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Construction and Evaluation of the Performance of a Horizontal Well in a Shallow, Thin Aquifer

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CONSTRUCTION AND EVALUATION OF THE PERFORMANCE OF A HORIZONTAL WELL IN A SHALLOW, THIN AQUIFER

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CONSTRUCTION AND EVALUATION OF THE PERFORMANCE

OF A HORIZONTAL WELL IN A SHALLOW,

THIN AQUIFER

This thesis is approved as a creditable, independent investigation by a candidate for the degree, Master of Science, and acceptable as meeting the thesis requirements for this degree; but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

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INTRODUCTION

For many years, man has sought means of securing water for both domestic and beneficial use from the underground supplies of the earth. Usually the method has involved the construction of a hole from the ground surface into or through the water bearing material below, the provision of curbing to protect the hole from the caving in of unconsolidated material, the development of the well so that the flow of water into it will be restricted as little as possible, and the pumping of the water to the surface or point of proposed use. Unfortunately the geology of one area is often different from the geology of other areas, and the occurrence of underground water varies immensely as to depth, areal extent, and usable quantity. As a result numerous problems have been encountered in the development of ground water supplies. One of the problems that has been encountered in South Dakota is how to construct a well economically in shallow, thin aquifers - a well that would yield sufficient water so as to be usable as a source of water for irrigation. A possible solution is suggested in the horizontal well.

Although the idea is not new, its application has been almost entirely confined to industrial and municipal developments, and usually to the immediate proximity of perennial streams. Consequently, the term most often used for these types of structures was infiltration galleries, and they were designed either to collect seepage from the stream or to intercept flow to the stream.[1] Basically, horizontal wells and infiltra-

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[1]Flinn, A.D., Weston, R.S., & Bogert, C.L., Waterworks Handbook, third edition, New York, McGraw-Hill Book Co., Inc., 1927, p. 260.

tion galleries are similar. Both are simply perforated conduits constructed horizontally in an aquifer, just as a vertical well is a conduit constructed vertically in an aquifer. Infiltration galleries have been successfully constructed from wood, metal, tile, and concrete at various locations throughout the world. Some have been constructed by penetrating the aquifer from the ground surface. Some have been constructed near streams and the water diverted to an infiltration field above them. Others have been projected laterally from vertical caissons. Most notable of the last group is the method of construction used by Ranney Method Water Supplies, Inc., of Columbus, Ohio. A large diameter caisson is sunk to the bottom of the aquifer. A concrete plug is poured in the bottom, and the caisson dewatered. The horizontal casing is then jacked out into the aquifer through specially prepared ports near the bottom of the caisson. As jacking proceeds, the aquifer is developed by the removal of fine sand, leaving a practically undisturbed, but improved, aquifer.

The initial construction cost of infiltration galleries has generally been higher than the cost of conventional vertical wells. Pumping and maintenance costs are usually less, however; and infiltration galleries usually intercept water more completely than vertical wells.[2] Galleries have been successful when constructed 5 to 10 feet below the water table at low stage. A notable example is the Des Moines gallery in the Raccoon River Valley of Iowa.[3] The great hazard in the use of

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[2] Ibid., p. 260.

[3] Ibid., p. 261.

infiltration galleries has been the decline of the water table. A vertical well can sometimes be made deeper, but a gallery can only be abandoned.

Since infiltration galleries have been effective as a means of securing large quantities of water from thin aquifers, the question has arisen as to whether or not a small horizontal well that would yield sufficient wa terfor irrigation could be constructed at a cost that would be within the budget of the family-sized farm in South Dakota. If so, a subsequent problem would be the development of some method of estimating the quantity of water a particular aquifer will yield to such a well.

The maximum limit of an expenditure for an irrigation well may be arbitrarily estimated at approximately $5.00 per gallon per minute of irrigation water. That would be a cost per irrigated acre of approximately $30.00. Of course, the production of high value crops will justify an even higher cost per acre for the well.

Probably the most economical method of construction of a horizontal well would be to excavate a hole extending from the ground surface to the bottom of the aquifer with a dragline, insert the completely prefabricated well, place a gravel envelope around the horizontal screen, and backfill with the previously excavated sand and gravel. Obviously, such a method is limited to aquifers whose bottoms are approximately 20 feet or less below the ground surface. The danger of cave-ins is probably the greatest hazard of such a construction process. If shoring should prove to be an absolute necessity, the cost would rise steeply.

Once the well is in place, some means must be provided for the removal of fine sand that is carried to, and deposited in, the horisontal casing during the development of the well. High pumping rates and surges in the pumping rate may loosen the fine sand and carry it all the way to the suction pipe of the pump so it can be lifted to the surface; but if such techniques do not keep the horizontal casing free of sediment, some method of physically removing that sediment must be devised.

After all these problems have been successfully overcome, there still remains the paramount question "How much water can be expected from a specific horisontal well?" A possible approach would be to relate a certain length of horisontal casing to a vertical well of some particular diameter. If such a relationship can be found, the performance of such a well in another place could be predicted from data collected during a conventional pump test.

This study will be confined to the details of the dragline construction of a pit; the placement of a horisontal well in a shallow, thin aquifer; and an actual pump test to learn if a certain length of horisontal casing may be equivalent to a vertical well of some particular radius.

SUMMARY OF GROUND WATER HYDROLOGY

At this point, a brief discussion will be made on a few of the principles of ground water hydrology and the terms that will be used herein will be defined. Water may occur below the surface in the rocks and unconsolidated materials that make up the earth's crust in any concentration from zero to complete saturation of the interstices, or voids, between the individual particles. The term "ground water" applies to the water in the zone of saturation. The upper surface of the zone of saturation may take the form of a free water surface or be confined below an impermeable layer. The free water surface in the unconfined form is called the water table. Some authorities[4] choose to identify any formation, group of formations, or part of a formation that is water bearing as an aquifer, whereas others[5] prefer to define an aquifer as any formation of permeable material that will yield water to gravity in appreciable quantities. Obviously the latter definition is a bit more restrictive than the former, but at the same time it recognizes the relative ease with which water may be secured in various areas. Consequently, by the latter definition a formation that would be classed an aquifer in one locality might not necessarily be classed as one in another locality.

The occurrence of aquifers throughout the world is not uniform in areal extent, depth, thickness, or the amount of water they will yield

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[4] Meinzer, O. E., Outline of ground-water hydrology, U. S. Geological Survey Water Supply Paper 494, 1923, p. 30.

[5] Ferris, J. G., Ground Water, In Wisler, C. O., & Brater, E. F., Hydrology, New York, John Wiley & Sons, Inc., 1956, p. 216.

to a well. Some areas may have several high yielding aquifers piled one above the other, whereas a neighboring area may have no aquifers at all or only low yielding ones, depending upon the geologic origin of the two regions. The amount of water that an aquifer will yield to gravity is limited not only by the physical properties of thickness and areal extent, but by its hydrologic properties as well. L. K. Wenzel[6] has singled out two hydrologic characteristics as being of the utmost importance - permeability and specific yield.

The permeability of a rock or soil with respect to water is its ability to transmit water under pressure.[7] Permeability varies greatly from one formation to another and may be different in different directions in a particular aquifer. Since permeability is such a variable property, an average value called the "coefficient of permeability" is usually used in ground water work. It is expressed quantitatively as "the rate of flow of water in gallons per day through a cross-sectional area of one square foot under a hydraulic gradient of one foot per foot at a temperature of 60°F."[8] Hydraulic gradient and pressure gradient are synonymous in ground water studies - the rate of change of pressure head per unit of distance.[9]

Under water table conditions the specific yield is the same as

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[6]Wenzel, L. K., The thiem method for determining permeability of water-bearing materials, U. S. Geological Survey Water-Supply Paper 679-A, p. 4.

[7]Meinzer, O. E., op. cit., p. 44.

[8]Ferris, J. G., op. cit., p. 206.

[9]Meinzer, O. E., op. cit., p. 38.

the "coefficient of storage" and is defined as the quantity of water, in cubic feet, that is yielded from a vertical column of aquifer whose base is 1 square foot when the water level drops 1 foot.[10] Coefficients of storage will usually range from small numbers to approximately 0.30 and are dimensionless.

In nature, a state of equilibrium has been established between the natural discharge from, and the natural recharge of, aquifers. The thickness is usually quite constant or cyclical, and so is the hydraulic gradient. If man constructs a well into the aquifer, the water level in the well will coincide with the water table adjacent to it until pumping is begun. When water is discharged from the well, the water level inside is lowered. This produces a pressure differential between the water levels inside and outside. Because of the high frictional resistance of the porous material, the water behaves more as a viscous fluid than as a liquid;[11] but even so, it tends to flow from places of high pressure to places of low pressure. Thus flow is induced toward and into the well.

During the early stages of pumping, much of the water that flows into the well comes from aquifer storage in the immediate vicinity of the well. As pumping continues, more and more pressure gradient is established, and the disturbance is noted in the aquifer farther and farther away. The free water surface is deflected down in the vicinity of the

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[10] Kazmann, R. G., Some field applications of water transmissibility and storage coefficients, Agricultural Engineering, 25:299, August 1944.

[11] Meinzer, O. E., General principles of ground water recharge, Economic Geology, 41:192, May 1946.

well. The dewatered volume takes on a shape similar to an inverted cone with its vertex at the water surface in the well. This cone of depression is described by Theis as:

> . . . the geometric solid included between the water table or other piezometric surface after a well has begun discharging and the hypothetical position the water table or other piezometric surface would have had if there had been no discharge by the well. . . . The vertical distance at any place between the hypothetical uninfluenced position of the piezometric surface and the actual surface after discharge has begun, that is, the lowering due to discharge, is the drawdown at that place caused by the discharge.[12]

As time progresses and discharge continues, the drawdown will continue to increase and the cone of depression will grow laterally. Also, a greater percentage of the discharge will be drawn from the flow induced by the hydraulic gradient produced by the well and a smaller percentage will be drawn from storage. When the flow from storage has diminished to zero, steady flow will have been established from the aquifer to the well in a quantity equal to the discharge of the well. The same quantity of water will be passing each of the infinite number of concentric cylinders that surround the well. The maximum distance from the well at which a disturbance is noted in the aquifer as a result of pumping is known as the radius of influence of the well at that discharge. This steady flow condition is a theoretical condition in most cases because equilibrium is seldom attained during any pump test. Theis makes this very clear when he states:

> Time is an essential variable in the description of the cone. The time rate of lateral growth of the cone is independent of the

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[12]Theis, C. V., The significance and nature of the cone of depression in ground-water bodies, Economic Geology, 33:891, December 1938.

> rate of discharge by the well and depends only on the physical characteristics of the aquifer. . . . Hence the cone of depression, as here defined, can have no definite limits short of at least one of the hydrologic boundaries of the aquifer.[13]

Although equilibrium is seldom established with reference to aquifer discharge, the cone of depression is one of the useful tools we have for determining aquifer characteristics and for estimating the safe yield. Another of the tools is the drawdown curve which is the drawdown observed in an observation well plotted against the logarithm of either time or the reciprocal of time since pumping started.

Figure 1. is a sketch of the effects produced by pumping a well under water table conditions and the symbols that will be used to identify certain specific values in this paper.

The hydraulic gradient at a certain point at some particular time is evidenced graphically by the slope of the cone of depression at the distance of the point from the well at that time. The rate of flow in any particular aquifer at any particular distance from the discharging well is proportional to the hydraulic gradient at that point. A steeper hydraulic gradient is required to induce as much flow in a media with a low permeability than in a media with a high permeability.

As a well is pumped and the water level is drawn down in the well, part of the aquifer is dewatered. In the case of thin aquifers, this dewatering is especially to our disadvantage because there is usually an appreciable decrease in the thickness of saturated aquifer that remains to supply the well. Even though an increase in the rate of discharge is

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[13] Ibid., p. 889.

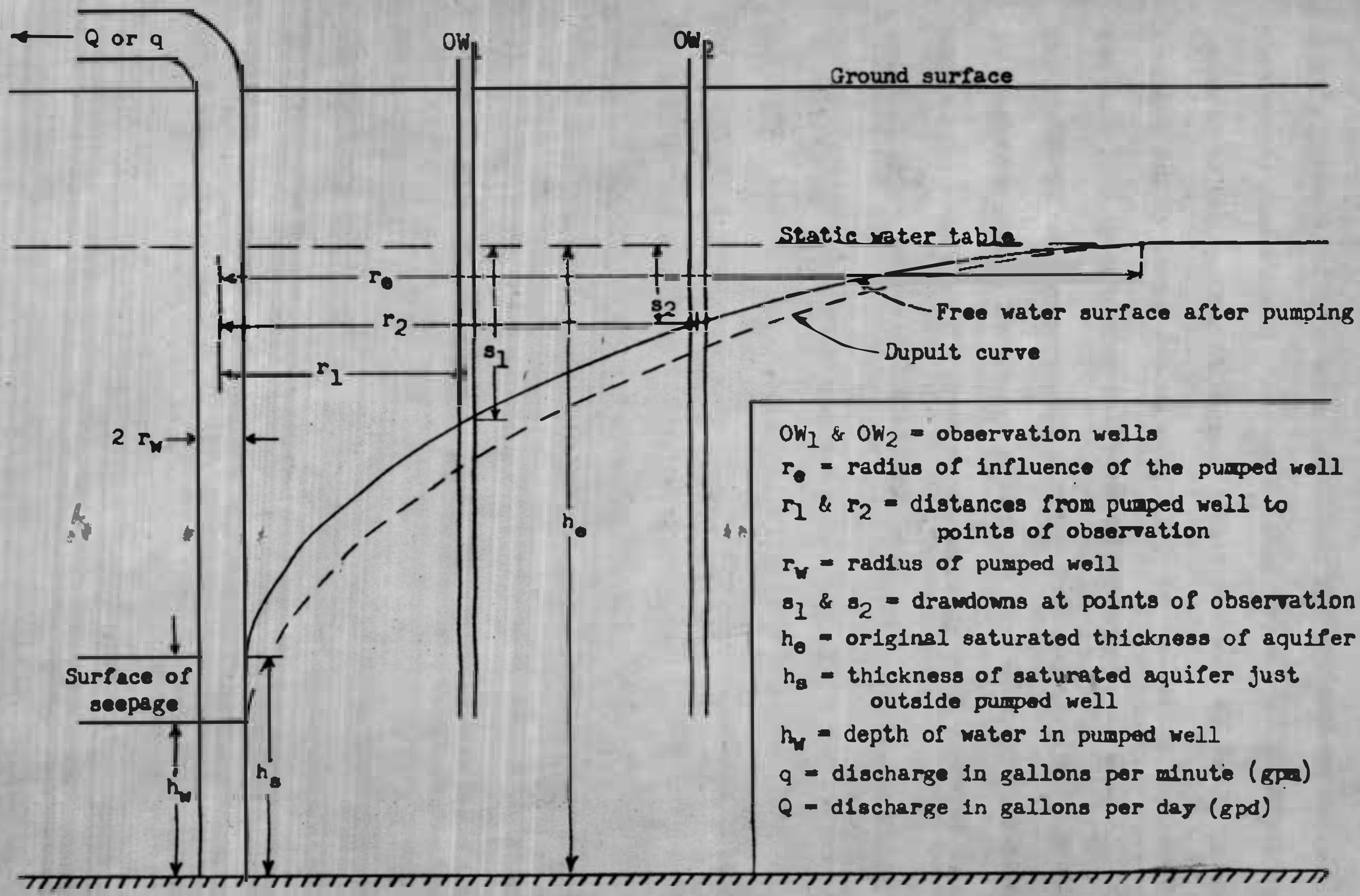

Figure 1. The Disturbance Produced in an Unconfined Aquifer by a Discharging Well.

usually accompanied by greater drawdowns, the two factors are not necessarily proportional. As the drawdown becomes relatively large as compared to the aquifer thickness, a unit increase in the hydraulic gradient can no longer induce enough flow to the well to compensate for the unit loss in saturated thickness of the aquifer as well as increase the discharge some proportional amount. Consequently, the increase in discharge usually becomes progressively less with each additional unit of drawdown when the drawdown is not very small compared to the original saturated aquifer thickness.

WORK OF OTHER INVESTIGATORS

The study of the movement of ground water can be divided into two major groups depending upon whether or not the hydrologic system is in a state of equilibrium. Equilibrium conditions are independent of time. Nonequilibrium conditions are constantly changing with time. Before man disturbed the ground water bodies, they were essentially in a state of equilibrium - the discharges and recharges were in balance, and the hydraulic gradients and water tables were relatively constant or cyclical. The pumping of a well creates an unbalance in this system that must be compensated for by a corresponding change somewhere in the original ground water body. The time it takes for nature to again establish equilibrium is difficult to determine, but it was in terms of this re-establishment of equilibrium that the first discoveries were made that gave us our initial insight into the laws of nature that govern ground water movements. It was not until 1935 that a means of handling the function, time, was devised, thus giving us a means of predicting what will have happened in an aquifer at any particular time even though the system is constantly changing.

The early discoveries pertaining to equilibrium conditions will first be described briefly. Then the nonequilibrium formulas that will be used in the analysis of this problem will be discussed.

The rate of flow of water through a porous media is proportional to the hydraulic gradient.[15] This is simply a restatement of Darcy's

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[15] Ferris, J. G., op. cit., p. 226.

law which was published in 1856 as a result of his studies of water percolating through filter sands. In the study of ground water, the quantity of water discharged is usually required instead of the velocity of flow; therefore, the equation stating Darcy's law of laminar flow is usually written

$$Q = PIA$$

where Q is the discharge in gallons per day (gpd); P is the coefficient of permeability of the aquifer in gallons per day per square foot; I is the hydraulic gradient in feet per foot; and A is the cross section of the area of flow in square feet.[16]

Equilibrium formulas:

In 1863, Jules Dupuit devised a formula for well discharge combining Darcy's law and the statement of continuity.[17] His formula was based on the assumptions that all discharge flows horizontally into the zone of influence of a well; that the flow through any concentric cylinder about a well is horizontal; that the hydraulic gradient at all points on the cylinder surface is equal to the slope of the free water surface at its intersection with the cylinder; and that steady flow exists.[18]

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[16] Ibid., p. 226.

[17] Peterson, D. F., Jr., Hydraulics of wells, Proceedings of American Society of Civil Engineers, Separate No. 708, June 1955, pp. 1-2.

[18] Peterson, D. F., Jr., Israelsen, O. W., Hansen, V. E., Hydraulics of wells, Utah Agricultural Experiment Station Bulletin 351 (Technical), 1952, pp. 5-14.

Dupuit's formula is as follows:

$$Q = K \frac{h_e^2 - h_w^2}{\log (r_e/r_w)}$$

where K is a constant that includes the permeability of the aquifer, log signifies the logarithm to the base 10, and the other symbols are the same as those identified in Figure 1. The chief objections to the use of this formula are that it is difficult to evaluate r_e and that the surface of seepage is completely ignored. However, it has been experimentally shown that it will give accurate estimates of Q.[19] The usual procedure in practice is to arbitrarily select a reasonable value for r_e to make the problem one of direct solution.

Babbitt and Doland[20] suggest a method of solving for both r_e and the aquifer characteristics using the Dupuit formula with observations made on one pumped well after two pump tests using two different rates of discharge. First, let re = CQ. The equation then becomes

$$Q = K \frac{h_e^2 - h_w^2}{\log (CQ/r_w)}.$$

By substituting the corresponding values of h_w and Q in the equation, two equations involving only C and K as unknowns are obtained. Solving simultaneously, C and K can be determined. Now r_e and Q can be computed for any particular drawdown. The drawback of this method lies in the need of two pump tests, on the same well, that have been continued until

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[19] Ibid., pp. 14-15.

[20] Babbitt, H. E., & Doland, J. J., Water Supply Engineering, fifty edition, New York, McGraw-Hill Book Co., Inc., 1955, p. 63.

equilibrium is established. It is questionable whether such conditions will actually be reached in practice unless the discharging well is very near to a recharge boundary.

In 1906, Gunter Thiem modified the Dupuit formula and applied it for the first time to the determination of the field coefficient of permeability using drawdowns obtained from two or more observation wells near a pumped well. The Thiem formula for the coefficient of permeability is[21]

$$P = \frac{527.7\ q \log \frac{r_2}{r_1}}{m' \ (s_1 - s_2)}$$

where q is the discharge in gallons per minute (gpm), r_1 and r_2 are distances from the pumped to the observation wells, s_1 and s_2 are drawdowns in the observation wells, log signifies the logarithm to the base 10, and m' is the average saturated thickness of the aquifer at the observation wells when equilibrium is reached. Although the Thiem formula is based upon the assumption that equilibrium has been re-established since pumping began (which usually never happens during a typical pump test), it has given quite reliable estimates of the field coefficient of permeability especially when r_1 and r_2 were not excessively large and the pumping time was several days. Such results are possible simply because the cone of depression assumes its equilibrium form near the pumped well quite soon after pumping begins. After this form is assumed, that portion of the cone simply lowers with time as the cone of depression extends laterally. Consequently, subsequent profiles of that portion of the cone are actually

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[21] Wenzel, L. K., op. cit., p. 10.

parallel. If the observation wells intersect the portion of the cone of depression that has reached equilibrium form, the quantity $(s_1 - s_2)$ will not change numerically with time.

The Thiem formula is also somewhat restricted as to its application because it is based on the assumptions that the permeability is constant for the entire aquifer and that the static water table is level.

__Nonequilibrium formula:__

In 1935 a formula was developed that described the behavior of ground water under the nonequilibrium conditions that were produced by discharging wells. Theis recognised that there was a similarity between the hydrologic conditions existing in an aquifer and the thermal conditions existing in a similar thermal system, and that the mathematical theory of heat conduction that had been developed by Fourier and others was largely applicable to hydraulic theory. The Theis nonequilibrium formula[22] is as follows:

$$s = \frac{114.6\ q}{T} \int_{1.87\ r^2 S/Tt}^{\infty} (e^{-u}/u)\ du$$

where s is the drawdown in feet, q is the discharge in gpm, r is the distance from the pumped to the observation well in feet, t is the time since pumping started in days, S is the coefficient of storage (as a decimal), and T is a new term called the coefficient of transmissibility of the aquifer.

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[22]The complete derivation of this formula as presented by Theis and Lubin is included in this paper as Appendix A.

> The term "coefficient of transmissibility" is here used to denote the product of Meinzer's coefficient of permeability and the thickness of the saturated portion of the aquifer; it quantitatively describes the ability of the aquifer to transmit water. Meinzer's coefficient of permeability denotes the character of the material; the coefficient of transmissibility denotes the analogous characteristic of the aquifer as a whole.[23]

The coefficient of transmissibility is defined quantitatively as the rate of flow of water in gallons per day through a vertical strip of the aquifer 1 foot wide and extending the full saturated height under a unit gradient at a temperature of 60°F.

The equation does not lend itself to direct solution for S and T. The following graphic solution is suggested by Theis.[24] The definite integral is a form of the exponential integral and is a function of the lower limit. Tables of the value of the definite integral for various values of the lower limit are available (Smithsonian Physical Tables, 8th rev. ed., table 32, 1933; the values to be used are those given for Ei(-X), with the sign changed). The value of the integral is given by the series[25]

$$\int_{1.87\, r^2 S/Tt}^{\infty} (e^{-u}/u)\, du = -0.577216 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots$$

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[23] Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage, Transactions American Geophysics Union, 16:520, August 1935.

[24] Wenzel, L. K., Methods for determining permeability of water-bearing materials, U. S. Geological Survey Water-Supply Paper 887, 1942, p. 89.

[25] Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage, Transactions American Geophysics Union, 16:520, August 1935.

when u = $(1.87\ r^2S/Tt)$. The exponential integral may be written W(u), which is usually read "well function of u"; and the formula becomes

$$s = \left[\frac{114.6\ q}{T}\right] W(u) \qquad \text{and} \qquad T = \frac{114.6\ q\ W(u)}{s} \qquad (1)$$

$$\frac{r^2}{t} = \left[\frac{T}{1.87\ S}\right] u \qquad \text{and} \qquad S = \frac{Ttu}{1.87\ r^2} \qquad (2)$$

For any particular aquifer and pump test, the quantities in the brackets will be constant. Therefore, the drawdown is related to $\frac{r^2}{t}$ in a manner that is similar to the relation between W(u) and u. When values of W(u) are plotted against corresponding values of u on logarithmic coordinate paper, a graph of the "type curve" is produced. If drawdown is plotted against corresponding values of $\frac{r^2}{t}$ on logarithmic coordinate tracing paper of the same scale as the type curve, another curve will be produced. If one curve is superimposed upon the other with the axes held parallel, there will be a point of best fit somewhere along the two curves. The coordinates of that match point are noted on both curves and those values of W(u), u, $\frac{r^2}{t}$, and s are used in equations (1) and (2) to determine the coefficients of transmissibility and storage. Table 1 and Figure 2 are the values of W(u) and u for the nonequilibrium formula and the plotted type curve respectively.

The nonequilibrium formula can be applied to cone of depression problems in other ways, too. If the coefficients of transmissibility and storage are known, the value of any one of the terms in either equation (1) or (2) can be computed for any particular combination of values for the other terms.

The nonequilibrium formula also lends itself to determination of

Table 1. Values of W(u) and u for the Nonequilibrium Formula.*

N \ u	$N\times10^{-15}$	$N\times10^{-14}$	$N\times10^{-13}$	$N\times10^{-12}$	$N\times10^{-11}$	$N\times10^{-10}$	$N\times10^{-9}$	$N\times10^{-8}$	$N\times10^{-7}$	$N\times10^{-6}$	$N\times10^{-5}$	$N\times10^{-4}$	$N\times10^{-3}$	$N\times10^{-2}$	$N\times10^{-1}$	N
1.0	[illegible]	31.6590	29.3564	27.0538	24.7512	22.4486	20.1460	17.8435	15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	[illegible]	[illegible]
1.1	[illegible]	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482	15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	[illegible]
1.2	[illegible]	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611	15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	[illegible]	[illegible]
1.3	[illegible]	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811	15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	[illegible]	[illegible]
1.4	[illegible]	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070	15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	[illegible]	[illegible]
1.5	[illegible]	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380	15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	[illegible]	[illegible]
1.6	[illegible]	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735	15.0709	12.7683	10.4657	8.1633	5.8621	3.5739	[illegible]	[illegible]
1.7	[illegible]	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128	15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	[illegible]	[illegible]
1.8	[illegible]	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557	14.9531	12.6505	10.3479	8.0455	5.7446	3.4581	[illegible]	[illegible]
1.9	[illegible]	31.0171	28.7145	26.4120	24.1094	21.8068	19.5042	17.2016	14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	[illegible]	[illegible]
2.0	[illegible]	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503	14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	[illegible]	[illegible]
2.1	[illegible]	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015	14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	[illegible]	[illegible]
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550	14.7524	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	[illegible]
2.3	[illegible]	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106	14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	[illegible]	[illegible]
2.4	[illegible]	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680	14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	[illegible]	[illegible]
2.5	[illegible]	30.7427	28.4401	26.1375	23.8349	21.5323	19.2298	16.9272	14.6246	12.3220	10.0194	7.7171	5.4167	3.1365	[illegible]	[illegible]
2.6	[illegible]	30.7035	28.4009	26.0983	23.7957	21.4931	19.1905	16.8880	14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	[illegible]	[illegible]
2.7	[illegible]	30.6657	28.3631	26.0606	23.7580	21.4554	19.1528	16.8502	14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	[illegible]	[illegible]
2.8	[illegible]	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138	14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	[illegible]	[illegible]
2.9	[illegible]	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788	14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	[illegible]	[illegible]
3.0	[illegible]	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449	14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	[illegible]	[illegible]
3.1	[illegible]	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121	14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	[illegible]	[illegible]
3.2	[illegible]	30.4958	28.1932	25.8907	23.5881	21.2855	18.9829	16.6803	14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	[illegible]	[illegible]
3.3	[illegible]	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495	14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	[illegible]	[illegible]
3.4	[illegible]	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197	14.3171	12.0145	9.7120	7.4097	5.1102	2.8379	.8147	[illegible]
3.5	[illegible]	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907	14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	.7942	[illegible]
3.6	[illegible]	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625	14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	.7745	[illegible]
3.7	[illegible]	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351	14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	.7554	[illegible]
3.8	[illegible]	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085	14.2059	11.9033	9.6007	7.2985	4.9993	2.7306	.7371	[illegible]
3.9	[illegible]	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825	14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	.7194	[illegible]
4.0	[illegible]	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572	14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	[illegible]	[illegible]
4.1	[illegible]	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325	14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	[illegible]	[illegible]
4.2	[illegible]	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084	14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	[illegible]	[illegible]
4.3	[illegible]	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3848	14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	[illegible]	[illegible]
4.4	[illegible]	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619	14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	[illegible]	[illegible]
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394	14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	.6253	[illegible]
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174	14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	.6114	[illegible]
4.7	32.4140	30.1114	27.8088	25.5062	23.2037	20.9011	18.5985	16.2959	13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	[illegible]	[illegible]
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748	13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	[illegible]	[illegible]
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542	13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	[illegible]	[illegible]
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340	13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	[illegible]	[illegible]
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142	13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	[illegible]	[illegible]
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948	13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	[illegible]	[illegible]
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758	13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	[illegible]	[illegible]
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4597	16.1571	13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	[illegible]	[illegible]
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387	13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	[illegible]	[illegible]
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207	13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	[illegible]	[illegible]
5.7	32.2211	29.9185	27.6159	25.3133	23.0108	20.7082	18.4056	16.1030	13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	[illegible]	[illegible]
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856	13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	[illegible]	[illegible]
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685	13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	[illegible]	[illegible]
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517	13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	[illegible]	[illegible]
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352	13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	[illegible]	[illegible]
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189	13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	[illegible]	[illegible]
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029	13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	[illegible]	[illegible]
6.4	32.1053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872	13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	.4197	[illegible]
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717	13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	.4115	[illegible]
6.6	32.0745	29.7719	27.4693	25.1667	22.8642	20.5616	18.2590	15.9564	13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	[illegible]	[illegible]
6.7	32.0595	29.7569	27.4543	25.1517	22.8491	20.5465	18.2439	15.9414	13.6388	11.3362	9.0337	6.7317	4.4351	2.1917	[illegible]	[illegible]
6.8	32.0446	29.7421	27.4395	25.1369	22.8343	20.5317	18.2291	15.9265	13.6240	11.3214	9.0189	6.7169	4.4204	2.1779	[illegible]	[illegible]
6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119	13.6094	11.3068	9.0043	6.7023	4.4059	2.1643	[illegible]	[illegible]
7.0	32.0157	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976	13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	[illegible]	[illegible]
7.1	32.0015	29.6989	27.3963	25.0937	22.7911	20.4885	18.1860	15.8834	13.5808	11.2782	8.9757	6.6737	4.3775	2.1376	[illegible]	[illegible]
7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694	13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	[illegible]	[illegible]
7.3	31.9737	29.6711	27.3685	25.0659	22.7633	20.4608	18.1582	15.8556	13.5530	11.2504	8.9479	6.6460	4.3500	2.1118	[illegible]	[illegible]
7.4	31.9601	29.6575	27.3549	25.0523	22.7497	20.4472	18.1446	15.8420	13.5394	11.2368	8.9343	6.6324	4.3364	2.0991	[illegible]	[illegible]
7.5	31.9467	29.6441	27.3415	25.0389	22.7363	20.4337	18.1311	15.8286	13.5260	11.2234	8.9209	6.6190	4.3231	2.0867	[illegible]	[illegible]
7.6	31.9334	29.6308	27.3282	25.0257	22.7231	20.4205	18.1179	15.8153	13.5127	11.2102	8.9076	6.6057	4.3100	2.0744	[illegible]	[illegible]
7.7	31.9203	29.6178	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022	13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	[illegible]	[illegible]
7.8	31.9074	29.6049	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893	13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	[illegible]	[illegible]
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766	13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	[illegible]	[illegible]
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640	13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	[illegible]	[illegible]
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516	13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	[illegible]	[illegible]
8.2	31.8574	29.5548	27.2523	24.9497	22.6471	20.3445	18.0419	15.7393	13.4367	11.1342	8.8317	6.5298	4.2346	2.0042	[illegible]	[illegible]
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272	13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	[illegible]	[illegible]
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152	13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	[illegible]	[illegible]
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034	13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	[illegible]	[illegible]
8.6	31.8098	29.5072	27.2046	24.9020	22.5995	20.2969	17.9943	15.6917	13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	[illegible]	[illegible]
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801	13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	[illegible]	[illegible]
8.8	31.7868	29.4842	27.1816	24.8791	22.5765	20.2739	17.9713	15.6687	13.3661	11.0636	8.7610	6.4593	4.1646	1.9393	[illegible]	[illegible]
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574	13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	[illegible]	[illegible]
9.0	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462	13.3437	11.0411	8.7386	6.4367	4.1423	1.9187	[illegible]	[illegible]
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352	13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	[illegible]	[illegible]
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6243	13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	[illegible]	[illegible]
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135	13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	[illegible]	[illegible]
9.4	31.7209	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028	13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	[illegible]	[illegible]
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922	13.2896	10.9870	8.6845	6.3828	4.0887	1.8695	[illegible]	[illegible]
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817	13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	[illegible]	[illegible]
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713	13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	[illegible]	[illegible]
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611	13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	[illegible]	[illegible]
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509	13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	[illegible]	[illegible]

*Table reproduced from U. S. Geological Survey Water-Supply Paper 887, facing p. 89

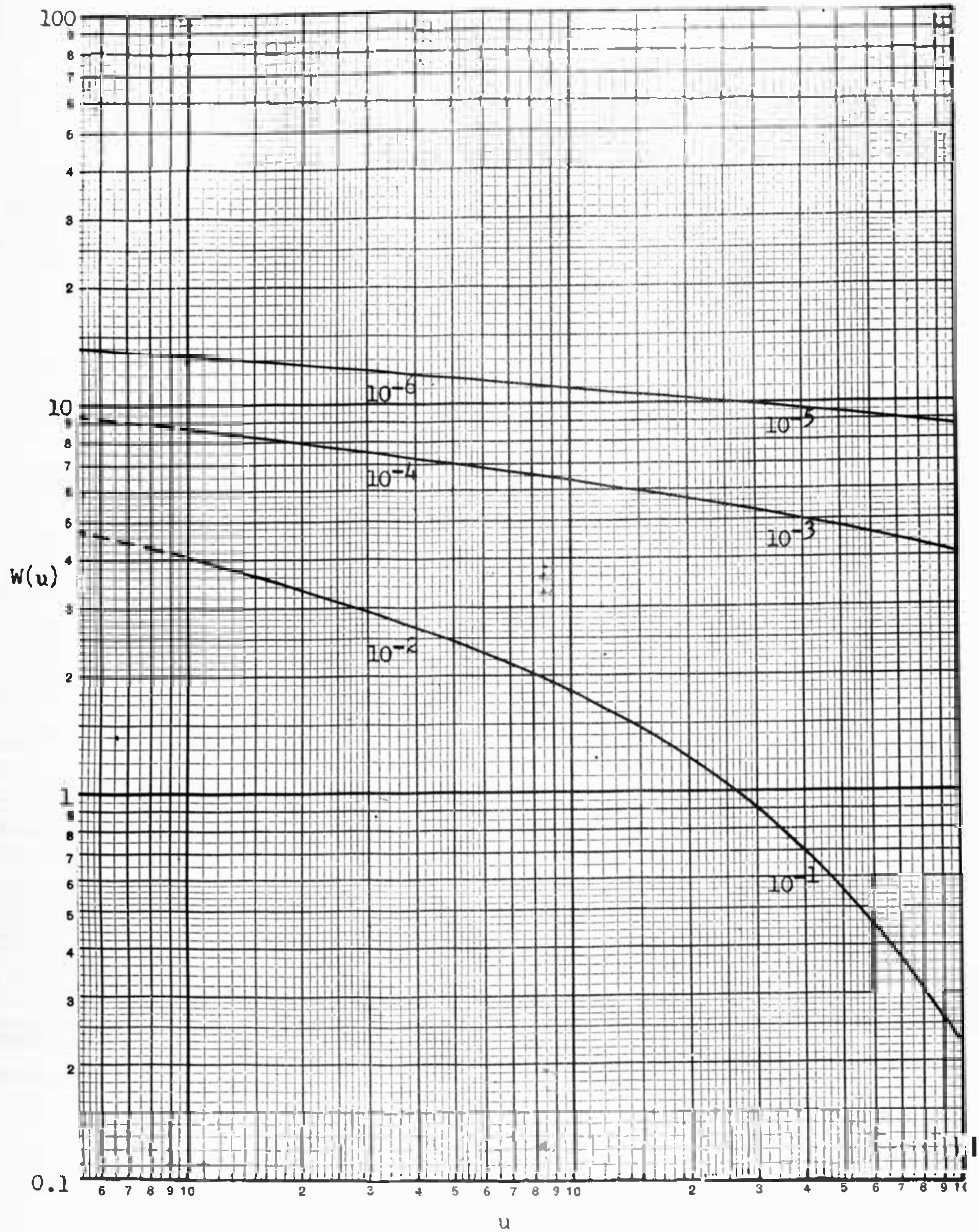

Figure 2. Type Curve for the Nonequilibrium Formula.

aquifer characteristics by using data collected during the recovery of a pumped well.[26] When the pumping of a well that has been discharging at a constant rate for a certain length of time is stopped and allowed to recover, the water level in the well will rise at the same rate as it would had pumping been continued at that constant rate and a recharge well with the same flow imposed on the system at the time the pumping was actually stopped. The net result would be the removal of no water from the aquifer. The residual drawdown at any time is the distance the water level in the well is below the static water table. The recovery at any time is the distance between the actual water level in the well and the level at which it would have been if pumping had been continued without recovery. The Theis nonequilibrum formula for the residual drawdown at any time becomes

$$s' = \frac{114.6\ q}{T}\left[\int_{1.87\ r^2S/Tt}^{\infty} \frac{e^{-u}}{u}\,du - \int_{1.87\ r^2S/Tt'}^{\infty} \frac{e^{-u}}{u}\,du\right]$$

which, for problems ordinarily encountered in ground water hydraulics, reduces to[27]

$$s' = \frac{114.6\ q}{T}\ \ln\frac{t}{t'}$$

where s' is the residual drawdown, t is the time since pumping began, and t' is the time since pumping stopped. (See figure 3.) (The complete

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[26]Ibid., p. 522.

[27]Ibid., p. 522.

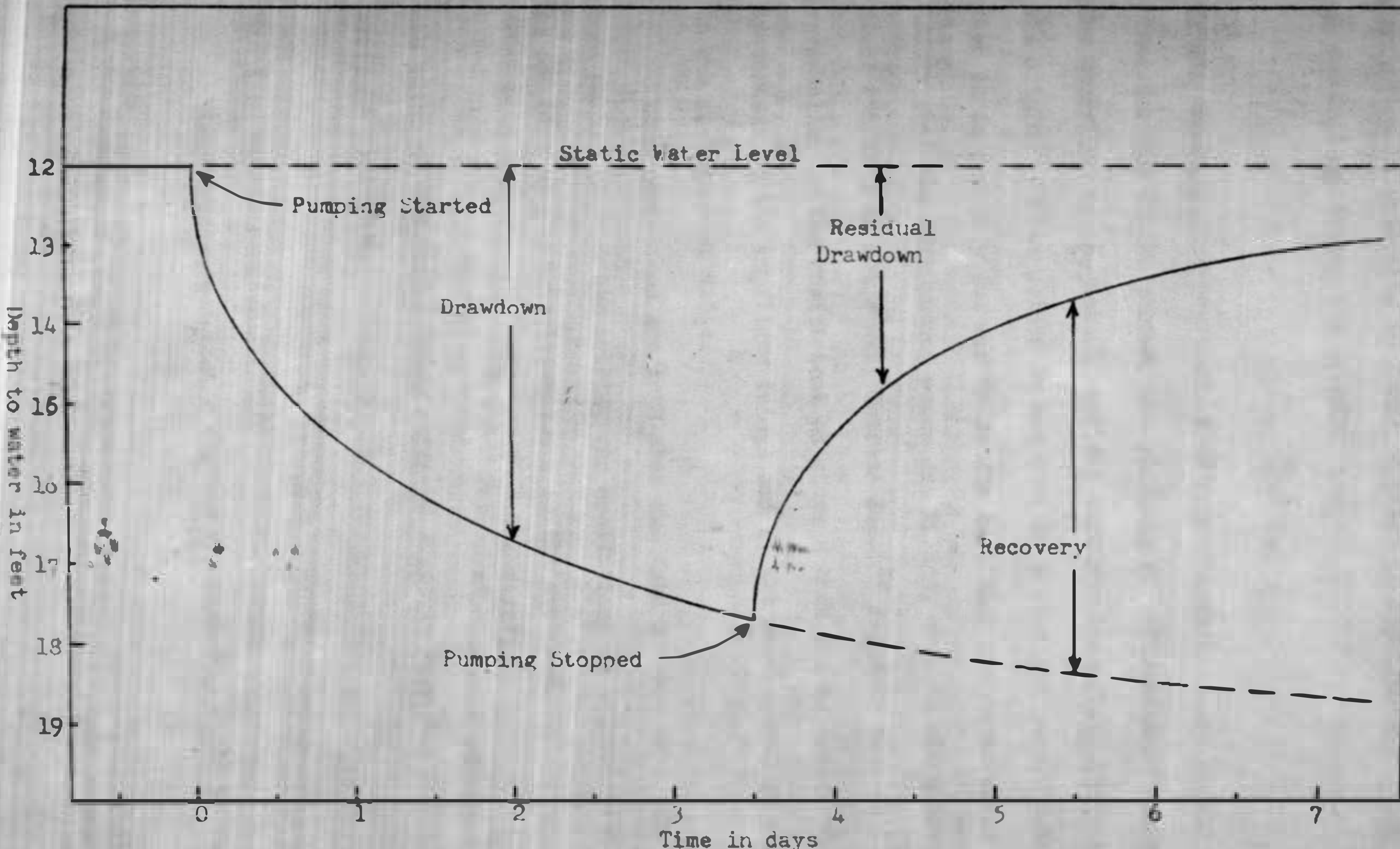

Figure 3. The Drawdown and Recovery of the Water Level in a Well.

derivation as presented by Theis is given in Appendix A.) When converted to common logarithms, the equation becomes

$$s' = \frac{264\,q}{T} \log \frac{t}{t'}$$

If the discharge and coefficient of transmissibility are considered constant for the entire period, the plotting of residual drawdown against the logarithm of $\frac{t}{t'}$ should yield a straight line that passes through the origin. If the points do not plot as a straight line, Theis suggests that it is probably due either to the fact that the water table rises faster than the surrounding pores are filled, or that the quantity ($1.87\ r^2 S/Tt'$) is not small; whereas Jacob[28] suggests that it is due to variability in the coefficient of storage that can be contributed to the hysteriesis of the capillary fringe and to the envelopment of air bubbles in the rising water table.

When conditions are such that the data do plot as a straight line that passes through the origin, the coefficient of transmissibility can be computed simply by solving for T in the equation

$$T = 264\,q\ \frac{\log\,(t/t')}{s'}$$

the value of $\frac{\log\,(t/t')}{s'}$ being obtained directly from the slope of the straight line plot.

Modified nonequilibrium formula:

Jacob recognized that a similar relationship could be derived for

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[28] Jacob, C. E., Recovery method of determining permeability, empirical adjustment for, U. S. Geological Survey, mimeographed report, November 10, 1945.

the change in the drawdown at one observation well as the time changed when the value of u was small.[29] The equation simply becomes

$$\Delta s = s_2 - s_1 = \frac{264\ Q}{T} \log \frac{t_2}{t_1}$$

where s_1 and s_2 are the drawdowns observed at times t_1 and t_2 respectively. This relationship is usually called the modified nonequilibrium formula. Obviously, if t_2 is ten times as great as t_1 the formula reduces to

$$\Delta s = \frac{264\ Q}{T} \quad \text{and} \quad T = \frac{264\ Q}{\Delta s}.$$

Therefore, if the drawdowns observed in one observation well are plotted against the logarithm of the corresponding time since pumping started in minutes; a straight line should be produced when t is large enough to make u be small. Small is defined by Brown[30] as a value that is less than 0.02 because the Theis type curve becomes a straight line for values of 1/u greater than 50 when it is plotted on semilogorithmic coordinate paper with W(u) on the arithmetic scale. (See figure 4.) Δs then, is the change in drawdown spanned by the straight line in any one log cycle.

The coefficient of storage can also be computed if the straight line is extended to intercept the zero-drawdown line. When the drawdown is zero, the equation

$$s = \frac{114.6\ Q}{T} \left[-\ln u - 0.5772 \right]$$

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[29] Ferris, J. G., op. cit., p. 242.

[30] Brown, Russel H., Selected procedures for analysing aquifer test data, Journal American Water Works Association, 45:858, August 1953.

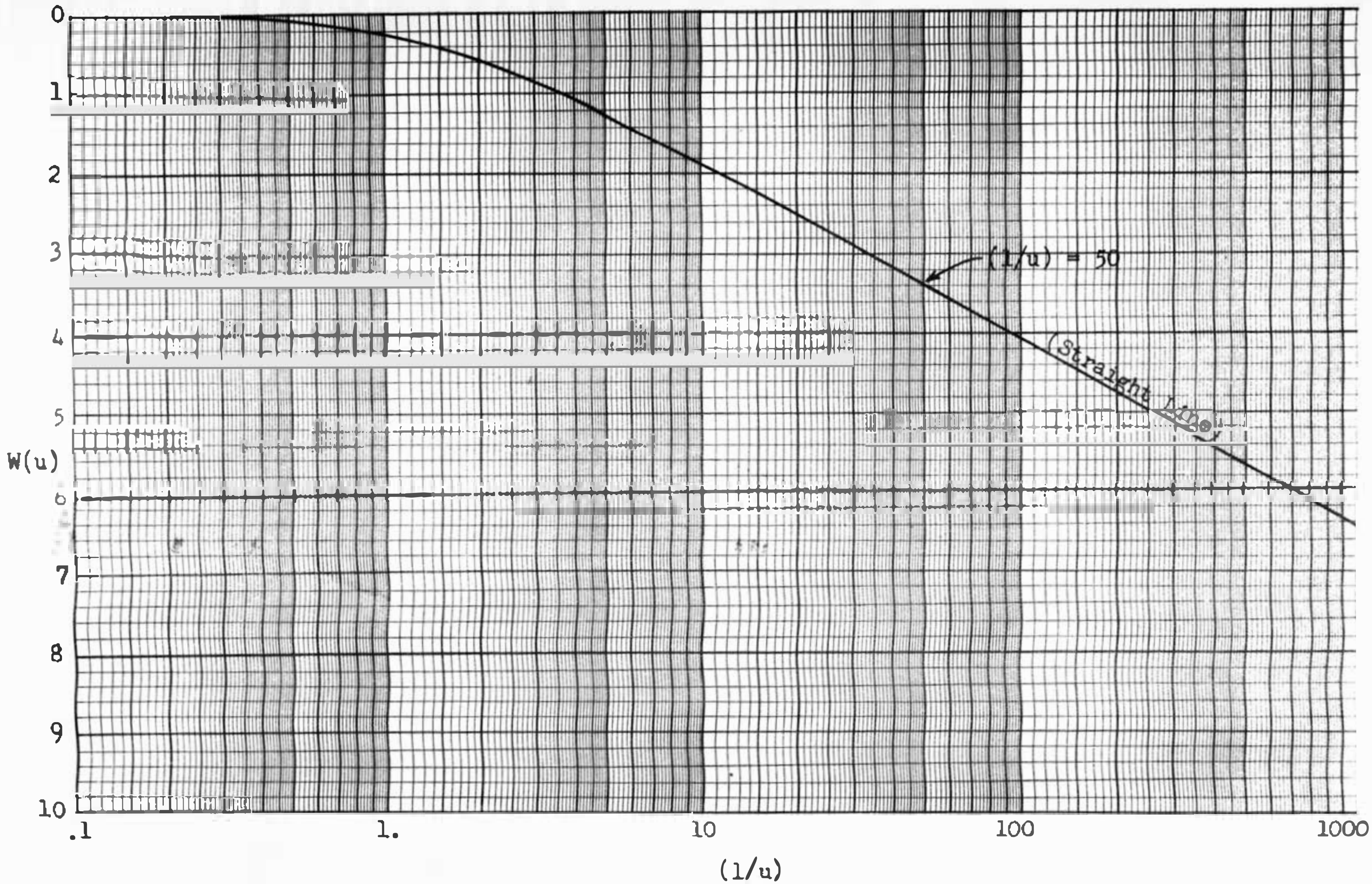

Figure 4. Type Curve Plotted on Semi-logarithmic Coordinate Paper.

reduces to

$$o = \ln (1/u) - 0.5772.$$

When

$$u = (1.87\ r^2 S/t_o\ T),\ \ln (T\ t_o/1.87\ r^2 S) = 0.5772.$$

Therefore,

$$\frac{T\ t_o}{1.87\ r^2 S} = e^{0.5772}$$

and

$$S = \frac{0.301\ T\ t_o}{r^2}$$

where t_o is the time intercept on the zero-drawdown axis, in days.[31]

Jacob[32] has also demonstrated that the coefficients of transmissibility and storage of an aquifer can be determined if the drawdowns are read simultaneously on a group of observation wells that are on line with the pumped well at some time after pumping is begun. He suggests, though, that the value of u should be less than 0.01. This time the drawdowns are plotted against the logarithms of their respective r values, and a straight line is drawn through the plotted points, rounding off the data. The formula for the coefficient of transmissibility in this case is

$$T = \frac{527.7\ q}{\Delta s} \log \frac{r_2}{r_1}.$$

As before, the logarithmic term becomes 1 when Δs is chosen as the

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[31]Ferris, J. G., op. cit., p. 244.

[32]Jacob, C. E., Notes on determining permeability by pumping tests under water-table conditions, U. S. Geological Survey, mimeographed report, June 1944.

change in drawdown spanned by the line in traversing one log cycle.

To determine the coefficient of storage, the line is again extended to intersect the zero-drawdown line and the radius of influence of the well is read at that intercept. The formula is the same as before

$$S = \frac{0.301\ T\ t_o}{r_e^2}$$

where r_e is the zero-drawdown intercept.

<u>Adjustment of test data for thin aquifers:</u>

One of the basic assumptions in the derivation of the formulas that have just been discussed is that the coefficient of transmissibility remains constant during the test. Under water table conditions, the drawdown that is produced to induce flow toward the well causes a reduction in the saturated thickness of the aquifer. If the observed drawdowns (y) are large as compared to the original saturated thickness of the aquifer (m), they must be reduced by the factor $(y^2/2m)$ or the computations will not give results that are representative of the existing conditions.[33] If the adjusted drawdowns are represented by s, the observed drawdowns by y, and the original saturated thickness of the aquifer by m, the formula for the adjusted drawdowns will be

$$s = y - (y^2/2m).$$

When dealing with thin aquifers, y is usually quite large compared to m; therefore, the adjustment will almost invariably need to be made before an analysis of the data is attempted.

[33]Ibid.

<u>Pump tests on horizontal wells:</u>

No evidence was found of a pump test ever having been conducted on a horizontal well consisting of one straight length of horizontal casing in a shallow, thin aquifer to determine aquifer characteristics.

ANALYSIS OF THE PROBLEM

There are, in general, two methods of increasing the expected yield of water from a well. One is to make the well deeper. That has the effect of not only increasing the effective coefficient of transmissibility, but also of permitting an increase in the hydraulic gradient that induces flow toward and into the well because the drawdown can be increased. When we are dealing with shallow, thin aquifers, the well usually penetrates the entire aquifer in the first place; consequently, making the well deeper is an impossibility.

The second method is to make the well larger in diameter. But to make a well two feet in diameter instead of only one foot does not double the yield. The increase is, in fact, much less than double. Let us refer again to the Dupuit equation and review the relationship of the various factors to each other:

$$Q = K \frac{h_e^2 - h_w^2}{\log \frac{r_e}{r_w}} = K \frac{(h_e + h_w)\ (h_e - h_w)}{\log \frac{r_e}{r_w}}$$

The term $(h_e - h_w)$ is actually the observed drawdown in the well. Obviously, if the permeability and the drawdown are held constant and the radius of the well is increased, the numerical value of the denominator will decrease. Since the lateral growth of the cone of depression is a function of time and this equation applies only to equilibrium conditions, r_e will remain constant. Therefore, Q will increase not directly as the radius of the well, but will be inversely proportional to the logarithm of $\frac{r_e}{r_w}$. If r_e for a particular case were 2000 feet, the diameter of a well would have to be changed from one foot to approximately 45 feet to double the yield. Such a solution would be neither economical nor practical.

Q is proportional to the permeability, and the thickness of the aquifer; but it is not necessarily proportional to the drawdown even though a hasty glance at the preceding equation might so indicate. The proportionality between Q and drawdown exists only when the drawdown is small compared to aquifer thickness. Under unconfined conditions the direct proportionality has usually ended by the time the drawdown becomes approximately 30% of the aquifer thickness.[34]

Since the depth of a well in a thin aquifer cannot be made greater to increase the yield, the only alternative is to increase its effective diameter if more water is needed. Representative figures can be used to simplify the problem. For instance, a particular thin aquifer has an original saturated thickness of 7 feet, a coefficient of transmissibility of 17,000 gallons per day per foot, a coefficient of storage of 0.225, and the required discharge to operate a particular irrigation system is 200 gallons per minute for a ten-day period of pumping. The maximum observed drawdown that may be produced is to be limited to 5.0 feet which when adjusted for thin aquifers is 3.22 feet ($s = y - \frac{y^2}{2m}$). How large must the well be? A solution of the Theis nonequilibrium formula for r results in a required radius of 147.7 feet. It is interesting to note in passing that a 12-inch diameter well in such an aquifer would produce

[34]Bennison, E. W., Ground Water, first edition, St. Paul 4, Minn., Edward E. Johnson, Inc., 1947, p. 208.

only 34.8 gallons per minute for that same ten-day interval. A 2-inch sand point would yield only 27.6 gallons per minute.

From the above considerations, it is apparent that the solution to such a problem is not a direct increase in the diameter. A battery of sandpoints connected as a unit and pumped as one well is a possible solution.[35] Using the method presented by Holman on a line of five 2-inch sandpoints spaced 20 feet apart in this same aquifer, the yield was computed for a ten-day period of pumping using the maximum drawdown. The results of these calculations are tabulated in Tables 2 and 3. These computations are based upon the assumptions that the drawdowns will be equal in all of the sandpoint wells. In practice this is an impossibility because of the friction losses in the connecting pipes. Such a system is also limited by the suction lift that can be developed by a pump that is common to all the sandpoints, and by extreme priming difficulties when the required lifts are in excess of 15 feet.

A review of the preceeding figures indicates that it may be quite impractical to attempt to enlarge a system of sandpoints enough to obtain the required discharge from such an aquifer, especially if all of the sandpoints are to be pumped by a common pumping unit.

The next logical approach to the problem would then be the principle of the horizontal well. This could be attacked from the angle of a vertical well of an effective radius to which a certain length of horizontal casing would be equivalent. In other words, the size of a vertical

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[35]Holman, Harold, Discharge from a sandpoint system for a thin aquifer in the sioux river area, Unpublished M.S. Thesis, Brookings, South Dakota, South Dakota State College Library, 1955.

Table 2. Values of u and W(u) for Vertical Wells of Various Radii - Case I. (T = 17,000 gallons per foot per day, S = 0.225, s = 3.22 feet, t = 10 days.)

Radius in feet	u	W(u)
0.0833	1.72×10^{-8}	17.30
20.0	9.90×10^{-4}	6.34
24.5	1.49×10^{-3}	5.93
40.0	3.96×10^{-3}	4.96
60.0	8.91×10^{-3}	4.15
80.0	1.58×10^{-2}	3.59

Table 3. Values of Combined W(u) and Discharge for Each of Five 2-inch Sandpoints Spaced 20 Feet Apart in a Line and the Discharge of the Sandpoint System and a 24.5 Foot Radius Well - Case I.

	Combined W(u)*	Discharge in gpm
Center sandpoint	39.9	12.0
Sandpoint 20 feet out	39.1	12.2
Sandpoint at end	36.3	13.2
The sandpoint system		62.8
24.5 foot radius well		80.5

*These values are combinations of values of W(u) from Table 2 according to the method presented by Holman.

well that would yield the same quantity of water as a certain length of horizontal casing if both types of well were constructed in the same aquifer. Such an estimate of effective radius is very difficult to make because flow into a horizontal well is likely not to be radial. The piezometric contours will likely approach an elipse in shape rather than a circle even in the theoretically perfect case of a homogeneous aquifer, and the cone of depression will quite likely take on different shapes at different stations along the horizontal.

To simplify the problem, a number of assumptions must be made: (1) that the aquifer is homogeneous; (2) that the horizontal screen will be placed at the bottom of the aquifer; (3) that the placement of the well does not disturb enough of the aquifer that there is an appreciable change in its characteristics; (4) that the static water level is, for all practical purposes, level; and (5) that flow approaches the horizontal in the same manner as it would approach a straight line battery of sand-points.

If all these conditions are met and an effective radius of $\frac{\text{length of horizontal}}{\pi}$ is assumed, estimates of the yield can be made. There is no magic involved in the selection of such an effective radius; it is simply the radius of a circle that has a circumference equal to twice the length of the horizontal. Obviously, the flow of water in the aquifer will approach the well from both sides, and the well should actually be fed by a length of aquifer equal to twice the length of the horizontal screen.

The use of the equivalent radius is easily demonstrated by an example. A well consisting of 77 feet of 8-inch horizontal casing is placed in an aquifer. The assumed equivalent radius then could be $\frac{77}{\pi}$ = 24.5 feet. Using the same aquifer characteristics as were previously used, the expected yield of a 24.5 foot radius well during a 10 day pumping period with maximum drawdown is shown in Table 3. Such computations would indicate that this type of a well would be more effective than a system of sandpoints under similar conditions if the assumptions are representative. Such reasoning seems logical, especially after an examination of Figure 5. The volume of sediments that is dewatered by the horizontal well is greater than the volume that is dewatered by the sandpoint system. It is also conceivable that the quantity of water that an aquifer will yield to the horizontal well should be greater than the quantity it will yield to the sandpoints because a much larger area of screen is in contact with the aquifer. All the water that is yielded to a sandpoint system must converge on one of the sandpoints, whereas much of the water that reaches the horizontal well can do so by following paths that are normal to the horizontal casing. The latter are shorter paths for much of the water.

Also, the Ranney Company often uses an estimated effective radius of 80% of the average length of their laterals when they use a complete system radiating all directions from the caisson.[36] By comparison, the assumption used herein is 31.8% for only one straight line instead of a number of radiating lines.

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[36]Klaer, Fred, Jr., Director of Research, Ranney Method Water Supplies, Inc., P. O. Box 5415, Columbus 19, Ohio, Information about ranney collectors, (Personal Conversation), 1957.

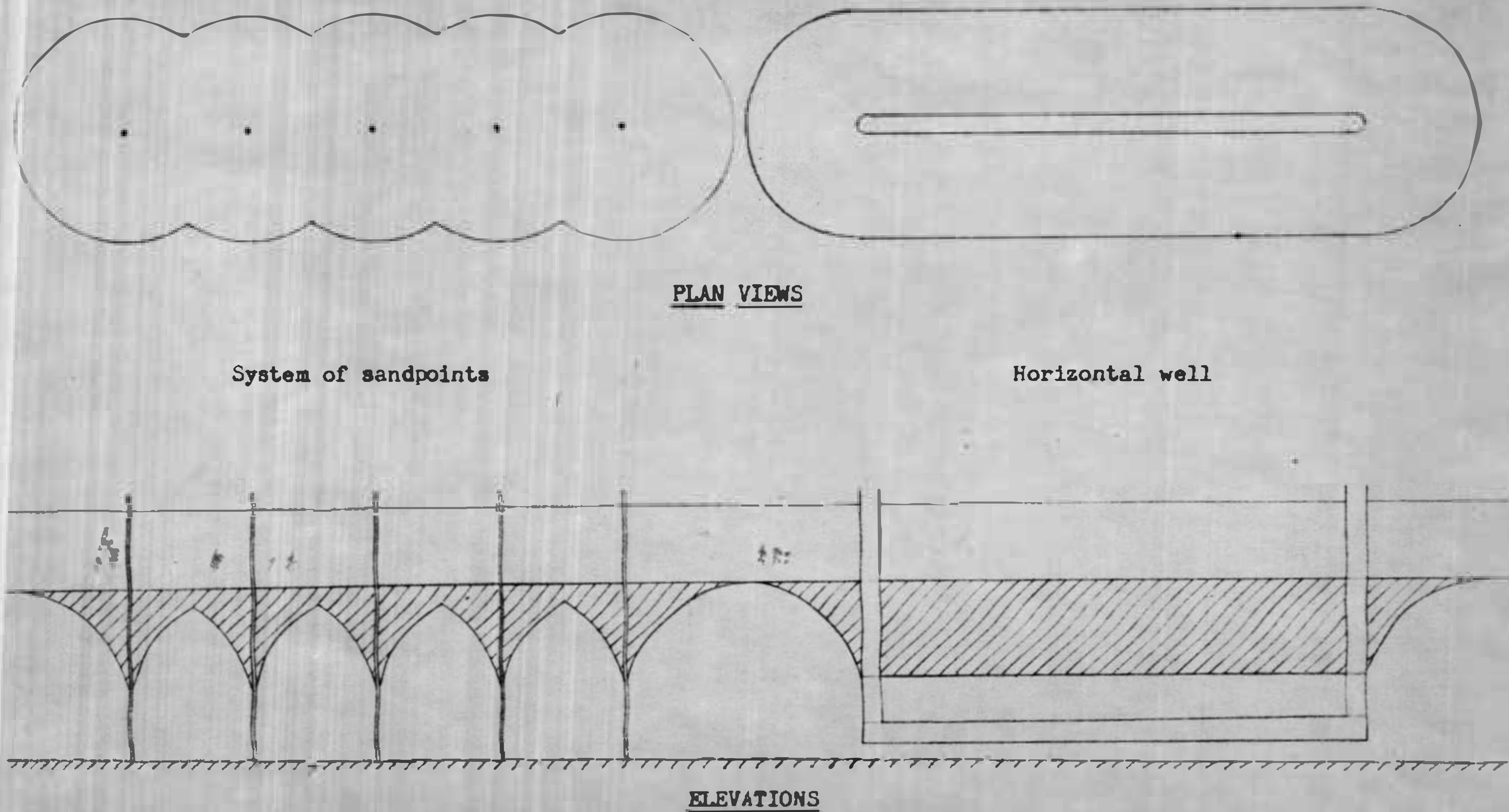

Figure 5. Elevation and Plan Views of the Dewatered Volume of Sediments Near a System of Sandpoints and a Horizontal Well.

The straight line of 2-inch sandpoints spaced 20 feet apart was purposefully chosen for comparison because the linear extent of such a system is 80 feet. The linear extent of the horizontal well was 77 feet. The yield of the horizontal well was estimated to be 80.5 gpm against 62.8 gpm for the sandpoint system. That would indicate a 28.2% advantage in favor of such a horizontal well.

Should such an advantage check out in practice and a horizontal well be economically feasible, horizontal wells might bring irrigation to a vast area that is presently plagued by frequent drouths, and that is blessed by shallow, thin aquifers.

CONSTRUCTION PROCEDURE

The site selected for the construction of the horizontal well was on a farm owned by Dean C. Austin located approximately 5 miles south of Brookings, South Dakota, in Brookings County. The well was constructed on the N½ of SW¼ of SW¼ of Sec. 24, T109N, R50W.

Two reasons prompted the selection of this location. First, both Austin and his tenant are deeply interested in irrigation; and second, the aquifer was thin enough that the problems so far discussed were of the utmost significance. The tenant is presently irrigating from a battery of sandpoints, but his program is seriously handicapped by the limited quantities of water which he is able to secure. The reasons for such a shortage of water have already been discussed. Also, this site is near the site used by Holman when he conducted his study of multiple sandpoint systems.

The aquifer itself is 7 feet thick, and it is covered by 11.5 feet of overburden of which all except the top 16 inches (approximately) is sand and gravel. The sands of the aquifer are quite fine and quite well graded as is evidenced by an effective size of 0.22 mm and a uniformity coefficient of 2.73.

A dragline contractor was engaged to construct a pit large enough to allow the placement of a prefabricated horizontal well consisting of 77 feet of 8-inch diameter perforated casing (horizontal) equipped with a plain 18-inch diameter riser at each end making a total of 80 feet of length. Such sediments as were encountered will stand nearly vertically when not influenced by ground water. In the range of ground water influence, the sediments are unstable and assume an angle of repose of approximately 2:1. The excavation was begun approximately 30 feet in width and the sides cut nearly vertical until the water table was reached. Then the excavation was continued in the middle allowing the sides to assume their own slope. It was assumed that such a method would entail the movement of the smallest possible volume of material.

The dragline moved back after the hole had been dug to the desired depth and width within its reach. Consequently, the pit was constructed from one end to the other. (See figure 6) Such a procedure is complicated by the problem of finding room to dump the excavated material within reach of the dragline. It is believed by the writer that a better plan for any future developments would be to have the dragline work crosswise to the proposed pit. By so doing, the excavated material could be dumped behind the dragline instead of beside it. The spoil material could be kept farther away. Work would still proceed from one end to the other, however.

During this construction process, the dragline operated in the water that seeped into the pit. The stirring of the water caused by the removal of each bucket of sediment seemed to cause siseable quantities of sand to be pulled toward the middle of the pit. It is possible that less material would have had to be removed if the water had been pumped down as low as possible in the pit. The pit could have been at least partially dewatered by the use of a flexible suction hose attached to a floating intake. Another advantage of working in only 1.5 or 2 feet of water instead of 7 feet is that the dragline operator could see what he

Figure 6. The Dragline Working on the Nearly Completed Pit.

was doing during nearly all of the digging operation.

Even though the banks were relatively stable, cave-ins were a constant threat. Working normal to the pit would probably have been an advantage here, too, because a little more slope could easily be put on the sides. Also, having the weight of the excavated material farther back should add to the stability of the banks.

While the digging was in process, the well was being prefabricated beside the pit. The 8-inch diameter horizontal screen was standard 14 gauge casing perforated with 3/16 inch by 1.25 inch slots that was purchased from Dempster Mills Pipe Company of Beatrice, Nebraska. These slots may sound excessively large for use in an aquifer with sand of such a small effective size; but a gravel envelope was to be installed. Klaer[37] suggested that the slots be 3/8 inch wide. With a gravel blanket, wide slots would be less likely to become plugged with sand than narrow slots. The Ranney Company uses a 3/8 inch slot exclusively, and remove an average of from 3 to 7 cubic feet of fine sand for each foot of 8 inch casing that is projected.[38] That results in a developed area about 8 feet in diameter around the lateral. No such elaborate aquifer development was anticipated for this well, but the principle of larger slots seemed to be as applicable here as in the larger, more elaborate developments.

A hole was cut in the side of two lengths of 18-inch diameter standard plain casing 8 inches up from the bottom and a length of the 8-inch diameter perforated casing was inserted, secured, and reinforced as is shown in Figure 7. Such a joint was constructed for each end and both ends were braced with an 11.5-foot 2x6 (wood) and bands as is shown in Figure 8. The completely assembled well is shown awaiting completion of the pit in Figure 9.

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[37] Ibid.

[38] Silitch, E. W., The place of ranney collectors in the water supply industry, Technical Report No. 3, Ranney Method Water Supplies, Inc., Columbus 19, Ohio, 1948.

The excessive turbidity (resistance to the passage of light) of the water in the pit made it necessary to check the condition of the bottom and the depth of the excavation with a weight suspended from the middle of a rope stretched across the pit. The final check is being made in Figure 9.

The dragline then moved to the side of the pit, picked up the well, and lowered it into place in the pit. Figure 10 and 11 show the well in transit to the pit. Such a structure is awkward, to say the least, and many helpers were needed to help guide the well into position.

Figure 7. The Reinforced Joint Between the 8-inch Casing and the 18-inch Riser.

Figure 8. The Reinforced and Braced Connection Between the 8-inch Casing and the 18-inch Riser.

The 18-inch riser was placed at both ends of the horizontal casing to serve a very special purpose. If fine sand should be carried into the well and deposited there, the benefits of a long horizontal casing would be lost; so some method of cleaning had to be provided. By using an 18-inch riser at each end, a portable roller could be inserted at each end of the horizontal and a sewer cleaning bucket could be worked back and forth to bring the sand to the end where it could be lifted out of the well. When the sewer cleaning bucket is pulled forward (to the left in Figure 12), the jaws are open so that sediments can enter the bucket. When it is pulled back (to the right in Figure 13), the jaws close and the load is retained in the bucket. The normal practice is to pull the load toward the discharge end of the conduit.

Figure 9. Making the Final Check of the Bottom of the Pit With a Weight Suspended From the Middle of a Rope.

Figure 10. The Completely Prefabricated Horizontal Well Being Moved to the Pit.

The portable roller is shown in Figures 14, 15, and 16. The wood block is a portion of an 18-inch diameter disk which distributes the horizontal thrust over a large portion of the wall of the riser. The roller is a piece of 5.5-inch diameter pipe filled with concrete to hold the hub in place. The long vertical pipe served as a handle and gives a convenient means of anchoring the roller to the top of the riser.

During the early stages of pumping the sewer cleaning bucket was operated continually by a man at each 18-inch riser to bring the sand to the suction pipe on the pump so that it could be discharged with the water. One man was able to pull the bucket each way during the development of the well. Very little sand entered the horizontal casing after about 5 hours of pumping. However, a 1/8 inch airplane cable was left threaded through the well. Should any sand accumulate in the future, the cable can be used to pull a rope through and the cleaning process can be repeated. It would be extremely difficult to clean a horizontal well without such a precaution or without access to both ends of the horizontal casing.

Figure 11. The Horizontal Well Being Lowered into the Pit. (A rainstorm added much discomfort to the anxiety of the moments.)

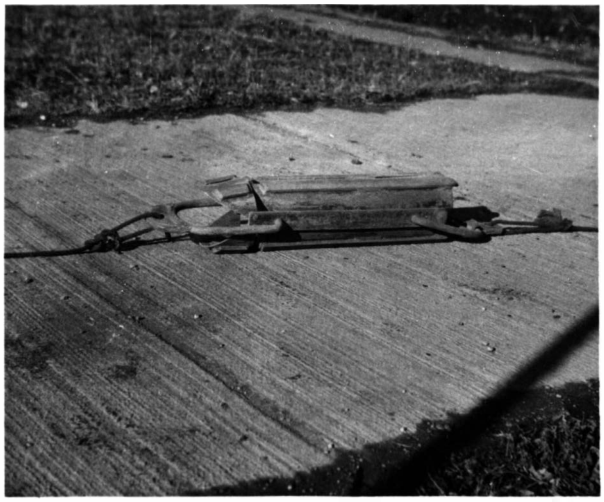

Figure 12. The Sewer Cleaning Bucket With the Jaws Open as it is Pulled to the Left to Get a Load.

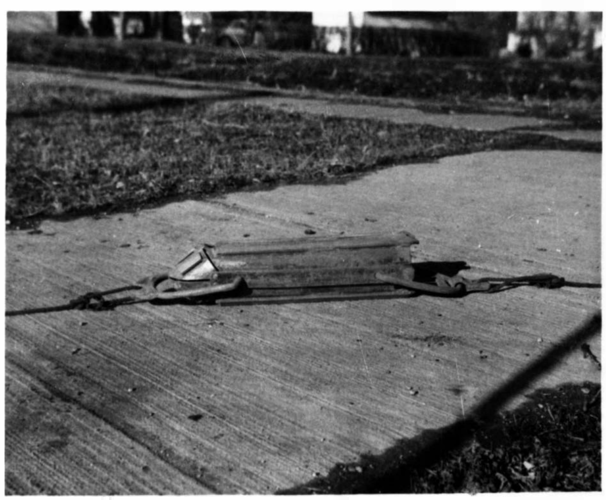

Figure 13. The Sewer Cleaning Bucket With the Jaws Closed to Hold a Load as it is Pulled to the Right.

Figure 14. Front View of the
Portable Roller

Figure 15. Top View of the
Portable Roller

Figure 16. Side View of the
Portable Roller

As soon as the well was placed in the pit, the dragline operator proceeded to place a pea-rock blanket over the horizontal casing. This gravel blanket was made approximately 1 foot thick and as uniform as possible under such conditions. Dewatering the pit might have had the same advantages here as have been previously mentioned.

Before backfilling was begun, the gravel envelope was checked by probing the bottom of the pool from a raft because the water was too turbid to allow a visual inspection. It appeared that the gravel envelope was quite uniform in thickness, but that a cave-in that occurred during the placement of the well had deposited some sand in the middle of the pit. Consequently, the middle of the horizontal casing was raised approximately ten inches. If this had not happened, a little more draw-down could have been safely developed in the subsequent pump test.

The pit was backfilled at the ends only, and only to about 1 foot above the water level. The entire hole was not filled because of two reasons. First, the pump could be placed lower, thereby reducing the suction lift required during development and testing. Second, by leaving the open pool in the middle of the pit, the gravel envelope could be visually inspected periodically during the development of the well.

Early in the development process, it became evident that the water level in the pool was higher than it was in the surrounding aquifer. Such a situation is not natural, since the free body of water was separated from the horizontal casing only by the gravel envelope. An inspection after the pool was emptied revealed a deposit of fine particles on the surface of the gravel. Apparently, the water was carrying the fine particles as it flowed to the well. The open pool was acting as a stilling basin and the sediments were forming a layer that was more impervious than the original aquifer.

This layer was removed and did not develope again to any appreciable extent. Apparently, the fine sediments were the result of a natural development; and had the pool not been left open, they would have been carried into the well and removed in the discharge water.

The cost of the materials for such a horizontal well as has been described was $450.00. The excavation cost $400.00. The cost of the labor required to prefabricate the joints between the 8-inch casing and the 18-inch risers and to build the portable rollers was $50.00 making a total cost of $900.00 for the unit.

THE PUMP TEST

A centrifugal pump powered by a gasoline engine[39] was used for the pump tests. The pumping unit was mounted at the east end of the well with the lower end of the suction pipe approximately level with the horizontal casing. (See Figure 17.) The discharge was measured by an elbow meter which consisted of a mercury-water differential manometer attached to the inside and outside of the bend of a standard 2-inch threaded elbow. The difference in pressure between the inside and outside of the bend is evidenced by an unbalance in the manometer. This unbalance can be converted to gallons per minute of discharge.[40] The first test was run at a discharge of 156 gallons per minute which is equivalent to a mercury-water unbalance of 6 inches. For the second test an unbalance of 2.5 inches was used which indicates a discharge of 100 gallons per minute.

The first pump test was handicapped by engine troubles that developed after 2.5 hours and the test had to be abandoned after 8 hours. However, the test did indicate that the pumping rate was too high and could not possibly have been sustained for 3 days.

Three weeks was allowed to ellapse before the second test was attempted. This time interval allowed the water table to recover from the disturbance created by the pumping. The testing was also done

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[39]Constructed Machinery Company, Rainmaster Model SS 5H 75, 5 in. 75 HP pumper.

[40]Roberson, J. A., The use and explanation of the elbow meter, Washington State Institute of Technology Bulletin 209, December 1950, p. 7.

Figure 17. The Pumping Unit Installed at the East Riser. (The arrow indicates the elbow portion of the elbow meter.)

during the fall of the year when the water table is generally changing from falling to rising stages. Not only was this near to the critical time as far as aquifer thickness is concerned, but it is also the time during which seasonal and regional changes are the least likely to cause measurable changes in this aquifer during a test. The discharge water was conveyed to a dry draw some 500 feet away from the well so that it could run away with as little danger as possible of establishing a boundary condition.

Figure 18. The Complete Elbow Meter.

The observation well for this second test was located 12.5 feet away from the well on a line perpendicular to the middle of the horizontal casing (Figure 19.) and consisted of two lengths of standard 8-inch diameter perforated well casing that was dug in by hand at the end of the first pump test to allow the use of the continuous recording stage recorder plus a 1¼-inch sandpoint that was driven down such that the top of the sandpoint screen coincided with the bottom of the 8-inch casing. Such an arrangement gave an observation well that penetrated the aquifer more than 4 feet with perforations over its entire length. Thus the decline of the free water surface could be recorded.

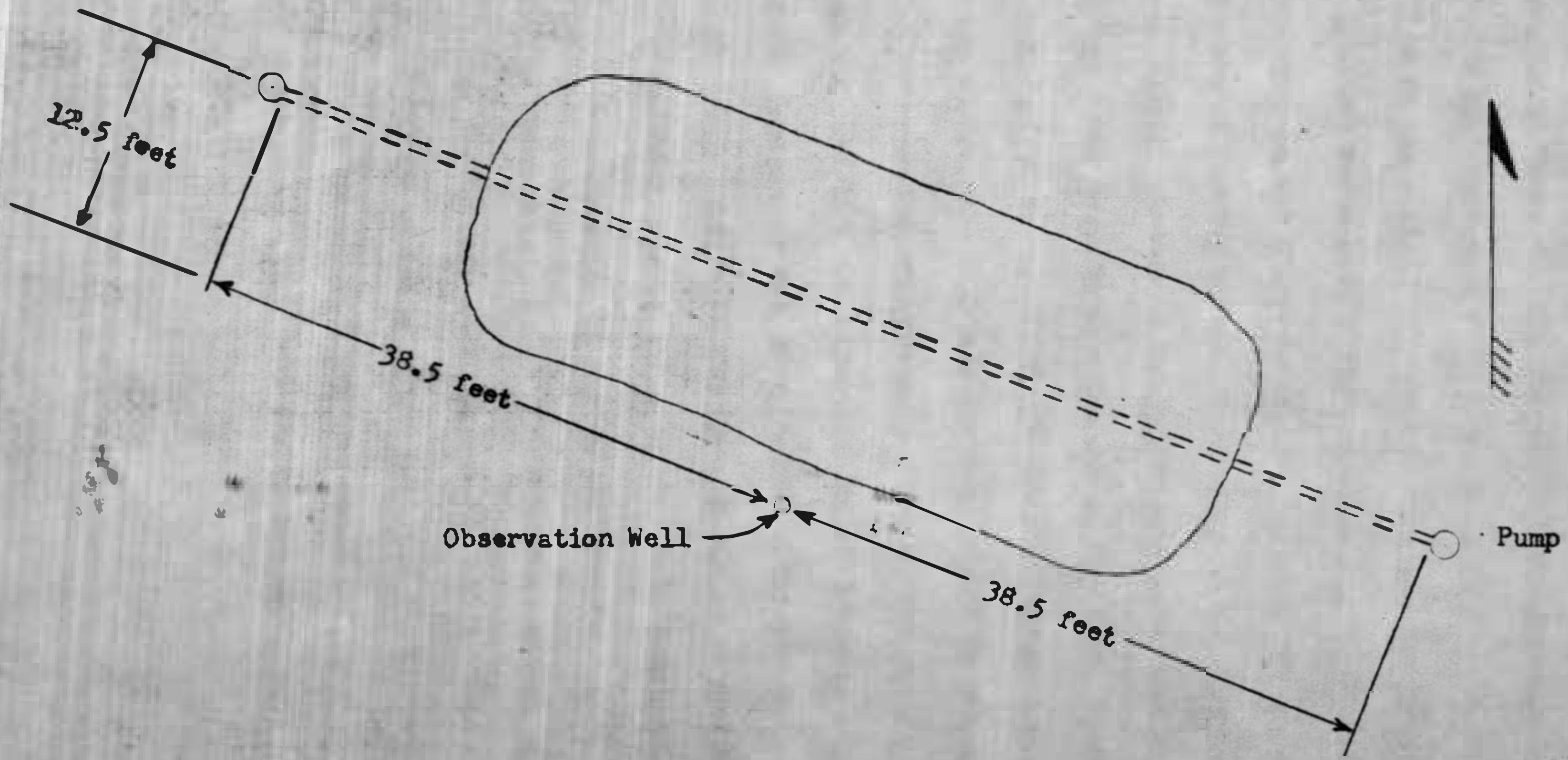

Figure 19. The General Arrangement of the Horizontal Well, the Observation Well, and the Surface of the Open Pool of Water.

The second pump test was accomplished without difficulty. The data are recorded in Table 4. The plotting of the drawdown versus $\frac{r^2}{t}$ curve (Figure 20.) indicates that there must be a variation in aquifer characteristics in the vicinity of the well because the points do not fit well to a smooth curve even though the discharge was held constant.

The readings on the observation well that were taken during the first few minutes of pumping were made by using an M-Scope because the smallest time interval that could be effectively read from the stage recorder chart was 5 minutes. Toward the end of the test, readings had to be taken by means of an M-Scope in the 1¼-inch sandpoint because the 8-inch diameter observation well was not deep enough to permit the use of the continuous recording stage recorder during the entire test.

The second pump test was continued for 67.3 hours. By the end of that time the drawdown in the east 18-inch riser was 4.70 feet. The elevation of the water surface in the 18-inch risers seemed to fluctuate slightly during the test. It is believed that this fluctuation was caused by slight irregularities in engine performance.

Changes in barometric pressure would have no effect upon the measurements taken during this test because the water table was exposed to the atmosphere in the vicinity of the well and observation well. Therefore, changes in atmospheric pressure would act equally on the water surface in the observation well and on the water table around the observation well.

Table 4. Discharge and Drawdown Data Collected During the Second Pump Test.

Date (Nov.)	Time of day	q (gpm)	t (min.)	Observation well* y	Observation well* $y^2/2m$	Observation well* s	in Pumped well* y
23	1400	0	0	0	0	0	
		100	0.5	.03	0	.03	
	1401	100	1.0	.06	0	.06	
		100	1.5	.08	0	.08	
	1402	100	2	.10	0	.10	
	1403	100	3	.12	0	.12	
	1404	100	4	.15	0	.15	
	1405	100	5	.17	0	.17	
	1406	100	6	.19	0	.19	
	1408	100	8	.21	0	.21	
	1410	100	10	.22	0	.22	
	1412	100	12	.23	0	.23	
	1414	100	14	.25	0	.25	
	1418	100	18	.28	.01	.27	
	1422	100	22	.32	.01	.31	
	1426	100	26	.34	.01	.33	
	1430	100	30	.37	.01	.36	
	1436	100	36	.40	.01	.39	
	1440	100	40	.43	.01	.42	
	1445	100	45	.45	.01	.44	
	1450	100	50	.47	.02	.45	
23	1500	100	60	.53	.02	.51	
	1510	100	70	.57	.02	.55	
	1520	100	80	.60	.03	.57	
	1530	100	90	.63	.03	.60	
	1550	100	110	.70	.03	.67	
	1610	100	130	.79	.04	.75	
	1630	100	150	.87	.05	.82	
	1700	100	180	.97	.07	.90	
	1730	100	210	1.09	.08	1.01	
	1800	100	240	1.17	.10	1.07	
	1900	100	300	1.35	.13	1.22	
23	2000	100	360	1.50	.16	1.34	

*All drawdowns are recorded in feet, m = 7 feet, s = $(y - y^2/2m)$, r = 12.5 feet.

Table 4. (Continued)

Date (Nov.)	Time of day	q (gpm)	t (min.)	Observation well*			in Pumped well*
				y	$y^2/2m$	s	y
23	2100	100	420	1.59	.18	1.41	
	2200	100	480	1.70	.21	1.49	
	2300	100	540	1.80	.23	1.57	
23	2400	100	600	1.91	.26	1.65	
24	0100	100	660	1.99	.28	1.71	
	0200	100	720	2.07	.31	1.76	
	0300	100	780	2.13	.32	1.81	
	0400	100	840	2.19	.34	1.85	
	0500	100	900	2.22	.35	1.87	
	0600	100	960	2.24	.36	1.88	
	0700	100	1020	2.27	.37	1.90	
24	0800	100	1080	2.31	.38	1.93	
	0900	100	1140	2.34	.39	1.95	
	1000	100	1200	2.37	.40	1.97	
	1100	100	1260	2.41	.41	2.00	
	1200	100	1320	2.43	.42	2.01	
	1300	100	1380	2.46	.43	2.03	
	1400	100	1440	2.49	.44	2.05	4.29
24	1600	100	1560	2.56	.47	2.09	
25	0100	100	2100	2.75	.54	2.21	
	0400	100	2280	2.84	.58	2.26	
	0900	100	2580	2.98	.63	2.35	4.48
	1120	100	2720	2.98	.63	2.35	
	1430	100	2910	3.05	.66	2.39	4.55
	1735	100	3095	3.11	.69	2.42	
	2015	100	3255	3.14	.70	2.44	
25	2400	100	3480	3.18	.72	2.46	4.70
26	0919	100	4039	3.18	.72	2.46	4.70

ANALYSIS OF RESULTS

The data were first analysed by the nonequilibrium method. Figure 20 was compared to Figure 2 to find the point of best fit between the curves when the coordinate axes of the 2 plots were held parallel. The respective coordinates of the match point on the 2 figures are listed on Figure 20. The coefficients of transmissibility and storage were computed and found to be 24,900 gallons per day per foot (gpd/ft.) and 0.506 respectively. Both of these coefficients are larger than was anticipated, probably as a result of the disturbance created in the aquifer by excavation, by the open pool of water and possibly by the shape of the well itself. The effect of the open pool on the storage coefficient should have been even more pronounced earlier in the pump test.

The data were then analysed in terms of the modified nonequilibrium method. The shape of the curve produced by plotting drawdown against the logarithm of time suggests that this is not a homogeneous aquifer. (See Figure 21.) It was felt that in spite of all the changes in the slope of the curve, the portion of the curve that would best represent the true aquifer conditions would be the portion represented by large values of t. Such a selection yields a coefficient of transmissibility of 24,900 gpd/ft and a coefficient of storage of 0.565. The similarity of the 2 sets of aquifer coefficients is to be expected. The fact that the coefficients of storage are not exactly alike is probably due to the choice of a line through the plotted points, on either Figure 20 or Figure 21. Since it is difficult to be sure which coefficient of storage is in error, an average of the two will be used - 0.535.

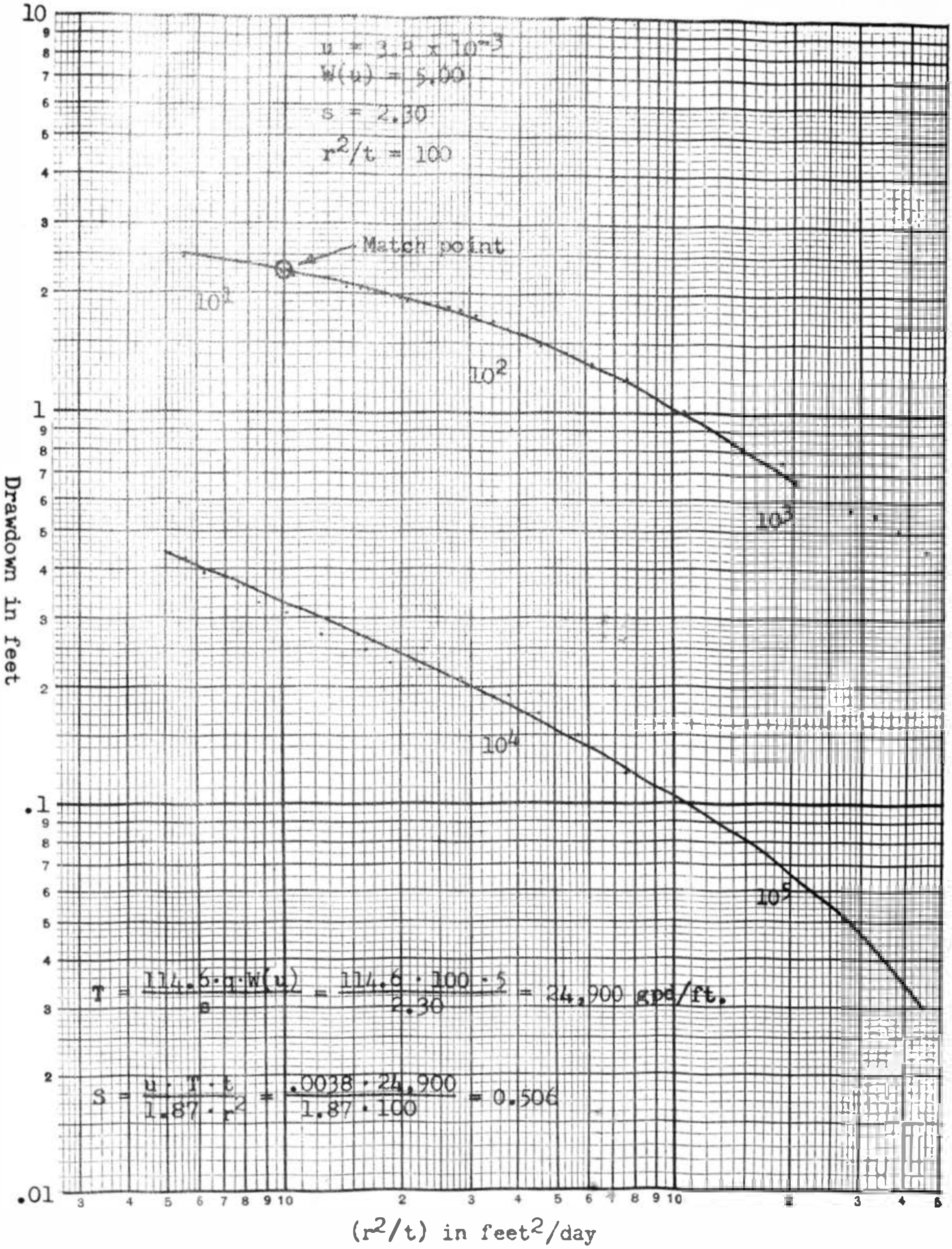

Figure 20. Drawdown Versus (r^2/t) Curve of the Data Collected During the Second Pump Test.

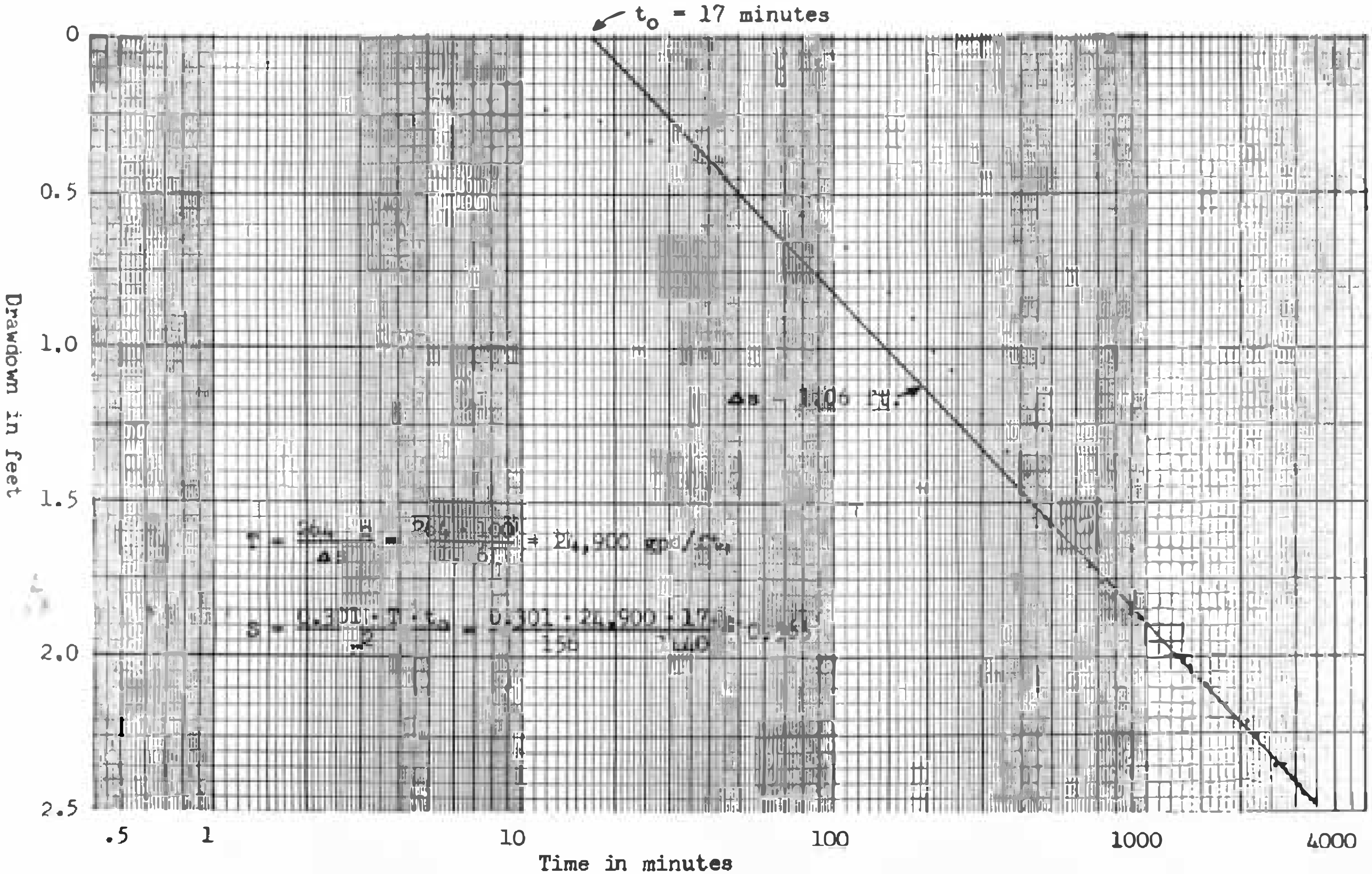

Figure 21. Drawdown Versus Time Curve for the Modified Nonequilibrium Method.

It has already been noted that the drawdown in the 18-inch riser below the pump was 4.70 feet just prior to the end of the test. If the coefficients of transmissibility and storage that were computed from the data collected during the second pump test conducted on the horizontal well are chosen as representative of the aquifer the computations of the discharge that can be expected from a system of five 2- inch sandpoints are tabulated in Tables 5 and 6 as Case II. These computations are based upon the same drawdown and the same period of pumping that produced the discharge of 100 gpm from the horizontal well.

Table 5. Values of u and W(u) for Vertical Wells of Various Radii - Case II. (T = 24,900 gallons per foot per day, S = 0.535, y = 4.70 feet, s = 3.12 feet, and t = 2.8 days.)

radius in feet	u	W(u)
0.0833	9.96×10^{-8}	15.55
20.0	5.75×10^{-3}	4.59
40.0	2.30×10^{-2}	3.22
60.0	5.17×10^{-2}	2.44
80.0	9.19×10^{-2}	1.90

When the computed discharge of 115.1 gpm for the sandpoint system is compared to the discharge of 100 gpm that was determined by pump test of the horizontal well, the indication is that the aquifer is capable of yielding water more readily to a system of five 2-inch sandpoints than to a horizontal well. Such a circumstance does not seem to be realistic,

Table 6. Values of Combined W(u) and Discharge for Each of Five 2-inch Sandpoints Spaced 20 Feet Apart in a Line - Case II.

	Combined W(u)*	Discharge in gpm
Center sandpoint	31.2	21.7
Sandpoint 20 feet out	30.4	22.3
End sandpoint	27.7	24.4
The sandpoint system		115.1

*These values are combinations of values of W(u) from Table 5 according to the method presented by Hoiman.

and causes the author to conclude that the nonequilibrium and modified nonequilibrium formulas cannot be applied directly to data collected during a pump test conducted on a horizontal well to determine aquifer characteristics. However, it is worth noting that the failure of the vertical well formulas to apply here may have been at least partially caused by the combined effects of the open pool and the amount of aquifer that was disturbed during the construction of the pit.

More realistic comparisons are obtained when the expected discharge of the sandpoint system is computed using the coefficients of transmissibility and storage that were computed from data collected during a previous pump test on one sandpoint well near this well, and the same drawdown that was observed here during this 2.8-day pump test. The computations are tabulated in Tables 7 and 8 as Case III.

A comparison of the calculated yield of 72.8 gpm for the sandpoint system and the pump test yield of 100 gpm for the horizontal well,

Table 7. Values of u and W(u) for Vertical Wells of Various Radii - Case III. (T = 17,000 gallons per foot per day, S = 0.225, y = 4.70 feet, s = 3.12 feet and t = 2.8 days.)

radius in feet	u	W(u)
0.0833	6.14×10^{-8}	16.03
20.0	3.34×10^{-3}	5.07
40.0	1.41×10^{-2}	3.70
60.0	3.18×10^{-2}	2.90
80.0	5.66×10^{-2}	2.35

Table 8. Values of Combined W(u) and Discharge for Each of Five 2-inch Sandpoints Spaced 20 Feet Apart in a Line - Case III.

	Combined W(u)*	Discharge in gpm
Center sandpoint	33.6	13.8
Sandpoint 20 feet out	32.8	14.1
End sandpoint	30.0	15.4
The sandpoint system		72.8

*These values are combinations of values of W(u) from Table 6 according to the method presented by Holman.

both for 2.8-day periods of pumping, is much more realistic than the comparison previously made using coefficients of transmissibility and storage obtained by a pump test of the horizontal well. This comparison gives an apparent advantage of 37.3% for the horizontal well over the sandpoint system. Once again it must be kept in mind that part of this apparent advantage may be caused by the effects of the open pool of water and the amount of the aquifer that was disturbed during construction of the pit. The writer believes that the apparent advantage in favor of the horizontal well would not have been as great if those two factors had not existed.

Now that it has been demonstrated that the coefficients of transmissibility and storage of 17,000 gallons per day per foot and 0.225 respectively appear to describe the aquifer in question more accurately than the coefficients determined by pumping the horizontal well, the radius of a vertical well that will yield 100 gpm of water for 2.8 days with a drawdown of 3.12 feet can be computed:

$$W(u) = \frac{T\,s}{114.6\,q} = \frac{17{,}000 \quad 3.12}{114.6 \times 100} = 4.62$$

$$u = 5.56 \times 10^{-3}$$

$$r^2 = \frac{T\,tu}{1.87\,S} = \frac{17{,}000 \cdot 2.8 \cdot 0.00556}{1.87 \cdot 0.225} = 629$$

$$r = 25.1 \text{ feet}$$

Therefore, the equivalent radius of this horizontal well is 25.1 feet, which is within 2.5% of the originally estimated equivalent radius.

It is now evident that this particular horizontal well is not capable of producing the desired 200 gpm of water from this aquifer for a 10-day period of pumping. In fact, the expected discharge for ten days of pumping based upon an equivalent vertical well with a radius of 25.1 feet, a coefficient of transmissibility of 17,000 gallons per day per foot, a coefficient of storage of 0.225, and an adjusted drawdown of 3.22 feet is 81.1 gallons per minute. This computed discharge results in a cost of $11.10 per gallon per minute for the well. Based upon the assumption that 6 gpm are required to irrigate one acre, the cost of the well per irrigated acre becomes $66.60.

Although such a cost per irrigated acre for a well can be defended economically, it is considerably higher than the usually expected costs involved in areas where vertical wells can be successfully used. Information that has been received indirectly from well drillers and other people working with irrigation in eastern South Dakota indicates that a vertical irrigation well usually costs about $15.00 per foot to construct and may average 100 feet in depth. Such wells generally produce from 600 to 1,000 gpm in the areas where they are used which results in a cost of from $1.50 to $2.50 per gallon per minute and a cost per irrigated acre of from $9.00 to $15.00. Consequently, the arbitrary estimation of $30.00 per irrigated acre as a probable maximum expenditure for a horizontal well seems to be a logical choice. People generally expect higher costs under adverse conditions and the problems to be overcome in the development of shallow, thin aquifers are numerous; but a cost of $66.60 per irrigated acre is high when compared to the $9.00 to $15.00 per irrigated acre for vertical wells in more fortunate areas. However, an even greater cost per irrigated acre for a well can be justified economically especially if high value crops (horticultural, etc.) are to be produced.

The quantity of water that a well can produce not only has a strong influence upon the cost of the water but is of the utmost importance in itself. This is especially true in the case of the diversified farms of eastern South Dakota and is illustrated by an example. Even though a well might be capable of producing 10 gpm at a cost of $1.00 per gallon per minute, it would not be practical for the irrigation of corn and alfalfa; whereas a well that is capable of producing 500 gpm at a cost of $5.00 per gpm may be both economical and practical for the irrigation of corn and alfalfa.

This particular horizontal well does not produce sufficient water to operate the proposed irrigation system. Consequently, its practicality is seriously impared regardless of the cost per gpm.

SUMMARY AND CONCLUSIONS

The horizontal well was constructed in a shallow, thin aquifer in the Sioux River Valley in Brookings County, South Dakota. The pump test data were analyzed by means of the nonequilibrium method and the modified nonequilibrium method to determine the aquifer characteristics. The coefficients of transmissibility and storage were found to be 24,900 gpd/ft. and 0.535 respectively. A previous pump test conducted near this well on a single 2-inch sandpoint well by another investigator indicated that the coefficients of transmissibility and storage for the aquifer were 17,000 gpd/ft. and 0.225 respectively.

The fact that the coefficients found by this test were so much higher than those found previously has caused the writer to question whether or not the existing vertical well formulas of Theis and Jacob can be applied directly to this horizontal well problem. Several comparisons were made between the performance of this horizontal well and a system of five 2-inch sandpoints spaced 20 feet apart in a line. The two systems are nearly equal in lineal extent and are somewhat comparable as to their general effect upon an aquifer while they are being pumped. The horizontal well has an advantage over the sandpoint system in that the plumbing and pipe friction can be kept to a minimum; but that advantage is opposed (1) by higher costs of construction, (2) by extensive aquifer disturbance during construction, and (3) by the fact that they are difficult to clean.

The following conclusions are offered:

1. A horizontal well consisting of 77 feet of standard 8-inch diameter perforated casing can be prefabricated and placed in an aquifer whose bottom is approximately 20 feet or less

below the ground surface by first excavating a pit of sufficient size and depth with a dragline and lowering the complete unit into the pit in one operation.

2. Although the pit can be successfully excavated in an aquifer that is 7 feet thick without dewatering the pit during the digging process, it appeared during the construction of this pit that dewatering would help by minimizing bank cave-ins caused by the agitation of the pool by the dragline bucket and by greatly increasing the ability of the dragline operator to see what he is doing after the water table is penetrated.

3. Such a horizontal well can be kept free of sediment by the use of a sewer cleaning bucket that can be operated inside the horizontal casing if each end is equipped with a riser and with a roller on which the rope may run. While the well is being pumped, the sewer cleaning bucket can be pulled back and forth in the horizontal casing by a man at each end of the well.

4. The coefficients of transmissibility and storage of the aquifer were higher when computed from data collected during the pump test of this horizontal well than when computed by another investigator from data collected during a pump test conducted on a sandpoint well. It appears that the nonequilibrium formula and modified nonequilibrium formula cannot be applied directly to horizontal well test data to determine aquifer characteristics because the use of these computed coefficients to calculate the discharge from a system of five 2-inch sandpoints spaced 20 feet apart in a line yields a larger discharge than the discharge of the horizontal well during this pump test. Such a circumstance appears to be improbable and may be caused partly by the open pool of water and by the disturbance caused in the aquifer by the construction of the pit.

5. During a 2.8-day period of pumping, this horizontal well yielded 100 gpm of water. Based upon the coefficients of transmissibility and storage that were previously computed from a test conducted on a sandpoint well in this aquifer, this 77-foot horizontal well would be equivalent to a vertical well with a radius of 25.1 feet.

6. The cost of this horizontal well resulted in a cost of $11.10 per gpm of water when based upon the computed 10-day yield. This cost for a well is higher than the cost usually encountered in the parts of eastern South Dakota where the aquifers are suited to the construction of vertical wells.

7. The horizontal well is capable of extracting greater quantities of water from a shallow, thin aquifer than a sandpoint system of approximately the same lineal extent.

This study has been limited to the construction and development of one horizontal well in a shallow, thin aquifer and to the analysis of data collected during one pump test. Consequently, the results obtained and the conclusions offered are representative of this one particular sample. To secure a more accurate analysis of the horizontal well, several tests on each of several horizontal wells should be conducted.

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APPENDIX A. DERIVATION OF THE NONEQUILIBRIUM FORMULA

The following is the derivation of the nonequilibrium formula as presented by C. V. Theis in Transactions American Geophysics Union 16: 519-524, August 1935. Theis credits C. I. Lubin with the actual mathematical derivation.

To the extent that Darcy's law governs the motion of ground water under natural conditions and under artificial conditions set up by pumping, an anology exists between the hydrologic conditions in an aquifer and the thermal conditions in a similar thermal system. Darcy's law is analogous to the law of the flow of heat by conduction, hydraulic pressure being analogous to temperature, pressure gradient to thermal gradient, permeability to thermal conductivity, and specific yield to specific heat. Therefore, the mathematical theory of heat conduction developed by Fourier and subsequent writers is largely applicable to hydraulic theory. . . .

The equation given by H. S. Carslaw (Introduction to the mathematical theory of the conduction of heat in solids, 2nd ed., p. 152, 1921) for the temperature at any point in an infinite plane with initial temperature zero at any time due to an "instantaneous line-source coinciding with the axis of z of strength Q" (involving two-dimensional flow of heat) is

$$v = (Q/4\pi kt)e^{-(x^2+y^2)/4kt} \tag{1}$$

where v = change in temperature at the point x, y at the time t; Q = strength of the source or sink -- in other words, the amount of heat added or taken out instantaneously divided by the specific heat per unit - volume; k = Kelvin's coefficient of diffusivity, which is equal to the coefficient of conductivity divided by the specific heat per unit-volume; and t = time.

The effect of a continuous source or sink of constant strength is derived from equation (1) as follows:

Let $Q = \phi(t')dt$;

then $$v_{(x,\ y,\ t)} = \int_0^t \left[\phi(t')/4\pi k(t-t')\right] e^{-(x^2+y^2)/4k(t-t')} \, dt'.$$

Let $\phi(t') = \lambda$, a constant; then

$$v_{(t)} = \lambda/4\pi k \int_0^t \left[e^{-(x^2+y^2)/4k(t-t')}/(t-t') \right] dt'.$$

Let $u = (x^2+y^2)/4k(t-t')$; then

$$v_{(t)} = \lambda/4\pi k \int_{(x^2+y^2)/4kt}^{\infty} \left[e^{-u}/(t-t') \right] \left[(x^2+y^2)/4k \right] \left[du/u^2 \right]$$

$$= \lambda/4\pi k \int_{(x^2+y^2)/4kt}^{\infty} \left[e^{-u}/u \right] du \qquad (2)$$

The definite integral, $\int_{(x^2+y^2)/4kt}^{\infty} (e^{-u}/u)du$, is a form of the exponential integral, tables of which are available (Smithsonian Physical Tables, 8th rev. ed., table 32, 1933; the values to be used are those given for Ei(-x), with the sign changed.) The value of the integral is given by the series

$$\int_x^{\infty} (e^{-u}/u)du = -0.577216 - \log_e x + x - \frac{x^2}{2\cdot 2!} + \frac{x^3}{3\cdot 3!} - \frac{x^4}{4\cdot 4!} + \cdots \qquad (3)$$

Equation (2) can be immediately adapted to ground water hydraulics to express the draw-down at any point at any time due to pumpint a well. The coefficient of diffusivity, is analogous to the coefficient of transmissibility of the aquifer divided by the specific yield. (The term "coefficient of transmissibility" is here used to denote the product of Meinzer's coefficient of permeability and the thickness of the saturated portion of the aquifer; it quantitatively describes the ability of the aquifer to transmit water. Meinzer's coefficient of permeability denotes the character of the material; the coefficient of transmissibility denotes the analogous characteristic of the aquifer as a whole.) The continuous strength of the sink is analogous to the pumping rate divided by the specific yield. Making these substitutions, we have

$$v = F/4T\pi \int_{r^2s/4Tt}^{\infty} (e^{-u}/u)\, du \qquad (4)$$

in which the symbols have the meanings given with equation (5). In equation (4) the same units must of course be used throughout. Equation (4) may be adapted to units commonly used

$$v = 114.6\ F/T \int_{1.87\ r^2 s/T\ t}^{\infty} (e^{-u}/u)\ du \qquad (5)$$

where v = the draw-down, in feet, at any point in the vicinity of a well pumped at a uniform rate; F = the discharge of the well, in gallons a minute; T = the coefficient of transmissibility of aquifer, in gallons a day, through each 1 foot strip extending the height of the aquifer, under a unit gradient -- that is the average coefficient of permeability (Meinzer) multiplied by the thickness of the aquifer; r = the distance from the pumped well to point of observation, in feet; s = the specific yield, as a decimal fraction; and t = the time the well has been pumped, in days.

Equation (5) gives the drawdown at any point around a well being pumped uniformly (and continuously) from a homogeneous aquifer of constant thickness and infinite areal extent at any time. The introduction of the function, time, is the unique and valuable feature of the equation. Equation (5) reduces to Thiem's or Slichter's equation for artesian conditions when the time of pumping is large. . . .

Theoretically, the equation applies rigidly only to water-bodies (1) which are contained in entirely homogeneous sediments, (2) which have infinite areal extent, (3) in which the well penetrates the entire thickness of the water-body, (4) in which the coefficient of transmissibility is constant at all times and in all places, (5) in which the pumped well has an infinitesimal diameter, and (6) applicable only to unconfined water-bodies -- in which the water in the volume of sediments through which the water-table has fallen is discharged instantaneously with the fall of the water-table. . . .

A useful corollary to equation (5) may be derived from an analysis of the recovery of a pumped well. If a well is pumped for a known period and then left to recover, the residual drawdown at any instant will be the same as if pumping of the well had been continued but a recharge well with the same flow had been introduced at the same point at the instant pumping stopped. The residual draw-down at any instant will then be

$$v' = 114.6\ F/T \left[\int_{1.87\ r^2 s/T\ t}^{\infty} (e^{-u}/u) du \ - \int_{1.87\ r^2 s/T\ t'}^{\infty} (e^{-u}/u) du \right] \qquad (6)$$

where t is the time since pumping started and t' is the time since pumping stopped.

In and very close to the well the quantity $(1.87\ r^2s/\mathcal{T}t')$ will be very small as soon as t' ceases to be small, because r is very small. In many problems ordinarily met in ground-water hydraulics, all but the first two terms of the series of equation (3) may be neglected, so that, if $Z = (1.87\ r^2s/\mathcal{T}t)$ and $Z' = (1.87\ r^2s/\mathcal{T}t)$ equation (6) may be approximately rewritten

$$v' = (114.6\ F/\mathcal{T})\left[-0.577 - \log_e Z + 0.577 + \log_e Z\ (\tfrac{t}{t'})\right]$$

$$= (114.6\ F/\mathcal{T})\ \log_e\ (t/t').$$

Transposing and converting to common logarithms, we have

$$\mathcal{T} = (264\ F/v')\ \log_{10}(\tfrac{t}{t'}) \qquad (7)$$

This equation permits the computation of the coefficient of transmissibility of an aquifer from an observation of the rate of recovery of a pumped well.

APPENDIX B. LIST OF SYMBOLS AND UNITS

C	a constant equal to (r_e/Q)
h_e	saturated thickness of the undisturbed aquifer in feet
h_s	thickness of saturated aquifer just outside a pumped well in feet
h_w	depth of water in a pumped well in feet
K	a constant that includes aquifer permeability
ln	natural logarithm (to the base e)
log	logarithm to the base 10
m	saturated thickness of the undisturbed aquifer in feet
m'	average saturated thickness in feet of an aquifer at 2 observation wells when equilibrium conditions exist during pumping
P	coefficient of permeability of an aquifer in gallons per day per square foot under a unit hydraulic gradient at a temperature of 60°F. (gpd/ft.^{2})
q	discharge in gallons per minute (gpm)
Q	discharge in gallons per day (gpd)
r	distance from pumped to observation well in feet
r_e	radius of influence of a pumped well - the distance in feet from a pumped well at which no disturbance is noted in the aquifer.
r_w	radius of the pumped well in feet
s	drawdown in feet as used in ground water formulas $s = (y - y^2/2m)$ for thin aquifers.
S	coefficient of storage of an aquifer as the ratio of the volume of water a unit volume of saturated aquifer will yield to gravity to the original unit volume (expressed as a decimal fraction)
t	time since pumping started in days
t'	time since pumping stopped in days