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1978

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Recommended Citation

Evenson, Paul, "Linear Regression Analysis Using a Programmable Pocket Calculator" (1978). *Agricultural Experiment Station Technical Bulletins*. 52.

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Linear Regression Analysis Using a Programmable Pocket Calculator

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Agricultural Experiment Station
South Dakota State University
Brookings, South Dakota

Linear Regression Analysis Using a Programmable Pocket Calculator

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Linear regression is an extremely useful "least squares" technique for fitting a linear equation to a set of data.

This program calculates means, sums of squares, and sums of cross-products of the dependent and independent values which are entered only once. It also calculates the slope and intercept of the line as well as the coefficients of determination and correlation. The program calculates a predicted dependent variable from a given independent variable and a predicted independent variable from a given dependent variable.

The program computes the reduction in sum of squares due to regression, residual sum of squares and degrees of freedom, variance about regression, standard error of the estimate, the standard deviation about the slope, and the t-test on the

slope of the line.

Confidence limits are calculated about the slope, about a predicted \hat{Y} , about a predicted future \hat{Y} , and about a predicted future \hat{y} made up of n_1 observations. This program was written for a Hewlett-Packard 97 programmable "pocket" calculator.

The symbol Y_i denotes the i^{th} dependent variable where i is the i^{th} pair and $i = 1, 2, \dots, n$. The symbol X_i denotes the i^{th} independent variable in the i^{th} pair.

The main hypothesis tested in this analysis is $H_0 : B = 0$, which says that the true slope of the regression line is equal to zero, or to say it another way, X has no influence on Y . The alternative hypothesis is $H_1 : B \neq 0$ or that X does influence Y . The null hypothesis ($H_0 : B = 0$) can be tested by several different methods.

Formulas Used in Regression Analysis

1. Means, sum of squares and sum of cross-products of dependent and independent variables.

a. mean of $X = \frac{\sum X_i}{n} = \bar{x}$

b. sum of squares of $X = \sum X_i^2 - \frac{(\sum X_i)^2}{n} = \sum x^2$

c. mean of Y = $\frac{\sum Y_i}{n} = \bar{y}$

d. sum of squares of Y = $\sum Y_i^2 - \frac{(\sum Y_i)^2}{n} = \sum y^2$

e. sum of cross-products = $\sum X_i Y_i - \frac{(\sum X_i)(\sum Y_i)}{n} = \sum xy$

2. Slope, intercept, coefficients of determination and correlation.

a. slope = $b = \frac{\sum xy}{\sum x^2}$

b. intercept = $a = \bar{y} - b\bar{x}$

c. coefficient of determination = $r^2 = \frac{(\sum xy)^2}{(\sum x^2)(\sum y^2)}$

d. correlation coefficient = $r = \sqrt{r^2}$

3. Prediction of dependent variable from a given independent variable.

$$\hat{Y} = a + bX$$

4. Prediction of independent variable from a given dependent variable.

$$X = \frac{\hat{Y} - a}{b}$$

5. Reduction in sum of squares due to regression, residual sum of squares and degrees of freedom, variance about regression, standard error of the estimate, standard deviation about the slope, and t-test on slope.

a. reduction in sum of squares due to regression = $\frac{(\sum xy)^2}{\sum x^2}$

b. residual sum of squares (Res. S.S.) and degrees of freedom (d.f.):

$$\text{Res. SS} = \sum y^2 - \frac{(\sum xy)^2}{\sum x^2} \quad \text{and d.f.} = n - 2$$

c. variance about regression = $s_{y \cdot x}^2 = \frac{\text{Res. SS}}{\text{d.f.}} = \frac{\sum y^2 - \frac{(\sum xy)^2}{\sum x^2}}{n - 2}$

d. standard error of the estimate = $s_{y \cdot x} = \sqrt{s_{y \cdot x}^2}$

e. standard deviation about the slope = $s_b = \frac{s_{y \cdot x}}{\sqrt{\sum x^2}}$

f. t-test on the slope $t = \frac{b}{s_b}$

6. Confidence limits about the slope =

$$CL(b) = b \pm t_{\alpha} \cdot s_b, \quad l_1 = b - t_{\alpha} \cdot s_b \quad \text{and} \quad l_2 = b + t_{\alpha} \cdot s_b$$

where t_{α} = Student's t at α -level of probability and $n-2$ df.

7. Confidence limits on \hat{Y} with a given value of $X =$

$$CL(\hat{Y}) = a + bX \pm t_{\alpha} \cdot s_{y \cdot x} \sqrt{\frac{1}{n} + \frac{(X-\bar{x})^2}{\Sigma x^2}}$$

where t_{α} = Student's t at $n-2$ df at α - level of probability.

8. Confidence limits on a future \hat{Y} with a given value of $X =$

$$CL(\text{future } \hat{Y}) = a + bX \pm t_{\alpha} \cdot s_{y \cdot x} \sqrt{1 + \frac{1}{n} + \frac{(X-\bar{x})^2}{\Sigma x^2}}$$

where t_{α} is the same as in 7.

9. Confidence limits on a future \hat{y} of sample size n_1 with a given value

$$\text{of } X = CL(\text{future } \hat{y}) = a + bX \pm t_{\alpha} \cdot s_{y \cdot x} \sqrt{\frac{1}{n_1} + \frac{1}{n} + \frac{(X-\bar{x})^2}{\Sigma x^2}}$$

where t_{α} is the same as in 7.

User Instructions

Linear Regression and Correlation

1 Clear $\bar{x} \dots \Sigma xy$ b, a, r^2, r $X \rightarrow \hat{Y}$ $\hat{Y} \rightarrow X$ 2

df, s_b, t $l_1 < b < l_2$ $l_1 < \hat{Y} < l_2$ $l_1 < \text{Fut. } \hat{Y} < l_2$ $l_1 < \text{Fut. } \hat{y} < l_2$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1.	Set switch to "Norm"		<input type="text"/> <input type="text"/>	
2.	Clear Registers		A <input type="text"/> <input type="text"/>	0.00
3.	Do 4-5 for each pair of data from $i = 1, 2, \dots, n$		<input type="text"/> <input type="text"/>	
4.	Enter i^{th} dependent variable	Y_i	\uparrow <input type="text"/> <input type="text"/>	Y_i
5.	Enter i^{th} independent variable	X_i	$\Sigma+$ <input type="text"/> <input type="text"/>	i
6.	In case of error do 7-8 for i^{th} incorrect entry		<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>	
7.	Enter k^{th} dependent variable	Y_k	\uparrow <input type="text"/> <input type="text"/>	Y_k
8.	Enter k^{th} independent variable	X_k	$\Sigma-$ <input type="text"/> <input type="text"/>	$i-1$
9.	Calculate means, sum of squares and sum of cross products		B <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>	\bar{x} Σx^2 \bar{y} Σy^2 Σxy
10.	Calculate slope (b), intercept (a), coefficient of determination (r^2), and correlation coefficient (r).		C <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>	b a r^2

(continued)

r

11.	\hat{Y} from a given value of X	X	D		\hat{Y}
12.	X from a given value of \hat{Y}	\hat{Y}	E		X
13.	Calculate reduction in sum of squares due to regression, $(\sum xy)^2/\sum x^2$; residual sum of squares, Res. SS; degrees of freedom, d.f.; variance about regression, $s_{y.x}^2$; standard error of the estimate, $s_{y.x}$; standard deviation about the slope, s_b ; and t-test on the slope, $t=b/s_b$.		f	a	$(\sum xy)^2/\sum x^2$ Res. SS df $s_{y.x}^2$ $s_{y.x}$ s_b t
14.	Calculate confidence limits about the slope	t_α	f	b	l_1 l_2
15.	Calculate confidence limits about \hat{Y} for a given value of X	X	f	c	\hat{Y} \pm value l_1 l_2
16.	Calculate confidence limits about a future \hat{Y} for a given value of X	X	f	d	\hat{Y} \pm value l_1 l_2
17.	Calculate confidence limits on future \hat{y} made up of n_1 observations at a given value of X	X n_1	†	e	X \hat{y} \pm value l_1 l_2

Example

Pair	Y	X
1	13.5	13.8
2	12.7	13.3
3	12.7	13.7
4	12.4	12.8
5	12.3	12.2
6	13.0	13.4
7	13.8	14.2
8	12.8	13.0

Operation	Output
1. Set switch to "Norm"	
2. A _____	Clears Registers GSBA
3. 13.5 † _____	13.50 Ent †
4. 13.8 $\Sigma+$ _____	13.80 $\Sigma+$
5. 12.7 † _____	12.70 Ent †
6. 13.3 $\Sigma+$ _____	13.30 $\Sigma+$
7. 12.7 † _____	12.70 Ent †
8. 13.7 $\Sigma+$ _____	13.70 $\Sigma+$

9.	12.4	†			12.40	Ent †
10.	12.8	Σ+			12.80	Σ+
11.	12.3	†			12.30	Ent †
12.	12.2	Σ+			12.20	Σ+
13.	13.0	†			13.00	Ent †
14.	13.4	Σ+			13.40	Σ+
15.	13.8	†			13.80	Ent †
16.	14.2	Σ+			14.20	Σ+
17.	12.8	†			12.80	Ent †
18.	13.0	Σ+			13.00	Σ+
19.	B					GSBB
				\bar{x}	13.30	***
				Σx^2	2.78	***
				\bar{y}	12.90	***
				Σy^2	1.88	***
				Σxy	1.98	***
20.	C					GSBC
				b	0.71	***
				a	3.43	***
				r^2	0.75	***
				r	0.87	***
21.	13.0	D		X	13.00	GSBD
				\hat{Y}	12.69	***
22.	13.0	E		\hat{Y}	13.00	GSBE
				X	13.44	***
23.	f	a				GSBa
				$(\Sigma xy)^2 / \Sigma x^2$	1.41	***
				Res. SS	0.47	***
				d.f.	6.00	***
				$s_{y \cdot x}^2$	0.08	***
				$s_{y \cdot x}$	0.28	***
				s_b	0.17	***
				t	4.24	***
24.	2.447	f	b		2.447	GSBb
				l_1	.030	***
				l_2	1.12	***

25.	13.0	f	c	_____	X	13.00	GSBc
					\hat{Y}	12.69	***
					\pm value	0.27	***
					C.L.	12.41	***
						12.96	***
26.	13.0	f	d	_____	X	13.00	GSBd
					\hat{Y}	12.69	***
					\pm value	0.74	***
					C.L.	11.95	***
						13.42	***
27.	13.0	†		_____	X	13.00	Ent †
28.	10.0	f	e	_____	n_1	10.00	GSBe
					\hat{Y}	12.69	***
					\pm value	0.35	***
					C.L.	12.34	***
						13.03	***

Note: $t = 4.24$ which exceeds $t_{.05} = 2.447$ at 6 d.f. Also, C.L. about the slope (0.30, 1.12) do not include zero. Therefore, we reject $H_0 : B = 0$ and accept $H_1 : B \neq 0$ and conclude that the independent variable does influence the dependent variable.

Example

GSBA	12.80	ENT†	13.00	GSBE	0.27	***	
13.50	ENT†	15.00	$\Sigma+$	13.44	***	12.41	***
13.80	$\Sigma+$	GSBB	GSBa		12.96	***	
12.70	ENT†	13.30	***	1.41	***	13.00	GSBd
13.30	$\Sigma+$	2.78	***	0.47	***	12.69	***
12.70	ENT†	12.90	***	6.00	***	0.74	***
13.70	$\Sigma+$	1.88	***	0.08	***	11.95	***
12.40	ENT†	1.98	***	0.28	***	13.42	***
12.80	$\Sigma+$	GSBC	0.17	***	13.00	ENT†	
12.30	ENT†	0.71	***	4.24	***	10.00	GSBe
12.20	$\Sigma+$	3.43	***	2.447	GSBb	12.69	***
13.00	ENT†	0.75	***	0.30	***	0.35	***
13.40	$\Sigma+$	0.87	***	1.12	***	12.34	***
13.80	ENT†	13.00	GSBD	13.00	GSBc	13.03	***
14.20	$\Sigma+$	12.69	***	12.69	***		

Program

001	*LBLA	21 21	061	PRTX	-14	121	+	-55	181	RTN	24
002	CLRG	16-53	062	JX	54	122	LSTX	16-63	182	*LBLc	21 16 15
003	P2S	16-51	063	PRTX	-14	123	+	-55	183	P2S	16-51
004	CLRG	16-53	064	STOI	35 46	124	PRTX	-14	184	STO0	35 00
005	CLX	-51	065	RTN	24	125	RTN	24	185	P2S	16-51
006	RTN	24	066	*LBLD	21 14	126	*LBLc	21 16 13	186	R4	-31
007	*LBLB	21 12	067	RCL0	36 00	127	STOD	35 14	187	STOD	35 14
008	P2S	16-51	068	x	-35	128	RCL1	36 01	188	RCL1	36 01
009	PCL5	36 05	069	RCL3	36 03	129	-	-45	189	-	-45
010	RCL4	36 04	070	+	-55	130	X2	53	190	X2	53
011	RCL9	36 09	071	PRTX	-14	131	RCL5	36 05	191	RCL5	36 05
012	=	-24	072	RTN	24	132	=	-24	192	=	-24
013	STO1	35 01	073	*LBLc	21 15	133	RCL9	36 09	193	RCL9	36 09
014	PRTX	-14	074	RCL3	36 03	134	1/X	52	194	1/X	52
015	X2	53	075	-	-45	135	+	-55	195	+	-55
016	RCL9	36 09	076	RCL0	36 00	136	JX	54	196	P2S	16-51
017	x	-35	077	=	-24	137	RCLc	36 15	197	RCL0	36 00
018	-	-45	078	PRTX	-14	138	x	-35	198	1/X	52
019	STO5	35 05	079	RTN	24	139	RCL4	36 04	199	P2S	16-51
020	PRTX	-14	080	*LBLa	21 16 11	140	x	-35	200	+	-55
021	RCL7	36 07	081	RCL7	36 07	141	RCLD	36 14	201	JX	54
022	RCL6	36 06	082	RCL8	36 08	142	GSBD	23 14	202	RCLc	36 15
023	RCL9	36 09	083	X2	53	143	X2Y	-41	203	x	-35
024	=	-24	084	RCL5	36 05	144	PRTX	-14	204	RCL4	36 04
025	STO2	35 02	085	=	-24	145	-	-45	205	x	-35
026	PRTX	-14	086	PRTX	-14	146	PRTX	-14	206	RCLD	36 14
027	X2	53	087	STOA	35 11	147	LSTX	16-63	207	GSBD	23 14
028	RCL9	36 09	088	-	-45	148	+	-55	208	X2Y	-41
029	x	-35	089	PRTX	-14	149	LSTX	16-63	209	PRTX	-14
030	-	-45	090	STO8	35 12	150	+	-55	210	-	-45
031	STO7	35 07	091	RCL9	36 09	151	PRTX	-14	211	PRTX	-14
032	PRTX	-14	092	2	02	152	RTN	24	212	LSTX	16-63
033	RCL8	36 08	093	-	-45	153	*LBLd	21 16 14	213	+	-55
034	RCL6	36 06	094	PRTX	-14	154	STOD	35 14	214	LSTX	16-63
035	RCL4	36 04	095	STOC	35 13	155	RCL1	36 01	215	+	-55
036	RCL9	36 09	096	=	-24	156	-	-45	216	PRTX	-14
037	=	-24	097	PRTX	-14	157	X2	53	217	RTN	24
038	x	-35	098	STOD	35 14	158	RCL5	36 05	218	R/S	51
039	-	-45	099	JX	54	159	=	-24			
040	STO8	35 08	100	PRTX	-14	160	RCL9	36 09			
041	PRTX	-14	101	STOE	35 15	161	1/X	52			
042	RTN	24	102	RCL5	36 05	162	+	-55			
043	*LBLC	21 13	103	JX	54	163	1	01			
044	RCL2	36 02	104	=	-24	164	+	-55			
045	RCL8	36 08	105	PRTX	-14	165	JX	54			
046	RCL5	36 05	106	STO6	35 06	166	RCLc	36 15			
047	=	-24	107	RCL0	36 00	167	x	-35			
048	PRTX	-14	108	X2Y	-41	168	RCL4	36 04			
049	STO0	35 00	109	=	-24	169	x	-35			
050	RCL1	36 01	110	PRTX	-14	170	RCLD	36 14			
051	x	-35	111	RTN	24	171	GSBD	23 14			
052	-	-45	112	*LBLb	21 16 12	172	X2Y	-41			
053	PRTX	-14	113	STO4	35 04	173	PRTX	-14			
054	STO3	35 03	114	RCL6	36 06	174	-	-45			
055	RCL8	36 08	115	x	-35	175	PRTX	-14			
056	X2	53	116	RCL0	36 00	176	LSTX	16-63			
057	RCL5	36 05	117	X2Y	-41	177	+	-55			
058	=	-24	118	-	-45	178	LSTX	16-63			
059	RCL7	36 07	119	PRTX	-14	179	+	-55			
060	=	-24	120	LSTX	16-63	180	PRTX	-14			

Published in accordance with an Act passed in, 1881 by the 14th Legislative Assembly, Dakota Territory, establishing the Dakota Agricultural College and with the Act of re-organization passed in 1887 by the 17th Legislative Assembly, which established the Agricultural Experiment Station at South Dakota State University.
300 printed at an estimated cost of 43 cents each—11-78gly—3033A