A PROCEDURE FOR NONPARTISAN DISTRICTING: DEVELOPMENT OF COMPUTER TECHNIQUES

JAMES B. WEAVER† and SIDNEY W. HESS††

In the landmark case of *Baker v. Carr* [1] the Supreme Court held that federal courts have jurisdiction to review the constitutionality of state legislative apportionments. The Court left many questions unresolved, most significantly what constitute appropriate standards for testing the constitutionality of apportionment and districting.[2] Sundry standards have since been suggested, ranging from equality of population [3] to the broad requirement that an apportionment and districting be rational—a consistent application of an intelligible policy.[4] But *Baker v. Carr* left unresolved another, less discussed issue, equally unsettled and increasingly important. Once having decided that a particular

†Director, Development Appraisal Department, Atlas Chemical Industries, Inc., Wilmington 99, Delaware; Registered Engineer, Delaware.

††Manager, Operations Research Section, Atlas Chemical Industries, Inc., Wilmington 99, Delaware.

The authors wish to acknowledge with thanks the encouragement and creative participation of Professor Ruth C. Silva, Pennsylvania State University; Professor John D. C. Little, Massachusetts Institute of Technology; Mr. Jack N. Whelan, E.I. duPont de Nemours & Co., Inc., especially for assistance in the computational aspects of this article; and the members of the group which initiated their interest, Wilmington's Committee of 39. See note 54 *infra*.

1. 369 U.S. 186 (1962).

2. It appears that the Court will examine the question of appropriate standards during the current term. A number of cases involving both congressional and legislative apportionment have been appealed. Some of these cases have already been argued. See N.Y. Times, Nov. 13, 1963, p. 1, col. 5 (city ed.); and N.Y. Times, Nov. 14, 1963, p. 29, col. 4 (city ed.). Others have been set down for argument. See 32 U.S.L. Week 3110 (Oct. 1, 1963). Several recently appealed cases have not yet been acted upon or set down for argument. *Ibid.*

3. See, *e.g.*, Mann v. Davis, 213 F. Supp. 577 (E.D. Va. 1962); Moss v. Burkhart, 207 F. Supp. 885 (W.D. Okla. 1962); Hanson, *Courts in the Thicket: The Problems of Judicial Standards in Apportionment Cases*, 12 Am. U.L. Rev. 51 (1963); American Civil Liberties Union Weekly Bulletin, Mar. 20, 1961, pp. 1-2.

4. See, *e.g.*, Israel, *On Charting a Course Through the Mathematical Quagmire: The Future of Baker v. Carr*, 61 Mich. L. Rev. 107 (1962); Bickel, *The Durability of Colegrove v. Green*, 72 Yale L.J. 39 (1962).

For other approaches to the troublesome problem of constitutional standards for apportionment, see Comment, *Baker v. Carr and Legislative Apportionments: A Problem of Standards*, 72 Yale L.J. 968 (1963); McCloskey, *The Supreme Court 1961 Term, Foreword: The Reapportionment Case*, 76 Harv. L. Rev. 54 (1962); Emerson, *Malapportionment and Judicial Power*, 72 Yale L.J. 64 (1962); Neal, *Baker v. Carr: Politics in Search of Law*, 1962 Sup. Ct. Rev. 252; Dixon, *Legislative Apportionment and the Federal Constitution*, 27 Law & Contemp. Prob. 329 (1962); McKay, *Political Thickets and Crazy Quilts*, 61 Mich. L. Rev. 645 (1963).

representation scheme is unconstitutional, courts must determine how to administer relief. Courts and commentators have agreed nearly unanimously that courts ought initially to refrain from granting direct relief and allow the state legislature another opportunity to reapportion in accord with the federal constitution.[5] Should such a response not be forthcoming, however, a court may be forced to grant direct relief. Again, a whole range of solutions may be open to the court.[6] This paper concerns one alternative, affirmative judicial apportionment and districting.[7]

5. See, *e.g.*, Sincock v. Duffy, 215 F. Supp. 169 (D. Del. 1963); Mann v. Davis, 213 F. Supp. 577 (E.D. Va. 1962); Comment, *Baker v. Carr and Legislative Apportionments: A Problem of Standards*, 72 YALE L.J. 968, 1033-35 (1962).

6. For cases in which courts have been disposed to grant direct relief, at least in the absence of legislative response, see Sims v. Frink, 205 F. Supp. 245 (M.D. Ala. 1962); Scholle v. Hare, 367 Mich. 176, 116 N.W.2d 350 (1962), *execution stayed pending appeal*, 31 U.S.L. WEEK 1018 (July 31, 1962).

Elections at large are a commonly mentioned form of direct relief. Indeed, it was this form of relief which appealed most to pre-*Baker v. Carr* commentators. See, *e.g.*, Lewis, *Legislative Apportionment and the Federal Courts*, 71 HARV. L. REV. 1057, 1087-90 (1958). Since *Baker v. Carr*, it has been applied by some courts. See, *e.g.*, Scholle v. Hare, *supra*; Mann v. Davis, *supra* note 5 (execution stayed by the district court pending appeal). The chief advantage of this form of relief is the tremendous incentive it would afford to legislators, whose political careers would be at stake in an election at large, to reapportion the state themselves. Moreover, an election at large, if it did occur, would result in a perfect application of the equal population principle applied to the entire state.

Yet there are reasons why a court might reject this form of direct relief and instead actively reapportion a state. Should the incentive for legislators to reapportion themselves be insufficient and consequently an election at large actually be held, there would be nearly insurmountable problems of administration, such as the printing of ballots with as many as one hundred candidates for each party. Moreover, a legislature elected at large would not conform to our usual notions of representation. See Black, *Inequities in Districting for Congress: Baker v. Carr and Colegrove v. Green*, 72 YALE L.J. 13, 15 (1962). Many other problems might arise. See Comment, *Baker v. Carr and Legislative Apportionments: A Problem of Standards*, 72 YALE L.J. 968, 1037-38 (1962).

7. While reference will be made throughout to *judicial* apportionment, the proposal made in this paper could be utilized by legislatures as well as courts.

There are two approaches which affirmative judicial apportionment can take. That advocated by Mr. Justice Clark in Baker v. Carr, 369 U.S. 186, 259-61 (1962), is to improve somewhat the existing unconstitutional apportionment and districting by eliminating some of the grossest disparities, while generally adhering to the existing district lines. The theory of this approach is that the resulting improvement in the apportionment and districting, together with the threat that the court will more thoroughly apportion and district the next time, will be sufficient to "break the stranglehold" of the minority areas in the legislature and after the next election to enable that body to apportion and district equitably. See Sims v. Frink, 208 F. Supp. 431 (M.D. Ala. 1962).

The other approach to active judicial reapportionment, and that with which this article is concerned, requires the court to reapportion the state without regard to existing district lines. This approach has the advantage not only of awarding underrepresented citizens prompt relief but also of ending the litigation, which by this time will usually have dragged on for an extended period of time.

Since redistricting usually affects the political balance of a legislature, a court undertaking affirmative apportionment and districting is likely to become the subject of highly partisan appeals and criticism. Such criticism may create the appearance that the court is acting from political motivation with the desire of benefiting a particular partisan interest.[8] To avoid this "political thicket,"[9] a court may desire to limit its own discretion in creating new legislative districts. One means of accomplishing this end could be to adopt a mechanical formula which makes the actual drafting of district lines non-discretionary once general principles of representation have been determined. It is at the stage of drafting district lines, after all, that the decisions having the most immediate political impact must be made—for example, when deciding whether a particular precinct should be included within district A or district B. If a court decided on this course, it might attempt it by basing its apportionment and districting on the principles of equal population and contiguousness—that is, territorial continuity.[10] Even when following these two principles, however, district lines can be drawn in many ways, each with different political repercussions.[11] Courts, therefore, may seek additional principles which when combined with contiguity and equal population more sharply limit judicial discretion in drawing district lines.

Legislatures also might find it useful to adopt a procedure which limits discretion in drawing district lines. Because of the volatile side-effects of alternative redistricting proposals, legislatures are frequently unable to adopt any representation scheme. Deadlocked legislatures might break the political impasse if agreement could be reached on a districting procedure which divorces the results reached from the claims of partisan interests.

Compactness is potentially a principle which, when combined with contiguity and equal population, could produce a non-discretionary districting procedure. Although the value of using compactness as a guiding principle has frequently been suggested,[12] no precise definition of the term has been gen-

8. Wisconsin v. Zimmerman, 205 F. Supp. 673, 209 F. Supp. 183 (W.D. Wis. 1962), is an example of an apportionment case in which the court became embroiled in a partisan debate and the subject of partisan attack. See, *e.g.*, N.Y. Times, June 20, 1962, p. 19, col. 4; N.Y. Times, July 4, 1962, p. 7, col. 1.

9. Colegrove v. Green, 328 U.S. 549, 556 (1946) (opinion of Frankfurter, J.).

10. A contiguous legislative district is one in which it is possible to travel between any two locations within it without leaving the district. The presence or absence of contiguity can be determined simply by glancing at a district map. There is no concept of "best" contiguity.

11. See Black, *Inequities in Districting for Congress: Baker v. Carr and Colegrove v. Green*, 72 YALE L.J. 13, 15-16 (1962). There is a vast literature on the interaction of politics and districting and the problem of the gerrymander, a deliberate manipulation of districting to maximize a partisan advantage. See, *e.g.*, JEWELL, POLITICS OF REAPPORTIONMENT 14-17, 27 (1962); GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER (1907); SCHMECKEBIER, CONGRESSIONAL APPORTIONMENT ch. IX (1941); note 31 *infra*.

12. Statutory requirements of contiguity and population equality in congressional districting were introduced by Congress in 1842. 5 Stat. 491 (1842). The requirement of

erally accepted. Usually compactness has been conceived as solely a geographic relationship, which might be mathematically expressed as requiring the maximization of the ratio of a district's area to its perimeter.[13] If this were the definition, the most compact district would be a circle, since it is in that figure that the maximum area within a given perimeter can be enclosed.[14] Although

compactness, while not defined, was added by the Reapportionment Act of 1901, 31 Stat. 734 (1901). This tripartite requirement (contiguity, compactness and population equality) was repeated in the Act of 1911, 37 Stat. 13 (1911), but was dropped in subsequent enactments. The current law is found at 46 Stat. 26 (1929), *as amended*, 2 U.S.C. § 2a (1958). See generally Wood v. Broom, 287 U.S. 1, 7 (1932); Celler, *Congressional Apportionment—Past, Present, and Future*, 17 LAW & CONTEMP. PROB. 268 (1952).

Representative Celler has sought in the past to have the three requirements re-enacted into law. H.R. 73, 86th Cong., 1st Sess. § 22(c) (1959) would have required congressional districts to be drawn up in "as compact form as practicable." See *Hearings Before Subcommittee No. 2 of the House Committee on the Judiciary*, 86th Cong., 1st Sess., ser. 10, at 22-23 (1959); Celler, *supra* at 274.

Despite Congress' omission of the requirement of compactness, as well as those of contiguity and population equality, most state constitutions maintain these three requirements for state legislative districting. EDWARDS, INDEX DIGEST OF STATE CONSTITUTIONS 627-35 (1959). Political scientists have also recommended that compactness be a requirement of districting. See, *e.g.*, Roeck, *Measuring Compactness as a Requirement of Legislative Apportionment*, 5 MIDWEST J. POL. SCI. 70 (1961); Vickery, *On the Prevention of Gerrymandering*, 76 POL. SCI. Q. 105 (1961); Krastin, *The Implementation of Representative Government in a Democracy*, 48 IOWA L. REV. 549, 570-72 (1963).

13. There have been few rigorous attempts to define "compactness"; in 1959, for instance, Congressmen seemed unable to explain what the term meant. *Hearings Before Subcommittee No. 2 of the House Committee on the Judiciary*, *supra* note 12, at 22-23. Where definitions have been given, however, they have usually been of the geographic nature referred to in the text. See, *e.g.*, Roeck, *supra* note 12; Vickery, *supra* note 12. *Cf.* Krastin, *The Implementation of Representative Government in a Democracy*, *supra* note 12, at 570-72. See also Professor Kallenbach's suggestion, the ratio of east-west measurement to north-south measurement, *Hearings Before Subcommittee No. 2 of the House Committee on the Judiciary*, *supra* at 64.

An interesting attempt to give different content to the term "compactness" has been made by Representative Celler:

> As to the requirement of compactness, such elements as economic and social interests of an area, its topography, means of transportation, the desires of the inhabitants as well as their elected representatives and finally the political factors should all be considered.

Celler, *supra* note 12, at 274. Representative Celler conceded to the state legislatures the task of pouring content into this political definition. *Ibid.*

14. A circle of one-mile perimeter encloses $1/4\,\text{pi} = 0.0795$ square miles. Regular hexagons, all of the same perimeter, would provide the highest area per unit of perimeter for geographical shapes that fit together in unlimited number. A regular hexagon of one-mile perimeter encloses 0.0722 square miles. Similar figures for a square and a triangle are 0.0625 square miles and 0.0621 square miles respectively.

One commentator has proposed a measure taking advantage of the circle's compactness. He would test the compactness of a legislative district by comparing the area of the district to the area of the smallest circle completely circumscribing the district. Roeck, *supra* note 12.

this definition is useful for comparison of already formulated districting plans, further study would be needed to determine whether it could be adapted to a procedure for creating districting proposals which are as compact as is possible consistent with other desired goals, such as contiguity and equal population.

As defined in this article, compactness is not solely a geographic measure. Because we are attempting to reflect at least to some extent popular interests [15] in districting and because population patterns may coincide with interest patterns, the principle of compactness is here defined as a measure of population as well as geographic concentration.[16] Under this definition a district's boundaries will not necessarily approach a circle as a limit as greater compactness is achieved. But constructing districts using this compactness definition will tend to locate districts of maximum compactness around centers of population, whereas, under prior definitions, "compact" districts would as likely divide population centers as respect them. And the expanded definition tends to favor districts coincident with communities of economic or other interests, insofar as these interests coincide with areas of high and low population densities.[17] Granted, comparison of districting plans by the proffered definition will require the use of more complex mathematical techniques than where only geography is considered. But use of this definition is fortunate in more than its tendency to favor interest-oriented plans; ready-made computer programs can be adapted to permit creation as well as comparison of districting plans based upon it.

Apportionment and Districting Procedures

Before courts or legislatures can draw district lines, certain decisions must be made.[18] The first of these, normally made in the state constitution, is the

15. The argument has been made that in our pluralistic society, a legislative body should reflect its various constituent communities—that districts should be drawn to encompass one particular "community" so as to reflect its interest in the legislature. See de Grazia, *General Theory of Apportionment*, 17 LAW & CONTEMP. PROB. 256 (1952). Some of this "reflective" philosophy is apparent in Representative Celler's statement, *supra* note 12, with its emphasis on social and economic factors, and in Wright v. Rockefeller, 211 F. Supp. 460, 465 (S.D.N.Y. 1962), where New York created a Manhattan congressional district that was nearly 100% Negro and Puerto Rican in population. Of course, it may be impossible to represent all possible diverse social interest groups and still adhere to the equal population principle, and attempts to account for such constituencies involve the courts to some degree in political matters.

16. Technically, the definition of compactness proposed here is a mathematical one, based on the moment of inertia principle. For this definition, see notes 34-40 *infra* and accompanying text.

17. For a more complete discussion of this phenomenon, see paragraph immediately preceding text accompanying note 40 *infra*.

18. These decisions can be made in one or more ways. Usually, some, if not all, will be made in the state constitution. Without a constitutional amendment, a legislature could not alter those decisions. Nor could a court, unless it found the state constitutional provisions violated the fourteenth amendment of the federal constitution. Some or all of these

number of legislative houses.[19] Next, the number of legislators in each house must be determined. Where this number is not fixed in the constitution, even a court might change it during apportionment and districting.[20] Perhaps most crucial is the decision as to principles of representation for each legislative house. If population, the most frequently mentioned principle, is adopted, then several population measurements are available: total population, population of voting age, population excluding aliens, or registered or actual voters. The choice among these population measures can have a considerable effect on the final pattern of representation.[21] In this article total population is assumed to be the appropriate population measure. Non-population principles of representation have also been used. Thus, area has been adopted by some states, as have such principles as community of economic or other interests.[22] The districting proposal as made in this paper assumes a choice of population equality, compactness, and contiguity as the only principles for representation, although modifications could probably be made to accommodate other principles.

---

decisions may be made by general statute or long-accepted practice. A legislature could then amend those statutes or ignore the practices while apportioning and districting. But a court, if it were apportioning, would probably feel constrained to apply the general statute, if not the consistent past practice, assuming neither violates the federal constitution.

19. All states have two houses except Nebraska, which has one. 12 Book of the States 29 (1958).

20. Only in Virginia and Washington is the number of legislators established by the legislature entirely apart from the constitution. In Maine and Rhode Island, however, the number of senators is set by a sliding scale based on population, as is the number of representatives in Connecticut. Many constitutions specify only ranges, minimum or maximum numbers, for one or both houses, or a ratio between the numbers in the houses. See National Municipal League, Compendium on Legislative Apportionment (1962).

Where a formula for determining the number of members is found to violate the fourteenth amendment of the United States Constitution, a court might choose a number of members differing from that derived from that formula. Such a situation would have faced the federal district court in WMCA v. Simon, 208 F. Supp. 368 (S.D.N.Y. 1962), had it found unconstitutional the challenged provision of the New York Constitution, a complicated formula for determining the number of senate seats. See N.Y. Const. art. 3, § 4.

21. See Silva, *The Population Base for Apportionment of the New York Legislature*, 32 Fordham L. Rev. 1 (1963); Committee of 39, Wilmington, Del., Notes and Statistics on Overlay Maps, pp. 1-2 (mimeographed materials). Two legislative houses could retain some difference in bases, and retain population as the primary principle of representation, if one house were based on total population and the other on, say, actual voters. See Silva, *Legislative Representation—With Special Reference to New York*, 27 Law & Contemp. Prob. 408, 409-14 (1962).

22. Michigan recently has amended its constitutions to recognize explicitly the principle of area. N.Y. Times, April 3, 1963, p. 24, col. 3. And in Wright v. Rockefeller, 211 F. Supp. 460, 465 (S.D.N.Y. 1962), the district court seemed to give credence to an argument upholding Manhattan's congressional districting on a "community" theory. See also note 15 *supra*.

Once principles of representation have been chosen it must be determined whether to respect boundaries of existing political units, such as cities or counties, in subsequent districting. If any such units are recognized, a number of legislators is then "apportioned" to each, using pre-established formulae reflecting some of the principles of representation adopted.[23] Such boundaries immediately restrict the population equality achievable through later districting and therefore may violate a constitutional standard of equal population, should one be adopted.[24] For example, even in tiny Delaware, which has but three counties, the apportionment of the thirty-five representatives among these counties on a population basis results in a minimum deviation between the counties of seven per cent in population-per-representative.[25]

23. Congress, for example, distributes representatives amongst states, in accordance with the principle of equal proportions, an adaptation of the equal population principle. New York uses a more complex formula recognizing pre-existing political units, in this case, counties. See Silva, *Apportionment in New York*, 30 FORDHAM L. REV. 581, 595-650 (1962). See note 55 *infra.*

In its technical sense, apportionment refers to the allocation of legislative seats among pre-defined units. Districting, on the other hand, is the process of drawing the geographic boundaries within those pre-defined units. Thus, Congress "apportions" representatives among the states while the states "district" by drawing congressional district lines.

24. For this reason the American Political Science Association recommends against recognition of such units. See ZELLER, AMERICAN STATE LEGISLATURES 46 (1954).

25. Since there are 35 representatives in the Delaware House, the ideal population for a legislative district in 1960 was 446,292 ÷ 35 = 12,751. But no solution giving effect to the county lines (*i.e.*, by requiring every district to be wholly within one county) could avoid an average population deviation per district of 7% (from the average in Kent to the average in Sussex):

Area	*Population*	*Pop. ÷ 12,751*	*No. of Reps. (to closest one-half)*	*Pop. ÷ No. of Reps.*	*% Deviation from 12,751*
New Castle Co.	307,446	24.1	24	12,810	—0.5%
Kent County	65,651	5.2	5	13,112	—2.8%
Sussex County	73,195	5.7	6	12,199	+4.3%
Total Delaware	446,292	35.0		12,751	

In fact, it is remarkable that these county populations, divided by 12,751, come even this close to assigning whole representatives to one county or the other.

The next important boundary in Delaware is the major city of Wilmington, which is within New Castle County. However, the same calculation indicates that a major deviation in population must be encountered unless one representative is assigned half to Wilmington, half to the remainder of New Castle County.

Wilmington	95,827	7.5	7½ @ 7 reps.	13,690	+7.4%
			@ 8 reps.	11,978	—7.1%
Other New Castle Co.	211,619	16.6	16½ @ 17 reps.	12,448	—2.4%
			@ 16 reps.	13,226	+3.7%

Acceptable district boundaries within the existing political units must also be determined so that the full set of inviolate lines is known. These boundaries might be county, ward or precinct lines, which normally follow such recognized features as highways, rivers or railroads.[26] A legislature, although probably not a court, could alter county and precinct boundaries while redistricting, if it so chose. As with the recognition of major political units, such minimum area restrictions may limit achievement of population equality. And since census data is usually not collected for smaller political units, such as election districts and precincts,[27] such restrictions may also make approximations necessary in estimating the degree of population equality.[28]

The final step in this process, to be dealt with at length in this paper, is the actual drawing of district lines. In the past, principles of representation chosen by legislatures have not significantly limited their freedom to do almost whatever they wished.[29] Exercising the available option, legislatures have generally

Since the county would be allocated only 24 representatives (7 + 17 or 8 + 16), based on 1960 population, either the city or the rest of the county must accept under-representation or both will have to share a representative with the other part of the county. If no legislator can be shared between Wilmington and New Castle County, a minimum deviation of 10% (7.4 + 2.4 or 7.1 + 3.7) is automatically required within this single county.

Likewise, further improvements are possible if county lines are not observed as the determining boundaries. The city of Milford (1960 population, 5,795) lies partly in Kent and partly in Sussex County. By considering the population of Milford entirely with Sussex County for apportionment purposes, the deviation between the two counties could be reduced below 2%.

	Population	*Pop. ÷ 12,751*	*No. of Reps.*	*Pop. ÷ No. of Reps.*	*Deviation From 12,751*
Kent less Milford (Part)	63,403	4.97	5	12,681	–0.6%
Sussex incl. all Milford	75,443	5.91	6	12,574	–1.4%

26. For example, the amendment to the Delaware Constitution approved January, 1963, and declared unconstitutional in Sincock v. Duffy, 215 F. Supp. 169 (D. Del. 1963), contained the following provision: "Each new Representative District shall, insofar as is possible, be formed of contiguous territory; shall be as nearly equal in population as possible to the other new districts being created within the existing Representative District; shall be bounded by ancient boundaries, major roads, streams, or other natural boundaries; and not be so created as to unduly favor any person or political party" (Section 5, in part, modifying Section 2A of the Constitution). In Delaware, several election districts are now merely lines on a map (as are most state boundaries) which do not coincide with the streets and communities developed since the election districts were first established.

27. United States Bureau of the Census, The Definition of Census Enumeration Districts by Local Authorities 2 (rev. ed., 1959) recognizes only the following political boundaries: congressional districts, counties, all incorporated communities, wards, and certain unincorporated communities. See also notes 42-44 *infra* and accompanying text.

28. If actual or registered voters were used as the basis of representation rather than total population, an exact count would be available.

29. See notes 8-11 *supra* and accompanying text.

districted on a partisan basis, often with the goal of re-electing as many incumbents as possible. While population, registration and voting data have been used to some extent, political goals are most frequently served.[30] Gerrymandering [31] remains a frequent complaint.

But courts and legislatures may in certain circumstances regard the making of such political judgments as undesirable when actually drafting district lines.[32] The proposal which follows assumes courts and legislatures have chosen contiguity, population equality and compactness as guiding principles before the districting stage is reached and have also decided to minimize the discretion exercised during districting itself. Application of this procedure creates alternative proposals and permits direct comparison of those and other proposals for adherence to these three principles. This comparison permits immediate rejection of proposals deviating more than others as to both population equality and maximum compactness, or deviating excessively in relation to one of those principles. Nevertheless, there may be several proposals not excludable by that test, and some discretion may be necessary to choose among those remaining.[33]

Development of a Compactness Measure

Consider the data points plotted as circles in Figure 1. On the graph, X might represent passage of time and Y population during consecutive periods. What is illustrated here is a common statistical problem. If there is some reason to believe a straight line will best fit a series of data points, one can take a straight edge and move it through the points to "eyeball" a good line. Better statistical practice is to utilize the "least-squares" technique.[34] This technique

30. See JEWELL, POLITICS OF REAPPORTIONMENT 14-17, 27 (1962); GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER 21, 100, 124 (1907). It has been suggested that the failure of state legislatures to establish equitable representation schemes, and the socio-economic effects of that failure, were among the chief factors leading to the Court's decision in Baker v. Carr, 369 U.S. 186 (1962). See Comment, *Baker v. Carr and Legislative Apportionments: A Problem of Standards,* 72 YALE L.J. 968, 979-81 (1963); Wheeler & Bebout, *After Reapportionment,* 51 NAT'L CIVIC REV. 246 (1962).

31. Gerrymandering is "the formation of election districts, on another basis than that of single and homogeneous political units as they existed previous to the . . . (redistricting), with boundaries arranged for partisan (or factional) advantage." GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER 21 (1907). There are two principal means of gerrymandering: (1) distributing the opposition party's vote among a number of districts so that it is diluted and the opposition can carry few, if any, districts; (2) concentrating the opposition's vote in a few districts so that it is dissipated in the form of large margins in these few districts. Districts can be absolutely equal in population and still be gerrymandered. *Id.* at 15-21. Nor will a gerrymandered district always be identifiable by its shape, in spite of the usual connotation of the term.

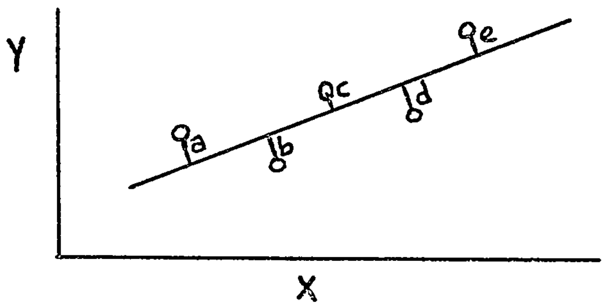
32. See notes 1-12 *supra* and accompanying text.

33. This is a methodological outline of the proposal put forth in this article. A more complete development of this procedure will follow in the subsequent text and notes.

34. CROXTON & COWDEN, APPLIED GENERAL STATISTICS 260-70 (2d ed. 1955); EZEKIEL & FOX, METHODS OF CORRELATION AND REGRESSION ANALYSIS 61-63 (3d ed. 1959); KEY, A PRIMER OF STATISTICS FOR POLITICAL SCIENTISTS 78-81 (1959).

mathematically locates the line which minimizes the sum of the squared distances[35] from the points to the line $(a^2 + b^2 + c^2 + d^2 + e^2)$.

FIGURE 1
"LEAST SQUARES" FIT OF LINE TO DATA



The least-squares line on a graph is analogous to the center of gravity in a physical body. Both are measures of average location. The former has points averaged into a line; the latter has weight of the body averaged into a central point.

The example of the center of gravity in a physical body brings us closer to a concept immediately relevant to the question of representation than dispersion about a line.[36] In studying the properties of rotating bodies, physicists find it useful to have a measure of the dispersion of the body's weight about an axis of rotation. This measure is called the moment of inertia.[37] The physical

35. The effect of squaring the distance may be clarified by a practical problem involving a least-squares solution. Assume two people live two miles apart, and it is desired to build a road perpendicular to the line between their houses. It is possible to put the road adjacent to one of the houses or equidistant between them. In either case the total man-miles from the highway are two: In the former case, one person two miles away $[(1 \times 2) + (1 \times 0) = 2]$, and in the latter, two people each one mile away $[(1 \times 1) + (1 \times 1) = 2]$. A decision rule which merely minimizes the total deviation would be indifferent to the choice. Intuitively, we can argue that the alternative represented by the latter best distributes man-miles over the whole community, while keeping the total at a minimum. It would also be the selection under a least-squares criterion, for the sum of the squared deviations in the former case would be $2^2 + 0^2 = 4$ while for the latter, it would be $1^2 + 1^2 = 2$.

36. Dispersion, or minimum variance, as applied to population equality, has also been proposed as a single measure of districting. PRAY & MANER, THE NEW PERSPECTIVE OF LEGISLATIVE APPORTIONMENT IN OKLAHOMA 27-28 (1962). A districting proposal embodying this procedure was submitted to a referendum in Oklahoma, but was defeated by a two-to-one margin. *Id.*

37. The moment of inertia of a mass about an axis of rotation is defined as the product of the mass and the square of the distance to the axis (the name was first suggested by Euler in 1765). It plays the same role in rotational motion that mass alone does in linear motion. In linear motion

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

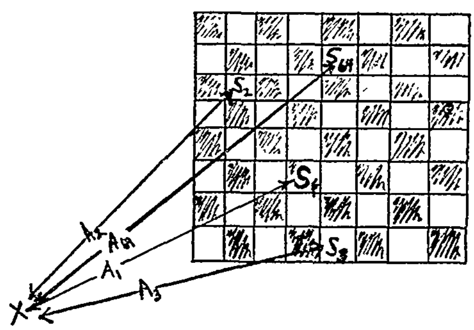
and

$$\text{Kinetic Energy} = 1/2 \times \text{Mass} \times (\text{Velocity})^2.$$

body consisting of the five points in Figure 1 has a moment of inertia about the illustrated line equal to $(a^2 + b^2 + c^2 + d^2 + e^2)$, assuming each point has a mass of one unit.[38]

If a body has only two dimensions and an axis of rotation is perpendicular to its plane, one can talk about the moment of inertia of the body about the point where the perpendicular axis intersects the plane. For purposes of computation the moment of inertia about that point X is defined as the weight of the body times the square of the distance from it to X. Thus, in Figure 2, the moment of inertia of the small shaded square labeled S_1 about point X is its weight W_1 times the distance A_1 squared—that is, $W_1A_1^2$. If the body is large with respect to the distance (the whole checkerboard in Figure 2, for example) its moment of inertia is computed for each of the segments of the body (each small square here) and added together to obtain the entire body's moment of inertia. This procedure is necessary because the small squares are different distances from X and may have different "weights" (*e.g.*, W_1, W_2, W_3, . . . W_{64}). Thus, the moment of inertia about X for the whole checkerboard is $(W_1A_1^2 + W_2A_2^2 + W_3A_3^2 + \ldots + W_{64}A_{64}^2)$.

FIGURE 2



For bodies of equal weight but differing distribution of the weight, the moment of inertia for an axis running through the center of gravity is smallest when the weight is concentrated at the center, *i.e.*, when the body is compact.[39]

In rotational motion

Torque = Moment of Inertia × Angular Acceleration

and

Kinetic Energy = 1/2 × Moment of Inertia × (Angular Velocity)^2.

That it takes more energy to stop a weight swirled about by a two foot string than one of equal weight swirled at an equal number of revolutions per minute (angular velocity) at the end of a one foot string is indicative of the fact that the moment of inertia of the first string and weight is greater.

38. SEARS, MECHANICS, HEAT AND SOUND 202-13 (2d ed. 1958); 1 SHORTLEY & WILLIAMS, PHYSICS 193-98 (1950); WHITE, MODERN COLLEGE PHYSICS 178-80 (3d ed. 1956).

39. See authorities cited in note 38 *supra*.

To grasp this idea, consider three objects: a bicycle wheel, a flat disk, and a top. If all have equal weight, the moment of inertia about the vertical axis through the center of each is least for the compact top, where weight is concentrated near the axis, and most for the wheel, where weight is concentrated at the rim, distant from the center. By the same token the moment of inertia about an axis of rotation for any particular object will be least if the axis passes through the body's center of gravity. For example, if the moment of inertia of the top were measured from a point one foot from the top's center of gravity, the calculation might give a result about equal to the value for the wheel calculated about its center of gravity. Thus, in the checkerboard of Figure 2, assuming the weight is equally distributed throughout, the moment of inertia is smallest about the checkerboard's center.

Moment of inertia provides a possible measure of compactness in legislative districting, involving both area and population. Assume that the checkerboard in Figure 2 represents one legislative district. By dividing the district into sixty-four rectangular blocks, it becomes possible to make a calculation analogous to moment of inertia about any point in the plane of the checkerboard. For each block, this calculation, which hereafter will also be called moment of inertia, would be the product of the block's population times the square of the distance between the block and that point. To obtain the moment of inertia about that point for the entire district, the moments of inertia for each block are summed. This figure will be smallest when the point about which the moment of inertia is calculated is the population center of the district—that is, the "center of population gravity." In Figure 2, if the people were distributed equally in each of the blocks, the "center of population gravity" would be at the center of the checkerboard. On the other hand, if there were a city in the upper left-hand corner and if the remaining blocks were sparsely populated, the population center would move from the center of the board toward that corner.

Now assume that the checkerboard in Figure 2 is to be divided into several legislative districts of equal population. If there is some concentration of population, one or more of these districts can be made relatively "compact"—its moment of inertia can be made small—by locating the legislative districts so that their population centers will be near the center of the population concentration. The remaining districts will be larger and have greater moments of inertia, however. To achieve a balance, and thereby guarantee that all the districts are somewhat compact, one could compare the summed moments of inertia of many different districting plans and choose that which produces the lowest sum. The compactness of a districting plan would then be defined as the sum of the moments of inertia of each proposed district about its own center of population gravity, the most compact plan being the one having the lowest sum.

Application of this definition of compactness would tend to discourage districts of extremely elongated shapes, since the farther a part of the district is from the population center of that district, the more it will add to the moment of inertia. Similarly it would tend to create districts the population centers of which coincide with areas of high population density, since the closer that high density area is to the population center of its district, the smaller will be the distance squared factor by which the population figure will be multiplied, and consequently the lower will be the moment of inertia. Since it is the sum of these moments of inertia of each district in the plan which is minimized, these phenomena are tendencies and not certainties.

Districting By the Compactness Measure

As already described, before districting is begun by this or any other method, certain choices must be made.[40] The number of houses, the number of legislators in and the principles of representation for each house, and the larger political units within which districting will occur must be determined. Finally, it must be decided what are the minimum units which must be wholly contained in any district. This last decision will both determine and depend upon the types of boundaries deemed acceptable for the final districts. In the example in Figure 2, these minimum units correspond to the sixty-four rectangular blocks. If population equality is a principle of representation, accuracy dictates that these units be ones for which population figures are available. Since census figures do not ordinarily provide population counts of precincts or election districts, these units are not apt for this purpose.[41] On the other hand, if equality of registered or actual voters were a basis of representation, then precinct or election units could be used.

The smallest unit of population count provided by the United States Census is the "enumeration district"[42] (hereinafter abbreviated ED), and these have been chosen as the minimum unit here. This unit has natural boundaries of the type usually desired for legislative districts (hereinafter abbreviated LD), such as rivers, highways, or railroads.[43] And since ED population averages under 1,000,[44] LDs will generally be sufficiently large to permit quite precise equalization of population even though each ED must be wholly contained within an LD.

40. See notes 18-28 *supra* and accompanying text.

41. See note 27 *supra*.

42. An enumeration district is a clearly defined geographic area, to be covered by one census enumerator during the decennial census. For large cities, population data per city block is also available. UNITED STATES BUREAU OF THE CENSUS, UNITED STATES CENSUSES OF POPULATION AND HOUSING, 1960: PRINCIPAL DATA-COLLECTION FORMS AND PROCEDURES (1961); UNITED STATES BUREAU OF THE CENSUS, THE DEFINITION OF CENSUS ENUMERATION DISTRICTS BY LOCAL AUTHORITIES (rev. ed. 1959).

Population statistics and maps of the enumeration districts are available for all areas within the United States, and may be purchased from the Bureau of the Census.

43. See note 26 *supra*.

44. In Delaware, the state to which the proposed formula will later be applied, the average population per ED is approximately 700: individual EDs range from 0 to 2200 in population.

Other units for which population data is available could also be used. Census tracts, which consist of several EDs, are readily adaptable to this formula. Counties could also be used where several counties are to be combined to create each LD. If larger minimum units are used, however, greater population inequalities will be necessitated and, consequently, constitutional standards may be violated.

Since districting by minimizing moment of inertia involves numerous calculations, application of this procedure by hand would require considerable time and introduce significant probability of arithmetic error.[45] To overcome these problems, we have used electronic computers, which very quickly perform the necessary calculations by applying an intricate set of logical rules—the computer "program"—to the data supplied them.[46] No available programs or computer techniques are known which will give a single, best answer to the districting problem, though such a solution seems possible if enough funds and efforts are put to the problem, especially considering the rapid advances in size and sophistication of available computers.

Despite a press of time and dearth of funds, it has still been possible largely to solve this problem through computers. The chosen measure of compactness makes it possible to take advantage of certain mathematical similarities between the redistricting problem and a problem already programmed on computers—that of assigning customer orders to specific warehouse locations so as to minimize freight costs.[47] This program, supplemented for this specific use by various additional steps and subcalculations, assigns EDs (customers) to LD centers (warehouses) in a manner minimizing moment of inertia (freight

45. Arithmetic error in reapportionment has been regularly troublesome. As early as 1790 when Thomas Jefferson made hand calculations for legislative apportionment and districting, several errors were carried over into the original congressional apportionment. SCHMECKEBIER, CONGRESSIONAL APPORTIONMENT Ch. VIII (1941). A recent opinion demonstrates similar problems exist today, even in a small state like Delaware:

> Upon analysis of plaintiffs' submissions the court has found a number of errors that illustrate vividly the extreme difficulty of apportioning a State in a mathematically correct and workable fashion.

Sincock v. Duffy, 215 F. Supp. 169, 194 (D. Del. 1963).

Automatic computation minimizes chance of such errors, in spite of many tedious manipulations required, and permits automatic checks of population totals and other figures at each stage.

46. For a general description of the operation of computers, see BERKELEY & WAINWRIGHT, COMPUTERS, THEIR OPERATION AND APPLICATION (1956); KOZMETSKY, ELECTRONIC COMPUTERS AND MANAGEMENT CONTROL (1956); CANNING, ELECTRONIC DATA PROCESSING FOR BUSINESS AND INDUSTRY (1956); GRABBE, RAMO & WOOLDRIDGE, HANDBOOK OF AUTOMATION, COMPUTATION, AND CONTROL, VOL. 2 (1958); STIBITZ & LARRIVEE, MATHEMATICS AND COMPUTERS (1957).

47. Mathematically, this is the transportation problem of linear programming and has been solved exactly in many kinds of applications. CHURCHMAN, ACKOFF & ARNOFF, INTRODUCTION TO OPERATIONS RESEARCH 279-98 (1957); GARVIN, INTRODUCTION TO LINEAR PROGRAMMING 85-104 (1960).

cost). This procedure will now be described in detail, followed by an illustrative example.[48]

The first step in this procedure is to feed the computer the data to which it will apply the formula, or "program." Since census data does not establish the location of individuals within each ED, all people are assumed to be located at the geographic center of their respective ED's. The centers of all the EDs within the unit to be districted are then located on a coordinate grid and the north-south, east-west coordinates of each ED center, along with its population, are fed into the computer.[49] This enables the computer to calculate the distance between any point and each of the ED centers, and in turn calculate the moment of inertia of the ED about any point. The number of LDs to be created, and their average population, must also be fed into the computer.

At this point it is necessary to make a set of initial guesses of the population centers of each LD (warehouse location), and then to feed the coordinates of those guesses into the computer. The computer assigns each ED (order) to an LD center in a way that minimizes the sum of the moments of inertia about the hypothesized centers for the entire unit being districted. A characteristic of the existing program requires exactly equal population in the LDs; therefore, the computer generally will assign parts of one or more EDs to different LDs.[50] To counteract this phenomenon, a supplementary computer program reunites split EDs so that the entire ED is assigned to the LD having the largest share of the ED's population. Based on this reassignment of EDs to LDs, the computer then calculates the population and moment of inertia of each LD and totals the moment of inertia of the entire unit districted.

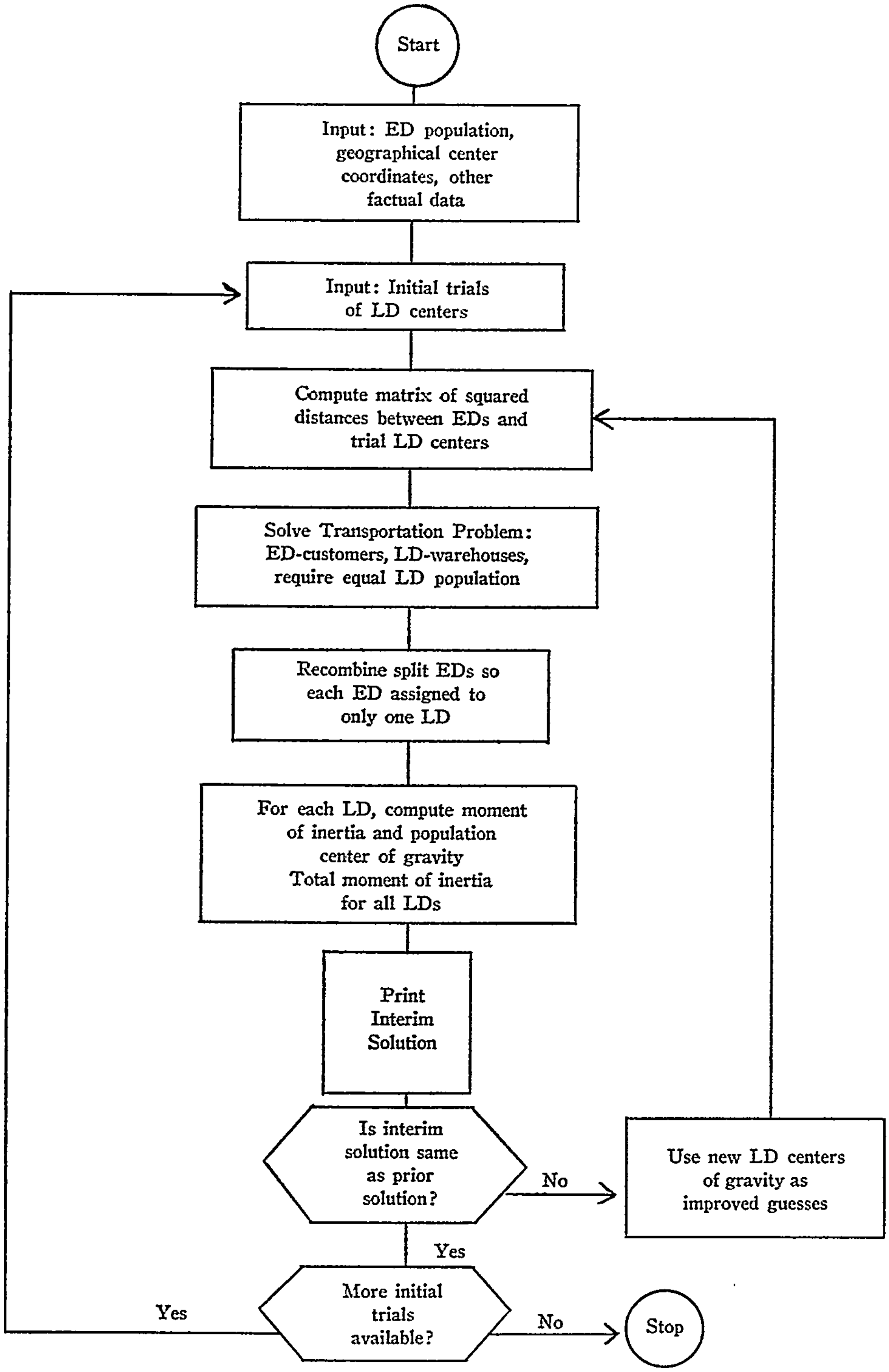
Since the assignment of split EDs to one LD will likely alter the population centers of the LDs affected, the computer is also directed to determine the actual LD population centers at this time. If the actual population centers of the LDs are different from the trial centers, the districting cannot be assumed the same as when districts are assimilated around the calculated centers. Consequently, the calculated centers are now used as new trial population centers for a new redistricting by the procedure just described. If this results in a different ED assignment to LDs, the actual population centers of these new LDs are again calculated and, if they are different from the ones previously calculated, the entire process is repeated. This procedure is continued until no change in ED assignment results from use of calculated as opposed to trial LD centers.

At this point, we have a series of possible districting plans with calculated population deviations and moments of inertia of each. Comparison of these

48. The following flow chart may be of some assistance in understanding this procedure. Except where indicated by arrows, the procedure flows vertically, step by step, down the chart. (See facing page, 303.)

49. See note 56 *infra*. In some areas the available coordinates of the ED centers will be so similar that it is possible to give only one set of coordinates, using the sum of the EDs' populations as the weight of the combined district.

50. It may be shown mathematically that the maximum number of EDs split will be one more than the number of LDs to be created. GARVIN, *op. cit. supra* note 47, at 87.



Start
Input: ED population, geographical center coordinates, other factual data
Input: Initial trials of LD centers
Compute matrix of squared distances between EDs and trial LD centers
Solve Transportation Problem: ED-customers, LD-warehouses, require equal LD population
Recombine split EDs so each ED assigned to only one LD
For each LD, compute moment of inertia and population center of gravity Total moment of inertia for all LDs
Print Interim Solution
Is interim solution same as prior solution?
No
Use new LD centers of gravity as improved guesses
Yes
More initial trials available?
Yes
No
Stop

will allow rejection of those plans which are inferior to others in both categories. The plans remaining are then outlined on a map to check for contiguity, for which the present computer program does not account, and the non-contiguous ones are rejected.[51] This entire process is then repeated using different initial guesses. There is no rule as to when trials should be stopped, but since additional trials can be promptly processed with high speed computers, a sufficient number should be used to obtain a good cross section of alternative districting plans.[52]

This procedure, supplemented by the manual rejection of solutions having both higher population deviations and greater moments of inertia than other possibilities, will eventually produce a collection of possible plans, among which there is no formalized process for choosing. Some of these plans can probably be rejected intuitively as being especially bad as regards one of two criteria. Thus, the plan with the lowest moment of inertia may have a district deviating as much as fifteen or twenty per cent from the average district population.[53] There may be a few plans which cannot be rejected on this basis, however. Among these, a court or legislature could decide to make an *ad hoc* judgment as to which is most desirable. Alternatively, it could decide beforehand to apply the plan having the lowest moment of inertia within some minimum population deviation, say ten per cent.

We have applied the above computational procedure to develop districting proposals for two of Delaware's three counties.[54] Following present practice,

51. The computer program could also be modified to check for contiguity, though the one outlined here does not do so.

52. Since the moment of inertia formula will tend to center LDs in areas of high population densities, it is natural that one's first sets of initial guesses will correspond with those areas. It is necessary, however, to include later sets of initial guesses which place LD centers in other areas. In Figure 3, for example, the best districting resulted from initial guesses different from the population centers.

53. Several population criteria have been suggested. The one used here measures the maximum population deviation as a percentage of the mean district population. Thus, if the mean district population in a state is 10,000 while the population of individual districts ranges from 8,000 to 11,000, the maximum deviation is 2,000, or 20%.

A second measure of population equality is the ratio of the most populous district to the least populous district. In the above hypothetical example, then, the most populous district (11,000) is 1.38 times as large as the least populous district (8,000). A third measure, the so-called Dauer-Kelsay scale, is the smallest percentage of a state's population that could elect a majority of the legislative body in question. See Dauer & Kelsay, *Unrepresentative States*, 44 NAT'L MUNIC. REV. 571 (1955).

For a more extensive discussion of various mathematical measures of population equality, see Goldberg, *The Statistics of Malapportionment*, 72 YALE L.J. 90, 96-101 (1962).

54. The interest of the authors, an engineer and an operations research analyst, developed from studies on reapportionment by a Wilmington, Delaware, civic group, The Committee of 39. The group gathered historical statistics, general information on apportionment criteria in other states, and applicable census and election statistics which were used by the court, the plaintiffs and the defendants in Sincock v. Duffy, 215 F. Supp. 169 (D. Del. 1963). The Committee, represented by Mr. Bruce Stargatt, petitioned for recognition as a friend of the court in late January, 1963. The possibility of districting via

each county was dealt with separately, so that county lines would not be crossed by the districting. The current number of thirty-five legislators was maintained and apportioned among Delaware's three counties according to the "method of equal proportions," which is now used to apportion seats in the United States Congress among the states after each decennial census.[55] Figure 3 and the following table of data illustrate the best districting solution obtained for Sussex County using this computer procedure.[56] Sussex County is not a good example for demonstrating this formula's tendency to center districts around areas of high population densities, since it is predominantly rural, containing no towns of sufficient size to create significant contrasts in population density within the county. The EDs in this county are shown in Map A, in Figure 3. Approximately twenty sets of initial guesses as to LD centers were examined. The set of centers which ultimately resulted in the best districts are shown in Map B. Map C shows the computer program's original assignment of EDs to LDs. By using the actual centers of population of these districts, another allocation of population was developed as shown in Map D. This was actually the best assignment.[57] As Map E indicates, the third trial, based on the actual population centers of Map D's districts, was slightly worse. The fourth trial gave no further change. Note that the county moment of inertia was improved by seven per cent between Maps C and D.

None of the other 19 initial trials yielded better districting solutions, although this does not prove that no better solution exists. In Sussex County, the contiguous districting with the lowest moment of inertia also showed the least population deviation. Consequently, it was not necessary to choose between several good solutions.

---

computer occurred to the senior author and the proposal herein was suggested and mathematically developed by the junior author.

Attempts were made to district all three Delaware counties by hand using rules similar to those which might be applied by the computer. See note 25 *supra*. When the computer formulation was developed, both Kent and Sussex Counties were subjected to many trials and single recommended plans established, according to the chosen principles of representation. These were far better than any hand solutions. Preliminary trials to district New Castle County on the computer have been made, using census tracts in order to minimize initial complexity. Further trials will be necessary to establish the best set of alternative plans. The Delaware plaintiffs have shown interest in the possibilities of this formula, but have not yet chosen to introduce it into the pending court case.

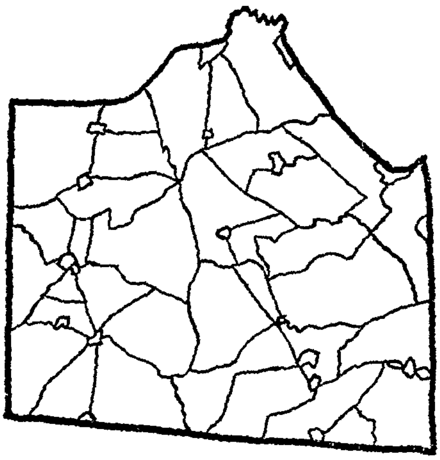
55. Schmeckebier, *The Method of Equal Proportions*, 17 LAW & CONTEMP. PROB. 302 (1952); SCHMECKEBIER, CONGRESSIONAL APPORTIONMENT 70 (1941) (quoting from report of a Committee of the National Academy of Sciences); Huntington, *A Survey of Methods of Apportionment in Congress*, S. Doc. No. 304, 76th Cong., 3d Sess. (1940).

56. The unit of distance used in moment of inertia is dictated by the map scale from which coordinates were read. Ours was scaled 0.288 inches to the mile. Therefore, to convert our values to a (man-miles)^2 moment of inertia, multiply by 12.1.

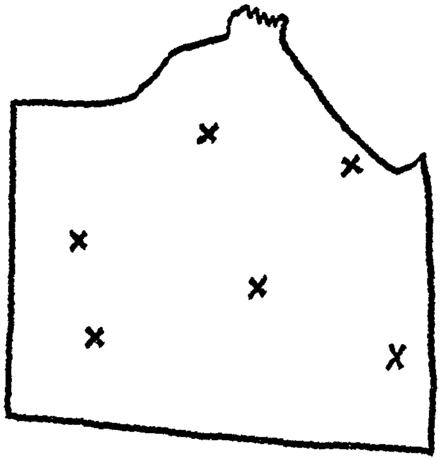
57. It should be pointed out that the computer-developed shape of districts is uneven due to borders of the EDs. If a smoother shape is desired, additional factors might be entered into the computer to penalize for deviations from a few major boundaries. This, of course, would further limit the achievable population equality. See note 25 *supra*.

FIGURE 3

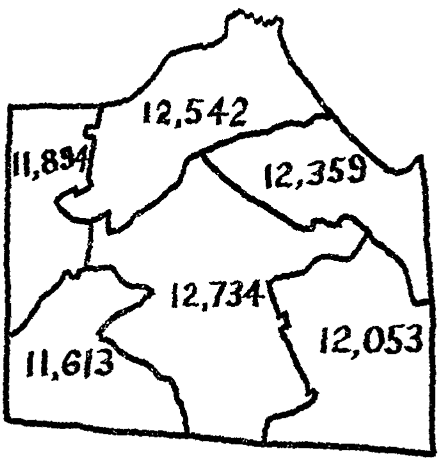
EXAMPLE OF DISTRICTING OF SUSSEX COUNTY



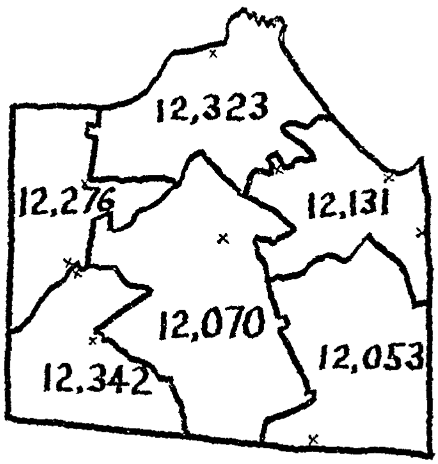
A



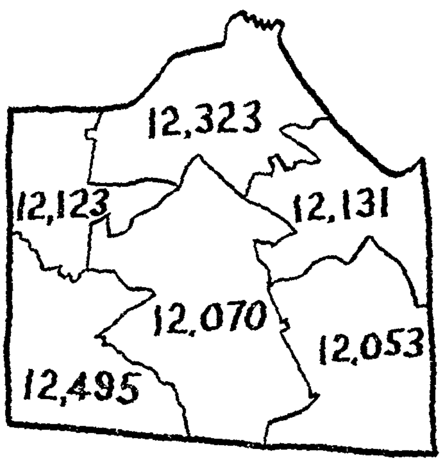
B



C



D



E

A—Location of U.S. Census Enumeration Districts

B—Set of initial guesses for the 6 legislative districts apportioned to Sussex

C—First assignment of population to legislative districts based on guessed centers
Maximum Deviation: 5%
Moment of Inertia: 143,774

D—Second trial — Improved assignment based on actual centers of first assignment (X's indicate towns of over 1000 population.)
Maximum Deviation: 1%
Moment of Inertia: 133,923

E—Third trial: slightly worse results
Maximum Deviation: 2%
Moment of Inertia: 133,992
(Fourth trial—no further change)

DATA FOR FIGURE 3

	Original Guessed Centers	*Trial 1*	*Trial 2*	*Trial 3*
LD #1				
X*	4.94"	4.58"	4.84"	4.84"
Y	7.28	8.08	8.20	8.20
Population		12,542	12,323	12,323
LD #2				
X	2.06	1.73	1.83	1.90
Y	4.62	4.93	5.05	5.08
Population		11,894	12,276	12,123
LD #3				
X	8.38	8.42	8.54	8.54
Y	6.75	6.17	6.04	6.04
Population		12,359	12,131	12,131
LD #4				
X	9.62	8.52	8.52	8.52
Y	2.62	1.93	1.93	1.93
Population		12,053	12,053	12,053
LD #5				
X	6.31	5.41	5.29	5.29
Y	3.75	3.88	3.83	3.83
Population		12,734	12,070	12,070
LD #6				
X	2.59	2.41	2.38	2.31
Y	2.41	2.11	2.22	2.22
Population		11,613	12,342	12,495
County Moment of Inertia		143,773.8	133,923.4	133,992.2
Total Population		73,195	73,195	73,195
Avg. 12,199				
Highest		12,734 +4%	12,342 +1%	12,495 +2%
Lowest		11,613 -5%	12,053 -1%	12,053 -1%

*X is E-W map coordinates in inches. (Map scale: 0.288 inches to the mile.)
Y is N-S map coordinates in inches.

CONCLUSION

This paper has proposed an objective, mathematically-based procedure for districting which produces contiguous districts nearly equal in population and more "compact" than other present methods can provide. It utilizes existing computer programming techniques to locate a given number of districts within a given area, by combining smaller areas of known population in accordance with selected principles of representation. Two of these, population and contiguity, are self-explanatory and measurable. In addition, the procedure recommended in this paper introduces a quantitative measure of compactness which tends to minimize perimeter and locate districts around population centers. By greatly reducing the number of choices that must be made, introduction of this third criteria assists the development of an impartial districting procedure. The proposed computer procedure will be of considerable usefulness to courts desiring to avoid partisan pressures and criticism when it must redistrict a state. Legislatures could also use it to avoid many compromises and delays. With the use of high-speed computers, good districting solutions may presently be calculated and chances for arithmetic error minimized.

The procedure as here reported is still in a state of development. The suggested program could be modified to accommodate principles in addition to contiguity, equal population, and compactness, if so desired. It can also be adapted to other problem situations, such as that of school districting in growing communities.[58] With sufficient effort, the computer program can probably be improved to produce "best" solutions which are not dependent on trial guesses as to LD population centers. Such a unique solution would have the least possible moment of inertia, given the prior assumptions. It might be useful in developing constitutional standards for apportioning and districting, although in itself it would not comprise a standard. Standards might be developed by first establishing certain principles of representation, among them compactness, which must be contained in any apportionment and districting, and then determining the constitutionally acceptable deviations from a quantitative norm for each principle. A "best" compactness solution could serve as a quantitative norm for the compactness principle. To make such standards a realistic alternative, we are working to create a computer program giving a "best" solution.[59] We urge others to do so as well. In the interim, the current proposal permits creation of superior districting proposals and a sound basis for comparison among these or other existing or proposed apportionment and districting plans.

58. The warehousing program, see note 47 *supra*, is particularly well adapted to adding a single warehouse (school) to an existing pattern, and determining orders (children) to be served from each location.

59. The authors have already formulated this particular problem statement mathematically as an integer programming problem. While the practical districting problem so formulated cannot now be solved, there is every reason to believe that efficient computer programs will be available to do so within one or two years.

For approaches to solving the analogous integer programming problem (the warehouse location problem) see Kuehn & Hamburger, *A Heuristic Program for Locating Warehouses*, 9 MANAGEMENT SCIENCE 643 (1963); BALINSKY & MILLS, A WAREHOUSE PROBLEM (1960); Baumol & Wolfe, *A Warehouse Location Problem*, 6 OPERATIONS RESEARCH 252 (1958); GOMORY, AN ALGORITHM FOR INTEGER SOLUTIONS TO LINEAR PROBLEMS (1958).

THE YALE LAW JOURNAL

VOLUME 73 DECEMBER 1963 NUMBER 2

CONTRIBUTORS TO THIS ISSUE

ELLEN A. PETERS. B.A. 1961, Swarthmore College; LL.B. 1954, Yale University.

JAMES B. WEAVER. B.S. 1944; M.S. 1953, Massachusetts Institute of Technology.

SIDNEY W. HESS. B.S. 1953, Massachusetts Institute of Technology; Ph.D. 1960, Case Institute of Technology.

KEITH S. ROSENN. B.A. 1960, Amherst College; LL.B. 1963, Yale University.

RALPH K. WINTER, JR., who contributed to Issue No. 1 is a Research Associate and lectures at Yale Law School.