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An Amplitude Modulated Radio Telemetry System for the Measurement of Physiological Temperatures and a Revised Analysis of R-C Phase-shift Transistor Oscillators

Carl Jerl Furchner

AN AMPLITUDE MODULATED RADIO TELEMETRY SYSTEM FOR THE MEASUREMENT OF PHYSIOLOGICAL TEMPERATURES AND A REVISED ANALYSIS OF R-C PHASE-SHIFT TRANSISTOR OSCILLATORS

BY

CARL JERL FURCHNER

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This thesis is approved as a creditable, independent investigation by a candidate for the degree, Master of Science, and acceptable as meeting the thesis requirements for this degree; but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Advisor

Head of the Major Department

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INTRODUCTION

In recent years, electronic instrumentation has been employed as a means of measuring physiological phenomena. One use of this instrumentation is the location of small radio transmitters within animals. In this case, the phenomena is sensed by the transducer associated with the transmitter. The information is then transmitted by electromagnetic waves to radio receivers outside the animal. During the transmission of the data, the animal is free in its usual environment and is not subject to restraint, excitement, or anesthetic.

The most common method of transmitting temperature data is by varying the carrier frequency of a small temperature sensitive oscillator located within the animal and recording the change of this frequency at the receiver. The main disadvantages of this system are: (1) drift in the carrier frequency from the original calibration; (2) complexity of receiving equipment; and (3) complexity in interpreting the received data. Reduction of the cited problems was believed to be possible by developing an amplitude modulated transmitting unit in which a temperature controlled audio oscillator would be used to amplitude modulate the carrier frequency oscillator. The variation in the audio frequency could then be recorded at the demodulated output of the receiver. The audio oscillator selected was an R-C phase shift transistor oscillator with three R-C combinations in the feedback network. The temperature transducer consisted of two thermistors which were used as resistive elements in the feedback network. The carrier frequency was generated by a separate transistor Colpitts oscillator.

Upon examination, the existing theory of single transistor R-C phase-shift oscillators was found to be limited by the assumptions of previous investigators. Modification of this theory resulted in a circuit that did not require such a high value of transistor h_{21} as the previous circuits. Equations for the prediction of frequency and required transistor current gain of this circuit were derived. This modification also made possible the operation of a single transistor R-C phase-shift oscillator using only two R-C sections in the feedback network.

REVIEW OF LITERATURE

Introduction

This study is concerned with the investigation of two separate areas which are involved in a temperature telemetry system. One area is the use of electronic instruments to measure and transmit the temperature of unrestrained animals. The other area concerns the theory and application of the transistor R-C phase-shift oscillator as an important link in a certain type of temperature telemetry system. Since the two areas are normally unrelated, the review of literature for each will be considered separately. Although the review is not complete, it is believed to contain the most important developments in both areas.

Temperature Telemetry of Unrestrained Animals

In June 1957, Jacobson and Mackay reported the use of a small radio transmitter that could be swallowed by humans.[1] During its passage through the gastro-intestinal tract the unit would transmit information from its particular location concerning the existing temperature and pressure. The unit, containing its own battery, was about three centimeters long. It operated at a frequency of about 400 kilocycles and radiated energy in the form of a changing magnetic field from the small oscillator coil within the capsule. The carrier frequency was varied by pressure changes which caused a movable iron core to change the inductance of the oscillator coil. The circuit was also biased to operate as a blocking oscillator, the frequency of which varied as the temperature

[1] B. Jacobson and R. Mackay, "Endoradiosonde," Nature, vol. 179, 1239-1240.

of the transistor changed. Thus, the transistor itself was used as the temperature transducer.

The temperature of incubating penquin eggs was investigated in March 1959 by Ekland and Charlton by using a transmitter inside an egg shell.[2] The temperature was transmitted by the pulse rate of the signal. The life of the unit was about 100 hours, and the receiving antenna was located about six feet from the egg. The pulse rate was measured by a counter at the receiver.

In February 1960, Busser proposed an implantable temperature telemeter which would have an external power supply.[3] The animal in which the unit was located would be surrounded by a low frequency power field. The unit would pick up some of the energy from the external field and use it to operate its own circuit to transmit information out of the animal. One of the proposed methods of using the external energy was to rectify the alternating current received and use it to charge a battery. The information would be transmitted by varying the carrier frequency with a temperature sensitive capacitor. The unit was to use a hollow ferrite core for the oscillator coil with the remaining parts being placed inside the core. The degree of success obtained with this proposed device has not yet been reported.

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[2] C. R. Ekland and F. E. Charlton, "Measuring the Temperature of Incubating Penquin Eggs," American Scientist, vol. 47, 80-86.

[3] J. H. Busser, "Implantable Temperature Telemeter for Laboratory Animals Having an Infinite Life Power Supply," Fourth Annual Meeting of the Biophysical Society, 1960.

A temperature telemetry unit of very long life was reported in 1960 by Essler.[4] This unit had a battery current drain of only 14 microamperes, which gave the unit a life of about one year of continuous transmission before it was necessary to replace the battery. This was by far the longest life yet obtained from an implantable unit which derived its power source entirely from its own batteries. The unit measured about two inches in length, and it transmitted information by variation of the carrier oscillator frequency with temperature. The signal was received by a loop surrounding the animal. The maximum diameter of the loop was about 10 feet. The temperature was sensed by a thermistor along with the temperature sensitivity of the transistor. The thermistor was connected in a half-bridge arrangement with a resistor. When the resistance of the thermistor changed, it changed the voltage applied to a back biased diode capacitor, which was across the oscillator coil. This arrangement caused a frequency shift of 1.5 kilocycles/degree centigrade for the temperature variations within the desired range of measurement. Thus, accuracy within about .1°C could be obtained. One of the difficulties encountered with this unit was the tendency of the oscillator to drift from its original calibration. The unit was succesafully used in dogs and cats to measure the changes in their temperature during a 24-hour period.

The successful operation of an implantable temperature telemetry

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[4] W. O. Essler, "Determination of Physiological Rhythms of Unrestrained Animals by Radio Telemetry," *Nature*, vol. 190, 90-91.

unit was reported in January, 1961, by England and Pasamanick.[5] This unit was about an inch long and had a battery current drain of 200 microamperes. The information was transmitted by varying the carrier frequency of the oscillator with the temperature.

R-C Phase-Shift Oscillators

The first significant report on a phase-shift oscillator using one tube was given by Ginzton and Hollingsworth in 1941.[6] A three or more R-C section phase-shifting network was connected between the output and input of an amplifier tube. Experimental circuits showed that changes in tube plate resistance or supply voltage had only a small effect on changing the frequency of oscillation. Typical designs which used this circuit as a variable frequency oscillator were discussed. It was found that when any of the phase-shift network elements were fixed while the others varied, the necessary amplification changed. Another tube was added to serve as an automatic volume control so that oscillation was barely maintained to give a pure sine wave. It was shown that the minimum amplification required to maintain oscillation was much lower for the four section R-C network than for the three section R-C network.

An article was published by Bacon in 1954 that showed a method in which there would be less attenuation in the phase-shift network

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[5] S. J. England and B. Pasamanick, "Radiotelemetry of Physiological Responses in the Laboratory Animal," Science, vol. 133, 106-107.

[6] E. L. Ginzton and L. M. Hollingsworth, "Phase-Shift Oscillators," Proceedings of the Institute of Radio Engineers, vol. 29, 43-48.

if identical R-C combinations were used.[7] His R-C network was used in a vacuum tube R-C phase-shift oscillator. If the value of the network terminating elements were either doubled or made half the value of the other network elements, depending upon the network configuration, the required amplifier gain would be less. This was essentially the addition of terminating sections to a ladder network where the phase-shift network was treated like a ladder.

A method of varying frequency with a change in voltage by using an R-C vacuum tube oscillator was reported by Uno in 1959.[8] This circuit used a completely different and more complex R-C arrangement than had previously been employed. Varistors were used as the resistors in the R-C network and the oscillation frequency was changed by varying the D.C. voltage impressed on the varistors.

In February, 1958, Garmash reported the successful operation of a single transistor R-C phase-shift oscillator using a four section R-C feedback network.[9] He emphasized the use of the feedback network as a current phasing device rather than the voltage phasing device of vacuum tube circuits. The shunting effect of the low input impedance of the transistor to the feedback network was shown to decrease attenuation in

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[7] W. Bacon, "Single-Stage Phase-Shift Oscillator," Wireless Engineer, vol. 31, 100-104, (England).

[8] M. Uno, "C-R Oscillator using Non Linear Resistors," Institute of Electrical Communications Engineers, vol. 42, 155-159, (Japan).

[9] E. N. Garmash, "R-C Oscillator with Single Junction Transistor," Telecommunications, vol. 9, 933-938, (English translation of Elektrosvyaz, Russia).

a voltage phasing network.

An analysis of the effect of the constant "k" in transistor R-C phase-shift networks was made in 1958 by Liubin.[10] The resistance of each successive R-C combination being "k" times greater than that preceding and each successive capacitor being "k" times smaller, serves to define the constant "k". He found that increasing "k" to a value greater than one caused the network attenuation to increase; whereas, it decreased in the vacuum tube case. He then concluded that in the case of transistors, making "k" greater than one was a disadvantage. He also constructed an experimental circuit using two transistors. In this circuit the capacitors were the shunting elements and the resistors were the series elements in the feedback R-C sections. This arrangement was found to be less stable than the shunting resistor configuration, but much higher frequencies could be obtained.

In 1958, Trokhimenko derived relationships for frequency and minimum required transistor current gain for a three R-C section oscillator using a single transistor.[11] He also indicated the effect of the various transistor parameters on the operation of the circuit.

In all of the previous reports, the effect of the biasing resistors on the circuit was neglected. Also, the effect which might arise

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[10] V. M. Lyubin, "Transistor R-C Oscillators with Phase Reversal," Radio Engineer, vol. 13, no. 2, 60-69, (English translation of Radiotekhnika, Russia).

[11] Y. K. Trokhimenko, "Single-Stage R-C Oscillator Circuit Using a Demiconductor Triode," Radio Engineer, vol. 13, no. 11, 57-68, (English translation of Radiotskhnika, Russia).

from a bypass capacitor around the emitter biasing resistor in a common emitter configuration was not considered in these reports.

AN AMPLITUDE-MODULATED TEMPERATURE TELEMETRY SYSTEM

Introduction

Investigation into several areas of physiology has long been hampered by the lack of suitable measuring devices. The accurate measurement of physiological phenomena without disturbance to the animal involved in the experiment has long been desired. During the past few years, several approaches have been made to various types of measurement. The primary objectives of this investigation are to determine the feasibility and possible advantages of a somewhat different approach to the problem of temperature telemetry. This approach is the use of an amplitude modulated device in which the audio frequency of the modulating signal is varied with temperature. Although many other applications are possible, the device constructed for this investigation was used to measure and determine the natural variation in the rumen temperature of a sheep during a 24-hour period. The abrupt changes in temperature due to drinking and eating were also observed.

Basic Similarities Among Temperature Telemetry Systems

There are many basic similarities between this telemetry system and others which have been employed to measure temperature. The information is transferred from the transmitter unit in the animal to the receiving antenna by means of a varying magnetic field. This type of field is used rather than electromagnetic radiation as a means of propagation. Electromagnetic radiation is not used because the small size of the transmitter unit does not lend itself to the construction of an efficient antenna for low frequencies. The frequencies used for transmission range from about 200 to 500 kilocycles. Low frequencies are employed because they are attenuated much less by animal tissue than high frequencies. The magnetic field is generated at the radio frequency oscillator coil in the transmitter unit. The changing radio frequency magnetic field induces a voltage in the receiving antenna. This antenna is composed of a one or more turn coil surrounding the immediate environment of the animal. The antenna is connected to a radio receiver by a lead which is made sufficiently long to isolate the animal from the observer and equipment.

The electronic circuitry and packaging of the transmitter unit is generally made as small as possible. The battery size, useful transmission life, and range of the transmitter unit must be compromised upon. The particular application of the unit determines the relative magnitudes of these considerations.

Advantages and Disadvantages of Other Temperature Telemetry Systems

Most temperature telemetry systems previously developed have operated on the principle of varying the frequency of a small temperature sensitive radio frequency oscillator. This method has the advantage of simple circuitry involving only one oscillator which uses few components in the transmitter unit. Since only one transistor is used, the current required to operate the circuit is much less than that of a more complex transmitter unit that requires more than one transistor.

However, whereas advantages lie in the simplicity of the transmitter unit of this system, there are also definite disadvantages due to the complexity of the receiving equipment. The operator of the system must understand the heterodyning principle and use a somewhat complicated procedure to interpret the received information. The possibility of human error exists to a greater extent with this method than it would in a system that presents information in a less complex manner. The complexity could also tend to discourage use of the system and cause a lack of confidence for investigators.

The change in position of an externally mounted unit on the animal will shift the distance of the oscillator coil from tissue. This can cause a change in capacitance effect which would shift the frequency of oscillation. The oscillators in these units have also been found to drift after an interval of continuous operation. Since the correct temperature interpretation depends upon the original calibration, any influence, other than temperature, that could shift the oscillator frequency would introduce error.

Description of the Amplitude Modulated Temperature Telemetry System

In the temperature telemetry system considered in this investigation, an amplitude modulated transmitting unit is used. The frequency of a resistance-capacitance phase-shift audio oscillator is varied with temperature by the use of thermistors in the feedback network. A diagram of the R-C phase shift audio oscillator used in this transmitter unit is shown in Figure 1. This audio oscillator is used to amplitude modulate the signal of a radio frequency oscillator. One transistor is required for each oscillator. Thus, the transmitter unit in this system requires more components and more complex circuitry than the previously

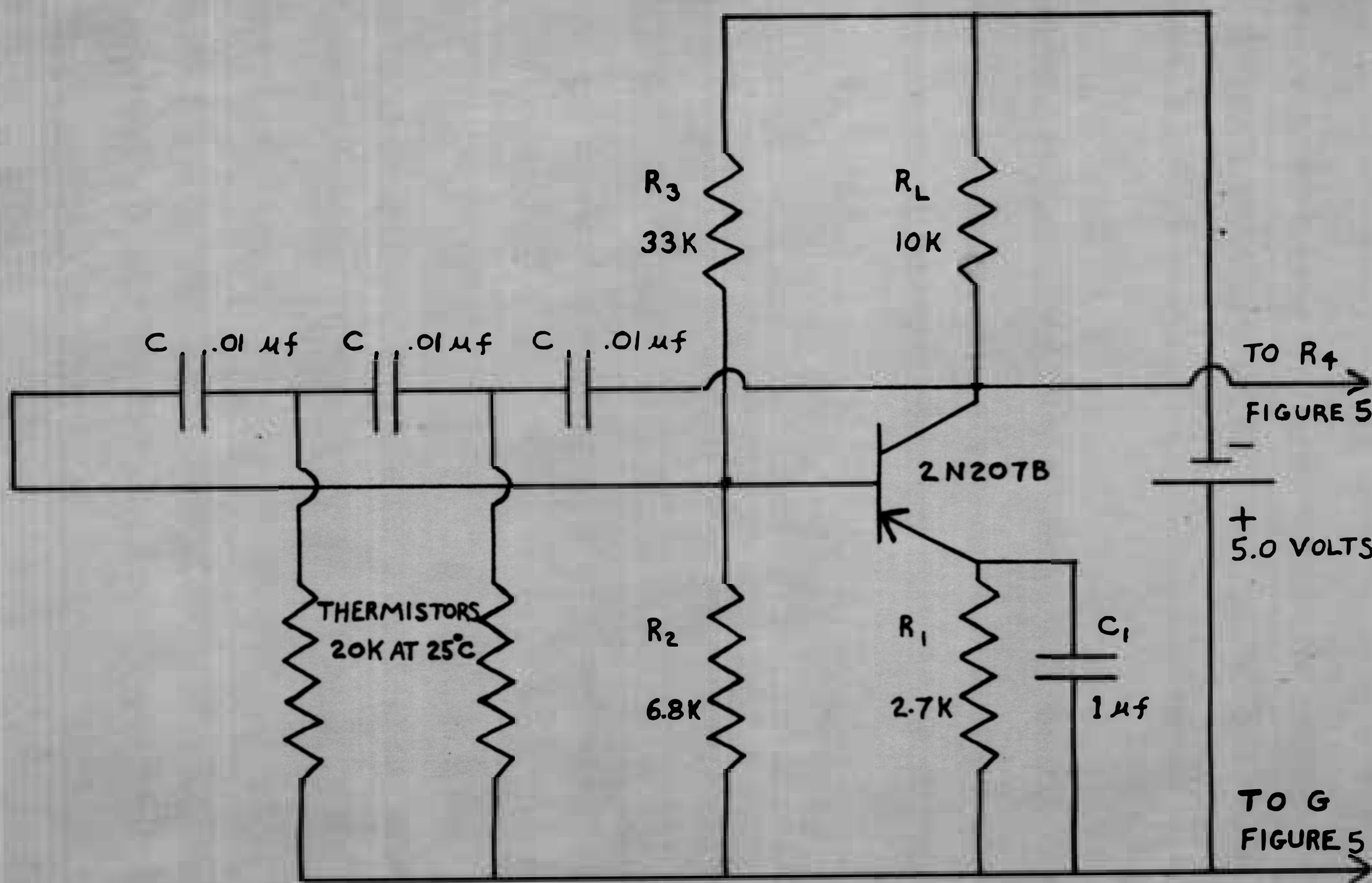

Figure 1. Audio Oscillator used in Transmitter Unit

described system. The amplitude modulated unit also requires more battery current than the other system. Therefore, the size of the amplitude modulated unit is somewhat larger. However, the interpretation of the information at the receiving equipment is relatively simple in the amplitude modulated system. The receiver is initially tuned to receive the signal from the radio frequency oscillator of the transmitter unit. The audio frequency corresponding to the temperature can then be observed at any instant by the investigator whose only requirement is to record the numbers displayed by a frequency counter.

One of the most difficult problems in developing this amplitude modulated system was the selection of a suitable audio oscillator to be used in the transmitter unit. It was desired that component sizes be kept at a minimum. Since the frequency of oscillation was to be in the audio range, inductors would have to be quite large. This made it desirable to avoid their use. A further disadvantage of using an inductor in the audio oscillator circuit was the fact that changes in the surrounding media could change the capacitance at the inductor windings. This would result in an undesired frequency shift. In this case, the animal tissue could change position with respect to the transmitter unit if the unit were mounted externally. A resistance-capacitance oscillator, commonly referred to as an R-C oscillator was chosen.

Application of the R-C Phase-Shift Oscillator to the System

The R-C oscillator selected operates on the principle of phase-shifting the output signal of an amplifier by successive R-C sections until the output signal is shifted 180 degrees at insertion into the amplifier input. The circuit will then oscillate, provided the gain of the amplifier is high enough to result in a loop gain of at least unity.[12] Since the maximum phase-shift of a single R-C combination is less than 90 degrees, it is obvious that a minimum of three R-C sections is required. Since only one frequency can be shifted 180 degrees in the R-C feedback network, this is the oscillation frequency of the circuit. Thus, by changing either the resistance or the capacitance in the feedback R-C sections, the 180 degree phase-shift would occur at another frequency and the oscillation frequency would change. It has also been found that as the number of R-C sections is increased, the attenuation in the R-C feedback network is decreased.[13] Thus, the gain requirements of the transistor are decreased as R-C sections are added to the feedback network. Since a three R-C section circuit is the minimum required number of sections, the gain requirements of the transistor in this arrangement are quite high. In addition, an excess of the minimum transistor current gain for oscillation is required for this application because the value of resistance in the R-C sections must vary over a wide range, and the voltage from the battery will tend to drop during continuous operation. Both of these variations will change the total oscillator loop gain. An effect that provided additional phase-shift was discovered while examining the operation and theory of the R-C phase-shift oscillator. This resulted in a circuit that did not require

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[12] J. D. Ryder, Engineering Electronics, p. 378, McGraw-Hill: New York, 1957.

[13] Ginzton and Hollingsworth, loc. cit.

such a high transistor h_{21} as had been necessary in previous circuits. In a separate section titled, "A Revised Analysis of Transistor R-C Phase-shift Oscillators," the effect which allows a lower required transistor gain is discussed. In view of this, the three R-C section oscillator was selected because of the fewer number of components it required.

An additional consideration which led to the selection of an R-C phase-shift oscillator was the simplicity with which it could lend itself to integration with a suitable temperature transducer. Also, the predicted frequency change over the intended range of operation was sufficient to insure the desired accuracy of temperature measurement. The temperature transducers used were thermistors which replaced the resistors in the R-C feedback sections. An experimental circuit was constructed, and the effects that changes in various circuit parameters had on the oscillator performance were observed. By this method, the required thermistor resistance range and the values of various components were determined. A Philco 2N207B transistor was used. This transistor was selected because of its high gain and small size. Its design is suited primarily for operation in the audio frequency range. In constructing the initial experimental circuit, the component values were based on circuits which had been constructed previously by others.[14] The available thermistor sizes, as well as their correspondence between resistance and temperature, were known. The upper and lower limits through which the resistance in the R-C feedback sections could be varied without stopping oscillation

[14] S. Schwartz, Selected Semiconductor Circuits Handbook, pp. 5-32, 5-33, John Wiley & Sons: New York, 1960.

were determined experimentally. Since the range of temperature variation to measure was known, it was then possible to choose the appropriate thermistor. The thermistor chosen had a valve of 1K at 25°C which decreased at a rate of about seven percent per degree as the temperature increased. Only the resistors in the R-C feedback sections which do not influence the transistor bias circuit are replaced by thermistors. This is to prevent the transistor operating point from varying with temperature. If this were not done, the temperature range through which the circuit could operate would be greatly reduced.

The minimum battery voltage at which the circuit would oscillate was about 4.0 volts. The battery voltage decided upon was 5.0 volts since this was a standard commercial rating among mercury batteries, and it was sufficiently above the minimum operational circuit voltage to allow for a drop in battery voltage due to continuous operation. The voltage discharge curves of the batteries under consideration showed that the service life of the battery ended when the battery voltage dropped to about 4.5 volts. It was desired to know how much this voltage change would affect the oscillation frequency of the unit. Most of the voltage change occurred near the end of the life of the battery. Frequency measurements were taken at various points along the expected operating range of the unit. The frequency of oscillation was varied by changing the value of resistance in the R-C feedback sections. At each point of measurement the frequency at 5.0 volts and 4.5 volts was observed. The results are shown in Figure 2. As the oscillation frequency is increased, the frequency shift due to the battery voltage drop

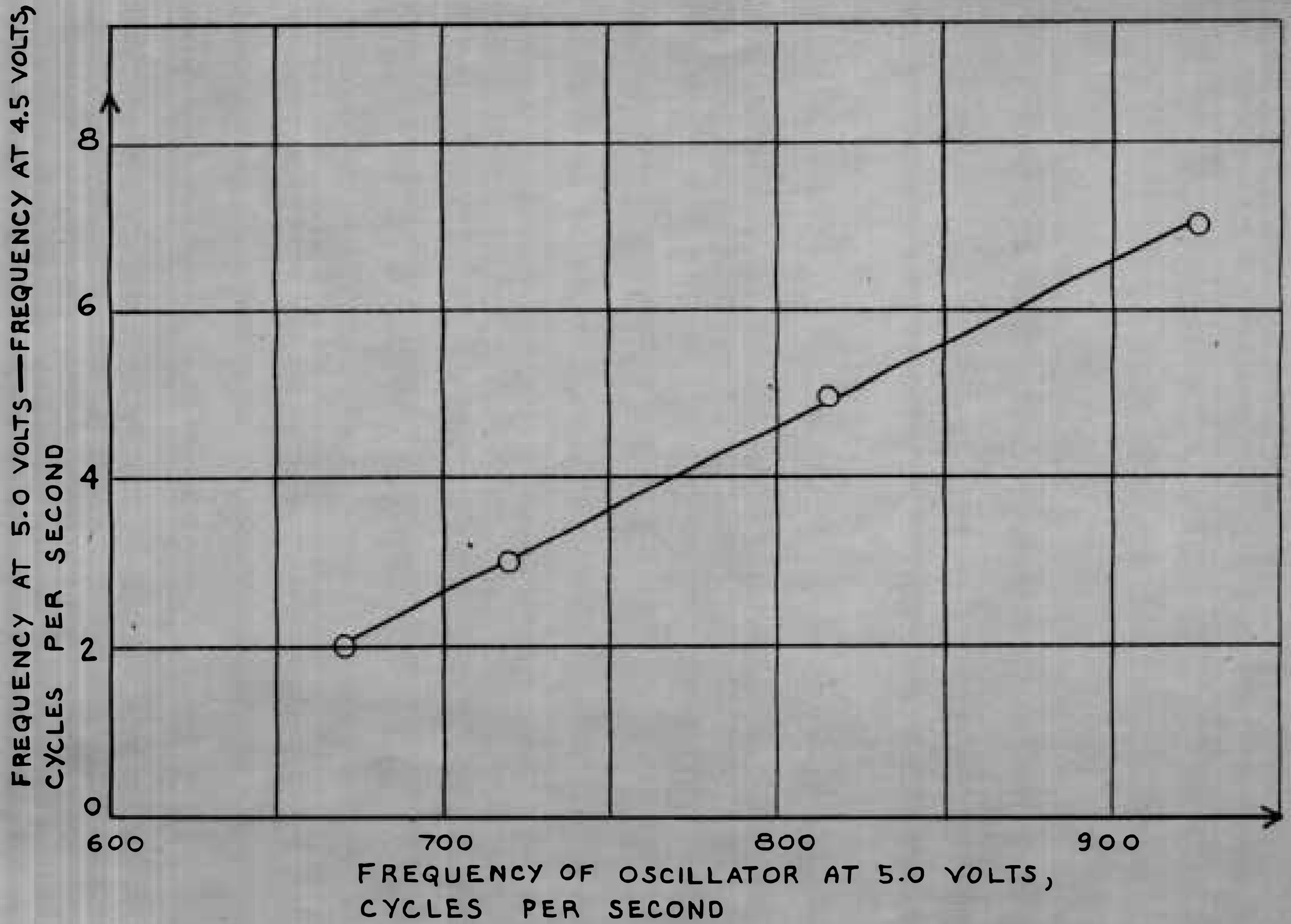

Figure 2. Effect of Battery Voltage Change on the Audio Oscillator's Frequency

is also increased. This indicates that most of the system error, due to battery voltage decrease in the transmitter unit, would be expected to occur at the high frequency region of the operating range.

The data for all curves (with the exception of the final data taken with the unit sealed) was taken with .01 uf capacitors in the R-C feedback sections instead of the .005 uf capacitors shown in Figure 1. The reason for the change is that in the final assembly of the transmitter unit, some of the .01 uf capacitors were damaged. The closest replacements available were the .005 capacitors. For a particular value of resistance in the R-C feedback sections, the replacements caused the oscillation frequency to nearly double that obtained by the .01 uf capacitors. However, the circuit was found to operate very satisfactorily throughout the desired frequency range by using the .005 uf capacitors. The curves which were found while using the .005 uf capacitors in the R-C feedback network are those in Figures 10, 11 and 13. All other figures represent data taken with .01 uf capacitors in the feedback network.

The frequency change due to varying the temperature of the phase-shift oscillator circuit is shown in Figure 3. This is seen to be a linear relationship until the temperature reaches about 45°C. Most of the frequency change is caused by the two thermistors in the R-C feedback network. However, the frequency change due to the temperature sensitivity of the other circuit components was investigated. The temperature sensitivity of the transistor was found to cause very little change in frequency. The only noticeable changes occurred above

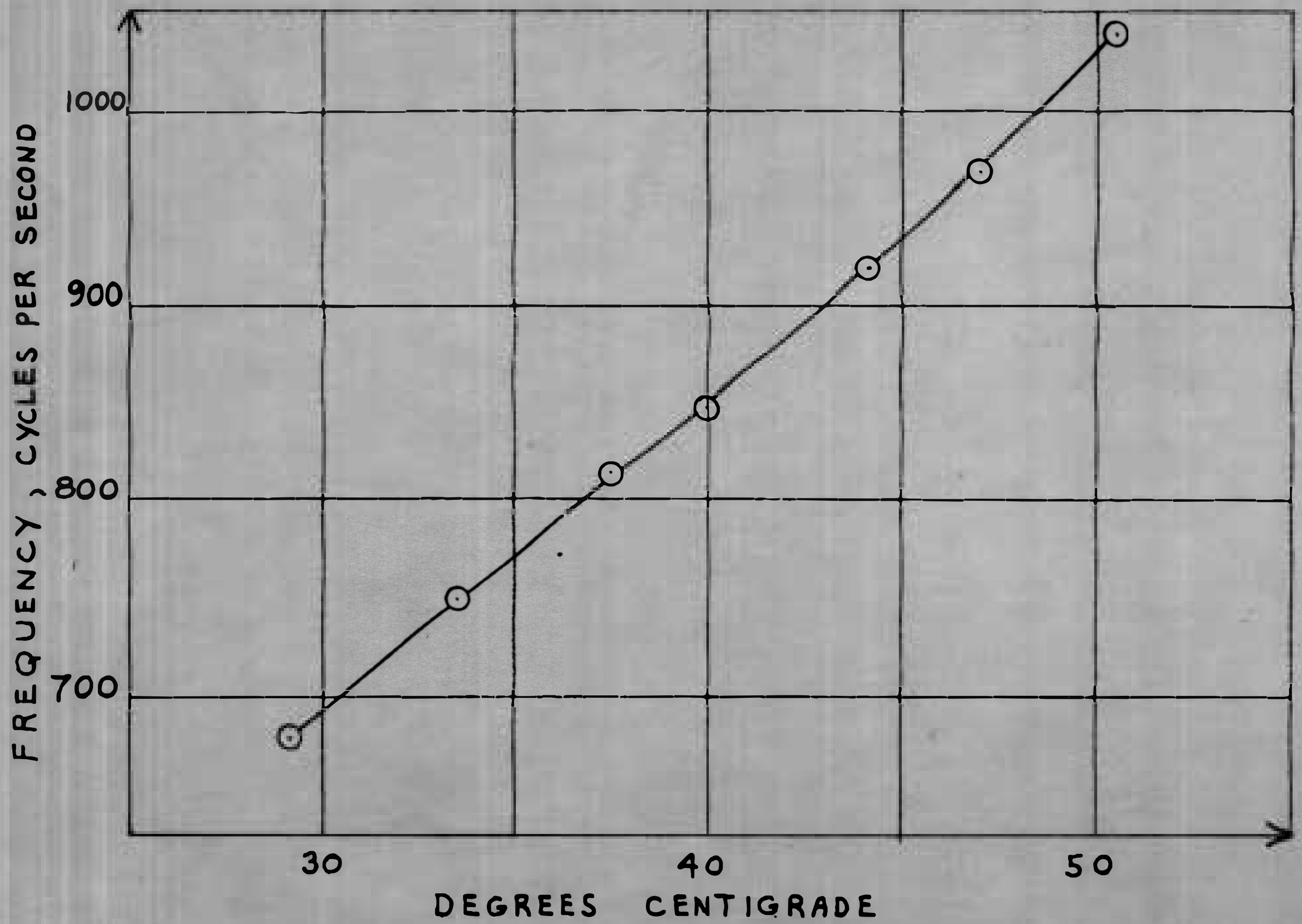

Figure 3. Effect of Temperature on the Frequency of the Complete Audio Oscillator Circuit

45°C, as can be seen in Figure 4. This measurement was taken by heating the transistor in an oven while holding the other circuit components at room temperature. The same procedure was used to determine the change in frequency due to varying the temperature of the bypass capacitor C_1. The effect of C_1 is shown by the dotted line in Figure 4. The frequency change caused by C_1 is a linear relationship with temperature, and it is a greater change than that caused by the transistor. The frequency change due to the other circuit components (excluding the thermistors, transistor, and C_1) was found to be negligible. Since this oscillator is not intended for application above 45°C, the only component other than the thermistors which could significantly influence the frequency-temperature relationship is C_1. The above conclusion assumes that there is a constant battery voltage.

Determination of the Complete Transmitter Unit

The Colpitts oscillator shown in Figure 5 was used in the transmitter unit to generate the radio frequency signal.[15] This is a standard type oscillator, and the reason for its choice is that it has a single, untapped coil. This type of coil greatly simplified the determination of the optimum coil size for this application.

Modulation of the radio frequency signal is achieved by injecting the output of the phase-shift audio oscillator into the base of the Colpitts oscillator. The path of the audio signal is seen in Figure 5

[15] R. F. Shea, Transistor Circuit Engineering, p. 226, John Wiley & Sons: New York, 1957.

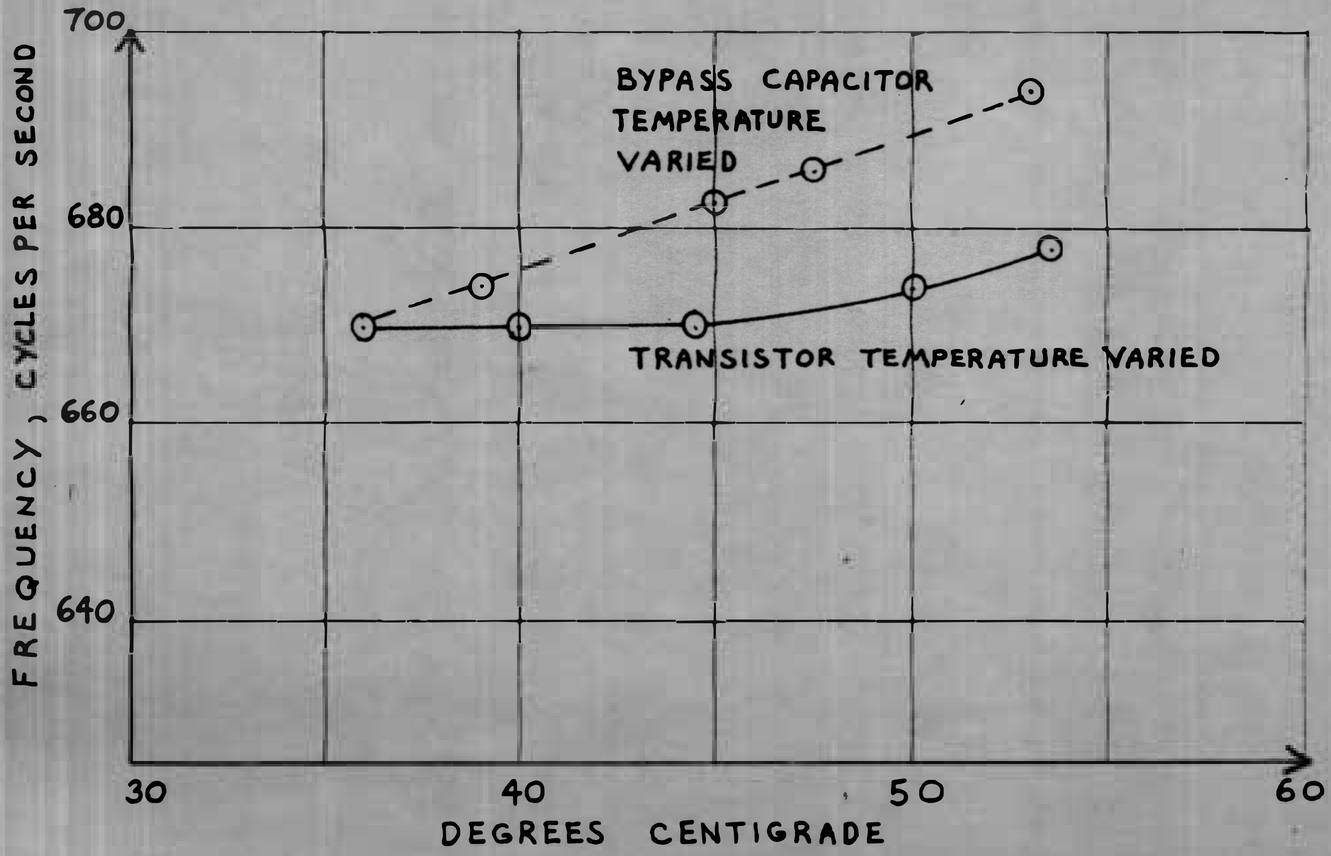

Figure 4. Effect of the Transistor and Bypass Capacitor Temperatures on the Frequency of the Audio Oscillator

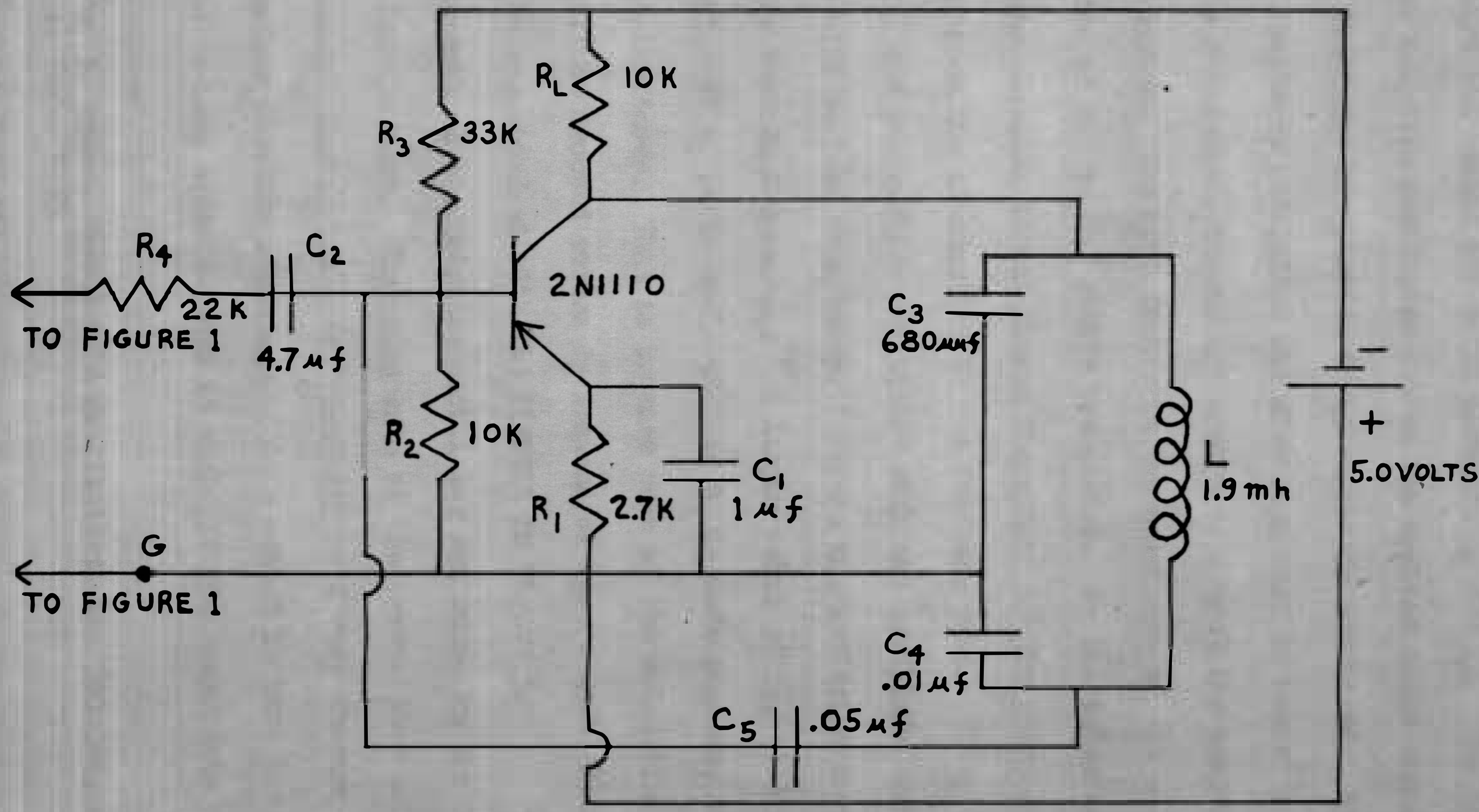

Figure 5. Radio Frequency Oscillator used in Transmitter Unit

to be through the series connected elements R_4 and C_2. The variation of bias in the Colpitts oscillator due to the applied audio signal causes the radio frequency oscillation to be amplitude modulated. A certain degree of frequency modulation was also suspected to occur[16]; but so far as could be determined, it was not enough to affect the usefulness of the telemetry system. The main factor in the selection of the value for R_4, is the value of h_{21} in the 2N207B transistor. As the value of R_4 becomes smaller, the shunting effect across the output of the phase-shift oscillator increases the attenuation in the feedback path. If R_4 is decreased far enough, the phase-shift oscillator will not oscillate because its loop gain will be less than unity.[17] Since the amplifier current gain is directly dependent upon h_{21},[18] a larger value of h_{21} will allow a smaller value of R_4 to be employed. As R_4 is made smaller, the modulation percentage increases. This is because more of the modulating signal voltage will be dropped across R_2 instead of R_4. A modulation percentage of about 40 was obtained with the circuit in Figure 5.

The coil of the Colpitts oscillator was wound on the outside of a hollow ferrite core. The components of the transmitter unit could then be mounted inside the core, resulting in a very compact arrangement. Another possible advantage of this method is that the Q of the coil would be less affected than it would if the components were mounted

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[16] W. A. Edson, Vacuum Tube Oscillators, pp. 393-396, John Wiley & Sons: New York, 1953.

[17] Ryder, loc. cit.

[18] Shea, op. cit., p. 74.

beside it. Metallic components mounted near the coil were found to lower the Q of the coil considerably. The core selected was constructed of Ceramag 27 material, which was manufactured by the Stackpole Carbon Company. This material was chosen because of its low loss and high permeability at the expected operating frequencies. The effect desired was for the flux to be confined inside the high permeability core and not to pass through the internally mounted components where losses could occur. The current in the coil of the radio frequency oscillator, at resonance, is proportional to the Q of the coil.[19] Since the magnetic flux of the coil is proportional to the current, a high Q coil is desired so that the strongest possible magnetic field may be produced. The Q of the coil used in the transmitter unit decreased from 93 to about 87 when the hollow core was completely filled with typical components. The greatest effect on the Q of the coil was found to occur from the battery which was to be mounted outside the core. The effect on the Q of the coil due to the distance of the battery from the end of the core is shown by the curve in Figure 6. A distance of about one centimeter was chosen as the mounting distance of the battery from the end of the coil. This distance was chosen as a compromise between lowering the Q of the coil and keeping the length of the transmitter unit as short as possible. The coil was hand wound on the core according to the dimensions given in Figure 7. The wire used was number 44 AWG Litz wire. One layer was wound with the turns spaced as closely as possible.

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[19] Everitt and Anner, Communication Engineering, pp. 160-161, McGraw-Hill: New York, 1956.

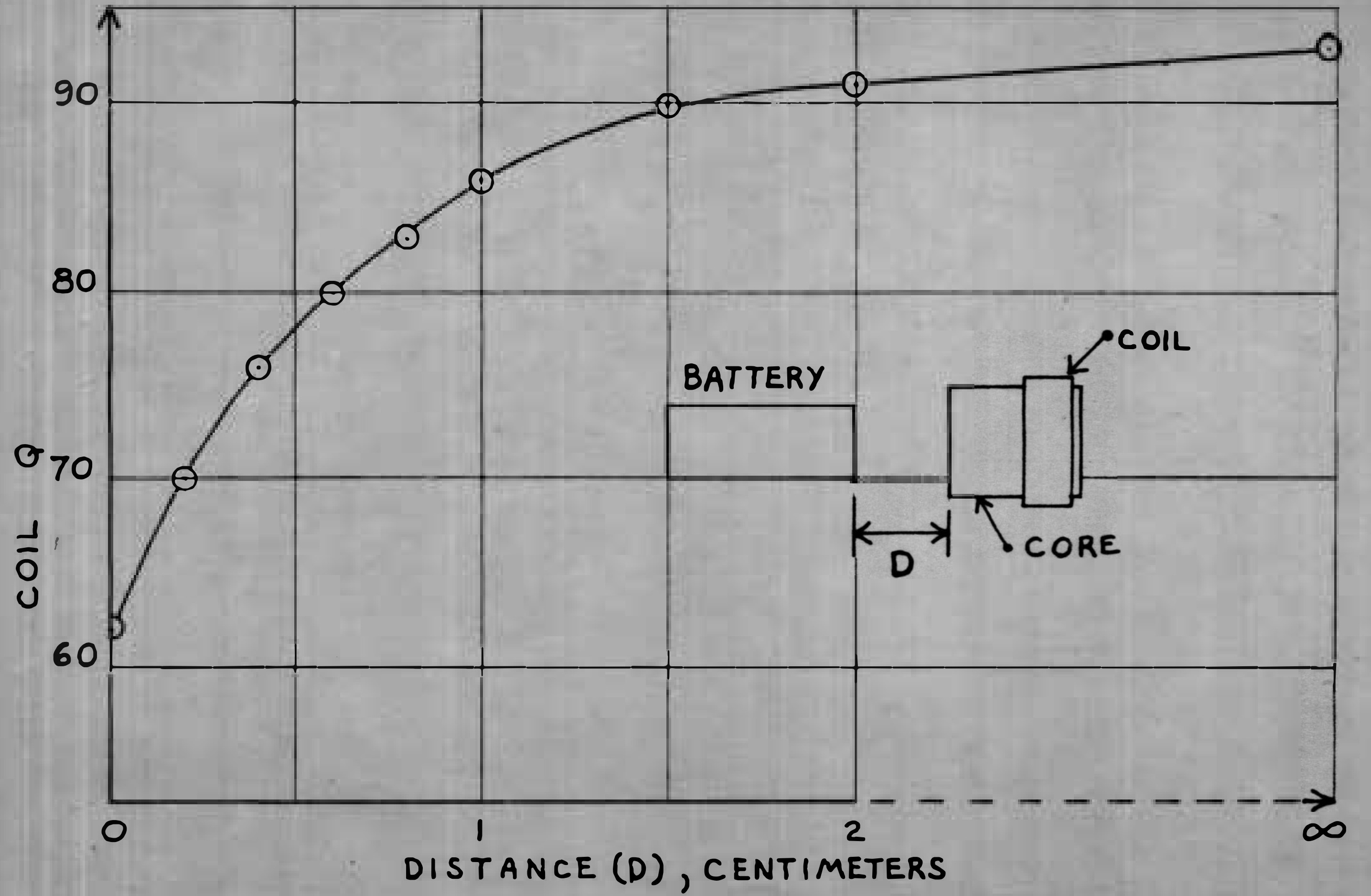


Figure 6. Effect of Battery Distance on Coil Q

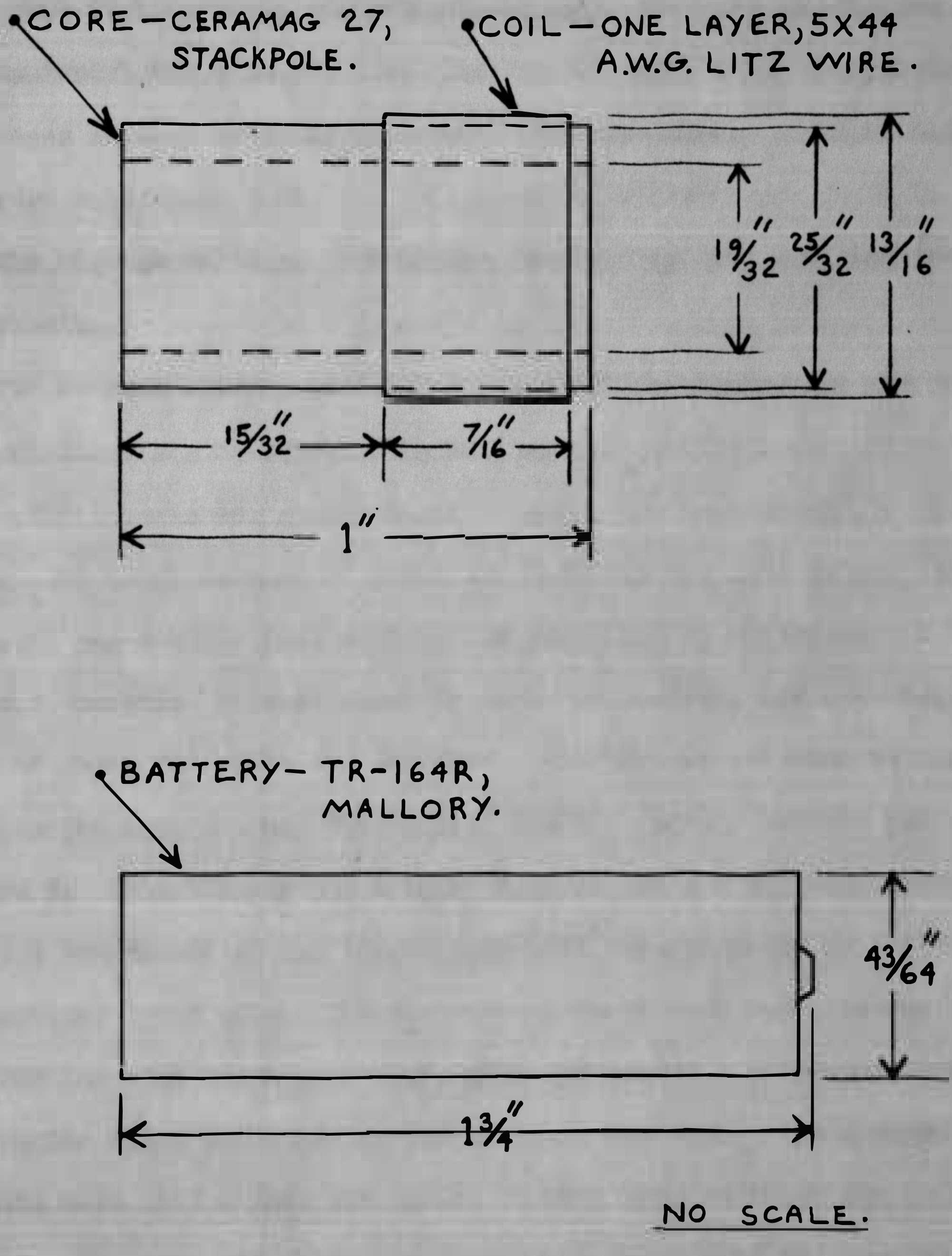


Figure 7. Dimensions of Core, Coil, and Battery

The size of the coil was determined by measuring the amount of voltage that it induced in a surrounding loop. The arrangement used for this measurement had a single loop that was 10 centimeters in diameter. The voltages induced by coils wound with various numbers of turns and layers were determined until the coil shown in Figure 7 was found to induce the highest voltage. The voltage induced by this coil was about 4.5 millivolts.

The current used by both the phase-shift oscillator and the Colpitts oscillator was measured. The phase-shift oscillator was found to use .25 milliamperes while the Colpitts oscillator used about .30 milliamperes. The total current of about .55 milliamperes used by the transmitter unit was a major influence in the selection of a battery. A continuous operating life of about 30 days was desired, and a battery voltage of about 5.0 volts was required. The battery selected to fulfill these requirements was the Mallory TR-164R mercury battery shown on Figure 7. This battery has a rated capacity of 500 milliampere-hours.

The components of the transmitter unit were enclosed by a circular machined nylon case. All the components of both oscillators, except for the coil and the battery, were mounted on a fiberglass board with circular disks attached perpendicular to the ends. The mounted components were slid inside the hollow ferrite core on which the coil was wound. The core was then slid inside the nylon case and secured at the end by screwing a threaded nylon disk into the end of the case. This disk had a hole in the center which exposed the 2N207B transistor and the two thermistors. The battery was slid into the other end of the nylon case and secured by a nylon disk. The dimensions of the mounting board, nylon case, and end disks are given in Figure 8. Photographs which show the placement of the components and the assembly of the transmitter unit are shown in Figure 9. The assembled unit was covered with three coats of Tygon series "k" plastic paint which acted as a waterproof seal.

Calibration of the Temperature Unit

Calibration was obtained by recording the frequency of the audio signal at various temperatures of the transmitter unit. The temperature of the transmitter unit was controlled by the use of a water-bath in which the unit was submerged. The calibration curve is shown in Figure 10. The change in frequency with temperature is seen to be about 30 cycles per degree centigrade. Since the frequency counter is accurate to .1 of a cycle, high accuracies are a possibility. However, due to a certain amount of drift in the transmitter unit's audio oscillator, an accuracy to about .1 of a degree centigrade was all that could be expected. This is sufficiently accurate for most physiological applications. Although the calibration curve includes the temperatures normally encountered in physiological studies, this curve could be extended to include much higher and lower temperatures. The transmitter unit was found to operate at the high and low temperatures of 70°C and 5°C. The high temperature was not exceeded because of the possibility of damaging the transistors from overheating. The radio frequency oscillator operated down to -5°C. It is believed that the audio oscillator stopped at 5°C because the thermistor resistance in the R-C sections became too

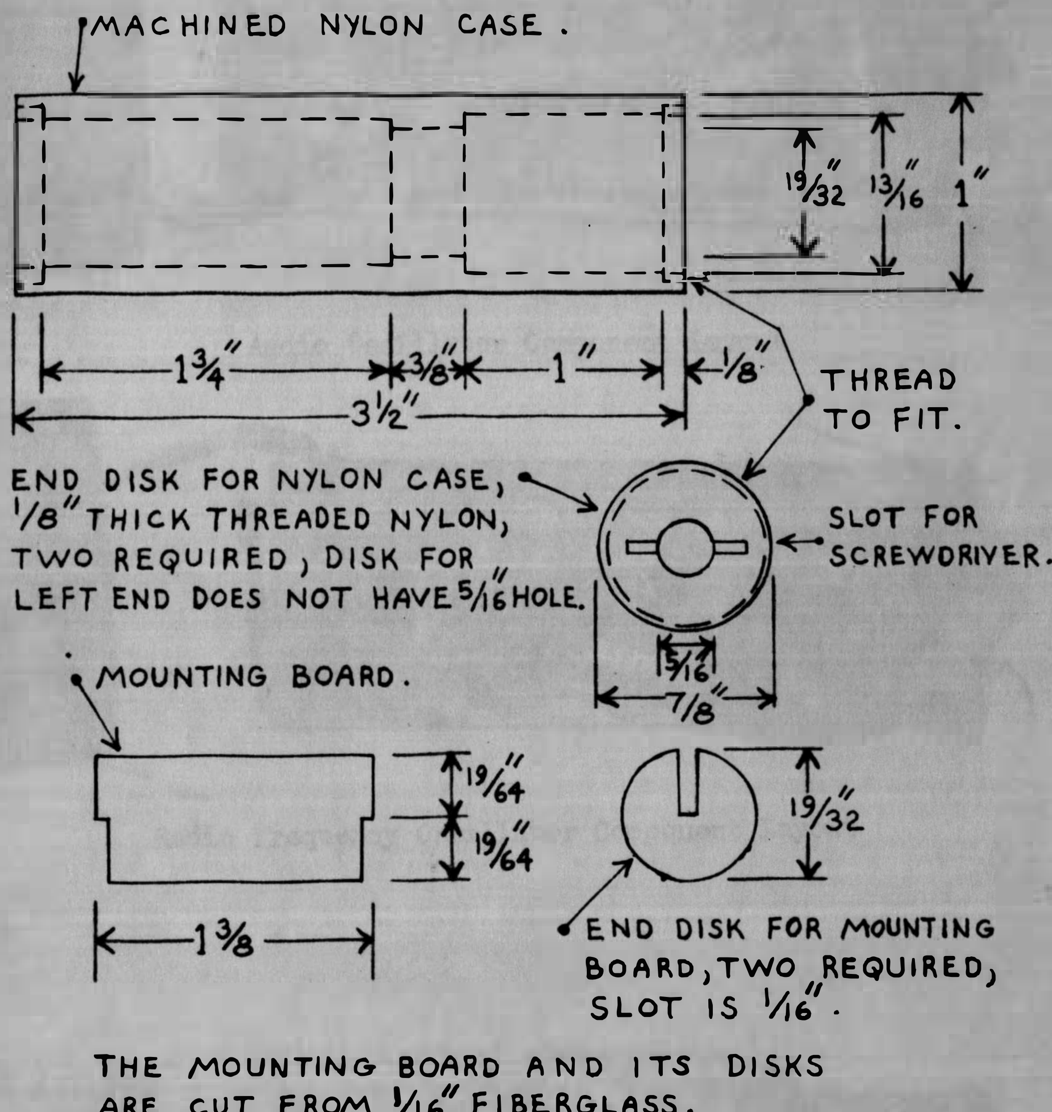
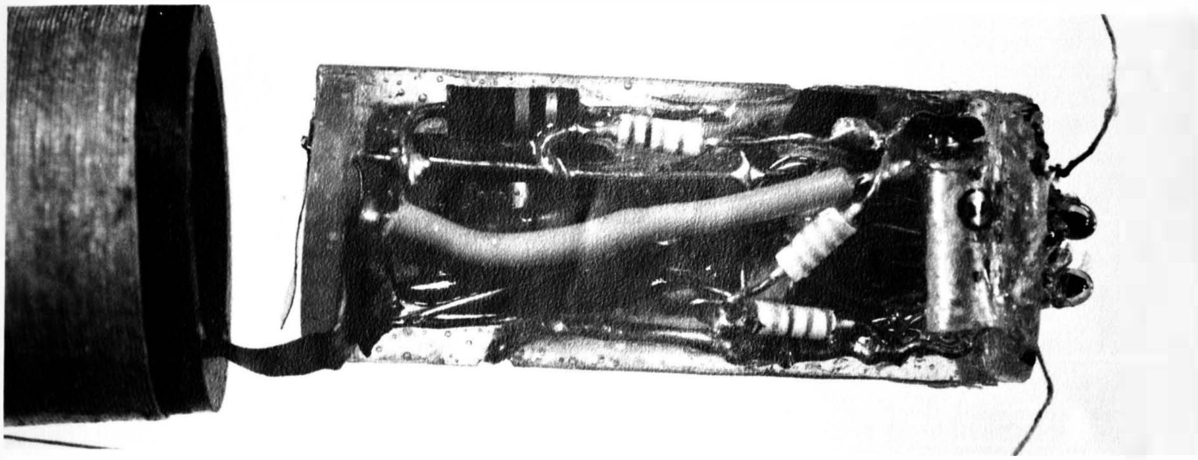
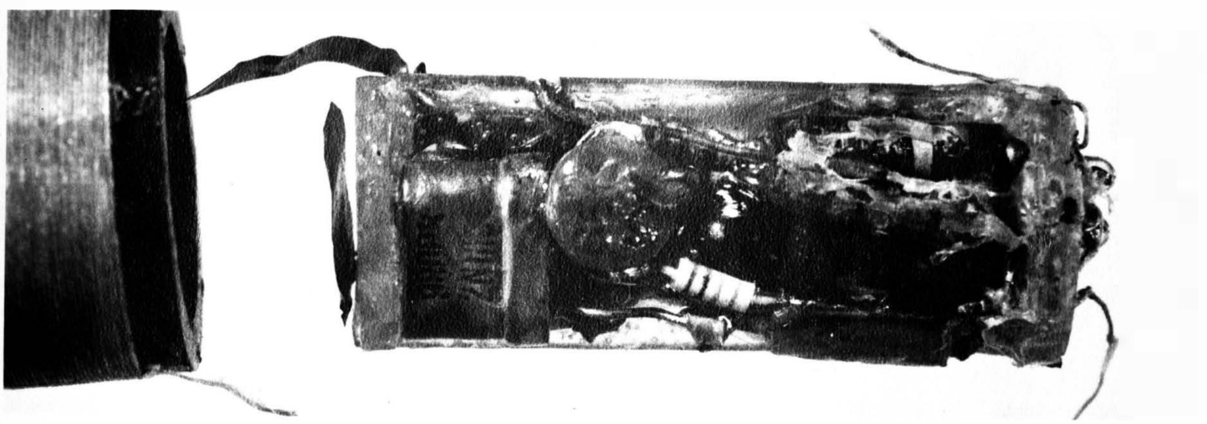


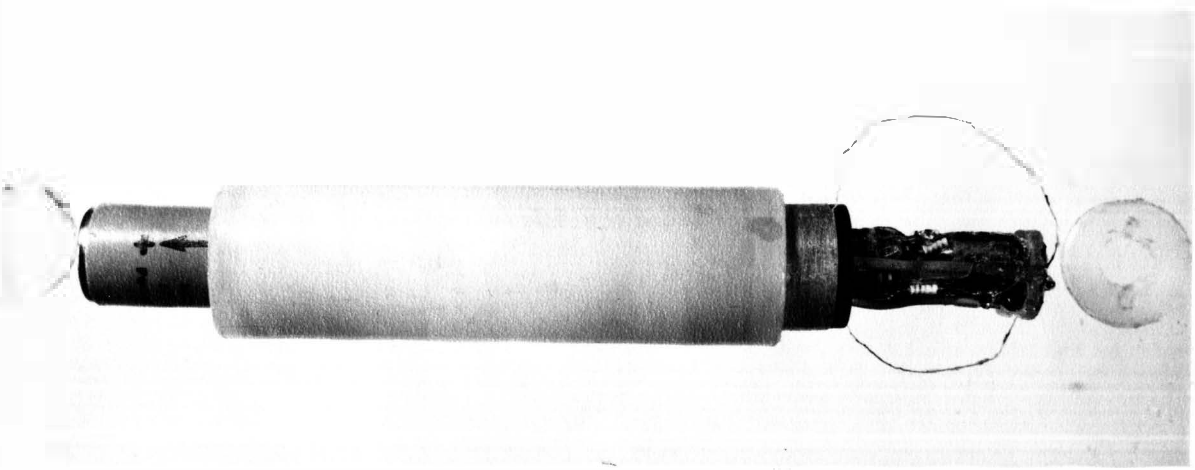
Figure 8. Dimensions of Mounting Board, Nylon Case, and End Disks



Audio Oscillator Component Layout



Radio Frequency Oscillator Component Layout



Transmitter Unit Assembly

Figure 9. Photographs of Component Layout and Unit Assembly

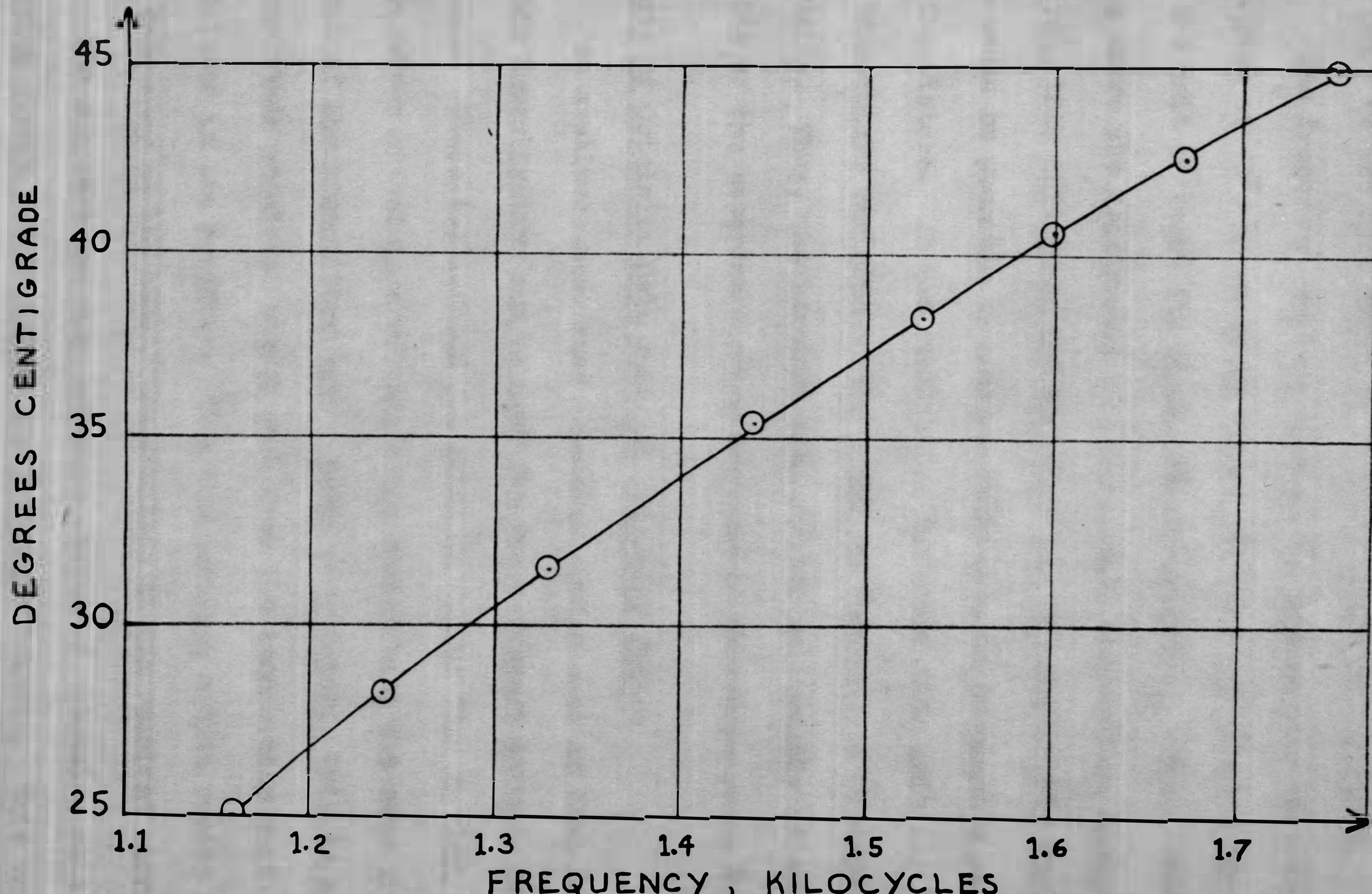


Figure 10. Calibration Curve of Transmitter Unit

high. If these thermistors were replaced by 10K at 25°C thermistors, the range of the unit might be extended considerably lower.

The frequency response time of the transmitter unit to a temperature step of 10°C is shown in Figure 11. Several minutes are required for the unit to reach its final, steady frequency. Thus, for applications where the measurement of large, rapid temperature changes is desired, this unit would not be accurate. A considerably faster response time would be possible by using a thinner layer of plastic paint to cover the thermistors. In the application for which this unit was intended, the temperature variations were small and the changes occurred quite gradually. Thus, the response time did not appreciably affect the results of the experiment which was done to determine rumen temperature.

Methods of Obtaining Data with the Telemetry System

An amplitude modulated telemetry system such as that developed in this investigation can be used for two different methods of obtaining data. These two methods are shown in Figure 12. In both methods, the receiver output is a voltage which varies with the same frequency as that of the transmitter unit's audio oscillator. This is because the amplitude modulated signal sent from the transmitter unit is demodulated in the receiver. Thus the receiver output varies with the same frequency as the modulation envelope of the received signal.

The simplest and most accurate method of obtaining data is that of making direct readings from the frequency counter. This method requires an observer to be present since the data must be read from the numbers displayed by the counter. The other method uses a recorder

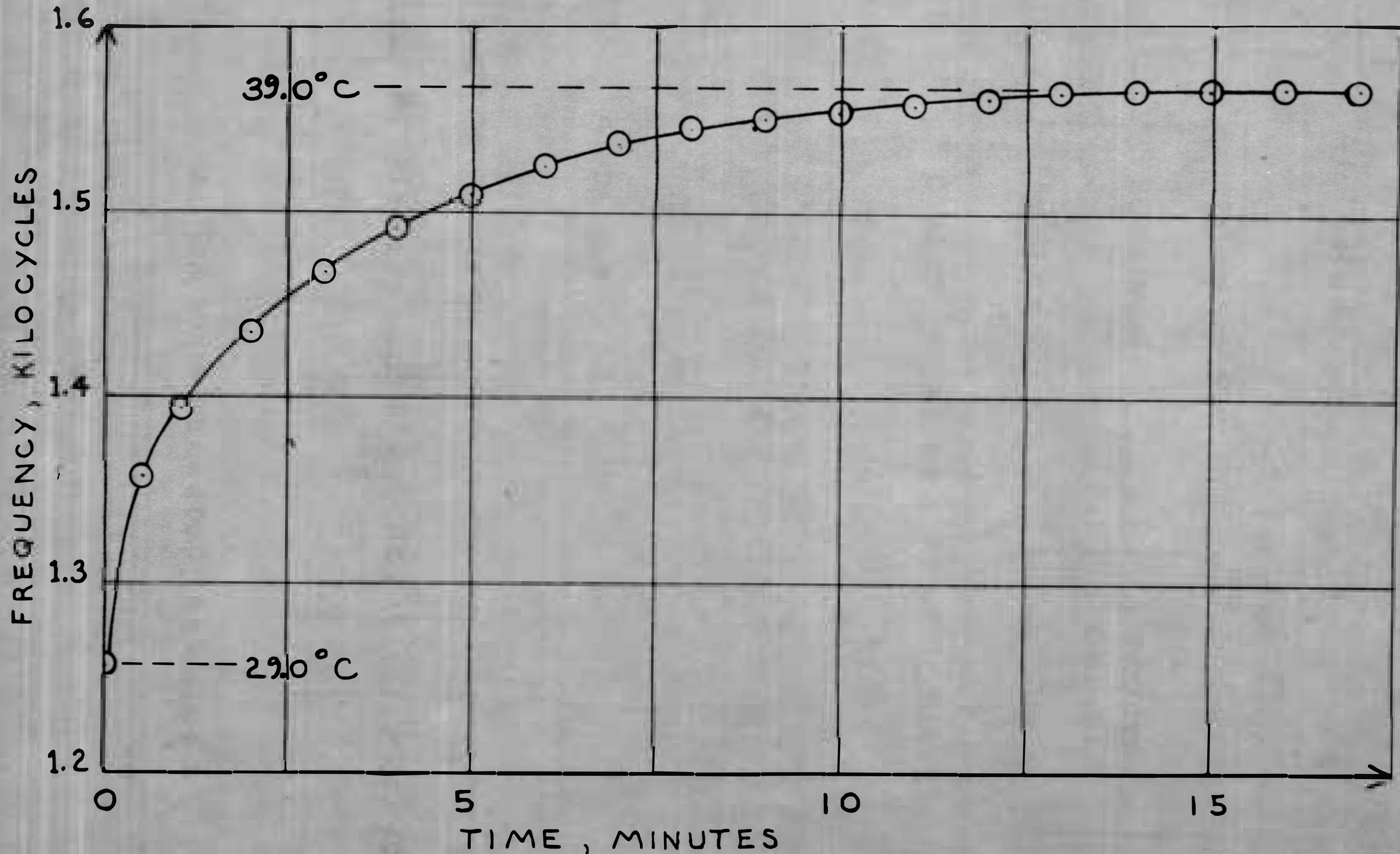
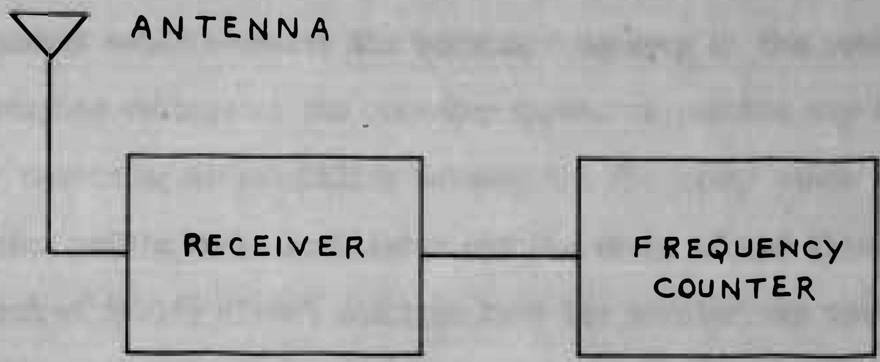
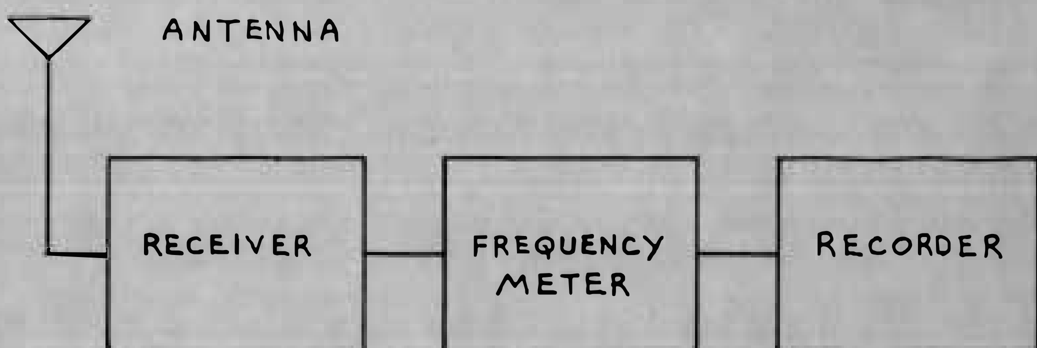


Figure 11. Frequency Response Time of Transmitter Unit



DIRECT READINGS FROM COUNTER



AUTOMATIC RECORDED READINGS

Figure 12. Methods of Obtaining Data from the Amplitude Modulated Temperature Telemetry System

which enables the observer to be absent while the data is being collected. The frequency meter converts the varying frequency at the receiver output into a varying voltage at the recorder input. A problem may be encountered in obtaining compatibility between the frequency range of the transmitter unit's audio oscillator and the range of the frequency meter. The method of taking direct readings from the counter was used to obtain the data for the following experiment.

EXPERIMENT TO DETERMINE RUMIN TEMPERATURE

The temperature telemetry system developed in this investigation was used to measure the temperature in the rumen of a sheep for a 72-hour duration. This duration provided three 24-hour periods during which the temperature rhythm of the rumen could be established. A temperature reading was taken from the frequency counter every 30 minutes for the 72-hour duration. The readings were averaged over the three 24-hour periods to give the result shown in Figure 13. The data that was taken during the 72-hour duration is shown in Appendix 1.

The transmitter unit was introduced into the rumen by an oral technique which avoided the trauma of surgical implantation. This accelerated the recovery to a normal physiological condition. After the unit was introduced into the rumen, the animal was given a 30-hour period to return to a normal physiological state and adjust to its new environment. The only deviation in the environment of the sheep, over the three 24-hour periods, was when the sheep's lamb was placed in the same cage. This was done during the first 24-hour period at 7:00 A. M. A total of 43-hours had elapsed between this time and the time when the sheep and lamb had been separated. The separation was an abrupt interruption in the normal physiological and psychological patterns of the animal. This was believed to be the cause of the slight elevation in temperature during the first 13 hours. The lamb remained with the sheep for the rest of the experiment. At 9:00 A. M. during each 24-hour period, the sheep was fed. During this period water was not included. The food remained in the cage for one hour before it was removed. At

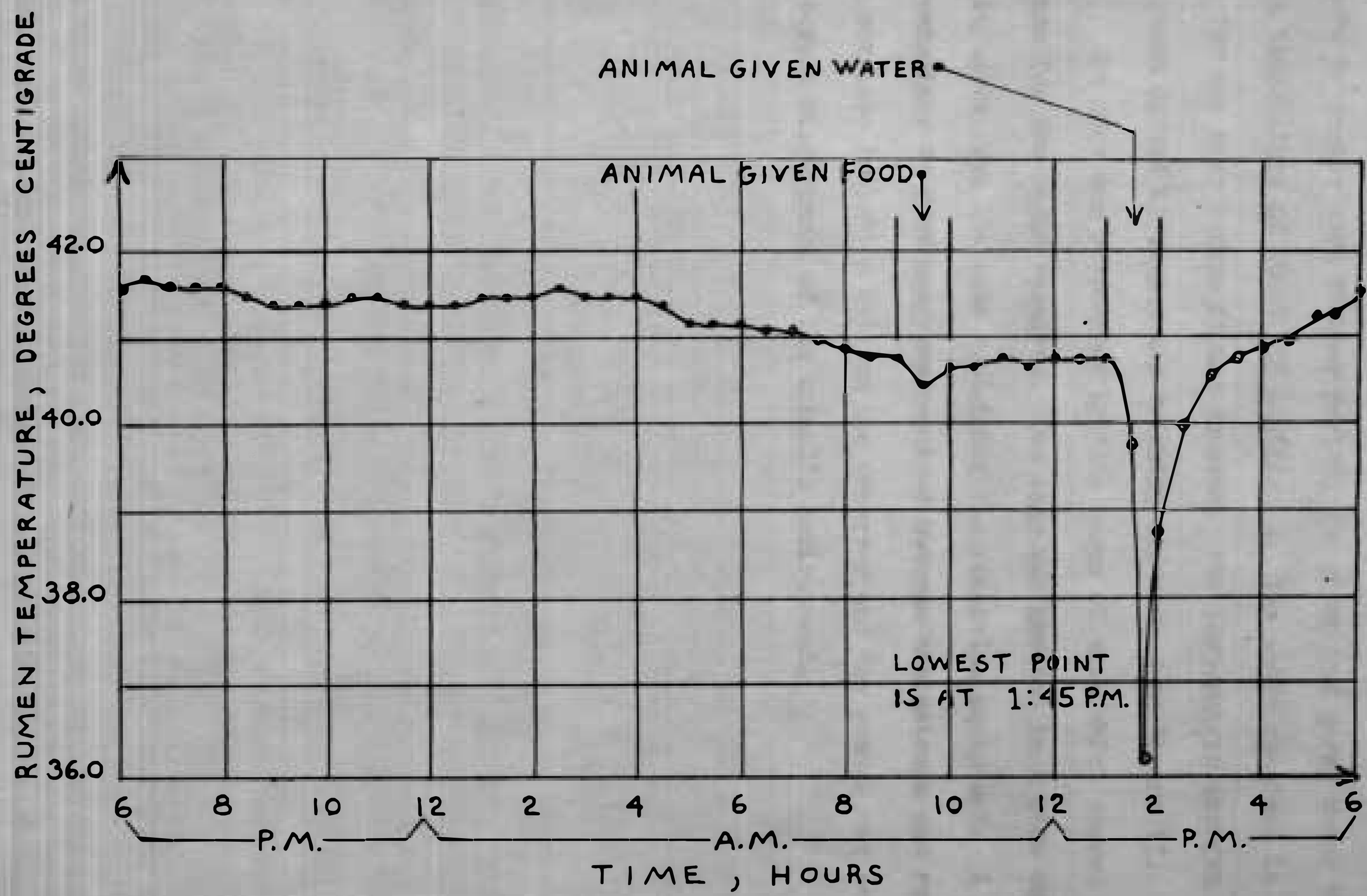


Figure 13. Temperature in the Rumen of a Sheep During a 24-Hour Period

1:15 P. M. during each 24-hour period, the sheep was given water which had a temperature of about 15° centigrade. The water remained in the cage for one hour before it was removed. The temperature changes in the rumen caused by eating and drinking are marked on Figure 13.

The cage was surrounded by five turns of wire which served as an antenna for the radio receiver. The cage was located in a room which was separate from the room containing the receiving equipment. A lead approximately 45 feet long was required between the antenna and receiver. This complete isolation between the observer and the animal was necessary to prevent disturbance of the animal's environment.

A REVISED ANALYSIS OF TRANSISTOR R-C PHASE-SHIFT OSCILLATORS

Three R-C Section Oscillator with the Phase-Shift Effect of C_1 Neglected

An analysis of the three R-C section phase-shift oscillator shown in Figure 14 will first be made by following a previous assumption of others.[9,10,11,15] This particular assumption is that the bypass capacitor C_1 forms a perfect A.C. shunt around R_1. Thus, in the equivalent circuit the parallel combination of C_1 and R_1 is replaced by a shorting connection. This would also be true in the equivalent circuit if the bias were obtained from an additional battery instead of the biasing resistor R_1. The phase shift oscillator under consideration assumes that the values of resistance and capacitance are identical for each R-C section.

The equivalent circuit of Figure 14, using y parameters to replace the transistor and admittances to replace the circuit resistors, is shown in Figure 15. In this circuit, y_{12} is omitted according to the conclusion in Appendix 2. The resistors R_2 and R_3 are combined by the relationship $Y_a = Y_2 + Y_3$. Nodal analysis of the circuit in terms of admittances and y parameters is used because it leads to fewer network equations than other methods which were considered. The other methods considered included using z and h parameters to represent the transistor and using loop equations instead of node equations.

Some of the circuit elements of Figure 15 can be combined to form the expressions:

$$Y_A = Y_a + y_{11}; \text{ and } Y_B = Y_L + y_{22}.$$

This leads to the simplified circuit shown in Figure 16. The nodes

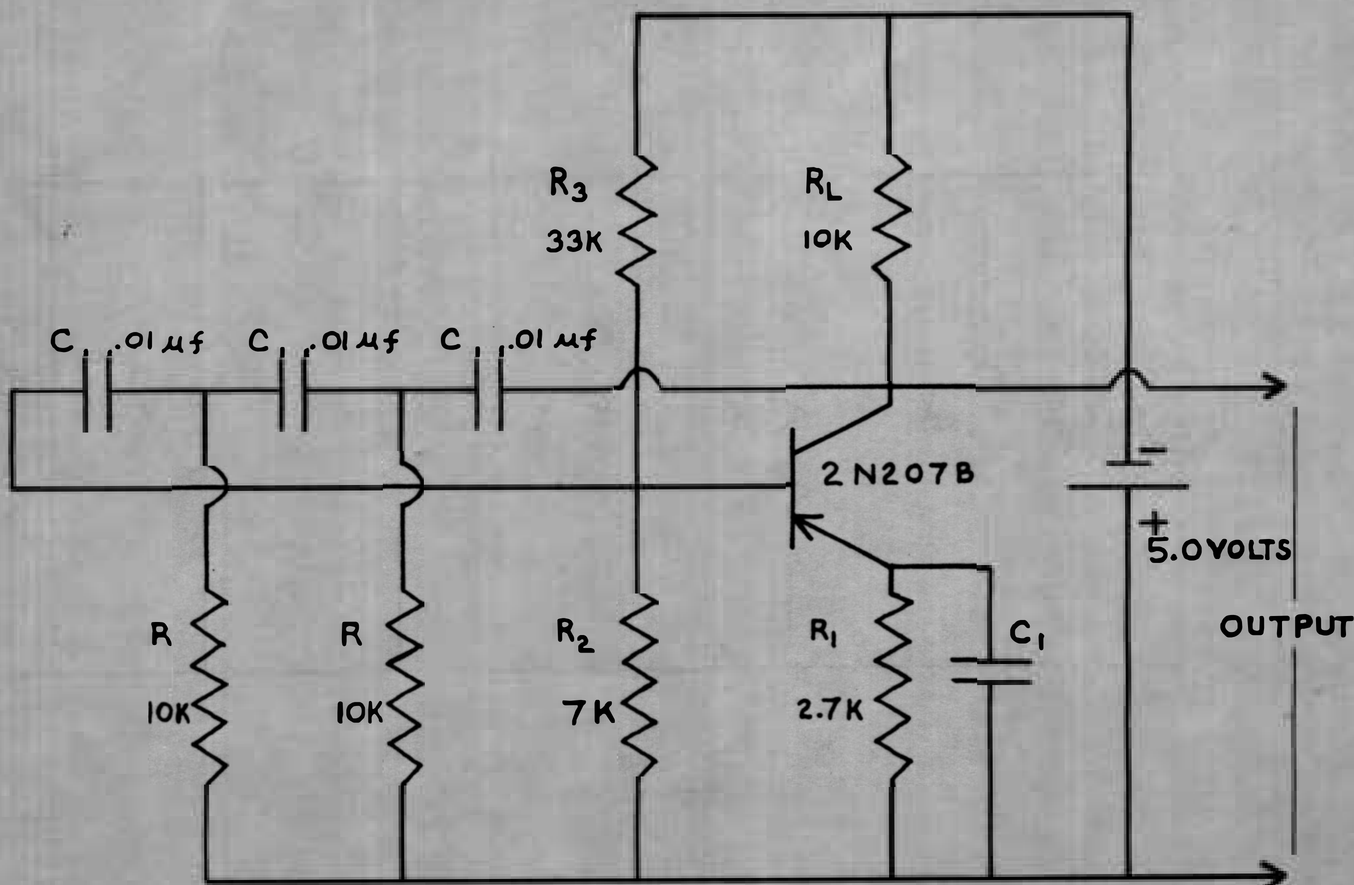


Figure 14. Three R-C Section Phase-Shift Oscillator

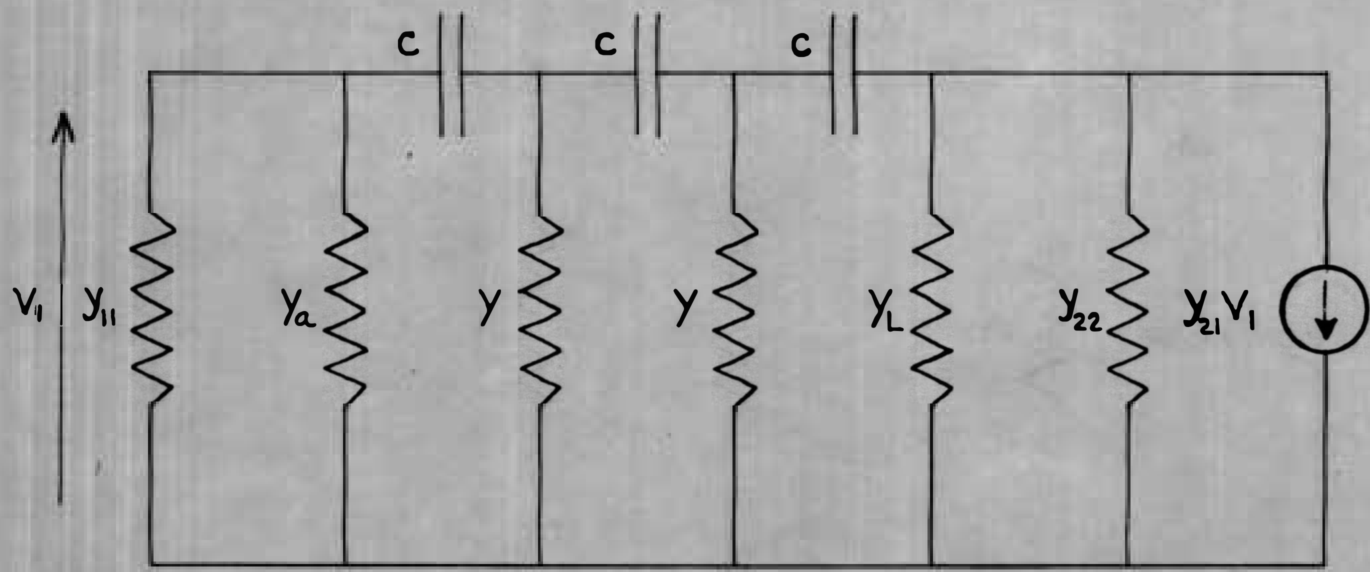


Figure 15. Equivalent Circuit of the Three R-C Section Phase-Shift Oscillator

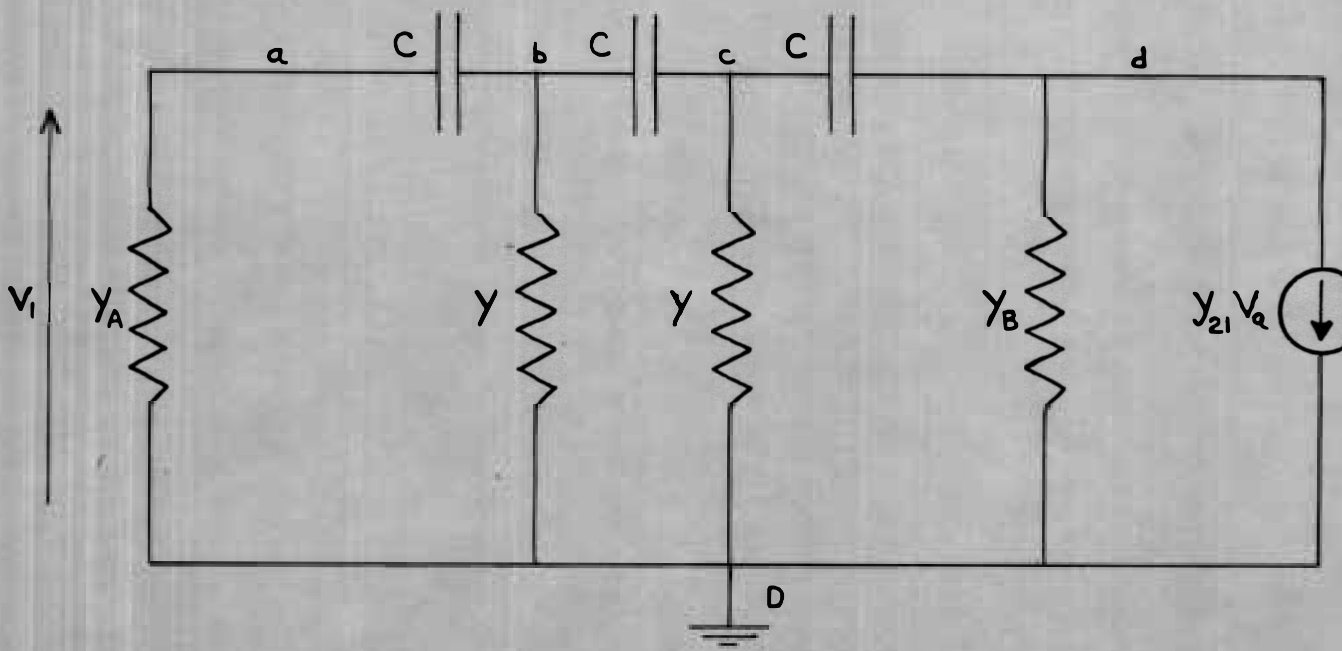


Figure 16. Simplified Equivalent Circuit of the Three R-C Section Phase-Shift Oscillator

are represented by a, b, c, and d; the reference is designated as D. The voltages at the respective nodes are V_a, V_b, V_c, and V_d. The operator, $S = jw$, is used to write the admittance of the capacitor as SC. The nodal equations of this circuit are:

$$0 = (Y_A + SC)\,V_a + (-SC)\,V_b + 0 + 0; \qquad (1a)$$

$$0 = (-SC)\,V_a + (Y + 2SC)\,V_b + (-SC)\,V_c + 0; \qquad (1b)$$

$$0 = 0 + (-SC)\,V_b + (Y + 2SC)\,V_c + (-SC)\,V_d; \text{ and} \qquad (1c)$$

$$0 = (y_{21})\,V_a + 0 + (-SC)\,V_c + (Y_B + SC)\,V_d. \qquad (1d)$$

The parameter y_{21} should not be interpreted as a driving function. It represents the forward short circuit transfer admittance as defined in Appendix 2. If the voltage V_1 were applied by an external driving function, then the representation of $y_{21}V_1$ would be that of a driving function in the transistor y parameter equivalent circuit. However, in this case the only external connection is that of an R-C feedback network and a battery. Since the equivalent circuit represents only those components necessary for A.C. analysis, the battery is not shown. The steady state solution of the circuit in oscillation is the desired result of the analysis. The transient solution with initial conditions is not considered. Thus, in considering the steady state solution of this circuit, there are no excitation functions.[20]

The system determinant Υ of the circuit can be written from equations 1a, 1b, 1c, and 1d as:

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[20] M. F. Gardner and J. L. Barnes, Transients in Linear Systems, p. 132, John Wiley & Sons: New York, 1942.

$$\mathbf{Y} = \begin{vmatrix} Y_A+SC & -SC & 0 & 0 \\ -SC & Y+2SC & -SC & 0 \\ 0 & -SC & Y+2SC & -SC \\ y_{21} & 0 & -SC & Y_B+SC \end{vmatrix} .$$

The system determinant would be the denominator in the solution for any of the node voltages.[21] However, all of the simultaneous equations are equated to zero which means that this is a homogeneous set of equations. Thus, nontrivial solutions exist only if $\mathbf{Y}$ is zero.[22] If $\mathbf{Y}$ is equated to zero and then expanded, the result is called the characteristic equation.[23] The values of S resulting from the solution of the characteristic equation are called characteristic values.[24] These characteristic values are the zeros of $\mathbf{Y}$. Expanding $\mathbf{Y}$ and collecting terms leads to:

$$\begin{aligned} \mathbf{Y} = {} & C^3(Y_B + 2Y + Y_A + y_{21})S^3 \\ & + C^2(Y^2 + 3YY_B + 3Y_AY_B + 3YY_A)S^2 \\ & + C(YY_B{}^2 + Y^2Y_A + 4YY_AY_B)S \\ & + Y^2\, Y_AY_B = 0. \end{aligned} \qquad (2)$$

Equation 2 can be expressed more simply by representing each coefficient of S by a symbol:

$$A_3S^3 + A_2S^2 + A_1S + A_0 = 0$$

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[21] M. E. Van Valkenburg, Modern Network Synthesis, pp. 9-11, John Wiley & Sons: New York, 1960.

[22] E. A. Guillemin, The Mathematics of Circuit Analysis, p. 17, John Wiley & Sons: New York, 1949.

[23] Ibid., p. 111.

[24] Ibid.

where,

$$A_3 = C^3(Y_B + 2Y + Y_A + y_{21});$$
$$A_2 = C^2(Y^2 + 3YY_B + 3Y_AY_B + 3YY_A);$$
$$A_1 = C\ (YY_B{}^2 + Y^2\ Y_A + 4YY_AY_B); \qquad \text{and}$$
$$A_0 = \quad Y^2\ Y_AY_B.$$

These coefficients can be simplified by assuming the load resistance R_L to be of the same value as R. The expression for Y_B then becomes $Y_B = Y + y_{22}$. The value of Y will normally be assumed great enough, as compared to y_{22}, so that the approximation $Y_B = Y$ can be used. The coefficients of the characteristic equation then become:

$$A_3 = C^3(3Y + Y_A + y_{21}); \tag{3a}$$
$$A_2 = C^2Y(6Y_A + 4Y); \tag{3b}$$
$$A_1 = CY^2(5Y_A + Y); \qquad \text{and} \tag{3c}$$
$$A_0 = Y^3\ Y_A. \tag{3d}$$

Applying the conclusions reached in Appendix 3 to this characteristic equation, the condition for sustained oscillation is that:

$$A_0A_3 = A_1A_2. \tag{4}$$

By substituting, the coefficient expressions 3a, 3b, 3c and 3d into equation 4 and multiplying, equation

$$C^3Y^3(3YY_A + Y_A{}^2 + y_{21}\ Y_A) = C^3Y^3(30Y_A{}^2 + 20YY_A + 6YY_A + 4Y^2) \tag{5}$$

is obtained. This equation can be simplified and solved for y_{21} to give:

$$y_{21} = 29Y_A + 23Y + 4\ \frac{Y^2}{Y_A}\ . \tag{6}$$

In Appendix 2, an expression for h_{21} in terms of y parameters is shown to be:

$$h_{21} = \frac{y_{21}}{y_{11}} .$$

The value of h_{21} at which sustained oscillations can occur is then:

$$h_{21} = 29 \frac{Y_A}{y_{11}} + 23 \frac{Y}{y_{11}} + 4 \frac{Y^2}{Y_A y_{11}} . \tag{7}$$

From the expression for instability developed in Appendix 3, equation 7 can be shown to give the minimum value of h_{21} at which the circuit will oscillate. The expression for instability of this circuit is:

$$h_{21} > 29 \frac{Y_A}{y_{11}} + 23 \frac{Y}{y_{11}} + 4 \frac{Y^2}{Y_A y_{11}} . \tag{8}$$

If the value of h_{21} is less than the right-hand expression, the circuit will not oscillate. However, if h_{21} is greater than the right-hand expression, the circuit will oscillate; but the resulting sinusoidal wave will be somewhat distorted. This expression for h_{21} was derived using the same basic approach to the circuit as used by Trokhimenko.[25] The final result is somewhat different, however, because Trokhimenko made the additional assumption that $Y_A = y_{11}$. This assumption does not seem to be justified for the general case and certainly not for the experimental circuits used in this study. The value of Y_A was over twice the value of y_{11} in the experimental circuits. These experimental circuits operated with a very low emitter current. The value of h_{11} has been found to increase as the emitter current decreases.[26] Thus, by the conclusions of Appendix 2, the value of y_{11} will decrease. By

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[25] Trokhimenko, op. cit., pp. 57-62.

[26] Shea, op. cit., pp. 47-48.

applying this result to relationship 8, the value required of h_{21} is seen to increase as the emitter current is decreased.

The minimum value of h_{21} is calculated for the circuit in Figure 14, based on the assumptions of the previous derivation. The value of C_1 is assumed large enough to form a perfect A.C. bypass around R_1. Thus, C_1 and R_1 do not appear in the equivalent circuit. For this approximation, C_1 should be at least 30 uf. The values of the circuit admittances to be used in the calculation are:

$$Y = (10)^{-4}\mho;\ Y_a = 1.73(10)^{-4}\mho;\ \text{and}\ Y_A = 3.07(10)^{-4}\mho.$$

A value of $y_{11} = 1.34(10)^{-4}\mho$ is chosen based on the results in Appendix 2. This leads to the value given for Y_A. Expression 8 is rewritten and followed by a numerical calculation based on the given values. The value of h_{21} is calculated as:

$$h_{21} > 29\frac{Y_A}{y_{11}} + 23\frac{Y}{y_{11}} + \frac{4Y^2}{Y_A y_{11}}$$

$$h_{21} > 66.5 + 17.2 + 1.0$$

$$h_{21} > 84.7.$$

Most transistors have a value of h_{21} which is less than 84.7. Thus, it would be difficult to find a transistor suitable for use in this circuit. An experimental circuit, having the same component values as given in Figure 14, was constructed. The 2N207B transistor has a large h_{21} when compared to most transistors. Of the 2N207B transistors available, the one with the largest h_{21} was connected into the circuit. The value of h_{21} for this transistor was calculated to be about 81. The circuit would not oscillate with this transistor or with another 2N207B transistor having a lower h_{21} of about 55.

If a transistor with a large enough h_{21} were available to enable the circuit to oscillate, the frequency of oscillation could be predicted by the relationship derived in Appendix 3. For a third degree characteristic equation, this relationship is:

$$f = \frac{1}{2\pi}\sqrt{\frac{A_0}{A_2}} \quad . \qquad (9)$$

Substituting into equation 9 the algebraic values of A_0 and A_2 from expressions 3d and 3b, leads to:

$$f = \frac{1}{2\pi} \frac{Y}{C} \sqrt{\frac{Y_A}{6Y_A + 4Y}} \quad . \qquad (10)$$

Three R-C Section Oscillator with the Phase-Shift Effect of C_1 Included

During the process of experimentation, the circuit in Figure 14 was found to oscillate if the bypass capacitor C_1 were reduced in value to about 1 uf. This was surprising since a decrease in C_1 would lower the gain of the amplifier. Lowering the gain of the amplifier would then make it more difficult to obtain the total loop gain of unity required for oscillation. The suspected explanation of this behavior was that a decrease in C_1 caused an additional amount of phase-shift in the feedback network. The addition of an R-C section to the feedback network would decrease the amount of phase-shift required by each section. This had been found to decrease the attenuation in the feedback network; and therefore, less amplifier gain was required to produce unity loop gain.[27]

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[27] Ginzton and Hollingsworth, op. cit., p. 47.

Thus, decreasing C_1 to about 1 uf evidently added another phase-shift section and lowered the attenuation in the feedback network. These suspicions were found to be true by experimentally noting the increase in phase-shift around the circuit open loop when C_1 was decreased. As C_1 was decreased, the amplifier gain was found to decrease, as had been expected. Evidently, the decrease in the feedback network attenuation was considerably greater than the decrease in amplifier gain. This conclusion then inferred that it would be possible to operate an R-C phase-shift oscillator with a much lower gain amplifier than had previously been expected. Since the current gain of a transistor amplifier is directly related to the value of h_{21},[28] the value of h_{21} required by the transistor is reduced.

Based on the above conclusions, a completely new analysis of the R-C phase-shift oscillator can be made. This analysis will use the assumption that C_1 has a definite effect on the equivalent circuit. However, it will be approximated that the reactance of C_1 is considerably smaller than the resistance of R_1. This enables R_1 to be neglected in the equivalent circuit. The approximation can be justified by using typical values from an experimental circuit. Let the value of R_1 be 2.7K and that of C_1 be 1 uf. If the frequency is assumed to be at 1000 cps, then the reactance of C_1 is 159Ω. Thus, in the parallel combination, the reactance of C_1 is 17 times less than the resistance of R_1; and R_1 can be neglected. The assumptions of the previous analysis in which y_{12} and y_{22} were neglected can apply here. The relationship $R_L = R$ can also be assumed. The equivalent circuit of Figure 14,

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[28] Shea, _op_. _cit_., p. 74.

including these assumptions, is shown in Figure 17. The objectives which are sought in the analysis of this circuit are: (1) the minimum value of h_{21} required for the circuit to oscillate; (2) the frequency of sustained oscillation; and (3) a relationship between C and C_1 that will enable the upper and lower limiting values of C_1 to be predicted.

The nodal equations of the circuit in Figure 17 can now be written. The relationship, $V_1 = V_e - V_a$, should be noted in writing y_{21} into the equations. The equations are:

$$0 = (y_{11} + Y_a + SC)V_a + (-SC)V_b + 0 + 0 + (-y_{11})V_e; \quad (11a)$$

$$0 = (-SC)V_a + (Y + 2SC)V_b + (-SC)V_c + 0 + 0; \quad (11b)$$

$$0 = 0 + (-SC)V_b + (Y + 2SC)V_c + (-SC)V_d + 0; \quad (11c)$$

$$0 = (y_{21})V_a + 0 + (-SC)V_c + (SC + Y)V_d + (-y_{21})V_e; \quad \text{and} \quad (11d)$$

$$0 = (-y_{11} - y_{21})V_a + 0 + 0 + 0 + (SC_1 + y_{11} + y_{21})V_e \,. \quad (11e)$$

In writing the coefficients of the node voltages into a determinant, the approximation that y_{21} is much greater than y_{11} will be made. This is justified by the values calculated in Appendix 2. The magnitude of y_{21} was found to be $0.73(10)^{-2}$ while that of y_{11} was $1.34(10)^{-4}$. Thus, the value of y_{11} has a negligible effect on the magnitude of the coefficients containing y_{21}; and y_{11} is neglected in these coefficients. The system determinant Υ of the circuit can now be written from equations 11a, 11b, 11c, 11d, and 11e as:

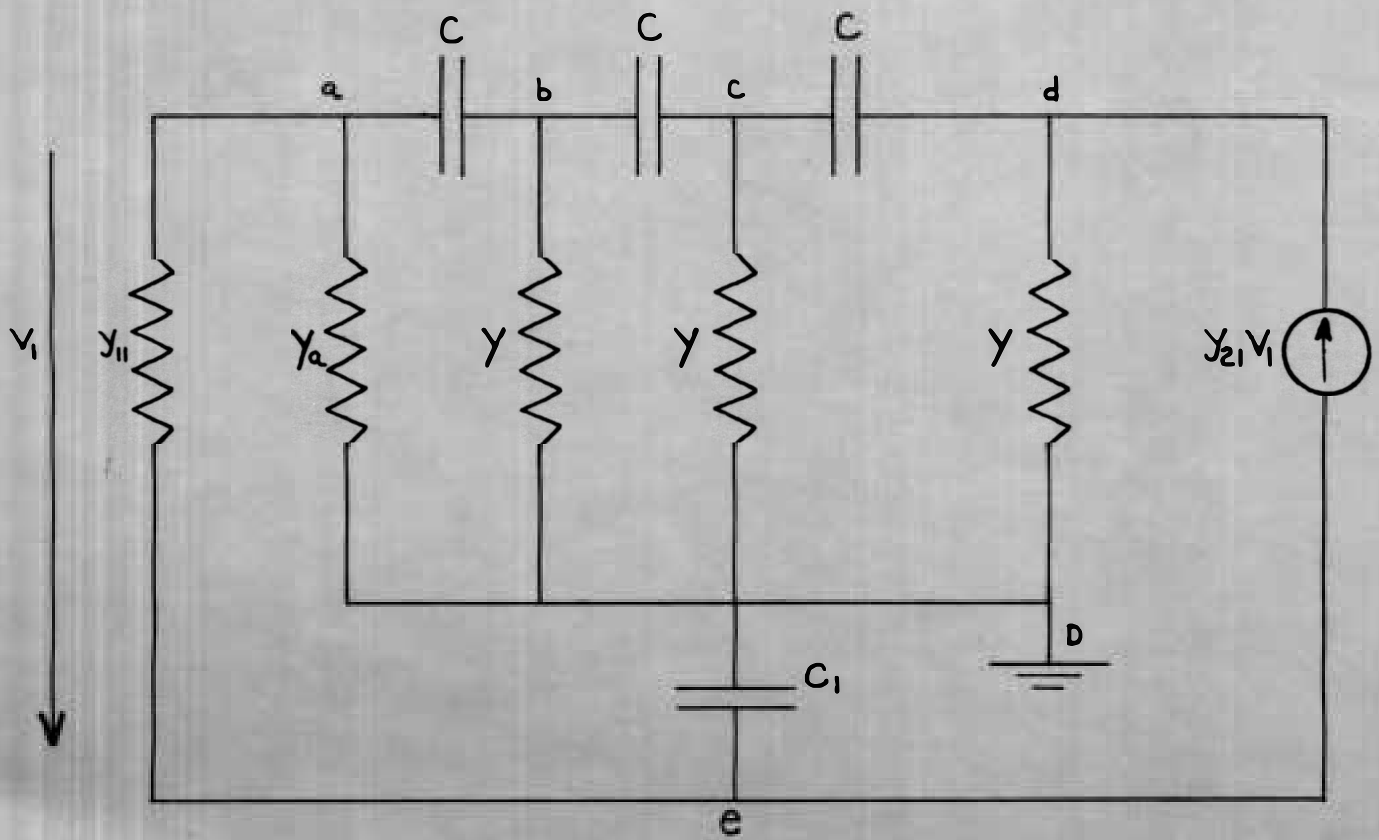

Figure 17. Equivalent Circuit of Phase-Shift Oscillator Including C_1

$$\mathbf{Y} = \begin{vmatrix} y_{11}+Y_a+SC & -SC & 0 & 0 & -y_{11} \\ -SC & Y+SC & -SC & 0 & \\ 0 & -SC & Y+2SC & -SC & 0 \\ y_{21} & 0 & -SC & Y+SC & -y_{21} \\ -y_{21} & 0 & 0 & 0 & SC_1+y_{21} \end{vmatrix} .$$

As shown previously, the characteristic equation of the system is obtained by expanding $\mathbf{Y}$ and then equating the result to zero.[29] Expanding $\mathbf{Y}$ and collecting terms leads to:

$$\begin{aligned} \mathbf{Y} = {} & C^3C_1\left[y_{11} + Y_a + y_{21} + 3Y\right] S^4 \\ & + C^2\left[C_1Y(6y_{11} + 6Y_a + 4Y) + Cy_{21}(2Y_a + 3Y)\right] S^3 \\ & + C\left[C_1Y^2(5y_{11} + 5Y_a + Y) + Cy_{21}Y(6Y_a + 4Y)\right] S^2 \\ & + \left[C_1Y^3(y_{11} + y_a) + Cy_{21}Y^2(5Y_a + Y)\right] S \\ & + Y^3Y_ay_{21} = 0. \end{aligned} \tag{12}$$

At this point, additional approximations will be made to reduce the number of terms in the coefficients of equation 12. The value of y_{21} will be considered sufficiently greater than $(y_{11} + Y_a + 3Y)$ that this sum of terms may be neglected in the coefficient of S^4. The approximation that $Y_a = Y_{11} = Y$ will also be made. Considerable error could possibly be introduced by this approximation, and caution must be used in its use. However, it enables the coefficients of equation 12 to be simplified to a large extent. The characteristic equation can be written as

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[29] Guillemin, op. cit., p. 111.

$$\mathbf{Y} = A_4S^4 + A_3S^3 + A_2S^2 + A_1S + A_0 = 0.$$

By using the approximations, the expressions for A_4, A_3, A_2, A_1, and A_0 may be simplified as:

$$A_4 = C^3C_1y_{21}; \quad (13a)$$

$$A_3 = C^2(16C_1Y^2 + 5CYy_{21}); \quad (13b)$$

$$A_2 = C(11C_1Y^3 + 10CY^2y_{21}); \quad (13c)$$

$$A_1 = 2C_1Y^4 + 6CY^3y_{21}; \text{ and} \quad (13d)$$

$$A_0 = y_{21}Y^4. \quad (13e)$$

As shown in Appendix 3, the relationship for sustained oscillation to occur in this system is that:

$$A_1(A_3A_2 - A_4A_1) - A_3^2A_0 = 0. \quad (14)$$

Expressions 13a, 13b, 13c, 13d and 13e can be substituted into equation 14 and the result arranged to give:

$$352Y^3C_1^3 + Y^2y_{21}C_1^2(1230C - 4C_1) + Yy_{21}^2CC_1(1230C - 24C_1) +$$
$$y_{21}^3C^2(275C - 36C_1) = 0. \quad (15)$$

The approximation that $y_{11} = Y$ will be used to obtain the expression:

$$h_{21} = \frac{y_{21}}{y_{11}} = \frac{y_{21}}{Y}. \quad (16)$$

Equation 15 can now be divided by Y^3, and expression 16 can be applied to give:

$$C^2(275C - 36C_1)h_{21}^3 + CC_1(1230C - 24C_1)h_{21}^2 +$$
$$C_1^2(1230C - 4C_1)h_{21} + 352C_1^3 = 0. \quad (17)$$

Equation 17 can be solved to give the minimum value of h_{21} required for oscillation if the values of C and C_1 are known. This theoretical prediction of h_{21} is an approximation that increases in error as the difference between Y and y_{11} increases. A greater degree of accuracy could be obtained by solving equation 14 directly from the values found for the coefficients of equation 12. However, this approach is quite laborious, and it will not be used in this case.

The approximate value of h_{21} required for oscillation in a circuit having the values C = .01 uf and C_1 = 1 uf will be determined. This calculation assumes that the approximations used in arriving at equation 17 are applicable to the circuit. Substituting the values of C and C_1 into equation 17, and then dividing this equation by the coefficient of h_{21}^3 gives:

$$h_{21}^3 + 35.1h_{21}^2 - 25.2(10)^2 h_{21} - 10.6(10)^4 = 0. \qquad (18)$$

The solution of equation 18 has one real root and two imaginary roots. The value of the real root was found to be $h_{21} = 52$. Using the circuit shown in Figure 14, oscillation was obtained by letting C_1 = 1 uf. A 2N207B transistor which had an $h_{21} = 55$ was used. The determination of h_{21} for this transistor is shown in Appendix 2. The oscillator's signal, as viewed on an oscilloscope, appeared to be perfectly sinusoidal. This result is in agreement with the prediction that the circuit would oscillate by using a transistor with an h_{21} of at least 52.

The relationship for the frequency of oscillation in a system having a fourth degree characteristic equation is shown in Appendix 3 to be:

$$f = \frac{1}{2\pi} \sqrt{\frac{A_1}{A_3}} . \qquad (18)$$

The expressions for A_1 and A_3 that will be used are those of equation 12. Substituting these coefficient expressions directly into equation 18 gives:

$$f = \frac{1}{2\pi C} \sqrt{\frac{C_1Y^3(y_{11} + Y_a) + Cy_{21}Y^2(5Y_a + Y)}{C_1Y(6y_{11} + 6Y_a + 4Y) + Cy_{21}(2Y_a + 3Y)}} . \qquad (19)$$

The frequency of the circuit in Figure 18 was varied by adjusting the value of the two variable resistors shown in the feedback network. The two resistors were always adjusted to identical values while R_L, the 10 K load resistor, remained fixed. The method used to vary the frequency of the phase-shift audio oscillator in the temperature telemetry unit used this method by replacing the two resistors with thermistors. The method that seemed to be the most suitable for utilizing this phase-shift oscillator as a variable frequency oscillator was that of varying the value of the two R-C section resistors. The method of varying frequency by changing the value of capacitance in the R-C section is not considered although this method may be of advantage in some applications.

A comparison between the frequency predicted by relationship 19 and the frequency experimentally obtained with the circuit of Figure 18 is shown in Figure 19. Relationship 19 was derived with the assumption that R_L and the two R-C section resistors were always equivalent. This is not true in the actual experimental circuit since R_L is held constant while the two R-C section resistors are varied. Although the above discrepancy exists between the theoretical and experimental assumptions, relationship 19 gives a useful approximation of the experimentally

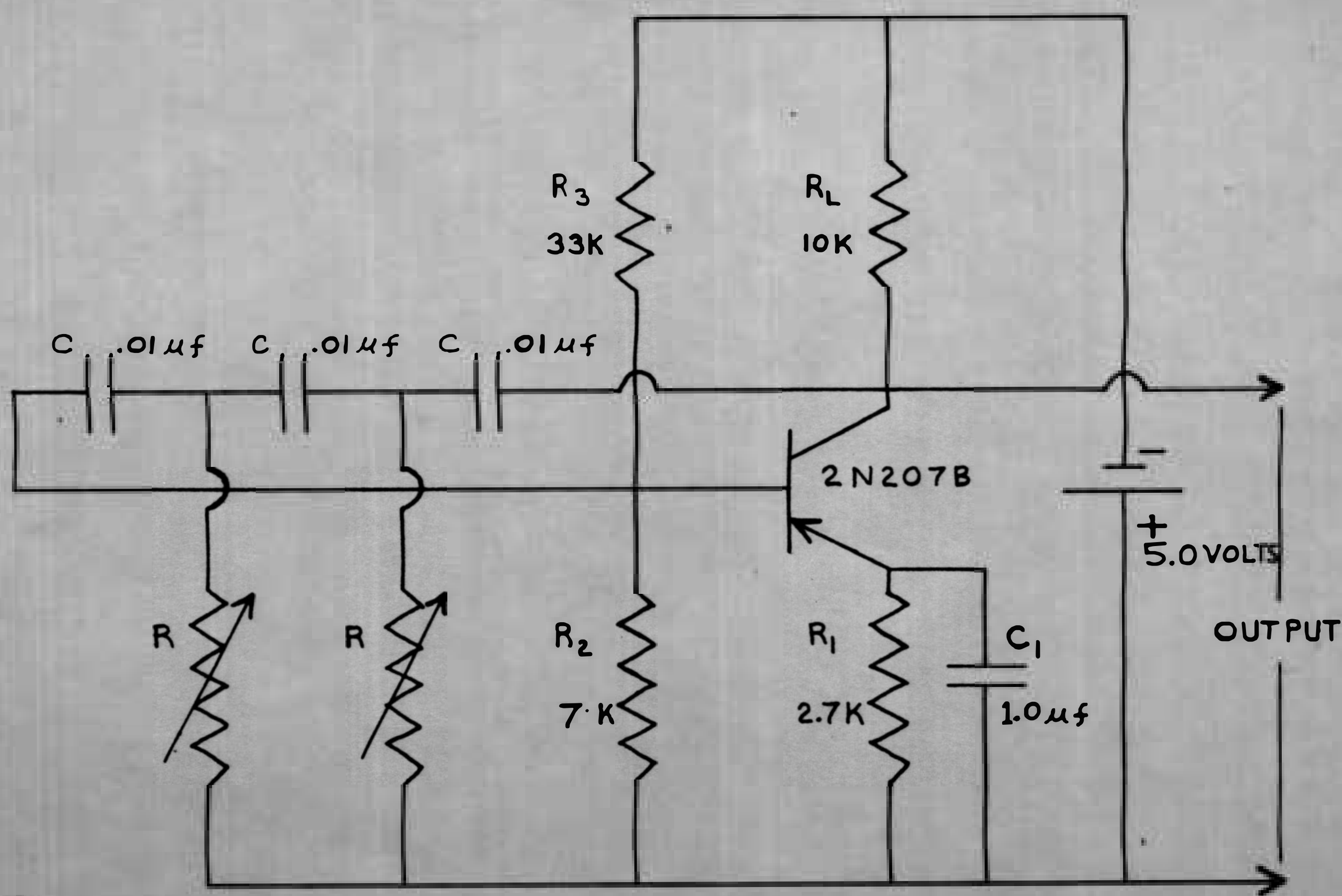

Figure 1[illegible]. Variable Frequency Phase-Shift Oscillator

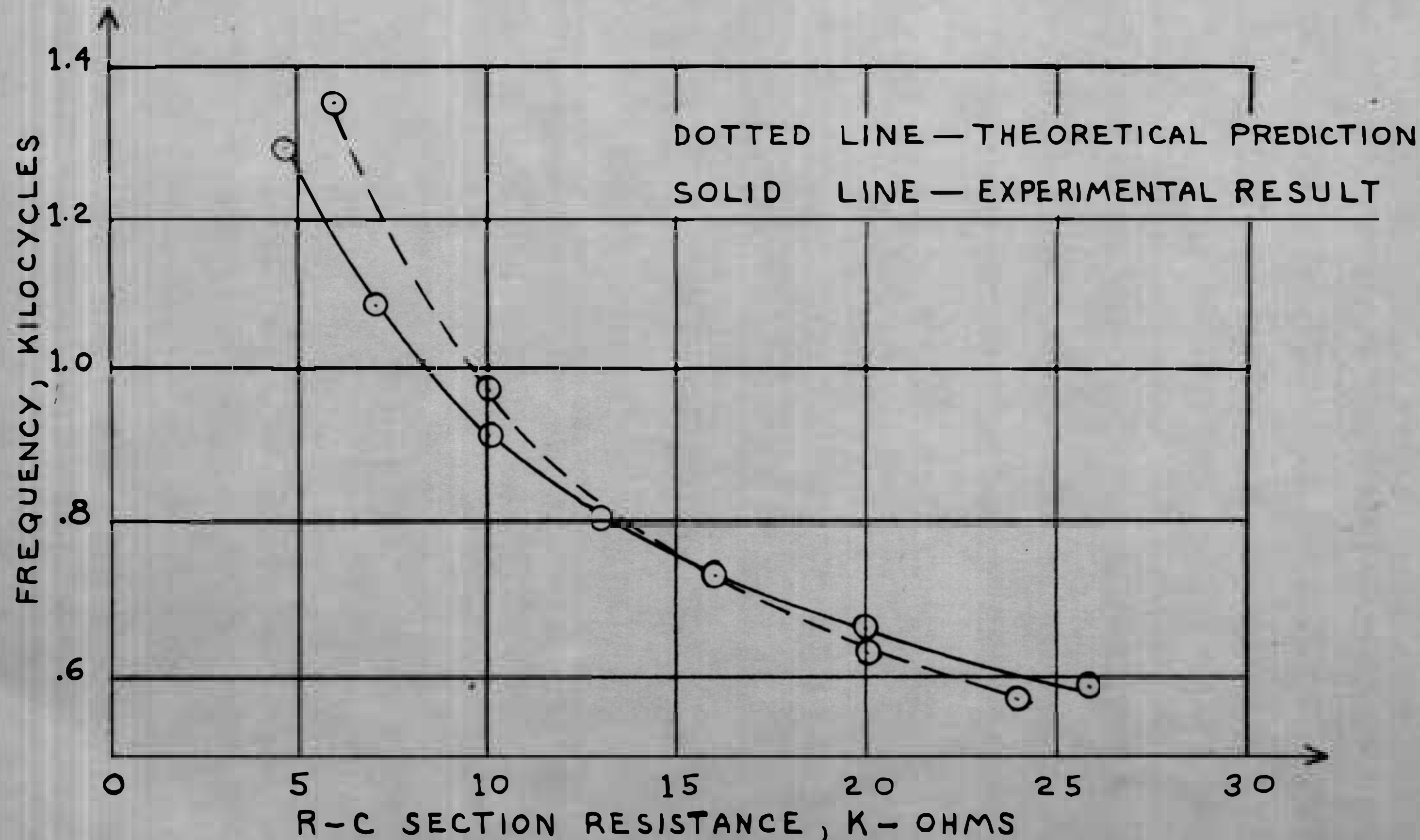

Figure 19. Theoretical and Experimental Frequency Variation in the Three R-C Section Phase-Shift Oscillator

determined frequency.

A relationship will now be derived which will be useful in predicting the upper and lower bounding values of C_1 between which oscillation can occur. A relationship between C and C_1 will be defined by the equation:

$$C = KC_1. \qquad (20)$$

Equation 20 is then substituted into equation 17. After regrouping, the relationship for oscillation becomes:

$$275h_{21}^3K^3 + (1230h_{21}^2 - 36h_{21}^3)K^2 + (1230h_{21} - 24h_{21}^2)K + (352 - 4h_{21}) = 0. \qquad (21)$$

Thus, for a transistor which has a known value of h_{21} the value of K can be determined by solving equation 21. Then if the value of C in the R-C feedback network is known, the values of C_1 that will allow the circuit to oscillate can be calculated. The value of h_{21} for the transistor in Figure 18 is 55. After substituting this value into equation 21, the result may be arranged to give:

$$K^3 - 4.97(10)^{-2}K^2 - 1.09(10)^{-4}K + 2.88(10)^{-6} = 0. \qquad (22)$$

Solving this equation leads to two positive roots and one negative root. All the roots are real, and the values of the positive roots are:

$$K = 5.08(10)^{-2}; \text{ and } K = .67(10)^{-2}.$$

The upper bound C_{1a} and the lower bound C_{1b} can now be found from equation 20 by using the value C = .01 uf. These bounds are:

$$C_{1a} = 1.5 \text{ uf}; \text{ and } C_{1b} = .197 \text{ uf}.$$

Thus, oscillation is predicted to occur only if C_1 is within these bounds.

The upper and lower bounding values of C_1 for the circuit in

Figure 18 were found experimentally by determining the family of curves shown in Figure 20. Each curve was obtained for a specific value of C_1 by varying the R-C section resistors and recording the frequency. The end of each curve represents the point just before the circuit will cease to oscillate. As can be seen on Figure 20, the allowable range of R-C section resistance decreases as C_1 is either increased or decreased from about 1 uf. The curves then narrow until they become the points indicated as C_{1a} and C_{1b} which are the experimentally determined upper and lower bounding values of C_1. The results obtained experimentally compare favorably with the theoretical predictions. The prediction of an approximate value for C_1 is useful in designing a circuit because the range of C_1 that will allow oscillation may be quite narrow. The best choice for C_1 is near the middle of the range, and a value of C_1 = 1 uf was found to give satisfactory results for the circuit in Figure 18.

<u>A Phase-Shift Oscillator Using Two R-C Sections</u>

Because of the additional phase-shift due to C_1, the operation of an oscillator with only two R-C feedback sections was suspected to be possible. The circuit shown in Figure 21 was constructed and found to operate. This oscillator was very stable, and its sinusoidal signal appeared undistorted.

An analysis of this circuit will be made by assuming that $R_L = R$. The transistor parameters y_{12} and y_{22} are neglected. The resistors R_2 and R_3 form a parallel combination in an equivalent circuit, and the value of this parallel combination can be represented by the admittance Y_a. The equivalent circuit of Figure 21 is shown in

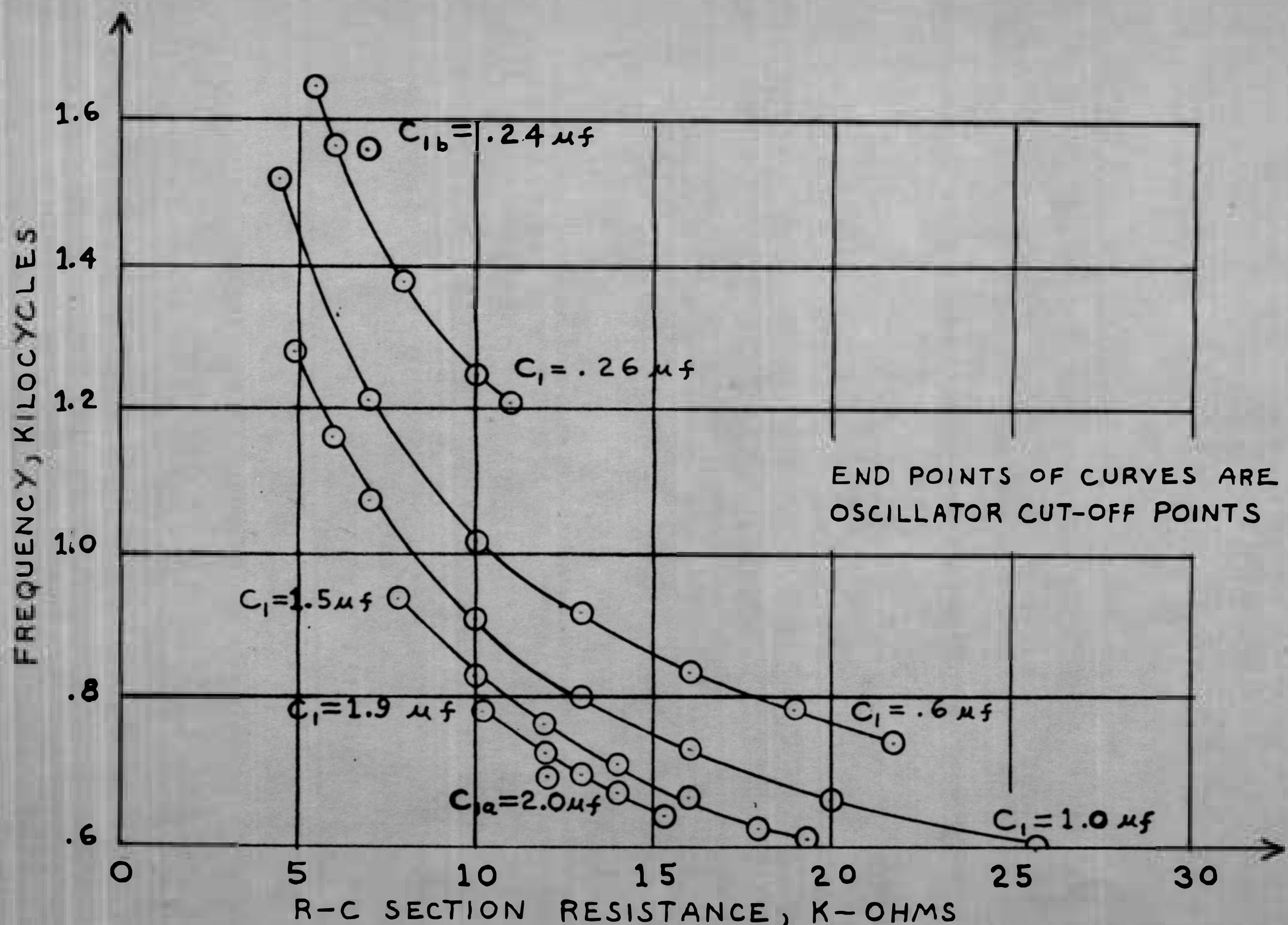

Figure 20. Family of Curves Showing the Effect of C_1

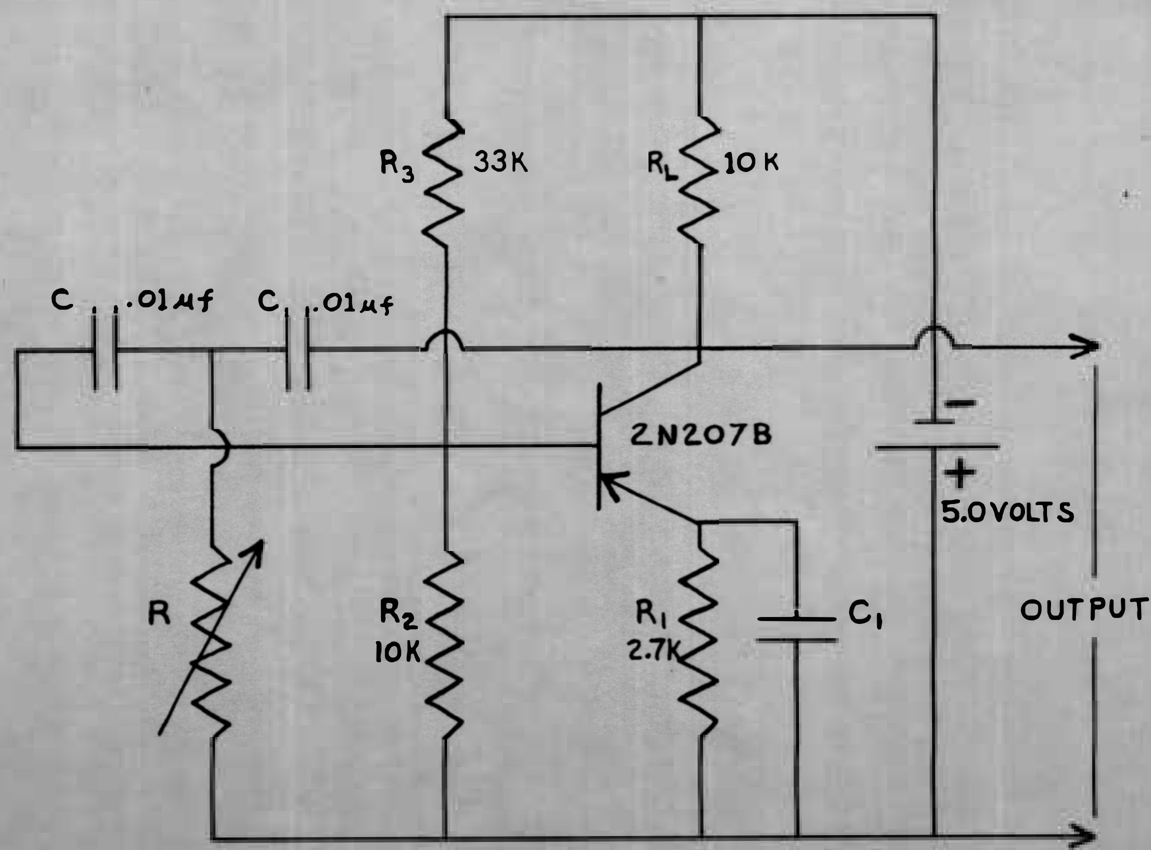

Figure 21. Two R-C Section Phase-Shift Oscillator

Figure 22.

The nodal equations of the circuit in Figure 22 can now be written. The relationship, $V_1 = V_d - V_a$, should be noted when writing y_{21} into the equations. The nodal equations are:

$$0 = (y_{11} + Y_a + SC)V_a + (-SC)V_b + 0 + (-y_{11})V_d; \qquad (23a)$$

$$0 = (-SC)V_a + (Y + SC)V_b + (-SC)V_c + 0; \qquad (23b)$$

$$0 = (y_{21})V_a + (-SC)V_b + (Y + SC)V_c + (-y_{21})V_d; \quad \text{and} \qquad (23c)$$

$$0 = (-y_{11} - y_{21})V_a + 0 + 0 + (SC_1 + y_{11} + y_{21})V_d. \qquad (23d)$$

The system determinant Y of the circuit can now be written from equations 23a, 23b, 23c, and 23d as:

$$\mathrm{Y} = \begin{vmatrix} (y_{11}+Y_a+SC) & (-SC) & 0 & (-y_{11}) \\ (-SC) & (Y+SC) & (-SC) & 0 \\ (y_{21}) & (-SC) & (Y+SC) & (-y_{21}) \\ (-y_{11}-y_{21}) & 0 & 0 & (SC_1+y_{11}+y_{21}) \end{vmatrix}.$$

As shown previously,[30] the characteristic equation of the system is obtained by equating Y to zero and expanding the determinant. Expanding Y and collecting terms leads to equation:

$$\begin{aligned} & C^2C_1\left[2Y + y_{11} + Y_a + y_{21}\right] S^3 \\ & + C\left[C_1Y(Y + 3y_{11} + 3Y_a) + C(2y_{11}Y + 2y_{21}Y + Y_ay_{11} + Y_ay_{21})\right] S^2 \\ & + Y\left[C_1Y(Y_a + y_{11}) + C(Yy_{11} + 2Y_ay_{11} + Yy_{21} + 3Y_ay_{21} + Y_ay_{11})\right] S \\ & + Y^2Y_a(Y_{11} + y_{21}) = 0. \end{aligned} \qquad (24)$$

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[30] Guillemin, op. cit., p. 111.

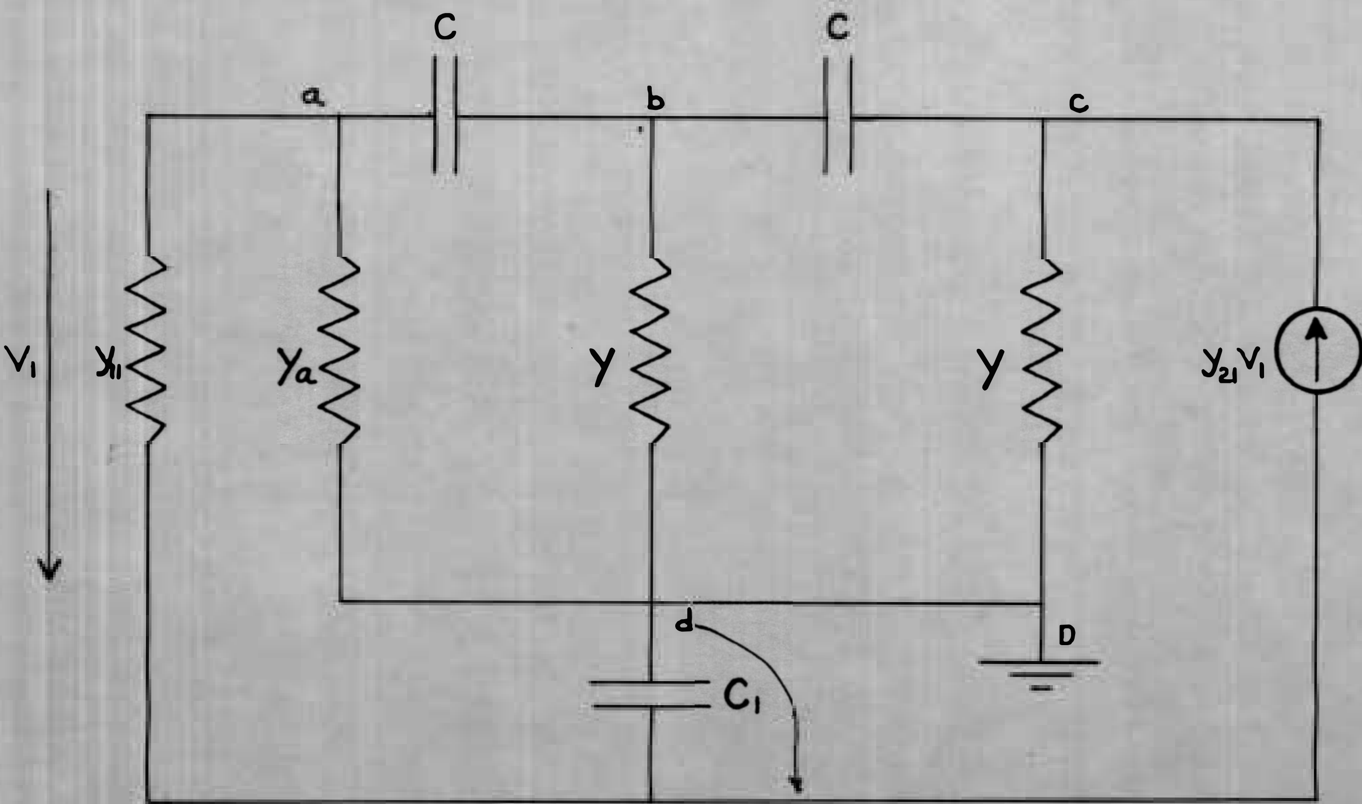

Figure 22. Equivalent Circuit of the Two R-C Section Phase-Shift Oscillator

Equation (24) can be written in the form:

$$A_3S^3 + A_2S^2 + A_1S + A_0 = 0.$$

The coefficients of the characteristic equation can be simplified by assuming that y_{21} is large compared to Y, Y_a, and y_{11}. By comparing the relative magnitudes of the terms in each coefficient of equation 24, the values of A_0, A_1, A_2, and A_3 may be simplified to give:

$$A_0 = Y^2Y_ay_{21}; \quad (25a)$$

$$A_1 = Y\left[C_1Y(Y_a + y_{11}) + Cy_{21}(Y + 3Y_a)\right]; \quad (25b)$$

$$A_2 = C\left[C_1Y(Y + 3y_{11} + 3Y_a) + Cy_{21}(2Y + Y_a)\right]; \text{ and} \quad (25c)$$

$$A_3 = C^2C_1y_{21}. \quad (25d)$$

The relationship for sustained oscillation to occur in this system was shown in Appendix 3 to be:

$$A_0A_3 - A_1A_2 = 0. \quad (4)$$

This expression will be used to predict the minimum value of h_{21} that is required for the circuit to oscillate. For this derivation, the approximation that $Y = Y_a = y_{11}$ will be made. Expressions 25a, 25b, 25c, and 25d can be simplified to give:

$$A_0 = Y^3y_{21}; \quad (26a)$$

$$A_1 = 2Y^2(C_1Y + 2Cy_{21}); \quad (26b)$$

$$A_2 = CY(7C_1Y + 3Cy_{21}); \text{ and} \quad (26c)$$

$$A_3 = C^2C_1y_{21}. \quad (26d)$$

Expressions 26a, 26b, 26c, and 26d are then substituted into equation 4 and the terms arranged to give:

$$C(12C - C_1)y_{21}^2 + 34CC_1Yy_{21} + 14C_1^2Y^2 = 0. \quad (27)$$

The approximation that $y_{11} = Y$ will be used to obtain the relationship:

$$h_{21} = \frac{y_{21}}{y_{11}} = \frac{y_{21}}{Y} .$$

Equation 27 can now be divided by Y^2 and arranged to obtain:

$$C(12C - C_1)h_{21}^{\ 2} + 34CC_1h_{21} + 14C_1^{\ 2} = 0. \qquad (28)$$

Substituting the values $C = .01$ uf and $C_1 = 1$ uf into equation 28 gives:

$$h_{21}^{\ 2} - 38.6h_{21} - 1590 = 0. \qquad (29)$$

The value of the negative root of equation 29 is disregarded since h_{21} cannot be negative. The positive root has the value $h_{21} = 64$. The circuit in Figure 21 would not oscillate by using the 2N207B transistor with an $h_{21} = 55$, but it did oscillate by using another transistor of the same type having an $h_{21} = 81$. This result would be expected on the basis of the theoretical prediction.

The relationship for the frequency of oscillation in a system having a third degree characteristic equation is:

$$f = \frac{1}{2\pi}\sqrt{\frac{A_0}{A_2}} .$$

By substituting expressions 25a and 25c for A_0 and A_2, the relationship for the frequency of oscillation becomes:

$$f = \frac{Y}{2\pi}\sqrt{\frac{Y_a y_{21}}{C\left[C_1Y(Y + 3y_{11} + 3Y_a) + Cy_{21}(2Y + Y_a)\right]}} . \qquad (30)$$

The frequency of the circuit in Figure 21 was varied by adjusting the value of the variable resistor in the feedback network while the 10K load resistor remained fixed. However, the variable resistor could be replaced by a thermistor. This would result in a circuit that could be used in temperature telemetry applications.

A comparison between the frequency predicted by equation 30 and the frequency found experimentally is shown in Figure 23. The derived relationship for frequency was based on the assumption that the load resistor and the variable resistor were equivalent. This is not true for the experimental case since the load resistance is held constant while the variable resistance changes. This discrepancy contributes to the error shown by Figure 23.

A relationship will now be derived that will be useful in predicting the upper and lower bounding values of C_1. Oscillation can occur only if C_1 is within these bounds. The expression:

$$C = KC_1 \qquad (20)$$

can be substituted into equation 28. After regrouping, the relationship for oscillation becomes:

$$12h_{21}^2K^2 + h_{21}(34 - h_{21})K + 14 = 0. \qquad (31)$$

The value of h_{21} for the transistor used in the circuit of Figure 21 is 81. After substituting this value of h_{21} into equation 31, the result may be simplified to give the expression:

$$K^2 - 4.82(10)^{-2}K + 1.77(10)^{-4} = 0. \qquad (32)$$

Solving equation 32 leads to two positive roots which are:

$$K = 4.41(10)^{-2}; \text{ and } K = .41(10)^{-2}.$$

Using the value $C = .01$ uf, the upper bound C_{1a} and lower bound C_{1b} can be found from equation 20. These bounds are:

$$C_{1a} = 2.44 \text{ uf; and } C_{1b} = .23 \text{ uf.}$$

The upper and lower bounds of C_1 for the circuit in Figure 21 were found

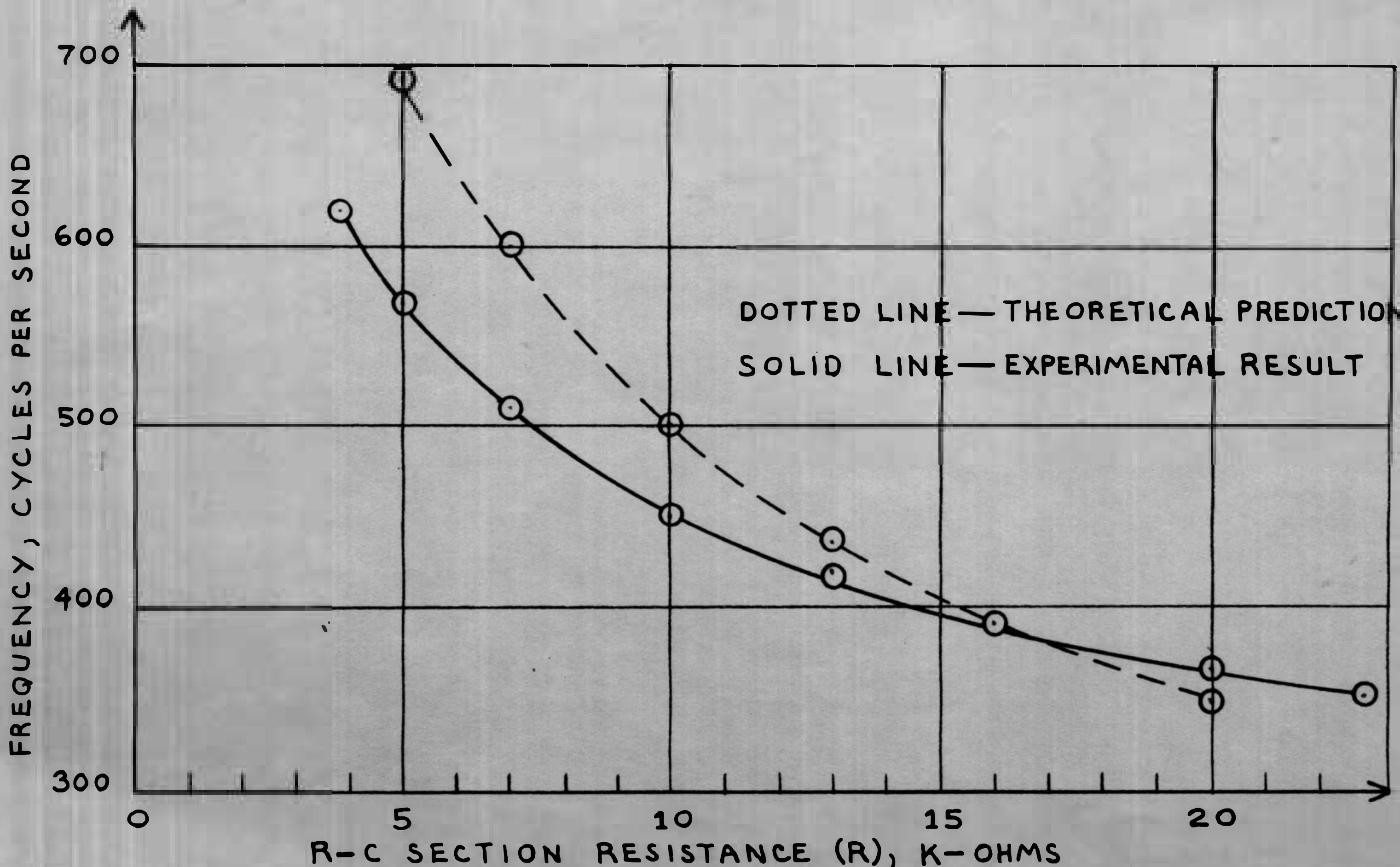

Figure 23. Theoretical and Experimental Frequency Variation in the Two R-C Section Phase-Shift Oscillator

experimentally to be:

$$C_{1a} = 2.6 \text{ uf; and } C_{1b} = .44 \text{ uf.}$$

The theoretical prediction gives a useful approximation of the value that C_1 must have so that the circuit will oscillate. The best choice for C_1 would be near the middle of the bounded range. For the circuit in Figure 21, a value of C_1 = 1 uf was found to give satisfactory results.

CONCLUSIONS

An amplitude modulated radio telemetry system was used successfully to measure the internal temperature of an animal. The system proved to be very easy for an investigator to use.

This temperature telemetry system is believed to be the first reported to accomplish:

(1) successful measurement of the internal temperature of an animal using an amplitude modulated radio telemetry system;

(2) successful measurement of the temperature in the rumen of a sheep during a 24-hour period without disturbing the animal or his environment;

(3) successful operation of a miniature transmitter unit with its components mounted within the core of the oscillator coil.

Since the audio frequency of the amplitude modulated transmitter unit is not sensitive to external media or mounting position, this unit could be mounted externally. The temperature could then be sensed by a miniature probe that contained only the two thermistors. The probe would be separated from the rest of the unit by several feet of thin plastic tubing, which would contain the thermistor wires. This arrangement would have excellent application to the continuous monitor of the rectal temperature of an unrestrained animal.

By using a higher gain transistor and/or an additional R-C section in the feedback network, the required battery voltage could possibly be reduced to 1.34 volts. This would enable the geometrical dimensions of the battery to be decreased, or it would allow a considerably longer operational life to be obtained from a battery of the present size. In addition, the value of the biasing resistors could probably be increased and adjusted so that the circuit would use less current.

Other than the thermistors, the 1 uf bypass capacitor was found to be the only component in the audio oscillator circuit that influenced the temperature data. A unit that is not temperature sensitive, except for its thermistors, is desirable. This would decrease the frequency response time of the unit and permit more accurate readings to be made with a thermistor probe application. Possibly a type of capacitor having similar geometrical dimensions but less temperature sensitivity, than the 1 uf tantalum capacitor of this unit, could be found.

Certain characteristics of a three R-C section phase-shift transistor oscillator were analyzed in this study. The oscillator was first considered to have a large enough bypass capacitor around the emitter resistor so that this biasing arrangement could be neglected in the equivalent circuit. The approach and the assumptions used in this study are believed to give a more accurate prediction of the required transistor h_{21} and the frequency of oscillation, than the predictions of previous investigators, especially when a low emitter current oscillator is involved. One of the most common previous methods of analysis is that given by Shea.[31] In Shea's analysis, the transistor is assumed to

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[31] Shea, *op. cit.*, pp. 222-226.

have zero input impedance. This assumption greatly simplifies the analysis by eliminating the transistor parameter h_{11} as well as the two biasing resistors connected to the base. However, according to the analysis used in this thesis, Shea's assumption is not valid, especially when applied to oscillators operating with a low emitter current.

By using a particular value of bypass capacitor around the emitter resistor, an additional phase-shift section was added to the feedback network. Thus, the bypass capacitor was used to serve a dual role. It served as an additional phase-shift section which would normally require two extra components, and it also functioned as an A.C. bypass around the emitter resistor. This enabled the three R-C section oscillator to operate with a much lower transistor h_{21} than it could without the additional phase-shift effect. The range of values within which the bypass capacitor must lie to produce this effect is quite narrow. An analysis of the three R-C section oscillator, using this effect, was made to predict:

(1) the value of the minimum transistor h_{21} required for the circuit to oscillate;

(2) the frequency of oscillation; and

(3) the range of values within which the bypass capacitor must lie to produce the desired phase-shift effect.

This is believed to be the first report of the phase-shift effect due to the bypass capacitor in an R-C phase-shift transistor oscillator. Thus, the analysis of this effect on circuit behavior is also original. The theoretical analysis proved to be sufficiently accurate to be considered a useful approximation of the behavior of experimental circuits. A certain amount of error was probably introduced into the theoretical predictions because of error in the experimental determination of the transistor parameters.

Because of the discovery of the phase-shift effect due to the bypass capacitor, the successful operation of a two R-C section phase-shift transistor oscillator was obtained. An analysis of the behavior of this circuit lead to the successful prediction of minimum h_{21}, oscillation frequency, and bypass capacitor range. This is believed to be the first reported successful operation of a two R-C section phase-shift transistor oscillator.

The method of analysis used in this study is applicable to R-C phase-shift oscillators using component values different from those used in the experimental circuits of this investigation. However, some of the approximations used in this study would probably have to be altered to fit the component values of another circuit.

APPENDIX I

FREQUENCY READINGS TAKEN DURING THE 72-HOUR DURATION

The temperature corresponding to each frequency reading can be obtained by using the calibration curve on Figure 10. The frequency reading at every half-hour interval is shown in Table I.

TABLE I. FREQUENCY READINGS CORRESPONDING TO RUMEN TEMPERATURE IN A SHEEP

Time	Frequency(CPS)
First 24-hour period	
6:00 P. M.	1632
6:30 P. M.	1633
7:00 P. M.	1633
7:30 P. M.	1633
8:00 P. M.	1634
8:30 P. M.	1637
9:00 P. M.	1637
9:30 P. M.	1636
10:00 P. M.	1631
10:30 P. M.	1636
11:00 P. M.	1633
11:30 P. M.	1630
12:00 M.	1628
12:30 A. M.	1634
1:00 A. M.	1636
1:30 A. M.	1636
2:00 A. M.	1635
2:30 A. M.	1634
3:00 A. M.	1638
3:30 A. M.	1641
4:00 A. M.	1632
4:30 A. M.	1629
5:00 A. M.	1626
5:30 A. M.	1626
6:00 A. M.	1624
6:30 A. M.	1619
7:00 A. M.	1619
7:15 A. M.	1611

TABLE I. CONTINUED

Time	Frequency(CPS)
7:30 A. M.	1611
8:00 A. M.	1606
8:30 A. M.	1599
9:00 A. M.	1597
9:30 A. M.	1572
10:00 A. M.	1585
10:30 A. M.	1593
11:00 A. M.	1595
11:30 A. M.	1595
12:00 M.	1599
12:30 P. M.	1600
1:00 P. M.	1602
1:30 P. M.	1601
2:00 P. M.	1602
2:30 P. M.	1604
3:00 P. M.	1607
3:30 P. M.	1610
4:00 P. M.	1615
4:30 P. M.	1619
5:00 P. M.	1619
5:30 P. M.	1619
Second 24-hour period	
6:00 P. M.	1619
6:30 P. M.	1621
7:00 P. M.	1619
7:30 P. M.	1620
8:00 P. M.	1622
8:30 P. M.	1619
9:00 P. M.	1614
9:30 P. M.	1614
10:00 P. M.	1616
10:30 P. M.	1618
11:00 P. M.	1621
11:30 P. M.	1620
12:00 M.	1621
12:30 A. M.	1622
1:00 A. M.	1624
1:30 A. M.	1627
2:00 A. M.	1632
2:30 A. M.	1634
3:00 A. M.	1635
3:30 A. M.	1632
4:00 A. M.	1631

TABLE I. CONTINUED

Time	Frequency(CPS)
4:30 A. M.	1622
5:00 A. M.	1614
5:30 A. M.	1614
6:00 A. M.	1614
6:30 A. M.	1615
7:00 A. M.	1611
7:30 A. M.	1611
8:00 A. M.	1611
8:30 A. M.	1606
9:00 A. M.	1609
9:30 A. M.	1605
10:00 A. M.	1606
10:30 A. M.	1608
11:00 A. M.	1610
11:30 A. M.	1605
12:00 N.	1606
12:30 P. M.	1606
1:00 P. M.	1608
1:30 P. M.	1515
2:00 P. M.	1513
2:30 P. M.	1543
3:00 P. M.	1584
3:30 P. M.	1592
4:00 P. M.	1604
4:30 P. M.	1613
5:00 P. M.	1619
5:30 P. M.	1622
Third 24-hour period	
6:00 P. M.	1628
6:30 P. M.	1631
7:00 P. M.	1627
7:30 P. M.	1624
8:00 P. M.	1623
8:30 P. M.	1615
9:00 P. M.	1610
9:30 P. M.	1609
10:00 P. M.	1612
10:30 P. M.	1611
11:00 P. M.	1612
11:30 P. M.	1609
12:00 M.	1607
12:30 A. M.	1607

TABLE I. CONTINUED

Time	Frequency(CPS)
1:00 A. M.	1608
1:30 A. M.	1611
2:00 A. M.	1607
2:30 A. M.	1609
3:00 A. M.	1606
3:30 A. M.	1603
4:00 A. M.	1605
4:30 A. M.	1604
5:00 A. M.	1603
5:30 A. M.	1602
6:00 A. M.	1599
6:30 A. M.	1596
7:00 A. M.	1596
7:30 A. M.	1595
8:00 A. M.	1592
8:30 A. M.	1592
9:00 A. M.	1591
9:30 A. M.	1590
10:00 A. M.	1590
10:30 A. M.	1587
11:00 A. M.	1586
11:30 A. M.	1589
12:00 N.	1592
12:30 P. M.	1591
1:00 P. M.	1590
1:30 P. M.	1592
2:00 P. M.	1500
2:30 P. M.	1577
3:00 P. M.	1584
3:30 P. M.	1591
4:00 P. M.	1587
4:30 P. M.	1586
5:00 P. M.	1590
5:30 P. M.	1594
6:00 P. M.	1595

APPENDIX II

DETERMINATION OF TRANSISTOR PARAMETERS

The transistor can be considered as a two-port device. A common method of representing the transistor two-port network is by using admittances to define the transistor parameters. The equivalent circuit of an admittance or y parameter two-port network is shown in Figure 24.[32] The equations for the y parameter network are:

$$I_1 = y_{11}V_1 + y_{12}V_2; \text{ and} \qquad (33a)$$

$$I_2 = y_{21}V_1 + y_{22}V_2. \qquad (33b)$$

The currents and voltages used in both the equivalent circuit and network equations are A.C. quantities. The y parameters can be defined from equations 33a and 33b as:[33]

$$y_{11} = I_1/V_1 \quad \text{when} \quad V_2 = 0; \qquad (34a)$$

$$y_{12} = I_1/V_2 \quad \text{when} \quad V_1 = 0; \qquad (34b)$$

$$y_{21} = I_2/V_1 \quad \text{when} \quad V_2 = 0; \text{ and} \qquad (34c)$$

$$y_{22} = I_2/V_2 \quad \text{when} \quad V_1 = 0. \qquad (34d)$$

Thus, y_{11} and y_{22} are defined as the input and output short-circuit admittances, respectively, whereas y_{12} and y_{21} are the reverse and forward short-circuit transfer admittances.[34]

Another type of parameter commonly employed to represent the

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[32] Shea, _op. cit._, p. 23.

[33] _Ibid._, p. 25.

[34] _Ibid._

transistor two-port device is the h or hybrid parameter. The equivalent circuit using h parameters is shown in Figure 25.[35] The equations for the h parameter network are:

$$V_1 = h_{11}I_1 + h_{12}V_2; \text{ and} \qquad (35a)$$

$$I_2 = h_{21}I_1 + h_{22}V_2. \qquad (35b)$$

The h parameters can be defined from equations 35a and 35b as:[36]

$$h_{11} = V_1/I_1 \quad \text{when} \quad V_2 = 0; \qquad (36a)$$

$$h_{12} = V_1/V_2 \quad \text{when} \quad I_1 = 0; \qquad (36b)$$

$$h_{21} = I_2/I_1 \quad \text{when} \quad V_2 = 0; \text{ and} \qquad (36c)$$

$$h_{22} = I_2/V_2 \quad \text{when} \quad I_1 = 0. \qquad (36d)$$

Thus, h_{11} is the input impedance with the output short-circuited; h_{12} is the backward voltage transfer ratio with the input open; h_{21} is the forward current transfer ratio with the output short-circuited; and h_{22} is the output admittance with the input open.[37]

An application where h parameters are extremely useful is in the experimental determination of their value. After the value of the h parameters are determined experimentally, their values can be converted to other types of parameters which may represent the same two-port network. The conversion from h to y parameters will be considered. Using equations 33a, 33b, 35a, and 35b, the relationships between h and y can be found as:[38]

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[35] Ibid., p. 23.

[36] Ibid., p. 25.

[37] Ibid.

[38] Ibid., p. 440.

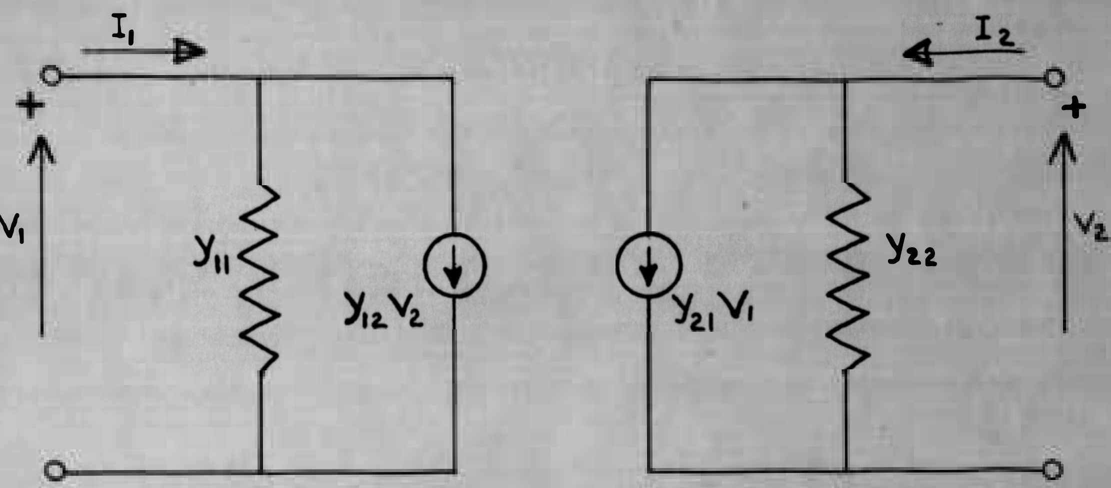

Figure 24. Equivalent Circuit Using y Parameters

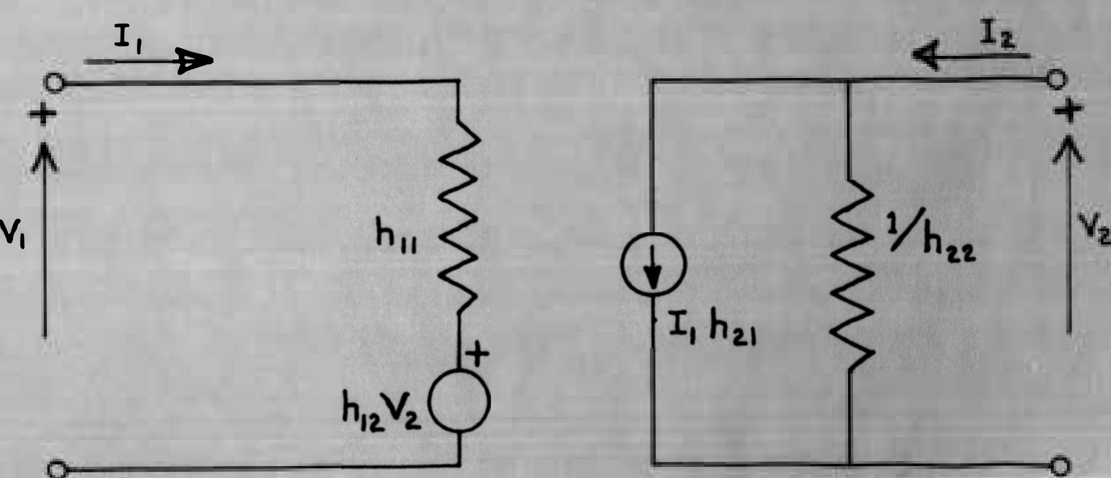

Figure 25. Equivalent Circuit Using h Parameters

$$y_{11} = 1/h_{11} \quad ; \qquad (37a)$$

$$y_{12} = -h_{12}/h_{11} \quad ; \qquad (37b)$$

$$y_{21} = h_{21}/h_{11} \quad ; \text{ and} \qquad (37c)$$

$$y_{22} = (h_{11}h_{22} - h_{12}h_{21})/h_{11}. \qquad (37d)$$

The transistor used in the experimental circuits was the Philco 2N207B. Two of these transistors were used during this study, and the h parameters for each were determined. The first step in determining the parameters was to find the operating point of each transistor in its particular circuit. The circuit shown in Figure 26 was used to determine the h parameters of each transistor. The particular equation 36a, 36b, 36c, or 36d, that defined the parameter being determined, was applied to the circuit in Figure 26. The quantity specified as zero was held constant at the transistor operating point while data was taken of the other two defining quantities. Sufficient data was taken so that a curve which included the operating point could be plotted. Holding at a constant value the quantity specified as zero, satisfies the defining equations 36a, 36b, 36c, and 36d because all defining quantities are A.C.

The curves for one transistor are given since the method used is identical for both transistors. Four curves were constructed as shown in Figures 27, 28, 29, and 30. Each curve was used to determine a particular h parameter by finding the slope of the curve at the operating point in accordance with the applicable equation 36a, 36b, 36c, or 36d. The resulting value of the h parameter is shown on the figure.

The y parameters are now calculated from the h parameters according to equations 37a, 37b, 37c, and 37d. The values of the y parameters

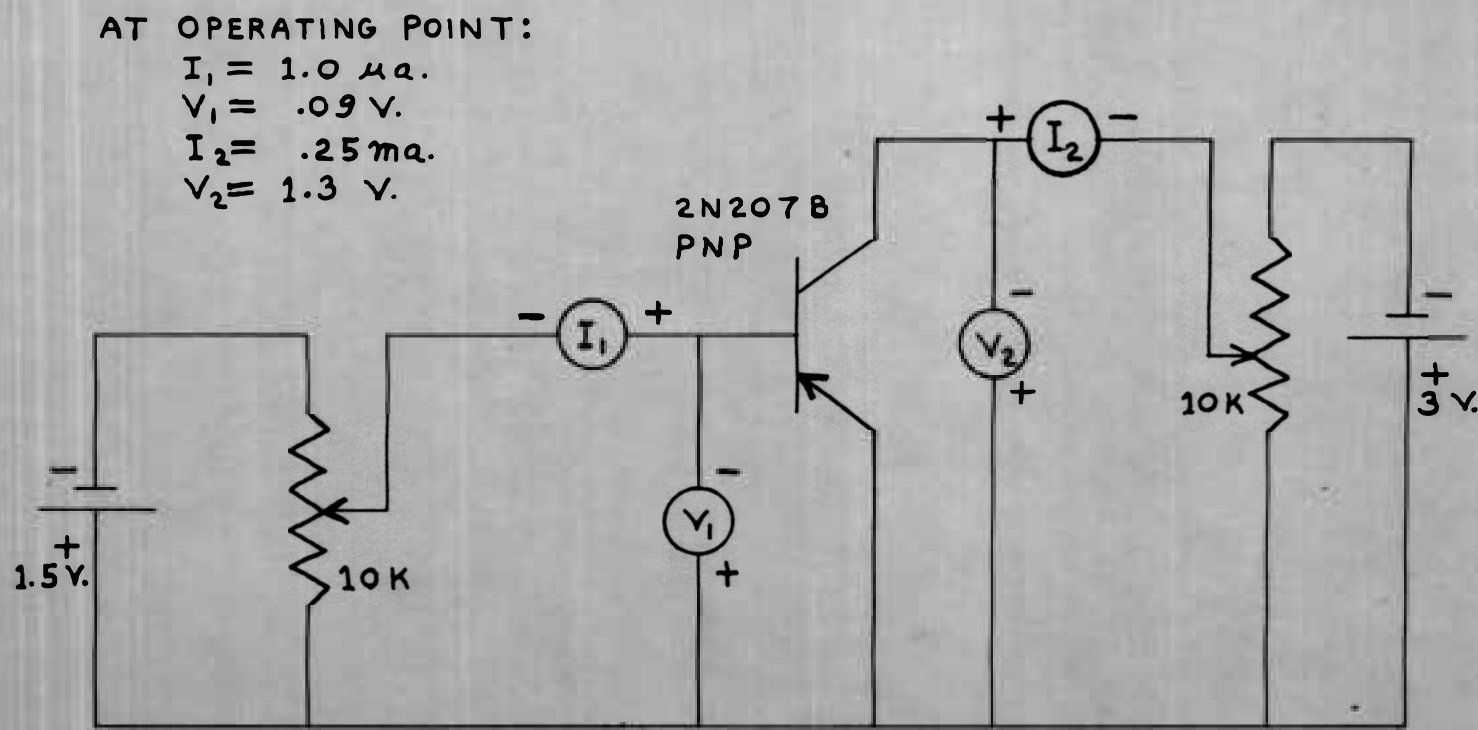

Figure 26. Circuit Used to Determine Transistor Parameters

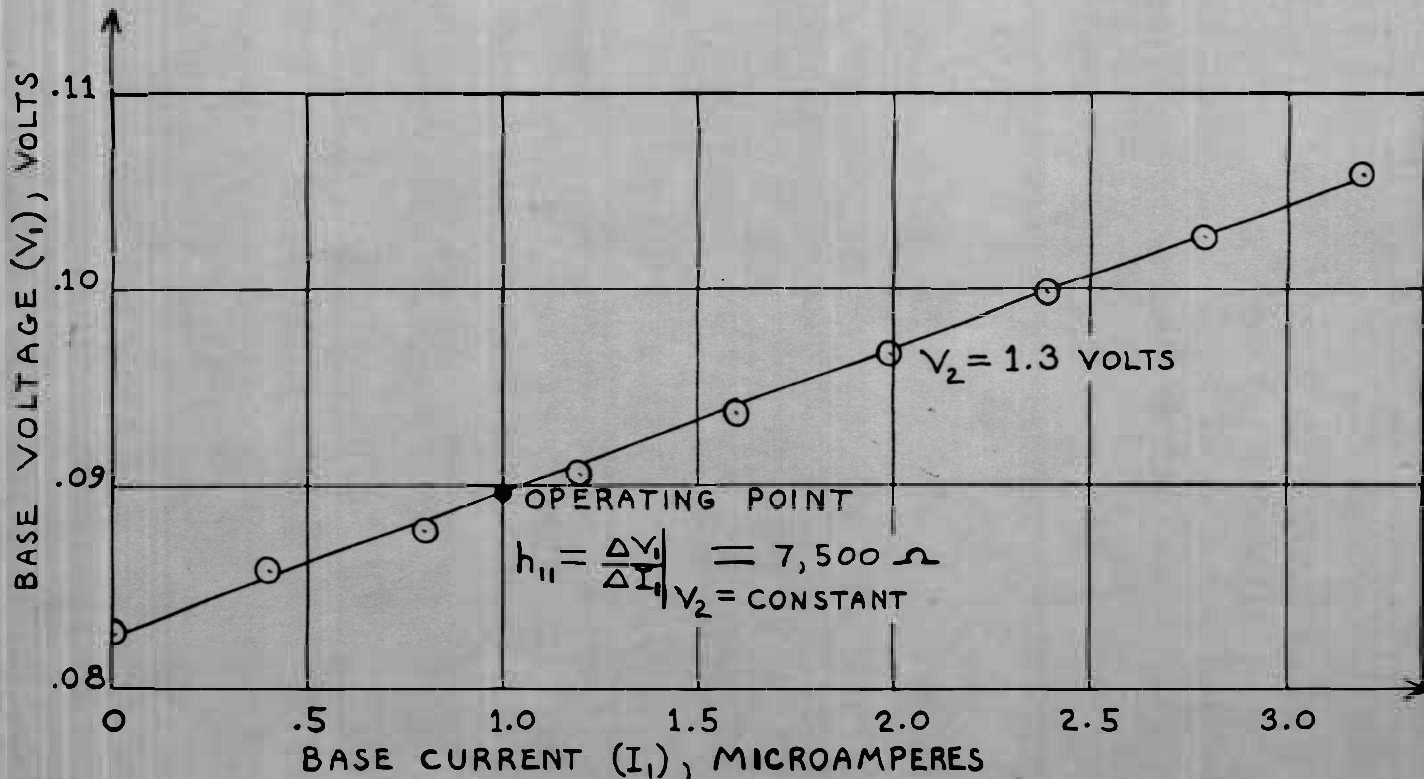

Figure 27. Curve Used to Determine h_{11}

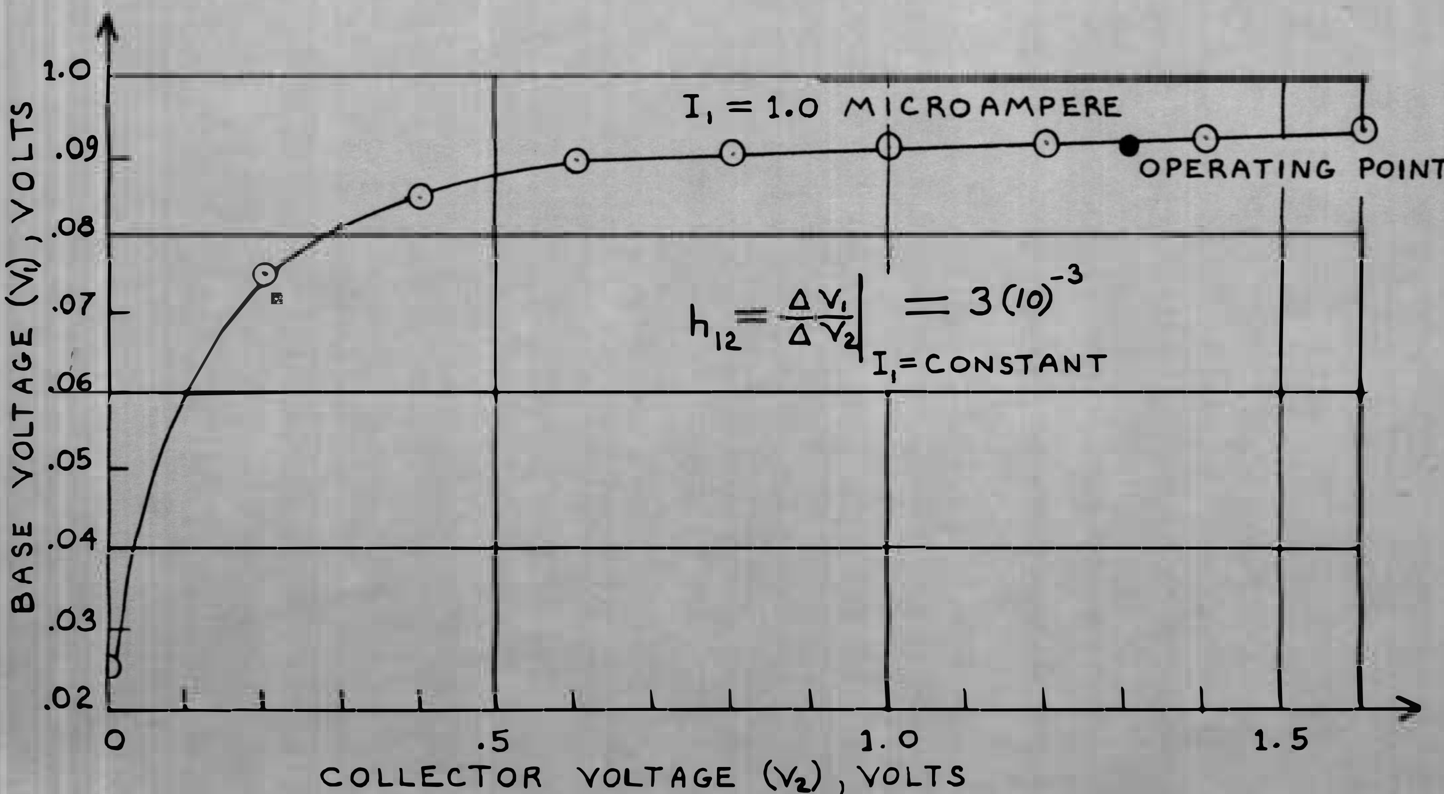

Figure 2[illegible]. Curve Used to Determine h_{12}

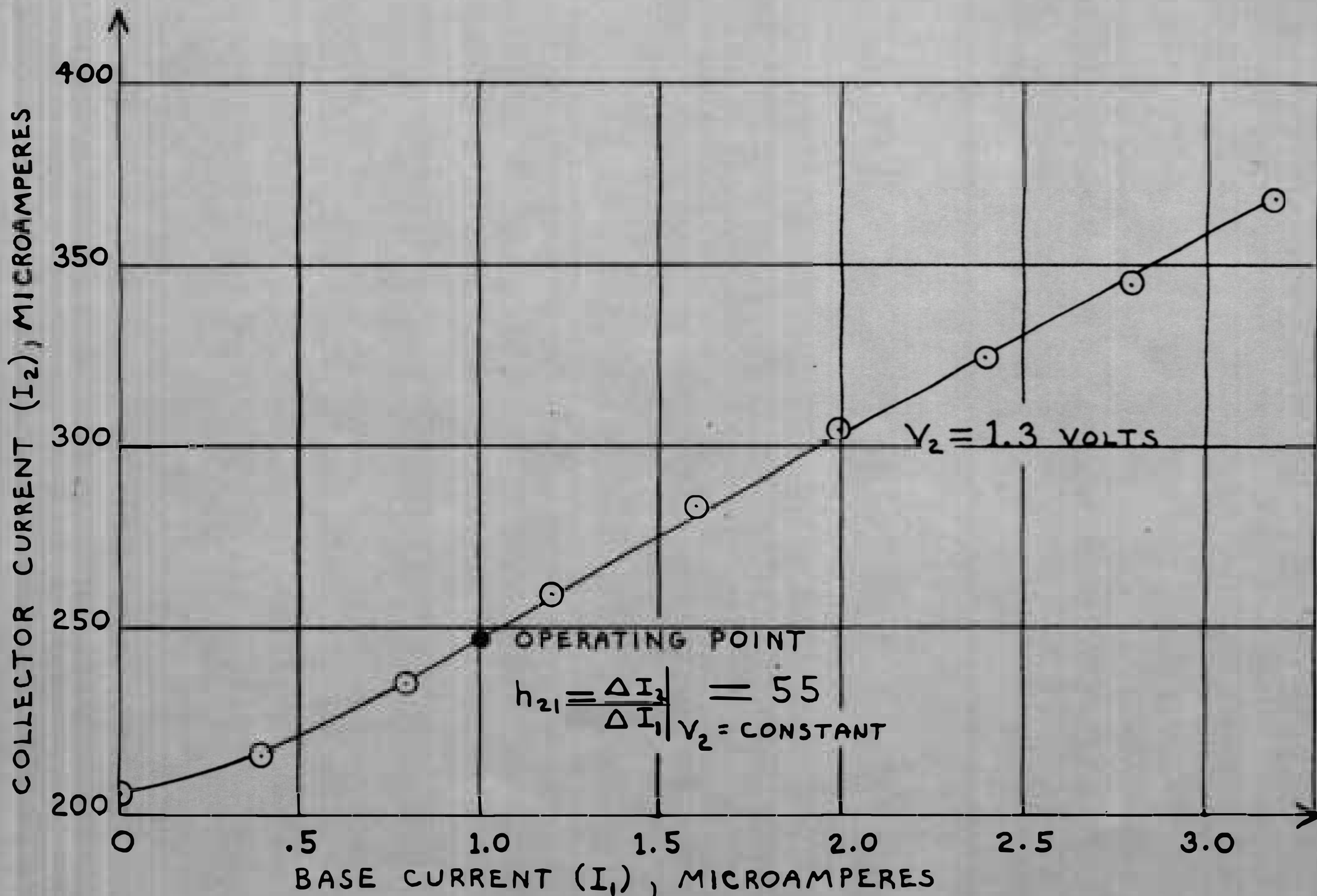

Figure 29. Curve Used to Determine h_{21}

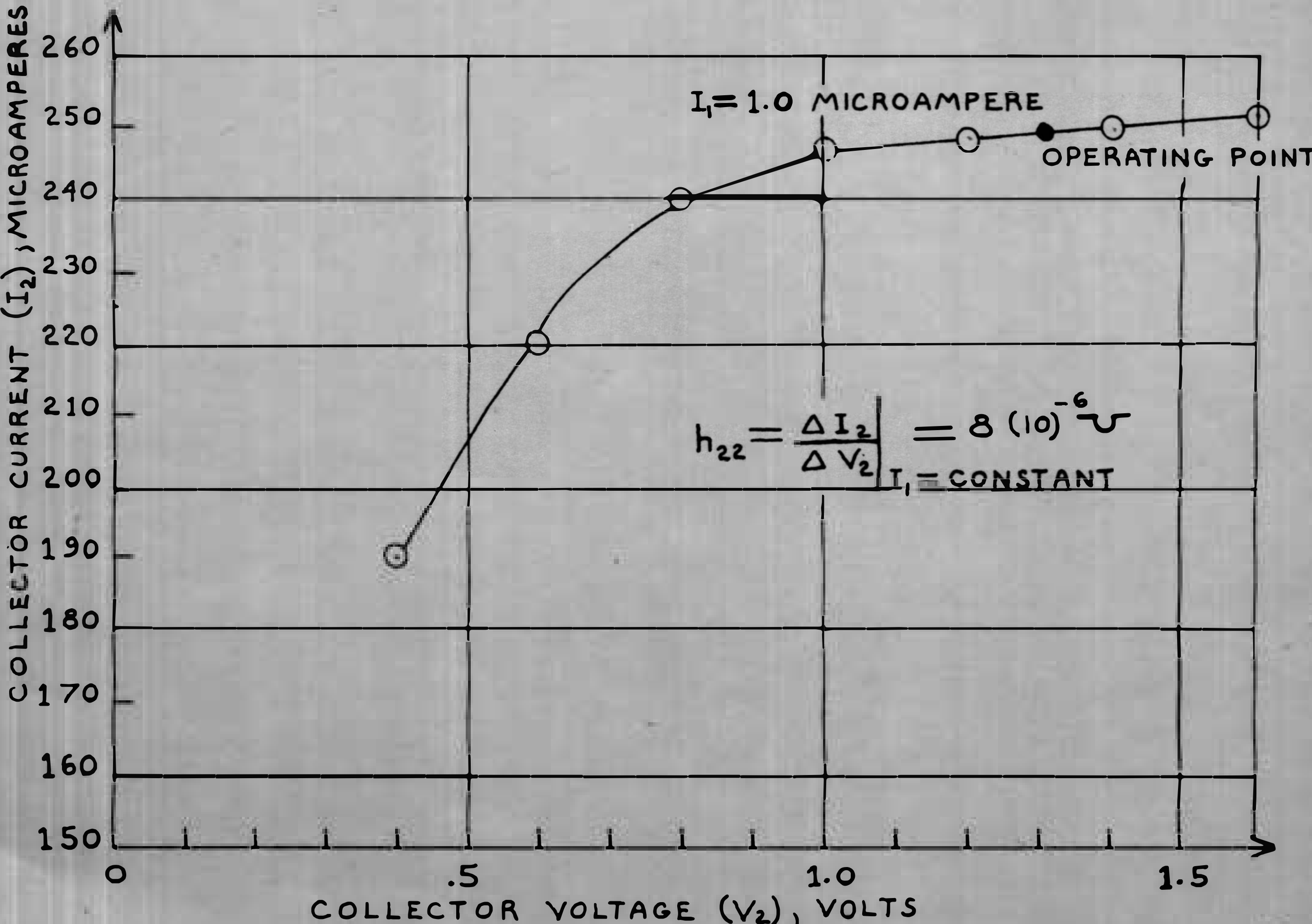

Figure 30. Curve Used to Determine h_{22}

are found to be:

$$y_{11} = 1.34(10)^{-4}\mho; \qquad y_{12} = 4(10)^{-7}\mho;$$

$$y_{21} = .73(10)^{-2}\mho; \text{ and } y_{22} = 8(10)^{-6}\mho.$$

The value of y_{12} is very small compared to the other parameters and it will be neglected in all the calculations in this study. The parameter y_{22} can usually be neglected unless the circuit load resistance becomes larger than it did in the circuits considered in this study.

The values of h_{11} and h_{21} for the other 2N207B transistor used in this study are:

$$h_{11} = 9{,}800\,\Omega; \quad \text{and} \quad h_{21} = 81.$$

The corresponding y parameters are then:

$$y_{11} = 1.02(10)^{-4}\mho; \quad \text{and} \quad y_{21} = .83(10)^{-2}\mho.$$

APPENDIX III

DETERMINATION OF STABILITY AND FREQUENCY RELATIONSHIPS

Routh's criterion is a test that, when applied to a finite polynomial with real coefficients, yields the numbers of roots of the polynomial with positive real parts and with zero real parts.[39] The stability of the oscillators considered in this study depend on the location of the zeros of the system characteristic equation, with zeros having positive real parts corresponding to an unstable system and with zeros having zero real parts corresponding to a sustained system oscillation.[40]

Routh's criterion will be applied to a system having a third degree characteristic equation of the form:

$$A_3S^3 + A_2S^2 + A_1S + A_0 = 0.$$

According to Routh's criterion, the coefficients of this polynomial are written in an array of the form:[41]

$$\begin{array}{c|ll} S^3 & A_3 & A_1 \\ S^2 & A_2 & A_0 \\ S^1 & \dfrac{A_1A_2 - A_0A_3}{A_2} & \\ S^0 & A_0 & \end{array}$$

The number of roots of the characteristic equation that have positive real parts is shown by the number of sign changes in the first column

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39G. C. Newton, L. A. Gould, J. F. Kaiser, Analytical Design of Linear Feedback Controls, p. 309, John Wiley & Sons: New York, 1957.

40Ibid., p. 310.

41Ibid.

of the array.[42] If A_0, A_1, A_2, and A_3 all exist and are all positive, then the only way that instability can occur is for $(A_1A_2 - A_0A_3)/A_2$ to be negative. Thus, for an unstable system to exist the relationship:

$$A_1A_2 - A_0A_3 < 0, \quad \text{or}$$

$$A_1A_2 < A_0A_3$$

must apply. If sustained oscillations are desired, the coefficients of the array must be such as to (1) make all the coefficients of one row of the array zero and (2) cause the auxiliary polynomial formed from the coefficients of the last nonvanishing row to have a pair of conjugate imaginary zeros.[43] The only way that all the coefficients of one row in the array can be zero is when $A_1A_2 = A_0A_3$.

The auxiliary polynomial's coefficients are the numbers in the last nonvanishing row. In this auxiliary polynomial S will appear only in even powers, the highest power being that of the S indicated at the left of the last nonvanishing row.[44] The roots of the equation formed by equating the auxiliary polynomial to zero are all roots of the original characteristic equation. If there are any roots with zero real parts they will be found among the roots of the auxiliary equation.[45] The auxiliary equation for this system is:

$$A_2S^2 + A_0 = 0.$$

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[42] Ibid.

[43] Gardner and Barnes, op. cit., p. 200.

[44] Ibid., p. 199.

[45] Ibid.

The roots of the auxiliary equation are then found to be a pair of conjugate imaginary zeros which are:

$$S = \pm j \sqrt{\frac{A_0}{A_2}} \; .$$

Since $S = j\omega$, where w is the angular frequency, the oscillation frequency of this system is:

$$f = \frac{1}{2\pi} \sqrt{\frac{A_0}{A_2}} \; .$$

The roots of the auxiliary equation can be shown to satisfy the system characteristic equation. By the substitution:

$$S = j\left(\frac{A_0}{A_2} \right)^{\frac{1}{2}} ,$$

the characteristic equation becomes:

$$A_3 j^3 \left(\frac{A_0}{A_2} \right)^{3/2} + A_2 j^2 \left(\frac{A_0}{A_2} \right)^{2/2} + A_1 j \left(\frac{A_0}{A_2} \right)^{1/2} + A_0 = 0;$$

then $-jA_3 \frac{A_0}{A_2} \left(\frac{A_0}{A_2} \right)^{1/2} - A_2 \frac{A_0}{A_2} + jA_1 \left(\frac{A_0}{A_2} \right)^{1/2} + A_0 = 0.$

For this system the relationship $A_1 A_2 = A_0 A_3$ must hold. Then the expression $A_1 = \frac{A_0 A_3}{A_2}$ can be substituted to give:

$$-jA_1 \left(\frac{A_0}{A_2} \right)^{1/2} - A_0 + jA_1 \left(\frac{A_0}{A_2} \right)^{2} + A_0 = 0.$$

The real and imaginary parts of the equation both reduce to zero showing that the positive root of the auxiliary equation satisfies the characteristic equation. By inspection, the negative root of the auxiliary equation can be seen to satisfy the characteristic equation.

Routh's criterion will next be applied to a system having a fourth degree characteristic equation of the form:

$$A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0.$$

The coefficients of the polynomial are written in an array of the form:

$$\begin{array}{c|lll} s^4 & A_4 & A_2 & A_0 \\ s^3 & A_3 & A_1 & \\ s^2 & \dfrac{A_2A_3 - A_1A_4}{A_3} & A_0 & \\ s^1 & A_1 - \dfrac{A_0A_3^2}{A_2A_3 - A_1A_4} & & \\ s^0 & A_0 & & \end{array}$$

If A_0, A_1, A_2, A_3, and A_4 all exist and are all positive, then the only way that instability can occur is when:

$$A_1 - \frac{A_0A_3^2}{A_2A_3 - A_1A_4} < 0, \text{ or}$$

$$A_1(A_2A_3 - A_1A_4) - A_0A_3^2 < 0.$$

For sustained oscillation to occur, the coefficients in one row of the array must be zero with the result that:

$$A_1(A_2A_3 - A_1A_4) - A_0A_3^2 = 0.$$

The auxiliary equation can be found from the array to be:

$$\frac{A_2A_3 - A_1A_4}{A_3} s^2 + A_0 = 0.$$

However, the equation giving the relationship for sustained oscillation in this system can be arranged to give:

$$\frac{A_2A_3 - A_1A_4}{A_3} = \frac{A_0A_3}{A_1} .$$

Then the auxiliary equation can be reduced to the more simple form:

$$\frac{A_0A_3}{A_1} S^2 + A_0 = 0, \quad \text{or}$$

$$A_3S^2 + A_1 = 0.$$

The roots of the auxiliary equation are then found to be a pair of conjugate imaginary zeros which are:

$$S = \pm j \sqrt{\frac{A_1}{A_3}} \quad .$$

Since $S = jw$, the oscillation frequency of this system is:

$$f = \frac{1}{2\pi} \sqrt{\frac{A_1}{A_3}} \quad .$$

The roots of the auxiliary equation can be shown to satisfy the system characteristic equation. By the substitution:

$$S = j(\frac{A_1}{A_3})^{1/2} ,$$

the characteristic equation becomes:

$$A_4j^4(\frac{A_1}{A_3})^{4/2} + A_3j^3(\frac{A_1}{A_3})^{3/2} + A_2j^2(\frac{A_1}{A_3})^{2/2} + A_1j(\frac{A_1}{A_3})^{1/2} + A_0 = 0,$$

then

$$\frac{A_4A_1^{\,2}}{A_3^{\,2}} - j\, A_1(\frac{A_1}{A_3})^{1/2} - \frac{A_2A_1}{A_3} + j\, A_1(\frac{A_1}{A_3})^{1/2} + A_0 = 0.$$

The imaginary parts then go to zero which leaves:

$$\frac{A_4A_1^{\,2}}{A_3^{\,2}} - \frac{A_2A_1}{A_3} + A_0 = 0, \text{ or}$$

$$A_4A_1^2 - A_2A_1A_3 + A_0A_3^2 = 0.$$

However, for this system the relationship $A_1(A_2A_3 - A_1A_4) = A_0A_3^2$ must hold for sustained oscillation to occur. This relationship can then be substituted to give:

$$A_4A_1^2 - A_2A_1A_3 + A_1(A_2A_3 - A_1A_4) = 0.$$

This equation, which represents the real part of the characteristic equation, then goes to zero. Thus the positive root of the auxiliary equation satisfies the characteristic equation. By inspection, the negative root of this auxiliary equation can also be seen to satisfy the characteristic equation.

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