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# **Machine Scheduling for Multitask Machining**

By

**Saleh Yavari**

A Thesis  
Submitted to the Faculty of Graduate Studies  
through the Industrial Engineering Graduate Program  
in Partial Fulfillment of the Requirements for  
the Degree of Master of Applied Science  
at the University of Windsor

Windsor, Ontario, Canada

2017

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# **Machine Scheduling for Multitask Machining**

by

**Saleh Yavari**

APPROVED BY:

---

N.Zamani

Department of Mechanical, Automotive and Materials Engineering

---

J.Urbanić

Department of Mechanical, Automotive and Materials Engineering

---

A.Azab, Co-Advisor

Department of Mechanical, Automotive and Materials Engineering

---

F.Baki, Co-Advisor  
Odette School of Business

December 8, 2017

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## ABSTRACT

Multitasking is an important part of today's manufacturing plants. Multitask machine tools are capable of processing multiple operations at the same time by applying a different set of part and tool holding devices. Mill-turns are multitasking machines with the ability to perform a variety of operations with considerable accuracy and agility. One critical factor in simultaneous machining is to create a schedule for different operations to be completed in minimum make-span. A Mixed Integer Linear Programming (MILP) model is developed to address the machine scheduling problem. The adopted assumptions are more realistic when compared with the previous models. The model allows for processing multiple operations simultaneously on a single part; parts are being processed on the same setup and multiple turrets can process a single operation of a single job simultaneously performing multiple depths of cut. A Simulated Annealing algorithm with a novel initial solution and assignment approach is developed to solve large instances of the problem. Test cases are presented to assess the proposed model and metaheuristic algorithm.

**Keywords:** Parallel Machining, Multitasking, Mill-turn, Mixed Integer Linear Programming, Scheduling, Simulated Annealing.

DEDICATION

*Dedicated to*

***My beloved father, mother, and sister.***

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## LIST OF ABBREVIATIONS/SYMBOLS

MILP	Mixed Integer Linear Programming
MU	Machining Unit
PML	Part Machining Location
NC	Numerical Control
CAPP	Computer Aided Process Planning
PMPS	Parallel Machine Planning System
CAD	Computer Aided Design
CAM	Computer Aided Manufacturing
GA	Genetic Algorithm
SA	Simulated Annealing



## CHAPTER 1: INTRODUCTION

### **1.1 Multitasking in metal cutting**

Mill-turn machines are machine tools with considerable abilities to machine different work-pieces simultaneously. Although the tendency to benefit from multitasking and parallel machine tools is increasing considerably in the industry, there has not been enough attention in the literature to the topic. Yip-Hoi and Dutta [1] provided a genetic algorithm for process planning operations on mill-turns. Norman & Bean [2] and Chiu et al.[3] modeled a small scale scheduling problem on parallel machine tools. In this thesis, multitask machining, parallel machining, simultaneous machining, or simply mill-turns are all used interchangeably.

Mill-turns are those machine tools that resemble lathes in structure and kinematics using spindles as work-piece holding devices, yet they carry live milling tools in their multiple turrets [4]. Mill-turns are one step ahead of traditional lathes and vertical mills in terms of agility and accuracy. These powerful machine tools offer some valuable features such as simultaneous machining of a single operation of a part with multiple turrets, which distinguishes them from other machines. Multitasking enables turning, milling, facing and drilling operations to be done on a single setup with minimum processing times [5-7]. In metal cutting, the final quality of products and the amount of time it takes to machine them are critical. A mill-turn machine can process parts, which might need three to four different machine tools to be turned into a finished product. Therefore, mill-turns offer a set of valuable features that make them a suitable choice for metal cutting and manufacturing industries.

## **1.2 Problem definition**

Mill-turns (parallel machines) are a class of machine tools, which have gained a lot of interest in recent years in the industry. Traditionally, CNC lathes were developed to deal with turning tasks and machining centers equipped with rotary cutting tools were designed to handle milling and drilling type of operations. With the application of multi-functional machine tools, not only it is possible to continuously process a single job by more than one cutting tool, it is also possible to machine more than one job at a time using multiple work holding and cutting tools simultaneously [7].

Mill-turns have different configurations with a variety of applications. If a process plan or a schedule is about to be created for a mill-turn, the configuration and category of the machine tool considered needs to be specified. With due attention to the capacity of these machine tools to agilely machine parts in a fast-paced environment, creating a schedule and sequence plan to fully present these capabilities is critical. Cycle time is a key factor in manufacturing facilities when satisfying a variety of demands with predefined quality standards in a minimum amount of time. It is crucial to benefit the most from the available production time and resources.

A feasible schedule plan is capable of reducing downtimes and will provide a procedure to complete the machining processes in an optimum amount of time. The following concepts are key to the understanding of multitask machining: Machining Unit (MU) and Part Machining Location (PML) [8].

### **1.2.1 Machining Unit (MU)**

Machining unit is used to hold different cutting tools on a parallel machine tool. In mill-turns, turrets are applied as machining tools [9, 10]. The turret itself is equipped with multiple cutting tools that can be mounted automatically for different machining purposes [10]. There could be two or more turrets present, depending on the configuration of the mill-turn machine.

The capacity of turrets in terms of holding cutting tools is also different from one machine tool to another. Rotation of turrets is another key feature in multitasking. Turrets

mounted on a mill-turn have limitations in terms of the rotational axis. Depending on the configuration of the machine tool, different machines have different rotational capacities. Machining units, which are capable to move along multiple axes have more flexibility in terms of performing operations on work-pieces [11, 12]. Machining units are usually located above and under the spindle. They can move along different axes. Upper turrets are commonly capable of moving along X, Y and Z axis, lower turrets though, usually have movements along X and Z axis. There are some configurations, which their upper turret have the capacity of B-axis rotation. Application of B-axis is a great feature when machining the most complicated parts that need a versatile tool orientation for the metal cutting processes [13-15]. In order to avoid overheating of cutting tools mounted on a turret, multitask machines are equipped with a coolant system, which is offered in different ways. The coolant system could be applied through the coolant fluid, it can also be through circulation of coolant inside the spindle or air blown system through nozzles directed towards the cutting tool and the work-piece [16]. Application of the coolant depends on the part being processed, the material used and the machine tool applied.

### **1.2.2 Part Machining Location (PML)**

Part machining location is a location on a mill-turn machine that holds the part for metal cutting purposes. A spindle or a chuck are common machining locations on a multitask machine [10]. Spindles have the duty to rotate the work-piece and provide enough energy for the metal cutting processes [17-19]. The common configurations of mill-turns have usually two spindles applied, main spindle and a sub spindle [3]. These spindles have access to each other so that if a specific part needs a certain type of machining process, it could be easily moved from one spindle to another [20]. Another application of applying the second spindle in parallel machining is using them as a tailstock. There are some parts, which are longer to be held by a single spindle and need a second spindle to act as a tailstock and support them while they are being machined. In some certain occasions, along with the application of tailstock, an extra support called steady rest is used to provide further supports while the part is being machined [21].

Spindles have different capacities in terms of loading parts. Depending on the need and configuration of the mill-turn, spindles might have different bar capacity, speed, and

power. Spindles of a mill-turn are fed through automatic bar feeders, which are one of the core components of a mill-turn in terms of accelerating the process of turning raw materials into the finished parts. Spindles and work holding supports are adjusted automatically and without the operator's interference. The movements of spindles and turrets are programmed through Computer Aided Manufacturing (CAM) software packages, which enable users to simulate the metal cutting procedure and therefore, eliminate the potential errors associated with the machining processes [22]. Figure 1 shows a sample part, which is being machined by one of the cutting tools mounted on a turret.

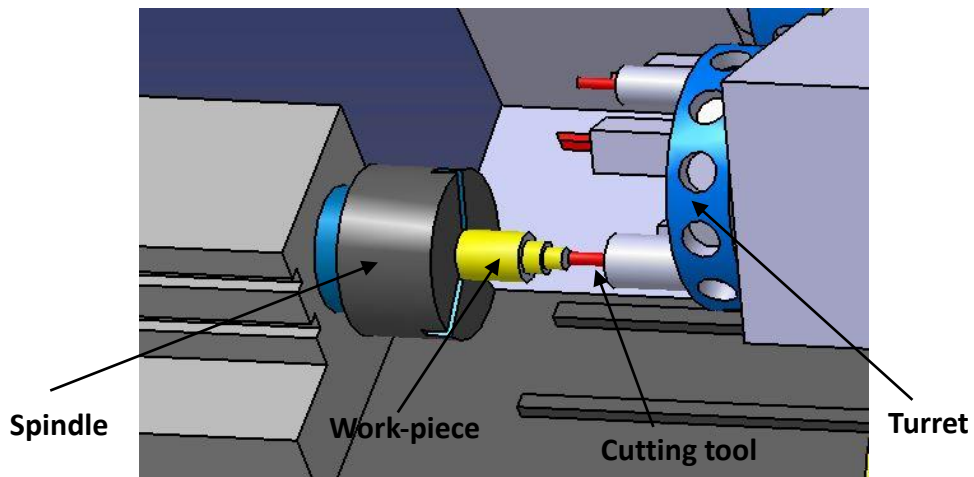


Figure 1: Work-piece held by a spindle and processed by a cutting tool mounted on a turret

### 1.3 Multitasking on mill-turns and its difference with transfer machines

Multitasking on mill-turns is consist of three individual concepts. The first concept is related to simultaneous machining of different parts or work-pieces on different spindles of multitasking machine tool. This feature enables the machine tool to hold more than one work-piece at a time and therefore, it can handle more than one machining process in a specific time slot. The machining procedure of these work-pieces could be totally different from each other. They could have a different schedule plan and yet be machined at the same time without any conflict or delays in their machining processes. The second concept is related to the simultaneous processing of different operations of a work-piece. This feature enables the machine tool to use its multiple machining units (turrets) to machine

the part and hence reduce the total time it takes to process it. Simultaneous machining of different operations of a part need some conditions to be met, these conditions along with their impact on the final schedule plan are discussed in detail in the remaining sections of this chapter. The third concept related to multitasking on mill-turns is the application of multiple machining units to process a single operation of a part. This important feature is also integrated into the structure of multitasking machine tools to accelerate the machining process and deliver high-quality parts in the shortest amount of time.

Transfer machines have some common features with mill-turns and there are some differences in terms of kinematic and structure. Both mill-turns and transfer machines have the capacity to process different parts at the same time but there are some differences that need to be discussed. Transfer machines are basically used to process a considerable number of the same category of parts [23]. These are the same identical parts that need a predefined machining procedures to be completed. The mechanism is such that the work-piece moves from a set of fixed part machining locations to another. On each station or work holding location, one or more specific type of machining operation is performed and then the part is moved to another location for further machining. The variety of parts machined on a transfer machine is very limited in comparison with mill-turns. Another major difference is that mill-turns are capable of simultaneous machining of work-piece on a single work holding location and excessive movements of parts are removed. Mill-turns are equipped with machining units that can approach the part from different angles and apply different machining operations based on the need. This important feature makes mill-turns a good fit for both small and large quantities of parts and increases the flexibility of the machine tool in compare with transfer machines.

## **1.4 Structure and functionality of mill-turns**

Mill-turns have two separate work holding locations built in their structure. These work holding locations have the duty to hold the work-piece for machining purposes and move the part from one location to another if certain types of machining processes are required. When machining procedure of a part is done, a work holding device or simply a sub spindle on a mill-turn machine could be applied to remove the part from the main spindle and lead it towards finished parts. Multitask machine tools usually apply two or more tool holding devices to enable them for application of some specific features like pinch turning and simultaneous machining of two different work-pieces. As discussed in section 1.2.1, the total number of machining units applied on a machine tool depends on the configuration of the machine and variety of parts to be processed. Turrets on a mill-turn machine have also live tooling capabilities to handle milling types of operations.

There are various configurations of mill-turns applied. Depending on the need, each of them could be used in a manufacturing plant. Most of the configurations have two spindles. However, in terms of turrets, they could be two or more each containing several cutting tools. The focus of this research is on scheduling operations on a mill-turn with dual-spindle and dual-turret. Each of these turrets can have access to both spindles; i.e., they could work separately or together on the same spindle.

## **1.5 Multi-axis machine tool features and specialties**

### **1.5.1 Definition and applications**

Multi-axis machining is a feature, which enables the machining unit on a multitask machine tool to process complicate parts [12]. Basically, traditional lathes are capable of machining the part in X, Y, and Z direction. However, multi-axis machine tools have the capability to process a work-piece with the application of up to five different axes. Machining unit on a multi-axis machine tool has more freedom to move in different directions with more agility and flexibility in comparison with conventional machine tools. This flexibility in terms of movements enhances the capacity of the machine tool to process work-pieces which are difficult to be machined. Furthermore, the capacity of the machining

unit to move along multiple directions accelerates the machining process and delivers parts with improved quality [24].

In order to tackle the problem of part complexity, mill-turns have the ability to apply a 5-axis machining feature. In 5-axis machining, we have the movement along the traditional linear X, Y and Z axis, and two rotational directions along two of these three axes. If the cutting tool has the capability to rotate around the X-axis, then we will have a rotational A-axis. If it is able to rotate around the Y-axis we will have the rotational B-axis and if it can rotate around Z-axis we have C-axis machining feature in addition to the three traditional directions [25, 26]. Application of machine tools with the advantage of moving along multiple axes creates a machining system that enables them to process complicate parts that cannot be machined on conventional machine tools [27]. It can also decrease the total time it takes to machine the part and therefore, enhance the efficiency of the process

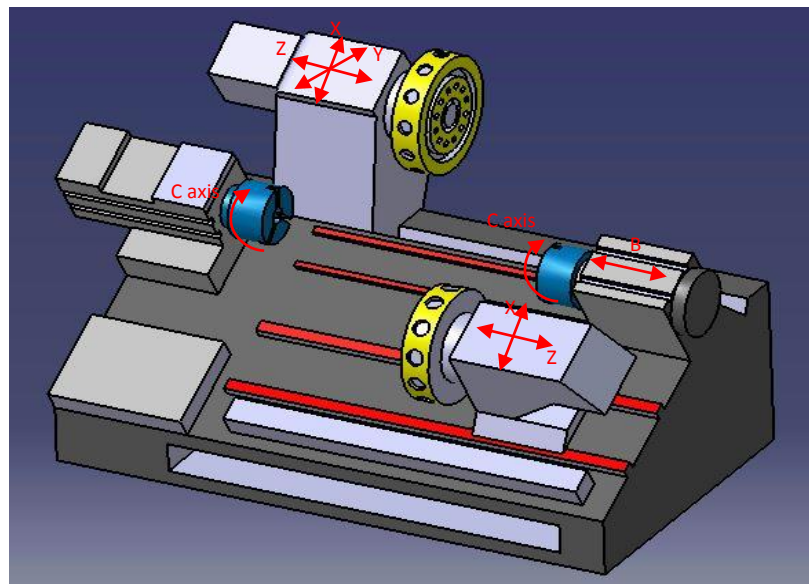


Figure 2: A dual-spindle dual-turret mill-turn

### **1.5.2 Live tooling**

One of the main features of mill-turns, which differentiate them from conventional lathes is that their turrets provide live tooling features. Turrets have the capability to be powered and have motors in their structure to be able to simply rotate the cutting tools mounted on them. In other words, they are capable of providing live tooling. If a turret has this capacity, it can handle those type of operations which need the rotation of the cutting tool. Live tooling is a great feature since it enhances the flexibility of the machine tool and therefore its performance. This feature is the logic behind the implementation of single setup concept. With the application of such an approach, cutting tools will be mounted by turrets based on the need and there is no need to further move the work-piece between machining centers.

### **1.5.3 Pinch turning**

Turrets are not only capable of processing different operations of a single job (work-piece), but they also have the ability to process a single operation of that job at the same time and hence, accelerate the process of machining [28, 29]. This feature is called pinch turning. The mechanism of pinch turning is such that one of the cutting tools is one depth of cut ahead of the other one [30]. The procedure is such that one cutting tool starts removing the material from outside diameter of the part and the other cutting tool is idle at the beginning, then after a very short hesitation, the other cutting tool starts the machining procedure from the opposite side of the part. This is a great feature since it reduces multiple movements of turrets and cutting tools to half and therefore, reduces processing times considerably. Figure 3 shows the described mechanism for the procedure of pinch turning.

Two turrets can also work simultaneously on a single milling operation and provide a pinch milling procedure. Pinch milling needs the two turrets to be equipped with live tooling system. Pinch milling considers restrictions of turrets in terms of rotating along different axis and space wise limitations, therefore, it is not common as pinch turning.



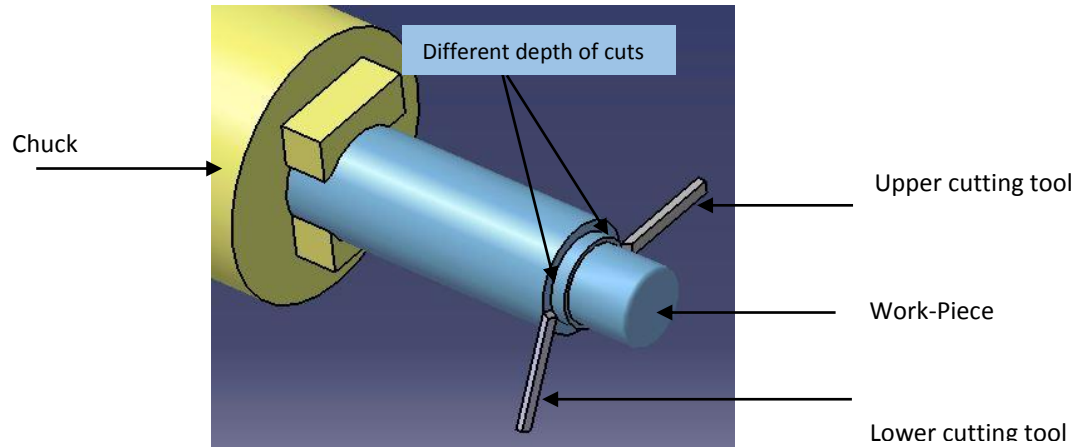


Figure 3: Application of pinch turning on a work-piece

#### 1.5.4 Operation mode

Operations processed on a mill-turn machine are affected by the rotation of work-piece and cutting tools mounted on turrets. There are two major types of operations, ones which need the rotation of work-piece such as turning and facing, and those which require the rotation of cutting tools such as milling and drilling. If two operations of the same job are of the same mode (rotation of either work-piece or cutting tool), these operations can be processed simultaneously on a single spindle. In other words, it is necessary for two operations to have the same mode to be machined simultaneously by two different turrets on a single spindle.

The mode of operations is in direct relation to the rotation of spindles. Turning operations need the rotation of the spindle and hence, the workpiece held by it, whereas milling operations need the rotation of the cutting tool and the spindle can be stationary. Therefore, these two operations cannot be processed simultaneously on a single part. However, in the world of multitask machining, turn-milling is a different type of processing, in which both the work-piece mounted in the spindle and the live milling tool rotates at the same time [31]. The reason to apply turn-milling instead of turning operations is that turn milling provides less contact time between the cutting tool and the part, therefore, there will be less heat produced during the machining process and the life of the

cutting tool will increase [32]. Figure 4 shows the procedure of turn-milling and how the cutting tool and work-piece are rotated.

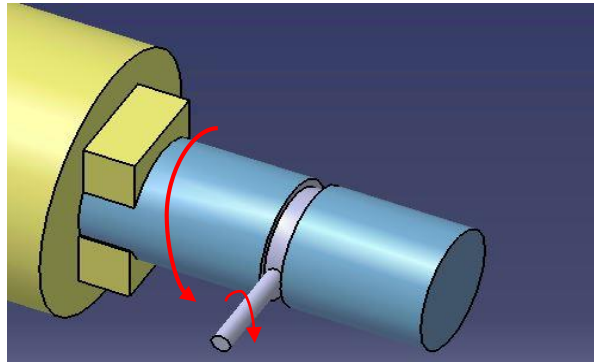


Figure 4: Application of turn-milling on a single work-piece

Limitations regarding the operation mode have a major effect on the final schedule plan proposed for processing operations of a single job on a machine tool. Figure 5 shows a sample part, which needs both turning and milling operations to be completely machined. As can be seen, four different segments of the machined sample part are specified. Features A and B, which include a hole drilling and a face milling are among those operations which need the rotation of the cutting tool. On the other hand, features C and D, which are processed through two different outside diameter turning operations, are among operations which need the rotation of the spindle and the cutting tool is stationary. Therefore, features B and C cannot be machined simultaneously, but features C and D could be processed at one time if there is an available free turret for both of them.

Another concept related to the mode of operations is that each of the presented features could be the result of multiple separate features. For instance, if we consider the feature A of the part presented in Figure 5, we can see that this feature is a drilled hole, which could be consist of four different operations; center drilling, drilling, boring and reaming. Each of these operations required to achieve the final drilled feature is considered as a separate operation and therefore, will affect the final schedule plan.

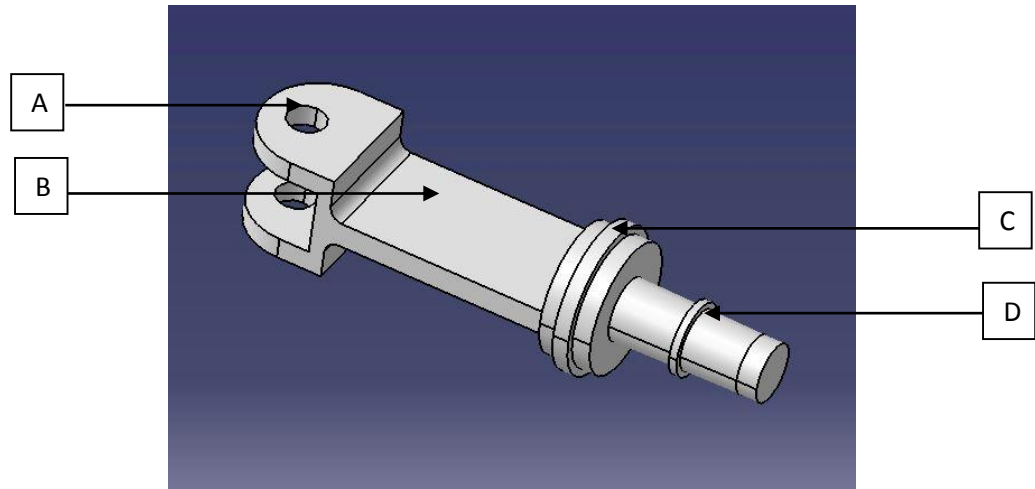


Figure 5: A sample part with milling and turning features

### 1.5.5 Hard turning

Hard turning is one of the key features of multi-axis machine tools. It is the process of cutting metals that are hardened to be used in some specific applications, and it could be a good replacement for grinding operations [33]. By application of hard turning, parts can be machined on the same setup and it provides a more agile and accurate type of machining in compare with grinding operations [34-36]. Mill-turns have the capability to apply hard turning, which is an important feature when integrating the concepts of single setup and a faster and more accurate machining process.

There are some points which should be considered when applying hard turning on a machine tool. The most important point is that the machine tool itself should be rigid enough to handle the process of hard turning. The rigidity of the hardened part and the cutting angle applied for this process creates a considerable amount of stress. Therefore, the machine tool itself should be capable of handling the procedure. Another key feature, which has to be considered is the cutting tool used for material removal. The cutting tool must be tough enough to stand high temperatures and not to get worn. The third element to be taken into account is the accurate control over the machining procedure and parameters. The material removal rate and the speed at which the spindle is turning has to be adjusted accordingly.

## 1.6 Process planning for parallel machines

General process planning and feature recognition of mill-turns have been introduced to create a connection between computer-aided design (CAD) and computer-aided manufacturing (CAM) systems. Process planning provides a set of procedures and instructions to accomplish the procedure of machining the part [37]. Based on the designed features and specifications of each part a process plan is created.

Process planning on mill-turns considers the detailed design specifications of a work-piece. It applies an automated computerized system, which is referred as CAPP, and specifies the following [8, 10]:

1/Types of machining operations/features required for the metal cutting procedures. At this level, different operations/features of a part, which should be processed to complete the procedure of machining the part are identified. This information is recorded in the system to be used in future steps of the planning module. When features are recognized, complementary data regarding the planning mode are integrated into the system. To do so, requirements such as precedence relationships between operations, the location where the work-piece is going to be processed and different operations needed to process a feature have to be considered.

2/Selection of proper cutting tool/machining unit and material removal rate. The tools considered for machining each feature should be selected based on that specific operation. Some extra features of cutting tools such as heat resistance and its rigidity should also be taken into account. There will be a link between the proper cutting tools used for machining procedures and the rate at which material will be removed to achieve a feasible planning scheme.

3/ Feasible sequence for processing each feature. A valid process plan considers the sequence in which different features of the part will be processed. In order to deliver a final product, operations required to complete a part have to be processed in a predefined sequence to reach the desired outcome.

4/Machining parameters, which affect the cutting procedures have to be specified. Attributes such as cutting speed and depth of cut are key parameters, which have to be considered. In addition, possible tool path for each feature needs to be defined.

### **1.7 Significance and research motivation**

Considering how mill-turns are fast and reliable processing parts in an accurate manner and a very short time makes them a suitable choice for many production units. They have many capabilities, which differentiate them from other milling or turning machine tools. Some of these features are listed below:

- Mill-turns have the privilege to handle both milling and turning operations on one setup [10]. This is an important feature because it can reduce handling of parts.
- Since spindles have access to each other, the movement of the work-pieces from one work holding location to another is done automatically. In case both sides of the part need to be machined, there is no need to stop the machine tool and change the setup [38].
- Automatic movements of parts remove the need for the operator's action [7] and therefore this could reduce the errors related to the operator.
- Parallel machining of a single part by different machining units, reduces the total make-span. This is an important feature when machining more parts in a very limited time. This feature can also reduce costs related to delays.
- Parallel machining of different parts at the same time is also a considerable feature when different products are planned to be machined and the total make-span of all parts is a key factor.

There has been a tremendous attention from manufactures to the applications of mill-turns and simultaneous machining. There is limited research done in this area and most of the key features and aspects are not fully covered. The main motivation of this thesis is to provide a detailed review of the applications and mechanism of parallel N.C machines and

to address the problem of scheduling operations on mill-turns by considering more realistic assumptions.

The second motivation of this thesis is to provide a mathematical model, which is capable enough to present most of the features and advantages regarding the application of parallel machines. Finally, since the problem at hand is computationally NP-complete [39], developing a creative algorithm to solve larger sets of the problem which is most likely to be the case, in reality, constitute the third motivation of this thesis.

## **1.8 Research outline**

The second section of this thesis contains a detailed literature review. In the third chapter, the Mixed Integer Linear Programming (MILP) model with an illustrated example is presented. The fourth chapter, explains the heuristic algorithm and steps involved in creating feasible solutions. The fifth chapter includes the case studies and computational results obtained through testing of the mathematical model and the simulated annealing algorithm. Finally, conclusions and future works are provided in the last chapter.

## CHAPTER 2: LITERATURE REVIEW

### 2.1 Problem statement

In this thesis, the problem of scheduling different Jobs on multitasking machine tools is considered. Jobs are simply products or parts, which consist of a different set of operations. The goal is to create a schedule for machining operations of different jobs such that minimum total completion time is achieved. The type of machines considered for the scheduling problem is multitasking machine tools known as mill-turns with dual-spindle and dual-turret configuration. The machine tools considered in this thesis have the capacity to perform multitasking on one part or multiple parts and therefore, they are different from transfer machines. The final schedule also determines the sequence of operations on different machine tools. Each operation has different processing times on different machines, and there are some constraints related to the capacity of each machine (turrets and spindles). Precedence relations among operations are considered as a vital part of the scheduling problem. Finally, the capacity of turrets and spindles in terms of simultaneous machining and concepts regarding the operation modes and single setup are taken into account.

### 2.2 Literature review

In spite of the growing demand in the industry to apply parallel machining and multitask machine tools, research done on scheduling so far is quite limited. The problem of scheduling and sequencing operations on mill-turns has not been considered in comparison with traditional parallel machine scheduling problems [40-42], which have gained more attention in the literature. Unique features and capabilities of mill-turns along with limitations will be explained in detail to further explore the physics of these machine tools and identify possible areas of improvement.

Miller, P. C [43] and Miska, K. H. [44] introduce a new class of machine tools, which are capable of machining parts in less amount of time and fewer setups. They fully discuss the similarities and differences between conventional lathes and mill-turns. They

also discuss the key concepts of parallel machining and different configurations of mill-turns.

Process planning and machining features have been considered by different authors in the literature [45-47]. Levin and Dutta [8], discuss different elements of parallel machining and multitasking, which affect the process planning for mill-turns. They compare variant process planning versus process planning using optimization methods and generative process planning. They also define the scope and development of Computer Aided Process Planning (CAPP) for mill-turn machines.

Yip-Hoi and Dutta [38] use the information available from CAD models to generate a valid process plan for machining different parts. To do so, they consider geometric models of work-pieces to be machined. Considering the geometric features of parts and in relation to CAD models, they present a scheme to determine the proper axis for machining parts and also define the most suitable turning volumes. Feature recognition has also been studied in the researches done by Tseng. Et al [48, 49], in the work of Li [50] and Dutta. et al [51].

Azab and Naderi [52] propose a variable neighborhood search metaheuristic for the problem of scheduling operations for simultaneous machining. The objective of the proposed algorithm is to reach the minimum completion time for all operations. They do not model the problem mathematically to define the capabilities and limitations of parallel machines. Furthermore, the capacity of the machine tool to process a specific operation of a single job by more than one cutting tool, mode of operations and concepts of single setup is not considered in their study.

Levin and Dutta [10] study a detailed CAPP for parallel machines. They introduce PMPS (Parallel Machine Planning System) for mill-turns to control removal of materials from parts through the use of proper cutting tools and then sequencing operations on machines. They also address concerns regarding tool path and parameters used for machining. Levin and Dutta divide PMPS into two different categories; preprocessor and planning module. In their defined preprocessor stage, permissible tolerances and depth of cuts, precedence relations between operations, types of operations and best locations to hold the part for machining purposes are determined. In the planning module defined by Levin and Dutta, different operations are sequenced regarding the precedence



relationships. The outcome of their PMPS system is a feasible sequence of operations with assigned turrets and spindles for each operation.

Yip-Hoi and Dutta [7] discuss important components of simultaneous machining, which are key to process planning. For that, they consider different properties of the part such as shape, size and approach direction. Another issue, which is considered in their work is exploring the most important components of mill-turns that are vital to the process planning inputs. To address that issue, they study a template configuration of mill-turns in detail and then they use instances of this template as an input for the process planning procedure.

Yip-Hoi & Dutta [1] suggest a genetic algorithm (GA) to address the problem of sequencing operations on mill-turns. They present a process plan to address geometric features and limitations regarding mill-turn machines. The objective of Yip-Hoi and Dutta's algorithm is to achieve an optimum completion time for all operations. The offered process plan, also considers the precedence relationships among operations, cutting tool approach directions and machining parameters. However, no mathematical model is proposed to further extend the problem for larger and more complicated instances.

Norman & Bean [2] present a small size model for the problem of scheduling mill-turns and consider a shop where one work-piece is machined and scheduled. The objective function is to minimize the total completion time. They assume a model in which all operations are pre-assigned to spindles. The capacity of machines in terms of applying multiple turrets for a single operation is not considered and since the model is considering only one job to be machined concepts regarding the single setup are absent.

Chiu et al. [3] solve the problem of sequencing and scheduling operations on parallel machines for a plant with only one part and one machine tool to be scheduled. These researchers solve a model that is not linear. In their study, operations are scheduled irrespective of the rotation of spindles and cutting tools and its effect on the final sequence. Similar to the research done by Norman & Bean [2], since only one job and one machine tool is considered for scheduling, no constraint is provided to prevent movement of part from one machine to another.

A Mixed Integer Linear Programming model is developed by Naderi and Azab [39]. They formulate a model for the problem of scheduling operations in a flexible

manufacturing cell to minimize the overall completion time. They present a mathematical model, which does not consider the mode of operations and how it could affect the number of operations processed at the same time on a single spindle. Also, the capacity of turrets to simultaneously process a specific operation of a single job on a single machine tool is not considered.

### **2.3 Gap analysis**

In order to summarize the related researches done in the area of multitasking and machine scheduling, a synthesis matrix has been provided. Table 1 presents these findings along with a brief description of the solution approach and assumptions, which are taken to tackle the scheduling/sequencing problem. According to the literature, unlike the general unrelated parallel machine scheduling problem [53-56] the research done to date in the area of scheduling mill-turns is limited to simplistic problems and assumptions where a few parts to be machined. However, this is not the case for most of the manufacturing plants in which there might be different parts and multiple machine tools used for processing them [57]. To the knowledge of the author, Naderi and Azab [39] were the only researchers who recently provided a mathematical model in which more than one workpiece with a different set of operations and machines were considered for the scheduling problem. They were also assuming each operation has different machining time on different machine tools.

The assumption of having multiple jobs in which each job has a different number of operations is a more realistic assumption in compare with traditional scheduling problems in the literature in which a limited number of fixed jobs and machine tools are considered. Moreover, the scheduling problem should fully resemble the features of these machines to generate a plan, which is fairly close to their mechanism in reality. Machining a part on a single setup and capacity of machine tools to process an operation with multiple turrets are among these features, which were not addressed in the work of Chiu et al. [3], Norman & Bean [2] and Naderi and Azab [39], who proposed a mathematical model to tackle the parallel machine scheduling problem.

Table 1: Synthesis Matrix

<b>Article</b>	<b>Problem tackled</b>	<b>Mathematical model</b>	<b>Objective function</b>	<b>Constraints</b>	<b>Solution approached</b>	<b>Remarks</b>
Levin and Dutta [8]	General process planning problem	-	-	Precedence relations, surface finish information, tolerances	An overall generalized comprehensive process planning	Theoretical issues related to process planning and sequencing operations on mill-turns
Yip-Hoi and Dutta [7]	Feature recognition	-	-	Precedence relations, geometric information, operation mode	A process planning system for parallel machines	Theoretical issues of CAPP related to mill-turns
Yip-Hoi and Dutta [38]	Calculation of machining axis and machining volumes	-	-	Machinable volumes, work-piece orientation, internal and external removal rate	A Computer Aided Process Planning approach for mill-turn machines	Extracting data from geometric models and using them to create feasible machining processes.

Article	Problem tackled	Mathematical model	Objective function	Constraints	Solution approached	Remarks
Levin and Dutta [10]	Computer Aided Process Planning problem	-	-	Precedence relations, earliest start and finish time of operations	A generative Computer Aided Process Planning system	Use of computers in the automation of process planning (CAPP) for parallel machine tools
Yip-Hoi and Dutta [1]	Scheduling	-	Min. C	Precedence relations, geometric information, operation mode, approach direction	Genetic Algorithm	Only one part is scheduled, multitasking on a single operation is not considered, no mathematical model
Azab and Naderi [52]	Scheduling	-	Min. Cmax	Precedence relations, capacity of each turret, capacity of each spindle	variable neighborhood search	Scheduling jobs without considering operation mode, concepts of single setup, multitasking on a single operation

Article	Problem tackled	Mathematical model	Objective function	Constraints	Solution approached	Remarks
Norman and Bean [2]	Scheduling	Schedule based	Min. Cmax	Precedence relations, capacity of turret and spindle, operation mode	Genetic Algorithm	A single job, multitasking on a single operation and concepts of single setup is not considered
Chiu et al. [3]	Scheduling	Sequence-based	Min. C	Precedence relations, capacity of each turret	Genetic Algorithm	Scheduling a job without considering: operation mode, single setup, and multitasking on a single operation
Naderi and Azab [39]	Scheduling	Schedule based	Min. Cmax	Precedence relations, capacity of each turret, capacity of each spindle	GA, SA, Iterated Greedy Algorithm, Artificial Immune algorithm, Iterated Local Search	Scheduling jobs without considering: operation mode, single setup, multitasking on a single operation
This Thesis	Scheduling	Schedule based	Min. Cmax	Precedence relations, capacity of each turret and spindle, multitasking on a single operation, operation mode	Simulated Annealing	This thesis considers all of the previously considered constraints, plus all the mentioned gap in the literature so far

## 2.4 Research contributions

There are many features, which should be considered when modeling mill -turns. Some of these features are basic concepts related to the mechanism of the mill-turns and constitute the overlap in previously proposed models in the literature. These concepts include precedence relationships, capacities of turrets, limitations of spindles, and constraints related to start and finish times of operations according to their processing times. There are some key areas that have not received enough attention in the literature such as the concept of the operation mode, pinch turning, and single setup.

Mode of operations and how it can affect the final schedule plan is considered in this work. The mode of operation and its relation with process planning of mill-turns could be found in some of the researches done in the literature but the concept of operation mode has not received enough attention in the literature for the scheduling problem. To the author's knowledge, the ability of multiple turrets to simultaneously process a specific operation of a single job is another area, which has not been considered in the literature yet. This feature is related to the application of pinch turning and its importance in multi-tasking and it is fully discussed in section 1.5.3.

Machining the product from raw material to a finished part in only one setup, which is known as "Done-in-One" concept, is another key factor, which has not gained attention so far. This feature removes the movement of parts from one machine tool to another and has a major effect on reducing costs related to part handling. These concepts are core features, which have a considerable effect on the feasibility of the final proposed plan. Each of them needs multiple constraints to resemble the mechanism of the machine and are discussed in detail in section three.

Therefore, the main contributions of this thesis can be summarized as follows:

- Creating a machine schedule in which the complete process of machining the part/job from raw material to a finished product is done on a single setup.
- Defining a rigorous mathematical model in which simultaneous machining of different operations of a single job is allowed based on concepts related to operations mode.
- Processing multiple operations of a single job on a single spindle simultaneously by using different turrets.
- Machining a single operation of a single work-piece (part) by multiple turrets at the same time.

## **2.5 Solution approaches compare to the literature**

The main focus of the previous studies done in the literature has been on computer-aided process planning and sequencing operations on mill-turns. Research done so far have mainly focused on feature recognition, machining parameters and sequencing operations with limited setups and available resources. However, there have been few attempts in the literature to provide a model in order to tackle scheduling problems, and if so, there have been few assumptions, which present the kinematics of mill-turns.

This thesis focuses on the scheduling problem with consideration of machine tool features, limitations, and capacities. Constraints applied to propose a model in this work could be divided into two sections. The first type considers parameters related to multitasking and specific features of mill-turns. The second type though fully presents the attributes related to the nature of tasks considered for scheduling. Precedence relations and type of each operation are among these features. The solution approached in this thesis could be compared with those research in the literature, which propose a mathematical model to tackle the scheduling problem. The proposed model is linear, therefore, optimality could also be achieved.

## CHAPTER 3: METHODOLOGY

### 3.1 Model

The problem of scheduling operations on mill-turns is addressed by developing a mixed integer linear programming model. The model optimizes the total make-span and creates a feasible solution for scheduling operations on spindles and machines. The model also assigns a feasible turret for processing each part. Solving the model creates an optimal solution to schedule different set of operations on available machine tools. The model also creates a sequence of operations for each job, taken into consideration all constraints and capabilities regarding the used parallel machine tools. The mixed integer linear programming model will consider:

- Jobs and operations with different processing times on different machine tools
- The number of turrets used for processing each operation of a specific job
- Capacity of spindles in terms of loading operations
- Capacity of spindles in terms of loading jobs
- Capacity of turrets in terms of machining multiple operations
- Restrictions regarding the operation mode
- Restrictions regarding precedence relations among operations
- Restrictions regarding assigning jobs and operations to a specific turret or spindle



### 3.1.1 Assumptions

In order to develop the mathematical model, the following assumptions were made.

1. Each operation is assigned to one machine.
2. On a machine tool, an operation is assigned one spindle.
3. Spindle engagement determines the number of active turrets (N) engaged in performing each operation, where  $N \leq 2$ .
4. If a spindle is loading two operations simultaneously, those operations should be the same type.
5. Each spindle can hold one job at a time.
6. Each job is loaded on one spindle at a time.
7. Each turret can process one operation of a job at a time.
8. Precedence relations among operations of each job must be satisfied.
9. No delay or pause is allowed between consecutive scheduled operations.
10. If a Job is assigned to a spindle as part of a setup on a machine tool, its operations will also be assigned to the same spindle of the same setup on the same machine tool. Parts are assumed to be processed on a single setup where only one spindle is used and no movement of parts between machines is allowed.
11. Application of sub spindle or tailstock is not required.

### 3.1.2 Nomenclature

$m$	total available machines	$i=1, 2 \dots m$
$n$	total jobs to be scheduled	$j=1, 2 \dots n$
$e$	spindle turret engagement	$e=1, 2$
$k_i$	count of spindles of machine $i$	$k=1, 2$
$h_i$	count of turrets of machine $i$	$h=1, 2$
$l_j$	count of operations of job $j$	$l=1, 2 \dots l_j$
$R_{j,l}$	precedence operations of operation $l$ of job $j$	
$S_{j,i}$	spindles, capable of loading job $j$ on machine $i$	
$T_{j,i}$	turrets, capable of processing job $j$ on machine $i$	
$P_{jli}$	machining time of operation $l$ of job $j$ on machine $i$	
$N_{jl}$	number of turrets used for processing each operation at a time	
$W_{jll'}$	binary parameter taking the value of 1 if operations $l$ and $l'$ of job $j$ are different types of operation and 0 otherwise	

### 3.1.3 Decision variables

#### Continuous decision variable

$St_{jl}$  start time of operation  $l$  of job  $j$

#### Binary decision variables

$B_{ji}$  1 if job  $j$  is processed on machine  $i$ , 0 otherwise

$B'_{jli}$  1 if operation  $l$  of job  $j$  is processed on machine  $i$ , 0 otherwise

$B''_{jlke}$  1 if operation  $l$  of job  $j$  is processed on spindle turret engagement  $e$  of spindle  $k$  of machine  $i$ , 0 otherwise

$B'''_{jlh}$  1 if operation  $l$  of job  $j$  is processed on turret  $h$  of machine  $i$ , 0 otherwise

$B''''_{jik}$  1 if job  $j$  is processed on spindle  $k$  of machine  $i$ , 0 otherwise

$Z_{jlj'}$  1 if operation  $l$  of job  $j$  is processed after operation  $l'$  of job  $j$ , 0 otherwise

$Z_{jlj'l'}$  1 if operation  $l$  of job  $j$  is processed after operation  $l'$  of job  $j'$ , 0 otherwise

### 3.1.4 Objective function and constraints

The objective function for the MILP model is to minimize the total completion times of all operations on all machines:

$$\text{Min } C_{max}$$

where  $C_{max}$  is the maximum completion time. The model considers all of the available machines and machining times of different operations and seeks for the best sequence of operations on each machine to minimize the make-span.

Eq. (1) and Eq. (2) are provided to ensure that each operation is processed by one spindle turret engagement, one spindle, and one machine. The parameter  $e$  defines the capacity of the spindle in terms of loading different operations. Assuming both of the available turrets work on a single operation, at any time, spindles have the capacity to load maximum two operations. It is necessary to provide a constraint for the model so that at any time each operation would only be scheduled on one machine and spindle. If there is only one operation being processed on a spindle, the parameter  $e$  takes the value of one and if two operations are simultaneously being processed on a single operation at a time, then  $e$  takes the value of two.

$$\sum_{i=1}^m \sum_{e=1}^2 \sum_{k \in S_{j,i}} B''_{jlike} = 1 \quad \forall j, l \quad (1)$$

$$\sum_{i=1}^m B'_{jli} = 1 \quad \forall j, l \quad (2)$$

Eq. (3) and Eq. (4) assign each job, which is consist of a different set of operations to one machine and spindle. As discussed in section 2.1, each job is consist of different operations and number of operations for each job is different with another. The model prevents scheduling of a single job at two different work-holding locations at a time. Hence, it is necessary to provide two different constraints, one for allocation of the job to the spindle (constraint 3) and the other for allocation of the job to machine (constraint 4).

$$\sum_{i=1}^m \sum_{k \in S_{j,i}} B''''_{jik} = 1 \quad \forall j \quad (3)$$

$$\sum_{i=1}^m B_{ji} = 1 \quad \forall j \quad (4)$$

Eq. (5) defines the number of turrets used for processing operations of each job.  $N$  is the parameter, which defines the number of turrets working on a specific operation. Some operations like turning, have the capacity to be processed by two turrets to achieve a faster pace in the machining procedure. Therefore,  $N$  is provided to define this major capacity of mill-turns, which has an important impact on the final schedule created.

$$\sum_{i=1}^m \sum_{h \in T_{j,i}} B'''_{jih} = N_{jl} \quad \forall j, l \quad (5)$$

Eq. (6) and Eq. (7) prevent simultaneous machining of two operations of the same job if they are not the same type of operations. This constraint is related to the limitations regarding the rotation of spindles. As discussed in section 1.5.4, some operations need the rotation of the work-piece and others need the rotation of the cutting tool, therefore, these two types of operations cannot be scheduled together and should not have an overlap in the final schedule created. Another point, which needs to be considered is that each feature needed to complete the machining process could itself be consist of multiple operations. In this work, each operation is considered separately and has its unique mode.

$$St_{jl} - St_{jl'} \geq B''_{jl'iker} \cdot P_{jl'i} - M(1 - Z_{jljl'}) - M(1 - B''_{jl'iker}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 1, e, e', i, k \in \{S_{j,i}\} \quad (6)$$

$$St_{jl'} - St_{jl} \geq B''_{jlike} \cdot P_{jli} - M(Z_{jljl'}) - M(1 - B''_{jl'iker}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 1, e, e', i, k \in \{S_{j,i}\} \quad (7)$$

Eq. (8-11) are provided to allow for simultaneous machining of two different operations of the same job only if they are the same type (either rotation of spindle or the cutting tool).  $W$  is the parameter, which defines if two operations are of the same type or not. If two operations of the same job are of the same type,  $W$  takes the value of zero otherwise it takes the value of one.

$$St_{jl} - St_{j'l'} \geq -M(1 - Z_{jlj'l'}) - M(1 - B''_{j'l'iker'}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 0, e, e', i, k \in \{S_{j,i}\} \quad e \neq e' \quad (8)$$

$$St_{j'l'} - St_{jl} \geq -M(Z_{jlj'l'}) - M(1 - B''_{j'l'iker'}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 0, e, e', i, k \in \{S_{j,i}\} \quad e \neq e' \quad (9)$$

$$St_{jl} - St_{j'l'} \geq B''_{j'l'iker'} \cdot P_{j'l'i} - M(1 - Z_{jlj'l'}) - M(1 - B''_{j'l'iker'}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 0, e, e', i, k \in \{S_{j,i}\} \quad e = e' \quad (10)$$

$$St_{j'l'} - St_{jl} \geq B''_{jlike} \cdot P_{jli} - M(Z_{jlj'l'}) - M(1 - B''_{j'l'iker'}) - M(1 - B''_{jlike})$$

$$\forall j, l < l' \ni W_{jll'} = 0, e, e', i, k \in \{S_{j,i}\} \quad e = e' \quad (11)$$

Eq. (12) and Eq. (13) are provided to prevent spindles from holding more than one job at any time.

$$St_{jl} - St_{j'l'} \geq B''''_{j'ik} \cdot P_{j'l'i} - M(1 - Z_{jlj'l'}) - M(1 - B''''_{j'ik}) - M(1 - B''''_{jik})$$

$$\forall j < j', l, j < n, l', i, k \in \{S_{j,i} \cap S_{j',i}\} \quad (12)$$

$$St_{j'l'} - St_{jl} \geq B''''_{jik} \cdot P_{ji} - M(Z_{jlj'l'}) - M(1 - B''''_{jik}) - M(1 - B''''_{jik})$$

$$\forall j < j', l, j < n, , l', i, k \in \{S_{j,i} \cap S_{j',i}\} \quad (13)$$

Eq. (14) and Eq. (15) state that each of the turrets used for machining purposes is capable of processing one operation at a time. There are two turrets considered for this problem. Each turret must work on one operation at a time so that the final schedule would be feasible.

$$St_{jl} - St_{j'l'} \geq B''''_{j'l'ih} \cdot P_{j'l'i} - M(1 - Z_{jlj'l'}) - M(1 - B''''_{j'l'ih}) - M(1 - B''''_{j'l'ih})$$

$$\forall j, l < l', i, h \in \{T_{j,i}\} \quad (14)$$

$$St_{j'l'} - St_{jl} \geq B''''_{jlih} \cdot P_{ji} - M(Z_{jlj'l'}) - M(1 - B''''_{j'l'ih}) - M(1 - B''''_{jlih})$$

$$\forall j, l < l', i, h \in \{T_{j,i}\} \quad (15)$$

Eq. (16) and Eq. (17) indicate that each turret has the capability to process one work-piece at a time.

$$St_{jl} - St_{j'l'} \geq B''''_{j'l'ih} \cdot P_{j'l'i} - M(1 - Z_{jlj'l'}) - M(1 - B''''_{j'l'ih}) - M(1 - B''''_{jlih})$$

$$\forall j < j', l, j < n, , l', i, h \in \{T_{j,i} \cap T_{j',i}\} \quad (16)$$

$$St_{j'l'} - St_{jl} \geq B''''_{jlih} \cdot P_{ji} - M(Z_{jlj'l'}) - M(1 - B''''_{j'l'ih}) - M(1 - B''''_{jlih})$$

$$\forall j < j', l, j < n, , l', i, h \in \{T_{j,i} \cap T_{j',i}\} \quad (17)$$

Eq. (18) is provided to satisfy the precedence relations between operations of a job. Operations of each job cannot be started unless their precedence has been processed before. For the simplicity of the problem, most of the scheduling problems in this area are considered to be linear. In the linear approach, operations have only one precedence. In this thesis, each operation could have more than one precedence, which is a more realistic assumption in compare with traditional assumptions.

$$St_{jl} - St_{jl'} \geq P_{jl'i} \quad \forall i, j, l < l' \in \{R_{j,l}\} \quad (18)$$

Eq. (19) and Eq. (20) are to assure that each job is assigned to a spindle and a turret which is capable of holding/processing that job. Turrets could have limitations due to the types of cutting tools mounted on them. This is the same for spindles, as they might have different rotational speed and bar feed capacities. Hence, there are some limitations related to the capacity of spindles and turrets, which should be considered in the proposed model.

$$B''_{jlike} = 0 \quad \forall j, l, i, e, k \notin \{S_{j,i}\} \quad (19)$$

$$B'''_{jih} = 0 \quad \forall j, l, i, h \notin \{T_{j,i}\} \quad (20)$$

Eq. (21-24) state that if a job is assigned to a machine its operations are also assigned to the spindles, turrets, and capacities of the same machine. These set of constraints are provided to apply the concepts related to the single setup and prevent movement of parts from one machine to another.

$$B'_{jli} \leq B_{ji} \quad \forall j, l, i \quad (21)$$

$$B''_{jlike} \leq B'_{jli} \quad \forall j, l, i, e, k \in \{S_{j,i}\} \quad (22)$$

$$B'''_{jih} \leq B'_{jli} \quad \forall j, l, i, h \in \{T_{j,i}\} \quad (23)$$

$$B''_{jlike} \leq B''''_{jik} \quad \forall j, l, i, e, k \in \{S_{j,i}\} \quad (24)$$

Eq. (25) determines the maximum completion time of each operation.

$$C_{max} \geq St_{jl} + P_{jli} \quad \forall j, l \quad (25)$$



Eq. (26-33) outline the decision variables.

$$St_{jl} \geq 0 \quad \forall j, l \quad (26)$$

$$B_{ji} \in \{0,1\} \quad \forall j, i \quad (27)$$

$$B'_{j\bar{i}} \in \{0,1\} \quad \forall j, l, i \quad (28)$$

$$B''_{jlike} \in \{0,1\} \quad \forall j, l, i, k, e \quad (29)$$

$$B'''_{jih} \in \{0,1\} \quad \forall j, l, i, h \quad (30)$$

$$B''''_{jik} \in \{0,1\} \quad \forall j, i, k \quad (31)$$

$$Z_{jlj' l'} \in \{0,1\} \quad \forall j, l, l' \quad (32)$$

$$Z_{jlj' l'} \in \{0,1\} \quad \forall j, l, j', l' \quad (33)$$

### 3.2 Explanation

The outcome of the model would be a feasible sequence and schedule for each operation of each job. A specific turret and spindle will be assigned to each operation and an optimum start time based on precedence relations will be determined. Moreover, the model will be flexible enough to generate plans for a different number of jobs and operations with multiple machine tools used for processing them. Regarding the mentioned constraints, the model will seek for the best assignments to optimize the make-span.

Figure 6 shows the relations among parameters and different parts of the scheduling problem at hand and how they are connected. As can be seen, each parameter is placed between two components in the diagram, and it defines the relationship between those two components. For instance,  $B'''_{jih}$  is the parameter, which explains the relation between each operation and the turret. If operation  $l$  of job  $j$  is processed by turret  $h$ , then  $B'''_{jih}$  takes the value of one, otherwise it takes the value of zero. The procedure is the same for the rest of the parameters shown in the diagram.

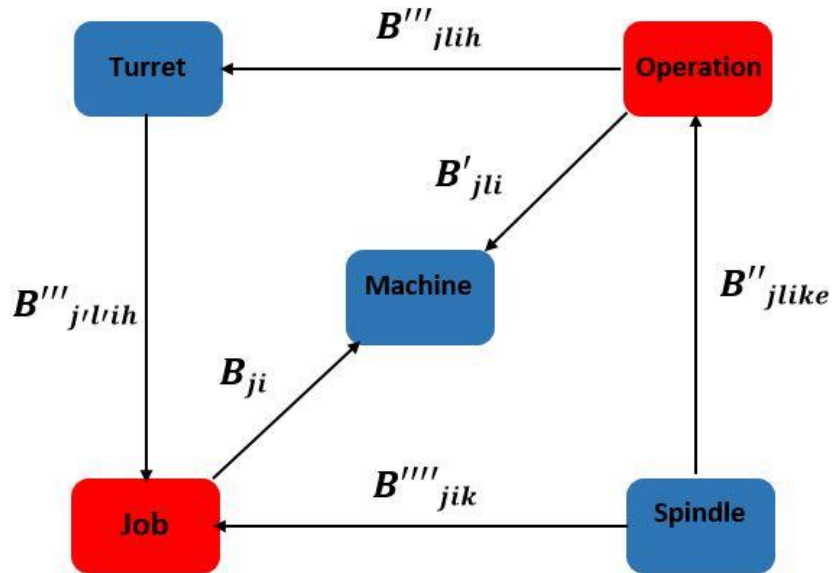


Figure 6: Relations between parameters and components of the system

### 3.3 Demonstrative example

It is assumed that there are totally three different jobs ( $n$ ) and each job has a different number of  $l$  operations. There are two machines ( $m$ ) assumed. One possible schedule for the problem is shown in Figure 7.

Each spindle is capable of holding a different series of jobs. For instance,  $S_{3,2} = \{\emptyset\}$ , which means there is no spindle on machine two which can hold job three. This is reflected in the final schedule and as can be seen, job three is scheduled on machine one. In this example, all turrets are assumed to be able to process all jobs for simplification purposes ( $T_{j,i} = \{1,2\}$ ), but the set of turrets capable of processing a particular type of job can vary and the model is capable of distinguishing them.

Precedence relations between operations ( $R_{ji}$ ) must be satisfied when a schedule plan is created. For instance, operation three of job two has a precedence, which is number two,  $R_{2,3} = \{2\}$ . If two operations of the same job do not have a necessary precedence to be satisfied, they might start at the same time or their schedule might overlap only if they have the same operation mode. In order to check the mode of operations, the model will refer to the data available for  $W_{jll'}$ . As shown in Figure 7, operations four and two of job one ( $O_{1,2}$  &  $O_{1,4}$ ) have an overlap because  $O_{1,2}$  &  $O_{1,4}$  do not have precedence relations with each other and also  $W_{1,2,4} = \{0\}$ , which means operations two and four of job one are the same type of operations. This mechanism is the same for operations three and five of job one ( $O_{1,3}$  &  $O_{1,5}$ ), and also operations three and four of job three ( $O_{3,3}$  &  $O_{3,4}$ ).

Considering the schedule plan for job two, it can be seen that operations two and three are processed by two turrets on spindle number one of machine two. This is the concept related to processing a single operation of a single job by multiple turrets. The information regarding the number of turrets needed for processing each operation is provided.  $N_{2,3} = \{2\}$  means operation number three of job two needs two turrets for machining. The parameter  $N$  for the rest of operations and jobs is one. Each job has a different processing time on each machine and the information regarding the processing times is provided in Table 2.

Another feature, which is included in the illustrated example is the single setup concept to complete all jobs without transferring them to other machines.

The total completion time for all operations on all machines ( $C_{\max}$ ) is 20 seconds. The model will seek for the best allocations according to the inputs provided to complete the machining process in the minimum amount of time.

The following data are assumed for a set of jobs and machines to be scheduled.

$$\begin{aligned}
 m=2 \quad n=3 \quad lj= \{5,3,4\} \\
 S_{1,1}=\{1\} \quad S_{1,2}=\{1,2\} \quad S_{2,1}=\{2\} \quad S_{2,2}=\{1,2\} \quad S_{3,1}=\{1,2\} \quad S_{3,2}=\{\emptyset\} \\
 T_{1,1}=\{1,2\} \quad T_{1,2}=\{1,2\} \quad T_{2,1}=\{1,2\} \quad T_{2,2}=\{1,2\} \quad T_{3,1}=\{1,2\} \quad T_{3,2}=\{1,2\} \\
 R_{1,1}=\{\emptyset\} \quad R_{1,2}=\{1\} \quad R_{1,3}=\{\emptyset\} \quad R_{1,4}=\{1\} \quad R_{1,5}=\{4\} \\
 R_{2,1}=\{\emptyset\} \quad R_{2,2}=\{1\} \quad R_{2,3}=\{2\} \\
 R_{3,1}=\{\emptyset\} \quad R_{3,2}=\{1\} \quad R_{3,3}=\{\emptyset\} \quad R_{3,4}=\{\emptyset\}
 \end{aligned}$$

Table 2: Processing times for the demonstrative example

job	machine	Operation				
		1	2	3	4	5
1	1	3	6	2	7	3
	2	3	9	4	3	6
2	1	4	8	6	-	-
	2	4	7	5	-	-
3	1	7	2	4	5	-
	2	8	5	8	6	-

$N_{1,1}=\{1\}$   $N_{1,2}=\{1\}$   $N_{1,3}=\{1\}$   $N_{1,4}=\{1\}$   $N_{1,5}=\{1\}$

$N_{2,1}=\{1\}$   $N_{2,2}=\{2\}$   $N_{2,3}=\{2\}$

$N_{3,1}=\{1\}$   $N_{3,2}=\{1\}$   $N_{3,3}=\{1\}$   $N_{3,4}=\{1\}$

$W_{1,1,2}=\{1\}$   $W_{1,2,3}=\{0\}$   $W_{1,3,4}=\{0\}$   $W_{1,4,5}=\{1\}$   $W_{1,2,4}=\{0\}$   $W_{1,3,5}=\{0\}$

$W_{2,1,2}=\{1\}$   $W_{2,2,3}=\{0\}$

$W_{3,1,2}=\{0\}$   $W_{3,2,3}=\{1\}$   $W_{3,3,4}=\{0\}$

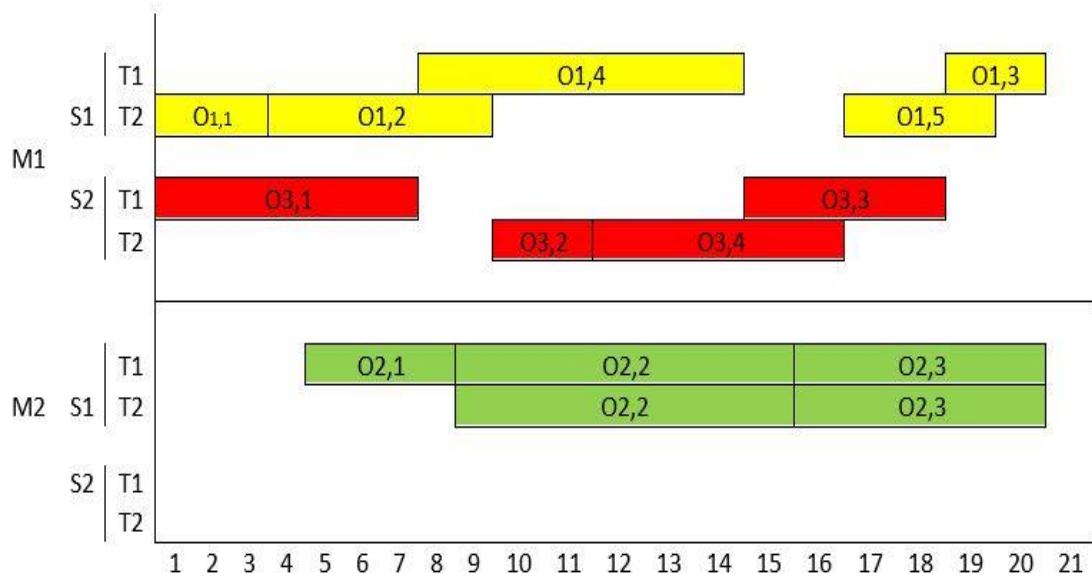


Figure 7: Resultant schedule for the example

## CHAPTER 4: METAHEURISTIC ALGORITHM

### 4.1 Simulated Annealing

#### 4.1.1 An outlook of SA

Simulated Annealing is an algorithm, which functions based on concepts related to annealing procedures and is used for optimization purposes [58]. SA has the privilege to use a framework that helps the algorithm to avoid getting surrounded by local improvements. It provides a scheme to accept worse solutions for the problem with a negative exponential distribution [59].

#### 4.1.2 Advantages of hill climbing over greedy algorithms

The working principle of greedy algorithms is such that they start the procedure of searching by considering an initial feasible solution for the problem and continue to look for better answers in compare with previous ones. Greedy algorithms repeat this procedure until no more enhancement is possible. In such a method, there is a high probability of not finding the global minima or maxima for the objective function. It is also possible that the search would get surrounded by local upgrades and it would be difficult to find the overall and best improvement. Therefore, at any time, greedy algorithms compare the current solution with the new solution found. If there is any improvement made over the new search than the new solution is considered as the best solution.

SA does not instantly reject solutions which are worse than the updated best solution and this feature is considered as an advantage of Simulated Annealing over greedy algorithms. SA not only considers the improved solutions but also takes into account the inferior values with a predefined probability function. Hill climbing method gives SA the capability to overcome getting looped in local optima (downhill) and gives it the chance to discover better answers to the problem [60].

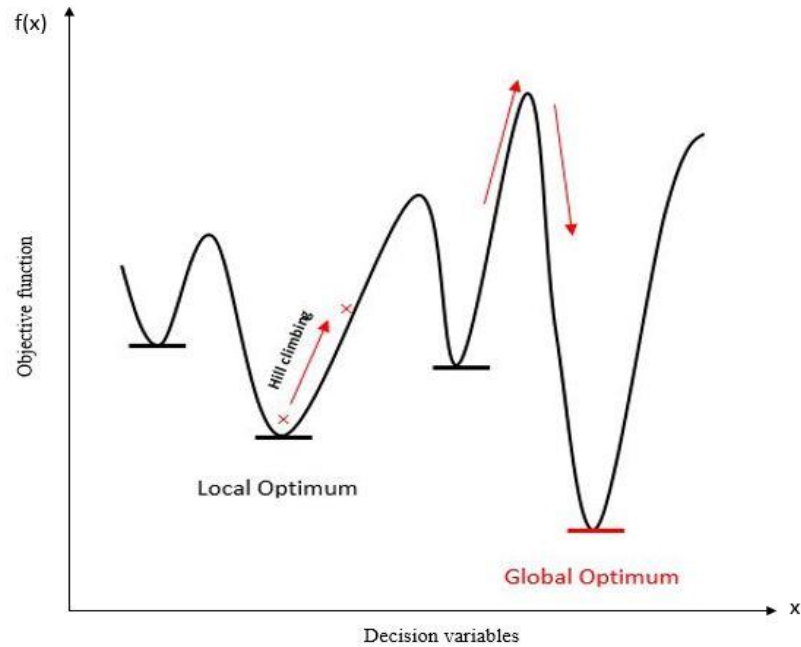


Figure 8: Hill climbing concept applied in SA

#### 4.1.3 The mechanism of Simulated Annealing

The first step in developing the SA algorithm is creating an initial feasible solution for the problem at hand. SA estimates the objective function based on the initial solution. After creating the initial solution, the next step is to adjust the initial temperature at which the annealing process is going to take place. When the temperature is adjusted, a neighborhood function is applied to create a new solution, which is also feasible. The newly created solution is assessed and the new objective function is calculated. To apply the neighborhood function move operators are used. Move operators basically create a new solution for the problem so that the algorithm can compare the new solution with the previous ones. If the objective value of the new solution is better than the previous one, then it is accepted and set as the best solution found so far. In case the new solution is worse than the previous one, then it is accepted by a negative exponential distribution to permit the concept of hill climbing and achieve better objective function outputs.

$$P = \exp(-\Delta f/T)$$

where  $\Delta f = f(x_{new}) - f(x)$ .

$T$  is the parameter related to the acceptance criteria. With an increment of  $T$ , worse solutions have lower chances to be accepted. An exponential function is used as the criteria to slowly reduce the temperature,  $T = \alpha.T$ , where  $\alpha$  (*alpha*) is considered as the rate at which temperature is decreased.



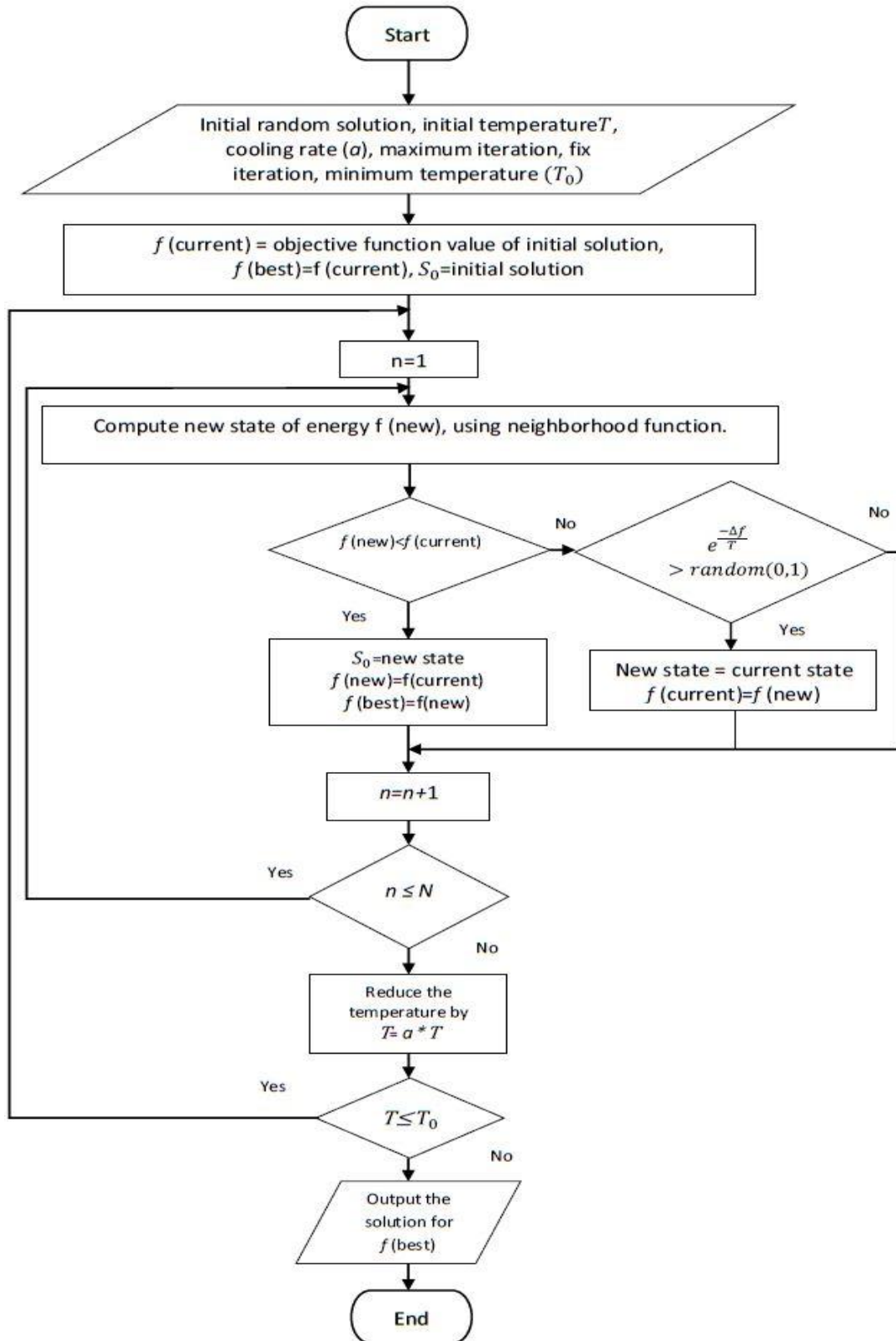


Figure 9: Simulated Annealing flowchart

## 4.2 Simulated annealing for multitasking scheduling

Simulated Annealing has been considered by different authors for the general machine scheduling problems [61-63], however, the use of SA for the problem of scheduling mill-turns is quite limited in the literature. The rest of this chapter discusses the relevant steps required to apply the metaheuristic algorithm for the problem at hand. A novel initialization method together with different dedicated approaches to implement the scheduling model has been described in details.

### 4.2.1 Initialization

An initial solution determines the first sequence at which operations will be scheduled. The assignment of operations to machines, spindles, and turrets are done through a set of directions. To tackle this problem, an initial random order of available jobs to be scheduled is created, then in order to apply the single setup concepts, each job is repeated to the number of its operations. For instance, if there are four jobs ( $n=4$ ) and each job has the following number of operations:  $l_j = \{3,5,3,4\}$ , one initial solution is  $\{4,4,4,4,2,2,2,2,3,3,1,1,1\}$ . Figure 10 shows the flowchart for the initialization phase of the proposed algorithm.

Upon creation of the initial solution, it is put into a matrix called  $M$ , which controls the assignment of operations on spindles, machine tools, and turrets.  $M$  has seven rows. The first row includes the list of operations. The feasible initial solution is placed in the first row of the matrix  $M$ . The number of columns in the matrix  $M$  is the total number of operations of all jobs. The second row of the matrix is filled gradually by the order in which operations are assigned. The order is not necessarily sequential since there might be some precedence involved among certain operations that would delay the order at which they are processed. The second row of the matrix, which defines the machining order is the last row to be completed. The algorithm is initiated in a way that after a machine, spindle, turret, start time and finish time is assigned to an operation, a machining order will be assigned to that specific operation. Figure 11 shows the flowchart for machining order in the matrix  $M$ . Figure 12 also shows the flowchart for identification of jobs ( $j$ ) based on their sequence in the initial solution in the matrix  $M$ .

The third row of the matrix  $M$  contains the machine number assigned to an operation. The spindle dedicated to an operation is placed in the fourth row, the turret(s) used for processing an operation in the fifth, and finally, start and finish times go to the sixth and seventh row. The matrix  $M$  has  $le$  columns.  $Le$  is the length of total operations of all jobs to be scheduled. Once the initial solution is created and operations are listed in the matrix  $M$ , the rest of the assignment procedure will start through a set of rules.

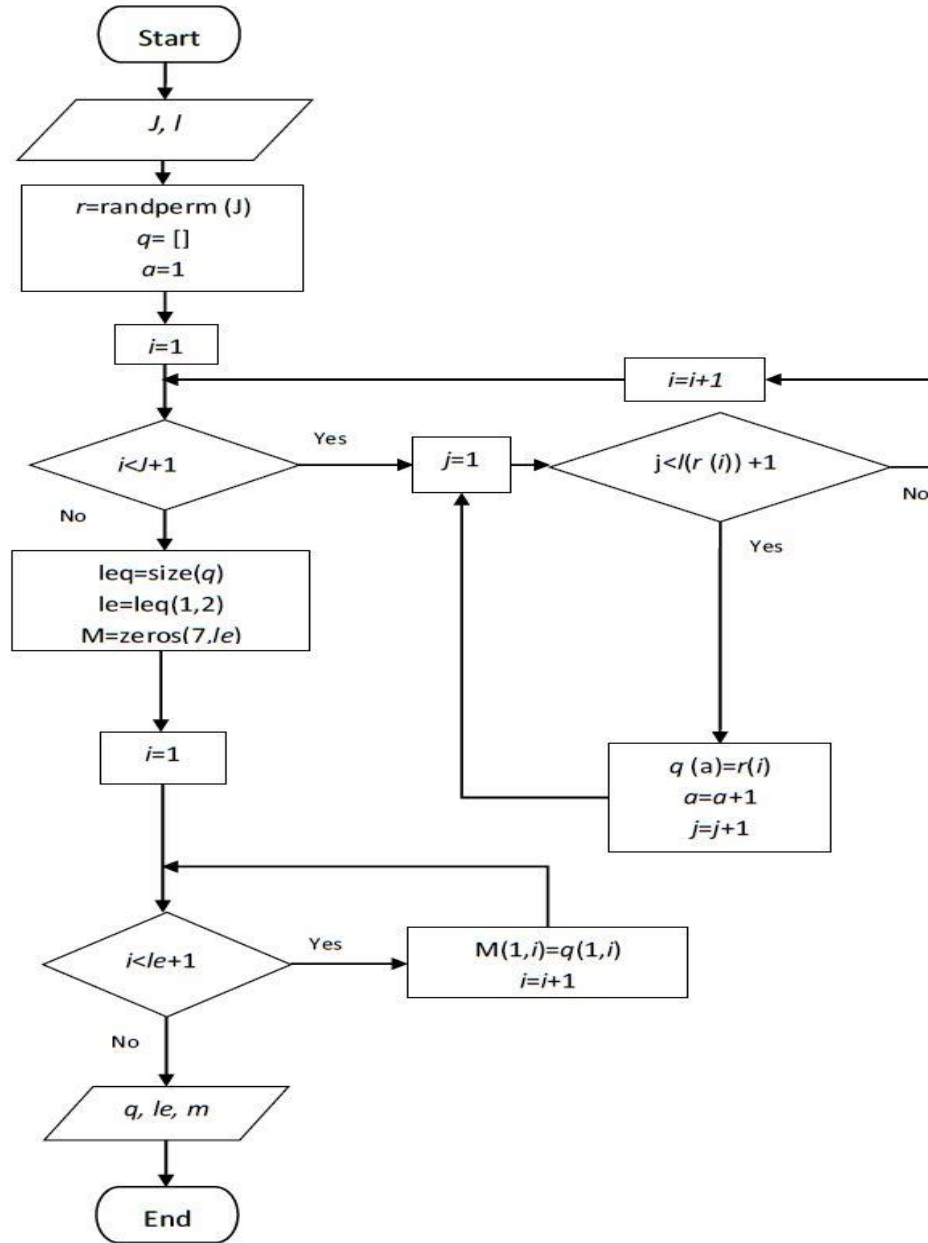


Figure 10: The initialization algorithm flowchart for multitasking scheduling

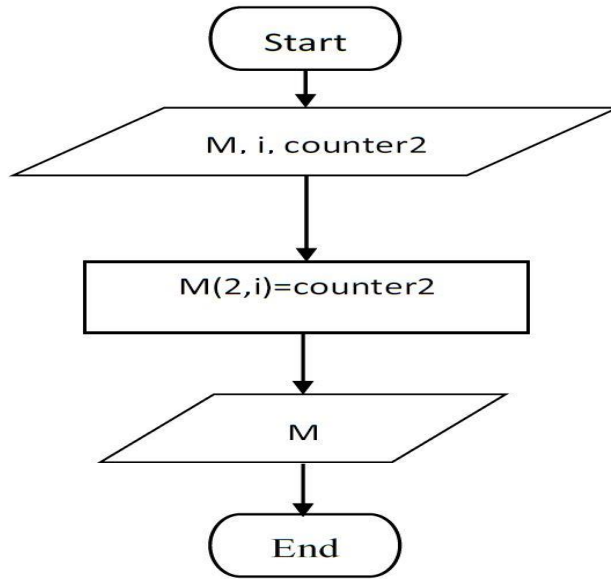


Figure 11: Machining order in the matrix  $M$

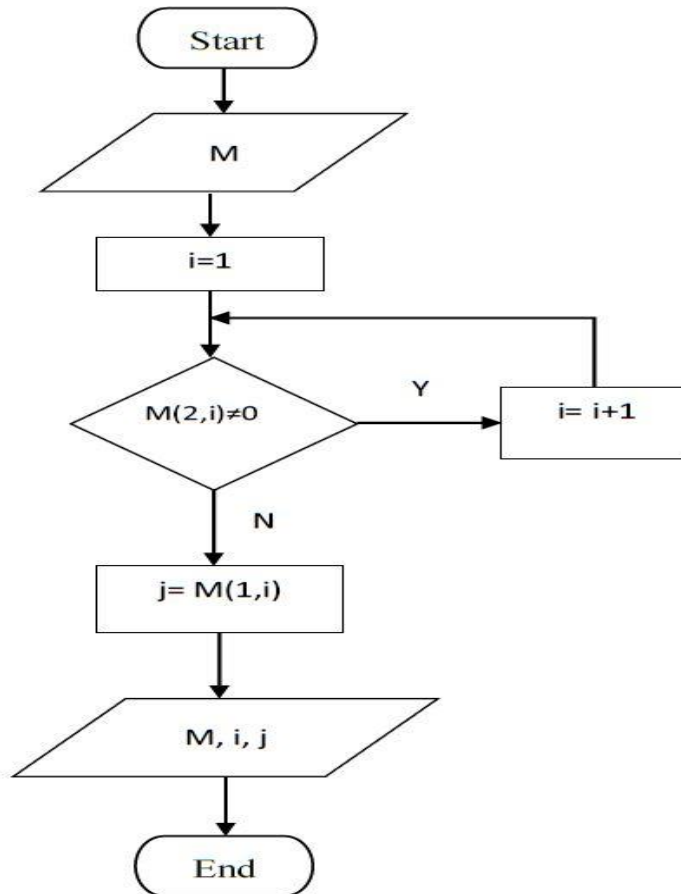


Figure 12: Identification of jobs based on their sequence in the initial solution

## **4.2.2 Neighborhood search**

For the proposed metaheuristic algorithm, three different move operators are applied as neighborhood search. Swap, reversion, and insertion. For the swap operator, two different jobs of the sequence are chosen and their position is changed. For instance, if there are six jobs and the initial order of the job numbers to be scheduled is  $A=\{5,2,4,1,3,6\}$ , after applying the swap operator the new solution would be  $B=\{5, \mathbf{3}, 4, 1, \mathbf{2}, 6\}$ . In this example, job numbers two and three are chosen randomly and then their position in the initial solution is changed and the new solution B is created.

In Reversion, two different random elements (job numbers) in the initial solution are selected, then the order of job numbers including the two jobs already selected are reversed. If we consider the previous sequence of jobs;  $A=\{5,2,4,1,3,6\}$ , after applying the reversion operator the new solution would be  $B=\{5, \mathbf{3}, \mathbf{1}, \mathbf{4}, \mathbf{2}, 6\}$ . In this example, the same job numbers are chosen (job two and job three) and their order including the two jobs is reversed.

For insertion, two random elements (jobs) are chosen, one of them is moved to the position right after the other. If we consider  $A=\{5,2,4,1,3,6\}$ , after applying insertion operator, the new solution would be  $B=\{5,4,1,3, \mathbf{2}, 6\}$ . As can be seen, job numbers two and three are chosen as before, job two is moved to the position right after job three and the new stream is shown as above. The method applied in this work, is a mixed operator, in which one random operator is chosen for creating a new solution.

## **4.2.3 Description of the assignment mechanism**

### **4.2.3.1 Precedence check**

In order to assign a machine, spindle and a turret to each operation, a set of procedures have to be followed. Operations are assigned one by one. The first step in the assignment process is to determine whether an operation is eligible to be machined or not. That is done by a precedence check before starting the assignment procedure. Based on the initial solution provided in the first row of the matrix, job numbers and operation numbers can be identified. Then the information regarding the precedence of an operation, which is about to be scheduled is extracted. If an operation does not have any precedence, then it

can be started, otherwise, the algorithm will seek for the precedence operation/operations and will start the assignment process with respect to them. By the time the precedence check is done, the algorithm will start the assignment process from the beginning and the procedure goes on. Figure 13 shows the mechanism for checking the precedence with respect to job numbers and operations associated with them.

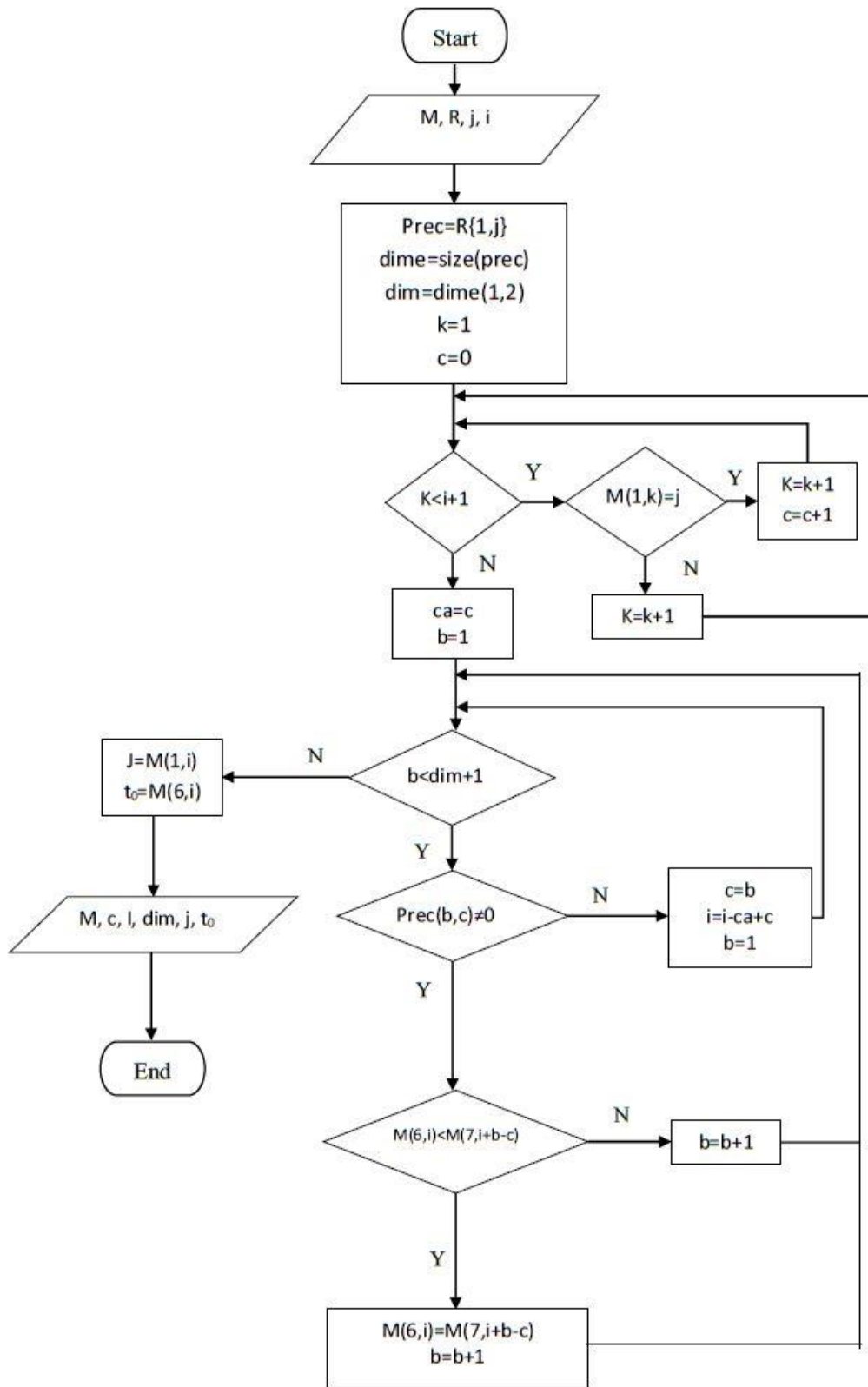


Figure 13: The procedure for checking the precedence relations

#### 4.2.3.2 Assignment of machines and spindles to operations

The first step when assigning a machine and spindle to an operation is to check if it is the first time that an operation of a job is going to be scheduled. This is an important procedure since it has a direct effect on the final assignment process. Figure 14 shows the steps to determine if it is the first time that a machine or spindle is assigned to an operation of a job.

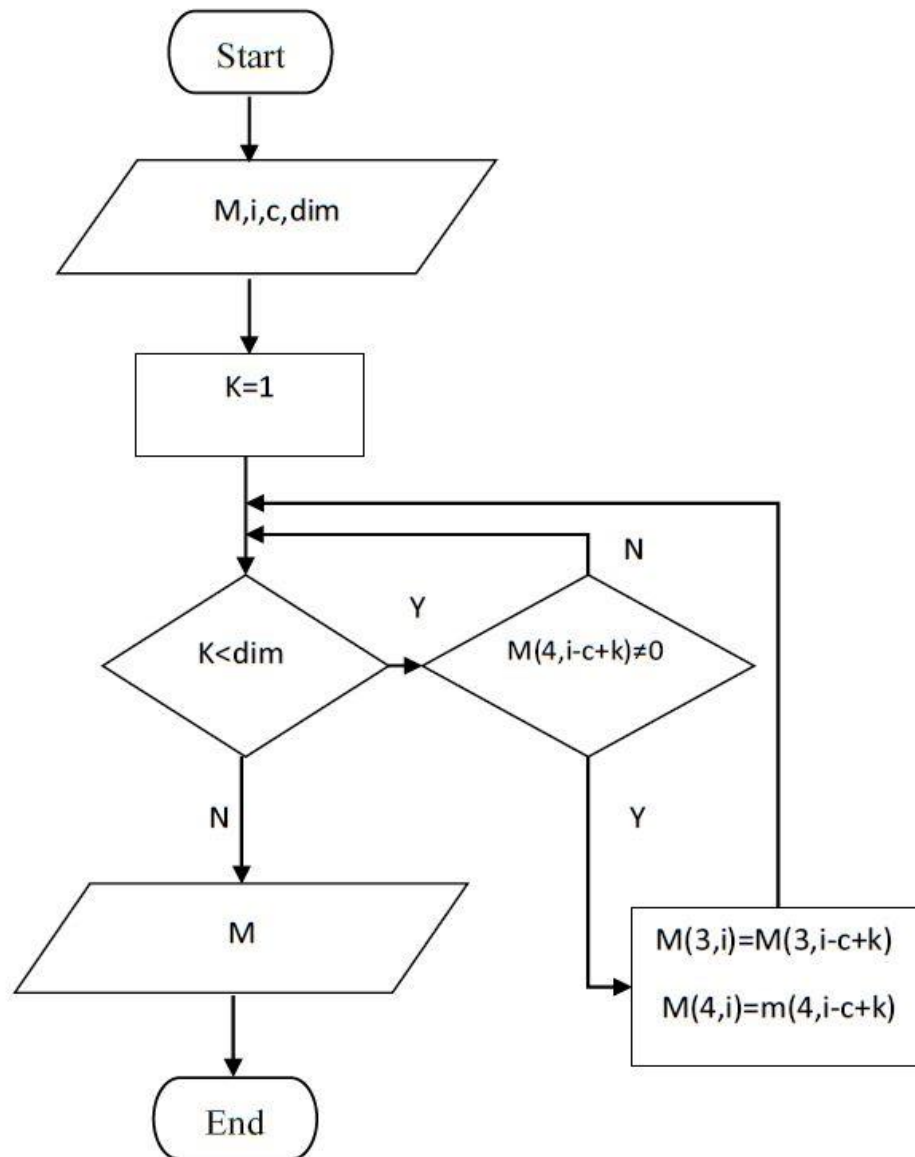


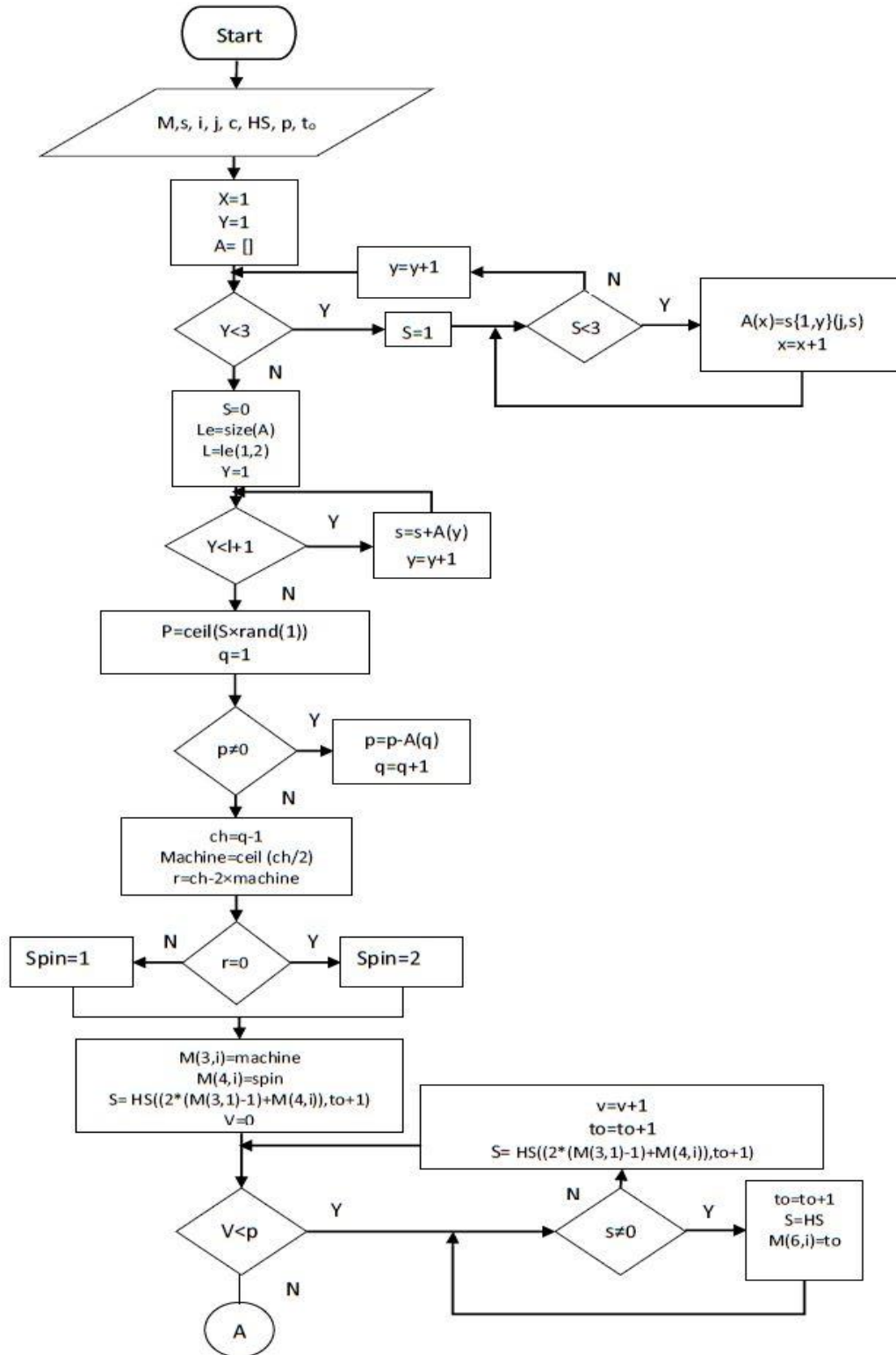
Figure 14: The procedure for checking the previous assignment of any operation of a job



If an operation does not have any precedence, a machine and a spindle, which is capable of loading that operation will be assigned to it. The machine assignment procedure takes into account the fact that whether or not the spindle is capable of holding a part. As for the first assignment, the algorithm will check the data available for the set of spindles ( $S$ ), which are permissible to hold the job. This information is sorted in a matrix called  $A$ . The matrix  $A$ , sorts all the permissible spindles for holding a work-piece. Based on this information, a random number is generated and a machine and spindle are assigned to that operation which is being considered.

Each spindle is capable of holding maximum two operations at a time, therefore, there should be a mechanism to record the assignment of operations on each spindle. To tackle this problem, another matrix called history of spindles or  $HS$  is created.  $HS$  keeps track of operations loaded on a spindle at a time and prevents the violation of constraints already created regarding the capacities of spindles. Figure 15 shows the flowchart for assignment of a machine and spindle to an operation if it is the first time that an operation of a job is being machined.

If a machine and a spindle have already been assigned to any operation of a job, the second operation of that job and the rest of them will go on the same setup. This procedure is applied to prevent movement of parts from one machine to another and to complete the machining process on a single setup. Figure 16 shows the procedure of assigning a machine and spindle to an operation when they have already been assigned to one of the operations of the same job. The checking procedure to make sure if an operation has been assigned previously is done every time that a new operation is about to be scheduled. Based on previous assignments, one of the aforementioned procedures is chosen and the process is carried out.



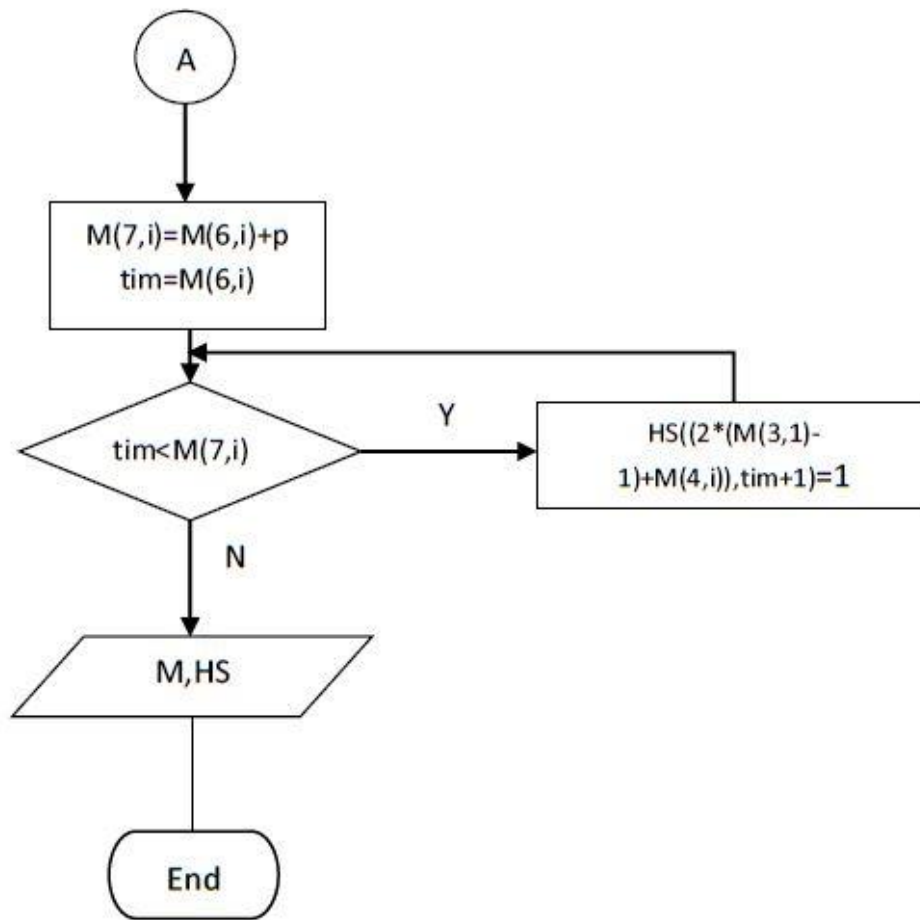


Figure 15: Flowchart for the first time assignment of a machine and spindle to an operation

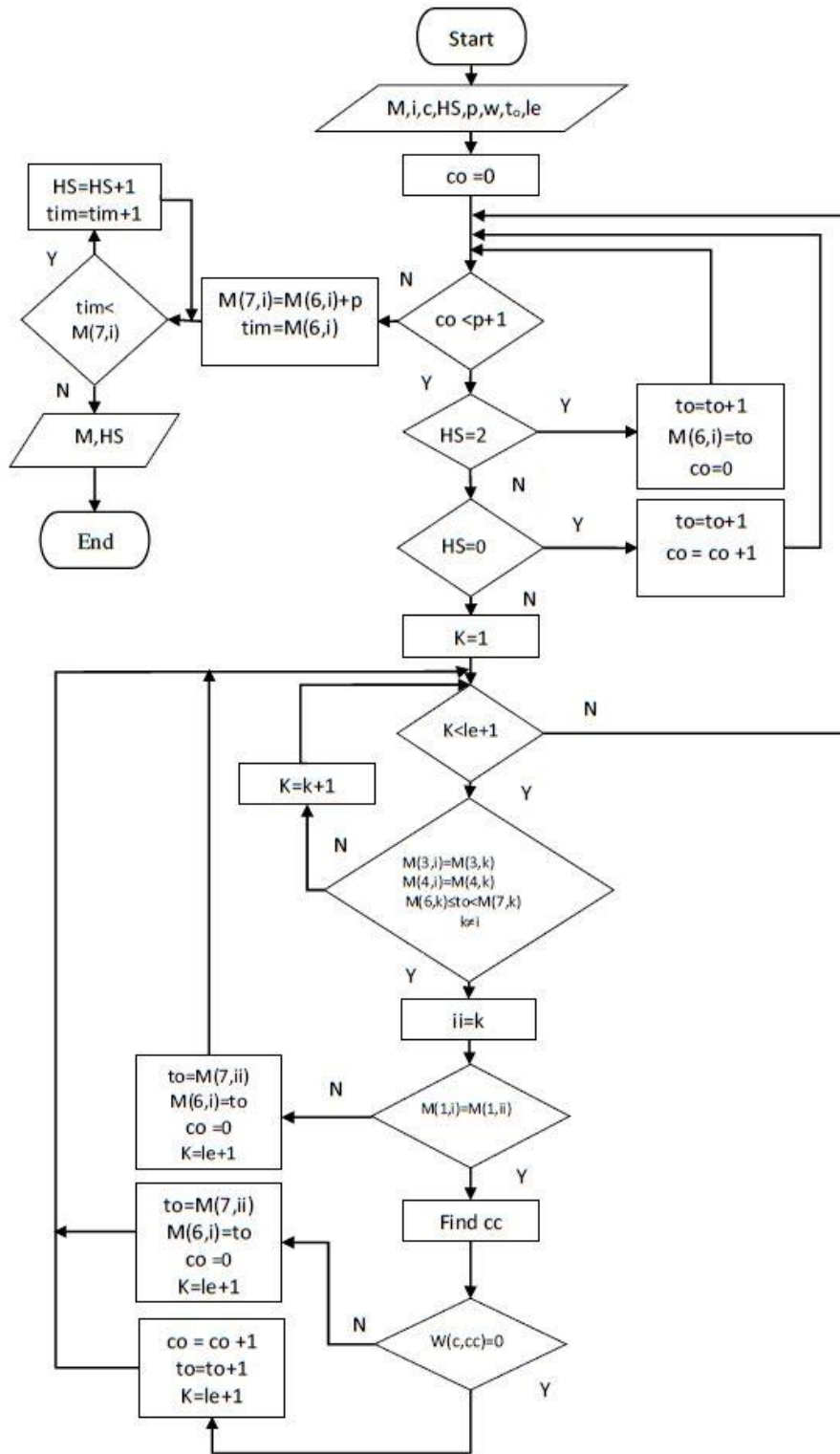


Figure 16: Flowchart for the second time assignment of a machine and spindle to an operation

#### 4.2.3.3 Assignment of turrets to operations

Assigning the number of turrets required for processing an operation is the next step after assignment of spindles and machines. There could be one or maximum two turrets assigned to each operation. Assignment of turrets to operations should take into account the feasibility of turret to process that job. The solution applied here is divided into different sections. The first case is when two turrets are needed for processing an operation. In this case, both turrets on at least one of the available machine tools should be capable to process that job. To keep track of the assignment procedure, and to make sure that at any time each turret is processing maximum one operation, a matrix called history of turrets or *HT* is created. Figure 17 shows the assignment of turrets to an operation if two turrets are needed for the machining process. The second scenario takes place when one turret is needed for processing an operation and only one of the two turrets on the machine tool is capable of processing that operation. In this case, the operation has no other choice rather than being machined by that specific turret, which is capable of processing the operation. Figure 18 shows the flowchart for assignment of the turret to an operation when only the first turret is able to process that part. Figure 19 shows the same procedure when the second turret is capable of processing the work-piece. The approach is the same as before however, the history of the second turret has to be updated accordingly. The last scenario occurs when one turret is needed to process the operation and there are two turrets available. In this case, each of the two turrets will be tested separately and then a comparison is made between them to identify which one is capable to complete the machining process in less amount of time. To do so, the completion time of processing the operation on each of the two turrets is calculated, and eventually, the one with earliest finish time is chosen. Figure 20 and Figure 21 show the assignment of the turret to an operation if both turrets are available for the machining process. Figure 22, shows the comparison between the two turrets to determine the earliest finish time between the two turrets.

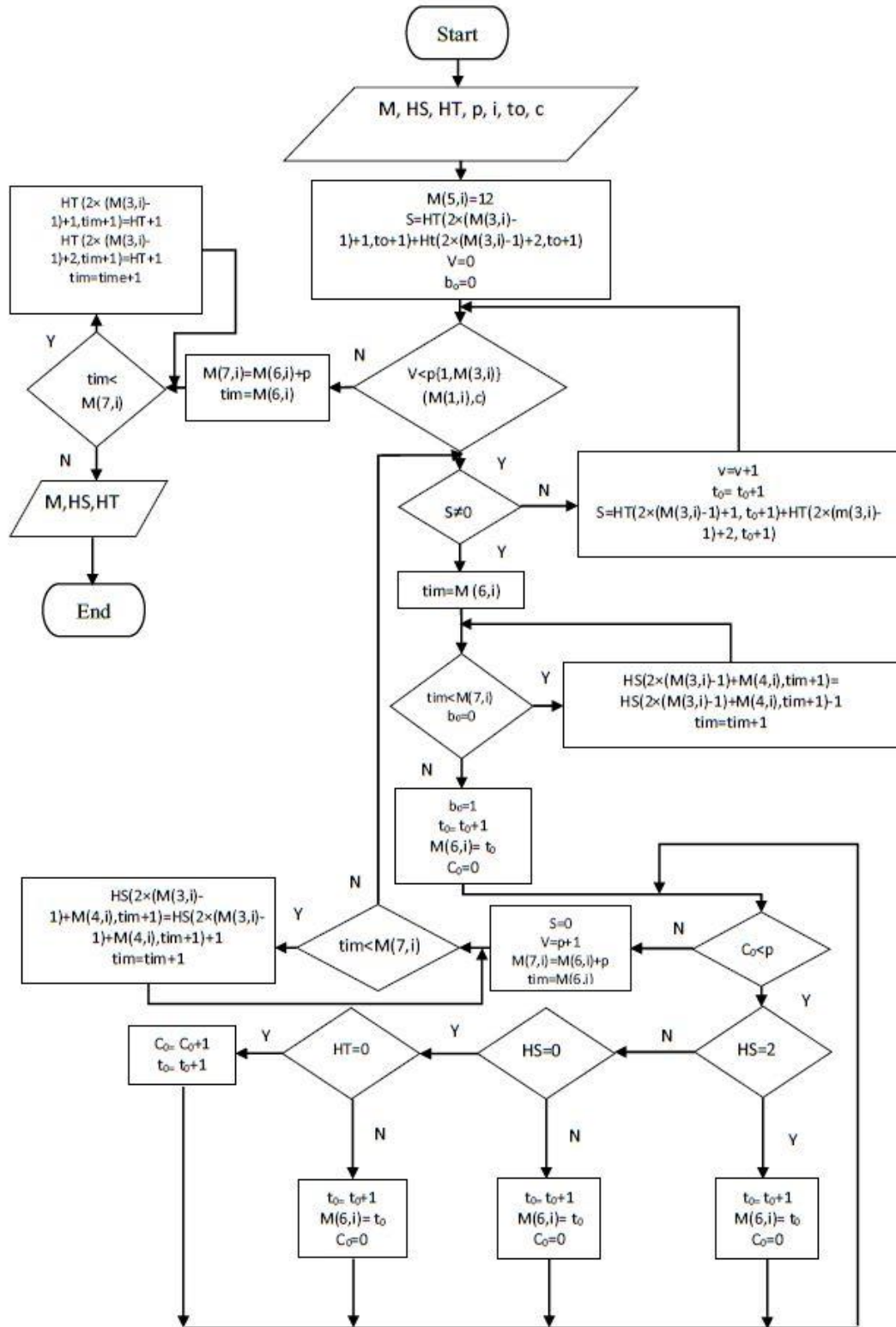
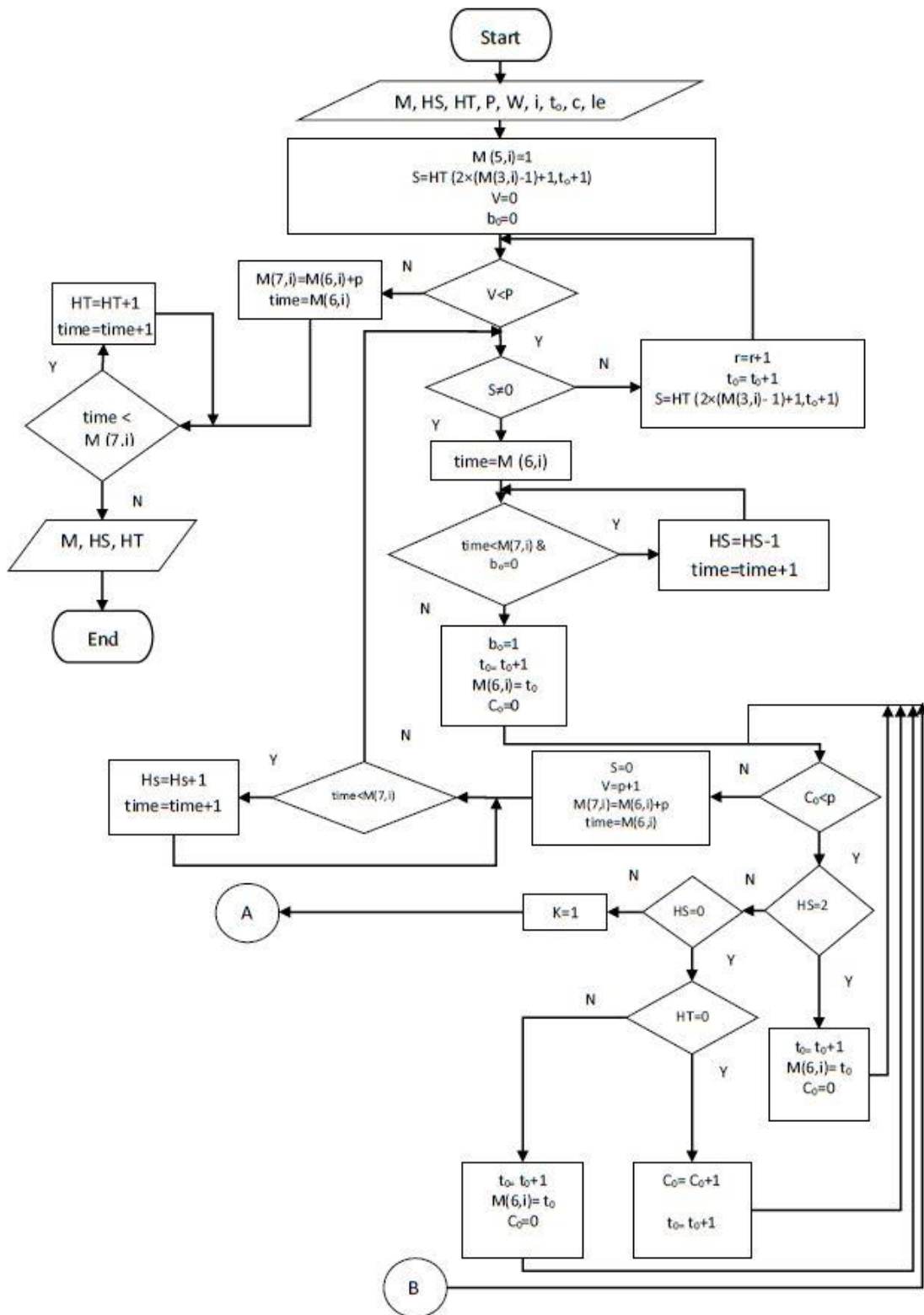


Figure 17: Flowchart for the assignment of turrets to an operation if two turrets are needed



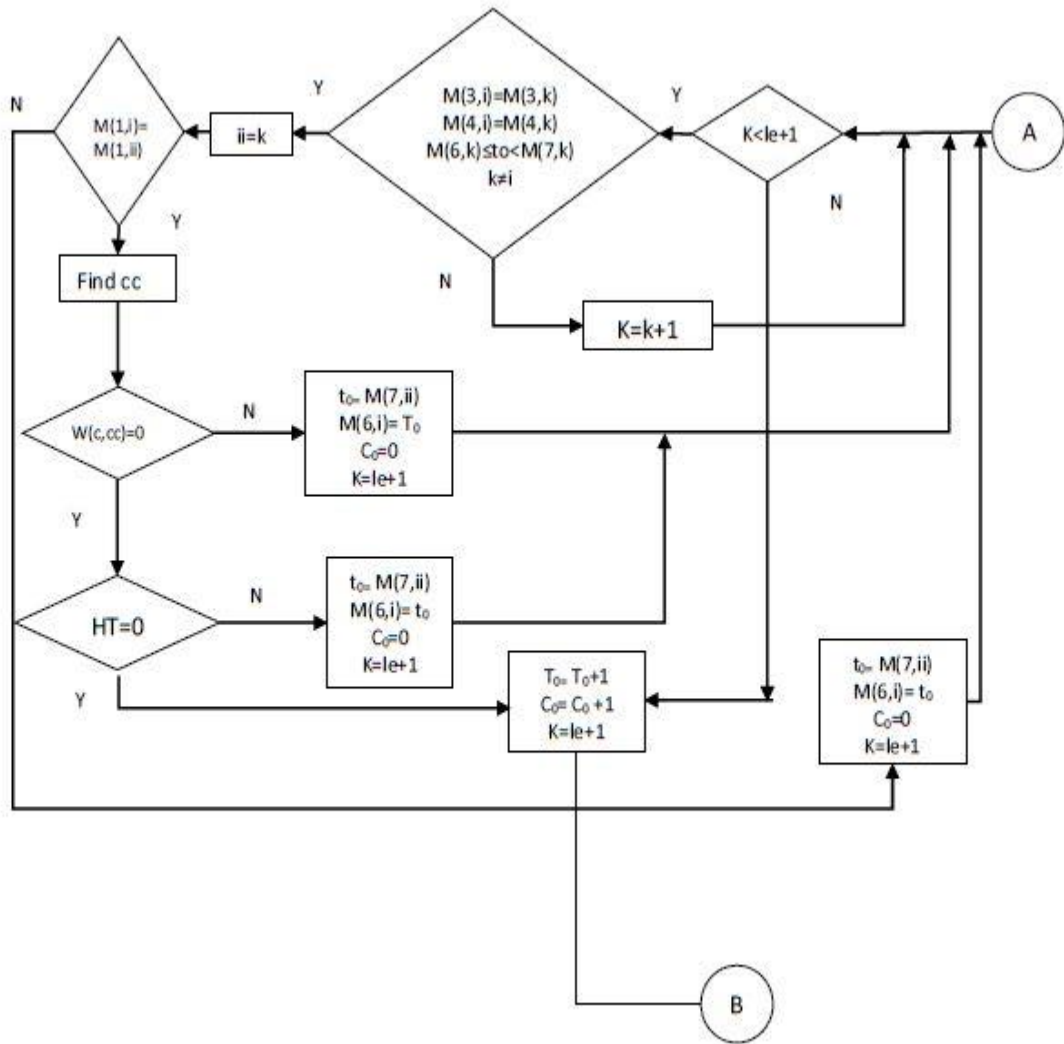
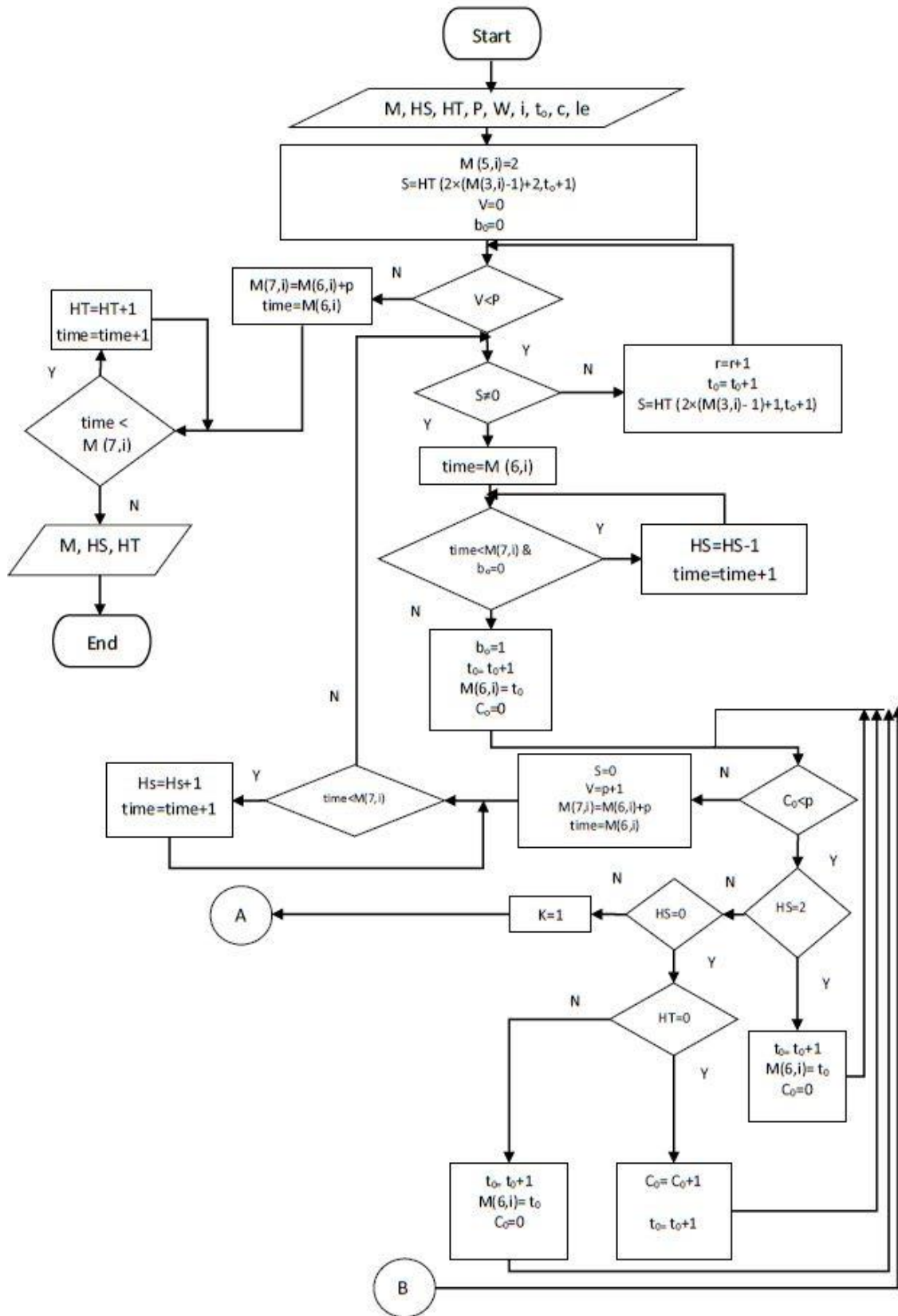


Figure 18: Flowchart for the assignment of turrets to an operation if one turret is needed and only the first turret is available





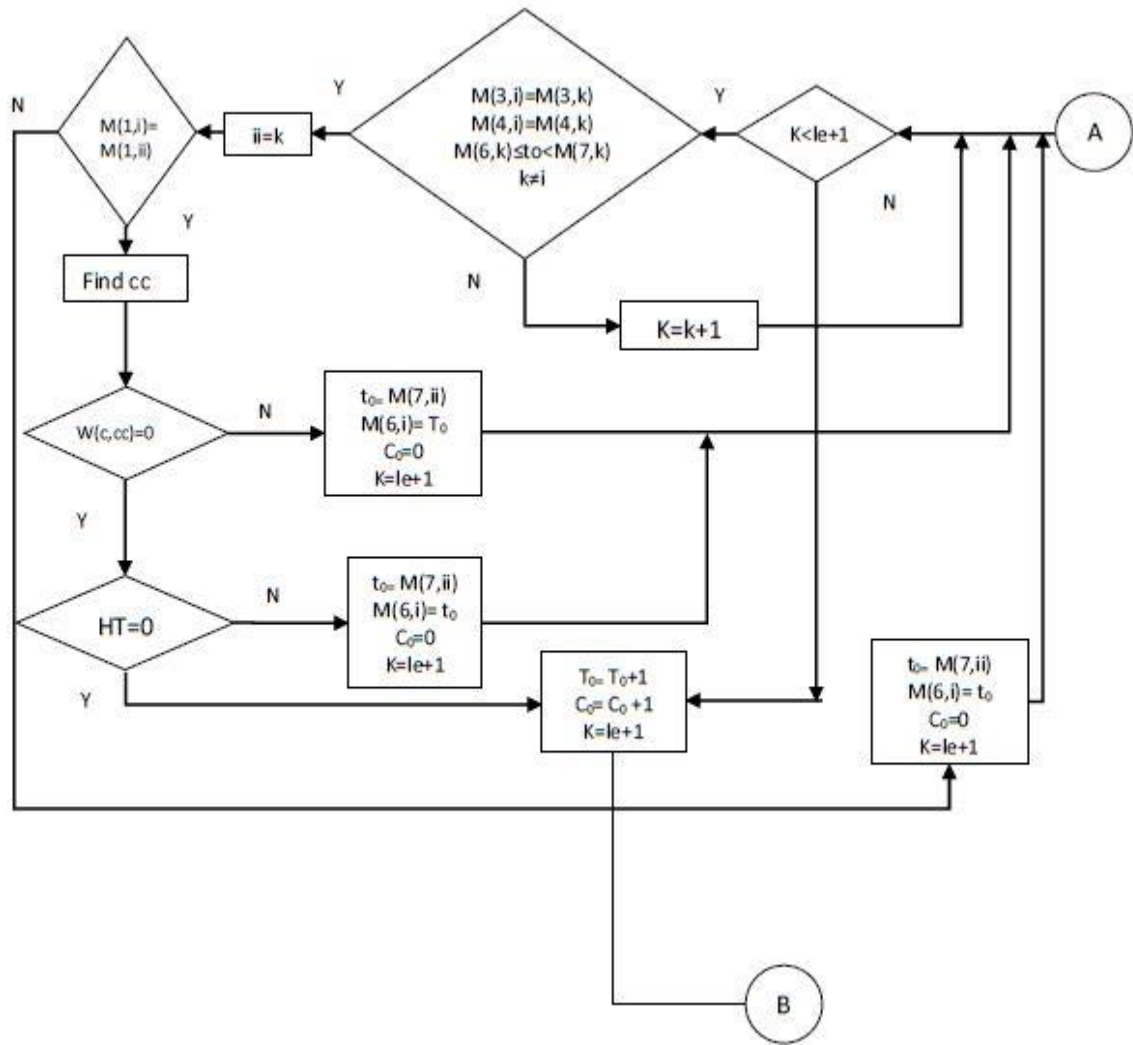
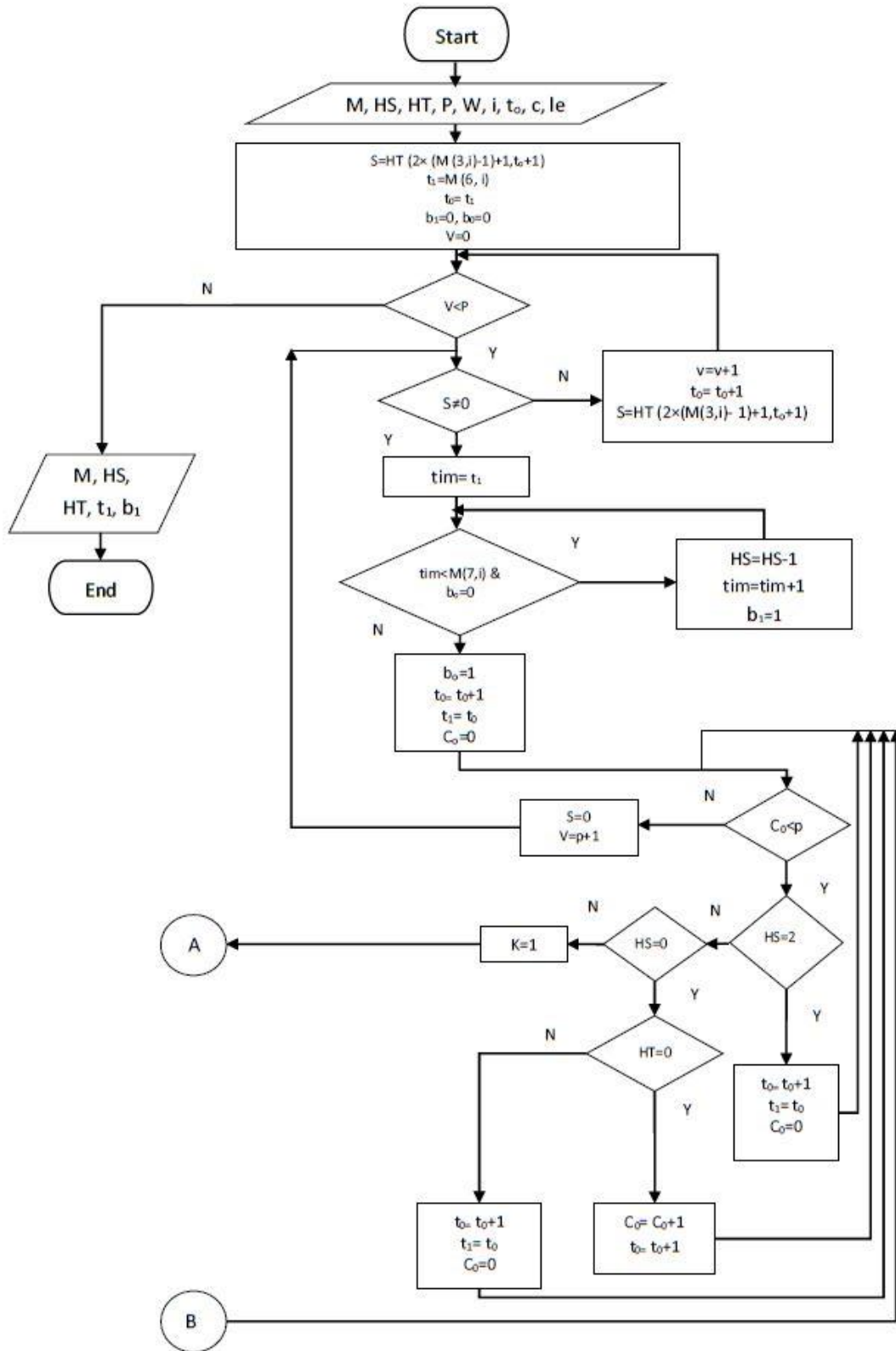


Figure 19: Flowchart for the assignment of turrets to an operation if one turret is needed and only the second turret is available



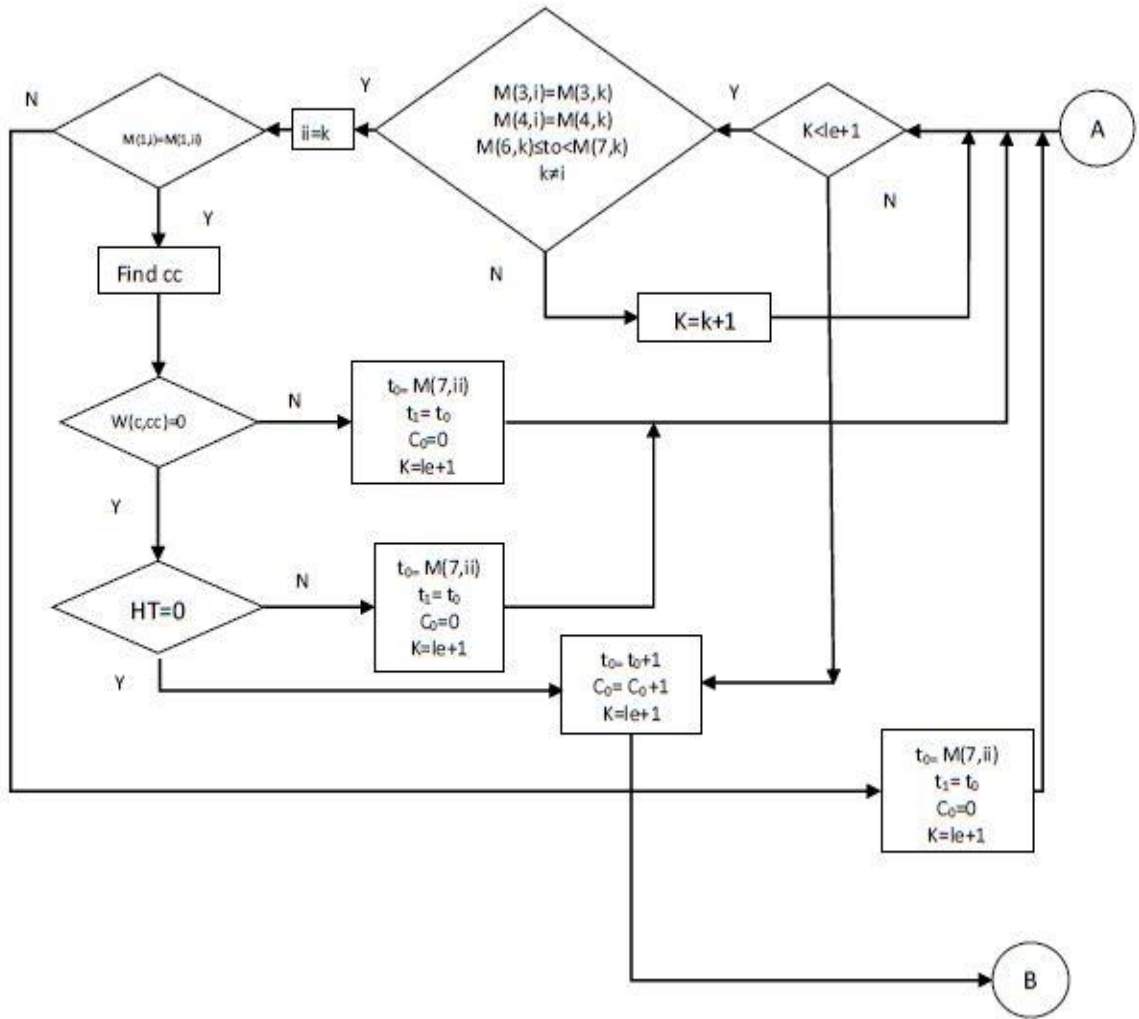
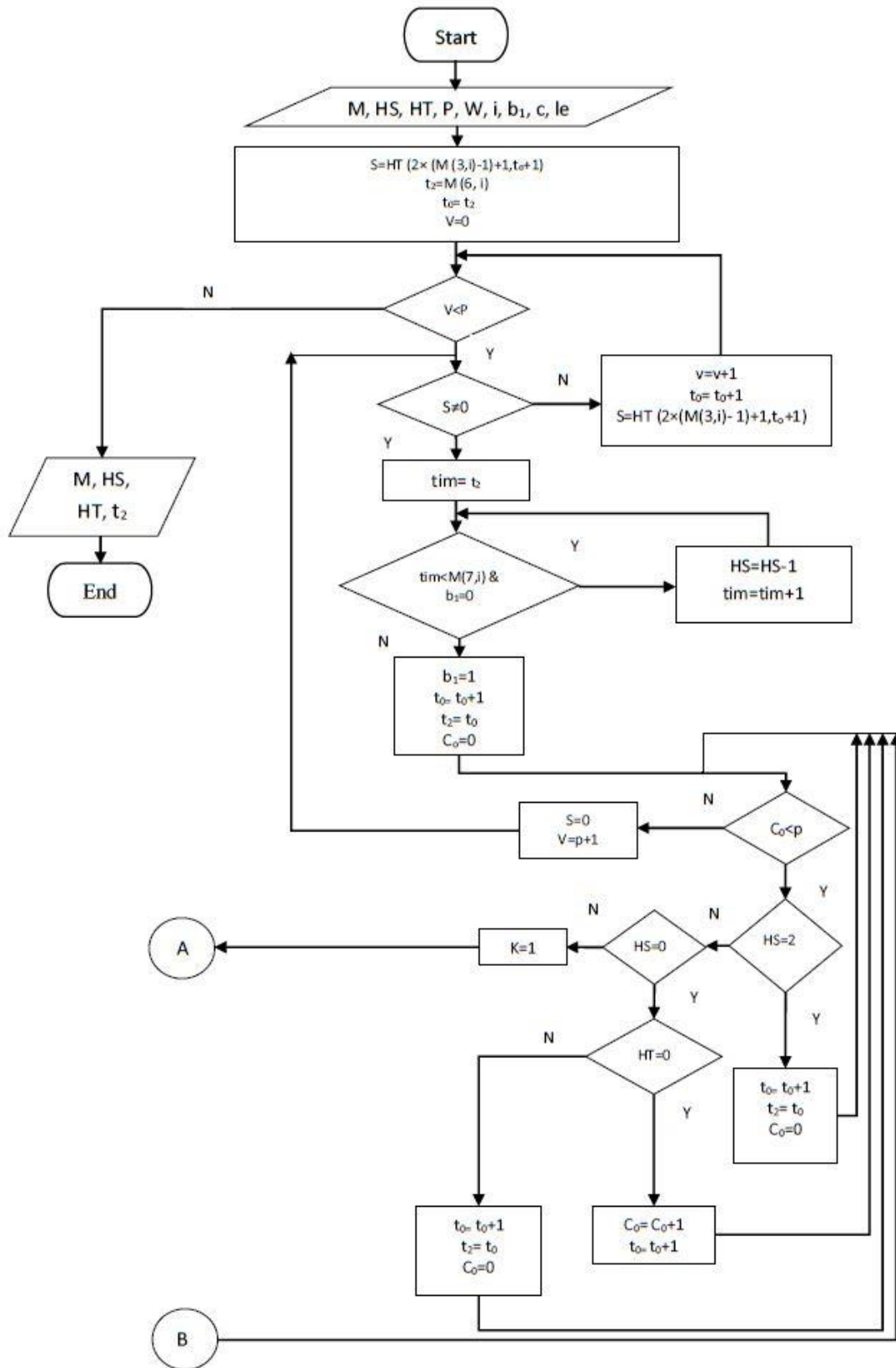


Figure 20: Flowchart for the assignment of turrets to an operation if one turret is needed and both turrets are available (testing the first turret)



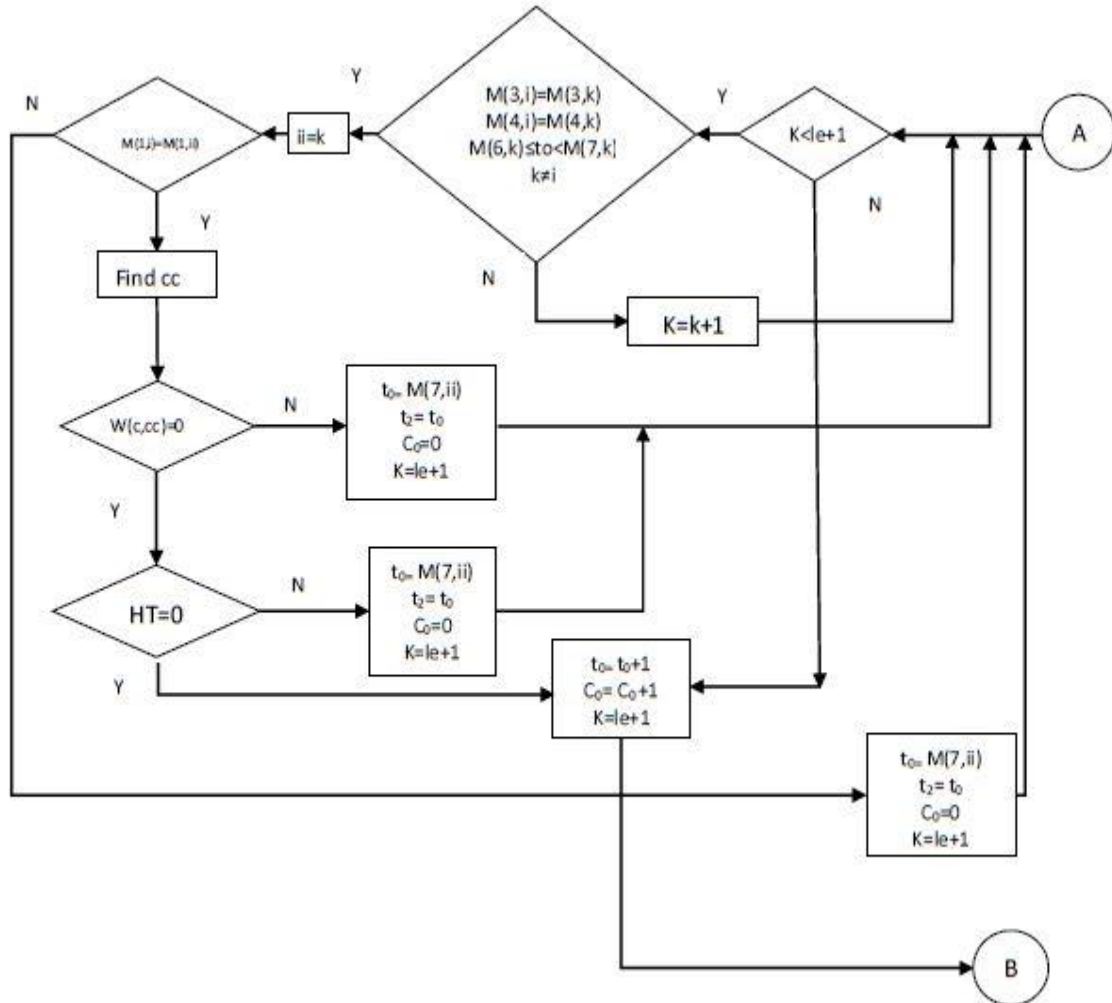


Figure 21: Flowchart for the assignment of turrets to an operation if one turret is needed and both turrets are available (testing the second turret)

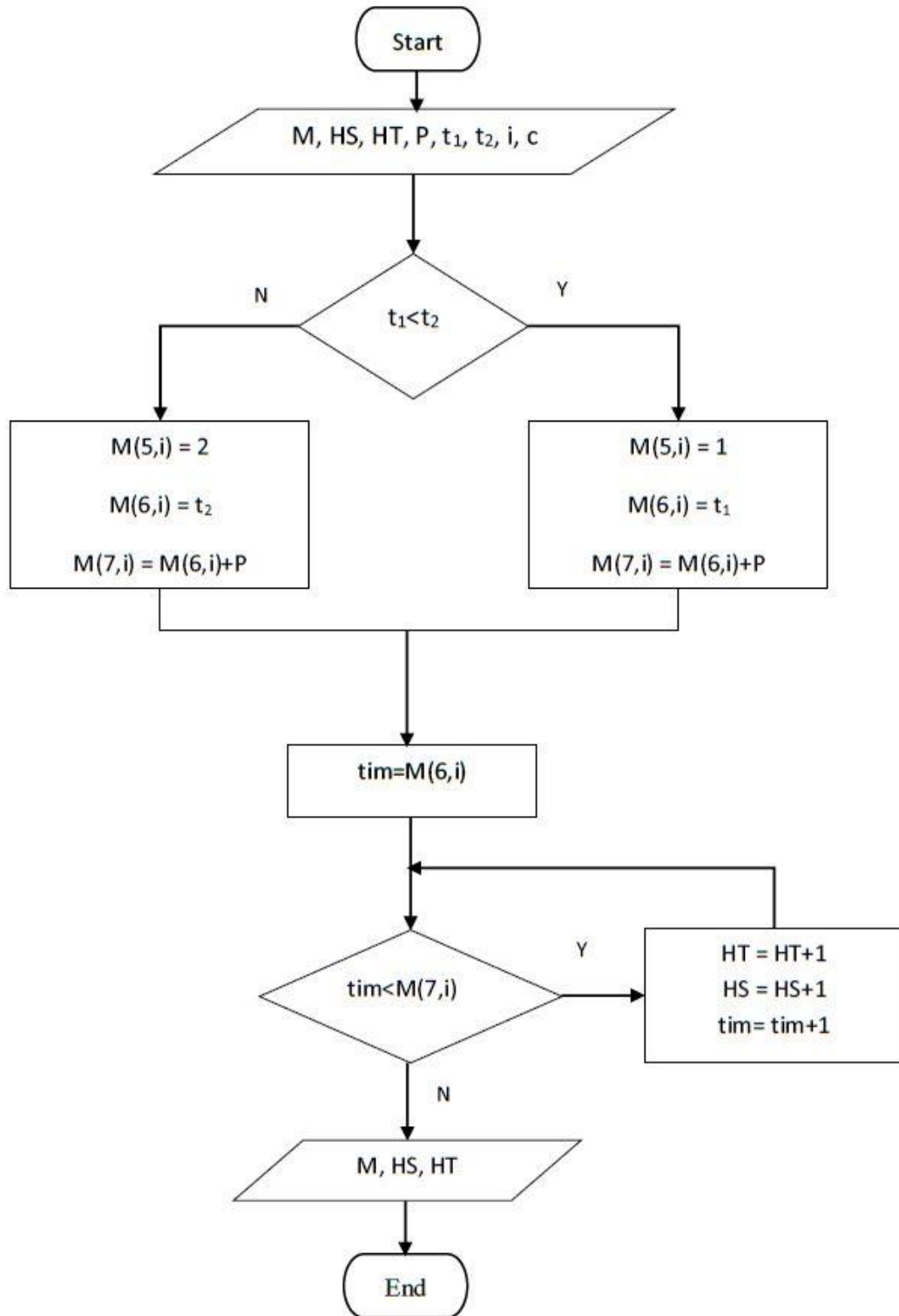


Figure 22: Flowchart for comparison between turrets to find the earliest completion time

Finally, a start and finish time is assigned to each operation. This is also the moment at which the order row takes a counter according to the order in which operations have been completed. Once assignment of an operation of a job is completely done, the algorithm starts the procedure from the next available operation, which has no precedence or its precedence operations are already done.



## CHAPTER 5: TEST CASES

### 5.1 Experimental data

In this chapter, small and large test cases are provided to evaluate the mathematical model and the Simulated Annealing algorithm. Small instances are used to assess the mechanism and complexity of the mathematical model as well as the algorithm. By testing larger instances of the problem, the performance of the proposed algorithm is assessed. FICO Xpress Optimization software version 7.6 is used to algebraically model the problem at hand. Simulated Annealing is executed in MATLAB version R2015a.

### 5.2 Assessment of the mathematical model

The mathematical model is assessed through the size of the problem and number of decision variables and constraints applied in the model. In order to determine the size complexity, it is assumed that there are  $n$  jobs where all jobs have the same number of  $l$  operations. It is assumed each job has two sets of precedence relations among its operations, and there are  $m$  machines in which all machines have dual spindles ( $S$ ) and dual turrets ( $T$ ). Each operation is assumed to need one turret for machining purposes ( $N=1$ ) and all operations are considered to be the same type ( $W=0$ ).

Table 3 shows different problem instances tested using FICO Xpress Optimization, for the size complexity of the model.

Table 3: Size complexity for the proposed model

<b>Job</b>	<b>Operation</b>	<b>Machine</b>	<b>Binary variables</b>	<b>Continuous variables</b>	<b>Constraints</b>
5	5	3	821	25	9,965
10	5	3	2,266	50	34,930
15	5	5	5,176	75	124,455
15	10	5	19,726	150	991,380
20	10	10	31,301	200	1,721,840

## 5.3 Assessment of the Simulated Annealing

### 5.3.1 Data analysis

Simulated Annealing is tested for small to larger instances. The results for small instances are compared with those obtained from the objective function of the mathematical model. For large instances, the results of the SA algorithm are compared with the previous works in the literature and the parameters used for large test cases are those used in the research of Naderi and Azab [39]. For small instances, four and five jobs are considered,  $n=\{4,5\}$ . For large instances, ten, fifteen and twenty jobs are selected,  $n=\{10,15,20\}$ . Each test case is tested for ten runs. The RPD method is used to analyze the metaheuristic algorithm. Relative Percentage Deviation (RPD) for the problem at hand is calculated as below:

$$RPD_{large\ instances} = \frac{Current\ objective\ function\ value - Best\ solution\ so\ far\ objective\ function\ value}{Best\ solution} \times 100$$

For small size instances, RPD considers the minimum solution obtained from the math model and compares it with the SA algorithm solutions.

$$RPD_{small\ instances} = \frac{SA\ Objective\ function\ value - Exact\ optimal\ objective\ function\ value}{Exact\ optimal\ objective\ function\ value} \times 100$$

There are four main parameters of SA considered to initialize the algorithm. These parameters include the inner loop, temperature, the rate at which the temperature is reduced and move operators. The tuned parameters are taken from the mill-turn machine scheduling problem designed in the research of Naderi and Azab [39]. A detailed description of applying SA parameters along with another sensitivity test regarding the total number of iterations will be explained further in section four of this chapter. To test the functionality of the SA, different instances of the machine scheduling problem are proposed. Large instances up to twenty jobs and five machines in which each job has a different set of operations are provided.

### 5.3.2 Simulated Annealing test case1

Table 4: Model parameters, test case 1

<b>Problem parameters</b>	<b>size</b>
job	4
machine	2
operation	3-5
spindle	2
turret	2
N	1,2
Processing time	2-9
W	0,1

Table 5: SA parameters, test case 1

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
$T_0$	500
$a$	0.97
Move operator	Mixed

Table 6: Computational results for test case 1

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	28.5	0.00	0
2	28.5	0.00	0
3	29	0.50	1.754386
4	28.5	0.00	0
5	28.5	0.00	0
6	29	0.50	1.754386
7	28.5	0.00	0
8	29	0.50	1.754386
9	28.5	0.00	0
10	28.5	0.00	0
<b>Average</b>	<b>28.65</b>	<b>0.15</b>	<b>0.526316</b>

Objective function obtained by the model = 28.5  
Best solution obtained = 28.5  
SDV= 0.25

### 5.3.3 Simulated Annealing test case 2

Table 7: Model parameters, test case 2

<b>Problem parameters</b>	<b>size</b>
job	5
machine	3
operation	2-5
spindle	2
turret	2
N	1,2
Processing time	5-15
W	0,1

Table 8: SA parameters, test case 2

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
$T_0$	500
$a$	0.97
Move operator	Mixed

Table 9: Computational results for test case 2

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	48.5	0.00	0
2	48.5	0.00	0
3	48.5	0.00	0
4	49.5	1.00	2.061856
5	48.5	0.00	0
6	49.5	1.00	2.061856
7	48.5	0.00	0
8	48.5	0.00	0
9	48.5	0.00	0
10	48.5	0.00	0
<b>Average</b>	<b>48.7</b>	<b>0.20</b>	<b>0.412371</b>

Objective function obtained by the model = 48.5  
 Best solution obtained = 48.5  
 SDV= 0.42

### 5.3.4 Simulated Annealing test case 3

Table 10: Model parameters, test case 3

<b>Problem parameters</b>	<b>size</b>
job	10
machine	3
operation	3-7
spindle	2
turret	2
N	1
Processing time	5-50
W	0,1

Table 11: SA parameters, test case 3

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 12: Computational results for test case 3

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	121	4.00	3.418803
2	123	6.00	5.128205
3	120	3.00	2.564103
4	117	0.00	0
5	117	0.00	0
6	129	12.00	10.25641
7	120	3.00	2.564103
8	119	2.00	1.709402
9	123	6.00	5.128205
10	117	0.00	0
<b>Average</b>	<b>120.6</b>	<b>3.60</b>	<b>3.076923</b>

Best solution obtained = 117  
SDV= 3.71

### 5.3.5 Simulated Annealing test case 4

Table 13: Model parameters, test case 4

<b>Problem parameters</b>	<b>size</b>
job	10
machine	5
operation	3-7
spindle	2
turret	2
N	1
Processing time	5-50
W	0,1

Table 14: SA parameters, test case 4

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 15: Computational results for test case 4

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	88	11	14.28571
2	82	5	6.493506
3	78	1	1.298701
4	77	0	0
5	82	5	6.493506
6	82	5	6.493506
7	81	4	5.194805
8	79	2	2.597403
9	86	9	11.68831
10	82	5	6.493506
<b>Average</b>	<b>59.8</b>	<b>3.8</b>	<b>6.103896</b>
Best solution obtained = 77			
SDV= 3.36			

### 5.3.6 Simulated Annealing test case 5

Table 16: Model parameters, test case 5

<b>Problem parameters</b>	<b>size</b>
job	15
machine	3
operation	3-7
spindle	2
turret	2
N	1
Processing time	5-50
W	0,1

Table 17: SA parameters, test case 5

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 18: Computational results for test case 5

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	214	1.00	0.469484
2	225	12.00	5.633803
3	213	0.00	0
4	214	1.00	0.469484
5	224	11.00	5.164319
6	216	3.00	1.408451
7	225	12.00	5.633803
8	228	15.00	7.042254
9	220	7.00	3.286385
10	224	11.00	5.164319
<b>Average</b>	<b>220.3</b>	<b>7.30</b>	<b>3.42723</b>
Best solution obtained = 213			
SDV= 5.59			

### 5.3.7 Simulated Annealing test case 6

Table 19: Model parameters, test case 6

<b>Problem parameters</b>	<b>size</b>
job	15
machine	5
operation	3-7
spindle	2
turret	2
N	1
Processing time	5-50
W	0,1

Table 20: SA parameters, test case 6

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 21: Computational results for test case 6

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	175	4.00	2.339181
2	186	15.00	8.77193
3	183	12.00	7.017544
4	182	11.00	6.432749
5	171	0.00	0
6	181	10.00	5.847953
7	171	0.00	0
8	187	16.00	9.356725
9	185	14.00	8.187135
10	176	5.00	2.923977
<b>Average</b>	<b>179.7</b>	<b>8.70</b>	<b>5.087719</b>

Best solution obtained = 171  
SDV= 6.019



### 5.3.8 Simulated Annealing test case 7

Table 22: Model parameters, test case 7

<b>Problem parameters</b>	<b>size</b>
job	20
machine	3
operation	3-7
spindle	2
turret	2
N	1
Processing time	5,50
W	0,1

Table 23: SA parameters, test case 7

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 24: Computational results for test case 7

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	317	6.00	1.92926
2	318	7.00	2.250804
3	321	10.00	3.215434
4	311	0.00	0
5	322	11.00	3.536977
6	318	7.00	2.250804
7	324	13.00	4.180064
8	312	1.00	0.321543
9	311	0.00	0
10	322	11.00	3.536977
<b>Average</b>	<b>317.6</b>	<b>6.60</b>	<b>2.122186</b>

Best solution obtained = 311  
SDV= 4.83

### 5.3.9 Simulated Annealing test case 8

Table 25: Model parameters, test case 8

<b>Problem parameters</b>	<b>size</b>
job	20
machine	5
operation	3-7
spindle	2
turret	2
N	1
Processing time	5-50
W	0,1

Table 26: SA parameters, test case 8

<b>SA parameters</b>	<b>size</b>
Inner loop iteration	50
T0	500
<i>a</i>	0.97
Move operator	Mixed

Table 27: Computational results for test case 8

<b>No.</b>	<b>SA Objective function value</b>	<b>Gap</b>	<b>RPD (%)</b>
1	254	3.00	1.195219
2	253	2.00	0.796813
3	256	5.00	1.992032
4	264	13.00	5.179283
5	264	13.00	5.179283
6	262	11.00	4.38247
7	251	0.00	0
8	265	14.00	5.577689
9	260	9.00	3.585657
10	257	6.00	2.390438
<b>Average</b>	<b>258.6</b>	<b>7.60</b>	<b>3.027888</b>

Best solution obtained = 251  
SDV= 5.08

## 5.4 Parameter tuning and sensitivity analysis

Simulated annealing will have a reasonable performance if the four parameters which are applied to the structure of the algorithm are well tuned. Maximum inner loop iteration or fix, annealing temperature ( $T$ ), temperature reduction rate ( $\alpha$ ) and move operators are these parameters. This work applies the tuned parameters, which have proved to have better performance for the problem of mill-turn scheduling in the recent literature. Naderi and Azab [39] provided three levels for each of the four parameters of the SA and concluded that the parameter fix has submitted minimum RPD for 50 iterations, the temperature has the lowest amount for 500 degrees and  $\alpha$  has less RPD for 0.97. Therefore, these parameters have been considered as the tuned parameters for the SA algorithm for the machine scheduling problem. As for stopping criteria of this work, SA is stopped after achieving 250 best minimum objective function values in each run. The gap is calculated as the difference between the objective value in each run and the minimum solution obtained in all runs.

With consideration of the mentioned parameters and in order to further test the functionality of the algorithm regarding the outer iteration loop, a sensitivity analysis is conducted. To do so, the four main parameters of the SA are set at their tuned level (fixed inner loop is set at 50, the temperature at 500, alpha at 0.97 and move operators are a mixed type) and the total outer loop is tested for different iterations. Table 28 shows the computational results obtained from running the SA for 250, 125 and 65 best minimum solution as the outer loop.

The obtained average RPDs are compared with the initial results obtained from running the SA with the same parameters but different stopping criteria and outer loop iteration. Figure 23 shows the variation in RPDs for the total 8 test cases run for different outer loop iterations. As can be seen, RPD tends to rise with a decrease in the total outer loop iterations. However, this is not true for all test cases. For instance, the RPD level for test four, with ten jobs and five machines has decreased by the change in the outer loop iterations from 250 to 125. It could be discussed that the initial solution of the problem will affect the inner and outer iterations required to find the near-optimum solutions. The initial solution created for this algorithm creates an initial random order of operations of each job

to be scheduled. Based on the initial order the first objective function value will be calculated and set as the best solution found so far. Therefore, the number of iterations required to find improved solutions in the neighborhood of the initial solution will affect the mechanism of the algorithm along with four other parameters that were already discussed in this section.

Table 28: Sensitivity of outer loop iteration for the proposed SA

Size complexity			Computational results		
Test No.	n	m	Average RPD /250 iteration	Average RPD /125 iteration	Average RPD /65 iteration
1	4	2	0.52632	0.70175	0.70175
2	5	3	0.41237	0.61856	0.41237
3	10	3	3.07692	3.33333	3.93162
4	10	5	6.10390	5.97403	7.14286
5	15	3	3.42723	3.89671	3.66197
6	15	5	5.08772	5.26316	5.55556
7	20	3	2.12219	2.66881	3.18328
8	20	5	3.02789	3.18725	4.10359
Average			2.97307	3.20545	3.58662

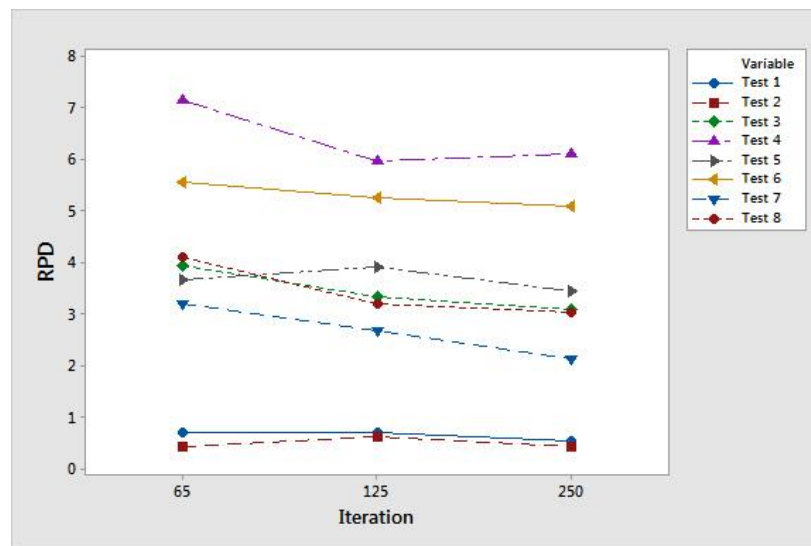


Figure 23: Variation of RPD for different test cases with change in the outer loop iteration

## 5.5 Evaluation of the results

A Simulated Annealing algorithm is developed the machine scheduling problem in multitask manufacturing. A novel assignment approach to fully present the mechanism of mill-turns and also to fulfill the constraints related to their kinematics, has been developed. The results of the implemented algorithm are provided in Table 29. To assess the functionality of the SA, a different set of jobs and machines are evaluated. Small and larger sets of the scheduling problem are tested and results are compared with the minimum of those obtained at each test case. There are totally eight test cases presented, with a different combination of machines and jobs. As can be seen, the SA algorithm provides the lowest RPD for small instances. The lowest RPD is obtained in the case with five jobs. As we go further, RPD tends to rise and picks in the case with ten jobs and five machines. Although the size increment of the problem has affected the average RPD, this is not the case for all test cases. Therefore, it can be concluded that the size complexity has not affected the performance of the proposed algorithm. Another point that can be obtained from the results is that RPD tends to rise with increment in the number of machines applied. The average of the RPDs obtained from the test cases is compared with those in the literature. The average results of the RPDs are better than the only SA algorithm applied so far for the problem of scheduling multitask machines.

The standard deviation is different from each test case to another. The test with four jobs has the lowest standard deviation and the standard deviation for the case with fifteen jobs and five machines stands at the top. This difference also specifies that the algorithm functions regardless of increment in the number of jobs considered for the scheduling problem. It can be seen that the proposed algorithm has lower SDV for the case with twenty jobs and three machines in comparison with the case with fifteen jobs and three machines and also the one with fifteen jobs and five machines. The average standard deviation for all test cases is less than 3.6 seconds, which is also a reasonable quantity for such an NP-complete problem.

Table 29: Computational results for Simulated Annealing

Size complexity			Computational results	
Test No.	n	m	SDV (sec)	Average RPD (%)
1	4	2	0.25000	0.52632
2	5	3	0.42164	0.41237
3	10	3	3.71782	3.07692
4	10	5	3.36815	6.10390
5	15	3	5.59861	3.42723
6	15	5	6.01941	5.08772
7	20	3	4.83506	2.12219
8	20	5	5.08156	3.02789
<b>Average</b>			<b>3.66153</b>	<b>2.97307</b>

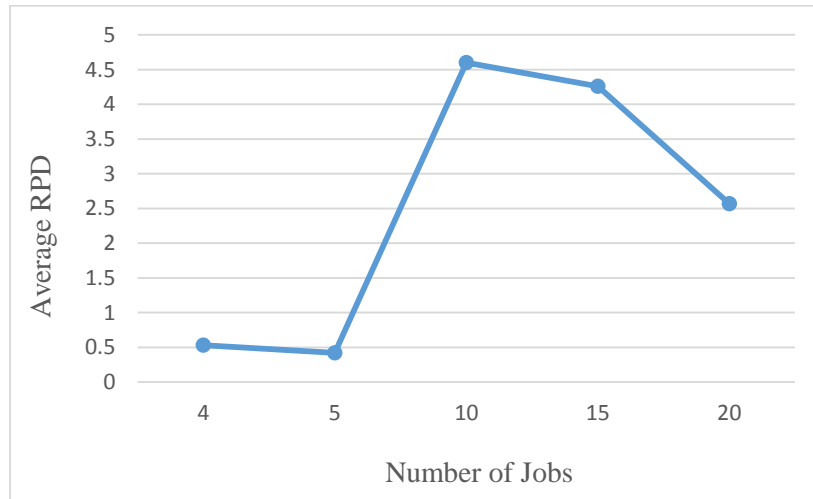


Figure 24: The number of jobs versus the average RPD of SA

## CHAPTER 6: CONCLUSION AND FUTURE WORK

### 6.1 Conclusion

In this thesis, operation scheduling on mill-turn machines is studied. The main focus of this thesis is to study applications and features of these multitask machine tools and present them in the form of an applicable model. To do so, at first, the kinematics and mechanism of parallel machine tools and specifically mill-turns are discussed in detail. To address the problem properly, a thorough review of the previous studies in the literature has been implemented in order to determine the possible gaps for improvements. According to the literature, the main focus of the limited researches done so far on mill-turns has been on computer-aided process planning and issues related to it. There have been few studies done on scheduling and presenting a model to tackle the problem algebraically. The conducted gap analysis also revealed that some critical features like the application of pinch turning, single setup, and definitions regarding the mode of operation are absent in the previously proposed models. Therefore, the mentioned gaps along with concepts, which were key to understanding multitask mill-turn machines and yet needed to be studied in detail, established the foundation of this thesis and the solutions approached.

In order to provide a model, which applies assumptions that are fairly close to the real-world situations, a few considerations were taken into account. First, the problem is modeled by developing a mixed integer linear mathematical model. The model is linear to provide solutions that are optimal. The aim of the presented model in this work is to visualize the kinematics of mill-turns and it applies the aforementioned assumptions. The outcome of the model is a schedule based plan, which also determines a part machining location and a dedicated machining unit used to process the part. The model is further tested for complexity in terms of decision variables and number of constraints applied.

A metaheuristic algorithm is developed to solve instances of the machine scheduling problem, which cannot be solved in a logical amount of time. The initialization phase considers the number of operations of each job. In order to create feasible solutions in the neighborhood of the initial solution, a mixed operator is used. All of the operations of each job are processed on the same machine tool so that the single setup concept would be applied. A novel approach is presented for the assignment and sequencing part of the

problem that is used in the simulated annealing metaheuristic algorithm. To test the application of SA, larger instances up to twenty jobs and five machines are applied. A sensitivity analysis was conducted to validate the stopping criteria, which was proposed for the algorithm. The results obtained from the analysis proved that combination of parameters and the stopping criteria, which were applied are a good fit for this problem and submit objective function values with minimum average RPDs. Test cases show that SA is capable of solving all instances provided, with the ability to solve some instances with less than one relative percentage deviation. Since there have been considerable enhances and changes in the structure of multitasking machine tools recently, the scheme provided in this work could also be used to further explore the process planning side of the problem.

## **6.2 Future work**

This thesis considers modeling and formulating a class of multitasking machine tools, which are among the commonly used mill-turn machines in the industry. A dual-spindle and dual-turret class of mill- turns was considered as a representative of a wide variety of these machine tools. Therefore, other configurations also need to be studied thoroughly. Dual-spindle three-turret mill-turns or dual-spindle four-turret configurations are among those settings, which can be further explored to develop a model that represents their specific kinematics and mechanisms. Furthermore, attributes related to machining parameters such as cutting speed and depth of cut and their effect on the scheduling problem could also be further considered to cover other perspectives of mill-turns, which have not been considered so far.



## REFERENCES/BIBLIOGRAPHY

1. Yip-Hoi, D. and D. Dutta, *A genetic algorithm application for sequencing operations in process planning for parallel machining*. IIE transactions, 1996. **28**(1): p. 55-68.
2. Norman, B.A. and J.C. Bean, *Scheduling operations on parallel machine tools*. Iie Transactions, 2000. **32**(5): p. 449-459.
3. Chiu, N.-C., S.-C. Fang, and Y.-S. Lee, *Sequencing parallel machining operations by genetic algorithms*. Computers & industrial engineering, 1999. **36**(2): p. 259-280.
4. Lee, Y.-S. and C.-J. Chiou, *Unfolded projection approach to machining non-coaxial parts on mill-turn machines*. Computers in industry, 1999. **39**(2): p. 147-173.
5. Crichigno Filho, J.M., *Prediction of cutting forces in mill turning through process simulation using a five-axis machining center*. The International Journal of Advanced Manufacturing Technology, 2012. **58**(1-4): p. 71-80.
6. She, C.-H. and C.-W. Hung, *Development of multi-axis numerical control program for mill—turn machine*. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 2008. **222**(6): p. 741-745.
7. Yip-Hoi, D. and D. Dutta. *Issues in computer-aided process planning for parallel machine tools*. in *Advances in Design Automation-1993, Proceedings of the 1993 ASME Design Automation Conference (Albuquerque, NM)*. 1993.
8. Levin, J.B. and D. Dutta, *Computer-aided process planning for parallel machines*. Journal of Manufacturing Systems, 1992. **11**(2): p. 79-92.
9. Battaia, O., et al., *Combinatorial techniques to optimally customize an automated production line with rotary transfer and turrets*. IIE Transactions, 2014. **46**(9): p. 867-879.
10. Levin, J., D. Dutta, and J. Bean, *PMPS: A prototype CAPP system for parallel machining*. TRANSACTIONS-AMERICAN SOCIETY OF MECHANICAL ENGINEERS JOURNAL OF MANUFACTURING SCIENCE AND ENGINEERING, 1996. **118**: p. 406-414.

11. Jun, C.-S., K. Cha, and Y.-S. Lee, *Optimizing tool orientations for 5-axis machining by configuration-space search method*. Computer-Aided Design, 2003. **35**(6): p. 549-566.
12. Tsutsumi, M. and A. Saito, *Identification and compensation of systematic deviations particular to 5-axis machining centers*. International Journal of Machine Tools and Manufacture, 2003. **43**(8): p. 771-780.
13. Bohez, E.L., et al., *A geometric modeling and five-axis machining algorithm for centrifugal impellers*. Journal of Manufacturing systems, 1997. **16**(6): p. 422-436.
14. Mori, M., M. Fujishima, and O. Yohei, *5 axis mill turn and hybrid machining for advanced application*. Procedia CIRP, 2012. **1**: p. 22-27.
15. Moriwaki, T., *Multi-functional machine tool*. CIRP Annals-Manufacturing Technology, 2008. **57**(2): p. 736-749.
16. Selek, M., et al., *Experimental examination of the cooling performance of Ranque-Hilsch vortex tube on the cutting tool nose point of the turret lathe through infrared thermography method*. International journal of refrigeration, 2011. **34**(3): p. 807-815.
17. Abele, E., Y. Altintas, and C. Brecher, *Machine tool spindle units*. CIRP Annals-Manufacturing Technology, 2010. **59**(2): p. 781-802.
18. Abele, E., et al., *Analyzing energy consumption of machine tool spindle units and identification of potential for improvements of efficiency*. Glocalized Solutions for Sustainability in Manufacturing, 2011: p. 280-285.
19. Guschinskaya, O., et al., *Minimizing makespan for multi-spindle head machines with a mobile table*. Computers & Operations Research, 2009. **36**(2): p. 344-357.
20. Battaïa, O., et al., *Equipment location in machining transfer lines with multi-spindle heads*. Journal of Mathematical Modelling and Algorithms, 2013: p. 1-17.
21. Dr. Jill Urbanic, p.c.
22. Sakal, R. and J. Chow, *An integrated intelligent process planning system for prismatic parts using PC-based CAD and CAM software packages*. The International Journal of Advanced Manufacturing Technology, 1994. **9**(3): p. 166-174.

23. Dolgui, A., N. Guschinsky, and G. Levin, *Graph approach for optimal design of transfer machine with rotary table*. International Journal of Production Research, 2009. **47**(2): p. 321-341.
24. Li, S.X. and R.B. Jerard, *5-axis machining of sculptured surfaces with a flat-end cutter*. Computer-Aided Design, 1994. **26**(3): p. 165-178.
25. Bohez, E.L., *Five-axis milling machine tool kinematic chain design and analysis*. International Journal of Machine Tools and Manufacture, 2002. **42**(4): p. 505-520.
26. Langeron, J.M., et al., *A new format for 5-axis tool path computation, using B-spline curves*. Computer-Aided Design, 2004. **36**(12): p. 1219-1229.
27. Lee, Y.-S., *Admissible tool orientation control of gouging avoidance for 5-axis complex surface machining*. Computer-Aided Design, 1997. **29**(7): p. 507-521.
28. Calleja, A., et al., *Turn-milling of blades in turning centres and multitasking machines controlling tool tilt angle*. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 2015. **229**(8): p. 1324-1336.
29. Lu, K., et al. *Industrial applications of chatter stability prediction and monitoring system for turning processes*. in *Mechatronics and Automation (ICMA), 2011 International Conference on*. 2011. IEEE.
30. Mr. Bob Hedrick, p.c.
31. Choudhury, S. and K. Mangrulkar, *Investigation of orthogonal turn-milling for the machining of rotationally symmetrical work pieces*. Journal of Materials Processing Technology, 2000. **99**(1): p. 120-128.
32. Savas, V. and C. Ozay, *The optimization of the surface roughness in the process of tangential turn-milling using genetic algorithm*. The International Journal of Advanced Manufacturing Technology, 2008. **37**(3): p. 335-340.
33. Tönshoff, H., C. Arendt, and R.B. Amor, *Cutting of hardened steel*. CIRP Annals-Manufacturing Technology, 2000. **49**(2): p. 547-566.
34. Byrne, G., D. Dornfeld, and B. Denkena, *Advancing cutting technology*. CIRP Annals-Manufacturing Technology, 2003. **52**(2): p. 483-507.

35. Klocke, F., E. Brinksmeier, and K. Weinert, *Capability profile of hard cutting and grinding processes*. CIRP Annals-Manufacturing Technology, 2005. **54**(2): p. 22-45.
36. König, W., A. Berktold, and K.-F. Koch, *Turning versus grinding—a comparison of surface integrity aspects and attainable accuracies*. CIRP Annals-Manufacturing Technology, 1993. **42**(1): p. 39-43.
37. Chang, T.C. and R.A. Wysk, *An introduction to automated process planning systems*. 1984: Prentice Hall Professional Technical Reference.
38. Yip-Hoi, D. and D. Dutta, *Data extraction from geometric models for process planning for parallel machines*. Journal of Manufacturing Systems, 1995. **14**(5): p. 307.
39. Naderi, B. and A. Azab, *Modeling and scheduling a flexible manufacturing cell with parallel processing capability*. CIRP Journal of Manufacturing Science and Technology, 2015. **11**: p. 18-27.
40. Fanjul-Peyro, L., F. Perea, and R. Ruiz, *Models and matheuristics for the unrelated parallel machine scheduling problem with additional resources*. European Journal of Operational Research, 2017. **260**(2): p. 482-493.
41. Low, C. and G.-H. Wu, *Unrelated parallel-machine scheduling with controllable processing times and eligibility constraints to minimize the makespan*. Journal of Industrial and Production Engineering, 2016. **33**(4): p. 286-293.
42. Rajkanth, R., C. Rajendran, and H. Ziegler, *Heuristics to minimize the completion time variance of jobs on a single machine and on identical parallel machines*. The International Journal of Advanced Manufacturing Technology, 2017. **88**(5-8): p. 1923-1936.
43. Miller, P.C., *Lathes turn to other tasks*. Tooling & Production, 1989. **54**(12): p. 54-60.
44. Miska, K.H., *Driven tools turn on turning centers*. Manufacturing Engineering, 1990. **104**(5): p. 63-66.
45. Date, N., P. Krishnaswami, and V.S.K. Motipalli, *Automated process planning method to machine A B-Spline free-form feature on a mill–turn center*. Computers & Industrial Engineering, 2009. **56**(1): p. 198-207.

46. Kim, Y.S., et al., *Geometric reasoning for mill-turn machining process planning*. Computers & industrial engineering, 1997. **33**(3-4): p. 501-504.
47. Waiyagan, K. and E.L. Bohez, *Intelligent feature based process planning for five-axis mill-turn parts*. Computers in Industry, 2009. **60**(5): p. 296-316.
48. Tseng, Y.-J. and S. Joshi, *Recognition of interacting rotational and prismatic machining features from 3-D mill-turn parts*. International Journal of Production Research, 1998. **36**(11): p. 3147-3165.
49. Tseng, Y.-J. and C.-C. Liu, *Concurrent analysis of machining sequences and fixturing set-ups for minimizing set-up changes for machining mill-turn parts*. International Journal of Production Research, 2001. **39**(18): p. 4197-4214.
50. Li, S. and J.J. Shah, *Recognition of user-defined turning features for mill/turn parts*. Journal of Computing and Information Science in Engineering, 2007. **7**(3): p. 225-235.
51. Dutta, D., et al. *Feature extraction and operation sequencing for machining on mill-turns*. in *CIE-4276, "Proceedings of the 1997 ASME Design Technical Conferences"*, Sacramento, ASME, New York. 1997.
52. Azab, A. and B. Naderi, *A variable neighborhood search metaheuristic for cellular manufacturing with multitask machine tools*. Procedia CIRP, 2014. **20**: p. 50-55.
53. Arroyo, J.E.C. and J.Y.-T. Leung, *Scheduling unrelated parallel batch processing machines with non-identical job sizes and unequal ready times*. Computers & Operations Research, 2017. **78**: p. 117-128.
54. Shahvari, O. and R. Logendran, *An enhanced tabu search algorithm to minimize a bi-criteria objective in batching and scheduling problems on unrelated-parallel machines with desired lower bounds on batch sizes*. Computers & Operations Research, 2017. **77**: p. 154-176.
55. Villa, F., E. Vallada, and L. Fanjul-Peyro, *Heuristic algorithms for the unrelated parallel machine scheduling problem with one scarce additional resource*. Expert Systems with Applications, 2017.
56. Yin, Y., et al., *Just-in-time scheduling with two competing agents on unrelated parallel machines*. Omega, 2016. **63**: p. 41-47.

57. Egbelu, P.J., *Planning for machining in a multijob, multimachine manufacturing environment*. Journal of Manufacturing Systems, 1986. **5**(1): p. 1-13.
58. Kirkpatrick, S., C.D. Gelatt Jr, and M.P. Vecchi, *Optimization by simulated annealing*, in *Spin Glass Theory and Beyond: An Introduction to the Replica Method and Its Applications*. 1987, World Scientific. p. 339-348.
59. Eglese, R., *Simulated annealing: a tool for operational research*. European journal of operational research, 1990. **46**(3): p. 271-281.
60. Corana, A., et al., *Minimizing multimodal functions of continuous variables with the "simulated annealing" algorithm* *Corrigenda for this article is available here*. ACM Transactions on Mathematical Software (TOMS), 1987. **13**(3): p. 262-280.
61. Glass, C., C. Potts, and P. Shade, *Unrelated parallel machine scheduling using local search*. Mathematical and Computer Modelling, 1994. **20**(2): p. 41-52.
62. Kim, D.-W., et al., *Unrelated parallel machine scheduling with setup times using simulated annealing*. Robotics and Computer-Integrated Manufacturing, 2002. **18**(3): p. 223-231.
63. Radhakrishnan, S. and J.A. Ventura, *Simulated annealing for parallel machine scheduling with earliness-tardiness penalties and sequence-dependent set-up times*. International Journal of Production Research, 2000. **38**(10): p. 2233-2252.

## APPENDICES

### Appendix 1: FICO Xpress codes for the proposed mathematical model

model "Optimization of Mill Turn machines"

uses "mmxprs"

Declarations

JOB = 1..4                      ! Number of jobs

MACHINE = 1, 2                ! Number of machines

E=1, 2

K: array (MACHINE) of a set of an integer!                Count of spindles of machine *i*

H: array (MACHINE) of a set of an integer!                Count of turrets of machine *i*

L: array (JOB) of a set of an integer!                      Count of operations of job *j*

R: dynamic array (JOB, range, range) of an integer! Precedence operations of operation *l*  
of job *j*

S: dynamic array (JOB, MACHINE) of a set of an integer! Spindles, capable of loading  
job  $j$  on machine  $i$

T: dynamic array (JOB, MACHINE) of a set of an integer! Turrets, capable of processing  
job  $j$  on machine  $i$

P: dynamic array (JOB, range, MACHINE) of real! Machining time of operation  $l$  of job  $j$   
on machine  $i$

N: dynamic array (JOB, range) of an integer! Number of turrets used for processing each  
operation at a time

W: dynamic array (JOB, range, range) of an integer! Binary parameter taking the value of  
1 if operations  $l$  and  $l'$  of job  $j$  are different types of operation and 0 otherwise

b:array (JOB,MACHINE) of mpvar !1 if job  $j$  is  
processed on machine  $i$ ,0 otherwise

b1: dynamic array (JOB, range, MACHINE) of mpvar !1 if  
operation  $l$  of job  $j$  is processed on machine  $i$ , 0 otherwise



b2: dynamic array (JOB, range, MACHINE, range, range) of mpvar           !1 if  
 operation  $l$  of job  $j$  is processed on machine  $I$ , spindle  $k$  capacity, 0 otherwise

b3: dynamic array (JOB, range, MACHINE, range) of mpvar                 !1 if  
 operation  $l$  of job  $j$  is processed on turret  $h$  of machine  $i$ , 0 otherwise

b4: dynamic array (JOB, MACHINE, range) of mpvar                         !1 if  
 operation  $l$  of job  $j$  is processed on machine  $i$  and turret  $h$ , 0 otherwise

x: dynamic array (JOB, range, JOB, range) of mpvar                       !1 if  
 operation  $l$  of job  $j$  is processed after operation  $l1$  of job  $j$

x1: dynamic array (JOB, range, JOB, range) of mpvar                     !1 if  
 operation  $l$  of job  $j$  is processed after operation  $l1$  of job  $j1$

s: dynamic array (JOB, range) of mpvar                                     !continuous  
 variable for the starting time of operation  $l$  of job  $j$

c: mpvar

end- declarations

Initializations from ' Optimization of Mill Turn machines.dat'

K H L R S T P N W

end- initializations

forall (j in JOB, i in MACHINE )create (b(j,i))

forall (j in JOB, l in L(j), i in MACHINE)create (b1(j,l,i))

forall (j in JOB,l in L(j), i in MACHINE ,k in S(j,i), e in E)create (b2(j,l,i,k,e))

forall (j in JOB, l in L(j), i in MACHINE, h in H(i))create(b3(j,l,i,h))

forall (j in JOB, i in MACHINE ,k in S(j,i))create (b4(j,i,k))

forall (j in JOB, l in L(j), jj in JOB , l1 in L(j)|l<l1)create (x(j,l,j,l1))

forall (j in JOB, l in L(j), j1 in JOB , l1 in L(j)|j<j1)create (x1(j,l,j1,l1))

forall (j in JOB, l in L(j))create (s(j,l))

forall (j in JOB, i in MACHINE ) b(j,i) is\_binary

forall (j in JOB, l in L(j), i in MACHINE) b1(j,l,i) is\_binary

forall (j in JOB,l in L(j), i in MACHINE , k in S(j,i), e in E) b2(j,l,i,k,e) is\_binary

forall (j in JOB, l in L(j), i in MACHINE, h in H(i)) b3(j,l,i,h) is\_binary

forall (j in JOB, i in MACHINE ,k in S(j,i)) b4(j,i,k)is\_binary

forall (j in JOB, l in L(j), jj in JOB , l1 in L(j)|l<l1) x(j,l,j,l1) is\_binary

forall (j in JOB, l in L(j), j1 in JOB , l1 in L(j)|j<j1) x1(j,l,j1,l1) is\_binary

forall (j in JOB, l in L(j)) s(j,l) is\_continuous

M = sum(j in JOB , l in L(j),i in MACHINE)P(j,l,i)

forall (j in JOB, l in L(j)) sum(i in MACHINE ,k in S(j,i),e in E) b2(j,l,i,k,e) = 1

forall (j in JOB) sum(i in MACHINE ,k in S(j,i)) b4(j,i,k) = 1

forall (j in JOB, l in L(j)) sum(i in MACHINE ) b1(j,l,i) = 1

forall (j in JOB ) sum(i in MACHINE ) b(j,i) = 1

forall (j in JOB, l in L(j)) sum(i in MACHINE ,h in T(j,i)) b3(j,l,i,h) = N(j ,l)

forall(j in JOB ,l in L(j),l1 in L(j) | W(j,l,l1)=1 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E) s(j,l)-s(j,l1) >= b2(j,l1,i,k,e)\*P(j,l1,i)-M\*(1-x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB ,l in L(j), l1 in L(j) | W(j,l,l1)=1 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E) s(j,l1)-s(j,l) >= b2(j,l,i,k,e)\*P(j,l,i)-M\*(x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB ,l in L(j), l1 in L(j) | W(j,l,l1)=0 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E |e<>e1) s(j,l)-s(j,l1) >= -M\*(1-x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB ,l in L(j), l1 in L(j) | W(j,l,l1)=0 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E |e<>e1 ) s(j,l1)-s(j,l) >= -M\*(x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB ,l in L(j), l1 in L(j) | W(j,l,l1)=0 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E |e=e1 ) s(j,l)-s(j,l1) >= b2(j,l1,i,k,e1)\*P(j,l1,i)-M\*(1-x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB ,l in L(j), l1 in L(j) | W(j,l,l1)=0 and l<l1 ,i in MACHINE,k in S(j,i),e in E,e1 in E |e=e1 ) s(j,l1)-s(j,l) >= b2(j,l,i,k,e)\*P(j,l,i)-M\*(x(j,l,j,l1))-M\*(1-b2(j,l1,i,k,e1))-M\*(1-b2(j,l,i,k,e))

forall(j in JOB , l in L(j),j1 in JOB, l1 in L(j) | j<j1 ,i in MACHINE, k in S(j,i)\*S(j1,i)) s(j,l)-s(j1,l1) >= b4(j1,i,k)\*P(j1,l1,i)-M\*(1-x1(j,l,j1,l1))-M\*(1-b4(j1,i,k))-M\*(1-b4(j,i,k))

forall(j in JOB , l in L(j), j1 in JOB, l1 in L(j) | j < j1 , i in MACHINE, k in S(j,i) \* S(j1,i))  
s(j1,l1) - s(j,l) >= b4(j,i,k) \* P(j,l,i) - M\*(x1(j,l,j1,l1)) - M\*(1 - b4(j1,i,k)) - M\*(1 - b4(j,i,k))

forall(j in JOB , l in L(j), j1 in JOB, l1 in L(j) | j < j1 , i in MACHINE, h in T(j,i) \* T(j1,i))  
s(j,l) - s(j1,l1) >= b3(j1,l1,i,h) \* P(j1,l1,i) - M\*(1 - x1(j,l,j1,l1)) - M\*(1 - b3(j1,l1,i,h)) - M\*(1 -  
b3(j,l,i,h))

forall(j in JOB , l in L(j), j1 in JOB, l1 in L(j) | j < j1 , i in MACHINE, h in T(j,i) \* T(j1,i))  
s(j1,l1) - s(j,l) >= b3(j,l,i,h) \* P(j,l,i) - M\*(x1(j,l,j1,l1)) - M\*(1 - b3(j1,l1,i,h)) - M\*(1 - b3(j,l,i,h))

forall(j in JOB , l in L(j), l1 in L(j) | l < l1 , i in MACHINE, h in T(j,i)) s(j,l) - s(j,l1) >=  
b3(j,l1,i,h) \* P(j,l1,i) - M\*(1 - x(j,l,j,l1)) - M\*(1 - b3(j,l1,i,h)) - M\*(1 - b3(j,l,i,h))

forall(j in JOB , l in L(j), l1 in L(j) | l < l1 , i in MACHINE, h in T(j,i)) s(j,l1) - s(j,l) >=  
b3(j,l,i,h) \* P(j,l,i) - M\*(x(j,l,j,l1)) - M\*(1 - b3(j,l1,i,h)) - M\*(1 - b3(j,l,i,h))

forall(j in JOB , l in L(j), l1 in L(j), i in MACHINE | exists (R(j,l1,l))) s(j,l) - s(j,l1) >=  
P(j,l1,i)

forall(j in JOB , l in L(j), i in MACHINE, k in S(j,i), e in E | k not in S(j,i)) b2(j,l,i,k,e) = 0

forall(j in JOB , l in L(j), i in MACHINE, h in T(j,i) | h not in T(j,i)) b3(j,l,i,h) = 0

forall(j in JOB , l in L(j), i in MACHINE) b1(j,l,i) <= b(j,i)

forall(j in JOB ,l in L(j), i in MACHINE,k in S(j,i), e in E)  $b2(j,l,i,k,e) \leq b1(j,l,i)$

forall(j in JOB ,l in L(j), i in MACHINE,k in S(j,i), e in E)  $b2(j,l,i,k,e) \leq b4(j,i,k)$

forall(j in JOB ,l in L(j), i in MACHINE, h in T(j,i))  $b3(j,l,i,h) \leq b1(j,l,i)$

forall(j in JOB ,l in L(j))  $s(j,l) \geq 0$

forall (j in JOB,l in L(j),i in MACHINE )  $c \geq s(j,l) + P(j,l,i)$

minimize (c)

forall(j in JOB, l in L(j)) writeln("start" , "(" ,j,l,"" , ": " ,getsol(s(j,l)))

forall(j in JOB, i in MACHINE)writeln("b" , "(" ,j,i,"" , ": " ,getsol(b(j,i)))

forall(j in JOB,l in L(j), i in MACHINE )

if(getsol(b1(j,l,i))> 0) then

writeln("b1" , "(" ,j,l,i,"" , ": " ,getsol(b1(j,l,i)))

end-if

forall(j in JOB,l in L(j), i in MACHINE,k in S(j,i),e in E)writeln("b2" , "(" ,j,l,i,k,e,"" , ": " ,getsol(b2(j,l,i,k,e)))

```
forall(j in JOB,l in L(j), i in MACHINE,h in T(j,i))writeln("b3" , "(" ,j,l,i,h,"" , " :  
",getsol(b3(j,l,i,h)))
```

```
forall (j in JOB, l in L(j), jj in JOB , l1 in L(j)|l<l1)
```

```
if (getsol(x(j,l,jj,l1))>0) then
```

```
writeln("x" , "(" ,j,l,jj,l1,"" , " : " ,getsol(x(j,l,jj,l1)))
```

```
end-if
```

```
end-model
```

## Appendix 2: Instances for test cases

### Test case 1:

```
P(1,1)=[ 5    2    8    5    0
         3    2    2    9    3
         2    3    9    0    0
         3    4    8    9    0];
```

```
P(1,2)=[ 6    4    2    6    0
         7    5    3.5  9    4
         7    6    3    0    0
         6    8    2    9    0];
```

```
S(1,1)=[1    1
         1    1
         0    1
         1    0];
```

```
S(1,2)=[0    0
         1    1
         0    1
         1    1];
```

```
T(1,1)=[1    0
         1    0
         0    1
         0    1];
```

```
T(1,2)=[1    1
         1    1
         1    1
         1    1];
```

```
N= [ 1    1    1    1    0
     1    2    2    1    2
     1    1    1    0    0
     1    1    1    1    0];
```



$$R(1,1) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,3) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,4) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,1) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,3) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

Test case 2:

```
P(1,1)=[ 15    12    8    15    0
         13    10.5  12    6    10
          6     7    10    0    0
         10     5     8     9     0
         10    14     0     0     0 ];
```

```
P(1,2)=[ 10     11    15     9     0
         13    14.5   12     9    10
          7     9.5    7     0     0
          6     14    12     5     0
         14     11     0     0     0 ];
```

```
P(1,3)=[ 7     15     9     6     0
         13     9     7     9     4
         12     5     6     0     0
          8     8    12     9     0
          7    10     0     0     0 ];
```

```
S(1,1) =[1     1
          1     1
          0     1
          1     0
          0     0
          ];
```

```
S(1,2) =[0     0
          1     1
          0     1
          1     1
          1     1];
```

```
S(1,3) =[1     1
          1     1
          1     1
          1     1
          0     1 ];
```

$$T(1,1) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix};$$

$$T(1,2) = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix};$$

$$T(1,3) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix};$$

$$N = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,1) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,3) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,4) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,5) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix};$$

$$W(1,1) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix};$$

$$W(1,3) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$W(1,4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix};$$

$$W(1,5) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix};$$

Test case 3:

```
P(1,1)=[ 15    20    8    44    13    11    8
         22    14    6     0     0     0     0
          8     8     9     5     0     0     0
         17    32    21     0     0     0     0
          5     9    12    23    12    40    22
         50    11    10     0     0     0     0
         14     6    19    39     8    13     0
         33    15     5     7     0     0     0
         12    33    44     0     0     0     0
         50    14    17     0     0     0     0];
```

```
P(1,2)=[ 21    12     9    20    34    10    5
         29    17     5     0     0     0     0
          8    18     8     6     0     0     0
         17    14    12     0     0     0     0
          6    18     6    43    11    12    18
          8    11    18     0     0     0     0
          7    23     5    23    33    10     0
         14     6    17    39     0     0     0
          7    20    17     0     0     0     0
         11     7     9     0     0     0     0];
```

```
P(1,3)=[ 14    14    17    16     5     8    11
         16     9     5     0     0     0     0
         13    34    20    11     0     0     0
         13    12     5     0     0     0     0
          7    29     9    20     9    39     8
         16     6    15     0     0     0     0
         20     9    13    19    12    14     0
         12    21     7    11     0     0     0
         43     9    16     0     0     0     0
         32     7    12     0     0     0     0];
```

```
S(1,1)=[1     1
         1     1
         1     1
         1     0
         1     1
         0     1
         1     1
         0     1
         1     1
         1     1
         1     0];
```

```

S(1,2) =[0    0
          1    1
          0    1
          1    1
          1    0
          0    1
          1    1
          1    0
          0    1
          1    1];

```

```

S(1,3) =[0    1
          1    1
          0    1
          1    1
          1    1
          0    0
          1    1
          0    1
          1    1
          1    0];

```

```

T(1,1) =[1    1
          1    0
          0    1
          0    1
          1    1
          1    1
          0    1
          0    1
          1    1
          1    1];

```

```

T(1,2) =[1    1
          1    1
          0    1
          1    1
          1    1
          1    0
          0    1
          0    1
          1    1
          1    1];

```

```

T(1,3) =[1    1
          1    0
          0    1
          0    1
          1    1
          1    0
          1    1
          0    1
          1    1
          1    1];

```

```

N= [ 1 1 1 1 1 1 1
      1 1 1 0 0 0 0
      1 1 1 0 0 0 0
      1 1 1 1 0 0 0
      1 1 1 1 1 1 1
      1 1 1 0 0 0 0
      1 1 1 1 1 1 0
      1 1 1 1 0 0 0
      1 1 1 0 0 0 0
      1 1 1 0 0 0 0 ];

```

```

R(1,1) =[0 1 0 0 1 0 0
          0 0 0 1 0 0 0
          0 0 0 1 0 0 0
          0 0 0 0 0 0 0
          0 0 0 0 0 0 0
          0 0 0 0 0 0 1
          0 0 0 0 0 0 0];

```

```

R(1,2) =[0 0 1
          0 0 1
          0 0 0];

```

```

R(1,3) =[0 0 0 0
          1 0 0 0
          0 0 0 1
          0 0 0 0];

```

```

R(1,4) =[0 0 0
          1 0 0
          0 0 0];

```

```

R(1,5) =[0 1 0 0 0 0 1
          0 0 0 0 1 1 1
          0 0 0 0 1 0 0
          0 0 0 0 0 0 0
          0 0 0 0 0 0 0
          0 0 0 0 0 0 0
          0 0 0 0 0 0 0];

```

```

R(1,6) =[0 0 1
          0 0 0
          0 0 0];

```

```

R(1,7) = [0  1  1  0  1  0
          0  0  0  0  0  0
          0  0  0  1  0  0
          0  0  0  0  0  0
          0  0  0  0  0  1
          0  0  0  0  0  0
          ];

```

```

R(1,8) = [0  1  0  1
          0  0  0  0
          0  0  0  0
          0  0  0  0];

```

```

R(1,9) = [0  0  0
          1  0  0
          0  0  0];

```

```

R(1,10) = [0  1  0
           0  0  0
           0  1  0];

```

```

W(1,1) = [0  0  0  0  0  0  0
          0  0  0  0  1  0  0
          0  0  0  0  0  1  0
          0  0  0  0  0  0  0
          0  1  0  0  0  0  0
          0  0  1  0  0  0  0
          0  0  0  0  0  0  0];

```

```

W(1,2) = [0  1  0
          1  0  0
          0  0  0];

```

```

W(1,3) = [0  0  0  0
          0  0  0  0
          0  0  0  0
          0  0  0  0];

```

```

W(1,4) = [0  1  0
          1  0  0
          0  0  0];

```



```

W(1,5) =[0    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    1    0    0    0
          0    0    1    0    0    1    0
          0    0    0    0    0    0    0
          0    0    0    1    0    0    0
          0    0    0    0    0    0    0];

```

```

W(1,6) =[0    0    1
          0    0    0
          1    0    0];

```

```

W(1,7) =[0    0    0    0    0    0
          0    0    0    0    0    1
          0    0    0    1    0    0
          0    0    1    0    0    0
          0    0    0    0    0    0
          0    1    0    0    0    0 ];

```

```

W(1,8) =[0    0    1    0
          0    0    0    0
          1    0    0    0
          0    0    0    0];

```

```

W(1,9) =[0    1    1
          1    0    0
          1    0    0];

```

```

W(1,10) =[0    0    0
           0    0    1
           0    1    0];

```

**Test case 4:**

```

P(1,1)=[ 5    20    8    0    0    0    0
          9    35    6    22    21    0    0
          7    8    17    13    0    0    0
          11   6    8    9    0    0    0
          50   5    12   5    24   0    0
          8    17    10   6    0    0    0
          7    6    19    0    0    0    0
          13   24   5    16   9    8    0
          18   33   7    17   6    12   8
          6    13   17   0    0    0    0];

```

```

P(1,2)=[ 17    12    5    0    0    0    0
         7    17    5   21   30    0    0
         8    18    6    6    0    0    0
        17    14   12    7    0    0    0
        11    9    6   19   11    0    0
        33   11    8    7    0    0    0
         7    17   18    0    0    0    0
        14    6   17    8    7    5    0
         7    6   17   11    5   10   15
        11   21    9    0    0    0    0];

```

```

P(1,3)=[ 7    14   17    0    0    0    0
        16   9    5   11   10    0    0
        13   33    8    7    0    0    0
        13   12    8   44    0    0    0
        19   14    9    7    9    0    0
         7   12   15    9    0    0    0
         8    9   13    0    0    0    0
        12    6   49   11    5    7    0
         5   18   16   13   11    6   10
        10    7    5    0    0    0    0];

```

```

P(1,4)=[ 7    14   17    0    0    0    0
        16   24    5   23   17    0    0
        34   23    8    7    0    0    0
        13   12    8    7    0    0    0
        37   14    9    7    9    0    0
         7    6   15    9    0    0    0
         8   19   13    0    0    0    0
        12    6   50   11    5    7    0
         5    9   16   13   11    6   10
        10   11    5    0    0    0    0];

```

```

P(1,5)=[ 17   20    8    0    0    0    0
         9   24    6   21   22    0    0
         7   34    9    8    0    0    0
        49   14    5   29    0    0    0
        15    5   44    5   12    0    0
        33   44   10   50    0    0    0
        13    6   27    0    0    0    0
        13   14   23    6   19    8    0
         7    8   18   17   17   34    8
         6    5   11    0    0    0    0];

```

```
S(1,1) =[0 1
          1 0
          0 1
          0 0
          1 1
          1 1
          1 1
          0 1
          1 1
          1 0];
```

```
S(1,2) =[0 1
          1 0
          0 1
          1 1
          1 1
          1 1
          1 0
          0 1
          0 1
          1 1];
```

```
S(1,3) =[0 1
          1 1
          1 0
          1 1
          0 1
          1 0
          0 1
          1 1
          1 1
          1 0];
```

```
S(1,4) =[1 1
          1 1
          0 1
          1 0
          0 1
          0 0
          1 1
          1 0
          1 1
          1 1];
```

```
S(1,5) =[0 1
          1 1
          1 1
          1 0
          1 1
          1 0
          1 1
          0 1
          0 1
          1 0];
```

```
T(1,1) =[1 1
          1 0
          1 1
          1 1
          1 1
          0 1
          0 1
          1 1
          1 1
          1 1];
```

```
T(1,2) =[1 1
          1 1
          0 1
          1 1
          1 1
          1 0
          0 1
          1 1
          1 1
          1 1];
```

```
T(1,3) =[1 1
          1 0
          1 1
          0 1
          1 1
          1 1
          0 1
          0 1
          1 1
          1 1];
```

```
T(1,4) =[1 1
          1 0
          0 1
          1 1
          1 1
          1 1
          0 1
          0 1
          1 1
          1 1];
```

```
T(1,5) =[1 1
          1 1
          0 1
          0 1
          1 1
          1 0
          1 1
          1 1
          1 1
          1 1];
```

```

N= [ 1 1 1 0 0 0 0
      1 1 1 1 1 0 0
      1 1 1 1 0 0 0
      1 1 1 1 0 0 0
      1 1 1 1 1 0 0
      1 1 1 1 0 0 0
      1 1 1 0 0 0 0
      1 1 1 1 1 1 0
      1 1 1 1 1 1 1
      1 1 1 0 0 0 0 ];

```

```

R(1,1) =[0 0 0
          1 0 0
          0 0 0];

```

```

R(1,2) =[0 0 0 0 0
          1 0 0 0 1
          0 0 0 0 0
          0 0 0 0 0
          0 0 0 0 0];

```

```

R(1,3) =[0 0 0 1
          0 0 0 1
          0 0 0 1
          0 0 0 0];

```

```

R(1,4) =[0 1 1 0
          0 0 0 1
          0 0 0 0
          0 0 0 0];

```

```

R(1,5) =[0 0 0 1 1
          1 0 0 0 1
          0 0 0 0 0
          0 0 0 0 0
          0 0 0 0 0];

```

```

R(1,6) =[0 1 0 0
          0 0 0 0
          0 0 0 1
          0 0 0 0];

```

```

R(1,7) =[0 0 0
          0 0 0
          1 0 0];

```

```

R(1,8) = [0    0    0    0    1    1
          0    0    0    0    1    1
          0    0    0    1    0    1
          0    0    0    0    0    0
          0    0    0    0    0    1
          0    0    0    0    0    0 ] ;

```

```

R(1,9) = [0    1    0    0    0    0    1
          0    0    1    0    0    0    1
          0    0    0    0    0    1    1
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0] ;

```

```

R(1,10) = [0    0    1
            0    0    0
            0    0    0] ;

```

```

W(1,1) = [0    0    0
           0    0    0
           0    0    0] ;

```

```

W(1,2) = [0    0    0    0    0
           0    0    0    0    0
           0    0    0    1    0
           0    0    0    0    0
           0    0    0    0    0] ;

```

```

W(1,3) = [0    1    0    0
           1    0    0    0
           0    0    0    0
           0    0    0    0] ;

```

```

W(1,4) = [0    0    0    0
           0    0    0    0
           0    0    0    0
           0    0    0    0] ;

```

```

W(1,5) = [0    0    0    0    0
           0    0    0    0    1
           0    0    0    0    0
           0    0    0    0    0
           0    1    0    0    0] ;

```

```

W(1,6) =[0    0    0    1
          0    0    0    0
          0    0    0    0
          1    0    0    0];

```

```

W(1,7) =[0    0    0
          0    0    0
          0    0    0];

```

```

W(1,8) =[0    0    0    0    0    0
          0    0    0    0    0    0
          0    0    0    0    0    0
          0    0    0    0    1    0
          0    0    0    1    0    0
          0    0    0    0    0    0
          ];

```

```

W(1,9) =[0    0    0    1    0    0    0
          0    0    0    0    1    0    0
          0    0    0    0    0    0    0
          1    0    0    0    0    0    0
          0    1    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0];

```

```

W(1,10) =[0    1    0
           1    0    0
           0    0    0];

```

### Test case 5:

```

P(1,1)=[ 29    12    8    39    0    0    0
          9     17    6    42    24    12    9
          14    8     19   15    45    20   17
          11   28    8     0     0     0     0
          5     5    12    5    12    0     0
          8     8    10   43    0     0     0
          7    16   19    0     0     0     0
          13   34    5     6     9     8     0
          7     8     7    17    6    12    8
          11    5    17    0     0     0     0
          22   21   12    8     0     0     0
          21    5    33    0     0     0     0
          10   47   33   28   31    7     8
          12   50   19    0     0     0     0
          20   11   13   18   14    0     0];

```

```

P(1,2)=[ 8    12    5    20    0    0    0
         7    17    27    14    10    17    5
         8    18    6     6    50    13    22
        17    14    12    0     0     0     0
        16    50    13    9     11    0     0
         6    11    18    17    0     0     0
        17    8     35    0     0     0     0
        14    6     17    8     7     5     0
         7    24    17    11    5    10    44
        11    7     9     0     0     0     0
        43    23    7     33    0     0     0
        21    17    18    0     0     0     0
         6    10    5     12    50    27    13
        23    31    20    0     0     0     0
        21    22    6     19    18    0     0];

```

```

P(1,3)=[ 17    14    17    16    0    0    0
         16    9     5    20    7    37    23
        13    8    32    7    21    24    43
        13    12    8     0     0     0     0
        47    14    9     7     9     0     0
         7     6    15    9     0     0     0
        22    50    13    0     0     0     0
        27    19    27    11    44    37    0
         5    20    16    13    36    6    10
        10    7     5     0     0     0     0
        10    5     7     8     0     0     0
        21    17    43    0     0     0     0
        19    11    12    33    41    6     6
        21    10    5     0     0     0     0
        42    20    8     21    18    0     0];

```

```

S(1,1) =[1    1
         0    1
         1    1
         1    0
         1    1
         0    0
         1    1
         0    1
         1    1
         1    0
         1    1
         0    1
         1    1
         0    1
         1    1];

```



```

S(1,2) =[0    0
          1    1
          0    1
          1    1
          1    0
          0    1
          1    1
          0    0
          0    1
          1    1
          1    1
          0    1
          1    1
          0    1
          0    1];

```

```

S(1,3) =[0    1
          1    1
          0    1
          1    0
          1    1
          0    0
          0    1
          0    1
          1    1
          1    0
          1    0
          1    0
          1    0
          1    1
          1    1];

```

```

T(1,1) =[1    1
          1    0
          0    1
          0    1
          1    1
          1    1
          0    1
          0    1
          1    1
          1    1
          1    1
          1    1
          0    1
          0    1
          1    0];

```

```

T(1,2) =[1    1
          1    1
          0    1
          1    1
          1    1
          1    0
          0    1
          0    1
          1    1
          1    1
          1    0
          1    0
          0    1
          0    1
          0    1];

```

```

T(1,3) =[1    1
          1    0
          0    1
          0    1
          1    1
          1    1
          0    1
          0    1
          1    1
          1    1
          1    1
          1    1
          1    1
          1    1
          0    1
          0    1];

```

```

N= [ 1    1    1    1    0    0    0
     1    1    1    1    0    0    0
     1    1    1    1    1    1    1
     1    1    1    0    0    0    0
     1    1    1    1    1    0    0
     1    1    1    1    0    0    0
     1    1    1    0    0    0    0
     1    1    1    1    1    1    0
     1    1    1    1    1    1    1
     1    1    1    0    0    0    0
     1    1    1    1    0    0    0
     1    1    1    0    0    0    0
     1    1    1    1    1    1    1
     1    1    1    0    0    0    0
     1    1    1    1    1    0    0];

```

$$R(1,1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,3) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,4) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,5) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,6) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,7) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$R(1,8) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,9) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,10) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,11) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,12) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,13) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,14) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,15) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,1) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

```

W(1,3) = [0    0    0    0    0    0    0
          0    0    0    1    0    1    0
          0    0    0    0    1    0    1
          0    1    0    0    0    0    0
          0    0    1    0    0    0    0
          0    1    0    0    0    0    0
          0    0    1    0    0    0    0];

```

```

W(1,4) = [0    0    1
          0    0    0
          1    0    0 ];

```

```

W(1,5) = [0    0    0    0    0
          0    0    0    0    0
          0    0    0    0    1
          0    0    0    0    0
          0    0    1    0    0];

```

```

W(1,6) = [0    0    0    0
          0    0    0    1
          0    0    0    0
          0    1    0    0];

```

```

W(1,7) = [0    1    0
          1    0    0
          0    0    0];

```

```

W(1,8) = [0    0    0    0    0    0
          0    0    1    0    0    0
          0    1    0    0    0    0
          0    0    0    0    1    0
          0    0    0    1    0    0
          0    0    0    0    0    0
          ];

```

```

W(1,9) = [0    0    0    0    0    0    0
          0    0    1    0    0    1    0
          0    1    0    0    1    0    0
          0    0    0    0    0    0    0
          0    0    1    0    0    0    0
          0    1    0    0    0    0    0
          0    0    0    0    0    0    0];

```

```

W(1,10) = [0    1    0
           1    0    0
           0    0    0];

```

$$W(1,11) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$W(1,12) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,13) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,14) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,15) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix};$$

**Test case 6:**

$$P(1,1) = \begin{bmatrix} 23 & 12 & 17 & 44 & 25 & 31 & 12 \\ 34 & 5 & 9 & 0 & 0 & 0 & 0 \\ 6 & 6 & 27 & 5 & 0 & 0 & 0 \\ 21 & 30 & 12 & 43 & 0 & 0 & 0 \\ 6 & 22 & 16 & 40 & 42 & 0 & 0 \\ 39 & 38 & 5 & 18 & 0 & 0 & 0 \\ 20 & 33 & 17 & 23 & 12 & 20 & 15 \\ 24 & 22 & 13 & 0 & 0 & 0 & 0 \\ 13 & 18 & 5 & 48 & 22 & 30 & 9 \\ 41 & 7 & 23 & 15 & 49 & 0 & 0 \\ 19 & 37 & 8 & 12 & 10 & 40 & 0 \\ 26 & 40 & 50 & 24 & 32 & 8 & 0 \\ 44 & 38 & 32 & 0 & 0 & 0 & 0 \\ 16 & 31 & 27 & 0 & 0 & 0 & 0 \\ 24 & 48 & 27 & 0 & 0 & 0 & 0 \end{bmatrix};$$

```

P(1,2)=[ 22   5   21   34   16   31   12
         14  47   9   0   0   0   0
          6  27  27   5   0   0   0
         41  30  12  43   0   0   0
          6   5  16  40  42   0   0
         39  20   5  18   0   0   0
         28  44  40  12  17  29  33
         24  22  12   0   0   0   0
          7  18  19  23  49  30   9
         48   7  10  20  33   0   0
         11  37  11  12  10   8   0
         26  46  18   6  10  21   0
         21  38  32   0   0   0   0
         16  31  27   0   0   0   0
         22  25  13   0   0   0  0];

```

```

P(1,3)=[ 24   14   20   28   5   8   11
         11   5   5   0   0   0   0
         13   8  23   7   0   0   0
         13  12   8  19   0   0   0
         31  18  23   7   9   0   0
          7  46  15   9   0   0   0
         25  16  17  22  14  46   8
         12   6   7   0   0   0   0
         18   9  16   6  11   6  10
          5  15  24  22   8   0   0
         27  12   7   8  31  29   0
         21  17  16   5  22  13   0
         19  39  12   0   0   0   0
         21  44  37   0   0   0   0
         24  33   8   0   0   0  0];

```

```

P(1,4)=[ 5   20   8   9   13  11   5
        38  23  17   0   0   0   0
          7  12   9   5   0   0   0
         11  45   8   9   0   0   0
          5   5  14  15  12   0   0
          8  34  33   6   0   0   0
          7   6  19  21  14  49   5
         13  30   5   0   0   0   0
         21   8   7  19   6  12   8
          6   6  17  43  10   0   0
         22  16  31   8  24  30   0
         21   5  27  41  28  15   0
         10  47  33   0   0   0   0
         12   5  41   0   0   0   0
         18  11   7   0   0   0  0];

```

```

P(1,5)=[ 6    38    10    35    13    46    15
         48    50    25    0     0     0     0
         11    43    45    13    0     0     0
         48    17    15    42    0     0     0
         15    34    27     7    33    0     0
           8    29    20    31    0     0     0
         49     5    32    33    12    20     9
         39    38    15     0     0     0     0
         46    44    29    17    39    33    21
         16    15    31    13    16     0     0
         27    34    43    23     8    19     0
         19    45    12    24    44    17     0
         26     9    19     0     0     0     0
         26    27    31     0     0     0     0
         32    15    10     0     0     0     0];

```

```

S(1,1) =[1    1
         1    1
         0    1
         1    0
         0    0
         1    0
         0    1
         0    0
         0    1
         1    0
         0    1
         0    1
         1    0
         0    1
         1    1];

```

```

S(1,2) =[0    0
         1    1
         0    0
         1    0
         1    0
         0    1
         1    1
         1    0
         0    1
         1    1
         0    1
         1    0
         1    1
         0    1
         1    1];

```



```
S(1,3) =[1 1
          0 1
          0 1
          1 1
          1 1
          0 0
          1 1
          1 1
          1 1
          1 0
          0 1
          1 1
          1 0
          0 1
          1 1];
```

```
S(1,4) =[1 0
          0 1
          1 1
          0 1
          1 0
          1 1
          1 0
          1 1
          0 1
          1 1
          1 0
          0 1
          1 1
          1 0
          0 1];
```

```
S(1,5) =[0 1
          0 1
          0 1
          1 1
          0 1
          0 1
          1 0
          0 0
          1 1
          1 0
          1 0
          0 1
          1 1
          0 1
          0 1];
```

```
T(1,1) =[0  0
          1  1
          1  1
          0  1
          1  1
          0  1
          1  1
          0  1
          1  0
          0  1
          1  0
          0  0
          1  1
          0  1
          1  0];
```

```
T(1,2) =[1  0
          1  1
          1  1
          1  1
          1  1
          1  0
          1  1
          0  1
          1  1
          1  0
          1  1
          1  0
          1  1
          1  1
          0  1];
```

```
T(1,3) =[0  1
          1  0
          0  1
          0  1
          1  1
          1  1
          0  1
          1  1
          1  1
          1  1
          0  1
          1  1
          1  1
          0  1
          1  1
          0  1
          0  1];
```

```

T(1,4) = [1    0
          0    0
          0    1
          1    0
          1    1
          1    0
          0    1
          0    1
          1    1
          0    1
          1    0
          1    0
          1    0
          1    1
          0    1];

```

```

T(1,5) = [0    1
          0    1
          0    1
          0    1
          1    1
          1    1
          1    1
          0    1
          1    1
          1    1
          1    0
          1    0
          1    1
          1    1
          0    1];

```

```

N= [ 1    1    1    1    1    1    1
     1    1    1    0    0    0    0
     1    1    1    1    0    0    0
     1    1    1    1    0    0    0
     1    1    1    1    1    0    0
     1    1    1    1    1    1    1
     1    1    1    0    0    0    0
     1    1    1    1    1    1    1
     1    1    1    1    1    0    0
     1    1    1    1    1    1    0
     1    1    1    0    0    0    0
     1    1    1    0    0    0    0
     1    1    1    0    0    0    0];

```

$$R(1,1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,3) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,4) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,5) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,6) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,7) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,8) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

```

R(1,9) =[0    0    0    0    1    1    1
          0    0    1    1    0    0    1
          0    0    0    1    0    0    0
          0    0    0    0    0    1    1
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0];

```

```

R(1,10) =[0    0    0    0    1
           0    0    1    1    0
           0    0    0    0    0
           0    0    0    0    0
           0    0    0    0    0];

```

```

R(1,11) =[0    1    0    0    1    0
           0    0    1    1    0    0
           0    0    0    0    0    0
           0    0    0    0    0    0
           0    0    0    0    0    1
           0    0    0    0    0    0];

```

```

R(1,12) =[0    0    1    0    0    1
           0    0    0    1    1    0
           0    0    0    0    0    0
           0    0    0    0    0    0
           0    0    0    0    0    0
           0    0    0    0    0    0];

```

```

R(1,13) =[0    1    0
           0    0    0
           0    0    0];

```

```

R(1,14) =[0    0    0
           0    0    0
           0    0    0];

```

```

R(1,15) =[0    0    1
           0    0    0
           0    0    0];

```

$$W(1,1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,3) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$W(1,4) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,5) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,6) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,7) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,8) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

```

W(1,9) = [0    0    1    0    0    0    0
          0    0    0    1    0    0    1
          1    0    0    0    0    0    0
          0    1    0    0    0    0    0
          0    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    1    0    0    0    0    0];

```

```

W(1,10) = [0    0    1    0    0
           0    0    0    1    0
           1    0    0    0    1
           0    1    0    0    0
           0    0    1    0    0];

```

```

W(1,11) = [0    0    0    0    1    0
           0    0    0    1    0    0
           0    0    0    0    0    0
           0    1    0    0    0    0
           1    0    0    0    0    0
           0    0    0    0    0    0];

```

```

W(1,12) = [0    0    0    0    0    0
           0    0    0    1    0    0
           0    0    0    0    0    1
           0    1    0    0    0    0
           0    0    0    0    0    0
           0    0    1    0    0    0];

```

```

W(1,13) = [0    1    0
           1    0    0
           0    0    0];

```

```

W(1,14) = [0    0    0
           0    0    1
           0    1    0];

```

```

W(1,15) = [0    0    1
           0    0    0
           1    0    0];

```

Test case 7:

```
P(1,1)=[ 14  7  36  0  0  0  0
         13  50  6  12  41  41  6
          5  16  41  9  6  0  0
         14  6  32  7  0  0  0
         44  39  32  0  0  0  0
         30  39  6  32  50  0  0
         21  20  41  8  40  12  21
          8  6  25  10  12  43  19
         12  37  26  5  0  0  0
          5  36  13  25  44  0  0
          9  20  44  0  0  0  0
         14  6  5  32  24  50  8
          6  6  7  13  3  7  0
          5  27  47  0  0  0  0
          7  6  6  33  38  0  0
         14  14  8  23  0  0  0
          5  13  36  8  0  0  0
         13  28  10  0  0  0  0
          5  8  24  7  6  34  6
         24  14  7  0  0  0  0];
```

```
P(1,2)=[ 10  8  8  0  0  0  0
         38  25  17  24  34  13  28
         15  17  21  28  25  0  0
         14  7  6  14  0  0  0
         40  30  21  0  0  0  0
         13  45  31  20  8  0  0
          8  22  13  16  25  50  33
         28  15  15  25  36  14  15
         12  8  14  18  0  0  0
         14  28  28  33  12  0  0
         45  20  6  0  0  0  0
         23  7  24  35  20  11  12
          5  20  13  17  5  7  0
         17  26  15  0  0  0  0
          6  46  23  12  15  0  0
         49  14  16  19  0  0  0
          8  13  13  6  0  0  0
         23  24  18  0  0  0  0
          5  37  41  8  14  14  10
         40  7  29  0  0  0  0];
```



```

P(1,3)=[ 5    23    22    0    0    0    0
         17    8    19    13    12    10    6
          5    11    15    9    22    0    0
          10   14    32    7    0    0    0
           7    5    12    0    0    0    0
          41    9    16    13    10    0    0
          11   14    5    8    20    41    33
           8    30    7    13    12    6    6
          29    7    6    33    0    0    0
           5    6    13    5    34    0    0
          19   17   40    0    0    0    0
          14    8    8    21    14    41    33
          17    6    30    33    18    13    0
          14   20   27    0    0    0    0
           5   13   32   18   41    0    0
          20   15   16   10    0    0    0
           5    8   19    7    0    0    0
          20   36   45    0    0    0    0
          10   49   25   40   29   18   36
          39   50   24    0    0    0    0 ];

```

```

S(1,1) =[0    1
          1    1
           0    1
           0    1
           1    1
           0    0
           1    1
           0    1
           1    1
           1    0
           1    1
           1    1
           1    0
           1    0
           0    1
           1    1
           0    0
           1    1
           0    1
           1    1];

```

```
S(1,2) =[0  0
          1  1
          0  1
          1  1
          1  0
          0  1
          1  1
          0  0
          0  1
          1  1
          1  1
          1  1
          1  1
          0  1
          1  1
          1  1
          1  0
          0  1
          0  0
          1  1
          1  1];
```

```
S(1,3) =[1  1
          0  1
          1  1
          1  0
          1  1
          0  1
          1  0
          0  1
          1  1
          1  0
          0  1
          1  1
          1  0
          0  0
          1  1
          0  1
          0  0
          1  1
          1  1
          0  1];
```

```
T(1,1) =[1 1
          1 0
          0 1
          0 1
          1 1
          1 1
          0 1
          0 1
          1 0
          1 1
          0 1
          1 1
          1 0
          1 1
          1 0
          1 0
          1 1
          1 1];
```

```
T(1,2) =[1 1
          1 1
          0 1
          1 1
          1 1
          1 0
          0 1
          0 1
          1 1
          0 1
          1 0
          1 1
          0 1
          1 1
          0 1
          1 1
          1 1
          1 1
          0 1
          1 0
          1 1];
```

```

T(1,3) = [1    0
          1    0
          1    1
          1    1
          1    1
          1    0
          0    1
          1    0
          0    1
          1    0
          1    0
          0    1
          0    1
          1    1
          0    1
          1    0
          0    1
          1    1
          1    0
          0    1];

```

```

N= [
      1    1    1    0    0    0    0
      1    1    1    1    1    1    1
      1    1    1    1    1    0    0
      1    1    1    1    0    0    0
      1    1    1    0    0    0    0
      1    1    1    1    1    0    0
      1    1    1    1    1    1    1
      1    1    1    1    1    1    1
      1    1    1    1    0    0    0
      1    1    1    1    1    0    0
      1    1    1    0    0    0    0
      1    1    1    1    1    1    1
      1    1    1    1    1    1    0
      1    1    1    0    0    0    0
      1    1    1    1    1    0    0
      1    1    1    1    0    0    0
      1    1    1    0    0    0    0
      1    1    1    1    1    1    1
      1    1    1    0    0    0    0];

```

$$R(1,1) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,2) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,3) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,5) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,6) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,7) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,8) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,9) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,10) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,11) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,12) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,13) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,14) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,15) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,16) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,17) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,18) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,19) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,20) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$W(1,1) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,3) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,4) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,5) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,6) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,7) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,9) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,10) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix};$$



$$W(1,11) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix};$$

$$W(1,12) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,13) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,14) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,15) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix};$$

$$W(1,16) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

$$W(1,17) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,18) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

```

W(1,19)=[0    0    0    0    1    0    0
          0    0    0    1    0    0    1
          0    0    0    0    0    0    0
          0    1    0    0    0    0    0
          1    0    0    0    0    0    0
          0    0    0    0    0    0    0
          0    1    0    0    0    0    0];

```

```

W(1,20)=[0    0    1
          0    0    0
          1    0    0];

```

Test case 8:

```

P(1,1)=[ 44    19    30    11    10    0    0
          33    22    18    0     0    0    0
          21    16    14    0     0    0    0
          28    23    13    29    33    7    11
          23     9    36    21    21    10    6
           8    11    13    22    50    0    0
          16    48    21    45    0     0    0
          10    13    33    30    10    19    36
          13    31    40    33    0     0    0
          16     7     8     0     0    0    0
          24    15    47    0     0    0    0
           7    21     7     0     0    0    0
          46    42    27    26    32    19    0
          48     5    25    13    21    45    34
          27     6    25    43    15    0     0
          27    12    19    13     5    50    0
          20    34    28     0     0    0    0
          46    13    28     0     0    0    0
          21    34    42    15    24    0     0
          10    25    41     7    10     0    0];

```

```

P(1,2)=[ 23   36   16    9   31    0    0
         13   35   33    0    0    0    0
          7   50    6    0    0    0    0
         23    6   24    5   29   48   30
          7   24   15   42    9    8   12
         31   27   44   13   34    0    0
         39   31   10   27    0    0    0
         19    8   18   17   16   38   35
         27   35   33   16    0    0    0
         15    7   40    0    0    0    0
         13   14   11    0    0    0    0
         27   37   47    0    0    0    0
         29   27   22   12   23   24    0
         15   33   12   20   12    8   10
          9   14   41   14   13    0    0
          7   34   10   32   33   21    0
         42   22   12    0    0    0    0
         40    9   21    0    0    0    0
         14   22   43   41   47    0    0
         46   17   31   12   41    0    0];

```

```

P(1,3)=[ 21    31    22    8    8    0    0
         32    12    6    0    0    0    0
          6    43   15    0    0    0    0
         16    18   32    7   21   11   21
         11    45    9   19   40   24   32
         39    13   16   13   10    0    0
          8    16    5    8    0    0    0
          9    17   11   21   12    6    6
         10    21    6    5    0    0    0
         18    43   21    0    0    0    0
         44    46   40    0    0    0    0
         35    22   32    0    0    0    0
         43    23    7    9   21    7    0
         21    18   41   23   18   37   33
         12    50   15   33   18    0    0
         10    6   27   21   48   13    0
         18    12   32    0    0    0    0
         35    34   16    0    0    0    0
         28    12   19   37    6    0    0
          6    36   21   29   33    0    0];

```

```

P(1, 4)=[ 5    23    22    12    9    0    0
          17    8     6     0     0    0    0
           5    12    15     0     0    0    0
          10    14    32     7    39    13    40
           7     5     9    19    26    19    45
          41    11    16    13    10     0     0
          11    14     5     8     0     0     0
           8     6    11    21    12     6     6
          29     7     6     5     0     0     0
          12     6    21     0     0     0     0
          12    17    23     0     0     0     0
          41    16    19     0     0     0     0
          14    21    41    34    12    25     0
          17    22     9    33    19    29    14
          14    20     6    21    44     0     0
           5    32    32     7    39    28     0
          20    15    16     0     0     0     0
          11    13    19     0     0     0     0
          20    36    45    33    45     0     0
          10    17    21    10    21     0    0];

```

```

P(1, 5)=[ 19    21    22    42     9     0     0
           8    50    43     0     0     0     0
          33    39     8     0     0     0     0
           5    15    11     7    50    12     5
          22     8    20    15    33     8    38
          31    14    31    13    10     0     0
           6    21     7     6     0     0     0
          17    50    18    50    12     6     6
          11    36    20    16     0     0     0
          10    32     7     0     0     0     0
          44    14    43     0     0     0     0
          12    20    19     0     0     0     0
          30     9    22    34    12    25     0
          23     7    40    31    12    40    16
          34    32    11    21    44     0     0
          19    45     5     7    33    30     0
          40    29    13     0     0     0     0
          18    10    22     0     0     0     0
          49    31    21    33    12     0     0
          31    48     7     5    21     0    0];

```

```
S(1,1) =[1 1
          1 0
          0 1
          1 1
          0 0
          0 1
          1 0
          0 1
          1 1
          0 0
          1 0
          0 0
          0 1
          1 0
          1 1
          1 0
          1 1
          0 0
          0 0
          0 0];
```

```
S(1,2) =[ 0 1
           0 1
           1 1
           0 0
           1 1
           0 0
           0 1
           0 1
           1 0
           0 1
           0 1
           1 1
           0 0
           0 1
           0 1
           1 0
           1 1
           0 0
           1 1
           0 1];
```

```
S(1,3) = [1 1
          1 0
          1 1
          1 0
          1 0
          0 0
          1 1
          1 0
          1 1
          1 0
          0 1
          1 0
          0 1
          1 0
          0 1
          1 0
          0 1
          1 0
          0 1
          1 0
          1 1];
```

```
S(1,4) = [1 0
          1 1
          0 1
          1 0
          1 1
          1 1
          1 0
          0 1
          0 1
          1 1
          1 1
          0 1
          1 0
          0 0
          1 1
          1 1
          0 0
          1 1
          0 1
          1 1];
```

```
S(1,5) = [ 1 0
           1 1
           1 0
           1 0
           0 0
           1 1
           0 0
           1 0
           1 0
           1 1
           0 1
           0 0
           1 0
           1 1
           1 0
           1 1
           0 1
           1 0
           0 1
           1 0];
```

```
T(1,1) = [1 1
           1 0
           1 1
           1 1
           1 1
           0 1
           0 1
           1 1
           0 1
           1 1
           1 1
           0 1
           1 0
           1 1
           0 1
           0 1
           1 0
           0 1
           1 1
           1 1];
```

```
T(1,2) =[1 1
          1 0
          0 1
          1 1
          0 1
          1 0
          0 1
          0 1
          0 1
          1 0
          0 1
          1 1
          0 1
          1 1
          0 1
          1 1
          1 1
          1 0
          1 1];
```

```
T(1,3) =[1 1
          1 0
          0 1
          1 0
          0 1
          1 1
          0 1
          1 1
          1 0
          1 1
          1 1
          0 1
          0 1
          1 1
          1 1
          1 1
          0 1
          1 0
          1 1
          1 1];
```



```

T(1,4) =[1  1
          1  1
          1  1
          1  0
          1  1
          1  1
          1  0
          0  1
          0  1
          1  1
          0  1
          0  1
          1  0
          0  0
          1  1
          1  1
          0  0
          1  1
          0  1
          1  1];

```

```

T(1,5) =[1  0
          1  1
          1  0
          1  0
          1  1
          1  1
          1  0
          0  1
          0  1
          1  1
          0  1
          0  1
          1  0
          1  1
          1  1
          1  1
          0  0
          1  1
          0  1
          1  1];

```

```

N=[
  1  1  1  1  1  0  0
  1  1  1  0  0  0  0
  1  1  1  0  0  0  0
  1  1  1  1  1  1  1
  1  1  1  1  1  1  1
  1  1  1  1  1  0  0
  1  1  1  1  0  0  0
  1  1  1  1  1  1  1
  1  1  1  1  0  0  0
  1  1  1  0  0  0  0
  1  1  1  0  0  0  0
  1  1  1  0  0  0  0
  1  1  1  1  1  1  0
  1  1  1  1  1  1  1
  1  1  1  1  1  0  0
  1  1  1  1  1  1  0
  1  1  1  0  0  0  0
  1  1  1  0  0  0  0
  1  1  1  1  1  0  0
  1  1  1  1  1  0  0];

```

```

R(1,1) =[0  0  0  1  1
          0  0  0  1  0
          1  0  0  0  0
          0  0  0  0  0
          0  0  0  0  0];

```

```

R(1,2) =[ 0  1  1
          0  0  0
          0  0  0 ];

```

```

R(1,3) =[ 0  0  0
          0  0  0
          1  0  0 ];

```

```

R(1,4) =[ 0  1  0  0  0  1  0
          0  0  0  1  0  0  0
          0  0  0  1  0  0  1
          0  0  0  0  0  0  1
          0  0  0  0  0  0  0
          0  0  0  0  0  0  0
          0  0  0  0  0  0  0];

```

```

R(1,5) =[ 0  0  0  0  0  0  1
          0  0  0  0  0  1  0
          0  0  0  1  0  1  0
          0  0  0  0  0  0  0
          0  0  0  0  0  0  0
          0  0  0  0  0  0  0
          0  0  0  0  0  0  0];

```

$$R(1,6) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,7) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,8) = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,9) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,10) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,11) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,12) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$R(1,13) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,14) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,15) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,16) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,17) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,18) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};$$

$$R(1,19) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R(1,20) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,1) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,3) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,4) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,5) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,6) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,7) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,9) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,10) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix};$$

$$W(1,11) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix};$$

$$W(1,12) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$W(1,13) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,14) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$W(1,15) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix};$$

$$\begin{aligned}
 W(1,16) = & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
 \end{aligned}$$

$$\begin{aligned}
 W(1,17) = & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix};
 \end{aligned}$$

$$\begin{aligned}
 W(1,18) = & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};
 \end{aligned}$$

$$\begin{aligned}
 W(1,19) = & \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};
 \end{aligned}$$

$$\begin{aligned}
 W(1,20) = & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix};
 \end{aligned}$$

## VITA AUCTORIS

NAME: Saleh Yavari

PLACE OF BIRTH: Karaj, Iran

YEAR OF BIRTH: 1988

EDUCATION: Karaj Azad University, B.Sc., Industrial  
Engineering, Karaj, Iran  
University of Windsor, M.Sc. Industrial and  
Manufacturing Engineering, Windsor, ON, 2017