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### Vibratory Feeder Bowl Analysis

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#### ABSTRACT

Vibratory feeder bowls are used to feed small parts into various stations. At each station the parts will be rejected or accepted depending on the orientation of the part. This analysis was for Royal Plastics, Inc. who was looking to increase the production by increasing the part feed rate of one of their vibratory feeder bowls. As the parts are orientated they will be fed to a new station and often used in robotic systems that produce larger assembled products. The scope of this analysis is to produce a detailed analysis of the vibration system required to move parts through the system. Using MATLAB, we were able to model the system and show how different spring configuration would affect the flow of the parts through the system. From this analysis, we were able in to increase production and reliability of the system.

#### INTRODUCTION

This research will be focused on improving the real time application of a vibratory feeder bowl as shown in Figure 1. Royal Plastics, Inc. approached us to improve the function of a feeder bowl in order to speed up the cycle time on the machine. This improvement will increase their productivity on the assembly line.



Figure 1: A Vibratory Feeder Bowl at Royal Plastics, Inc.

Our initial analysis was to determine how we could increase the vibration of the feeder bowl in order to feed parts at a faster rate to the next step in production. A simple way of increasing the velocity would be to reduce the amount of resistance applied it the bowl. In order to obtain the spring constant applied to the motion of the bowl we will determine the correct combination of steel plates used in the mounts. We will do this by modeling all spring, damping, and forces applied to the vibratory feeder bowl through modeling.

Motion of the feeder bowl is limited to the radial direction with one degree of freedom (DOF). We can model this to show that all springs (steel plates) are in parallel and at an identical radius from the center of the bowl. Simplifying the model will give us a single spring constant. The dampening from the springs will be assumed to be negligible and the only dampening on the feed bowl would be viscous effects from the air. Using this information, the entire feeder bowl was then broken down into a simple one DOF model, as shown in Figure 2. This model contains a single spring constant,  $K_{eq}$ , and a single viscous dampener,  $C_{eq}$ .

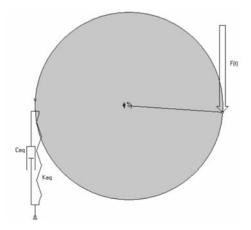


Figure 2: Simplified Free Body Diagram of the Feeder Bowl

#### **METHODS**

The spring constant is represented as equation (1). The spring constant is modeled so simply because each spring will deflect the same amount, which is the definition of a parallel spring.

$$k_{T} = k_{1} + k_{2} + k_{3} + k_{4} \tag{1}$$

The viscous dampener can be shown as equation (2). Since the viscous dampener is air this was just shown as a simple c.

$$c = \frac{\pi * \mu * D^2 * (l-h)}{(2d)} + \frac{\pi * \mu * D^3}{(32*h)}$$
(2)

Where:

µ is the viscosity of the airD is the assumed diameter of the affected aird is the diameter of the feeder bowlh is the height of the air from the bottom of the feeder bowll is the length of the affected air on the sides of the feeder bowl

For the equation of motion, we start out with Newton's Second Law of motion, equation (3). Then the moments created by the excitation force, spring force, and damping force were set into equation (3) and the results are shown in equation (4).

$$\Sigma M_0 = I_0 * \ddot{\theta}$$

$$I_0 * \ddot{\theta} = -c * \dot{x} * r - k_T * x * r + F(t) * r$$
(3)

$$\begin{aligned} x &= r * \theta \\ \dot{x} &= r * \dot{\theta} \end{aligned} \tag{4}$$
$$I_0 * \ddot{\theta} &= -c * r^2 * \dot{\theta} - k_T * r^2 * \theta + F(t) * r \end{aligned}$$

Where:

 $M_o$  is the resultant moment about point O

 $I_o$  is the mass moment of inertia about point O

 $F_{(t)}$  is the excitation force

 $\theta$  is rotational angle of the feeder bowl

r is the radius of the feeder bowl

Assuming that F (t) = Fo cos ( $\omega^*$ t) and the negative terms are moved to the left side of the equation and the results of such equation will be shown in equation (5). This equation will represent the equation of motion for the vibration analysis of the feed bowl mechanism. (5)

$$I_0 * \ddot{\theta} + c * r^2 * \dot{\theta} + k_T * r^2 * \theta = F_0 \cos(\omega * t) * r$$
 The

mass moment of inertia, and the natural frequency,  $\omega_n$ , can be calculated by equation (6) and equation (7) respectively.

$$I_0 = \frac{1}{2}m * r^2$$
(6)

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} \tag{7}$$

Substituting equation (6) into equation (5) would yield equation (8) for equation of motion.

$$\left(\frac{1}{2}m*r^2\right)*\ddot{\theta}+(c*r^2)*\dot{\theta}+(k_T*r^2)*\theta=F_0\cos\left(\omega*t\right)*r$$
(8)

These equations will be used to find the steady state response as well as the transient response. Once we find our information we will know whether the system is undamped, under-damped, over-damped, or critically damped. Then we will know which total response equation and what other information needs to be found for a working model to be constructed.

$$k_{eq} = k_T * r^2 \tag{9}$$

$$m_{eq} = \frac{1}{2}m * r^2$$
 (10)

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{(k_T * r^2)}{(\frac{1}{2}m * r^2)}} = \sqrt{\frac{k_T}{\frac{1}{2}m}} = \sqrt{\frac{1.15e7 N/m}{\frac{1}{2}*(40kg)}} = 756.71 \ rad/s \tag{11}$$

$$c_{eq} = c * r^2$$

$$\zeta = \frac{c_{eq}}{2*\omega_n * m_{eq}} = \frac{(c*r^2)}{2*\omega_n * m_{eq}} = \frac{(3.11e - 6Nm * .2286m^2)}{2*756.71 \ rad + 1.045 \ kg \ m^2} = 1.03e - 10 \tag{12}$$

Where:

 $k_{eq}$  is the equivalent stiffness

 $m_{eq}$  is the equivalent mass moment of inertia

 $\zeta$  is the dampening ratio

Due to  $\zeta$  (the dampening ratio) being relatively close to zero and thus causing no considerable change in total displacement we will now assume the vibratory feeder bowl to be undamped.

Undamped system of equations:

$$\theta(t) = \theta_{h}(t) + \theta_{p}(t) \tag{13}$$

In equation (13),  $\theta_h(t)$  is the homogenous solution,  $\theta_p(t)$  is the particular solution, and  $\theta(t)$  is the total solution. Under normal operating conditions, the particular solution dominates the vibration responses. We may ignore the homogeneous solution. From reference [1], we have the particular solution as follows:

$$\theta_{\rm p}(t) = \Theta \, \ast \cos(\omega t) \tag{14}$$

$$\Theta = \frac{F_0}{\sqrt{(k - m\omega^2)^2}} = \frac{F_0 * r/k_{eq}}{1 - (\frac{\omega}{\omega_n})^2} = \frac{\frac{20000N * .2286m}{1.15e7N * m}}{1 - (\frac{376.99 \text{ rad/s}}{756.71 \text{ rad/s}})^2} = .04445 \text{ rad}$$
  

$$\theta_p(t) = .04445 * \cos(377t)$$
  

$$\theta(t) = \theta_p(t)$$
(15)  

$$\theta(t) = .04445 * \cos(377t)$$

In equations (14) and (15), is the angular magnitude of the feeder bowl and  $\omega$  is the frequency of the excitation force.

#### RESULTS

We used the equation (15) to plot the vibration response curves of the feeder bowls under two different spring configurations. Both curves are plotted by Matlab by using the codes given in the appendix. Figure 3 represents the curve for the original spring configuration before the redesign, and Figure 4 shows the curve for the redesigned spring configuration.

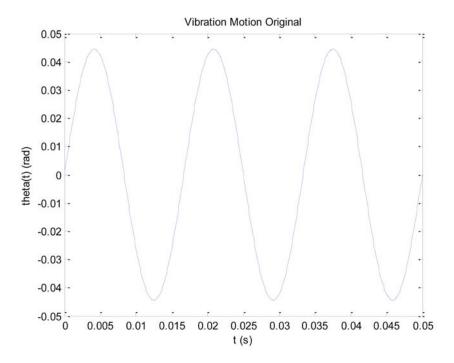


Figure 3. Motion of the Feeder Bowl with Original Spring Configuration

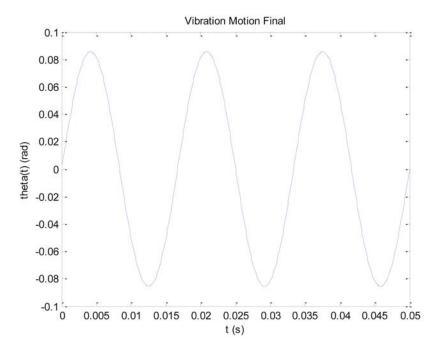


Figure 4. Motion of the Feeder Bowl with Redesigned Spring Configuration

#### DISCUSSION

Using equation (15) we obtain the vibration response of the feeder bowl under the excitation force, and can now better understand the effects of the spring constant on the velocity and displacement in terms of  $\theta$  for the feeder bowl. Through analysis of the system using Matlab, we were able to change the total spring constant and determine what sizes of the hot rolled steel plates were necessary to achieve maximum displacement without causing damage to the machine.

The current feeder bowl design will only allow .09 rad of motion. This is based on the distance measured between the electromagnet providing the force and the bowl. We relate this distance x to  $\theta$  by the equation x=r\* $\theta$ . By analyzing Figure 3 we find our total displacement is currently .04445 rad.

Since we only have two sizes of steel plates in the current design to remove and make adjustments with, we will start by removing one of the thicker plates. This change in the spring constant only yielded 1/3 of the total desired change in maximum displacement. By deduction and reasoning we removed three large plates and were able to produce Figure 4. The figure shows the new displacement in radians that would occur by removing three large plates from the mounting legs.

Our desired  $\theta$  was .09 and by analysis of the graphs this change in the spring constant shows to be exactly what we desired. With the new spring setup we are able to maximize the vibration of the feeder bowl without causing damage to the spring. We now plan on taking the results of our study and recommendations to Royal Plastics, Inc. Hopefully our findings will benefit them and their business.

#### LIMITATIONS

The major limitation for the accuracy of the project was that the material dampening was not taken into effect. The material is stiff with internal friction that damps the system, but that analysis is beyond the scope of this analysis.

#### REFERENCES

[1] Rao, S. S. Mechanical Vibrations 4th Edition Pearson/Prentice-Hall, 2004

#### **APPENDIX**

% Vibratory Feeder Bowl

format long

m= 40; %mass of bowl E=  $2.07*10^{11}$ ; %modulus of elasticity I=.13743; %length of springs (meters) width=.077; %width of springs (meters) t1=.00965; %thickness of spings (meters) t2=.00647; %thickness of spings (meters) w=60\*2\*pi; %frequency D= .4572; %diameter of bowl d= .1524; %air around bowl (horizontal) h= .1524; %height of bowl 11= .2032; %air around bowl (vertical) u= 1.82\*10^-5; %viscosity of air x=.001; %max displacement radius=.2286; %radius of bowl

%moment of Inertia for springs I1= (width\*t1^3)/12; I2= (width\*t2^3)/12; %mild steel hot rolled spring equivilence

```
k1=(3*E*I1)/l^3;
k2 = (3*E*I2)/l^3;
kT=5*k1+k2;
keq=kT*radius^2;
meq=.5*m*radius^2;
wn= (keq/(meq))^{(1/2)}; %rad/s
cc=2*meq*wn;
r=w/wn;
ceq = (pi*u*(D^2)*(11-h))/(2*d)+(pi*u*(D^3))/(32*h);
z=ceq/cc;
F0=20000;
Theta = (F0/keq)/((1-r^2)^2)^{(1/2)};
for i = 1:501;
t(i)=(i-1)/10000;
theta(i)=Theta*sin(w*t(i));
thetadot(i)=Theta*w*sin (w*t(i));
end
plot (t,theta, 'k');
```

```
xlabel ('t (s)');
ylabel ('theta (rad)');
```