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Paul Evenson

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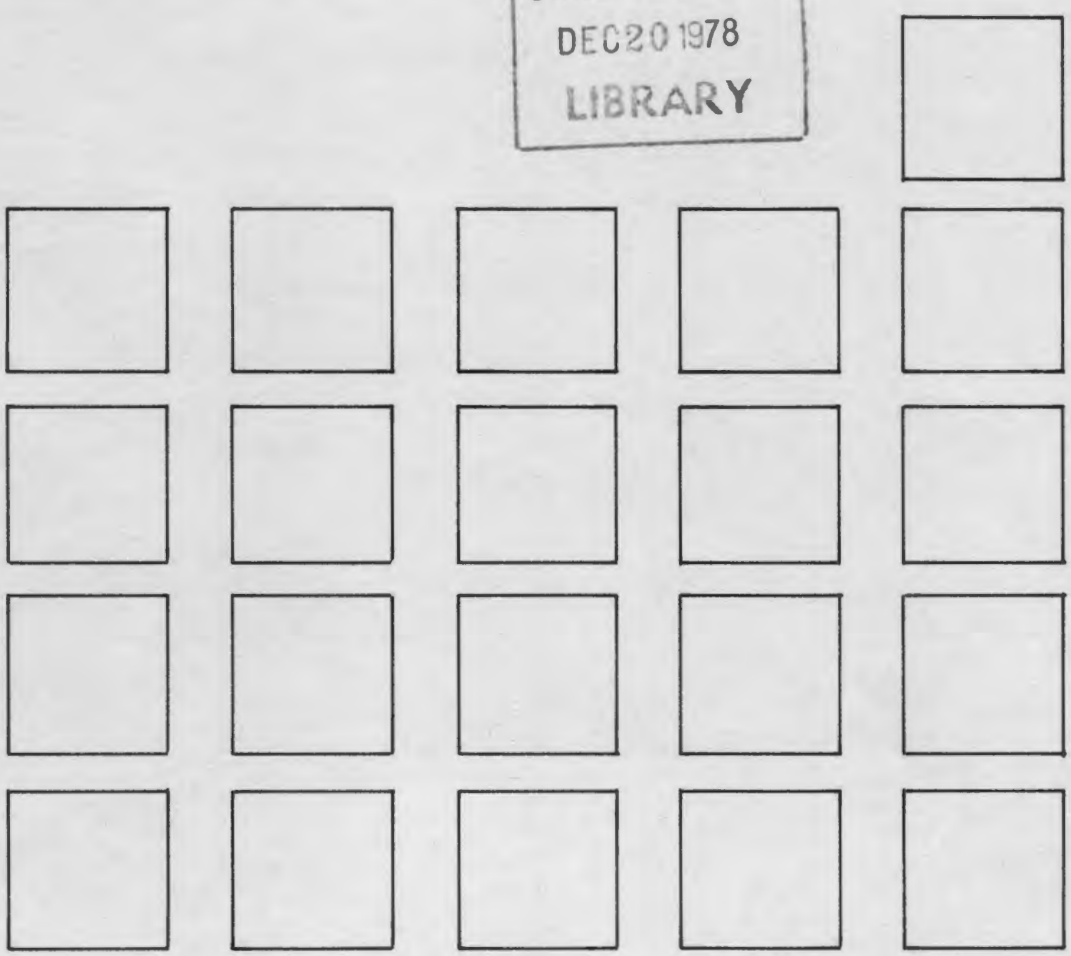


Agricultural Experiment Station
South Dakota State University
Brookings South Dakota 57007

Calculation of the Two-Way Analysis of Variance (ANOVA) with Subsampling Using a Programable Pocket Calculator

By: Paul D. Evenson
Assoc. Professor
Plant Science and Statistics

The two-way ANOVA with subsampling is one of the more popular experimental designs in statistics. It has an advantage over the standard two-way ANOVA, because the existence of an interaction between rows and column can be detected.

A program is described in this paper which requires the input of each piece of data once, and the input of row by column totals once. The calculator computes means and totals for each cell within the rows and columns, as well as means for each row and column.

The program also computes the complete ANOVA, the F-tests for all four possible models, and the coefficient of variation. An unlimited number of subsamples and rows and up to ten columns can be handled by this program, which was written for the Hewlett-Packard 97 programable "pocket" calculator.

The two-way ANOVA with subsampling tests the homogeniety of row means and column means and tests for the existence of an interaction between rows and columns.

If there are significant differences among row means, reject the null hypothesis that means of populations (μ_i's) of row factors are equal ($H_o : \mu_1 = \mu_2 = \ldots = \mu_r$), and accept the alternative hypothesis that means of populations of row factors are unequal ($H_1 : \mu_1 \neq \mu_2 \neq \ldots \neq \mu_r$). In the same manner, if there are significant differences among column means, reject $H_o : \mu_1 = \mu_2 = \ldots = \mu_c$ and accept $H_1 : \mu_1 \neq \mu_2 \neq \ldots \neq \mu_c$. Also, test the hypothesis of no interaction. If there is a significant interaction, reject this hypothesis and accept the one that says there is an interaction.

The symbol X_{ijk} denotes the variable that is the $k^{\underline{th}}$ subsample in $i^{\underline{th}}$ row and $j^{\underline{th}}$ column, where $k = 1, 2, 3, \ldots, s$; $i = 1, 2, 3, \ldots, r$; and $j = 1, 2, 3, \ldots, c$. A dot (.) in place of a subscript means that the variables have been summed across that subscript, e.g., $\sum_k X_{ijk} = X_{ij}$.

Formulas for Calculation of Row by Column Totals and Means and for the Calculation of Row Means and Column Means

1. row by column totals = $X_{ij.}$
2. row by column means = $\bar{x}_{ij.} = \frac{X_{ij.}}{s}$
3. row means = $\bar{x}_{i..} = \frac{X_{i..}}{sc}$
4. column means = $\bar{x}_{.j.} = \frac{X_{.j.}}{sr}$

Formulas for Calculation of the Two-Way ANOVA with Subsampling

1. correction term = $C = \frac{X_{...}^2}{src}$
2. total sum of squares = $SS_T = \sum_{i,j,k} X_{ijk}^2 - C$
3. row sum of squares = $SS_R = \frac{\Sigma X_{i..}^2}{sc} - C$
4. column sum of squares = $SS_C = \frac{\Sigma X_{.j.}^2}{sr} - C$
5. row by column sum of squares = $SS_{RC} = \frac{\Sigma X_{ij.}^2}{s} - C - SS_R - SS_C$
6. sampling error sum of squares = $SS_E = \sum_{i,j}\left(\Sigma_k X_{ijk}^2 - \frac{X_{ij.}^2}{s}\right)$
   $= SS_T - SS_R - SS_C - SS_{RC}$

Note: d.f. = degrees of freedom

M.S. = mean square = $\frac{SS}{df}$

M.S.E. = mean square expectations

σ^2 = variance of a population

s^2 = variance of a sample

Table 1. ANOVA (Both Row and Column Effects Fixed)

Source	df	SS	MS	F	MSE
Total	rcs-1	SS_T	----		
Row	r-1	SS_R	$\frac{SS_R}{r-1} = MS_R$	$\frac{MS_R}{MS_E}$	$\sigma^2 + sc\sigma_R^2$
Column	c-1	SS_C	$\frac{SS_C}{c-1} = MS_C$	$\frac{MS_C}{MS_E}$	$\sigma^2 + sr\sigma_C^2$
R x C	(r-1)(c-1)	SS_{RC}	$\frac{SS_{RC}}{(r-1)(c-1)} = MS_{RC}$	$\frac{MS_{RC}}{MS_E}$	$\sigma^2 + s\sigma_{RC}^2$
Sampling Error	rc (s-1)	SS_E	$\frac{SS_E}{rc(s-1)} = MS_E$		σ^2

Table 2. ANOVA (Row Effects Fixed and Column Effects Random)

Source	df	SS	MS	F	MSE
Total					
Row		SAME		$\frac{MS_R}{MS_{RC}}$	$\sigma^2 + s\sigma_{RC}^2 + sc\sigma_R^2$
Column				$\frac{MS_C}{MS_E}$	$\sigma^2 + sr\sigma_C^2$
R x C	AS	TABLE 1		$\frac{MS_{RC}}{MS_E}$	$\sigma^2 + s\sigma_{RC}^2$
Sampling Error					σ^2

Table 3. ANOVA (Row Effects and Column Effects Random)

Source	df	SS	MS	F	MSE
Total	SAME				
Row				$\frac{MS_R}{MS_{RC}}$	$\sigma^2 + s\sigma^2_{RC} + sc\sigma^2_R$
Column	AS			$\frac{MS_C}{MS_{RC}}$	$\sigma^2 + s\sigma^2_{RC} + sr\sigma^2_C$
R x C		TABLE 1		$\frac{MS_{RC}}{MS_E}$	$\sigma^2 + s\sigma^2_{RC}$
Sampling Error					σ^2

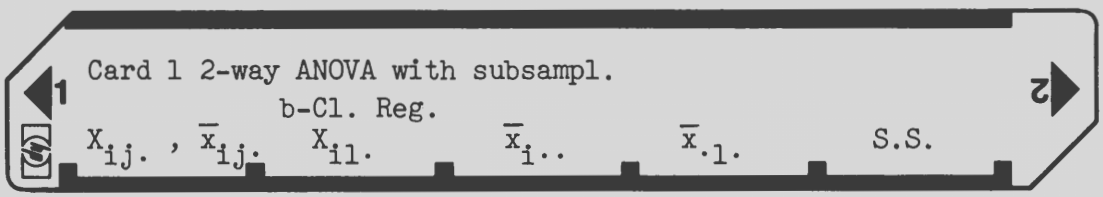
Table 4. ANOVA (Row Effects Random and Column Effects Fixed)

Source	df	SS	MS	F	MSE
Total					
Row	SAME			$\frac{MS_R}{MS_E}$	$\sigma^2 + sc\sigma^2_R$
Column				$\frac{MS_C}{MS_{RC}}$	$\sigma^2 + s\sigma^2_{RC} + sr\sigma^2_C$
R x C		TABLE 1		$\frac{MS_{RC}}{MS_E}$	$\sigma^2 + s\sigma^2_{RC}$
Sampling Error					σ^2

Calculation of Coefficient of Variation (C.V.)

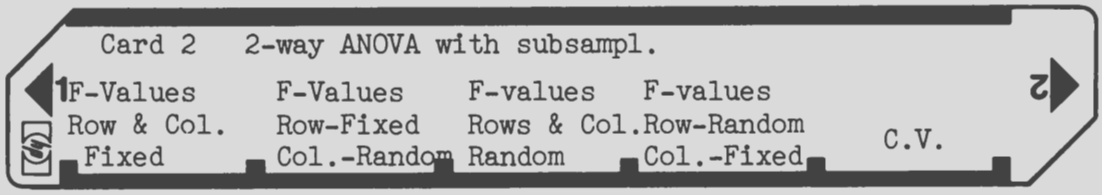
$$CV = \frac{\sqrt{MS_E}}{\bar{x}...} \times 100 \text{ where } \bar{x}... = \frac{X...}{src}$$

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Set Print Mode to Norm			
1.	Insert Card 1			
2.	Initialize		f b	0.00
3.	Do 4 ~ 5 for each ij^{th} cell			
4.	Calculate S.S. for X_{ijk} in each ij^{th} cell			
	from k = 1, 2, ..., s	X_{ijk}	Σ^+	k
5.	Calculate total and mean for each ij^{th} cell		A	$X_{ij.}$
				$\bar{x}_{ij.}$
6.	Do 7 ~ 9 for each row from i = 1, 2, ..., r			
7.	Enter total for il^{th} cell	$X_{il.}$	B	Count
8.	Enter $X_{ij.}$ for successive cells in row i			
	from j = 2, 3, ..., c	$X_{i2.}$	R/S	
		$X_{i3.}$	R/S	
		⋮	⋮	
		$X_{ic.}$	R/S	
9.	Calculate mean for the i^{th} row		C	$\bar{x}_{i..}$
10.	Calculate mean for column 1		D	$\bar{x}_{.1.}$
11.	Calculate mean for successive columns from			
	j = 2, ..., c		R/S	$\bar{x}_{.2.}$
			⋮	⋮
			R/S	$\bar{x}_{.r.}$
12.	Calculate Sum of Squares (S.S.)		E	Total SS
				Row SS
				Column SS
				R x C SS
				S.Error SS

User Instructions

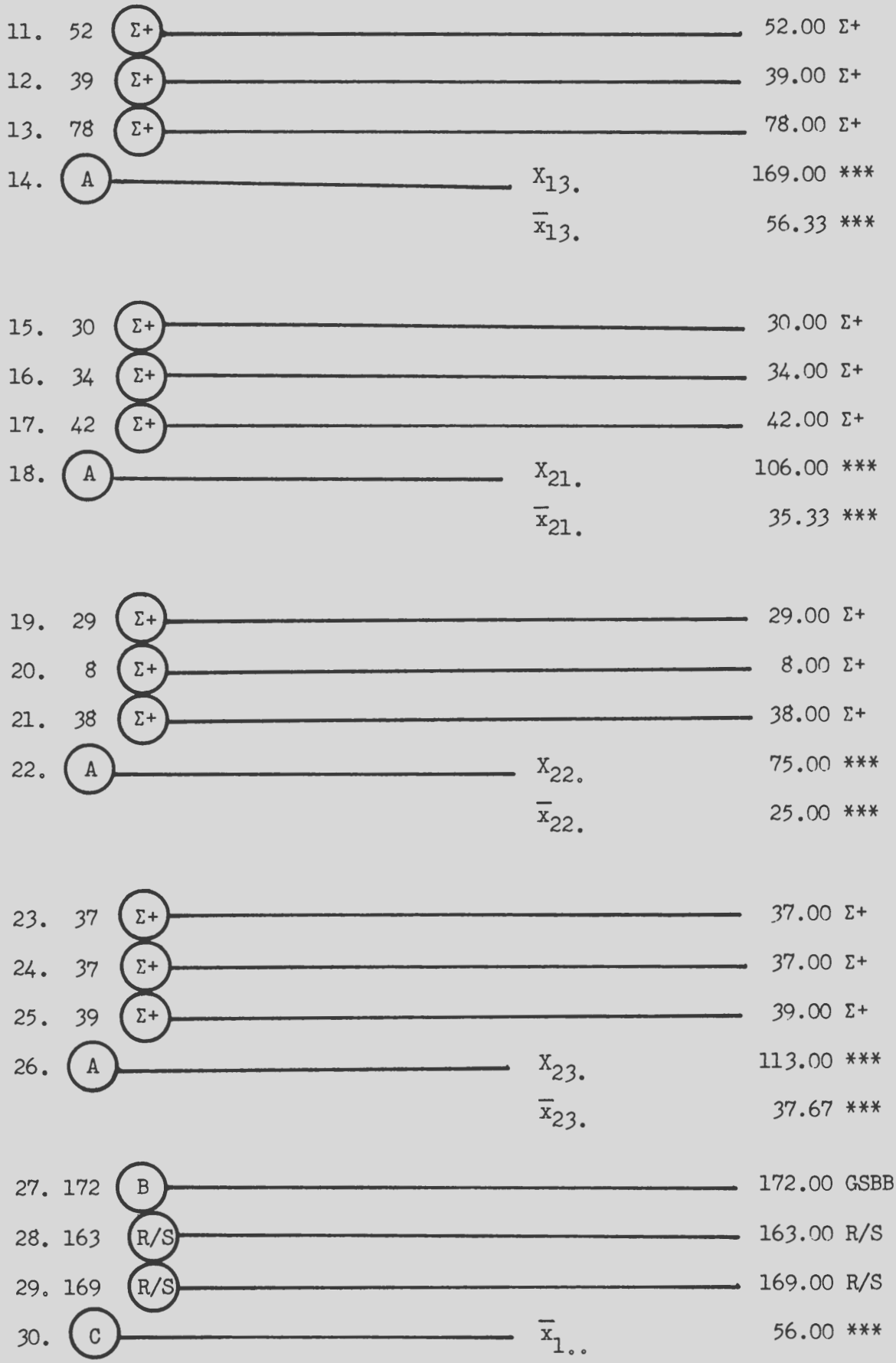


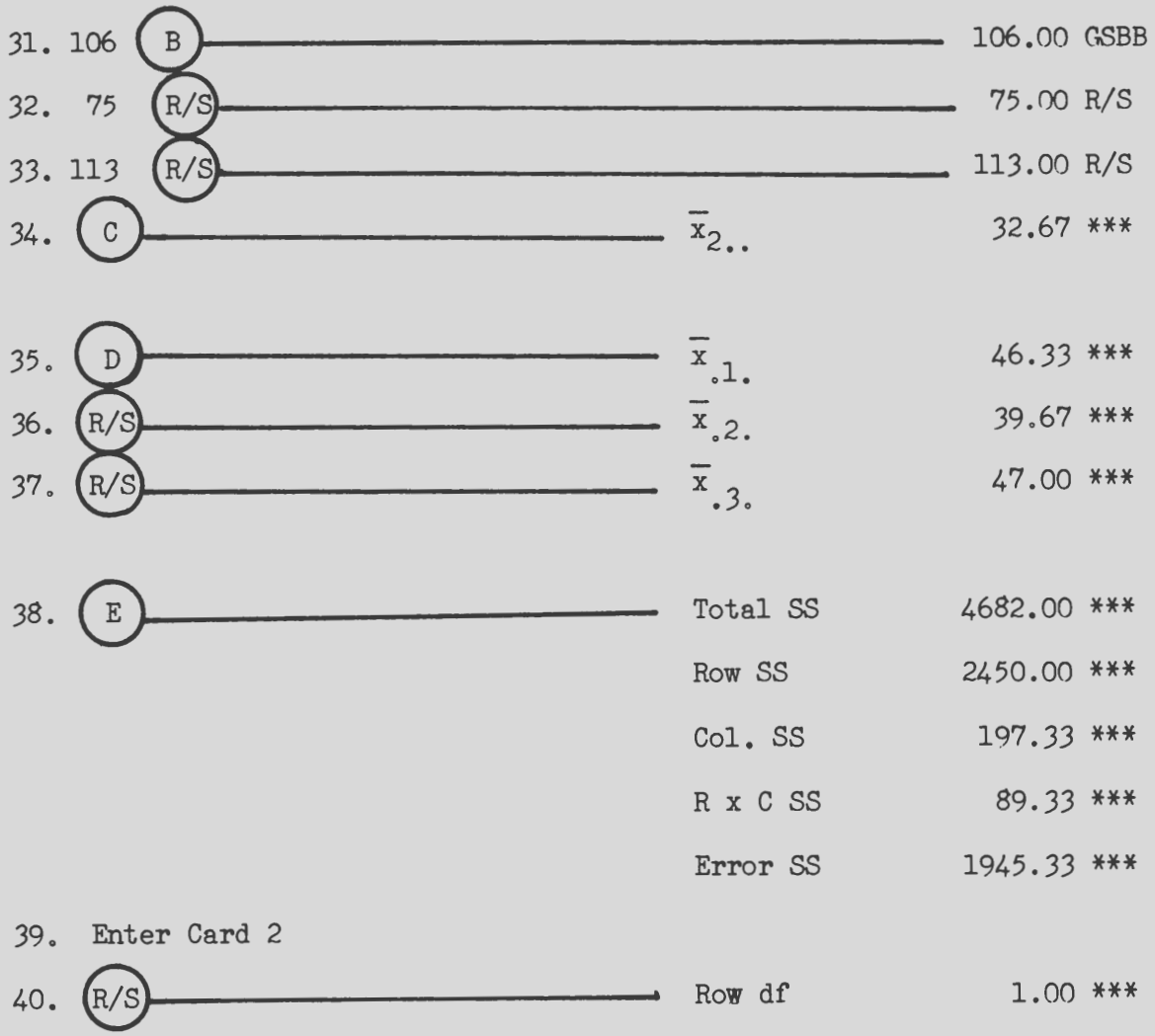
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter Card 2				
2.	Calculate degrees of freedom (df)		R/S		Row d.f.
	and mean squares (MS)				Row M.S.
					Column df
					Column MS
					R x C df
					R x C MS
					Error df
					Error MS
3.	F-values where all factors are fixed		A		F (Row)
					F (Col.)
					F (R x C)
4.	F-values where Rows are fixed and Columns		B		F (Row)
	are random				F (Col.)
					F (R x C)
5.	F-values where all factors are random		C		F (Row)
					F (Col.)
					F (R x C)
6.	F-values where Rows are random and Columns		D		F (Row)
	are fixed				F (Col.)
					F (R x C)
7.	Coefficient of Variation (C.V.)		E		C.V.

EXAMPLE

Rows \ Columns	1	2	3
	76	57	52
1	54	53	39
	42	53	78
	30	29	37
2	34	08	37
	42	38	39

	Keystrokes		Output
1.	Enter Card 1		
2.	f b		0.00 GSBb
3.	76 Σ+		76.00 Σ+
4.	54 Σ+		54.00 Σ+
5.	42 Σ+		42.00 Σ+
6.	A	$X_{11.}$	172.00 ***
		$\bar{x}_{11.}$	57.33 ***
7.	57 Σ+		57.00 Σ+
8.	53 Σ+		53.00 Σ+
9.	53 Σ+		53.00 Σ+
10.	A	$X_{12.}$	163.00 ***
		$\bar{x}_{12.}$	54.33 ***


11. 52 Σ+ 52.00 Σ+
12. 39 Σ+ 39.00 Σ+
13. 78 Σ+ 78.00 Σ+
14. A $X_{13.}$ 169.00 ***
$\bar{x}_{13.}$ 56.33 ***
15. 30 Σ+ 30.00 Σ+
16. 34 Σ+ 34.00 Σ+
17. 42 Σ+ 42.00 Σ+
18. A $X_{21.}$ 106.00 ***
$\bar{x}_{21.}$ 35.33 ***
19. 29 Σ+ 29.00 Σ+
20. 8 Σ+ 8.00 Σ+
21. 38 Σ+ 38.00 Σ+
22. A $X_{22.}$ 75.00 ***
$\bar{x}_{22.}$ 25.00 ***
23. 37 Σ+ 37.00 Σ+
24. 37 Σ+ 37.00 Σ+
25. 39 Σ+ 39.00 Σ+
26. A $X_{23.}$ 113.00 ***
$\bar{x}_{23.}$ 37.67 ***
27. 172 B 172.00 GSBB
28. 163 R/S 163.00 R/S
29. 169 R/S 169.00 R/S
30. C $\bar{x}_{1..}$ 56.00 ***


31. 106
B
106.00 GSBB
32. 75
R/S
75.00 R/S
33. 113
R/S
113.00 R/S
34.
C
$\bar{x}_{2..}$
32.67 ***
35.
D
$\bar{x}_{.1.}$
46.33 ***
36.
R/S
$\bar{x}_{.2.}$
39.67 ***
37.
R/S
$\bar{x}_{.3.}$
47.00 ***
38.
E
Total SS
4682.00 ***
Row SS
2450.00 ***
Col. SS
197.33 ***
R x C SS
89.33 ***
Error SS
1945.33 ***
39. Enter Card 2
40.
R/S
Row df
1.00 ***
Row MS
2450.00 ***
Col. df
2.00 ***
Col. MS
98.67 ***
R x C df
2.00 ***
R x C MS
44.67 ***
S. Error df
12.00 ***
S. Error MS
162.11 ***

41.	(A)	R & C - Fixed	F - Row	15.11 ***
			F - Col.	0.61 ***
			F - R x C	0.28 ***
42.	(B)	R - Fixed	F - Row	54.85 ***
		C - Random	F - Col.	0.61 ***
			F - R x C	0.28 ***
43.	(C)	R & C - Random	F - Row	54.85 ***
			F - Col.	2.21 ***
			F - R x C	0.28 ***
44.	(D)	R - Random	F - Row	15.11 ***
		C - Fixed	F - Col.	2.21 ***
			F - R x C	0.28 ***
45.	(E)		CV	28.72 ***

Note: Under all four conditions, we would reject $H_o : \mu_1 = \mu_2 = \ldots = \mu_r$ and accept $H_1 : \mu_1 \neq \mu_2 \neq \ldots \neq \mu_r$, and we could not reject the hypotheses $H_o : \mu_1 = \mu_2 = \ldots = \mu_c$ and that there is no row by column interaction.

Example

Card 1

```
        GSBb
 76.00   Σ+
 54.00   Σ+
 42.00   Σ+
        GSBA
172.00  ***
 57.33  ***

 57.00   Σ+
 53.00   Σ+
 53.00   Σ+
        GSBA
163.00  ***
 54.33  ***

 52.00   Σ+
 39.00   Σ+
 78.00   Σ+
        GSBA
169.00  ***
 56.33  ***

 30.00   Σ+
 34.00   Σ+
 42.00   Σ+
        GSBA
106.00  ***
 35.33  ***

 29.00   Σ+
  8.00   Σ+
 38.00   Σ+
        GSBA
 75.00  ***
 25.00  ***

 37.00   Σ+
 37.00   Σ+
 39.00   Σ+
        GSBA
113.00  ***
 37.67  ***

172.00 GSBB
163.00  R/S
169.00  R/S
        GSBC
 56.00  ***

106.00 GSBB
 75.00  R/S
113.00  R/S
        GSBC
 32.67  ***
```

```
        GSBD
 46.33  ***
        R/S
 39.67  ***
        R/S
 47.00  ***
        GSBE

4682.00 ***
2450.00 ***
 197.33 ***
  89.33 ***
1945.33 ***
```

Card 2

```
        R/S

   1.00 ***
2450.00 ***

   2.00 ***
  98.67 ***

   2.00 ***
  44.67 ***

  12.00 ***
 162.11 ***
        GSBA
  15.11 ***
   0.61 ***
   0.28 ***
        GSBB
  54.85 ***
   0.61 ***
   0.28 ***
        GSBC
  54.85 ***
   2.21 ***
   0.28 ***
        GSBD
  15.11 ***
   2.21 ***
   0.28 ***
        GSBE
  28.72 ***
```

Card 1 of Program

001	*LBLb	21 16 12	061	R/S	51	121	RCL0	36 00	181	RCLI	36 46
002	CLRG	16-53	062	ST+4	35-55 04	122	GSBc	23 16 13	182	RCL1	36 01
003	P≠S	16-51	063	GSBa	23 16 11	123	R/S	51	183	×	-35
004	CLRG	16-53	064	R/S	51	124	RCL1	36 01	184	÷	-24
005	CLX	-51	065	ST+5	35-55 05	125	GSBc	23 16 13	185	P≠S	16-51
006	RTN	24	066	GSBa	23 16 11	126	R/S	51	186	RCL1	36 01
007	*LBLA	21 11	067	R/S	51	127	RCL2	36 02	187	-	-45
008	P≠S	16-51	068	ST+6	35-55 06	128	GSBc	23 16 13	188	PRTX	-14
009	RCLA	36 11	069	GSBa	23 16 11	129	R/S	51	189	STO4	35 04
010	RCL5	36 05	070	R/S	51	130	RCL3	36 03	190	RCLC	36 13
011	+	-55	071	ST+7	35-55 07	131	GSBc	23 16 13	191	RCLI	36 46
012	STOA	35 11	072	GSBa	23 16 11	132	R/S	51	192	÷	-24
013	RCLB	36 12	073	R/S	51	133	RCL4	36 04	193	RCL1	36 01
014	RCL4	36 04	074	ST+8	35-55 08	134	GSBc	23 16 13	194	-	-45
015	+	-55	075	GSBa	23 16 11	135	R/S	51	195	RCL3	36 03
016	STOB	35 12	076	R/S	51	136	RCL5	36 05	196	-	-45
017	RCLC	36 13	077	ST+9	35-55 09	137	GSBc	23 16 13	197	RCL4	36 04
018	RCL4	36 04	078	GSBa	23 16 11	138	R/S	51	198	-	-45
019	X²	53	079	RTN	24	139	RCL6	36 06	199	PRTX	-14
020	+	-55	080	*LBLC	21 13	140	GSBc	23 16 13	200	STO5	35 05
021	STOC	35 13	081	P≠S	16-51	141	R/S	51	201	RCLE	36 15
022	RCLD	36 14	082	RCL1	36 01	142	RCL7	36 07	202	PRTX	-14
023	RCL9	36 09	083	1	01	143	GSBc	23 16 13	203	R/S	51
024	+	-55	084	+	-55	144	R/S	51			
025	STOD	35 14	085	STO1	35 01	145	RCL8	36 08			
026	RCL5	36 05	086	RCL4	36 04	146	GSBc	23 16 13			
027	RCL4	36 04	087	RCL9	36 09	147	R/S	51			
028	PRTX	-14	088	RCLI	36 46	148	RCL9	36 09			
029	RCL9	36 09	089	×	-35	149	GSBc	23 16 13			
030	÷	-24	090	RCL1	36 01	150	RTN	24			
031	PRTX	-14	091	÷	-24	151	*LBLE	21 15			
032	X²	53	092	÷	-24	152	SPC	16-11			
033	RCL9	36 09	093	PRTX	-14	153	RCLA	36 11			
034	STOI	35 46	094	RCL4	36 04	154	RCLB	36 12			
035	×	-35	095	X²	53	155	RCLD	36 14			
036	-	-45	096	ST+0	35-55 00	156	÷	-24			
037	RCLE	36 15	097	0	00	157	STO0	35 00			
038	+	-55	098	STO4	35 04	158	X²	53			
039	STOE	35 15	099	P≠S	16-51	159	RCLD	36 14			
040	0	00	100	SPC	16-11	160	×	-35			
041	STO4	35 04	101	RTN	24	161	STO1	35 01			
042	STO5	35 05	102	*LBLc	21 16 13	162	-	-45			
043	STO9	35 09	103	P≠S	16-51	163	PRTX	-14			
044	SPC	16-11	104	ST+6	35-55 06	164	STO2	35 02			
045	RTN	24	105	X²	53	165	P≠S	16-51			
046	*LBLa	21 16 11	106	ST+7	35-55 07	166	RCL0	36 00			
047	Σ+	56	107	LSTX	16-63	167	RCLI	36 46			
048	RTN	24	108	RCLI	36 46	168	RCL9	36 09			
049	*LBLB	21 12	109	RCL1	36 01	169	RCL1	36 01			
050	ST+0	35-55 00	110	×	-35	170	÷	-24			
051	GSBa	23 16 11	111	÷	-24	171	STO8	35 08			
052	R/S	51	112	PRTX	-14	172	×	-35			
053	ST+1	35-55 01	113	P≠S	16-51	173	÷	-24			
054	GSBa	23 16 11	114	RTN	24	174	P≠S	16-51			
055	R/S	51	115	*LBLD	21 14	175	RCL1	36 01			
056	ST+2	35-55 02	116	P≠S	16-51	176	-	-45			
057	GSBa	23 16 11	117	0	00	177	PRTX	-14			
058	R/S	51	118	STO6	35 06	178	STO3	35 03			
059	ST+3	35-55 03	119	STO7	35 07	179	P≠S	16-51			
060	GSBa	23 16 11	120	P≠S	16-51	180	RCL7	36 07			

Card 2 of Program

001	SPC	16-11
002	RCL3	36 03
003	P≠S	16-51
004	RCL1	36 01
005	1	01
006	-	-45
007	PRTX	-14
008	÷	-24
009	PRTX	-14
010	P≠S	16-51
011	STO6	35 06
012	SPC	16-11
013	RCL4	36 04
014	P≠S	16-51
015	RCL8	36 08
016	1	01
017	-	-45
018	PRTX	-14
019	÷	-24
020	PRTX	-14
021	P≠S	16-51
022	STO7	35 07
023	SPC	16-11
024	RCL5	36 05
025	P≠S	16-51
026	RCL1	36 01
027	1	01
028	-	-45
029	RCL8	36 08
030	1	01
031	-	-45
032	×	-35
033	PRTX	-14
034	÷	-24
035	PRTX	-14
036	P≠S	16-51
037	STO8	35 08
038	SPC	16-11
039	RCLE	36 15
040	P≠S	16-51
041	RCL1	36 46
042	1	01
043	-	-45
044	RCL9	36 09
045	×	-35
046	PRTX	-14
047	÷	-24
048	PRTX	-14
049	P≠S	16-51
050	STO9	35 09
051	RTN	24
052	*LBLA	21 11
053	RCL6	36 06
054	RCL9	36 09
055	÷	-24
056	PRTX	-14
057	RCL7	36 07
058	RCL9	36 09
059	÷	-24
060	PRTX	-14
061	RCL8	36 08
062	RCL9	36 09
063	÷	-24
064	PRTX	-14
065	RTN	24
066	*LBLB	21 12
067	RCL6	36 06
068	RCL8	36 08
069	÷	-24
070	PRTX	-14
071	RCL7	36 07
072	RCL9	36 09
073	÷	-24
074	PRTX	-14
075	RCL8	36 08
076	RCL9	36 09
077	÷	-24
078	PRTX	-14
079	RTN	24
080	*LBLC	21 13
081	RCL6	36 06
082	RCL8	36 08
083	÷	-24
084	PRTX	-14
085	RCL7	36 07
086	RCL8	36 08
087	÷	-24
088	PRTX	-14
089	RCL8	36 08
090	RCL9	36 09
091	÷	-24
092	PRTX	-14
093	RTN	24
094	*LBLD	21 14
095	RCL6	36 06
096	RCL9	36 09
097	÷	-24
098	PRTX	-14
099	RCL7	36 07
100	RCL8	36 08
101	÷	-24
102	PRTX	-14
103	RCL8	36 08
104	RCL9	36 09
105	÷	-24
106	PRTX	-14
107	RTN	24
108	*LBLE	21 15
109	RCL9	36 09
110	√x	54
111	RCL0	36 00
112	÷	-24
113	1	01
114	0	00
115	0	00
116	×	-35
117	PRTX	-14
118	RTN	24
119	R/S	51

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