3-28-2002

Do Zeno's Arguments Challenge Aristotle's Account of Motion?

Rose Cherubin
George Mason University, rcherubi@gmu.edu

Rose Cherubin, George Mason University

Do Zeno's Arguments Challenge Aristotle's Account of Motion?

Presented to the Society for Ancient Greek Philosophy
At the Seattle meeting of the Pacific Division, APA, March 2002

What is the relationship between the arguments that Aristotle and Simplicius attribute to Zeno of Elea, and the account of motion that Aristotle presents in the *Physics*? Do the considerations that Aristotle raises in *Physics* Z.9 overcome the arguments about motion that he attributes to Zeno? Do they show the Zenonian arguments to be inapplicable or ill formed? Or do considerations that Zeno raises in the discussions attributed to him instead undermine Aristotle's account of motion? Do they undermine the possibility of physics as *epistēmē*? And why does Aristotle not treat Zeno's arguments about magnitude and plurality in his account of motion? After all, motion involves distances and multiple positions and times.

What is at stake here is *phusikē* as *epistēmē*: If it can be shown that his conception of motion is incoherent or self-contradictory, then there is for Aristotle small prospect of a science of physics, an account of *phusis* through its *archai*. If it can be shown that problems of the sort Zeno raised with respect to plurality and magnitude will surface in the account of motion that Aristotle proposes, then we shall need to ask whether or to what extent this undermines the possibility of an *epistēmē* of *phusis*.

I. Background

I have referred to "the arguments attributed to Zeno" because we have no complete statement of any of Zeno's arguments. Even where we have what seem to be quotations from Zeno, namely in the arguments about magnitude and plurality reported by Simplicius, the conclusions are generally not quoted; they are paraphrased, interpreted, or perhaps provided by Simplicius. Most importantly, there are no quotations that might indicate what Zeno intended his arguments ultimately to show, what points he meant them to serve.

Would Zeno have concluded, for example, that if contrary implications follow from the assumptions that he took to be the basis of the claim that there are multiple things, that therefore there could not be multiple things? That is, did he hold that something whose account is self-contradictory cannot be?[1] Or did he instead hold that an alternate account or framework is needed, or that a further explanation might be sought? If he did conclude that there could not be multiple things, would he infer from this - as he is often taken to have done - that what is must be one? That would not follow immediately from the claim that there could not be multiple things, much less from the notion that supposing that there are multiple things leads to contradictions. It is frequently claimed that Zeno intended to show that what is is one and motionless; but I think we must look carefully to see whether we should be justified in making such a claim.

It is worth noting that in none of the quotations from his work does Zeno say that what is is (or must be) one, that what is does not (or cannot) move; nor even that there are not multiple or moving things. This in itself does not show that he did not say or mean these things, but it is also worth noting that Aristotle and Simplicius do not establish, in the *Physics* and its commentary respectively, that he said those things[2]. In fact, Aristotle does not name Zeno, in *Physics* A.2-3, as having been one of those who claim that what is is one and immovable, whereas he does include Parmenides and Melissus in that group. Zeno's remarks on plurality and magnitude might place him in the unenumerated group of "those who inquire whether being/what is (to on) is one and

immovable" at 184b25-185a2, a point I will discuss shortly; but to inquire into the nature and number of to on is not the same as to say that what is is one and immovable.

Many modern commentators - e.g. Russell, Owen, Benacerraf, Grünbaum, Faris, Curd[3] - infer that Zeno tried to show that what is is one and that motion must be impossible. According to Simplicius, Zeno "showed in each of his attempts (*epicheirēmata*) that by saying that many [things] are, opposites (opposite things) follow" (in *Phys.* 139,5; DK B2). The "opposite" results would be e.g. that everything must be both so small as to lack magnitude and indefinitely large. A further assumption is required, though, in order to pass from this result to the further conclusion that there cannot be multiple things. That assumption is that if asserting the existence of something X has contradictory or inconsistent implications, then X cannot be, or at least cannot be as we had said it was. We have no evidence that Zeno accepted this further assumption.[4] Neither can we conclude, on the basis of the extant fragments, that Zeno meant to assert that motion is impossible. The fragments show only that common assumptions about what motion involves result in inconsistencies.[5] So we need not assume that Zeno meant to deny motion and/or plurality, and we certainly need not assume beyond this that he meant to assert that what is is one and unmoving.

However, Aristotle argues in *Metaphysics* Γ.3-4 that in order to understand anything through inquiry, we must hold that nothing can both belong and not belong to a thing in the same respect at the same time. In order for us to investigate *phusis*, we must not hold that any aspect of it involves a contradiction or inconsistency. Whether what we must suppose in order to study *phusis* is true unconditionally or tout court is another matter, and one not investigated in the study of *phusis*.

II. Zeno's Appearances in Aristotle's *Physics*

Aristotle does not quote or paraphrase any of Zeno's arguments concerning plurality or magnitude in the *Physics*.[6] A remark at 184b25ff. suggests why this is the case. Aristotle says that "to examine if what is is one and immovable is not to examine about *phusis*; for just as the geometer has no argument against the person who denies (abrogates) the principles (*archai*) of geometry, but the argument is either of another *epistēmē* or of things common to all *epistēmai*, in that way so too in the case concerning principles; for a principle yet is not, if only one and one in this way is. For a principle is a principle of some thing or things." Zeno's rehearsals of the implications of the claim that there are multiple things would seem to fall under the heading of "examining if what is is one". Therefore they would not be part of physics, nor would they be compatible with it.

In contrast, Zeno's arguments concerning motion can be discussed in a work on the study and *epistēmē* of physics because they can be understood or used in a way that takes them as invoking (and not directly examining) the supposition that there are multiple things. Whether Zeno meant the arguments concerning motion to contribute to the undermining of the thesis that there are many things (as philosophers such as Owen have suggested[7]) is another matter. The arguments concerning motion can be considered as separable from the investigation of plurality and magnitude, for example in the sense that they can be advanced by someone who believes that there are multiple things with magnitude. The claim that there is motion presupposes plurality (or states, places, and/or things), but the converse is not true.

By Aristotle's time the term '*geōmetria*' had come to invoke, at the most fundamental level, entities that could never be observed: points without magnitude, lines effectively without width, planes effectively without thickness, and so on. The characteristics of points, lines, planes, and so on are given us by axioms and definitions. Points, lines, angles, and the axioms and definitions that express their properties, are at least some of the *archai* of geometry to which Aristotle refers[8]. Is

what is true of geometry also true of physics, or does Aristotle mean to imply that it is? Aristotle leaves open the possibility that at least some of the fundamental entities for physics - which is generally thought to study the observable world - are in fact not the kind of thing that could ever be observed. He seems to imply that any science of physics must accept that at least some of the sources of observable things (points, nows, infinite divisibility) are known only through the axioms and definitions we pose.

Aristotle thus offers the following reasons for putting aside arguments concerning plurality when discussing *phusis*. First, the study of *phusis* as *epistēmē* requires one to search for *archai* (*Physics* A, 184a10ff), and if we say that only one thing is, then we cannot identify a source and a thing of which it is a source (because that is - at least conceptually - two things, or some actual division; 185a3-5). Second, Aristotle notes in Book B that things that "are by *phusis*" appear to have a principle of motion and of standstill; this implies that in speaking of what is "by *phusis*" (what is "by nature") one takes as given that *archai* are involved. Put another way, closer to the statement of 184b25ff., the study of geometry requires that one hold that there are lines, planes, circles, points, and so on, of a particular kind. These would be, or exhibit, principles or sources of geometry. One who denies that such things are cannot study geometry. One who inquires as to whether there are such things is making an inquiry that is outside the province of geometry. One who sought proof before engaging in the study of geometry that the principles of geometry were real, proof that there are points without magnitude, one-dimensional lines that were the shortest distance between two points, and so on, would be delayed (at best) in his or her geometric endeavors. Now, the geometer cannot, by means of geometry, prove that the principles of geometry are real (whatever that would mean), or that they are valid and are in effect across all contexts or independent of geometry. Similarly, the "physicist" (the person who studies *phusis* "scientifically") cannot by means of physics prove that the things we must suppose, the things we must take as given (e.g. that there are multiple things, some of which are movable), in order to say that there is *phusis* and in order to study that, are real or valid or in effect. Physics does not address the question at all. Aristotle makes no pronouncement as to whether Zeno's arguments concerning plurality and magnitude are valid in their own right, or appropriate for the investigation of things other than *phusis* (such as, e.g., the sorts of things investigated in the *Metaphysics*).

Let us consider some implications of Aristotle's analogy at 184a25-b5 between inquiring into whether what is is one and immovable, and failing to accept the *archai* of geometry. First, we should note that Aristotle does not say that it is necessary or desirable to accept the *archai* of geometry (except for purposes of studying geometry and anything that requires its use), nor that the principles of geometry are known to be true, accurate, valid, etc. across all contexts, or at all. He only points out that a student of geometry must accept these principles, and that their fundamental meaning and validity are not things that are or can be investigated by or within geometry. One needs to accept the principles of geometry in order to study and to use geometry, and to accept those of physics in order to study or to use physics. But Aristotle does not add, "And we must accept the principles of geometry because they are known to be as we say they are, across all contexts," or "And we must accept the principles of geometry because geometry is useful/beautiful/good/etc." He says nothing about whether or why or under what circumstances or in what contexts he thinks the principles of geometry or of physics should be accepted, and these subjects investigated. He says nothing about his assessment of the value or the meaning of geometry or of physics; he gives no reasons for accepting or not accepting the principles of either one, beyond interest in each subject. He does not say that the principles of physics are known to be as we say they are (e.g. that things are and move, or that motion is as we take it to be), across all

contexts; he says only that "for us" (presumably, we who would seek *epistēmē peri phuseōs*, 184a14-15), "let it be hypothesized (*hupokeisthē*) that the things that are by nature, either all or some of them, are moving things" (185a12-13). He gives no further information to suggest a limitation of the parallel between physics and geometry; and indeed Aristotle does not say that acceptance of the principles of physics is needed or desirable for anything outside of the study of *phusis*. The question of whether acceptance of these principles is necessary or desirable outside of the study of physics is a question that is beyond the bounds of physics. So too is the question of whether it is necessary, desirable, or justifiable to accept the principles and study physics.

In other words, Aristotle does not say that Zeno is wrong to examine at all whether there are multiple or moving things; nor does he say that a proper conclusion for such an examination would be to say that there are indeed multiple or moving things. That examination is not part of the study of *phusis* as such. The study of *phusis* does not require or proceed from proof that there are multiple things or motion. Aristotle only says that one who examines whether what is is one and immovable cannot at the same time be studying *phusis*. To study physics, then, one must sacrifice the investigation of the truth, validity across all contexts, meaning, and implications of the assumptions that multiple things and motion are.

Importantly, if such an assumption contains or is based on any contradictions or inconsistencies, or if any contradictions and inconsistencies must follow from the use of that assumption in conjunction with others fundamental to physics, then these contradictions and inconsistencies will be embedded in the findings of physics. Aristotle argues in *Metaphysics* Γ.3 that the "most certain" (*bebaiotatē*, 1005b17) of all *archai*, that which one must have in order to understand anything of beings (1005b15-17), and the *archē* that is the *archē* of all axioms (1005b33-34), is a principle of non-contradiction: "It is not possible for the same thing both to belong to the same entity and not belong to the same entity at the same time and in the same respect" (1005b19-20). In that case if the fundamental assumptions of the study of nature result in contradictions or inconsistencies, then the same assumptions that make it possible to search for or to pursue the *epistēmē* of physics also undermine physics as *epistēmē*. The meanings of terms used in physics will then depend on contradictions or inconsistencies.

In order to examine this potential problem we must first consider what Aristotle says about physics as *epistēmē* in the first sentences of the *Physics*, and then look at his response to Zeno's remarks about motion in Z9.

III. Status of the *archai* of *phusis*

Aristotle began the *Physics* with a call to try to determine things concerning the principles or sources (*ta peri tas archas*) pertaining to the *epistēmē* concerning *phusis*. We should make this attempt, he says, because "we think" (*oiometha*) we know a thing when we are aware of relevant first causes and principles and elements, and on this basis in the case of all inquiries (*methodoi*) concerned with principles or causes *epistasthai* and *eidenai* follow on the awareness of these. For Aristotle, then, if the inquiry into *phusis* studies something that has causes and principles and elements, the standard way of trying to find knowledge (*gignōskein*, 184a12) about that something is to try to become acquainted with these principles, causes, and so on. The way to try to attain an *epistēmē* concerning *phusis*, if it is attainable at all, will be to try to become acquainted with the principles, causes, and elements of *phusis*. It is not yet clear, from what Aristotle has said, that there is an *epistēmē* of *phusis*, or that there could be. As for whether *phusis* has principles or sources, it follows that we must take it to have some if physics as *epistēmē* is to be taken to be possible, as noted above.

It is in the context of inquiry concerning *phusis* that Aristotle enjoins us to "let it be hypothesized" (185a12-13) that at least some things move or are moved; and if we hold this hypothesis we must take it that there are principles (to account for distinctions among things, for varieties of motion, differences of place, etc.). Why should we hypothesize this? "It is clear" from *epagōgē* (185a13). That remark is explicated in Book B at 192b8-15, 192b20-23, and 193a1-9: the things that are said to be by *phusis* appear (*phainetai*) to differ from other things by way of having a principle of moving and of standing still. *Phusis* would then be a cause and a principle of being moved and of being still (*ēremein*) in those things to which it belongs primarily and not accidentally. That there is *phusis*, or that *phusis* is, is simply *phaneros*, visible or manifest: it appears (*phainetai*). It cannot, Aristotle says, be proven, in that it would have to be proven from what is *aphanos*, unapparent or not manifest. That attempt at proof would encounter the problem that we would not be able to be sure that the terms we were using referred to or accounted for real visible things (193a8-9).

Thus Aristotle does not prove, or attempt to prove, that there are multiple things that move. Rather, he starts from the notion that there are multiple things that move, as a hypothesis fundamental to the possibility of an *epistēmē* of physics. He goes on to defend the hypothesis against potential objections that arise in response to its implications. It is in that regard that he takes on the arguments and conceptions he attributes to Zeno.

IV. Zeno and *Physics* Z

Book Z is the book of the *Physics* in which Aristotle most comprehensively addresses Zeno's work on motion. It begins with a conditional: "If continuity, contact, and succession are such as defined earlier [sc. in Z3]...then it is not possible for a continuous thing to be made of indivisibles" (231a21-24). Thus a line will not consist of points, nor will a magnitude; and time will not consist of "nows" (moments, instants). Nor will anything be in motion in (within) a now; and no point or other indivisible will be in motion, except accidentally.

Aristotle does not say, "Since continuity, contact, etc. are this way," but rather, "If they are as they were defined (*diōristai*)". He is treating these relationships (continuity, contact, etc.) not as though he knows that he is presenting an adequate and accurate account of what is, but instead as if he is presenting a hypothesis that invokes a set of proposed fundamental definitions.[9]

Thus we may look at Book Z as being in part an exploration of the implications, the successes, the failings, and the "cost" of the use of these definitions or distinctions in the study of motion. The definitions or distinctions offered, specifically, were that things are said to be continuous (*suneches*) if their extremities (*eschata*) are one; to be touching (*haptesthai*) if they are together (i.e. in one place; not in distinct places); and in succession (*ephexes*) if there is no thing of the same kind between them (Book E, 226b18-227a3).

Aristotle does not explain exactly what extremities are, but it would seem that extremities in this sense could not themselves have extremities, or else contact that is not continuity could not be said to occur. This would seem to suggest, though Aristotle does not say it, that extremities would be points, or else boundaries that lack thickness. That Aristotle would understand extremities in this way is also suggested by the analogy to time, the extremities of whose intervals are indivisible moments or nows.

Aristotle goes on in Z1 to argue that if a line is continuous and a point is indivisible, then (contrary to the assertions of modern geometry and of the physics it is used to model) a line cannot be made of points; similarly, if time is continuous (as argued in E4, 228b1-11), it cannot consist of nows or moments. In so far as a two-dimensional or three-dimensional object or shape is seen as

continuous (as it would have to be if it is to be called one object or shape), it too will be infinitely divisible, i.e. divisible into things which are always divisible (231b15-18), and this would seem to imply that it cannot consist of boundaries within boundaries etc. Motions too will be continuous (228a20-b11), as well as objects in motion (235a6-7) and changes (235b24-25).

How, then, are points related to lines and magnitudes, and nows to intervals of time? Aristotle says that we say (234a23) that a now is an indivisible thing that is "in time" (*en tōi chronōi*). And there will have to be such a thing, according to Aristotle, given what has been said, because an indivisible thing must be the limiting extremity of two successive intervals, that is, the point where one leaves off and the other begins. For if the two extremities were not one, the two intervals, e.g. past and future, could not be in succession, and if they were separated there would be time between them. If what separated or distinguished the two intervals were to be an interval of time, then the two intervals would overlap (so that, as Aristotle says, part of the past would be in the future and vice-versa), or part of the dividing interval will be in each of the divided ones; and the now will not be all the same thing. These alternatives Aristotle finds to be unacceptable, and so the now is taken to be an indivisible thing dividing - and, equally, "in" - two time intervals.[10]

The case of points and lines or magnitudes should be analogous to that of nows and time intervals; Aristotle says at 233a15-20 that "the divisions will be the same both for the time and for the magnitude". So if intervals or lengths of magnitude that are successive must be divided by something indivisible, that something would be a point, and the point would be "in" or "on" the interval or magnitude - just not "in" it as a part.

Then if time is not composed of nows, Aristotle says, Zeno's "Arrow" argument fails. Aristotle does not give a clear account of this argument, but says that its result was that a flying arrow is motionless (*akinēton*, 239b7) and that its result was that a flying arrow stands [still] (*hestēken*, 239b30).[11] Certainly, being motionless is not necessarily the same as standing [still] numbers are motionless but do not stand still), but Aristotle has already noted that when a thing that by nature moves (*to pephukos kineisthai*) is not moving in the ways it naturally does, we say (*legomen*) that it is resting (234a31-33).

The argument of the Arrow seems to say that if the arrow at any moment "is over" (extends over, *ēi kata*) what is equal [to the arrow in length or perhaps volume], then at all moments during a given interval it is "over" something equal to its length. At each now, if it is over a length equal to its own, then (and this Aristotle does not state) it is not moving. (If it were moving, it would be traveling "over" a distance greater than its length during any interval of time, or so we say for any interval.) Therefore, Zeno is supposed to have argued, if the arrow is not moving at any now in the interval, it is not moving during the interval as a whole, even as we say it is flying.

This conclusion depends, as Aristotle notes, on the assumption that the interval is composed of nows. This assumption is false if the conceptions Aristotle has accepted are true, so he rejects the Arrow argument. According to Aristotle, if we do not claim that time is composed of nows, we do not have to conclude that the arrow does not move, even though we say that it does not move (or rest, 234a31-34) in any instantaneous now. On the conception Aristotle offers, time is full of nows but not divisible into them. Time is divisible only into divisible intervals, which are not reducible to or deducible from nows, or vice-versa. Distance, traveled by the arrow, is divisible not into points, but into intervals or segments that are themselves always divisible. Points do not move, except incidentally[12], and nothing moves within a point. Intervals or magnitudes are not reducible to or deducible from points, nor points reducible to or deducible from intervals or magnitudes. Thus for Aristotle the Arrow argument he attributes to Zeno fails in two ways. First, if the argument says that a thing that does not move in any now during an interval does not in fact move during the

interval, then the argument fails in that it wrongly assumes that the interval is composed of nows. Second, if the Arrow argument says that a thing that does not move at any now during an interval must be at rest in every now in the interval, then the argument fails because nothing either moves or rests in a now.

V. Implications

If we put aside Zeno's arguments concerning the inconsistent implications of multiplicity and magnitude, we can begin to seek *epistēmē* concerning *phusis*. But we will only attain that knowledge if the *archai* of *phusis* are coherent and do not have inconsistent or contradictory implications. As it turns out, accepting the assumptions that make it possible to study *phusis*, accepting a physics based on Aristotle's account of time, length or magnitude, and motion, comes at the price of striking limitations on possibilities for knowledge. That price includes both a limitation on precision and an inconsistency in fundamental principles.

The first problem is this. Aristotle has shown that on his proposed understanding of nows and intervals, it is incorrect to say that an interval is composed of nows. He has shown that an argument that portrays an interval as composed of nows - as he says Zeno's arguments do - comes to an incorrect conclusion about a moving thing. But if nows are indivisible and not parts of intervals, and if intervals are infinitely divisible, there will be no way to determine exactly when a motion or a resting begins or ends. As Aristotle remarks at 237b21-2, "in whatever [sc. change or state of change] a thing may be, it is not in it as a first [change or state of change]." Whatever is changing (motion being a species of change) must also have changed; and whatever has changed must also have been changing (237a9-20). Everything that is in motion must have moved before (earlier in the same movement) (236b32-33), and there is no first time at which anything that is [changing by] coming to a standstill is coming to a standstill (239a1).[13]

That is, not only can we not identify the exact point or instant at which something began to move, or was moving, or ceased to move, or began to cease to move; but we also cannot say that there was such a point or instant. In fact, it seems that we must say that there was not such a point or instant. Yet we say that motions have definite beginnings and endings, and it was precisely through saying this that we arrived at the conclusion that they cannot be said to have definite beginnings or endings. The situation is similar for distance; there would seem to be no first point that a moving object crosses, no last point that it crosses before stopping. In effect, Zeno's arguments about magnitude, the ones Aristotle had put aside as not falling within the province of physics, have resurfaced in Aristotle's account of motion. We can say that something begins and continues to move, on this account, as long as we do not - and as long as we cannot - say when or where it was first moving or changing, or even that there was an initial point or instant of the motion or change. Similarly, if we say that one object is moving past another, we will not be able to specify the first point on each that passed any of the other, nor the first point on each that was passed.

Zeno's arguments about plurality and magnitude, as presented by Simplicius, take the form of dilemmas, exploring e.g. the consequences of the leading part of a thing having magnitude and of its having zero magnitude. Aristotle does not present Zeno's arguments about motion in dilemma form. Could there be another horn to the Arrow argument, for example? According to Aristotle, Zeno considers the case of time being composed of durationless nows, and that is why his argument fails. But what would happen if one took as a hypothesis instead that time was composed of infinitely divisible intervals, as Aristotle proposes that we take it to be?[14]

One result that would follow is that there would be no interval in which the arrow was over a space exactly equal to its resting length (recall that Zeno seems to have suggested that everything is supposed to be "over" a space equal to its length), although the arrow does not grow. This is consistent with Aristotle's conclusion that there is no "first" time at which a thing is moving, nor a "first" point or interval over which it moves (238b36-239a2). We need to be able to say, of a moving thing, that it started here and went there, and so on; and on Aristotle's account, we cannot. This second horn of the dilemma does not show that motion does not exist, but it does show a fundamental incommensurability and a contradiction that are an important "cost" of physics.

Simplicius describes Zeno's arguments about plurality as showing that if we say that many [things] are, saying opposite[s] [things] results. Similarly, as we have seen, if we assume that things move, opposite[s] result. For physics, we wind up saying that the moving arrow both must and cannot extend over a length equal to its own at each now; or else we must say that the arrow travels at once through both an interval of time and a series of nows that do not add up to or reduce to an interval of time. We must say that at least some movements begin and end, but we must also say that there is no first time at which a thing began or ceased to move.[15]

Then if we cannot determine a first part of a motion or a first moment or interval of motion, our precision in physical inquiry will be limited (asymptotically). This leads to a question: given this apparently approximative character (in relation to the world we say we study in physics), what exactly does physics tell us, and what does it tell us about?

Beyond the lack of precision, there is another "cost" in physics that we must address, namely that of the principles. Research based on the fundamental hypotheses of physics can be carried out even if those hypotheses are flawed or problematic; the meaning and adequacy of the results might need scrutiny, but research can be conducted. But if physics is to be an *epistēmē* or any kind of understanding, it must have coherent and non-contradictory *archai* (according to the remarks on non-contradiction in *Metaphysics* Γ.3-4). Does physics as Aristotle portrays it have coherent and non-contradictory principles? Aristotle provides no clear answer to this, but only points out that this is not a question asked within physics. Even if physics "works", in the sense of Aristotle's account of motion allowing theorems that have or seem to have predictive success, the meaning of the findings of physics will be affected by the nature and applicability of the underlying assumptions.

Zeno explored the fact that conceptions of unity, plurality, motion, distance, and time all seem to invoke both continuity and division or limitation, intervals and points or moments or boundaries. On Aristotle's account of motion, intervals and points, moments, or boundaries are not interreducible and cannot be derived from one another. Yet they are either *archai* of *phusis* or necessary to such *archai*, for two reasons. First, because if there is a principle there is plurality; and second, because physics deals with that which can be said to move or change, and such things, as Aristotle says, seem to involve the notion of distinction as well as that of continuity, in both space and time (at least).

Zeno was right, I think, if he suggested that incommensurabilities, contradictions, and incoherences arise from the notion of things that are distinct and infinitely divisible. And if we are to take it that there are to be multiple things, we must take it that they are distinct (or they would not be multiple) and infinitely divisible (or we would not be able to take it that they were, or that they had content). But then the limit point of a thing (or a time) cannot be found in its last part, because there is no such thing as a last part, and the limit will not be part of the thing. At the same time no series of division-points will exhaust a thing. Yet points are said to be "on" or "in" the thing or time interval. If points were derivable from intervals or if they composed intervals, we

could not say that there were things at all. In this way the *archai* of physics are or invoke incommensurable entities, and in some sense contradictories.

Zeno reveals a further incoherence connected with Aristotle's portrayal of motion. Motions must be said to begin and to end, but on Aristotle's account nothing is in motion as something "first"; whatever is said to be in motion must also be said to have been in motion. Line segments or distance intervals must be taken to have endpoints; but no point can be taken to be the first or last in the segment or interval. These sound like Zenonian "opposite things" resulting from Aristotle's hypothesis. Aristotle could say that points and intervals are said to be "in" lines and segments in different senses of "in", but this does not solve the problem. It only means that the claim that there are spatial things, temporal intervals, and motion requires an equivocation.

Put another way, if we cannot identify a beginning of a motion we cannot account for the transition from rest to motion, or for the difference between rest and motion. This recalls Zeno's Dichotomy argument (*Physics* 239b10-14 and 263a4-b9). In any case it makes the *archai* of physics incoherent, or even contradictory.

As Aristotle said, this is not a problem for physics to study. It does seem to undermine the possibility of physics being an *epistēmē*. Aristotle does not say that an *epistēmē* concerning *phusis* has been achieved, and perhaps the *Physics* can be seen as in part an attempt to see whether that achievement is possible. But in undermining the possibility of physics being an *epistēmē*, the problems I have mentioned also undermine the possibility of our being able to gain demonstrable knowledge of the observable world through physics. The unit of magnitude or time, which makes physical inquiry possible, also undermines that inquiry.

If an *epistēmē* gives knowledge of *archai*, and a case arises in which the purported *archai* invoked by some body of learning are found to contradict one another or to "lead to saying opposite things" (as in Simplicius' description of what Zeno was trying to show about common assumptions), will that body of learning qualify as an *epistēmē*? I would think not, for on the basis of contradictory *archai*, anything could be derived. And if from the *archai* in combination opposite things are found to follow, it seems that we will not have *epistēmē* either: anything could be accounted for, and anything contradicted.

But does a contradiction, or "opposites follow[ing]", with respect to a thing mean that the thing is not? (We have seen that there is no evidence that Zeno thought that a contradiction or "opposite conclusions" meant this.) In an *epistēmē*, or toward the development of an *epistēmē*, we must say that whatever has contradictory results or a contradictory nature or account cannot be. As Aristotle notes, though, this must be accepted axiomatically in order to seek *epistēmē*. For that reason, it cannot be proved, or disproved. It is part of what makes proofs possible, and nothing will be called an *archē* if this principle is not accepted. This says nothing about what must be, independent of our conceptions or unconditionally, but only states what we must take to be the case if proofs and *epistēmē* are to be possible. The question then arises as to whether or on what basis or under what conditions we can say that there are *archai*, and *epistēmai*, at all. Acknowledging this question has important consequences concerning the relationship between philosophy and scientific thought. Zeno's investigations of the unit and of motion implied such a question, and Aristotle's approach to those investigations, in the *Physics*, suggests that he was aware of this.

Endnotes

1. Hegel asserts in section 89 of the *Logic* (Hegel 1975, 133) that "Zeno, who first showed the contradiction native to motion, concluded that there is no motion." As we will see, this sort of assumption about Zeno is implicit in a number of other important modern interpretations of his work.
2. One passage in Aristotle's *Physics* may say that Zeno argued for there being no motion. At 239b11-14, Aristotle describes one of Zeno's arguments (logoi) concerning motion as being "*peri tou mē kineisthai.*" Aristotle says that he had treated this argument earlier; this was at 233a2, where he does not say that Zeno intended to show that motion was impossible. He would be attributing this intention to Zeno, though, if he held that Zeno insisted that any motion required a moving object to touch or traverse an infinite or indefinite number of points. Yet it is not clear whether the *mē kineisthai* that Aristotle refers to is the conclusion of Zeno's whole argument, or only one part of a dilemma, or something that Zeno will follow to a further conclusion that Zeno intends to serve a larger point (following from the untenability of the saying that motion is not, or from the assumptions that made motion require crossing or touching an infinite or indefinite number of points). The phrase *peri tou mē kineisthai* appears as a paraphrase or an inference and not a quotation. Aristotle also suggests in the *Topics* (160b8-10) that there is an argument of Zeno's to the effect that moving is impossible, and then again in the *Sophistical Refutations* (179b20-21) and in the *Prior Analytics* (65b15-19) that Zeno argued that moving is not (that there is no moving). In none of these cases, however, does Aristotle elaborate, or identify the argument of Zeno's. Simplicius claims that Zeno intended to show that all things are one (in *Phys.* 138, 20-22); but he provides no quotations from Zeno that show this intention.
3. Russell 1970, 45 and 47; Benacerraf 1970, 103; Owen 1970, 140f.; Grünbaum 1970, 165; Faris 1996, 108-115; Curd 1998, 172n118. In contrast, Barnes 1982 observes that in the extant fragments Zeno "argues 'If P, then Q', where Q states some absurdity; but he does not explicitly infer the falsity of P.....he does not use reductio ad absurdum as a technique for disproof" (236). Barnes sees Zeno as not much more than an "eristic disputant" (236) whose "aims were critical, not constructive" (294). While I agree with Barnes that Zeno's surviving fragments contain no conclusions about what is, and that they do not offer any alternatives to the absurdities they turn up, I will argue that Zeno does make a substantial positive contribution to philosophy and science.
4. In DK B2/in *Phys.* 139,5 Simplicius says that Zeno showed that "in saying that many [things] are, saying the opposites (opposite things) follows," *tēi polla einai legonti sumbainei ta enantia legein.* This specific mention of saying may suggest that Zeno's focus was linguistic or epistemological.
5. Glazebrook 2001 argues that "Zeno's paradoxes of motion demonstrate that mathematizing nature results in absurdity" (205), not that motion is impossible (195). Glazebrook shows that neither Zeno nor Aristotle would hold that there could be a mathematical science of physics. I will argue here that Zeno's paradoxes also pose serious or possibly insurmountable obstacles to the development of any science of *phusis*, mathematical or otherwise.
6. Aristotle mentions Zeno's Millet Seed argument at 250a19-25 with respect to movement of air.
7. Owen 1970, 141. In order to get from the (possibly non-Zenonian) claim that there cannot be multiple things to the conclusion that there must be just one thing, we must make the further assumption that unity and multiplicity of beings of the kinds we usually conceive are our only

alternatives. But another alternative might be to suggest that our conception of what it is to be a thing within *phusis* is flawed.

8. At *Posterior Analytics* 76a31-b16, Aristotle names points, lines, magnitude, and the straight as *archai* for geometry. He also says geometry also posits or takes (*lambanein*) the meaning of that which is incommensurable or irrational (*alogos*), the broken or inflected (*to keklasthai*, apparently referring to lines bent or "broken back" at a surface), and inclining or verging (*neuein*). See also Heath 1981, vol. 1, 336-337.
9. As definitions, these cannot be proven to be correct or valid, although they could perhaps be shown to be inadequate if they should be found to fail to cover something important about the phenomena they purport to delineate.
10. In Δ3, 210a14-24, Aristotle enumerates eight ways in which one thing is said to be "in" (*en*) another. He does not identify which if any of these ways he has in mind at 234a22-24 where when he speaks of nows as being "in time."
11. Shamsi 1994 and Faris 1996 (chapters 4 and 5) makes careful studies of possible reconstructions of the argument, with somewhat different conclusions as to the details of its implicit premises and the senses of its explicit ones. What I will say here does not turn on the acceptance of either of these readings.
12. That is, a point is said to move if the segment of magnitude that it is "in" or "on" moves.
13. This must mean that there is no first "now" and no first point on a path of motion. See Lear 1981, 92-93.
14. Brochard 1954 proposes that the four paradoxes of motion form a dilemma (4). The Arrow begins from the supposition that space and time are composed of indivisibles, and shows that on this supposition, to say that a thing moves across an interval results in incoherences and contradictions. On reaching these results, one might wish to explore the supposition that space and time are infinitely divisible. The Dichotomy, Brochard holds, traces what would follow from that supposition, and like the Arrow it treats the case of a single object moving across an interval.
15. After having argued at 237b19-22 that something cannot be in a state of change as "something first," Aristotle refers at 238b31-32 to a "first time in which something is coming to a stop." Then at 238b36-239a2, he repeats that there is no "first" [interval of time] in which a thing in motion is moving or in which a thing coming to a stop is coming to a stop. Bolotin 1993 (331-333) suggests a way of understanding these remarks such that they do not form a contradiction: where a "completed change is not preceded by other completed changes, it also makes sense to speak of a first or primary time during which something is changing" (333). This is a time during which something is changing, not a time at which something changes: it will make some sense to say that there is a first or last interval during which a change is taking place, but not to say that there is a first or last point or instant. We must also note the approximative character of the sort of first and last intervals Aristotle will allow. If the starter's pistol goes off at noon, and at 12:00:01 the runners are visibly in motion, certainly the first interval of the race can be said to be 12:00:00-12:00:01. But we cannot identify the last now when a runner was not moving, nor the first now when the running had begun.

Works Cited

Apostle, H.G., trans. 1966. *Aristotle's Metaphysics*. Grinnell, Iowa: The Peripatetic Press.

_____, trans. 1969. *Aristotle's Physics*. Grinnell, Iowa: The Peripatetic Press.

_____, trans. 1981. *Aristotle's Posterior Analytics*. Grinnell, Iowa: The Peripatetic Press.

Aristotle. 1964. *Analytica Priora et Posteriora*. Edited by W.D. Ross, with preface and appendices by L. Minio-Paluello. Oxford Classical Texts. Oxford: Oxford University Press.

_____. 1958. *Topica et Sophistici Elenchi*. Edited by W.D. Ross. Oxford Classical Texts. Oxford: Oxford University Press.

_____. 1957. *Metaphysica*. Edited by W. Jaeger. Oxford Classical Texts. Oxford: Oxford University Press.

_____. 1950. *Physica*. Edited by W.D. Ross. Oxford Classical Texts. Oxford: Oxford University Press.

Barnes, Jonathan. 1982. *The Presocratic Philosophers*. Revised ed. London: Routledge.

Benacerraf, Paul. 1970. "Tasks, Super-Tasks, and the Modern Eleatics." In Salmon 1970, 103-129. Originally published in *Journal of Philosophy* 59 (1962): 765-784.

Bolotin, David. 1993. "Continuity and Infinite Divisibility in Aristotle's *Physics*." *Ancient Philosophy* 13: 323-340.

Brochard, V. 1954. *Etudes de philosophie ancienne et de philosophie moderne*. 2d ed. Edited and with introduction by Victor Delbos. Paris: Vrin.

Curd, Patricia. 1998. *The Legacy of Parmenides: Eleatic Monism and Later Presocratic Thought*. Princeton: Princeton University Press.

Diels, Hermann. 1951. *Die Fragmente der Vorsokratiker*. 6th ed. Edited by Walther Kranz. Zurich and Hildesheim: Weidmann.

Faris, J.A. 1996. *The Paradoxes of Zeno*. Avebury Series in Philosophy. Aldershot, U.K.: Avebury.

Glazebrook, Trish. 2001. "Zeno Against Mathematical Physics." *Journal of the History of Ideas* 62: 193-210.

Grünbaum, Adolf. 1970. "Modern Science and Refutation of the Paradoxes of Zeno." In Salmon 1970, 164-175. Originally published in *The Scientific Monthly* 81 (1955): 234-239.

Heath, Thomas. 1981. Reprint. *A History of Greek Mathematics*. New York: Dover Publications. Original edition, Oxford: Clarendon Press, 1921.

Hegel, G.W.F. 1975. *Logic*. 3d ed. Translated by William Wallace. Oxford: Clarendon Press.

Lear, Jonathan. 1981. "A Note on Zeno's Arrow." *Phronesis* 26: 91-104.

Owen, G.E.L. 1970. "Zeno and the Mathematicians." In Salmon 1970, 139-163. Originally published in *Proceedings of the Aristotelian Society* n.s. 58 (1957-58): 199-122.

Russell, Bertrand. "The Problem of Infinity Considered Historically." In Salmon 1970, 45-58. Originally published in Bertrand Russell, *Our Knowledge of the External World* (New York: W.W. Norton, 1929), 182-198.

Salmon, Wesley, ed. 1970. *Zeno's Paradoxes*. Indianapolis and New York: The Bobbs-Merrill Company, The Library of Liberal Arts.

Shamsi, F.A. 1994. "A Note on Aristotle, *Physics* 239b5-7: What Exactly Was Zeno's Argument of the Arrow?" *Ancient Philosophy* 14: 51-72.

Vlastos, Gregory. 1993. "Zeno's Race Course." In *Studies in Greek Philosophy*, vol. 1: *The Presocratics*, edited by Daniel W. Graham (Princeton: Princeton University Press). Originally published in *Journal of the History of Philosophy* 4 (1966): 95-108.