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An Iamblichean Derivation Theory and Its Relation to Speusippean and Aristotelian Doctrine

by

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In the De Communi Mathematica Scientia (14.18-17.29) Iamblichus advances a theory explaining the derivation of mathematical numbers and the mathematical magnitudes of line, plane, and solid through the operation of three principles, two material and one formal. According to this theory, the principle of Multiplicity (ἀρχὴ τοῦ πλήθους), functioning as a receptacle for the One and, like moist, pliant matter, receiving its impress, produces in combination with it the mathematical numbers (τὸ πρῶτον γένος ἀριθμῶν- 15.16). Although the result of any single such impression is actually a unit (μονάς- cf. 17.14-15), the principle of Multiplicity is apparently constrained by some characteristic in its nature peculiar to itself to reproduce the One as two units, in the formation of the first number. This first number must be the number 2, not 1, because a unit is not reducible to component factors, and because Iamblichus expressly says that the contribution of the material principle to each number is its quantitative aspect, its property of being divisible into the units which compose it, whereas its qualitative aspect, its determinate nature as a single number in the number series, it receives from the One (15.17-23). The magnitudes of line, plane, and solid, however, as entities with extension cannot according to the theory be generated by the impress of the One on the principle of Multiplicity, for if the material principle of the magnitudes were the same as that for numbers, all products of the combination of the One with matter would be generically the same, i.e., would be numbers or lines or planes or solids alone (16.15 - 17.1). Moreover, a single but multi-differentiated matter cannot be posited to account for the different genera of numbers, on the one hand, and of lines, planes, and solids, on the other, because such a matter would in strictness not be a principle or non-composite entity, the very fact of its differentiation implying a formal element and a matter prior to it as principles to account for its differentiated divisions (17.1 - 13). The theory therefore posits a second kind of matter (cf. 17.13 - 27), analogous to but not identical with the first, as the material cause of magnitudes. This matter (really, a principle of extension, though Iamblichus does not name it) contributes position and dimensionality to the mathematical lines, planes and solids, combining with the One to produce in the point the analogue of the unit produced in formalization of the first matter by the One. It is clear, moreover, that the theory could no more have advanced the point as the first magnitude than the unit as the first number, for the second matter must give dimension to the magnitudes (the point has position but not dimension), and the point, as ontological analogue of the unit, must in fuction be analogous to it as well. The first magnitude implied in the details of the theory given by Iamblichus, therefore, must be the line, consisting of 2 component points and bearing the same formal relationship to the One among magnitudes as does 2 among numbers; for the same reason, the plane must be the analogue of 3 and the solid of the number 4. Since each magnitude is just a system of points that is differentiated from each of the other systems by the number of points composing it, however, each magnitude is probably to be regarded simply as a number in extenso, line as the number 2, plane as the number 3, and solid as the number 4. Thus, in one sense, the theory implies that the existence of each magnitude, as a system of points increased by one beyond that of its proximate predecessor in the magnitudinal series, is necessarily implicated in the existence of the antecedent system, so that just as line (a system of 2 points [1 plus 1]) cannot exist without the point, plane (a system of 3 points [2 plus 1]) cannot exist without line, or solid (a system of 4 points [3 plus 1]) without plane. But, in another sense, since as a single and determinate entity in the series each magnitude is just the number of the points which are its matter, its existence is also necessarily implicated in the existence of its numerical formal element, so that line cannot exist without the number 2, plane without 3, or solid without 4.

There would seem to be no doubt that this doctrine was suggested to Iamblichus (or to the formulator of the source Iamblichus may here be using) by the speculations of Speusippus, who, according to Aristotle, "making his initial principle the One, thought that there were even more substances [than did Plato] and that there were principles for each kind of substance, one for numbers, another for magnitudes, and another again for soul."[1] It comports exactly with what is known to be Speusippean, as distinct from Platonic and Xenocratean, theory in treating mathematical rather than ideal or idea-numbers as the basic entities of reality, in asserting that the One and πλῆθος are the principles of number, and in allowing to magnitudes a material principle that is the analogue of πλῆθος.[2] Speusippus' influence, moreover, is to be detected in the stress the doctrine lays upon the point in the generation of magnitudes, and upon the point as the analogue of an element involved in the generation of numbers.[3] Lastly, the theory that beauty and ugliness and good and evil are not to be found among the principles; that πλῆθος is not evil and the One not good, but that these qualities appear only when a state of nature is reached in the generative process that is beyond the stage of principles and first numbers and magnitudes, i.e., when the sensibles, whose properties may be good or evil, beautiful or ugly, are formed--this theory is uniquely Speusippean among Platonist theories of the good,[4] and the fact that Iamblichus presents it (at De Comm. 15.23 - 16.14) as part of a more general doctrine concerning the generation of numbers and magnitudes is the clearest kind of confirmation that the ultimate authority for that doctrine was Speusippus.

On the other hand, the Iamblichean version of the doctrine deviates from what are known to have been Speusippus' views in several important respects. Iamblichus' formal principle for magnitudes, we have seen, is identical with his formal principle for numbers--the One; but Speusippus' formal principle for magnitudes is the point.[5] Iamblichus' point is the analogue of his unit or monad, is derived, like the latter, from the combination of formal and material principles, and functions as the basic component of his magnitudes; Speusippus' point, however, is the analogue of his formal principle, the One, is underived, and cannot function as component of his magnitudes since it is their formal cause. And, whereas the Iamblichean numbers and magnitudes are in a sense aggregates of units or of points, the Speusippean numbers and magnitudes are not in any sense such aggregates.[6]

What, then, could have made Iamblichus or his source for this revised Speusippean doctrine alter the strict ontological theory in such a way as to strip it of its formal cause for magnitudes and to turn its incomposite numbers and magnitudes into aggregates of component units and points? Could he have been conscious of "deficiencies" in the theory as Speusippus had propounded it, and was he attempting, in revising the theory, to reconcile it with the objections of some critic who had pointed out those "deficiencies"?

The answer to both parts of this question is, very probably, Yes. At De Comm. 16.18 - 17.13 Iamblichus presents a dilemma that arises if one posits either a single (i.e., homogeneous) or a multiple (i.e., single but multi-differentiated) material principle for numbers and magnitudes: if the principle be single, then numbers, lines, planes, and solids will be the same and will have no differentia to distinguish them one from another, since their formal and material principles will be the same; if the principle be multiple, however, with specific divisions for numbers, lines, planes and solids, then its multiplicity working against the unifying effect of the formal principle will produce disconnected entities, and the numbers, lines, planes, and solids which result will have no unity of relationship to one another, the more complex being deprived of derivability from the simpler. Now this dilemma, applicable to a theory which posits a single material principle for numbers and magnitudes, ought to be no less applicable to Speusippus' theory which posited two material principles, one for the mathematical numbers and another for the mathematical magnitudes. And that it actually was expressly applied to Speusippus' material principle for magnitudes and could, thus, be applied to his material principle for numbers is proved by Met. $1085^{a}32$ - $1085^{b}4$ where Aristotle says that if the principle is single and homogeneous, lines, planes, and solids will be the same, and if it is multiple, either its parts imply one another, in which case the magnitudes will be identical, or there is no implication, in which case the magnitudes will be disconnected. This dilemma, then, constitutes the "deficiencies" of the Speusippean system for Iamblichus or his source: given the One and πλῆθος as the principles of number, and the Point and analogue of πλῆθος as the principles of magnitude, any single number or magnitude so produced should not be different from any other number or magnitude so produced (2 should be same as 3, for example, or line the same as plane); and if one number or magnitude is different from another, then different and distinct portions of πλῆθος or its analogue ought to function as distinct material principles for each distinct number or magnitude, but in that case there can be no possibility of deriving one number or magnitude from any other and thus the unity of the number--or magnitude-series is destroyed.

If the possibility that such reasoning could have been identically and independently conducted by Aristotle and Iamblichus be put to one side, there is every likelihood that the source of these "deficiencies" in the system as Iamblichus viewed it was just Aristotle's critique of Speusippus in Met. $1085^{a}32$ - $1085^{b}4$. And that Iamblichus does reflect this critique in the De Comm. seems an unescapable conclusion when one compares his "solution" for the dilemma--to make neither number nor magnitude the first product of the principles, but entities that are prior to number and magnitude, the unit for numbers and the point for magnitudes, thus allowing the numbers and magnitudes to be differentiated from one another in terms of the number of their component units or points and, at the same time, by virtue of the homogeneity of their material components to be interconnected and derivable, more complex from simpler--when one compares this "solution" with that to be found suggested in a passage immediately following Met. $1085^{a}32$-$^{b}4$, namely, in Met. $1085^{b}12$-34. There Aristotle, in attacking Speusippus for his failure to account for the generation of a number's unit and of a magnitude's point, argues that, since his unit cannot be the same as his formal principle for number, the One, and since his point cannot be the same as his formal principle for magnitude, the Point, unit and point must therefore be the products respectively of the One and πλῆθος and of the Point and the analogue of πλῆθος. He then goes on to point out the difficulties involved in so regarding the origin of unit and point, arguing that if they derive from πλῆθος and its analogue they ought not to be unities, limited, irreducible, and indivisible. In here imputing to Speusippus a failure to account for the units and points of his mathematical numbers and magnitudes, Aristotle, by tacitly overlooking the fact that these abstract mathematicals were conceived by Speusippus to be unit-less and point-less,[7] tacitly treats the latter as though he were one of the Pythagoreans, for whom numbers were just aggregates of units and whose magnitudes were just aggregates of numbered atomic points[8]. Such Pythagoreanization of Speusippus could only have been welcomed by Iamblichus as a solution for the problem of derivability and differentiation, for he was himself an avowed Pythagorean and knew Speusippus as a Platonist who had been a careful student of Pythagorean theory and as one who had written a book on the Pythagorean numbers.[9] The second aspect of Aristotle's critique, however, must have struck him as a serious difficulty that could not be ignored and that demanded alteration of the system to meet it; for to argue that the ultimate units and points composing the numbers and magnitudes cannot be atomic is to argue that the Pythagoreans too are wrong in positing such atomic entities as the ultimate constituents of the universe. Accordingly, Iamblichus meets this difficulty by degrading the Point from its place among Speusippus' formal principles (οὐ γὰρ μία γε μόνον στιγμή ἐστιν αὕτη, says Aristotle [Met. 1085b29], implying that the Point in so far as it is a point [i.e., an extended entity] must be divisible) and leaving the One as the single formal principle to function, in opposition to the pluralizing effect of πλῆθος and its analogue in production of the units and points, as the force rendering the latter single and indivisible.

Aristotle's critique at Met. 1085b12-34, then, serves to explain the Pythagoreanized version of Speusippus' doctrine, in which numbers are aggregates of units and magnitudes aggregates of numbered atomic points, that is set forth at De Comm. 14.18 - 17.29. It is possible, of course, that Iamblichus did not derive this "solution" for the "deficiencies" of the strict doctrine immediately from the Metaphysics; he may have derived it, for example, from a late Pythagorean handbook or found it already ascribed to Speusippus or to the Pythagoreans in one of the works of the doxographers.[10] But, in any case, what is of consequence for our problem is the fact that he could not have derived the Pythagoreanized version of Speusippus in the De Comm. from the works of Speusippus himself or even from one of Aristotle's early works (if it be assumed that Aristotle in his so-called Platonist period could have advanced the strict Speusippean doctrine as his own), for the version that Iamblichus gives implies the critique of Speusippus in the Metaphysics and an interpreter of that critique who, in terms of it, gave to the doctrine the form that it has in the De Comm.

Footnotes

1 *Met.* 1028b21-24.

2 For Speusippus' rejection of ideal and idea-numbers in favor of mathematical numbers, see Aristotle, *Met.* 1028b21-24; 1075b37-1076a4 (cf. Ross's note *ad loc.*, *Aristotle's Metaphysics, a Revised Text with Introduction and Commentary*, II [Oxford, Clarendon 1924] 405); 1080b14-16; 1083a20-24; 1086a2-5; 1090a7-15 (cf. Ross's note *ad loc.*, *op. cit.* p. 480); 1090a25-28. For τὸ ἕν and τὸ πλῆθος as his principles of number, see *Met.*, 1028b21-24; 1091b22-25; 1092a35-1092b1; 1087b6-9; 1085b4-6 (cf. P. Lang, *De Speusippi Academici Scriptis, Accedunt Fragmenta* [Bonn, 1911], frags. 42g, 48a, 48b, 48c, pp. 74 and 77). For the material principle of magnitudes as the analogue of πλῆθος, see *Met.* 1085a32-34 (cf. E. Zeller, *Die Philosophie der Griechen*, II.1 [5th ed.], p. 1002, n. 2, and Ross, *op. cit.*, p. lxxiii.).

3 See *Met.* 1085a32-33 and *Topics* 108b7-31 (cf. Ross, *op. cit.*, p. lxxiii and H. Cherniss, *Aristotle's Criticism of Plato and the Academy*, I [Baltimore, The Johns Hopkins Press 1944], 131, n. 82.).

4 For this theory, see *Met.*, 1072b30-34; 1075a36-37; 1091a33-b1; 1092a11-15. N.B. that the theory appears to have been a *Pythagorean* tenet, as well (1072b31; cf. Lang, *op. cit.*, frags. 35c and 37a.

5 See *supra*, n. 3.

6 Speusippus' numbers and magnitudes are transcendental, self-subsistent entities, immovable and unique. They are χωριστά and take in Speusippus' system the place of the ideas in Plato's. As such they must be *incomposite*, and if incomposite, then not combinations (as numbers) of units or (as magnitudes) of points. P. Merlan (From Platonism to Neoplatonism [M. Nijhoff 1953], 88-112, who contends that *De Comm.* 14.18-17.29 is Speusippus' doctrine unalloyed, has observed neither the incompatibility of Speusippus' incomposite and of Iamblichus' composite numbers and magnitudes nor the fact that Speusippus' doctrine of *two* distinct formal principles for numbers and magnitudes respectively is incompatible with Iamblichus' doctrine of a *single* formal principle for both kinds of substance. Kohnke, following Merlan (*Gnomon*, 27 [1955], pp. 160-161), also thinks that Iamblichus in this passage is reproducing unadulterated Speusippean doctrine, but is inclined to believe that this reproduction is achieved through the intermediacy of Posidonius.

7 See *supra*, n. 6.

8 Other passages in the treatises show that Aristotle believed the Speusippean and Pythagorean systems to be related. We have already cited *Met.* 1072b30-34 and *Eth. Nic.* 1096b5-7=Lang, frag. 37a. Cf. *Met.* 1080b14-22, where the Pythagoreans and Speusippus are coupled together as alike believing in mathematical number only.

9 Lang, frag. 4, N.B. 53.3-6.

10 Theophrastus seems to have been the originator of the erroneous tradition which ascribed derivation of mathematical magnitudes from numbers to the Phythagoreans. Aristotle never attributes this doctrine to the latter by name, limiting it to the *Platonists*, but a careless reading of those passages of the

Metaphysics in which the doctrine is discussed and in which critiques of the Platonists in general and of Speusippus in particular are intermingled with critiques of the Pythagoreans (e.g. *Met.* 1001^{b} 26-1002^{b}11) in all probability contributed to the generation of the error. That Theophrastus was the first to propound the doctrine as Pythagorean is indicated by Cicero's statement at *Lucullus*, 37.118 ("Pythagorei ex numeris et *mathematicorum initiis* proficisci volunt omnia"), if the phrase *mathematicorum initiis* denotes, as I believe it goes, the elements or principles of mathematical lines, planes, and solids; for Cicero's source here is Clitomachus, and the latter is repeating the testimony of Theophrastus in the φυσικῶν δόξαι (cf. Diels, *Doxographi Graeci*, Berolini et Lipsiae, apud Walter de Gruyter et socios, 1929, p. 119-121). The same error was made, whether independently or derivatively, by the commentator Alexander (cf. his note on *Met.* 1002^{a}11 [*In Met. Comm.* 230.12 Hayduck] in which he interprets οἱ ὕστεροι as meaning both Plato and the Pythagoreans); and he is followed in modern times by Bonitz and Ross, who think that οἱ ὕστεροι of *Met.* 1002^{a}11 either are Pythagoreans alone or include Pythagoreans (cf. Ross's notes on 1002^{a}11 and 27, *op. cit.*, p. 248). H. Cherniss has brilliantly demonstrated, however, that the critique of mathematical magnitudes in *Met.* 1001^{b}26-1002^{b}11 cannot fit the Pythagoreans (*Aristotle's Criticism of Presocratic Philosophy* [Baltimore, The Johns Hopkins Press 1935], 40-43 and *op. cit.* [*supra*, note 3], n. 83, pp. 132-134).