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# Calculation of the Two-Way Analysis of Variance (ANOVA) Using a Programable "Pocket" Calculator

Paul Evenson

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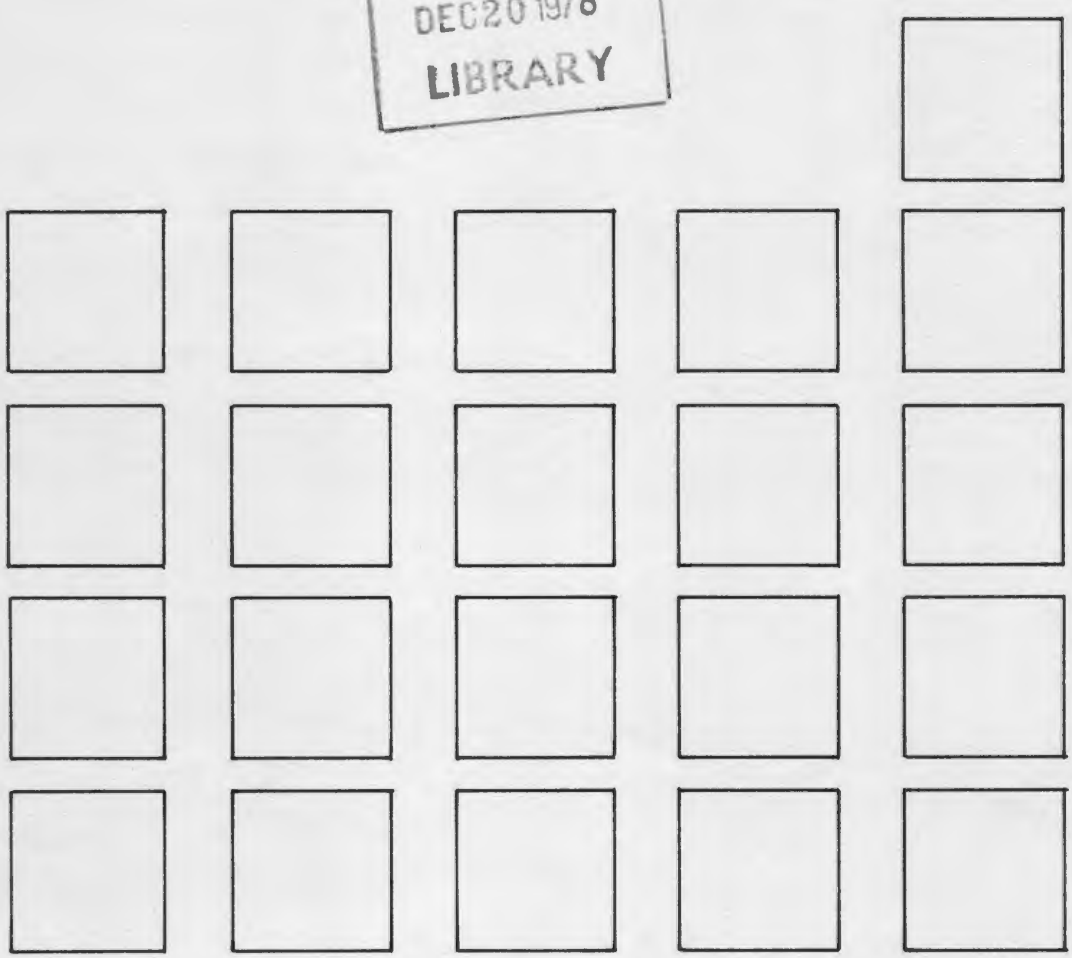

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# Calculation of the Two-Way Analysis of Variance (ANOVA) Using a Programable Pocket Calculator

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The two-way ANOVA is one of the most used experimental designs in statistics. A program is described in this paper which requires the input of each piece of data only once.

The calculator computes the means of all rows and columns, the complete ANOVA including F-tests, the components of variance, and the coefficient of variation. An unlimited number of rows and up to ten columns can be handled by this program, which was written for the Hewlett Packard 97 programable "pocket" calculator.

The two-way ANOVA tests the homogeniety of row means and column means. If there are significant differences among row means, we will reject the null hypothesis that means of populations ($\mu_i$'s) of row factors are equal ($H_o$ : $\mu_1 = \mu_2 = \ldots = \mu_r$), and accept the alternative hypothesis that means of populations of row factors are unequal ($H_1$ : $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_r$). In the same manner, if there are significant differences among column means, we reject $H_o$ : $\mu_1 = \mu_2 = \ldots = \mu_c$ and accept $H_1$ : $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_c$.

The $X_{ij}$ denotes the variable in the $i^{\underline{th}}$ row and the $j^{\underline{th}}$ column, where i = 1, 2, 3, ..., r and j = 1, 2, 3, ..., c. A dot (.) in place of a subscript means that the variables have been summed across the subscript, e.g., $\sum_i X_{ij} = X.j$

## Formulas for Calculation of Row and Column Means

1. row means = $\bar{x}_{i.} = \frac{X_{i.}}{c}$

2. column means = $\bar{x}_{.j} = \frac{X.j}{r}$

# Formulas for Calculation of Two-Way ANOVA

1. correction term = $C = \frac{X_{..}^2}{cr}$

2. total sum of squares = $SS_T = \Sigma X_{ij}^2 - C$

3. row sum of squares = $SS_R = \frac{\Sigma X_{i.}^2}{c} - C$

4. column sum of squares = $SS_c = \frac{\Sigma X_{j.}^2}{r} - C$

5. error sum of squares = $SS_E = SS_T - SS_R - SS_C$

Note: d.f. = degrees of freedom

M.S. = mean square = $\frac{SS}{df}$

M.S.E. = mean square expectations

$\sigma^2$ = variance of population

$s^2$ = variance of a sample

ANOVA

| Source | df | SS | MS | F | MSE |
|---|---|---|---|---|---|
| Total | rc-1 | $SS_T$ | -- | -- | -- |
| Row | r-1 | $SS_R$ | $\frac{SS_R}{r-1} = MS_R$ | $\frac{MS_R}{MS_E}$ | $\sigma^2 + c\sigma_R^2$ |
| Column | c-1 | $SS_C$ | $\frac{SS_C}{c-1} = MS_C$ | $\frac{MS_C}{MS_E}$ | $\sigma^2 = r\sigma_C^2$ |
| Error | (r-1)(c-1) | $SS_E$ | $\frac{SS_E}{(r-1)(c-1)} = MS_E$ | | $\sigma^2$ |

# Formulas for Calculation of Variance Components

$$\sigma^2 = MS_E$$

$$\sigma_C^2 = \frac{MS_C - MS_E}{r}$$

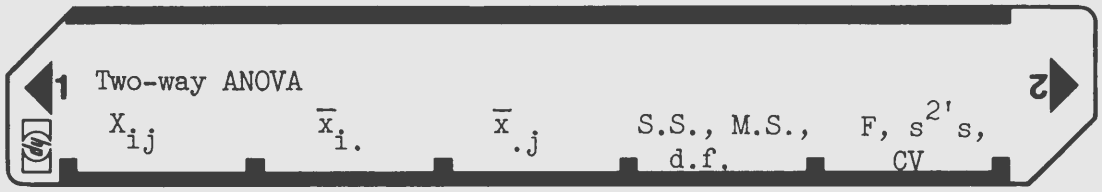
$$\sigma_R^2 = \frac{MS_R - MS_E}{c}$$

# Formula for Calculation of Coefficient of Variation (C.V.)

$$C.V. = \frac{\sqrt{MS_E}}{\bar{x}_{..}} \times 100$$

where $\bar{x}_{..} = \frac{X_{..}}{rc}$

# User Instructions




| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
|---|---|---|---|---|
| 1. | Set print mode switch to "Norm" | | | |
| 2. | Initialize | | f c | |
| | Do 3 ~ 5 for rows i = 1, 2, ..., r | | | |
| 3. | Input $X_{i1}$ | $X_{i1}$ | A | Count |
| 4. | Input $X_{ij}$ for successive values in the $i^{th}$ | | | |
| | row for columns j = 2, 3, ..., c | $X_{ij}$ | R/S | Count |
| 5. | Calculate row mean | | B | $\bar{x}_{i.}$ |
| 6. | Calculate mean for column 1 | | C | $\bar{x}_{.1}$ |
| 7. | Calculate means for successive columns from | | | |
| | column 2, 3, ..., c | | R/S | $\bar{x}_{.2}$ |
| | | | R/S | $\bar{x}_{.3}$ |
| | | | | : |
| | | | | $\bar{x}_{.c}$ |
| 8. | Calculate sum of squares (SS) | | | |
| | Mean Squares (MS) and Degrees of Freedom (d.f.) | | D | Total SS |
| | | | | Row SS |
| | | | | Col. SS |
| | | | | Res. SS |
| | | | | |
| | | | | Row df |
| | | | | Row MS |
| | | | | |
| | | | | Col. df |
| | | | | Col. MS |
| | | | | |
| | | | | Res. df |
| | | | | Res. MS |
| | | | | |
| 9. | Calculate F-values | | E | F (row) |
| | | | | F (col.) |
| | | | | |
| 10. | Calculate components of variance | | R/S | $s_B^2$ |
| | | | | $s_C^2$ |
| | | | | $s^2$ |
| | | | | |
| 11. | Coefficient of Variation (CV) | | R/S | CV |

EXAMPLE

| i \ j | Columns 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Row 1 | 7 | 6 | 8 | 7 |
| Row 2 | 2 | 4 | 4 | 4 |
| Row 3 | 4 | 6 | 5 | 3 |

| | | | | Outputs: |
|---|---|---|---|---|
| 1. | Set Print Mode Switch to "NORM" | | | |
| 2. | f | c | Initialize | GSBc |
| 3. | 7 | A | | 7.00 GSBA |
| 4. | 6 | R/S | | 6.00 R/S |
| 5. | 8 | R/S | | 8.00 R/S |
| 6. | 7 | R/S | | 7.00 R/S |
| 7. | | B | | GSBB |
| | | | $Row_1$ Mean | 7.00 *** |
| 8. | 2 | A | | 2.00 GSBA |
| 9. | 4 | R/S | | 4.00 R/S |
| 10. | 4 | R/S | | 4.00 R/S |
| 11. | 4 | R/S | | 4.00 R/S |
| 12. | | B | | GSBB |
| | | | $Row_2$ Mean | 3.50 *** |
| 13. | 4 | A | | 4.00 GSBA |
| 14. | 6 | R/S | | 6.00 R/S |
| 15. | 5 | R/S | | 5.00 R/S |
| 16. | 3 | R/S | | 3.00 R/S |
| 17. | | B | | GSBB |
| | | | $Row_3$ Mean | 4.50 *** |
| 18. | | C | | GSBC |
| | | | $Col_1$ Mean | 4.33 *** |
| 19. | | R/S | | R/S |
| | | | $Col_2$ Mean | 5.33 *** |

| | | | |
|---|---|---|---|
| 20. | (R/S) | | R/S |
| | | $Col_3$ Mean | 5.67 *** |
| 21. | (R/S) | | R/S |
| | | $Col_4$ Mean | 4.67 *** |
| 22. | (D) | | GSBD |
| | | Total SS | 36.00 *** |
| | | Row SS | 26.00 *** |
| | | Col. SS | 3.33 *** |
| | | Res. SS | 6.67 *** |
| | | Row df | 2.00 *** |
| | | Row MS | 13.00 *** |
| | | Col. df | 3.00 *** |
| | | Col. MS | 1.11 *** |
| | | Residual df | 6.00 *** |
| | | Residual MS | 1.11 *** |
| 23. | (E) | | GSBE |
| | | F (Row) | 11.70 *** |
| | | F (Col.) | 1.00 *** |
| 24. | (R/S) | | R/S |
| | | $s_R^2$ | 2.97 *** |
| | | $s_C^2$ | - 5.666666667 - 09 *** |
| | | $s^2$ | 1.11 *** |
| 25. | (R/S) | | R/S |
| | | CV | 21.08 *** |

Note: Reject $H_o : \mu_1 = \mu_2 = \ldots = \mu_r$ and accept $H_1 : \mu_1 \neq \mu_2 \neq \ldots \neq \mu_r$ with respect to row means. We cannot reject $H_o : \mu_1 = \mu_2 = \ldots = \mu_c$ with respect to column means.

# Example

```
                GSBC
           7.00 GSBA
           6.00  R/S
           8.00  R/S
           7.00  R/S
                GSBB
           7.00  ***
           2.00 GSBA
           4.00  R/S
           4.00  R/S
           4.00  R/S
                GSBB
           3.50  ***
           4.00 GSBA
           6.00  R/S
           5.00  R/S
           3.00  R/S
                GSBB
           4.50  ***
                GSBC

           4.33  ***
                 R/S
           5.33  ***
                 R/S
           5.67  ***
                 R/S
           4.67  ***
                GSBD

          36.00  ***
          26.00  ***
           3.33  ***
           6.67  ***

           2.00  ***
          13.00  ***

           3.00  ***
           1.11  ***

           6.00  ***
           1.11  ***
                GSBE

          11.70  ***
           1.00  ***
                 R/S

           2.97  ***
-5.666666667-09  ***
           1.11  ***
                 R/S
          21.08  ***
```

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# PROGRAM

```
001  *LBLc  21 16 13
002   CLRG     16-53
003   P≠S      16-51
004   CLRG     16-53
005   CLX        -51
006   RTN         24
007  *LBLa  21 16 11
008   P≠S      16-51
009   ST+0  35-55 00
010    X²         53
011   ST+1  35-55 01
012   RCL2     36 02
013      1        01
014      +       -55
015   STO2     35 02
016   P≠S      16-51
017   RTN         24
018  *LBLA     21 11
019   ST+0  35-55 00
020   GSBa  23 16 11
021    R/S        51
022   ST+1  35-55 01
023   GSBa  23 16 11
024    R/S        51
025   ST+2  35-55 02
026   GSBa  23 16 11
027    R/S        51
028   ST+3  35-55 03
029   GSBa  23 16 11
030    R/S        51
031   ST+4  35-55 04
032   GSBa  23 16 11
033    R/S        51
034   ST+5  35-55 05
035      5        05
036   GSBa  23 16 11
037    R/S        51
038   ST+6  35-55 06
039   GSBa  23 16 11
040    R/S        51
041   ST+7  35-55 07
042   GSBa  23 16 11
043   ST+8  35-55 08
044   GSBa  23 16 11
045    R/S        51
046   ST+9  35-55 09
047   GSBa  23 16 11
048    RTN        24
049  *LBLB     21 12
050   P≠S      16-51
051   RCL3     36 03
052      1        01
053      +       -55
054   STO3     35 03
055   RCL0     36 00
056   RCL2     36 02
057   RCL3     36 03
058      ÷       -24
059      ÷       -24
060   PRTX       -14
061   RCL0     36 00
062   ST+4  35-55 04
063    X²         53
064   ST+5  35-55 05
065      0        00
066   STO0     35 00
067   P≠S      16-51
068   RTN         24
069  *LBLb  21 16 12
070   P≠S      16-51
071   ST+6  35-55 06
072    X²         53
073   ST+7  35-55 07
074   LSTX     16-63
075   RCL3     36 03
076      ÷       -24
077   PRTX       -14
078   P≠S      16-51
079   RTN         24
080  *LBLC     21 13
081    SPC      16-11
082   RCL0     36 00
083   GSBb  23 16 12
084    R/S        51
085   RCL1     36 01
086   GSBb  23 16 12
087    R/S        51
088   RCL2     36 02
089   GSBb  23 16 12
090    R/S        51
091   RCL3     36 03
092   GSBb  23 16 12
093    R/S        51
094   RCL4     36 04
095   GSBb  23 16 12
096    R/S        51
097   RCL5     36 05
098   GSBb  23 16 12
099    R/S        51
100   RCL6     36 06
101   GSBb  23 16 12
102    R/S        51
103   RCL7     36 07
104   GSBb  23 16 12
105    R/S        51
106   RCL8     36 08
107   GSBb  23 16 12
108    R/S        51
109   RCL9     36 09
110   GSBb  23 16 12
111    RTN        24
112  *LBLD     21 14
113    SPC      16-11
114    P≠S      16-51
115   RCL1     36 01
116   RCL4     36 04
117    X²         53
118   RCL2     36 02
119      ÷       -24
120   STO4     35 04
121      -       -45
122   PRTX       -14
123   STO1     35 01
124   RCL5     36 05
125   RCL2     36 02
126   RCL3     36 03
127      ÷       -24
128      ÷       -24
129   RCL4     36 04
130      -       -45
131   PRTX       -14
132   STO5     35 05
133   RCL7     36 07
134   RCL3     36 03
135      ÷       -24
136   RCL4     36 04
137      -       -45
138   PRTX       -14
139   STO7     35 07
140   RCL1     36 01
141   RCL5     36 05
142      -       -45
143   RCL7     36 07
144      -       -45
145   PRTX       -14
146   STO8     35 08
147    SPC      16-11
148   RCL5     36 05
149   RCL3     36 03
150      1        01
151      -       -45
152   PRTX       -14
153      ÷       -24
154   STO5     35 05
155   PRTX       -14
156    SPC      16-11
157   RCL7     36 07
158   RCL2     36 02
159   RCL3     36 03
160      ÷       -24
161      1        01
162      -       -45
163   PRTX       -14
164      ÷       -24
165   STO7     35 07
166   PRTX       -14
167    SPC      16-11
168   RCL8     36 08
169   RCL2     36 02
170   RCL3     36 03
171      ÷       -24
172      1        01
173      -       -45
174   RCL3     36 03
175      1        01
176      -       -45
177      x       -35
178   PRTX       -14
179      ÷       -24
180   PRTX       -14
181   STO8     35 08
182    RTN        24
183  *LBLE     21 15
184    SPC      16-11
185   RCL5     36 05
186   RCL8     36 08
187      ÷       -24
188   PRTX       -14
189   RCL7     36 07
190   RCL8     36 08
191      ÷       -24
192   PRTX       -14
193    R/S        51
194    SPC      16-11
195   RCL5     36 05
196   RCL8     36 08
197      -       -45
198   RCL2     36 02
199   RCL3     36 03
200      ÷       -24
201      ÷       -24
202   PRTX       -14
203   RCL7     36 07
204   RCL8     36 08
205      -       -45
206   RCL3     36 03
207      ÷       -24
208   PRTX       -14
209   RCL8     36 08
210   PRTX       -14
211    R/S        51
212    ƒX         54
213   RCL6     36 06
214   RCL2     36 02
215      ÷       -24
216      ÷       -24
217      1        01
218      0        00
219      0        00
220      x       -35
221   PRTX       -14
222    RTN        24
223    R/S        51
```