MULTI-MEMBER ELECTORAL DISTRICTS—DO THEY VIOLATE THE "ONE MAN, ONE VOTE" PRINCIPLE†

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One must be ever aware that the Constitution forbids "sophisticated as well as simple-minded modes of discrimination." . . . However complicated or sophisticated an apportionment scheme might be, it cannot, consistent with the Equal Protection Clause, result in a significant underevaluation of the weight of the votes of certain of a State's citizens merely because of where they happen to reside.[1]

Multi-member electoral district plans, in which two or more representatives are elected at large from a single district, are now in use in legislative systems in a large majority of the fifty states.[2] However, such systems contain inherent inequalities in representation which may be of constitutional magnitude and which heretofore have escaped the attention of scholars, legislators, and the Supreme Court. Now that *Reynolds v. Sims* and its companion cases require both houses of state legislatures to be apportioned substantially on a population basis,[3] the legislatures and the courts will be reevaluating multi-mem-

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1. Reynolds v. Sims, 377 U.S. 533, 563 (1964) (Warren, C.J.); WMCA, Inc. v. Lomenzo, 377 U.S. 633, 653 (1964) (Warren, C.J.) (companion cases).

2. Up-to-date figures are difficult to obtain as systems change in efforts to conform to the reapportionment mandate. As of 1960, all of the state senators from Alaska and Arizona and some senators in sixteen other states were elected from multi-member districts. For the election of members of the lower house, Illinois, Maryland, and Washington used multi-member districts exclusively, and thirty-five states employed mixed systems of single and multi-member districts. Silva, *Compared Values of the Single- and the Multi-Member Legislative District*, 17 WEST POL. Q. 504, 506 (1964). As of March 1962, 2704 out of 5883 seats in the Nation's lower houses were filled from multi-member districts. These represent over 45% of the seats and they were distributed over three-fourths of the states. DAVID & EISENBERG, STATE LEGISLATIVE REDISTRICTING: MAJOR ISSUES IN THE WAKE OF JUDICIAL DECISION 20 (1962); See Klain, *A New Look at the Constituencies: The Need for a Recount and a Reappraisal*, 49 AM. POL. SCI. REV. 1105, 1107 (1955).

3. Reynolds v. Sims, *supra* note 1, at 568 (Ala.) ("We hold that, as a basic constitu-

ber district plans. If, despite their simplistic mathematical appeal and other possible advantages, such plans fail to provide substantially equal representation for all constituents, they will have to be abandoned. The object of this article is to use established techniques of mathematics and political science to analyze multi-member electoral district systems in terms of representation and voting power within the limits established by the Supreme Court. It will show that no mixed system of single and multi-member districts—or system using multi-member districts of different sizes—can provide substantially equal representation or voting power for all citizens. This holds true even in cases of simplified mathematical models which follow the Supreme Court in ignoring many complicating political realities and treating electoral systems merely in terms of the voting rules and the relative numbers of constituents.

This article will briefly examine the "one man, one vote" principle and present a method of measuring effective voting power with which different electoral and representative systems can be evaluated. The measure will be used to analyze systems employing multi-member districts of unequal size and to demonstrate why such arrangements necessarily produce inherent inequities in the voting power and representation of citizen-voters. It will also indicate how this method of analysis may be used in addition to measure the impact of other factors which are also relevant to voting power and effective representation. Finally, the article will explain why several recent Supreme Court decisions have not foreclosed the issue of the constitutionality of multi-member district systems. The multi-member district systems of Arkansas, Georgia, Hawaii, Texas, and Wyoming will be used as examples to demonstrate the extent to which actual systems may disadvantage portions of the electorate.

I. The Method, Its Basis and Its Objective

So far the Supreme Court has effectively limited its consideration of the inequalities of voting power and effective representation in apportionment plans to the relative number of constituents in each electoral district. The Court appears to have given little weight to the strength or cohesiveness of the political parties, the presence and distribution of major ethnic blocs, the power structure within the legislative body

tional standard, the Equal Protection Clause requires that the seats in both houses of a bicameral state legislature must be apportioned on a population basis"); WMCA v. Lomenzo, *supra* note 1 (N.Y.); Maryland Comm. for Fair Representation v. Tawes, 377 U.S. 656 (1964); Davis v. Mann, 377 U.S. 678 (1964) (Va.); Roman v. Sincock, 377 U.S. 695 (1964) (Del.); Lucas v. Forty-Fourth General Assembly, 377 U.S. 713 (1964) (Colo.).

(committees and their chairmen, etc.) and other factors which obviously would affect the ability of a citizen-voter to pick a representative of his choice and to have some influence on which bills pass.[4] The analysis presented here deliberately goes no further. It also considers all voters to be equal in a statistical sense and does no more than study in a more precise way the effects of their distribution among different electoral districts.

This analysis does not pretend to be a complete picture of the representative system and the conclusions do not necessarily reflect all political realities. Rather it is an attempt to measure more accurately in the Supreme Court's own terms the theoretical effects of multi-member districts. If the Court's consideration of such issues continues on this limited level of abstraction, the analysis and its conclusions should serve as a basis for decision. If, on the other hand, the Court decides to take other factors into account, the simple mathematical models presented here may be modified to include the additional assumptions, and similar methods of analysis may be used to present a better picture of legislative realities and to aid the courts in their decisions.

For all of their talk about "one man, one vote," the Justices of the Supreme Court have not stated with sufficient precision the decisive factor which must be "substantially equal" or the way in which it is to be measured and compared. Instead, they have approved one method —that of substantially equally populous districts electing legislators casting one vote—and have refused to invalidate others without proof of inequalities. (See VI *infra*.) One may construe the reapportionment decisions strictly, limiting their holdings to particular circumstances, and conclude that only the relative ability of citizen-voters to vote for their legislators need be equalized.[5] On the other hand the opinions may be read more broadly, giving effect to their underlying purpose

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4. Burns v. Richardson, 384 U.S. 73 (1966); Fortson v. Dorsey, 379 U.S. 433 (1965); cases cited note 3 *supra*; Wesberry v. Sanders, 376 U.S. 1 (1964); Gray v. Sanders, 372 U.S. 368 (1963); see Baker v. Carr, 369 U.S. 186 (1962). The Court has indicated, however, that some weight may be given to other legitimate legislative considerations in considering whether the deviations from the equal population standard are constitutionally permissible. See, *e.g.*, Reynolds v. Sims, *supra* note 1 at 578-81.

5. See, *e.g.*, Baker v. Carr, *supra* note 4, at 188 (complaint alleges "debasement of . . . votes"); *id.* at 208 ("a citizen's right to vote free of arbitrary impairment"); *id.* at 242 ("the question is the extent to which a State may weigh one person's vote more heavily than it does another's." (Douglas, J. concurring)). The debasement of the votes of citizens in the populous counties, the Court said, was similar to the "dilution [caused] by a false tally, . . . a refusal to count votes from arbitrarily selected precincts, . . . and a stuffing of the ballot box." *Id.* at 208. (Citations omitted.) In Reynolds v. Sims, *supra* note 1, at 566, the Court clearly stated "we conclude that the Equal Protection Clause guarantees the opportunity for equal participation by all voters in the election of state legislators."

and many expressions of dicta,[6] to hold that only systems which provide substantially equally effective representation—at least in theory—are constitutional.[7]

The method of analysis proposed in this article does not depend upon one particular interpretation of the reapportionment decisions. On the contrary, it is possible to develop and analyze a mathematical model based upon either criteria. Because the two principles are so closely related, the two models are complementary and not antagonistic; both lead to the conclusion that inequities are inherent in the multi-member districting.

One might wonder, however, whether the two concepts are not only complementary, but also equivalent. Doesn't equality in voting power among citizen-voters automatically produce equal representation? The answer, as will be shown, is that it will do so only if the elected representatives are theoretically equal; *i.e.*, are elected in the same fashion and have the same voting power in the legislature. Conversely, if the elected representatives are not mathematically identical, then there can be serious inequalities in representation even if each district con-

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6. See, *e.g.*, Reynolds v. Sims, *supra* note 1, at 568 ("the Equal Protection Clause demands no less than substantially equal state representation for all citizens. . . ."); WMCA, Inc. v. Lomenzo, *supra* note 1, at 636 ("as a result [of the misapportionment], the Plaintiff's votes are not as effective in either house of the legislature as the votes of other citizens . . . ."); *id.* at 654 ("[because of a] built-in bias against voters living in the states' more populous counties. . . the legislative representation accorded to the urban and suburban area becomes proportionately less as the population of those areas increases."); Maryland Comm. for Fair Representation v. Tawes, *supra* note 3, at 673 ("the proper, and indeed indispensable, subject for judicial focus in a legislative apportionment controversy is the overall representation accorded to the state's voters, in both houses of a bicameral state legislature.").

7. Professor Robert G. Dixon, George Washington Law School, puts it in these words: These are not right-to-vote cases, even though voting is involved. They are representation cases; *i.e.*, they are cases concerning the most interesting, the most complex, the most baffling aspect of any democratic political system, namely, the ascertainment of public feeling on innumerable public policy issues through the medium of periodic, partisan selection of district delegates to a multimembered representative assembly.

Dixon, *Reapportionment Perspectives: What is Fair Representation*, 51 A.B.A.J. 819 (1965), reprinted in 111 CONG. REC., 10601 (daily ed. May 19, 1965).

The effective difference between these two approaches was well stated by Miller and Stokes:

Broadly speaking, the constituency can control the policy actions of the Representative in two alternative ways. The first of these is to choose a Representative who so shares its views that in following his own convictions he does his constituents' will. . . . The second means of constituency control is for the Congressman to follow his (at least tolerably accurate) perceptions of district attitude in order to win reelection.

Constituency Influence in Congress, 57 AM. POL. SCI. REV. 45, 50-51 (1963).

tains equal numbers of citizen-voters.[8] Equal voting power is not syn-
onymous with equal representation.

In the most common legislative situation all representatives are
equal, at least in terms of a simplified mathematical model. Each repre-
sents a single district and has the same number of votes as every
other representative.[9] Given these two conditions, a necessary and
sufficient condition for equal representation is that each district con-
tain substantially the same number of people. In effect, this has been
the holding of recent reapportionment cases which have been confined
to situations involving mathematically identical representatives. But
equality in district population alone does not insure equal representa-
tion if the representatives themselves are in some way unequal (that
is, if the conditions assumed above are changed). As a simple example,
consider a state divided into twenty-one districts of substantially equal
population. The citizens would not be equally represented if the legis-
lators from the ten even-numbered districts could each cast two votes
while those from the remaining eleven destricts could cast only one
vote. Likewise, the residents of the odd-numbered districts would be
underrepresented, despite the equality in district population, if each
even-numbered district elected two representatives at large while the
other districts each elected only one representative. A less obvious
example is if representatives from the ten even-numbered districts auto-
matically become members of a committee which has absolute power
over which bills are put to a vote.

Equal representation, then, demands more than mere equality of
voting power at the citizen-voter level (*i.e.*, substantially equally popu-

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8. There is some uncertainty concerning the numbers which are to be used in calculat-
ing the people of a district who are entitled to equal representation. It is not certain
whether the measure should be based on the number of inhabitants, residents, voters,
etc. *Cf.* Davis v. Mann, *supra* note 3; Burns v. Richardson, *supra* note 4, at 90-97. For
simplicity, this article will use the words "citizen-voters," "voters," "population," "people
represented," etc. interchangeably to indicate that class of people who should be counted
in determining effective representation, without considering the relative merits of using
any particular class as a measure. Where the words "larger" or "smaller" are used with
respect to districts, the reference is to population and not to geographic size.

9. Naturally, no two legislators are ever exact equivalents. Some will be more effective
because seniority will have given them powerful committee positions. Others may be
more persuasive speakers, have more or better political connections, or be in closer
touch with constituents. However, for the sake of constructing a mathematical model to
determine the minimum requirements for equality of representation, they may be said to
be equal or mathematically equivalent if they are elected in identical manners and
have the same voting power. In other words, they are theoretically equal if they are
distinguished only by what are normally regarded as personal characteristics and not
because of inequalities in the electoral or representative system.

lous districts). If the electoral districts have substantially equal popula-
tions, equal representation also requires that all districts elect the same
number of representatives who are equal in the legislature. Conversely,
if the elected representatives are mathematically equivalent, then rep-
resentation will be equal if and only if the districts are of substantially
equal population. This much has been clear without mathematical
analysis. Serious questions arise, however, when the districts are of
substantially unequal populations and the legislators are also unequal
in their voting power, method of election, or both. Two examples of
this are weighted voting[10] and multi-member districts. To resolve
such questions, it is necessary to devise a test of voting power or effec-
tive representation (depending on which interpretation is accepted)
which will include both factors—the ability of the citizen-voter to
elect or affect his representatives and the ability of representatives to
affect the outcome of legislative decisions through voting—in a single
all-inclusive measure.

II. THE MEASURE OF VOTING POWER

A measure of voting power may most easily be developed by consider-
ing a system of direct representation such as voting in a private club
or stockholders' meeting. If all of the members of a given body can
cast only one vote, if no member has a veto, and if there are no other
procedural devices whereby one member has a built-in advantage, then
all members obviously have equal voting power. However, if the mem-
bers do not all cast an equal number of votes, or if one or more have
some type of veto, or if a small committee of only a few members has
absolute control over what measures may be voted upon, the voting
power of the members cannot be equal. Here the solution is not obvi-
ous and a clearly defined measure is necessary.

The ability to cast a vote does not always carry with it voting power
in a meaningful sense. Moreover, the number of votes each participant
may cast is not always an accurate indication of voting strength. For
example, if in a three-man body A has 3 votes, B has 1 vote, and C
has 1 vote, B and C have no effective voting power because they can-
not affect the outcome in any way. A has an absolute majority and can
pass or defeat any measure regardless of the wishes of B and C. The
ratio of voting power of A to B is not 3 to 1, as might be expected
from a simple comparison of the numbers of votes, but rather 3

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10. For an application of the ideas in this article to weighted voting situations, see
Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 RUTGERS L. REV.
317 (1965); notes 19-22, 33 *infra* and note 44 and accompanying text *infra*.

to 0. A has all of the voting power and B and C have no voting power at all.

Consider a second situation in which A, B, and C each have 1 vote. They have equal voting power because a combination of any two of the three can pass or defeat any proposal and it makes no difference which two of the three join together. Now suppose that A and B each may cast 100 votes while C still casts only 1 vote. At first it might seem that A and B each have a greater voting power than C. Yet this is untrue. No change has been made in the relative ability of A, B, or C to affect the outcomes. Any two of the three may still pass or defeat any proposal. The majority of A plus B, with 200 votes, is no more powerful than the majority of A plus C, with only 101 votes, because each constitutes a majority with the power to pass or defeat proposals. No proposal which would have passed before will fail to pass under the new system; a bill defeated under the old system will have the same fate under the new. Since the results in all cases and under all circumstances are the same, effective voting power has not changed. The change in the distribution of votes has had no effect on the effective voting power.

The purpose of any voting system is to allow each participant some chance, however small, to affect the decisions of the group. Where all have an equal vote, each has an equal chance to affect the outcome. Power in a voting body, in the abstract mathematical sense, is the ability to affect outcomes by participating in the voting process. It is reasonable, therefore, to base a measure of voting power on the relative ability of each member to affect the outcome of the group's decisions through his vote. For any particular body there are a definite number of different possible voting combinations—a number which represents all of the possible ways the members can vote. In many of these combinations an individual may not be able to affect the group's decision by changing his vote. In some the vote is close enough that a change in an individual member's vote will alter the outcome. His relative ability to do this serves as an index of his voting power. By examining all voting members in all possible voting combinations, it is possible to derive a measure of overall voting effectiveness.

There are a number of reasons why a member may not be able to affect the outcome with his vote in a particular voting combination. In the first place, he may have no voting power in any voting combination, as in the case of B and C above who have one vote each while A has three. No change in the vote of B or C could ever change the outcome. Secondly, if some members have relatively large amounts

of voting power, a less powerful member may have a chance to affect the outcome only when all of the larger powers are deadlocked. Finally, even in a relatively evenly balanced body, any individual member can affect the outcome only if the difference between the minority and majority votes is small.

A measure of a member's overall effectiveness (*i.e.*, voting power) may be obtained by examining all of the situations in which he can affect the outcome. After all, voting power can only be measured by its effects or potential effects, and the one significant effect of a member's voting power is to affect group decisions.[11] Because *a priori* all voting combinations are equally possible, any objective measure of voting power must treat them as equally significant. To put the same thought another way, in drafting or evaluating a particular system, no one can say beforehand which combinations will occur most often, or which will occur with respect to more important issues. The best a constitutional draftsman or judge seeking equal voting power can do is to satisfy himself that the system allows each voting member an opportunity to affect the outcome in an equal number of equally likely voting combinations.

A fair, reasonable, and objective measure of voting power may be obtained as follows. Construct a table of all possible voting combinations for a given body. Determine in which combinations each voting member can alter the outcome by changing his vote. The total for each member is a relative measure of his voting power. More explicitly, in a case in which there are N voters, each acting independently and each capable of influencing the outcome only by means of his votes, the ratio of the voting power of voter X to the voting power of voter Y is the same as the ratio of the number of possible voting combinations of the entire voting body in which X can alter the outcome by changing his vote to the number of combinations in which Y can alter the outcome by changing his vote.

This measure meets certain basic requirements. It is based on commonly accepted notions of the purpose and mechanics of voting situations; it is completely objective and depends only on factors which can

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11. As previously indicated, a legislator in practice may have political power far in excess of that which comes solely from his right to vote. Moreover, he may exercise power in many ways other than by voting; *e.g.*, where he influences political appointments. Yet, the purpose of this study is not to produce an all-conclusive estimate of a legislator's power but rather to see if the legislative system itself gives him or his constituents an unfair advantage. To achieve this purpose, a possible approach, and the one which so far has been followed by the courts, is to ignore individual differences between persons casting votes.

be readily and accurately determined; and it can be used to make mean-
ingful calculations in actual voting situations,[12] as well as the two
simple examples which have been discussed.[13] Finally, it is substan-
tially in accord with a measure of voting power which has been gen-
erally accepted in the fields of mathematics[14] and political science[15]
and which has been used by others to analyze voting power in the
Electoral College,[16] Congress and stockholders meetings,[17] the French

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12. The measure is based upon commonly accepted notions of the purpose of voting
systems because it focuses on the ability of the individual voter to affect the outcome of
decisions. It makes no unreasonable assumptions about the legislative process because
it assumes only that *a priori* no particular voting combination is more important or sig-
nificant than any other. It is completely objective because it depends only on the rules
governing the voting situation (number of votes per legislator, veto power, etc.) and
not on past performances or individual characteristics of legislators. Finally, it yields a
measure from which meaningful calculations can and have been made in actual voting
situations. See notes 16-22 *infra.*

13. In a three-man body where A has 3 votes and B and C each have 1 vote, there are
8 possible voting combinations. In all 8 a change in the vote of A will change the deci-
sion of the body. In no combination will B or C be able to alter the outcome by chang-
ing a vote. This demonstrates, as was already obvious, that A has all the power and B
and C have none.

In a three-man body where A and B each have 100 votes and C has 1, there are also
8 possible voting combinations. A can change the outcome in 4 of these, B can change
the outcome in 4 combinations, and C can change the outcome in 4 combinations. This
demonstrates, as was previously shown, that A, B and C are equally powerful. For addi-
tional examples and discussions, see Banzhaf, *supra* note 10.

14. See, *e.g.,* KEMENY, SNELL & THOMPSON, INTRODUCTION TO FINITE MATHEMATICS 74,
108 (1957); Shapley, *Solutions of Compound Games,* in ADVANCES IN GAMES THEORY 267
(Dresher, Shapley & Tucker ed. 1964); Riker, *A Test of the Adequacy of the Power Index,*
4 BEHAVIORAL SCIENCE 120-31 (1959); Shapley, *Simple Games: An Outline of the Descrip-
tive Theory,* 7 BEHAVIORAL SCIENCE 59 (1962); Shapley, *A Value for N-Person Games,* 2
ANNALS OF MATHEMATICAL STUDIES 307 (No. 28, 1953).

15. See, *e.g.,* DAVID, GOLDMAN & BAIN, THE POLITICS OF NATIONAL PARTY CONVENTIONS
174-75 (1960); SCHUBERT, QUANTITATIVE ANALYSIS OF JUDICIAL BEHAVIOR, ch. 4 (1959);
SCHUBERT, JUDICIAL BEHAVIOR (1964); Krislov, *Theoretical Attempts at Predicting Judicial
Behavior,* 79 HARV. L. REV. 1573 (1966); Krislov, *Power and Coalition in a Nine-Man Body,*
AM. BEHAVIORAL SCI. (Apr. 1963); Riker, *Some Ambiguities in the Notion of Power,* 58
AM. POL. SCI. REV. 341 (1964); Riker, *Bargaining in a Three-Person Game,* a paper deliv-
ered at 1966 Annual Meeting of the American Political Science Association Sept. 6-10,
1966; Schubert, *The Power of Organized Minorities in a Small Group,* 9 AD. SCIENCE Q.
133 (1964); Shapley & Shubik, *A Method for Evaluating the Distribution of Power in a
Committee System,* 48 AM. POL. SCI. REV. 787 (1954); and notes 16-22 *infra.*

16. Mann & Shapley, *The A Priori Voting Strength of the Electoral College,* in GAME
THEORY AND RELATED APPROACHES TO SOCIAL BEHAVIOR (Shubik ed. 1964); MANN & SHAP-
LEY, VALUES OF LARGE GAMES IV: EVALUATING THE ELECTORAL COLLEGE BY MONTE CARLO
TECHNIQUES (RAND Corp. Memo RM-2651, Sept. 19, 1960); MANN & SHAPLEY, VALUES OF
LARGE GAMES VI: EVALUATING THE ELECTORAL COLLEGE EXACTLY (RAND Corp. Memo RM-
3158-PR, May 1962).

17. Riker & Niemi, *The Stability of Coalitions on Roll Calls in the House of Repre-
sentatives,* 56 AM. POL. SCI. REV. 58-65 (1962); Shapley & Shubik, *supra* note 15.

Assembly,[18] New York City's Board of Estimate,[19] and weighted vot-
ing situations in general.[20] The measure presented here has previously
been used to analyze weighted voting in the New Jersey Senate, Nassau
County, N.Y.,[21] and Herkimer County, N.Y., where it formed the basis
for a decision against weighted voting.[22]

In most of the studies mentioned above, the measure has been used
to analyze voting power in systems of direct representation or by treat-
ing systems of indirect representation in terms of the voting power of
elected representatives. This is quite reasonable; although a legislator
may serve as a spokesman, a lobbyist, and an ombudsman, he is first
and foremost a voting member of a deliberative voting body. However,
to study the distribution of voting power in multi-member district
systems in terms of the "one man, one vote" principle, it is necessary
to apply the measure to the individual citizen-voter.[23]

18. Riker, *supra* note 14, at 120-21.

19. Krislov, *The Power Index, Reapportionment and the Principle of One Man, One
Vote*, 1965 M.U.L.L. 37.

20. *Ibid.*; Riker & Shapley, *Weighted Voting: A Mathematical Analysis for Instru-
mental Judgments*, presented at the Christmas 1965 meeting of the American Society for
Political and Legal Philosophy, in New York City, and will appear in the 1966 volume
of NOMOS, the Society yearbook, also reprinted by RAND Corp. in P-3318 (1966).

21. Banzhaf, *supra* note 10. The study demonstrated that the Nassau County system of
weighted voting, widely cited in support of weighted voting systems, was unconstitutional
because 3 of the 6 members of the Board of Supervisors could not affect the decisions
of the Board with their votes. Plans are now under way to change the system. See
NASSAU COUNTY COMM. ON GOVERNMENTAL REVISION, MAJORITY REPORT ON ARTICLE I
(June 28, 1965); CONCURRING REPORT OF BOARD OF SUPERVISORS (June 30, 1965); N.Y. Times,
July 8, 1965, p. 63, col. 1.

The measure of voting power used by the author is substantially the same as the other
mentioned above. Both define voting power in terms of the ability of the voter to affect
the decisions of the group. Both measure it by counting the number of voting arrange-
ments in which a voter can cast a decisive or critical vote. The difference is that the
Shapley-Shubik index of power is based on the number of permutations, rather than
combinations, in which each legislator can cast a decisive vote. According to Riker and
Shapley, however, there are no significant qualitative differences between the two meas-
ures and the results of the analysis presented herein are the same with either definition.
See Riker & Shapley, *supra* note 20.

22. Morris v. Board of Supervisors of Herkimer County, 50 Misc. 2d 929 (Sup. Ct. 1966)
(Cardamone, J.), in which the author was called upon to make a mathematical analysis
of a proposed reapportionment plan. In finding the plan unacceptable, the court cited
the results of the study and the author's earlier article; this article was also presented
to the court in draft form. See also Davis v. Board of Supervisors of the County of
Clinton, Supreme Court of New York, Clinton County, August 22, 1966 (Soden, J.);
Graham v. Board of Supervisors, Supreme Court of New York, County of Erie, June 28,
1966 (Kelly, J.); WMCA, Inc. v. Lomenzo, 246 F. Supp. 953, 959-60 (S.D.N.Y. 1965) (Levet,
J. dissenting); Barzelay & Carocci v. Board of Supervisors, Supreme Court of New York,
Onondaga County.

23. The rights protected by the Constitution are those of the citizen-voter and not

III. VOTING POWER AND MULTI-MEMBER DISTRICTS

It is possible to analyze legislative systems in terms of voting power by applying the measure just developed to determine if each citizen-voter has a substantially equal chance to affect the election of his representative. If all of the districts in a state have, for example, 10,000 voters, it is easy to see without any calculation that all voters have the same voting power in the Supreme Court's terms. In the abstract model a citizen of district A will have the same chance, however small, to affect the election of a legislator from A as the citizen of B will have to affect the election of the representative from B. Moreover, one can see, again without calculations, that if A becomes more populous than B, the chance of any citizen of A to affect the election of his representative decreases. Hence, in terms of the definition, and in accordance with the reapportionment decisions, one concludes that his voting power (or weight or strength) has decreased. As long as there are only single-member districts, it is evident that all citizens will have an equal ability to affect the election of their legislators only if the population of the districts is the same.

But suppose the population of A grows to 30,000 while B and the other districts still have approximately 10,000 voters. Instead of dividing district A into three smaller ones, it is decided to allow district A to elect 3 representatives at large who will each have the same vote as the legislators from the smaller districts. Do all voters still have equal voting power? Proponents of such systems seem to regard the equality of voting power as self-evident. They would reason that a voter's chance of affecting an election decreases when the district grows from 10,000 to 30,000 but that the opportunity to vote for three candidates restores him to parity with a voter in a district of 10,000.[24] Un-

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those of the legislator. The legislator's voting power is significant only insofar as it affects his constituents. Therefore, the contitutional issues can only be resolved by extending the analysis to the voter himself.

24. Compare, for example, the explanation presented by the defendants in *Fortson v. Dorsey* to justify the Georgia multi-member district system:

The challenged method of electing senators in this case does not produce any mathematical devaluation of the vote. For example, let us compare the status of a Fulton County voter with one who resides in a rural district electing a single senator. The Fulton voter is a part of an electorate which is approximately seven times larger than the electorate of which the rural voter is a part, however, the Fulton County voter has the right to vote for seven senators whereas the rural voter may only vote for one. Theoretically, the rural voter would have a greater influence upon his single senator than the Fulton voter would have upon any one of his seven senators, but the latter's aggregate influence upon each of his senators would equal the rural voter's influence upon his single senator. In other words, the Fulton voter has an advantage in being able to vote for seven senators, but this advantage is offset by

fortunately, there is a serious flaw in this theoretical justification of the system, for voting power does not vary—even in theory—as the inverse of (1 divided by) the district population.

A few simplified examples will demonstrate that the ability of the individual citizen-voter to affect the election of his representative(s) does not vary inversely with population. Consider the four small districts in a given state illustrated below, each electing one legislator.

District	Population
H	3
I	5
J	7
K	9

In each case it is assumed that the principal contest is between two major candidates and that each voter can cast one vote for each legislative position in his district.[25] With three voters in a district and only two possible choices, there are eight possible voting combinations and each voter can change the outcome of the election by changing his vote in four of them. Thus in District H each resident casts a critical or

his being a part of the large electorate necessary to support the representation of seven senators.
Brief for the Appellant, p. 18, Fortson v. Dorsey, 379 U.S. 433 (1965); Jurisdictional Statement, pp. 12-13. *Id.*; Petition for Writ of Certiorari, pp. 11-12, *Id.*

25. In the vast majority of elections in multi-member districts, the contest is between only two major political parties. Silva, *Relation of Representation and the Party System to the Number of Seats Apportioned to a Legislative District,* 17 WEST. POL. Q. 742, 742-45 (1964); Silva, *supra* note 2. Votes for third parties or splinter candidates are usually only a small percentage of the total cast. Moreover, it is probably true that people casting these votes do so as a sign of protest or strength rather than as a genuine attempt to affect the election of the "candidate." In what appears to be the general rule, each major party nominates as many candidates as there are legislators to be chosen from a given district.

In some states, candidates are by law or by custom nominated for particular legislative positions within a given multi-member district. See, *e.g.,* O'Shields v. McNair, 254 F. Supp. 708, 713-14 (D.S.C. 1966); Ga. Laws, Sept.-Oct. 1962, Extra Sess., pp. 7-31; GA. CODE ANN. § 47-102 (9) (Cum. Supp. 1963). For example, if a district elects three representatives, each major party will support one candidate for seat one, one for seat two, and one for seat three. The voter thus has only an either-or choice with respect to each position. The effective results are probably the same even where the candidates run at large from the district.

The analysis would be different under various hybrid voting systems such as cumulative voting, proportional representation, or limited voting. However, elections for 98 out of 99 state legislative bodies, including both single and multi-member district states, use the familiar single-ballot plurality vote system. Silva, *Relation of Representation and the Party System to the Number of Seats Apportioned to a Legislative District,* 17 WEST. POL. Q. 742 (1964).

determinative vote 50% of the time. In District I with five voters, there are thirty-two possible voting combinations and each voter can cast a critical vote in twelve of them. Thus a voter in District I can cast a critical vote 37.5% of the time. With seven votes, there are 128 voting combinations and each voter casts a critical vote in forty of these. The percentage is thus 31% for District J. For District K, the corresponding figures are 512 voting combinations, 140 in which any individual voter can cast a critical vote, and a percentage of about 28%.[20] The

26. Let the three voters in district H be A, B, and C. A may vote in 2 ways. In each of these 2 cases, B could also vote 2 ways. Finally, in each of these 4 cases, C could vote in either of two ways. In general, a body of N persons voting yea or nay has 2^N different possible voting combinations.

The table below shows the 8 voting combinations possible in the district. It also shows in which combinations each voter casts a critical vote. It should be noted that often more than one voter may cast a critical vote in the same combination in the sense that a change by either would affect the outcome. Thus in combination 2 a change in the vote of either A or B would change the outcome of the district's majority vote from pass (P) to fail (F).

	Possible Voting Combinations			Measure Passes or Fails	Voter Casts a Critical Vote		
	A	B	C		A	B	C
1	Y	Y	Y	P			
2	Y	Y	N	P	X	X	
3	Y	N	Y	P	X		X
4	Y	N	N	F		X	X
5	N	Y	Y	P		X	X
6	N	Y	N	F	X		X
7	N	N	Y	F	X	X	
8	N	N	N	F			

Y = Yea   P = Pass   X means that the par-
N = Nay   F = Fail   ticular legislator can
change the outcome in
that combination.

In a district of N voters, there are 2^N different voting combinations. For an individual voter to be able to cast a critical vote, the other voters in the district must be equally divided. The formula for the number of combinations which can be formed by M persons divided into two equal groups is

$$\frac{M!}{\left(\frac{M}{2}!\right) \cdot \left(\frac{M}{2}!\right)}$$

The exclamation point (!) indicates a factorial. It means that the number it follows is to be multiplied by every positive integer smaller than itself (e.g., $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$). In calculating the number of times each person can cast a critical vote, the fraction must

following chart compares these figures of actual voting effectiveness with those which would result if the ability of each voter to affect his legislator's election actually varied inversely with population (as proponents of multi-member districts suggest).

District	Population	Actual	Number of Combinations (in %) in Which Voter Can Affect Outcome Calculated From Inverse Ratio Hypothesis
H	3	50.0%	33.3%
I	5	37.5%	20.0%
J	7	31.1%	14.3%
K	9	27.4%	11.1%

This table shows that voting effectiveness actually decreases more slowly with increasing population than the inverse ratio hypothesis would suggest.

Of course, the population figures in these four examples are unrealistically low. They were chosen merely to illustrate the effect which is described above and to demonstrate the fallacy of the inverse ratio hypothesis which is used to justify multi-member electoral systems with different sized districts. In real electoral systems the populations of each district will be in the thousands or tens of thousands. With numbers this large actual numerical calculations are difficult if not impossible.[27] However, as the number of voters becomes larger, the percentage of times in which each citizen-voter can affect the outcome of

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be multiplied by 2 to account for both the yea and nay votes. Calculations for the four examples are presented below.

District	Population	Possible Combinations	Number of Times Each Casts Critical Vote	Percent Influence
H	3	$2^3 = 8$	$2 \times \dfrac{2!}{(1!) \cdot (1!)} = 4$	$4/8 = 50\%$
I	5	$2^5 = 32$	$2 \times \dfrac{4!}{(2!) \cdot (2!)} = 12$	$12/32 = 37.5\%$
J	7	$2^7 = 128$	$2 \times \dfrac{6!}{(3!) \cdot (3!)} = 40$	$40/128 = 31\%$
K	9	$2^9 = 512$	$2 \times \dfrac{8!}{(4!) \cdot (4!)} = 140$	$140/512 = 28\%$

27. For example, in a district of only 101 citizens, there are 2,535,301,200,456,458,802, 993,406,410,752 different possible voting combinations. The factorial of fifty (50!), one of the other numbers which would enter into the calculations, is approximately equal to three followed by sixty-four zeros. These numbers are too large even for large electronic computers to work with.

the election can be approximated with a great deal of accuracy. The use of a standard mathematical technique of approximation demonstrates that the voter's ability to affect the election of his legislator decreases as the inverse of the square root of the population of the district[28] rather than as the simple inverse of the population.

In electoral systems employing different sized multi-member districts, residents of the larger districts thus have more voting power than those of less populous districts. Although the individual voter's ability to influence the election of his representative decreases as the square root of the population, each district is entitled under the system to a number of representatives proportional to the population. Thus, as the districts get more populous, they are given more representatives

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28. Consider a district made up of $N + 1$ citizen-voters, where N is an even number. The total number of ways in which each citizen-voter could vote yea or nay on a particular issue is $2^N + 1$ which is equal to $2 \cdot 2^N$. Each would be able to cast a critical vote only where the other N citizen-voters were equally divided into two groups; $\dfrac{N}{2}$ voting yea and $\dfrac{N}{2}$ voting nay. This, as previously indicated, can happen in $\dfrac{2 \cdot N!}{\left(\left[\dfrac{N}{2}\right]!\right) \cdot \left(\left[\dfrac{N}{2}\right]!\right)}$ ways. Thus, an individual citizen-voter would be critical in determining the outcome of a vote in the following fraction of combinations.

$$\frac{2 \cdot N!}{\left(\left[\dfrac{N}{2}\right]!\right) \cdot \left(\left[\dfrac{N}{2}\right]!\right)} \div (2 \cdot 2^N) = \frac{N!}{2^N \cdot \left(\left[\dfrac{N}{2}\right]!\right) \cdot \left(\left[\dfrac{N}{2}\right]!\right)}$$

The factorial of large numbers may be very closely approximated by the following formula which is known as Stirling's formula:

$$m! = e^{-m} \cdot m^m \cdot \sqrt{2\pi m}$$

where e and π are well known mathematical constants. Substituting this value into the previous formula by allowing $\dfrac{N}{2}$ to equal m, the fraction becomes:

$$\frac{e^{-2m} \cdot (2m)^{2m} \cdot \sqrt{4\pi m}}{2^{2m} \cdot e^{-m} \cdot m^m \cdot \sqrt{2\pi m} \cdot e^{-m} \cdot m^m \cdot \sqrt{2\pi m}}$$

By expanding the numerator into separate multiplicative terms and cancelling common terms in the numerator and denominator, the result is that the individual citizen-voter is critical in $\dfrac{1}{\sqrt{2\pi N}}$ combinations.

Where m is greater than 100, Stirling's formula gives a result which is accurate to within 0.1% and becomes more accurate as m increases. SOKOLNIKOFF & REDHEFFER, MATHEMATICS OF PHYSICS AND MODERN ENGINEERING 644-45 (1958).

than are necessary to compensate for the decrease in the individual citizen-voter's voting effectiveness. As an example, if one district is four times the population of another, it will have four times the number of representatives although each citizen-voter has half (one divided by the square root of four) the voting power (per representative) of the citizen-voters in the smaller district. Thus residents of the larger district will be overcompensated in voting power with respect to the residents of the smaller district by 100%. If the original ratio of district populations is nine to one, the overrepresentation is by a factor of 200%.

It should be noted that these inequalities in voting power are an inherent problem even in a theoretical model in which districts are exact multiples of each other in population and all considerations other than relative populations are ignored. In actual situations where populations only approximate this "ideal," the inequalities will tend to be magnified. If other realities of political life which may affect actual voting power—such as political parties, pressure groups, ethnic blocs, campaign strategies, voter apathy, etc.—are considered in addition, their influence will be superimposed upon the inequalities revealed in the simplified model. It is highly unlikely that one or more of these would exactly counterbalance the effect which has been demonstrated.

IV. REPRESENTATION AND MULTI-MEMBER DISTRICTS

The broader interpretation of the reapportionment decisions is that equality in the power to vote and thereby to affect the election of legislators is only a means to an end, a part of a larger and more complicated problem. The problem is how to afford each voter equal representation under a system in which one or more legislators act for him in a legislative body. So far the courts have held that one way this can be accomplished is by dividing the state into districts of substantially equal population, allowing each district to select one legislator, and allowing each legislator one vote in the chamber. It is an open question whether voters can be equally represented where districts of different populations elect different numbers of legislators to the same body.

To answer the question it is necessary to consider the role and function of the legislator. Here one may broadly distinguish between two contrasting hypotheses. One, which is often called the Burkean (or republican) model, assumes a legislator who acts for the whole area being governed without consideration for the particular interests of his constituents. He decides issues according to what he feels is best for the whole state, based either upon his assessment of their majority

wishes or his own best judgment. In contrast there is the delegate (or democratic or representative) model, which assumes that a legislator acts as the delegate of his constituents. On each issue he is presumed to act according to what he believes to be the will of the majority in his district.[29] In effect the votes of all of the citizens are "funneled" into the legislature through their representative.

Certainly both theories are gross simplifications and are subject to criticism for this and other reasons. Evidence suggests that neither model is entirely satisfactory in general but that in many respects the representative theory provides a reasonable approximation to legislative situations.[30] Furthermore, in terms of effective representation, only the delegate model need be considered. To the extent that legislators act according to the Burkean model, it is of little relevance what size districts they represent. Each acts for all of the people and does not attempt to reflect the particular wishes of his constituents. If one legislator has more or less than the average number of constituents, how is anyone advantaged or harmed? To this extent the purpose of the elective system would appear to be simply to select the requisite number of wise and able men to act together to make judgments for the people of the state. There is no evidence to show that the ability to select such men depends closely on population or that a basic constitutional right is involved.[31]

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29. See Riker & Shapley, *supra* note 20, at 23-31. See generally SABINE, HISTORY OF POLITICAL PHILOSOPHY 610 (1950); WAHLKE, EULAU, BUCHANAN & FERGUSON, THE LEGISLATIVE SYSTEM 273 ff. (1962).

30. See, *e.g.*, Cnuddle & McCrone, *The Linkage Between Constituency Attitudes and Congressional Voting Behavior: A Causal Model*, 60 AM. POL. SCI. REV. 66, 69-70 (1966):
   This analysis indicates that constituencies do not influence civil rights roll calls in the House of Representatives by selecting Congressmen whose attitudes mirror their own. Instead, Congressmen vote their constituencies' attitudes (as they perceive them) with a mind to the next election.

(The authors report "influence coefficients" of 88%.); Kornberg, *Perception and Constituency Influence on Legislative Behavior*, 19 WEST. POL. Q. 285, 291 (1966):
   [S]ubstantial constituency control over legislative leaders has by now come to be regarded as a factual truth as well as a normative principle. . . . The taking of the Burkean-Trustee role style is apparently a luxury . . . . The relatively small number who actually took the role of Trustees suggests that empirical reality (the requirement of being reelected) precludes the taking of this role regardless of the status attached to it.

(Of their sample, 15% said that they acted according to the Burkean role, 49% said that they followed the delegate-representative model, and 36% adopted an intermediate position); FROMAN, CONGRESSMEN AND THEIR CONSTITUENCIES (1963); MILLER & STOKES, REPRESENTATION IN THE AMERICAN CONGRESS (Prentice-Hall, in press); WAHLKE, EULAU, BUCHANAN & FERGUSON, *supra* note 29, at 281; Miller & Stokes, *Constituency Influence in Congress*, 57 AM. POL. SCI. REV. 45, 45-46 (1963).

31. It might be argued that the more populous constituencies will tend to produce

On the other hand, at least part of the time a legislator is supposed to act as a delegate. Certainly American voters want and expect a legislator who will execute their wishes on certain issues, and punish those who deviate too far from the delegate model. While acting purely in a representative capacity, a legislator will cast his vote as he thinks the majority of his constituents would vote if they had the opportunity. However infrequently a legislator may in fact try to act as a delegate and however imprecise his estimate of his constituents' wishes, the reapportionment decisions construed broadly outlaw systems which deny voters an equal chance to have their wishes reflected in the votes of their representative(s). No one can say *a priori* which legislators will be best able to perform this function nor which will take their obligations most seriously. All that a legislative draftsman or judge can do is to insure that the elective and representative system itself does not tend to make it more likely that some citizens will be better able to have their wishes reflected than others. Thus, in constructing a mathematical model, which must of necessity ignore many of the real problems of the system,[32] one may hypothesize the representative to be no more than a vehicle for reflecting as best he can the votes of his constituents on certain issues. In such a model representative system, each legislator would in effect poll his district on each issue and cast his vote according to the majority vote. For the limited purpose of establishing the outer boundaries of a fair representative system, it seems reasonable to assume this type of legislator as an oversimplified model.[33]

more able legislators simply because the number of qualified candidates will be higher. However, this hardly seems to rise to the level of a constitutionally protected right demanding reapportionment to within 15%.

32. What little is known about how legislators' votes are influenced tends to cast doubt on any theory which would have a constituent's ability to affect his representative's vote depend solely on the population of the district. Such a theory would ignore party alliances, ethnic blocs, regional differences and interests, lobbying, influence peddling, and other realities of political life. Yet, so far, the Supreme Court has looked no further than population figures in deciding reapportionment cases. Moreover, the justification offered for multi-member district systems also depends upon such a theory. If influence and representation cannot with some reasonable degree of accuracy be approximated by such a theory, then the justification fails and multi-member district systems should be abandoned. On the other hand, if any such numerical theory can give even a reasonable approximation to political reality, it is submitted that the analysis contained herein is at least mathematically consistent and therefore more likely to be correct than the inverse ratio theory offered as justification for such systems.

33. The courts can go only so far in protecting the rights of citizens to equal representation. Some factors which this model necessarily ignores are no doubt beyond the competence of the courts to correct. Others are so imperfectly understood that theorizing would be impractical.

This model does not assume that all legislators from a given multi-member district,

With this as a model, it can be shown mathematically why in the usual reapportionment case single-member districts must be of equal population to guarantee equality of representation. Consider as an example a legislative body representing three districts, A, B, and C, each containing three voters (a_1,a_2,a_3; b_1,b_2,b_3; c_1,c_2,c_3). Each district of three voters is represented by a legislator who casts his vote in accordance with the majority vote of his constituents. Each of the three representatives has one vote in the legislative body.

Looking first at the legislative body, with three persons voting and a choice of only yea or nay, there are eight different possible voting combinations.[34] Each legislator will be "critical"—*i.e.*, be able to alter the result by changing his vote—in four of these cases.[35] The same situation exists at the citizen-voter level. The citizens of each district can cast their votes in eight different ways (voting combinations) and each citizen-voter will be able to cast a critical vote in his district in four of these.[36] The true issue, however, is how well each citizen-voter is actually represented in the legislature. In other words, considering all of the combinations in which all nine of the votes could be cast, in how many combinations will any individual voter be able to affect the outcome through the medium of his representative's vote? The answer is that he can cast a critical vote in his district in half of the district's voting combinations and the resulting vote on behalf of the district, as cast by its representative, is critical in half of the legislative voting combinations. Combining these two figures, each voter can cast a critical vote in one-fourth of the total voting combinations. Put another way, if all of the other citizen-voters do not change their original positions on any given issue, any given citizen-voter will be able to change the legislative decision through the medium of his representative's vote in one-fourth of the possible voting combinations by changing his vote.[37]

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because they are each supposed to be responsive to the same constituency, will as a result tend to vote as a bloc. Representatives from the same district may of course differ in their assessment of their constituents' wishes and may tend to vote according to the delegate model with respect to different issues. If, however, there is some tendency for them to vote as a bloc, the result will resemble to some extent a weighted voting situation with additional inherent difficulties. See notes 10, 19-22 *supra*, and note 44 and accompanying text *infra*.

34. See note 26 *supra*.

35. See table in note 26 *supra*.

36. A table similar to the one in note 26 could be constructed to show the voting combinations of the citizen-voters in district A by replacing A, B, and C in the table with a_1, a_2, and a_3 (for district A). Thus, for example, a_1 would cast a deciding vote in combinations 2, 3, 6, and 7.

37. It would also be possible to demonstrate this by constructing a table of the 512

To show how this method may be applied to demonstrate the inequities in a classic case of malapportionment in which there are single-member districts of substantially unequal population, consider the case where the number of constituents in district A has grown to 5. Representative A, like representatives B and C, can still cast a critical vote in one-half of the legislative decisions. However, at the district level, each resident of A can now cast a vote which will be critical with respect to his representative's vote in twelve out of thirty-two possible voting combinations.[38] Because of the increase in population, the votes of residents of district A have less effect on their representative's vote yet his vote is no more effective in the legislature. In terms of overall effective representation, the citizen-voters in district A can cast a vote which will be critical with respect to legislative decisions in only six thirty-seconds of the voting combinations (one-half times twelve thirty-seconds) while residents of the other two districts still may cast decisive votes in one-fourth (eight thirty-seconds) of the combinations. Thus, using this technique, it is possible to demonstrate mathematically exactly why and by how much a citizen is disadvantaged by being a resident of an over-populated district in the usual single-member district electoral system. It may likewise be used to determine whether multi-member district systems provide equally effective representation for all citizens.

Consider for purposes of illustration two districts in a state using multi-member districts of unequal size.[39] District S has 10,000 people and elects one legislator. District L has 40,000 voters and has four legislators elected at large in the same house. The voting power of all of the legislators in the body is the same because each can cast only one vote. District L's four representatives, taken together, are four

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possible voting combinations and examining each with respect to each citizen-voter.

The reader who is mathematically inclined may wish to compare this result with that of a system of direct representation in which the nine citizens decide issues by a simple majority vote. Any indivdual voter would then be critical only in a voting combination in which the other 8 voters were equally divided. This could occur in 140 different ways out of a total of 512. Thus, in the idealized system of indirect representation illustrated here, each citizen-voter has less chance of influencing the outcome than he would have under a system of direct representation. However, under either arrangement, the voting powers of all members are equal.

38. These figures may be obtained by constructing a table similar to that in note 26 *supra.* The thirty-two possible voting combinations can be examined and the ability of each voter to affect the outcome noted. For example, using the same numbering system as before, voter a_1 casts a critical vote in combinations 4, 6, 7, 10, 11, 13, 20, 22, 23, 26, 27 and 29. See also the calculations for district I in the same footnote.

39. This includes systems containing both single and multi-member districts as single-member districts are degenerate forms of the more general multi-member district.

times as powerful as District S's single legislator. Proponents will argue that the voters are equally represented because each of District L's legislators represents four times the number of people that district S's legislator does: 4 divided by 40,000 equals 1 divided by 10,000; there is one representative for every 10,000 people. Despite its simplistic appeal, this argument is fallacious.　·

There is no justification here for simple division; the resulting product has no meaning. Although at first glance it might seem to be a logical operation, there is no mathematical theory to support it. The issue is effective representation. In terms of the model and the definition of voting power, the question is whether the relative ability, however small, of the people in districts L and S to affect the outcome of legislative decisions through the medium of their legislator's votes is substantially equal. It has already been shown that in large districts the ability to affect the outcome of an election between two candidates varies as one divided by the square root of the population. The mathematics and the result is the same here even though the model is different (because it is based on a broader interpretation of the Court's opinions). The representative in the model votes according to the majority vote of an imaginary poll of his constituents. The choice once again is twofold: instead of choosing between candidates X and Y, the people vote yea or nay on given propositions.[40] Therefore, in mathematical terms the problem is identical and the ability of constituents to affect the outcome of legislative decisions through the medium of their representative(s) also varies inversely as the square root of the population. People in District L have more representation than they are entitled to. By giving them more legislators than the diminution in their relative effectiveness required, the overall effect is to give them twice as much voice in the legislature as residents of S. Under either analysis (*i.e.*, proceeding from either reading of the Supreme Court's opinions), these inequities are inherent in the system. Whether analyzed in terms of voting power or representation, systems using different-sized multi-member districts contain hidden inequalities.

Abstract mathematical analysis may suggest that the results are equally abstract and that the conclusions are primarily of academic interest. The following analysis of multi-member district systems in

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40. In actual practice, people often choose various positions on any given issue. However, these can always be broken down, as the legislator must eventually do, into a series of yes or no answers to a number of specific legislative proposals.

Arkansas, Georgia, Hawaii, Texas, and Wyoming—each of which has recently been involved in reapportionment litigation[41]—demonstrates the extent of the inequalities which may exist in many states today. The simplified mathematical model in each case assumes that all districts electing one legislator have the same population and that the populations of the other districts are exact multiples. Because populations are not divided with mathematical precision, the actual inequalities will vary somewhat from the figures presented in each table but these variations will be small compared with the inequalities shown in the tables. In each table minus signs (—) indicate underrepresentation while plus signs (+) indicate over-representation (with respect to each of the two averages).

ARKANSAS (House)

Number of Legislators Per District	Number of Districts In State	Effective Representation (as %) Compared With Single-Member District	Percent Deviation in Effective Representation With Respect to Average (Per District)*	Percent Deviation in Effective Representation With Respect to Average (Per Voter)**
1	17	100%	— 30%	— 46%
2	15	141%	— 1%	— 23%
3	6	173%	+ 22%	— 6%
4	3	200%	+ 41%	+ 9%
5	2	224%	+ 58%	+ 22%
13	1	361%	+ 154%	+ 96%

* The average effective representation, computed on a per district basis, is 142% compared with a single-member district. Approximately 73% of the districts are below this average in effective representation.

** The average effective representation, computed on a per voter basis, is 184% compared with a single-member district. Approximately 65% of the voters are below this average in effective representation.

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41. See Yancey v. Faubus, 251 F. Supp. 998 (E.D. Ark. 1965), aff'd sub. nom. Crawford County Bar Ass'n v. Faubus, 383 U.S. 271 (1966); Fortson v. Dorsey, 379 U.S. 433 (1965) (Georgia, figures cited obtained from briefs); Holt v. Richardson, 240 F. Supp. 724 (D. Hawaii 1965), aff'd sub. nom. Burns v. Richardson, 384 U.S. 73 (1966); Kilgarlin v. Martin, 252 F. Supp. 404 (S.D. Tex. 1966); Schaefer v. Thomson, 251 F. Supp. 450 (D. Wyo. 1965), aff'd sub. nom. Harrison v. Schaefer, 383 U.S. 269 (1966).

Two different averages and deviations therefrom have been computed. The first is an average per district, treating each district as the significant entity, and is obtained by multiplying each district by its corresponding effective representation and dividing by the total number of districts. The fourth column indicates deviations from this average. The second is an average per citizen-voter, treating each citizen-voter as the significant entity, and is obtained by multiplying the number of voters in each district (distributed as assumed) by his corresponding effective representation and dividing by the total number of voters in the state. Deviations from this second average are presented in the last column.

GEORGIA (Senate)

Number of Legislators Per District	Number of Districts In State	Effective Representation (as %) Compared With Single-Member District	Percent Deviation in Effective Representation With Respect to Average (Per District)*	Percent Deviation in Effective Representation With Respect to Average (Per Voter)**
1	23	100%	— 14%	— 30%
2	4	141%	+ 22%	— 1%
3	2	173%	+ 49%	+ 21%
7	1	264%	+ 127%	+ 84%

* The average effective representation, computed on a per district basis, is 116% compared with a single-member district. Approximately 77% of the districts are below this average in effective representation.

** The average effective representation, computed on a per voter basis, is 143% compared with a single-member district. Approximately 70% of the voters are below this average in effective representation.

HAWAII (Senate)

Number of Legislators Per District	Number of Districts In State	Effective Representation (as %) Compared With Single-Member District	Percent Deviation in Effective Representation With Respect to Average (Per District)*	Percent Deviation in Effective Representation With Respect to Average (Per Voter)**
1	1	100%	— 42%	— 46%
2	1	141%	— 18%	— 24%
3	2	173%	0%	— 6%
4	4	200%	+ 16%	+ 8%

* The average effective representation, computed on a per district basis, is 173% compared with a single-member district. Approximately 25% of the districts are below this average in effective representation.

** The average effective representation, computed on a per voter basis, is 185% compared with a single-member district. Approximately 36% of the voters are below this average in effective representation.

TEXAS (House)

Number of Legislators Per District	Number of Districts In State	Effective Representation (as %) Compared With Single-Member District	Percent Deviation in Effective Representation With Respect to Average (Per District)*	Percent Deviation in Effective Representation With Respect to Average (Per Voter)**
1	52	100%	— 22%	— 48%
2	9	141%	+ 9%	— 27%
3	4	173%	+ 34%	— 10%
4	3	200%	+ 55%	+ 4%
5	1	224%	+ 74%	+ 17%
6	2	245%	+ 90%	+ 28%
7	1	264%	+ 105%	+ 37%
8	1	283%	+ 119%	+ 47%
10	1	316%	+ 145%	+ 65%
14	1	374%	+ 190%	+ 95%

* The average effective representation, computed on a per district basis, is 129% compared with a single-member district. Approximately 69% of the districts are below this average in effective representation.

** The average effective representation, computed on a per voter basis, is 192% compared with a single-member district. Approximately 55% of the voters are below this average in effective representation.

WYOMING (Senate)

Number of Legislators Per District	Number of Districts In State	Effective Representation (as %) Compared With Single-Member District	Percent Deviation in Effective Representation With Respect to Average (Per District)°	Percent Deviation in Effective Representation With Respect to Average (Per Voter)°*
1	10	100%	— 21%	— 34%
2	4	141%	+ 11%	— 7%
3	1	173%	+ 36%	+ 14%
4	1	200%	+ 57%	+ 32%
5	1	224%	+ 76%	+ 47%

* The average effective representation, computed on a per district basis, is 127% compared with a single-member district. Approximately 59% of the districts are below this average in effective representation.

** The average effective representation, computed on a per voter basis, is 152% compared with a single-member district. Approximately 60% of the voters are below this average in effective representation.

V. THE RESULTS, AN ASSESSMENT AND OTHER APPLICATIONS

It is important to keep these results in their proper perspective. No attempt has been made to evaluate all the advantages and disadvantages of multi-member district systems. Furthermore there is no suggestion that this article presents a realistic picture of their actual operations which of course would involve factors which are not present in these simple mathematical models. Finally this article is by no means an indictment of all multi-member district systems.

According to this theory there are no inherent inequalities where all multi-member districts elect the same number of legislators. Where the disparity in size is not too great the inherent inequalities in voting power and representation may be tolerable in view of other socially desirable ends which might not otherwise be achievable. Finally, means may be found to counterbalance the inequalities which such plans generate.[42]

Political realists will be happy to know that the basic method presented here may also be used as an analytic tool to evaluate the effect of other factors affecting voting and representation. Thus parties may be able to present courts with a more accurate analysis of reapportionment plans (and even persuade them to enlarge their areas of consideration), politicians may be able to make better informed judgments, and political scientists may use this tool to dissect existing and proposed systems (floterial districts, cumulative voting, limited voting, proportional representation, etc.).

In the models it was assumed that all voters were equal and that each was equally likely to vote one way as the other. This is by no means the only possible assumption and the model may be varied to simulate the effect of additional factors. In terms of voting power, one might assume that certain blocs of voters would have a tendency to vote for one party or the other, and then describe the results in terms of statistical probabilities. Such studies are not unknown.[43] Alternatively one could consider the effect of third party candidates by allowing each model voter a three-way choice. Finally one might empirically determine the voting patterns of various ethnic and/or socioeconomic groups, replace the mass of voters whose voting habits were presumed identical with those whose voting records are statisti-

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42. It has been suggested that under this analysis one way of eliminating the inequalities would be to allocate legislators to multi-member districts in proportion to the square root of their populations. This is hardly a workable solution. It would probably run afoul of the most commonly applied reapportionment standards and be unacceptable to a large number of persons. The author sees no easy solution but feels that the possibility of counterbalancing should not be dismissed.

43. It can be shown, for example, that if all of the districts for a given house are single-member, if no more than two parties receive at least 90 percent of the popular vote, and if the electoral strength of the two parties are equally distributed throughout the state, the ratio of the seats won by the parties is approximately equal to the CUBE of the state-wide popular vote each receives. Kendall & Stuart, *The Law of Cubic Proportion in Election Results*, 1 BRITISH J. OF SOCIOLOGY 183 (1950). For a comparison of the formula with actual results, see BUTLER, THE ELECTORAL SYSTEM IN BRITAIN 191-200 (1953); LAKEMAN & LAMBERT, VOTING IN DEMOCRACIES 48-49 (1955); MACKENZIE, FREE ELECTIONS 52 (1958).

cally known, and calculate the voting power of the different blocs and their individual members.

In terms of representation, one might attempt to set up a table of relative influences upon a legislator (*e.g.*, the relative persuasive power of a newspaper editorial, a telegram, a hand-written letter, a form letter, etc.) and from that calculate the relative abilities of various interest groups to affect his vote through different allocations of their resources. If it were possible to make some estimate of an individual representative's "domination by the party" and his own ability in turn to affect "party decisions" and thereby to affect the votes of other legislators, a voter could estimate whether he could increase his chances of affecting legislative decisions by voting for the "boss controlled" candidate over the "fighting independent."

Naturally the method outlined here, being analytical and not empirical, cannot provide any new facts about the representative process. It can, however, provide new ways of looking at known information, better ways of analyzing the information, and more accurate methods of predicting political outcomes.

The analysis presented in this article has direct application to two other important electoral systems: weighted voting (including fractional voting) and the Electoral College. An earlier study by the author revealed that weighted voting systems do not in fact allocate effective voting power among legislators in proportion to the number of votes each may cast (*i.e.* in proportion to their respective populations). The present analysis, though directed primarily towards multi-member districts, applies to any system of representation employing districts of substantially unequal population. Whether voting power in the legislature is distributed among several representatives elected at large from each district or rests in only one legislator from each district who casts multiple votes, the analysis and the results are the same; voters in the more populous districts are overrepresented in proportion to the square root of their populations. Thus weighted voting systems suffer from two separate inherent defects; an inability to distribute voting power among legislators in proportion to the number of votes each can cast (which of course affects the effective representation of the citizen-voters) and in addition a separate built-in bias in voting power and effective representation in favor of the more populous districts. These two analyses, moreover, are entirely consistent; in fact, the second answers a question which was deliberately left open in the first.[44]

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44. See Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis,* 19 RUTGERS L. REV. 317, 319 n. 22 (1965).

This technique of analysis may also be used to calculate the relative abilities of voters in different states to affect the election of the President of the United States through the medium of presidential electors. Because the electors vote only once and almost invariably reflect the wishes of the majority of the voters they represent, the assumptions implicit in the delegate model are clearly justified. It can be shown that the relative ability of citizen-voters to affect the election of the President under the state unit-vote (winner take all in each state) system is proportional to the number of electors per state divided by the square root of the state's population base. Thus, in the 1964 election a voter in New York has over 2.3 times as much chance to affect the election as a voter in Delaware. This is despite the so-called "small state bias" which is said to result from allowing each state at least three electors regardless of population (New York had approximately one elector per 390,000 people; Delaware had approximately one elector per 149,000 people). Leave was recently sought by Delaware to file an original suit in the Supreme Court, based in part upon the author's analysis, to invalidate the unit-vote rule while leaving the Electoral College intact. Leave was not granted, apparently for reasons of procedure.[45] Moreover, it is by no means certain that action short of a constitutional amendment could equalize the effective voting power of citizens of all of the states and the District of Columbia in the presidential election.

VI. THE SUPREME COURT AND MULTI-MEMBER DISTRICTS

The Supreme Court has considered several reapportionment cases involving multi-member districts and they are occasionally cited to show the Court's unqualified endorsement of such systems. This is far from correct. The Supreme Court has refused to strike down such plans in the past either because the plans were improperly attacked and/or because the record did not sustain the attackers' contentions.

In *Fortson v. Dorsey*,[46] the petitioners contended that voters in the most populous multi-member electoral district (Fulton County) were discriminated against because residents of any one of the seven residence areas in the district (which were used "merely as the basis of

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45. See Brief for Plaintiff, State of Delaware v. State of New York, No. 28 Original (filed July 28, 1966); 35 U.S. WEEK 3138 (1966). The results reported in the papers cited in note 16 *supra* are not relevant here because they involve inequalities resulting from the distribution of voting power from the point of view of the states as "players" (members of a voting body) and not the effect upon the individual citizen-voters.

46. 379 U.S. 433 (1965).

residence for candidates, not for voting or representation")[47] had to join with residents of the other residence areas in electing senators under the system of countywide elections at large. The court below granted petitioner's motion for summary judgment on the bare record but the Supreme Court reversed because "it is not accurate to treat a senator from a multi-district county as the representative of only that district within the county wherein he resides."[48] As to petitioners' contention that the plans minimized the strength of political minorities in the populous areas, the Court held that the record was insufficient to afford a basis for decision.[49]

The Court expressly limited its ruling to a reversal of a grant of summary judgment:

> We treat the question as presented in that context, and our opinion is not to be understood to say that in all instances or under all circumstances such a system as Georgia has will comport with the dictates of the Equal Protection Clause.[50]

Strictly speaking, in reversing a grant of a motion for summary judgment, the Court held only that these plaintiffs were not entitled to the relief on the meager record before it. Even a broader reading would establish only that voters of the more populous counties do not suffer a disadvantage in representation—a result consistent with the analysis presented herein.

In *Burns v. Richardson*,[51] the lower court struck down a plan utilizing multi-member districts because, among other things, they felt that residents of the least populous districts had a built in advantage in representation. The Supreme Court vacated, holding that the record did not support this conclusion:

> We conclude . . . that H.B. No. 987 . . . has not been shown to fall short of federal standards. . . . [T]he demonstration that a particular multi-member scheme effects an invidious result must appear from the evidence in the record. . . . That demonstration was not made here.[52]

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47. *Id.* at 438.

48. *Ibid.*

49. "But appellees never seriously pressed this point below and offered no proof to support it, the District Court did not consider or rule on its merits, and in oral argument here counsel for appellees stressed that they do not rely on this argument. The record thus does not contain any substantiation of the bold assertion in appellees' brief. Since, under these circumstances, the issue has 'not been formulated to bring it into focus, and the evidence has not been offered or appraised to decide it, our holding has no bearing on that wholly separate question.' " *Id.* at 439.

50. *Id.* at 438-39.

51. 384 U.S. 73 (1966).

52. *Id.* at 86, 88.

Once again they indicated their willingness to reconsider the question where a proper demonstration of the alleged inequalities is made:

> We direct the District Court . . . to adopt the plan . . . until . . . constitutional deficiencies in the interim plan are shown. . . . There may, for example, be merit in the argument that by encouraging block voting multi-member districts diminish the opportunity of a minority party to win seats. But such effects must be demonstrated by evidence.[53]

The Supreme Court has also affirmed without opinion at least four other cases involving multi-member districts.[54] However, in none were arguments similar to those presented in this article made to the courts and therefore they cannot be considered precedent to the contrary. In effect the Supreme Court is still waiting to be shown. It is hoped that this article may help to shed some light on this largely unexplored field.

VII. CONCLUSION

The Supreme Court has made "one man, one vote" the law of the land and has held that both houses of state legislatures must meet the new standard. In issuing the edict, it has not indicated clearly whether equal voting power for all citizens was the right being protected or whether it was only a means to the more important end of equal representation. Finally, it has refused to look beyond population statistics in finding wrongs and fashioning remedies. Proponents of multi-member district systems have argued that the plans pass muster under these standards and have attempted to justify them in terms of the simplistic mathematics which has so far characterized reapportionment opinions. This article has analyzed the systems in these terms and under these limitations and has found them wanting.

Under predefined conditions, voting power is susceptible of precise mathematical analysis. To the extent that the theoretical model approximates the Court's simplified picture of the election of legislators—*i.e.*, where the Court considers only the relative number of voters and the rules of the elective system—it can be shown that legislative systems employing districts electing different numbers of representatives inequitably allocate greater voting power to voters in the most populous districts. This discrimination, which is inherent in all such systems, is proportional to the square root of the district population and may easily reach the magnitude of a constitutional deprivation.

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53. *Id.* at 86, 88 n.14.

54. Burnette v. Davis, 382 U.S. 42 (1966) (per curiam); Thornton v. Davis, 382 U.S. 42 (1966) (per curiam); Harrison v. Schaefer, 383 U.S. 269 (1966) (per curiam); Crawford County Bar Ass'n v. Faubus, 383 U.S. 271 (1966) (per curiam).

The Court for the most part has restricted its considerations of equal representation to the population and the number of legislators from each district and has granted relief only where the representative system itself discriminated against various constituencies. An analysis of representation in these limited terms has been presented which is consistent with certain fundamental beliefs about the functions of a legislative system. As a test of the theory and as an example, it can be used to describe in quantitative terms the rationale of the reapportionment decisions. Applied to systems of unequal size multi-member districts, the analysis shows that such systems inevitably create inequities in representation proportional to the square root of the district population. The inequalities are thus of the same form and magnitude as those revealed in the distribution of voting power.

No attempt has been made to present in this article an all-inclusive analysis of voting power or effective representation. The models have deliberately been limited to the factors which the courts have so far considered relevant. If they continue to limit their scrutiny to these factors, this analysis should enable them to more carefully consider and evaluate the effects of such systems. If they expand their considerations in the quest for equal representation to other factors which are known to affect voting power and effective representation, perhaps this analysis can serve as a starting point from which a more inclusive model can be developed. Whatever else it does, this article should shift the burden of proof back to the proponents of such systems. Without some further and more persuasive explanation of how such systems are supposed to provide equal voting power and effective representation to all of a state's citizens, the courts should strike them down as an inconclusive experiment with the constitutional rights of citizens in cases where the inequalities outweigh any legitimate social interests which might be served.[55]

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