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Deflection During Prestressing of an Unbonded Prestressed Concrete Beam

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DEFLECTION DURING PRESTRESSING
OF AN UNBONDED PRESTRESSED CONCRETE BEAM

by

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This thesis is approved as a creditable independent investigation by a candidate for the degree, Master of Science, and acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

FOREWORD

In recent years great strides have been made towards improving the quality and strength of both concrete and steel. Reliable concrete with a compressive strength of 5000 to 12,000 psi and suitable steel with a tensile strength of over 200,000 psi can be obtained.[1] If high strength concrete and high strength steel can be used together satisfactorily a substantial reduction in materials will result. This is partly due to the fact that the materials are stronger and partly due to the fact that dead load is reduced.

In designing a plain reinforced concrete beam it is assumed that about one-third of the concrete (the part in compression) is effective in resisting moment. The other two-thirds of the concrete serve mainly to encase the reinforcing steel in the tension portion of the beam. From the above facts it appears that much of the added expense of producing stronger concrete would be wasted.[1]

It might be assumed that high strength steel could be used to advantage in plain reinforced concrete. However, the high working stresses in such steel cause it to deform much more than the surrounding concrete, and cracking of the concrete occurs.

Prestressed concrete is a type of reinforced concrete construction in which both high strength concrete and high strength steel can be utilized satisfactorily. Prestressed concrete dates back at least to 1886, when a patent was issued to P. H. Jackson of San Francisco. In 1928, R. E. Dill of Alexandria, Nebraska, obtained a patent for the

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1. Goff, L., "Prestressed Concrete - A New Frontier", Engineering News-Record, September 1, 1949, Vol. 143, No. 9

construction of prestressed concrete units. Also in 1928, Eugene Freysinet filed his fundamental patent in France; he demonstrated that high-strength steel is essential to the success of prestressed concrete.

European countries have used and developed prestressed concrete to a higher degree than we have because they have a scarcity of materials. Accounts show that they have used prestressed concrete in the construction of items ranging from bridges to bath tubs. These countries have conducted and are still conducting considerable research in this field.

In the United States materials do not contribute nearly as much to the cost of construction as they do in Europe. For this reason there has not been the interest in prestressed concrete here that exists in Europe. However, the few structures built here have proven their worth, and at present interest is mounting. In the United States prestressed concrete has been used to construct bridges, floor slabs, beams, tanks, pipe, and fence posts.

United States engineers and architects find it difficult to design structures of prestressed concrete because of a lack of information on design procedures. There is also a lack of construction methods that fit into our economic condition of relatively high labor cost as compared to cost of material. There is a definite need for more research on prestressed concrete in the United States. Following is a quotation from a paper prepared by the Portland Cement Association entitled "Review of Research in Prestressed Concrete":

> In the development of any new idea, research in some form is necessary to uphold the theories and to prove that the new idea is sound. With prestressed concrete we have a new idea entering the American construction field. Unfortunately, there is a dearth of American

research data to substantiate the design theories. Emphasis is placed on the need for American research data for two reasons: this data would be more conclusive and definite regarding American materials; American engineers are generally reserved in their attitude toward the results of foreign research. This lack of information may delay and minimize the effectiveness of promotion of prestressed concrete because of a reluctance to accept and apply new theories until thorough tests have been made. For this reason, American laboratories and research organizations in this field should make an effort to supply the answers necessary to properly establish prestressed concrete as a competitive construction material.

ACKNOWLEDGMENTS

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Appreciation is extended to the John A. Roebling's Sons Company for their interest in the author's thesis and for the data they supplied.

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INTRODUCTION

The problem to be considered in this thesis is the deflection that occurs in unbonded prestressed concrete beams while the steel is being tensioned. Unbonded (sometimes called post-tensioned) prestressed beams have no bond between the prestressing steel and the concrete. It has been reported that this deflection, for some conditions, is not as calculated.* Following are some conclusions reported to the author of this thesis: (These conclusions were drawn from observations of beams tested for various purposes.)

1. During the tensioning of the steel prestressed concrete beams with only unbonded reinforcing have upward bow or upward deflection that is greater than calculated.
2. The excessive deflection becomes particularly noticeable after the stress in the beam reaches about one-third of the ultimate strength of the concrete.
3. Beams of the same type as in number 1, but with a small amount of plain reinforcing in the form of a mild steel mesh or a few small diameter mild steel rods, have deflections in line with calculated values.

In considering item number two above it seems possible that the change in shape of the beam as it deflects might have some effect on the action of the beam. Generally, the loads acting on a beam are

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*See Appendix A, Page 63.

perpendicular to the axis of the beam and are of a constant value. The forces acting on a prestressed concrete beam from the prestressing steel are of a different nature than the usual forces. From the above facts it seems worthwhile to investigate the possibility that a change in shape of the beam would alter the effect of the prestressing force.

In considering item number 3 above it appears possible that a non-uniform distribution of stress is the cause of the excessive deflection. The reason for this is that the small amount of plain reinforcing, properly placed, probably would tend to give a more uniform distribution of stress in the concrete. If a concentration of stress occurred at some point in the beam the effect would be much the same as if a material of lower modulus of elasticity was at that point. In other words, it would have greater strain than the material in the rest of the beam. A condition of this sort might be accompanied by excessive deflection. If a prestressed concrete beam with unbonded steel and with no plain reinforcing was built and tested, the shape of the deflection curve might reveal the presence of stress concentrations if they occurred and were of sufficient magnitude to cause excessive deflection.

There are at least two courses of action open for investigating the cause of the unpredicted deflections of unbonded prestressed concrete beams. One is the testing of beams in the laboratory for the specific purpose of studying the deflections. This would give more conclusive data because the observations upon which the previously mentioned conclusions were drawn were not observations of tests made specifically for studying deflections. Another course of action is to investigate usual design formulas in order to determine if an error is being made when using these formulas. In studying the problem at hand the author decided to follow each of these courses of action.

The investigation of usual design formulas indicated that these formulas are theoretically sound. It was found that various changes in the effect of the prestressing force occurred as the beam deflected, but the changes cancelled each other.

The testing of beams in the laboratory indicated that satisfactory prediction of deflection for the prestressing operation can be made if the proper value of E_c is used. Using measured values of strain and calculated values of stress it was found that E_c for an unbonded prestressed concrete beam with no plain reinforcing was appreciably lower than the generally accepted value. ($E_c = 1000\ f'_c$ is generally used for designing concrete structures.) In the same manner it was found that the value of E_c in an unbonded prestressed concrete beam with a small amount of plain reinforcing was more nearly equal to 1000 f'_c.

INVESTIGATION OF FORMULAS USED IN DESIGN

Commonly Used Design Procedure

The method of superposition has been used quite generally in the design of prestressed beams. Essentially, this method consists of the following:

1. Calculate stress due to the prestressing force.
2. Calculate stress due to dead load.
3. Calculate stress due to live load.
4. Add the proper combinations of 1, 2 and 3 to determine whether or not the resulting stresses will be satisfactory.

A simple example will be worked in order to illustrate more clearly the method and to point out the formulas involved. Figure 1 shows the beam. A calculation of stresses at the center line will be made for prestressing force, dead load, and live load.

Stress in concrete from prestressing force:

Direct stress $= \frac{T}{A}$

T = Total prestressing = 121,000 lb.

A = Total crossectional area = 108 sq. in.

Direct stress $= \frac{121000}{108} = 1120$ psi

Bending stress $= \frac{Mc}{I}$ (stress at outer fibers)

M = Center line moment from prestressing force

M = Te = (121,000)(1.9) = 230,000 in. lb.

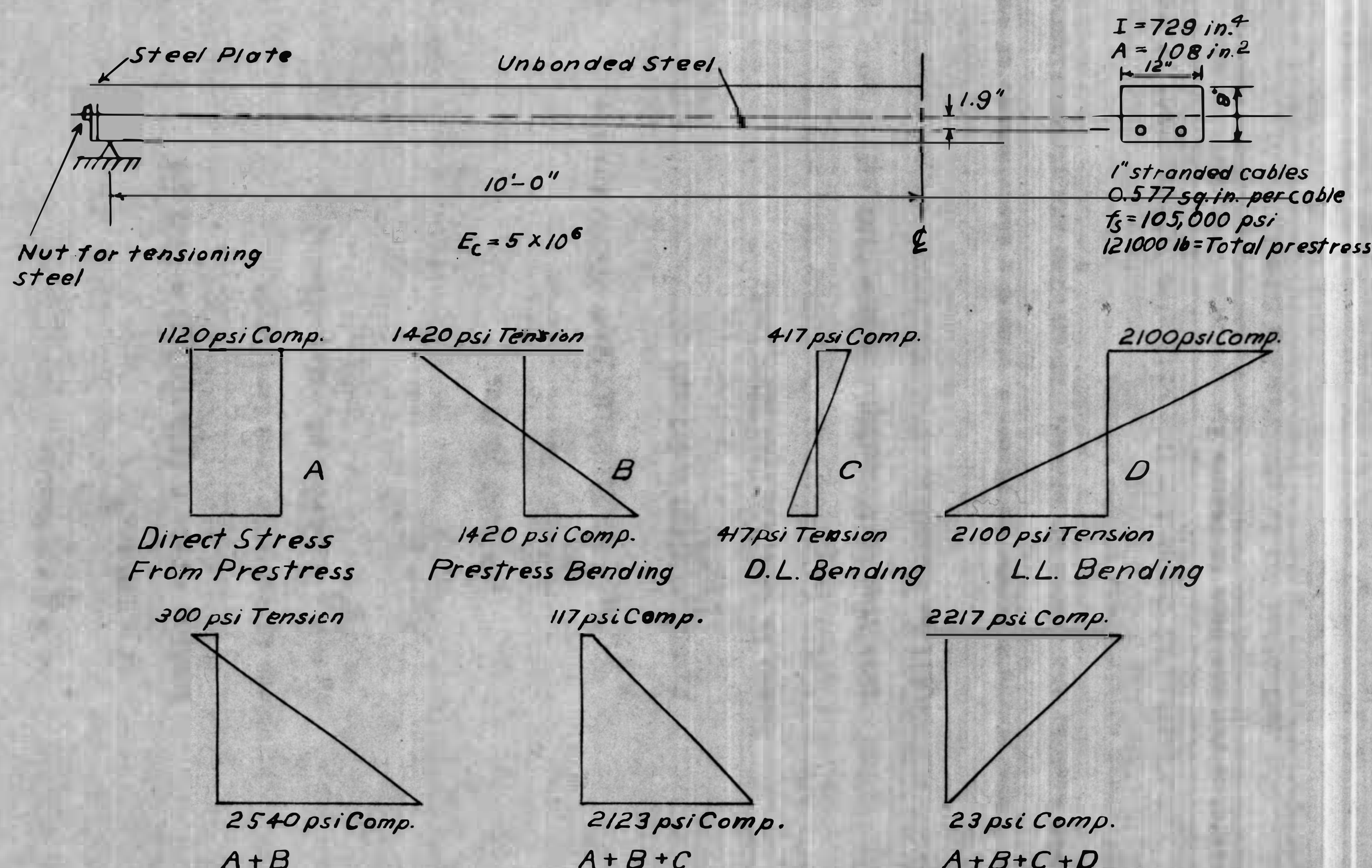

Fig. 1. STRESSES IN AN UNBONDED PRESTRESSED CONCRETE BEAM

$c = \frac{d}{2} = \frac{9}{2}$ in.

$I = 729$ in.^{4}

Bending stress $= \frac{(230,000)(9)}{(729)(2)} = 1420$ psi

Stress in concrete from dead load:

$f_c = \frac{M_{DL}}{I} c$ (stress at outer fibers)

$M_{DL} = \frac{wL^2}{8}$

$w = 9.38$ lb./in.

$L = 240$ in.

$M_{DL} = \frac{(9.38)(240)(240)}{8} = 67,500$ in. lb.

$f_c = \frac{(67,500)(9)}{(729)(2)} = 417$ psi

Stress in concrete from live load:

A live load moment at the center line such that live load bending stress = 2100 psi will be assumed.

The stress diagrams in Figure 1 show the distribution of stress from each of the causes of stress. Shown also are stress diagrams for different combinations of the stress.

Deflection at the center line will also be determined by superposition for the beam in Figure 1.

Deflection due to prestressing force:

$y = \frac{5TeL^2}{48E_cI}$*

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*See Appendix A, Page 64.

$T = 121{,}000$ lb.
$e = 1.9$ in.
$L = 240$ in.
$E_c = (5)(10^6)$
$I = 729$ in.^{4}

$$y = \frac{(5)(121{,}000)(1.9)(240)(240)}{(48)(5)(10^6)(729)}$$

$y = 0.379$ in. up

Deflection due to dead load:

$$y = \frac{5wL^4}{384E_cI}\text{*}$$

$w = 9.38$ lb./in.

$$y = \frac{(5)(9.38)(240)(240)(240)(240)}{(384)(5)(10^6)(729)}$$

$y = 0.111$ in. down

Deflection resulting from prestressing force and dead load:

$y = 0.379 - 0.111 = 0.268$ in. up

The deflection calculations illustrated here represent the necessary calculations for conditions that would exist at the time the steel is tensioned. There are other considerations; such as, plastic flow in the concrete, shrinkage in the concrete, and creep in the steel. These things will effect stress conditions and deflections over a period of time. Unbonded prestressing operations are completed in a comparatively short time (sometimes a few minutes). Therefore, it seemed improbable that plastic flow in the concrete, shrinkage in the concrete, and creep in the steel would have an appreciable effect on the deflections being considered in this paper.

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*See Appendix A, Page 64.

Discussion of Commonly Used Design Formulas

The formulas used for calculating stress and deflection due to dead load and live load are formulas that have been found to give satisfactory results for beams of most any material and type. In the derivation of these formulas no account is taken for the change in shape of the beam as it deflects. But, since these formulas are applied in the same manner as for any other beam, it seems logical that they should give satisfactory results when used to design prestressed concrete beams.

The formulas used to determine the effect of prestressing force have not been used to the extent that the dead load and live load formulas have. The prestressing formulas also do not take into account the change in shape of the beam as it deflects. Therefore, an investigation of the prestressing formulas was made.

As shown by the example problem, the moment in the beam due to prestressing force is calculated by multiplying the tension in the steel by the offset of the steel from the central axis of the beam. There is another way of calculating the moment at any section due to prestress, and the forces involved are shown in Figure 2.[1] This method gives results that are identical to results obtained by the method used in the illustrative problem. This approach to the problem also does not take into consideration any change in the effect of the prestressing force as the beam changes shape when the steel is being tensioned. This

1. Parme, Alfred L. and Paris, George H., "Designing for Continuity in Prestressed Structures", ACI Journal, September, 1951, Proceedings V. 48.

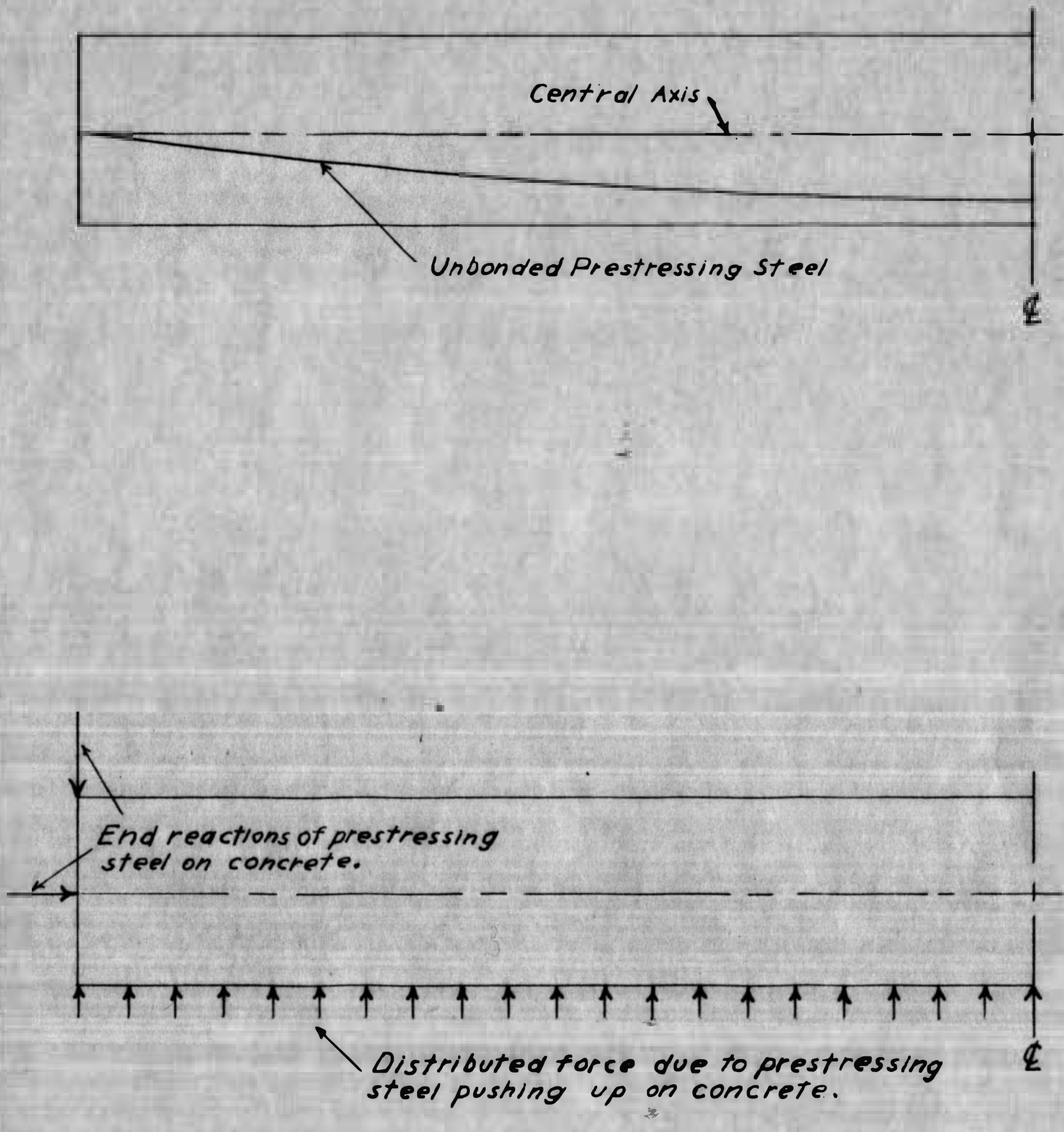


Fig. 2. PRESTRESSING FORCE ON CONCRETE

latter method is mentioned here because the free body diagram involved will be used in the following investigation.

<u>Investigation of the Change in Effect of Prestressing Force Due To a Change in Shape of the Beam</u>

The author will now attempt to show whether or not the change in shape of an unbonded prestressed concrete beam will cause an appreciable change in the effect of the prestressing force. First, a case will be considered in which there is a parabolic placement of the steel. The steel will intersect the end of the beam at the central axis of the beam.

Referring to Figure 3a:

$$y_c = -e + kx_1^2$$

when $x_1 = L/2$, $y_c = 0$

$$0 = -e + \frac{kL^2}{4}$$

$$k = \frac{4e}{L^2}$$

$$y_c = -e + \frac{4e}{L^2} x_1^2$$

y_c = offset of cable from central axis of beam

To transfer the origin to point O let $x_1 = \frac{L}{2} - x$

$$y_c = -e + \frac{4e}{L^2} \left(\frac{L^2}{4} - Lx + x^2\right)$$

$$y_c = -e + e - \frac{4ex}{L} + \frac{4ex^2}{L^2}$$

$$y_c = \frac{4ex^2}{L^2} - \frac{4ex}{L}$$

Referring to Figure 3b:

$$y_d = y + y_e$$

$$y_d = y + \frac{4ex^2}{L^2} - \frac{4ex}{L}$$

y_d = offset of cable from undeflected central axis

Referring to Figure 4:

$$F_y = T_x \tan \theta$$

$$(F + \Delta F)_y = T_x \tan (\theta + \Delta\theta)$$

$$\Sigma Y = 0$$

$$-T_x \tan \theta - A\Delta x + T_x \tan (\theta + \Delta\theta) = 0$$

$$\frac{T_x \left[\tan (\theta + \Delta\theta) - \tan \theta\right]}{\Delta \theta} = \frac{A\Delta x}{\theta}$$

Taking the limit as $\Delta\theta$ approaches zero

$$T_x \sec^2\theta = A \frac{dx}{d\theta}$$

$$\sec^2\theta = 1 + \left(\frac{dy_d}{dx}\right)^2$$

$$\frac{d\theta}{dx} = \frac{d^2y_d}{dx^2} \Big/ \left[1 + \left(\frac{dy_d}{dx}\right)^2\right]$$

$$T_x \frac{d^2y_d}{dx^2} = A$$

$$\frac{d^2y_d}{dx^2} = \frac{A}{T_x}$$

$$y_d = y + \frac{4ex^2}{L^2} - \frac{4ex}{L}$$

$$\frac{dy_d}{dx} = y' + \frac{8ex}{L^2} - \frac{4e}{L}$$

$$\frac{d^2y_d}{dx^2} = y'' + \frac{8e}{L^2} = \frac{A}{T_x}$$

$A = T_x(y'' + \frac{8e}{L^2})$ = vertical force per unit horizontal distance

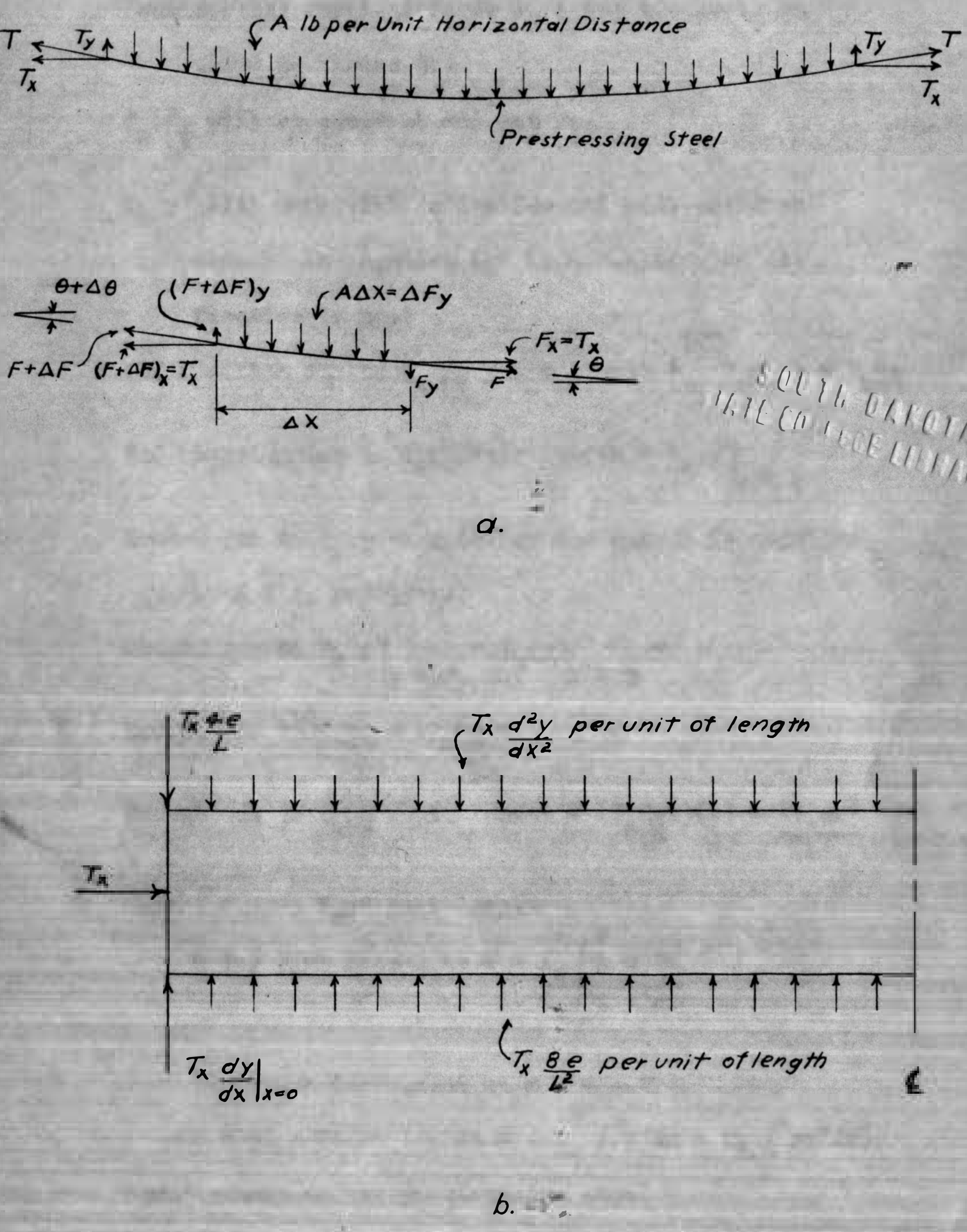
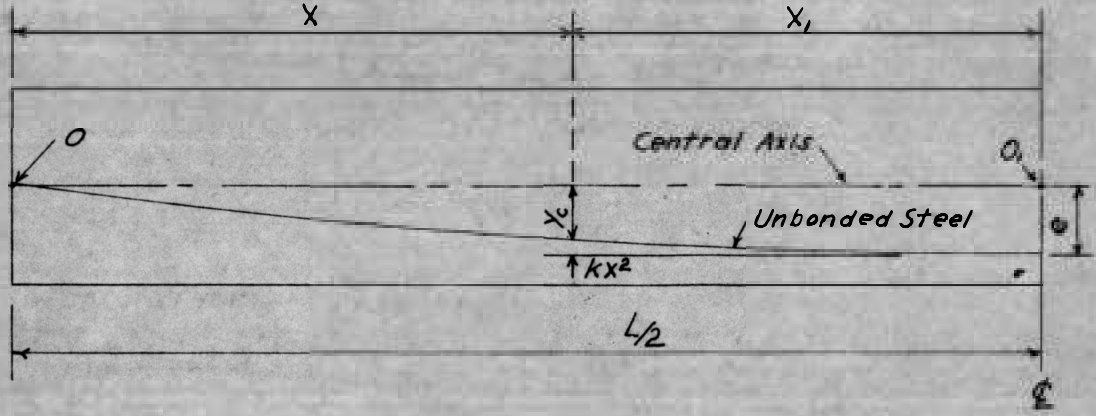


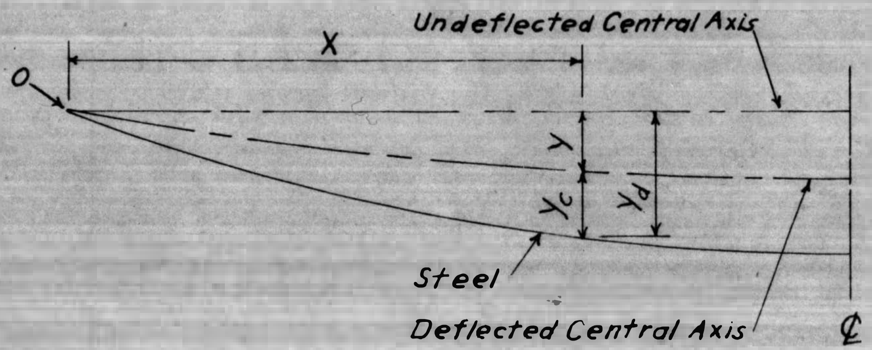
Fig. 4. FREE BODY DIAGRAMS FOR PRESTRESS



Note

Rectangular cross-section

a.



b.

Fig. 3. PARABOLIC PLACEMENT OF STEEL

Consider the vertical force as being two parts as shown in Figure 4b.

$T_x \frac{8e}{L^2}$ will be constant and act up.

$T_x y''$ will vary with deflection and will act down when y'' is negative (y'' is negative when deflection is up.)

End reaction due to $T_x \frac{8e}{L^2}$ /unit length $= \int_0^{L/2} T_x \frac{8e dx}{L^2} = T_x \frac{4e}{L}$

End reaction due to $T_x y''$/unit length $= T_x \, y' \Big|_{x=0}$

Moment due to $T_x = -T_x y$ (since the moment is negative when y is positive.)

Moment due to $T_x \, y' \Big|_{x=0} = T_x \, y' \Big|_{x=0} x$

Moment due to $T_x \frac{4e}{L} = -T_x \frac{4e}{L} x$

Moment due to $T_x \frac{8e}{L^2}$ /unit length $= (T_x \frac{8ex}{L^2})(\frac{x}{2}) = \frac{4T_x ex^2}{L^2}$

Moment due to $T_x y''$/unit length

$\bar{x}$ for $T_x y''$ from 0 to $x = \frac{T_x \int_0^x x \, y'' dx}{T_x \int_0^x x \, y'' dx}$

Lever arm for moment at $x = x - \bar{x}$

$M = (x - \bar{x}) \, T_x \int_0^x y'' dx = (xT_x \int_0^x y'' dx - T_x \int_0^x xy'' dx)$

Total moment due to prestressing steel =

$$-T_x y + T_x y' \Big|_{x=0} x - \frac{T_x 4ex}{L} + \frac{T_x 4ex^2}{L^2} + xT_x \int_0^x y'' dx - T_x \int_0^x xy'' dx$$

Let $y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$

$$y' = a_1 + 2a_2x + \dots + na_nx^{n-1} + \dots$$

$$y'' = 2a_2 + (2)(3)a_3x + \dots(n-1)(n)a_nx^{(n-2)} + \dots$$

$$xT_x\int_0^x y''dx = xT_x\int_0^x\Big[2a_2 + (2)(3)a_3x + \dots(n-1)(n)a_nx^{(n-2)} + \dots\Big]\,dx$$

$$= xT_x\left[2a_2x + 3a_3x^2 + \dots\frac{(n-1)}{(n-1)}(n)a_nx^{(n-1)} + \dots\right]_0^x$$

$$= xT_x\left[2a_2x + 3a_3x^2 + \dots na_nx^{(n-1)} + \dots\right]$$

$$xT_x\int_0^x y''dx = T_x\,(2a_2x^2 + 3a_3x^3 + \dots na_nx^n + \dots)$$

$$T_x\int_0^x xy''dx = T_x\int_0^x\Big[2a_2x + (2)(3)a_3x^2 + \dots(n-1)(n)a_nx^{(n-1)} + \dots\Big]\,dx$$

$$= T_x\left[\frac{2a_2x^2}{2} + (2)\frac{(3)}{3}a_3x^3 + \dots(n-1)\frac{(n)}{n}a_nx^n + \dots\right]_0^x$$

$$T_x\int_0^x xy''dx = T_x\left[a_2x^2 + 2a_3x^3 + \dots(n-1)a_nx^n + \dots\right]$$

$$T_xy'\big|_{x=0}\,x = T_xa_1x$$

Moment from dead load $= \frac{wLx}{2} - \frac{wx^2}{2}$

Equation of elastic curve with dead load and prestress acting:

$EIy'' = M$ (M = moment due to prestressing and dead load)

$$EI\left[2a_2 + \dots + (n-1)(n)a_nx^{(n-2)} + \dots\right] = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$- T_x\,(a_0 + a_1x + \dots + a_nx^n + \dots)$$

$$+ T_xa_1x - \frac{T_x4ex}{L} + \frac{T_x4e}{L^2}x^2$$

$$+ T_x(2a_2x^2 + \dots + na_nx^n + \dots)$$

$$- T_x\,(a_2x^2 + \dots + (n-1)a_nx^n + \dots)$$

$$= \frac{wL}{2}x - \frac{wx^2}{2} - T_xa_0 - \frac{T_x4ex}{L} + \frac{T_x4e}{L^2}x^2$$

$$EI\left[2a_2 + \dots + (n-1)(n)a_n x^{(n-2)} + \dots\right] = \frac{wLx}{2} - \frac{wx^2}{2} - T_x a_0$$

$$- \frac{T_x 4ex}{L} + \frac{T_x 4ex^2}{L^2}$$

$$0 = \frac{wLx}{2} - \frac{wx^2}{2} - T_x a_0 - \frac{T_x 4ex}{L} + \frac{T_x 4ex^2}{L^2} - EI\left[2a_2 + \dots + \right.$$

$$\left.(n-1)(n)a_n x^{(n-2)} + \dots\right]$$

Collecting coefficients of like powers of x and setting them equal to 0.

$$-2\,EIa_2 - T_x a_0 = 0$$

$$-2\cdot3\,EIa_3 + \frac{wL}{2} - \frac{T_x 4e}{L} = 0$$

$$-3\cdot4\,EIa_4 - \frac{w}{2} + \frac{T_x 4e}{L^2} = 0$$

$$-4\cdot5\,EIa_5 = 0$$

a_0 must $= 0$ because $y = 0$ when $x = 0$

So $a_2 = 0$

$$a_3 = \frac{wL}{2\cdot2\cdot3EI} - \frac{4eT_x}{2\cdot3EIL}$$

$$a_4 = \frac{4eT_x}{3\cdot4EIL^2} - \frac{w}{2\cdot3\cdot4EI}$$

a_5 etc. $= 0$

$$y = a_1x + \left(\frac{wL}{12EI} - \frac{2}{3}\frac{T_x e}{EIL}\right)x^3$$

$$+ \left(\frac{T_x e}{3EIL^2} - \frac{w}{24EI}\right)x^4$$

$$y' = a_1 + 3\left(\frac{wL}{12EI} - \frac{2}{3}\frac{T_x e}{EIL}\right)x^2 + 4\left(\frac{T_x e}{3EIL^2} - \frac{w}{24EI}\right)x^3$$

at $x = L/2 \quad y' = 0$

$$0 = a_1 + \frac{wL^3}{16EI} - \frac{T_x eL}{2EI} + \frac{T_x eL}{6EI} - \frac{wL^3}{48EI}$$

$$a_1 = \frac{wL^3}{48EI} - \frac{wL^3}{16EI} + \frac{T_x eL}{2EI} - \frac{T_x eL}{6EI}$$

$$a_1 = \frac{wL^3}{EI}\left(\frac{1}{48} - \frac{1}{16}\right) + \frac{T_x eL}{EI}\left(\frac{1}{2} - \frac{1}{6}\right)$$

$$a_1 = -\frac{wL^3}{24EI} + \frac{T_x eL}{3EI}$$

$$y = \left(\frac{T_x eL}{3EI} - \frac{wL^3}{24EI}\right)x + \left(\frac{wL}{12EI} - \frac{2}{3}\frac{T_x e}{EIL}\right)x^3 + \left(\frac{T_x e}{3EIL^2} - \frac{w}{24EI}\right)x^4$$

Proof

$$y' = a_1 + \left(\frac{wL}{4EI} - \frac{2T_x e}{EIL}\right)x^2 + \left(\frac{4T_x e}{3EIL^2} - \frac{w}{6EI}\right)x^3$$

$$y'' = \left(\frac{wL}{2EI} - \frac{4T_x e}{EIL}\right)x + \left(\frac{4T_x e}{EIL^2} - \frac{w}{2EI}\right)x^2$$

$$EI\, y'' = \left(\frac{wL}{2} - \frac{4T_x e}{L}\right)x + \left(\frac{4T_x e}{L^2} - \frac{w}{2}\right)x^2$$

$$M = \frac{wLx}{2} - \frac{wx^2}{2} - \left(\frac{T_x^2 eL}{3EI} - \frac{wT_x L^3}{24EI}\right)x - \left(\frac{wT_x L}{12EI} - \frac{2}{3}\frac{T_x^2 e}{EIL}\right)x^3$$

$$- \left(\frac{T_x^2 e}{3EIL^2} - \frac{T_x w}{24EI}\right)x^4 + \left(\frac{T_x^2 eL}{3EI} - \frac{T_x wL^3}{24EI}\right)x - \frac{4\,T_x e\,x}{L}$$

$$+ \left(\frac{wT_x L}{4EI} - \frac{2T_x^2 e}{EIL}\right)x^3 + \left(\frac{4T_x^2 e}{3EIL^2} - \frac{wT_x}{6EI}\right)x^4 + \frac{8T_x e}{L^2}x^2$$

$$- \left(\frac{wT_x L}{6EI} - \frac{4T^2 e}{3EIL}\right)x^3 - \left(\frac{T_x^2 e}{EIL^2} - \frac{wT_x}{8EI}\right)x^4 - \frac{8T_x e}{2L^2}x^2$$

$$M = \left(\frac{wL}{2} - \frac{4T_x e}{L}\right)x + \left(\frac{4T_x e}{L^2} - \frac{w}{2}\right)x^2 = EI\, y''$$

For practical purposes $T_x = T$

At $x = L/2$

$$y = \frac{TeL^2}{6EI} - \frac{wL^4}{48EI} + \frac{wL^4}{96EI} - \frac{TeL^2}{12EI} + \frac{TeL^2}{48EI} - \frac{wL^4}{384EI}$$

$$y = \frac{TeL^2}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{48}\right) + \frac{wL^4}{EI}\left(-\frac{1}{48} + \frac{1}{96} - \frac{1}{384}\right)$$

$$y = \frac{5TeL^2}{48EI} - \frac{5wL^4}{384EI}$$

The above expression for deflection at the center line is identical to the expression obtained by the usual method.* This then shows that for the case considered the change in shape of the beam produces no change in the effect of prestress force. There remains to be considered the cases where the prestressing steel intersects the end of the beam below the central axis and above the central axis. The case of the steel being at a constant offset, as shown in Figure 5a, will next be considered.

Referring to Figure 5b

Let y = deflection at any point

Let origin be at left reaction

$A = T_x\, y''$

$T_y = T_x y' \big|_{x=0}$

Moment due to $T_x = -T_x(y + e)$

Moment due to $T_y = T_x y' \big|_{x=0}\, x$

Moment due to $A = xT_x \int_0^x y''\,dx - T_x \int_0^x xy''\,dx$

Total moment from prestressing steel $= -T_x(y+e) + T_x y' \big|_{x=0}\, x$

*See Appendix A, Page 64.

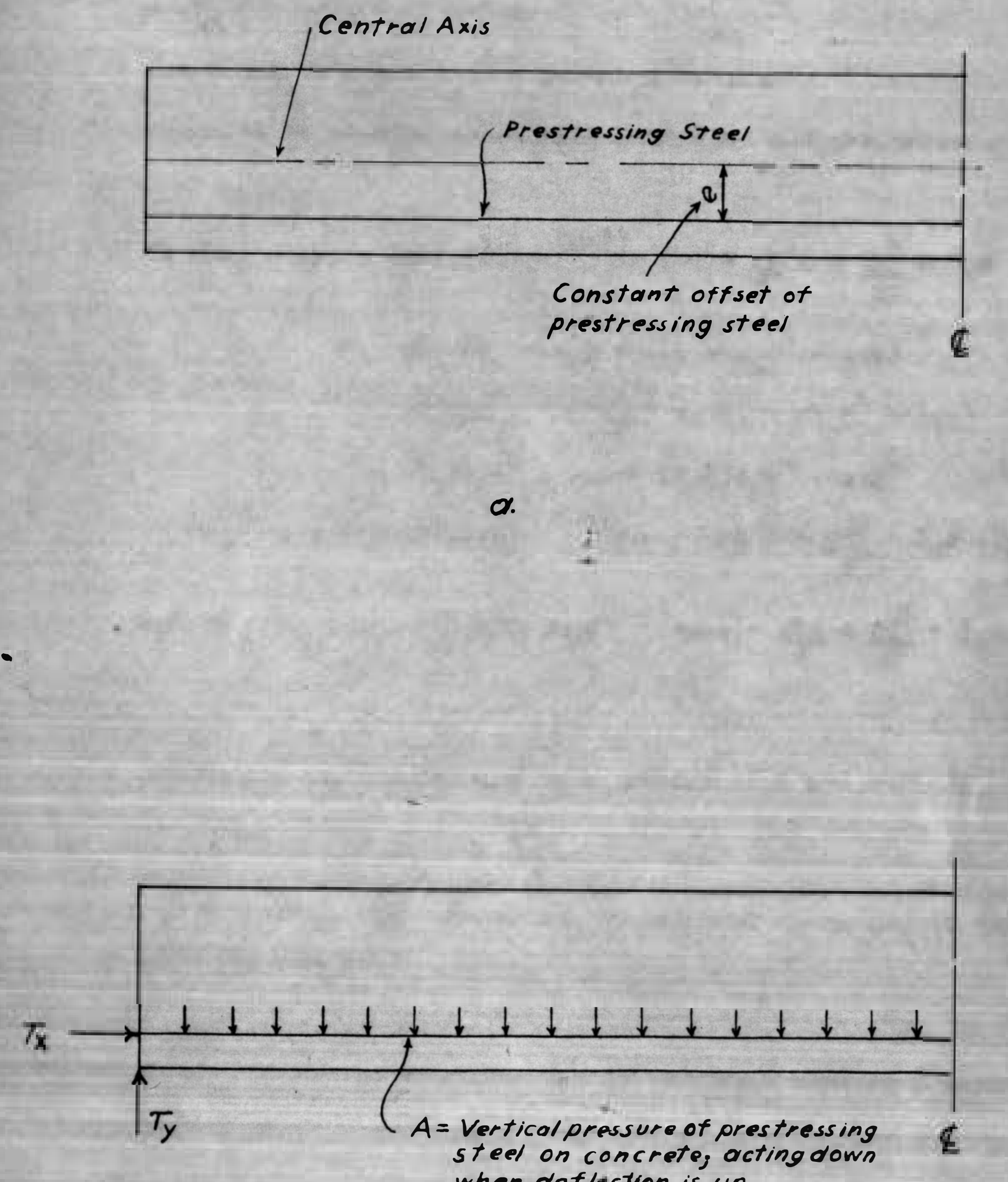


Fig. 5. CONSTANT OFFSET OF PRESTRESSING STEEL

$+ xT_x \int_0^x y''dx - T_x \int_0^x xy''dx$

Let $y = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$

Equation of elastic curve with dead load and prestress acting

$$EI\left[2a_2 + \dots + (n-1)(n)a_n x^{(n-2)} + \dots\right] = \frac{wL}{2}x - \frac{wx^2}{2} - T_x e$$

$$-T_x(a_0 + a_1 x + \dots + a_n x^2 + \dots)$$

$$+T_x a_1 x + T_x(2a_2 x^2 + \dots + na_n x^n + \dots)$$

$$-T_x\left[a_2 x^2 + \dots + (n-1)a_n x^n + \dots\right]$$

$$EI\left[2a_2 + \dots + (n-1)(n)a_n x^{(n-2)} + \dots\right] = \frac{wL}{2}x - \frac{wx^2}{2} - T_x e - T_x a_0$$

$$0 = EI\left[2a_2 + \dots + (n-1)(n)a_n x^{(n-2)} + \dots\right] - \frac{wLx}{2} + \frac{wx^2}{2} + T_x e + T_x a_0$$

Collecting coefficients of like powers of x and setting them equal to 0.

$2EIa_2 + T_x e + T_x a_0 = 0$

$2 \cdot 3EIa_3 - \frac{wL}{2} = 0$

$3 \cdot 4EIa_4 + \frac{w}{2} = 0$

$4 \cdot 5EIa_5 = 0$

$a_0 = 0$ because $y = 0$ when $x = 0$

$a_2 = -\frac{T_x e}{2EI}$

$a_3 = +\frac{wL}{2 \cdot 2 \cdot 3EI}$

$a_4 = -\frac{w}{2 \cdot 3 \cdot 4EI}$

$$y = a_1x - \frac{T_xe}{2EI}x^2 + \frac{wL}{2\cdot2\cdot3EI}x^3 - \frac{w}{2\cdot3\cdot4EI}x^4$$

$$y' = a_1 - \frac{T_xex}{EI} + \frac{wLx^2}{4EI} - \frac{wx^3}{2\cdot3EI}$$

$y' = 0$ when $x = L/2$

$$0 = a_1 - \frac{T_xeL}{2EI} + \frac{wL^3}{16EI} - \frac{wL^3}{48EI}$$

$$a_1 = +\frac{T_xeL}{2EI} - \frac{wL^3}{16EI} + \frac{wL^3}{48EI} = +\frac{T_xeL}{2EI} - \frac{wL^3}{24EI}$$

$$y = \left(-\frac{wL^3}{24EI} + \frac{T_xeL}{2EI}\right)x - \frac{T_xe}{2EI}x^2 + \frac{wL}{12EI}x^3 - \frac{wx^4}{24EI}$$

For $x = L/2$; and letting $T_x = T$

$$y = -\frac{wL^4}{48EI} + \frac{TeL^2}{4EI} - \frac{TeL^2}{8EI} + \frac{wL^4}{96EI} - \frac{wL^4}{384EI}$$

$$y = \frac{wL^4}{EI}\left(\frac{-1}{48} + \frac{1}{96} - \frac{1}{384}\right) + \frac{TeL^2}{EI}\left(+\frac{1}{4} - \frac{1}{8}\right)$$

$$y = \frac{TeL^2}{8EI} - \frac{5wL^4}{384EI}$$

The above expression for deflection is observed to be identical to the expression obtained by the usual methods.*

Discussion & Conclusions of Investigation of Formulas Used in Design

It appears from the two foregoing examples that the change in shape of an unbonded prestressed concrete beam as it deflects will have no effect upon the action of the prestressing force. A deflection up or down of the beam will change the magnitude of the vertical forces and will change the lever arm of the thrust on the end of the beam. The net effect, however, is zero. In the derivations shown it will be

*See Appendix A, Page 65.

noted that the terms involving moment due to a change in shape of the beam cancel. Upon inspecting Figure 4b, the development leading up to the expression for moments due to prestressing force, and the manner in which terms cancel out, it can be seen that for any similar case the same terms will cancel. It is the opinion of the author of this paper that the formulas that have been used to calculate deflections of unbonded prestressed beams are theoretically sound.

TESTING OF BEAMS IN THE LABORATORY

Description and Design of Test Beams

As a means of determining the extent to which unexpected deflections in unbonded prestressed concrete beams occur and to detect the cause of such deflections, beams were built and tested in the laboratory. As previously stated, if the unbonded prestressed beam had in it a small amount of plain reinforcing, the deflections were observed to be in line with calculated values. In view of this fact two beams were built. One beam had only prestressed reinforcing. The other beam was identical, but it had in addition a small amount of plain reinforcing.

It was decided to use a beam with a ten-foot span and a six-inch by six-inch cross-section. The concrete was designed for 5000 psi ultimate strength at 28 days. In determining the amount of steel required for prestressing it was decided to limit the compressive stress in the concrete to about 2500 psi. The prestressing steel was hot rolled mild steel (S.A.E. 1020). The steel used for plain reinforcing was mild steel wire with a diameter of 0.142 inches.

Ordinarily high strength steel should be used for prestressing concrete. High strength steel is stretched much more than low strength steel in reaching the working stress. In time plastic flow and shrinkage of the concrete will practically nullify the prestressing action of low strength steel. High strength steel, because it is stretched farther, is not affected as much by plastic flow and shrinkage of the concrete. For the tests made in this instance it was believed that the time needed for testing would not be great enough to cause an appreciable loss in prestress from shrinkage and plastic flow of the concrete if low strength steel was used. It was found when these beams were tested that no appreciable loss in prestressing force occurred.

To determine the quantity and placement of prestressing steel the procedure used was as shown in Appendix A. Following are the calculations:

1. Live load moment (M_{LL}) will be assumed great enough to cause a bending stress of about 2500 psi.

2 & 3. Omitted because the dimensions of the beam were arbitrarily selected.

4. Dead load moment at center line.

$$w = \frac{150}{(4)(12)} = 3.12 \text{ lb./in.}$$

$$M_{DL} = \frac{wL^2}{8} = \frac{(3.12)(120)(120)}{8}$$

$$M_{DL} = 5,620 \text{ in. lb.}$$

5. Omitted because dead load stresses are considered in step number 6.

6. Section modulus $= \frac{I}{c} = S$

$$I = \frac{bd^3}{12} = \frac{(6)^4}{12} = 108 \text{ in.}^4$$

$$c = d/2 = \frac{6}{2} = 3 \text{ in.}$$

$$S = \frac{108}{3} = 36 \text{ in.}^3$$

$$(1)\ \frac{T}{A} + \frac{Te}{S} - \frac{M_{DL}}{S} = \frac{M_{LL}}{S}$$

T = Total prestressing force

A = Concrete area = (6)(6) = 36 sq. in.

e = Offset of center of gravity of steel at center line of beam.

S = 36 in.^{3}

M_{DL} = 5620 in.lb.

$\frac{M_{LL}}{S}$ = 2500 psi

$$(1)\ \frac{T}{36} + \frac{Te}{36} - \frac{5620}{36} = 2500$$

$$(2)\ \frac{T}{A} - \frac{Te}{S} + \frac{M_{DL}}{S} = 0$$

$$(2)\ \frac{T}{36} - \frac{Te}{36} + \frac{5620}{36} = 0$$

Solution of equations (1) and (2) gives:

T = 45,000 lb.

e = 1.125 in.

The value for e is satisfactory, but the value for T would require too much steel for convenient placing. The yield point for S.A.E. 1020 steel is 45,000 psi. Therefore, well over a square inch of steel would be required.

After several trials, a value of T equal to 36,000 was selected. Then, using equation (2), e = 1.156 inches and, using equation (1), $\frac{M_{LL}}{S}$ = 2000 psi.

It was decided to use four 5/8 inch smooth round bars for the prestressing steel. The results of step

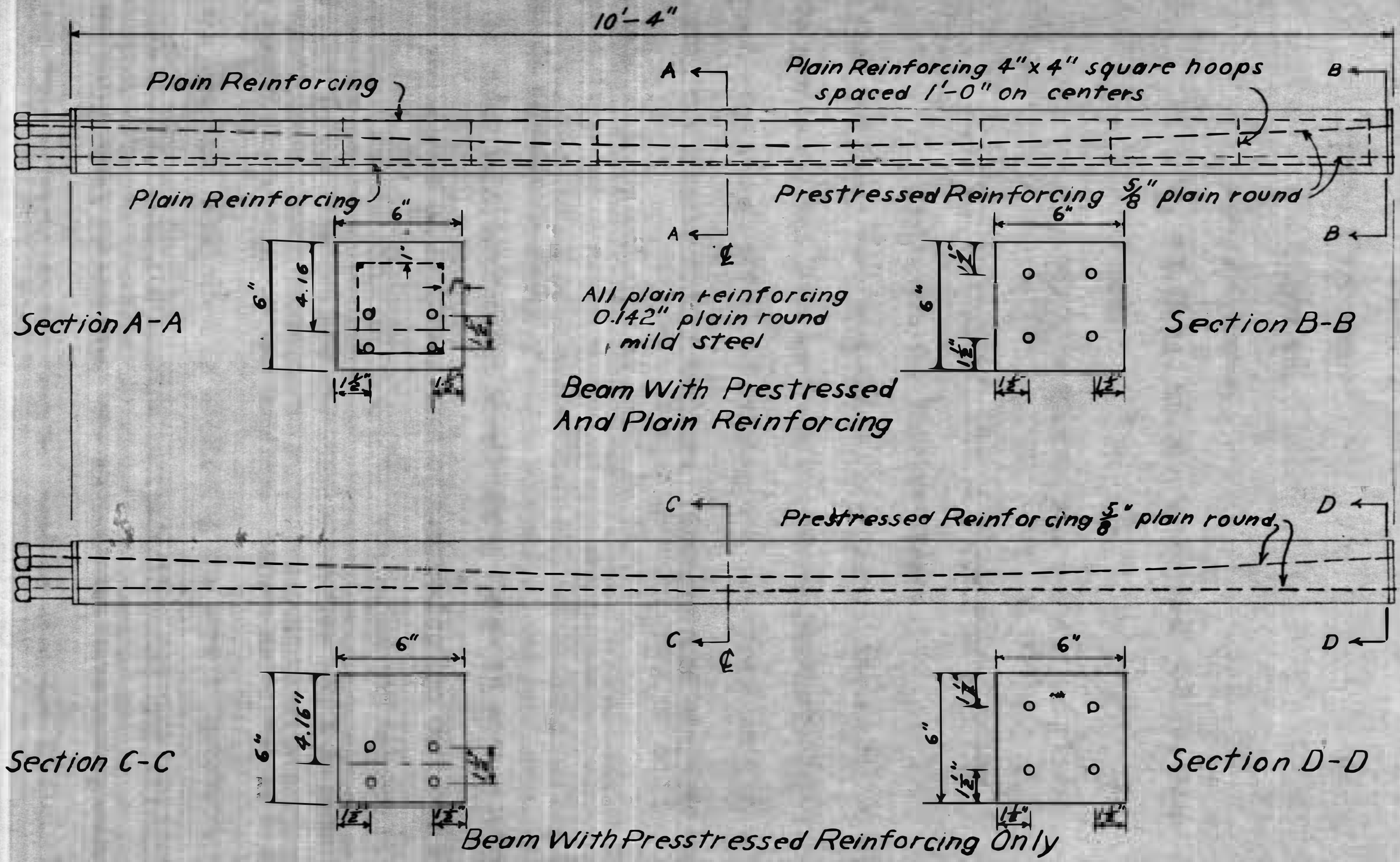


Fig. 6, STEEL PLACEMENT AND DIMENSIONS OF TEST BEAMS number 6 are then:

$T = 36{,}000$ lb.

$e = 1.16$ inches

$A_s = 1.25$ sq. in. = area of four 5/8 in. round bars.

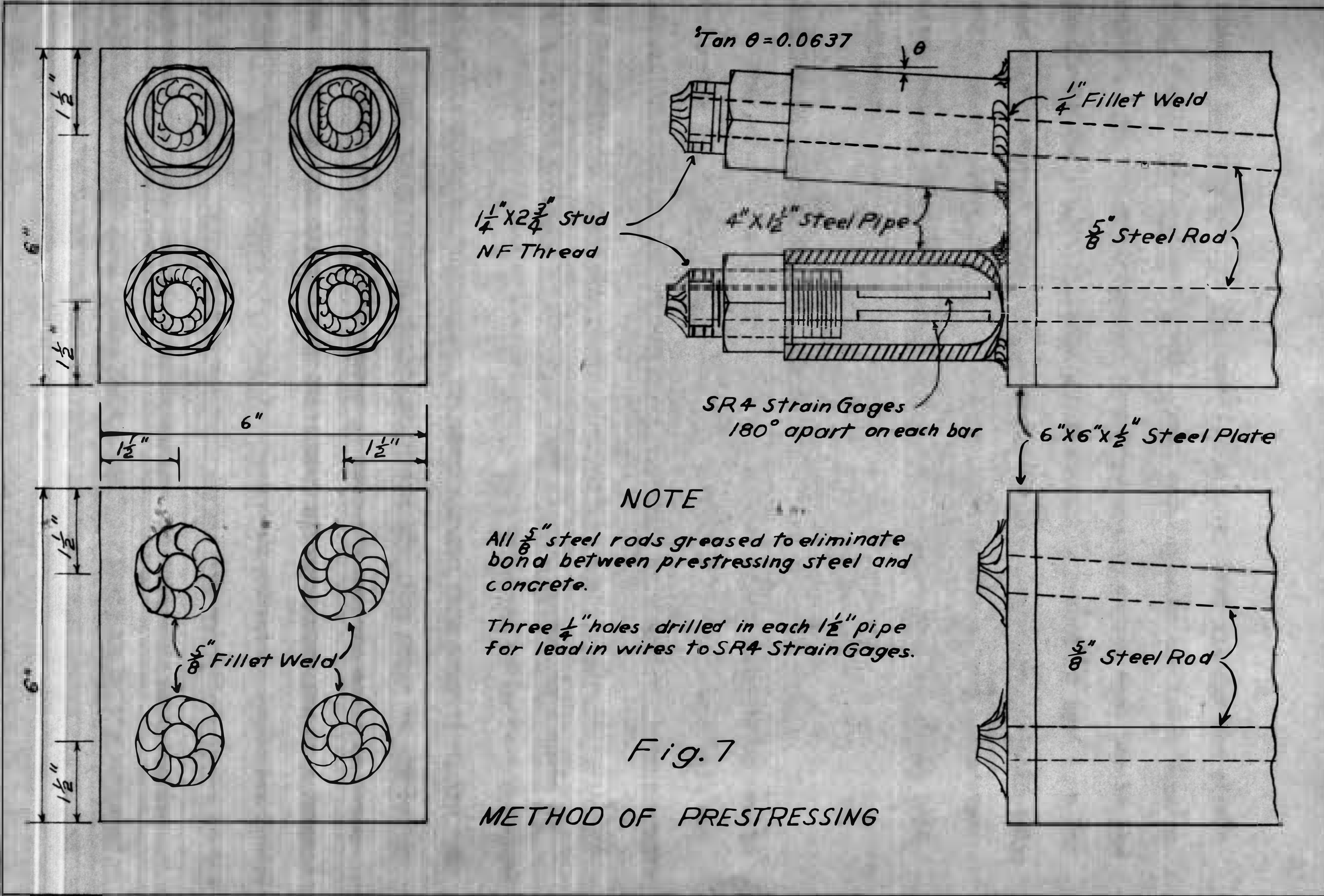
$$f_s = \frac{36{,}000}{1.25} = 29{,}200 \text{ psi.}$$

7,8,9, and 10 omitted because these steps are not applicable for this problem.

When the beams were tested the prestressing force was raised to 40,000 pounds and the live load was increased until cracking occurred. The most significant effect of raising the prestress was that the top fibers in the beam went into tension. Dimensions of the beam and placement of the steel were as shown in Figure 6.

Method of Tensioning Bars

In order to determine the prestressing force in each steel rod SR4 strain gages were used. The method of tensioning the bars and location of the gages was as shown in Figure 7. Two gages 180 degrees apart were used on each bar so that the average reading could be used and thus cancel the effects of any bending. Before using the bars the gages on each one were calibrated by tensioning the bars in a testing machine. Each bar was put into the machine individually, and load was applied in one-thousand pound increments. The maximum load was nine-thousand pounds. The calibration data can be found in Appendix B, Table I. The relationship between strain and load for each bar was determined by



NOTE

All 5/8" steel rods greased to eliminate bond between prestressing steel and concrete.

Three 1/4" holes drilled in each 1 1/2" pipe for lead in wires to SR4 Strain Gages.

Fig. 7

METHOD OF PRESTRESSING

finding the equation of the straight line of best fit for the strain load data. The equation of this line was found by the least squares method. The calculations and equations are shown for each bar in Appendix B, Table II. The average strain for any desired load in a bar can be determined from the equations for the strain load relationship. While prestressing the beams the bars were tensioned until predetermined values of average strain were indicated on the SR4 strain gage indicator.

Methods of Observing Beam Action

The primary purpose of the tests was to study deflections and to determine the cause of any irregular action of the beams in this respect. Deflections were measured by stretching a wire over pins mounted as shown in Figure 9. Steel scales were taped to the side of each beam so that movement of the wire with respect to the beam could be observed. The scales were calibrated in hundredths of an inch and were spaced as shown in Figure 9. SR4 strain gages were mounted on the bottom of each beam as shown in Figure 9 and 10. The gages on the bottom of the beams were put on in pairs so that the average could be taken for each pair and thus cancel the effect of any deflection in a lateral direction. This would also supply a means of detecting excessive lateral deflection. It was suspected that stress concentrations might be the cause of excessive vertical deflections. If this were true the deflection data and strain gage data should indicate it by the shape of the deflection curve and distribution of stress throughout the length of the beam.

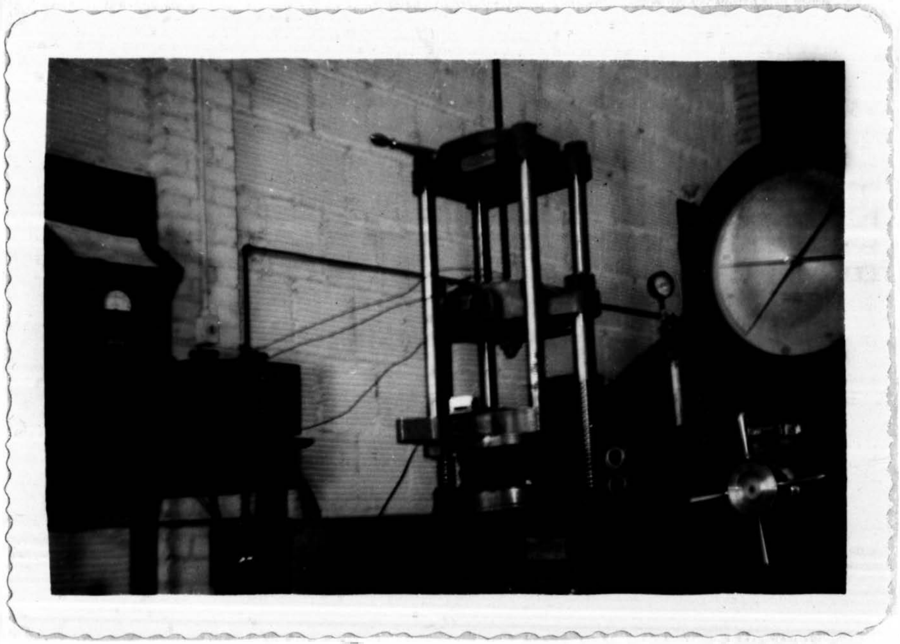


Fig. 8. Calibrating a Prestressing Bar.

<u>Construction & Preparation for Testing</u>

The test beams were cast, cured, and tested in the Civil Engineering Materials Laboratory at South Dakota State College. The prestressing units were made up in the Engineering Machine Shop at South Dakota State College by the author. Forms were made of two-by-eight-inch wood as shown in Figure 11.

Concrete for the beams was designed for a one-inch slump and a twenty-eight day ultimate compressive strength of 5000 psi. A Concrete Laboratory class designed and mixed the concrete. The aggregate was crushed quartsite with a three-quarter-inch maximum size and washed sand. Three batches of concrete were mixed for each beam. The material in each batch was as follows:

10.95 lb. water
42.61 lb. sand
25.11 lb. Type I Portland Cement
71.94 lb. crushed rock

A five-by-ten inch test cylinder was taken from each batch of concrete (making six test cylinders for the two beams). A vibrator was used in casting both the beams and test cylinders.

The beam with only prestressed reinforcing was cast on February 25, 1952, and the beam with prestressed and plain reinforcing was cast on February 26, 1952. The timing on casting was such that the first beam could be prepared for testing and then tested while the second beam was being prepared for testing. This procedure assured identical curing time for the two beams. The forms were stripped from the sides of the beams on the first day after casting each beam. Melted paraffin was then brushed on the top and sides of the beams. The test cylinders

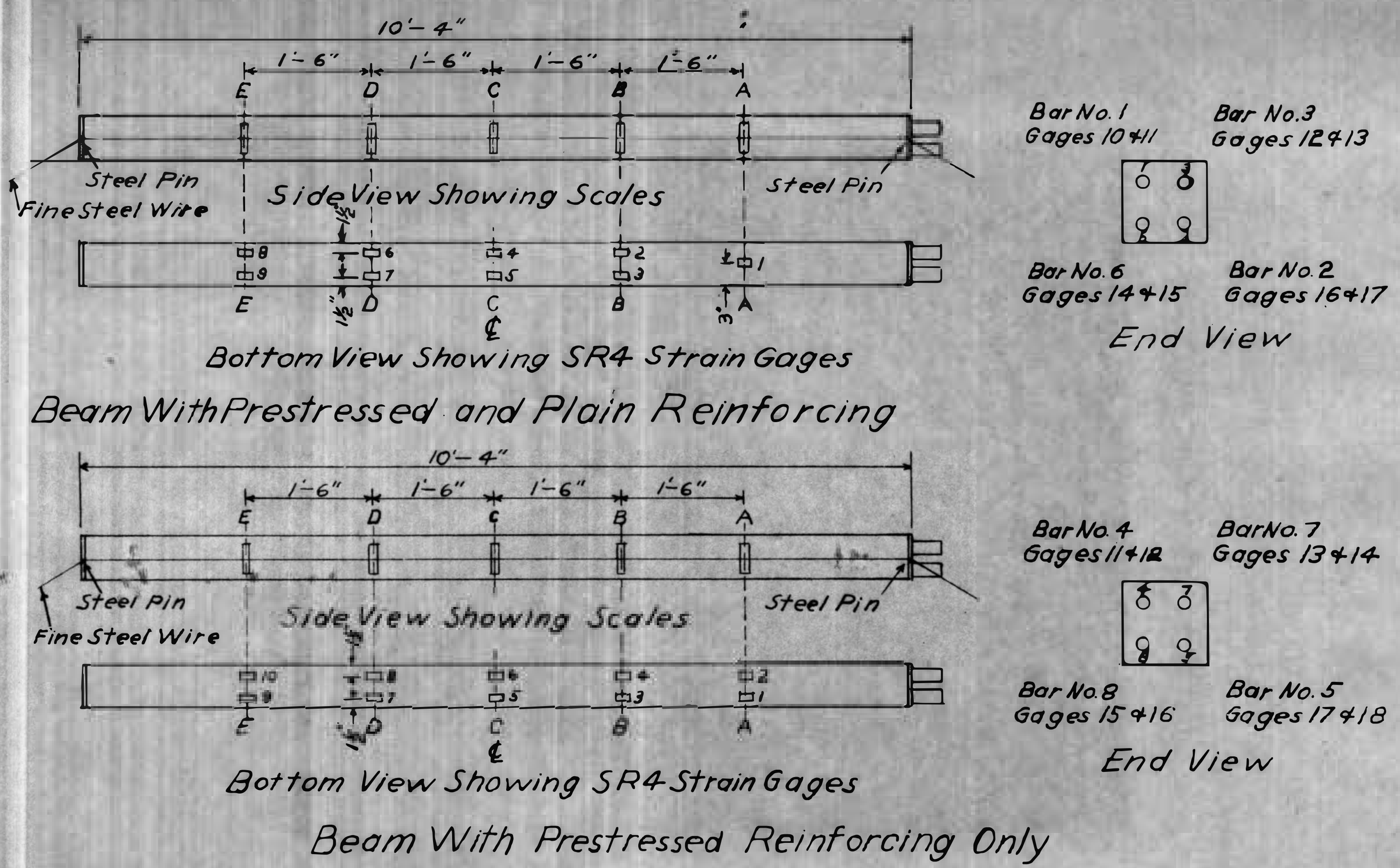


Fig. 9. DEFLECTION AND STRAIN MEASURING DEVICES

Fig. 10. SR4 Strain Gages Attached to Bottom of Beam. Gages used for measuring deformation of concrete.

Fig. 11. Prestressing Unit Positioned in Form.

were taken from their forms at the time the side forms were removed from each beam. The test cylinders were also coated with melted paraffin. The beams and test cylinders were left in the laboratory to cure.

On the twenty-seventh day after casting the beams were turned over, and the bottom was allowed to dry for one day. Also the paraffin was stripped from the cylinders on the twenty-seventh day after casting. At twenty-eight days the spots where SR4 strain gages were to be attached were given a liberal coat of SR4 cement. One cylinder from each beam was also prepared in the same manner for applying SR4 strain gages. The SR4 strain gages were cemented to the beams on the twenty-ninth day. A small sand bag was placed on each gage and left for three hours. The sand bags were then removed and the gages allowed to dry over night. Testing of the beams was started on the thirtieth day, and two test cylinders from each beam were tested for ultimate strength that day. The cylinders with SR4 strain gages were tested for modulus of elasticity on the thirty-first day after casting of the second beam. The modulus of elasticity test had to be done at this time because one SR4 gage on one cylinder had to be used as a compensating gage for the test of the other cylinder.

Testing Procedure

The first phase of the testing of each beam was the prestressing operation. I beams were set on the floor for supports at each end of the test beam. Eighteen SR4 strain gages were in use on one beam and seventeen on the other. A switching unit especially made for this purpose was used to connect the gages to the strain indicating unit. The prestressing was applied in four-thousand pound increments (one-thousand pounds per bar for each increment). The force in each bar was determined

Fig. 12. Concrete Cylinders With SR4 Strain Gages Attached. These cylinders tested in compression in order to determine E_c.

Fig. 13. Prestressing. Note: Scales taped to side of beam for measuring deflection; concrete cylinder with SR4 strain gage wired in as compensating gage for SR4 strain gages on the beam.

from the SR4 strain gages on the bars. The strain indicator was read for each gage on the bottom of the beam for each increment of prestressing force. All SR4 strain gage readings and deflection readings were recorded for each increment of prestressing force.

Loading of the beams was the next phase of the testing. A concentrated load was applied at the center of each beam with a beam testing machine. The increments of load were as shown in Appendix B, Tables XIV and XV. At zero load a reading was taken on each SR4 strain gage and on the scales. All gages were again read and deflection readings taken and recorded for each load increment.

The time intervals involved and any comments were recorded at the time of test and are given in the various tables of test data. (Appendix B)

Investigation of the Properties of the Crossection

In order to study the test results it was considered desirable to determine the effect of the reduced concrete area due to the space taken by the steel. The effect of the plain reinforcing was also calculated. In ordinary concrete design work these effects are neglected. It seems desirable to consider them in this case since the use of low strength steel required a larger steel area than would normally be used.

An equation will now be derived for calculating the moment of inertia at any point in the beam which had prestressed reinforcing only.

Referring to Figure 15:

$$I_{x-x} = \frac{bh^3}{12} + A\bar{y}^2 - (I_{z-z} + A'd^2)$$

Fig. 14. Loading Test.

I_{z-z} = moment of inertia for holes about z-z

$I_{z-z} = I_0 + A'd'^2$

$I_0 = \frac{\pi r^4}{4}$ (for one hole)

$I_0 = 0.03$ in.^{4} (for four holes)

$A' = (4)(0.3068) = 1.227$ sq. in. = area of four 5/8 in. holes

$d' = \frac{e'}{2}$

e' varies parabolically from center line to end of beam.

e' = 1.5 in. at center line of beam

e' = 3 in. at end of beam.

$e' = 1.5 + kx^2$

$3 = 1.5 + k(62)^2$

$1.5 = 3844k$

$k = \frac{1.5}{3844}$

$e' = 1.5\ (1 + \frac{x^2}{3844})$

$d' = \frac{1.5}{2}\ (1 + \frac{x^2}{3844}) = (0.75)(1 + \frac{x^2}{3844})$

$d'^2 = \left[.75 + \frac{(.75)x^2}{3844}\right]^2 = 0.5625 + \frac{0.5625}{1922}\ x^2 + \frac{0.5625x^4}{14776336}$

$d'^2 = 0.5625 + 0.0002927x^2 + 0.000,000,038,070x^4$

$I_{z-z} = 0.03 + 1.227(0.5625 + 0.0002927x^2 +$
$0.000,000,038,07x^4)$

$I_{z-z} = 0.03 + 0.690 + 0.000,359x^2 + 0.000,000,046,7x^4$

Sum of the moments of areas about y-y = (0)(bh) +

$$A'y_c = A_{net}\,\bar{y}$$

$\bar{y} = \dfrac{A'y_c}{A_{net}}$

$y_c = 1.16 - kx^2$

y_c = 1.16 in. at center line of beam.

y_c = 0 at end of beam.

$0 = 1.16 - k(62)^2$

$k = \dfrac{1.16}{3844}$

$y_c = 1.16 - \dfrac{1.16}{3844}x^2 = 1.16 - 0.000{,}301x^2$

$\dfrac{A'}{A_{net}} = \dfrac{1.227}{34.77}$

$\bar{y} = \dfrac{(1.16)(1.23)}{34.8} - \dfrac{(1.16)(1.23)x^2}{(34.8)(3844)}$

$\bar{y} = 0.041{,}0 - 0.000{,}010{,}7x^2$

$\bar{y}^2 = 0.00168 - 0.000{,}000{,}877x^2 + 0.000{,}000{,}000{,}115x^4$

$A\bar{y}^2 = 0.060{,}5 - 0.000{,}003{,}16x^2 + 0.000{,}000{,}004{,}14x^4$

$d = e + \bar{y} = 1.156 - 0.000{,}301x^2 + 0.0408 - 0.000{,}010{,}6x^2$

$d = 1.20 - 0.000{,}312x^2$

$d^2 = 1.44 - 0.000{,}748x^2 + 0.000{,}000{,}097{,}3x^4$

$A'd^2 = 1.77 - 0.000{,}920x^2 + 0.000{,}000{,}120x^4$

$I_{x-x} = 108 + 0.060{,}5 - 0.000{,}003{,}16x^2 + 0.000{,}000{,}004{,}14x^4$

$\quad - 0.72 - 0.000{,}359x^2 - 0.000{,}000{,}046{,}7x^4$

$\quad - 1.77 + 0.000{,}920x^2 - 0.000{,}000{,}120x^4$

$I_{x-x} = 105.6 + .000{,}558x^2 - 0.000{,}000{,}163x^4$

Using the above equations for I_{x-x} it is found that I at the center of the beam equals 105.6 and I at the end of the beam equals 105.3. This represents a change of about 0.4%. For all practical purposes the moment of inertia of the beam can be considered as 105 throughout the length of the beam.

The equation for $\bar{y}$ shows that at the center line $\bar{y}$ equals 0.041 inches. The lever arm for the prestressing force at the center line of the beam would normally be considered as 1.16 inches in this case. Adding 0.041 inches represents a change of about 3.5% in the moment due to prestress. Although this does not represent an extremely large error if neglected, it will be considered in deriving an equation for the deflection of the beam.

The moment of inertia of the cross-section of the beam with plain reinforcing will be calculated next. This beam had in it four steel wires as shown in Figure 6. These wires were bonded to the concrete, so the increase in moment of inertia will be determined by the transformed section.

Referring to Figure 15 and Figure 16:

$$I_{x-x} = I_{y-y} + A\bar{y}^2 - (I_{z-z} + A'd^2)$$

Assume $n = \frac{E_s}{E_c} = 10$

Area of one wire $= \frac{\pi d^2}{4} = \frac{(3.14)(0.142)(0.142)}{4}$

$= 0.0158$ sq. in.

Transformed area of four wires = (4)(10-1)(0.0158) =

0.569 sq. in.

Neglecting I_0 for the transformed steel area

$$I_{y-y} = \frac{bd^3}{12} + A''d^2$$

$A'' = 0.569$ sq. in. = transformed wire area

$d = 2$ in.

$$I_{y-y} = 108 + (0.569)(4) = 110.3 \text{ in.}^4$$

Summing moments of areas about y-y

$$\Sigma M = (0)bh + A'y_c + (0.569)(0) = A_{net}\bar{y}$$

$$\bar{y} = \frac{A'y_c}{A_{net}}$$

$$A_{net} = 36 + 0.569 - 1.23 = 35.3$$

$$\bar{y} = \frac{(1.16)(1.23)}{35.3} - \frac{(1.16)(1.23)x^2}{(35.3)(3840)}$$

$$\bar{y} = 0.0404 - 0.000,010,5x^2$$

$$\bar{y}^2 = 0.00164 - 0.000,000,850x^2 + 0.000,000,000,110x^4$$

$$A = 36 + 0.569 = 36.6$$

$$A\bar{y}^2 = 0.060 - 0.000,003,11x^2 + 0.000,000,004,03x^4$$

$$I_{z-z} = 0.03 + 0.690 + 0.000,359x^2 + 0.000,000,046,7x^4$$

$$d = y_c + \bar{y} = 1.20 - 0.000,312x^2$$

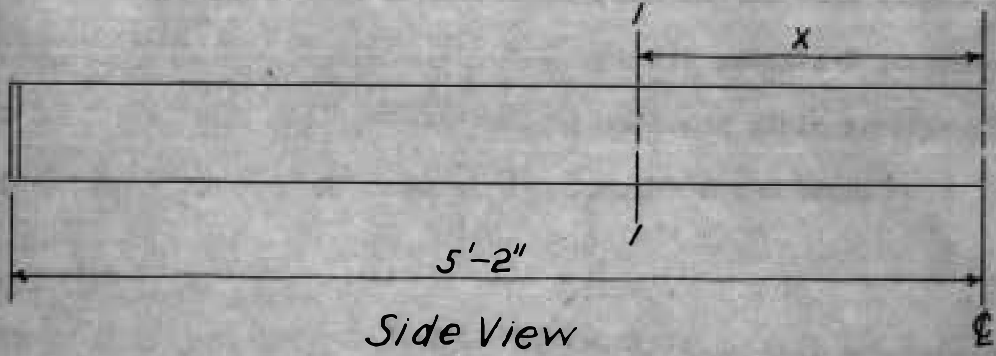
$$d^2 = 1.44 - 0.000,750x^2 + 0.000,000,097,4x^4$$

$$A'd^2 = 1.77 - 0.000,923x^2 + 0.000,000,120x^4$$

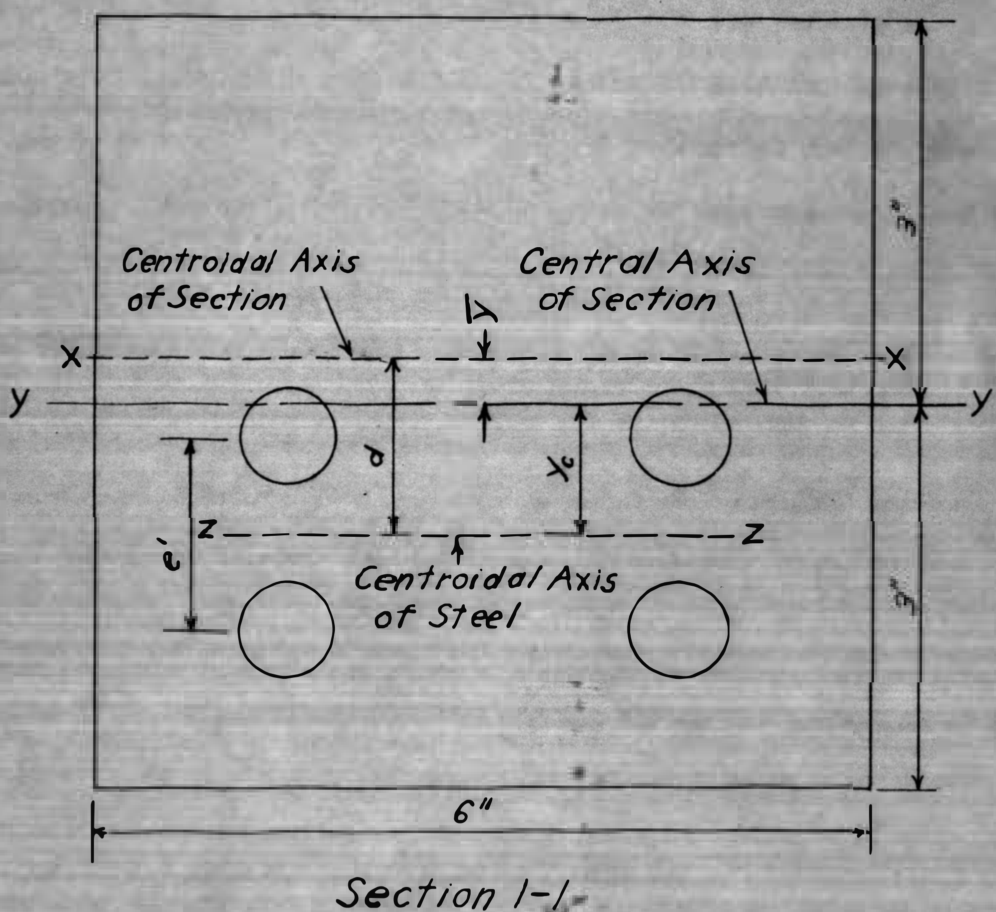
$$I_{x-x} = 110.3 + 0.060 - 0.000,003,11x^2 + 0.000,000,004,03x^4$$
$$- 0.72 - 0.000,359x^2 - 0.000,000,046,7x^4 - 1.77 + 0.000,923x^2$$
$$- 0.000,000,120x^4$$

$$I_{x-x} = 107.9 + 0.000,561x^2 - 0.000,000,163x^4$$

Using the equation above for I_{x-x} it is found that I at the center



Side View



Section 1-1

Fig. 15

line of the beam equals 107.9 and I at the end of the beam equals 107.6. This represents a change of about 0.4%. For all practical purposes the moment of inertia of the beam with prestressed and plain reinforcing can be considered as 108 throughout the length of the beam.

Equation for Deflection

When the beams were prestressed the deflection that occurred was due to prestress alone. The full dead load was acting initially because the beams were supported at each end before the bars were tensioned. The deflection shown in the tables of data for prestressing should be compared with calculated values of deflection due to prestress alone. The formula for deflection due to prestress will now be derived.

$$y'' = \frac{M}{E_c I}$$

$$M = -T\,d$$

$$d = y_c + \bar{y}$$

$$d = 1.20 - 0.000{,}312x^2$$

$$E_c I y'' = -1.20T + T\,0.000{,}312x^2$$

$$E_c I y' = -1.20Tx + \frac{T\,0.000{,}312x^3}{3} + C_1$$

$$y' = 0 \text{ when } x = 0$$

$$C_1 = 0$$

$$E_c I y = \frac{-1.20}{2}Tx^2 + T\,\frac{0.000{,}312x^4}{12} + C_2$$

$$y = 0 \text{ when } x = \frac{L}{2}$$

$$0 = \frac{-1.20}{8}\,TL^2 + T\,\frac{0.000{,}312}{192}\,L^4 + C_2$$

$$C_2 = \frac{-0.000{,}312}{192} TL^4 + \frac{1.20}{8} TL^2$$

$$E_c Iy = \frac{-1.20}{2} Tx^2 + \frac{0.000{,}312}{12} Tx^4 + \frac{1.20}{8} TL^2 - \frac{0.000{,}312 TL^4}{192}$$

L = 124 inches

$$y = 0.6\frac{Tx^2}{EI} + 0.000{,}026\frac{Tx^4}{EI} + 2310\frac{T}{EI} - 384\frac{T}{EI}$$

$$y = \frac{T}{EI}(-0.6x^2 + 0.000{,}026x^4 + 1930)$$

Where "x" is measured from the center of the beam.

For T = 40,000 lb. and I = 105 and E_c = 5,000,000 the deflection at the center of the beam should be 0.147 inches up.

If y_c were used instead of d in the derivation of the formula for deflection, the equation would be:

$$y = \frac{T}{E_c I}(-0.58x^2 + 0.000{,}025{,}1x^4 + 1860)$$

For T = 40,000 lb. and I = 105 and E_c = 5,000,000 the deflection at the center of the beam should be 0.142 inches up.

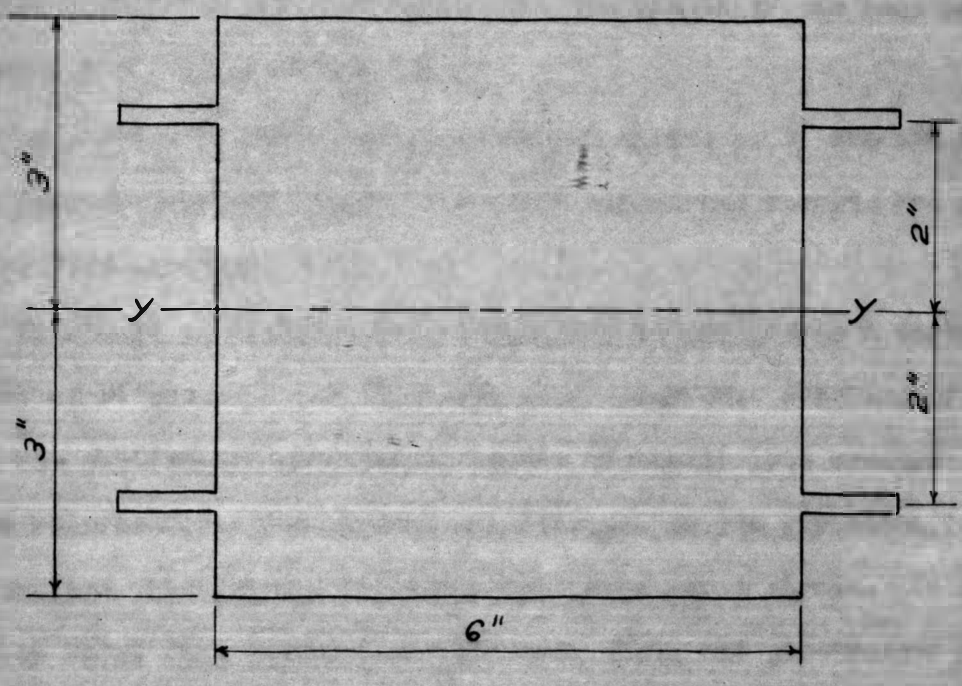
The difference in deflection when considering the lever arm of the prestress force as y_c rather than d would be about 3.5%.

Comparison of Calculated and Measured Deflections

The ultimate compressive strength of the concrete was found to be very nearly 5000 psi. (See Appendix B, Table IIIA.) A generally accepted value for modulus of elasticity of concrete is $1000f'_c$. For concrete with a compressive strength of 5000 psi, E_c would be 5,000,000 psi. In this instance the calculated deflections during prestressing, using E_c = 5,000,000 psi, would be as shown in Figure 17. The formula for deflection that was derived for the beams tested was used in calculating deflection. Also shown in this figure are the measured deflections for each beam tested. The beam with only prestressed reinforcing deflected about 20% more than calculated, and the beam with plain and prestressed reinforcing deflected about 7% less than calculated. This one test would tend to bear out the belief that unbonded prestressed beams with no plain reinforcing deflect more than calculated when the beam is being prestressed. It would also indicate that beams with a small amount of additional plain reinforcing have deflections more in line with calculated values. As data is accumulated from any past and future tests of this sort more decisive conclusions can be drawn. The author of this paper was unable to find any reports of deviations from calculated deflections of prestressed concrete beams other than those previously mentioned.

<u>Study of Prestressing Data</u>

The data obtained from the prestressing of the beams was studied in order to detect reasons for the deviations from calculated deflections. The secant modulus of elasticity of the concrete, as calculated from the test cylinder data, ranges from about 5,170,000 for a stress of 49 psi. to 3,330,000 for a stress of 2463 psi. If E_c = 3,330,000 is used to calculate deflection the deflection at a prestress of 40,000 pounds would be about 0.23 inches. This value is higher than either



Transformed Section
Showing Transformed Area of Plain Reinforcing

Fig. 16

measured value. The fiber stress in the bottom of the beams was calculated to be about 2500 psi at 40,000 pounds prestress, but the stress decreases to a value of about zero in the top fibers. It is reasonable to conclude that the actual modulus of elasticity in the concrete in the beam would be somewhere between 3,330,000 and 3,170,000. However, the value of E_c (or, in other words, the ratio between stress and strain) may not be constant throughout the length of the beam and throughout the depth of the beam.

As a means of calculating the actual values of E_c the SR4 strain gage readings were utilized. The calculations and results are given in Tables VIII to XIII.

It should be noted at this point that in both beams a certain amount of sticking of the bars must have occurred. Sticking of the bars was particularly noticeable in the beam with both prestressed and plain reinforcing. The bonding was detected by the variation in strain at sections which should theoretically have equal strain. It should also be noted that in the beam with both plain and prestressed reinforcing bar No. 6 had to have its nut tightened excessively to tension it for the last increment of prestress. This would indicate that bar No. 6 had suddenly broken loose. Bar No. 6 was a lower bar and would have a greater effect on strains than one of the upper bars. An inspection of strain data for this beam will indicate that on the last increment of prestress the strains became fairly uniform in their variation throughout the length of the beam.

Deflections at the center line were calculated for a prestress force equal to the final value in each beam and using E_c equal to the calculated values in Tables VIII and XI. The calculations are as follows:

For beam with only prestressed reinforcing:

$$y = \frac{T}{EI}(1930)$$

$T = 39,700$ lb.

$E = 3,960,000$ psi

$I = 105$ in.^{4}

$$y = \frac{(39,700)(1930)}{(3,960,000)(105)} = 0.18 \text{ in.}$$

$y = 0.18$ in.* (Table VI)

For beam with plain and prestressed reinforcing:

$$y = \frac{T}{EI}(1930)$$

$T = 39,900$ lb.

$E = 5,450,000$ psi

$I = 108$

$$y = \frac{(39,900)(1930)}{(5,450,000)(108)} = 0.13 \text{ in.}$$

$y = 0.13$ in. (Table VII)

It will be noted that the above calculated values of deflection check exactly with measured values. The fact that the values check exactly is not particularly significant in view of the fact that the error in reading deflections could have been 0.01 inch. The agreement of

*This value was obtained by subtracting 0.01 from the final value of 0.19 in. in order to compensate for plastic flow that occurred during the 1½ hour break in prestressing.

deflections calculated in this manner with measured deflections indicates that the actual value of E_c in an unbonded prestressed concrete beam would have to be known in order to make accurate predictions of deflections during prestressing.

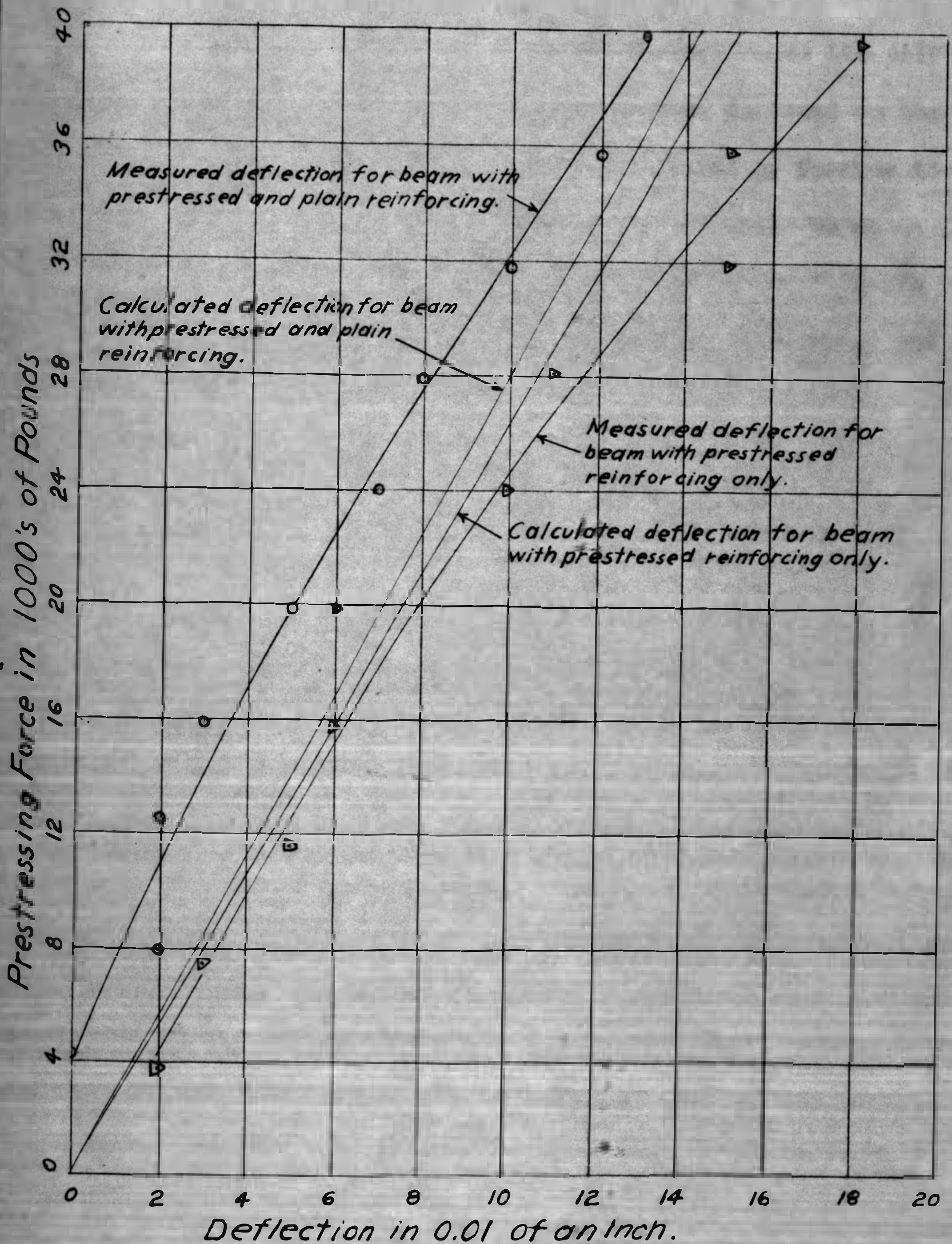
An inspection of Tables VIII to XIII indicates that the value of E_c is consistently lower in the beam with only prestressed reinforcing. The reason for a decrease in E_c in any case is the occurance of plastic flow in the concrete. Evidently the small amount of plain reinforcing adds enough resistance to plastic flow in the concrete to keep the value of E_c up to approximately 1000 f'_c.

Results of Loading Test

The results of observations on the loading of the beams are given in Appendix B, Tables XIV to XVII. It is interesting to note that the cracks which appeared upon loading closed when load was removed. (If the beams had been reinforced with only plain reinforcing there would have been much greater values of permanent deflection and the cracks would not have closed.) The loading tests were not particularly important in connection with the problem studied in this paper. They were conducted with the thought that the data thus obtained would be useful for some other purpose if not used in this paper.

Conclusion Drawn From Testing Beams in the Laboratory

It is the conclusion of the author of this paper that the deviation from calculated deflection while prestressing an unbonded prestressed concrete beam with no plain reinforcing is due to plastic flow



CALCULATED AND MEASURED DEFLECTION CURVES

Fig. 17

of the concrete in the beam. The plastic flow evidences its self by a reduced value of E_c. This conclusion, of course, is based on the tests described herein and would have to be substantiated by further tests and observations of beams used in structures. It would be convenient to have an expression or factor with which the value of E_c could be determined for beams and other members of various proportions and stress conditions.

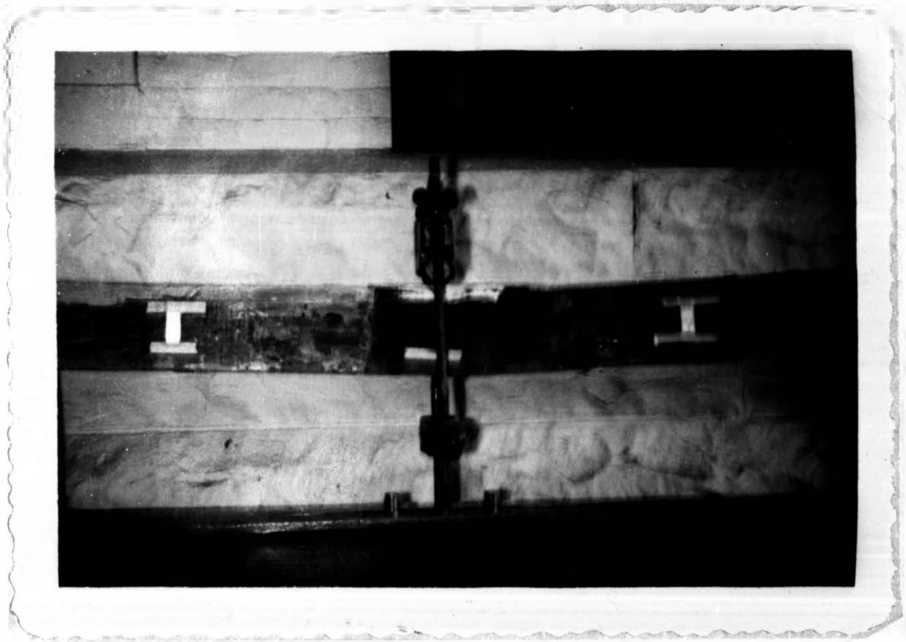


Fig. 18. Beam Deflected Under Load. Note crack at right of loading yoke. Crack closed when load was reduced in spite of partial crushing of concrete in top of beam.

SUMMARY

This thesis gives an account of the author's investigation of the deflection that occurs in an unbonded prestressed concrete beam during prestressing. This deflection has been reported to be greater than calculated if the beam had no plain reinforcing in it, and this deflection has been reported to be in line with calculated values if the beam contained a small amount of plain reinforcing. Two courses of action were followed in order to determine the reason for the way in which these beams react to the prestressing force. One course of action was to investigate the formulas used to calculate deflection of an unbonded prestressed concrete beam. The other course of action was to test beams in the laboratory in order to observe the action of unbonded prestressed concrete beams and to detect the reason for any deviations from calculated deflection during prestressing.

The investigation of formulas took into account the change in prestressing moment due to a change in shape of the beam as it deflects. This investigation indicated that, while the change in shape of the beam as it deflected changed different components of moment due to the prestressing force, the net effect of a change in shape of the beam was zero. This was true because the magnitude and sign of such changes in moment were such that they cancelled each other. It was the conclusion of the author that the change in shape of the beam need not be considered and that the formulas now in use are theoretically sound.

The results of the beam tests indicated that unbonded prestressed concrete beams with no plain reinforcing do deflect appreciably more

than calculated during the prestressing operation. It was also found that the deflection of an unbonded prestressed concrete beam with a small amount of plain reinforcing had deflection which was in better agreement with calculated values when the prestress was applied. These two findings depend on the use of 1000 f'_c for modulus of elasticity in the concrete. Calculations were made involving measured strain, measured deflection, and calculated stress. These calculations showed good correlation of the quantities involved. It was concluded that if the proper value of E_c is used a satisfactory prediction of the deflection can be made. The actual value of E_c in the beam with no plain reinforcing was about 20% lower than 1000 f'_c. The actual value of E_c in the beam with a small amount of plain reinforcing was about 10% higher than 1000 f'_c. It is the belief of the author that plastic flow of the concrete is the cause of the reduced value of E_c in the beam with no plain reinforcing. Evidently the small amount of plain reinforcing in the other beam resisted plastic flow enough so that E_c for this beam was closer to 1000 f'_c. As more data is compiled on this subject a trend may be established in regard to the choice of E_c.

NOMENCLATURE

A_{net} - Area of concrete minus A_s and including concrete area for transformed section where applicable.

A_s - Steel area.

a - Depth of beam.

b - Width of beam.

c - Distance from centroidal axis to outer fibers.

d - Offset of steel from centroidal axis.

d' - e'/2

E - Modulus of elasticity.

E_c - Modulus of elasticity of concrete.

E_s - Modulus of elasticity of steel.

e - Offset of steel from central axis at center line of beam.

e' - Vertical distance between steel bars at any point along length of beam.

f_c - Concrete stress.

f'_c - Ultimate compressive strength of concrete.

f_s - Steel stress.

ft. - Feet.

I - Moment of inertia.

in. - Inches.

L - Span length

lb. - pounds.

M - Moment.

M_{DL} - Dead load moment.

NOMENCLATURE (cont.)

S - Section modulus = I/c

S_b - Section modulus with respect to the bottom.

S_t - Section modulus with respect to the top.

sq. - Square.

T - Total prestressing force.

w - Pounds per unit length.

y - Deflection of beam at any point.

y_c - Offset of steel from central axis at any point.

y_d - Offset of steel from undeflected position of central axis at any point.

$\bar{y}$ - Offset of centroidal axis from central axis at any point.

y' - dy/dx

y'' - $\frac{d^2y}{dx^2}$

M_{LL} - Live load moment.

n - E_s/E_c

psi - Pounds per square inch.

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APPENDIX A

Design Procedure for Prestressed Concrete Beam on a Simple Span.*

1. Compute live load moment. (Live load moment is defined as the moment due to all dead and live loads applied after member has been prestressed.)

2. Compute the required section modulus by dividing the live load moment by the allowable concrete working stress.

3. Design a concrete beam having a section modulus equal to or greater than that found in Step 2. Section moduli are found by using the whole cross section of the concrete but neglecting the steel. The steel area is so small that the error in making this assumption is negligible.

4. Compute the dead load moment caused by the weight of the beam in the form in which it is to be prestressed.

5. Find the dead load stresses at the top and bottom fibres caused by this moment.

6. Find the necessary prestressing force (P) required and the eccentricity (e) of application. (1) Concrete must not go into tension; therefore under dead load the bottom fibres must have a compressive stress equal to or greater than the tension stress which will result from the live load. (2) Similarly the stress in the top fibres under dead load must not be less than 0. Using the above statements (1) and (2) write equations for the dead load stresses in the top and bottom fibres at the point of maximum live load moment. These stresses are due to P, Pe and the live and dead load moments.

For stress in bottom fibre

$$(1)\ \frac{P}{A} + \frac{Pe}{S_b} - \frac{M_{DL}}{S_b} = \frac{M_{LL}}{S_b}$$

in which $\frac{P}{A}$ is a compressive stress due to the prestressing load (P) applied at the ends of the beam; $\frac{Pe}{S_b}$ is a compressive stress due to the eccentricity of the prestressing force; $\frac{M_{DL}}{S_b}$ is a tension stress due to

*Set forth by John A. Roebling's Sons Company, Bridge Division, Trenton, New Jersey.

the dead load and $\frac{M_{DL}}{S_b}$ is the tension stress due to the live load.

For stress in top fibre = 0

$$(2)\ \frac{P}{A} - \frac{Pe}{S_t} + \frac{M_{DL}}{S_t} = 0$$

in which $\frac{P}{A}$ is a compressive stress due to the prestressing load (P) applied at the ends of the beam; $\frac{Pe}{S_t}$ is a tension stress due to the eccentricity of the prestressing force; and $\frac{M_{DL}}{S_t}$ is the compressive stress due to the dead load.

Substitute numerical values for all known quantities in the Equations (1) and (2). This leaves (P) and (e) unknown. Solve the equations simultaneously for (P) and (e).

7. Using an allowable strand stress of 105,000 lb. per sq. in., figure the area of steel prestressing strand required.

8. It is recognized that there is a loss of stress in the prestressing steel between the time the structure is prestressed and the time all plastic flow of concrete has taken place. In a properly designed structure using strands, this loss of stress never exceeds 20,000 lb. per sq. in. For this reason the strands are pulled up to an initial tension of 125,000 lb. per sq. in. and we know this stress will never drop below 105,000 lb. per sq. in. Therefore, 105,000 lb. per sq. in. is the figure used to determine the strands required. Since the initial force is applied at the stress of 125,000 lb. per sq. in., we must check the stresses in the top and bottom fibres for this new value of (P) due to initial prestressing. If these stresses are not within the allowable limits, the design of the structure will have to be altered slightly to keep the stresses within the allowable limits specified.

The trial and error part of the preceding design can be partially eliminated as follows:

$$\frac{\text{Initial Prestress} = 125{,}000 \text{ psi}}{\text{Final Prestress} = 105{,}000 \text{ psi}} = 1.19 \text{ from}$$

which Initial prestress (Pi) = 1.19 Final prestress (P).

In item 6 the stress in the bottom fibre as found by Equation (1) will reach zero sooner under the condition of final prestress than under the condition of initial prestress so this equation is correct as written. The stress in the top fibre will reach tension sooner under the condition of initial prestress than under the condition of final prestress. Therefore, this equation should be written for the initial prestress condition as follows:

$$(2A)\quad \frac{1.19\,P}{A} - \frac{1.19\,Pe}{S_t} + M_{DL} = 0$$

Now by substituting numerical values in Equations (1) and (2A), we can solve them simultaneously for P and e.

Find strand area as in item 7.

The values for P and e as found above give the greatest economy of strand. Sometimes the e found by this method will be so great that it will place the strand outside the concrete of the member. If this is the case, place the strand in the member with the greatest e possible for the given shape. Substitute this e with the other known values in Equation (1) and solve for P.

9. Plot stress diagrams:

For
Initial prestress plus dead load
Initial prestress plus dead load plus live load

Final prestress plus dead load
Final prestress plus dead load plus live load

10. Find horizontal shear and from this compute diagonal tension. This stress will usually be maximum on the center of gravity of the concrete section at the point of greatest shear. This point in a simple span is at the supports. If this computed tension stress is greater than .03 f_c (ultimate strength of concrete), then provide stirrups to take care of the excess. If this stress is less than the allowable, no stirrups are required. To find horizontal shear and the diagonal tension, use the following formulas:

Horizontal Shear (S_s) formula

$$S_s = \frac{V}{I_c \times t} \times a'\bar{y} \qquad \left(\frac{VQ}{I_b}\right)$$

in which
V = Total shear in beam due to dead and live loads minus shear carried by cables.
I_c = Moment of inertia of entire section about center of gravity of concrete.

t = Thickness of section at point under consideration.
Is the area (a') above the point under consideration times the distance from that point to the center of gravity of that area.

Diagonal tension (f_t) formula

$$f_t = \frac{f't}{2} \pm \sqrt{(S_s)^2 + \left(\frac{f't}{2}\right)^2}$$

in which

S_s = Horizontal shear as computed above.
f't = Horizontal stress at point under consideration. Since prestressed concrete is not allowed to go into tension f't is always a compressive stress.

When the point under consideration is the center of gravity of the concrete, section f_t is $\frac{P}{A_c}$.

When it is some other point, f't can be found by plotting a stress diagram for the beam at the point under consideration (similar to item 9) and reading the stress.

APPENDIX A

The following paragraph is taken from a letter dated August 21, 1951. This letter was received by the author of this thesis from Mr. H. Kent Preston, Engineer, Bridge Division, John A. Roebling's Sons Company, Trenton, New Jersey.

> "In reply to question No. 2 in your letter, our experience to date indicates that beams built of plain concrete have excessive deflections during prestressing, whereas beams built with a relatively small amount of reinforcing steel have a deflection which is in line with computed deflections. The excessive deflection is particularly noticeable when the stress in the beam reaches one third of the ultimate and higher. The purpose of this test would be to verify the above statements. We have never conducted a test on two identical beams but have reached the above conclusions on the basis of a number of different tests which were conducted for other purposes."

APPENDIX A

Derivation, by double integration, of equation for deflection of unbonded prestressed concrete beam:

Referring to Figure 3a:

With origin at point "O"

$$y_e = \frac{4ex^2}{L^2} - \frac{4ex}{L}$$

$M = T\left(\frac{4ex^2}{L^2} - \frac{4ex}{L}\right)$ = moment due to prestressing force.

$$y'' = \frac{M}{EI}$$

$$y''EI = T\left(\frac{4ex^2}{L^2} - \frac{4ex}{L}\right)$$

$$y'EI = T\left(\frac{4ex^3}{3L^2} - \frac{4ex^2}{2L}\right) + C_1$$

$y' = 0$ when $x = L/2$

$$0 = T\left(\frac{4eL}{24} - \frac{4eL}{8}\right) + C_1$$

$$C_1 = \frac{TeL}{3}$$

$$EIy' = T\left(\frac{4ex^3}{3L^2} - \frac{4ex^2}{2L}\right) + \frac{TeL}{3}$$

$$EIy = T\left(\frac{4ex^4}{12L^2} - \frac{4ex^3}{6L}\right) + \frac{TeL}{3}x + C_2$$

$y = 0$ when $x = 0$

$$0 = 0 + C_2$$

$$EIy = T\left(\frac{4ex^4}{12L^2} - \frac{4ex^3}{6L}\right) + \frac{TeL}{3}x$$

At $x = L/2$

$$EIy = \frac{TeL^2}{48} - \frac{TeL^2}{12} + \frac{TeL^2}{6}$$

$y = \frac{5TeL^2}{48EI}$ = deflection at center line due to prestressing force.

$y = \frac{-5wL^4}{384EI}$ = deflection at center line due to dead load.

Total $y = \frac{5TeL^2}{48EI} - \frac{5TeL^2}{384EI}$ = combined deflection due to prestress force and dead load for the case considered. (Fig. 5a)

Referring to Figure 5a:

$Y_c = e$

$M = -Te$

$y'' = \frac{M}{EI}$

$y''EI = -Te$

$y'EI = -Tex + C_1$

$y' = 0$ when $x = L/2$

$0 = -\frac{TeL}{2} + C_1$

$C_1 = \frac{TeL}{2}$

$y'EI = -Tex + \frac{TeL}{2}$

$yEI = \frac{-Tex^2}{2} + \frac{TeLx}{2} + C_2$

$y = 0$ when $x = 0$

$0 = 0 + C_2$

$yEI = \frac{-Tex^2}{2} + \frac{TeLx}{2}$

At $x = L/2$

$yEI = \frac{-TeL^2}{8} + \frac{TeL^2}{4}$

$y = \frac{TeL^2}{8EI}$ = deflection due to prestress force alone.

Total $y = \frac{TeL^2}{8EI} - \frac{5wL^4}{384EI}$ = combined deflection due to prestress and dead load for the case considered. (Fig.5a)

APPENDIX B

TABLE I CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 1

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-8-1015	0	0	0	0	A
0	0-6-1769	0	0	0	0	B
1000	0-8- 852	0	-163	163	108	A
1000	0-6-1716	0	- 53	53		B
2000	0-8- 742	0	-273	273	217	A
2000	0-6-1608	0	-161	161		B
3000	0-8- 641	0	-374	374	317	A
3000	0-6-1510	0	-259	259		B
4000	0-8- 536	0	-479	479	423	A
4000	0-6-1402	0	-367	367		B
5000	0-8- 430	0	-585	585	528	A
5000	0-6-1299	0	-470	470		B
6000	0-8- 328	0	-687	687	633	A
6000	0-6-1190	0	-579	579		B
7000	0-8- 220	0	-795	795	741	A
7000	0-6-1083	0	-686	686		B
8000	0-8- 117	0	-898	898	846	A
8000	0-6- 975	0	-794	794		B
9000	0-8- 12	0	-1003	1003	950	A
9000	0-6- 872	0	-897	897		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 2

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-12- 89	0	0	0	0	A
0	0- 8- 197	0	0	0		B
1000	0-10-1920	-2000	+1831	169	95	A
1000	0- 8- 177	0	- 20	20		B
2000	0-10-1810	-2000	+1721	279	198	A
2000	0- 8- 80	0	- 117	117		B
3000	0-10-1700	-2000	+1611	389	300	A
3000	0- 6-1987	-2000	+1790	210		B
4000	0-10-1588	-2000	+1499	501	403	A
4000	0- 6-1895	-2000	+1696	304		B
5000	0-10-1474	-2000	+1385	615	507	A
5000	0- 6-1798	-2000	+1601	399		B
6000	0-10-1360	-2000	+1271	729	613	A
6000	0- 6-1700	-2000	+1503	497		B
7000	0-10-1240	-2000	+1151	849	723	A
7000	0- 6-1600	-2000	+1403	597		B
8000	0-10-1137	-2000	+1048	952	825	A
8000	0- 6-1500	-2000	+1303	697		B
9000	0-10-1025	-2000	+ 936	1064	932	A
9000	0- 6-1398	-2000	+1201	799		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 3

Load lb.	Reading	$T_2-T_1(1000)$	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-8- 260	0	0	0	0	A
0	0-8-1090	0	0	0	0	B
1000	0-8- 304	0	+ 44	- 44	100	A
1000	0-8- 845	0	- 245	245		B
2000	0-8- 213	0	- 47	47	204	A
2000	0-8- 729	0	- 361	361		B
3000	0-8- 122	0	- 138	138	306	A
3000	0-8- 617	0	- 473	473		B
4000	0-8- 30	0	- 230	230	415	A
4000	0-8- 490	0	- 600	600		B
5000	0-6-1934	-2000	+1674	326	518	A
5000	0-8- 380	0	- 710	710		B
6000	0-6-1839	-2000	+1579	421	625	A
6000	0-8- 261	0	- 829	829		B
7000	0-6-1740	-2000	+1480	520	729	A
7000	0-8- 153	0	- 937	937		B
8000	0-6-1640	-2000	+1380	620	835	A
8000	0-8- 40	0	-1050	1050		B
9000	0-6-1543	-2000	+1283	717	939	A
9000	0-6-1930	-2000	+ 840	1160		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 4

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-6-1350	0	0	0	0	A
0	0-8-1078	0	0	0		B
1000	0-6-1240	0	- 110	110	106	A
1000	0-8- 976	0	- 102	102		B
2000	0-6-1138	0	- 212	212	210	A
2000	0-8- 870	0	- 208	208		B
3000	0-6-1042	0	- 308	308	312	A
3000	0-8- 763	0	- 315	315		B
4000	0-6- 940	0	- 410	410	418	A
4000	0-8- 653	0	- 425	425		B
5000	0-6- 837	0	- 513	513	523	A
5000	0-8- 546	0	- 532	532		B
6000	0-6- 733	0	- 617	617	625	A
6000	0-8- 445	0	- 633	633		B
7000	0-6- 630	0	- 720	720	729	A
7000	0-8- 340	0	- 738	738		B
8000	0-6- 525	0	- 825	825	833	A
8000	0-8- 238	0	- 840	840		B
9000	0-6- 421	0	- 929	929	938	A
9000	0-8- 132	0	- 946	946		B

TABLE I (cont.)

CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 5

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-12- 965	0	0	0	0	A
0	0- 8-1625	0	0	0		B
1000	0-12- 850	0	- 115	115	92	A
1000	0- 8-1557	0	- 68	68		B
2000	0-12- 742	0	- 223	223	203	A
2000	0- 8-1443	0	- 182	182		B
3000	0-12- 636	0	- 329	329	308	A
3000	0- 8-1338	0	- 287	287		B
4000	0-12- 533	0	- 432	432	409	A
4000	0- 8-1240	0	- 385	385		B
5000	0-12- 427	0	- 538	538	513	A
5000	0- 8-1137	0	- 488	488		B
6000	0-12- 320	0	- 645	645	614	A
6000	0- 8-1042	0	- 583	583		B
7000	0-12- 208	0	- 757	757	721	A
7000	0- 8- 940	0	- 685	685		B
8000	0-12- 100	0	- 865	865	821	A
8000	0- 8- 848	0	- 777	777		B
9000	0-10-1990	-2000	+1025	975	929	A
9000	0- 8- 743	0	- 882	882		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 6

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0-12-1419	0	0	0	0	A
0	0- 6- 48	0	0	0	0	B
1000	0-12-1357	0	- 62	62	120	A
1000	0- 4-1870	-2000	+1822	178		B
2000	0-12-1236	0	- 183	183	231	A
2000	0- 4-1770	-2000	+1722	278		B
3000	0-12-1108	0	- 311	311	350	A
3000	0- 4-1660	-2000	+1612	388		B
4000	0-12- 990	0	- 429	429	459	A
4000	0- 4-1560	-2000	+1512	488		B
5000	0-12- 865	0	- 554	554	574	A
5000	0- 4-1455	-2000	+1407	593		B
6000	0-12- 742	0	- 677	677	686	A
6000	0- 4-1353	-2000	+1305	695		B
7000	0-12- 615	0	- 804	804	802	A
7000	0- 4-1249	-2000	+1201	799		B
8000	0-12- 494	0	- 925	925	916	A
8000	0- 4-1142	-2000	+1094	906		B
9000	0-12- 366	0	-1053	1053	1034	A
9000	0- 4-1034	-2000	+ 986	1014		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 7

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Avg. strain	Gage
0	0- 8- 260	0	0	0	0	A
0	0-10-1850	0	0	0		B
1000	0- 8- 156	0	- 104	104	111	A
1000	0-10-1733	0	- 117	117		B
2000	0- 8- 51	0	- 209	209	216	A
2000	0-10-1628	0	- 222	222		B
3000	0- 6-1949	-2000	+1689	311	322	A
3000	0-10-1518	0	- 332	332		B
4000	0- 6-1840	-2000	+1580	420	431	A
4000	0-10-1408	0	- 442	442		B
5000	0- 6-1735	-2000	+1475	525	538	A
5000	0-10-1300	0	- 550	550		B
6000	0- 6-1628	-2000	+1368	632	646	A
6000	0-10-1191	0	- 659	659		B
7000	0- 6-1523	-2000	+1263	737	752	A
7000	0-10-1083	0	- 767	767		B
8000	0- 6-1415	-2000	+1155	845	859	A
8000	0-10- 978	0	- 872	872		B
9000	0- 6-1308	-2000	+1048	952	964	A
9000	0-10- 875	0	- 975	975		B

TABLE I (cont.) CALIBRATION DATA FOR SR4 STRAIN GAGES MOUNTED ON PRESTRESSING BARS

Bar No. 8

Load lb.	Reading	T_2-T_1(1000)	D_2-D_1	Strain in micro in./in.	Ave. strain	Gage
0	0- 6-1450	0	0	0	0	A
0	0- 8- 344	0	0	0		B
1000	0- 6-1136	0	- 314	314	122	A
1000	0- 8- 413	0	+ 69	- 69		B
2000	0- 6- 998	0	- 452	452	228	A
2000	0- 8- 339	0	- 5	5		B
3000	0- 6- 880	0	- 570	570	332	A
3000	0- 8- 250	0	- 94	94		B
4000	0- 6- 747	0	- 703	703	440	A
4000	0- 8- 168	0	- 176	176		B
5000	0- 6- 616	0	- 834	834	557	A
5000	0- 8- 65	0	- 279	279		B
6000	0- 6- 499	0	- 951	951	660	A
6000	0- 6-1975	-2000	+1631	369		B
7000	0- 6- 383	0	-1067	1067	771	A
7000	0- 6-1870	-2000	+1526	474		B
8000	0- 6- 274	0	-1176	1176	879	A
8000	0- 6-1763	-2000	+1419	581		B
9000	0- 6- 170	0	-1280	1280	981	A
9000	0- 6-1662	-2000	+1318	682		B

TABLE II LOAD STRAIN RELATIONSHIP FOR CALIBRATION OF SR4 STRAIN GAGES ON PRESTRESSING BARS LEAST SQUARES METHOD

Bar No. 1

Load	Ave. Strain	Load^2	(Ave. Strain)(Load)
1000	108	1,000,000	108,000
2000	217	4,000,000	434,000
3000	317	9,000,000	951,000
4000	423	16,000,000	1,692,000
5000	528	25,000,000	2,640,000
6000	633	36,000,000	3,798,000
7000	741	49,000,000	5,187,000
8000	846	64,000,000	6,768,000
9000	950	81,000,000	8,550,000
45000	4763	285,000,000	30,128,000

$$a = \frac{(9)(30,128,000) - (45,000)(4763)}{(9)(285,000,000) - (45,000)(45,000)} = 0.105$$

$$b = \frac{(285,000,000)(4763) - (45,000)(30,128,000)}{(9)(285,000,000) - (45,000)(45,000)} = 3.14$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.105 Load + 3.14

Bar No. 2

Load	Ave. Strain	Load^2	(Ave. Strain)(Load)
1000	95	1,000,000	95,000
2000	198	4,000,000	396,000
3000	300	9,000,000	900,000
4000	403	16,000,000	1,612,000
5000	507	25,000,000	2,535,000
6000	613	36,000,000	3,678,000
7000	723	49,000,000	5,061,000
8000	825	64,000,000	6,600,000
9000	932	81,000,000	8,388,000
45000	4596	285,000,000	29,265,000

$$a = \frac{(9)(29,265,000) - (45,000)(4596)}{(9)(285,000,000) - (45,000)(45,000)} = 0.105$$

$$b = \frac{(285,000,000)(4596) - (45,000)(29,265,000)}{(9)(285,000,000) - (45,000)(45,000)} = -13.1$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.105 Load - 13.1

TABLE II (cont.) LOAD STRAIN RELATIONSHIP FOR CALIBRATION OF SR4 STRAIN GAGES ON PRESTRESSING BARS
LEAST SQUARES METHOD

Load	Ave. Strain	Load²	(Ave. Strain)(Load)
		Bar No. 3	
1000	100	1,000,000	100,000
2000	204	4,000,000	408,000
3000	306	9,000,000	918,000
4000	415	16,000,000	1,660,000
5000	518	25,000,000	2,590,000
6000	625	36,000,000	3,750,000
7000	729	49,000,000	5,103,000
8000	835	64,000,000	6,680,000
9000	939	81,000,000	8,451,000
45000	4671	285,000,000	29,660,000

$$a = \frac{(9)(29,660,000) - (45,000)(4671)}{(9)(285,000,000) - (45,000)(45,000)} = 0.105$$

$$b = \frac{(285,000,000)(4671) - (45,000)(29,660,000)}{(9)(285,000,000) - (45,000)(45,000)} = -6.42$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.105 Load − 6.42

Load	Ave. Strain	Load²	(Ave. Strain)(Load)
		Bar No. 4	
1000	106	1,000,000	106,000
2000	210	4,000,000	420,000
3000	312	9,000,000	936,000
4000	418	16,000,000	1,672,000
5000	523	25,000,000	2,615,000
6000	625	36,000,000	3,750,000
7000	729	49,000,000	5,103,000
8000	833	64,000,000	6,664,000
9000	938	81,000,000	8,442,000
45000	4694	285,000,000	29,708,000

$$a = \frac{(9)(29,708,000) - (45,000)(4694)}{(9)(285,000,000) - (45,000)(45,000)} = 0.104$$

$$b = \frac{(285,000,000)(4694) - (45,000)(29,708,000)}{(9)(285,000,000) - (45,000)(45,000)} = 1.72$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.104 Load + 1.72

TABLE II (cont.) LOAD STRAIN RELATIONSHIP FOR CALIBRATION OF SR4 STRAIN GAGES ON PRESTRESSING BARS LEAST SQUARES METHOD

Bar No. 5

Load	Ave. Strain	$Load^2$	(Ave. Strain)(Load)
1000	92	1,000,000	92,000
2000	203	4,000,000	406,000
3000	308	9,000,000	924,000
4000	409	16,000,000	1,636,000
5000	513	25,000,000	2,565,000
6000	614	36,000,000	3,684,000
7000	721	49,000,000	5,047,000
8000	821	64,000,000	6,568,000
9000	929	81,000,000	8,361,000
45000	4610	285,000,000	29,283,000

$$a = \frac{(9)(29,283,000) - (45,000)(4610)}{(9)(285,000,000) - (45,000)(45,000)} = 0.104$$

$$b = \frac{(285,000,000)(4610) - (45,000)(29,283,000)}{(9)(285,000,000) - (45,000)(45,000)} = -7.19$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.104 Load - 7.19

Bar No. 6

Load	Ave. Strain	$Load^2$	(Ave. Strain)(Load)
1000	120	1,000,000	120,000
2000	231	4,000,000	462,000
3000	350	9,000,000	1,050,000
4000	459	16,000,000	1,836,000
5000	574	25,000,000	2,870,000
6000	686	36,000,000	4,116,000
7000	802	49,000,000	5,614,000
8000	916	64,000,000	7,328,000
9000	1034	81,000,000	9,306,000
45000	5172	285,000,000	32,702,000

$$a = \frac{(9)(32,702,000) - (45,000)(5172)}{(9)(285,000,000) - (45,000)(45,000)} = 0.114$$

$$b = \frac{(285,000,000)(5172) - (45,000)(32,702,000)}{(9)(285,000,000) - (45,000)(45,000)} = 4.50$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.114 Load + 4.50

TABLE II (cont.) LOAD STRAIN RELATIONSHIP FOR CALIBRATION OF SR4 STRAIN GAGES ON PRESTRESSING BARS LEAST SQUARES METHOD

Bar No. 7

Load	Ave. Strain	Load^2	(Ave. Strain)(Load)
1000	111	1,000,000	111,000
2000	216	4,000,000	432,000
3000	322	9,000,000	966,000
4000	431	16,000,000	1,724,000
5000	538	25,000,000	2,690,000
6000	646	36,000,000	3,876,000
7000	752	49,000,000	5,264,000
8000	859	64,000,000	6,872,000
9000	964	81,000,000	8,676,000
45000	4839	285,000,000	30,611,000

$$a = \frac{(9)(30,611,000) - (45,000)(4839)}{(9)(285,000,000) - (45,000)(45,000)} = 0.107$$

$$b = \frac{(285,000,000)(4839) - (45,000)(30,611,000)}{(9)(285,000,000) - (45,000)(45,000)} = 3.00$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.107 Load + 3.00

Bar No. 8

Load	Ave. Strain	Load^2	(Ave. Strain)(Load)
1000	122	1,000,000	122,000
2000	228	4,000,000	456,000
3000	332	9,000,000	996,000
4000	440	16,000,000	1,760,000
5000	557	25,000,000	2,785,000
6000	660	36,000,000	3,960,000
7000	771	49,000,000	5,397,000
8000	879	64,000,000	7,032,000
9000	981	81,000,000	8,829,000
45000	4970	285,000,000	31,337,000

$$a = \frac{(9)(31,337,000) - (45,000)(4970)}{(9)(285,000,000) - (45,000)(45,000)} = 0.108$$

$$b = \frac{(285,000,000)(4970) - (45,000)(31,337,000)}{(9)(285,000,000) - (45,000)(45,000)} = 11.6$$

Ave. Strain = (a) Load + b

Ave. Strain = 0.108 Load + 11.6

TABLE III DATA FOR MODULUS OF ELASTICITY OF CONCRETE IN TEST BEAMS

Test Cylinder No. 1

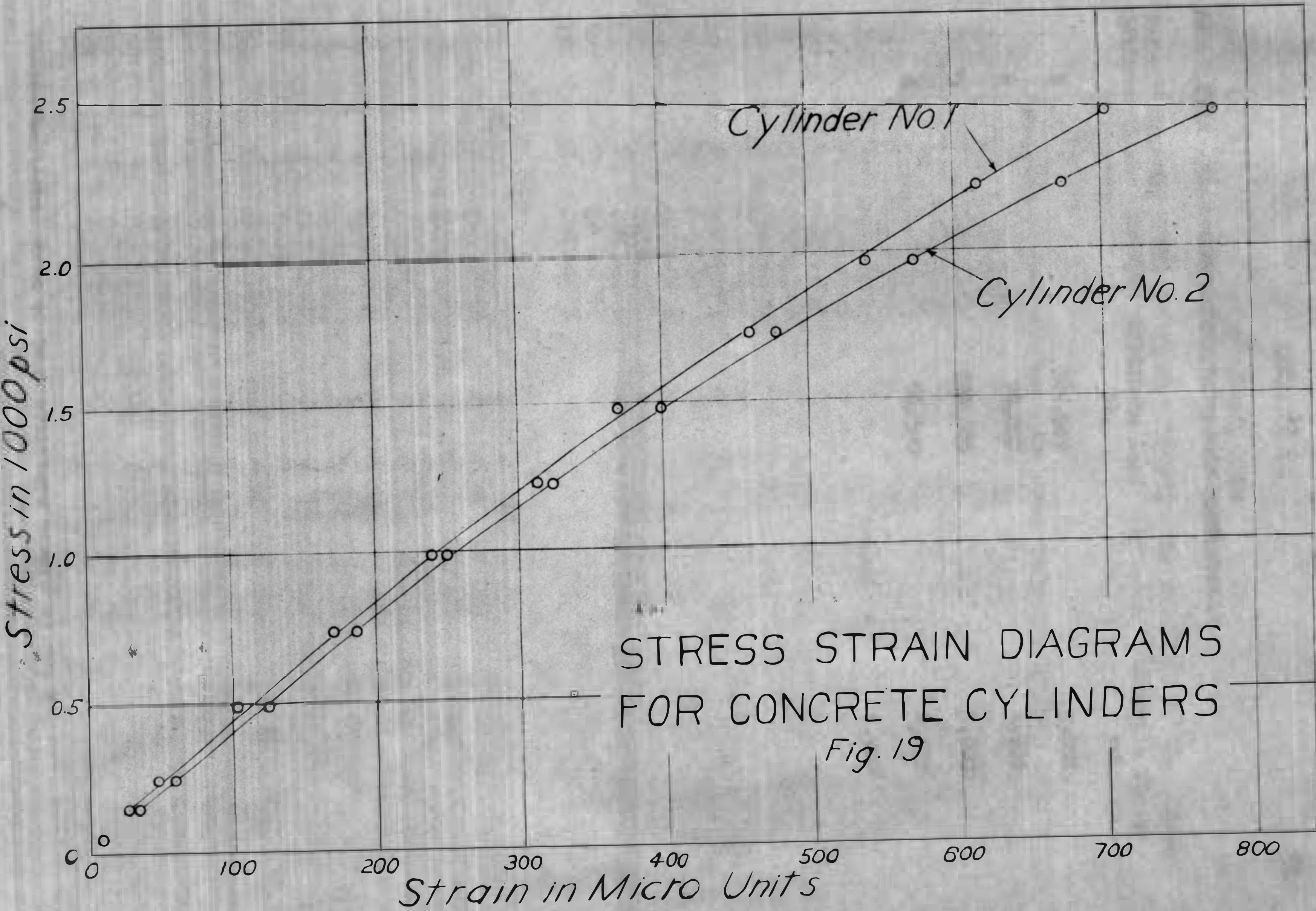
Gage No.	Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Stress psi
1	0	0-10-1829	0	0	0	0	0
2	0	0-10-1142	0	0	0		
1	1,000	0-10-1812	0	- 17	17	9	49
2	1,000	0-10-1142	0	- 0	0		
1	3,000	0-10-1787	0	- 42	42	27	148
2	3,000	0-10-1131	0	- 11	11		
1	5,000	0-10-1760	0	- 69	69	49	246
2	5,000	0-10-1114	0	- 28	28		
1	10,000	0-10-1698	0	-131	131	104	493
2	10,000	0-10-1065	0	- 77	77		
1	15,000	0-10-1623	0	-206	206	172	739
2	15,000	0-10-1005	0	-137	137		
1	20,000	0-10-1555	0	-274	274	240	985
2	20,000	0-10- 937	0	-205	205		
1	25,000	0-10-1481	0	-348	348	314	1232
2	25,000	0-10- 865	0	-279	279		
1	30,000	0-10-1412	0	-417	417	389	1478
2	30,000	0-10- 782	0	-360	360		
1	35,000	0-10-1342	0	-487	487	460	1724
2	35,000	0-10- 710	0	-432	432		
1	40,000	0-10-1269	0	-560	560	540	1970
2	40,000	0-10- 623	0	-519	519		
1	45,000	0-10-1189	0	-640	640	616	2217
2	45,000	0-10- 551	0	-591	591		
1	50,000	0-10-1109	0	-720	720	704	2463
2	50,000	0-10- 455	0	-687	687		
1	60,000	0-10- 940	0	-889	889	865	2956
2	60,000	0-10- 300	0	-842	842		

TABLE III (cont.)

DATA FOR MODULUS OF ELASTICITY OF CONCRETE IN TEST BEAMS

Test Cylinder No. 2

Gage No.	Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Stress psi
1	0	0-10- 612	0	0	0	0	0
2	0	0-10-1040	0	0	0		
1	1,000	0-10- 597	0	- 15	15	10	49
2	1,000	0-10-1036	0	- 4	4		
1	3,000	0-10- 549	0	- 63	63	35	148
2	3,000	0-10-1033	0	- 7	7		
1	5,000	0-10- 540	0	- 108	108	60	246
2	5,000	0-10-1029	0	- 11	11		
1	10,000	0-10- 421	0	- 191	191	126	493
2	10,000	0-10- 990	0	- 50	50		
1	15,000	0-10- 346	0	- 266	266	187	739
2	15,000	0-10- 932	0	- 108	108		
1	20,000	0-10- 270	0	- 342	342	251	985
2	20,000	0-10- 880	0	- 160	160		
1	25,000	0-10- 190	0	- 422	422	325	1232
2	25,000	0-10- 813	0	- 227	227		
1	30,000	0-10- 102	0	- 510	510	399	1478
2	30,000	0-10- 752	0	- 288	288		
1	35,000	0-10- 15	0	- 597	597	479	1724
2	35,000	0-10- 680	0	- 360	360		
1	40,000	0- 8-1900	-2000	+1288	712	572	1970
2	40,000	0-10- 609	0	- 431	431		
1	45,000	0- 8-1789	-2000	+1177	823	674	2217
2	45,000	0-10- 515	0	- 525	525		
1	50,000	0- 8-1646	-2000	+1034	966	777	2463
2	50,000	0-10- 452	0	- 588	588		
1	60,000	0- 8-1430	-2000	+ 818	1192	961	2956
2	60,000	0-10- 310	0	- 730	730		



STRESS STRAIN DIAGRAMS
FOR CONCRETE CYLINDERS
Fig. 19

TABLE IIIA ULTIMATE STRENGTH OF CONCRETE*

Cylinder No.	Ultimate Load lb.		Ultimate Stress psi
3	101,000		5,050
4	94,000		4,700
5	100,000		5,000
6	96,500		4,830
		Average =	4,900

*Cross-sectional area of test cylinders equals 20 sq. in.

TABLE IV

PRESTRESSING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
Tensioning of bars started 2:00 P.M., March 24, 1952							
1	0	0-10-1100					
2	"	0-10-1120					
3	"	0-10-1540					
4	"	0-12- 180					
5	"	0-10-1273					
6	"	0-10-1670					
7	"	0-12- 188					
8	"	0-10- 899					
9	"	0-10-1943					
10	"	0-12- 194					
11	"	0-14-1049					
12	"	0-12-1282					
13	"	0-14-0126					
14	"	0-10-0553					
15	"	0-14-0963					
16	"	0-14-0049					
17	"	0- 8-1351					
18	"	0-12-0853					
1	4000	0-10-1070	0	- 30	- 30	- 35	
2	"	0-10-1080	0	- 40	- 40		
3	"	0-10-1505	0	- 35	- 35	- 35	
4	"	0-12- 145	0	- 35	- 35		
5	"	0-10-1190	0	- 83	- 83	- 57	
6	"	0-10-1640	0	- 30	- 30		
7	"	0-12- 168	0	- 20	- 20	- 16	
8	"	0-10- 887	0	- 12	- 12		
9	"	0-10-1930	0	- 13	- 13	- 11	
10	"	0-12- 185	0	- 9	- 9		
11	"	0-14-1248	0	+ 199	+ 199	+ 104	980
12	"	0-12-1290	0	+ 8	+ 8		
13	"	0-14-0425	0	+ 299	+ 299	+ 104	940
14	"	0-10- 462	0	- 91	- 91		
15	"	0-14-1378	0	+ 415	+ 415	+ 118	980
16	"	0-12-1870	-2000	+1821	- 179		
17	"	0- 8-1260	0	- 91	- 91	+ 89	920
18	"	0-12-1122	0	+ 269	+ 269		
					Total prestress =		3820

TABLE IV

PRESTRESSING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	8000	0-10-1038	0	- 62	- 62	- 72	
2	"	0-10-1039	0	- 81	- 81		
3	"	0-10-1460	0	- 80	- 80	- 80	
4	"	0-12- 100	0	- 80	- 80		
5	"	0-10-1075	0	- 198	- 198	- 138	
6	"	0-10-1592	0	- 78	- 78		
7	"	0-12- 132	0	- 56	- 56	- 45	
8	"	0-10- 865	0	- 34	- 34		
9	"	0-10-1892	0	- 51	- 51	- 42	
10	"	0-12- 162	0	- 32	- 32		
11	"	0-14-1270	0	+ 221	+ 221	+ 165	1570
12	"	0-12-1390	0	+ 108	+ 108		
13	"	0-14- 595	0	+ 469	+ 469	+ 210	1950
14	"	0-10- 505	0	- 48	- 48		
15	"	0-14-1462	0	+ 499	+ 499	+ 220	1930
16	"	0-12-1990	-2000	+1941	- 59		
17	"	0- 8-1368	0	+ 17	+ 17	+ 197	1960
18	"	0-12-1229	0	+ 376	+ 376		
					Total prestress =		7410

Tensioning of bars stopped at 5:00 P.M., March 24 and continued at 8:30 A.M., March 25, 1952.

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	8000	0-10-1000	0	- 100	- 100	- 111	
2	"	0-10- 998	0	- 122	- 122		
3	"	0-10-1420	0	- 120	- 120	- 107	
4	"	0-12-0087	0	- 93	- 93		
5	"	0-10-1032	0	- 241	- 241	- 176	
6	"	0-10-1560	0	- 110	- 110		
7	"	0-12- 80	0	- 108	- 108	- 84	
8	"	0-10- 840	0	- 59	- 59		
9	"	0-10-1850	0	- 93	- 93	- 85	
10	"	0-12- 118	0	- 76	- 76		
11	"	0-14-1264	0	+ 215	+ 215	+ 158	1500
12	"	0-12-1382	0	+ 100	+ 100		
13	"	0-14- 589	0	+ 463	+ 463	+ 203	1870
14	"	0-10- 497	0	- 56	- 56		
15	"	0-14-1455	0	+ 492	+ 492	+ 214	1870
16	"	0-12-1985	-2000	+1936	- 64		
17	"	0- 8-1358	0	+ 7	+ 7	+ 189	1890
18	"	0-12-1225	0	+ 370	+ 370		
					Total prestress =		7130

TABLE IV (cont.)

PRESTRESSING DATA

BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	12,000	0-10- 968	0	- 132	- 132	- 151	
2	"	0-10- 950	0	- 170	- 170		
3	"	0-10-1375	0	- 165	- 165	- 158	
4	"	0-12- 50	0	- 150	- 150		
5	"	0-10- 950	0	- 323	- 323	- 242	
6	"	0-10-1510	0	- 160	- 160		
7	"	0-12- 48	0	- 140	- 140	- 111	
8	"	0-10- 818	0	- 81	- 81		
9	"	0-10-1818	0	- 125	- 125	- 113	
10	"	0-12- 93	0	- 101	- 101		
11	"	0-14-1410	0	+ 361	+ 361	+ 302	2900
12	"	0-12-1525	0	+ 243	+ 243		
13	"	0-14- 688	0	+ 562	+ 562	+ 324	3000
14	"	0-10- 638	0	+ 85	+ 85		
15	"	0-14-1650	0	+ 687	+ 687	+ 326	2910
16	"	0-14- 15	0	- 34	- 34		
17	"	0- 8-1501	0	+ 150	+ 150	+ 297	2920
18	"	0-12-1296	0	+ 443	+ 443		
					Total prestress	=	11730
1	16,000	0-10- 932	0	- 168	- 168	- 194	
2	"	0-10- 900	0	- 220	- 220		
3	"	0-10-1330	0	- 210	- 210	- 209	
4	"	0-10-1972	-2000	+1792	- 208		
5	"	0-10- 872	0	- 401	- 401	- 307	
6	"	0-10-1458	0	- 212	- 212		
7	"	0-12- 10	0	- 178	- 178	- 144	
8	"	0-10- 790	0	- 109	- 109		
9	"	0-10-1780	0	- 163	- 163	- 144	
10	"	0-12- 70	0	- 124	- 124		
11	"	0-14-1468	0	+ 419	+ 419	- 414	3980
12	"	0-12-1690	0	+ 408	+ 408		
13	"	0-14- 685	0	+ 559	+ 559	+ 424	3940
14	"	0-10- 841	0	+ 288	+ 288		
15	"	0-14-1880	0	+ 917	+ 917	+ 432	3890
16	"	0-12-1997	-2000	+1948	- 52		
17	"	0- 8-1648	0	+ 297	+ 297	+ 403	3940
18	"	0-12-1362	0	+ 509	+ 509		
					Total prestress	=	15830

TABLE IV (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	20,000	0-10- 890	0	- 210	- 210	- 243	
2	"	0-10- 845	0	- 275	- 275		
3	"	0-10-1270	0	- 270	- 270	- 279	
4	"	0-10-1893	-2000	+1713	- 287		
5	"	0-10- 795	0	- 478	- 478	- 379	
6	"	0-10-1390	0	- 280	- 280		
7	"	0-10-1968	-2000	+1780	- 220	- 182	
8	"	0-10- 755	0	- 144	- 144		
9	"	0-10-1735	0	- 208	- 208	- 181	
10	"	0-12- 40	0	- 154	- 154		
11	"	0-14-1620	0	+ 571	+ 571	+ 516	4940
12	"	0-12-1742	0	+ 460	+ 460		
13	"	0-14- 755	0	+ 629	+ 629	+ 531	4940
14	"	0-10- 985	0	+ 432	+ 432		
15	"	0-14-1980	0	+1017	+1017	+ 544	4920
16	"	0-14- 120	0	+ 71	+ 71		
17	"	0- 8-1763	0	+ 412	+ 412	+ 520	5070
18	"	0-12-1480	0	+ 627	+ 627		

Total prestress = 19870

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	24,000	0-10- 840	0	- 260	- 260	- 301	
2	"	0-10- 778	0	- 342	- 342		
3	"	0-10-1196	0	- 344	- 344	- 366	
4	"	0-10-1792	-2000	+1612	- 388		
5	"	0-10- 702	0	- 571	- 571	- 466	
6	"	0-10-1310	0	- 360	- 360		
7	"	0-10-1900	-2000	+1712	- 288	- 240	
8	"	0-10- 708	0	- 191	- 191		
9	"	0-10-1672	0	- 271	- 271	- 233	
10	"	0-12-0000	0	- 194	- 194		
11	"	0-14-1803	0	+ 754	+ 754	+ 625	5990
12	"	0-12-1778	0	+ 496	+ 496		
13	"	0-14- 980	0	+ 854	+ 854	+ 640	5950
14	"	0-10- 978	0	+ 425	+ 425		
15	"	0-14-1978	0	+1015	+1015	+ 656	5960
16	"	0-14- 345	0	+ 296	+ 296		
17	"	0- 8-1802	0	+ 451	+ 451	+ 625	6080
18	"	0-12-1652	0	+ 799	+ 799		

Total Prestress = 23980

APPENDIX B

TABLE IV (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	28,000	0-10- 800	0	- 300	- 300	- 350	
2	"	0-10- 720	0	- 400	- 400		
3	"	0-10-1142	0	- 398	- 398	- 434	
4	"	0-10-1710	-2000	+1530	- 470		
5	"	0-10- 645	0	- 628	- 628	- 523	
6	"	0-10-1252	0	- 418	- 418		
7	"	0-10-1856	-2000	+1668	- 332	- 278	
8	"	0-10- 675	0	- 224	- 224		
9	"	0-10-1638	0	- 305	- 305	- 230	
10	"	0-10-1970	-2000	+1776	- 224		
11	"	0-14-1793	0	+ 744	+ 744	+ 730	7020
12	"	0-12-1998	0	+ 716	+ 716		
13	"	0-14-1132	0	+1006	+1006	+ 747	6950
14	"	0-10-1040	0	+ 487	+ 487		
15	"	0-16- 120	+2000	- 843	+1157	+ 774	7050
16	"	0-14- 440	0	+ 391	+ 391		
17	"	0- 8-1965	0	+ 614	+ 614	+ 730	7090
18	"	0-12-1698	0	+ 845	+ 845		
					Total Prestress =		28110
1	32,000	0-10- 765	0	- 335	- 335	- 397	
2	"	0-10- 661	0	- 459	- 459		
3	"	0-10-1099	0	- 451	- 451	- 502	
4	"	0-10-1627	-2000	+1447	- 553		
5	"	0-10- 598	0	- 675	- 675	- 577	
6	"	0-10-1192	0	- 478	- 478		
7	"	0-10-1816	-2000	+1628	- 372	- 316	
8	"	0-10- 640	0	- 259	- 259		
9	"	0-10-1602	0	- 341	- 341	- 301	
10	"	0-10-1934	-2000	+1740	- 260		
11	"	0-14-1900	0	+ 851	+ 851	+ 825	7930
12	"	0-14- 81	+2000	-1201	+ 799		
13	"	0-14-1260	0	+1134	+1134	+ 855	7960
14	"	0-10-1138	0	+ 585	+ 585		
15	"	0-16- 338	+2000	- 625	+1375	+ 880	8040
16	"	0-14- 433	0	+ 384	+ 384		
17	"	0-10- 90	+2000	-1261	+ 739	+ 819	7950
18	"	0-12-1751	0	+ 898	+ 898		
					Total Prestress =		31880

Tensioning of bars stopped 12:00 noon, March 25 and continued 1:15 P.M., March 25.

TABLE IV (cont.)

PRESTRESSING DATA

BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	$T_2 - T_1$ x(1000)	$D_2 - D_1$	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	32,000	0-10- 757	0	- 343	- 343	- 407	
2	"	0-10- 650	0	- 470	- 470		
3	"	0-10-1089	0	- 451	- 451	- 514	
4	"	0-10-1604	-2000	+1424	- 576		
5	"	0-10- 583	0	- 690	- 690	- 590	
6	"	0-10-1180	0	- 490	- 490		
7	"	0-10-1805	-2000	+1617	- 383	- 323	
8	"	0-10- 636	0	- 263	- 263		
9	"	0-10-1597	0	- 346	- 346	- 307	
10	"	0-10-1927	-2000	+1733	- 267		
11	"	0-14-1895	0	+ 846	+ 846	+ 822	7900
12	"	0-14- 80	+2000	-1202	+ 798		
13	"	0-14-1255	0	+1129	+1129	+ 855	7960
14	"	0-10-1134	0	+ 581	+ 581		
15	"	0-16- 330	+2000	- 633	+1367	+ 874	7980
16	"	0-14- 430	0	+ 381	+ 381		
17	"	0-10- 86	+2000	-1265	+ 735	+ 816	7920
18	"	0-12-1750	0	+ 897	+ 897		
					Total Prestress		31760
1	36,000	0-10- 720	0	- 380	- 380	- 450	
2	"	0-10- 600	0	- 520	- 520		
3	"	0-10-1045	0	- 495	- 495	- 565	
4	"	0-10-1545	-2000	+1365	- 635		
5	"	0-10- 540	0	- 733	- 733	- 633	
6	"	0-10-1138	0	- 532	- 532		
7	"	0-10-1768	-2000	+1580	- 420	- 355	
8	"	0-10- 610	0	- 289	- 289		
9	"	0-10-1562	0	- 381	- 381	- 339	
10	"	0-10-1897	-2000	+1703	- 297		
11	"	0-16- 8	+2000	-1041	+ 959	+ 926	8910
12	"	0-14- 175	+2000	-1107	+ 893		
13	"	0-14-1248	0	+1122	+1122	+ 957	8910
14	"	0-10-1345	0	+ 792	+ 792		
15	"	0-16- 300	+2000	- 573	+1427	+ 985	9000
16	"	0-14- 592	0	+ 543	+ 543		
17	"	0-10- 217	+2000	-1134	+ 866	+ 933	9040
18	"	0-12-1852	0	+ 999	+ 999		
					Total Prestress =		35860

TABLE IV (cont.) PRESTRESSING DATA

BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	40,000	0-10- 678	0	- 422	- 422	- 502	
2	"	0-10- 538	0	- 582	- 582		
3	"	0-10- 985	0	- 555	- 555	- 648	
4	"	0-10-1440	-2000	+1260	- 740		
5	"	0-10- 480	0	- 795	- 795	- 697	
6	"	0-10-1070	0	- 600	- 600		
7	"	0-10-1710	-2000	+1522	- 478	- 406	
8	"	0-10- 566	0	- 335	- 335		
9	"	0-10-1518	0	- 425	- 425	- 386	
10	"	0-10-1848	-2000	+1654	- 346		
11	"	0-16- 148	+2000	- 901	+1099	+1035	9940
12	"	0-16- 252	+2000	-1030	+ 970		
13	"	0-14-1380	0	+1254	+1254	+1066	9930
14	"	0-10-1430	0	+ 877	+ 877		
15	"	0-16- 305	+2000	- 658	+1342	+1088	9960
16	"	0-14- 882	0	+ 835	+ 835		
17	"	0-10- 350	+2000	-1021	+ 979	+1024	9860
18	"	0-12-1922	0	+1069	+1069		

Total Prestress = 39690

Tensioning of bars completed 2:30 P.M., March 25

TABLE V

PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
	Tensioning of bars started at 9:00 A.M., March 27, 1952						
1	0	0-12-0080					
2	"	0-10-0874					
3	"	0-10-1425					
4	"	0-10-1558					
5	"	0-10-1350					
6	"	0-10-0650					
7	"	0-10-1455					
8	"	0-10-1190					
9	"	0-10-1642					
11	"	0-12-1390					
12	"	0-14-0608					
13	"	0-14-0345					
14	"	0-12-1115					
15	"	0- 8-1000					
16	"	0-16-0188					
17	"	0- 8-1940					
18	"	0-14-0612					
1	4,000	0-12-0052	0	- 28	- 28	- 28	
2	"	0-10-0832	0	- 42	- 42	- 31	
3	"	0-10-0832	0	- 20	- 20		
4	"	0-10-1545	0	- 13	- 13	- 22	
5	"	0-10-1320	0	- 30	- 30		
6	"	0-10-0635	0	- 15	- 15	- 11	
7	"	0-10-1448	0	- 7	- 7		
8	"	0-10-1242	0	+ 52	+ 52	+ 28	
9	"	0-10-1645	0	+ 3	+ 3		
11	"	0-12-1710	0	+ 320	+ 320	+ 111	1030
12	"	0-14-0510	0	- 98	- 98		
13	"	0-14-0590	0	+ 245	+ 245	+ 110	1100
14	"	0-12-1090	0	- 25	- 25		
15	"	0- 8-0788	0	- 212	- 212	+ 121	1020
16	"	0-16-0643	0	+ 455	+ 455		
17	"	0- 8-1992	0	+ 52	+ 52	+100	1080
18	"	0-14-0760	0	+ 148	+ 148		

Total Prestress = 4230 lb.

TABLE V (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	8,000	0-12-0032	0	- 48	- 48	- 48	
2	"	0-10-0795	0	- 79	- 79	- 62	
3	"	0-10-1380	0	- 45	- 45		
4	"	0-10-1513	0	- 25	- 25	- 37	
5	"	0-10-1300	0	- 50	- 50		
6	"	0-10-0630	0	- 20	- 20	- 16	
7	"	0-10-1443	0	- 12	- 12		
8	"	0-10-1192	0	+ 2	+ 2	+ 3	
9	"	0-10-1645	0	+ 3	+ 3		
11	"	0-12-1832	0	+ 442	+ 442	+ 207	1940
12	"	0-14-0580	0	- 28	- 28		
13	"	0-14-0692	0	+ 347	+ 347	+ 214	2090
14	"	0-12-1195	0	+ 80	+ 80		
15	"	0- 8-0865	0	- 135	- 135	+ 221	1900
16	"	0-16-0766	0	+ 578	+ 578		
17	"	0-10-0132	+2000	-1808	+ 192	+ 195	1980
18	"	0-14-0810	0	+ 198	+ 198		
					Total Prestress =		7910
1	12,000	0-12-0012	0	- 68	- 68	- 68	
2	"	0-10-0762	0	- 112	- 112	- 86	
3	"	0-10-1365	0	- 60	- 60		
4	"	0-10-1528	0	- 30	- 30	- 48	
5	"	0-10-1284	0	- 66	- 66		
6	"	0-10-0622	0	- 28	- 28	- 23	
7	"	0-10-1458	0	- 17	- 17		
8	"	0-10-1192	0	+ 2	+ 2	+ 2	
9	"	0-10-1643	0	+ 1	+ 1		
11	"	0-12-1947	0	+ 557	+ 557	+ 316	2980
12	"	0-14-0683	0	+ 75	+ 75		
13	"	0-14-0840	0	+ 495	+ 495	+ 364	3520
14	"	0-12-1348	0	+ 233	+ 233		
15	"	0- 8-0888	0	- 112	- 112	+ 344	2980
16	"	0-16-0988	0	+ 800	+ 800		
17	"	0-10-0265	+2000	-1675	+ 325	+ 299	3000
18	"	0-14-0885	0	+ 273	+ 273		
					Total Prestress =		12480

TABLE V (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	16,000	0-10-1985	-2000	+1905	- 95	- 95	
2	"	0-10-0715	0	- 159	- 159	- 123	
3	"	0-10-1338	0	- 87	- 87		
4	"	0-10-1510	0	- 48	- 48	- 79	
5	"	0-10-1240	0	- 110	- 110		
6	"	0-10-0600	0	- 50	- 50	- 40	
7	"	0-10-1426	0	- 29	- 29		
8	"	0-10-1184	0	- 6	- 6	- 5	
9	"	0-10-1638	0	- 4	- 4		
11	"	0-14-0055	+2000	-1335	+ 665	+ 419	3970
12	"	0-14-0780	0	+ 172	+ 172		
13	"	0-14-0860	0	+ 515	+ 515	+ 412	3980
14	"	0-12-1424	0	+ 309	+ 309		
15	"	0- 8-0965	0	- 35	- 35	+ 452	3920
16	"	0-16-1128	0	+ 940	+ 940		
17	"	0-10-0385	+2000	-1555	+ 445	+ 411	4040
18	"	0-14-0988	0	+ 376	+ 376		

Total Prestress = 15910

Tensioning of bars interrupted at 11:20 A.M. and continued at 1:00 P.M., March 27, 1952.

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	16,000	0-10-1990	-2000	+1910	- 90	- 90	
2	"	0-10-0710	0	- 164	- 164	- 127	
3	"	0-10-1336	0	- 89	- 89		
4	"	0-10-1507	0	- 51	- 51	- 81	
5	"	0-10-1239	0	- 111	- 111		
6	"	0-10-0599	0	- 51	- 51	- 40	
7	"	0-10-1427	0	- 28	- 28		
8	"	0-10-1187	0	- 4	- 4	- 3	
9	"	0-10-1640	0	- 2	- 2		
11	"	0-14-0051	+2000	-1339	+ 661	+ 416	3940
12	"	0-14-0779	0	+ 171	+ 171		
13	"	0-14-0854	0	+ 509	+ 509	+ 408	3940
14	"	0-12-1421	0	+ 306	+ 306		
15	"	0- 8-0961	0	- 39	- 39	+ 446	3870
16	"	0-16-1119	0	+ 931	+ 931		
17	"	0-10-0382	+2000	-1558	+ 442	+ 405	3990
18	"	0-14-0980	0	+ 368	+ 368		

Total Prestress = 15740

TABLE V (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	20,000	0-10-1952	-2000	+1872	- 128	- 128	
2	"	0-10-0654	0	- 220	- 220	- 172	
3	"	0-10-1302	0	- 123	- 123		
4	"	0-10-1480	0	- 78	- 78	- 129	
5	"	0-10-1170	0	- 180	- 180		
6	"	0-10- 550	0	- 100	- 100	- 77	
7	"	0-10-1401	0	- 54	- 54		
8	"	0-10-1164	0	- 26	- 26	- 23	
9	"	0-10-1622	0	- 20	- 20		
11	"	0-14-0150	+2000	-1240	+ 760	+ 517	4900
12	"	0-14-0881	0	+ 273	+ 273		
13	"	0-14-0895	0	+ 550	+ 550	+ 512	4940
14	"	0-12-1589	0	+ 474	+ 474		
15	"	0- 8-1145	0	+ 145	+ 145	+ 568	4940
16	"	0-16-1179	0	+ 991	+ 991		
17	"	0-10-0470	+2000	-1470	+ 530	+ 515	5030
18	"	0-14-1112	0	+ 500	+ 500		
					Total Prestress =		19810
1	24,000	0-10-1918	-2000	+1838	- 162	- 162	
2	"	0-10-0600	0	- 274	- 274	- 215	
3	"	0-10-1270	0	- 155	- 155		
4	"	0-10-1442	0	- 116	- 116	- 193	
5	"	0-10-1080	0	- 270	- 270		
6	"	0-10-0481	0	- 169	- 169	- 127	
7	"	0-10-1370	0	- 85	- 85		
8	"	0-10-1135	0	- 55	- 55	- 52	
9	"	0-10-1593	0	- 49	- 49		
11	"	0-14-0259	+2000	-1131	+ 869	+ 614	5820
12	"	0-14- 967	0	+ 359	+ 359		
13	"	0-14-1020	0	+ 675	+ 675	+ 617	5930
14	"	0-12-1673	0	+ 558	+ 558		
15	"	0- 8-1333	0	+ 333	+ 333	+ 680	5920
16	"	0-16-1215	0	+1027	+1027		
17	"	0-10-0518	+2000	-1422	+ 578	+ 609	5930
18	"	0-14-1252	0	+ 640	+ 640		
					Total Prestress =		23600

TABLE V (cont.) PRESTRESSING DATA

BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	28,000	0-10-1872	-2000	+1792	- 208	- 208	
2	"	0-10-0540	0	- 334	- 334	- 265	
3	"	0-10-1230	0	- 195	- 195		
4	"	0-10-1400	0	- 158	- 158	- 261	
5	"	0-10-0987	0	- 363	- 363		
6	"	0-10-0406	0	- 244	- 244	- 235	
7	"	0-10-1350	0	- 125	- 125		
8	"	0-10-1100	0	- 90	- 90	- 84	
9	"	0-10-1565	0	- 77	- 77		
11	"	0-14-0360	+2000	-1030	+ 970	+ 730	30
12	"	0-14-1098	0	+ 490	+ 490		
13	"	0-14-1222	0	+ 877	+ 877	+ 721	6920
14	"	0-12-1680	0	+ 565	+ 565		
15	"	0- 8-1495	0	+ 495	+ 495	+ 811	7070
16	"	0-16-1315	0	+1127	+1127		
17	"	0-10-0630	+2000	-1310	+ 690	+ 714	6950
18	"	0-14-1349	0	+ 737	+ 737		
					Total Prestress =		27850
1	32,000	0-10-1850	-2000	+1750	- 250	- 250	
2	"	0-10-0485	0	- 391	- 391	- 312	
3	"	0-10-1195	0	- 232	- 232		
4	"	0-10-1359	0	- 199	- 199	- 323	
5	"	0-10-0904	0	- 446	- 446		
6	"	0-10-0335	0	- 315	- 315	- 241	
7	"	0-10-1288	0	- 167	- 167		
8	"	0-10-1063	0	- 127	- 127	- 120	
9	"	0-10-1630	0	- 112	- 112		
11	"	0-14-0467	+2000	- 923	+1077	+ 841	7990
12	"	0-14-1213	0	+ 605	+ 605		
13	"	0-14-1474	0	+1129	+1129	+ 831	7970
14	"	0-12-1647	0	+ 532	+ 532		
15	"	0- 8-1658	0	+ 650	+ 650	+ 901	7860
16	"	0-16-1340	0	+1152	+1152		
17	"	0-10-0789	+2000	-1151	+ 849	+ 829	8020
18	"	0-14-1420	0	+ 808	+ 808		
					Total Prestress =		31840

TABLE V (cont.)

PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	36,000	0-10-1780	-2000	+1700	- 300	- 300	
2	"	0-10-0420	0	- 454	- 454	- 364	
3	"	0-10-1152	0	- 273	- 273		
4	"	0-10-1313	0	- 245	- 245	- 385	
5	"	0-10-0825	0	- 525	- 525		
6	"	0-10-0263	0	- 387	- 387	- 302	
7	"	0-10-1239	0	- 216	- 216		
8	"	0-10-1023	0	- 167	- 167	- 162	
9	"	0-10-1485	0	- 157	- 157		
11	"	0-14- 579	+2000	- 811	+1189	+ 939	8920
12	"	0-14-1296	0	+ 688	+ 688		
13	"	0-14-1440	0	+1095	+1095	+ 937	8980
14	"	0-12-1894	0	+ 779	+ 779		
15	"	0- 8-1940	0	+ 940	+ 940	+1020	8950
16	"	0-16-1288	0	+1100	+1100		
17	"	0-10- 900	+2000	-1040	+ 960	+ 919	8830
18	"	0-14-1489	0	+ 877	+ 877		
					Total Prestress =		35680
1	40,000	0-10-1720	-2000	+1640	- 360	- 360	
2	"	0-10-0344	0	- 530	- 530	- 429	
3	"	0-10-1097	0	- 328	- 328		
4	"	0-10-1259	0	- 299	- 299	- 462	
5	"	0-10-0725	0	- 625	- 625		
6	"	0-10-0172	0	- 478	- 478	- 378	
7	"	0-10-1178	0	- 277	- 277		
8	"	0-10-0974	0	- 216	- 216	- 211	
9	"	0-10-1457	0	- 205	- 205		
11	"	0-14-0690	+2000	- 700	+1300	+1057	10,000
12	"	0-14-1422	0	+ 814	+ 814		
13	"	0-14-1430	0	+1085	+1085	+1043	10,000
14	"	0-14-0115	+2000	-1000	+1000		
15*	"	0-10-0200	+2000	- 800	+1200	+1141	10,000
16*	"	0-16-1270	0	+1082	+1082		
17	"	0-10-1013	+2000	- 927	+1073	+1031	9,900
18	"	0-14-1600	0	+ 988	+ 988		
					Total Prestress =		39,900

Tensioning of bars completed at 5:00 P.M., March 27, 1952.

*Nut on bar with gages 15 & 16 had to be turned two or three complete revolutions in going from 8950 lb. to 10,000 lb.

TABLE V (cont.) PRESTRESSING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Approx. Total Prestress lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
Following data taken at 8:00 A.M., March 28, 1952.							
1	40,000	0-10-1710	-2000	+1630	- 370	- 370	
2	"	0-10-0280	0	- 494	- 494	- 433	
3	"	0-10-1054	0	- 371	- 371		
4	"	0-10-1220	0	- 338	- 338	- 507	
5	"	0-10-0674	0	- 676	- 676		
6	"	0-10-0115	0	- 535	- 535	- 420	
7	"	0-10-1151	0	- 304	- 304		
8	"	0-10- 970	0	- 220	- 220	- 215	
9	"	0-10-1432	0	- 218	- 210		
11	"	0-14-0660	+2000	- 730	+1270	+1033	9820
12	"	0-14-1403	0	+ 795	+ 795		
13	"	0-14-1406	0	+1061	+1061	+1026	9810
14	"	0-14-0105	+2000	-1010	+ 990		
15	"	0-10-0181	+2000	- 819	+1181	+1122	9830
16	"	0-16-1250	0	+1062	+1062		
17	"	0-10- 996	+2000	- 944	+1056	+1005	9720
18	"	0-14-1565	0	+ 953	+ 953		
						Total Prestress =	39180

TABLE VI DEFLECTION DATA FOR PRESTRESSING BEAM WITH PRESTRESSED REINFORCING ONLY

Section	Prestress Force lb.	Reading in.	Defl. in.	Prestress Force lb.	Reading in.	Defl. in.
A	0	7.12		23980	7.06	.06
B	"	10.21		"	10.13	.08
C	"	1.29		"	1.18	.11
D	"	4.13		"	4.04	.09
E	"	4.30		"	4.24	.06
A	3820	7.11	.01	28110	7.05	.07
B	"	10.20	.01	"	10.11	.10
C	"	1.27	.02	"	1.17	.12
D	"	4.12	.01	"	4.05	.10
E	"	4.29	.01	"	4.22	.08
A	7410	7.11	.01	31880	7.03	.09
B	"	10.19	.02	"	10.08	.13
C	"	1.26	.03	"	1.13	.16
D	"	4.11	.02	"	3.98	.15
E	"	4.28	.02	"	4.19	.11
	*			**		
A	7130	7.11	.01	31760	7.02	.10
B	"	10.18	.03	"	10.07	.14
C	"	1.25	.04	"	1.13	.16
D	"	4.10	.03	"	3.97	.16
E	"	4.28	.02	"	4.18	.12
A	11730	7.10	.02	35860	7.03	.09
B	"	10.17	.04	"	10.08	.13
C	"	1.23	.06	"	1.13	.16
D	"	4.08	.05	"	3.98	.15
E	"	4.27	.03	"	4.20	.10
A	15830	7.08	.04	39690	7.01	.11
B	"	10.15	.06	"	10.05	.16
C	"	1.22	.07	"	1.10	.19
D	"	4.07	.06	"	3.96	.17
E	"	4.26	.04	"	4.18	.12
A	19870	7.08	.04			
B	"	10.15	.06			
C	"	1.22	.07			
D	"	4.07	.06			
E	"	4.25	.05			

*15½ hour break in tensioning of bars.
** 1 hour 15 minute break in tensioning of bars.

TABLE VII DEFLECTION DATA FOR PRESTRESSING BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Section	Prestress Force lb.	Reading in.	Defl. in.	Prestress Force lb.	Reading in.	Defl. in.
A	0	10.48		23600	10.43	.05
B	"	1.45		"	1.38	.07
C	"	4.50		"	4.43	.07
D	"	7.41		"	7.35	.06
E	"	4.60		"	4.55	.05
A	4230	10.48	.00	27850	10.42	.06
B	"	1.44	.01	"	1.37	.08
C	"	4.50	.00	"	4.42	.08
D	"	7.40	.01	"	7.33	.08
E	"	4.59	.01	"	4.54	.06
A	7910	10.47	.01	31840	10.41	.07
B	"	1.43	.02	"	1.35	.10
C	"	4.48	.02	"	4.40	.10
D	"	7.39	.02	"	7.32	.09
E	"	4.58	.02	"	4.53	.08
A	12480	10.46	.02	35680	10.40	.08
B	"	1.42	.03	"	1.33	.12
C	"	4.48	.02	"	4.38	.12
D	"	7.39	.02	"	7.30	.11
E	"	4.58	.02	"	4.52	.08
A	15910	10.46	.02	39900	10.39	.09
B	"	1.42	.03	"	1.32	.13
C	"	4.47	.03	"	4.37	.13
D	"	7.38	.03	"	7.29	.12
E	"	4.58	.02	"	4.51	.09
	*			**		
A	15740	10.46	.02	39180	10.38	.10
B	"	1.41	.04	"	1.31	.14
C	"	4.47	.03	"	4.35	.15
D	"	7.38	.03	"	7.28	.13
E	"	4.57	.03	"	4.50	.10
A	19810	10.45	.03			
B	"	1.40	.05			
C	"	4.45	.05			
D	"	7.36	.05			
E	"	4.56	.04			

* 1 hour and 20 minute break in tensioning of bars.
** 15 hour break after completion of tensioning.

TABLE VIII CALCULATED E_c AT CENTER LINE
BEAM WITH ONLY PRESTRESSED REINFORCING

$$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$$ (Stress in bottom fibers)

$M = 1.20T$ in. lb.

$C = 3.04$ in.

$I = 102$ in.^{4}

$A_{net} = 34.8$ sq. in.

$$f_c = \frac{(1.20)(3.04)T}{102} + \frac{T}{34.8} = 0.0358T + \frac{T}{34.8}$$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain micro units	Unit Strain* micro units	$E_c \times 10^{-6}$
3820	247	57	57	4.33
7410	478	138	138	3.46
** 7130	460	176	176	2.61
11730	756	242	204	3.71
15830	1020	307	269	3.79
19870	1280	379	341	3.75
23980	1550	466	428	3.62
28110	1810	523	485	3.73
31880	2060	577	539	3.82
*** 31760	2050	590	552	3.72
35860	2320	633	582	3.99
39690	2560	697	646	3.96

*Unit strain adjusted for the strain that occurred during the 15-hour break. Unit strain at T = 7410 was 138; unit strain at T = 7130 was 176. The difference = 38. 38 was subtracted from each value of unit strain below T = 7130. For each value of unit strain below T = 31760 another 13 was subtracted.

**15 hour time lapse

***1 hour time lapse

TABLE IX CALCULATED E_c AT SECTION B-B
BEAM WITH ONLY PRESTRESSED REINFORCING

$$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$$ (Stress in bottom fibers)

$M = 1.10\ T$ in. lb.

$C = 3.04$ in.

$I = 102$ in.^{4}

$A_{net} = 34.8$ sq. in.

$$f_c = \frac{(1.10)(3.04)T}{102} + \frac{T}{34.8} = 0.0328T + \frac{T}{34.8}$$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain* micro units	$E_c \times 10^{-6}$
3820	235	35	6.71
7410	456	80	5.70
** 7130	439	107	4.10
11730	720	131	5.50
15830	973	182	5.35
19870	1220	252	4.84
23980	1470	339	4.34
28110	1730	407	4.25
31880	1970	475	4.15
*** 31760	1950	487	3.70
35860	2210	526	4.20
39690	2440	609	4.00

*Values of unit strain adjusted in the same manner as for Table VIII.
** 15 hour time lapse
*** 1 hour time lapse

TABLE X CALCULATED E_c AT SECTION A-A
BEAM WITH ONLY PRESTRESSED REINFORCING

$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$ (Stress in bottom fibers)

$M = 0.80\ T$ in. lb.

$C = 3.03$ in.

$I = 102$ in.^{4}

$A_{net} = 34.8$ sq. in.

$$f_c = \frac{(0.80)(3.03)T}{102} + \frac{T}{34.8} = 0.0238T + \frac{T}{34.8}$$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain* micro units	$E_c \times 10^{-6}$
3820	201	35	5.74
7410	389	72	5.40
** 7130	375	111	3.38
11730	614	112	5.48
15830	830	155	5.35
19870	1050	204	5.15
23980	1260	262	4.81
28110	1480	311	4.76
31880	1680	358	4.69
*** 31160	1670	368	4.54
35860	1890	401	4.71
39690	2090	453	4.61

* Values of unit strain adjusted in the same manner as for Table VIII.
** 15 hour lapse
*** 1 hour lapse

TABLE XI CALCULATED E_c AT CENTER LINE
BEAM WITH PLAIN AND PRESTRESSED REINFORCING

$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$ (Stress in bottom fibers)

$M = 1.20\ T$ in. lb.

$C = 3.04$ in.

$I = 105$ in.^{4}

$A_{net} = 35.3$ sq. in.

$$f_c = \frac{(1.20)(3.04)T}{105} + \frac{T}{35.3} = 0.0347T + \frac{T}{35.3}$$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain micro units	$E_c \times 10^{-6}$
4230	267	22	12.1
7910	520	37	14.1
12480	788	48	16.4
15910	1000	79	12.7
* 15740	990	81	12.2
19810	1250	129	9.69
23600	1490	193	7.72
27850	1750	261	6.70
31840	2000	323	6.20
35680	2250	385	5.85
39900	2520	462	5.45
** 39180	2470	507	4.87

* 1½ hour time lapse
** 15 hour time lapse

TABLE XII CALCULATED E_c AT SECTION B-B
BEAM WITH PLAIN AND PRESTRESSED REINFORCING

$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$ (Stress in bottom fibers)

$M = 1.10\ T$ in. lb.

$C = 3.04$ in.

$I = 105$ in.^{4}

$A_{net} = 35.3$ sq. in.

$$f_c = \frac{(1.10)(3.04)T}{105} + \frac{T}{35.3} = 0.0318T + \frac{T}{35.3}$$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain micro units	$E_c \times 10^{-6}$
4230	255	31	8.23
7910	497	62	8.02
12480	752	86	8.74
15910	955	123	7.75
* 15740	945	127	7.43
19810	1190	172	6.92
23600	1420	215	6.61
27850	1670	265	6.30
31840	1910	312	6.13
35680	2150	364	5.91
39900	2400	429	5.83
** 39180	2360	433	5.45

* 1½ hour time lapse
** 15 hour time lapse

TABLE XIII CALCULATED E_c AT SECTION A-A
BEAM WITH PLAIN AND PRESTRESSED REINFORCING

$f_c = \frac{MC}{I} + \frac{T}{A_{net}}$ (Stress in bottom fibers)

$M = 0.80\ T$ in. lb.

$C = 3.03$ in.

$I = 105$ in.^{4}

$A_{net} = 35.3$ sq. in.

$f_c = \frac{(0.80)(3.03)}{105}T + \frac{T}{35.3} = 0.0231T + \frac{T}{35.3}$

$E_c = f_c$/unit strain

T lb.	f_c psi	Unit strain micro units	$E_c \times 10^{-6}$
4230	218	28	9.09
7910	407	48	8.49
12480	645	68	9.45
15910	819	95	8.62
* 15740	808	90	8.98
19810	1020	128	7.97
23600	1210	162	7.46
27850	1440	208	6.93
31840	1640	250	6.56
35680	1840	300	6.13
39900	2050	360	5.69
** 39180	2020	370	5.46

* 1½ hour time lapse
** 15 hour time lapse

TABLE XIV

LOADING DATA

BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
Loading test started at 4:00 P.M., March 25, 1952							
1	0	0-10- 670	0				
2	"	0-10- 520	0				
3	"	0-10- 979	0				
4	"	0-10-1403	0				
5	"	0-10- 459	0				
6	"	0-10-1051	0				
7	"	0-10-1700	0				
8	"	0-10- 567	0				
9	"	0-10-1514	0				
10	"	0-10-1845	0				
11	"	0-16- 132	+2000	- 917	+1083	+1025	9840
12	"	0-14- 249	+2000	-1033	+ 967		
13	"	0-14-1570	0	+1244	+1244	+1058	9860
14	"	0-10-1424	0	+ 871	+ 871		
15	"	0-16- 288	+2000	- 675	+1325	+1076	9860
16	"	0-14- 875	0	+ 826	+ 826		
17	"	0-10- 323	+2000	-1028	+ 972	+1016	9840
18	"	0-12-1913	0	+1060	+1060		
						Total Prestress =	39400
1	665	0-10- 701	0	+ 31	+ 31	+ 35	
2	"	0-10- 558	0	+ 38	+ 38		
3	"	0-10-1037	0	+ 58	+ 58	+ 62	
4	"	0-10-1468	0	+ 65	+ 65		
5	"	0-10- 549	0	+ 90	+ 90	+ 94	
6	"	0-10-1149	0	+ 98	+ 98		
7	"	0-10-1759	0	+ 59	+ 59	+ 51	
8	"	0-10- 610	0	+ 43	+ 43		
9	"	0-10-1540	0	+ 26	+ 26	+ 30	
10	"	0-10-1878	0	+ 33	+ 33		
11	"	0-16- 130	0	- 2	- 2	- 3	- 30
12	"	0-14- 245	0	- 4	- 4		
13	"	0-14-1570	0	0	0	0	
14	"	0-10-1424	0	0	0		
15	"	0-16- 288	0	0	0	+ 2	+ 20
16	"	0-14- 878	0	+ 3	+ 3		
17	"	0-10- 328	0	+ 5	+ 5	+ 5	+ 50
18	"	0-12-1917	0	+ 4	+ 4		

Total change in prestress = + 40

Total prestress = 39,400 + 40 = 39,440

TABLE XIV
(cont.)

LOADING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	1165	0-10- 730	0	+ 60	+ 60	+ 65	
2	"	0-10- 589	0	+ 69	+ 69		
3	"	0-10-1082	0	+ 103	+ 103	+ 110	
4	"	0-10-1520	0	+ 117	+ 117		
5	"	0-10- 620	0	+ 161	+ 161	+ 170	
6	"	0-10-1229	0	+ 178	+ 178		
7	"	0-10-1800	0	+ 100	+ 100	+ 90	
8	"	0-10- 646	0	+ 79	+ 79		
9	"	0-10-1560	0	+ 46	+ 46	+ 52	
10	"	0-10-1902	0	+ 57	+ 57		
11	"	0-16- 130	0	- 2	- 2	- 4	- 40
12	"	0-14- 245	0	- 6	- 6		
13	"	0-14-1569	0	- 1	- 1	- 2	- 20
14	"	0-10-1422	0	- 2	- 2		
15	"	0-16- 290	0	+ 2	+ 2	+ 4	+ 40
16	"	0-14- 880	0	+ 5	+ 5		
17	"	0-10- 350	0	+ 7	+ 7	+ 7	+ 70
18	"	0-12-1920	0	+ 7	+ 7		

Total change in prestress = + 50

Total prestress = 39,400 + 50 = 39,450

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	1665	0-10- 755	0	+ 93	+ 93	+ 96	
2	"	0-10- 619	0	+ 99	+ 99		
3	"	0-10-1135	0	+ 156	+ 156	+ 167	
4	"	0-10-1580	0	+ 177	+ 177		
5	"	0-10- 700	0	+ 241	+ 241	+ 251	
6	"	0-10-1312	0	+ 261	+ 261		
7	"	0-10-1848	0	+ 148	+ 148	+ 131	
8	"	0-10- 680	0	+ 113	+ 113		
9	"	0-10-1583	0	+ 69	+ 69	+ 77	
10	"	0-10-1950	0	+ 85	+ 85		
11	"	0-16- 129	0	- 3	- 3	- 9	- 90
12	"	0-14- 235	0	- 14	- 14		
13	"	0-14-1365	0	- 5	- 5	- 5	- 50
14	"	0-10-1420	0	- 4	- 4		
15	"	0-16- 299	0	+ 11	+ 11	+ 13	+ 120
16	"	0-14- 890	0	+ 15	+ 15		
17	"	0-10- 340	0	+ 17	+ 17	+ 16	+ 150
18	"	0-12-1927	0	+ 14	+ 14		

Total change in prestress = + 130

Total prestress = 39,400 + 130 = 39,530

TABLE XIV (cont.)

LOADING DATA

BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	2165	0-10- 780	0	+ 110	+ 110	+ 120	
2	"	0-10- 650	0	+ 130	+ 130		
3	"	0-10-1190	0	+ 211	+ 211	+ 225	
4	"	0-10-1642	0	+ 239	+ 239		
5	"	0-10- 800	0	+ 341	+ 341	+ 345	
6	"	0-10-1400	0	+ 349	+ 349		
7	"	0-10-1890	0	+ 190	+ 190	+ 169	
8	"	0-10- 715	0	+ 148	+ 148		
9	"	0-10-1608	0	+ 94	+ 94	+ 101	
10	"	0-10-1953	0	+ 108	+ 108		
11	"	0-16- 125	0	- 7	- 7	- 8	- 80
12	"	0-14- 240	0	- 9	- 9		
13	"	0-14-1362	0	- 8	- 8	- 6	- 60
14	"	0-10-1420	0	- 4	- 4		
15	"	0-16- 312	0	+ 24	+ 24	+ 28	+ 260
16	"	0-14- 907	0	+ 32	+ 32		
17	"	0-10- 354	0	+ 31	+ 31	+ 29	+ 280
18	"	0-12-1940	0	+ 27	+ 27		

Total change in prestress = + 400

Total prestress = 39,400 + 400 = 39,800

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	2665	0-10- 805	0	+ 135	+ 135	+ 148	
2	"	0-10- 680	0	+ 160	+ 160		
3	"	0-10-1250	0	+ 271	+ 271	+ 296	
4	"	0-10-1723	0	+ 320	+ 320		
5	"	0-10- 980	0	+ 521	+ 521	+ 485	
6	"	0-10-1500	0	+ 449	+ 449		
7	"	0-10-1935	0	+ 235	+ 235	+ 210	
8	"	0-10- 751	0	+ 184	+ 184		
9	"	0-10-1630	0	+ 116	+ 116	+ 126	
10	"	0-10-1980	0	+ 135	+ 135		
11	"	0-16- 120	0	- 12	- 12	- 14	- 130
12	"	0-14- 233	0	- 16	- 16		
13	"	0-14-1360	0	- 10	- 10	- 8	- 70
14	"	0-10-1418	0	- 6	- 6		
15	"	0-16- 315	0	+ 25	+ 25	+ 43	+ 400
16	"	0-14- 935	0	+ 60	+ 60		
17	"	0-10- 375	0	+ 52	+ 52	+ 47	+ 450
18	"	0-12-1955	0	+ 42	+ 42		

Total change in prestress = + 650

Total prestress = 39,400 + 650 = 40,050

APPENDIX B

TABLE XIV (cont.)

LOADING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	3165	0-10- 851	0	+ 161	+ 161	+ 178	
2	"	0-10- 715	0	+ 195	+ 195		
3	"	0-10-1309	0	+ 330	+ 330	+ 374	
4	"	0-10-1820	0	+ 417	+ 417		
5	"	0-10-1220	0	+ 761	+ 761	+ 659	
6	"	0-10-1608	0	+ 557	+ 557		
7	"	0-10-1980	0	+ 280	+ 280	+ 252	
8	"	0-10- 790	0	+ 225	+ 225		
9	"	0-10-1651	0	+ 137	+ 137	+ 148	
10	"	0-12- 3	+2000	-1842	+ 158		
11	"	0-16- 119	0	- 13	- 13	- 18	- 170
12	"	0-14- 230	0	- 19	- 19		
13	"	0-14-1356	0	- 14	- 14	- 13	- 120
14	"	0-10-1412	0	- 12	- 12		
15	"	0-16- 267	0	- 21	- 21	+ 52	+ 480
16	"	0-14-1000	0	+ 125	+ 125		
17	"	0-10- 400	0	+ 77	+ 77	+ 66	+ 630
18	"	0-12-1968	0	+ 55	+ 55		

Total change in prestress = + 820

Total prestress = 39,400 + 820 = 40,220

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	3665	0-10- 860	0	+ 190	+ 190	+ 210	
2	"	0-10- 750	0	+ 230	+ 230		
3	"	0-10-1575	0	+ 394	+ 394	+ 461	
4	"	0-10-1930	0	+ 527	+ 527		
5	"	0-10-1705	0	+1244	+1244	+ 911	
6	"	0-10-1629	0	+ 578	+ 578		
7	"	0-12- 20	+ 2000	-1680	+ 320	+ 291	
8	"	0-10- 828	0	+ 261	+ 261		
9	"	0-10-1680	0	+ 166	+ 166	+ 171	
10	"	0-12- 20	+ 2000	-1825	+ 175		
11	"	0-16- 113	0	- 19	- 19	- 20	- 190
12	"	0-14- 229	0	- 20	- 20		
13	"	0-14-1350	0	- 20	- 20	- 18	- 170
14	"	0-10-1409	0	- 15	- 15		
15	"	0-16- 214	0	- 74	- 74	+ 56	+ 520
16	"	0-14-1061	0	+ 186	+ 186		
17	"	0-10- 435	0	+ 112	+ 112	+ 92	+ 880
18	"	0-12-1984	0	+ 71	+ 71		

Total change in prestress = +1040

Total prestress = 39,400 + 1040 = 40,440

TABLE XIV (cont.)

LOADING DATA
BEAM WITH PRESTRESSED REINFORCING ONLY

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	4665	0-10- 901	0	+ 251	+ 251	+ 265	
2	"	0-10- 818	0	+ 298	+ 298		
3	"	0-10-1498	0	+ 519	+ 519	+ 863	
4	"	0-12- 610	+2000	- 793	+1207		
5	"	0-18-1770	+8000	+1311	+9311	+4898	
6	"	0-10-1535	0	+ 484	+ 484		
7	"	0-12- 100	+2000	-1600	+ 400	+ 359	
8	"	0-10- 885	0	+ 318	+ 318		
9	"	0-10-1720	0	+ 206	+ 206	+ 210	
10	"	0-12- 58	+2000	-1787	+ 213		
11	"	0-16- 109	0	- 23	- 23	- 26	- 250
12	"	0-14- 220	0	- 29	- 29		
13	"	0-14-1340	0	- 30	- 30	- 27	- 250
14	"	0-10-1400	0	- 24	- 24		
15	"	0-16- 190	0	- 98	- 98	+ 80	+ 740
16	"	0-14-1133	0	+ 258	+ 258		
17	"	0-10- 580	0	+ 257	+ 257	+ 199	+1910
18	"	0-14- 55	+2000	-1860	+ 140		
				Total change in prestress =			+2150
1	0	0-10- 760	0	+ 90	+ 90	+ 103	
2	"	0-10- 635	0	+ 115	+ 115		
3	"	0-10-1109	0	+ 130	+ 130	+ 184	
4	"	0-10-1640	0	+ 237	+ 237		
5	"	Gage No. 5 broken by crack					
6	"	0-10-1173	0	+ 122	+ 122		
7	"	0-10-1799	0	+ 99	+ 99	+ 85	
8	"	0-10- 638	0	+ 71	+ 71		
9	"	0-10-1589	0	+ 75	+ 75	+ 80	
10	"	0-10-1930	0	+ 85	+ 85		
11	"	0-16- 90	0	- 42	- 42	- 41	- 390
12	"	0-14- 210	0	- 39	- 39		
13	"	0-14-1329	0	- 41	- 41	- 43	- 400
14	"	0-10-1380	0	- 44	- 44		
15	"	0-14-1719	-2000	+1431	- 569	- 331	-3070
16	"	0-14- 682	0	- 193	- 193		
17	"	0-10- 149	0	- 174	- 174	- 207	-1990
18	"	0-12-1673	0	- 240	- 240		
				Total change in prestress =			-5850

Total change in prestress = +2150 (first loading, above)

Total prestress = 39,400 + 2150 = 41,550
Crack in bottom at center line 2¼" into beam.
Ultimate load between 4665 and 5000 lb.

Total prestress = 39,400 - 5850 = 33,550
Crack closed so that it was almost not visible.
Loading test completed at 5:00 P.M., March 25, 1952.

TABLE XV

LOADING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Loading test started at 11:00 A.M., March 28, 1952

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	0	0-10-1711	0				
2	"	0-10-0277	0				
3	"	0-10-1053	0				
4	"	0-10-1212	0				
5	"	0-10-0661	0				
6	"	0-10-0115	0				
7	"	0-10-1153	0				
8	"	0-10-0973	0				
9	"	0-10-1457	0				
11	"	0-14-0658	+2000	- 732	+1268	+1030	9820
12	"	0-14-1400	0	- 792	- 792		
13	"	0-14-1405	0	+1060	+1060	+1024	9820
14	"	0-14-0103	+2000	-1012	+ 988		
15	"	0-10-0180	+2000	- 820	+1180	+1117	9740
16	"	0-16-1242	0	-1654	-1654		
17	"	0-10-0971	+2000	- 969	+1031	+ 980	9460
18	"	0-14-1540	0	+ 928	+ 928		

Total prestress = 38840

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	700	0-10-1744	0	+ 33	+ 33	+ 33	
2	"	0-10-0350	0	+ 73	+ 73	+ 65	
3	"	0-10-1110	0	+ 57	+ 57		
4	"	0-10-1300	0	+ 88	+ 88	+ 98	
5	"	0-10-0769	0	+ 108	+ 108		
6	"	0-10-0185	0	+ 70	+ 70	+ 69	
7	"	0-10-1220	0	+ 67	+ 67		
8	"	0-10-1005	0	+ 32	+ 32	+ 36	
9	"	0-10-1476	0	+ 39	+ 39		
11	"	0-14-0657	0	- 1	- 1	- 1	- 10
12	"	0-14-1400	0	0	0		
13	"	0-14-1405	0	0	0	0	0
14	"	0-14-0103	0	0	0		
15	"	0-10-0180	0	0	0	+ 1	+ 10
16	"	0-10-1243	0	+ 1	+ 1		
17	"	0-10- 973	0	+ 2	+ 2	+ 1	+ 10
18	"	0-14-1540	0	0	0		

Total change in prestress = + 10

Total prestress = 38,840 + 10 = 38,850

TABLE XV (cont.)

LOADING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	1700	0-10-1795	0	+ 82	+ 82	+ 82	
2	"	0-10-0459	0	+ 182	+ 182	+ 157	
3	"	0-10-1185	0	+ 132	+ 132		
4	"	0-10-1420	0	+ 208	+ 208	+ 267	
5	"	0-10-0986	0	+ 325	+ 325		
6	"	0-10-0300	0	+ 185	+ 185	+ 171	
7	"	0-10-1310	0	+ 157	+ 157		
8	"	0-10-1051	0	+ 78	+ 78	+ 88	
9	"	0-10-1534	0	+ 97	+ 97		
11	"	0-14-9652	0	- 6	- 6	- 5	- 50
12	"	0-14-1397	0	- 3	- 3		
13	"	0-14-1403	0	- 2	- 2	0	0
14	"	0-14-0104	0	+ 1	+ 1		
15	"	0-10-0180	0	0	0	+ 2	+ 20
16	"	0-16-1245	0	+ 3	+ 3		
17	"	0-10-0980	0	+ 9	+ 9	+ 10	+ 100
18	"	0-14-1551	0	+ 11	+ 11		
			Total change in prestress =				+ 70
1	2700	0-10-1846	0	+ 135	+ 135	+ 135	
2	"	0-10-0577	0	+ 300	+ 300	+ 250	
3	"	0-10-1252	0	+ 199	+ 199		
4	"	0-10-1520	0	+ 308	+ 308	+ 514	
5	"	0-10-1380	0	+ 719	+ 719		
6	"	0-10-0457	0	+ 322	+ 322	+ 280	
7	"	0-10-1391	0	+ 238	+ 238		
8	"	0-10-1098	0	+ 125	+ 125	+ 138	
9	"	0-10-1587	0	+ 150	+ 150		
11	"	0-14-0650	0	- 8	- 8	- 8	- 80
12	"	0-14-1392	0	- 8	- 8		
13	"	0-14-1400	0	- 5	- 5	- 4	- 40
14	"	0-14-0100	0	- 3	- 3		
15	"	0-10-0180	0	0	0	+ 3	+ 30
16	"	0-16-1248	0	+ 6	+ 6		
17	"	0-10-0991	0	+ 20	+ 20	+ 28	+ 270
18	"	0-14-1575	0	+ 35	+ 35		
			Total change in prestress =				+ 180

Total prestress = 38,840 + 180 = 39,020

TABLE XV LOADING DATA
(cont.) BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	5200	0-10-1875	0	+ 162	+ 162	+ 162	
2	"	0-10-0641	0	+ 364	+ 364	+ 301	
3	"	0-10-1290	0	+ 237	+ 237		
4	"	0-10-1536	0	+ 324	+ 324	+ 829	
5	"	0-12-0001	+2000	- 666	+1334		
6	"	0-10-0511	0	+ 396	+ 396	+ 340	
7	"	0-10-1437	0	+ 284	+ 284		
8	"	0-10-1118	0	+ 145	+ 145	+ 154	
9	"	0-10-1610	0	+ 173	+ 173		
11	"	0-14-0648	0	- 10	- 10	- 10	- 100
12	"	0-14-1390	0	- 10	- 10		
13	"	0-14-1400	0	- 0	- 0	- 2	- 20
14	"	0-14-0100	0	- 3	- 3		
15	"	0-10-0182	0	+ 2	+ 2	+ 5	+ 40
16	"	0-16-1250	0	+ 8	+ 8		
17	"	0-10-1003	0	+ 32	+ 32	+ 41	+ 390
18	"	0-14-1590	0	+ 50	+ 50		

Total change in prestress = + 310

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	5700	0-10-1900	0	+ 189	+ 189	+ 189	
2	"	0-10-0710	0	+ 433	+ 433	+ 353	
3	"	0-10-1325	0	+ 272	+ 272		
4	"	0-10-1520	0	+ 308	+ 308	+1429	
5	"	0-12-1210	+2000	+ 549	+2549		
6	"	0-10-0589	0	+ 474	+ 474	+ 386	
7	"	0-10-1450	0	+ 297	+ 297		
8	"	0-10-1134	0	+ 161	+ 161	+ 177	
9	"	0-10-1630	0	+ 193	+ 193		
11	"	0-14-0644	0	- 14	- 14	- 13	- 120
12	"	0-14-1388	0	- 12	- 12		
13	"	0-14-1397	0	- 8	- 8	- 10	- 100
14	"	0-14-0092	0	- 11	- 11		
15	"	0-10-0188	0	+ 8	+ 8	+ 10	+ 90
16	"	0-16-1254	0	+ 12	+ 12		
17	"	0-10-1021	0	+ 50	+ 50	+ 59	+ 560
18	"	0-14-1608	0	+ 68	+ 68		

Total change in prestress = + 430

Total prestress = 38,840 + 430 = 39,270

TABLE XV (cont.) LOADING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force
1	4200	0-10-1924	0	+ 213	+ 213	+ 213	
2	"	0-10-0770	0	+ 493	+ 493	+ 398	
3	"	0-10-1355	0	+ 302	+ 302		
4	"	0-10-1490	0	+ 278	+ 278	+1592	
5	"	0-12-1566	+2000	+ 905	+2905		
6	"	0-10-0638	0	+ 523	+ 523	+ 402	
7	"	0-10-1433	0	+ 280	+ 280		
8	"	0-10-1144	0	+ 171	+ 171	+ 187	
9	"	0-10-1640	0	+ 203	+ 203		
11	"	0-14-0640	0	- 18	- 18	- 18	- 170
12	"	0-14-1382	0	- 18	- 18		
13	"	0-14-1392	0	- 13	- 13	- 13	- 120
14	"	0-14-0090	0	- 13	- 13		
15	"	0-10- 204	0	+ 24	+ 24	+ 26	+ 230
16	"	0-16-1270	0	+ 28	+ 28		
17	"	0-10-1055	0	+ 82	+ 82	+ 90	+ 860
18	"	0-14-1638	0	+ 98	+ 98		

Total change in prestress = + 800

Total prestress = 38,840 + 800 = 39,640

Hair line crack at center line

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force
1	4700	0-10-1938	0	+ 227	+ 227	+ 227	
2	"	0-10-0820	0	+ 543	+ 543	+ 433	
3	"	0-10-1375	0	+ 322	+ 322		
4	"	0-10-1470	0	+ 258	+ 258	+1299	
5	"	0-12-1000	+2000	+ 339	+2339		
6	"	0-10-0680	0	+ 565	+ 565	+ 414	
7	"	0-10-1415	0	+ 262	+ 262		
8	"	0-10-1148	0	+ 175	+ 175	+ 197	
9	"	0-10-1645	0	+ 208	+ 208		
11	"	0-14-0636	0	- 22	- 22	- 21	- 200
12	"	0-14-1380	0	- 20	- 20		
13	"	0-14-1388	0	- 17	- 17	- 19	- 180
14	"	0-14-0082	0	- 21	- 21		
15	"	0-10-0255	0	+ 75	+ 75	+ 76	+ 670
16	"	0-16-1318	0	+ 76	+ 76		
17	"	0-10-1260	0	+ 289	+ 289	+ 177	+1690
18	"	0-14-1605	0	+ 65	+ 65		

Total change in prestress = +1980

Total prestress = 38,840 + 1980 = 40,820

TABLE XV (cont.) LOADING DATA
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Gage No.	Center Line Load lb.	SR4 Reading	T_2-T_1 x(1000)	D_2-D_1	Strain in micro in./in.	Strain Ave.	Prestress Force lb.
1	0	0-10-1810	0	+ 109	+ 109	+ 109	
2	"	0-10-0491	0	+ 214	+ 214	+ 156	
3	"	0-10-1150	0	+ 97	+ 97		
4	"	0-10-1385	0	+ 173	+ 173	+ 211	
5	"	0-10-1009	0	+ 248	+ 248		
6	"	0-10-0240	0	+ 125	+ 125	+ 110	
7	"	0-10-1243	0	+ 95	+ 95		
8	"	0-10-1031	0	+ 58	+ 58	+ 57	
9	"	0-14-1492	0	+ 55	+ 55		
11	"	0-14-0580	0	- 78	- 78	- 82	- 780
12	"	0-14-1315	0	- 85	- 85		
13	"	0-14-1327	0	- 78	- 78	- 86	- 820
14	"	0- 8-0010	0	- 95	- 95		
15	"	0-16-1975	-2000	+1795	- 205	- 208	-1830
16	"	0-16-1032	0	- 210	- 210		
17	"	0-10-0975	0	+ 4	+ 4	- 293	-2790
18	"	0-14- 950	0	- 590	- 590		

Total change in prestress = -6220

Total prestress = 38,840 - 6220 = 32,620

Loading test completed 12:00 noon, March 28, 1952. Crack closed so that it was almost not visible.

TABLE XVI DEFLECTION DATA FOR LOADING BEAM WITH PRESTRESSED REINFORCING ONLY

Section	Center Line Load lb.	Reading in.	Defl. in.	Center Line Load lb.	Reading in.	Defl. in.
A	0	7.01		3665	7.14	-.13
B	"	10.06		"	10.27	-.21
C	"	1.12		"	1.36	-.24
D	"	3.98		"	4.20	-.22
E	"	4.18		"	4.33	-.15
A	665	7.03	-.02	4665	7.25	-.24
B	"	10.09	-.03	"	10.43	-.37
C	"	1.16	-.04	"	1.55	-.43
D	"	4.03	-.05	"	4.36	-.38
E	"	4.20	-.02	"	4.42	-.24
A	1165	7.04	-.03	5165	7.40	-.39
B	"	10.10	-.04	"	10.69	-.63
C	"	1.17	-.05	"	1.88	-.76
D	"	4.04	-.06	"	4.61	-.63
E	"	4.22	-.04	"	4.56	-.38
A	1665	7.07	-.06	0	7.06	-.05
B	"	10.14	-.08	"	10.13	-.07
C	"	1.22	-.10	"	1.19	-.07
D	"	4.08	-.10	"	4.06	-.08
E	"	4.24	-.06	"	4.22	-.04
A	2165	7.09	-.08			
B	"	10.17	-.11			
C	"	1.25	-.13			
D	"	4.11	-.13			
E	"	4.26	-.08			
A	2665	7.11	-.10			
B	"	10.21	-.15			
C	"	1.29	-.17			
D	"	4.14	-.16			
E	"	4.28	-.10			
A	3165	7.12	-.11			
B	"	10.23	-.17			
C	"	1.32	-.20			
D	"	4.17	-.19			
E	"	4.30	-.12			

TABLE XVII DEFLECTION DATA FOR LOADING
BEAM WITH PRESTRESSED AND PLAIN REINFORCING

Section	Center Line Load lb.	Reading in.	Defl. in.	Center Line Load lb.	Reading in.	Defl. in.
A	0	10.38		4700	10.63	-.25
B	"	1.31		"	1.69	-.38
C	"	4.36		"	4.83	-.47
D	"	7.27		"	7.67	-.40
E	"	4.52		"	4.76	-.24
A	700	10.40	-.02	0	10.44	-.06
B	"	1.35	-.04	"	1.40	-.09
C	"	4.40	-.04	"	4.47	-.11
D	"	7.31	-.04	"	7.37	-.10
E	"	4.53	-.01	"	4.57	-.05
A	1700	10.44	-.06			
B	"	1.40	-.09			
C	"	4.46	-.10			
D	"	7.36	-.09			
E	"	4.57	-.05			
A	2700	10.48	-.10			
B	"	1.46	-.15			
C	"	4.53	-.17			
D	"	7.43	-.16			
E	"	4.62	-.10			
A	3200	10.50	-.12			
B	"	1.50	-.19			
C	"	4.58	-.22			
D	"	7.46	-.19			
E	"	4.63	-.11			
A	3700	10.53	-.15			
B	"	1.55	-.24			
C	"	4.64	-.28			
D	"	7.52	-.25			
E	"	4.67	-.15			
A	4200	10.57	-.19			
B	"	1.61	-.30			
C	"	4.71	-.35			
D	"	7.53	-.26			
E	"	4.71	-.19			