replenishing

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[2]John F. Magee, Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., New York, 1958, p. 317.

warehouse inventories is as follows:

$$q(i) = \sum_{k=1}^{U+1} d(i+k) - \sum_{k=1}^{U} q(i-k) - \left[I(i) - I^* \right]$$

where U = lead time (in periods)

    q(i) = amount ordered at the end of period i, available at the

        beginning of period i + U + 1

    d(i) = forecast demand for period i

    I(i) = inventory at the end of period i

    I* = planned inventory level

Under this rule, the warehouse would place an order in each period
equal to anticipated requirements over the lead time plus the reorder
period, less the amount on order, plus the amount by which the desired
inventory on hand and on order exceeds actual. This rule was set up
on the assumption that there is no cost of changing the size of order
from period to period. For example, suppose we are setting up a
scheme to control the operating hours of a packaging line. The demand
forecast for the coming thirteen weeks (expressed in hours of line
operation) is shown in Table 3. Demand in a given week might vary
±18 hours from forecast, and over two weeks it might vary ±25 hours
from forecast.

    Production is to be adjusted weekly, and because of the work
notice to employees, it takes one week for a decision (to change the
production levels) to become effective. Thus, the sum of the lead time
and review interval equals two weeks. The inventory fluctuations will
be equal to the fluctuations between actual and forecast demand during

Table 3

Thirteen-week Demand Forecast,
Hours of Production Per Week

Week	Forecast demand (hours of production)	Week	Forecast demand (hours of production)
1	21.0	8	36.5
2	24.5	9	42.5
3	24.5	10	52.0
4	28.0	11	54.5
5	28.0	12	45.5
6	31.5	13	35.5
7	31.5		
		Total 13 weeks	455.0

this interval. Since these fluctuations can run up to $\pm$ 25 hours over a two-week span, planned inventories cannot be less than the equivalent of 25 hours of production.

Let us suppose that there were the equivalent of 38 hours in in-inventory. Then, the production requirements over the period

= (demand) + (minimum planned inventory) - (on-hand inventory)

= 455 + 25 - 38

= 442 hours or 34 hours per week.

A check will show that a uniform planned rate of 34 hours per week will meet the forecast. The cumulative forecast demand, planned production, and planned inventories, which might be arrived at, are shown in Table 4. It should be noted that this plan is in terms of production

Table 4

Thirteen-week Operating Plan

Week	Cumulative forecast + demand	Planned weekly production	Planned cumulative production + inventory	Planned inventory
Opening stock	--	--	38.0	--
1	21.0	34.0	72.0	51.0
2	45.5	34.0	106.0	60.5
3	70.0	34.0	140.0	70.0
4	98.0	34.0	174.0	76.0
5	126.0	34.0	208.0	82.0
6	157.5	34.0	242.0	84.5
7	189.0	34.0	276.0	87.0
8	225.5	34.0	310.0	84.5
9	268.0	34.0	344.0	76.0
10	320.0	34.0	378.0	58.0
11	374.5	34.0	412.0	37.5
12	420.0	34.0	446.0	26.0
13	455.0	34.0	480.0	25.0

hours, not physical product units. Since one week's notice to employees is required, production for the first two weeks is fixed by the original plan. However, at the end of the first week, the first review period, we have a chance to adjust the production rate. Suppose that the demand during the first week was equivalent to only seventeen hours, four hours less than the forecast demand of twenty-one hours. Then, if production were on plan, the inventory at the end of the week would be equivalent to fifty-five hours. Now, the control rule will be as follows:

Production level

in the third week = preliminary budget for the week being

planned

+ the amount by which originally budgeted

production during the intervening period

(week 2) exceeds scheduled amount

+ the amount by which budgeted inventory on

hand exceeds actual

= 34.0 + (34.0 - 34.0) + (51.0 - 55.0)

= 30.0 hours

Then the production level planned for the third week would be 30.0 hours. The full difference between forecast and actual sales in the first week was taken up by an adjustment in production in the third week.

The important problems of production control are somewhat dependent upon the industry and the company under consideration. The

types of data available, the types of data necessary, the character-
istics of the processing or the manufacturing operation, the service
demanded by the customer, the characteristics of the product, etc.,
will vary from one situation to another.

In the processing industries there are instances where raw
materials cannot be stored but the finished product is capable of al-
most indefinite storage. An example is the canning of vegetables and
fruit. In other situations the raw materials are capable of relatively
long-term storage but the finished product is not. An example of this
situation is the ready-mix concrete plant. Still other cases present
a problem of limited procurement periods.

Similar situations are present in the manufacturing and service
industries. The grocery store is an example of a service industry
where goods with both long and short storage lives can be purchased.

These factors determine where the major emphasis on production
control must be placed. Examination of the production control require-
ments in a continuous manufacturing operation reveals that the emphasis
should be placed on the availability of the correct kind and quantity
of raw materials at the appropriate times, the prevention of bottle-
necks in the production line, and the removal of the finished product
from the line and its distribution to the point of storage or sale.
Much of the control is built into the production line.

In intermittent manufacturing, other problems arise. In such an
activity, there is no predetermined manufacturing process. Usually, a
new and different process is required for each order. Stoppages or

shortages at a limited number of points do not stop the entire flow of production. Since each item is built to a specific order, the finished product is usually shipped directly to the customer. In this type of manufacturing, the responsibility for balancing the production operations falls to the production control group. In continuous manufacturing this responsibility lies with the engineering group that designs the manufacturing process. Once established, it remains the same until major product changes or equipment replacements occur.

The major advantages of a successful production control program may be listed as follows:

1. Requested delivery dates are met. This means satisfied customers and more future orders.

2. Shop foremen are assisted in solving their production schedule problems by trained production control specialists who furnish records, reports, and other clerical services.

3. Available manpower and equipment are more thoroughly and efficiently utilized. Production activities tend to be leveled, eliminating costly peaks and depressions in the volume of work.

4. Raw material, work-in-process, and finished-good inventories are maintained at an optimum level which minimizes material shortages without needlessly tying up working capital in excessive inventories.

5. Production bottlenecks are foreseen in time to be avoided.

6. Employees are seldom required to wait for tools or materials not readily available before starting the next job.

7. Work-in-process is not permitted to accumulate on the shop floor while waiting for the next operation.

8. Material procurement from outside vendors is carefully planned so that errors and rush orders seldom occur.

9. Production goals are attained in an orderly manner. The personal satisfaction and high morale in employees who have experienced a job well done is one of the most important attributes of a progressive organization.

Material Management

To any manufacturing enterprise, material is one of the most important factors in the operation. It is the substance from which the finished products and all of their constituent parts are made. It is the material which is subjected to the various factory processes as it travels from the beginning of the manufacturing cycle to the end. The economic importance of the material varies from one product to another, but it is not at all uncommon to find products in which materials constitute more than one-half of the total cost of production.

Material control is the term generally given to the functions of:

1. Placing orders with the Purchasing Department for materials needed in the manufacturing process in sufficient time and in sufficient quantity to meet scheduled requirements.

2. Storing materials for manufacturing until they are ready to be used.

3. Maintaining adequate records and controls on these materials

to avoid the waste of invested capital in:

a) Overstocked or understocked storerooms.

b) Improper inventory turnover.

c) Lost, damaged, stolen, deteriorated, or obsolete materials.

In certain types of industrial organizations, the material-control activity may include other functions, such as those of purchasing or receiving.

The first step in understanding the function of material control is to understand the classifications of inventory. In the broad sense of the word, "inventory" refers to all materials owned for the purpose of development or manufacturing a saleable product or service. For the manufacturing industry, such materials may be used directly by inclusion in the end product, or they may be used indirectly to facilitate the manufacturing operations and controls.

Following is a list of several kinds of inventory accounts which may be found in a typical manufacturing company:

1. Raw Materials. These are direct materials obtained from some outside source, and are destined for further processing before becoming a part of the end product.

2. Detail Parts. These are also known as component parts or piece parts. These parts are the finished elements which will be assembled to make up the final or end product. Details or components may be purchased as stock items from an outside source, or they may be parts that were manufactured from raw materials in the company's manufacturing departments.

3. Work-in-Process. This consists of all direct materials that exist in some transitory condition between the raw material or detail-parts state, and the finished-good state. They may be progressing from one operation to another. They may be temporarily held in a storage bank waiting for the next operation, or they may be undergoing a manufacturing operation.

4. Finished Goods. These are the parts, units, or assemblies that have completed the manufacturing cycle, and are ready to be delivered to the customer or to a distributing agency.

5. Indirect Items. These are used in the manufacturing process but do not form a part of the finished product. Such supplies are usually purchased. They include cutting and lubricating oils, cleaning or pickling solutions, waste and wiping rags, janitor's supplies, office supplies, construction materials, repair parts from machine and equipment, other maintenance items, etc.

Material control, as one of the major functions of production planning and control, is generally divided into two areas of functional responsibility: planned control and physical control. Planned control has the responsibility of maintaining a constantly available supply of required raw materials, purchased parts, and supplies. Physical control has the primary responsibility for the proper receipt, storage, protection, and issuance of materials from inventory and for establishing appropriate controls for safeguarding those materials.

Planned Control

Planned control is the activity of constantly maintaining an

adequate but not excessive supply of each of the raw materials, purchased parts, and supplies that are required for the manufacture of the product. It is the making of daily decisions of "how much" and "when" to order these materials to keep the supply pipeline filled at all times. These are replenishment decisions which are combined with the interpretation of usage trends and some anticipation of the usage that may occur during delivery time. Lead time is an important influence on these replenishment decisions. This is the total amount of time required to procure a material--from the time a purchase requisition is written by the material control department until the material is received in the plant. It is the responsibility of the purchasing department to provide material control with a realistic statement of lead times for all production materials and supplies.

In order to maintain a constant supply of materials, it is necessary to maintain inventory records which will permit analysis by showing the facts required for a replenishment decision. These are receipts, usage, inventory balance, assignments, available inventory balance, a summary of usage for each month in the past, order quantity, number of orders to be placed per year, lead time, and inventory reserve expressed in number of days or weeks of supply. These facts should be shown on one record to simplify the making of a decision. When the decision is made, it is the obligation of material control to place the purchase requisition with the purchasing department.

An important part of the planned control of materials is the verification of records. It is essential that the inventory balances,

average usage, and lead times be verified periodically to assure order-
ing the correct quantity at the right time and to prevent depletion.

Physical Control

The physical control of production materials is accomplished in
the two major plant activities of receiving and storeskeeping. Al-
though the storeskeeping activity will sometimes include receiving, its
major responsibilities are for storage, protection, issue and physical
count.

The receiving department has the responsibility for making an
accurate count and correct statement of all materials that are received
in the plant.

The function of the storeskeeping department is to carry out the
storage, protection, and issuance of production materials and supplies
in accordance with the plan developed by the planned-control section of
material control.

In order to secure fully the advantages of proper materials con-
trol it is necessary to think through and set up policies, procedures,
and a suitable organizational structure. Unless this is properly done
the various phases will not be coordinated into the most effective oper-
ation. Some of the steps for an effective system of materials control
are as follows:[3]

    1. Set policies necessary to guide the materials and inventory

        control program.

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[3]Gordon B. Carson, Production Handbook, The Ronald Press
Company, New York, 1958, p. 4.3.

2. Determine the most appropriate organization structure to carry out these policies.

3. Establish the basis for materials control according to the method of manufacturing and the type of material.

4. Plan the availability status of each class of material and modify control methods to suit the value classification.

5. Set up records and procedures for properly ordering materials required and for controlling same.

6. Establish auxiliary procedures, including standardization of materials and parts.

7. Establish a procedure for physical verification of records.

8. Provide storage and physical handling facilities.

9. Provide and train manpower for effective operation of the system.

The responsibility for material inventory policies lies on top management. Unless the broad and basic policies are laid down by top management, the detailed operating policies and procedures necessary for effective provision and control of materials cannot be developed properly by the manufacturing organization.

Policies set up by top management will vary with the type of industry, type of company, characteristics of the product line, and the current state of business. The current financial condition of the particular company and the amount of capital available also may cause variations in policy.

For example, in some of the process industries, such as steel

and chemicals, where the price of raw materials procured has a certain known effect on the profit-and-loss statement, the company may engage in a certain amount of speculative or forward buying. Since this is a vital factor in the profit of the business, the decision as to when and how much raw material to buy is usually made by top management. In job shop operations, on the other hand, the usual procedure is to do as little speculative or forward buying as possible.

There are many advantages for a company in properly providing for and controlling materials. Some of these are:

1. Reducing the possibility of nonaccomplishment of customer delivery promises. This is one of the most important factors in maintaining customer good will and sales position and therefore the ultimate profits of the company.

2. Reducing the possibility of shutting down production lines or other manufacturing activities by not having material on hand.

3. Reducing material waste due to theft, breakage, deterioration, spoilage, and obsolescence.

4. Reducing the cost of manufacturing by having proper parts on hand when needed so that it is not necessary to substitute other parts or material.

Companies which have established strong materials control activities under the direction of integrated production planning and control departments have usually reduced inventories, storage times, and handling costs, and have often increased their volume of output.

CHAPTER IV

ECONOMIC PRODUCTION QUANTITY

Deciding how many items to make for stock at one time is one of the most common and still frequently unresolved questions of inventory management that businessmen face. It happens also to be a question that has received early and continuous attention in the literature of inventory control over the period since 1920.[1] Because it is so frequently found, the lot-size problem serves as a useful starting point to discuss inventory functions and methods for analyzing them.

The problem arises because of the need to produce or purchase in quantities greater than will be sold or used at the moment. Thus, businessmen buy raw materials in sizable quantities in order to reduce the costs connected with purchasing and control, to obtain a favorable price and to minimize handling and transportation costs. They replenish factory in-process stocks of parts in sizable quantities to avoid, where possible, the cost of equipment set-up and clerical routine. Likewise, finished stocks maintained in warehouses usually come in shipments substantially greater than the amount sold in one order, the motive being, in part, to avoid equipment set-up and paper-work cost and, in the case of field warehouses, to minimize shipping costs.

In all these cases the practice of replenishing in sizable

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[1] John F. Magee, Production Planning and Inventory Control, McGraw-Hill Book Co., Inc., New York, 1958, p. 44.

quantities compared with the typical usage quantity means that inventory has to be carried; it makes it possible to spread fixed costs (e.g. set-up and clerical costs) over many units and thus to reduce the unit cost. However, one cannot carry this principle too far, for if the replenishment orders become too large, the resulting inventories get out of line, and the capital and handling costs of carrying these inventories more than offset the possible saving in production, transportation, and clerical costs. Here is the matter again, of striking a balance between these conflicting conditions.

Even though formulas for selecting the optimum lot size appeared in the literature as early as 1920, few companies make any attempt to arrive at an explicit quantitative balance of inventory and change-over or set-up costs. Franklin G. Moore[2] has concluded that their use is declining for these reasons:

> Most companies today probably no longer compute economic-lot sizes. At the Hawthorne plant of the Western Electric Company, for example, a whole department was once engaged in computing economic lots. Today it has disappeared and very little, if any, such computation is carried on. Perhaps part of the reason for the general decline in interest in economic lots has been the cost of computation. Furthermore, only approximate reliability of a computed answer can be counted on, since it is often impossible to forecast future needs and the possibility of obsolescence; also economic lots have only transitory validity because changes occur in demand, costs and other factors.

> General operational policies play an important part in actual lot size determination today. Quantities larger than the economic lot may be produced during temporary slack periods

[2]Franklin G. Moore, Production Control, McGraw-Hill Book Co., New York, 1951, pp. 178-183.

in order to level out production. Present and prospective price trends are important and play a part. The financial position of the company may limit inventory investment regardless of economic lots. Equipment limitations may force short runs to permit a variety of items to be produced on the machines available. Management may not know that production is being carried on in economic lots or may not fully appreciate the costliness of uneconomic lots.

If setting up is done by special set-up men, the machine operator must be put on other work while the machine is being set up. Often work of equal calibre is not available, and the operator is idle or is used on lower grade work. Short runs cause extra costs in getting jigs, fixtures, or patterns from storage and returning them. These costs are rarely charged to the order. In some companies the accounting procedures charge set-up costs also to an overhead account rather than to orders. This practice of charging machine accessory handling time and set-up time to over-head accounts reduces the reported unit cost on short runs and tends to misinform management as to the costliness of uneconomic lots.

The computation of optimal lot sizes is not a serious difficulty if electronic computers are available. Another objection raised by Moore is that the optimal lot-size formulas do not take into account enough relevant factors.

Despite the difficulty in measuring costs and indeed because of such difficulty, it is worthwhile to look at the lot-size problem explicitly formulated. The value of an analytic solution does not rest solely on one's ability to plug in precise cost data to get an answer. Even when the data available for use are crude, an analytic solution often helps in clarifying questions of principle.

The lot-size decision rules are used to find the total cost as a function of the aggregate inventory and the sales rates. This function may be added to the overtime, payroll and other quadratic cost functions needed to find the production and employment scheduling

rules. The production rule then determines the total inventory level which serves as the constraint on the lot sizes of the individual products.

Lot Size Model--No Stockouts

First we will consider a relatively simple inventory-control problem in order to make the general approach as clear as possible. The problem is to determine the optimum lot sizes when the demand for each item is deterministic and is constant (independent of time).[3] When the inventory of a product declines to a specified level, an immediate order for production of a lot is placed. The only unknown is the size of the lot. Furthermore, it will be assumed that the entire quantity produced is delivered as a single package, i.e., it never happens that an order is split so that part of it arrives at one time and part of it at another time. We shall imagine that the item can be inventoried indefinitely, and that it will never become obsolete. Then it is convenient to assume that the system will continue to operate for all time and the system is never out of stock when a demand occurs.

The basis of inventory theory is to write an appropriate cost equation that includes all possible costs such as set-up costs, raw material, labor, storage and so on. Further, we proceed to minimize this total cost equation. Thus:

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[3]C. C. Holt, Decision Rules for Allocating Inventories to Lots, The Journal of Industrial Engineering, Vol. 9, no. 1, January-February 1958,

Number of lots to be manufactured per year $= \dfrac{Y_i}{Q_i}$

(for symbols, see Appendix A)

$\therefore$ Total yearly set-up cost $= \dfrac{Y_i}{Q_i} \cdot S_i$ (1)

where $i = 1, 2, ---- p$

We will assume that on-hand inventory at the time of arrival of a procurement is zero, i.e., the system just runs out of stock as the new procurement arrives. In other words we are assuming that the sales or withdrawals from inventory are made uniformly over each period, the inventory of product i will go from a maximum of Q_i to a minimum of zero as shown in Figure I. Thus:

The average level of inventory $= \dfrac{Q_i}{2}$

The average value of inventory for product i $= \dfrac{Q_i V_i}{2}$
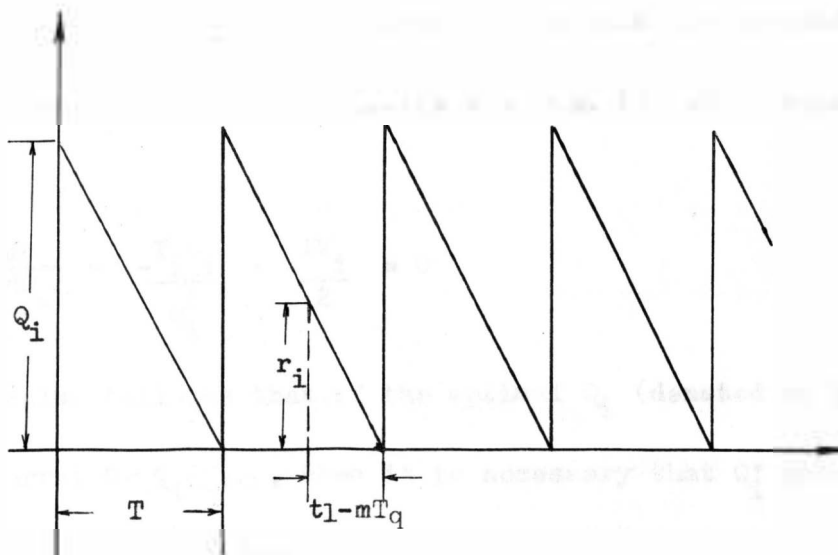
Figure I. Units in inventory as a function of time

Hence the total carrying cost $= I \cdot \dfrac{Q_i V_i}{2}$ \hfill (2)

where $i = 1, 2, \text{----}, p$.

Total variable cost = Total set-up cost + Total carrying cost

$$C_i = \frac{Y_i}{Q_i} \cdot S_i + I \cdot \frac{Q_i V_i}{2} \tag{3}$$

where $i = 1, 2, \text{----}, p$.

We can write the total variable cost for all products as:

$$C = \sum_{i=1}^{p} c_i = \sum_{i=1}^{p} \frac{Y_i S_i}{Q_i} + \frac{I}{2} \cdot \sum_{i=1}^{p} Q_i V_i \tag{4}$$

Our object is to find the values of $Q_i > 0$ which minimize the total cost "C". We can minimize this quantity in a number of ways:

1. We can take the derivative of total cost with respect to Q_i and set the quantity equal to zero (that is $\dfrac{dc}{dQ_i} = 0$) in order to deter-mine the point at which zero slope and minimum total cost occurs. Assuming that demand is continuous, Q_i can also be treated as a continuous variable. Differentiating equation (3) with respect to Q_i, we get:

$$\frac{dc}{dQ_i} = \frac{-Y_i S_i}{Q_i^2} + \frac{IV_i}{2} = 0 \tag{5}$$

The calculus tells us that if the optimal Q_i (denoted by Q_i^*) lies in the interval $0 < Q_i < \infty$, then it is necessary that Q_i^* should satisfy the above equation. Thus:

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i}} \quad \text{where } i = 1, 2, \text{----}, p. \tag{6}$$

This is the minimum-cost production order quantity. This can be seen by differentiating the equation (5) again.

$$\frac{d_2 C}{dQ_i^2} = \frac{2Y_i S_i}{Q_i^3}$$

The above quantity is positive for all $Q_i > 0$ and hence the Q_i determined from equation (5) yields the absolute minimum value of C.

    2. We can use a graphical method. This requires plotting each cost component.
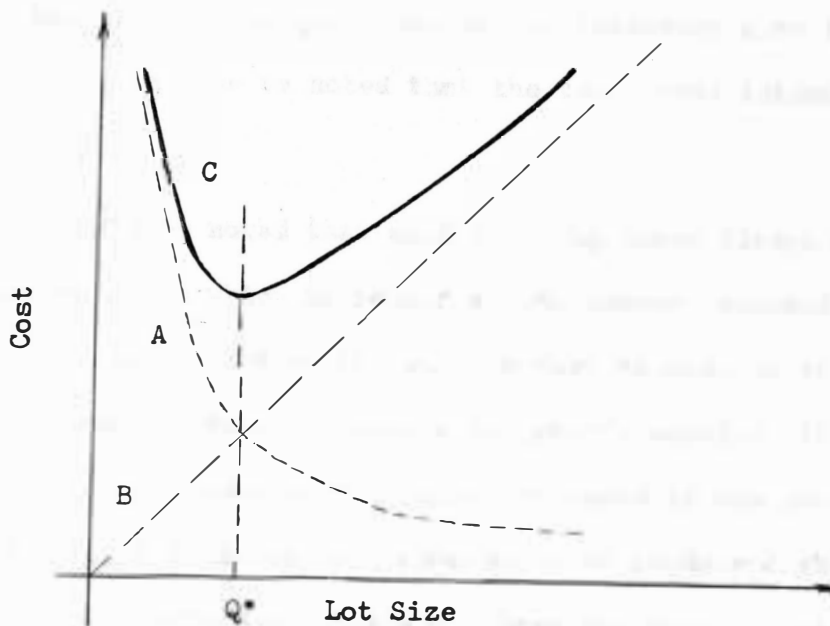
Figure II. Cost as a function of lot size

Figure II illustrates the inventory manager's dilemma.[4] The curve "A"
represents the set-up cost, which decreases as the number of parts pro-
duced on one run increases. Curve "B" represents the carrying cost,
which increases as the number of parts produced on one run increases.
Curve "C" represents the Total cost, i.e., the sum of A and B. The
objective of minimizing the total cost is fulfilled by selecting the
strategy of producing Q_i^* parts on one run where Q_i^* is determined from
the minimum of the total cost curve, C. The figure shows clearly that
the Q_i^* which yields the minimum is unique. It might be noted that the
optimal Q_i^* occurs at the points where the slope of the set-up cost
curve is the negative of the slope of the inventory carrying cost
curve. It should also be noted that the two curves intersect at the
point $Q_i = Q_i^*$.

It should be noted that such opposing costs always exist. If
there were no costs which increased as the number produced in one set-
up increased (curve B) then it would be most reasonable to produce an
enormous amount in advance, perhaps ten year's supply. If there were
no costs which increased as the number produced in one set-up decreased
(curve A) then it would be most reasonable to produce each part as it
was needed. The unreasonableness of these two possibilities in almost

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[4]H. Chestnut, T. F. Kavanagh, and J. E. Mulligan, Applying
Control to Inventory Management, Control Engineering, Vol. 9, no. 9,
September, 1962, p. 96.

all cases is due to the existence of both kinds of costs.

So, the decision problem in question is solved as soon as the two curves in question are obtained and summed to get the total cost curve C. The shape of the curves given in Figure II is arbitrary and is only meant to illustrate the general situation. For each specific problem the actual shape of these curves must be determined.

3. We could use trial and error methods, by substituting different values of Q_1 into the total cost equation until the minimum total cost was obtained.

4. Further, another method to minimize the total cost is as follows: The minimum total cost will occur, for this equation, when the total carrying cost is equal to the total set-up cost. This is an application of the marginal principle which is basic to most economic thinking. An economist, if presented this problem, would immediately set out to find the value of Q_1 for which the marginal cost of set-up equalled the marginal cost of carrying stock in inventory. He would, upon solving for Q_1, get the same result as equation (6).

Determination of Re-order Point

Let t_1 = lead time (in units of time), the interval of time elapsing between the point of time at which the item is triggered for production and the point of time at which the production of it is completed and it becomes available.

Tq = lot time, the interval of time within which a lot is consumed by sales.

m = largest integer less than or equal to $\dfrac{t_1}{T_q}$.

Then, $T_q = \dfrac{Q_i^*}{Y_i}$

Thus, if we place an order for production of product i when the on-hand inventory reaches the level

$$r_i = Y_i \, (t_1 - mT_q)$$
$$= Y_i t_1 - mQ_i^*$$
$$= D_1 - mQ_i^* \tag{7}$$

Where $D_1 = Y_i t$ is the lead time demand (i.e., the number of units demanded from the time a production order is placed until it arrives). The on-hand inventory will be zero at the time production is completed. The number r_i is called the reorder point; each time the on-hand inventory in the system reaches r_i an order for the production of Q_i^* units is placed. This is illustrated graphically in Figure I.

The problem of determining how to operate the system has now been solved. The reorder point, given by equation (7) tells us when an order for production should be placed.

Lot-size Model in Case of a Finite Production Rate

The assumption that the order is received and placed into inventory all at one time is often not true in manufacturing runs. The formula derived in the previous case, i.e., $Q_i^* = \sqrt{\dfrac{2Y_i s_i}{IV_i}}$ assumes the general inventory pattern shown in Figure I where the order quantity Q_i^*

is received into inventory all at one time. The inventory is then drawn down at the usage rate, subsequent orders being placed with sufficient lead time so that their receipt coincides with zero (or minimum stock) inventory.

For many manufacturing situations the production of the total order quantity Q_i^* takes place over a period of time and the parts go into inventory, not in one large batch, but in smaller quantities, as production continues. This results in an inventory pattern similar to Figure III.
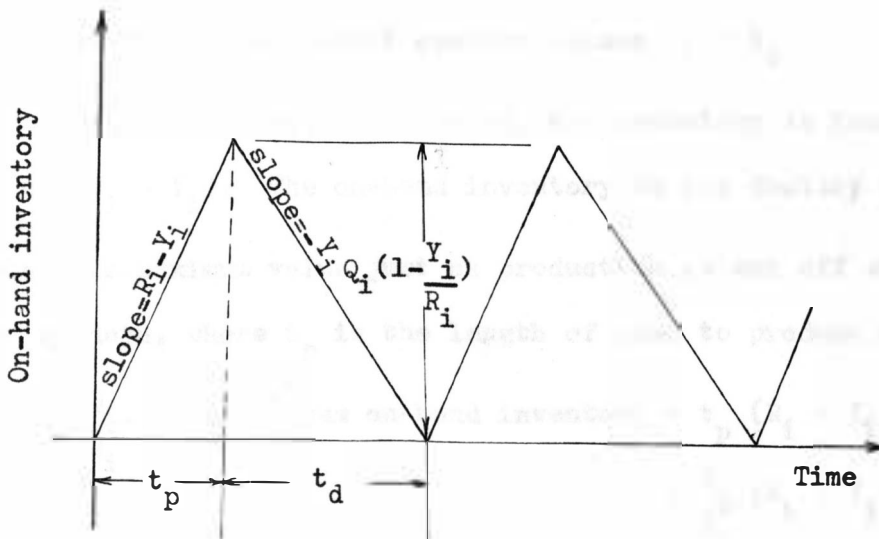
Figure III. Inventory balance when Q^* is received over a period of time

Thus the inventory, in this case, will not increase by the full amount of order or run quantity at one time. It will increase slowly over the course of the run, while production going into inventory exceeds usage going out. It will reach a maximum at the end of the run. If the length of the run extends over a substantial period, for example if the

item is in production half the time, more or less, this can have a substantial effect on the maximum inventory build-up and on the size of the economical run quantity. This can be seen as follows:

Let Y_i = sales rate in units per day for product i.

R_i = production rate in units per day for product i.

It is supposed that demands are deterministic and are incurred at the factory warehouse at a rate of Y_i units per day. Once the factory is set up to produce a lot, it will be imagined that the production rate is R_i units per day (independent of the size of the lot). It is quite clear that the system cannot operate unless $R_i > Y_i$.

During the production period, the inventory is increasing at a rate of $(R_i - Y_i)$. The on-hand inventory in the factory warehouse reaches its maximum value just as production is cut off at the factory after t_p days, where t_p is the length of time to produce a lot and is equal to $\frac{Q_i}{R_i}$. The maximum on-hand inventory

$$= t_p (R_i - Y_i)$$
$$= \frac{Q_i}{R_i} (R_i - Y_i)$$
$$= Q_i (1 - \frac{Y_i}{R_i})$$

The average on-hand inventory $= \frac{Q_i}{2} (1 - \frac{Y_i}{R_i})$ (assuming that the system just runs out of stock as the new procurement arrives). Let t_d = time required to deplete the on-hand inventory at the warehouse. Then,

$$t_d = \frac{Q_i}{Y_i} \left(1 - \frac{Y_i}{R_i}\right).$$ The length of the cycle $= T = t_p + t_d$

$$= \frac{Q_i}{R_i} + \frac{Q_i}{Y_i} \left(1 - \frac{Y_i}{R_i}\right)$$

$$= \frac{Q_i}{Y_i}$$

Set-up cost $= \frac{Y_i}{Q_i} \cdot S_i$ where $i = 1, 2, ----, p.$

Inventory carrying cost $= I \cdot V_i \quad \frac{Q_i}{2} \left(1 - \frac{Y_i}{R_i}\right)$

The average total cost of set-up and holding inventory for product i:

$$C_i = \frac{Y_i}{Q_i} S_i + IV_i \cdot \frac{Q_i}{2} \left(1 - \frac{Y_i}{R_i}\right) \tag{8}$$

Differentiating the above equation with respect to Q_i and setting the

derivative equal to zero, we get the optimal value Q_i (denoted by Q_i^*).

Thus:

$$\frac{dc_i}{dQ_i} = -\frac{Y_i S_i}{Q_i^2} + \frac{IV_i}{2} \cdot \left(1 - \frac{Y_i}{R_i}\right) = 0$$

which has the unique positive solution

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i} \left(\frac{R_i}{R_i - Y_i}\right)} \tag{9}$$

If Y_i is almost equal to R_i, then Q_i^* becomes very large, approaching infinity as the difference between Y_i and R_i approaches zero. This result makes sense. In effect it states: if the demand rate is as great as the production rate, then run the process continuously. On

the other hand, if R_i is very much greater than Y_i, i.e., $R_i >>> Y_i$, then Q_i^* given by equation (9) will be equal to that given by equation (6) in the previous case. This result is also reasonable.

In many circumstances, of course, the total order quantity is produced in a relatively short time compared with the time between runs. In such a case also, the difference between equations (6) and (9) is negligible (since $\dfrac{Y_i}{R_i}$ is close to zero). The utility of this model can be seen by considering a numerical example.

Example:

A manufacturer for automobile accessories makes several parts. These parts are supplied to the customers from the factory warehouse. One particular part has the following properties: The demand rate can be assumed to be known with certainty and to be constant at 1,500 units per year. The fixed cost of set-up for each production run is $8.33. The cost per unit is $5.00. Inventory carrying charge is 0.02 per dollar per year. The production rate is 48 units per day. A period of 20 days is required from the time that a production requisition is received at the factory until finished units begin to come off the production line. It is desired to determine the optimal lot size and the warehouse re-order point based on the assumption that stockouts are not permitted. Assume the number of working days in a year to be 250. The optimal lot size, Q^*, would be:

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i} \left(\frac{R_i}{R_i - Y_i}\right)}$$

$$= \sqrt{\frac{2\,(8.33)\,(1,500)}{(0.02)\,(5)} \times \frac{(48)\,(250)}{(48)\,(250) - (1,500)}}$$

$$= 534 \text{ units}$$

A computer program written in the form shown in Appendix B can be used for solving this problem. The time required to produce this lot

$$= t_p$$

$$= \frac{Q_i}{R_i}$$

$$= \frac{535}{48} = 11.15 \text{ days}$$

The time between runs:

The on-hand inventory in warehouse reaches a maximum value of

$$Q_i^* \left(1 - \frac{Y_i}{R_i}\right)$$

$$= 535 \left(1 - \frac{1,500}{48 \times 250}\right)$$

$$= 468.125 \text{ units}$$

The demand per day $= \dfrac{1,500}{250}$

$$= 6 \text{ units}$$

Therefore, the time required to deplete the on-hand inventory at the warehouse $= t_d$

$$= \frac{468.125}{6}$$

$$= 78.02 \text{ days}$$

Therefore, total cycle time $= t_p + t_d$

$$= 11.15 + 78.02$$

$$= 89.17 \text{ days}$$

that is, there would be one run every 90 days. The average annual cost of set-up and holding inventory is:

$$c_i = \frac{Y_i S_i}{Q_i^*} + IV_i \cdot \frac{Q_i^*}{2} \left(1 - \frac{Y_i}{R_i}\right)$$

$$= \frac{(1,500)(8.33)}{534} + (0.02)(5.0)\left(\frac{534}{2}\right)\left(1 - \frac{1,500}{12,000}\right)$$

$$= \$46.76$$

Since the lead time = 20 days, it follows that the re-order point based on the on-hand inventory is:

$$r_i^* = (\text{lead time})(\text{demand rate})$$

$$= (20)(6)$$

$$= 80 \text{ units}$$

CONSTRAINTS

General Explanation

Inventories are seldom composed of a single item. Usually, many different items are carried in stock. Even for a single item, it is not unusual to have many associated stock-keeping units. For example, in the category "screws," a typical manufacturer's inventory will include various lengths, diameters, number of threads to an inch, wood screws, machine screws, brass screws, steel screws, and so on. In the same way a department store will carry many different sizes, colors,

materials, and styles of socks, and the supermarket stocks a great variety of soups and soaps.

We can, if we have enough information, obtain the optimal lot size for each stock-keeping unit. This would give us the minimum over-all total cost system. However, two factors intervene:

1. It costs money to study inventories and to develop policies for each item. From the point of view of a break-even chart, the cost of the inventory study increases fixed costs. The savings obtained from the study decrease variable costs. The result must represent a sufficient return on the capital invested in the inventory study to make this investment preferable to alternative investments in bonds and stocks, machinery, or additional personnel. Because this criterion underlies all inventory studies, companies seldom undertake inventory studies of all items that are needed. Instead, the items are divided into categories, frequently called a-b-c. The "a" items represent the highest dollar-volume group. The "b" and "c" groups are proportion- ately lesser participants in dollar volume. Figure IV shows a typical a-b-c breakdown.[5] We see that only 25 per cent of the total number of items carried contributes over 75 per cent of the total dollar volume for this example. Because savings to be realized are a function of the dollar volume, it is clear why "a" items should be singled out for at- tention before the others.

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[5] M. K. Starr, _Production Management_, Prentice-Hall, Inc., New Jersey, 1964, p. 322.
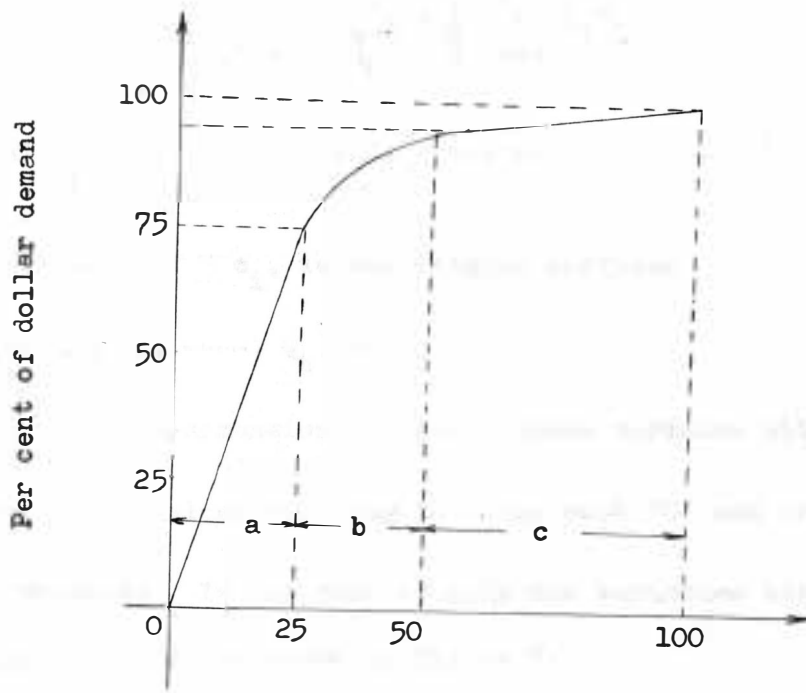
Figure IV. A representative a-b-c breakdown

2. The company's resources are limited. It is frequently un-
reasonable to carry the total average dollar inventory that the indi-
vidual item's optimal policies would require. The capacity of the
ordering department may be over-taxed; storage facilities may be filled
to capacity; the amount of capital invested in inventory may exceed the
amount that the company has available. These limitations, if they
exist, require a modification of inventory policy. That is, the theo-
retical system's optimal is not feasible because it violates other
practical system's constraints.

Let us consider equations (4) and (6) which represent the total
variable cost and optimal lot size. That is,

$$C = \sum_{i=1}^{p} c_i = \sum_{i=1}^{p} \frac{Y_i S_i}{Q_i} + \frac{I}{2} \sum_{i=1}^{p} Q_i V_i \qquad (4)$$

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i}} \qquad i = 1, 2, \text{----}, p. \qquad (6)$$

For any value C of $\sum c_i$, we can imagine surfaces

$$c \ (Q_1, \ Q_2, \ \text{----}, \ Q_p) = C$$

of Q_i values in p-dimensional space. These surfaces will constitute the locus of Q_i values yielding the same cost "C" and are called constraint surfaces. In the case of only two variables they constitute C = constant lines, as shown in Figure V.
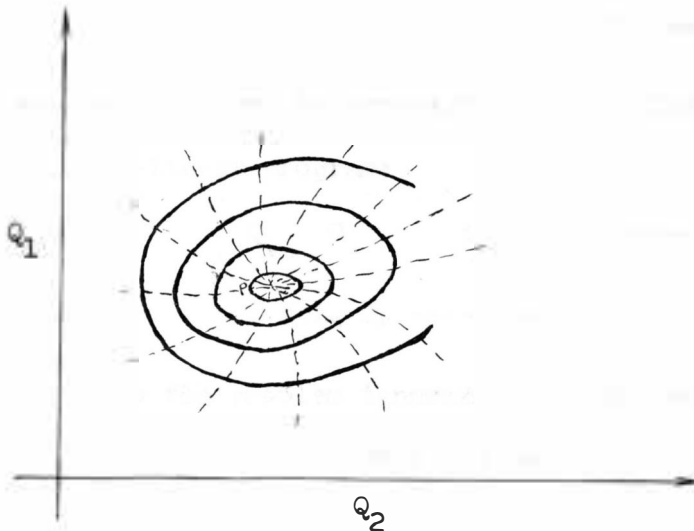
Figure V. Iso-cost curves for pairs of lot sizes

In this figure, the solid lines represent the c $(Q_1, \ Q_2)$ = C curves. In other words they connect pairs of lot-size points which yield equal costs. These are sometimes referred to as Iso-cost lines and the

dotted lines represent their orthogonal trajectories, showing the direction of the minimum variation of "C" in the plane Q_1, Q_2. The curve $c_1 = C^*$ is reduced to the stationary point P (Q_1^*, Q_2^*) where C^* is the minimum cost obtained by substituting Q_i^* for Q_i in equation (4). Thus the point P is the pair of lot sizes Q_1^*, and Q_2^* that minimize total cost. The Iso-cost lines represent higher and higher costs as one goes farther and farther from P.

Now let us consider a group of m constraints.

$$G_j (Q_i) = 0 \qquad i = 1, 2, \text{-----}, p. \qquad (10)$$
$$j = 1, 2, \text{-----}, m.$$

These constraints will be represented by surfaces in the Q_i space. We want to find the absolute minimum of C in the region O Q_1 , $i = 1, 2, \text{-----}, p.$, subject to constraint (10). Thus, we are confronted with a non-linear program:

$$\text{Min (C)} \qquad Q_1 \quad O \qquad i = 1, 2, \text{----}, p.$$
$$G_j (Q_i) = 0 \qquad i = 1, 2, \text{----}, p., j = 1, 2, \text{----}, m.$$

First we will solve the problem ignoring the constraint, i.e., we minimize over each Q_i separately. This yields:

$$Q_i = \frac{2Y_i S_i}{I V_i} \qquad i = 1, 2, \text{----}, p.$$

If the Q_i given by this equation satisfy constraint (10), then these Q_i are optimal and the constraint, in such a case, is said to be inactive. On the other hand, if the Q_i given by the above equation do

not satisfy (10), then the constraint is active and these Q_i's are not optimal. To find the optimal Q_i, the Lagrange multiplier technique can be used.[6]

In cases where the constraints are stated in the form of in-equations, we must add to the corresponding "G" function an appropriate slack variable.

We form a new function:

$$J = C + \sum_{j=1}^{m} \theta_j G_j (Q_i)$$

$$= \sum_{i=1}^{p} \frac{Y_i S_i}{Q_i} + \frac{I}{2} \sum_{i=1}^{p} Q_i V_i + \sum_{j=1}^{m} \theta_j G_j (Q_i) \qquad (11)$$

where the parameters θ_j are called Lagrange multipliers. Then the set of Q_i, i = 1, 2, ----, p., which yield the absolute minimum of C subject to constraint (10) are solutions to the set of equations:

$$\frac{\partial J}{\partial Q_i} = - \frac{Y_i S_i}{Q_i^2} + \frac{I}{2} \cdot V_i + \theta_j \cdot G_j = 0 \qquad (12)$$

$$\frac{\partial J}{\partial \theta_j} = G_j (Q_i) = 0 \qquad (13)$$

where i = 1, 2, ----, p.

j = 1, 2, ----, m.

Equations (12) and (13) give the stationary points. Out of these, we

[6] G. Hadley and T. M. Whitin, Analysis of Inventory Systems, Prentice-Hall, Inc., New Jersey, pp. 433-437.

have to find the one that satisfies the constraints and renders "C" minimum. From equation (12) we get

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i + 2\theta_j^* \cdot G_j}} \tag{14}$$

where θ_j^* is the value of θ_j such that the Q_i^* of (14) satisfy the equation (13). The procedure of determining the conditions satisfied by the Q_i, $i = 1, 2, ----, p.$, that minimize C subject to the constraints $G_j(Q_i) = 0$, by forming the function "J" and setting the partial derivatives of "J" with respect to Q_i and θ_j equal to zero, is referred to as the method of Lagrange multipliers.

This approach, which requires computation of all the stationary points, becomes extremely difficult as the number of Q_i variables becomes very great and with each additional constraint which is imposed on the system. It must be admitted that there is no "step-by-step" method for non-linear programs which would permit finding the optimum by means of an algorithm of the kind used in the simplex method of linear programming.

Let us take a few examples to show the ways in which inventory problems with constraints may appear.

Case-1:

Where there is an upper limit to the total investment in inventory. Consider a company which produces and stocks five items. The management desires never to have an investment in inventory of more than $3,000. The items are produced in lots. The demand rate for each

item is constant and can be assumed to be deterministic. No back
orders are to be allowed. The pertinent data for the various products
are given in Table 5. The carrying charge on each item is I = 0.12.

Table 5

Item	1	2	3	4	5
Demand rate (units per year) Y_i	600	900	2,400	12,000	18,000
Variable cost (dollars per unit) V_i	3	10	5	5	1
Set-up cost per lot (dollars) S_i	10	10	10	10	10

Neglecting the constraint, the optimal lot sizes are given by:

$$Q_1 = \sqrt{\frac{2Y_i S_i}{IV_i}}$$

$$Q_1 = \sqrt{\frac{2 \times 600 \times 10}{0.12 \times 3}}$$

$$= 183 \text{ units}$$

$$Q_2 = \sqrt{\frac{2 \times 900 \times 10}{0.12 \times 10}}$$

$$= 123 \text{ units}$$

$$Q_3 = \sqrt{\frac{2 \times 2,400 \times 10}{0.12 \times 5}}$$

$$= 283 \text{ units}$$

$$Q_4 = \sqrt{\frac{2 \times 12,000 \times 10}{0.12 \times 5}}$$

$$= 632 \text{ units}$$

$$Q_5 = \sqrt{\frac{2 \times 18,000 \times 10}{0.12 \times 1}}$$

$$= 1,732 \text{ units}$$

Since the average inventory for each item is simply one-half the order size, the optimal average inventory investment will be:

$$C = \frac{1}{2}(183)(3) + \frac{1}{2}(123)(10) + \frac{1}{2}(283)(5) + \frac{1}{2}(632)(5) + \frac{1}{2}(1732)(1)$$

$$= \$4,043$$

But due to shortage of working capital, the company cannot afford to maintain the indicated optimal average inventory investment of $4,043. What then should be done? The cash limit prevents the use of the individual item optimal lot-size quantities. How should these be changed to minimize the total cost under this restriction? The restriction in this case can be written mathematically as

$$\frac{1}{2} \sum_{i=1}^{p} V_i Q_i \leq 3,000$$

We can use equation (14) for calculating the optimal Q_i's under this situation,

$$q_i^* = \sqrt{\frac{2Y_i S_i}{IV_i + 2 \, \theta_j^* \cdot G_j}}$$

By analogy we see that $G_j = V_i$, then

$$Q_i^* = \sqrt{\frac{2Y_i S_i}{V_i(I + 2 \theta^*)}} \qquad i = 1, 2, 3, 4, 5. \tag{15}$$

Since we are considering only one constraint, there will be only one Lagrange multiplier. Then, we have to find the value of θ^*:

$$\frac{1}{2} \cdot \sum_{i=1}^{5} \sqrt{\frac{2Y_i S_i}{V_i(I+2\theta^*)}} \cdot V_i = 3,000$$

$$\sum_{i=1}^{5} \sqrt{\frac{2Y_i S_i V_i}{I+2\theta^*}} = 6,000$$

Substituting the values, we have

$$\sqrt{\frac{2 \times 600 \times 10 \times 3}{0.12 + \theta^*}} + \sqrt{\frac{2 \times 900 \times 10 \times 10}{0.12 + \theta^*}} + \sqrt{\frac{2 \times 2,400 \times 10 \times 5}{0.12 + \theta^*}}$$

$$+ \sqrt{\frac{2 \times 12,000 \times 10 \times 5}{0.12 + \theta^*}} + \sqrt{\frac{2 \times 18,000 \times 10 \times 1}{0.12 + \theta^*}} = 6,000$$

$$\sqrt{0.06 + \theta^*} = 0.314621$$

or $\theta^* = 0.048837$

Then, the optimal Q_i are given by equation (14)

$$Q_1^* = \sqrt{\frac{2 \times 600 \times 10}{3(0.12 + 2 \times 0.048837)}}$$

= 136 units

$$Q_2^* = \sqrt{\frac{2 \times 900 \times 10}{10(0.12 + 2 \times 0.048837)}}$$

= 91 units

$$Q_3^* = \sqrt{\frac{2 \times 2,400 \times 10}{5(0.12 + 2 \times 0.048837)}}$$

= 210 units

$$Q_4^* = \sqrt{\frac{2 \times 12,000 \times 10}{5(0.12 + 2 \times 0.048837)}}$$

= 470 units

$$Q_5^* = \sqrt{\frac{2 \times 18,000 \times 10}{1(0.12 + 2 \times 0.048837)}}$$

= 1,286 units

The minimum cost of set-ups and holding inventory for the five items in the absence of any constraint on investment in inventory is

$$C = \sum_{i=1}^{5} (\frac{Y_i S_i}{Q_i}) + \frac{I Q_i V_i}{2}$$

$$= (\frac{600 \times 10}{183} + \frac{0.12 \times 183 \times 3}{2}) + (\frac{900 \times 10}{123} + \frac{0.12 \times 123 \times 10}{2})$$

$$+ (\frac{2,400 \times 10}{283} + \frac{0.12 \times 283 \times 5}{2}) + (\frac{12,000 \times 10}{632} + \frac{0.12 \times 632 \times 5}{2})$$

$$+ (\frac{18,000 \times 10}{1,732} + \frac{0.12 \times 1,732 \times 1}{2})$$

$$= \$970.00$$

The corresponding minimum cost in the presence of the constraint is

$$C = \sum_{i=1}^{5} (\frac{Y_i S_i}{Q_i^*} + \frac{I V_i Q_i^*}{2})$$

$$= (\frac{600 \times 10}{136} + \frac{0.12 \times 3 \times 136}{2}) + (\frac{900 \times 10}{91} + \frac{0.12 \times 10 \times 91}{2})$$

$$+ (\frac{2,400 \times 10}{210} + \frac{0.12 \times 5 \times 210}{2}) + (\frac{12,000 \times 10}{470} + \frac{0.12 \times 5 \times 470}{2})$$

$$+ (\frac{18,000 \times 10}{1,286} + \frac{0.12 \times 1 \times 1,286}{2})$$

$$= \$1,013.00$$

It will be seen that the total cost has gone up as compared to the optimal policy neglecting the constraint. However, for an increase in total cost of only $43.00 the company has accomplished its purpose of cutting its average inventory investment by $1,043. The program written to achieve this objective is given in Appendix C.

While inventory was limited in this example in terms of inventory value, identical treatment would be used on problems in which

available space, number of units, or any other linear function of lot sizes was limited.

Case-2:

Now we shall consider a case where there are two constraints imposed on the system. Let these constraints be on

1. Warehouse capacity

2. Availability of machine time

Warehouse space limitations, as we have seen in the previous case, are linear restrictions on the lot sizes. Another common restriction on lot sizes is the availability of machine time. In addition to the actual cost of set-up, a certain amount of time is required for set-ups during which production is stopped and it should be noted that this is a non-linear restriction.

Restriction on the warehouse capacity results in a reduction of lot sizes. Consequently set-ups are increased in number and set-up time requirements are thereby increased as smaller lots require more frequent set-ups than do larger lots, leaving less time for production. Similarly, if we want to reduce the number of set-ups, we will have to increase the lot sizes thereby increasing the space requirements. We now require that lot sizes change in such a way that warehouse space and set-up time requirements are both reduced.

Let F be the total available space.

f_i be the space occupied by the product i.

T_s be the total time available for set-ups.

t_{si} be the time required to set-up for product i.

Since the average inventory level of each product equals half the lot size, the average space required by the product "i" will be $\frac{1}{2} \cdot f_i \cdot Q_i$. Thus the total space requirement $= \frac{1}{2} \sum_{i=1}^{p} f_i \cdot Q_i$. The constraint can then be written as

$$\frac{1}{2} \sum_{i=1}^{p} f_i \cdot Q_i \leq F \tag{16}$$

Similarly, since the average number of set-ups for product $i = \frac{Y_i}{Q_i}$, we can write the constraint as

$$\sum_{i=1}^{p} \frac{t_{si} Y_i}{Q_i} \leq T_s \tag{17}$$

We know that the total variable cost is given by equation (4), that is:

$$C = \sum_{i=1}^{p} \frac{Y_i S_i}{Q_i} + \frac{1}{2} \sum_{i=1}^{p} Q_i V_i$$

We wish to find the minimum cost satisfying the inequations (16) and (17).

To find the optimal solution we shall make use of a special method suggested by Beckmann,[7] which is essentially an adaptation of the technique of Lagrangian multipliers. Let us introduce two parameters λ and μ such that:

$$\lambda < 0 \text{ for } F - \frac{1}{2} \sum f_i Q_i = 0 \tag{18}$$

[7]C. W. Churchman, R. L. Ackoff, and E. L. Arnoff, Introduction to Operations Research, John Wiley & Sons, Inc., New York, 1957, p. 261.

$$\lambda = 0 \text{ for } F - \tfrac{1}{2} \sum f_i Q_i > 0 \tag{19}$$

$$\mu < 0 \text{ when } T_s - \sum \frac{t_{si} Y_i}{Q_i} = 0 \tag{20}$$

$$\mu = 0 \text{ when } T_s - \sum \frac{t_{si} Y_i}{Q_i} > 0 \tag{21}$$

Since $F - \tfrac{1}{2} \sum f_i Q_i < 0$ and $T_s - \sum \frac{t_{si} Y_i}{Q_i} < 0$ is not admissible, it

not need be considered. Then, $\lambda (F - \tfrac{1}{2} \sum f_i Q_i)$ and $\mu (T_s - \sum \frac{t_{si} Y_i}{Q_i})$

are both identically equal to zero in the domain where the constraint
is satisfied and we can add it to "C" without changing its value in
this domain. This, then gives us:

$$C = \sum_{i=1}^{p} \frac{Y_i S_i}{Q_i} + \frac{I}{2} \sum_{i=1}^{p} Q_i V_i + \lambda (F - \tfrac{1}{2} \sum_{i=1}^{p} f_i Q_i)$$

$$+ \mu (T_s - \sum \frac{t_{si} Y_i}{Q_i}) \tag{22}$$

Taking the derivative of this equation with respect to Q_i, we obtain:

$$\frac{dC}{dQ_i} = - \frac{Y_i S_i}{Q_i^2} + \frac{IV_i}{2} - \frac{\lambda f_i}{2} + \mu \cdot \frac{t_{si} Y_i}{Q_i^2} \quad i = 1, 2, \text{----}, p. \tag{23}$$

Setting the derivative equal to zero and solving for the optimum Q_i^*, we

obtain:

$$Q_i^* = \sqrt{\frac{2Y_i (S_i - \mu t_{si})}{IV_i - \lambda f_i}} \quad i = 1, 2, \text{----}, p. \tag{24}$$

The method consists of computing Q_i for $\lambda = 0$ and $\mu = 0$, and

substituting the values thus found into (16) and (17). If the constraints are satisfied, we have the solution we are looking for; if it is not, we assign to $(-\lambda)$ and $(-\mu)$ increasingly large positive values, and tabulate the result. With these values, we construct the curves λ = constant and μ = constant. Next, we interpolate for the values of λ and μ that surround the first point of contact with the area in which the constraints are satisfied.

To illustrate the procedure mentioned above, we will consider a numerical example.

Let there be two products P_1 and P_2 for which the following data are given in Table 6.

Table 6

Product	Y_i (units)	V_i ($)	S_i ($)	f_i (cu. ft.)	t_i (hours per lot)
P_1	200	10,000	100,000	1.0	40
P_2	800	8,000	245,000	1.0	5

$I = 0.025$

The constraints are:

F = Total available space = 1,500 cubic feet.

T = Total available time = 20 hours/month.

The optimal lot sizes in the absence of constraints are given by

$$Q = \sqrt{\frac{2Y_i S_i}{I V_i}}$$

$$Q_1 = \sqrt{\frac{2 \times 200 \times 100,000}{0.025 \times 10,000}}$$

$$= 400 \text{ units}$$

$$Q_2 = \sqrt{\frac{2 \times 800 \times 245,000}{0.025 \times 8,000}}$$

$$= 1,400 \text{ units}$$

We can write the constraints in the following manner:

$$Q_1 + Q_2 \leq 1,500$$

$$\frac{(200)(40)}{Q_1} + \frac{(800)(5)}{Q_2} \leq 20$$

$$\text{or} \quad \frac{8,000}{Q_1} + \frac{4,000}{Q_2} \leq 20$$

Using the formula (16) to find the optimal lot sizes under the given conditions, we obtain:

$$Q_1^* = \sqrt{\frac{2(200)(100,000 - 40\mu)}{(0.025)(10,000) - \lambda}}$$

$$= 126.5 \sqrt{\frac{2,500 - \mu}{250 - \lambda}}$$

Similarly, $Q_2^* = \sqrt{\frac{2(800)(245,000 - 5\mu)}{(0.025)(8,000) - \lambda}}$

$$= 89.4 \sqrt{\frac{49,000 - \mu}{200 - \lambda}}$$

Using these equations we can calculate Q_1^* and Q_2^* for different values of μ and λ. These are shown in Table 7. These pairs of lot sizes are then plotted in Figure VI. It should be noted that the set-up time restriction $\frac{8,000}{Q_1} + \frac{4,000}{Q_2} = 20$ represents a Hyperbola. We see that the

Table 7

$-\mu \backslash -\lambda$	0	50	100	150	200	250	300	350	400	450
0	400 / 1400	365 / 1250	338 / 1142	315 / 1058	298 / 970	283 / 933	270 / 884	258 / 843	248 / 808	239 / 776
1,000	474 / 1413	432 / 1265	400 / 1155	374 / 1070	353 / 1000	335 / 942	319 / 894	305 / 852	294 / 816	283 / 784
2,000	537 / 1428	490 / 1276	454 / 1167	424 / 1080	400 / 1009	379 / 952	362 / 903	346 / 860	333 / 824	321 / 792
3,000	593 / 1440	542 / 1290	502 / 1176	469 / 1089	442 / 1020	419 / 960	400 / 911	383 / 870	367 / 833	355 / 799
4,000	645 / 1455	590 / 1304	545 / 1188	510 / 1101	480 / 1029	456 / 971	435 / 920	416 / 877	400 / 840	385 / 806
5,000	693 / 1470	632 / 1315	585 / 1200	546 / 1110	516 / 1039	490 / 980	467 / 928	447 / 885	426 / 847	414 / 815
6,000	738 / 1484	673 / 1327	624 / 1211	583 / 1120	550 / 1048	522 / 990	497 / 938	477 / 894	457 / 856	441 / 822
7,000	780 / 1496	711 / 1340	658 / 1222	616 / 1131	582 / 1057	552 / 997	526 / 946	504 / 902	484 / 864	466 / 829
8,000	820 / 1508	748 / 1350	684 / 1230	649 / 1140	611 / 1067	580 / 1003	553 / 955	529 / 910	508 / 870	491 / 836

Figure VI. Pairs of lot sizes that satisfy warehouse space and set-up time restrictions

solution $Q_1 = 400$ and $Q_2 = 1,400$ does not satisfy the restrictions and therefore the point P representing these lot sizes lies outside the shaded area where both the conditions are satisfied. Thus, the shaded area contains the pairs of lot sizes which satisfy both restrictions. We can see from Table 7 that we cannot get a point in the shaded area if $\lambda = 0$ and $\mu = 0$. Hence both must be negative, and so

$$T_s - \sum \frac{t_{si} Y_i}{Q_i} = 0$$

$$F - \frac{1}{2} \sum f_i Q_i = 0$$

Now, we construct the curves $\lambda = $ constant and $\mu = $ constant. Next, we interpolate for the values of λ and μ that surround the first point of contact with the area in which the conditions are satisfied. From

Figure VI, we may observe that this intersection takes place at about $Q_1 = 500$, $Q_2 = 1,000$.

Checking our results, we see that these lot sizes require $(1)(500) + (1)(1,000) = 1,500$ cubic feet of warehouse space. Furthermore, the set-up time required is

$$\frac{8,000}{500} + \frac{4,000}{1,000} = 16 + 4 = 20 \text{ hours}$$

Hence, the lot sizes that minimize cost and still satisfy the restrictions on available warehouse space and set-up time are

$$Q_1^* = 500$$

$$Q_2^* = 1,000$$

From these values of Q_1^* and Q_2^*, we can calculate the implicit values of λ and μ

$$500 = 126.5 \sqrt{\frac{2,500 - \mu}{250 - \lambda}}$$

and $1,000 = 89.4 \sqrt{\frac{49,000 - \mu}{200 - \lambda}}$

Solving these two equations for λ and μ, we have

$$\lambda = -231.5$$

$$\mu = -4937.5$$

It may be seen in Figure VI that the values of λ and μ lie between -200 and -250, and $-4,000$ and $-5,000$ respectively. These calculated values lie within those limits.

A computer program for this type of problem is given in Appendix D.

CHAPTER V

COMPUTING METHODS IN INVENTORY

For the first time, Industrial Engineers have a tool, the computer. This tool will allow them to fulfill their charter, which was established by such men as Taylor and Gilbreth, to aid management in exerting efficient control over men, materials, and machines.[1]

During the last 13 years, the computer has become one of the most accepted tools of modern industry. The adoption by industry of this device has resulted in tremendous changes in the traditional practice of business management. Its ability to accept and manipulate data at electronic speeds has made it possible to create large scale information systems, which not only replace clerical labor, but also provide the information for management decision making. For instance, the United States Steel Company Magazine of January, 1958 reported that data processing equipment saved the Corporation an estimated 100,000 engineering man-hours during 1957 alone and looked hopefully to increased savings in the future. Some of the advantages for applying computers to engineering calculations are:[2]

1. Information can be obtained faster, hence earlier.

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[1]Richard L. Smith, Impact of Computers on the Practice of Industrial Engineering, The Journal of Industrial Engineers, Vol. 15, no. 5, September-October 1964, pp. 277-280.

[2]William A. Smith, Engineers and Computists, The Journal of Industrial Engineers, Vol. 12, no. 2, March-April 1961, pp. 94-96.

2. More useful information can be analyzed or assembled.

3. Mathematical models can be more accurate.

4. Alternative solutions can be more readily achieved.

5. Standardization of approach can be achieved.

6. Reliability of results is greater than for manual methods.

7. Better methods are available to inexperienced personnel.

8. Drudgery of engineering work can be lightened.

Development of Information Systems

Computers are not in themselves an information system. Rather, the computer is but one important component in a system that generally contains human as well as electronic and mechanical components. When the right configuration of these components interacts to handle the information flow of an organization, an information system is formed. The task of bringing these components together should be the prime task of the Industrial Engineer.

An approach, which industry is finding useful, required to develop such systems embodies a concept of "total systems." The philosophy of the total systems concept has been stated by Peter Drucker:[3]

The whole of a system is not necessarily improved if one particular function or part is improved or made more efficient. In fact, the system may well be damaged, thereby, or even destroyed. In some cases, the best way to strengthen the system may be to weaken a part; for what matters in all systems is the performance of the whole.

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[3]Peter Drucker, Thinking Ahead, Harvard Business Review, Vol. 33, no. 1, January-February 1959, p. 67.

The relationship and value of this concept to an industrial organization are apparent. Any complex organization is a system of many functions, all acting together in their respective ways to carry out the purpose of the whole. The separate functions of an organization, such as marketing, manufacturing, and engineering, are all recognizable individually and as a part of the organization. Yet, it is only when the relationship of each individual part to all of the others has been defined that their true role may properly be assessed in terms of the purpose of the total organization. In summary, the total system concept is essentially a philosophy and requires strict enforcement in order to be applied effectively. In the design of an information system, the total systems approach embodies the following three steps:

1. System requirements.

2. System design specifications

3. System implementation.

Although these steps could be applied independently to any subfunction within a business, maximum benefits are derived when they are applied to the business as a whole within the total system concept.

The "systems requirements" phase of the total systems approach is primarily concerned with the formulation of the problem and consists of the following:

1. Decision criteria.

2. The objectives of the system.

3. A definition of the system requirements relating to the system components, which are:

a. Management

b. Men

c. Materials

d. Machines

e. Operating environment

The formulation of systems requirements is the most important step of the total systems approach. Incorrect solutions, no solutions, or at least inefficiencies in finding a solution will occur unless the system components stated above are described accurately. The establishment of systems requirements is a time-consuming task and requires constantly directed attention to the needs of the total system. "Systems design" involves three substeps in the total systems approach:

1. Hypothesis of solutions.

2. Evaluation of solutions.

3. Selection and detailed specification of the best solution.

Each of the steps of the systems-design phase relies on the availability and accuracy of the systems requirements information obtained in phase one. "Systems implementation" relates to two often-overlooked substeps in the total systems approach, namely:

1. Implementation of the best solution.

2. Monitoring of the implemented solution.

In this phase, theory is checked against practical considerations, and modifications are made to correct inconsistencies in the implemented system. Monitoring of the implemented system is also important because systems requirements change with time and the system must be

modified accordingly.

Mechanizing Production and Inventory

The capabilities of electronic data-processing equipment can often be utilized to perform major portions of the clerical work involved in a production and inventory control system. This is not to say that every production and inventory control system needs to be mechanized to be effective or that every production and inventory control system will be effective if it is mechanized. Nor is it intended to intimate that punched-card and electronic equipment is the only effective means of mechanizing production and inventory control functions. There are other devices and techniques like bookkeeping machines, duplicating techniques, and Gantt-type charts, which have their own attractions.

Why should companies mechanize their production and inventory control systems? What are the key objectives they can accomplish more easily with punched-card or electronic equipment? Some of the important objectives are as follows:

1. Mathematical abilities of the machines can be utilized to handle the extremely large number of additions, subtractions, and multiplications required to control inventories and production.

2. A mechanized system can print out information which it retains or has produced in mechanical form in order to prepare various lists, reports, instructions, requisitions, and purchase orders which are needed.

3. Machines can perform certain kinds of "thinking operations":

a. They can detect automatically certain situations requiring attention and report only on these.

b. They can be made to act more precisely (sometimes with the help of the mathematical techniques of operations research) by giving consideration to a number of factors which can only be treated generally when computers are not available.

4. The clerical work of a production and inventory control system can be integrated more closely with the clerical work involved in related areas like purchasing, production reporting and incentive payroll, order entry and so on, to make them part of a continuous clerical process.

5. A mechanized system can be used to shorten the time required to alter production patterns as sales requirements change.

The production and inventory control system in use in a specific company must be geared to the needs of that company if it is to be successful. Because sales patterns and production facilities vary so greatly, there is, probably, no such thing as a "typical" system. However, a mechanized system would perform all the following major functions and perhaps more:

1. Maintenance of inventory records.

2. Determination of gross requirements for purchased and manufactured items.

3. Determination of net requirements for purchased and manufactured items.

4. Development of manufacturing schedules for plants, departments, shops, and machines.

A master card or tape file is prepared to show the parts requirements for each product. This information is worked down through subassemblies into the basic purchased and manufactured parts. It is updated as a result of engineering and specification changes. The master file will usually contain the following:

1. Part number (or assembly number).

2. Part name.

3. Pieces required.

4. Source—whether purchased or manufactured.

5. Lead time.

6. Supplier name.

7. Department manufacturing and time allowance.

8. Raw material required to be purchased and quantity needed.

9. Location in stockroom.

The mathematical computation involved in determining gross requirements can be extremely numerous. The increasing speed of the more highly powered computing equipment can, therefore, shorten the time elapsing between release of the final schedule and determination of requirements. In arriving at net production or purchase requirements for parts, several additional factors should be given consideration, such as:

1. Quantities on hand.

2. Quantities on order (for purchase or manufacture).

3. Desired minimum inventory.

4. Economic order quantity.

Matching the gross requirements for parts with the amounts on hand and
on order and the other factors relating to availability will make it
possible for a net-requirements schedule to be produced. This can be
done mechanically if the data concerning inventory on hand, outstanding
orders, etc., are maintained in mechanical form. The machine system
will then produce not only the schedules of net requirements but also
many of the documents necessary to take action:

1. Manufacturing orders.

2. Requisitions to purchase.

3. Schedules of revised delivery dates.

The mechanized system will also compute the raw material re-
quirements, gross and net, along the same procedural lines. Figures
VII - X[4] show how an electronic system might handle some of the major
production and inventory control operations.

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[4] E. D. Lucas, <u>Automatic Production Inventory Control</u>, Control
Engineering, Vol. 2, no. 9, September 1955, pp. 68-73.
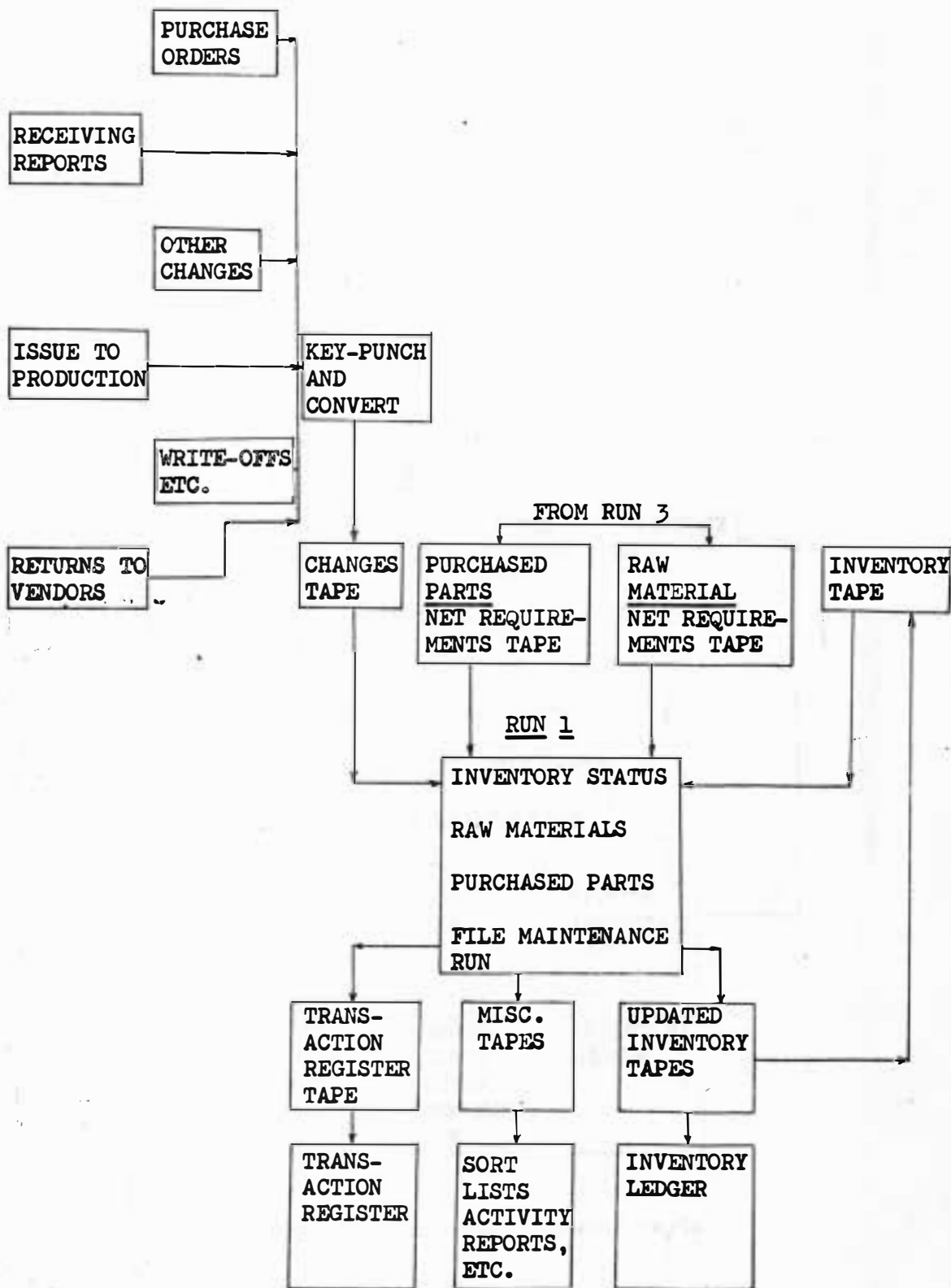
FIGURE VII.   Inventory Control--Raw Materials and Purchased Parts
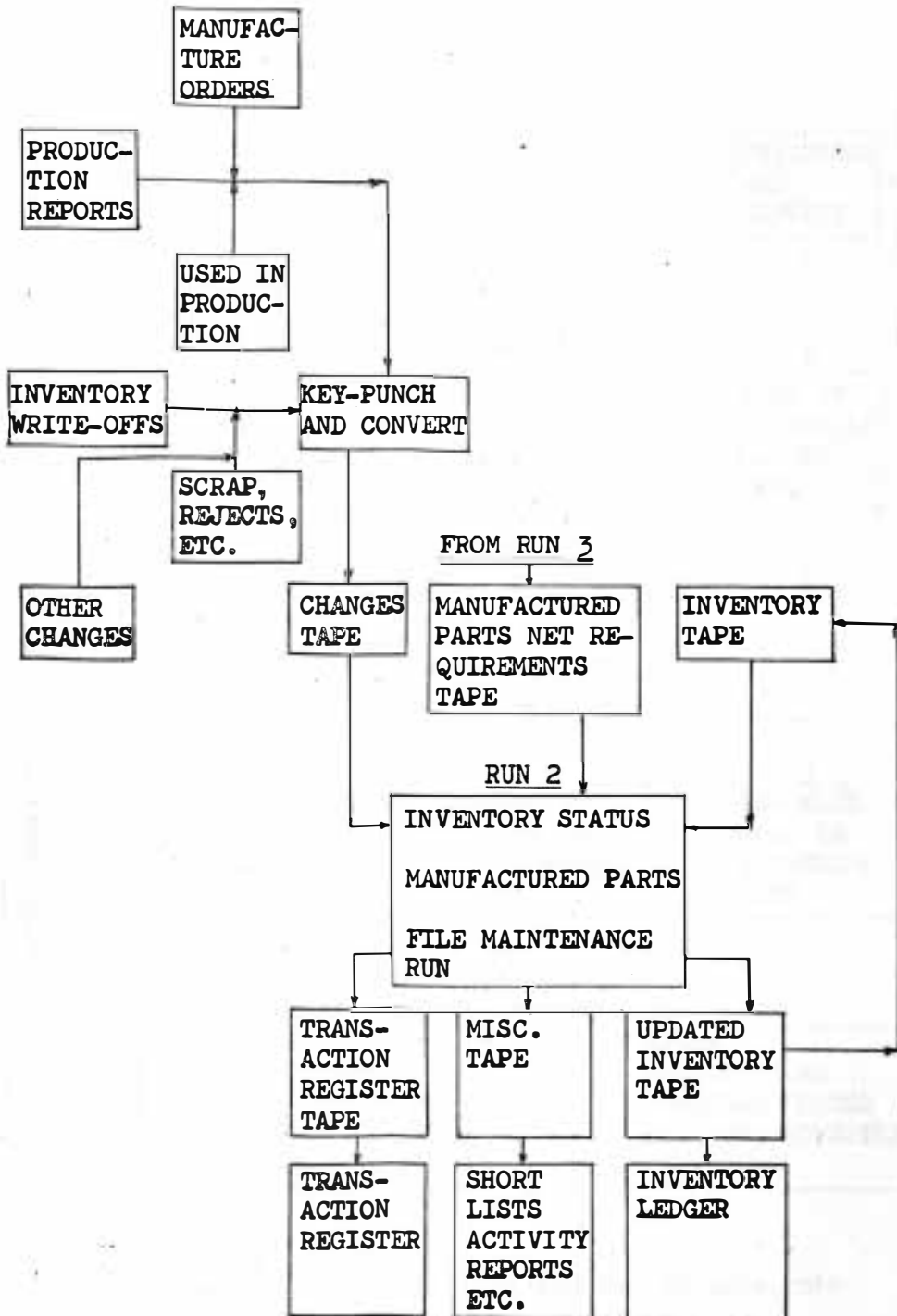
FIGURE VIII.   Inventory Control--Manufactured Parts

SALES FORECAST

FINISHED GOODS INVENTORY RECORD

PRODUCTION SCHEDULE

KEY-PUNCH AND CONVERT

BILL OF MATERIALS

KEY-PUNCH AND CONVERT

NEW PRODUCTS AND ENGINEERING CHANGES

PRODUCTION SCHEDULE TAPE

FROM RUNS 1 & 2

UPDATED INVENTORY TAPE

BILL OF MATERIALS MASTER TAPE

RUN 3

DETERMINATION OF GROSS AND NET REQUIREMENTS

PURCHASED PARTS REQUIREMENT TAPE

MANUFAC-TURED PARTS REQUIREMENT TAPE

RAW MATE-RIAL RE-QUIREMENT TAPE

TO RUN 2

LISTS REQUISITIONS PURCHASE ORDERS DELIVERY SCHEDULE REVISIONS

LISTS REQUISITIONS PURCHASE ORDERS DELIVERY SCHEDULE REVISIONS

FIGURE IX. Determination of Gross and Net Requirements

RECORDED ON ANY OR ALL
OF THESE INPUT DEVICES

OUTPUT
FORMS

COMPUTER CONTINUALLY
REVISES MASTER FILES
AND ISSUES REPORTS

```
┌──────────────┐        ┌──────────────┐                                    ┌──────────┐  ┌──────────┐   ┌──────────────┐
│ ISSUES OR    │        │ ELECTRIC     │                                    │ PAPER    │──│ ELECTRIC │   │ PARTS &      │
│ SHIPPING     │        │ TYPEWRITER   │        ┌──────────────┐            │ TAPE     │  │ TYPE-    │   │ MATERIAL     │
│ REPORTS      │        └──────────────┘        │ HIGH SPEED   │            │ PUNCH    │──│ WRITERS  │   │ REQUISI-     │
├──────────────┤                                │              │            └──────────┘  └──────────┘   │ TIONS        │
│ RECEIPTS OR  │        ┌──────────────┐        │ GENERAL PURPOSE           ┌──────────┐  ┌──────────┐   ├──────────────┤
│ REPORTS FROM │        │ PUNCHED PAPER│        │              │            │ MAG-     │──│ LINE     │   │ PURCHASE     │
│ INCOMING     │        │ TAPE         │        │ DIGITAL      │            │ NETIC    │  │ PRINTERS │   │ REQUISI-     │
│ INSPECTION   │        └──────────────┘        │              │            │ TAPE     │──│          │   │ TIONS        │
├──────────────┤                                │ COMPUTER     │            └──────────┘  └──────────┘   ├──────────────┤
│ ORDERS       │        ┌──────────────┐        │              │                                         │ EXPEDIT-     │
│ FROM         │        │ PUNCHED      │        └──────────────┘                                         │ ING IN-      │
│ CUSTOMERS    │        │ BUSINESS     │                                                                  │ STRUC-       │
├──────────────┤        │ CARDS        │                                    ┌──────────┐                 │ TIONS        │
│ INQUIRIES    │        └──────────────┘                                    │ HIGH     │                 ├──────────────┤
│ FROM         │                                                            │ SPEED    │                 │ PRODUC-      │
│ CUSTOMERS    │        ┌──────────────┐                                    │ PRINTERS │                 │ TION         │
├──────────────┤        │ MAGNETIC     │                                    └──────────┘                 │ ORDERS       │
│ INQUIRIES    │        │ TAPE         │                                                                  ├──────────────┤
│ FROM         │        └──────────────┘                                    ┌──────────┐                 │ SHIPPING     │
│ SUPPLIERS    │                                                            │ PUNCHED  │                 │ ORDERS       │
├──────────────┤                                                            │ CARDS    │                 ├──────────────┤
│ INQUIRIES    │                                                            └──────────┘                 │ ACCOUNT-     │
│ FROM         │                                                                                         │ ING &        │
│ MANAGEMENT   │                                                                                         │ STATIS-      │
└──────────────┘                                                                                         │ TICAL        │
                       ┌──────────┐ ┌──────────┐ ┌──────────┐ ┌──────────┐                               │ DATA         │
                       │ PARTS    │ │ MATERIALS│ │PURCHASING│ │PRODUCTION│                               ├──────────────┤
                       │ INVENTORY│ │ INVENTORY│ │ SCHEDULE │ │ SCHEDULE │                               │ ANSWERS      │
                       └──────────┘ └──────────┘ └──────────┘ └──────────┘                               │ TO IN-       │
                                                                                                         │ QUIRIES      │
                                                                                                         └──────────────┘
```

MASTER FILES ON MAGNETIC TAPE MACHINES OR
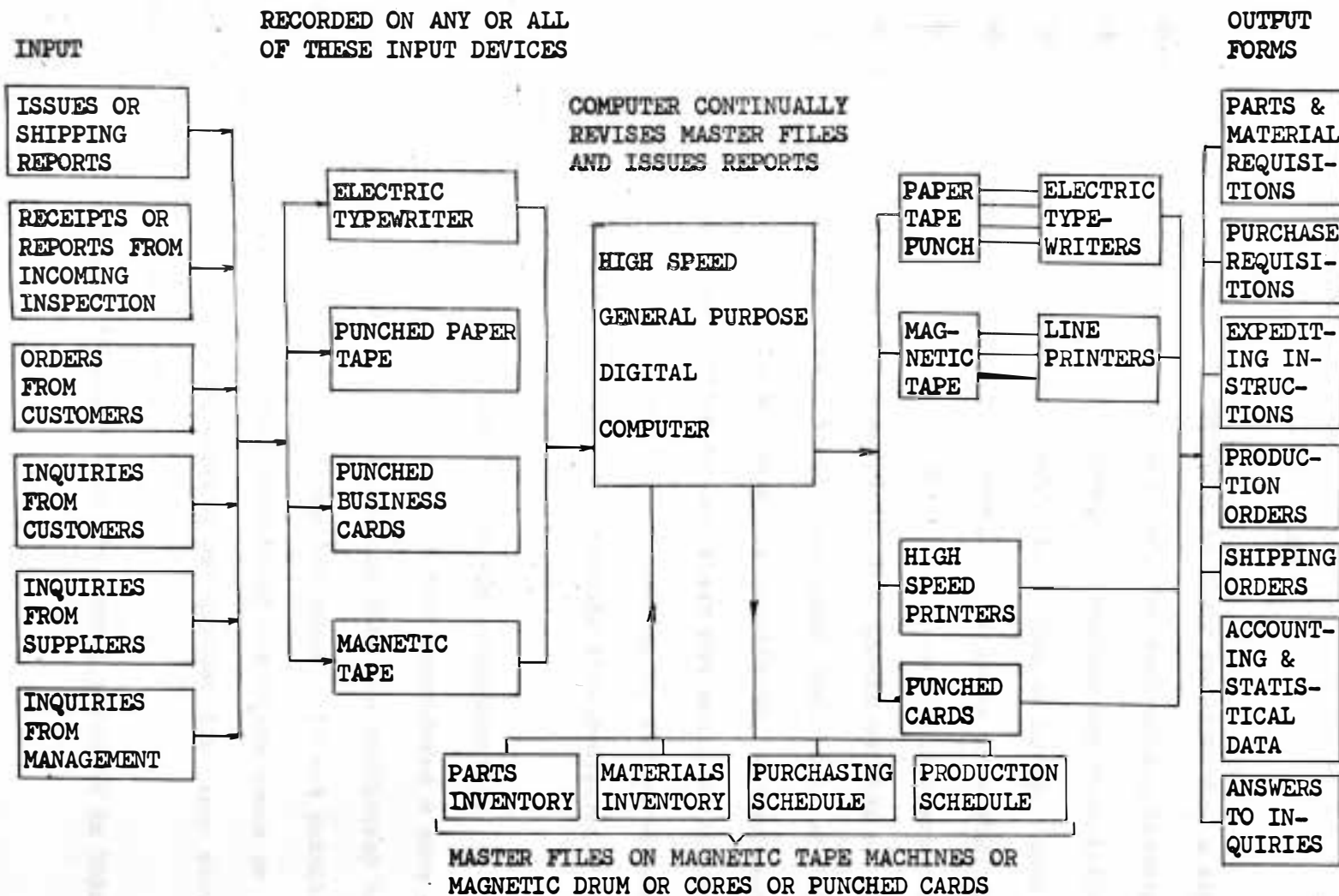MAGNETIC DRUM OR CORES OR PUNCHED CARDS

FIGURE X.   General Production and Inventory Control System

CHAPTER VI

CONCLUSIONS

In order to make the approach as clear as possible, a simple mathematical model was considered first, for controlling inventory in situations where it was not necessary to consider any restrictions on production facilities, storage facilities, time or money. When such restrictions are introduced in situations involving more than one product, it is necessary to allocate the limited available resources among the products. The method suggested in this thesis enables us to determine how much of each item to produce under the specified restrictions. From the results of the examples considered, we may summarize the effects of the restrictions on lot sizes and costs as follows:

1. Compared to the unrestricted condition, the warehouse restriction lowers lot sizes, while the machine time restriction raises lot sizes.

2. Each restriction increases costs independently.

It is interesting to note that we have considered a case where neither warehouse capacity nor the machine time was sufficient to permit us to use the unrestricted optimum lot sizes. It was possible to find a solution without acquiring additional warehouse space or machinery. The work should be carried out further for cases where the number of restrictions is more.

In conclusion it is hoped that the method proposed in this

thesis will contribute to the understanding and practical solution of production and inventory problems.

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APPENDIX A

LIST OF SYMBOLS

Y_i = demand for product i (units)

S_i = set-up cost per lot for product i ($1 lot)

V_i = cost per unit of product i ($1 unit)

I = inventory carrying factor (expressed as a percentage of total inventory investment)

Q_i = lot size for product i

Q_i^* = minimum-cost production order quantity for product i

c_i = total variable cost for product i

C = total variable cost for all products

p = number of products

θ_j = Lagrange multipliers

j = number of constraints

F = total available space

f_i = space occupied by the product i

T_s = total time available for set-up

T_q = lot time

t_{si} = time required to set-up for product i

t_l = lead time

D_l = lead time demand

r_i = re-order point

APPENDIX B

```
1   FORMAT (1H0,16X,8HLOT SIZE,6X,22HTOTAL VARIABLE COST ($) )

2   FORMAT (1H0,13X,F8.1,12X,F10.2)

3   FORMAT(1H0,5X,13HOPT LOT SIZE=,I5,18HMIN TOT VAR COST=$,F10.2)
    BIGNR=999999.9
    DPY=1500.0
    STC=8.33
    UHC=0.02
    CPU=5.0
    PDR=12000.0
    PUNCH1
    DO 4 IQ=1,9999
    Q=IQ
    TVC=DPY*STC/Q+UHC*CPU*Q/2.0*(1.0-DPY/PDR)
    PUNCH 2,Q,TVC
    IF(TVC-BIGNR)4,5,5
4   BIGNR=TVC
5   IQ=IQ-1
    PUNCH3,IQ,BIGNR
    DO 6 I=1,10
    Q=Q+1.
    TVC=DPY*STC/Q+UHC*CPU*Q/2.0*(1.0-DPY/PDR)
6   PUNCH 2,Q,TVC
    END
OPT LOT SIZE = 534      TOT VAR COST = 346.76
END OF PROGRAM AT STATEMENT 0006 + 00 LINES
```

APPENDIX C

```
1 FORMAT(3HL 1,3X,3HL 2,3X,3HL 3,3X,3HL 4,3X,3HL 5,3X,6HCOST $)

2   FORMAT(5F6.0)

3   FORMAT(8HTHETA = ,F10.8)

4   FORMAT(F5.0,3X,F5.0,3X,F5.0,3X,F5.0,2X,F6.0,3X,F10.2)

    DIMENSION DPY(5),CPU(5),STC(5),Q(5)

    READ 2,DPY(1),DPY(2),DPY(3),DPY(4),DPY(5)

    READ 2,CPU(1),CPU(2),CPU(3),CPU(4),CPU(5)

    READ 2,STC(1),STC(2),STC(3),STC(4),STC(5)

    UHC=0.12

    TS=0.

    DO 10 I=1,5

10  TS=TS+SQRT(2.*DPY(I)*CPU(I)*STC(I) )

    TH=(TS*TS-36.E6*UHC)/(2.*36.E6)

    PUNCH 3,TH

    DO 11 I=1,5

11  Q(I)=SQRT(2.*DPY(I)*STC(I)/(CPU(I)*(UHC+2.*TH)))

    C=0.

    DO 12 I=1,5

12  C=C+DPY(I)*STC(I)/Q(I)+UHC*CPU(I)*Q(I)/2.

    PUNCH 1

    PUNCH 4,Q(1),Q(2),Q(3),Q(4),Q(5),C

    END
```

```
600.      900.      2400.     12000.    18000.
3.        10.       5.        5.        1.
10.       10.       10.       10.       10.
```

THETA = .04883786

L 1	L 2	L 3	L 4	L 5	Cost $
136.	91.	210.	470.	1286.	1013.03

END OF PROGRAM AT STATEMENT 0012 + 02 LINES

APPENDIX D

```
1   FORMAT(7H-LAMBDA,6X,3H-MU,5X,5HLOT 1,5X,5HLOT 2,3X,10HVAR COST *)

2   FORMAT(F6.0,3X,F6.0,2F10.1,3X,F10.2)
    UHC=.025
    UHC1=.025
    UHC2=.025
    DPY1=200.
    DPY2=800.
    CPU1=10000.
    CPU2=8000.
    STC1=100000.
    STC2=245000.
    F1=1.
    F2=1.
    TMC1=40.
    TMC2=5.
    PUNCH1
      DO 10 I=1,19
      X=I*25-25
      DO 7 J=1,17
      Y=J*500-500
    Q1=SQRT((2.*DPY1*(STC1+Y*TMC1))/(UHC1*CPU1+X*F1))
    Q2=SQRT((2.*DPY2*(STC2+Y*TMC2))/(UHC2*CPU2+X*F2))
    IF(Q1+Q2-1500.)5,5,7
5   IF(8000./Q1+4000./Q2-20.)9,9,7
```

```
9   TVC=DPY1*STC1/Q1+UHC*Q1/2.*CPU1+DPY2*STC2/Q2+UHC*Q2/2.*CPU2
    PUNCH 2,X,Y,Q1,Q2,TVC
7   CONTINUE
10  CONTINUE
    END
```

C   C   MARCH 17,1966

-LAMBDA	-MU	LOT 1	LOT 2	VAR COST *
250.	5500.	506.0	984.3	400328.12
275.	6000.	509.0	962.5	402807.74
275.	6500.	523.7	966.8	403062.15
300.	6500.	511.7	942.3	405274.18
300.	7000.	525.7	946.6	405477.22
300.	7500.	539.4	950.8	405724.42
325.	7000.	514.1	923.8	407720.09
325.	7500.	527.5	927.9	407875.27
325.	8000.	540.5	932.0	408071.01
350.	7500.	516.4	906.5	410140.00
350.	8000.	529.2	910.5	410250.53
375.	8000.	518.5	890.5	412529.94

END OF PROGRAM AT STATEMENT 0010 + 00 LINES