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Optimization of a Dual-Channel Retailing System with Customer Returns

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By

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DECLARATION OF CO-AUTHORSHIP / PREVIOUS PUBLICATION

I. Co-Authorship Declaration

I hereby declare that this thesis is a joint research wherein all key ideas, primary contributions, experimental designs, data analysis and interpretations were performed by the author and Dr. Guoqing Zhang as the advisor.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.

II. Declaration of Previous Publication

This thesis includes four original papers that have been previously submitted for publication in peer reviewed journals, as follows:

Thesis Chapter	Publication title/full citation	Publication status
Chapter 3	Radhi, M., Zhang, G., Pricing Policies for a Dual-Channel Retailer with Cross-Channel Returns, Computers and Industrial Engineering.	*Submitted*
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Chapter 6	Radhi, M., Zhang, G., Inventory Policies for a Dual-Channel Retailer with Customer Returns, Production and Operations Management.	*Submitted*

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ABSTRACT

A plethora of retailers have begun to embrace a dual-channel retailing strategy wherein items are provided to consumers through both an online store and a physical store. As a result of standards and competitive measures, many retailers provide buyers who are unhappy with their purchases with the ability to achieve a full refund. In a dual-channel retailing system, full reimbursements can be done through what is called a cross-channel return, when a buyer purchases a product from an online store and returns it to a physical store. They can also be done through what is called a same-channel return, when a buyer purchases a product from a physical store and returns it back to the physical store, or purchases a product from an online store and returns it back to the online store. No existing research has examined all common types of customer returns in the context of a dual-channel retailing system. Be notified that the practice of cross-returning an item purchased from the physical store back to the online store is not common. Thus, it is not considered in this dissertation.

We first study the optimal pricing policies for a centralized and decentralized dual-channel retailer (DCR) with same- and cross-channel returns. We consider two factors: the dual-channel retailer's performance under centralization with unified and differential pricing schemes, and the dual-channel retailer's performance under decentralization with the Stackelberg and Nash games. How dual-channel pricing behaviour is impacted by customer preference and rates of customer returns is discussed. In this study, a channel's sales requests is a linear function of a channel's own pricing strategy and a cross-channel's pricing strategy.

The second problem is an extension of the first problem. The optimal pricing policies and online channel's responsiveness level for a centralized and decentralized dual-channel retailer with same- and cross-channel returns are studied. Indeed, the online store is normally the distribution centre of the enterprise and is not limited to the customers in its neighbourhood. Also, the online store experiences a much higher return rate compared to the physical store. Thus, it has the capability and the need to optimize its responsiveness to customer returns along with its pricing strategy. A channel's sales requests, in the second problem, is a linear function of a channel's own price, a cross-channel's price, and the online store's responsiveness level.

The third problem studies the dilemma of whether or not to allow unsatisfactory online purchases to be cross-returned to the physical store. If not properly considered, those returns may create havoc to the system and a retailer might overestimate or underestimate a channel's order quantity. Therefore, we study and compare between four

different strategies, and propose models to determine optimal order quantities for each strategy when a dual-channel retailer offers both same and cross-channel returns. Several decision making insights on choosing between the different cross-channel return strategies and some properties of the optimal solutions are presented.

From the retailer's perspective of outsourcing the e-channel's management to a third party logistics and service provider, we finally study three different inventory strategies, namely transaction-based fee, flat-based fee, and gain sharing. For each strategy, we find both channels' optimal inventory policies and expected profits. The performances of the different strategies are compared and the managerial insights are given using analytical and numerical analysis.

Methodologies, insights, comparative analysis, and computational results are delivered in this dissertation for the above aforementioned problems.

DEDICATION

My earnest gratitude and respect to my honourable parents Tomader Mansoury and Hassan Radhi for their unconditional love and endless care. My purest form of love to my extraordinary wife Marwa for her everlasting support, devotion and patience. My deepest affection to my kids Feras and Kenan through whom I can have better patience, more strength and higher hope.

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CHAPTER 1: INTRODUCTION

1.1 Dual-Channel Retailing Strategy

The rapid development of the Internet, the growth of third-party logistics and service providers (herein called providers), and the existence of several competitive players have inspired enterprises to adopt a dual-channel retailing strategy. According to Ryan et al. (2013), such a strategy offers products through both physical stores (sometimes called retail, brick-mortar, traditional, conventional or offline stores) and online stores (sometimes called e-tail, click or electronic stores). The dual-channel retailing strategy was first introduced by Sears in 1925 when it initiated a physical store beside a catalogue store (Zhang et al. 2010). About 42% of merchants in the different industries implement the dual-channel retailing strategy (Dan et al. 2012). 80% of all US retailers and almost every top US retailer use more than one channel to sell products and services (Zhang et al. 2010). For example Wal-Mart, Toys“R”Us, Target, IBM, Hewlett–Packard, Nike, Pioneer Electronics, Dell, Costco, Kmart, Barnes and Noble, Kohl’s, and Cisco System are all dual-channel retailers. Those DCRs are, sometimes, referred to as click-and-mortar companies. In the coming few years, it is expected that dual-channel retailing will be the dominant retailing strategy for all type of enterprises (Chen et al. 2012).

Adopting a dual-channel retailing strategy offers retail businesses several advantages and disadvantages. For example, retailers may use the strategy as a mechanism to segment consumers. Shoppers who prefer to save time and transportation costs purchase products online after viewing products’ descriptions. They are separated from those who prefer to purchase products in person after touching and feeling the items (Lu and Liu 2013). The use of a dual-channel retailing strategy, allows retailers to reach wider segments of customers and increase revenue (Ryan et al. 2013). Since the Internet is cost effective and has low entry barriers, the use of an online channel can reduce operational retail costs (Chen et al. 2012; Hua et al. 2010; Zhang et al. 2010). Moreover, using a dual-channel retailing strategy allows retailers to satisfy increasing customer demands for multiple channels through which to shop. Thus, using a dual-channel retailing strategy increases customer loyalty and satisfaction (Zhang et al. 2010). Providing an online channel adds flexibility to each retailer’s supply chain (Chiang and Monahan 2005). This flexibility allows customers to view a product’s description online and purchase it at a physical store, order a product online and pick it up from a physical store, or purchase a product online and return it to a physical store. Furthermore, having an online channel facilitates stock-out substitutions for consumers, as retailers can provide easy access to their inventory levels (Chian 2010). A dual-channel retailer can use a differential pricing strategy to direct customer traffic depending on an enterprise's best interest (Zhang et al. 2010). For example, if revenue to weight ratio is low, then the enterprise can set a higher price in the online channel to avoid high shipping and return cost. Coordinated channels allow retailers to collect wide information about their customers' behaviour. Thus, marketing activities can be easily performed and uncertainty can be efficiently reduced.

On the other hand, the dual-channel retailing strategy is normally associated with sales substitution or cannibalization that may form competition between stores. If stores are not coordinated, then this competition may lower a retailer's total profitability (Ryan et al. 2013). Moreover, selling through the online channel imposes operational difficulties when it comes to receiving large packaged items and then repackages customer orders in smaller cartoons (Zhang et al. 2010).

1.2 Customer Returns

In today's market, many North American retailers implement a full refund policy. The policy increases the number of customer purchase returns, as it allows customers to return purchased products for numerous reasons. For example, a product can be returned to a merchant because it is defective, a wrong order, or a wrong product, because the customer did not like it, or because the purchase was impulsive. However, a full refund policy allows retailers to increase customer loyalty, provide customer satisfaction, boost sales, and/or comply with country legislations.

Additionally, customer returns have risen due to a growth in online selling channels. Online customers do not see, touch, or feel products prior to purchasing them. In these cases, a full refund policy "can be an indicator of product quality" (Akcay et al. 2013; Chen and Grewal 2013). Therefore, products purchased online are expected to have higher return rates. For example, fashion products purchased in person through physical stores can have return rates as high as 35%. In contradistinction, fashion products purchased online can have return rates as high as 75% (Akcay et al. 2013; Mostard and Teunter 2006; Vlachos and Dekker 2003). Furthermore, online returns may lead to an annual reduction in revenue between \$1.5 and \$2.5 billion dollars (Li et al. 2013). In the United States and Canada, yearly returns to merchants total between \$100 and \$10 billion dollars of products, respectively (Akcay et al. 2013; Chen and Bell 2009; Su 2009).

According to the Center of Logistic at the University of Nevada, Wal-Mart, alone, process returns worth of $6 billion dollars every year (Chen & Bell 2012). While a significant amount of returned apparel is of good quality and can be resold several times without a recovery process, electronic returns cannot be resold as new and must instead be sold as open-box items (Akcay et al. 2013). In general, defective returns constitute only 5% of all customer returns (Akcay et al. 2013; Su 2009).

Moreover, many retailers (for example, Wal-Mart or Toys“R”Us) allow same-channel returns, wherein an item purchased from one of their physical stores is returned to one of their physical stores, or an item purchased from their online store is returned to their online store. However, many also allow cross-channel returns, wherein a product purchased from their online store may be returned to one of their physical stores. Allowing cross-channel returns is vital for online stores as such a policy increases sales and customer satisfaction and allow physical stores to create additional cross-selling opportunities (Zhang et al. 2010; Cao and Li 2015). Notice that the process of purchasing items from the physical stores and return them to the online stores is not common. Thus, it is not considered in this work.

1.3 Research Objective

The objective of this dissertation is to analyze the problem of customer returns under the dual-channel retailing system. The main goal is to investigate the different cross-channel return polices, to study selected outsourcing options for the management of the online store, and to develop proper methodologies of inventory control management, price management and responsiveness level management.

1.4 Research Methodologies

We will now explain some of the tools and methodologies that have been used throughout this dissertation.

Unconstrained non-linear programming: the unconstrained non-linear programming is the process of minimizing or maximizing a non-linear function without considering any constraint. The problem might be unbounded or has several critical points if the function is positive semi-definite, negative semi-definite or indefinite. In all aforementioned cases the optimization process is impossible or difficult. Thus, to simplify the prediction of systems’ behavior and grantee the existence of a sole optimal solution, we normally condition parameters so that our functions are either positive definite (i.e. there exist one global minima) or negative definite (i.e. there exist one global maxima).

We use the second derivative test to check the concavity or convexity of single variable, two variables, or three variables functions as the following:

For single variable functions:

1. Assume that the second derivative of the function f is continues on $\mathbb{R}$.
2. The function's "Hessian" matrix is $H = [f_{xx}]$.

Let $D = |f_{xx}|$.

(a) If $D > 0$, then the function f posses a global minimum at (a); where $f_x(a) = 0$.

(b) If $D < 0$, then the function f posses a global maximum at (a); where $f_x(a) = 0$.

For two variables functions:

1. Assume that the second derivative of the function f is continues on $\mathbb{R}^2$.
2. The function's "Hessian" matrix is $H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$.

Let $D_1 = |f_{xx}|$ and $D_2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$.

(a) If $D_1 > 0$ and $D_2 > 0$, then f posses a global minimum at (a, b); where $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

(b) If $D_1 < 0$ and $D_2 > 0$, then f posses a global maximum at (a, b); where $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

For three variables functions:

1. Assume that the second derivative of the function f is continues on $\mathbb{R}^3$.
2. The function's "Hessian" matrix is $H = \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix}$.

Let $D_1 = |f_{xx}|$, $D_2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ and $D_3 = \begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix}$.

(a) If $D_1 > 0$, $D_2 > 0$ and $D_3 > 0$, then f posses a global minimum at (a, b, c); where $f_x(a, b, c) = 0$, $f_y(a, b, c) = 0$ and $f_z(a, b, c) = 0$.

(b) If $D_1 < 0$, $D_2 > 0$ and $D_3 < 0$, then f posses a global maximum at (a, b, c); where $f_x(a, b, c) = 0$, $f_y(a, b, c) = 0$ and $f_z(a, b, c) = 0$.

Nash games: the Nash game is a non-cooperative game that is widely used in supply chain management. It is used in a static environment where active competitors simultaneously choose their decisions in isolation. Let us consider two competitors where players' decisions affect each player's payoff. Thus, if player-1's decision is p_1 and player-2's decision is p_2, then player-1's utility is $u_1(p_1, p_2)$ and player-2's utility is $u_2(p_1, p_2)$. The objective of each player is to maximize his/her own utility. Assuming u_1 and u_2 are differentiable, then the following conditions are necessary for the existence of Nash equilibriums:

$$\frac{\partial u_1(p_1,p_2)}{\partial p_1} = 0$$

$$\frac{\partial u_2(p_1,p_2)}{\partial p_2} = 0$$

To find the equilibrium points, the above system of equations should be solved simultaneously. Thus, for as long as the competitor adheres to Nash equilibrium, a player will generate a lower payoff when deviating from the equilibrium point. If each utility function is concave with respect to the player's own strategy, i.e. $u_1(p_1, p_2)$ is concave in p_1 and $u_2(p_1, p_2)$ is concave in p_2, then there exists at least one Nash equilibrium in the game. To test for the Nash equilibrium uniqueness, we construct the hessian matrix as the following:

$$H = \begin{bmatrix} \frac{\partial^2 u_1}{\partial p_1^2} & \frac{\partial^2 u_1}{\partial p_1 \partial p_2} \\ \frac{\partial^2 u_2}{\partial p_2 \partial p_1} & \frac{\partial^2 u_2}{\partial p_2^2} \end{bmatrix}.$$

If $(-1)^n|H|$ is positive, then a unique Nash equilibrium is guaranteed. n in the previous relationship is the number of strategies and it equals two in our example.

Stackelberg games: the Stackelberg game is another non-cooperative game that is widely used in supply chain management where one competitor dominants the other. The game is dynamic in nature and the events occur in two stages. The dominant competitor is the leader and is allowed to initiate the game or choose his/her strategy first. Based on the revealed strategy, the dominated competitor or the follower end the game or chooses his/her strategy second. Knowing that both players are rational, the leader should use the backward induction and choose his/her decision based on the follower's best response function or strategy. Again, let us consider two competitors where players' decisions affect each player's payoff. If the leader's decision is p_l and follower's decision is p_f, then the leader's utility is $u_l(p_l, p_f)$ and the follower utility is $u_f(p_l, p_f)$. The objective of each player is to maximize his/her own utility. Assuming u_1 and u_2 are differentiable, then the following conditions are necessary for the existence of Stackelberg equilibriums:

$$\left.\frac{\partial u_f(p_l,p_f)}{\partial p_f}\right|_{p_f=p_f^*(p_l)} = 0$$

$$\left.\frac{\partial u_l\left(p_l,p_f^*(p_l)\right)}{\partial p_l}\right|_{p_l=p_l^*} = 0$$

If $u_f\left(p_l, p_f\right)$ is concave in p_f, then there is a unique solution for the follower. However, if $u_l\left(p_l, p_f^*(p_l)\right)$ is concave in p_l, then there is a unique solution for the leader and we can guarantee the existence of a unique Stackelberg equilibrium.

Stochastic programming: the purpose of stochastic programming is to seek optimal decisions that best suit all applicable random parameters. Thus, with stochastic programming one may minimize the system's expected cost or maximize the system's expected profit.

1.5 Dissertation Organization

This dissertation is organized as follows: literature is reviewed in Chapter 2. Pricing policies for a dual-channel retailer is discussed in Chapter 3. Chapter 4 is an extension of Chapter 3 and it discusses pricing and responsiveness level decisions under a dual-channel retailing system. The optimal cross-channel return policy for a dual-channel retailer is studied in Chapter 5. The optimal outsourcing strategy for managing the e-channel is then discussed in Chapter 6. Finally, conclusion and future works are discussed in Chapter 7.

CHAPTER 2: LITERATURE REVIEW

2.1 Literature on Dual-Channel Systems

This chapter sheds the light upon two streams of literature. The first stream addresses dual-channel systems under two settings: dual-channel supply chain setting and dual-channel retailing setting. The second stream addresses customer returns under four settings: single retailer setting, manufacturer-retailer setting, two competitive retailers setting, and dual-channel retailing setting.

2.1.1 Dual-Channel Supply Chains

Considerable research works have analyzed systems that contain a manufacturer (or supplier) that sells a single product to customers through both a manufacturer-owned online store and an independent retail store(s) (Figure 2.1). Several different types of competition take place between the two channels, including competition in price (Balakrishnan et al. 2014; David and Adida 2015; Ryan et al. 2013), competition in services (Lu and Liu 2013; Dan et al. 2012), and competition in product availability (Takahashi et al. 2011; Chiang 2010; Chiang and Monahan 2005). Price competition may take a horizontal form, between an online store and a physical store; take a vertical form, between an upper echelon (i.e. manufacturer) and a lower echelon (i.e. retailer); or take both forms (David and Adida 2015; Lu and Liu 2013; Ryan et al. 2013; Dan et al. 2012). According to Chiang (2010), vertical competition enhances price double marginalization inefficiency, while horizontal competition enhances cannibalization. Literature shows that a great deal should be exerted to coordinate both channels in order to diminish, eliminate or, even, reverse the negative effect of competition.

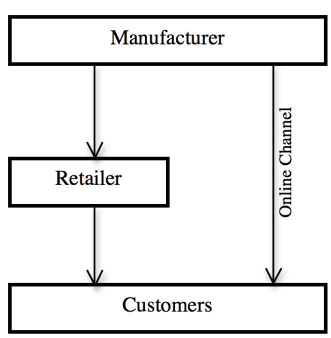


Figure 2.1: Dual-Channel Supply Chain (Chiang and Monahan 2005)

Existing literature shows that decentralized systems are a representation of a situation wherein each channel seeks to maximize its own profit in the presence of cannibalization. Thus, a manufacturer competes by selecting a wholesale price, an online price, a production quantity, a service level, and/or a delivery lead time. A retailer competes by selecting retail price, an acquisition quantity, and/or a service level. This competition is primarily modeled as a manufacturer-Stackelberg game (e.g. David and Adida 2015; Lu and Liu 2013; Ryan et al. 2013; Dan et al. 2012; Huang et al. 2012; Chen et al. 2012; Hua et al. 2010; Bin et al. 2010; Chiang 2010; Yao et al. 2009; Cai et al. 2009). However, it is sometimes modeled as a Nash game (e.g. Lu and Liu 2013; Ryan et al. 2013; Chiang 2010; Cai et al. 2009). A few existing papers have also modeled this competition as a retailer-Stackelberg game (e.g. Cai et al. 2009).

In a coordinated or centralized duopolistic system, each player maximizes its own profit. However, it is done within the boundaries of a contract. David and Adida (2015) examined the equilibrium quantities and prices of a supplier selling items through a self-owned online store and N different retailers. They proposed a linear quantity discount contract to coordinate the system wherein the amount of discount offered to a retailer store is linearly related to the quantity ordered. Ryan et al. (2013) studied the performance of a system under a modified revenue sharing contract in which a retailer shares part of its revenue with a manufacturer. They also studied the performance of a system under a gain/loss sharing contract in which a manufacturer shares part of its gain_or loss with a retailer. Moreover, a two-part tariff contract was studied by Chen et al. (2012). In such a contract a retailer is charged a lump sum fee and a wholesale price equivalent to production costs. Chen et al. (2012) also studied a profit-sharing agreement wherein channels share their profits with competitors' channels for better system performance. Furthermore, an (a, d) contract was examined by Bin et al. (2010). In such a contract, a retailer pays a franchise fee of the value d to a manufacturer. In turn the manufacturer offers the retailer a portion, a, of its total revenue. Chiang (2010) implemented an inventory and direct revenue sharing contract wherein a retailer shares part of its inventory cost with a manufacturer. The manufacturer then shares part of its revenue with the retailer. Finally, Cai et al. (2009) compared a system's performance with and without price discount contracts. They assumed prices were either consistent (i.e., channels' prices were equal) or inconsistent (i.e., channels' prices were not necessarily equal).

Some of the findings that are related to contracts implementation are stated next. Ryan et al. (2013) claim that unless coordinated, loss of retailer's profit and significant increase in manufacturer's profit can occur due to the introduction of online channel. Similarly, Bin et al. (2010) argue that competition between channels enhances system's performance if contracts are used and a retailer will experience lower selling costs. They, also, stated that higher information uncertainty in the retailer channel leads manufacturer to adopt a menu of contracts, while lower uncertainty leads to a single contract. Cai et al. (2009) found that contract coordination and consistent pricing strategy reduce conflict and increase profitability in a system where customers are considered to be either brand loyal or retailer loyal. On the other hand, Chen et al. (2012) claim that coordination through complementary contracts such as two-part tariff and profit sharing is not always profitable to the system.

A fully coordinated or centralized monopolistic system uses a sole decision maker to maximize the system's total profit (e.g. Huang et al. 2012; Hua et al. 2010; Yao et al. 2009; Dumrongsiri et al. 2008). Bin et al. (2010) used both centralization and the principle agent method to examine an asymmetry in information that occurs within the dual-channel system when demand is stochastic. Chiang (2010) argued that a monopolistic dual-channel supply chain outperforms both_duopolistic and uncoordinated dual-channel supply chains. Yao et al. (2009) optimized the inventory system of a dual-channel supply chain with a single period newsvendor model. They compared three different management styles. In the first style, the inventory levels of both channels were managed by one central manager. In the second style, each channel managed its own inventory level. In the third style, the retailer managed its own inventory level while the online store's inventory level was managed by a third-party logistics provider.

Different than most papers in dual-channel supply chain literature, Chiang and Monahan 2005, Takahashi et al. 2011, and Chiang 2010 study Markovian systems that capture competition in product availability. Portion of a channel's loyal customers will cross fill when products are not available in their preferred channel (stuck out base substitution). Chiang and Monahan (2005) claim that the increase of this portion might harm profitability as it could increase total inventory related costs. They argue that retailers and manufacturers are, almost, always better off in adopting dual-channel supply chain. In there modeling, they capture holding and lost sale costs and apply one-for-one inventory control policy. In contrast, Takahashi et al. (2011) have included production and delivery setup costs and found that one-for-one inventory control policy is not an appropriate policy for a dual-channel supply chain.

2.1.2 Dual-Channel Retailing Systems

Other papers in the literature have examined the situation wherein a dual-channel retailer offers the same product in both self-owned online and physical stores (Figure 2.2). Yan et al. (2010), Yan (2010), and Yan (2008) all studied Nash, online-Stackelberg and retailer-Stackelberg games to model the price competition that stems from operating a dual-channel retailing system. Each of the studies stated that the Stackelberg games always outperform the Nash game. Yan et al. (2010) argue that by using profit sharing to integrate the dual-channel retailing system, conflict is eliminated, coordination is improved, and both channels generate more profit. Yan (2010) stated that higher brand differentiation better handle conflict especially when consumers are less price sensitive and market size is large. Additionally, Berger et al. (2006) examined the profit enhancement induced by a dual-channel retailer that integrates the advertisement efforts of both online and physical stores. They found that higher integration leads both channels to generate higher profits.

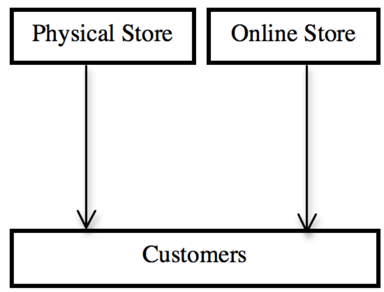


Figure 2.2: Dual-Channel Retailing System

Using branch-and-bound algorithm, Mahar et al. (2009) examine the positive impact that a real time information capability has on a dual-channel retailer. Such a capability allows the dynamic assignment policy to replace the common static assignment policy. Based solely on the proximity, the static policy pre-identifies the e-fulfilment location responsible to serve a certain customer. On the other hand, based on both proximity and real time inventory position, the dynamic policy identifies the e-fulfilment location. It is found that such a policy might decrease total cost (holding, backorder and shipping) for up to 8.2%.

One may note that each study presented above examined the possible coordination strategies or policies between different competing channels in a dual-channel system. None of the papers have considered customer returns and the impact that the returns have had on dual-channel retailing systems. Papers that have considered customer returns are reviewed next.

2.2 Literature on Customer Returns

Before we review the papers under the different settings, it is adequate to examine the researchers findings in regard to the different refund policies. A large body of work on customer returns has examined a refund policy that is exogenously determined as a full refund or a Money Back Guarantee (MBG) (Reimann 2016; Chang and Yeh 2013; Chen and Bell 2013; Choi et al. 2013; Akcay et al. 2013; Wang et al. 2010; You et al. 2010;

Vlachos and Dekker 2003). Other papers have compared a system's performance with no refund policy to a system's performance with a full refund policy (Chen and Grewal 2013; Choi et al. 2013; Chen and Bell 2012; Chen and Zhang 2011). Several papers that conducted such a comparison also examined a partial refund policy (Chen and Grewal 2013; Li et al. 2013; Chen and Bell 2012; Chen and Zhang 2011; Su 2009; Yalabik et al. 2005). Hsiao and Chen (2012) found that the optimal refund policy may exceed the full price of the item. Su (2009), Chen and Bell (2009), and Yalabik et al. (2005) all argued that a full refund policy is not optimal as it overwhelms retailing systems. In contrast, Chen and Zhang (2011) argued that a full refund policy may be optimal in the presence of competition. However, Hu et al. (2014) and Su (2009) claimed that the optimal refund policy depends upon the refunded product's salvage value. According to Li et al. (2013), retailers should offer either a lenient return policy with a low quality and a low price or a strict return policy with a high quality and a high price. Their choice depends upon customer sensitivity in regards to price, return policy, and quality. Moreover, Yu and Goh (2012) stated that retailers should enforce a return policy that takes the nature of products and their condition upon return into consideration. Akcay et al. (2013) encouraged retailers to reduce the number of returns they receive by controlling selling prices and enforcing a refund policy with restocking fees. However, Hu and Li (2012) argued that offering a manufacturer buyback price equivalent to the retailer's refund price is the optimal coordinating mechanism.

2.2.1 Single Retailer Systems

Additionally, many papers have considered a retailer faces returns from unsatisfied customers (Figure 2.3). Reimann (2016) considered a retailing system where refurbished returns can be used to satisfy demand that exceeds the order quantity. Akcay et al. (2013) studied a system wherein customers could differentiate between a new sell and a resell but their product valuation was uncertain. Yu and Goh (2012) examined a retailer facing eight different scenarios. The eight scenarios had several combinations that consisted of whether or not returns occurred within a grace period, whether or not returns were accompanied by a penalty, and whether or not returns were recoverable. While their research did not consider a resell option for returns, it was considered that recoverable items could be salvaged in a secondary market. Additionally, according to You et al. (2010), a single selling period can be divided into N countable sub-periods. Each sub-period is associated with a probability of return. Chen and Bell (2009) did not allow "as good as new" returns to be sold in the same period in which they were sold, but did allow them to be salvaged in a single-period setting or resold in the following period in a multi-period setting. Wang et al. (2010) investigated a system wherein customer returns could be resold several times. Inventory holding cost and deterioration over time is incorporated in their model. Selling periods were divided into three sub-periods: a period in which sales consumed both new and returned stocks, a period in which sales only consumed returned stocks, and a period in which there were only returns, not sales. Li et al. (2013), Hsiao and Chen (2012), and Mukhopadhyay and Setaputra (2007) discussed the interrelationship between price, refund policy, and quality. Li et al. (2013) define quality as product consistency with the online description, while Hsiao and Chen (2012) identify product defects, misfit, and unconformity as quality risks that face customers. Choi et al. (2013) and Mukhopadhyay and Setoputro (2005) examined a system in which demand is linearly dependant on price, refund policy, and modularity level, while return is linearly dependant on refund policy. They claim that with higher level of modularity, retailer should charge higher price regardless of the refund policy applied. Vlachos and Dekker (2003) studied six different systems according to whether or not returns could be resold in the primary market, whether or not resalable returns needed a recovery process, and whether or not the needed recovery process was associated with fixed or variable costs. The objective of the paper is to maximize the profit by selecting the optimal ordering quantity where resalable returns can be resold once a season. Mostard et al. (2005) and Mostard and Teunter (2006) examined a system wherein a resalable return could be resold an infinite number of times until it became non- resalable by not having demand to consume it or by being returned after the end of the selling season. Mollenkopf et al. (2007) studied the effect of customer return related services on the Internet shoppers' perceived loyalty and the key drivers that positively impact the return experience. The paper Yalabik et al. (2005) is an extension of the work done by Davis et al. (1998). They classify customers into matched and mismatched with product valuation $v1$ and $v2$, respectively. Post purchase the customer will consume a portion α of the original value of the product. Ultimately, customers will keep the product if the utility of keeping the product is more than the utility of returning it. The previous papers considered single retailer systems where customers can return unsatisfactory purchases to retailers. None of those papers studied the effect of customer returns on dual-channel retailers.

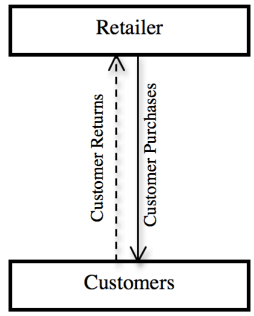


Figure 2.3: A Single Retailer System with Customer Returns

2.2.2 Manufacturer-Retailer Systems

Several other papers considered a system in which retailers and manufacturers handled customer returns through the use of contractual agreements and information sharing (Figure 2.4). Chang and Yeh (2013) studied returns from customer to retailer and from retailer to manufacturer under centralized and decentralized settings. Returns could not be resold in the same selling season during which they were returned due to seasonal length constraints. Additionally, Chen and Bell (2013), Hu and Li (2012), Chen and Bell (2011), and Chen (2011) examined the effect of sharing return information with a manufacturer on the profitability of the system. Chen (2011) claims that not sharing return information is a better strategy if the product is mature. Otherwise, and in consistence with Chen and Bell (2011) work, not sharing information will negatively impact both retailers and manufacturer. Hu and Li (2012), investigate a system under Stackelberg game with demand and customer valuation uncertainty. Based on the system's acceptability of returns, four scenarios have been recognized. Moreover, Hu et al. (2014) studied a consignment contract between a vendor and a retailer in which the vendor owned the inventory placed in the retailer store. They examined both retailer-managed and vendor-managed consignment inventory systems. Finally, Su (2009) studied four contracts that may be used to resolve conflicts that arise between retailers and manufacturers due to customer returns: buy back, differential buy back, return to manufacturer, and rebate contracts. The later is found to outperform all other contracts as long as manufacturer can monitor sales. The previous papers considered returns from customer to manufacturer, from customers to retailer and/or from retailer to manufacturer under manufacturer-retailer settings. The effect of customer returns on dual-channel retailers was not considered in any of the above papers.

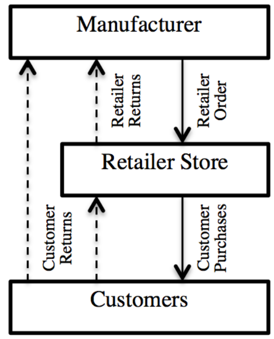


Figure 2.4: A Manufacturer-Retailer System with Customer Returns

2.2.3 Two Competitive Retailers Systems

Several other papers have considered customer returns when two retailers compete in the same market (Figure 2.5). Chen and Bell (2012) examined a system with two customer behaviours: return-sensitive customers willing to pay more and enjoy the privilege of returning a product if it is a mismatch, and price-sensitive customers willing to pay less and keep the product if it is a mismatch. Both a returnable channel and a non-returnable channel are thus considered in Chen and Bell's study. Furthermore, Chen and Grewal (2013) studied Stackelberg and Nash competitions in situations wherein a new channel competes with a well-established retailer that offers a full refund policy. Additionally, Chen and Zhang (2011) studied Stackelberg and Nash competitions between two retailers that both offered a full refund policy. Balakrishnan et al. (2014) studied the browse and switch behaviour exerted by consumers on the brick and mortar stores. The effect of such behaviour on system's profits and prices are examined when returns are allowed for online purchases only. None of the above works considered dual-channel retailers. Contrary to practice, they also assumed returns to be non-resalable.

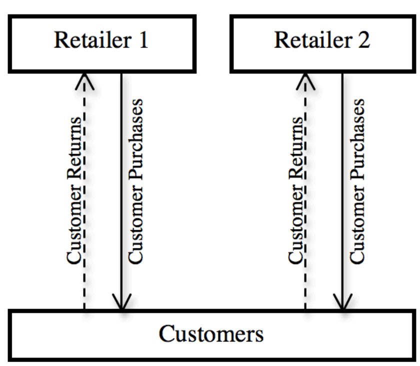


Figure 2.5: Two Competing Retailers with Customer Returns

2.2.4 Dual-channel Retailing Systems

Widodo et al. (2010) and Widodo et al. (2009) studied both Nash and Stackelberg competitions between a retailer's physical and online channels (Figure 2.6). Contrary to practice, they studied returns that were only allowed for online purchases. Online customers were allowed to return items to the online store (a same-channel return) or the physical store (a cross-channel return). Also, the two studies assumed that returns could be exchanged but not refunded. One may note that none of the above two papers has collectively considered all common forms of customer returns a dual-channel retailer may experience. There is thus a research gap in this area.

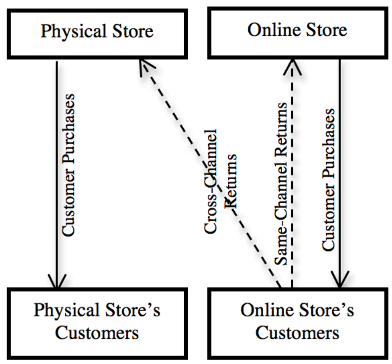


Figure 2.6: A Dual-Channel Retailer with Online Store's Customer Returns

CHAPTER 3: PRICING POLICIES

3.1 Introduction and Motivation

Since coordination coincides with high costs and imposes operational difficulties, most multi-channel retailers (i.e., Target, Nike, Kmart, Barnes and Noble, Jo-Ann Fabric and Craft Stores, and Kohl's) use decentralized teams to run their stores (Zhang et al. 2010; Yan et al. (2010); Neslin and Shankar 2009; Yan 2008; Berger et al. 2006; Webb and Hogan 2002; Schoenbachler and Gordon 2002). Also, a variety of managerial skills are needed for different channels; thus, some retailers outsource the management of unfamiliar or newly opened channels to a third party. An example is Toys"R"Us, which outsources the management of its online channel to Amazon (Berger et al. 2006).

Many often believe that decentralizing the dual-channels reduces market shares due to cannibalization. This will in turn spark competition and trigger a price war that may harm the parties involved. If decentralization is uncoordinated, its resulting competition may lower supply chain profitability (Ryan et al. 2013; Yan et al. 2010; Steinfield 2004; Schoenbachler and Gordon 2002; Webb and Hogan 2002). Furthermore, as Webb and Hogan (2002) have stated, "goal incompatibility" (between physical stores and online stores, for example) is an inevitable result of decentralization. Channels may generate internal conflict due to scarce resources (for example, a tight budget or few customers) or tight objectives (for example, a targeted revenue and profit). They define competition as goal-centered behaviour and conflict as opponent-centered behaviour. Webb and Hogan's research supports this; they found that 66% of 50 interviewed retail businesses viewed channel conflict as the most troublesome issue that is faced when they run dual-retailing channels. The competition associated with such conflict may cause channels to limit cooperation and inspire customers to change companies due to confusion and agitation (Steinfield 2004). These limitations may be so intense that one channel may sabotage another. For example, Levi Strauss and Best Buy had terminated their online stores after a few years of their first operational trial due to internal competition (Yan 2010; Falk et al. 2007). Consequently, companies such as Wal-Mart Stores, Gap, and the Home Depot have successfully integrated their dual channels under a sole decision maker to maximize their total profits (Yan 2008).

Several dual-channel retailers offer both same- and cross-channel returns, whether their operating channels are integrated or not. Zhang et al. (2010) have stated that if cross-channel returned items are not offered at a physical store, then the items must be shipped to the online store. Otherwise the ownership of such items is transferred to the physical store. This is done by conducting an inventory transfer that is subjected to the retailer's internal rules. The retailer's policy and practice of having cross-channel returns can be acquired through partial integration. As Cao and Li (2015) have stated, channels will only have full integration when prices align to meet the retailer's goals and objectives. That is to say, cross-channel returns do not contradict the fact that channels may still undergo price competition.

Many studies have considered competition and possible coordination strategies between dual channels, which are owned by either the same retailer or different enterprises. The customer returns topic has also been thoroughly studied in single retailer or two retailers systems. However, few papers have studied customer returns under a dual-channel retailing system. As stated before, there is no work that has collectively considered all common forms of customer returns for both types of stores of a dual-channel retailer. Also, there is no published paper that has studied the impact of cross-channel returns on both stores of a DCR, especially when those returns are resalable. For example, the effect of cross-channel returns on channels' pricing policies and inventory management has not being studied yet. Thus, Chapter 3 studies a dual-channel retailer with both return mechanisms and investigates optimal pricing policies. Both centralization with differential and unified pricing schemes and competition in regards to theoretical game frameworks are addressed.

When determining prices in a Stackelberg game, the online store leads and the physical store follows. However, in a Nash game, the physical store and the online store determine optimal prices simultaneously. This study provides several contributions to existing literature in three ways. First, it collectively considers all common forms of customer returns for a dual-channel retailer. As stated previously, purchasing an item from a physical store and returning it to an online store is not a common practice and, thus, it is not considered in this work. Second, it addresses dual-channel competition from a game theoretic perspective. Third, it compares dual-channel retailer's total performance under centralization with unified pricing scheme, and Stackelberg and Nash games.

3.2 Model Formulation

This chapter considers merchants that run both a physical store and an online store. It examines two coordination schemes: one in which channels are managed collectively in a centralized setting and one in which channels are managed competitively in a decentralized setting. Customers may receive a full refund for purchases returned within a merchant-specified time period. The probability that a product purchased from a physical store is returned to a physical store is $0 \leq r \leq 1$. The probability that a product purchased from an online store is returned to an online store is $0 \leq w \leq 1$. The probability that a product purchased from an online store is cross-returned to a physical store is $0 \leq v \leq 1$ (Figure 3.1). The assumption of ratios for returns has been implemented in literature before, such as in works by Chen and Grewal (2013), Mostard and Teunter (2006), Mostard et al. (2005), Vlachos and Dekker (2003), and many more.

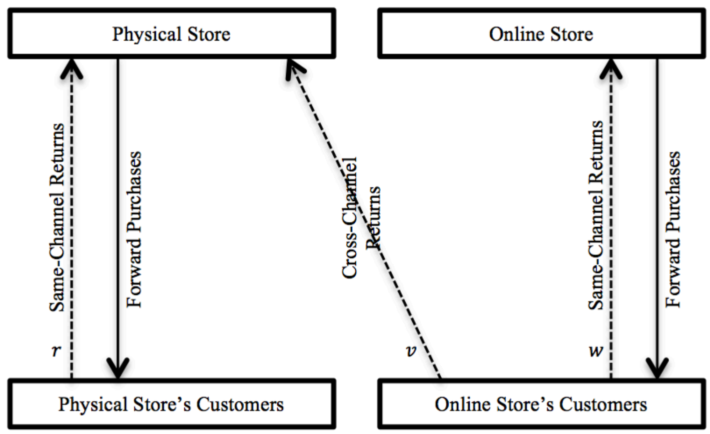


Figure 3.1: A Dual-Channel Retailer with Same- and Cross-Channel Returns

Akcay et al. (2013) have stated that apparel is often returned "as good as new"; thus, apparel can be resold several times during a single period. Therefore, for a returned product to be resalable it must be returned in its original packaging and condition. We assume that a returned product has a resalability rate of k_r if the item was purchased from and returned to a physical store, k_o if the item was purchased from and returned to an online store, and k_{or} if the item was purchased from an online store but cross-returned to a physical store.

We assume that all same-channel resalable returns can be resold for ε times from their original channels. Regardless of the number of times an item is sold in the online store, all cross-channel resalable returns can be resold for ε times from the physical store. According to Vlachos and Dekker (2003), if a resalable returned product takes a relatively long time to be placed on a store's shelf from the moment it is purchased, then one may assume it can only be resold once during a selling season (i.e. $\varepsilon = 1$). In contradistinction, if a resalable returned product takes a relatively short time to be placed on a store's shelf from the moment it is purchased (i.e., if it has a lead time of zero), then one may assume it can be resold an infinite number of times (i.e. $\varepsilon = \infty$) during a selling season until it is permanently sold, cross-channel returned, or returned but not resalable (Mostard and Teunter 2006; Mostard et al. 2005). Therefore, two cases are studied: a general case where returns are assumed to be resold ε number of times; $\varepsilon \in [1, \infty)$, and a more simplified special case where returns are assumed to be resold infinitely; $\varepsilon = \infty$.

Each returned item is associated with a return collection cost of the value d. If an item is returned as not resalable or as resalable after the end of the selling season, then its salvage value, s, is acquired by selling the item in a secondary market. The unit's salvage value must be less than or equal to the unit's purchasing cost $s \leq c$; otherwise the profit function would be unbounded above. Items that are purchased from or returned to the online store will cost the store a per-unit shipping expense of t.

The parameter α represents the base level of sales, or the sales level when items are offered to customers free of charge (Chen et al. 2012; Huang et al. 2012). If $0 \leq \theta \leq 1$ is the degree of customer preference for the physical store, then $\alpha_r = \alpha\theta$ is the physical store's base level of sales. Similarly, if $1 - \theta$ is the degree of customer preference for the online store, then $\alpha_o = \alpha(1 - \theta)$ is the online store's base level of sales.

Several papers have considered customer preference in their studies. For example, Lu and Liu (2013) argue that customer preference for a certain channel induces dominance and profitability to that channel and the degree of that preference greatly affects the equilibrium prices. According to Hua et al. (2010), different products lead to different degrees of customer preference for the physical store. For example, products that are customized, require a high level of examination prior to being purchased (such as used cars, clothes, shoes, or eyeglasses), or require after-sale services (such as electronics) better-fit physical stores. In contradistinction, products that do not require a high level of examination in regards to their quality level prior to being purchased, standardized, or mature (such as books and CDs) better fit online stores. Hua et al. (2010) also stated that customer preference for the online store is directly affected by lead-time and product type. According to Ryan et al. (2013) the positive impact of coordination is magnified when customers tend to stick to their preferable channel even in the existence of price differentiation. Opposite to the general perception that customer preference for a certain channel is the most important driver of demand in that channel, services provided to customers (e.g. customer support, presale advice, in-store advertising and promotions, technical and shopping assistance, and return services) will be the key factor in driving demand up or down (Zhang et al. 2010). As customers are becoming more attached to dual-channel retailing, a new competing player in the market may increase customer base for the incumbent, thus, increasing the channel's profitability (Huang and Swaminathan 2009).

β is an ownership price sensitivity that measures the rate at which sales are affected by a channel's own price. γ is the cross-price sensitivity that reflects the degree of cannibalization between two channels. A channel's cross-price sensitivity has a lesser effect on sales than a channel's ownership-price sensitivity, which is $\gamma \leq \beta$. D_r and D_o denote total customer sales within the physical store and the online store, respectively. Therefore, physical and online store sales functions are given, respectively, as:

$D_r = \alpha_r - \beta p_r + \gamma p_o$ and (3.1)

$D_o = \alpha_o - \beta p_o + \gamma p_r$. (3.2)

Linear sales functions in a dual-channel system were utilized in Ryan et al. (2013), Huang et al. (2012), Chen et al. (2012), Bin et al. (2010), and more.

Q_ε^r and Q_ε^o are the order quantities placed at the physical store and the online store at the beginning of the selling season, respectively. Since the studied retailing system allows customer returns and a portion of those returns to be resold in the same selling season, then it is intuitive to see that a channel's order quantity is lower than its total sales. Thus, if resalable returns can be resold ε number of times, then an online store will sell its order quantity (Q_ε^o), all of its same-channel first time resalable returns ($wk_oQ_\varepsilon^o$), all of its same-channel second time resalable returns ($w^2k_o^2\, Q_\varepsilon^o$), and so on. Thus,

$$D_o = Q_\varepsilon^o(1 + wk_o + (wk_o)^2 + \cdots + (wk_o)^\varepsilon) = Q_\varepsilon^o \textstyle\sum_0^\varepsilon (wk_o)^n \quad (3.3)$$

The order quantity is as follows:

$$Q_\varepsilon^o = \frac{D_o}{\sum_0^\varepsilon (wk_o)^n} \quad (3.4)$$

Due to the ratio v, a quantity of vD_o is cross-returned from the online store to the physical store. A portion, k_{or} of this quantity, is resaleable and can be resold ε number of times to satisfy part of the physical store's total sales D_r. Thus, the physical store will sell its order quantity (Q_ε^r), all of its same-channel first time resalable returns of ($rk_rQ_\varepsilon^r$), all of its same-channel second time resalable returns ($r^2k_r^2Q_\varepsilon^r$), and so on. Similarly, it will sell the quantity ($vk_{or}D_o$) and all of its same-channel first time resalable returns ($rk_rvk_{or}D_o$), all of its same-channel second time resalable returns ($r^2k_r^2vk_{or}D_o$), and so on. Thus,

$D_r = Q_\varepsilon^r(1 + rk_r + \cdots + (rk_r)^\varepsilon) + vk_{or}D_o(1 + rk_r + \cdots + (rk_r)^{\varepsilon-1}) =$

$$Q_\varepsilon^r \sum_0^\varepsilon (rk_r)^n + vk_{or}D_o \sum_1^\varepsilon (rk_r)^{n-1} \tag{3.5}$$

The order quantity is as follows:

$$Q_\varepsilon^r = \frac{D_r - vk_{or}D_o \sum_1^\varepsilon (rk_r)^{n-1}}{\sum_0^\varepsilon (rk_r)^n} \tag{3.6}$$

Notice that the term $vk_{or}D_o \sum_1^\varepsilon (rk_r)^{n-1}$ is conditioned to be less than or equal to D_r (i.e. $Q_\varepsilon^r \geq 0$); otherwise the physical store would be overwhelmed by cross-channel returns that would allow the store to start its selling season without any quantity ordered from the supplier. Such a case is unrealistic; thus, its analytical complications are omitted from the calculations.

Assume that resalable returns can be resold several times in a selling season. If return rate, resalability rate and sales requests are not staggeringly high, then ε is safely assumed to be infinity (i.e. $\varepsilon = \infty$). The previous assumption will greatly simplify the calculation of equilibrium points. Under such a case equations (3.3), (3.4), (3.5), and (3.6) are modified respectively as the following:

$$D_o = \frac{Q_{\varepsilon=\infty}^o}{1 - wk_o}, \tag{3.7}$$

$$Q_{\varepsilon=\infty}^o = (1 - wk_o)D_o, \tag{3.8}$$

$$D_r = \frac{Q_{\varepsilon=\infty}^r + vk_{or}D_o}{1 - rk_r}, \text{ and} \tag{3.9}$$

$$Q_{\varepsilon=\infty}^r = (1 - rk_r)D_r - vk_{or}D_o. \tag{3.10}$$

To comprehend, let us assume that the apparel industry is being studied. If the selling season consists of four months and the unsatisfied customers posses sold items for two weeks before reimbursing, then it is expected that $\varepsilon = 8$. Assume that $w = 0.5$ and $k_o = 0.7$, then using both equations (3.3) and (3.7) an ordered quantity of 5000 can satisfy up to 7692. The answers are closely similar and thus the special case (i.e. $\varepsilon = \infty$) can be used to model the problem.

The following two sections examine the integration of a dual-channel retailer under a centralized management using two pricing strategies: differential pricing mode and uniform pricing mode. They also examine online and physical stores' equilibriums when the stores use two different competitive pricing schemes: Stackelberg game and Nash game. In the Stackelberg game, a retailer's online store leads. It announces its selling price first and is followed by its physical store. However, in the Nash game both channels are equally powerful in price determination. Thus, they set their price strategies simultaneously. Table 3.1 presents a summary of the notations used in Chapter 3.

Notation	Description
$\boldsymbol{r}$	Probability an item purchased from the physical store is returned to the physical store
$\boldsymbol{w}$	Probability an item purchased from the online store is returned to the online store
$\boldsymbol{v}$	Probability an item purchased from the online store is cross-returned to a physical store
$\boldsymbol{k_r}$	Probability an item purchased from and returned to the physical store is resalable
$\boldsymbol{k_o}$	Probability an item purchased from and returned to the online store is resalable
$\boldsymbol{k_{or}}$	Probability an item purchased from the online store and cross-returned to the physical store is resalable
$\boldsymbol{c}$ & $\boldsymbol{s}$	Unit purchasing cost and salvage value, respectively
$\boldsymbol{d}$ & $\boldsymbol{t}$	Return collection and shipping costs, respectively
$\hat{\boldsymbol{c}}$	Amount a physical store pays to an online store for every cross-channel return in the decentralization scheme
$\boldsymbol{D_r}$ & $\boldsymbol{D_o}$	Retail and online stores' total sales including returns, respectively
$\boldsymbol{Q_\varepsilon^r}$ & $\boldsymbol{Q_\varepsilon^o}$	Quantities ordered by retail and online stores, respectively
$\boldsymbol{\alpha}, \boldsymbol{\alpha_r}$ & $\boldsymbol{\alpha_o}$	Enterprise, physical store and online store base levels of sale, respectively
$\boldsymbol{\theta}$	Customer preference for the physical store
$\boldsymbol{\beta}$ & $\boldsymbol{\gamma}$	Ownership price and cross-price sensitivities of a channel, respectively
$\boldsymbol{p_r}$ & $\boldsymbol{p_o}$	Retail and online store's prices, respectively
$\boldsymbol{\pi_\varepsilon^{C_r}}, \boldsymbol{\pi_\varepsilon^{C_o}}$ & $\boldsymbol{\pi_\varepsilon^{C}}$	Physical store, online store, and enterprise profits in the centralized case, respectively
$\boldsymbol{\pi_\varepsilon^{D_r}}$ & $\boldsymbol{\pi_\varepsilon^{D_o}}$	Physical store and online store profits in the decentralized case, respectively
$\boldsymbol{\varepsilon}$	Number of times a resalable return can be resold in a selling season

Table 3.1: Third Chapter's Notations

3.3 Centralized Dual-Channel Retailing System

This section studies pricing policies in a centralized system wherein a retailer's physical and online stores are vertically integrated. One may assume the existence of a central decision maker who pursues the maximum total supply chain profit (π_ε^C). The central decision maker simultaneously determines the physical store's price, p_r, and the online store's price, p_o, to meet the retailer's goals and objectives.

The online store's profit function is modeled as the following:

$$\pi_\varepsilon^{C_o} = D_o\left[(1-w-v)p_o - t - w(d+t) + w(1-k_o)s + s\frac{(wk_o)^{\varepsilon+1}}{\sum_0^\varepsilon (wk_o)^n}\right] - Q_\varepsilon^o c. \tag{3.11}$$

A portion from D_o, $(1-w-v)$, is a final sale and contributes positively. Every sold item contributes negatively due to the shipped cost t paid by the store. A w portion from D_o is returned to the online store and contributes negatively due to collection and shipping costs. A portion of $w(1-k_o)$ from D_o is salvaged and contributes positively as it is returned as non-resalable. The term $s\frac{(wk_o)^{\varepsilon+1}}{\sum_0^{\varepsilon}(wk_o)^n}$ assures the salvaging of an item that ends up being returned as resalable after the end of the selling season. Notice that this term will be zero when $\varepsilon=\infty$. The second term is the ordering cost for the quantity assigned to the online store.

The physical store's profit function is modeled as:

$$\pi_{\varepsilon}^{C_r}=D_r\left[(1-r)p_r-rd+r(1-k_r)s+s\frac{(rk_r)^{\varepsilon+1}}{\sum_0^{\varepsilon}(rk_r)^n}\right]+vD_o\left[-d+(1-k_{or})s+s\frac{k_{or}(rk_r)^{\varepsilon}}{\sum_0^{\varepsilon}(rk_r)^n}\right]-Q_{\varepsilon}^{r}c. \tag{3.12}$$

In the first term, a portion of $(1-r)$ from D_r is a final sale and contributes positively, a portion of r from D_r is returned to the physical store and contributes negatively due to its collection cost, and a portion of $r(1-k_r)$ from D_r is salvaged and contributes positively as it is returned as a non-resalable item. The term $s\frac{(rk_r)^{\varepsilon+1}}{\sum_0^{\varepsilon}(rk_r)^n}$ assures the salvaging of items that end up being resalable returns after the end of the selling season when the system experience no cross-channel returns. This term will be zero when $\varepsilon=\infty$. In the second term, a portion of v from D_o is cross-returned to the physical store and contributes negatively due to its collection cost. A portion of $v(1-k_{or})$ from D_o is salvaged and contributes positively as it is cross-returned as a non-resalable item. The term $s\frac{k_{or}(rk_r)^{\varepsilon}}{\sum_0^{\varepsilon}(rk_r)^n}$ considers the increment in salvaged resalable returns at the physical store when the system experience cross-channel returns. Again, the term will be equivalent to zero when $\varepsilon=\infty$. The third term is the ordering cost for the items assigned to the physical store.

The total supply chain profit function can be modeled by adding functions (3.11) and (3.12) as the following:

$$\pi_{\varepsilon}^{C}=\pi_{\varepsilon}^{C_o}+\pi_{\varepsilon}^{C_r}=D_o\left[(1-w-v)p_o-t-w(d+t)+w(1-k_o)s+s\frac{(wk_o)^{\varepsilon+1}}{\sum_0^{\varepsilon}(wk_o)^n}-vd+v(1-k_{or})s+vs\frac{k_{or}(rk_r)^{\varepsilon}}{\sum_0^{\varepsilon}(rk_r)^n}\right]+D_r\left[(1-r)p_r-rd+r(1-k_r)s+s\frac{(rk_r)^{\varepsilon+1}}{\sum_0^{\varepsilon}(rk_r)^n}\right]-Q_{\varepsilon}^{o}c-Q_{\varepsilon}^{r}c. \tag{3.13}$$

By replacing the quantity Q_{ε}^{r} and Q_{ε}^{o} with their functions, the total supply chain profit can be transformed into the following:

General Case (i.e. $\boldsymbol{\varepsilon\in[1,\infty)}$):

Using the formulas (3.4) and (3.6), one gets:

$$\pi_\varepsilon^C = D_o\left[(1-w-v)p_o - t - w(d+t) + w(1-k_o)s + \frac{s(wk_o)^{\varepsilon+1}-c}{\sum_0^\varepsilon (wk_o)^n} + v\left((1-k_{or})s - d + \frac{k_{or}\left(c\sum_1^\varepsilon (rk_r)^{n-1} + s(rk_r)^\varepsilon\right)}{\sum_0^\varepsilon (rk_r)^n}\right)\right] + D_r\left[(1-r)p_r - rd + r(1-k_r)s + \frac{s(rk_r)^{\varepsilon+1}-c}{\sum_0^\varepsilon (rk_r)^n}\right]. \quad (3.14)$$

Especial Case (i.e. $\boldsymbol{\varepsilon = \infty}$):

With the formulas (3.8) and (3.10), one obtains:

$$\pi_{\varepsilon=\infty}^C = D_o[(1-w-v)p_o - t - w(d+t) + w(1-k_o)s - (1-wk_o)c + v(ck_{or} + (1-k_{or})s - d)] + D_r[(1-r)p_r - rd + r(1-k_r)s - c(1-rk_r)]. \quad (3.15)$$

One may reformulate profit functions (3.13) and (3.14) as the following:

$$\pi_\varepsilon^C = D_o(Jp_o - B_\varepsilon) + D_r(Ip_r - A_\varepsilon) = D_o\left(REV_\varepsilon^{C_o}\right) + D_r\left(REV_\varepsilon^{C_r}\right). \quad (3.16)$$

Where;

$I = 1 - r, J = 1 - w - v, A_\varepsilon = rd - r(1-k_r)s - \frac{s(rk_r)^{\varepsilon+1}-c}{\sum_0^\varepsilon (rk_r)^n}$,

$B_\varepsilon =$

$t + w(d+t) - w(1-k_o)s - \frac{s(wk_o)^{\varepsilon+1}-c}{\sum_0^\varepsilon (wk_o)^n} - v\left((1-k_{or})s - d + \frac{k_{or}\left(c\sum_1^\varepsilon (rk_r)^{n-1} + s(rk_r)^\varepsilon\right)}{\sum_0^\varepsilon (rk_r)^n}\right)$,

$A_{\varepsilon=\infty} = rd - r(1-k_r)s + c(1-rk_r)$, and

$B_{\varepsilon=\infty} = t + w(d+t) - w(1-k_o)s + (1-wk_o)c - v(ck_{or} + (1-k_{or})s - d)$.

Notice that $REV_\varepsilon^{C_o}$ and $REV_\varepsilon^{C_r}$ is the revenue generated by satisfying a single sale from the online store and physical store, respectively. Thus, the optimal solution is subjected to the following constraints:

$D_o \geq 0, D_r \geq 0, \left(REV_\varepsilon^{C_o}\right) \geq 0, \left(REV_\varepsilon^{C_r}\right) \geq 0$, and $Q_\varepsilon^r \geq 0$.

Section 3.3.1 presents an analysis of a situation wherein a central decision maker adopts a differential pricing strategy or does not add any constraint to prices. Section 3.3.2 studies a situation wherein a central decision maker adopts a unified pricing strategy or constrains prices so that they are equal.

3.3.1 Dual-Channel Retailing System under the Differential Pricing Strategy

It has been argued that differential pricing is the optimal strategy when higher prices are assigned to the channel with the highest operational costs (Zhang et al. 2010 and Yan 2008). Neslin et al. (2006) have also argued in favour of differential pricing, but with higher prices assigned to the channel with the fewest price-sensitive customers.

However, several other authors have argued that a unified pricing strategy is not optimal for a dual-channel retailer and that a channel's pricing strategy should be proportional to its customer's preference and its provided services (Chen et al. 2012; Dan et al. 2012; Hua et al. 2010). Thus, this section investigates the effect customer preference and rates of return have on pricing policies when a sole manager chooses to run its enterprise using the differential pricing strategy.

Proposition 3.1

i. $\frac{\partial \pi_{\varepsilon}^{C}}{\partial w} < 0$ *and* $\frac{\partial \pi_{\varepsilon}^{C}}{\partial r} < 0$

ii. *The profit function (3.16) is strictly and jointly concave in* p_o *and* p_r*, given that* $4\beta^2 IJ \geq \gamma^2 (I+J)^2$*. The system will perform at its best with the online store's optimal price of* p^{C_o} *and the physical store's optimal price of* p^{C_r}*:*

$$p^{C_r} = \frac{(\gamma J\alpha_o - \gamma^2 A_{\varepsilon})(I+J) + \gamma\beta B_{\varepsilon}(I-J\,) + 2J(\beta I\alpha_r + \beta^2 A_{\varepsilon})}{4JI\beta^2 - \gamma^2 (I+J)^2}. \tag{3.17}$$

$$p^{C_o} = \frac{(\gamma I\alpha_r - \gamma^2 B_{\varepsilon})(I+J) - \gamma\beta A_{\varepsilon}(I-J) + 2I(\beta J\alpha_o + \beta^2 B_{\varepsilon})}{4JI\beta^2 - \gamma^2 (I+J)^2}. \tag{3.18}$$

From (3.17) and (3.18) we get:

$$\frac{\partial p^{C_r}}{\partial \theta} = \frac{\alpha J(2\beta I - \gamma(I+J))}{4JI\beta^2 - \gamma^2(I+J)^2}, \frac{\partial p^{C_o}}{\partial \theta} = -\frac{\alpha I(2\beta J - \gamma(I+J))}{4JI\beta^2 - \gamma^2(I+J)^2}, \left|\frac{\partial p^{C_r}}{\partial \theta}\right| - \left|\frac{\partial p^{C_o}}{\partial \theta}\right| = \frac{\alpha\gamma(I+J)(I-J)}{4JI\beta^2 - \gamma^2(I+J)^2}$$

If $p^{C_r} = p^{C_o}$*, then*

$$\bar{\theta} = \frac{\alpha J(2\beta J - \gamma(I+J)) - \gamma\beta(I-J)(A_{\varepsilon} + B_{\varepsilon}) + 2\beta^2(IB_{\varepsilon} - JA_{\varepsilon}) + \gamma^2(I+J)(A_{\varepsilon} - B_{\varepsilon})}{\alpha(4\beta IJ - \gamma(I+J)^2)}. \tag{3.19}$$

The proofs for Proposition 3.1 and all other propositions can be found in the appendix. The first part of the proposition indicates that returns impose difficulty and loss on the system. The assumption that the same-channel return can infinitely generate salvage value with each sale does not impose the superiority of a system with returns over a system without returns. Since this is true for the worst-case scenario $\varepsilon = \infty$, then it will be true for the general case. Thus, the proof for the later is omitted.

The condition stated in Proposition 3.1 may not apply if γ is very close to β and the total return rate of a channel is much higher than what it is for the other channel. Those cases are less likely to occur since γ is expected to be much less than β. Also, a channel with excessive total return rate will, most likely, be eliminated or its return policy will, at least, be changed.

The optimal price for a certain channel will not always increase as customers' preference for that channel increases. It could instead increase or decrease depending on the signs $2\beta I - \gamma(I+J)$ and $2\beta J - \gamma(I+J)$ for the physical store and the online store, respectively. That is, managers should not assume that higher customer preference for a certain channel drives prices in that channel up; they must first consider customer returns. Compare the above result with the fact that a higher base level of demand in a single sale channel leads to a higher selling price. Also, if the physical store has a high level of customer preference (i.e., if there is a higher base level of demand for the physical store), then when customer returns are not considered, the physical channel should have a higher selling price than the online channel (Dan et al. 2012 and Hua et al. 2010).

As θ increases, the online store is found to have a higher corresponding rate of change in its optimal price than the physical store if $w + v < r$, an identical rate if $w + v = r$, and a lower rate if $w + v > r$. If $\theta = \bar{\theta}$ and $0 \le \bar{\theta} \le 1$, then it is optimal for both channels to have a similar pricing strategy. Note that $\bar{\theta}$ will mostly lie out of range if $\frac{\partial p^{C_r}}{\partial \theta}$ and $\frac{\partial p^{C_o}}{\partial \theta}$ are either positive or negative. From the above proposition, one may observe that customer preference for a certain channel has a significant impact on the optimal prices of channels.

3.3.2 Dual-Channel Retailing System under the Unified Pricing Strategy

Webb and Lambe (2007) have stated that pricing strategy causes most of the conflicts that arise between channels. In addition, several authors have stated that one may avoid customer confusion and retain a business's image by using a unified price across all channels (Neslin and Shankar 2009; Webb and Lambe 2007). Consequently, 80% of all multichannel retailers choose to unify their pricing strategies across all channels (Ofek et al. 2009). Thus, this section investigates the effect that customer preference and rates of return have on pricing policies when a sole manager choses to run its enterprise with a unified pricing strategy. Due to the added constraint (i.e., $p_r = p_o = p$), it is trivial that the profit generated by the unified pricing strategy is less than or equal to the profit generated by the differential pricing strategy.

Proposition 3.2

If $p_r = p_o = p$, the profit function is strictly concave in p. Thus, there is a unique optimal solution of p^{C_u} that derives the maximum system's profit π^{C_u}.

$$p^{C_u} = \frac{1}{2}\left(\frac{(\beta-\gamma)(A_\varepsilon+B_\varepsilon)+\alpha_o J+\alpha_r I}{(\beta-\gamma)(J+I)}\right). \tag{3.20}$$

From (3.20) we get $\frac{\partial p^{C_u}}{\partial \theta} = \frac{\alpha}{2}\left(\frac{I-J}{(\beta-\gamma)(J+I)}\right)$.

The optimal price will increase as θ increases under the condition $r < v + w$, and will decrease as θ increases under the condition $r > v + w$. Intuitively, the change in θ has no effect on the dual-channel retailer's pricing strategy when $r = v + w$. One may notice that the decision to increase or decrease the unified price solely depends on the values r, v, and w. This places an emphasis on customer returns when one selects pricing policies for dual-channel retailing systems.

3.4 Decentralized Dual-Channel Retailing System

According to Zhang et al. (2010), "most retail corporations manage their channels in a decentralized fashion and many of them maintain separate teams of inventory management." Falk et al. (2007) claim that integration may not be optimal if it is associated with a high implementation cost. As previously stated, a failure to centralize or integrate a dual-channel retailer will trigger price and service competition that is normally initiated by cannibalization. Notice that a cross-channel return policy allows online stores to increase both sales and customer satisfaction and allows physical stores to create cross-selling opportunities. Assume that $\hat{c}$ is the amount a physical store pays to an online store for every cross-channel return. If $\hat{c}$ is constructed fairly, then it is of all channels' best interest to accept such a return policy. Thus, there is no contradiction between having a cross-channel return as an accepted practice and the fact that competition takes place between channels.

The performance of the competing channels is studied using a sequential game, namely the Stackelberg game, discussed in Section 3.4.1, and a simultaneous game, namely the Nash game, discussed in Section 3.4.2. Yan et al. (2010), Yan (2010), and Yan (2008) have stated that Target, Nike, and Kohl's are all good candidates for Stackelberg competition. They have also stated that a Stackelberg game always outperform a Nash game. Similarly, Lu and Liu (2013) have argued that a Stackelberg game influences the profitability of channels more effectively than a Nash game. In a competitive environment, each channel forms its own decision in isolation to maximize its individual profit. One may assume that all sales function parameters, return rates, cost parameters, and decision rules are known to both competitors.

Due to decentralization, the profit functions below are constructed in a manner similar to formulas (3.11) to (3.16), with the exception that $\hat{c}$ is included in the formulation.

General Case (i.e. $\boldsymbol{\varepsilon} \in [\mathbf{1}, \infty)$):

$$\pi_{\varepsilon}^{D_o} = D_o\left[(1-w-v)p_o - t - w(d+t) + w(1-k_o)s + v\hat{c} + \frac{s(wk_o)^{\varepsilon+1}-c}{\sum_0^{\varepsilon}(wk_o)^n}\right]. \quad (3.21)$$

$$\pi_{\varepsilon}^{D_r} =$$
$$D_r\left[(1-r)p_r - rd + r(1-k_r)s + \frac{s(rk_r)^{\varepsilon+1}-c}{\sum_0^{\varepsilon}(rk_r)^n}\right] + vD_o\left[\frac{k_{or}\left(c\sum_1^{\varepsilon}(rk_r)^{n-1}+s(rk_r)^{\varepsilon}\right)}{\sum_0^{\varepsilon}(rk_r)^n} + \right.$$
$$\left.(1-k_{or})s - \hat{c} - d\right]. \quad (3.22)$$

Special Case (i.e. $\boldsymbol{\varepsilon = \infty}$):

$$\pi_{\varepsilon=\infty}^{D_o} = D_o[(1-w-v)p_o - t - w(d+t) + w(1-k_o)s + v\hat{c} - (1-wk_o)c]. \quad (3.23)$$

$$\pi_{\varepsilon=\infty}^{D_r} = D_r[(1-r)p_r - rd + r(1-k_r)s - (1-rk_r)c] + vD_o(ck_{or} + (1-$$
$$k_{or})s - \hat{c} - d). \quad (3.24)$$

One may reformulate the above profit functions as the following:

$$\pi_{\varepsilon}^{D_o} = D_o(Jp_o - G_{\varepsilon}) = D_o\left(REV_{\varepsilon}^{D_o}\right). \quad (3.25)$$

$$\pi_{\varepsilon}^{D_r} = D_r(Ip_r - A_{\varepsilon}) + vD_oF_{\varepsilon} = D_r\left(REV_{\varepsilon}^{D_r}\right) + vD_oF_{\varepsilon}. \quad (3.26)$$

Where;

$G_{\varepsilon} = t + w(d+t) - w(1-k_o)s - v\hat{c} - \frac{s(wk_o)^{\varepsilon+1}-c}{\sum_0^{\varepsilon}(wk_o)^n}$,

$F_{\varepsilon} = \frac{k_{or}\left(c\sum_1^{\varepsilon}(rk_r)^{n-1}+s(rk_r)^{\varepsilon}\right)}{\sum_0^{\varepsilon}(rk_r)^n} + (1-k_{or})s - \hat{c} - d$,

$G_{\varepsilon=\infty} = t + w(d+t) - w(1-k_o)s - v\hat{c} + (1-wk_o)c$, and

$F_{\varepsilon=\infty} = ck_{or} + (1-k_{or})s - \hat{c} - d$.

F_{ε} represents the savings or losses the physical store makes by accepting each cross-channel return. One may subject the optimal solution to the following constraints:

$D_o \geq 0, D_r \geq 0, \left(REV_{\varepsilon}^{D_o}\right) \geq 0, \left(REV_{\varepsilon}^{D_r}\right) \geq 0$, and $Q_{\varepsilon}^{r} \geq 0$.

Since each channel aims to maximize it own profit in the competitive setting, the online store may over estimate the value of cross-channel returns $\hat{c}$. In return, the physical store may stop cooperating with the online store. Such a lack of cooperation may create havoc in the system and cause unnecessary practices such as returning all cross-channel returns back to the online store at its own expense. Therefore, the following condition on the value of $\hat{c}$ should be satisfied:

General Case (i.e. $\boldsymbol{\varepsilon \in [1, \infty)}$):

$$\hat{c} \leq \frac{k_{or}\left(c\sum_1^{\varepsilon}(rk_r)^{n-1}+s(rk_r)^{\varepsilon}\right)}{\sum_0^{\varepsilon}(rk_r)^n} + (1-k_{or})s - d. \quad (3.27)$$

To better comprehend, assume that $\varepsilon = 1$. Thus, relationship (3.27) becomes:

$$\hat{c} \leq \frac{k_{or}c}{1+rk_r} + \frac{k_{or}srk_r}{1+rk_r} + (1-k_{or})s - d. \tag{3.28}$$

The right-hand side of the above relationship represents how a physical store should consider a cross-channel return. The first term denotes the physical store's valuation of a resalable cross-channel return. Since an item purchased by the physical store at the beginning of the selling season can satisfy $(1 + rk_r)$ sales, it is worth a value of c. In contradistinction, since a resalable cross-channel return can only satisfy one sale it is worth a value of $\frac{c}{1+rk_r}$. The second term calculates the increase in salvaged resalable returns at the end of the selling season caused by each resalable cross-channel return. The third term denotes the physical store's gain, due to salvaging, from a non-resalable cross-channel return. The fourth term denotes the physical store's loss, due to the collection cost, from each cross-channel return.

Special Case (i.e. $\boldsymbol{\varepsilon} = \infty$):

$$\hat{c} \leq ck_{or} + (1-k_{or})s - d. \tag{3.29}$$

Due to the assumption that an item owned by a physical store is infinitely sold until it is permanently sold or returned but not resalable, both an item purchased by the physical store at the beginning of the selling season and a resalable cross-channel return can satisfy $\frac{1}{1-rk_r}$ sales. Thus, they are both worth a value of c.

3.4.1 Dual-Channel Retailing System under the Stackelberg Game

In contrast to the physical store, forming a customer base for the online store is not limited to the store's neighbourhood. Also, due to the advancement in cellular phones and IT, customers of a dual-channel retailer may always check the prices of an online store before they conduct their purchases from a physical store. Additionally, online stores are normally considered to be the distribution centers of enterprises. Therefore, they can start the selling season before their competitors. For the aforementioned facts, the online store is considered to have more price influence on customers compared to the physical store. Thus, a retailer's online store will lead and its physical store will follow. In this game, the physical store optimizes its performance based on the online store's optimal price. The online store optimizes its performance based on the physical store's best response function.

Proposition 3.3.

i. $\frac{\partial \pi_{\varepsilon}^{D_o}}{\partial w} < 0$ *and* $\frac{\partial \pi_{\varepsilon}^{D_r}}{\partial r} < 0$

ii. *Given the online store's optimal price, the physical store's profit function (3.26) is strictly concave in* p_r. *Given the physical store's optimal price function, the online store's profit function (3.25) is strictly concave in* p_o. *Thus, the physical store's maximum profit of* π^{S_r} *and the online store's maximum profit of* π^{S_o} *are generated by selecting the unique physical store's optimal price of* p^{S_r} *and online store's optimal price of* p^{S_o}.

$$p^{S_r}(p^{S_o}) = \frac{1}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_\varepsilon}{I} + \frac{F_\varepsilon v\gamma}{\beta I} + \frac{\gamma}{\beta} p^{S_o}\right). \tag{3.30}$$

$$p^{S_o} = \frac{G_\varepsilon}{2J} + \frac{\alpha_o \beta}{(2\beta^2-\gamma^2)} + \frac{\alpha_r \gamma}{2(2\beta^2-\gamma^2)} + \frac{A_\varepsilon \beta\gamma}{2I(2\beta^2-\gamma^2)} + \frac{v\gamma^2 F_\varepsilon}{2I(2\beta^2-\gamma^2)}. \tag{3.31}$$

From (3.30) and (3.31), one gets:

$$\frac{\partial p^{S_o}}{\partial \theta} = -\frac{\alpha}{2}\left(\frac{2\beta-\gamma}{2\beta^2-\gamma^2}\right) < 0, \frac{\partial p^{S_r}}{\partial \theta} = \frac{\alpha}{4\beta}\left(\frac{4\beta^2-2\beta\gamma-\gamma^2}{2\beta^2-\gamma^2}\right) > 0, \left|\frac{\partial p^{S_o}}{\partial \theta}\right| - \left|\frac{\partial p^{S_r}}{\partial \theta}\right| = \frac{\alpha\gamma^2}{4\beta(2\beta^2-\gamma^2)} > 0,$$

$$\frac{\partial p^{S_r}(p^{S_o})}{\partial p^{S_o}} = \frac{\gamma}{2\beta} > 0.$$

If $p^{S_o} = p^{S_r}$, *then*

$$\breve{\theta} = \frac{2\beta(2\beta^2-\gamma^2)}{\alpha(8\beta^2-\gamma^2-4\beta\gamma)}\left(\frac{G_\varepsilon}{J} - \frac{A_\varepsilon}{I} - \frac{G_\varepsilon\gamma}{2J\beta} + \frac{4I\beta\alpha+2A_\varepsilon\beta\gamma-2I\alpha\gamma-A_\varepsilon\gamma^2+2v\gamma^2F_\varepsilon}{2I(2\beta^2-\gamma^2)} - \frac{v\gamma^3F_\varepsilon}{2\beta I(2\beta^2-\gamma^2)} - \frac{F_\varepsilon v\gamma}{\beta I}\right). \tag{3.32}$$

The above relationships indicate that a physical store's optimal price will increase as θ increases, while an online store's optimal price will decrease as θ increases. The follower's (physical store's) pricing strategy is always less affected by the change in θ than the leader's (online store's) pricing strategy. If the online store's best response of p^{S_o} increases by a single unit, then the physical store's best response of p^{S_r} will increase by half a unit at the most. Dan et al. (2012) came to a similar conclusion for a dual-channel system without customer returns. This in fact shows how much control the leader has over the follower, especially when customer returns are allowed. Similar to Dan et al. (2012), Chen et al. (2012), and Hua et al. (2010) a $\breve{\theta}$ exists, such that if $\theta = \breve{\theta}$, then the pricing strategies in both channels are similar. If customer preference for the physical store is lower than the threshold ($\theta < \breve{\theta}$), then the selling price in the physical store is lower than the selling price in the online store ($p^{S_r} < p^{S_o}$). If customer preference for the physical store is higher than the threshold ($\theta > \breve{\theta}$), then the selling price in the physical store is higher than the selling price in the online store ($p^{S_r} > p^{S_o}$). For example, when remanufactured or used items are offered for sale, customers are most likely eager to verify the quality of the offered items before completing a purchase. Thus higher prices should be offered in the physical store. While changing the θ value may impose a different outcome on the decentralized setting than on the centralized setting, in both cases it significantly impacts pricing decisions.

3.4.2 Dual-Channel Retailing System under the Nash Game

In a dual-channel Nash game, online and physical stores are equally powerful. The market has no price leader. Thus, prices are selected simultaneously in both channels. In this game, each store optimizes its performance given the rival's price.

Proposition 3.4

A unique Nash equilibrium exists under the physical store's price, p^{N_r}, and the online store's price, p^{N_o}. Under equilibrium, the physical store generates a profit of π^{N_r} and the online store generates a profit of π^{N_o}.

$$p^{N_r} = \left(\frac{1}{4\beta^2-\gamma^2}\right)\left(2\beta\alpha_r + \gamma\alpha_o + \frac{4\beta^3 G_\varepsilon}{J\gamma} + \frac{2\beta^2 A_\varepsilon}{I} + \frac{2v\beta\gamma F_\varepsilon}{I}\right) - \frac{\beta G_\varepsilon}{J\gamma}. \quad (3.33)$$

$$p^{N_o} = \left(\frac{1}{4\beta^2-\gamma^2}\right)\left(2\beta\alpha_o + \gamma\alpha_r + \frac{2\beta^2 G_\varepsilon}{J} + \frac{\beta\gamma A_\varepsilon}{I} + \frac{v\gamma^2 F_\varepsilon}{I}\right). \quad (3.34)$$

From (3.33) and (3.34), one gets:

$\frac{\partial p^{N_r}}{\partial\theta} = \left(\frac{1}{4\beta^2-\gamma^2}\right)\left(2\beta\alpha - \gamma(\alpha-1)\right)$. Since $(\alpha-1)\approx\alpha$, then $\frac{\partial p^{N_r}}{\partial\theta} \approx \left(\frac{\alpha}{4\beta^2-\gamma^2}\right)(2\beta-\gamma) > 0$.

$\frac{\partial p^{N_o}}{\partial\theta} = \left(\frac{1}{4\beta^2-\gamma^2}\right)\left(\gamma\alpha - 2\beta(\alpha-1)\right)$. Since $(\alpha-1)\approx\alpha$, then $\frac{\partial p^{N_o}}{\partial\theta} \approx \left(\frac{-\alpha}{4\beta^2-\gamma^2}\right)(2\beta-\gamma) < 0$.

$\left|\frac{\partial p^{N_r}}{\partial\theta}\right| - \left|\frac{\partial p^{N_o}}{\partial\theta}\right| = \left(\frac{2\alpha-1}{4\beta^2-\gamma^2}\right)(2\beta-\gamma) > 0.$

If $p^{N_r} = p^{N_o}$, then

$$\tilde{\theta} = \frac{1}{2\alpha}\left(\alpha + \frac{\beta G_\varepsilon}{J} - \frac{\beta A_\varepsilon}{I} - \frac{v\gamma F_\varepsilon}{I}\right). \quad (3.35)$$

As θ increases, the physical store's optimal price will increase and the online store's optimal price will decrease. Different than the Stackelberg game, the online store's pricing strategy is less affected by the change in θ than the physical store's pricing strategy. Also, there exists a $\tilde{\theta}$, such that if $\theta = \tilde{\theta}$, then the pricing strategies in both stores are similar. If customer preference for the physical store is higher than the threshold ($\theta > \tilde{\theta}$), then the selling price in the physical store is higher than the selling price in the online store ($p^{N_r} > p^{N_o}$). If customer preference for the physical store is lower than the threshold ($\theta < \tilde{\theta}$), then the selling price in the physical store is lower than the selling price in the online store ($p^{N_r} < p^{N_o}$). Due to functions' complexity, it is difficult to carry on a comparison between a channel's price and profitability under the different games. Thus, the comparison is done in the sensitivity analysis.

3.5 Sensitivity Analysis

This numerical study aims to provide several key managerial insights by answering the following questions: Does a unified pricing strategy under centralized management have a higher total profit than competing dual channels? If not, under what conditions is this statement not correct? The latter's answer leads to the following question: Under what competition setting and conditions is the total performance best? How does a channel's pricing strategy compare to different cases? Which case will be the most affected if returns can be resold several times in a selling season? This study uses the following parameters:

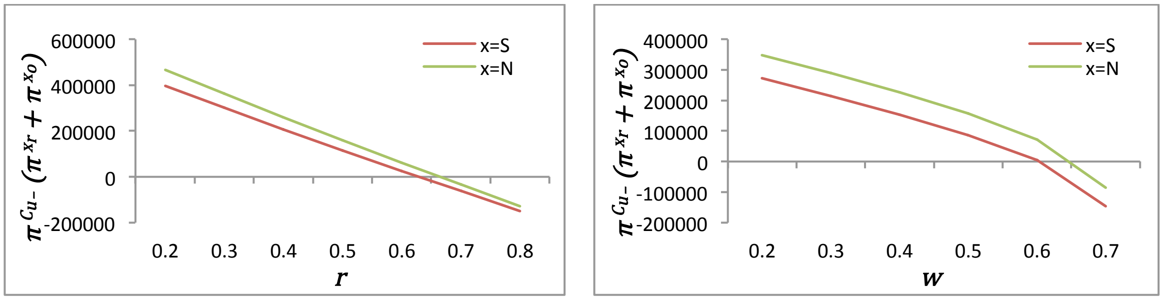
$c = 30,\ s = 10,\ d = 2,\ t = 4,\ r = 0.2,\ w = 0.2,\ v = 0.2,\ k_r = 0.6,\ k_o = 0.4,\ k_{or} = 0.4, \theta = \{0.45, 0.65\}, \gamma = 5, \beta = 10, \alpha = 15k, \varepsilon = \{1, \infty\}$,

$\hat{c}_{\varepsilon=1} = \frac{k_{or}(c+srk_r)}{1+rk_r} + (1-k_{or})s - d$, & $\hat{c}_{\varepsilon=\infty} = ck_{or} + (1-k_{or})s - d$.

The parameter $\varepsilon = 1$ is used throughout this sensitivity analysis except in Section 3.5.4 where the change in retailer's performance is tracked and compared.

3.5.1 Total System Performance under Unified Pricing Strategy and Competition

If the centralization process eliminates conflict by including the unification of selling prices across all channels (Yan 2010), then an enterprise may be better off with uncoordinated channels. As presented in Figure 3.2a, when customer preference for the physical store, θ, and the physical store's rate of return, r, are sufficiently high, competition between channels leads to a better retailer's total performance. Similarly, when customer preference for the online store, $1-\theta$, and the online store's same-channel rate of return, w, are sufficiently high, an enterprise should encourage competition rather than coordination (Figure 3.2b). Indeed, embracing a sole price will reduce channel conflict but deprive the system of agility. That is, it is difficult for an enterprise to divert sales from a high return-rate channel to a low return-rate channel. It should be noted that centralization with a differential pricing strategy has not been considered in this section. Similar to the findings of Yan (2008, 2010) and of Yan et al. (2010), such a setting will lead to the best system performance for all applicable parameters, especially when coordination cost is not considered.

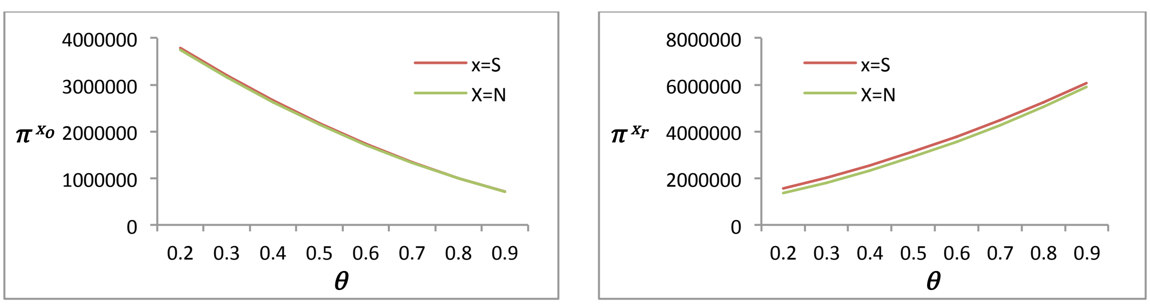


(a) high θ (b) low θ

Figure 3.2: Total profit comparison between centralization with unified pricing strategy and decentralization

3.5.2 Channel's Performance and Pricing Strategy under Competition Schemes

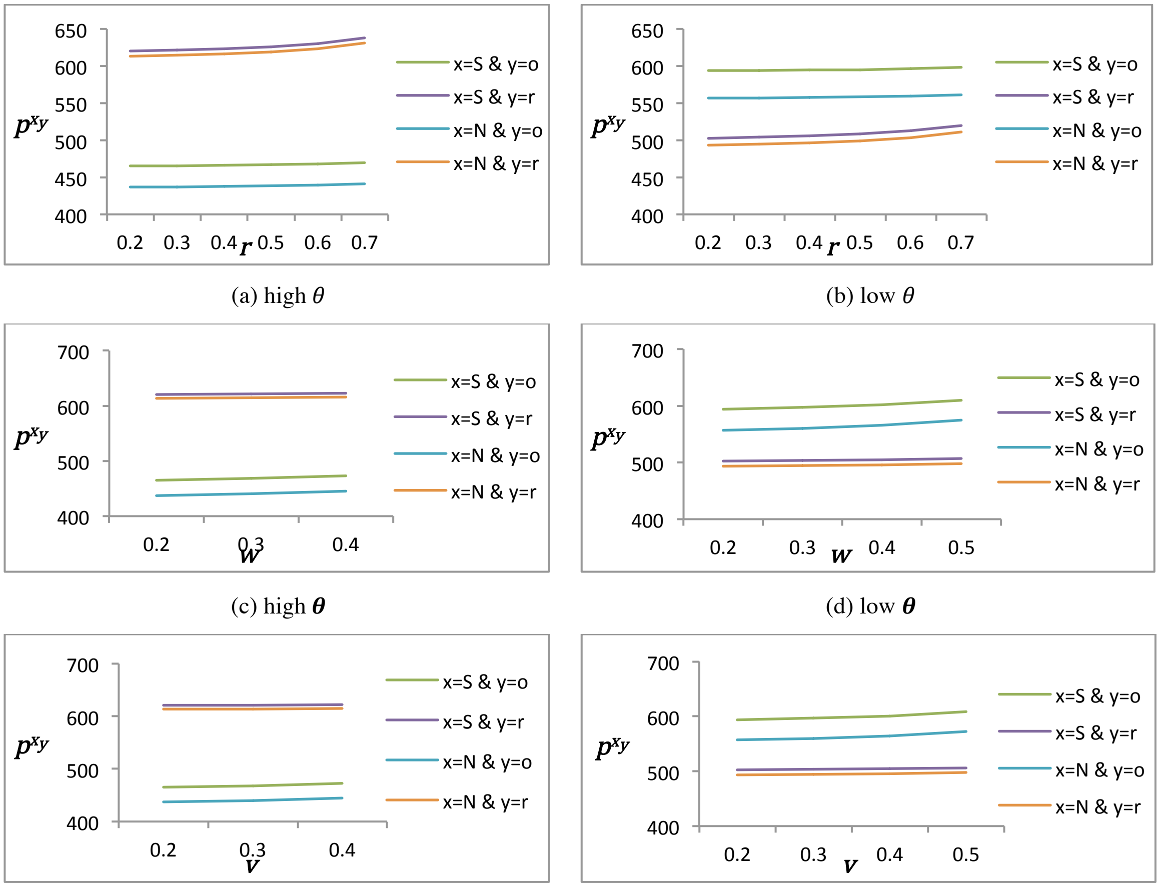
This section will compare between the Stackelberg game and the Nash game. Similar to the findings of Yan et al. (2010), Yan (2010), and Yan (2008), the Stackelberg competition has better channels' profits and thus system performance than the Nash competition (Figure 3.3). It also induces higher equilibrium prices compared to Nash competition (Figure 3.4). This finding is intuitive, since Stackelberg game imposes a higher coordination level between channels. Additionally, both channels are equally powerful in the Nash game. This implies that there will be increased price competition. This provides an explanation for why the channels prices in a Nash game are lower than the channels prices in a Stackelberg game. Thus, enterprises should consider employing the Stackelberg scheme rather than Nash scheme to set channel prices in a competitive market. However, this statement can't be generalized on all possible parameters sets, as it is not analytically proven.



(a) (b)

Figure 3.3: Comparison of a channel's profit under different competition schemes

Under the competition schemes, one may notice that the impact a return rate has on stores' prices is not profound. A channel will not drastically increase its selling price as it experiences a higher return rate. Intuitively, such an increase will cause a channel to lose sales in favour of the competing channel (Figure 3.4).



(a) high θ (b) low θ

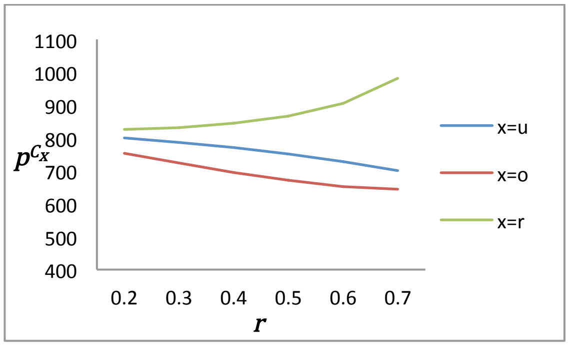
(c) high $\boldsymbol{\theta}$ (d) low $\boldsymbol{\theta}$

(e) high $\boldsymbol{\theta}$ (f) low $\boldsymbol{\theta}$

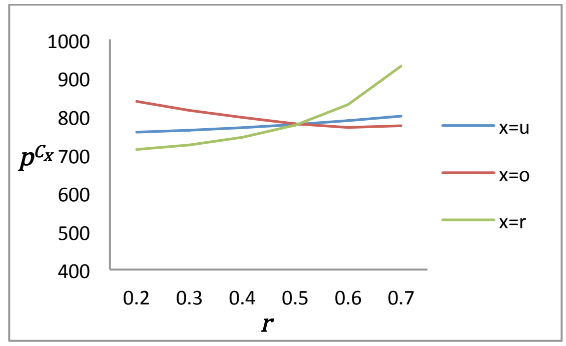
Figure 3.4: Return rates' effect on physical and online store pricing strategies in competition schemes

3.5.3 Pricing Strategies under Centralized Management

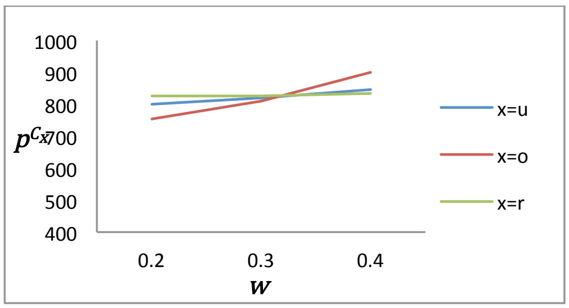
A comparison of Figures 3.4 and 3.5 shows that channels have higher selling prices when coordinating rather than when decentralizing. Since the prices under the two settings are not equal, it can be stated that decisions in a decentralized setting deviate from the overall system's perspective. Indeed, coordination eliminates price competition, providing a chance for both channels to increase prices. Similarly, Yan et al. (2010) have indicated that differential prices set by a sole manager are higher than those set by competing channels. In contrast, Ryan et al. (2013) have indicated that coordination increases total retailer's profit, but at the same time decreases the prices of both channels.



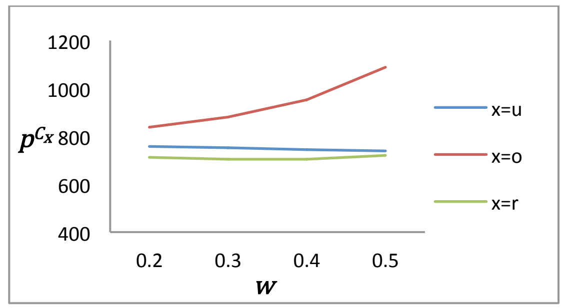
(a) high θ



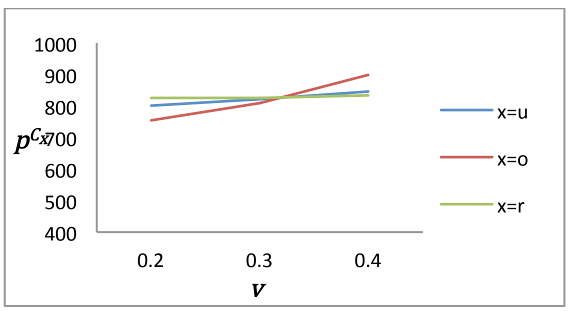
(b) low θ



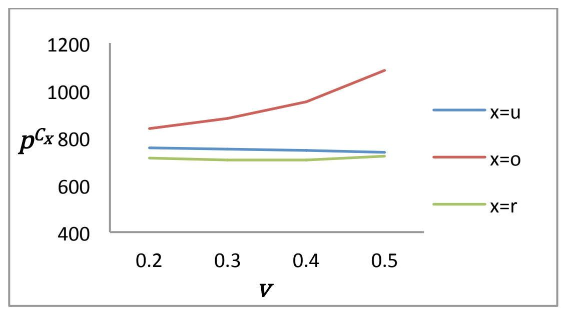
(c) high $\boldsymbol{\theta}$



(d) low $\boldsymbol{\theta}$



(e) high $\boldsymbol{\theta}$



(f) low $\boldsymbol{\theta}$

Figure 3.5: Return rates' effect on physical and online pricing strategies under centralization schemes

As r increases within the differential case, the physical store's price increases in an attempt to decrease the negative effect of return. Consequently, the physical store's sales will decline. The online store should decrease the selling price to attract more customers and to shift part of the lost sales from the physical store to the online store (Figure 3.5 a and b). Under an extremely high r, the physical store can be used as a show room and most purchases can be directed to the online store. When w or v increase, channel prices are set such that sales shifts from the online store to the physical store (Figure 3.5 c to f). If w and/or v are extremely high, then the online store can serve as an information channel and the physical store can serve as a transaction channel (Neslin and Shankar 2009; and Steinfield 2004). Both channels operate in coordination to fulfill organizational-level goals rather than channel-level goals. Indeed, the compensation system in the centralized case should not depend upon the channel's profitability. It should instead depend upon the degree of coordination and the total supply chain's profitability.

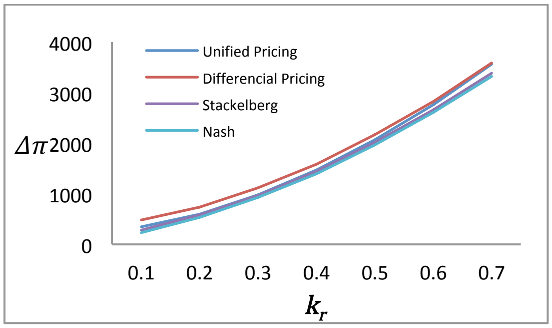
Compare the above pricing strategy to that of both Stackelberg and Nash games wherein a store has no intention of losing customers in favour of the competing store (Figure 3.4). For example, as r increases, a slight increase in the retail price is implemented. The online store will consider this increase in the rival's price as an opportunity to increase the channel's price and to generate more profit. The findings in this section support Baal's (2014) hypothesis that "the higher the degree of harmonization, the greater the degree of cannibalization."

In the case of unification, it is difficult to mitigate the customer returns problem by shifting sales from one channel to another due to the pricing policy used. It has been found that the unified price should decrease if the rate of return for the channel with high customer preference increases. In contradistinction, if the rate of return for the channel with low customer preference increases, the unified price will increase (Figure 3.5).

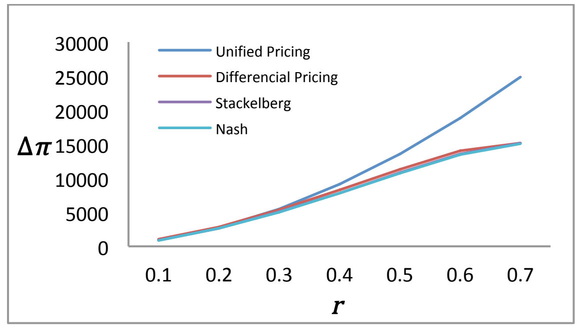
3.5.4 Retailer's Performance under Single Resalability and Infinite Resalability

When returns can be resold several times during a selling season, the quantity that is needed is reduced while profit is increased. Thus, this section measures the increase in total profitability for the different cases when infinite resalability is applicable. If the change is high, then it is probably worth altering the return policy (e.g., by reducing the return time limit) or investing in the reverse supply chain (e.g., investing in collection, shipping, fixing, and/or repackaging processes).

As the resalability rate (k_r) increases, the profitability of the system increases. One may notice that all cases have experienced almost the same changes to $\Delta\pi$ (Figure 3.6 a). The system will behave in a similar way when v, w, k_o or k_{or} change. However, the unified pricing strategy experiences the highest change in profitability when the rate of return, r, increases. With a high r, $\Delta\pi$ scored \$25,000 in the unified pricing case and \$15,000 in all other cases (Figure 3.6 b). Thus, there is a need to shorten the length of time that a return stays off shelves, especially when an enterprise uses the unified pricing strategy.



(a) high θ



(b) high θ

Figure 3.6: Increase in profitability for different cases when resalable returns can be resold infinitely

3.6 Managerial Insights

This chapter has several implications in regards to the pricing strategies of a dual-channel retailer wherein both same- and cross-channel returns have been considered. It has examined several insights related to the centralization of a dual-channel retailer under unified pricing or differential pricing schemes. It has also examined insights related to the decentralization of a dual-channel retailer under Stackelberg or Nash games.

It has been found that when customer preference for the physical channel is higher than a threshold value, then the retailer's set price should be higher in the physical channel than in the online channel. The threshold is defined as $\breve{\theta}$ and $\tilde{\theta}$ in the Stackelberg, and Nash games, respectively. Such a situation may occur when products have been custom designed or when remanufactured or used items have been offered for sale. Consequently, customers are more likely to verify the design or quality of the offered items before completing their purchase. However, when customer' preference for the physical channel is lower than the threshold value, the retailer's set price should be lower in the physical channel than in the online channel. Such a situation may occur when the products that have been offered for sale do not require a high level of examination in regards to their design or quality level before being purchased. For example, products that are standardized or mature (such as books and CDs) better fit this category. In a centralized situation, the threshold value is defined as $\bar{\theta}$. Under valid, but unusual parameters, $\bar{\theta}$ could be higher than one, which means that the online store should always be priced higher than the physical store. Additionally, $\bar{\theta}$ could be less than zero, which means that the online store should always be priced lower than the physical store.

Centralization with a differential pricing scheme may cause a retailer to significantly shift sales from one channel to another, leaving the first channel with virtually no customers. Such a management style imposes hardship on the retailer when it comes to tailoring a compensation program that is fair for both channels and dependent on coordination level rather than on sales. As a result, many retailers centralize decision-making under a unified pricing strategy. Thus, retailers should be aware of several important issues related to the price unification process. This strategy, compared to all strategies studied in Chapter 3, has the highest positive correlation between profit and number of times an item can be resold in a single selling season, especially when the online store has a low customer preference and the physical store has a high same-channel return rate. In this situation, the retailer may adopt a more stringent return policy when it comes to the trial period, or may consider increasing the capability of the reverse supply chain by investing more in the collection, shipping, fixing, and/or repackaging processes. Additionally, when under a centralized dual-channel retailer and a unified price strategy, a retailer's profit is not always higher than when under a decentralized dual-channel retailer. Thus, the retailer should be careful in regards to encouraging or discouraging competition between channels. For example, if a channel experiences a sufficiently high customer preference and a same-channel rate of return, then it is better for the retailer to encourage competition rather than coordination. This could occur in the apparel industry, for example, wherein customers are increasingly inclined to use online stores despite having up to a 75% chance that they will return their purchases due to size or material mismatches.

Prices in the Nash game when neither channel is a price leader, are lower than in the Stackelberg game, when the online channel is the price leader. Our study shows that both channels are worse off in the Nash game. This can be attributed to the fact that higher competition induces lower prices. Consequently, one may argue that Nash game leads to a lower total retailer's profitability. These results indicate that game schemes have a significant impact on retailer payoffs, and that the schemes have a substantial influence on sales and customer welfare.

3.7 Conclusion

Chapter 3 has studied the effect that same- and cross-channel customer returns have on a dual-channel retailer wherein an enterprise runs both a physical store and an online store. The results confirm that accounting for both types of returns is very important when calculating channels' optimal prices. Closed form formulas were assigned to the optimal unified price and the differential prices set by centralized management. The optimal prices for the competing channels were derived using the Stackelberg game and the Nash game.

It has been found that customer preference for a certain channel greatly affects the pricing strategy of that channel. For example, the optimal price for a channel facing competition will increase as customer preference for that channel also increases. Unlike common perception, the optimal price for a channel under centralized management and a differential pricing strategy could either increase or decrease when customer preference for that channel increases. The previous setting may not possess a customer preference value that causes both channels to optimally be priced equally. When the physical store's rate of return is less than all online store's rates of returns, then the optimal unified price will increase as customer preference for the physical store increases.

From the numerical example it has been observed that the prices set by centralized management are higher than those set by competing channels. When compared to Stackelberg competition, Nash competition imposes lower pricing strategies for both channels and lower total supply chain profitability. When a channel faces high rate of returns and high customer preference, then the retailer should promote competition over coordination with price unification. Having the ability to resell returns several times rather than once in a selling season increases the total profitability of the system. This has been found to be the highest increment when using a unified pricing strategy and when a channel with high customer preference experiences a high rate of returns.

CHAPTER 4: PRICES AND RESPONSIVENESS LEVEL DECISIONS

4.1 Introduction and Motivation

Only a few works addressed in Chapter 2 considered the service level provided by a certain channel and its effect on a system's performance. According to Ramanathan (2011), services provided by a retailer can be categorized into two groups: pre-purchase services and post-purchase services. Hua et al. (2010) have considered a post-purchase service level offered by an online channel. The responsiveness of the forward supply chain was an important decision variable in their study. However, Dan et al. (2012) have studied comprehensive service levels (pre-purchase and post-purchase) offered by a physical store. Due to the nature and quantity of returns, one of the online store's most important post-purchase services is its responsiveness level to customer returns. The level greatly influences customer demands, customer loyalties, service expenses, and return reselability (Ramanathan 2011; Dan et al. 2012).

As stated earlier, there is no work that has collectively considered all common forms of customer returns experienced by a dual-channel retailer. Hence, the impact of resalable cross-channel returns on both stores has not being studied previously. In addition, no published paper that has investigated online store's ability to better handle returns and, thus, optimize returns' resalability rate. Accordingly, this chapter is an extension of Chapter 3. It discusses dual-channel retailers with both return types, and examines optimal pricing policies. Additionally, it studies the optimal responsiveness level for online stores when handling product returns, as responsiveness influences online and offline channel sales and online store resalability rate. While theoretical game frameworks are addressed in regards to competition between channels, unified and differential pricing strategies are addressed in regards to integration.

The study in this chapter makes four important contributions to the existing body of work, as shown below.

- All common forms of customer returns that may arise in a dual-channel retailing system are examined. As stated earlier, purchasing an item from a physical store and returning it to an online store is not a common practice and, thus, it is not considered in this work.

- Various theoretical games are used to investigate competition that may arise between the channels of a dual-channel retailer.
- The ability to resell a same-channel online return is related to the responsiveness level provided by the store.
- A comparative study between the total dual-channel retailer's performance under the different schemes is provided.

4.2 Model Formulations

As stated before, the online store is normally the distribution centre of the enterprise and is not limited to customers in its neighbourhood. Also, the online store experiences a much higher return rate compared to the physical store. Thus, it has the capability and the need to optimize its resalability rate. In this chapter k_o is assumed to be responsiveness level dependent and can be estimated by the linear function $k_o(e) = a + be$. a measures the online store's resalability rate if the store fully utilizes its current reverse supply chain capability or responsiveness level. For example, if the online store fully utilizes the employees, buildings, tools, systems, and contracts from previous selling seasons, then it would have a resalability rate of a. We assume here that customers do not acquire knowledge on products from previous seasons. Otherwise, it is imperative to increase resalability rate as they gain more knowledge. This assumption can be satisfied when dealing with fashionable apparel items wherein customers' return behaviour can be forecasted from earlier periods and items are regularly redesigned. On the other hand, b represents the sensitivity of the online store's resalability rate to the change in responsiveness level. In another word, it measures the increase (decrease) in resalability rate when the store invests in (underutilize) responsiveness level. Two cases are studied: a case $\varepsilon = 1$ where returns are assumed to be resold once and a case $\varepsilon = \infty$ where returns are assumed to be resold infinitely.

To simplify our problem setting, the models proposed in this paper do not consider the salvage value of leftovers, i.e. non-resalable returns or resalable items that were returned after the end of the selling season. A change in responsiveness level (an increase or a decrease) would cause the online store to bear a cost of $c(e) = \eta e^2/2$. The previous function is strictly convex and has the property $\partial^2 c(e)/\partial e^2 > 0$. If $e > 0$, then the online store's management would invest more in the channel's reverse supply chain to increase its responsiveness when handling returns. For example, the management could initiate a quality control department to check and fix returns if possible. This will increase resalability and boost sales in the online store due to the fact that the store will have a reputation not to send damaged items. Additionally, providing pickup services may ease the return process. It will also allow the online store to increase its resalability by providing a safer packaging for returns. Management could also invest in a department that performs data collection and analysis. With such a capability, customers are offered more suitable products for their needs. For example, a customer buys shoes online regularly and returns uncomfortable ones. When data are collected and analysed, better suggestions are given to the customer. Consequently, if a return is to be made by this customer, most likely it is material or fabric mismatch. This will decrease the likelihood that the return is not resalable. However, if $e < 0$, then the online store's management would underutilize the online store's current reverse supply chain capability to decrease its responsiveness level. Such a technique could be used to shift sales from an online store to a physical store when channels are run collectively.

The total overall sales functions (3.1) and (3.2) have been changed to accommodate the effect of responsiveness level as follows:

$$D_r = \alpha_r - \beta p_r + \lambda p_o - \rho e. \tag{4.1}$$

$$D_o = \alpha_o - \beta p_o + \lambda p_r + \rho e. \tag{4.2}$$

ρ is the sensitivity to responsiveness level. It measures the rate at which sales are affected by the responsiveness level set by the online store.

The next two sections investigate the integration of a dual-channel retailer using two pricing strategies: uniform pricing and differential pricing. They also consider the equilibrium decisions of the online and physical stores at times when the stores are undergoing Stackelberg and Nash competitive schemes. In the Stackelberg scheme, the online store leads, declaring its selling price and responsiveness level first and is then followed by the physical store. In the Nash scheme, online and physical stores are commensurate in power. They therefore make their decisions simultaneously. A summary of this chapter's notations is showing in Table 4.1 below.

Notation	Description
$\boldsymbol{r}$	Probability an item purchased from the physical store is returned to the physical store
$\boldsymbol{w}$	Probability an item purchased from the online store is returned to the online store
$\boldsymbol{v}$	Probability an item purchased from the online store is cross-returned to a physical store
$\boldsymbol{k_r}$	Probability an item purchased from and returned to the physical store is resalable
$\boldsymbol{k_o}$	Probability an item purchased from and returned to the online store is resalable

$\boldsymbol{k_{or}}$	Probability an item purchased from the online store and cross-returned to the physical store is resalable
$\boldsymbol{a}$	Online store's current resalability rate
$\boldsymbol{e}$	Online store's responsiveness level
$\boldsymbol{b}$ & $\boldsymbol{\rho}$	Sensitivity of online store's resalability rate and a channel's sales to responsiveness level, respectively
$\boldsymbol{c}$	Unit purchasing cost
$\boldsymbol{d}$ & $\boldsymbol{t}$	Return collection and shipping costs, respectively
$\boldsymbol{\hat{c}}$	Amount a physical store pays to an online store for every cross-channel return in the decentralization scheme
$\boldsymbol{D_r}$ & $\boldsymbol{D_o}$	Retail and online stores' total sales including returns, respectively
$\boldsymbol{Q_\varepsilon^r}$ & $\boldsymbol{Q_\varepsilon^o}$	Quantities ordered by retail and online stores, respectively
$\boldsymbol{\alpha}$, $\boldsymbol{\alpha_r}$ & $\boldsymbol{\alpha_o}$	Enterprise, physical store and online store base levels of sale, respectively
$\boldsymbol{\theta}$	Customer preference for the physical store
$\boldsymbol{\beta}$ & $\boldsymbol{\gamma}$	Ownership price and cross-price sensitivities of a channel, respectively
$\boldsymbol{p_r}$ & $\boldsymbol{p_o}$	Retail and online store's prices, respectively
$\boldsymbol{\pi_\varepsilon^{C_r}}$, $\boldsymbol{\pi_\varepsilon^{C_o}}$ & $\boldsymbol{\pi_\varepsilon^{C}}$	Physical store, online store, and enterprise profits in the centralized case, respectively
$\boldsymbol{\pi_\varepsilon^{D_r}}$ & $\boldsymbol{\pi_\varepsilon^{D_o}}$	Physical store and online store profits in the decentralized case, respectively
$\boldsymbol{\varepsilon}$	Number of times a resalable return can be resold in a selling season

Table 4.1: Fourth Chapter's Notations

4.3 Centralized Dual-Channel Retailing System

This section investigates the optimal decisions of a dual-channel retailer under a centralized management. The physical store and online store are integrated vertically. It may be assumed that a central decision maker is used with an objective to achieve the retailer's total possible profit. He or she decides the price for the physical store, which is p_r, the price for the online store, which is p_o, and the responsiveness level for the online store, which is e, at the same time.

The online store's and physical store's profit functions are modeled respectively as follows:

$$\pi_\varepsilon^{C_o} = D_o\big((1-w-v)p_o - t - w(d+t)\big) - Q_\varepsilon^o c - \tfrac{1}{2}\eta e^2. \tag{4.3}$$

$$\pi_\varepsilon^{C_r} = D_r\big((1-r)p_r - rd\big) - D_o v d - Q_\varepsilon^r c. \tag{4.4}$$

A description of the above profit functions has been omitted due to their similarity to the profit functions presented in Chapter 3. The retailer profit function can be modeled as

$\pi_\varepsilon^C = \pi_\varepsilon^{C_o} + \pi_\varepsilon^{C_r} = D_o(Jp_o - t - w(d+t) - vd) + D_r(Ip_r - rd) - Q_\varepsilon^o c - Q_\varepsilon^r c - \frac{1}{2}\eta e^2.$ (4.5)

Where $I = 1 - r$ and $J = 1 - w - v$. By replacing the quantity Q_ε^o, the quantity Q_ε^r, and the resalability k_o with their functions, the retailer profit function π_ε^C is transformed into the equations below.

Selling Resalable Returns Once ($\boldsymbol{\varepsilon = 1}$)

Based on formulas (3.4) and (3.6) when $\varepsilon = 1$, one has

$$\pi_{\varepsilon=1}^C = D_o\left(Jp_o - B_{\varepsilon=1} - \frac{c}{1+wa+wbe}\right) + D_r(Ip_r - A_{\varepsilon=1}) - \frac{1}{2}\eta e^2. \quad (4.6)$$

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

With formulas (3.8) and (3.10), one obtains

$$\pi_{\varepsilon=\infty}^C = D_o(Jp_o - B_{\varepsilon=\infty} + wbec) + D_r(Ip_r - A_{\varepsilon=\infty}) - \frac{1}{2}\eta e^2. \quad (4.7)$$

Where

$B_{\varepsilon=1} = t + w(d+t) + vd - \frac{cvk_{or}}{1+rk_r}$, $A_{\varepsilon=1} = rd + \frac{c}{1+rk_r}$,

$B_{\varepsilon=\infty} = t + w(d+t-ac) + vd - cvk_{or} + c$, and $A_{\varepsilon=\infty} = rd + c(1 - rk_r)$.

One may reformulate (4.6) and (4.7) as

$$\pi_\varepsilon^C = D_o\left(REV_\varepsilon^{C_o}\right) + D_r\left(REV_\varepsilon^{C_r}\right) - \frac{1}{2}\eta e^2. \quad (4.8)$$

Notice that $REV_\varepsilon^{C_o}$ and $REV_\varepsilon^{C_r}$ is the revenue generated by satisfying a single sale from the online store and physical store, respectively. Thus, the optimal solution is subjected to the constraints

$D_o \geq 0, D_r \geq 0, \left(REV_\varepsilon^{C_o}\right) \geq 0, \left(REV_\varepsilon^{C_r}\right) \geq 0, Q_\varepsilon^r \geq 0$, and $-a/b \leq e \leq (1-a)/b$.

Section 4.3.1 presents an analysis of a situation wherein the central decision maker chooses a differential pricing strategy or does not constraint prices to be equal. Section 4.3.2 examines a scenario in which a central decision maker uses a unified pricing strategy or sets equal prices in both stores.

4.3.1 Dual-Channel Retailing System under the Differential Pricing Strategy

This section of the work examines a situation in which a centralized management decides to operate its business while utilizing a pricing approach that is differential.

Proposition 4.1

The profit function (4.7) is strictly and jointly concave in p_o, p_r, and e, given that $4IJ\beta^2 - \lambda^2(J+I)^2 > 0$ and $\left|H_{\varepsilon=\infty}^{C_d}\right| < 0$. Similarly, the profit function (4.6) is strictly and jointly concave in p_o, p_r, and e within the feasible region, given that $4IJ\beta^2 - \lambda^2(J+I)^2 > 0$ and $\left|H_{\varepsilon=1}^{C_d}\right| < 0$.

The conditions in this proposition guarantee the existence of a unique equilibrium point. If those conditions were not satisfied, then the profit functions (4.6) and (4.7) could have several equilibrium points. This would make the prediction of the system's behaviour difficult in practice. Throughout this work one or two conditions have been set in a similar fashion to guarantee the existence of a sole equilibrium point. As stated in Chapter 3, the condition $4IJ\beta^2 - \lambda^2(J+I)^2 > 0$ may not be satisfied if cross-price sensitivity is very close to ownership price sensitivity, and if one channel experiences much higher return rates than the other.

The online store's optimal price ($p_\varepsilon^{C_o}$) and responsiveness level ($e_\varepsilon^{C_d}$) and the physical store's optimal price ($p_\varepsilon^{C_r}$) are then found for the cases $\varepsilon = \infty$ and $\varepsilon = 1$, respectively.

Selling Resalable Returns Infinitely ($\varepsilon = \infty$)

If one sets $\frac{\partial \pi_{\varepsilon=\infty}^{C}}{\partial p_o} = 0$, $\frac{\partial \pi_{\varepsilon=\infty}^{C}}{\partial p_r} = 0$, and $\frac{\partial \pi_{\varepsilon=\infty}^{C}}{\partial e} = 0$ and then solves the first-order conditions simultaneously, one gets

$$e_{\varepsilon=\infty}^{C_d} = \frac{\left\{\begin{matrix}\left(4\beta^2IJ-\lambda^2(J+I)^2\right)(wbc\alpha_o+\rho A_{\varepsilon=\infty}-\rho B_{\varepsilon=\infty})+(J\rho-\beta wbc)\left(2I\beta(J\alpha_o+\beta B_{\varepsilon=\infty}-\lambda A_{\varepsilon=\infty})+\lambda(J+I)(I\alpha_r+\beta A_{\varepsilon=\infty}-\lambda B_{\varepsilon=\infty})\right)\\ -(\rho I-\lambda wbc)\left(2J\beta(I\alpha_r+\beta A_{\varepsilon=\infty}-\lambda B_{\varepsilon=\infty})+\lambda(J+I)(J\alpha_o+\beta B_{\varepsilon=\infty}-\lambda A_{\varepsilon=\infty})\right)\end{matrix}\right\}}{\left\{\begin{matrix}(4\beta^2IJ-\lambda^2(J+I)^2)(\eta-2\rho wbc)-(J\rho-\beta wbc)\left(I\rho\left(2\beta J-\lambda(J+I)\right)-wbc\left(2I\beta^2-\lambda^2(J+I)\right)\right)\\ -(\rho I-\lambda wbc)\left(J\rho\left(2\beta I-\lambda(J+I)\right)+\beta\lambda wbc(I-J)\right)\end{matrix}\right\}}, \tag{4.9}$$

$$p_{\varepsilon=\infty}^{C_o} = \frac{2I\beta(J\alpha_o+\beta B_{\varepsilon=\infty}-\lambda A_{\varepsilon=\infty})+\lambda(J+I)(I\alpha_r+\beta A_{\varepsilon=\infty}-\lambda B_{\varepsilon=\infty})+\left(I\rho\left(2\beta J-\lambda(J+I)\right)-wbc\left(2I\beta^2-\lambda^2(J+I)\right)\right)e_{\varepsilon=\infty}^{C_d}}{4\beta^2IJ-\lambda^2(J+I)^2}, \text{ and} \tag{4.10}$$

$$p_{\varepsilon=\infty}^{C_r} = \frac{2J\beta(I\alpha_r+\beta A_{\varepsilon=\infty}-\lambda B_{\varepsilon=\infty})+\lambda(J+I)(J\alpha_o+\beta B_{\varepsilon=\infty}-\lambda A_{\varepsilon=\infty})-\left(J\rho\left(2\beta I-\lambda(J+I)\right)+\beta\lambda wbc(I-J)\right)e_{\varepsilon=\infty}^{C_d}}{4\beta^2IJ-\lambda^2(J+I)^2}. \tag{4.11}$$

Selling Resalable Returns Once ($\boldsymbol{\varepsilon = 1}$)

If one sets $\frac{\partial \pi_{\varepsilon=1}^{C}}{\partial p_o} = 0, \frac{\partial \pi_{\varepsilon=1}^{C}}{\partial p_r} = 0$ and $\frac{\partial \pi_{\varepsilon=1}^{C}}{\partial e} = 0$, then

$$\left(J\rho - \frac{\beta wbc}{(1+aw+bwe)^2}\right)p_o - \left(\rho I - \frac{\lambda wbc}{(1+aw+bwe)^2}\right)p_r + \frac{wb\alpha_o c}{(1+aw+bwe)^2} - \rho\left(B_{\varepsilon=1} - A_{\varepsilon=1} + \frac{(1+aw)c}{(1+aw+bwe)^2}\right) - \eta e = 0. \tag{4.12}$$

$$p_{\varepsilon=1}^{C_o}(e) = \frac{\left(2I\beta^2-\lambda^2(J+I)\right)\left(B_{\varepsilon=1}+\frac{c}{1+aw+bwe}\right)+I(J+I)\lambda(\alpha_r-\rho e)+2IJ\beta(\alpha_o+\rho e)+(J-I)\beta\lambda A_{\varepsilon=1}}{4IJ\beta^2-\lambda^2(J+I)^2}. \tag{4.13}$$

$$p_{\varepsilon=1}^{C_r}(e) = \frac{\lambda\beta(I-J)\left(B_{\varepsilon=1}+\frac{c}{1+aw+bwe}\right)+2IJ\beta(\alpha_r-\rho e)+J(J+I)\lambda(\alpha_o+\rho e)+\left(2J\beta^2-\lambda^2(J+I)\right)A_{\varepsilon=1}}{4IJ\beta^2-\lambda^2(J+I)^2}. \tag{4.14}$$

Substitute (4.14) and (4.13) in (4.12). If the restrictions in Proposition 4.1 are satisfied, then the first-order condition in (4.12) has at most one real root for the responsiveness level e between $-a/b$ and $(1-a)/b$. This root is the optimal responsiveness level $e_{\varepsilon=1}^{C_d}$.

One may substitute the value of $e_{\varepsilon=1}^{C_d}$ in (4.13) and (4.14) to get the online store's optimal price $p_{\varepsilon=1}^{C_o}$ and the physical store's optimal price $p_{\varepsilon=1}^{C_r}$.

4.3.2 Dual-Channel Retailing System under the Unified Pricing Strategy

This section examines a scenario in which a sole manager makes the decision to operate its business using a unified pricing approach. Because of an added constraint (i.e., $p_r = p_o = p$), the unified pricing scheme's generated profit is equal to or less than that of a differential pricing scheme.

Proposition 4.2

The profit functions $\pi_{\varepsilon=\infty}^{C}$ is strictly and jointly concave in p and e, given that $\left|H_{\varepsilon=\infty}^{C_u}\right| > 0$. Similarly, The profit functions $\pi_{\varepsilon=1}^{C}$ is strictly and jointly concave in p and e within the feasible region, given that $\left|H_{\varepsilon=1}^{C_u}\right| > 0$.

Similar to proposition 4.1, one should confirm that η is high enough to satisfy the above condition so that one may have a single equilibrium point rather than multiple points. The retailer's optimal unified price ($p_{\varepsilon}^{C_u}$) and online store's responsiveness level ($e_{\varepsilon}^{C_u}$) are then found for the cases $\varepsilon = \infty$ and $\varepsilon = 1$, respectively.

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

If one sets $\frac{\partial \pi_{\varepsilon=\infty}^{C}}{\partial p} = 0$ and $\frac{\partial \pi_{\varepsilon=\infty}^{C}}{\partial e} = 0$ and then simultaneously solve the first-order conditions, one gets

$$e_{\varepsilon=\infty}^{C_u} = \frac{\left(2(\beta-\lambda)(I+J)\right)(\alpha_o wbc+\rho A_{\varepsilon=\infty}-\rho B_{\varepsilon=\infty})-\left(wb(\beta-\lambda)c+\rho(I-J)\right)\left(I\alpha_r+J\alpha_o+(\beta-\lambda)(B_{\varepsilon=\infty}+A_{\varepsilon=\infty})\right)}{\rho(J-I)-wb(\beta-\lambda)c+\left(2(\beta-\lambda)(I+J)\right)(\eta-2\rho wbc)}. \tag{4.15}$$

$$p_{\varepsilon=\infty}^{C_u} = \frac{I\alpha_r+J\alpha_o+(\beta-\lambda)(B_{\varepsilon=\infty}+A_{\varepsilon=\infty})+(\rho(J-I)-wb(\beta-\lambda)c)e_{\varepsilon=\infty}^{C_u}}{2(\beta-\lambda)(I+J)}. \tag{4.16}$$

Selling Resalable Returns once ($\boldsymbol{\varepsilon = 1}$)

If one sets $\frac{\partial \pi_{\varepsilon=1}^{C}}{\partial p} = 0$ and $\frac{\partial \pi_{\varepsilon=1}^{C}}{\partial e} = 0$, then

$$\left(\frac{1}{2(J+I)}\left(B_{\varepsilon=1} + A_{\varepsilon=1} + \frac{c}{1+aw+bwe} + \frac{J(\alpha_o+\rho e)+I(\alpha_r-\rho e)}{(\beta-\lambda)}\right)\right)\left(\rho(J-I) - \frac{wb(\beta-\lambda)c}{(1+aw+bwe)^2}\right) - \rho\left(B_{\varepsilon=1} - A_{\varepsilon=1} + \frac{c}{1+aw+bwe}\right) + \frac{wb(\alpha_o+\rho e)c}{(1+aw+bwe)^2} - \eta e = 0. \tag{4.17}$$

$$p_{\varepsilon=1}^{C_u}(e) = \frac{1}{2(J+I)}\left(B_{\varepsilon=1} + A_{\varepsilon=1} + \frac{c}{1+aw+bwe} + \frac{J(\alpha_o+\rho e)+I(\alpha_r-\rho e)}{(\beta-\lambda)}\right). \tag{4.18}$$

The first-order condition in (4.17) has at most one real root for the responsiveness level e within the range $e \in [-a/b, (1-a)/b]$. If found, then this is the optimal responsiveness level $e_{\varepsilon=1}^{C_u}$. Substitute the value of $e_{\varepsilon=1}^{C_u}$ in (4.18) to get the dual-channel's optimal unified price $p_{\varepsilon=1}^{C_u}$.

4.4 Decentralized Dual-Channel Retailing System

This section investigates the optimal decisions of a dual-channel retailer under a decentralized management. Thus, each channel makes an isolated decision so that it may achieve its greatest individual profit. A Stackelberg game, discussed in Section 4.4.1, has been used to examine the performance of channels that are sequentially competing. Also, a Nash game, discussed in Section 4.4.2, has been used to examine the performance of channels that are simultaneously competing. Apart from the inclusion of $\hat{c}$, the profit functions below have been constructed in a manner similar to the profit functions in the centralization scheme.

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

$$\pi_{\varepsilon=\infty}^{D_o} = D_o(Jp_o - G_{\varepsilon=\infty} + wbec) - \frac{1}{2}\eta e^2. \tag{4.19}$$

$$\pi_{\varepsilon=\infty}^{D_r} = D_r(Ip_r - A_{\varepsilon=\infty}) + vD_oF_{\varepsilon=\infty}. \tag{4.20}$$

Selling Resalable Returns Once ($\boldsymbol{\varepsilon = 1}$)

$$\pi_{\varepsilon=1}^{D_o} = D_o\left(Jp_o - G_{\varepsilon=1} - \frac{c}{1+wa+wbe}\right) - \frac{1}{2}\eta e^2. \tag{4.21}$$

$$\pi_{\varepsilon=1}^{D_r} = D_r(Ip_r - A_{\varepsilon=1}) + vD_oF_{\varepsilon=1}. \tag{4.22}$$

Where

$G_{\varepsilon=\infty} = t + w(d + t - ac) - v\hat{c} + c, F_{\varepsilon=\infty} = ck_{or} - \hat{c} - d,$

$G_{\varepsilon=1} = t + w(d + t) - v\hat{c},$ *and* $F_{\varepsilon=1} = \frac{ck_{or}}{1+rk_r} - \hat{c} - d.$

The profit functions of the above may be reformulated as

$$\pi_{\varepsilon}^{D_o} = D_o\left(REV_{\varepsilon}^{D_o}\right) - \frac{1}{2}\eta e^2. \tag{4.23}$$

$$\pi_{\varepsilon}^{D_r} = D_r\left(REV_{\varepsilon}^{D_r}\right) + vD_oF_{\varepsilon}. \tag{4.24}$$

F_{ε} represents the savings or losses the physical store makes by accepting each cross-channel return. The optimal solution is to be subjected to the following constraints: $D_o \geq 0, D_r \geq 0, Q_{\varepsilon}^r \geq 0, \left(REV_{\varepsilon}^{D_o}\right) \geq 0, \left(REV_{\varepsilon}^{D_r}\right) \geq 0,$ and $-a/b \leq e \leq (1-a)/b.$

4.4.1 Dual-Channel Retailing System under the Stackelberg Game

Under a Stackelberg game, the online stores have more price influence on customers compared to the physical store. Consequently, online stores lead and physical stores follow. As the leader, an online store will determine the selling price, $p_{\varepsilon}^{S_o}$, and the responsiveness level, e_{ε}^{S}, first. As a follower, a physical store will determine its selling price, $p_{\varepsilon}^{S_r}$, based on the online store's optimal decisions. We analyze first the case wherein a returned item can be resold several time in a selling season (i.e. case $\varepsilon = \infty$), then the case wherein a returned item can be resold only one time in a selling season (i.e. case $\varepsilon = 1$).

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$):

Given p_o and e in (4.20), then the physical store's profit function ($\pi_{\varepsilon=\infty}^{D_r}$) is concave on ($p_r$). This is due to the fact that $\frac{\partial^2 \pi_{\varepsilon=\infty}^{D_r}}{\partial p_r^2} = -2I\beta < 0$. Thus, if we set $\frac{\partial \pi_{\varepsilon=\infty}^{D_r}}{\partial p_r} = 0$, then we get the physical store's optimal price function

$$p_{\varepsilon=\infty}^{S_r}(p_o, e) = \frac{1}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=\infty}}{I} + \frac{v\lambda F_{\varepsilon=\infty}}{\beta I} + \frac{\lambda}{\beta} p_o - \frac{\rho}{\beta} e\right). \tag{4.25}$$

If (4.25) is substituted into (4.19), one get the online store's profit $\pi_{\varepsilon=\infty}^{D_o}$ as a function of p_o and e

$$\pi_{\varepsilon=\infty}^{D_o} = \left(\alpha_o - \left(\frac{2\beta^2-\lambda^2}{2\beta}\right) p_o + \frac{\lambda}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=\infty}}{I} + \frac{v\lambda F_{\varepsilon=\infty}}{\beta I}\right) + \left(\frac{2\beta-\lambda}{2\beta}\right) \rho e\right) (J p_o - G_{\varepsilon=\infty} + wbec) - \frac{1}{2}\eta e^2. \tag{4.26}$$

Proposition 4.3

The above online store's profit function (4.26) is strictly and jointly concave in p_o and e under the condition $|H_{\varepsilon=\infty}^{S}| > 0$.

The condition in proposition 4.3 assures the existence of a unique equilibrium point. If it is satisfied, one can find the online store's optimal price ($p_{\varepsilon=\infty}^{S_o}$) and responsiveness level ($e_{\varepsilon=\infty}^{S}$) by solving the set of equations $\frac{\partial \pi_{\varepsilon=\infty}^{D_o}}{\partial p_o} = 0$ and $\frac{\partial \pi_{\varepsilon=\infty}^{D_o}}{\partial e} = 0$. Thus

$$p_{\varepsilon=\infty}^{S_o} = \frac{G_{\varepsilon=\infty}}{2J} + \frac{\beta\alpha_o}{(2\beta^2-\lambda^2)} + \frac{\lambda\alpha_r}{2(2\beta^2-\lambda^2)} + \frac{\beta\lambda A_{\varepsilon=\infty}}{2I(2\beta^2-\lambda^2)} + \frac{v\lambda^2 F_{\varepsilon=\infty}}{2I(2\beta^2-\lambda^2)} + \left(\frac{(2\beta-\lambda)\rho}{2(2\beta^2-\lambda^2)} - \frac{wbc}{2J}\right) e_{\varepsilon=\infty}^{S}. \tag{4.27}$$

$$e_{\varepsilon=\infty}^{S} = \frac{\left\{\begin{array}{c}\frac{\rho(2\beta-\lambda)}{4\beta(2\beta^2-\lambda^2)}\left[2J\beta\alpha_o + J\lambda\alpha_r + \frac{J\beta\lambda A_{\varepsilon=\infty}}{I} + \frac{Jv\lambda^2 F_{\varepsilon=\infty}}{I} - G_{\varepsilon=\infty}(2\beta^2-\lambda^2)\right] + \\ \frac{wbc}{4}\left[2\alpha_o + \frac{\lambda\alpha_r}{\beta} + \frac{\lambda A}{I} + \frac{v\lambda^2 F_{\varepsilon=\infty}}{\beta I} + \frac{\lambda^2 G_{\varepsilon=\infty}}{J\beta} - \frac{2\beta G_{\varepsilon=\infty}}{J}\right]\end{array}\right\}}{\eta - \frac{J\rho^2(2\beta-\lambda)^2}{4\beta(2\beta^2-\lambda^2)} - \frac{\rho wbc(2\beta-\lambda)}{2\beta} - \frac{w^2 b^2 c^2 (2\beta^2-\lambda^2)}{4\beta J}}. \tag{4.28}$$

Substitute (4.27) and (4.28) into (4.25) to get the physical store's optimal price $p_{\varepsilon=\infty}^{S_r}$.

Selling Resalable Returns once ($\boldsymbol{\varepsilon = 1}$):

Given p_o and e in (4.22), the physical store's profit function ($\pi_{\varepsilon=1}^{D_r}$) is concave on ($p_r$). This is due to the fact that $\frac{\partial^2 \pi_{\varepsilon=1}^{D_r}}{\partial p_r^2} = -2I\beta < 0$. Thus, if one sets $\frac{\partial \pi_{\varepsilon=1}^{D_r}}{\partial p_r} = 0$, one gets the physical store's optimal price function

$$p_{\varepsilon=1}^{S_r}(p_o, e) = \frac{1}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=1}}{I} + \frac{v\lambda F_{\varepsilon=1}}{\beta I} + \frac{\lambda}{\beta} p_o - \frac{\rho}{\beta} e\right). \tag{4.29}$$

(4.29) is substituted into (4.21) to get the online store's profit as a function of p_o and e:

$$\pi_{\varepsilon=1}^{D_o} = \left(\alpha_o - \left(\frac{2\beta^2-\lambda^2}{2\beta}\right)p_o + \frac{\lambda}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=1}}{I} + \frac{v\lambda F_{\varepsilon=1}}{\beta I}\right) + \left(\frac{2\beta-\lambda}{2\beta}\right)\rho e\right)\left(Jp_o - G_{\varepsilon=1} - \frac{c}{1+wa+wbe}\right) - \frac{1}{2}\eta e^2. \tag{4.30}$$

Proposition 4.4.

The aforementioned online store's profit function (4.30) is concave in p_o, but not jointly concave in p_o and e.

Proposition 4.4 indicates that one may not find the optimal values of p_o *and* e by using the first-order conditions. Thus, one can deal with it using the two-stage optimization technique, i.e., we first find the optimal value of $p_{\varepsilon=1}^{S_o}$ for a given e, and then find the optimal responsiveness level $e_{\varepsilon=1}^{S}$ that maximizes $\pi_{\varepsilon=1}^{D_o}$. Set $\frac{\partial \pi_{\varepsilon=1}^{D_o}}{\partial p_o} = 0$ in (4.30) to get the online store's optimal price as a function of e, i.e., $p_{\varepsilon=1}^{S_o}(e)$. Thus,

$$p_{\varepsilon=1}^{S_o}(e) = \frac{G_{\varepsilon=1}}{2J} + \frac{c}{2J(1+wa+wbe)} + \left(\frac{\beta}{2\beta^2-\lambda^2}\right)\left(\alpha_o + \frac{\lambda\alpha_r}{2\beta} + \frac{\lambda A_{\varepsilon=1}}{2I} + \frac{v\lambda^2 F_{\varepsilon=1}}{2\beta I} + \rho e - \frac{\lambda\rho}{2\beta}e\right). \tag{4.31}$$

If (4.31) is substituted into (4.30), then one gets the online store's profit as a function of e.

$$\pi_{\varepsilon=1}^{D_o} = \left(\frac{\alpha_o}{2} - \left(\frac{2\beta^2-\lambda^2}{2\beta}\right)\left(\frac{G_{\varepsilon=1}}{2J} + \frac{c}{2J(1+wa+wbe)}\right) + \frac{\lambda\alpha_r}{4\beta} + \frac{\lambda A_{\varepsilon=1}}{4I} + \frac{v\lambda^2 F_{\varepsilon=1}}{4\beta I} + \frac{\rho e}{2} - \frac{\lambda\rho}{4\beta}e\right)\left(\left(\frac{J\beta}{2\beta^2-\lambda^2}\right)\left(\alpha_o + \frac{\lambda\alpha_r}{2\beta} + \frac{\lambda A_{\varepsilon=1}}{2I} + \frac{v\lambda^2 F_{\varepsilon=1}}{2\beta I} + \rho e - \frac{\lambda\rho}{2\beta}e\right) - \frac{G_{\varepsilon=1}}{2} - \frac{c}{2(1+wa+wbe)}\right) - \frac{1}{2}\eta e^2. \tag{4.32}$$

Now we can start the second stage, i.e., we find the optimal $e_{\varepsilon=1}^{S}$ that maximizes $\pi_{\varepsilon=1}^{D_o}$ in (4.32). We differentiate $\pi_{\varepsilon=1}^{D_o}$ in (4.32) with respect to e, which yields the first-order condition

$$\left(\frac{cwb}{2(1+wa+wbe)^2} + \rho\left(1-\frac{\lambda}{2\beta}\right)\left(\frac{J\beta}{2\beta^2-\lambda^2}\right)\right)\left(\alpha_o + \frac{\lambda\alpha_r}{2\beta} + \frac{\lambda A_{\varepsilon=1}}{2I} + \frac{v\lambda^2 F_{\varepsilon=1}}{2\beta I} + \rho e\left(1-\frac{\lambda}{2\beta}\right) - \left(\frac{2\beta^2-\lambda^2}{J\beta}\right)\left(\frac{G_{\varepsilon=1}}{2} + \frac{c}{2(1+wa+wbe)}\right)\right) - \eta e = 0. \tag{4.33}$$

One may notice that the first-order condition in (4.33) has at most three real roots for the responsiveness level e. Compare the values of $\pi_{\varepsilon=1}^{D_o}$ at all e roots that lie within the range $[-a/b, (1-a)/b]$. The one at which (4.32) is the largest is the optimal responsiveness level $e_{\varepsilon=1}^{S}$. Substitute the value of $e_{\varepsilon=1}^{S}$ in (4.31) to get the online store's optimal price $p_{\varepsilon=1}^{S_o}$. Substitute the values of $e_{\varepsilon=1}^{S}$ and $p_{\varepsilon=1}^{S_o}$ in (4.29) to get the physical store's optimal price $p_{\varepsilon=1}^{S_r}$.

4.4.2 Dual-Channel Retailing System under the Nash Game

Under a dual-channel Nash game, physical and online stores are commensurate in power and the market does not have a leader. As a result, the channels make decisions simultaneously. In this game, the performance of each store is optimized, dependent upon the rival's decisions.

Proposition 4.5

Under infinite resalability (i.e. case $\varepsilon = \infty$), there is a unique Nash equilibrium if the two conditions $\left|\widetilde{H_{\varepsilon=\infty}^{N}}\right| > 0$ and $\left|H_{\varepsilon=\infty}^{N}\right| < 0$ *are satisfied. Similarly, under one resalability (i.e. case $\varepsilon = 1$), there exist a unique Nash equilibrium at the most within the feasible region if the two conditions* $\left|\widetilde{H_{\varepsilon=1}^{N}}\right| > 0$ and $\left|H_{\varepsilon=1}^{N}\right| < 0$ *are satisfied.*

Next, the online store's optimal price ($p_{\varepsilon}^{N_o}$) and responsiveness level (e_{ε}^{N}) are found, as well as the physical store's optimal price ($p_{\varepsilon}^{N_r}$) for the cases $\varepsilon = \infty$ and $\varepsilon = 1$, respectively.

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

One may set $\frac{\partial \pi_{\varepsilon=\infty}^{D_o}}{\partial p_o} = 0$, $\frac{\partial \pi_{\varepsilon=\infty}^{D_o}}{\partial e} = 0$ and $\frac{\partial \pi_{\varepsilon=\infty}^{D_r}}{\partial p_r} = 0$. When one simultaneously solves the three equations, one gets

$$e_{\varepsilon=\infty}^{N} = \frac{IJ\rho\left(2\beta J\alpha_o - G_{\varepsilon=\infty}(2\beta^2-\lambda^2)\right) + J\lambda(I\alpha_r + \beta A_{\varepsilon=\infty} + \lambda v F_{\varepsilon=\infty})(J\rho + \beta wbc) + \beta wbc\left[2IJ\beta\alpha_o - IG_{\varepsilon=\infty}(2\beta^2-\lambda^2)\right]}{IJ\eta(4\beta^2-\lambda^2) - IJ^2\rho^2(2\beta-\lambda) + IJ\lambda\rho wbc(\lambda+\beta) - I\beta w^2b^2c^2(2\beta^2-\lambda^2)}. \tag{4.34}$$

$$p_{\varepsilon=\infty}^{N_o} = \frac{2I\beta(J\alpha_o + \beta G_{\varepsilon=\infty}) + J\lambda(I\alpha_r + \beta A_{\varepsilon=\infty} + \lambda v F_{\varepsilon=\infty}) + I(2\beta(J\rho - \beta wbc) - J\lambda\rho)e_{\varepsilon=\infty}^{N}}{4IJ\beta^2 - IJ\lambda^2}. \tag{4.35}$$

$$p_{\varepsilon=\infty}^{N_r} = \frac{2J\beta(I\alpha_r + \beta A_{\varepsilon=\infty} + \lambda v F_{\varepsilon=\infty}) + I\lambda(J\alpha_o + \beta G_{\varepsilon=\infty}) - I(\lambda\beta wbc + 2J\beta\rho - J\lambda\rho)e_{\varepsilon=\infty}^{N}}{4IJ\beta^2 - IJ\lambda^2}. \tag{4.36}$$

Selling Resalable Returns once ($\boldsymbol{\varepsilon = 1}$)

If one sets $\frac{\partial \pi_{\varepsilon=1}^{D_o}}{\partial p_o} = 0$, $\frac{\partial \pi_{\varepsilon=1}^{D_o}}{\partial e} = 0$ and $\frac{\partial \pi_{\varepsilon=1}^{D_r}}{\partial p_r} = 0$. One gets

$$p_{\varepsilon=1}^{N_o}(e) = \left(\frac{2\beta}{4\beta^2-\lambda^2}\right)\left(\alpha_o + \frac{\lambda A_{\varepsilon=1}}{2I} + \frac{\lambda\alpha_r}{2\beta} + \rho e - \frac{\lambda\rho e}{2\beta} + \frac{v\lambda^2 F_{\varepsilon=1}}{2I\beta} + \frac{\beta G_{\varepsilon=1}}{J} + \frac{c\beta}{J(1+wa+wbe)}\right). \tag{4.37}$$

$$p_{\varepsilon=1}^{N_r}(e) = \frac{A_{\varepsilon=1}}{2I} + \frac{v\lambda F_{\varepsilon=1}}{2I\beta} + \frac{\alpha_r}{2\beta} - \frac{\rho e}{2\beta} + \left(\frac{\lambda}{4\beta^2-\lambda^2}\right)\left(\alpha_o + \frac{\lambda A_{\varepsilon=1}}{2I} + \frac{\lambda\alpha_r}{2\beta} + \rho e - \frac{\lambda\rho e}{2\beta} + \frac{v\lambda^2 F_{\varepsilon=1}}{2I\beta} + \frac{\beta G_{\varepsilon=1}}{J} + \frac{\beta c}{J(1+wa+wbe)}\right). \tag{4.38}$$

$$\lambda\left(\frac{wb\beta + J\rho}{4\beta^2-\lambda^2}\right)\left(\alpha_r + \frac{\beta A_{\varepsilon=1}}{I} + \frac{v\lambda F_{\varepsilon=1}}{I} - \rho e\right) - \left(\frac{wb(2\beta^2-\lambda^2) - 2\beta J\rho}{4\beta^2-\lambda^2}\right)\left(\alpha_o + \rho e + \frac{\beta}{J}\left(G_{\varepsilon=1} + \frac{c}{(1+wa+wbe)}\right)\right) - \rho G_{\varepsilon=1} + \frac{c}{1+wa+wbe}\left(wb\frac{\alpha_o + \rho e}{1+wa+wbe} - \rho\right) - \eta e = 0. \tag{4.39}$$

The first-order condition in (4.39) has at most one real root for the responsiveness level e within the range $e \in [-a/b\,, (1-a)/b]$. If one substitutes the value of $e^{N}_{\varepsilon=1}$ in (4.37) and (4.38), then one gets the online and physical store's optimal prices $p^{N_o}_{\varepsilon=1}$ and $p^{N_r}_{\varepsilon=1}$, respectively.

4.5 Sensitivity Analysis

In this section the relationships between a physical store's price, an online store's price and responsiveness level, and the total system's profit in both centralized and decentralized systems are studied. A comparison is made between responsiveness decisions, pricing strategies, and total system profits in the two settings. All parameters for this section were chosen such that the constraints and the theorems' conditions were satisfied in order to make the models feasible and meaningful. It was found that the customer preference for a certain channel and the different return rates experienced by a dual-channel retailer make a significant impact on responsiveness and pricing decisions.

For this analysis the following parameters $r = \{0.2, 0.4\}$, $w = 0.1$, $v = 0.1$, $c = 30, d = 2$, $t = 4$, $k_r = 0.2, k_{or} = 0.2$, $a = 0.2$, $b = 0.05$, $\alpha = 5k$, $\theta = 0.5, \beta = 10$, $\gamma = \{1,5\}$, $\rho = 1$, and $\eta = 300$ were used. Additionally, a situation wherein returns can be resold several times in a selling season (i.e. $\varepsilon = \infty$) has been used. A firm's physical store will pay the firm's online store the highest amount possible for cross-channel returns (i.e. $\hat{c} = ck_{or} - d = 14$ as per Eq. 3.29 when $s = 0$). Thus, apart from the cross-selling opportunities associated with such a policy or practice, the physical store does not gain any financial advantage (i.e. $F_{\varepsilon=\infty} = 0$).

The impact that customer preference for the physical store imposes on the prices of the dual-channel retailer is shown in Figure 4.1. One may compare the results of the aforementioned figure to all propositions given in Chapter 3. In addition, one may find that the online store's responsiveness level decreases (increases) when customer preference for the physical store increases (decreases) under all studied schemes.

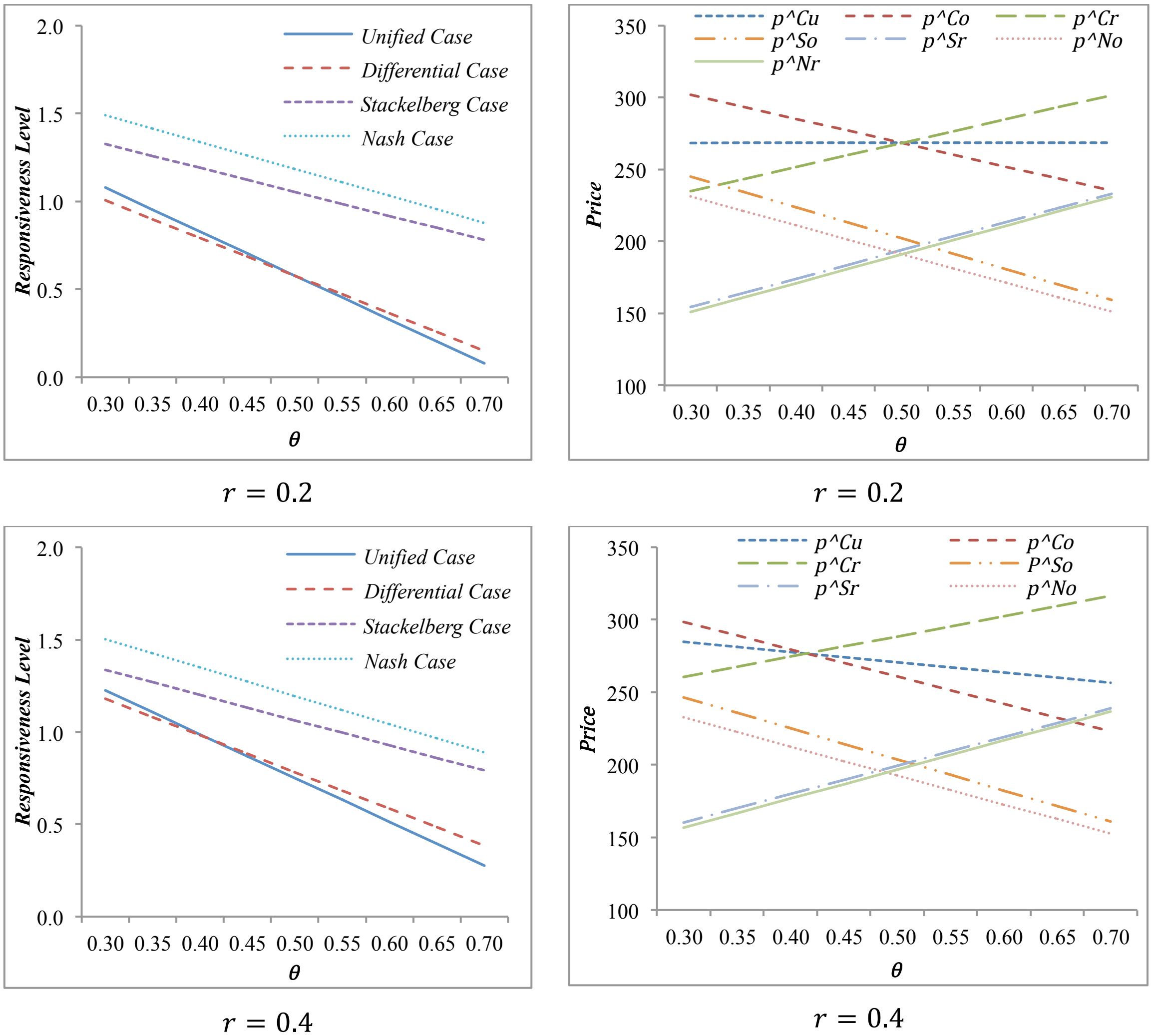
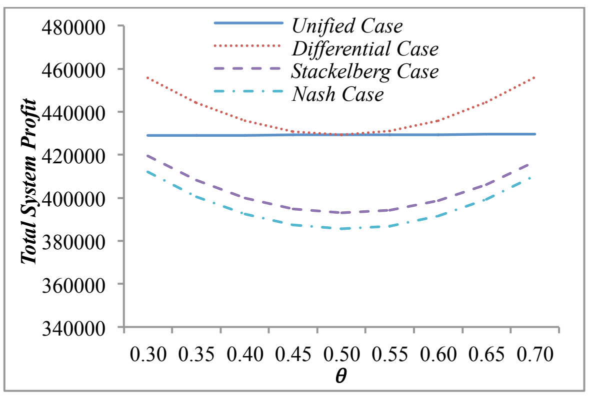


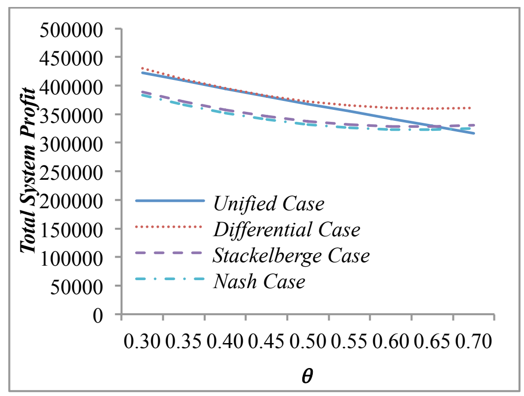
Figure 4.1: Effect that $\boldsymbol{\theta}$ and $\boldsymbol{r}$ have on the equilibrium prices and responsiveness level

To proceed, a reader is referred to Chapter 3 in regard to the customer preference thresholds that cause all channels to be priced equally. To further explore, it should be noted that when under decentralization, this threshold (i.e., $\theta \approx 0.5$, as used in the example) is not noticeably affected by the values of the different return rates experienced by the system. However, under a centralized management with a differential pricing scheme, when $r-(v+w)$ increases (decreases), then this threshold decreases (increases). This occurs as a result of increasing (decreasing) the physical store's prices and decreasing (increasing) the online store's prices under all θ values, in an attempt to switch the sales from the channel experiencing an overwhelmingly high return rates to the channel experiencing lower return rates. It should be noted that increasing (decreasing) the online store's responsiveness level also helps in this switch. This should be compared to decentralized situations wherein competing stores have no intention to switch sales from one channel to another.

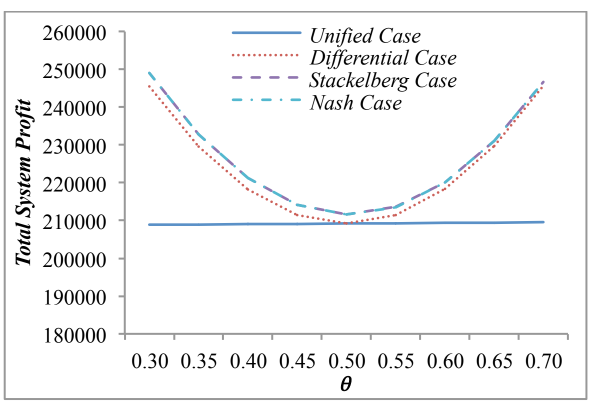
One may also notice that centralized management sets a lower responsiveness level compared to decentralised management. Consequently, customers are offered better return services when their purchases are conducted through the competing channels than when their purchases are conducted through the integrated channels. In addition, it is noticed that the online store offers a higher responsiveness level under the Nash game than under the Stackelberg game. Indeed, the online store's market power is lower under the former scheme. Thus, it can be said that the less dominant a channel is in the market, it should offer a higher responsiveness level so that it may strengthen its market position. It should be noted that this study does not model a competition in the responsiveness level between the active channels.



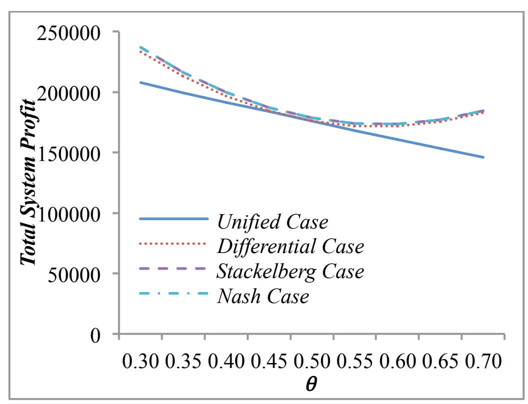
$\lambda = 5$ and $r = 0.2$



$\lambda = 5$ and $r = 0.4$



$\lambda = 1$ and $r = 0.2$



$\lambda = 1$ and $r = 0.4$

Figure 4.2: Effect that $\boldsymbol{\theta}$, $\boldsymbol{r}$ and $\boldsymbol{\lambda}$ have on total system profit

Figure 4.2 shows that when the return rate for purchases conducted through the physical store is equivalent to the return rates for purchases conducted through the online store (i.e., when $r = v + w$), then the total performance of the dual-channel retailer under all schemes except the unification scheme is better when customers has a high preference for one channel over the other. To elucidate, when both channels are equally burdened with customer returns, then the total dual-channel retailer performance is symmetrical along customer preference, and the lowest profits occur around the center value ($\theta \approx 0.5$). It should be noted that customer preference for a certain channel has a minimal effect on the total performance of a system under a unification scheme. This is intuitive since it has also a minimal effect on the pricing strategy of this scheme under the condition $r = v + w$ (Figure 4.1).

However, when the return rate for purchases conducted through the physical store is higher than the return rates for purchases conducted through the online store, then the total performance of the dual-channel retailer, under all schemes, is better when customers prefer the online store over the physical store. Conversely, when the return rate for purchases conducted through the physical store is lower than the return rates for purchases conducted through the online store, then the total performance of the dual-channel retailer is better when customers prefer the physical store over the online store. Generally speaking, the system performs better when customers prefer the channel that causes low return rates over the channel that causes high return rates. Thus, from an enterprise or top-level perspective, certain measures can be taken to positively increase customer preference for the less troublesome channel (e.g., by changing the presentation in the store to create a better shopping experience). Certain technologies can also be used to decrease the return rate of the channel with higher customer preference level (e.g., by using smart phone applications to help shoe shoppers identify their shoe sizes). Additionally, centralized management, as aforementioned, may use the differential pricing strategy and an appropriate responsiveness level to give a low return rate channel more appeal than a high return rate channel.

When λ is low, then the dual channels have a low degree of cannibalization or customers of a certain channel do not respond well to the prices of the competing channel. This could happen when the channels' customers are highly segregated. Consequently, customers would tend to choose another retailer if their preferred channel were to not meet their price expectations. It has been found that in such situations, competition (regardless of the channels' dominance) generates a slightly higher total profit for the system than integration. When the dual channels have a high degree of cannibalization, then a centralized management with a differential pricing strategy will always out perform decentralization. This is also true for a centralized management with a unified pricing scheme that is under low observed return rates. In contradistinction, as r becomes higher ($r \geq 0.4$), all decentralization schemes tend to perform better than a unified pricing strategy when customer preference for the physical store is high. Similarly, when v and/or w are high, then competition is better than centralization with unification when the customer preference for the online store is high. Generally speaking, competition will outperform centralization with a unified pricing scheme when the channel with highest customer preference experiences high return rates.

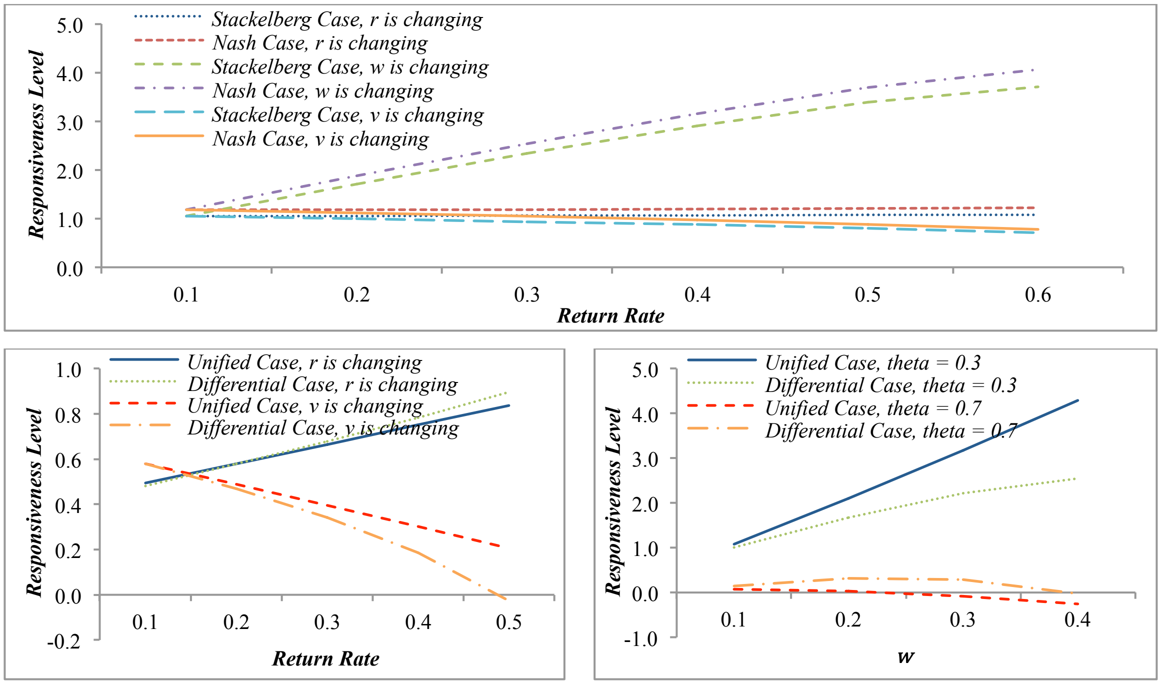


Figure 4.3: Effect that different return rates have on responsiveness level

Using Figure 4.3, the diverse impact that different return rates have on responsiveness level has been explored. In decentralized settings, the optimal responsiveness level stays unchanged when the physical store's return rate (r) increases. This is trivial since such an increase has no effect on the physical store's competing channel (i.e., the online store). In contradistinction, v's increase indicates that online customers become increasingly inclined to return purchases to the physical store. In this situation, fewer customers benefit from the responsiveness provided by the online channel. Thus, the online channel's management reduces responsiveness level as v increases. Intuitively, when w increases, the online channel's management should increase its responsiveness level to enhance both sales and resalability at the same time. It should be noted that the system will behave as indicated above under all values of θ.

In centralized situations, no competition exists between channels. Thus, encouraging customers to conduct purchases from the more profitable channel is a goal the enterprise would work for. When r increases, the system's sole management increases the responsiveness level, encouraging customers to switch to the online store in an effort to decrease the negative effect of returns in the physical store. When v increases, management should decrease this level to avoid unnecessary spending on the online store's responsiveness.

When w increases, the system's sole management faces a question as to whether it should increase the online store's responsiveness level to better handle return and increase sales or decrease its responsiveness level to switch customers to the physical store and decrease the negative effect of return. It has been noted that with the online channel's low customer preference level of $\theta = 0.7$, there is no need to increase its responsiveness level in order to boost resaleability. Thus, in general the responsiveness level has a steady to decreasing trend. In contradistinction, when the customer preference level for the online channel is high $(\theta = 0.3)$, then the system's sole management faces high return quantities. It is thus important to increase resalability by increasing responsiveness level. Since the management with a differential pricing strategy can better use prices to encourage or discourage the use of one channel over the other, it can be said that the unified pricing strategy has a higher dependence on responsiveness level to conduct a similar task.

4.6 Conclusion

Chapter 4 has studied the effect that customer returns have on a dual-channel retailer when deciding on prices and responsiveness level. Similar to Chapter 3, this chapter has studied dual-channel integration under unified and differential pricing strategies and dual-channel competition under the Stackelberg, and Nash games. Returns have been assumed to be resold either once or several times in a selling season.

Through numerical examples and a sensitivity analysis the effect that customer preference for a certain channel and that customer return rates have on a dual-channel retailer's pricing and responsiveness decisions have been studied. For example, an increase in customer preference for an online channel encourages both centralized and competing online managements to increase the channel's responsiveness level to better handle a higher amount of returns. Additionally, the change in the return rates triggers a different responsiveness level reaction when under integration than when under competition. For instance, while the increase in the physical store's same-channel return rate does not trigger any response from the competing online management, it forces the centralized management to increase responsiveness level in order to switch sales to the online store. Moreover, while the competing online management simply increases responsiveness level as the online store's same-channel return rate increases, the centralized management faces challenges. The latter must choose between increasing online store's responsiveness level to better handle online returns, and decreasing online store's responsiveness level to switch customers to the physical store and decrease the negative effect of online returns. Intuitively, both a competitive and an integrated online channel will decrease responsiveness when the rate of cross-channel returns increases. Moreover, it has been found that there is a negative correlation between the responsiveness level of a competitive online store and the online store's dominance or market power.

It has been found that a dual-channel retailer generates a higher total profit when its customers prefer a low return rate(s) channel. Consequently, when channels experience similar return rates, a higher total profit is generated when customers greatly prefer any one channel over another. When a dual-channel retailer experiences a degree of high cannibalization, then competition can only generate a higher total profit than integration with price unification when the channel with the high customer preference level experiences high return rate(s). Otherwise, centralization schemes always generate a higher total profit. However, when the dual-channel retailer experiences a degree of cannibalization that is low enough, then all competition schemes outperform all integration schemes.

CHAPTER 5: OPTIMIZING CROSS-CHANNEL RETURN POLICY

5.1 Introduction and Motivation

The cross-channel return policy positively impacts online stores' sales and customer satisfaction, and provides physical stores with additional cross-selling opportunities. Nonetheless, such a policy creates disruption in the supply chains of enterprises (Zhang et al. 2010; Cao and Li 2015). Consequently, several important questions arise regarding such a policy. For example, under what conditions is it ideal to permit cross-channel returns? If cross-channel returns are allowed, when is it optimal to have physical stores ship all cross-channel returns back to online stores, and when is it optimal to have the physical stores retain cross-returned items for coming sales? How should the order quantity decisions be rectified if a dual-channel retailer faces both same-channel returns and cross-channel returns?

Despite an intensive effort, we could not find any paper that analytically studies the different cross-channel return strategies and their impact on dual-channel order quantities. Thus, this study has three contributions: first, we study four return strategies while simultaneously considering same- and cross-channel returns in a dual-channel retailing system; second, we propose mathematical models to determine the optimal order quantities for each strategy under uncertain demand; third, we present decision making insights by comparing the impacts of the different strategies that would help retailers to choose the suitable solution for their specific business environment.

According to practice, the first strategy is that cross-channel returns are allowed with the condition that they are regularly shipped back to the original point of purchase. In another word, they are allowed without shifting inventory from an online store to a physical store. The second strategy is that cross-channel returns are not allowed or banned. The third and fourth strategies are that there is a transfer of ownership or a shift of inventory for cross-channel returns from the online store to the physical store under decentralized management and centralized management, respectively.

5.2 Model Formulation

This work considers a dual-channel retailer that operates noncompeting physical and online stores. A full refund is granted for purchases returned in accordance to the retailer's rules and conditions. There is a $0 \leq r \leq 1$ chance that a customer of the retailer's physical store will return the purchased product to the physical store. There is a $0 \leq w \leq 1$ chance that a customer of the retailer's online store will return the purchased product to the online store. There is a $0 \leq v \leq 1$ chance that a customer of the online store will prefer to return a purchased product to the physical store if cross-channel returns are permissible. If not, the customer will have to return the purchased item to the online store (Figure 5.1).

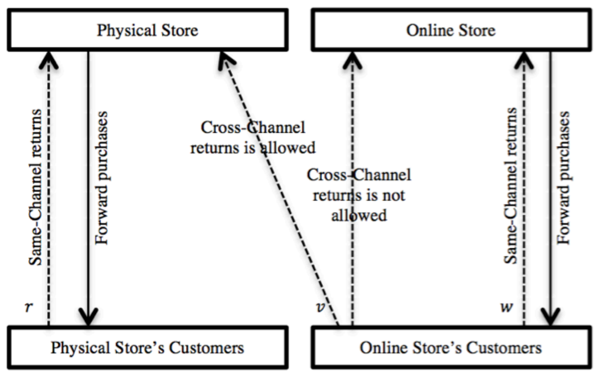


Figure 5.1: A Dual-Channel Retailer with All Common Forms of Customer Returns

Similar to Chapter 3, we assume that an item purchased from and returned to the physical store has a resalability rate of k_r, an item purchased from and returned to the online store has a resalability rate of k_o, and an item purchased from the online store and returned to the physical store has a resalability rate of k_{or}. This Chapter examines a general case where resalable returns can be resold countable number of times in a selling season, i.e. $\varepsilon \in [1, \infty)$, and a special case where resalable returns can be resold infinitely in a selling season, i.e. $\varepsilon = \infty$. We give resalable returns a selling priority over unsold items. It is worth noting that this priority assumption is only required for classification and simplification purposes. Since both resalable returns and unsold items are sold at the same price, the decision not to apply this prioritization will impose no change on the system's profitability.

Items are sold for an exogenous unified price of p in both channels. A sale request that is not satisfied costs the enterprise a shortage value of g whether it will be permanently sold or returned. This assumption is realistic since missing a sale request greatly affects a retailer's reputation and its customers' loyalty. Additionally, a returned item should be processed for inventory level correction and reimbursement purposes. Thus, it is associated with a return collection cost of d. If an item is returned as not resalable or resalable after the end of the selling season, then it is salvaged in a secondary market for a value of s. The unit's salvage value is less than or equal to the unit's purchasing cost $s < c$; otherwise the profit function is unbounded. Under the third strategy, the physical store pays a value of $\hat{c}$ to the online store to transfer the ownership of each cross-channel return. Items that are purchased from or returned directly to the online store will cost a per-unit shipping expense of t_{co}. Items that are shipped collectively from the physical store to the online store will cost a per-unit shipping expense of t_{ro}.

Due to customer returns and the different associated types of costs, the profit functions are constructed using the expected revenue generated by satisfying a single sale rather than the selling price of the item. Thus, the expected revenue generated by satisfying a single sale in the physical store should satisfy the condition $p_r > c$. Otherwise, the system generates losses by conducting the business. Similarly, depending on the strategy in use (i), the expected revenue generated by satisfying a single sale in the online store should satisfy the condition $p_o^i > c$. We remark that p_r and p_o^i have their highest values (i.e. $p_r = p_o^i = p$) when the system experiences no returns (i.e. $r = w = v = 0$).

Each channel faces a total sales request that is random and independent of the other channel. Thus, we assume that the online store's total sales request (x_o) has a probability density function (PDF) of $f_o(x_o)$, a cumulative distribution function (CDF) of $F_o(x_o)$, and a mean of μ_o, while the physical store's total sales request (x_r) has a probability density function (PDF) of $f_r(x_r)$, a cumulative distribution function (CDF) of $F_r(x_r)$, and a mean of μ_r. The objective of the channel's (or channels') manager is to maximize the profit of the store (or stores) by selecting the optimal order quantity (or quantities). Chapter's notations are presented in Table 5.1 below.

Notation	Description
$\boldsymbol{r}$	Probability an item purchased from the physical store is returned to the physical store
$\boldsymbol{w}$	Probability an item purchased from the online store is returned to the online store
$\boldsymbol{v}$	Probability an item purchased from the online store is preferably cross-returned to a physical store
$\boldsymbol{k_r}$	Probability an item purchased from and returned to the physical store is resalable
$\boldsymbol{k_o}$	Probability an item purchased from and returned to the online store is resalable
$\boldsymbol{k_{or}}$	Probability an item purchased from the online store and cross-returned to the physical store is resalable
$\boldsymbol{p}$	Channels' selling price
$\boldsymbol{g}$	Shortage cost
$\boldsymbol{c}$	Unit purchasing cost
$\boldsymbol{d}$	Return collection cost

$\boldsymbol{s}$	Salvage value
$\hat{\boldsymbol{c}}$	Amount the physical store pays to the online store to transfer the ownership of each cross-channel return
$\boldsymbol{t_{co}}$	Per unit shipping cost from a customer to the online store
$\boldsymbol{t_{ro}}$	Per unit shipping cost from the physical store to the online store
$\boldsymbol{x_o}$	Online store's total sales; where $f_o(\boldsymbol{x_o})$ & $F_o(\boldsymbol{x_o})$ are $\boldsymbol{x_o}$'s PDF and CDF, respectively
$\boldsymbol{x_r}$	Physical store's total sales; where $f_r(x_r)$ & $F_r(x_r)$ are x_r's PDF and CDF, respectively
$\boldsymbol{Q_r}$ & $\boldsymbol{Q_o}$	Quantities ordered by physical and online stores, respectively
$\boldsymbol{\pi_r^i}$ & $\boldsymbol{\pi_o^i}$	Physical store, and online store profits, respectively
$\boldsymbol{i}$	Cross-channel return strategy used $i = \{co, ro, D, C\}$
$\boldsymbol{\varepsilon}$	Number of times a resalable return can be resold in a selling season

Table 5.1: Fifth Chapter's Notations

5.3 Ship all Cross-Channel Returns Back to Online Store ($\boldsymbol{i = ro}$)

Under this practice, items purchased from a retailer's online store and cross-returned to its physical store should be shipped back to their original point of purchase. The physical store in this strategy should only act as a facilitator, and thus all costs associated with cross-channel returns (e.g. collection and shipping costs) are paid by the online store. We assume that the time it takes an item to be shipped from a customer to the online store by a third-party logistics provider (3LP) is equivalent to the time it takes an item to be shipped from the physical store to the online store after being cross-channel returned. Thus, we use the same online store resalability rate (k_o) for both types of returns.

If resalable returns can be resold ε number of times in a selling season, then the online store's ordered quantity Q_o can satisfy a total sales request of up to $Q_o\big(1 + (w+v)k_o + \cdots + \big((w+v)k_o\big)^{\varepsilon}\big) = Q_o \sum_0^{\varepsilon}\big((w+v)k_o\big)^n = \eta_\varepsilon Q_o$. Similarly, the physical store's ordered quantity Q_r can satisfy a total sales request of up to $Q_r(1 + rk_r + \cdots + (rk_r)^{\varepsilon}) = Q_r \sum_0^{\varepsilon}(rk_r)^n = \delta_\varepsilon Q_r$.

For the special case where resalable returns can be resold an infinite number of times, i.e. $\varepsilon = \infty$, in a selling season, the online store's ordered quantity Q_o can satisfy a total sales request of up to $Q_o\big(1 + (w+v)k_o + \cdots + \big((w+v)k_o\big)^{\infty}\big) = \frac{Q_o}{1-(w+v)k_o} = \eta_{\varepsilon=\infty} Q_o$. Similarly, the physical store's ordered quantity Q_r can satisfy a total sales request of up to $Q_r(1 + rk_r + \cdots + (rk_r)^\infty) = \frac{Q_r}{1-rk_r} = \delta_{\varepsilon=\infty} Q_r$. A similar procedure was used in works such as Mostard and Teunter (2006) and Mostard et al. (2005).

The expected revenue generated by satisfying a single sale from the online store is $p_o^{ro} = -t_{co} + (1 - v - w)p - w(d + t_{co} - (1 - k_o)s) - v(2d + t_{ro} - (1 - k_o)s) + \frac{s}{\eta_\varepsilon}\left((w + v)k_o\right)^{\varepsilon+1}$. In the first term, every sold item contributes negatively due to the shipped cost t_{co} paid by the store. In the second term, there is a $(1 - v - w)$ chance that the item is a final sale and contributes positively due to the selling price. In the third term, there is a w chance that the item is returned to the online store and contributes negatively due to the collection cost, negatively due to the shipping cost, and positively due to salvaging non-resalable returns. In the fourth term, there is a v chance that the item is cross-returned to the physical store and contributes negatively due to the collection cost at both stores, negatively due to the shipping cost, and positively due to salvaging non-resalable returns. The fifth term assures the salvaging of an item that ends up being returned as resalable after the end of the selling season. Notice that this term will be zero when $\varepsilon = \infty$. Similarly, the expected revenue generated by satisfying a single sale from the physical store is $p_r = (1 - r)p - rd + r(1 - k_r)s + \frac{s}{\delta_\varepsilon}(rk_r)^{\varepsilon+1}$. Due to the similarity, a detailed explanation of the relationship has been omitted. Under this strategy, the decision of one channel does not impose any changes to the optimal decision of the other channel. Thus, each channel maximizes its own profit function in isolation.

The profit function for the online store can be constructed as

$$\pi_o^{ro} = \int_0^{\eta_\varepsilon Q_o} \left\{x_o p_o^{ro} + s\left(Q_o - \left(\frac{1}{\eta_\varepsilon}\right)x_o\right)\right\} f_o(x_o)dx_o + \int_{\eta_\varepsilon Q_o}^{\infty} \{\eta_\varepsilon Q_o p_o^{ro} - g(x_o - \eta_\varepsilon Q_o)\} f_o(x_o)dx_o - cQ_o. \tag{5.1}$$

Since priority is given to resales rather than first sales and the items are sold several times until they become non-resalable, the online store's total sales request of x_o can be satisfied by the quantity $(1/\eta_\varepsilon)x_o$. Accordingly, the first term includes the expected revenue and salvage value of items that have never been sold before when the sales request is less than $\eta_\varepsilon Q_o$. Notice that the expected salvage value for non-resalable returns and those items that end up being returned as resalable after the end of the selling season are included in the expected revenue (i.e. p_o^{ro}). The second term depicts the expected revenue and shortage value when the sales requests are more than $\eta_\varepsilon Q_o$. The third term is the cost of purchasing inventory for the online store.

The profit function for the physical store is constructed as

$$\pi_r^{ro} = \int_0^{\delta_\varepsilon Q_r} \left\{x_r p_r + s\left(Q_r - \left(\frac{1}{\delta_\varepsilon}\right)x_r\right)\right\} f_r(x_r)dx_r + \int_{\delta_\varepsilon Q_r}^{\infty} \{\delta_\varepsilon Q_r p_r - g(x_r - \delta_\varepsilon Q_r)\} f_r(x_r)dx_r - cQ_r. \tag{5.2}$$

The physical store's profit function is constructed similarly to the online store's profit function. Thus, a detailed explanation of it has been omitted.

Proposition 5.1

The expected profit function π_o^{ro} is strictly concave on Q_o. Thus, a unique maximum exists at Q_o^{ro*}.

$$Q_o^{ro*} = \left(\frac{1}{\eta_\varepsilon}\right) F_o^{-1}\left(\frac{p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - c}{p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s}\right). \tag{5.3}$$

The expected profit function π_r^{ro} is strictly concave on Q_r. Thus, a unique maximum exists at Q_r^{ro*}.

$$Q_r^{ro*} = \left(\frac{1}{\delta_\varepsilon}\right) F_r^{-1}\left(\frac{p_r\delta_\varepsilon + g\delta_\varepsilon - c}{p_r\delta_\varepsilon + g\delta_\varepsilon - s}\right). \tag{5.4}$$

Assuming that the total sales request of each store follows a uniform distribution is sufficiently general to capture the effect that same- and cross-channel returns have on both stores. Such a generality made the application of uniform distribution common in the field of supply chain management (Yao et al. 2009). Furthermore, the distribution is bounded and tractable. Hu and Li (2012) stated that sales uncertainty for the apparel industry is best described by a uniform distribution.

Thus, the total sales request of the online store (x_o) is distributed between 0 and b_o, i.e. $x_o \sim U[0, b_o]$. Additionally, the total sales request of the physical store (x_r) is distributed between 0 and b_r, i.e. $x_r \sim U[0, b_r]$. Accordingly, the optimal quantity and maximum profit for the online and physical stores when sales are uniformly distributed are given below.

$$Q_o^{ro*} = \frac{b_o(p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - c)}{\eta_\varepsilon(p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s)}. \tag{5.5}$$

$$\pi_o^{ro*} = \frac{b_o}{2\eta_\varepsilon}\left\{\frac{(p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - c)^2}{p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s} - g\eta_\varepsilon\right\}. \tag{5.6}$$

$$Q_r^{ro*} = \frac{b_r(p_r\delta_\varepsilon + g\delta_\varepsilon - c)}{\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)}. \tag{5.7}$$

$$\pi_r^{ro*} = \frac{b_r}{2\delta_\varepsilon}\left\{\frac{(p_r\delta_\varepsilon + g\delta_\varepsilon - c)^2}{p_r\delta_\varepsilon + g\delta_\varepsilon - s} - g\delta_\varepsilon\right\}. \tag{5.8}$$

Under the condition that $b_o = b_r$, the magnitude in which a channel's order quantity and profitability differ from the other channel is determined by return rates and return processes (Equations 5.5 – 5.8). Also, one may notice that a channel's optimal order quantity and profit is similar to that of a classical newsvendor model. In fact, they become identical when a retailer does not permit customer returns (i.e. $r = w = v = 0$ and $p_r = p_o^i = p$).

5.4 No Cross-Channel Returns ($i = co$)

This strategy requests that all online customers return unsatisfactory items to the location the items were purchased from. Thus, cross-channel returns are not allowed. According to Steinfield (2004), more than 90% of all examined retailers and more than 60% of all examined department stores ban cross-channel returns. Due to the added constraint in the online store's return policy, some customers may develop channel intolerance and stop purchasing from the online store. Thus, a fixed portion of $1 - \alpha$ of the online store's total sales request x_o is lost. Consequently, the store loses an expected shortage value of $\int_0^\infty g(1-\alpha)x_o f_o(x_o)dx_o$ just by embracing this strategy. The observed total sales request for the online store is defined as $y_o = \alpha x_o$. We assume that multiplying the random variable x_o with the constant α will not change the general shape of the distribution. Such a property exists in the uniform and normal distributions. Consequently, $\alpha\mu_o$, $f_\alpha(y_o)$, and $F_\alpha(y_o)$ are the mean, PDF, and CDF for the online store's observed total sales request, respectively. Note that the parameter α might be thought of as the portion of the online store's customers who are loyal and willing to stay with the channel even when the store's policies change.

While the expected revenue generated by satisfying a single sale from the online store becomes $p_o^{co} = -t_{co} + (1 - v - w)p - w(d + t_{co} - (1 - k_o)s) - v(d + t_{co} - (1 - k_o)s) + \frac{s}{\eta_\varepsilon}\left((w + v)k_o\right)^{\varepsilon+1}$, it is unchanged in the physical store. The profit function for the online store can be formulated as

$$\pi_o^{co} = -\int_0^\infty g(1-\alpha)x_o f_o(x_o)dx_o + \int_0^{\eta_\varepsilon Q_o}\left\{y_o p_o^{co} + s\left(Q_o - \left(\frac{1}{\eta_\varepsilon}\right)y_o\right)\right\} f_\alpha(y_o)dy_o + \int_{\eta_\varepsilon Q_o}^\infty \{\eta_\varepsilon Q_o p_o^{co} - g(y_o - \eta_\varepsilon Q_o)\} f_\alpha(y_o)dy_o - cQ_o. \tag{5.9}$$

Since this strategy has no effect on the physical store, the store's optimal quantity and maximum profit stay unchanged – i.e., $Q_r^{co^*} = Q_r^{ro^*}$ and $\pi_r^{co^*} = \pi_r^{ro^*}$.

Proposition 5.2

The expected profit function π_o^{co} is strictly concave on Q_o. Thus, a unique maximum exists at $Q_o^{co^*}$.

$$Q_o^{co^*} = \left(\frac{1}{\eta_\varepsilon}\right) F_\alpha^{-1}\left(\frac{p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - c}{p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - s}\right). \tag{5.10}$$

From the previous proposition one may find the optimal quantity and maximum profit when the online store's total sales request is uniformly distributed. Thus,

$$Q_o^{co^*} = \frac{\alpha b_o(p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - c)}{\eta_\varepsilon(p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - s)}, \text{ and} \tag{5.11}$$

$$\pi_o^{co*} = \frac{\alpha b_o}{2\eta_\varepsilon}\left\{\frac{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-c)^2}{p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-s} - \frac{g\eta_\varepsilon}{\alpha}\right\}. \tag{5.12}$$

Based on formulas (5.5) and (5.11), when α is higher than a threshold of $\dot{\alpha}$, the online store's order quantity under policy $i = co$ is higher than its order quantity under policy $i = ro$, i.e. $Q_o^{co*} > Q_o^{ro*}$, where $\dot{\alpha}$ is calculated as

$$\dot{\alpha} = \frac{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-s)(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-c)}{(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-s)(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-c)}. \tag{5.13}$$

Similarly, when α is higher than a threshold of $\ddot{\alpha}$, the online store's maximum profit under policy $i = co$ is higher than its maximum profit under policy $i = ro$, i.e. $\pi_o^{co*} > \pi_o^{ro*}$, where $\ddot{\alpha}$ is formulated as

$$\ddot{\alpha} = \frac{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-s)(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-c)^2}{(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-s)(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-c)^2} = \dot{\alpha}\,\frac{(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-c)}{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-c)}. \tag{5.14}$$

Since $\pi_r^{co*} = \pi_r^{ro*}$, the value of π_o^{i*} identifies which strategy, i.e. $i = ro$ or $i = co$, generates a higher retailer profit. Furthermore, t_{ro} is uniquely defined in strategy $i = ro$. Consequently, one can easily use it for a comparison of the above-stated strategies. If the derivative for $\ddot{\alpha}$ with respect to t_{ro} is taken, then the outcome is a negative term as per relationship (5.15). That is to say, as the shipping cost for each item moved from the physical store to the online store increases, the superiority of strategy $i = co$ over strategy $i = ro$ occurs at a lower value of α.

$$\frac{\partial\ddot{\alpha}}{\partial t_{ro}} = -v\eta_\varepsilon\,\frac{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-s)(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-c)\big((p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-s)+(c-s)\big)}{(p_o^{co}\eta_\varepsilon+g\eta_\varepsilon-c)^2(p_o^{ro}\eta_\varepsilon+g\eta_\varepsilon-s)^2}. \tag{5.15}$$

Corollary 5.1: $1 \geq \dot{\alpha} \geq \ddot{\alpha} \geq 0$

From the previous corollary we notice that the online store will not order a higher quantity under policy $i = co$ than under policy $i = ro$ unless the former policy is more profitable than then the latter. That is to say, $Q_o^{co*} > Q_o^{ro*}$ if and only if $\pi_r^{co*} > \pi_r^{ro*}$. Additionally, due to the effect that α has on Q_o^{co*}, managers may jump to a false conclusion that it is lower than Q_o^{ro*}. Therefore, observing the costs associated with each strategy is essential to make such a claim.

5.5 Perform Inventory Shift for Cross-Channel Returns Under a Decentralized Management ($i = D$)

Zhang et al. (2010) indicated that if cross-channel returns are not offered at the physical store, then they must be shipped to the online store. Otherwise they could be claimed by the physical store through an inventory-shift process that is subject to the retailer's internal rules. Channels might have not aligned their objectives at that point or it might be too expensive to do so; thus each channel seeks to maximize its own profit regardless of the other channel's performance. To transfer the ownership of a cross-channel return, the physical store pays a value of $\hat{c}$ to the online store. For simplicity, the physical store will use the expected, rather than the observed, total amount of cross-channel returns (Q_{or}) to conduct its analysis. A portion k_{or} of Q_{or} is resalable and can be used to satisfy some of the sales requested through the physical store. We assume that all parameters and decision rules are known to both supply chain members. Therefore Q_{or} can be correctly estimated by the physical store.

If resalable returns can be resold ε number of times, then Q_o can satisfy a total sales request in the online store of up to $Q_o(1 + wk_o + \cdots + (wk_o)^\varepsilon) = Q_o \sum_0^\varepsilon (wk_o)^n = \lambda_\varepsilon Q_o$. Notice that the online store normally starts the selling season before the physical store does and most of its sales occur at the beginning of the season. Thus, we may assume that all cross-channel returns reach the physical store at the beginning of its selling season. Consequently, the quantity ordered from the supplier (Q_r) and the resalable cross-channel returns of ($k_{or}Q_{or}$) can satisfy a total sales request in the physical store of up to $(Q_r + k_{or}Q_{or})(1 + rk_r + \cdots + (rk_r)^\varepsilon) = (Q_r + k_{or}Q_{or}) \sum_0^\varepsilon (rk_r)^n = \delta_\varepsilon(Q_r + k_{or}Q_{or})$.

For the special case where resalable returns can be resold an infinite number of times, i.e. $\varepsilon = \infty$, the ordered quantity of Q_o can satisfy a total sales request in the online store of up to $Q_o(1 + wk_o + \cdots + (wk_o)^\infty) = \frac{Q_o}{1-wk_o} = \lambda_{\varepsilon=\infty} Q_o$. However, the quantity $(Q_r + k_{or}Q_{or})$ can satisfy a total sales request in the physical store of up to $(Q_r + k_{or}Q_{or})(1 + rk_o + \cdots + (rk_o)^\infty) = \frac{Q_r + k_{or}Q_{or}}{1-rk_r} = \delta_{\varepsilon=\infty}(Q_r + k_{or}Q_{or})$.

While the expected revenue generated by satisfying a single sale from the physical store is unchanged, the expected revenue generated by satisfying a single sale from the online store becomes

$$p_o^D = -t_{co} + (1 - v - w)p - w(d + t_{co} - (1 - k_o)s) + v\hat{c} + \frac{s}{\lambda_\varepsilon}(wk_o)^{\varepsilon+1}. \tag{5.16}$$

The profit function for the online store is formulated as

$$\pi_o^D(Q_o) = \int_0^{\lambda_\varepsilon Q_o} \left\{x_o p_o^D + s\left(Q_o - \left(\frac{1}{\lambda_\varepsilon}\right)x_o\right)\right\} f_o(x_o)dx_o + \int_{\lambda_\varepsilon Q_o}^{\infty} \{\lambda_\varepsilon Q_o p_o^D - g(x_o - \lambda_\varepsilon Q_o)\} f_o(x_o)dx_o - cQ_o. \tag{5.17}$$

The expected quantity of all cross-channel returns is estimated as

$$Q_{or} = v\int_0^{\lambda_\varepsilon Q_o} x_o\, f_o(x_o)dx_o + v\int_{\lambda_\varepsilon Q_o}^{\infty} \lambda_\varepsilon Q_o f_o(x_o)dx_o. \tag{5.18}$$

The profit function for the physical store is formulated as

$$\pi_r^D(Q_r) = \int_0^{\delta_\varepsilon(Q_r+k_{or}Q_{or})} \left\{x_r p_r + s\left(Q_r + k_{or}Q_{or} - \left(\frac{1}{\delta_\varepsilon}\right)x_r\right)\right\} f_r(x_r)dx_r + \int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} \{\delta_\varepsilon(Q_r + k_{or}Q_{or})p_r - g\big(x_r - \delta_\varepsilon(Q_r + k_{or}Q_{or})\big)\} f_r(x_r)dx_r - cQ_r - \big(\hat{c} + d - s(1 - k_{or})\big)Q_{or}. \tag{5.19}$$

In function (5.16), we assumed that the reimbursement process for all cross-channel returns was done through the original point of purchase (i.e., the online store and not the physical store). Due to the similarity between function (5.17) and online store's previous profit functions, a detailed explanation has been omitted. Equation (5.18) has two parts. The first part captures the expected number of cross-channel returns if the online store's sales request is below $\lambda_\varepsilon Q_o$. The second part captures the expected number of cross-channel returns if the online store's sales request is above $\lambda_\varepsilon Q_o$. Finally, in constructing the profit function for the physical store, i.e. formula (5.19), we considered several issues. The physical store will have to pay a value of $\hat{c}$ to the online store for all expected cross-channel returns, pay a collection cost for all expected cross-channel returns, and collect a salvage value for all expected non-resalable cross-channel returns. The physical store will also use all resalable cross-channel returns to satisfy some of the sales requested through its channel. Otherwise, the profit functions (5.19) and (5.2) were constructed similarly.

Proposition 5.3

The expected profit function π_o^D is strictly concave on Q_o. Thus, a unique maximum exists at $Q_o^{D^*}$.

$$Q_o^{D^*} = \left(\frac{1}{\lambda_\varepsilon}\right) F_o^{-1}\left(\frac{p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - c}{p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s}\right). \tag{5.20}$$

The expected profit function π_r^D is strictly concave on Q_r. Therefore, a unique maximum exists at $Q_r^{D^*}$, where $Q_r^{D^*}$ is conditioned to be positive.

$$Q_r^{D^*} = \left(\frac{1}{\delta_\varepsilon}\right) F_r^{-1}\left(\frac{p_r\delta_\varepsilon + g\delta_\varepsilon - c}{p_r\delta_\varepsilon + g\delta_\varepsilon - s}\right) - k_{or}Q_{or}. \tag{5.21}$$

When a channel's total sales request follows a uniform distribution, using the above functions one may derive the online store's optimal order quantity, the online store's maximum profit, the physical store's optimal order quantity, and the physical store's maximum profit, respectively.

$$Q_o^{D^*} = \frac{b_o(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - c)}{\lambda_\varepsilon(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s)}. \tag{5.22}$$

$$\pi_o^{D^*} = \frac{b_o}{2\lambda_\varepsilon}\left\{\frac{(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - c)^2}{p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s} - g\lambda_\varepsilon\right\}. \tag{5.23}$$

$$Q_r^{D^*} = \frac{b_r(p_r\delta_\varepsilon + g\delta_\varepsilon - c)}{\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)} - \frac{vk_{or}b_o(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - c)\big((p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s) + (c - s)\big)}{2(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s)^2}. \tag{5.24}$$

$$\pi_r^{D^*} = \frac{b_r}{2\delta_\varepsilon}\left\{\frac{(p_r\delta_\varepsilon+g\delta_\varepsilon-c)^2}{(p_r\delta_\varepsilon+g\delta_\varepsilon-s)} - g\delta_\varepsilon\right\} + \frac{vb_o\big((s-\hat{c}-d)+k_{or}(c-s)\big)(p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-c)\big((p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-s)+(c-s)\big)}{2(p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-s)^2}. \tag{5.25}$$

Comparing the profit function (5.25) to the profit function (5.8), one may notice that the physical store will incur losses by cooperating with this strategy if the amount paid to the online store for each cross-channel return, i.e. value $\hat{c}$, is more than $(s-d)+k_{or}(c-s)$. Under such a condition the physical store should refuse to perform the inventory shift or to be part of this strategy. However, it is vital that the online store calculate the minimum $\hat{c}$ that will make this policy more appealing than the previous two policies, i.e. $\pi_o^{D^*} > \pi_o^{i^*}$, where $i = \{ro, co\}$. By equating the profit function (5.23) to both profit functions (5.6) and (5.12), one may derive the relationship

$$\lambda_\varepsilon^2(p_o^D)^2 + \lambda_\varepsilon\left(g\lambda_\varepsilon - 2c - \frac{2\lambda_\varepsilon}{b_o}\pi_o^{i^*}\right)p_o^D + \left((s - g\lambda_\varepsilon)\frac{2\lambda_\varepsilon}{b_o}\pi_o^{i^*} - g\lambda_\varepsilon(2c-s) + c^2\right) = 0. \tag{5.26}$$

The function (5.26) is convex in terms of p_o^D and has at most two real positive roots $\left\{\underline{p_o^D}, \overline{p_o^D}\right\}$. Thus, the online store generates a higher profit under policy $i = D$ than under policy $i = \{ro, co\}$, if the value of $\hat{c}$ drives p_o^D to be higher than $\overline{p_o^D}$.

Consider the retailer's profit under policy $i = D$, i.e. $\pi_t^{D^*}$. It may be calculated by summing both the online store's profit function (5.23) and the physical store's profit function (5.25). If the derivative for $\pi_t^{D^*}$ with respect to $\hat{c}$ is taken, then one gets

$$\frac{\partial \pi_t^{D^*}}{\partial \hat{c}} = \frac{v^2\lambda_\varepsilon b_o\big(s-\hat{c}-d+k_{or}(c-s)\big)(c-s)^2}{(p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-s)^3}. \tag{5.27}$$

Notice that the sign of the previous derivative is dependent on the value of $\hat{c}$. Since the physical store will not be involved in policy $i = D$ if the value of $\hat{c}$ is higher than $s - d + k_{or}(c-s)$, one may say that $\frac{\partial \pi_t^{D^*}}{\partial \hat{c}} > 0$ and $\pi_t^{D^*}$ score the highest profit when $\hat{c} = s - d + k_{or}(c-s)$. It is worth noting that the highest value of $\pi_t^{D^*}$ requires the physical store to not generate any financial benefit out of this strategy.

When v and k_{or} are non-zeroes, then it can be noted that $Q_r^{D^*} < Q_r^{ro^*}$ and $Q_r^{D^*} < Q_r^{co^*}$. It is intuitive to say that physical stores will have higher order quantities under strategies that do not include shifts in inventories. If we take the derivative of $Q_o^{D^*}$ in equation (5.22) with respect to $\hat{c}$, then one may notice that it is strictly positive (please refer to function (5.28), given below). In contrast, if we take the derivative of $Q_r^{D^*}$ in equation (5.24) with respect to $\hat{c}$, then one may notice that it is strictly negative (please refer to function (5.29), given below).

$$\frac{\partial Q_o^{D^*}}{\partial \hat{c}} = \frac{vb_o(c-s)}{\left(p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-s\right)^2}, \text{ and} \tag{5.28}$$

$$\frac{\partial Q_r^{D^*}}{\partial \hat{c}} = -\frac{v^2\lambda_\varepsilon k_{or}b_o(c-s)^2}{\left(p_o^D\lambda_\varepsilon+g\lambda_\varepsilon-s\right)^3}. \tag{5.29}$$

This indicates that the online store is encouraged to order a higher quantity when the value of $\hat{c}$ increases. Accordingly, the expected total amount of cross-channel returns will increase, allowing the physical store to decrease its order quantity.

5.6 Perform Inventory Shift for Cross-Channel Returns Under a Centralized Management ($\boldsymbol{i = C}$)

It is well known that centralization is expensive and requires a significant amount of effort. Therefore, it is important that the enterprise realize whether it enhances profits or it is not vital for the enterprise. However, under this practice the objectives are aligned and a sole decision-maker maximizes the total profit of the supply chain. The internal transactions between channels can thus be excluded from the analysis, as they do not induce any change to the system's profit. Thus, an inventory shift is still conducted, but no payment is required for cross-channel returns. The expected revenue generated by satisfying a single sale from the online store changes to $p_o^C = -t_{co} + (1-v-w)p - w(d+t_{co}-(1-k_o)s) - v(d-(1-k_{or})s) + \frac{s}{\lambda_\varepsilon}(wk_o)^{\varepsilon+1}$. The total profit of the retailer is formulated as below. Due to similarity, no explanation is needed to comprehend its terms.

$$\begin{aligned}\pi^C = {} & \int_0^{\lambda_\varepsilon Q_o}\left\{x_o p_o^C + s\left(Q_o - \left(\frac{1}{\lambda_\varepsilon}\right)x_o\right)\right\} f_o(x_o)dx_o + \int_{\lambda_\varepsilon Q_o}^{\infty}\{\lambda_\varepsilon Q_o p_o^C - g(x_o-\lambda_\varepsilon Q_o)\}f_o(x_o)dx_o + \\ & \int_0^{\delta_\varepsilon(Q_r+k_{or}Q_{or})}\left\{x_r p_r + s\left(Q_r + k_{or}Q_{or} - \left(\frac{1}{\delta_\varepsilon}\right)x_r\right)\right\} f_r(x_r)dx_r + \int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty}\{\delta_\varepsilon(Q_r + \\ & k_{or}Q_{or})p_r - g\left(x_r - \delta_\varepsilon(Q_r+k_{or}Q_{or})\right)\}f_r(x_r)dx_r - c(Q_o+Q_r)\end{aligned} \tag{5.30}$$

Proposition 5.4

The expected profit function π^C is strictly and jointly concave on Q_o and Q_r, and thus a unique global maximum exists at $Q_o^{C^*}$ and $Q_r^{C^*}$, where $Q_r^{C^*}$ is conditioned to be positive.

$$Q_o^{C^*} = \left(\frac{1}{\lambda_\varepsilon}\right)F_o^{-1}\left(\frac{p_o^C\lambda_\varepsilon+g\lambda_\varepsilon+vk_{or}c\lambda_\varepsilon-c}{p_o^C\lambda_\varepsilon+g\lambda_\varepsilon+vk_{or}c\lambda_\varepsilon-s}\right). \tag{5.31}$$

$$Q_r^{C^*} = \left(\frac{1}{\delta_\varepsilon}\right)F_r^{-1}\left(\frac{p_r\delta_\varepsilon+g\delta_\varepsilon-c}{p_r\delta_\varepsilon+g\delta_\varepsilon-s}\right) - k_{or}Q_{or}. \tag{5.32}$$

If uniform distributions are used for total sales requests, then one may use the above functions to derive the online store's optimal order quantity, the physical store's optimal order quantity, and the retailer's maximum profit, respectively.

$$Q_o^{C^*} = \frac{b_o(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - c)}{\lambda_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - s)}. \tag{5.33}$$

$$Q_r^{C^*} = \frac{b_r(p_r\delta_\varepsilon + g\delta_\varepsilon - c)}{\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)} - \frac{vk_{or}b_o(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - c)\left((p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}c\lambda_\varepsilon - s) + (c - s)\right)}{2(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - s)^2}. \tag{5.34}$$

$$\pi^{C^*} = \frac{b_o}{2\lambda_\varepsilon}\left\{\frac{(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - c)^2}{p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - s} - g\lambda_\varepsilon\right\} + \frac{b_r}{2\delta_\varepsilon}\left\{\frac{(p_r\delta_\varepsilon + g\delta_\varepsilon - c)^2}{p_r\delta_\varepsilon + g\delta_\varepsilon - s} - g\delta_\varepsilon\right\}. \tag{5.35}$$

Similar to before, when v and k_{or} are non-zeros, then $Q_r^{C^*} < Q_r^{ro^*}$ and $Q_r^{C^*} < Q_r^{co^*}$. Next we compare the online store's optimal order quantity under strategies $i = D$ and $i = C$ – i.e. equations (5.22) and (5.33). One may notice that by comparing between the values p_o^D and $p_o^C + vk_{or}c$ we can identify under what policy the online store will have a higher ordered quantity. The latter is written as $-t_{co} + (1 - v - w)p - w(d + t_{co} - (1 - k_o)s) + v\left(s - d + k_{or}(c - s)\right) + \frac{s}{\lambda_\varepsilon}(wk_o)^{\varepsilon+1}$.

Since $\hat{c} \leq s - d + k_{or}(c - s)$, then $p_o^D \leq p_o^C + vk_{or}c$. Therefore, the online store's ordered quantity under policy $i = D$ is at most as high as its ordered quantity under policy $i = C$, i.e. $Q_o^{D^*} \leq Q_o^{C^*}$. Notice that $Q_o^{D^*} = Q_o^{C^*}$ when $\hat{c} = s - d + k_{or}(c - s)$. In a similar fashion, one may compare the physical store's optimal order quantity under policies $i = D$ and $i = C$, i.e. equations (5.24) and (5.34). Following the above logic one may find that the physical store's ordered quantity under policy $i = C$ is at most as high as its ordered quantity under policy $i = D$, i.e. $Q_r^{D^*} \geq Q_r^{C^*}$. One may notice that they are equivalent under the condition $\hat{c} = s - d + k_{or}(c - s)$. One may also note that under this condition policy $i = D$ performs as ideally as policy $i = C$, i.e. $\pi_t^{D^*} = \pi^{C^*}$.

Next we compare the retailer's performance under strategies $i = C$ and $i = ro$. One may calculate $\pi_t^{ro^*}$ by summing both the online store's profit function (5.6) and the physical store's profit function (5.8). By equating π^{C^*}in (5.35) to the calculated $\pi_t^{ro^*}$ one derives the relationship

$$\eta_\varepsilon^2(p_o^{ro})^2 + \eta_\varepsilon\left(2g\eta_\varepsilon - 2c - \frac{\eta_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - c)^2}{\lambda_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - s)}\right)p_o^{ro} + (g\eta_\varepsilon - c)^2 - (g\eta_\varepsilon - s)\frac{\eta_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - c)^2}{\lambda_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + vk_{or}\lambda_\varepsilon c - s)} = 0. \tag{5.36}$$

The function (5.36) is convex in terms of p_o^{ro} and has at most two real positive roots $\left\{\underline{p_o^{ro}}, \overline{p_o^{ro}}\right\}$. Thus, the retailer generates a higher profit under strategy $i = ro$ than under strategy $i = C$ if $p_o^{ro}(t_{ro}) < \overline{p_o^{ro}}$. Notice that we used the parameter t_{ro} to compare the two strategies above, because it solely effects the strategy $i = ro$. Thus, if the online store's management can lower t_{ro} so that p_o^{ro} drops below $\overline{p_o^{ro}}$, then the channel may not need to lose its inventory for the physical store.

We then consider the retailer's profit under strategy $i = co$, i.e. $\pi_t^{co^*}$. It is calculated by summing both the online store's profit function (5.12) and the physical store's profit function (5.8). By equating π^{C^*} to $\pi_t^{co^*}$ one may find the threshold at which α is higher; then $\pi_t^{co^*} > \pi^{C^*}$.

5.7 Sensitivity Analysis and Managerial Insights

In this analysis we compare a store's optimal quantity, a store's performance, and a retailer's performance under the different strategies. We also derive important managerial insights that form general guidelines for managers running dual-channel retailing systems. To conduct our analysis we consider the following parameters: $r = 0.1$, $w = 0.1$, $v = 0.3$, $k_r = 0.6$, $k_o = 0.4$, $k_{or} = 0.4$, $p = 100$, $c = 30$, $t_{ro} = 4$, $t_{co} = 5$, $d = 3$, $s = 5$, $g = 40$, $b_o = 300$, $\alpha = 0.995$, $b_r = 300$ and $\varepsilon = 1$.

Under strategies $i = ro$ and $i = co$ the physical store should order the same quantities, i.e. $Q_r^{ro^*} = Q_r^{co^*}$. This is intuitive since the system does not permit shifting the inventories of cross-channel returns and the physical store may only assist in shipping those returns to their original point of purchase, i.e. online store. However, when the system permits the inventory shift process, then the physical store will order lower quantities under strategies $i = D$ and $i = C$ than under strategies $i = ro$ and $i = co$, i.e. $Q_r^{D^*} \leq Q_r^{ro^*}$, $Q_r^{D^*} \leq Q_r^{co^*}$, $Q_r^{C^*} \leq Q_r^{ro^*}$, and $Q_r^{C^*} \leq Q_r^{co^*}$. This is comprehensible since the physical store may use resalable cross-channel returns to satisfy some of its sales requests.

As mentioned before, the majority of online stores' return policies request that customers return all unsatisfactory purchases to the online store, i.e. that they apply strategy $i = co$. This in turn imposes a constraint that many customers dislike. Thus, only a portion of α will conduct their purchases from the channel. If the portion is higher than the threshed of $\dot{\alpha}$ defined in equation (5.13), i.e. $\alpha > \dot{\alpha}$, then $Q_o^{co^*} > Q_o^{ro^*}$. Thus, one may conclude that the reduction in sales requests does not always induce a lower order quantity for the online store under strategy $i = co$ than under strategy $i = ro$.

We then compare the online store's order quantity when the system does not permit an inventory shift, i.e. strategies $i = ro$ and $i = co$, and its order quantity when the system permits an inventory shift, i.e. strategies $i = D$ and $i = C$. It is noted that the order quantity under any no-inventory shift strategy is lower than the order quantity under any inventory shift strategy. That is to say, $Q_o^{D^*} \geq Q_o^{ro^*}$, $Q_o^{D^*} \geq Q_o^{co^*}$, $Q_o^{C^*} \geq Q_o^{ro^*}$, and $Q_o^{C^*} \geq Q_o^{co^*}$. Although this is not analytically proven, several tests were conducted under different parameter sets. The notion is logical since the inventory shift strategies cause the online store to lose items for the physical store, and thus order a higher quantity at the beginning of the season. Not doing so might leave the store starving for items to satisfy the sales request with. Thus, it is crucial that a dual-channel retailer define the strategy of handling cross-channel returns prior to the beginning of the selling season, and that it orders the optimal quantity for a channel accordingly.

When the system permits an inventory shift, one should compare a channel's optimal quantity under strategies $i = D$ and $i = C$. As indicated in Section 5.6, $Q_o^{D^*}$ is at most equivalent to $Q_o^{C^*}$, while $Q_r^{D^*}$ is at least equivalent to $Q_r^{C^*}$. That is to say, $Q_o^{D^*} \leq Q_o^{C^*}$ and $Q_r^{D^*} \geq Q_r^{C^*}$. In (5.27) and (5.28) one may note that $Q_o^{D^*}$ increases and $Q_r^{D^*}$ decreases when $\hat{c}$ increases. Thus as the amount paid to the online store for each cross-channel return increases, the store will be encouraged to order more as the negative consequences of such type of return is diminished. As this happens, the expected amount of cross-channel returns will increase, which will allow the physical store to reduce its order quantity. Conversely, as $\hat{c}$ decreases, the store will order less as the negative consequences of this type of return are intensified. As a result, the expected number of cross-channel returns will decrease, forcing the physical store to increase its order quantity.

We next study the channel's performance and the retailer's performance when strategies $i = ro$ and $i = co$ are followed. Under the earlier strategies, the physical store only assists in returning cross-channel returns to the online store. Thus, its measurable performance under both strategies is the same, i.e. $\pi_r^{ro^*} = \pi_r^{co^*}$. Note that there could be an increase in the physical store's sales and thus in the profit under strategy $i = ro$ due to the cross-channel selling opportunity. That is to say, the online customer might return an item purchased from the online store to the physical store and at the same time purchase an item that is appealing to him or her. Such a purchase is mostly impulsive and not planned. According to Neslin and Shankar (2009), a physical store may experience up to a 20% increase in sales if the online store's policy allows for cross-channel returns. Thus, another study in the field of dual-channel retailing and customer returns might include cross-channel selling opportunities in the system analysis.

Regarding the profitability of the online store and the retailer, they are higher under strategy $i = co$ than under strategy $i = ro$ if $\alpha > \ddot{\alpha}$, they are lower if $\alpha < \ddot{\alpha}$, and they are equal if $\alpha = \ddot{\alpha}$. Therefore, managers should pay a careful attention to the value of α. This value is controllable as the online store can invest in several other reverse supply chain options. For example, the service of picking up items from customers' locations at their chosen times may increase α drastically. Indeed, such a service may be costly to the online channel, and switching to another strategy could be a wiser decision. Thus, studying the benefits and drawbacks of the provided services are vital to the success of the enterprise. Nonetheless, one of the parameters that has a clear effect on the threshold $\ddot{\alpha}$ and can be controlled by the enterprise is the value t_{ro}. As per relationship (5.15), $\ddot{\alpha}$ increases as t_{ro} decreases. Thus, if t_{ro} is too small, then it might be hard for strategy $i = co$ to outperform strategy $i = ro$. This could be the case when the online store performs the function of a distribution center and ships items to the physical store on a regular basis. Thus, empty trucks could return to the online store and be used freely (e.g. $t_{ro} = 0$) to ship cross-channel returns to their original points of purchase.

For the physical store to prefer strategy $i = D$ over the no-inventory shift strategies, i.e. $\pi_r^{D^*} > \pi_r^{ro^*}$ and $\pi_r^{D^*} > \pi_r^{co^*}$, the amount paid to the online store for each cross-channel return ($\hat{c}$) should be less than $s - d + k_{or}(c - s)$. If $\hat{c}$ is higher than the indicated value, then the physical store will benefit more from satisfying its entire inventory needs from the supplier. If they are equivalent, then the store will not generate any extra measurable profit or loss under the strategy, i.e. $\pi_r^{D^*} = \pi_r^{ro^*} = \pi_r^{co^*}$. Note that based on the parameters given above, $s - d + k_{or}(c - s) = 12$.

For the online store to prefer strategy $i = D$ over the no-inventory shift strategies, i.e. $\pi_o^{D^*} > \pi_o^{ro^*}$ and $\pi_o^{D^*} > \pi_o^{co^*}$, the amount paid by the physical store for each cross-channel return should be sufficient. The minimum $\hat{c}$ that allows for $\pi_o^{D^*} > \pi_o^{ro^*}$ is calculated next. By applying formula (5.29) and using the above parameters, one gets $0.097\hat{c}^2 + 26.30\hat{c} - 213.53 = 0$. Solving the quadratic equation reveals that $\hat{c} = \{-278, 7.9\}$. Thus, when $\hat{c} > 7.9$, $\pi_o^{D^*} > \pi_o^{ro^*}$. By applying the same formula and procedure for strategy $i = co$ one finds that when $\hat{c} > 9$, $\pi_o^{D^*} > \pi_o^{co^*}$. The above outcomes can also be found in Figure 5.2 when $\hat{c}$ changes from 5 to 12.

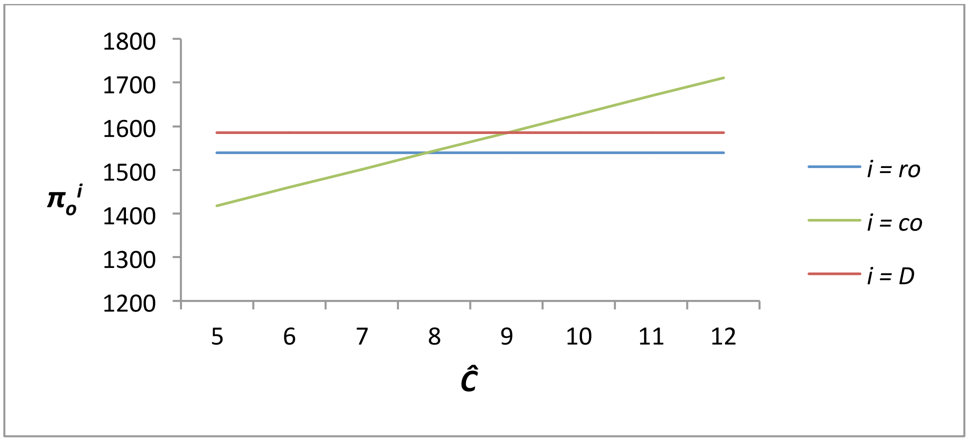


Figure 5.2: $\hat{c}$'s Effect on Online Stores' Profits

In (5.27), one may notice that as $\hat{c}$ increases within the range $[0, s - d + k_{or}(c - s))$, the retailer's profit under strategy $i = D$, i.e. $\pi_t^{D^*}$, increases. Conversely, as $\hat{c}$ increases within the range $(s - d + k_{or}(c - s), \infty]$, $\pi_t^{D^*}$ decreases (Figure 5.3). At the point that $\hat{c} = s - d + k_{or}(c - s)$, the strategy performs at its full potential and is equivalent to strategy $i = C$, i.e. $\pi_t^{D^*} = \pi_t^{C^*}$, $Q_r^{D^*} = Q_r^{C^*}$, and $Q_o^{D^*} = Q_o^{C^*}$. According to our parameters, one may note that the difference in the retailer's performances $\pi_t^{C^*} - \pi_t^{D^*}$ is insignificant. Note that under different parameters the difference may increase to a significant level.

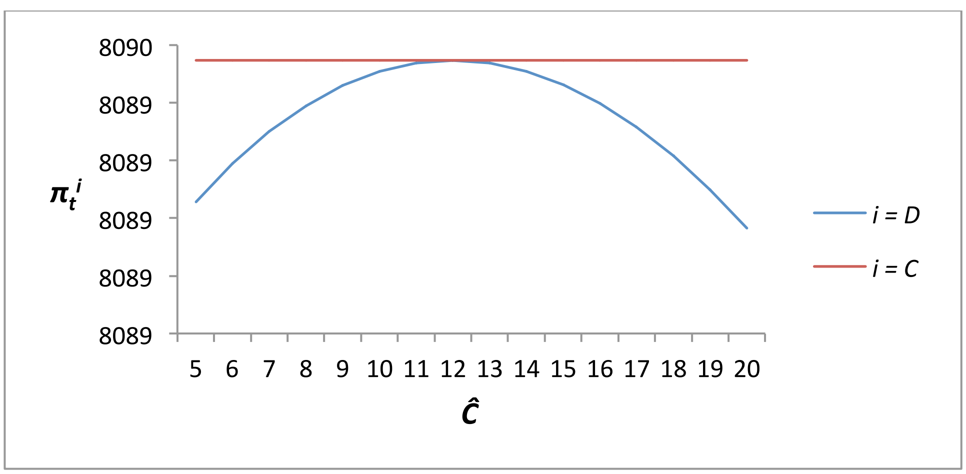


Figure 5.3: $\hat{c}$'s Effect on Retailers' Profits

Thus, if our focus is aimed at the total performance rather than at a channel's performance, the physical store should pay the maximum value of $\hat{c}$ to the online store.

Consequently, there will be no tangible profit increase for the physical store under strategy $i = D$. This could be considered a form of channel cooperation. One would need to remind the physical store that the online store would also be involved in this cooperation indirectly. For example, cross-channel returns increase the physical store's sales, as stated before. Moreover, items' descriptions posted on the online store's website would positively impact the physical store's sales and, eventually, its profit. Indeed, many physical store customers have a desire to purchase certain products only after going through the items' descriptions. Additionally, it is known that centralization is expensive and requires a significant amount of effort. Since the two strategies $i = D$ and $i = C$ are equivalent in performance under the highest value of $\hat{c}$, then we see no point to centralization.

The rates k_o and k_{or} induce tangible and contradicting effects on the superiority of inventory shift strategies, i.e. $i = D$ and $i = C$, over no-inventory shift strategies, i.e. $i = ro$ and $i = co$. In Figure 5.4, one may notice that as k_o increases the no-inventory shift strategies generate higher profits and start to perform better than the inventory shift strategies at a similar value of $k_o \approx 0.525$. Conversely, as k_{or} increases, the inventory shift strategies generate higher profits and start to perform better than the no-inventory shift strategies at a similar value of $k_{or} \approx 0.525$. Notice that both rates, to some extent, can be controlled by the enterprise. For example, simple measures such as sending emails to customers and asking for their feedback regarding a product's look and functionality may be used. A return form and a return procedure may be attached to that email. Such an approach might positively increase the resalability rates k_o and k_{or}. Additionally, the online store could be provided with a refabricating facility to fix damaged items and thus increase the online store's resalability rate of k_o. Indeed, the enterprise must weigh the financial benefits and drawbacks of such measures to better judge their effectiveness.

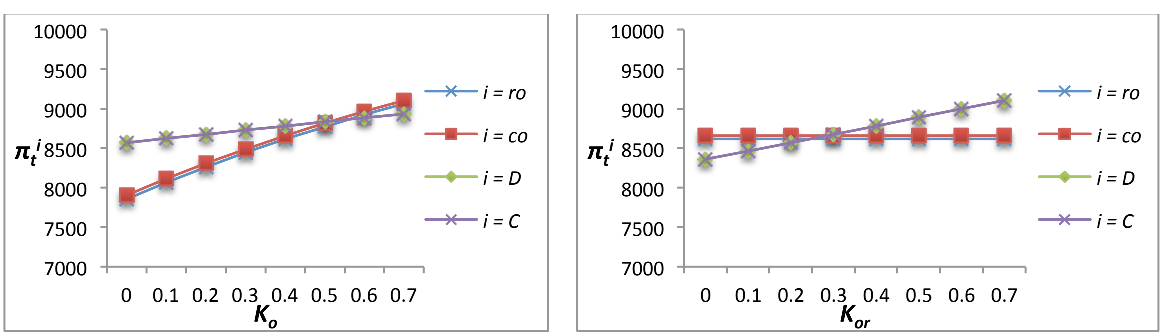


Figure 5.4: $\boldsymbol{k_o}$'s and $\boldsymbol{k_{or}}$'s Effects on Retailer's Profit

5.8 Conclusion

Identifying the optimal order quantity through the use of the newsvendor model is a tangible problem in the field of supply chain management. As customers are able to return their purchases and a high percentage of those returns are considered as good as new and resalable, many retailers seem to overestimate their order quantities. Thus, researchers have expanded the newsvendor model to include customer returns. Today's retailing businesses normally run both a physical store and an online store to meet customers' desires to shop through multiple channels. The complications that customer returns impose on the optimal quantities of a dual-channel retailing system have not yet been sufficiently studied. Customers can return items to the items' original points of purchase. They also can cross-return online purchases to physical stores. Cross-channel returns may destabilize the retailing system and retailers may simultaneously overestimate one channel and underestimate another. Thus, in Chapter 5 we developed newsvendor models that consider both same- and cross-channel returns experienced by a dual-channel retailer running a physical store and an online store. Four strategies that may be used to handle cross-channel returns were studied. The first strategy was to collect all cross-channel returns on a regular basis and to send them back to the online store ($i = ro$). The second strategy was to ban cross-channel returns ($i = co$). The third and fourth strategies were to transfer the ownership of all cross-channel returns to the physical store under decentralized ($i = D$) and centralized ($i = C$) management styles, respectively. Our results confirmed that selecting the right policy might noticeably increase a system's performance and profitability. To simplify the analysis of the different strategies, uniform distributions were assigned to sales requests in both channels.

The optimal order quantity for the physical store (the online store) under any inventory shift strategy is at most (least) equivalent to its order quantity under any no-inventory shift strategy. Additionally, the order quantity for the physical store (the online store) under strategy $i = D$ is at least (most) equivalent to its order quantity under strategy $i = C$. If customer loyalty to a retailer's online store exceeds a threshold of $\dot{\alpha}$, then the online store will order a higher quantity under strategy $i = co$ than under strategy $i = ro$.

If customer loyalty to the retailer's online store exceeds a threshold of $\ddot{\alpha}$, then the online store will generate a higher profit under strategy $i = co$ than under strategy $i = ro$. This threshold increases as the cost of shipping each unit from the physical store to the online store decreases. The physical store is encouraged to be part of the decentralized inventory shift strategy if the amount paid to the online store for each cross-channel return ($\hat{c}$) is less than $s - d + k_{or}(c - s)$. However, the online store is encouraged to be part of this strategy if $\hat{c}$ derives p_o^D to be higher than $\overline{p_o^D}$. Both the decentralized and centralized inventory shift strategies have exact quantities and total performance when $\hat{c}$'s value is equivalent to $s - d + k_{or}(c - s)$.

CHAPTER 6: OPTIMAL ONLINE STORE'S OUTSOURCING STRATEGY

6.1 Introduction and Motivation

Customer returns have a massive effect on a DCR's business functions, especially on the field of operations management. The failure to satisfy customers' orders and the expenses associated with customer returns were among the major reasons for unsuccessful e-tail attempts. Certainly, potential unsatisfied customers may exhaust the enterprise with forward and reverse logistics. It is common for e-tail stores to heavily invest on marketing activities, while exerting much less efforts on managing customers' orders and returns. Indeed, an elegant online store is useless if it is not able to deliver goods as promised, deal with customer returns, and still generate profit. Therefore, successful management of channels' inventory levels when customer returns are allowed is vital for the DCR existence.

Over the past years, third party logistics and service providers (herein called providers) have drastically developed their competences to support services such as transportation, warehousing, inventory management, fleet management, production management, and many others. The management of a retailer-provider relationship is a complex task and literature has reported conflicting results with regard to outsourcing (Hartmann and de Grahl 2012). However, successful outsourcing for services can boost a retailer's competitiveness and improve its performance.

In today's market, several common inventory, logistics and warehousing strategies are being used by DCRs. For example, a number of firms pile their selling items in a single warehouse, i.e. a big store or a distribution center, from where both e-tail and retail stores demand fulfillment are satisfied. Such businesses include The Home Depot, and Wal-Mart Stores (Yan 2008). Other DCRs use two different locations or two different inventory piles in the same location to fulfill demand from each channel. According to Yao et al. (2009), Penny is implementing such a business strategy to avoid the intermixing of inventory items for both stores. Other firms adopt such a strategy to avoid the expensive and difficult coordination process between channels (Zhang et al. 2010; Yan 2008; Berger et al. 2006).

It is common to have selected e-tail store-related logistics and warehousing tasks outsourced to one or several providers. Such outsourcing strategy allows a firm to improve its services and concentrate on its main competency, while exploiting external resources and expertise. For example, around 70% of the retailing firms outsource the transportation needs to third party logistics providers (Lei et al. 2006). According to Min

(2013), outsourcing policies range from a simple per service charge (most common) to a real partnership with gain sharing (least common). Modern examples of this partnership are the alliance between several logistics providers and Sheetz Corporation as well as the alliance between Transplace and AutoZone Inc. According to Lei et al. (2006), profit sharing is the vital compensation policy that enabled the success of the previous partnerships. Also, Toys“R”Us had formed a well known alliance with Amazon.com to provide inventory management, site development, order fulfilment, and customer services for duration of ten years (Berger et al. 2006). Amazon.com, in this case, housed the inventory of Toys“R”Us in its distribution centers.

Research on outsourcing logistical activities to a third party logistics and service provider has been growing noticeably in recent years. For example, Giri, and Sarker (2017) examined the supply chain’s performance under a manufacturer, a third party logistics service provider, and multiple independent retailers facing price sensitive and uncertain demand. Buyback and revenue sharing contracts are implemented to coordinate the decentralized supply chain wherein production disruption may take place in the manufacturing facility. He et. al (2016) studied several strategies that can be implemented between the manufacturer’s and retailer’s online stores. They assumed demand to be influenced by price, national advertising effort, and logistics service level. Jiang et al. (2014) investigated decision and coordination in a system consisting of a manufacturer, a third-party logistics provider, and two competing retailers wherein the product distribution functions are implemented. Cai et. al (2013) considered a producer providing a fresh product to a distant distributor who sells it to end customers. The product is transported through a third-party logistics provider where both quantity and quality may deteriorate during the transportation process. They assumed market demand to be stochastic, and sensitive to selling price and product freshness. Liu et al. (2013) investigated the fairest revenue-sharing coefficient for a two-echelon system consisting of a logistics integrator and provider, and a three echelon system consisting of a logistics integrator, provider and subcontractor. Through the use of profit distribution equity, a fair entropy function is introduced and the fairest coefficient is measured. Lei et al. (2006) studied the impact that coordination and pricing policies have on system’s profitability when the cost function of the logistics provider is concave in nature. Jharkharia and Sankar (2007) proposed two methodologies that can assist in the selection process of a third-party logistics and service provider: preliminary screening of the available providers, and analytic network process-based final selection. Fabbe-Costes et al. (2009) examined the logistics providers’ role in supporting supply chain integration and clients’ performance. Lim (2000) developed a game-theoretic model that examines the contractual agreement between a third party logistics buyer and a third party logistics provider. The paper assumes that service quality and service cost to be known to the logistics provider only. Chen et al. (2001) analyzed third party warehousing contracts wherein demand on space is uncertain. A contract is set by an initial space commitment and it can be modified for a certain number of times. Alp et al. (2003) studied designing the parameters of a specific transportation contract that may be used between a transporter and a manufacturer. They claimed that vehicle dispatching, inventory control, and contract value define the contract-designing problem. Their proposed model minimizes the total cost of the manufacturer by considering various parameters combinations. Although its importance in the modern retailing systems, none of the above papers have considered customer returns and their impact on the interactions between a retailer and a third-party logistics and service provider.

However, there is no work that has considered and compared the different inbound/outbound outsourcing options for the online store when a DCR faces customer returns. Thus, a profound task in Chapter 6 is to determine the inventory level of both stores of a DCR when all common forms of customer returns are allowed. This paper considers the case when customers are split in their preference between the two channels and the system's total demand is random. It, also, investigates three different outsourcing policies a DCR may implement with a third-party logistics and service provider as the following:

(1) *Transaction-based fee strategy*: In this strategy, the DCR fulfills retail and e-tail stores' customer demand from two inventory piles or two different locations. With regard to the e-tail store's orders and returns, the firm may depend on its fleet to perform the required logistic activities or hire a provider with a transaction-based fee compensation policy. In such a policy, a dollar amount is paid to the provider per delivered or returned order, without a significant amount of upfront fee. Since this policy lacks a long-term retailer-provider agreement, the fee is assumed to be exogenous to the system and, thus, there exist a sole decision maker within this strategy.

(2) *Flat-based fee strategy*: In this case, the firm only fulfills the retail store's orders from the traditional store. However, a provider is outsourced to manage the e-tail store's orders. As a Stackelberg leader, the provider charges the retailer a quantity dependent flat fee for the entire selling season. As a Stackelberg follower, the retailer decides the order quantity for both stores. Since the retailer is the one making the inventory decision on the e-tail store,

then this strategy is similar to that of the vendor-managed inventory (VMI) strategy.

(3) *Gain sharing strategy*: Similar to the previous strategy, the retailer fulfills the retail store's orders from the traditional store and the provider fulfills the online orders from the e-tail store. On top of the quantity dependent seasonal fee paid by the retailer, the partners strengthen their strategic alliance by sharing the profit of the e-tail store.

6.2 Model Formulation

This research considers a dual-channel retailer with both traditional retail and e-tail stores selling a single item that is perishable or has a short life cycle. Thus, our problem may be modeled using the single period or newsvendor modeling technique. Customers may purchase the product through the DCR's retail store or e-tail store. System's total demand is assumed to be stochastic. A portion θ of buyers prefers to conduct their purchases through the retail store, while a portion $\bar{\theta} = 1 - \theta$ of buyers prefers to conduct their purchases through the e-tail store. Since customers prefer a certain channel over another, their unmet demand from that channel is considered to be lost. This is rational for customers that are channel loyal. For example, e-tail store's customers may enjoy making their purchases online due to store's all day availability and convenience. They are more likely to switch to another online store if the requested items are not available. On the other hand, retail store's customers may seek to touch and feel the product before conducting their purchases. Thus, they are more likely to switch to another product or retail store if their requested items are not available. We assume that there is no transhipment between channels if a product is out of stock due to the shortage of the selling season.

We also assume that system's demand is drawn from a certain distribution and in accordance to customer preference it splits between the two channels. Therefore, if the observed system's demand is x, then the retail store's demand and the e-tail store's demand will be θx and $\bar{\theta} x$, respectively. Demand split has appeared in papers related to the topic of dual-channel supply chain (For example, in Chiang and Monahan 2005, Yao et. al 2009). Also, $f(x)$ is the probability density function of total system's demand. Since the Uniform distribution is tractable and bounded, then we assume that total demand is Uniformly distributed, i.e., $x \sim U[a, b]$. $x = a$ represents minimum possible total demand, while $x = b$ represents maximum possible total demand. The uniform distribution is general enough to capture variability in a short selling season with little or no demand history. This is why it is commonly used in the supply chain literature.

We assume here that channel's selling prices are exogenously determined due to market competitiveness. Thus, p_r and p_e are the unit sale price for retail store and e-tail store, respectively. Unmet demand from the retail store costs the channel a shortage value of g_r, while unmet demand from the e-tail store costs the channel a shortage value of g_e.

Buyers are provided the opportunity to acquire a full refund of their purchases if the items are returned within a time-period specified by the firm. We assume the following: a ratio of r_r from the retail store's total demand will be returned back to the retail store in person, a ratio of r_{er} from the e-tail store's total demand will be cross-returned back to the retail store in person, and a ratio of r_e from the e-tail store's total demand will be returned back using the e-tail store's logistic services. Due to the seasonal length constraint, we assume that returned items may not be resold again in the same selling season. However, they can be salvaged in a secondary market with a value of s_r.

We assume that an unsold item can be salvaged for a value higher than that of a returned item, i.e. $s_n > s_r$. This assumption is satisfactory since salvagers (i.e. secondary market customers or the manufacturer) can, normally, distinguish between sold and unsold items. Both salvage values are assumed to be less than the unit's purchasing cost, i.e. $w > s_n > s_r$. If this is not the case, then the profit function will be unbounded above. Due to the economy of scale, a provider incurs a lower forward/return handling cost compared to a retailer who uses his own fleet or pays the provider a transaction fee for each logistical service performed, i.e. $h_l < h_r$.

Indeed, many DCRs fail in their first attempt to establish successful e-businesses that can balance between the services provided to customers and deal with the negative consequences of customer returns. Therefore, the inventory strategy implemented when customer returns are allowed is a vital decision to be made from the retailer's perspective. This work offers inventory related decision-making models for a DCR aiming to maximize channels' profits wherein customer returns are permitted and demand is uncertain. For suitability, we refer to the retailer as he, and to the provider as she. Next, we study a centralized case where the retailer uses his own logistics fleet or uses a provider with a transaction-based fee compensation strategy. Table 6.1 presents a summary of the notations used in Chapter 6.

Notation	Description
$\boldsymbol{r_e}$	Probability an item purchased from a retail store is returned to the retail store
$\boldsymbol{r_e}$	Probability an item purchased from an e-tail store is returned to the e-tail store
$\boldsymbol{r_{er}}$	Probability an item purchased from an e-tail store is cross-returned to a retail store
$\boldsymbol{w}$	An item wholesale price or purchasing cost
$\boldsymbol{p_r}$ & $\boldsymbol{p_e}$	Unit sale price for retail store and e-tail store, respectively
$\boldsymbol{s_n}$ & $\boldsymbol{s_r}$	Unit salvage value for new items and open items, respectively. Note that $w > s_n > s_r$
$\boldsymbol{h_r}$ & $\boldsymbol{h_l}$	Forward/reverse per unit transaction handling cost incurred by retailer and provider, respectively. Note that $h_r > h_l$
$\boldsymbol{g_r}$ & $\boldsymbol{g_e}$	Unit shortage cost for the retail channel and e-tail channel, respectively
$\boldsymbol{x}$	Total system's demand
$\boldsymbol{f(x)}$	Probability density function of total system's demand
$\boldsymbol{\theta}$	Customer preference for the retail store
$\boldsymbol{\bar{\theta} = 1 - \theta}$	Customer preference for the e-tail store
$\boldsymbol{Q_r}$ & $\boldsymbol{Q_o}$	Quantities ordered for retail and e-tail stores, respectively
$\boldsymbol{\pi_i^r}$ & $\boldsymbol{\pi_i^e}$	Retail store and e-tail store profits, respectively
$\boldsymbol{\pi_i^L}$	Third party logistics and service provider's profit
$\boldsymbol{F_i}$	Flat fee or seasonal fee paid to the provider at the beginning of the selling season
$\boldsymbol{i}$	Strategy used to manage the e-tail store; where $i = \{1,2,3\}$

Table 6.1: Sixth Chapter's Notations

6.3 Transaction-Based Fee Strategy ($\boldsymbol{i = 1}$)

In this setting, the provider is not a key player, and the retailer fully controls the decision making process of the whole system. Therefore, there is no strategic alliance between the retailer and the provider. Channels' demands are satisfied from segregated inventories within the same facility or from two different stores. Companies, such as Penny, use segregation to avoid items intermingling, and, thus, assure satisfying e-tail store's accepted orders. E-tail orders and returns are shipped from or back to the facility using the retailer's own fleet or using a provider where she is paid a service fee per delivery. Thus, the retailer optimizes two objective functions and the decision variables here are retail store's inventory level Q_r and e-tail store's inventory level Q_e.

The revenue generated by satisfying a single demand from the e-tail store is $p_1^e = (1 - r_e - r_{er})p_e + (r_e + r_{er})s_r - (1 + r_e)h_r$. In the first term, there is a $(1 - r_e - r_{er})$ chance that a sold item is a final sale and contributes positively due to the selling price. In the second term, there is a $(r_e + r_{er})$ chance that a sold item is returned back to the retailer and contributes positively due to the salvage value. The third term measures the expected handling or shipping cost acquired when a single customer demand is met. Similarly, the expected revenue generated by satisfying a single demand from the retail store is $p_i^r = (1 - r_r)p_r + r_r s_r$. Due to the similarity, a detailed explanation of the previous relationship has been omitted. For the e-tail and retail stores to generate profits, the following conditions are essential $p_1^e > w$ and $p_i^r > w$. The total expected profit for the retail store is modeled in Eq. (6.1):

$$\pi_i^r(Q_r) = \int_a^{\frac{Q_r}{\theta}} (\theta x p_i^r + (Q_r - \theta x)s_n)f(x)dx + \int_{\frac{Q_r}{\theta}}^{b} (Q_r p_i^r - (\theta x - Q_r)g_r)f(x)dx - wQ_r \tag{6.1}$$

The first term in Eq. (6.1) is the expected profit when the channel's demand θx is less than or equal to the order quantity Q_r. It considers the expected revenue generated by the channel and the salvage value for unsold items. The second term is the expected profit when the channel's demand θx is more than the order quantity Q_r. This is the expected revenue generated by satisfying orders excluding the shortage cost for unsatisfied demand at the retail store. The third term is the purchasing cost associated with the inventory level Q_r. Similarly, the total expected profit for the e-tail store could be modeled as:

$$\pi_1^e(Q_e) = \int_a^{\frac{Q_e}{\bar{\theta}}} (\bar{\theta} x p_1^e + (Q_e - \bar{\theta} x)s_n)f(x)dx + \int_{\frac{Q_e}{\bar{\theta}}}^{b} (Q_e p_1^e - (\bar{\theta} x - Q_e)g_e)f(x)dx - wQ_e \tag{6.2}$$

Given the aforementioned expected profit functions, we find the optimal inventory levels as follows:

Proposition 6.1

The optimal order quantity for the retail store and the e-tail store are given respectively as follows:

$$Q_i^r = \theta\left(b - \frac{(b-a)(w - s_n)}{p_i^r + g_r - s_n}\right)$$

$$Q_1^e = \bar{\theta}\left(b - \frac{(b-a)(w - s_n)}{p_1^e + g_e - s_n}\right)$$

Proposition 6.1 gives closed-form solutions of the retail and e-tail stores' optimal order quantities. If all customers prefer the retail store (i.e. $\theta = 1$), and returns are not allowed by the store (i.e. $r_r = 0$), then optimal order quantity is similar to that of the classical newsvendor problem. Also, Proposition 6.1 shows that the variation in Q_i^r and Q_1^e when θ changes are different. It is interesting to see that as customer preference to the retail store, i.e. θ, increases, the inventory level in the retail store increases at a rate of

$\left(b - \frac{(b-a)(w-s_n)}{p_i^r+g_r-s_n}\right)$ and the inventory level in the e-tail store decreases at a rate of $\left(b - \frac{(b-a)(w-s_n)}{p_1^e+g_e-s_n}\right)$. We can, also, state that as θ increases the total retailer's inventory level, i.e. $Q_i^r + Q_1^e$, changes in correspondence to the rates in both channels. That is to say, total inventory level will increase as θ increases if $p_1^e + g_e < p_i^r + g_r$, will decrease as θ increases if $p_1^e + g_e > p_i^r + g_r$, and will stay unchanged if $p_1^e + g_e = p_i^r + g_r$.

6.4 Flat-Based Fee Strategy ($i = 2$)

In this section, we consider another outsourcing strategy that requires a higher involvement of the provider. Presently, several companies outsource e-tail store's orders fulfillment to a provider, because they lack the competency needed to handle small and unstable customer demand. Normally, those providers have validated their competency and acquired fairly noticeable reputation in the market. Such outsourcing strategy allows retailers to avoid capital investment while improving flexibility, productivity, and customer satisfaction. For example, HP has a warehouse in Memphis that is used to fulfill e-tail store's customer orders. The management of the entire warehouse (i.e. facility layout, and orders transaction processing, picking up, labeling, bar coding/RFID, packaging and packing) and the execution of orders delivery were outsourced to FedEx. This form of integration enabled the retailer to better use the provider's efficient operation and economy of scale. Therefore, reducing the unit handling cost for sales conducted over the Internet. Another way to apply this strategy is when the provider is used to carryout the inventory of the e-tail store in her own distribution center. An example for that is Global Sports outsourcing Kmart.com (Yao et al. 2009). When customer returns are allowed, then the provider is responsible for the fulfillment of e-tail store's orders and returns. The retailer in return pays a seasonal fee that is correlated to the size of the business. Rationally, this business size can, also, be correlated to the number of units the provider will manage, i.e. Q_e. We notify here that the retailer still decides on the quantity ordered for the e-tail store.

In this strategy, both the retailer and provider select their decisions in isolation to maximize their individual profits. Despite the fact that outsourcing is referred to as a 'strategic alliance', the partners involved may have contradicting interests. Consequently, it is vital for the firm to compare between the negative consequences stemmed from double marginalization and those incurred by the high handling fees. We need to point out here that it is not always advantageous for a firm to outsource a logistics provider when she can be hired for a single service instead. In this study, we assume that the retailer and provider undergo a Stackelberg competition where the provider is the leader and the retailer is the follower. The game's decisions sequences are as the following:

(1) In order to maximize her expected profit, the provider decides first on the flat fee (F) paid by the retailer in response to the management of each online item. This is done given the expected response function of the e-tail store that is run by the DCR. We assume here that the provider has full knowledge of the parameters associated with customer demand and return behavior in the e-tail store.

(2) In response to the provider's decision, the retailer decides on the order quantity of the e-tail store, i.e. Q_e, to maximize his own expected profit.

A similar procedure has been implemented by Giri and Sarker (2017) study wherein the retailer places the order size in response to the service charge set by the provider. Notice that this strategy has no effect on the decision process carried at the retail store. Also, the revenue generated by satisfying a single demand from the e-tail store becomes $p_2^e = (1 - r_e - r_{er})p_e + (r_e + r_{er})s_r$.

6.4.1 The Retailer's Problem

The expected profit for the e-tail store is formulated as the following:

$$\pi_2^e(Q_e|F) = \int_a^{\frac{Q_e}{\theta}} (\bar{\theta}xp_2^e + (Q_e - \bar{\theta}x)s_n)f(x)dx + \int_{\frac{Q_e}{\theta}}^b (Q_e p_2^e - (\bar{\theta}x - Q_e)g_e)f(x)dx - (w + F)Q_e \tag{6.3}$$

Due to similarity, a detailed explanation of the previous profit function is omitted. In the coming proposition we find the best response function of the e-tail store (the Stackelberg follower) given F.

Proposition 6.2

The optimal order quantity for the e-tail store is as follows:

$$Q_2^e = \bar{\theta}\left(b - \frac{(b-a)(w + F - s_n)}{p_2^e + g_e - s_n}\right)$$

From Proposition 6.2, one may notice that for as long as the seasonal fee F is less than the term $\left(\frac{b(p_2^e+g_e-s_n)}{(b-a)} - (w - s_n)\right)$, then as θ increases the inventory level in the e-tail store decreases at a rate of $\left(b - \frac{(b-a)(w+F-s_n)}{p_2^e+g_e-s_n}\right)$. One may, also, notice that the satisfaction of such a condition promotes a positive inventory level in the e-tail store. Corollary 6.1 proofs that this is always the case under the Stackelberg competition.

6.4.2 The Provider's Problem

The profit function of the provider (the Stackelberg leader) can be modeled as the following:

$$\pi_2^L(F) = Q_e F - \int_a^{\frac{Q_e}{\bar{\theta}}} \bar{\theta} x c f(x) dx - \int_{\frac{Q_e}{\bar{\theta}}}^{b} Q_e c f(x) dx \tag{6.4}$$

The first term in Eq. (6.4) is the total seasonal fee paid by the retailer to acquire the provider's logistical and handling services. The second term is the expected handling expenditure when the e-tail store's demand is less than or equal to the order quantity Q_e. The third term is expected handling expenditure when the e-channel's demand is more than the channel's order quantity. Notice that $c = h_l(1 + r_e)$ is the expected handling cost incurred by the provider when a single customer demand is met. In the next proposition we find the provider's optimal policy given the retailer's best response function.

Proposition 6.3

The optimal seasonal fee for each item handled by the provider is as follows:

$$F_2 = \frac{\frac{b}{b-a}(p_2^e + g_e - s_n)^2 - (w - s_n)(p_2^e + g_e - s_n - c)}{2\left(p_2^e + g_e - s_n - \frac{c}{2}\right)}$$

Proposition 6.3 gives a closed-form solution of the optimal seasonal charge for each e-tail store's item managed by the provider. One may notice that this charge is influenced by the revenue acquired when a single online request is met, purchasing cost, salvaging value, shortage cost, return rates, and handling costs. Nonetheless, it is interesting to see that customer preference to a certain channel has no effect on this charge from the provider's perspective.

Corollary 6.1: $F_2 < \frac{b(p_2^e + g_e - s_n)}{(b-a)} - (w - s_n)$

Corollary 6.1 shows that the optimal seasonal fee does not prevent the retailer from conducting business with the logistics provider. In another word, the e-tail store's inventory level is always positive, i.e. $Q_2^e > 0$, when the involved parties undergo Stackelberg competition.

6.5 Gain Sharing Strategy ($i = 3$)

This section studies a scheme wherein the parties involved further strengthen their alliance or partnership. The study Hartmann and de Grahl (2012) has identified that sharing of benefits and burdens are among the most important factors in the formation of retailer-provider successful alliance. It reflects the parties' willingness to accept hardships and success. In this compensation policy, each partner has a specific role to perform and they eventually share the total revenue of the e-channel. On top of that, the provider may charge the retailer a seasonal fee for each item assigned to the e-channel. An example of this strategy is the long-term alliance that was established by Toys "R" Us and Amazon.com during the year of 2001. Toys "R" Us was responsible of identifying, buying and managing inventory, while Amazon.com was responsible of developing site, fulfilling customer orders, conducting customer services and carrying inventory in her own distribution center. Under the terms of their agreement, Amazon.com will be rewarded with fixed payments, per unit payments, and a share of the total revenue.

According to Min (2013), due to the substantial risk involved in the gain sharing compensation policy, it is scarcely being used despite the possible enhancement in the firms' long-term performance, productivity and profitability. Another reason that caused the low popularity of the gain sharing policy is the difficulty in specifying a share for each partner that will govern a win-win situation. Also, the performance stress exerted upon the provider when such a partnership is offered may leave the management reluctant to accept it. The aforementioned facts give this work a tangible potential to help retailers form a successful alliance with their logistics providers.

We assume here that all costs and revenue associated with the e-tail store is shared between partners. The retailer's share is $\emptyset$, while the provider's share is $1-\emptyset$. The value of $\emptyset$ is influenced by each partner's market position and, thus, negotiation power. It, also, reflects each partner's share in the responsibilities needed for the success of the e-channel. On top of that, the retailer is charged a seasonal fee for each item managed by the provider.

Notice that the retail store's inventory decision is similar to what it is in the previous sections. However, $p_3^e = (1-r_e-r_{er})p_e + (r_e+r_{er})s_r - (1+r_e)h_l$ is the revenue generated by satisfying a single sale at the e-tail store. Again, we apply Stackelberg competition when the provider is the game leader and the retailer is the game follower. The retailer's and the provider's profit functions from the e-channel are $\pi_3^e(Q_e|F) = \emptyset\pi_{gs} - FQ_e$ and $\pi_3^L(F) = (1-\emptyset)\pi_{gs} + FQ_e$, respectively, where:

$$\pi_{gs} = \int_a^{\frac{Q_e}{\bar{\theta}}} (\bar{\theta}xp_3^e + (Q_e - \bar{\theta}x)s_n)f(x)dx + \int_{\frac{Q_e}{\bar{\theta}}}^b (Q_e p_3^e - (\bar{\theta}x - Q_e)g_e)f(x)dx - wQ_e \tag{6.5}$$

Proposition 6.4

The optimal order quantity for the e-tail store and the optimal seasonal fee for each item handled by the provider are given, respectively, as:

$$Q_3^e = \bar{\theta}\left(b - \frac{(b-a)\left(\frac{F}{\emptyset} + w - s_n\right)}{p_3^e + g_e - s_n} \right)$$

$$F_3 = \frac{\emptyset^2}{\emptyset + 1}\left(\frac{b(p_3^e + g_e - s_n)}{(b-a)} - (w - s_n) \right)$$

Proposition 6.4 considers the optimal decisions when the compensation policy includes both gain sharing and seasonal fee. We notify readers that the two partners can extract a higher surplus out of the e-channel by centralizing the decision-making process and, thus, eliminating double marginalization and all forms of retailer-provider competition. In the fully centralized case, the seasonal fee becomes an internal parameter. Thus, there is a single decision to be made and that is the channel's optimal inventory level. It can be easily calculated using the above stated formula given that $F = 0$. Consequently, centralization promotes a higher inventory level compared to decentralization. For centralization to be applicable, retailer's share from the e-channel should be lowered such that each player is guaranteed a higher surplus.

Corollary 6.2: $F_3 < \emptyset\left(\frac{b(p_3^e + g_e - s_n)}{(b-a)} - (w - s_n) \right)$

Similar to Corollary 6.1, the aforementioned corollary shows that the gain sharing contract does not prevent the retailer from conducting business with the logistic provider. Therefore, the e-tail store's inventory level is always positive under such a setting, i.e. $Q_3^e \geq 0$.

Corollary 6.3

i. $\frac{\partial F_3}{\partial \emptyset} = \frac{(\emptyset + 2)F_3}{\emptyset(\emptyset + 1)} > 0.$

ii. $\frac{\partial Q_3^e}{\partial \emptyset} = -\frac{\bar{\theta}}{(\emptyset + 1)^2}\left(\frac{b(p_3^e + g_e - w) + a(w - s_n)}{p_3^e + g_e - s_n} \right) < 0.$

iii. $F_3 \leq F_2$.

Corollary 6.3 indicates that as the provider's share from the gain of the e-channel increases, i.e. $\emptyset$ decreases, the seasonal fee decreases. In another word, a more attractive compensation policy causes a lower initial payment required for services. Consequently, the retailer is encouraged to increase the order quantity in an attempt to increase his own profitability. This, in turns, will positively reflect back on both the provider's performance and the e-channel's performance, as it is yet to come in Corollary 6.5. Due to the same logic stated above, the seasonal fee when the provider shares the e-channel's gain, i.e. F_3, is lower than the seasonal fee when the provider does not share the e-channel's gain, i.e. F_2.

Corollary 6.4

i. *It is always true that* $Q_2^e \leq Q_3^e$.

ii. *If the condition* $h_l \leq h_r - \frac{(p_2^e+g_e-s_n)(b(p_1^e+g_e-w)+a(w-s_n))}{(w-s_n)(b-a)(1+r_e)}$ *holds, then* $Q_1^e \leq Q_2^e$. *Otherwise,* $Q_1^e > Q_2^e$.

iii. *If the condition* $\emptyset \geq \frac{(b-a)(w-s_n)(p_3^e-p_1^e)}{(b(p_1^e+g_e-w)+a(w-s_n))(p_3^e+g_e-s_n)}$ *holds, then* $Q_1^e \geq Q_3^e$. *Otherwise,* $Q_1^e < Q_3^e$.

Compared to flat-based fee compensation policy, revenue sharing offers a more attractive partnership opportunity for the provider and leads to a lower seasonal payment. This low initial burden on the retailer encourages the e-store's management to further increase the inventory level, i.e. $Q_2^e \leq Q_3^e$. Also, strategically partnering with a provider allows the retailer to increase his e-tail store's initial stocking for as long as she has a high logistical efficiency, i.e. low h_l cost. Consequently, the firm is expected to satisfy a higher demand from the e-channel, become more responsive, and provide better services to customers. However, if the provider does not posses high enough logistical efficiency and her share from the e-channel is low, then the e-tail store's initial stocking may be the highest when the retailer forms no partnership with the provider. Therefore, inventory management in a system with a retailer and a provider is greatly dependent on the type of strategy used, and the efficiency of the logistics and service provider.

6.6 Managerial insight with numerical analysis

The aim of this numerical analysis is to gain additional insights with regard to the optimal policies. The optimal profits for the different strategies will be compared. Also, the change in the optimal profits will be studied under various market conditions particularly those assumed to be exogenous to our model. The results may generate several managerial insights into the avenue of the retailer-provider compensation policies. In this analysis, we normalize the demand such that $x \sim U[0,1]$. This will greatly simplify the different mathematical expressions and yet offer reliable insights. The optimal decision variables and profits are simplified accordingly in Table 6.1 below. Unless otherwise stated the following parameters are used: $r_e = 0.15$, $r_{er} = 0.15$, $r_r = 0.1$, $p_e = 60$, $p_r = 60$, $h_r = 15$, $h_l = 2$, $w = 15$, $s_n = 10$, $s_r = 5$, $g_e = 5$, $g_r = 5$, $\emptyset = 0.7$, $\theta = 0.5$.

Strategy i	Transaction based fee Strategy $i = 1$	Flat-based fee strategy $i = 2$	Gain sharing strategy $i = 3$
Q_i^e	$\bar{\theta}\left(\frac{p_1^e + g_e - w}{p_1^e + g_e - s_n}\right)$	$\bar{\theta}\left(\frac{p_2^e + g_e - w - F}{p_2^e + g_e - s_n}\right)$	$\frac{\bar{\theta}}{\emptyset + 1}\left(\frac{p_3^e + g_e - w}{p_3^e + g_e - s_n}\right)$
F_i	---	$\frac{(p_2^e + g_e - s_n)^2 - (w - s_n)(p_2^e + g_e - s_n - c)}{(2p_2^e + 2g_e - 2s_n - c)}$	$\frac{\emptyset^2}{\emptyset + 1}(p_3^e + g_e - w)$
π_i^e	$\frac{\bar{\theta}}{2}\left(\frac{(p_1^e + g_e - w)^2}{p_1^e + g_e - s_n} - g_e\right)$	$\frac{\bar{\theta}}{2}\left(\frac{(p_2^e + g_e - w - c)^2(p_2^e + g_e - s_n)}{(2p_2^e + 2g_e - 2s_n - c)^2} - g_e\right)$	$\emptyset\frac{\bar{\theta}}{2}\left(\frac{(p_3^e + g_e - w)^2}{(\emptyset + 1)^2(p_3^e + g_e - s_n)} - g_e\right)$
π_i^L	---	$\frac{\bar{\theta}(p_2^e + g_e - w - c)^2}{2(2p_2^e + 2g_e - 2s_n - c)}$	$\frac{\bar{\theta}}{2}\left(\frac{(p_3^e + g_e - w)^2}{(\emptyset + 1)(p_3^e + g_e - s_n)} - (1 - \emptyset)g_e\right)$

Table 6.2: Optimal decision variables and profits when demand is uniformly distributed between zero and one

6.6.1 Effect of Retailer's Share on the Provider's Performance

Corollary 6.5

i. π_3^L *increases as* $\emptyset$ *decreases.*

ii. $\frac{\partial(\pi_3^e + \pi_3^L)}{\partial\emptyset} = -\frac{\bar{\theta}\emptyset}{(\emptyset+1)^3}\frac{(p_3^e + g_e - w)^2}{(p_3^e + g_e - s_n)} < 0.$

iii. $\pi_2^L < \pi_3^L.$

To be realistic, we assume that the retailer's profit is positive, i.e. $\pi_3^e \geq 0$, given any split value $(\emptyset)$. The contrary occurs, i.e. $\pi_3^e < 0$, when the e-channel's shortage value is extremely high. Corollary 6.5 and Figure 6.1 show the increase in the provider's performance as she attains a higher portion of the e-store's gain, i.e. a lower $\emptyset$ value. As indicated in Corollary 6.3, this not only supported by the increase in her own share, but also by the increase in the e-channel's total performance. Corollary 6.5, also, indicates that the provider strictly generates a higher profit under gain sharing strategy compared to flat fee strategy, i.e. $\pi_3^L > \pi_2^L$. With this in mind, she can have her mined set even before the negotiation process starts. Indeed, with gain sharing strategy the provider is expected to be more involved in preforming the different logistical tasks, however, her gain is expected to be worthy especially when we know that it is, probably, from the retailer's best interest not to attain extremely high share.

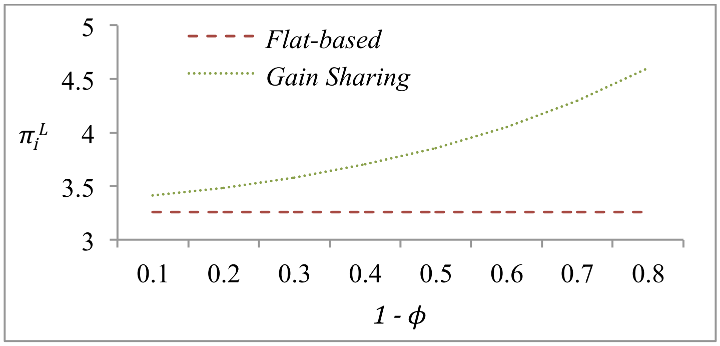


Figure 6.1: Effect of retailer's share on logistics provider

6.6.2 Effect of retailer's share on his performance

Corollary 6.6

i. $\frac{\partial^2 \pi_3^e}{\partial \emptyset^2} = -\frac{\bar{\theta}(2-\emptyset)(p_3^e+g_e-w)^2}{(\emptyset+1)^4(p_3^e+g_e-s_n)} < 0.$

ii. *If there exist a root, i.e.* $\widetilde{\emptyset}$, *for the function* $\emptyset^3 A + \emptyset^2 B - \emptyset C + D = 0$ *within the range* $[0,1]$, *then* $\pi_2^e > \pi_3^e$ *on the range* $\left[0, \widetilde{\emptyset}\right]$ *and* $\pi_2^e \leq \pi_3^e$ *on the range* $\left[\widetilde{\emptyset}, 1\right]$. *Otherwise,* $\pi_2^e \geq \pi_3^e$ *at all values of* $\emptyset$.

The concavity test in the previous corollary shows that it may not be optimal for the retailer to earn the highest possible e-store's revenue share. The retailer needs to consider the pros and cons of having high share and high flat seasonal fee verses somewhat fare share and a lower fee. To find the value $\emptyset$ that enhances the retailer's profit the most, the following relationship is solved: $\frac{\partial \pi_3^e}{\partial \emptyset} = 0$ or $g_e(p_3^e + g_e - s_n)(\emptyset + 1)^3 - (1 - \emptyset)(p_3^e + g_e - w)^2 = 0$. The aforementioned expression has at the most a sole root, i.e. $\widehat{\emptyset}$, within the range $\emptyset = [0,1]$. If found and $\emptyset > \widehat{\emptyset}$, then it is from all partners' interests to lower the retailer's share to the value of $\widehat{\emptyset}$. However, if $\emptyset < \widehat{\emptyset}$, then a contradicting outcomes take place when the retailer's share changes. If not found, then contradictions with regard to splitting the revenue always exist and each player is better off with a higher share. Part *ii* of Corollary 6.6 indicates that the retailer should be careful when it comes to choosing his proper strategy. Although the flat fee strategy may seem quite unpredictable from the retailer's point of view, it could be his best strategy even with high share allocation when undergoing the gain sharing strategy. This, in fact, may trigger conflict between the parties involved that could be eliminated by centralizing the decision process and redefining a new split that guarantees a win-win situation for both partners. From the above analysis and Figure 6.2, we remark that if there exist a value $\widetilde{\emptyset}$ with in the range $[0,1]$, then there exist a value $\widehat{\emptyset}$ with in the range $\left[\widetilde{\emptyset}, 1\right]$.

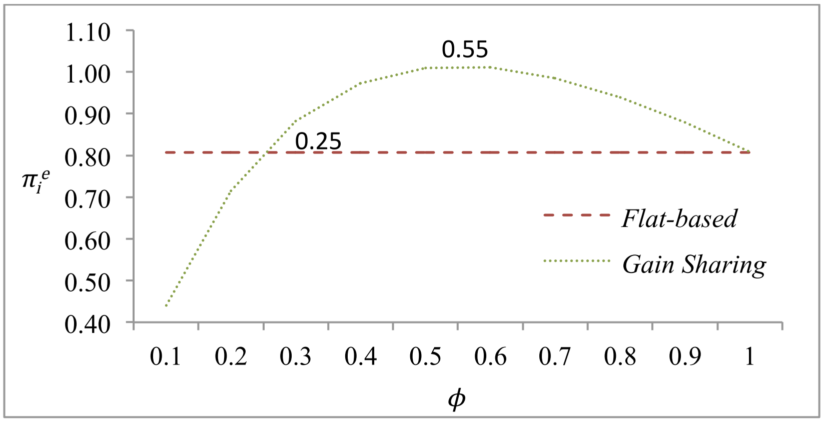


Figure 6.2: Effect of retailer's share on retailer

6.6.3 Effect of return rates on partners' performances and choices

Both forms of returns, i.e. same-channel and cross-channel returns, negatively affect the parties involved. For example, the retailer's profit, from the e-store, decreases as r_e increases due to the burden exerted on the online channel. In an effort to mitigate the losses caused by returns, the retailer should reduce the e-store's inventory level. Interestingly, the provider should, also, decrease the seasonal fee (Figure 6.3 a). Since she is required to exert higher operational and logistical efforts due reverse shipping and handling, one may think the opposite is true. Notice that the increase in the seasonal fee is responded by a further decrease in the inventory level, which may drastically decrease the provider's financial efficiency. It is, also, noticed that customer returns have profound effect on the retailer's choice when it comes to his optimal partnership setting. If the e-store experiences low return rates, then he may perform well under the transaction-based fee strategy. In contrast, if the store experiences high return rates, then a higher logistical involvement and support are needed (Figure 6.3 b). Thus, with low return rates, the loss caused by double marginalization exceeds the handling expenditure paid the retailer. One may find that the opposite is true when the e-channel experiences high return rates. In relation to what indicated above, Hartmann and de Grahl (2012) confirmed that not all partnerships developed with logistics and service providers improve retailers' performances, at least in the short run. To clarify the concept, a different set of variables have been used as follows: $r_e = variable, r_{er} = 0, r_r = 0.1, p_e = 75, p_r = 60, h_r = 20,$ $h_l = 2, w = 7, s_n = 5, s_r = 2, g_e = 2, g_r = 2, \emptyset = 0.7, \theta = 0.5.$

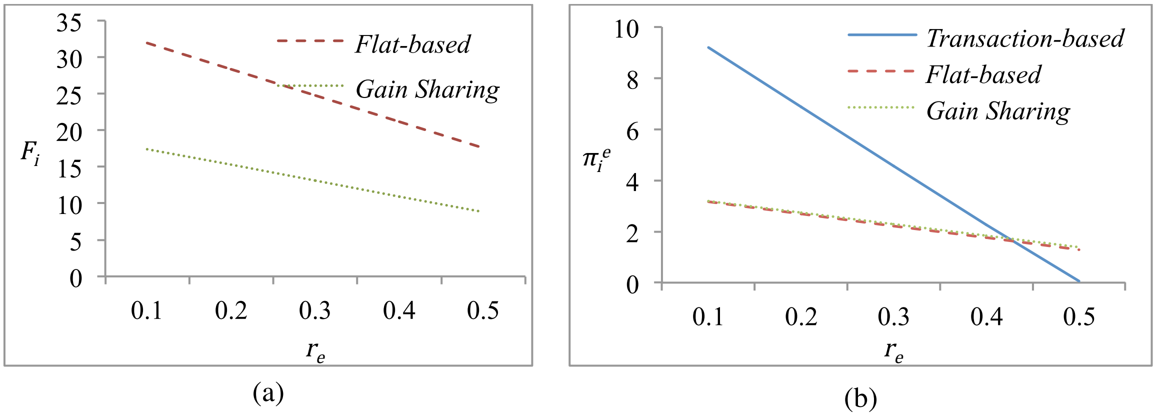


Figure 6.3: Effect of e-tail store's same-channel return rate on partners' performances

However, with the decline in trade barriers, decline in obstacles related to crossing national borders, and increase in transportation and handling efficiencies, current retailing systems are heading towards further globalization. Also, return rates for purchases conducted through online stores are staggeringly high in several documented cases, i.e. up to 70%. Due to the aforementioned facts, there is a tangible demand to have competent and responsive distribution systems that can handle an increasingly complex supply chains. Thus, a logistics and service provider may become an essential partner for a DCR in the coming near future.

6.6.4 Effect of customer preference on the retailer's profitability

Hua et al. (2010) stated that different products lead to different degrees of customer preference for the different channels. Products that are customized, require a high level of examination prior to being purchased, or require after-sale services better-fit retail stores. In contradistinction, products that do not require a high level of examination, standardized, or mature better-fit online stores. They, also, indicated that customer preference for the e-tail store is directly affected by the services provided to customers (e.g. shipping lead time, customer support, technical and shopping assistance, and return services). When hiring a competent logistics and service provider all the aforementioned services are expected to improve. Thus, it is essential for business owners to understand that such a decision may noticeable decrease customer preference to the retail store. Definitely, the retailer's profitability from the e-store may increase (Figure 6.4 a); nonetheless, he may suffer a decline in his total performance due to customers shifting from a more profitable channel to a less profitable one (Figure 6.4 b). For example, public acceptance for purchasing grocery goods through the online channel is increasing. To accommodate such an acceptance, several grocery stores offer pickup points for online orders. However, they have not yet taken tangible leaps towards the implementation of reliable logistical services due to the expected reduction in their total profits.

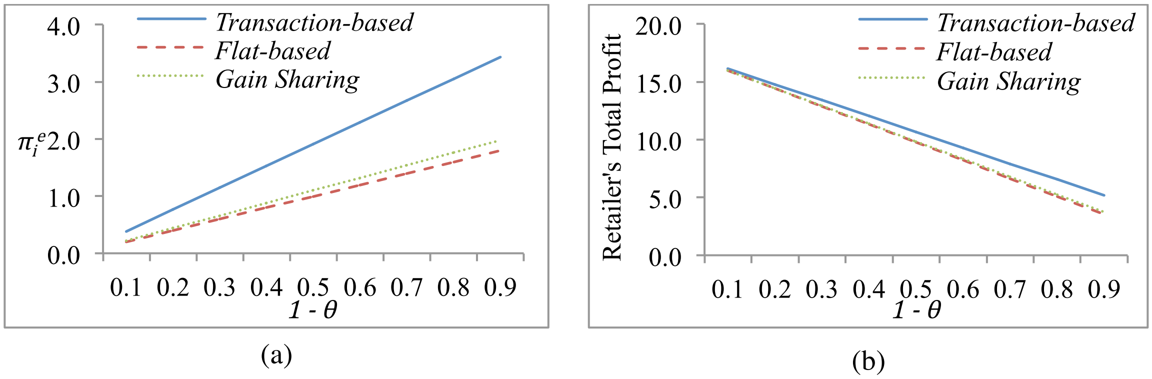


Figure 6.4: Effect of customer preference

6.7 Conclusion

Today's retailing businesses normally run both a retail store and an e-tail store to meet customers' desires to shop through multiple channels. Many of those retailers offer full refunds for unsatisfied purchases and that led to a drastic increase in customer returns especially in the e-channel. Customers can return items to the items' original points of purchase or can cross return the e-channel items to the retail channel. Also, many dual-channel retailers focus on their core competencies while outsourcing e-stores' logistical activities to third-party logistics and service providers. Therefore, we studied three strategies that can be used to handle orders and customer returns under the context of a dual-channel retailer. The first strategy is the transaction-based fee, where a retailer handles all logistical activities using his own fleet or pays a logistics and service provider a per deliver fee. The second strategy is the flat-based fee, where a retailer hand over the logistical operation of his e-tail store to a logistics and service provider for a flat fee per managed item. In the third strategy the provider manages the operations of the e-channel and gains a share of the channel's revenue and a fee per managed item. Under each strategy, we present a profit maximization model to select the optimal inventory level in each channel while considering both same- and cross-channel returns. To make the analysis of the different strategies tractable, the retailer demand is assumed to follow the uniform distribution.

Our study shows that as customer preference to the retail store increases, the inventory level increases in the retail store and decreases in the e-tail store with different proportions. Those proportions, greatly, depend on channels' prices, channels' total return rate and the e-channel's logistical effort. Consequently, the total order quantity for both channels changes as customer preference for a certain channel changes. That is to say, running a dual channel retailing system would not just divide the inventory level between the channels, but would also change its level. We, also, compared the e-channel's performance under the aforementioned strategies. We found it to be better under the flat-based fee or gain sharing strategies when the channel's total return rate is high. However, we found it to be better under the transaction-based fee when the channel's total return rate is low. While the logistics provider always preforms better under the gain sharing strategy compared to the flat-based fee strategy, the retailer's performance greatly depends on his share from the e-channel's gain.

CHAPTER 7: CONCLUSION AND FUTURE RESEARCH

7.1 Conclusion

Many retailers are adopting a dual-channel retailing strategy in which products are offered through two channels: physical stores and online stores. Due to regulations or competitive measures, such a strategy allows customers who find a purchase unsatisfactory to obtain a full refund through a same-channel return (i.e., purchasing an item from a physical store and returning it to the same physical store or purchasing an item from an online store and returning it to the same online store) or a cross-channel return (i.e., purchasing an item from an online store and returning it to a physical store). Therefore, the objective of this dissertation is to analyze the problem of customer returns under a dual-channel retailing system. The main goal is to investigate the different cross-channel return practices or polices, examine several outsourcing options for the management of the online store's operations, and to develop proper methodologies of inventory control management, price management and responsiveness level management.

In Chapter 3 we captured competition and goals conflict that may develop between stores due to cannibalization in sales, scarce resources, or tight objectives. The optimal pricing policies for a centralized and decentralized dual-channel retailer with same- and cross-channel returns are studied. We considered two factors: dual-channel retailer performance under centralization with unified and differential pricing schemes, and dual-channel retailer performance under decentralization with the Stackelberg and Nash games. In this study, a channel's sales requests was modeled as a linear function of a channel's own pricing strategy and a cross-channel's pricing strategy.

Due to the high number of customer returns, one of the online store's important post-purchase services is the responsiveness level of its reverse supply chain. This level greatly influences customer demand, customer loyalty, service expenses, and return resalability. Also, an online store may fulfill the role of a retailer's distribution center. It is not limited to a certain geographical neighbourhood and it is less constrained to area compared to a physical store. Therefore, it has the capability and the need to optimize its responsiveness level along its pricing strategy. Consequently, Chapter 4 is an extension of Chapter 3 and it helps retailers expand their horizon a step further when dealing with customer returns in a dual-channel context. In Chapter 4, a channel's sales requests was modified so that it is a linear function of a channel's own price, a cross-channel's price, and the online store's responsiveness level. While theoretical game frameworks are addressed in regards to competition between channels, unified and differential pricing strategies are addressed in regards to integration.

Identifying the optimal order quantity through the use of the newsvendor model is a tangible problem in the field of supply chain management. As customers are able to return their purchases and a high percentage of those returns are considered as good as new and resalable, many retailers seem to overestimate their order quantities. Thus, researchers have expanded the newsvendor model to include customer returns. However, the complications that customer returns impose on the optimal quantities of a dual-channel retailer have not yet been sufficiently studied. In this context, cross-channel returns may destabilize the retailing system and retailers may simultaneously overestimate one channel and underestimate another. Thus, in Chapter 5 we developed newsvendor models that considered both same- and cross-channel returns experienced by a dual-channel retailer. Four strategies that may be used to handle cross-channel returns were studied. The first strategy was to collect all cross-channel returns on a regular basis and to send them back to the online store. The second strategy was to ban cross-channel returns. The third and fourth strategies were to transfer the ownership of all cross-channel returns to the physical store under decentralized and centralized management styles, respectively.

Due to the continuously growing global recession, many dual-channel retailers focus on their core competencies while outsourcing e-stores' logistical activities to third-party logistics and service providers. In Chapter 6, we studied three strategies that can be used to handle orders and customer returns under the context of a dual-channel retailing system. The first strategy is the transaction-based fee, where a retailer handles all logistical activities using his own fleet or pays a logistics and service provider a per deliver fee. The second strategy is the flat-based fee, where a retailer hand over the logistical operation of his e-tail store to a logistics and service provider for a flat fee per managed item. In the third strategy the provider manages the operations of the e-channel and gains a share of the channel's revenue and a fee per managed item. Under each strategy, we present a profit maximization model to select the optimal inventory level in each channel while considering both same- and cross-channel returns.

7.2 Future Research

In Section 7.2 we will consider some of the possible avenues researchers may take to expand the knowledge of multi-channel retailers who are offering customer returns as a competitive business strategy.

Multiple items: This dissertation considers a retailer that offers a single item to customers. In reality, a retailing business contains thousands of items under the same return policy. Thus, considering a system with multiple items would be a beneficial extension of this work. The decision process of selecting the ideal cross-channel return policy would be more realistic as it considers customers' behaviours with regards to the whole retailing system.

Multiple physical stores: In this study one physical store is considered to represent the offline channel. However, retailers may posses several physical stores in the same geographical region. Purchases from a physical store can be cross-returned to another physical store in the same city in a regular bases. The assumptions in this study could be extended to include such a notion.

Two-echelon system: A supplier of a dual-channel retailer was not considered in problem formulation of this study. Therefore, it is interesting to see Chapter 5 extended to include a two-echelon system where the retailer study the option of retuning unsatisfactory items directly to the supplier. Indeed, such a return process will deprive the retailer from being able to resell returns. The retailer should clarify under what contract or agreement this process is optimal.

Partial refund policy: Several researchers support a partial refund rather than a full refund policy. Such a policy can be implemented by charging unsatisfied customers a restocking fee or shipping fee. Thus, another beneficial extension of this study is to optimize the refund policy along the other decision variables in each channel. It is trivial to notice that return rates and total sales are policy dependant. For example, if unsatisfactory online purchases should be shipped to the online store at the expense of customers, then such a return policy might provoke customers to return items to the physical store instead. Thus, an increase in the rate of cross-channel returns might be experienced. Also, a noticeable reduction in the online store's total sales will be observed.

System with more uncertain parameters: Today's business environment is full of uncertainties. Thus, extending Chapter 3 and 4 to include sales uncertainty could be helpful to retailers. Chapters 3 – 6 may also be extended to include uncertainty in the different return rates. Also, uncertain sales in Chapters 5 and uncertain demand in Chapters 6 are assumed to follow uniform distributions wherein all possible outcomes are equally likely. However, this may not be the case and a distribution with a most likely outcome will be more realistic. Therefore, future works may conduct similar studies with triangular or normal distributions.

Sales distribution over a selling season: In this dissertation, resalable returns are those that were returned before the end of the selling season and in their original packaging and condition. Since this study does not include the distribution of sales over a selling season, there might be no demand to consume some of those returns. The problem is even amplified when cross-channel returns are considered. Thus, it is helpful to see such an impact on the optimal cross-channel return policy.

Dealing with returns: Having the ability to resell returns several times in a selling season is not applicable to a wide rage of products. Thus, more realistic assumptions and models could be used to better study customer returns. For example, a multi-period setting could be investigated. Indeed, demand range and variability along that range is different from a period to another as customers develop more knowledge about a product. Also, customer acceptance to a certain product may not be known at the beginning of the selling season. Thus, attempting to resell a return could be related to the popularity of the product especially when the retailer is limited in space.

Stock-out substitution: One of the limitations in this study is not considering stock-out substitution where a channel's customers seek to purchase from the other channel when stock-out occurs. It is interesting to see the effect of such a fact on cross-channel return policies. Also, this substitution may have a noticeable impact on the retailer and provider behaviours, especially when customers are product loyal rather than channel loyal.

APPENDICES

Appendix A. Proof of CHAPTER 3's propositions

A.1 Proof of Proposition 3.1 (Part i) and Proposition 3.3 (Part i)

First proof must be provided that $p_o > c$ and $p_r > c$. Since revenue that is generated by satisfying a single sale should be positive and for the worst-case scenario ($\hat{c} = ck_{or} + (1 - k_{or})s - d$ and $s = c$), the following is true.

From (3.15 and 3.23), $p_o > \frac{t+w(d+t)-w(1-k_o)s+(1-wk_o)c-v(ck_{or}+(1-k_{or})s-d)}{(1-w-v)}$.

For the worst case scenario, $p_o > \frac{t+w(d+t)+vd}{(1-w-v)} + c$, thus $p_o > c$.

From (3.15 and 3.24), $p_r > \frac{rd-r(1-k_r)s+(1-rk_r)c}{(1-r)}$.

For the worst case scenario, $p_r > \frac{rd}{(1-r)} + c$, thus $p_r > c$.

The first condition for the profit functions is then taken with respect to the rate of same-channel-return (w or r).

$$\frac{\partial \pi^C_{\varepsilon=\infty}}{\partial w} = \frac{\partial \pi^{D_o}_{\varepsilon=\infty}}{\partial w} = D_o[s + ck_o - (p_o + d + t + sk_o)]$$

$$\frac{\partial \pi^C_{\varepsilon=\infty}}{\partial r} = \frac{\partial \pi^{D_r}_{\varepsilon=\infty}}{\partial r} = D_r[s + ck_r - (p_r + d + sk_r)]$$

For the worst-case scenario, assume that s is as high as c, and d and t are as low as zero. Thus,

$$\frac{\partial \pi^C_{\varepsilon=\infty}}{\partial w} = \frac{\partial \pi^{D_o}_{\varepsilon=\infty}}{\partial w} = D_o[c - p_o] < 0$$

$$\frac{\partial \pi^C_{\varepsilon=\infty}}{\partial r} = \frac{\partial \pi^{D_r}_{\varepsilon=\infty}}{\partial r} = D_r[c - p_r] < 0$$

A.2 Proof of Proposition 3.1 (Part ii)

Substitute sales functions into (3.16). Thus,

$$\frac{\partial^2 \pi^C_\varepsilon}{\partial p_r^2} = -2\beta I < 0$$

$$\frac{\partial^2 \pi^C_\varepsilon}{\partial p_o^2} = -2\beta J < 0$$

$$\frac{\partial^2 \pi^C_\varepsilon}{\partial p_r p_o} = \frac{\partial^2 \pi^C_\varepsilon}{\partial p_o p_r} = \gamma(I + J)$$

$$\frac{\partial^2 \pi^C_\varepsilon}{\partial p_r^2}\frac{\partial^2 \pi^C_\varepsilon}{\partial p_o^2} - \left(\frac{\partial^2 \pi^C_\varepsilon}{\partial p_r p_o}\right)^2 = 4\beta^2 IJ - \gamma^2(I + J)^2.$$

Thus, if $4\beta^2 IJ \geq \gamma^2(I + J)^2$, then the function is strictly and jointly concave in p_r and p_o.

$\frac{\partial \pi_\varepsilon^C}{\partial p_r} = 0$, thus $p^{C_r} = \frac{(\gamma J\alpha_o - \gamma^2 A_\varepsilon)(I+J) + \gamma\beta B_\varepsilon(I-J) + 2J(\beta I\alpha_r + \beta^2 A_\varepsilon)}{4JI\beta^2 - \gamma^2(I+J)^2}$

$\frac{\partial \pi_\varepsilon^C}{\partial p_o} = 0$, thus $p^{C_o} = \frac{(\gamma I\alpha_r - \gamma^2 B_\varepsilon)(I+J) - \gamma\beta A_\varepsilon(I-J) + 2I(\beta J\alpha_o + \beta^2 B_\varepsilon)}{4JI\beta^2 - \gamma^2(I+J)^2}$

A.3 Proof of Proposition 3.2

Substitute sales functions into (3.16) with $p_r = p_o = p$. Thus,

$\frac{\partial^2 \pi_\varepsilon^C}{\partial p^2} = -2(\beta - \gamma)(I + J) < 0.$

Thus, the profit function is strictly concave in p. Therefore,

$\frac{\partial \pi_\varepsilon^C}{\partial p} = 0$, thus $p^{C_u} = \frac{1}{2}\left(\frac{(\beta-\gamma)(B_\varepsilon + A_\varepsilon) + \alpha_o J + \alpha_r I}{(\beta-\gamma)(J+I)}\right).$

A.4 Proof of Proposition 3.3 (Part ii)

Substitute sales functions into (3.25) and (3.26). Thus,

Given p_o

From (3.26), $\frac{\partial^2 \pi_\varepsilon^{D_r}}{\partial p_r^2} = -2\beta I < 0.$ Thus, $\pi_\varepsilon^{D_r}$ is concave in p_r.

$\frac{\partial \pi_\varepsilon^{D_r}}{\partial p_r} = 0$, thus $p^{S_r}(p_o) = \frac{1}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_\varepsilon}{I} + \frac{F_\varepsilon v\gamma}{\beta I} + \frac{\gamma}{\beta}p_o\right)$

After substituting the value of $p^{S_r}(p_o)$ into (3.25), $\frac{\partial^2 \pi_\varepsilon^{D_o}}{\partial p_o^2} = \frac{J(\gamma^2 - 2\beta^2)}{\beta} < 0$ because $\beta \geq \gamma$.

Thus, $\pi_\varepsilon^{D_o}$ is concave in p_o and there exist a unique Stackelberg equilibrium.

$\frac{\partial \pi_\varepsilon^{D_o}}{\partial p_o} = 0$, thus $p^{S_o} = \frac{G_\varepsilon}{2J} + \frac{\alpha_o\beta}{(2\beta^2 - \gamma^2)} + \frac{\alpha_r\gamma}{2(2\beta^2 - \gamma^2)} + \frac{A_\varepsilon\beta\gamma}{2I(2\beta^2 - \gamma^2)} + \frac{v\gamma^2 F_\varepsilon}{2I(2\beta^2 - \gamma^2)}$

Substitute p^{S_o} into $p^{S_r}(p_o)$ gives

$p^{S_r} = \frac{1}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_\varepsilon}{I} + \frac{\gamma G_\varepsilon}{2\beta J} + \frac{\alpha_o\gamma}{(2\beta^2 - \gamma^2)} + \frac{\alpha_r\gamma^2}{2\beta(2\beta^2 - \gamma^2)} + \frac{A_\varepsilon\gamma^2}{2I(2\beta^2 - \gamma^2)} + \frac{v\gamma^3 F_\varepsilon}{2\beta J(2\beta^2 - \gamma^2)} + \frac{F_\varepsilon v\gamma}{\beta I}\right)$

A.5 Proof of Proposition 3.4

Substitute sales functions into (3.25) and (3.26). Thus,

$\frac{\partial^2 \pi_\varepsilon^{D_r}}{\partial p_r^2} = -2I\beta < o$

$\frac{\partial^2 \pi_\varepsilon^{D_o}}{\partial p_o^2} = -2J\beta < o$

$\frac{\partial^2 \pi_\varepsilon^{D_o}}{\partial p_o \partial p_r} = J\gamma$

$\frac{\partial^2 \pi_\varepsilon^{D_r}}{\partial p_r \partial p_o} = I\gamma$

$\frac{\partial^2 \pi_\varepsilon^{D_r}}{\partial p_r^2}\frac{\partial^2 \pi_\varepsilon^{D_o}}{\partial p_o^2} - \frac{\partial^2 \pi_\varepsilon^{D_r}}{\partial p_r \partial p_o}\frac{\partial^2 \pi_\varepsilon^{D_o}}{\partial p_o \partial p_r} = IJ(4\beta^2 - \gamma^2) > 0$

Since $\beta > \gamma$, then there exists a unique Nash equilibrium for the game.

$\frac{\partial \pi_{\varepsilon}^{D_r}}{\partial p_r} = 0$, thus $p_r(p_o) = \frac{\beta A_{\varepsilon} + I\alpha_r + v\gamma F_{\varepsilon} + I\gamma p_o}{2\beta I}$

$\frac{\partial \pi_{\varepsilon}^{D_o}}{\partial p_o} = 0$, thus $p_o(p_r) = \frac{\beta G_{\varepsilon} + J\alpha_o + J\gamma p_r}{2\beta J}$

By simultaneously solving the two equations, an equilibrium point is reached. Thus,

$p^{N_r} = \left(\frac{1}{4\beta^2 - \gamma^2}\right)\left(2\beta\alpha_r + \gamma\alpha_o + \frac{4\beta^3 G_{\varepsilon}}{J\gamma} + \frac{2\beta^2 A_{\varepsilon}}{I} + \frac{2v\beta\gamma F_{\varepsilon}}{I}\right) - \frac{\beta G_{\varepsilon}}{J\gamma}$

$p^{N_o} = \left(\frac{1}{4\beta^2 - \gamma^2}\right)\left(2\beta\alpha_o + \gamma\alpha_r + \frac{2\beta^2 G_{\varepsilon}}{J} + \frac{\beta\gamma A_{\varepsilon}}{I} + \frac{v\gamma^2 F_{\varepsilon}}{I}\right).$

Appendix B. Proof of CHAPTER 4's propositions

B.1 Proof of Proposition 4.1

Selling Resalable Returns Infinitely ($\varepsilon = \infty$)

Substitute sales function into (4.7). Thus,

$$H_{\varepsilon=\infty}^{C_d} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o \partial p_r} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_r \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_r^2} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_r \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial e \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial e \partial p_r} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta J & \lambda(J+I) & -\beta wbc + \rho J \\ \lambda(J+I) & -2\beta I & \lambda wbc - \rho I \\ -\beta wbc + \rho J & \lambda wbc - \rho I & -\eta + 2\rho wbc \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o^2}\right| = -2\beta J < 0$$

$$\begin{vmatrix} \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_o \partial p_r} \\ \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_r \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=\infty}^C}{\partial p_r^2} \end{vmatrix} = 4IJ\beta^2 - \lambda^2(J+I)^2$$

$$\left|H_{\varepsilon=\infty}^{C_d}\right| = (-\eta + 2\rho wbc)(4\beta^2 JI - \lambda^2(J+I)^2) - 2\lambda(J+I)(\beta wbc - \rho J)(\lambda wbc - \rho I) + 2\beta(J(\lambda wbc - \rho I)^2 + I(\beta wbc - \rho J)^2)$$

Thus, under the conditions $4IJ\beta^2 - \lambda^2(J+I)^2 > 0$ and $\left|H_{\varepsilon=\infty}^{C_d}\right| < 0$ the profit function is strictly and jointly concave in p_o, p_r and e. As stated earlier, the value of η can be associated with the cost of signing new contracts, hiring more employees, and/or incorporating more reliable technologies. Consequently, it is realistic that η will derive the value of $\left|H_{\varepsilon=\infty}^{C_d}\right|$ to be negative.

Selling Resalable Returns Once ($\varepsilon = 1$)

Substitute sales function into (4.6). Thus,

$$H_{\varepsilon=1}^{C_d} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o \partial p_r} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_r \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_r^2} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_r \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial e \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial e \partial p_r} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta J & \lambda(J+I) & -\frac{\beta wbc}{(1+aw+bwe)^2} + \rho J \\ \lambda(J+I) & -2\beta I & \frac{\lambda wbc}{(1+aw+bew)^2} - \rho I \\ -\frac{\beta wbc}{(1+aw+bew)^2} + \rho J & \frac{\lambda wbc}{(1+aw+bew)^2} - \rho I & -\eta - \frac{2w^2b^2cD_o}{(1+aw+bew)^3} + \frac{2\rho wbc}{(1+aw+bew)^2} \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o^2}\right| = -2\beta J < 0$$

$$\begin{vmatrix} \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_o \partial p_r} \\ \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_r \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=1}^C}{\partial p_r^2} \end{vmatrix} = 4IJ\beta^2 - \lambda^2(J+I)^2$$

$$\left|H_{\varepsilon=1}^{C_d}\right| = \left(-\eta - \frac{2w^2b^2cD_o}{(1+aw+bew)^3} + \frac{2\rho wbc}{(1+aw+bew)^2}\right)(4\beta^2 JI - \lambda^2(J+I)^2) - 2\lambda(J+I)\left(\frac{\beta wbc}{(1+aw+bew)^2} - \rho I\right)\left(\frac{\lambda wbc}{(1+aw+bew)^2} - \rho I\right) + 2\beta\left(J\left(\frac{\lambda wbc}{(1+aw+bew)^2} - \rho I\right)^2 + I\left(\frac{\beta wbc}{(1+aw+bwe)^2} - \rho J\right)^2\right)$$

It is required that $4IJ\beta^2 - \lambda^2(J+I)^2 > 0$. Also, if D_o is set to be zero and $\left|H_{\varepsilon=1}^{C_d}\right|$ is calculated to be negative for all values of $e \in [-a/b, (1-a)/b]$, then profit function is strictly and jointly concave in p_o, p_r and e within the feasible region. Since the value of η is normally high, the above conditions are satisfied naturally when virtual system parameters are used.

B.2 Proof of Proposition 4.2

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

Substitute sales function into (4.7), and set $p_r = p_o = p$. Thus,

$$H_{\varepsilon=\infty}^{C_u} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=\infty}^{C}}{\partial p^2} & \frac{\partial^2 \pi_{\varepsilon=\infty}^{C}}{\partial p \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=\infty}^{C}}{\partial e \partial p} & \frac{\partial^2 \pi_{\varepsilon=\infty}^{C}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2(\beta - \lambda)(I + J) & -wb(\beta - \lambda)c - \rho(I - J) \\ -wb(\beta - \lambda)c - \rho(I - J) & -\eta + 2\rho wbc \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi_{\varepsilon=\infty}^{C}}{\partial p^2}\right| = -2(\beta - \lambda)(I + J) < 0$$

$$\left|H_{\varepsilon=\infty}^{C_u}\right| = 2(\beta - \lambda)(I + J)(\eta - 2\rho wbc) - \left((\beta - \lambda)wbc + \rho(I - J)\right)^2$$

Thus, under the condition $\left|H_{\varepsilon=\infty}^{C_u}\right| > 0$ the profit function is strictly and jointly concave in p and e.

Selling Resalable Returns Once ($\boldsymbol{\varepsilon = 1}$)

Substitute sales function into (4.6), and set $p_r = p_o = p$. Thus,

$$H_{\varepsilon=1}^{C_u} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=1}^{C}}{\partial p^2} & \frac{\partial^2 \pi_{\varepsilon=1}^{C}}{\partial p \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=1}^{C}}{\partial e \partial p} & \frac{\partial^2 \pi_{\varepsilon=1}^{C}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2(\beta - \lambda)(I + J) & -\frac{wb(\beta-\lambda)c}{(1+aw+bew)^2} - \rho(I - J) \\ -\frac{wb(\beta-\lambda)c}{(1+aw+bew)^2} - \rho(I - J) & -\eta - \frac{2cw^2b^2D_o}{(1+aw+bew)^3} + \frac{2\rho wbc}{(1+aw+bew)^2} \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi_{\varepsilon=1}^{C}}{\partial p^2}\right| = -2(\beta - \lambda)(I + J) < 0$$

$$\left|H_{\varepsilon=1}^{C_u}\right| = 2(\beta - \lambda)(I + J)\left(\eta + \frac{2cw^2b^2D_o}{(1+aw+bew)^3} - \frac{2\rho wbc}{(1+aw+bew)^2}\right) - \left(\frac{(\beta-\lambda)wbc}{(1+aw+bew)^2} + \rho(I - J)\right)^2$$

If D_o is set to be zero and $\left|H_{\varepsilon=1}^{C_u}\right|$ is calculated to be positive for all values $e \in [-a/b, (1 - a)/b]$, then the profit function is strictly and jointly concave in p and e within the feasible region.

B.3 Proof of Proposition 4.3

From (4.26).

$$H_{\varepsilon=\infty}^{S} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=\infty}^{D_o}}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=\infty}^{D_o}}{\partial p_o \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=\infty}^{D_o}}{\partial e \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=\infty}^{D_o}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -J\left(\frac{2\beta^2-\lambda^2}{\beta}\right) & -\frac{wbc(2\beta^2-\lambda^2)}{2\beta} + J\rho\left(1 - \frac{\lambda}{2\beta}\right) \\ -\frac{wbc(2\beta^2-\lambda^2)}{2\beta} + J\rho\left(1 - \frac{\lambda}{2\beta}\right) & -\eta + \rho wbc\left(\frac{2\beta-\lambda}{\beta}\right) \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi_{\varepsilon=\infty}^{D_o}}{\partial p_o^2}\right| = -J\left(\frac{2\beta^2-\lambda^2}{\beta}\right) < 0$$

$$\left|H_{\varepsilon=\infty}^{S}\right| = J\left(\frac{2\beta^2-\lambda^2}{\beta}\right)\left(\eta - \rho wbc\left(\frac{2\beta-\lambda}{\beta}\right)\right) - \left(\frac{wbc(2\beta^2-\lambda^2)}{2\beta} - J\rho\left(1 - \frac{\lambda}{2\beta}\right)\right)^2$$

Thus, under the conditions $\left|H_{\varepsilon=\infty}^{S}\right| > 0$ the online store's profit function $\pi_{\varepsilon=\infty}^{D_o}$ is strictly and jointly concave in p_o and e.

B.4 Proof of Proposition 4.4

From (4.30):

$$H_{\varepsilon=1}^{S} = \begin{bmatrix} \frac{\partial^2 \pi_{\varepsilon=1}^{D_o}}{\partial p_o^2} & \frac{\partial^2 \pi_{\varepsilon=1}^{D_o}}{\partial p_o \partial e} \\ \frac{\partial^2 \pi_{\varepsilon=1}^{D_o}}{\partial e \partial p_o} & \frac{\partial^2 \pi_{\varepsilon=1}^{D_o}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -J\left(\frac{2\beta^2-\lambda^2}{\beta}\right) & -\frac{wbc(2\beta^2-\lambda^2)}{2\beta(1+wa+wbe)^2} + \frac{J\rho(2\beta-\lambda)}{2\beta} \\ -\frac{wbc(2\beta^2-\lambda^2)}{2\beta(1+wa+wbe)^2} + \frac{J\rho(2\beta-\lambda)}{2\beta} & \left(\begin{array}{c} \frac{\rho wbc(2\beta-\lambda)}{\beta(1+wa+wbe)^2} - \eta - \\ \left(\alpha_o - p_o\left(\frac{2\beta^2-\lambda^2}{2\beta}\right) + \frac{\lambda}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=1}}{I} + \frac{v\lambda F_{\varepsilon=1}}{I\beta}\right) + \frac{e\rho(2\beta-\lambda)}{2\beta}\right)\left(\frac{2cw^2b^2}{(1+wa+wbe)^3}\right) \end{array}\right) \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o^2}\right| = -J\left(\frac{2\beta^2-\lambda^2}{\beta}\right) < 0$$

$$|H^S_{\varepsilon=1}| = J\left(\frac{2\beta^2-\lambda^2}{\beta}\right)\left(\eta + \left(\alpha_o - p_o\left(\frac{2\beta^2-\lambda^2}{2\beta}\right) + \frac{\lambda}{2}\left(\frac{\alpha_r}{\beta} + \frac{A_{\varepsilon=1}}{I} + \frac{v\lambda F_{\varepsilon=1}}{I\beta}\right) + \frac{e\rho(2\beta-\lambda)}{2\beta}\right)\left(\frac{2cw^2b^2}{(1+wa+wbe)^3}\right) - \frac{\rho wbc(2\beta-\lambda)}{\beta(1+wa+wbe)^2}\right) - \left(\frac{wbc(2\beta^2-\lambda^2)}{2\beta(1+wa+wbe)^2} - \frac{J\rho(2\beta-\lambda)}{2\beta}\right)^2$$

If p_o is high enough, then $|H^S_{\varepsilon=1}|$ could be negative. Thus, the online store's profit function $\pi^{D_o}_{\varepsilon=1}$ is indefinite with respect to p_o and e and hence is not strictly and jointly concave in p_o and e.

B.5 Proof of Proposition 4.5

Selling Resalable Returns Infinitely ($\boldsymbol{\varepsilon = \infty}$)

Substitute sales functions into (4.19) and (4.20). Thus,

$$H^N_{\varepsilon=\infty} = \begin{bmatrix} \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o \partial e} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o \partial p_r} \\ \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial e \partial p_o} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial e^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial e \partial p_r} \\ \frac{\partial^2 \pi^{D_r}_{\varepsilon=\infty}}{\partial p_r \partial p_o} & \frac{\partial^2 \pi^{D_r}_{\varepsilon=\infty}}{\partial p_r \partial e} & \frac{\partial^2 \pi^{D_r}_{\varepsilon=\infty}}{\partial p_r^2} \end{bmatrix} = \begin{bmatrix} -2J\beta & J\rho - \beta wbc & J\lambda \\ J\rho - \beta wbc & -\eta + 2\rho wbc & \lambda wbc \\ I\lambda & -I\rho & -2I\beta \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o^2}\right| = -2J\beta < 0$$

$$\begin{vmatrix} \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial p_o \partial e} \\ \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial e \partial p_o} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=\infty}}{\partial e^2} \end{vmatrix} = \left|\widetilde{H^N_{\varepsilon=\infty}}\right| = 2J\beta(\eta - 2\rho wbc) - (J\rho - \beta wbc)^2$$

$$\left|\frac{\partial^2 \pi^{D_r}_{\varepsilon=\infty}}{\partial p_r^2}\right| = -2I\beta < 0$$

$$|H^N_{\varepsilon=\infty}| = -IJ\left((4\beta^2 - \lambda^2)\eta + \rho\lambda\big((J\rho - \beta wbc) + 2\lambda wbc\big)\right) + 2IJ\beta\rho wbc(4\beta - \lambda) + 2I\beta(J\rho - \beta wbc)^2 + I\lambda^2 wbc(J\rho - \beta wbc)$$

If $\left|\widetilde{H^N_{\varepsilon=\infty}}\right| > 0$, then the online store's profit function is concave on the store's own decision variables. Intuitively, the physical store's profit function is concave on the store's own decision variable. If $|H^N_{\varepsilon=\infty}| < 0$, then there exist a unique Nash equilibrium.

Selling Resalable Returns Once ($\boldsymbol{\varepsilon = 1}$)

Substitute sales functions (4.21) and (4.22). Thus,

$$H^N_{\varepsilon=1} = \begin{bmatrix} \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o \partial e} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o \partial p_r} \\ \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial e \partial p_o} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial e^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial e \partial p_r} \\ \frac{\partial^2 \pi^{D_r}_{\varepsilon=1}}{\partial p_r \partial p_o} & \frac{\partial^2 \pi^{D_r}_{\varepsilon=1}}{\partial p_r \partial e} & \frac{\partial^2 \pi^{D_r}_{\varepsilon=1}}{\partial p_r^2} \end{bmatrix} = \begin{bmatrix} -2J\beta & J\rho - \frac{\beta wbc}{(1+wa+wbe)^2} & J\lambda \\ J\rho - \frac{\beta wbc}{(1+wa+wbe)^2} & -\eta - \frac{2cw^2b^2D_o}{(1+wa+wbe)^3} + \frac{2wb\rho c}{(1+wa+wbe)^2} & \frac{\lambda wbc}{(1+wa+wbe)^2} \\ I\lambda & -I\rho & -2I\beta \end{bmatrix}$$

$$\left|\frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o^2}\right| = -2J\beta < 0$$

$$\begin{vmatrix} \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o^2} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial p_o \partial e} \\ \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial e \partial p_o} & \frac{\partial^2 \pi^{D_o}_{\varepsilon=1}}{\partial e^2} \end{vmatrix} = \left|\widetilde{H^N_{\varepsilon=1}}\right| = 2J\beta\left(\eta + \frac{2cw^2b^2D_o}{(1+wa+wbe)^3} - \frac{2wb\rho c}{(1+wa+wbe)^2}\right) - \left(J\rho - \frac{\beta wbc}{(1+wa+wbe)^2}\right)^2$$

$$\left|\frac{\partial^2 \pi^{D_r}_{\varepsilon=1}}{\partial p_r^2}\right| = -2I\beta < 0$$

$$|H^N_{\varepsilon=1}| =$$

$$-IJ\left((4\beta^2 - \lambda^2)\left(\eta + \frac{2cw^2b^2D_o}{(1+wa+wbe)^3}\right) + \lambda\rho\left(J\rho + \frac{wbc(2\lambda-\beta)}{(1+wa+wbe)^2}\right)\right) + \frac{2IJ\beta\rho wbc(4\beta-\lambda)}{(1+wa+wbe)^2} + \left(2I\beta J\rho - \frac{Iwbc(2\beta^2-\lambda^2)}{(1+wa+wbe)^2}\right)\left(J\rho - \frac{\beta wbc}{(1+wa+wbe)^2}\right)$$

If D_o is set to be zero and $\left|\widetilde{H_{\varepsilon=1}^{N}}\right|$ is calculated to be positive for all values $e \in [-a/b, (1-a)/b]$, then the online store's profit function is strictly and jointly concave in p_o and e within the feasible region. Also, the physical store's profit function is concave in p_r. If $|H_{\varepsilon=1}^{N}| < 0$ under all values $e \in [-a/b, (1-a)/b]$ and $D_o = 0$, then there exist a unique Nash equilibrium at the most within the feasible region.

Appendix C. Proof of CHAPTER 5's propositions and corollaries

C.1 Proof of Proposition 5.1

The profit function (5.1) is reformulated as:

$$\pi_o^{ro}(Q_o) = \left(p_o^{ro} - \frac{s}{\eta_\varepsilon}\right)\mu_o + (s-c)Q_o - \left(p_o^{ro} + g - \frac{s}{\eta_\varepsilon}\right)\int_{\eta_\varepsilon Q_o}^{\infty} x_o f_o(x_o)dx_o +$$
$$(p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s)Q_o \int_{\eta_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o$$

Then,

$$\frac{\partial \pi_o^{ro}}{\partial Q_o} = (s-c) + (p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s)\int_{\eta_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o$$

$\frac{\partial^2 \pi_o^{ro}}{\partial Q_o^2} = -\eta_\varepsilon(p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon - s)f_o(\eta_\varepsilon Q_o)$. Since $p_o^{ro} > s$ and $\eta_\varepsilon \geq 1$, then $\frac{\partial^2 \pi_o^{ro}}{\partial Q_o^2} < 0$

A unique optimal solution for profit function (5.1) exists. If $\frac{\partial \pi_o^{ro}}{\partial Q_o} = 0$, then

$$Q_o^{ro*} = \left(\frac{1}{\eta_\varepsilon}\right)F_o^{-1}\left(\frac{p_o^{ro} + g - \frac{c}{\eta_\varepsilon}}{p_o^{ro} + g - \frac{s}{\eta_\varepsilon}}\right)$$

Additionally, profit function (5.2) is reformulated as

$$\pi_r^{ro}(Q_r) = \left(p_r - \frac{s}{\delta_\varepsilon}\right)\mu_r + (s-c)Q_r - \left(p_r + g - \frac{s}{\delta_\varepsilon}\right)\int_{\delta_\varepsilon Q_r}^{\infty} x_r f_r(x_r)dx_r + (p_r\delta_\varepsilon +$$
$$g\delta_\varepsilon - s)Q_r \int_{\delta_\varepsilon Q_r}^{\infty} f_r(x_r)dx_r$$

Then,

$\frac{\partial \pi_r^{ro}}{\partial Q_r} = (s-c) + (p_r\delta_\varepsilon + g\delta_\varepsilon - s)\int_{\delta_\varepsilon Q_r}^{\infty} f_r(x_r)dx_r$, and

$\frac{\partial^2 \pi_r^{ro}}{\partial Q_r^2} = -\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)f_r(\delta_\varepsilon Q_r)$. Since $p_r > s$ and $\delta_\varepsilon \geq 1$, then $\frac{\partial^2 \pi_r^{ro}}{\partial Q_r^2} < 0$

A unique optimal solution for profit function (5.2) exists. If $\frac{\partial \pi_r^{ro}}{\partial Q_r} = 0$, then

$$Q_r^{ro*} = \left(\frac{1}{\delta_\varepsilon}\right)F_r^{-1}\left(\frac{p_r + g - \frac{c}{\delta_\varepsilon}}{p_r + g - \frac{s}{\delta_\varepsilon}}\right)$$

C.2 Proof of Proposition 5.2

The profit function (5.5) is reformulated as:

$$\pi_o^{co}(Q_o) =$$
$$\left(\left(p_o^{co} - \frac{s}{\eta_\varepsilon}\right)\alpha - g(1-\alpha)\right)\mu_o + (s-c)Q_o - \left(p_o^{co} + g - \frac{s}{\eta_\varepsilon}\right)\int_{\eta_\varepsilon Q_o}^{\infty} y_o f_\alpha(y_o)dy_o +$$
$$(p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - s)Q_o \int_{\eta_\varepsilon Q_o}^{\infty} f_\alpha(y_o)dy_o$$

Then,

$\frac{\partial \pi_o^{co}}{\partial Q_o} = (s-c) + (p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - s)\int_{\eta_\varepsilon Q_o}^{\infty} f_\alpha(y_o)dy_o$, and

$\frac{\partial^2 \pi_o^{co}}{\partial Q_o^2} = -\eta_\varepsilon(p_o^{co}\eta_\varepsilon + g\eta_\varepsilon - s)f_\alpha(\eta_\varepsilon Q_o)$. Since $p_o^{co} > s$ and $\eta_\varepsilon \geq 1$, then $\frac{\partial^2 \pi_o^{co}}{\partial Q_o^2} < 0$.

A unique optimal solution for profit function (5.5) exists. If $\frac{\partial \pi_o^{co}}{\partial Q_o} = 0$, then

$$Q_o^{co*} = \left(\frac{1}{\eta_\varepsilon}\right) F_\alpha^{-1}\left(\frac{p_o^{co}+g-\frac{c}{\eta_\varepsilon}}{p_o^{co}+g-\frac{s}{\eta_\varepsilon}}\right)$$

C.3 Proof of Corollary 5.1

For $1 \geq \ddot{\alpha} \geq \dot{\alpha} \geq 0$ to hold true, then from (5.14) the relationship $p_o^{ro} \geq p_o^{co}$ is true. Thus, $x = p_o^{ro}\eta_\varepsilon + g\eta_\varepsilon \geq y = p_o^{co}\eta_\varepsilon + g\eta_\varepsilon$. Based on x and y values, equation (5.13) can be rewritten as: $\dot{\alpha} = \frac{(y-s)(x-c)}{(x-s)(y-c)} \leq 1$. The previous relationship is only satisfied when $x \leq y$. That is a contradiction with our previous assumption. Thus, $1 \geq \dot{\alpha} \geq \ddot{\alpha} \geq 0$ at all reasonable values.

C.4 Proof of Proposition 5.3

The profit function (5.7) is reformulated as:

$$\pi_o^D(Q_o) = \left(p_o^D - \frac{s}{\lambda_\varepsilon}\right)\mu_o + (s-c)Q_o - \left(p_o^D + g - \frac{s}{\lambda_\varepsilon}\right)\int_{\lambda_\varepsilon Q_o}^{\infty} x_o f_o(x_o)dx_o + (p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s)Q_o\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o$$

Then,

$\frac{\partial \pi_o^D}{\partial Q_o} = (s-c) + (p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s)\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o$, and

$\frac{\partial^2 \pi_o^D}{\partial Q_o^2} = -\lambda_\varepsilon(p_o^D\lambda_\varepsilon + g\lambda_\varepsilon - s)f_o(\lambda_\varepsilon Q_o) < 0$. Since $p_o^D > s$ and $\lambda_\varepsilon \geq 1$, then $\frac{\partial^2 \pi_o^D}{\partial Q_o^2} < 0$.

A unique optimal solution for the profit function (5.7) exists. If $\frac{\partial \pi_o^D}{\partial Q_o} = 0$, then

$$Q_o^{D^*} = \left(\frac{1}{\lambda_\varepsilon}\right) F_o^{-1}\left(\frac{p_o^D+g-\frac{c}{\lambda_\varepsilon}}{p_o^D+g-\frac{s}{\lambda_\varepsilon}}\right)$$

The profit function (5.9) is reformulated as:

$$\pi_r^D(Q_r) = \left(p_r - \frac{s}{\delta_\varepsilon}\right)\mu_r + (s-c)Q_r + (s-\hat{c}-d)Q_{or} - \left(p_r + g - \frac{s}{\delta_\varepsilon}\right)\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} x_r f_r(x_r)dx_r + (p_r\delta_\varepsilon + g\delta_\varepsilon - s)(Q_r + k_{or}Q_{or})\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r$$

$\frac{\partial \pi_r^D}{\partial Q_r} = (s-c) + (p_r\delta_\varepsilon + g\delta_\varepsilon - s)\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r$, and

$\frac{\partial^2 \pi_r^D}{\partial Q_r^2} = -\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)f_r\big(\delta_\varepsilon(Q_r + k_{or}Q_{or})\big)$. Since $p_r > s$ and $\delta_\varepsilon \geq 1$, then $\frac{\partial^2 \pi_r^D}{\partial Q_r^2} < 0$.

A unique optimal solution for the profit function (5.9) exists. If $\frac{\partial \pi_r^D}{\partial Q_r} = 0$, then

$$Q_r^{D^*} = \left(\frac{1}{\delta_\varepsilon}\right) F_r^{-1}\left(\frac{p_r+g-\frac{c}{\delta_\varepsilon}}{p_r+g-\frac{s}{\delta_\varepsilon}}\right) - k_{or}Q_{or}$$

C.5 Proof of Proposition 5.4

The profit function (5.12) is reformulated as:

$$\pi^C(Q_o,Q_r) = \left(p_o^C - \frac{s}{\lambda_\varepsilon}\right)\mu_o + \left(p_r - \frac{s}{\delta_\varepsilon}\right)\mu_r + (s-c)Q_o + (s-c)Q_r + sk_{or}Q_{or} - \left(p_o^C + g - \frac{s}{\lambda_\varepsilon}\right)\int_{\lambda_\varepsilon Q_o}^{\infty} x_o f_o(x_o)dx_o + (p_o^C\lambda_\varepsilon + g\lambda_\varepsilon - s)Q_o\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o - \left(p_r + g - \frac{s}{\delta_\varepsilon}\right)\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} x_r f_r(x_r)dx_r +$$
$$(p_r\delta_\varepsilon + g\delta_\varepsilon - s)(Q_r + k_{or}Q_{or}) \int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r.$$

Then,

$$\frac{\partial \pi^C}{\partial Q_r} = (s-c) + (p_r\delta_\varepsilon + g\delta_\varepsilon - s)\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r,$$

$$\frac{\partial^2 \pi^C}{\partial Q_r^2} = -\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)f_r\big(\delta_\varepsilon(Q_r + k_{or}Q_{or})\big),$$

$$\frac{\partial \pi^C}{\partial Q_o} = (s-c) + vk_{or}\lambda_\varepsilon s\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o + (p_o^C\lambda_\varepsilon + g\lambda_\varepsilon - s)\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o + vk_{or}\lambda_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)\left(\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o\right)\left(\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r\right),$$ and

$$\frac{\partial^2 \pi^C}{\partial Q_o^2} =$$
$$-\lambda_\varepsilon(p_o^C\lambda_\varepsilon + g\lambda_\varepsilon + svk_{or}\lambda_\varepsilon - s)f_o(\lambda_\varepsilon Q_o) -$$
$$vk_{or}\lambda_\varepsilon^2(p_r\delta_\varepsilon + g\delta_\varepsilon - s)\left(f_o(\lambda_\varepsilon Q_o)\left(\int_{\delta_\varepsilon(Q_r+k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r\right) + \delta_\varepsilon k_{or}v\left(\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o\right)^2 f_r\big(\delta_\varepsilon(Q_r + k_{or}Q_{or})\big)\right).$$

Since p_r & $p_o^C > s$ and δ_ε & $\lambda_\varepsilon \geq 1$, then $\frac{\partial^2 \pi^C}{\partial Q_r^2} < 0$ & $\frac{\partial^2 \pi^C}{\partial Q_o^2} < 0$.

Also, $\frac{\partial^2 \pi^C}{\partial Q_r \partial Q_o} = -vk_{or}\lambda_\varepsilon\delta_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)f_r\big(\delta_\varepsilon(Q_r + k_{or}Q_{or})\big)\left(\int_{\lambda_\varepsilon Q_o}^{\infty} f_o(x_o)dx_o\right)$.

Thus,

$\frac{\partial^2 \pi^C}{\partial Q_r^2}\frac{\partial^2 \pi^C}{\partial Q_o^2} - \left(\frac{\partial^2 \pi^C}{\partial Q_r \partial Q_o}\right)^2 = \delta_\varepsilon\lambda_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)f_o(\lambda_\varepsilon Q_o)f_r\big(\delta_\varepsilon(Q_r + k_{or}Q_{or})\big)\Big((p_o^C\lambda_\varepsilon +$

$g\lambda_\varepsilon + svk_{or}\lambda_\varepsilon - s) + vk_{or}\lambda_\varepsilon(p_r\delta_\varepsilon + g\delta_\varepsilon - s)\left(\int_{\delta_\varepsilon(Q_r + k_{or}Q_{or})}^{\infty} f_r(x_r)dx_r\right)\Big)$.

Since $\frac{\partial^2 \pi^C}{\partial Q_r^2}\frac{\partial^2 \pi^C}{\partial Q_o^2} - \left(\frac{\partial^2 \pi^C}{\partial Q_r \partial Q_o}\right)^2$ is positive, then the profit function is strictly and jointly concave on Q_r and Q_o, and a unique global maximum exists.

If $\frac{\partial \pi^C}{\partial Q_r} = \frac{\partial \pi^C}{\partial Q_o} = 0$, then

$Q_r^{C^*} = \left(\frac{1}{\delta_\varepsilon}\right)F_r^{-1}\left(\frac{p_r + g - \frac{c}{\delta_\varepsilon}}{p_r + g - \frac{s}{\delta_\varepsilon}}\right) - k_{or}Q_{or}$ and $Q_o^{C^*} = \left(\frac{1}{\lambda_\varepsilon}\right)F_o^{-1}\left(\frac{p_o^C + g + vk_{or}c - \frac{c}{\lambda_\varepsilon}}{p_o^C + g + vk_{or}c - \frac{s}{\lambda_\varepsilon}}\right)$.

Appendix D. Proof of CHAPTER 6's propositions and corollaries

D.1 Proof of Proposition 6.1

We can rework function (6.1) to be as follows:

$$\pi_i^r(Q_r) = \theta(p_i^r - s_n)\frac{a+b}{2} - (w - s_n)Q_r + \frac{p_i^r + g_r - s_n}{2(b-a)}\left(2bQ_r - \frac{Q_r^2}{\theta} - \theta b^2\right).$$

Thus, $\frac{\partial^2 \pi_i^r}{\partial Q_r^2} = -\frac{p_i^r + g_r - s_n}{\theta(b-a)} < 0$ and the function is strictly concave in Q_r. By solving $\frac{\partial \pi_i^r}{\partial Q_r} = 0$, one may find the optimal inventory level Q_i^r.

Also, we can rework function (6.2) to be as follows:

$$\pi_1^e(Q_e) = \bar{\theta}(p_1^e - s_n)\frac{a+b}{2} - (w - s_n)Q_e + \frac{p_1^e + g_e - s_n}{2(b-a)}\left(2bQ_e - \frac{Q_e^2}{\bar{\theta}} - \bar{\theta} b^2\right).$$

Thus, $\frac{\partial^2 \pi_1^e}{\partial Q_e^2} = -\frac{p_1^e + g_e - s_n}{\bar{\theta}(b-a)} < 0$ and the function is strictly concave in Q_e. By solving $\frac{\partial \pi_1^e}{\partial Q_e} = 0$, one may find the optimal inventory level Q_1^e.

D.2 Proof of Proposition 6.2

For the e-tail store decision we rewrite equation (6.3) as the following:

$$\pi_2^e(Q_e|F) = \bar{\theta}(p_2^e - s_n)\frac{a+b}{2} - (w + F - s_n)Q_e + \frac{p_2^e + g_e - s_n}{2(b-a)}\left(2bQ_e - \frac{Q_e^2}{\bar{\theta}} - \bar{\theta} b^2\right).$$

Given F, $\frac{\partial^2 \pi_2^e}{\partial Q_e^2} = -\frac{p_2^e + g_e - s_n}{\bar{\theta}(b-a)} < 0$ and the function is strictly concave in Q_e. By solving $\frac{\partial \pi_2^e}{\partial Q_e} = 0$, one may find the best response function for the e-tail store Q_2^e.

D.3 Proof of Proposition 6.3

Equation (6.4) can be rewritten as $\pi_2^L(F) = Q_e F - \bar{\theta} c\mu - \frac{c}{2(b-a)}\left(2bQ_e - \frac{Q_e^2}{\bar{\theta}} - \bar{\theta} b^2\right)$.

Thus, $\frac{\partial^2 \pi_2^L}{\partial F^2} = -2\frac{\bar{\theta}(b-a)}{(p_e^2 + g_e - s_n)^2}\left(p_e^2 + g_e - s_n - \frac{c}{2}\right)$. Knowing the following: $c = h_l(1 + r_e)$ and $h_r > h_l$, one may conclude that $p_e^2 - \frac{c}{2} > p_e^1 > w > S_n$. Therefore, $\frac{\partial^2 \pi_2^L}{\partial F^2} < 0$ and the function is strictly concave in F. By solving $\frac{\partial \pi_2^L}{\partial F} = 0$, one may find the optimal seasonal fee F_2.

D.4 Proof of Corollary 6.1

We assume the opposite is true, thus $F_2 > \frac{b(p_2^e + g_e - s_n)}{(b-a)} - (w - s_n)$. By substituting the value of F_2 in the previous inequality and performing basic algebra we get $\frac{b}{b-a} < \frac{w - s_n}{p_2^e + g_e - s_n - c}$. Since $c = h_l(1 + r_e)$ and $h_r > h_l$, thus, $p_2^e - c > p_1^e > w > S_n$. Thus,

$\frac{b}{b-a} \geq 1$, and $\frac{w-s_n}{p_2^e+g_e-s_n-c} < 1$. Therefore, there is a clear contradiction and Corollary 6.1 is always true.

D.5 Proof of Proposition 6.4

The e-tail channel's profit function, i.e. equation (6.5), can be rewritten as:

$\pi_{gs} = (p_3^e - s_n)\bar{\theta}\mu + (s_n - w)Q_e + \frac{p_3^e+g_e-s_n}{2(b-a)}\left(2bQ_e - \frac{Q_e^2}{\bar{\theta}} - b^2\bar{\theta}\right)$.

Given F, $\frac{\partial^2\pi_3^e}{\partial Q_e^2} = -\frac{\emptyset}{\bar{\theta}}\left(\frac{p_3^e+g_e-s_n}{b-a}\right) < 0$ and the function is strictly concave in Q_e. By solving $\frac{\partial\pi_3^e}{\partial Q_e} = 0$, one may find the best response function for the e-tail store Q_3^e.

Substitute the value of Q_3^e in the function π_3^L. Since, $\frac{\partial^2\pi_3^L}{\partial F^2} = -\frac{\bar{\theta}(b-a)(\emptyset+1)}{\emptyset^2(p_3^e+g_e-s_n)} < 0$, then function is strictly concave in F. By solving $\frac{\partial\pi_3^L}{\partial F} = 0$, one may find the optimal seasonal fee F_3.

D.6 Proof of Corollary 6.2

We assume the opposite is true, thus $F_3 > \emptyset\left(\frac{b(p_3^e+g_e-s_n)}{(b-a)} - (w - s_n)\right)$. By substituting the value of F_3 in the previous inequality and performing basic algebra we get $\emptyset > \emptyset + 1$. Therefore, there is a contradiction and Corollary 6.2 is always true.

D.7 Proof of Corollary 6.3

We proof this corollary by contradiction. Thus, assume that the opposite is true, i.e. $F_3 \geq F_2$. Since $\frac{\partial F_3}{\partial\emptyset}$ is positive, then F_3 will have its highest value at $\emptyset = 1$. By substituting the values of F^2 and F^3 into the inequality $F_3 \geq F_2$ when $\emptyset = 1$, one may find the following relationship:

$(p_3^e + g_e - s_n)\left(p_2^e + g_e - s_n - \frac{c}{2}\right) \geq (p_2^e + g_e - s_n)^2 + \frac{(w-s_n)(b-a)c}{2b}$. Since $p_2^e \geq p_3^e$, then it is a necessity that $LHS \leq RHS$, and, therefor, the inequality $F_3 \leq F_2$ always holds true.

D.7 Proof of Corollary 6.4 (Part ii)

We can easily proof this corollary by contradiction. Assume that the opposite is true, i.e. $Q_2^e \geq Q_3^e$. Since $\frac{\partial Q_3^e}{\partial\emptyset}$ is negative, then Q_3^e will have its lowest value at $\emptyset = 1$. By substituting the values of Q_2^e and Q_3^e into the inequality $Q_2^e \geq Q_3^e$ when $\emptyset = 1$, one may find the following relationship: $b(p_3^e + g_e - w) < -a(w - s_n)$. It is trivial to see that this is not true and, thus, the inequality $Q_2^e \leq Q_3^e$ always holds true.

D.8 Proof of Corollary 6.5 (Part i)

We assume that $\pi_3^e \geq 0$ under all values of $\emptyset = [0,1]$. That is to say the e-tail store's shortage value is not unrealistically high, i.e. $g_e < (p_3^e + g_e - w)^2/4(p_3^e + g_e - s_n)$. Thus, $\frac{\partial \pi_3^L}{\partial \emptyset} = \frac{\overline{\theta}}{2}\left(g_e - \frac{(p_3^e+g_e-w)^2}{(\emptyset+1)^2(p_3^e+g_e-s_n)}\right) < 0$.

D.9 Proof of Corollary 6.5 (Part ii)

Under the above condition, we can easily use contradiction to prove that $\pi_2^L < \pi_3^L$. We assume the opposite is true, i.e. $\pi_2^L > \pi_3^L$. Since $\frac{\partial \pi_3^L}{\partial \emptyset} < 0$, then π_3^L will have its lowest value at $\emptyset = 1$. Substituting $\emptyset = 1$ into the relationship $\pi_2^L > \pi_3^L$ and performing simple algebraic manipulation leads to $2 < 1$. That is a contradiction and, thus, our statement is correct.

D.10 Proof of Corollary 6.6

We know that π_3^e is concave on $\emptyset$. Also, we can easily prove that $\pi_3^e \geq \pi_2^e$ at $\emptyset = 1$. Assume the opposite is true and substitute $\emptyset = 1$ into the relationship $\pi_3^e \leq \pi_2^e$ to get $\left((1 + r_e)h_l\right)^2 \leq 0$. That is a contradiction and, thus, our statement is true. We now set $\pi_2^e = \pi_3^e$ to find the formula $\emptyset^3 A + \emptyset^2 B - \emptyset C + D = 0$. Since $\partial\pi_3^e/\partial\emptyset < 0$ and $\partial\pi_2^e/\partial\emptyset = 0$, the previous formula has at the most a single root that satisfies the condition $0 \leq \emptyset \leq 1$ where;

$A = g_e(p_3^e + g_e - s_n)(2p_2^e + 2g_e - 2s_n - c)^2$

$B = (p_3^e + g_e - s_n)\left(g_e(2p_2^e + 2g_e - 2s_n - c)^2 + (p_3^e + g_e - w)^2(p_2^e + g_e - s_n)\right)$

$C = (p_3^e + g_e - w)^2((p_3^e + g_e - s_n)^2 + (p_2^e + g_e - s_n)^2) + g_e(p_3^e + g_e - s_n)(2p_2^e +$
$2g_e - 2s_n - c)^2$

$D = (p_3^e + g_e - s_n)((p_3^e + g_e - w)^2(p_2^e + g_e - s_n) - g_e(2p_2^e + 2g_e - 2s_n - c)^2)$

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