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Reply [to "Comment on 'Dynamics of diffusive bubble growth in magmas: Isothermal case' by A. A. Proussevitch, D. L. Sahagian, and A. T. Anderson"]

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Reply

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Sparks [this issue] has made several insightful and important comments on our paper [*Proussevitch et al.*, 1993]. We are glad to have this opportunity to respond for the purposes of clarifying some of the model conditions and numerical techniques employed in our work, as well as for elaborating on the interpretation of our model results. Sparks is quite correct in suggesting that the present calculations must be thoroughly understood before more complex models can be reliably constructed.

In general, the interpretation of our model results in the context of real volcanic systems is necessarily limited by the conditions imposed on the model. One condition was that of instantaneous decompression, while another required the variation of each parameter independently of all others, so that its effect on the results could be most uniquely quantified. An example of the artificial conditions imposed for our simple model is the variation of viscosity while holding diffusivity, initial volatile concentration, and temperature constant. Clearly, this is not a natural scenario, but it determines the effect of viscosity alone. In real systems, these parameters are not independent but are rather a part of a complex set of interactions and positive and negative feedbacks. Our continuing work is directed at quantifying the nature of these relationships (for various decompression histories) and should provide results which will be applicable to to a broader range of natural systems.

A major point raised by Sparks is the interpretation of the "time delay" indicated in our model results. He calls this a period of "accelerating growth" which may be a more appropriate term for the phenomenon under some conditions. We agree that the sigmoidal shape of the bubble growth curve warrants further discussion. It should be noted that in many cases with basaltic melts, the "normal" parabolic growth curve is observed when plotted on linear axes (not logarithmic), in agreement with "classical" results [*Scriven*, 1959]. However, in other cases, the sigmoidal curve is real and may shed some light on the processes of early degassing and bubble growth. In our paper, we did not venture to explain the cause of the sigmoidal pattern other than to suggest that surface tension pressure may play a role for bubbles close to nuclear size. In this case, there is a time delay caused by elevated bubble pressure artificially maintaining "almost" equilibrium with a high concentration of volatiles in the melt. In the case of larger bubbles, "accelerating growth" may be a more appropriate term, since surface tension becomes relatively small, but viscous resistance to growth becomes important, as suggested by Sparks. However, as he indicates, the accelerating growth phase is observed at lower viscosities than expected. We have considered his suggestion that the initially small surface area inhibits the transfer of gas from the melt into the bubble and appreciate his drawing attention to this potentially important effect. This effect was built in to our model formulation, but we did not explore the implications of this in our paper. In response to Sparks' suggestion, we have run test cases for small bubbles with and without the effects of viscosity and surface tension to determine the role of the artificially small bubble size relative to oversaturation pressure. We use a different criterion for viscosity than does Sparks because of the finite melt volume between bubbles. His $4\mu(dr/dt)/r$ is valid for a single bubble in an infinite melt. We used $4\mu(dr/dt)(1/r - r^2/s^3)$ which reduces to Sparks' criterion for infinite s (s is separation distance of adjacent bubbles).

The results of our first test with basalt indicate that there is no discernable difference in the early growth history for bubbles with radius differences of a factor of 2 (surface area factor of 4). It is important here to distinguish growth of radius from growth of surface area and growth of volume, as well as to recognize the measure of growth. We have plotted only radial growth in our paper. If volumetric growth were plotted, the "classic" curves would appear quite different. (We mention this here, although it is clear that Sparks did not misunderstand our plots.) The measure of growth we have used is an absolute scale (in meters) rather than a percentage of initial size. This allows direct intercomparisons of model results.

For very small (near critical) initial bubble size, the surface area effect could be inferred to be important if the results for large and small viscosity and surface tension show similar growth curves. As can be seen in Figure 1, the curve for low (near zero) viscosity and low surface tension begins its growth much earlier than the one for geologically reasonable viscosity and surface tension with the same geometry. Furthermore, in additional model runs (not illustrated) we found that the difference in growth rate between the two cases is greater for higher diffusivity. This suggests that viscosity more severely inhibits bubble growth when the growth rate is higher (larger diffusivity). We also found that for low values of viscosity and surface tension, there is no time delay (no accelerating growth phase) for any trial value of diffusivity (Figure 2), and the curves for different diffusivities have the

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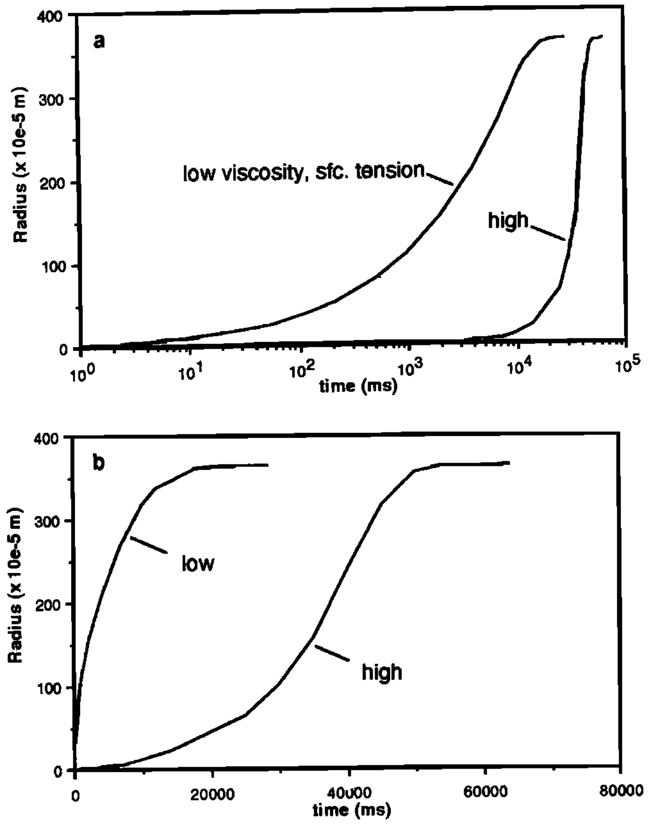

Figure 1. Effect of viscosity and surface tension on early bubble growth. The "high" case has normal magmatic viscosity and surface tension of 10^6 Pa s and 0.32 J/m^2, respectively. The "low" case has 10^{-5} Pa s and 10^{-6} J/m^2, respectively (essentially zero). (a) When plotted on a logarithmic timescale, the curves show a clear difference in growth pattern. (b) A linear time scale highlights the absence of an "accelerating growth phase" for unrealistically low viscosity and surface tension but its presence for normal conditions. This suggests that the accelerating growth phase is not dependent upon geometry but is controlled by viscosity and/or surface tension. Both curves had the following values for relevant parameters: Initial bubble radius, 10^{-5} m; diffusivity, 10^{-10} m^2/s. All curves in this and following figures have the following: ambient Pressure, 0.1 Mpa; initial dissolved volatile concentration, 0.5%.

same relationship as those in Figure 16 of *Proussevitch et al.* [1993]. We interpret this to indicate that for low viscosity and surface tension, small bubble size (surface area) does not limit growth even for diffusivities as high as 10^{-10} or low as 10^{-14} m^2/s. Bubble growth appears to be completely controlled by diffusivity in the absence of viscosity or surface tension.

In order to isolate the effect of surface tension and viscosity, we conducted several model runs varying only surface tension. The results indicate that surface tension is important for near-critical bubbles (Figure 3), but is not a factor otherwise. Viscosity is also a factor, but only when it is high. An important comparison can be made between Figures 2 and 3. Figure 2b displays no accelerating growth phase, while Figure 3b does. The only differences between the low surface tension case in Figure 3b and the intermediate diffusivity case in Figure 2b are viscosity and initial bubble size. The former had normal viscosity (10^6 Pa s), while the latter had low viscosity (10^{-4} Pa s). The initial bubble size for the former was 10^{-5} m (10 μm), while that for the latter was 4.4×10^{-6} m (0.44 μm). Since the larger initial bubble size case shows an accelerating growth phase while the smaller does not, these results are in direct contradiction to those that would have been expected if geometry were more important than rheology.

Additional model runs with very small initial bubble sizes show no dependence of early growth history on initial bubble size. A further test with rhyolite (Figure 4) produced identical results for model runs whose only difference was initial bubble radii of 2×10^{-8} m (0.02 μm) and 5×10^{-8} m (0.05 μm). At these sizes, it would have been expected that the surface area limitation would be most evident, but it is not observed in the results. A model run with a larger initial bubble size shows

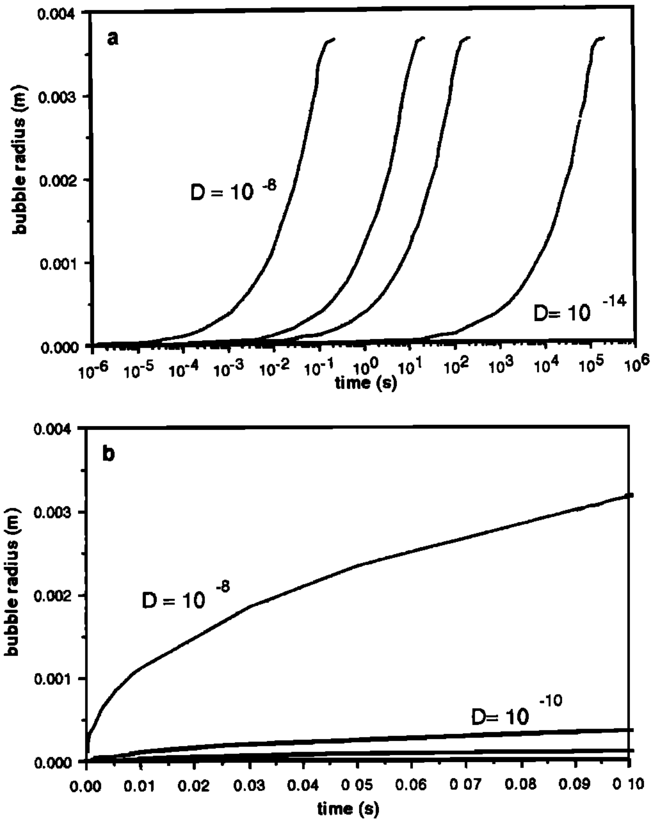

Figure 2. Effect of diffusivity for negligible viscosity and surface tension. (a) In model runs with all parameters the same except for diffusivity, it was found that without the inhibiting effects of viscosity and surface tension, the family of growth curves for rhyolite are self-similar, as in the case for basalt. This indicates that rapid growth rates (even with very high diffusivities) are unimpeded under these conditions. (b) When plotted with linear timescale, it is evident that there is no time delay (accelerating growth phase) even for the highest diffusivity (10^{-8} m^2/s). Growth rate was clearly impeded in model runs with normal rhyolitic viscosity and surface tension [*Proussevitch et al.*, 1993, Figure 15]. This is one illustration of the importance of viscosity and surface tension to bubble growth. All curves had the following values for parameters: initial bubble radius, 4.4×10^{-7} m (0.44 μm); viscosity, 10^{-4} Pa s; surface tension, 10^{-6} J/m^2.

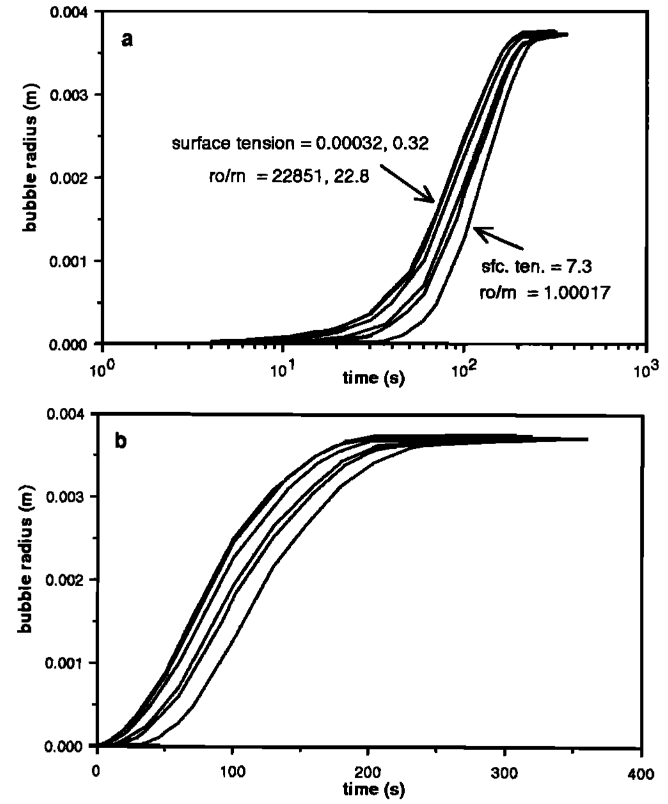

Figure 3. Effect of surface tension alone on bubble growth in rhyolite. Surface tension only plays a role when the bubble size is close to critical. For bubbles with initial radius 10 μm, surface tension is important if it is greater than 1.0 J/m^2. Six curves are plotted with values of 0.00032, 0.32, 3.2, 6.4, 6.9, and 7.3 J/m^2. The smaller two values lead to overlapping curves. We interpret these results to indicate that the time delay (accelerating growth phase) for bubbles near critical size is controlled at least in part by surface tension. (a) Logarithmic timescale. (b) Linear timescale. All curves had the following values for parameters: ambient pressure, 0.1 MPa; initial bubble radius, 10^{-5} m (10 μm); viscosity, 10^6 Pa s; diffusivity, 10^{-11} m^2/s.

larger radii at early times but has no less time delay or period of accelerating growth and joins the other curves after a short time (1 s).

As a result of these additional investigations as suggested by Sparks, we must conclude that small surface area is not a growth-limiting factor, even for critically small bubbles in the range of parameters used for rhyolite (or basalt) where surface tension pressure and/or viscosity is large. It should be noted that the conditions used in this test are not realistic for most natural systems (in which instantaneous external decompression is not achieved), but the results do illuminate an important process in early bubble growth. We anticipate that the results of our continuing study will lead to more realistic interpretations than possible at present.

In his point regarding bubble size, Sparks states that the majority of bubbles in typical pumice and ash (on a volumetric basis) are 10^{-4} m rather than 10^{-3} m in size [*Sparks and Brazier*, 1982; *Whitham and Sparks*, 1986]. By size, Sparks presumably means diameter of cylindrical vesicles. We do not disagree with Sparks' statement but wish to emphasize three points: (1) Our computations refer to a simplified geometry (spherical rather than cylindrical bubbles); (2) our model bubbles are uniform in size, whereas a wide range of vesicle sizes exist in natural materials, thus increasing the vesicle surface area per bulk vesicularity; and (3) it may be argued whether ash, which makes up a large mass fraction of many silicic pyroclastic deposits, has the same vesicle size distribution as coerupted pumice. The recent work by Fisher et al. [1993] states "[the ash flow] consists of 67% elongate, thin, platy shards, and 15% pumice lapilli. The elongate shards are fragments of bubble walls broken from a highly viscous melt containing abundant large (200-350 μm diameter), elongate vesicles." The equivalent spherical diameter of a 200-μm-diameter and 2-mm-long vesicle (aspect ratio of 10) is about 500 μm. In future work it will be important to investigate the significance of various bubble sizes and size distributions on the evolution of bubbly magma. This can be accomplished in our modeling scheme by including a spatially varying separation between bubbles.

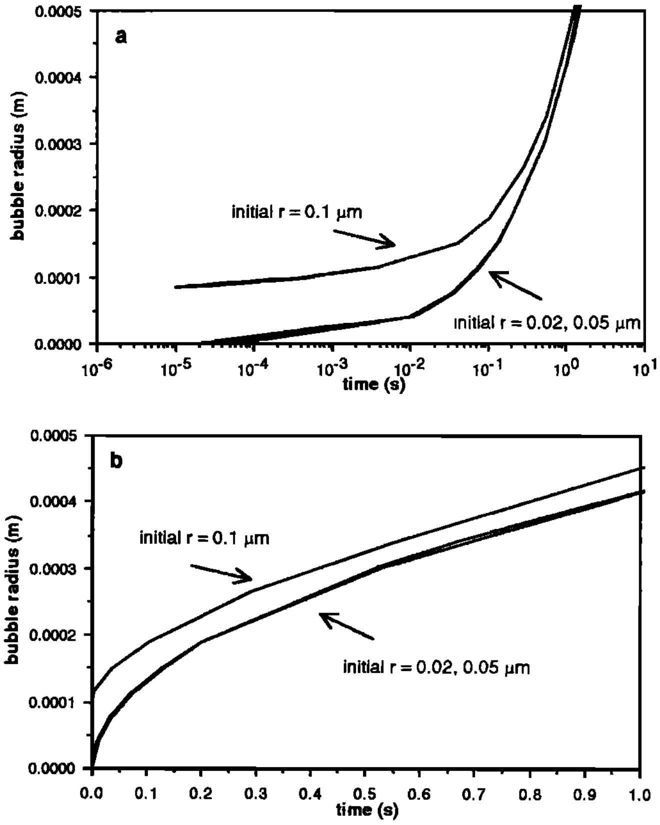

Figure 4. Initial stages of bubble growth for very small bubbles. If bubble surface area were a limiting effect, then the duration of the accelerating growth phase would be dependent on initial bubble size. (a) Model results show that this is not the case even for bubbles as small as 0.02 μm and 0.05 μm. A much larger bubble (0.1 μm) has a "head start" but joins the other curves within about 1 second. (b) The same results plotted on a linear time scale show that there is in fact no accelerating growth phase at all even with these very small bubbles. We attribute this to the low surface tension and viscosity in these model runs. All curves had the following values of parameters: ambient pressure, 0.1 MPa; viscosity, 10^{-3} Pa s; diffusivity, 10^{-11} m^2/s; surface tension, 10^{-6} J/m^2.

Sparks raises an interesting point regarding the effect of diffusivity on bubble growth. He is correct in his understanding of the effect of viscosity on bubble growth. In Figure 16 of *Proussevitch et al.* [1993], the slopes of the various curves appear similar as a result of the log scale on the time axis. Higher diffusivity causes proportionally greater growth rate. As correctly indicated by Sparks, viscosity does not become important until higher viscosities than used in Figures 15 or 16 (as indicated in our Figure 13). The higher nonlinear growth rate for higher diffusivities in Figure 15 actually arises from the effect of the rapid rate of radius increase creating an elevated volatile concentration gradient in the melt in the vicinity of the bubble wall. This is a positive feedback which was accounted for in our model, but which we did not discuss in detail. An interesting comparison can be made in this regard between Figures 15 (rhyolite) and 16 (basalt). For rhyolite there was a high dissolved volatile concentration leading to a very rapid rate of growth (note graph scales and slope of 10^{-10} case). For basalt, there was slow growth. Thus the concentration gradient in the vicinity of the rhyolite bubble wall was elevated by the kinematics of bubble growth, but that of the basalt was not. In the latter case, the rate of bubble growth appears to be directly and solely limited by diffusion. We attribute the difference between Figures 15 and 16 to this effect rather than simply to viscous resistance.

Sparks questions the convergence of the model results for small bubble sizes. Indeed this is an important concern which we did not discuss in our paper. In our convergence tests, we found convergence of model results for arbitrarily small bubble sizes. This was planned for in the original model formulation because of concerns of singularities in the concentration gradient near and at the bubble wall. This required an exponential gridding scheme which resulted in convergence for the smallest bubbles modelled. For bubbles with 0.02 μm initial radius, we varied the grid spacing parameters by 2 orders of magnitude to degrade resolution below that used in our published model runs. The resulting curves were completely overlapping, so we did not include an illistration of this.

We appreciate S. Sparks calling attention to unexplored or unexplained details of our model results. We hope that this simple model will be a reliable basis for more realistic model.

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