4-20-2007

Aristotle on Mathematical Existence

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ARISTOTLE ON MATHEMATICAL EXISTENCE

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Presented to the Society for Ancient Greek Philosophy at the Central Division meeting of the American
Philosophical Association, April, 2007

Do mathematical objects exist in some realm inaccessible to our senses? It may be tempting to deny this. For one thing, how we could come to know mathematical truths, if such knowledge must arise from causal interaction with non-empirical objects? However, denying that mathematical objects exist *altogether* has unsettling consequences. If you deny the existence of mathematical objects, then you must reject all claims that commit you to such objects, which means rejecting much of mathematics as it is standardly understood. For, as David Papineau (1990) vividly puts it, it is doublethink to deny that mathematical objects exist but to continue to believe, for example, that there are two prime numbers between ten and fifteen.

Two current responses to this problem are literalism and fictionalism. Both literalists and fictionalists deny the existence of a world of mathematical objects distinct from the empirical world. But they differ markedly in this denial. Literalists argue that mathematical objects simply exist in the empirical world; on this account, mathematical assertions assert true beliefs about perceivable objects. Fictionalists, on the other hand, hold that, strictly speaking, mathematical objects do not exist at all, and so exist in neither the empirical world nor in some realm distinct from the empirical world. They argue that mathematical objects are not actual objects but rather harmless fictions; on this account, mathematical assertions do not assert true beliefs about the world but merely fictional attitudes.

Although these two positions are apparently quite opposed to one another, they nonetheless have been both ascribed to Aristotle. Indeed, as I'll argue, Aristotle's philosophy of mathematics exhibits some of the features characteristic of literalism and some of the features characteristic of fictionalism. However, Aristotle's position also exhibits features interestingly different from both literalism and fictionalism.

The paper comes in three parts. In the first part, I'll quickly survey the variety of descriptions which Aristotle uses to characterize the relation between mathematical objects and the perceivable world. This will help to explain how apparently opposed positions have been ascribed to Aristotle. In the second part, I'll discuss literalism in contemporary philosophy of mathematics, the ascription of literalism to Aristotle and the points of agreement and disagreement between Aristotle and literalists. In the third and final part of the paper, I'll discuss fictionalism in contemporary philosophy of mathematics, the ascription of fictionalism to Aristotle and the points of agreement and disagreement between Aristotle and fictionalists.

1

Aristotle holds that mathematical objects are not something which exist apart from the physical world. Consider, for example, *Phys*. 2.2:

> The next point to consider is how the mathematician differs from the physicist. Obviously physical bodies contain surfaces, volumes, lines, and points, and these are the subject matter of mathematics (193b23-25).

But if mathematical objects do not exist apart from sensible substances, how are we to take this? Are they themselves perceivable objects? Or do mathematical objects not exist at all and talk of mathematical objects is merely a convenient (but, strictly speaking, false) way of talking about the sensible world? It will be helpful to begin by gathering together the relevant passages and classifying the descriptions of this relation between mathematical objects and the sensible world. There are three classes of descriptions.

The first class of descriptions is the use of a 'qua'-operator as the adverbial modification of a verb of consideration.[1] One such use of *qua* is in a negative description with the object of consideration sensible (or mobile or physical) things. Thus Meta. M.3 (1077b20ff.):

> clearly it is possible that there should also be both propositions and demonstrations about sensible magnitudes, not however *qua* sensible but *qua* possessed of certain definite qualities.... and in the case of mobiles there will be propositions and sciences, which treat them however not *qua* mobile but only *qua*

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[1]*theorein*: Meta. E.1 (1026a10), K.3 (1061a35ff.), M.3 (1078a24ff.). Cf. *noein*: De An. III.7 (431b15); *skopein*: Phys. II.2 (194a10); *pragmateusthei*: Phys. II.2 (193b31); *zetein*: Meta. E.1 (1025b1); *endechesthai peri*: Meta. M.3 (1977b21). With the use of *hos*, not *he*, De An. III.7 (431b12).

bodies, or again only *qua* planes, or only *qua* lines, or *qua* divisibles, or *qua* indivisibles having position, or only *qua* indivisibles.[2]

The second class of descriptions is where the sensible is abstracted from the mathematical. Thus Meta. K.3 (1061a28-33):

the mathematician investigates abstractions for before beginning his investigation he strips off (*perielon*) all the sensible qualities, e. g. weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties, and leaves only the quantitative and continuous.[3]

The third class of description is where the mathematical is separated from the sensible. Thus Phys. II.2 (193b31-34): "the mathematician, though he too treats of these things [the properties of the earth and the world], ... separates (*chorizei*) them; for in thought (*te noesei*) they are separable (*chorista*) from motion."[4] In separation descriptions the verb of separation is regularly qualified in some way. For example, in the Phys. II.2 passage quoted above, 'separable' (*chorista*) is qualified with the dative, 'in thought' (*te noesei*).[5] An especially interesting case of separation terminology is the description at Meta. M.3 (1078a21-22): "[the mathematician] studies what is not separate by separating."

      These descriptions present, at first blush, an ambiguous picture of Aristotle's view of mathematical existence. Is the subject matter of mathematics properties or material entities? If mathematical abstraction is the elimination of non-mathematical *properties*, the subject matter of mathematics would seem to be physical or material objects considered as if they did not have certain properties. Consideration descriptions with the object of consideration sensible might support this view: in the Meta M.3 (1077b20ff.) passage, quoted above, the mathematician is represented as considering sensible things (but not as sensible) and mobile things (but not as mobile). Those who take a literalist interpretation of Aristotle's philosophy of mathematics, and so hold that mathematics studies in part a distinctly mathematical matter contained in the physical world, tend to emphasize these descriptions, as we'll see.

      On the other hand, if abstraction is the elimination of such features of sensible substances as their *matter*, the subject matter of mathematics would seem to be certain mathematical *properties* of sensible things⸺properties such as triangularity. Those who take a fictionalist interpretation of Aristotle's philosophy of mathematics tend to emphasize these descriptions, as we'll see. The ambiguity between the two pictures apparently presented by these descriptions is noted by Mueller (1971, 162ff.) and Annas (1976, 30), and is one reason why both literalism and fictionalism has been ascribed to Aristotle. I'll next address each ascription in turn.

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[2]Translations based on those collected in McKeon (1941). Cf. Phys. II.2 (194a9-12): "Geometry investigates physical lines but not *qua* physical." Also Meta. K.3. (1061a34).

[3]Two compounds of *hairein* are used. First, compounds with *peri* are used to describe the 'stripping off' of the nonmathematical properties of sensible objects to leave only the mathematical. Other compounds--for example, *ana-hairein* ('extract': Phys. I.4) and *dia-hairesis* ('division': Phys. III.6)--are used in senses unrelated to mathematical abstraction. Cf. Meta. Z.3 (1029a11). The second compounds are compounds with *apo*; this is a rare use in verb form. APo. I.5 (74a37-b1). Also Meta. Z.3 (1029a16), a passage whose credibility for the ascription of any view to Aristotle I will draw into question later. There are other uses of *apahiresis* unrelated to mathematical abstraction: for example, Meta. D.22 (1022b31), with *apahiresis* associated with privation. More common are substantive phrases such as *ta ex aphaireseos legomena*, used in explicit apposition with *ta mathematika*, De Caelo (299a14-18); perhaps not referring to strictly mathematical abstract objects, APo. I.18 (81b3); as *ta en aphairesei legomena*, De An. III.7 (431b12-13), III.8 (432b5); as *ta ne aphairesei onta*, De An. III.4 (429b21); as *ta di' aphaireseos estin* in apposition with *ta mathematika* and contrasted with *ex empeirias*, NE (1042a18). The various *ta ... legomena* constructions are ambiguous. Ross translates as "the so-called ..."; an alternative reading is "the things said as a result of ...".

[4]to separate (*chorizein*) from matter (*hyle*), De An. III.4 (429b21); without an indirect object, Meta. M.3 (quoted below); cf. *apotemonenai*, Meta. G.1 (1003a24-5); *perigrapsomenai*, Meta. E.1.

[5]Hardie and Gaye's translation of this dative phrase as "in thought," suggests a locative sense. However, this is a rare use of the dative (see Sonneschein §434); the sense may be instead instrumental. More common is the use of the Greek *he*, or 'qua', discussed above.

I'll begin by briefly considering literalism in recent philosophy of mathematics. I have noted that mathematical statements, as standardly interpreted, commit us to the existence of mathematical objects. In contemporary philosophy of mathematics the most pressing difficulty with such commitment is epistemological. As Benacerraf (1973) framed the issue, if there is a transcendent world of mathematical entities, it is unclear how such a world could cause our knowledge of it. The difficulty, then, is to reconcile ontological commitment and a causal account of knowledge.

Mathematical literalists[6] accept that mathematics commits us to the existence of mathematical objects and attempt to avoid the epistemological difficulties resulting from this commitment by arguing that we indeed do have perceptual knowledge of these mathematical objects. On this account then, we simply are in causal interaction with mathematical objects. Penelope Maddy, for example, argues that we simply perceive sets.[7] She calls such set-theoretical realism Aristotelian, not Platonic in part "since sets, on the view [she is] concerned with, are taken to be individuals or particulars, not universals."[8] Donald Gillies has endorsed some of Maddy's views and also the designation of these views as Aristotelian.[9] Gillies writes that it seems "highly plausible to claim that sets exist in the material world. Examples of naturally occurring sets would be: the stars of a galaxy, the planets of the solar system.... If sets exist in the material world, then it seems reasonable to suppose that we might on occasion perceive a set with our senses."[10]

The ascription of literalism to Aristotle has textual support. Recall that, in passages such as 193b23-25, quoted above, Aristotle asserts that mathematical objects are part of the physical world. But can Aristotle mean that all mathematical objects whatsoever are physical? The set-theoretical literalism of Maddy and Gillies may be a plausible position, but extending literalism to other branches of mathematics is problematic. For physical objects lack the exactitude characteristic of many kinds of mathematical objects. It seems, for example, that we do not encounter perfectly straight lines in the physical world. The ascription of literalism to Aristotle, then, saddles him with an implausible view.

The best developed literalist interpretation of Aristotle's philosophy of mathematics is to be found in Mueller (1969). Mueller resolves the problem that most mathematical properties of sensible substances lack the exactitude characteristic of the subject matter of mathematics by arguing that Aristotle's claim that the physical world contains mathematical objects is merely the claim that the physical world contains a matter of pure extension—whose only features are length, width and depth—and that this is also the basis of geometric objects. So, on Mueller's view, Aristotle does not claim that all mathematical objects are contained in the physical world. Rather, he holds that the physical and mathematical realms overlap. Although physical lines and triangles lack the exactitude characteristic of geometric objects, the physical world shares with mathematics the precise extensional features of length, width and depth.

Mueller's view is not obviously incorrect; and the Phys. 2.2 passage quoted above might be read so to lend some support to it. However, an unattractive result of this view is that Aristotle's claim that the physical world contains mathematical objects is severely restricted. The physical world only contains only a small part of geometry.[11] There is a view ascribable to Aristotle, however, which allows that much of geometry is contained in the world without ascribing to Aristotle the implausible view that there are in the physical world exact geometric figures. Aristotle carefully distinguishes between individuals and universals within all categories. This individual triangle may be imperfect but it is an individual falling under a sortal, triangularity, which perfectly exhibits the exactitude expected of mathematics. After all, the universal is common to all instances of triangles and, if the universal triangle lacked the exactitude of mathematical triangles, it would be limited as to what individuals could be instances of it.

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[6]The term is Chihara's (1990, 3ff.).

[7]Maddy (1980, 178ff.); also Maddy (1990).

[8]Maddy (1980, 163).

[9]Gillies (1992, 266ff.).

[10]Gillies (ms., 9). Gillies believes that sets are perceptible since observation is theory-laden. Set-theoretic literalism has received some critical attention; see, for example, Chihara (1990, 194-215).

[11] In response, Lear (1982) argues that it is not so implausible to ascribe to Aristotle the view that there are in fact exact mathematical objects such as triangles in the world. This allows much more of mathematics to be contained in the physical world than Mueller's view allows. However, there are unattractive results of this view as well. For although the physical world contains most of geometry, in Lear's view, standard geometry is a part of a mere sliver of the physical world.

It's perhaps Aristotle's view that both the individual property of being this particular imperfect triangle and the perfect universal of triangularity are present in the individual sensible substance which has the property of being triangular. This view has the result that mathematics studies a significant number of mathematical properties of sensible substances. Mueller (1970, 162-3) canvasses this proposal and objects to it on the grounds that universals lack exactitude and that Greek math does not study universals but relies on spatial intuitions about representative individuals, not universals. Mueller's example in defense of the claim that universals lack exactitude defends a somewhat different point: Mueller (1970, 162) notes that "circularity does not touch straightness in a point or in any other way," but this would support the claim that mathematicians study representative individuals, not universals. Mathematical properties of sensible substances are universals but mathematicians treat them as if they were representative individuals as part of the mathematical fiction, of which I'll say more below. Generally, the question of mathematical practice is different from the question of the relation between mathematical objects and the sensible world.[12]

However, if this is Aristotle's view, then he would reject the literalist view that we can perceive mathematical objects. For the subject matter of mathematics are certain universal properties of sensible substances, and Aristotle holds that we do not directly perceive universals. We come to grasp universals rather through a process of induction beginning with perception.[13]

3

I turn to fictionalism in contemporary philosophy of mathematics, the ascription of fictionalism to Aristotle and the points of agreement and disagreement between Aristotle and fictionalists. As I've noted, if you deny the existence of mathematical objects, then it seems that you must reject all claims that commit you to such objects, which means rejecting most of mathematics as standardly understood. Contemporary mathematical fictionalists such as Hartry Field accept this consequence. According to mathematical fictionalism, mathematicians make the fictitious assumption that mathematical objects exist: such an assumption, they admit, is false; but the fiction, they assure us, is harmless and useful. Fictionalists disarm the apparent commitment to mathematical objects in mathematical statements by showing how in principle these statements could be rewritten into synonymous statements which do not have problematic ontological commitments. One strategy takes the form of a reduction to quantificational statements. Although the nature of these quantifiers is controversial, I will present an example using existential quantifiers: this is the simplest case. Consider the equation

(A) 2+3=5

A reductionist reading of this equation, with the numerical quantifier $(\exists n)$ an abbreviation for a sequence of n distinct existential quantifiers, would translate (A) as follows:

(B) $(\forall V)(\forall W)[(\exists 2x)(Vx)\&(\exists 3x)(Wx)\&\sim(\exists x)(Vx\&Wx)\supset(\exists 5x)(Vx\vee Wx)]$.

That is, the equation 2+3=5 can be read as saying merely that if there are two Vs and three different Ws, then there will be five things which are V or W. Where the arithmetical equation mentions abstract objects, the quantificational statement is free of such reference. Such quantificational statements are elephantine: this is partly why mathematics is a *useful* fiction.

Jonathan Lear and others have ascribed mathematical fictionalism to Aristotle.[14] The ascription has some initial plausibility. As we've seen, Aristotle sometimes describes the relation between mathematical objects and the sensible world in ways which suggest fictionalism. For example, Aristotle claims that mathematicians separate mathematical properties in thought. And this sounds rather like the claim that mathematical objects don't exist but

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[12] Mueller raises some other objections to the proposal. Mueller objects that universals aren't fully real; but Aristotle holds that neither universals nor mathematical objects are unqualified beings; on this, see below. Finally, Mueller objects that, if mathematical objects are universals, then Aristotle cannot distinguish between legitimate mathematical separation and illegitimate separation of Platonic Forms. But Aristotle objects to the separation of Platonic Forms partly because they separate what is even conceptually embodied. Aristotle uses the well-known contrast between the snub and the concave at Phys. 2.2 (194a7) to illustrate this point.

[13] See, for example, De An. 2.5 (417b23).

[14] See Lear 1982. Edward Hussey (1991) interprets mathematical objects as representational objects. Representational objects are fictive objects with just those properties characterizing the class of physical objects represented. For the original exposition on representational objects, see Kit Fine 1985. This form of fictionalism entails the rejection of the principle of bivalence; Aristotle would resist such a result (except possibly for future contingent statements).

mathematicians make the fictitious assumption that they do. Is this the right picture for Aristotle's philosophy of mathematics?

Aristotle holds that there are a variety of different kinds of entities: individual substances such as you and me, universal substances such as humanity, and individuals and universals among such other categories as qualities and quantities are all among things that have an ontological status. Of all these, only individual substances have their ontological status independently of standing in a relation to some other kind of entity. All other entities have their ontological status in virtue of standing in a relation to some individual substance or other.[15] Mathematical objects have their ontological status in virtue of being the properties of sensible substances. I believe that this is what Aristotle means when he describes mathematical objects as existing "qualifiedly" at *Metaphysics* 1077B16, contrasting this with the unqualified existence enjoyed by sensible individual substances, and what he means by claiming, at 1078a21-22, quoted in the first section, that mathematical objects are not separate from sensible substances.

For Aristotle, ontological dependence is closely connected to predicability. The predicability of an expression suggests that the referent of that expression is ontologically dependent on another entity; the impredicability of an expression, on the other hand, suggests that the referent of that expression is ontologically independent. Expressions referring to individual substances are the only expressions which cannot be predicated of another entity. Expressions referring to other kinds of entities are predicable of individual substances. This is Aristotle's methodology in the *Categories*: the predicability or impredicability of an expression provides a rationale for classifying the referent of that expression as ontologically dependent or independent.

However, within mathematical discourse, certain mathematical objects play this role. Entities which are, strictly speaking, the properties of sensible substances are, in mathematics, the subjects of predications. Consider a mathematical claim such as 'This triangle has interior angles equal to 180 degrees'. Here a mathematical property is predicated of a subject which cannot be predicated of another mathematical entity. The impredicability of the subject, within mathematical discourse, suggests that the referent of the expression is ontologically independent with respect to other mathematical entities. I propose that this is, according to Aristotle, the conceit of mathematics—a conceit which resembles mathematical fictionalism insofar as the mathematician treats mathematical objects with an ontological status they in fact lack. However, where contemporary mathematical fictionalists hold that mathematics treats what does not in fact exist as if it does exist, Aristotle holds that mathematics treats what exists qualifiedly as if it exists unqualifiedly. I believe that this is what Aristotle means when he says, in the 1078a21-22 passage mentioned above, that the mathematician separates what, strictly speaking, is not separate from sensible substances.

§

I'll draw a few conclusions. Somewhat like the literalist, Aristotle holds that mathematical objects exist in the physical world as the properties of sensible substances, although it may be that Aristotle would deny that mathematical objects are themselves perceivable. Somewhat like the fictionalist, on the other hand, Aristotle holds that mathematics ascribes to mathematical properties of sensible substances an ontological status they in fact lack; unlike the fictionalist, Aristotle does not believe that this requires that we deny that mathematical objects exist.

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[15] Aristotle holds that substances, alone of the categories, are separate: see, for example, 185a31-2, 1029a27-8. I discuss the evidence for taking this to mean that substances are ontologically independent from all other entities, in my prior work.

Works Cited:

Annas, J. (1976) *Aristotle's Metaphysics Books M and N*. Oxford: Oxford University Press.

Benacerraf, P. (1973) "Mathematical truth," *Journal of Philosophy* 70, 661-80.

Bostock, D. (1996) *Aristotle's Metaphysics Books* Z-Q. Oxford: Oxford University Press.

Charlton, W. (1992) *Aristotle's Physics Books I and II*. Oxford: Clarendon.

Chihara, C. (1990) *Constructibility and Mathematical Existence*. Oxford: Oxford University Press.

Field, H. (1980) *Science without Numbers*. Oxford: Blackwell.

Fine, K. (1985) *Reasoning with Arbitrary Objects*. Oxford: Blackwell.

Gillies, D. (1992) "Do we need mathematical objects?" *British Journal for the Philosophy of Science*. 43, 263-78.

-------------- (ms.) "An empiricist philosophy of mathematics and its implications for the history of mathematics." Unpublished.

Hussey, E. (1991) "Aristotle on mathematical objects," *Peri ton mathematon.* [*Apeiron* 24.4] ed. I. Mueller. pp. 105-34.

King, H. R. (1956) "Aristotle without *prima materia*," *Journal for the History of Ideas* 17, 370-89.

Lear, J. (1982) "Aristotle's philosophy of mathematics," *The Philosophical Review* 91, 161-92.

Maddy, P. (1980) "Perception and mathematical intuition," *Philosophical Review*. 89, 163-96.

-------------- (1990) *Realism in Mathematics*. Oxford: Oxford University Press.

McKeon, R. (1941) *The Basic Works of Aristotle*. New York: Random House.

Mueller, I. (1970) "Aristotle on geometrical objects," *Archiv für Geschichte der Philosophie.* 52, 156-71. Reprinted in Barnes, Schofield and Sorabji (edd.) *Articles on Aristotle*, vol. 3. London: Duckworth. 1979.

-------------- (1990) "Aristotle's doctrine of abstraction in the commentators," *Aristotle Transformed.: The ancient commentators and their influence.* ed. R. R. K. Sorabji. London: Duckworth. 463-80.

Papineau, D. (1990) "Knowledge of mathematical objects," *Physicalism in Mathematics*, ed. A. Irvine. Kluwer.

Robinson, H. M. (1974) "Prime matter in Aristotle," *Phronesis* 19, 168-88.

Ross, W. D. (1924)*Aristotle's Metaphysics*. 2 vols. Oxford: Oxford University Press. 2nd edn. 1956.

Schofield, M. (1972) "*Metaph*. Z 3: some suggestions," *Phronesis* 17, 97-101.

Sorabji, R. R. K. (1983) *Matter, Space and Motion.* London: Duckworth.