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The Concept of Abstraction

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The Conception of Abstraction

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Philosophers deal with abstractions. Being reflective, they also have come up with theories about what these abstractions are. Aristotle is no exception, and indeed gave what came to be a canonical account of abstraction.[1] Here I shall investigate what Aristotle thinks abstraction is. I shall conclude that Aristotle views abstraction as *selective attention*.

As its very name suggests, abstracting (ἀφαιρέω) consists in taking away something from an object. The root verb, αἱρέω, suggests additionally a sense of grasping or of choosing, of taking for oneself something of what lies ready to hand.[2]

These lexical meanings leave open a wide range of conceptions of 'abstraction'. Does the abstraction consist in taking out something and discarding the rest? Or does it consist in taking away something and keeping what is left? We can call the first one the selection view, and the latter the subtraction view. The Greek gives an ambiguity between the two because 'ἀφαίρεσις', being a verbal noun, could be derived from the active form 'ἀφαιρεῖν', which generally does have the sense of 'removal', or from the middle form 'ἀφαιρεῖσθαι', which generally has the sense of 'take away for oneself' or 'steal'. On linguistic grounds of common usage, the selective reading of 'ἀφαίρεσις' has the advantage, as the middle voice forms are far more common than the active voice forms. Yet, as Aristotle is a philosopher, and philosophy stretches or distorts the ordinary usage of language, the philological evidence does not settle the issue. For that, we must turn to Aristotle's texts.

Origins of Aristotle's Theory

Originally, so some have speculated, Aristotle may have developed his conception of abstraction (ἀφαίρεσις) in order to have an alternative to Platonism.[3] Such an abstraction theory claims to provide a way to distinguish and recognize the different aspects of things, both universal and (perhaps) singular, but without granting any of these an independent, substantial existence *in re*, such as Plato's theory of Forms asserts. On this account, we can consider an object with respect to some of its attributes, take them out, and thereby create a new abstract thing consisting in that object in only those respects. We can then use this new thing as a subject in its own right. Yet we have only one object, the original substance existing *in re*, although with many attributes. In contrast, for Plato, everything that is a subject in its own right is an object existing independently. In this way, the doctrine of abstraction lies at the heart of Aristotle's metaphysical enterprise, to construct a theoretical alternative to Platonism.

In Aristotle's case, there is a natural basis for some such abstractions as opposed to others: some are scientific, like genera and *differentiae*; others are sophistical, like "musical Coriscus". [Metaph. 1026b17-8] In making the scientific abstractions, we isolate the proper subjects for the statements being made about real attributes of individual substances. In this way, we can start from sense perceptions of individuals and arrive at sciences of universals like

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[1] Julius Weinberg, <u>Abstraction, Relation, and Induction</u> (Madison, 1965), p. 5.

[2] LSJ s. v. aφaιreέw and aιρeέw.

[3] Wolfgang Wieland, <u>Die Aristotelische Physik</u> (Göttingen, 1962), p. 197 n. 12.

numbers, plane figures, and motion. Yet we are still talking about the real individual substances, not some fictitious, transcendent Forms, existing over and in addition to those individuals.

In accord with this approach, Aristotle explains how the universal is abstracted from the particular in his account of perception and thought. [An. III.4; Metaph. I.1; Phys. I.1] Likewise, he speaks of "cutting off a part of being" and making a science about it. [Metaph. 1003a24-5] Physics concerns substances *qua* movable; geometry considers substances *qua* figure. [Metaph. 1026a7-10; 1061a28-1062b11; 1077b22-1078a21] We start with the individual substances given in sense perception and then isolate aspects of them, the *abstracta*, for study in particular sciences.

Aristotle seems to recognize several types of these *abstracta*. First, he does clearly recognize universals in all the categories. The sciences study universals: not only species and genera of substance like dog, rose, plant and animal, but also those from other categories, like square, figure, sight, perception, justice and virtue. As items in the categories exist and further as the sciences study only things that exist [An. Po. 89b31-5], clearly Aristotle holds these universal species and genera to exist in reality. Yet, if Aristotle is to avoid Platonism, it is thereby quite likely that he holds these universals, or our knowledge of them, somehow to be abstracted from singular things.

Second, Aristotle might recognize also singular *abstracta*, like mathematical objects.[4] For not only do scientists need to speak of number, triangle, bird, redness, and walking in general. They also need to speak of particular instances of 'two' in '2 + 2 = 4', of the particular triangles used in the diagram of a geometrical proof bisecting a square on the diagonal, and of more than one bird in the mating process. The mathematical particulars at least do not seem to be sense objects.[5] In modern terms, they seem to be tokens of a universal type. In support of this interpretation, Aristotle speaks of an intelligible matter and not of perceptible matter, providing a basis for having more than a single instance of a type of mathematical object. Thus he seems to be indicating that there can be several instances of the same species, differing in number, even when there is no corporeal matter to differentiate. [Cf. Metaph. 1036a2-12; 1059b14-6.][6] These instances are particulars of some type. For they are composed of matter and form, and, being singulars, are not definable. Aristotle seems to state clearly that some mathematical objects are individuals. [Metaph. 1036a2-3] But, if they are singular, they are individuals quite differently than the sensible individuals are.[7]

Whether these intelligible particulars be taken as universal or as particular, they are going to create complications for a theory of abstraction, especially if the mathematical objects cannot be physical, strictly speaking. For a diagram would then be a token of a type of sign signifying a

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[4] Ian Mueller, "Aristotle's Doctrine of Abstraction in the Commentators," in Aristotle Transformed, ed. R. Sorabji (Ithaca, 1990), pp. 463-4.

[5] Although some have argued that Aristotle or some Aristotelian commentators took geometry to be about the particular figures and diagrams perceived by the senses. See Mueller's article for a general discussion.

[6] C. D. C. Reeve, Substantial Knowledge, pp. 62-3, also recognizes both universal and particular intelligible matter, which I shall discuss more below.

[7] Unless Aristotle holds that these individuals are abstracted directly from perceptions of individual substances. On this account, e.g., when I see a particular bronze sphere, I also upon abstraction have an individual sphere, the mathematical object. So too when I see the iron sphere I see another individual sphere. Also, looking at the spheres, I have upon abstraction an individual 2, an individual mathematical object. Cf. Simplicius, in Cat. 124,28-125,2. Yet, even so, if we are to have items in mathematics for which we have no exemplars *in re*, such as very large numbers or very complex geometrical figures, we still cannot reduce mathematical individuals to perceptible individuals.

mathematical object. These tokens too have a certain universality: it is not merely the ones here: '2 + 2 = 4', on this particular page that are being discussed. Rather, when I write that equation, the marks on the page are signs not only of themselves but also of some other tokens or token types. In order to have that equation, we need two instances of the number two, each represented by an instance of the numeral '2'. We can then see why Aristotle would think that mathematical objects need to have some sort of intelligible matter, in order to have many instances of the same species (or type) of number.[8] So mathematical objects have special problems, which I shall bracket here. Still, clearly Aristotle thinks that they too are "abstracted" from our sense experience of the world.

Aristotle thinks also that the things thus abstracted are objects existing *in re* that are in some sense independent of our thought. For the universal *abstracta* include the species and genera, the secondary substances that are the objects of science. To be sure, Aristotle does say that, if the individual, primary substances did not exist, neither would these secondary substances or universal accidents. [<u>Cat.</u> 2b6^b-6^c] Still, he does not deny that these species and genera exist really. So he seems to be saying that these abstract objects exist *in re*, but not independently of and separately from their concrete individuals, the primary substances. Mind (*noûs*) makes these items separate in thought by separating them off from the whole sense perceptions of individuals. One might then think that for Aristotle these *abstracta* are mere concepts, artifacts of the human mental process with no real correlates.[9] That is, on human, pragmatic grounds, we might focus on certain features of individual things in a particular science. Still, such grounds do not give any assurance that this science does more than to provide a useful, heuristic model nor that its objects have more than a conventional unity.

Nevertheless, Aristotle has a different view. As he recognizes that universal substances and accidents exist *in re*, he is assuming that these *abstracta* have a real basis. In performing at least certain abstractions, the scientific ones, we are asserting or presupposing the real existence of common structures of individuals *in re*. In our sciences, we may then be said to be "recognizing" (ἀναγιγνώσκειν) certain aspects of real things that apply in fact to more than a single individual in a basic sense of the word. That is, we are "re-cognizing", or representing again in thought, what already has a basis to be distinguished *in re*. A science becomes then more than a mere model; it becomes a "theory" (θεωρία) in an original, literal sense: of observing or looking at real structures existing in the world.[10]

We have then two basic phenomena or data about Aristotle's conception of abstraction. First, a process of abstraction is not supposed to create or presuppose new objects existing *in re* over and about the individual substances given in sense perception. Aristotle does not take abstract objects to be real objects *sui generis*. The species man does not exist *in re* over and above the individual human beings. Second, the abstract objects themselves do seem to include the universal substances and accidents, the universal species and genera asserted to exist and studied by scientists. So, on the one hand, abstract objects are not "real", and, on the other, they are objective.

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[8] Moreover, as the equation itself can be stated or written in many particular speech acts or writing acts, the numeral itself will need to have some way to have many instances, just as we can have many repetitions of the same statement (lo¢goV), as when we all utter the same true sentence in a chorus. Yet Aristotle does not seem to pursue this issue much, although some medieval Aristotelians like Ockham did, in subdivisions f material supposition.

[9] So J. Klein, <u>Greek Mathematical Thought and the Development of Algebra</u> (Cambridge, Mass, 1968), pp. 100-13.

[10] Cf. Deborah Modrak, <u>Aristotle's Theory of Language and Meaning</u> (Cambridge, 2001), p. 96: "…objects in the world…present themselves as concrete individuals and simultaneously as exemplifications of universals."

We see this tension exemplified in Aristotle's account of substance in the Metaphysics. There again, he does not want the substantial forms to be separate, universal objects, existing independently from individual substances. At the same time, he wants them to be "objective", to represent ('re-present') structures present in these real individuals, not merely in our conventional thought. Aristotle wants objective universal structures but only individuals *in re*. That is, Aristotle takes substantial forms to be abstract objects. Aristotle uses abstraction to explain how we can come to know universals from having sense perceptions, to give an account of mathematical objects without positing universals *in re*, and to discuss the universal features of what it is to be an individual substance without relapsing, he thinks, into Platonism.

These explanations lie at the very core of Aristotle's thought. Abstraction lies at the very core of these explanations. Accordingly, if we can but get clear on the structure of the sort of abstraction that he is using, we can gain insight into his theory as well as an increased ability to evaluate it.

The Meaning of 'Abstraction' (Ἀφαίρεσις)

The general discussion so far might suggest thinking of abstraction as extraction. Aristotle does speak of "cutting off a part of being" and making a science about it. Such talk suggests that we are cutting out, or extracting, certain aspects from the object and erecting them as separate objects. Yet this sort of extraction cannot be 'extraction' in the usual sense, though. E.g., when I "extract" a splinter from my foot, or gold from the ore, I end up with a pair of independent, individual substances: the splinter and my wounded foot, and the gold and the slag. If abstract objects were "abstracted" in this way, they would indeed have a separate existence over and above the individual substances from which they are abstracted.[11] Thus Aristotle's 'abstraction' would have to be thought of as a type of extraction where the items being extracted do not have a separate, independent existence. Consequently, it is not clear how helpful viewing abstraction as extraction is.

Accordingly, John Cleary has suggested that, rather, Aristotle conceives "abstraction" (ἀφαίρεσις) as a process of subtraction.[12] Here the individual substance remains, and we merely subtract everything that does not pertain to the respects stated. In support of his view, he notes that in the Topics Aristotle contrasts the method of "ἀφαίρεσις" with that of "πρόσθεσις", which at the time had the common meaning of 'addition' in the arithmetical sense. [Top. 118b10-9; 140a33-b15; 152b10-6] Plato too, he says, seems to use 'addition' and 'subtraction' in this sense. [Phaed. 95C; Euthyd. 296b; Cart. 393d; Prm. 131d; 158c] Aristotle himself contrasts the natural scientist's use of "addition" with the mathematician's use of "subtraction". [Cael. 299a14-8; Phys. 193b22-194a12; An. 403b9-19; Metaph. 1077b9-11]

Indeed, Cleary objects to calling 'ἀφαίρεσις' 'abstraction' altogether, partly because this translation suggests a conception of extraction, and partly because Aristotle does not view the process as psychological or epistemological, as in the later discussions of "abstraction" in Locke and Berkeley. For on their account of abstraction we make up general concepts or signs for our

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[11] Theodore Scaltsas, Substance and Universals in Aristotle's *Metaphysics* (Cornell, 1994), pp. 11-2; 34; 116, suggest that abstraction generates two objects. However he focuses on the abstraction of matter and form from a substance, and there we have a form, capable of definition, and, with the ultimate if not the proximate matter, an indefinite stuff. So unlike subtraction abstraction does not yield two equally definite things.

[12] John Cleary, "On the Terminology of 'Abstraction' in Aristotle," Phronesis, Vol. 32 (1985), pp. 18-9; Aristotle and Mathematics (Leiden, 1995), pp. 304; 309-14.

convenience after having experiences of individual existing *in re*.[13] The things abstracted may have use for us but need not reflect real structures in reality: they may be far removed from the "secret springs" of physical objects.[14] In contrast, Aristotle holds the things abstracted to reflect reality.

Cleary insists that 'ἀφαίρεσις' does not signify the way by which we come to have a certain sort of knowledge. Rather, it is the way by which the primary subjects for each science are isolated: it is that by which we "chop off a piece of being" so as to make it the proper subject of a special science. We do this by subtracting or removing attributes from the totality of those constituting an experienced object until we get a primary subject. However, although we do the paring down, still the process is not so much a merely psychological process by which we come to have perception and science, as an objective process by which we come to be aware of the attributes and types of individual substances. That is, although abstraction is a mental process, it is grounded upon real distinctions between aspects of things in the world. Other, non-rational animals also make abstractions in their sense perceptions, memories, and imaginings, although they do not make the ultimate abstractions whereby rational beings can locate the proper subjects for science, the universals. Cleary then sees that for Aristotle abstraction proper is primarily an ontological process whereby we locate and isolate the primary subjects for each science from our perceptions of individual substances with their full array of attributes—not a way by which we come to know the objects that we are locating and isolating in a peculiarly human, conventional way of knowing.[15]

Cleary's main evidence for Aristotle's not viewing 'ἀφαίρεσις' as an epistemological process whereby we acquire knowledge of objects lies in this passage:

> Now it is also evident that, if some [type of] perception is lacking, it is necessary also that some [type of] knowledge is lacking, if indeed we learn either by induction or by demonstration, where demonstration is from the universals and induction from the particulars, and it is impossible to contemplate the universal if not through induction (for since also those said from abstraction will be able to be made familiar through induction, because [or: that[16]] some things belong to each genus, even if not separate, *qua* each such thing [sc., the genus]), it is impossible for those who do not have the [type of] perception to make the induction [literally: be led to, sc., have the induction made for them]. For perception is of the singulars: for it is not possible to take knowledge of them: for neither from the universals without induction, nor through induction without perception. [An. Po. 81a38-b9]

The main points of the passage are clear: we have no acquaintance with singulars except through sense perception. We may then come to become acquainted with universals through induction on the singulars once acquired.[17] Then we may come to have knowledge of universals through performing demonstrations on these universals. So all knowledge comes from, or depends upon, sense perceptions, directly or indirectly. [Eth. Nic. 1139b27-31] As Cleary stresses, Aristotle does not say here that we perceive or know anything through abstraction. Rather, we come to grasp "even the things said from abstraction" through induction. Consequently, abstraction

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[13] Locke, An Essay Concerning Human Understanding, II11.9; IV.7.9; Berkeley, Principles of Human Knowledge, Introduction §§15-6.

[14] Hume, An Enquiry Concerning Human Understanding, V.1.

[15] Cleary, Aristotle and Mathematics, p. 308. His account agrees mostly with Jonathan Lear, "Aristotle's Philosophy of Mathematics," Philosophical Review Vol. 91 (1982), p. 168.

[16] I agree with Cleary, "On the Terminology of 'Abstraction' in Aristotle," p. 15, that either translation is possible.

[17] Jonathan Barnes, trans. & comm., Aristotle's Posterior Analytics, First Edition (Oxford, 1975), p. 161, notes that Aristotle claims here only that induction can make abstractions familiar to us, not that it alone can do so. He claims that Aristotle argues for that stronger claim at An. 432a3-6.

appears to be a process different from induction or demonstration. Its products are "the things said from abstraction". [81b3]

This phrase (τὰ ἐξ ἀφαιρέσεως λεγόμενα ἔσται δι' ἐπαγογῆς γνώριμα ποιεῖν [81b3]). may appear ambiguous: it may signify what is said from abstraction, sc., statements made as a result of abstraction, or the objects that we are now able to talk about as a result of the abstraction. Yet the dilemma of: words or objects? is misleading. For, as I have argued elsewhere, as Aristotle wants in his scientific language, an isomorphism between the words and the objects, what is said will match the actual properties of those objects. So we may as well take the phrase realistically, to mean the objects signified by such subject terms as 'triangle' and 'sphere'. Indeed, as Aristotle takes "the things said from abstraction" to provide the objects for the mathematical sciences, and science concerns only what exists *in re*, he is committed to a realistic views of these things. Accordingly, I shall henceforth call the "things said by abstraction" 'abstract objects'.

Also, we might see two possible ways of understanding 'from' (ἐκ) in "the things said from abstraction". On one reading, we would be inventing abstract objects, by treating aspects of real objects as if they were real, independent objects, without their really existing as such. On another reading, we would be discovering real abstract objects. The former gives a nominalist reading; the latter a Platonist. As Aristotle insists that he rejects Platonist accounts of abstract objects, like the objects of mathematics, we should take the first reading. Yet, given that Aristotle speaks of cutting off parts of being and of secondary substances existing in their own right, he does seem to want these *abstracta* to be extracted so as to constitute independent objects, albeit derivative, dependent ones. So the nominalism will be a "realistic" nominalism.

Aristotle has what I shall call a *transcendent* sort of abstraction. For the abstraction goes beyond the original objects perceived so as to generate, or at any rate to recognize, new objects. We perceive individual things and then via abstraction are able to know the universal objects of mathematics. These new objects have quasi-independence if not a real independence. For, as they serve as the objects of the sciences, they are the most intelligible objects of the things that are. Abstract terms are more than mere *façons de parler*.

Aristotle says here that these abstract objects become familiar to us through induction. Induction is a process whereby simple apprehension, via noûs, of the things apprehended is achieved. [An. Po. 100b3-15] So we become directly acquainted with these objects apprehended by induction. Then induction makes us able to apprehend and know abstract objects. The abstraction would have to serve a function other than enabling us to apprehend abstract objects, as Cleary maintains.

Aristotle implies at 81b4-5 (whether we take the 'ὅτι' at 81b4 to indicate the reason or to indicate the content of what has become familiar to us) also that each genus has some of the things said by abstraction given by induction. An abstract object belongs to a genus not in the way that a separate thing, sc., an individual substance, does. Rather each belongs to one "*qua* each such thing," i.e., *qua* itself. [81b5] Thus number belongs to discrete quantum and to quantum *qua* number; likewise number belongs to two *qua* two, or to two *per se* (καθ' αὑτό), *qua* number. Neither numbers nor even individual numbers exist *in re* as separate substances. Still, we may legitimately treat them as if they were separate individuals and put them under a genus, so as to have a science of arithmetic.

Posterior Analytics I.18 does then give us strong grounds not to view abstraction as a merely psychological process. It also gives us strong grounds not to identify abstraction with induction. Yes it does not follow, as Cleary seems to say, that the induction is not a type of abstraction. It could be that induction is one application of a process of abstraction, where abstraction could

have other applications. This text by itself does not resolve this issue. For instance, take induction as the process whereby the universals arise from the relevant singulars, and the abstraction used to generate the abstract, proper objects of mathematics as the process whereby universals inseparable *in re* in the individual substance and even *in intellectu* initially come to be treated as if they were separate. E.g., we might start off with individual physical objects and then via induction come to the general concept of body. Such a body would have color and shape (in general). Yet we may then "abstract" and treat the color and the shape as if they were separate, even though these universals necessarily go together. A non-rational animal could not make the final abstraction, Aristotle might say, although it can have experience and general notions ("primitive universals" as in Phys. 184a24-5; An. Po. 100a16) via some less ultimate processes of abstraction.

Again, should we agree with Cleary and translate 'ἀφαίρεσις' as 'subtraction'? This translation has the advantage that we can see the parallel with 'addition' clearly. Cleary seems to dislike the use of 'abstraction' because it, like 'extraction', suggests that the item to be abstracted already lies there ready to hand, and needs be only plucked out, like a raisin in a pudding. Rather, we should understand 'ἀφαίρεσις' to indicate a process whereby we take the object and pare away, or subtract, attributes until we arrive at the abstract object desired.

I see several problems with this approach. First, as we do not know all the items to be subtracted, the analogy with mathematical subtraction breaks down. I can fix upon only the numerical or geometrical attributes to an individual substance by stipulating, '*qua* number' or '*qua* shape'. I do not thereby list all the items to be subtracted and then see what is left. The process of subtraction generates two things, two numbers, the number subtracted and the remainder, each of which can be known determinately. In contrast abstraction generates one abstract object and an indefinite residue.[18]

Again, taking the abstraction process as one of subtraction or paring away makes an individual substance something like an uncarved block, ready to be shaped according to the whim of the sculptor. Yet Aristotle seems to view the abstract objects apprehended to have a real basis in the individual substance. For science is of real beings. Remember that Aristotle holds that both individuals and universals exist *in re*. For he says that both the primary substances and the secondary substances, the universal substances, exist *in re*. To be sure, he does say that the existence of the latter depends on the existence of the appropriate singular substances, which are primary. Still the universal substances exist nonetheless. Likewise, Aristotle admits that universal accidents exist. Apart from saying so in the Categories, Aristotle needs them in order to have science. For *propria* and *differentiae* are in accidental categories, and these *per se* accidents, along with substances, serve as the main items discussed in science.[19]

Consequently, the 'subtraction' interpretation has its problems too. Just as Aristotle appropriates many geometrical terms in his theory of syllogistic (like 'term' and 'figure') and demonstration, but uses them differently or at any rate extends their usage, so too he may be doing likewise in his use of 'ἀφαίρεσις'. I am inclined to admit that 'ἀφαίρεσις' does end up having the negative function or result of eliminating, or paring away, all those attributes that do not agree with the aspect specified. Yet we need not do this in advance. Rather, we subject the

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[18] Theodore Scaltsas, Substance and Universals in Aristotle's *Metaphysics* (Cornell, 1994), pp. 11-2; 34; 116, suggest that abstraction generates two objects. However he focuses on the abstraction of matter and form from a substance, and there we have a form, capable of definition, and, with the ultimate if not the proximate matter, an indefinite stuff. So unlike subtraction abstraction does not yield two equally definite things.

[19] On the status of *differentiae* and *propria*, see Bäck. On Predication, **pp.** .

predications presented to a test, namely whether they agree with the aspect specified. Then, if they pass that test, we admit them into this particular scientific discourse; if they do not pass, then we eliminate or "subtract" them. However, unlike arithmetical subtraction, we need not specify, in advance or all at once, all the predications, all the items to be removed. We need only to look at those attributes of which we have come to be aware, and require that those that do not pass the test of relevance be excluded. We need not "subtract" all possible irrelevant attributes. Accordingly, I shall opt for the traditional translation of 'abstraction' for 'ἀφαίρεσις' to signify a process *sui generis*. Too, although we do not have the same problem, of not being able to specify all the objects to be added, perhaps it is best, to emphasize that the mathematical use is only an analogy, also to translate 'πρόσθεσις' not as 'addition' but as 'combination' or 'synthesis'.[20]

I do concede, however, that at times Aristotle does use 'ἀφαίρεσις' in the sense of mathematical subtraction. [E.g., Metaph. 1061b20; 1023b13-5; 1024a27]. Here we can indeed think of abstraction as removal. [Cf. (ps.) Alexander, in Metaph. 427,18.] (Ps.) Alexander suggests that 'ἀφαίρεσις' means subtraction in the category of quantum strictly speaking but only metaphorically so in other categories. [in Metaph. 423,36-9] Perhaps this is the solution. For the mathematical conception of subtraction applies in full force only to quantities. To avoid ambiguity I think it better not to have two uses of the same term, and so will continue to call the non-quantitative "subtraction" 'abstraction'.

Abstraction as Selective Attention

In order to mark off an abstract object, like 'two' or 'number', we must be able to specify the aspect that we wish to separate off. We specify an aspect like number so as to generate abstract objects. We then look at our sense perceptions, examine the phenomena, to see what content they have under this aspect. As Lear puts it, we "filter" our experience in order to get at what we have chosen to find relevant. We do not invent the phenomena, but do choose what we want to notice. Hence I suggest conceiving abstraction as selective attention.[21]

Construing abstraction as selective attention has the advantage of unifying the two different sorts of abstraction that Alain de Libera finds in Aristotle: 1) the sort in the mathematical sciences, of taking the form from the matter [in effect, what I have called 'extraction'] and 2) subtracting as opposed to adding on attributes.[22] Selective attention performs both functions.

Likewise, taking abstraction as selective attention provides a common basis for the different views about Aristotle's theory of mathematical objects distinguished by Mueller.[23] It leaves open the question whether the *abstracta* are universal or singular (or even some other

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[20] C. D. C. Reeve, Substantial Knowledge (Indianapolis, 2000), p. 40, translates 'πρόσθεσις' as "positing", with "abstraction" for 'ἀφαίρεσις'. But this seems too far removed from the mathematical background of the two terms.

[21] C. C. Taylor, "Berkeley's Theory of Abstract Ideas," Philosophical Quarterly, Vol. 28 (1978) claims that Locke takes abstraction to be selective attention. Kenneth Winkler, Berkeley (Oxford, 1989), pp. 40-2, denies this claim, but then argues that Berkeley "discovered" the view that abstraction is selective attention. Robin Rollinger, Meinong and Husserl on Abstraction and Universals (Amsterdam, 1993), p. 13 n. 21, has likewise used 'selective attention' to characterize Meinong's view, although not in the same sense.

This interpretation of Aristotle would make him fit in not too badly with work on perception and cognition in modern psychology. See, e.g., Dana Ballard, "On the Function of Visual Representation," in Perception ed. K. Akins (Oxford, 1996), p. 116-9.

[22] Alain de Libera, L'art des généralités (Paris, 1999), p. 30.

[23] Ian Mueller, "Aristotle's Doctrine of Abstraction in the Commentators," pp. 464-5.

option[24]). It allows for mathematical objects' being either *abstracta* of the physical objects themselves, as Lear and Cleary take them, or of certain features of extension as such, underlying physical objects, as in Mueller's view.[25] For this pure extension itself would be an *abstractum*, on which we then perform another abstraction operation. Indeed, we can classify these different interpretations according to what the abstraction is performed upon and what features are being abstracted.

Thinking of abstraction as selective attention has another advantage. For it gives the intellect, and even the sense organs, an active role in locating these structures in its sense experience: it must "attend" to those features. Still, as I shall stress below, selective attention need not be a self-conscious, deliberate process. View 'attention' then as a sort of 'aiming at'. Aristotle himself seems to have this sort of conception when he attributes ὄρεξις to all animals able to perceive and imagine. [An. 413b23] We can translate 'ὄρεξις' as 'desire', but only 'desire' in a basic sense in which all animals can be said to "desire" food when they move towards a source of food. I mean 'attention' in the definition of 'abstraction' in this way too. Again, selectivity also need not imply any sort of deliberation or even of thought. Indeed, the sense organs themselves interact with the environment so as to be responsive to only certain types of stimuli as input. So they respond to stimuli "selectively" without any consciousness or choice being required.[26] (Likewise in modern science particles respond selectively to different sorts and quanta of forces.) This interpretation will fit nicely with Aristotle's psychology, particularly with the recursive abstractions constituting the perceptual and cognitive processes.
As opposed to the modern empiricists, Aristotle does not view abstraction as a merely human psychological operation. To be sure, he takes abstraction to be a psychological operation. Still for him psychological operations are just as real as other natural operations. So too Aristotle puts perception and knowledge in the same category as colors and shapes. For Aristotle we shall see abstraction naturalized. To this extent we can agree that Aristotle holds human mental experience is the mirror of nature: it is not mirror but part of nature. Yet, as it is a part of nature, it will reflect, and reflect upon, other natural phenomena.

John N. Martin claims that in antiquity abstraction ('ἀφαίρεσις') in the general sense has two aspects: it conserves something while taking something else away. He goes on to claim that 'ἀφαίρεσις' came to acquire two special meanings: roughly, one Aristotelian and one Platonist: the former consisting in the process of subtraction, or, as I prefer to think of it, in selective attention; the latter in the inverse relation of construction.

Martin takes Aristotle to have a specialized sense of abstraction as concept formation, which is vaguer than the general one, as Aristotle has no theory of conceptual abstraction.[27] However, he says, Porphyry and Boethius made the process explicit. I would say that that Aristotle's commentators were merely restating his views—as Martin himself goes on to imply. Moreover, so I shall be arguing (elsewhere), Aristotle takes both mathematical abstraction and

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[24] As discussed above *re* types and tokens.

[25] "Aristotle on Geometric Objects," in Articles on Aristotle, Vol. 3, ed. J. Barnes et al. (London, 1979).

[26] Of course, in the case of animals, certain types of selective attention may require consciousness. My conception of selective attention agrees with Victor Caston, "Aristotle on Consciousness," Mind, Vol. 111.4 (2002), p. 759. "…Aristotle cannot plausibly mean that animals are continually aware of such changes as a result of deliberately observing them and directing their intention towards them." I.e., not introspection; rather: "not unaware" [Phys. 244b12-245a2; cf. 437a26-9; 447a15-7] in "an unobtrusive way".

[27] So too Leen Spruyt, "Agent Intellect and Phantasms," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), p. 126-7.

conceptual abstraction as different applications of the abstraction operation, for which Aristotle does offer a theory.

Martin claims that Plotinus and Proclus, following the Pythagoreans and Plato, have a different, special sense of 'abstraction'. In their ontology they construct the more complex things from the more basic ones, ultimately the One, by *adding* features on to it.[28] Martin holds that going in reverse, so as to break down composites would be subtraction or ἀφαίρεσις.

> Abstraction is the epistemic converse of the process of physical composition…the mental process of reversion to the One. Ontologically, the Chain of Being proceeds downwards through the process of causation, but the Understanding remounts backwards from the bottom to the top. The process of remotion is called abstraction.[29]

Martin does not want to attribute the mathematical or Aristotelian sense of abstraction to Plotinus on account of the standard Hegelian complaint that then the One, arrived at via abstraction, would have less content than the beings emanating from it.[30] Rather, the One is the set of all things, with the things emanating from it its "smaller effect sets".[31]

I see some problems with Martin's claim that the Platonists had another conception of 'ἀφαίρεσις'. First, he offers little textual support in favor of this view *re* the occurrences of 'ἀφαίρεσις. What textual support there is can be explained by the general, mathematical use of 'ἀφαίρεσις', common to both Platonists and Aristotelians, where 'ἀφαίρεσις' just means subtraction, contrasted with addition. It's just that what is left for the Platonists once the differentiations and divisions of the lower genera are removed is a whole or One embracing them all. Moreover, 'ἀνάλυσις' in the Prior Analytics etc. seems to mean what Martin is taking 'ἀφαίρεσις' to mean. Alexander of Aphrodisias says that "analysis is the rendering of every composite into its highest principles, and is the way back to the highest principles from the last conclusions." [in An. Pr. 7,14-8] Second, Martin gives a false etymology for 'ἀφαιρέω': as coming from 'φέρω', while in fact it comes from 'αἱρέω', 'to take' or 'to choose'.[32]

Francesco Coniglione has a much more convincing account of the difference between Platonist and Aristotelian abstraction:[33] Unlike Aristotle, Plato did not derive universals as common elements from perceptions of individuals. For Plato abstraction is the process of leaving out all the imperfections of the exemplars of Forms and ascending to the Forms themselves. [Resp. 525c] Abstraction thus becomes a purifying, intellectual process for apprehending Forms via being reminded of them by sense perception. The Forms themselves are causal principles governing the behavior of their instances. In contrast, Aristotle denies that mathematics can be applied to astronomy. [Cf. Metaph. 997b] In the modern period, scientists like Galileo, Descartes and Newton returned to Platonism so as to construct idealized objects like point masses and frictionless bodies by which to formulate laws of nature.[34] "Only by creating fictitious, ideal entities and then descending from them by means of experiment and approximation to the "roughness of experience" is it possible to combine mathematics and

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[28] John N. Martin, Themes in Neoplatonic and Aristotelian Logic, (Hampshire, 2004), pp. xi-xiii; 37-9.

[29] John N. Martin, Themes in Neoplatonic and Aristotelian Logic, (Hampshire, 2004), p. 163.

[30] John N. Martin, Themes in Neoplatonic and Aristotelian Logic, (Hampshire, 2004), pp. 40; 115 n. 58.

[31] John N. Martin, Themes in Neoplatonic and Aristotelian Logic, (Hampshire, 2004), p. 45.

[32] John N. Martin, Themes in Neoplatonic and Aristotelian Logic, p. xiii n. 8.

[33] Francesco Coniglione, "Between Abstraction and Idealization," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), pp. 70-80.

[34] Cf. Ernst McMullin, "Galilean Idealization," Studies in the History and Philosophy of Science Vol. 16 (1985); Amos Funkenstein, Theology and the Scientific Imagination (Princeton, 1986), p. 89.

reality."[35] Later philosophers took up this conception: Descartes and Leibniz (despite their protests), Hegel, Cassirer, Lotze, Husserl.[36]

On Coniglione's account Aristotle and Plato do not have difference conceptions of abstraction proper. In both cases we have selective attention: some things are selected; others omitted. Rather, they differ in what they take to be the results of the abstraction process: on the one hand, universals; on the other, reminders of universals.

Despite the differences between Platonic and Aristotelian uses of abstraction, we can find both uses of 'abstraction' in Aristotle anyway. Abstraction as selective attention concerns the process whereby the *abstracta* are generated; the abstracta themselves are "ideal objects". In constructing a universal from singulars, Aristotle at best has to go with what holds for the most part, and ignore *im-perfections* etc. He comes up with his universal species, genera, properties, principles from what holds for the most part. Aristotle somehow gets to perfect geometrical shapes and lines, which have no instances in the actual things *in re*.[37]

When we look in detail, so far as possible, at how Aristotle views universals to be constructed, we shall then find Aristotle having a view of abstraction as selective attention where the content is somewhat idealized: its imperfections stripped away.

Let me close by mentioning other conceptions of abstraction current today. I have already mentioned the modern empiricist way, of using abstraction pretty much like Aristotle except for restricting it to the psychological and withholding it from the ontological.

Another usage distinguishes 'abstraction' from 'exclusion'. Thus Descartes says:

> There is a great difference between abstraction (*abstraction*) and exclusion (*exclusion*). If I said simply that the idea of which I have of my soul does not represent it to me as being dependent on the body and identified with it, this would be merely an abstraction, from which I could form only a negative argument which would be unsound. But I say that the idea represents to me as a substance which can exist even though everything belonging to the body be excluded from it, from which I form a positive argument, and conclude that it can exist without the body.[38]

This abstraction amounts to the Aristotelian conception though. Abstract objects for Aristotle do not exist independently from their bases as separate substances. Descartes has introduced 'exclusion' for "abstract" objects that are such separate substances. Some later philosophers like Stout have similar views.[39]

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[35] Francesco Coniglione, "Between Abstraction and Idealization," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), p. 72.

[36] G. Hegel, Science of Logic, Werke, Vols. 5-6 (Frankfurt, 1969), pp. 258-9; E. Cassirer, Substance and Function, and Einstein's Theory of Relativity, trans. W. & M. Swabey (Chicago, 1923), Chapter I; Determinism and Indeterminism in Modern Physics, trans. O. Benfay (New Haven, 1956), p. 83; E. Husserl.,Logical Investigations, Vol. 1, trans. J. Findlay (London, 1970), II; Lotze, Logik (Leipzig, 1880), §14; pp. 151-2. Cf. Francesco Coniglione, "Between Abstraction and Idealization," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), pp. 81-2; Robin Rollinger, "Hermann Lotze on Abstraction and Platonic Ideas," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), pp. 151-2.

[37] Alexander, in Metaph. 52,15-25; Mueller, "Aristotle on Geometric Objects," in Articles on Aristotle, Vol. 3, ed. J. Barnes et al. (London, 1979), p. 465.

[38] "Letter to [Mesland]," May 2, 1644 [AT, Vol. IV, p. 120; CSM, p. 236]. Cf. Justin Skirry, "Descartes's Conceptual Distinction and its Ontological Import," Journal of the History of Philosophy, Vol. 41.1 (2003).

[39] G. F. Stout, "Alleged Self-Contradictions in the concept of Relations, Proceedings of the Aristotelian Society, Vol. 2 1901-2, p. 13. Cf. Maria van der Schaar, "The Red of a Rose," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), p. 208.

Another, more modern usage, aggravating and documented by Angelelli, makes 'abstract' amount to 'incorporeal' or 'universal': not existing in space-time.[40] This use seems to have its roots in the Platonist, idealized sense of 'abstraction', but has now lost the Platonism and assumed, explicitly or implicitly, a materialism or a positivism. Thus the ideal objects, particularly those used in scientific theory, no longer are taken as more real than their exemplars, but rather are taken as mere shadows of them or heuristic devices for our knowledge of them. All it would take to give 'abstract' this sense is for people to take the Platonist, idealizing usage of abstraction and to add on a materialist attitude that only physical singulars accessible to sense perception have reality in the robust sense.

Frege has not only (1) the traditional, Aristotelian use of 'abstraction' but also two more.[41] (2) He suggests but finally rejects *definition by abstraction*. Here a term is introduced in the context of an equivalence relation, of the form of Hume's Law: if we have a relation '$\Phi(\xi,\zeta)$' that is commutative and associative then we can write instead of it '$\S\xi, = \S\zeta$'.[42] To use the classic example from the <u>Grundlagen</u>: the number of F's ↔ the number of G's iff the F's and the G's are equinumerous:[43] The point is that the term introduced, '§' or 'number', is completely uninterpreted. Its only content comes from this equivalence, the "definition by abstraction". So then we "look around" and see if we can use the term in some interpretation useful to us.[44] Frege ended up rejecting this method because it leaves the term completely undefined for things that cannot be put into the equivalence. This is the Caesar problem: in the definition of 'number', as Caesar cannot be put into the relation of equinumerosity, it is left open whether Caesar is or is not a number. So we cannot rule out that Caesar is a number and hence whether we are right in our interpretation for 'number' when the domain is our ordinary world.[45]

Frege also discusses and ridicules (3) a "magical" sort of abstraction, where different things are made identical by abstracting away all their differences. He was objecting here to mathematicians like Cantor and Ballue, who wanted to generate a set of identical units to use as

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[40] Ignacio Angelelli, "Frege and Abstraction," <u>Philosophia Naturalis</u>, Vol. 21 (1984)**,** p. 462: Ignacio Angelelli, "Adventures of Abstraction," in <u>Idealization XI</u>: <u>Historical Studies on Abstraction and Idealization</u>, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), pp. 18-25.

[41] Ignacio Angelelli, "Frege and Abstraction," p. 459; "Adventures of Abstraction," in <u>Idealization XI</u>: <u>Historical Studies on Abstraction and Idealization</u>, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), p. 17.

[42] Gottlob Frege, "Letter to Russell," July 28, 1902, in <u>Wissenschaftlicher Briefwechsel</u> (Hamburg, 1976), quoted in Angelelli, "Frege and Abstraction," p. 458.

[43] <u>Grundlagen,</u> Second Edition, ed. & trans. J. L. Austin (Oxford, 1953), p. 56.

[44] Rudolf Carnap, <u>Meaning and Necessity</u> (Chicago, 1956), pp. 1; 117; Ignacio Angelelli, "Abstraction, Looking-around and Semantics," <u>Studia Leibnitiana</u>, Vol. 8 (1979), pp. 108-23.

[45] <u>Grundgesetze</u>, Vol. 1 §10; <u>Grundlagen</u>, §§55-6; 65. Cf. Michael Dummett, The <u>Interpretation of Frege's Philosophy</u> (Cambridge, Mass., 1981), p. 402: "Frege has laid down that the value-range of a function f is the same as that of a function g...just in case f and g have the same value for every argument." Frege then says that this does not suffice "to determine uniquely the reference of every value-range term." "...for an object not given as a value-range, we have no means of deciding whether it is a value-range ..."

Frege's method of definition by abstraction is having a current renaissance though. Cf. Kit Fine, <u>The Limits of Abstraction</u> (Oxford, 2002); the articles by Fine and Wright in Matthias Schirn, <u>The Philosophy of Mathematics</u>, (Oxford, 1998); Crispin Wright, <u>Frege's Conception of Numbers as Objects</u> (Aberdeen, 1983); Crispin Wright, "On the Philosophical Significance of Frege's Theorem," in Heck, 1997, pp. 201-244; Crispin Wright, "Is Hume's Principle Analytic?," repr. in M. Schirn, ed.:, <u>Frege: Importance and Legacy</u>, (Berlin, 1996).

numbers.[46] He goes so far as to style this sort of abstraction as a miraculous divine force, "Shiva", beyond the comprehension of ordinary mortals.[47]

For Frege "ordinary abstraction" (1) consists in comparing objects and taking the ones in which they agree so as to arrive at a concept under which all the individuals fall, "Now this concept has neither the properties abstracted from nor those common to" the individuals.[48] So too the concept 'female mammal' does not bear young or give milk, although the objects that are female mammals do.

In contrast, the "divine" abstraction (3) sticks to the level of the original individuals, but takes them now as stripped of some of those properties. Frege ridicules this sort of procedure often.[49] He criticizes Husserl for using a type of numerical abstraction that makes "things absolutely identical without changing them."[50] But, Frege insists, this is possible only in "the washtub of the mind". He objects that "the way of considering an object, and the abstractions performed in the mind of a subject, seem to be being taken for qualities of the object."[51] If we consider Jupiter, he says, as an isolated object, it still does not lose its shape, mass or gravitational relations. It would be silly, he says, to think that the mental act of abstraction creates a new object, an impoverished Jupiter if you like.[52] So too he writes,

> By abstraction the logician acquires the concept pea, and to him it does not usually matter whether he has a handful more or less. The individual peas remain completely unchanged in the process and are not thereby transformed into the concept pea or replaced by it, but continue to exist beside it. The present process is much more marvelous: each individual pea divests itself entirely of its nature as a pea, but—and this is the most marvelous part--continues nevertheless to have a shadowy being separate from its fellow peas and without fusing with them.[53]

Frege objects that the abstract peas, now stripped of all difference, have no right to claim any plurality of objects. Rather what is abstracted is the general concept of pea.

I have described Frege's views in some detail because they have some relevance to how we understand Aristotle. Aristotle insists that we are not creating transcendent, magical objects via abstraction. When the geometer abstracts from physical objects to consider them only as spheres and lines, she is not creating new individual substances, Aristotle says. Yet she is treating them "as if" they were independent substances. Moreover, the objects so considered are hypostasized so as to be subjects and not, as with Frege's reputable abstraction (1), unsaturated concepts of objects (in the formal language: predicate functions of individual constants). That is,

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46 Claire Ortiz Hill, "Abstraction and Idealization in Husserl and Cantor prior to 1895," in Idealization XI: Historical Studies on Abstraction and Idealization, ed. F. Coniglione, R. Poli, & R. Rollinger (Amsterdam, 2004), pp. 222-3; 234.

47 "Draft Towards a Review of Cantor's…." in Posthumous Writings, ed. H. Hermes et al. (Chicago, 1979), p. 69.

48 "Draft Towards a Review of Cantor's…." in Posthumous Writings, p. 71.

49 "Review of Husserl's Philosophy of Arithmetic," in Collected Papers, ed. B. MacGuiness (Oxford, 1984), pp. 204-5 (also in Kleine Schriften, ed. I. Angelelli (Hildesheim, 1967).

50 Husserl claims to base number on a type of abstraction different from Locke and Aristotle: we get concept of a number from taking a set of like elements and retaining each "only insofar as it is a something…" Husserl, Philosophie der Arithmetik (The Hague, 1970), pp. 88-92; 165-6; "On the Concept of Number," in Husserl: Shorter Works, ed. P McCormick & F. Ellison (Notre Dame, 1981), pp. 16-7.

51 "Whole Numbers," in Collected Papers, ed. B. MacGuiness (Oxford, 1984), p. 231 (also in Kleine Schriften, ed. I. Angelelli (Hildesheim, 1967).

52 "Whole Numbers," in Collected Papers, ed. B. MacGuiness (Oxford, 1984), p. 232.

53 "Schubert's Numbers," in Collected Papers, ed. B. MacGuiness (Oxford, 1984), p. 254 (also in Kleine Schriften, ed. I. Angelelli.

unlike Frege, Aristotle allows for these *abstracta* to have properties of the same types as those that the original substances have. Thus not only is the Cube in Mecca cubical but so too is the cube studied in geometry. In contrast, Frege rejects Aristotle's antepredicamental rule, that the predicates of the predicates of an object are predicates of the object.[54] Frege holds that the predicates of an object are concepts, and their predicates are higher-order predicates not predicated of the object. Universals of the sort that Aristotle allows are objects formed by abstraction. Like Frege, Aristotle will reject the magical abstractions (3) leading us to Plato's transcendent Forms. Yet, by ending up with objects and not concepts, Aristotle might have become a bit too magical for Frege's taste. I shall not be able to discuss here how Aristotle works his magic.

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[54] Cf. Ignacio Angelelli, <u>Studies on Gottlob Frege and Traditional Philosophy</u> (Dordrecht, 1967), pp. 52-3; Allan Bäck, <u>Aristotle's Theory of Predication</u> (Leiden, 2000), pp. 178-85**.**