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# A NOISE ESTIMATION SCHEME FOR BLIND SPECTRUM SENSING USING EMD

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# **A NOISE ESTIMATION SCHEME FOR BLIND SPECTRUM SENSING USING EMD**

BY

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THESIS

Submitted to the University of New Hampshire  
in Partial Fulfillment of  
the Requirement for the Degree of

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in  
Electrical Engineering

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On April 10, 2017

Original approval signatures are on file with the University of New Hampshire Graduate School.

## DEDICATION

I dedicate my dissertation work to my family. A special feeling of gratitude to my loving parents, **my father Prof. Medhat Nasr** and **my mother Dr. Abir Ali** whose words of encouragement and push for tenacity ring in my ears. My sister Dr. Shaza and my brother Mostafa. They have always been there to cheer me up and stood by me through the good times and bad. They are my greatest sources of inspiration to go through tough times while keeping my head high.

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## LIST OF ACRONYMS

<b>SPTF</b>	Spectrum Policy Task Force
<b>FCC</b>	Federal Communications Commission
<b>CR</b>	Cognitive Radio
<b>ED</b>	Energy Detector
<b>SU</b>	Secondary Users
<b>PU</b>	Primary Users
<b>SS</b>	Spectrum Sensing
<b>EMD</b>	Empirical Mode Decomposition
<b>IMF</b>	Intrinsic Mode Function
<b>GSM</b>	Global System for Mobile
<b>PSD</b>	Power Spectral Density
<b>PDF</b>	Probability Density Function
<b>ADC</b>	Analog to Digital Converter
<b>ROC</b>	Receiver Operating Characteristics
<b>CFAR</b>	Constant False Alarm Rate
<b>CDR</b>	Constant Detection Rate

<b>CBD</b>	Covariance Based Detection
<b>EBD</b>	Eigenvalue Based Detection
<b>MME</b>	Maximum Minimum Eigenvalue
<b>MAE</b>	Minimum Average Eigenvalue
<b>HHT</b>	Hilbert Huang Transform

# **ABSTRACT**

## **A NOISE ESTIMATION SCHEME FOR BLIND SPECTRUM SENSING USING EMD**

by

Amr Nasr

University of New Hampshire, May 2017

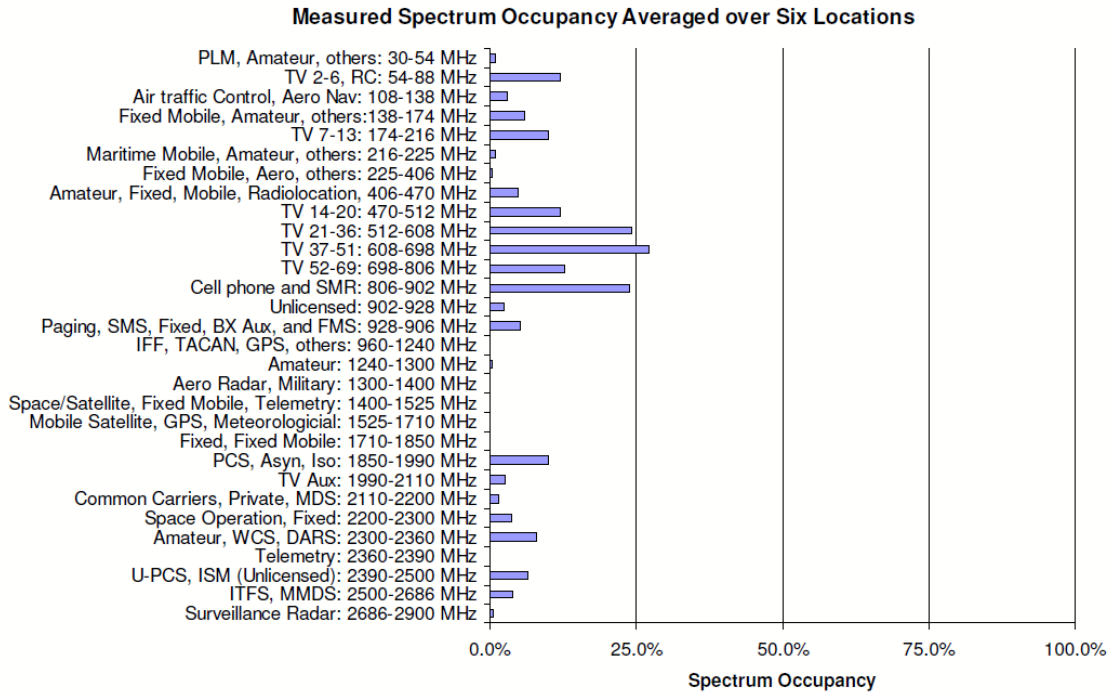
The scarcity of spectral resources in wireless communications, due to a fixed frequency allocation policy, is a strong limitation to the increasing demand for higher data rates. One solution is to use underutilized spectrum. Cognitive Radio (CR) technologies identify transmission opportunities in unused channels and avoid interfering with primary users. The key enabling technology is the Spectrum Sensing (SS). Different SS techniques exist, but techniques that do not require knowledge of the signals (non-coherent) are preferred. Noise estimation plays an essential role in enhancing the performance of non-coherent spectrum sensors such as energy detectors. In this thesis, we present an energy detector based on the behavior of Empirical Mode Decomposition (EMD) towards vacant channels (noise-dominant). The energy trend from the EMD processed signal is used to determine the occupancy of a given band of interest. The performance of the proposed EMD-based detector is evaluated for different noise levels and sample sizes. Further, a comparison is carried out with conventional spectrum sensing techniques to validate the efficacy of the proposed detector and the results revealed that it outperforms the other sensing methods.

# Chapter 1

## Introduction

### 1.1 Motivation

Although frequency spectrum is considered a limited natural resource due to near total allocation, measurements show that often there are moments when it is underutilized [1]. Additionally, with the increasing emergence of new wireless products and the explosive development of mobile internet applications, the demands on RF spectrum have been constantly increasing. In recent years, it has become evident that there will not be enough spectrum exclusively available for all wireless systems currently under development; the Spectrum Policy Task Force (SPTF) within the Federal Communications Commission (FCC) has reported that localized temporal and geographic spectrum utilization efficiency ranges from 15% to 85% [2]. In another experiment as shown in Figure 1.1, the maximal occupancy of the spectrum from 30 MHz to 3 GHz (in New York city) has been reported to be only 13.1%, with average occupancy (over six locations) of 5.2% [3].



**Figure 1.1** Spectrum occupancy measurement results averaged over six locations [3].

The fact that the spectrum is under-utilized opens the possibility that technologies could be created to use frequency spectrum more efficiently. To address this issue, regulatory entities are now considering more flexible spectrum management policies than traditional fixed frequency allocations [4]. These policies give Secondary Users (SU) opportunities to access the spectrum of incumbents, also called Primary Users (PU), and can then improve the spectrum usage efficiency.

Cognitive Radio (CR) is a technology whose primary purpose is to equip the radio terminal with some form of artificial intelligence. The artificial intelligence will allow the CR to autonomously detect which is the best service the user needs and what radio resources to use based on the context of utilization.

One promising application of cognitive radio is dynamic spectrum access, in which a CR utilizes information about the spectrum usage to access empty spectrum. To achieve this goal without causing harmful interference to the PUs, Spectrum Sensing (SS) plays a key role in detecting the holes (vacant channels) for a given band [5]. There are various kinds of SS techniques i.e. Energy Detection (ED) which is used widely for spectrum sensing purposes due to its non-coherent nature and low computational complexity [6].

Energy Detectors (ED) work by comparing the energy of the channel under test to a predefined threshold to determine if the channel is occupied [7]. However, the ED threshold is a function of noise power which is assumed at a prior time or estimated through measurements in nearby channels. Consequently, misestimating the noise power might result in a severe degradation in the detector performance [8]. In practical scenarios, noise variance can vary over wireless channels and, coupled with thermal noise at the receiver front-end, lead to what is called noise uncertainty [9].

More recently, Empirical Mode Decomposition (EMD) has been proposed as a detection method for wireless applications [10,11, 12]. EMD is an adaptive and blind technique that decomposes time-series signals into a set of modes called Intrinsic Mode Functions (IMFs) [13]. Like ED, EMD techniques behave require no prior information about the signal characteristics for detection and decomposition respectively.

Roy and Doherty used EMD, in general, to enhance the detection of weak signals in the presence of noise [14]. This technique is dependent on a characteristic of EMD that would require calculations that may not make it practical for real-time sensing.



To detect non-stationary and non-linear signals, Bektas et al. proposed a spectrum sensing algorithm using relative entropy [15]. However, this method also requires many calculations to determine the classifier for separating a signal from noise.

Based on previous work, one single detector technique that has the best performance for all scenarios is a bit challenging research topic. In this thesis, we address this problem by proposing, that has better performance than other existing detection techniques without requiring prior information of the transmitted signal.

## **1.2 Thesis Objective**

Vast segments of the frequency spectrum are reserved for primary (licensed) users. These legacy users often underutilize their reserved spectrum thus causing bandwidth waste. The unlicensed (secondary) users can take advantage of this fact and exploit the spectral holes (vacant spectrum segments). Since spectrum occupancy is transient in nature it is imperative that the spectral holes are identified as fast and accurate as possible.

The problem with the most spectrum detection techniques i.e. with the energy detector (ED), is that it requires the knowledge of the noise variance to correctly set the test threshold to meet a selected false-alarm probability. In practice, the noise variance must be estimated by some estimation procedure, which is subject to various errors that are introduced by the detection device and environment, e.g., temperature, humidity, device aging, radio interference, etc. It has been found that the ED is sensitive to the accuracy of the estimated noise variance [16].

Our goal is to enhance the performance of the non-coherent spectrum sensors such as energy detectors. Thus, the major focus of this thesis is to evaluate and implement a method that can result in better estimation of the received signal's noise energy which is essential for energy detection based schemes.

Our proposed method exploits the ability of EMD to decompose a signal in to components, which can be analyzed for channel detection.

### **1.3 Thesis Outline**

The remainder of this thesis is organized as follows: In Chapter 2 the structure, functionalities, and potential applications of cognitive radios are presented. Traditional blind spectrum sensing techniques in the literature are introduced, and the advantages and disadvantages are summarized.

In Chapter 3, our proposed method “energy detector based on the behavior that Empirical Mode Decomposition (EMD)” is presented. But first, we provide background on the EMD operation and a description of the system model. Further, a comparison is carried out with conventional spectrum sensing techniques to validate the efficacy of our proposed method. Finally, Chapter 4 concludes the dissertation and present the future work.

# Chapter 2

## Literature Review

In this chapter, an overview of spectrum sensing, which is one of the fundamental prerequisites for the successful deployment of cognitive radio networks, is presented along with background of the related technologies. Cognitive Radio (CR) networks enable secondary users to monitor the spectrum usage of the primary user and utilize the spectrum when it is not used. A review of blind spectrum sensing in cognitive radio is addressed in this chapter. Emphasis is put on performance in the low signal to noise ratio range. In this Chapter, it is shown how methods relying on traditional sample-based estimation methods, such as the energy detector, suffer at low SNRs. The end of the chapter includes a comparative analysis of the fundamentals of spectrum blind sensing techniques along with their performance results.

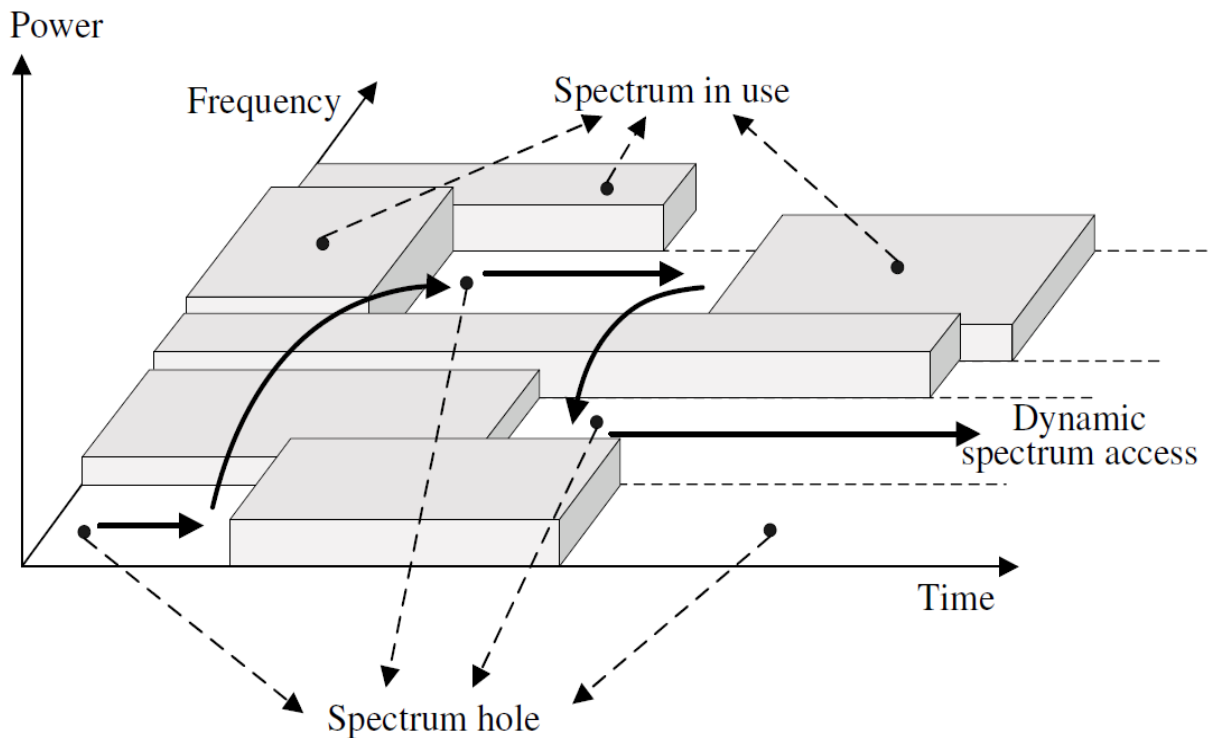
### 2.1 Background

The goal of new wireless technologies is to increase the capacity and speed of the transmission data due to the high demand (see Sec 1.1). Another important feature of these new technologies is that they improve the spectrum efficiency by adapting to changes in the transmission conditions. To fulfill these needs and requirements, the idea of Cognitive Radio (CR) was born.

## 2.1.1 Cognitive Radio

“Cognitive radio is viewed as a novel approach for improving the utilization of a precious natural resource: the radio electromagnetic spectrum.” -S. Haykin [9]

To improve spectrum utilization and provide high bandwidth to mobile users, the next generation communication networks (xG) [2] program was developed to implement spectrum policy intelligent radios, also known as cognitive radios [17],



**Figure 2.1** Illustration of spectrum holes and the concept of dynamic spectrum access

[18].

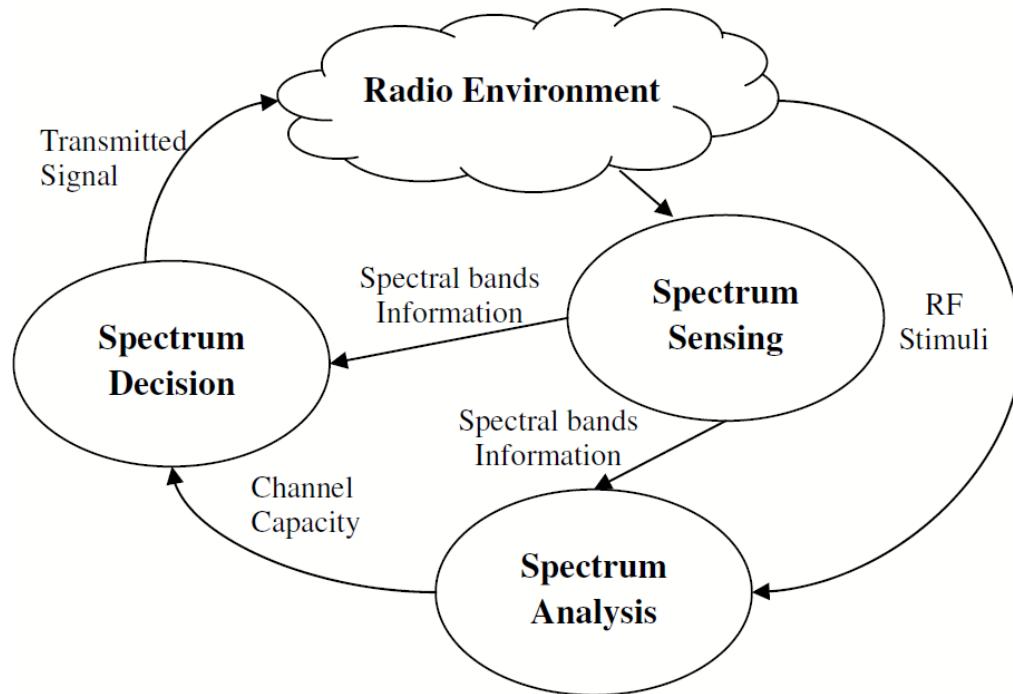
by dynamic spectrum access techniques as shown in Figure 2.1. Furthermore, the IEEE has organized a new working group, known as the Wireless Regional Area Network, (WRAN, IEEE 802.22), for cognitive radio techniques to allow sharing of geographically unused television (TV) spectrum on a non-interfering basis [19, 39].

### **2.1.1.1 Cognitive Radio Functionalities**

The term cognitive radio was first coined by Mitola in [17] and has the following formal definition as [9]: “Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e. outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- highly reliable communications whenever and wherever needed;
- efficient utilization of the radio spectrum. ”-S. Haykin

From the definition, the two main characteristics of cognitive radio can be summarized as cognitive capability and reconfigurability [2]. The former enables the cognitive radio to interact with its environment in a real-time manner and intelligently determine appropriate communication parameters based on Quality of Service (QoS) requirements.



**Figure 2.2** The cognitive capability of cognitive radio enabled by a basic cognitive cycle [2].

These tasks can be implemented by a basic cognitive cycle: spectrum sensing, spectrum analysis, and spectrum decision as shown in Figure 2.2 [2].

- Spectrum sensing: either by cooperating (using more than one sensing technique) or not, the cognitive radio nodes regularly monitor the RF environment. To improve the spectral usage efficiency, cognitive radio nodes should not only find spectrum holes by sensing some particular spectrum, but also monitor the whole spectral band.
- Spectrum analysis: the characteristics of the spectral bands that are sensed through spectrum sensing are estimated. The estimation results, e.g., capacity, and reliability, will be delivered to the spectrum decision step.

- Spectrum decision: according to the spectrum characteristics analyzed above, an appropriate spectral band will be chosen for a particular cognitive radio node. Then the cognitive radio determines new configuration parameters, e.g., data rate, transmission mode, and bandwidth of the transmission.

### **2.1.1.2 Potential Applications**

Since cognitive radio is aware of the RF environment and is capable of adapting its transmission parameters to the RF spectrum environment, cognitive radios and the concepts of cognitive radio can be applied to a variety of wireless communication environments, especially in commercial and military applications. A few of applications are listed below:

- Coexistence of wireless technologies [21]: Cognitive radio techniques were primarily considered for reusing the spectrum that is currently allocated to the TV broadcast service. WRAN users can take advantage of broadband data delivery by the opportunistic usage of the underutilized spectrum. Additionally, the dynamic spectrum access techniques will play an important role in full interoperability and coexistence among diverse technologies for wireless networks.

- Military networks [20, 21]: In military communications, bandwidth is often at a premium. By using cognitive radio concepts, military radios can not only achieve substantial spectral efficiency on a noninterfering basis, but also reduce implementation complexity for defining the spectrum allocation for each user. Furthermore, military radios can obtain benefits from the opportunistic spectrum access function supported by the

cognitive radio [21]. For example, the military radios can adapt their transmission parameters to use Global System for Mobile (GSM) bands, or other commercial bands when their original frequencies are jammed.

- Heterogeneous wireless networks [21, 22]: From a user's point of view, a cognitive radio device can dynamically discover information about access networks, e.g. WiFi and GSM, and makes decisions on which access network is most suitable for its requirements and preferences. Then the cognitive radio device will reconfigure itself to connect to the best access network. When the environmental conditions change, the cognitive radio device can adapt to these changes. The information as seen by the cognitive radio user is as transparent as possible to changes in the communication environment.

With Cognitive Radio being used in several applications, the area of spectrum sensing has become more important. Spectrum sensing is responsible for detecting holes in the spectrum while simultaneously taking advantage of the opportunity without causing interference to PUs.

### **2.1.2 State of the Art Spectrum Sensing Techniques**

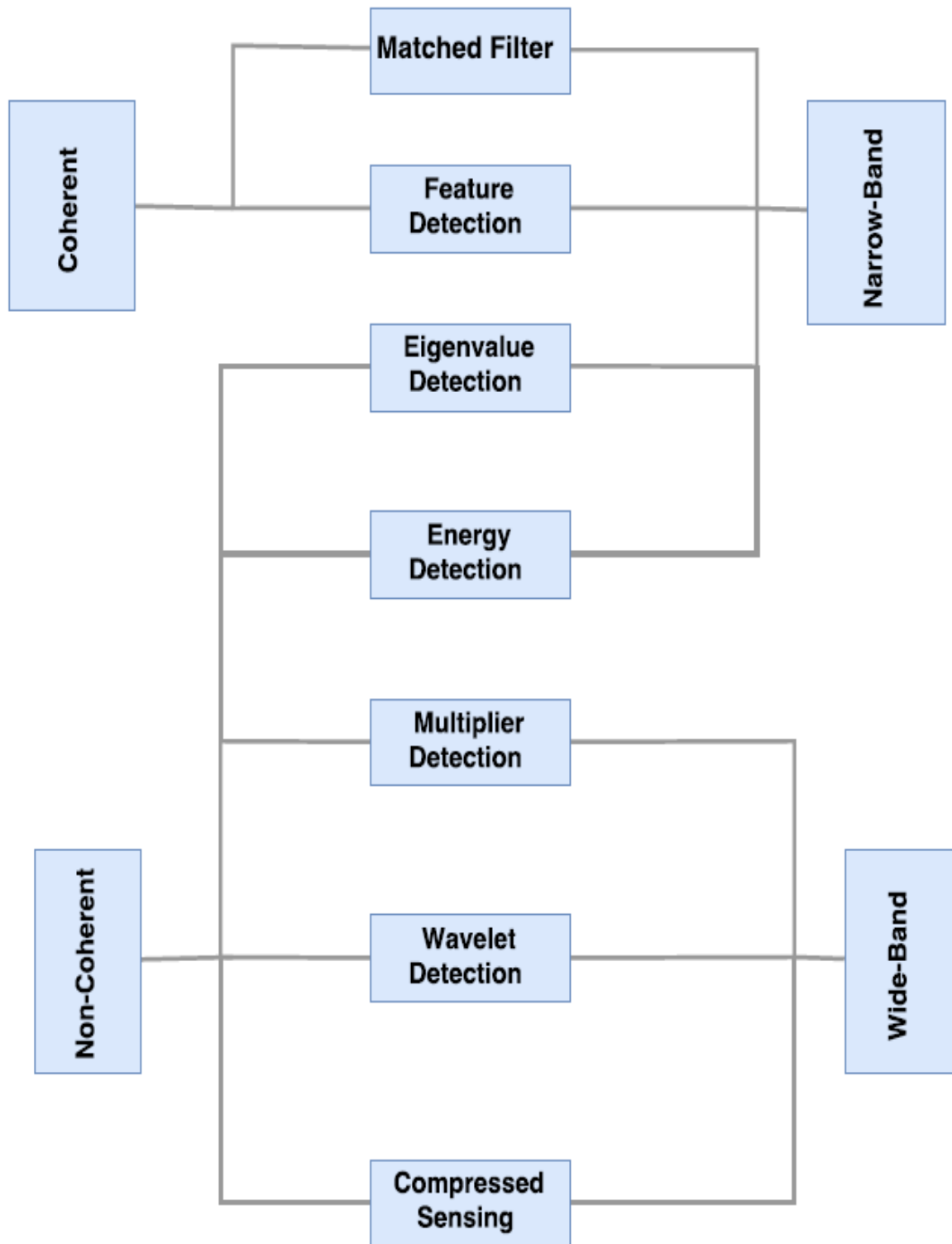
One of the most prominent features of CR networks will be the ability to switch between radio access technologies, transmitting in different portions of the radio spectrum as unused frequency band slots become available [2, 9]. This spectrum sensing feature is one of the fundamental requirements for transmitters to adapt to varying channel quality, network congestion, interference and service requirements [2,9]. Sensing techniques are further broken down into four broad categories. The first two broad



categories are coherent and non-coherent [2]. In coherent detection, prior knowledge of the PU signals is needed. In non-coherent detection, prior knowledge of PU signals is not required [2]. The other categories, based on the bandwidth requirements for sensing, are the narrowband and wideband detection techniques. The classifications of spectrum sensing techniques are shown in Figure 2.3 [23].

## **2.2 The General Spectrum Sensing Problem**

There are several algorithms available for spectrum sensing, each with its own set of advantages and disadvantages that depends on the specific scenario. Ultimately, a spectrum sensing device must be able to give a general picture of the medium over the entire radio spectrum. This allows the CR network to analyze all degrees of freedom (time, frequency and space) to predict the spectrum usage.

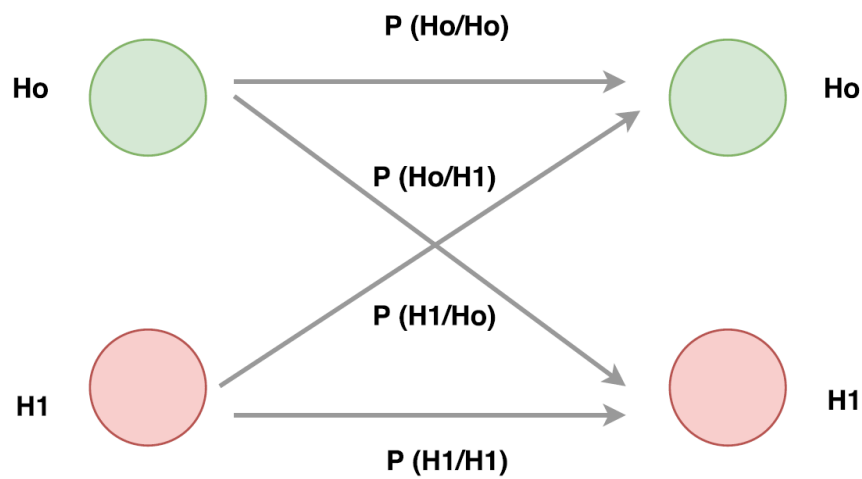


**Figure 2.3** Flow chart of the classification of spectrum sensing techniques.

(The focus will be on the non-coherent detection techniques)

## 2.2.1 Fundamentals of Spectrum Sensing Techniques

Spectrum sensing is essentially wireless signal detection. In a nutshell, signal detection can be described as a method for identifying the presence of a signal in a noisy environment. Analytically, signal detection can be reduced to a simple identification problem, formalized as a hypothesis test [7]:



**Figure 2.4** Hypothesis test statistics with all possible outcome.

$$x(k) = \begin{cases} n(k), & H_0 \\ h(k) s(k) + n(k), & H_1 \end{cases}$$

where  $x(k)$  refers to the complex received signal at time  $k$ ,  $s(k)$  the signal to be detected,  $h(k)$  represents the channel response and  $n(k)$  is additive white Gaussian noise (AWGN) in the channel.  $H_0$  refers to the null hypothesis which represents the sensed channel with the absence of the primary user signal.  $H_1$  represents the presence of the signal. The outcome of spectrum sensing techniques is the hypothesis and the probabilities of detection and false alarm can be determined from this framework.

## 2.2.2 Performance Criteria

The performance of spectrum sensing techniques can differ in different scenarios. Hence, it is imperative to evaluate and choose the most adequate scheme for a given scenario. Different characteristics that can be used to evaluate the sensing algorithms are discussed in this section.

- **Probability of false alarm:** It is the probability that the detector declares the presence of the PU, when the PU is absent. Considering a binary hypothesis test. There are two types of errors that can be made, type I and type II errors, respectively. A type I error is made if  $H_1$  is accepted when  $H_0$  is true. The probability of making a type I error is often called the probability of false alarm ( $P_{fa} = \text{Prob} \{ \text{Decision} = (H_1 / H_0) \}$ ), which is a significant design parameter since false alarms lead to missing spectral opportunities. Therefore, controlling the probability false alarm is crucial for efficient spectrum usage.

- **Probability of missed detection:** It is the probability that the detector declares the absence of PU, when the PU is present. A type II error is made if  $H_0$  is accepted when  $H_1$  is true. Missed detection probability ( $P_m = \text{Prob} \{ \text{Decision} = (H_0 / H_1) \}$ ) also, called type II error, comes about because of the probability of missed detection and can lead to collisions with the PU transmission and hence, reduced rate for both the PU and SU, respectively. Establishing distributions of decision statistics helps in controlling the probabilities of missed detection and false alarm.

Based on the previous stated two points, it worth mention that on the whole, a CR system ought to satisfy constraints on both the false alarm and missed detection probability [24]. Designing a detection rule brings about a trade-off between both probabilities. Nevertheless, if the detectors behave reasonably, as the number of samples increases, both constraints may be satisfied by selecting the number of samples to be big enough [25]. For implementation, it is advantageous to have the schemes whose threshold and performance may be set analytically. In a practical scenario, the probability of detection and the samples required to achieve a given detection probability will have to be determined experimentally because of variables, such as the fading channel, channel errors, and noise power uncertainty affecting their observations [25].

- **Signal-to-Noise-Ratio (SNR):** Type I and type II errors are linked to each other through sensing time, SNR, and detection threshold. The SNR at the SUs depends on the PU transmitted power and the spectrum environment. The detection performance improves with an increase in the SNR.

## 2.3 Spectrum Sensing Techniques

Different sensing techniques are designed to overcome many of the obstacles that make the detection process difficult. For example, this obstacle could be: the typical low signal to noise ratio (SNR) in the transmissions; fading and multi-path in wireless communications; the unstable noise level in the channel; the need of a low sensing time, and more. Many detection methods exist, but not all of them are independent from these

factors so they cannot be efficacious. In this section the most important blind detection techniques are presented.

## **2.4 Energy Detector (ED)**

### **2.4.1 Introduction**

The Energy Detection (ED) technique is a popular narrow band detection method that is widely employed in the literature. ED compares the signal energy received in a certain frequency band to a properly set decision threshold. If the signal energy lies above the threshold, the band is declared to be occupied. Otherwise, the band is assumed to be idle and could be accessed by secondary users.

Owing to the generality of its operating principle, the performance of energy detection would not be expected to depend on the type of primary signal being detected, thus it is known as non-coherent spectrum sensing [9, 21, 26]. The major drawback of this method is that it has poor detection performance under low SNR scenarios and cannot differentiate between the signals from PUs and the interference from other cognitive radios. In following section, we will focus on a brief background of the ED, but we will elaborate more on those recent research activities and their associated problems and solutions.

### **2.4.2 Literature Review ED**

In the initial ED presentation by Urkowitz in his classical article [7], the author discussed ED as a binary hypothesis test for signal detection in white Gaussian noise

environments. Digham et al [27] further investigated Urkowitz's work for unknown deterministic signals received over Rayleigh and Nakagami fading channels.

The main goal of ED-based detection is to decide between two hypotheses:  $H_1$  indicates the presents of a signal and  $H_0$  indicates its absence. The rest of the major outlines of the ED characteristics were presented by Urkowitz and Digham et al [27] and they are:

- I. ED is a blind detection technique, therefore any prior information related to the desired signal to be detected is not required. Some information is assumed to be known such as the spectral region, specified by central frequency ( $f_c$ ) and bandwidth (B), to which the transmitted signal is confined but any distinguishing parameters / factors are not provided.
- II. The reconstruction of the received signal during a specific sensing duration time (T) is based on the finite number of the collected samples of a bandlimited signal. The measured energy from the received signal is then normalized by the noise power spectral density ( $N_0/2$ ) to provide a dimensionless test metric.

The primary signal presence or absence is determined based on the comparison between the test metric and a specific threshold.

In their work, Urkowitz et al assumed the noise is white Gaussian and that the noise Power Spectral Density (PSD) is known. This is a drawback of ED, and in some papers ED is considered a semi-blind sensing technique. All the previous stated fundamental

assumptions have been further investigated in recent literature from various points of view.

In the next section, a brief description of the conflicts in the reported papers and journals are presented and the important assumptions in ED are highlighted. In addition, a representation of the unsolved problems regarding the validity of a Gaussian approximation to test statistic distribution is discussed [28].

#### **2.4.2.1 Different Approaches in Determining Test Statistic**

The decision metric in ED is based on the energy content of the received signal at CR. The problem lies in the ambiguity in defining the exact test statistic for ED in the literature.

To establish the test statistic some researchers, such as Urkowitz [7] and Digham et al [27], use a specific technique which normalizes energy in the received samples by the noise variance. Others like Zeng et al [29] and Zhuan et al [30] calculate the average energy of the received signal by scaling the energy in the received samples by the number of the observed samples so a decision on the presence or absence of desired signal could be made. Another method of calculating the test statistics is shown in the work of Sonnenchein and Fishman [31]. It is based on the unscaled version of energy content in the received samples.

The various methods of determining the probability of detection and probability of false alarm are often a source of confusion for any new researchers starting in this field.



### 2.4.2.2 Spectrum Sensing in Special Conditions

During the performance analysis of any spectrum sensing technique, the source of uncertainties is usually due to the channel fading effect and noise uncertainty. Getting exact and accurate knowledge of noise power level is extremely difficult because the noise power is frequently changing with time and location. The assumption that the exact noise power  $\widehat{\sigma}_n^2$  is already known or can be calculated is not realistic. A more appropriate method is to estimate the noise variance  $\widehat{\sigma}_n^2$  using this relation  $\widehat{\sigma}_n^2 = \alpha \sigma_n^2$ .  $\alpha$  represents the noise uncertainty factor with a given upper bound B (in dB), given that  $B = \sup \{10 \log \alpha\}$  defines the noise uncertainty bound. This was defined in the literature as “energy detector’s inherent noise uncertainty” until Tandra and Sahai [8, 32] identified it as SNR wall.

In the literature, various noise uncertainty models have been presented. For example, Sonnenchein and Fishman [31] tried to find the minimum required SNR by using peak to peak uncertainty in the noise power to achieve given detection and false alarm probabilities. Zeng et al [36] assumed that noise uncertainty factor (in dB) is uniformly distributed. The effect of the noise uncertainty on the reliability of ED-based sensing was studied at very low SNR values. At the end of the Zeng et al paper, a cooperative sensing technique was proposed to increase the performance efficiency.

As evident from above, the literature has covered large area of the noise uncertainty problem in ED as well as proposing some solutions. Most of the work has a detailed discussion on the ED performance over fading channels; others proposed cooperative sensing to improve the detection performance in a fading environment.

### 2.4.3 ED Structure

Figure 2.5 shows an energy detector block diagram. Initially, the received signal passes through a band-pass filter to remove all out-of-band signals based on the frequency of interest, which is centered around  $f_c$  and spanning over bandwidth  $W$ . Then  $x(t)$  (the filtered signal) is sampled/digitized by an Analog to Digital Converter (ADC). At the next stage, a simple squaring device followed by an accumulator gives the energy content in  $N$  samples of  $x(k)$ , which represents the test statistic for ED. Finally, the calculated energy content of the received signal (decision metric  $u$ ) is compared with a threshold,  $\lambda$ , to decide if the sensed frequency band is vacant ( $H_0$ ) or occupied ( $H_1$ ).

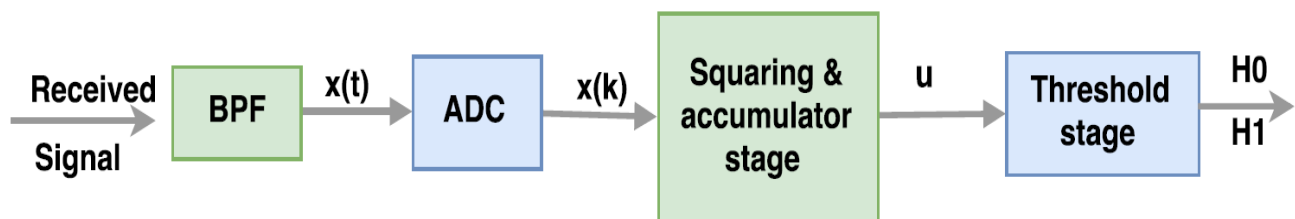


Figure 2.5 ED block diagram.

#### 2.4.3.1 The Limitation on The Performance of ED

As shown in Section 2.2, the performance of any spectrum sensing detector can be measured by the probability of detection and the probability of false alarm. The probability of detection,  $P_d$ , is given by

$$P_d = \Pr(\text{signal is detected} / H_1) = \Pr(u > \lambda / H_1)$$

$$= \int_{\lambda}^{\infty} f(u / H_1) du \quad (2.1)$$

The probability of false alarm,  $P_{fa}$ , is given by

$$\begin{aligned} P_{fa} &= \Pr(\text{signal is detected} / H_0) = \Pr(u > \lambda / H_0) \\ &= \int_{\lambda}^{\infty} f(u / H_0) du \end{aligned} \quad (2.2)$$

where  $f(u / H_i)$  refers to the Probability Density Function (PDF) of the test statistic under hypothesis  $H_i$  with  $i = 0,1$ . Based on the above equations (2.1) and (2.2), increasing the ED performance can be done by maximizing  $P_d$  while minimizing  $P_{fa}$ . A method to clarify the relationship between  $P_d$  &  $P_{fa}$  would be to plot  $P_d$  vs  $P_{fa}$  using Receiver Operating Characteristics (ROC), which is considered an important performance indicator. ROC is a graphical plot that clarifies the performance of a detector as its threshold is varied.

When using the ROC method, caution is necessary when determining the threshold  $\lambda$ . For further clarification, to obtain the minimum  $P_{fa}$ , the threshold  $\lambda$  should be kept high but that will end up in a greatly decreased  $P_d$ . Similarly, to obtain maximum  $P_d$ , threshold  $\lambda$  should be kept low but that will end up in  $P_{fa}$  exceeding the acceptable limits.

Thus, this tradeoff will be handled very carefully while setting the ED threshold. In practice, if a certain spectrum re-use probability of unused spectrum is targeted,  $P_{fa}$  is fixed to a small value (e.g.  $\leq 5\%$ ) and  $P_d$  is maximized. This is referred to as the Constant False Alarm Rate (CFAR) detection principle. However, if in CR it is required to guarantee

a given non-interference probability,  $P_d$  is fixed to a high value (e.g.  $\geq 95\%$ ) and  $P_{fa}$  is minimized. This requirement is known as Constant Detection Rate (CDR) principle.

### 2.4.3.2 Mathematical Ground for $P_{fa}$

To have an accurate calculation for  $P_{fa}$  and  $P_d$ , it is necessary to have the exact test statistic distribution ( $P_{fa}$  &  $P_d$  depend on the conditional PDFs of the test statistic) [28].

To calculate  $P_{fa}$  the first step is to start from the null hypothesis (see Sec (2.1)):

$$\begin{aligned}
 H_0: \left(\frac{1}{\sigma_n^2}\right) u &= \sum_{k=1}^N \left(\frac{1}{\sigma_n^2} n(k)\right)^2 \\
 &= \sum_{k=1}^N (y(k))^2 \text{ where } y(k) \sim N(0,1) \\
 &\sim \chi_N^2
 \end{aligned} \tag{2.3}$$

Where  $n(k)$  refers to Gaussian with zero mean,  $\sigma_n^2$  refers to the variance. The result of the summation of squares of  $N$  random variables will give the test statistic  $u$ .

When  $u$  is divided by  $\sigma_n^2$  it is said to have a central Chi-square distribution with  $N$  degrees of freedom. The  $P_{fa}$  equation can be reformulated using the fact that  $f\left(\left(\frac{1}{\sigma_n^2}\right) u / H_0\right) = \chi_N^2$

and  $\lambda$  refers to the threshold

$$P_{fa} = \Pr\left(\left(\frac{1}{\sigma_n^2}\right) u > \left(\frac{1}{\sigma_n^2}\right) \lambda / H_0\right)$$

$$\begin{aligned}
&= \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} f\left(\left(\frac{1}{\sigma_n^2}\right) u/H_0\right) du \\
&= Q_{x_N^2}\left(\frac{\lambda}{\sigma_n^2}\right)
\end{aligned} \tag{2.4}$$

$Q_{x_N^2}\left(\frac{\lambda}{\sigma_n^2}\right)$  represents the right-tail probability for a  $x_N^2$  random variable as given by Kay in

[33]. From the PDF of  $x_N^2$ , the exact expression for  $P_{fa}$  is

$$P_{fa} = \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} f\left(x^{\frac{N}{2}-1} * \frac{e^{-\frac{x}{2}}}{(x^{\frac{N}{2}} \Gamma(\frac{N}{2}))}\right) du \tag{2.5}$$

For simplicity assume  $\frac{N}{2} = m$ ,  $\frac{x}{2} = y$ . From the definition of incomplete Gamma function

$$\Gamma(\alpha, \beta) = \int_{\beta}^{\infty} x^{\alpha-1} e^{-x} dx \tag{2.6}$$

will get

$$\begin{aligned}
P_{fa} &= 1/\Gamma(m) \int_{\frac{\lambda}{\sigma_n^2}}^{\infty} (y^{m-1} * e^{-y}) dy = \Gamma(m, \frac{\lambda}{\sigma_n^2}) / \Gamma(m) \\
&= F_m\left(\frac{\lambda}{2\sigma_n^2}\right)
\end{aligned} \tag{2.7}$$

The final output matches what was derived by Digham et al in [27], and by Ghasemi and Sousa in [34], with a slight difference that both used the scaled ED test statistic as

$$u_{scl} = \frac{u}{\sigma_n^2}$$

### 2.4.3.3 Mathematical Background for ED Threshold

From the previous  $P_{fa}$  equation (2.7), The ED threshold for constant false alarm rate can be derived:

$$\lambda = 2\sigma_n^2 F_m^{-1}(P_{fa}) \quad (2.8)$$

From this equation (2.8), the threshold will have a variable value which is a function of noise variance ( $\sigma_n^2$ ), number of observed samples,  $N$ , and the targeted constant false alarm probability.

### 2.4.4 Pros and Cons of Energy Detectors

From the comparison in Table 2.1 is clear that the simplicity and low computational complexity of ED are its key favorable aspects that have motivated most of the recent work in Spectrum Sensing (SS) for CR toward enhanced energy detection techniques [35]. ED needs to estimate only the noise power to set its threshold and does not require any information on primary user transmission characteristics. This makes energy detection a semi-blind technique. Besides that, it is shown to be an optimal technique for detecting Independent Identically Distributed (IID) primary user transmissions especially when PU signal features are unknown to CR.

**Table 2.1** Comparison of advantages and disadvantages of energy detector techniques.

<b>Pros</b>	<b>Cons</b>
<ul style="list-style-type: none"> <li>+ Implementation simplicity</li> <li>+ Low computational complexity</li> <li>+ Optimal for detecting IID primary signals</li> <li>+ Semi-blind (No a priori PU signal information required)</li> </ul>	<ul style="list-style-type: none"> <li>– Non-robust (Threshold strongly depends on noise uncertainties)</li> <li>– Low accuracy/reliability                             <ul style="list-style-type: none"> <li>• Unable to differentiate interference from PU signal and noise</li> <li>• Poor performance under low SNR (due to shadowing and multipath fading)</li> <li>• Inability to detect spread spectrum signals</li> </ul> </li> <li>– Inefficient for detecting correlated primary signals</li> <li>– More susceptible to hidden terminal problem</li> </ul>

On the other hand, ED faces some problems which are addressed in [8] and some hidden assumptions in conventional ED are unveiled more recently in [28]. The main problem of ED is uncertainty in the decision threshold that produces optimal sensing results, since it depends on the accurate estimation of the noise power which changes temporally and spatially. Sensing results based on ED have limited reliability as energy observations are unable to differentiate between primary and secondary user signals, which appears as a cost of semi-blind signal detection. This may result in false detection of PU signal triggered by other unintended signals. All these factors characterize ED with less robustness and low accuracy/reliability.

## 2.5 Eigenvalues Based Detection

### 2.5.1 Introduction to Eigenvalue Detection

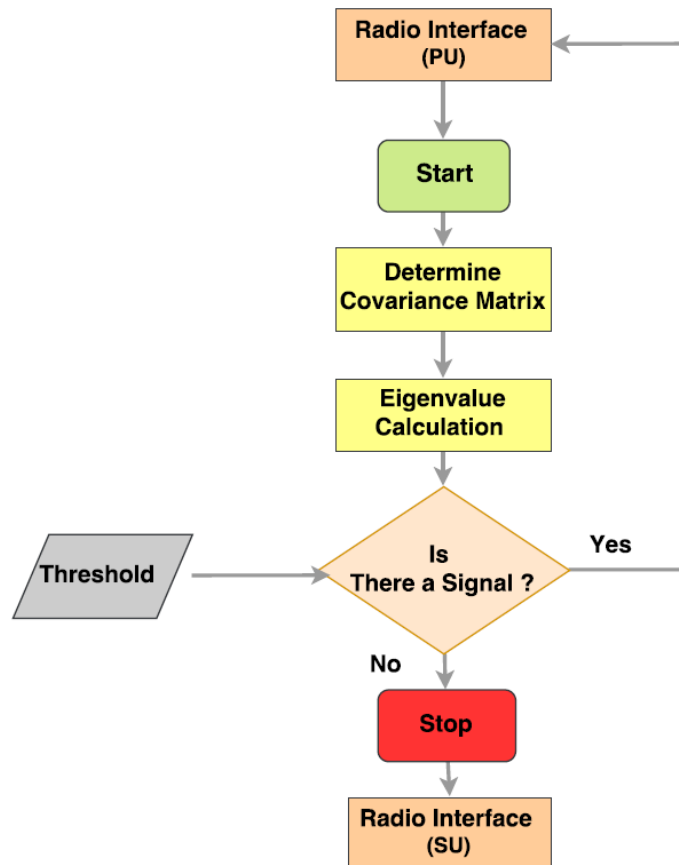
Covariance Based Detection (CBD) is a group of narrowband spectrum sensing techniques that exploit aspects of statistical covariance matrixes; a received signal and noise can have different characteristics [36, 34, 37]. The distinguishing properties can be used to detect whether a primary user's signal exists or not. From the CBD a new detection technique was born known as Eigenvalue Based Detection (EBD). EBD is based on the analysis of eigenvalues of the covariance matrix and its key aspect that it can reduce the computational complexity, as compared to other blind algorithms [38, 33, 52]. Different EBD schemes use different characteristics of the covariance matrix eigenvalues.

EBD techniques are blind because they do not require any a priori information of PU signals and/or the transmission channel. Thus, it overcomes problematic noise uncertainty encountered by other detectors. The different detection methods are:

1. Max-min eigenvalue detection in which the test statistic is defined as ratio of maximum to minimum eigenvalues of the received signal covariance matrix.
2. Energy with min eigenvalue, in which the test statistic is defined as ratio of average power of received signal to the minimum eigenvalue of the covariance matrix.
3. Max eigenvalue detection, in which the test statistic is given by the maximum eigenvalue of the signal covariance matrix.



In Figure 2.6 the main components of the EBD method is illustrated. The sampled signal comes from the radio system interface, from which the covariance matrix is calculated. The eigenvalues of the matrix and subsequent-maximum-minimum ratios are determined;



**Figure 2.6** Eigenvalue-based spectrum sensing technique flow chart.

The next section will cover the eigen-analysis of the covariance matrix, which includes further explanation of the covariance. The analysis of EBD along with its mathematical model are presented.

## 2.5.2 Eigen-Analysis of the Autocovariance Matrix

The eigenvalue analysis of the autocovariance matrix is necessary to better explain the detection algorithm.

It is assumed that a random process  $x(n)$  is wide-sense stationary and is a linear combination of  $m$  basic components  $s_i(n)$  is given by

$$x(n) = \sum_{i=1}^m a_i s_i(n), \quad (2.9)$$

Since the sequence observed is  $y(n) = x(n) + w(n)$ , where  $w(n)$  is a complex additive white Gaussian noise sequence with spectral density  $\sigma^2$ , the  $M \times M$  autocovariance matrix for  $y(n)$  can be expressed as

$$C_{yy} = C_{xx} + \sigma_w^2 I \quad (2.10)$$

where  $C_{xx}$  is the autocovariance matrix for the signal  $x(n)$ ,  $\sigma_w^2 I$  is the autocovariance matrix for the noise and  $M$  is the dimension of the covariance matrix.

Note that if  $M > m$ , then  $C_{xx}$  which is of dimension  $M \times M$  is not of full rank.

Now, let us perform an eigen-decomposition of the matrix  $C_{yy}$ . Let the eigenvalues be ordered in decreasing value with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$  and let the corresponding eigenvectors be denoted as  $\{v_i, i = 1, \dots, M\}$ . It is assumed that the eigenvectors are normalized so that  $v_i^H \cdot v_i = \delta_{ij}$  ( $H$  denotes the conjugate transpose). In the absence of noise, the eigenvalues  $\lambda_i, i = 1, 2, \dots, m$  are nonzero while  $\lambda_{m+1} = \lambda_{m+2} = \dots = \lambda_M = 0$ . Thus, the eigenvectors  $v_i, i = 1, 2, \dots, m$  span the signal subspace.

These vectors are called principal eigenvectors and the corresponding eigenvalues are called principal eigenvalues.

In the presence of noise, the eigen-decomposition separates the eigenvectors in two sets. The set  $v_i$ ,  $i = 1, 2, \dots, m$ , which are the principal eigenvectors, span the signal subspace, while the set  $v_i$ ,  $i = m+1, \dots, M$ , which are orthogonal to the principal eigenvectors, are said to belong to the noise subspace. It follows that the signal  $x(n)$  is simply linear combinations of the principal eigenvectors.

Finally, the variances of the projections of the signal on the principal eigenvectors are equal to the corresponding eigenvalues of the covariance matrix. So, the principal eigenvalues are the power factors in the new signal space.

### 2.5.3 Data / Mathematical Model

During a particular time interval, the frequency band may be occupied by only one primary user. The frequency band of interest has a central frequency  $f_c$  and bandwidth  $W$ . In this thesis, a non-cooperative spectrum sensing scheme is considered where the sensing work is completed by only one secondary user (only one transmitting source, only one receiver).

For signal detection, two hypotheses  $H_1$  &  $H_0$  given as (see Sec 2.2.1)

$$H_0 : x(n) = w(n), \quad n = 0, 1, \dots \quad (2.11)$$

$$H_1 : x(n) = h(k) s(n - k) + w(n), \quad (2.12)$$

where  $x(n)$  denotes the discrete signal at the secondary receiver,  $s(n)$  is the primary signal seen at the receiver,  $h(k)$  is the channel response and  $w(n)$  are the noise samples.

Considering a sub-sample  $M$  of consecutive outputs and defining

$$\hat{x}(n) = [x(n), x(n-1), \dots, x(n-M+1)]^T \quad (2.13)$$

$$\hat{w}(n) = [w(n), w(n-1), \dots, w(n-M+1)]^T \quad (2.14)$$

$$\hat{s}(n) = [s(n), s(n), \dots, s(n-N_1-M+1)]^T \quad (2.15)$$

Now, from the previously given equation the estimated received signal at the SU receiver can be written as

$$\hat{x}(n) = H \hat{s}(n) + \hat{w}(n) \quad (2.16)$$

where  $H$  is the channel response matrix of size  $M \times (N+M)$ , defined as

$$H = \begin{bmatrix} h(0) & \dots & h(N_j) & 0 \\ \vdots & \ddots & \vdots & \\ 0 & \dots & h(0) & h(N) \end{bmatrix} \quad (2.17)$$

Considering the statistical properties of the transmitted signal and channel noise, let us assume that the noise is white and that the noise and the transmitted signal are uncorrelated.

Let  $R$  be the sample correlation matrix of the received signal, that is

$$R = (1/N_s) \sum_{n=M}^{M-1+N_s} \hat{x}(n) \hat{x}^T(n) \quad (2.18)$$

where  $N_s$  is the number of collected samples at the receiver side. If  $N_s$  is large, based on the assumptions made earlier (see eq. (2.10) and (2.16)), the sample covariance matrix ( $R$ ) can be written as

$$R \approx E [\hat{x}(n) \hat{x}^T(n)] = H R_s H^T + \sigma_w^2 I_M \quad (2.19)$$

where  $R_s$  is the statistical correlation matrix of the input signal,  $R_s \approx E [\hat{s}(n) \hat{s}^T(n)]$ ,  $\sigma_w^2$  is the variance of the noise,  $I_M$  denotes an  $M \times M$  identity matrix.

### 2.5.3.1 Mathematical Model of MME and MAE

From the sample covariance matrix  $R_s$  (see eq. (2.18)), Zeng et .al [39] proposed two detection methods, one based on the ratio of the Maximum eigenvalue to Minimum Eigenvalue (MME); the other is based on the ratio of the Average Eigenvalue to Minimum eigenvalue (MAE).

Let  $\hat{\lambda}_{\max}$  and  $\hat{\lambda}_{\min}$  be the maximum and the minimum eigenvalues of  $R$ , and  $\hat{\rho}_{\max}$  and  $\hat{\rho}_{\min}$  be the maximum and the minimum eigenvalues of  $H R_s H^T$ .

Then, in this case

$$\hat{\lambda}_{\max} = \hat{\rho}_{\max} + \sigma_w^2 \quad (2.20)$$

$$\hat{\lambda}_{\min} = \hat{\rho}_{\min} + \sigma_w^2 \quad (2.21)$$

Obviously,  $\hat{\rho}_{\min}$  can equal  $\hat{\rho}_{\max}$  only if  $\mathbf{H}\mathbf{R}_s\mathbf{H}^T$  equal  $\delta\mathbf{I}_M$  where  $\delta$  is a positive number. In practice, when signal is present, it is very unlikely that  $\mathbf{H}\mathbf{R}_s\mathbf{H}^T = \delta\mathbf{I}_M$  (due to dispersive channel and/or oversampling and/or correlation among the signal samples).

Hence, from the equations in (2.20) & (2.21), and if there is no signal, the ratio of  $\left(\frac{\hat{\lambda}_{\max}}{\hat{\lambda}_{\min}}\right)$  will equal 1, otherwise,  $\left(\frac{\hat{\lambda}_{\max}}{\hat{\lambda}_{\min}}\right)$  will be greater than 1. Therefore, finding the ratio  $\left(\frac{\hat{\lambda}_{\max}}{\hat{\lambda}_{\min}}\right)$  is considered the core idea of the Maximum-Minimum Eigenvalue (MME) based detection technique.

Note that: The max to min eigenvalue ratio of covariance matrix serves as the test statistic, which is then compared to a threshold to form decisions. In the eigenvalue-based methods, the expression for the decision threshold has been derived based on random matrix theory to make a hypothesis test for signal detection. In most of the eigenvalue-based detection schemes proposed so far in the literature, both the threshold value and the probabilities of detection and false alarm are calculated based on the asymptotical (limiting) distributions of eigenvalues that is mathematically tractable and less complex.

#### **2.5.4 Pros and Cons of Eigenvalue-Based Detection**

Since the statistical covariance matrices of the received signal and noise are usually different, eigenvalue-based detection technique makes full use of this property to detect whether the primary user exists or not.

The performance of such detectors was measured against the ED and has shown significant performance gains. These methods overcome the noise uncertainty problem and perform globally better than ED in both AWGN and Rayleigh faded channels.

**Table 2.2** Comparison of advantages and disadvantages of eigenvalue-based detection technique

<b>Pros:</b>	<b>Cons:</b>
<ul style="list-style-type: none"> <li>+ Non-coherent</li> <li>+ Sensing performance is highly reliable, can detect signals with low SNR.</li> <li>+ Robust to noise uncertainty</li> </ul>	<ul style="list-style-type: none"> <li>- Moderate computational complexity.</li> <li>- Performance degrades for uncorrelated PU signals</li> </ul>

Favorable aspects of eigenvalue-based detection: Generally, EBD does not require any information about the primary signal or noise. However, if some a priori information about primary signal correlation becomes available, this may assist in choosing corresponding elements in the sample covariance matrix thus making the decision test statistic more efficient. Most importantly, EBD does not need noise power estimation as the threshold is related to  $P_{fa}$  and sample size  $N$  of the received signal at the CR only. It achieves better performance for highly correlated signals.

Limitations of EBD: Performance of covariance-based detection strongly depends on the statistics of the received primary signal, which degrades if the primary signal tends to be uncorrelated.

To sum up, it is obvious from the discussion on advantages and disadvantages of the ED and eigenvalue-based detection techniques that no single detector has the best performance for all scenarios.

## **2.6 Empirical Mode Decomposition (EMD)**

In the next section, we will introduce details of the Empirical Mode Decomposition (EMD) to give theoretical background for our proposed method presented in Chapter 3. We will explain the method of applying EMD to a signal, we accompany that with an example and at the end of the section a comparison analysis of the EMD vs. Wavelet is presented.

### **2.6.1 Introduction to EMD**

EMD algorithm (also known as Hilbert Huang Transform (HHT)) is a method of deconstructing a signal in the time domain. It can be compared to other analysis methods like Fourier Transforms and wavelet decomposition. But, in contrast to other common transforms, EMD is an empirical algorithm, which is applied to a data set, rather than a theoretical function.

EMD filters out functions (also known as oscillatory modes) which form a complete and nearly orthogonal basis for the original signal. The functions, known as Intrinsic Mode Functions (IMFs), are therefore sufficient to describe the signal, even though they are not necessarily orthogonal. The reasons are described in Huang et al, "...the real meaning



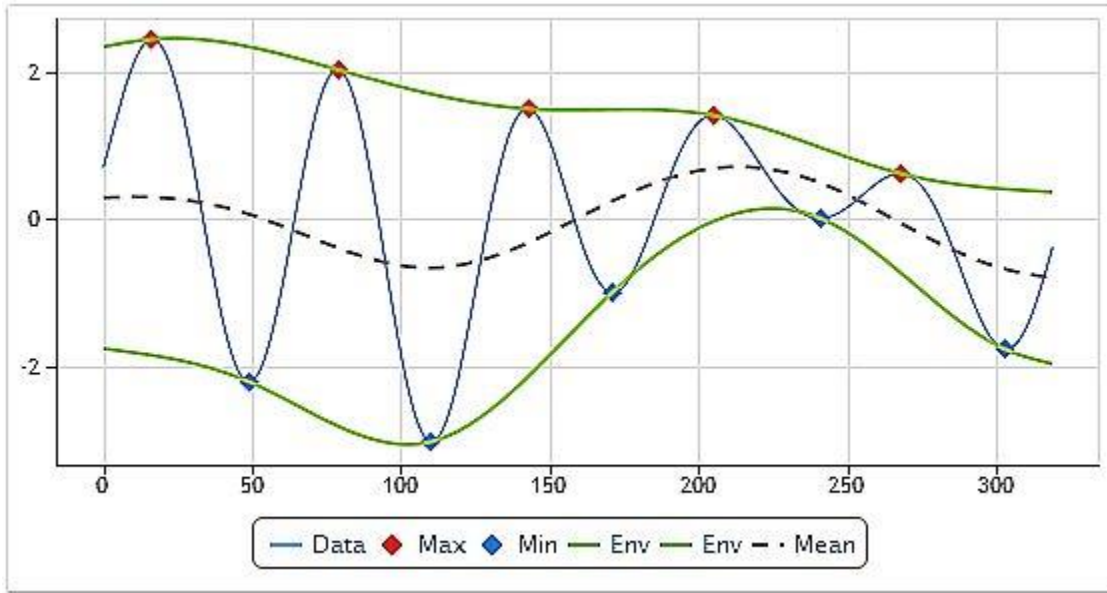
here applies only locally. For some special data, the neighboring components could certainly have sections of data carrying the same frequency at different time durations. But locally, any two components should be orthogonal for all practical purposes" [40].

Obtaining IMFs from real world signals is important because natural processes often have multiple causes, and each of these causes may happen at specific time intervals. This type of data is evident in an EMD analysis, but quite hidden in the Fourier domain or in wavelet coefficients.

### **2.6.2 Sifting Process**

The essential step of the Empirical Mode Decomposition is the sifting process shown in Figure 2.7. The sifting process basically extracts scales of the signal. Assume a signal with  $Q$  minima and  $P$  maxima. The sifting process starts with determining the extrema of the signal,  $x(t)$ , where  $S_{\max}$  represents the sets of points which are interpolated to form the upper envelope of the signal,  $\hat{x}_{\max}$ . Similarly, the minima of the signal,  $S_{\min}$  are interpolated to form the minimum envelope  $\hat{x}_{\min}$ .

Then the average envelope is calculated,  $(\hat{x}_{\max} + \hat{x}_{\min}) / 2$ , and later this envelope is subtracted from the original signal  $x(t)$  giving the first iteration of the sifting process known as  $x_j^k(t)$  where  $k$  refers to the iteration, and  $k = 1$  for the first iteration.

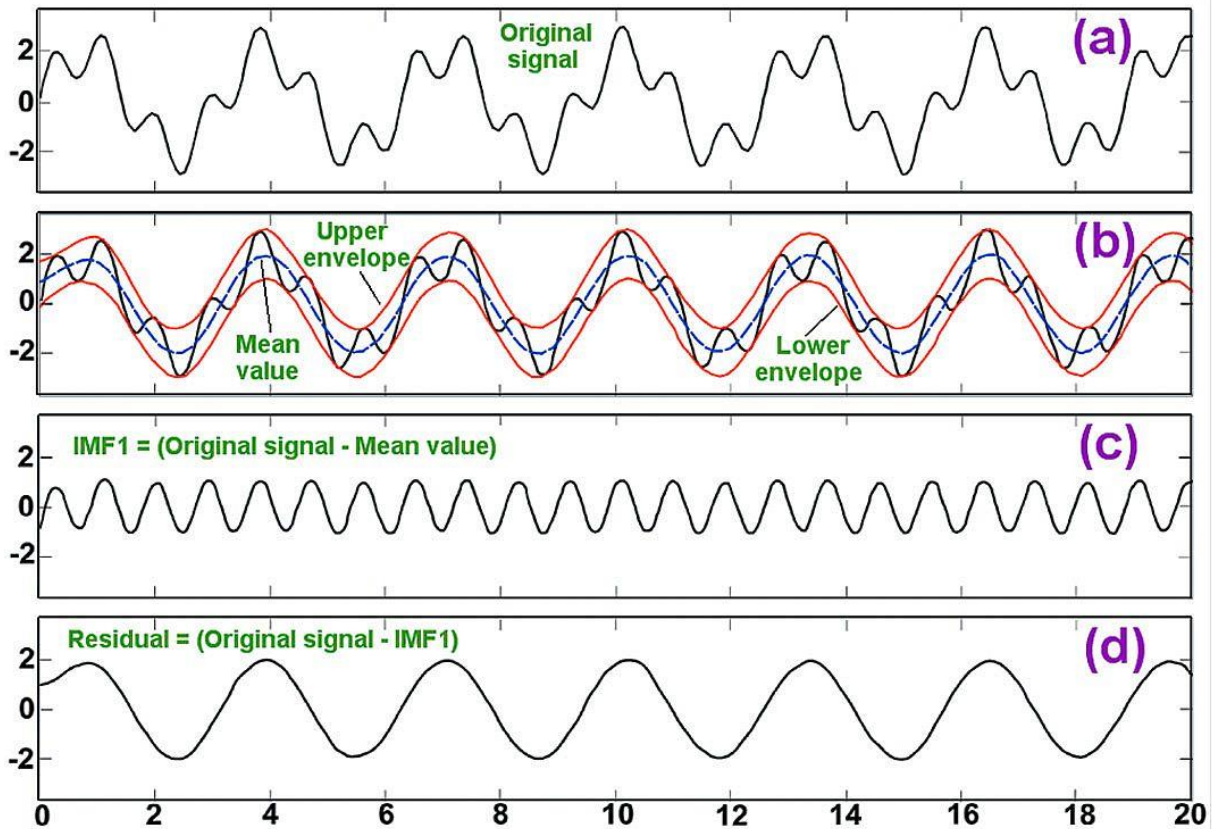


**Figure 2.7** Sifting process (Basic idea) [45].

The iteration on  $k$  is continued until  $\langle x_j^k \rangle = 0$  and the number of extrema of  $x_j^k$  should not be less than the number of zero-crossing. For simplicity, it is possible to drop the term  $K$  and write the resulting function as  $x_j$ . Figure 2.8 represents an example for this case. The output from the first sifting process is the first IMF, where  $j = 1$ . Based on this, the function  $x_1^r = x(t) - x_1(t)$  is created, and the sifting process is repeated, giving  $x_2(t)$ , which is the second IMF. The IMF's are generated until the residue signal.

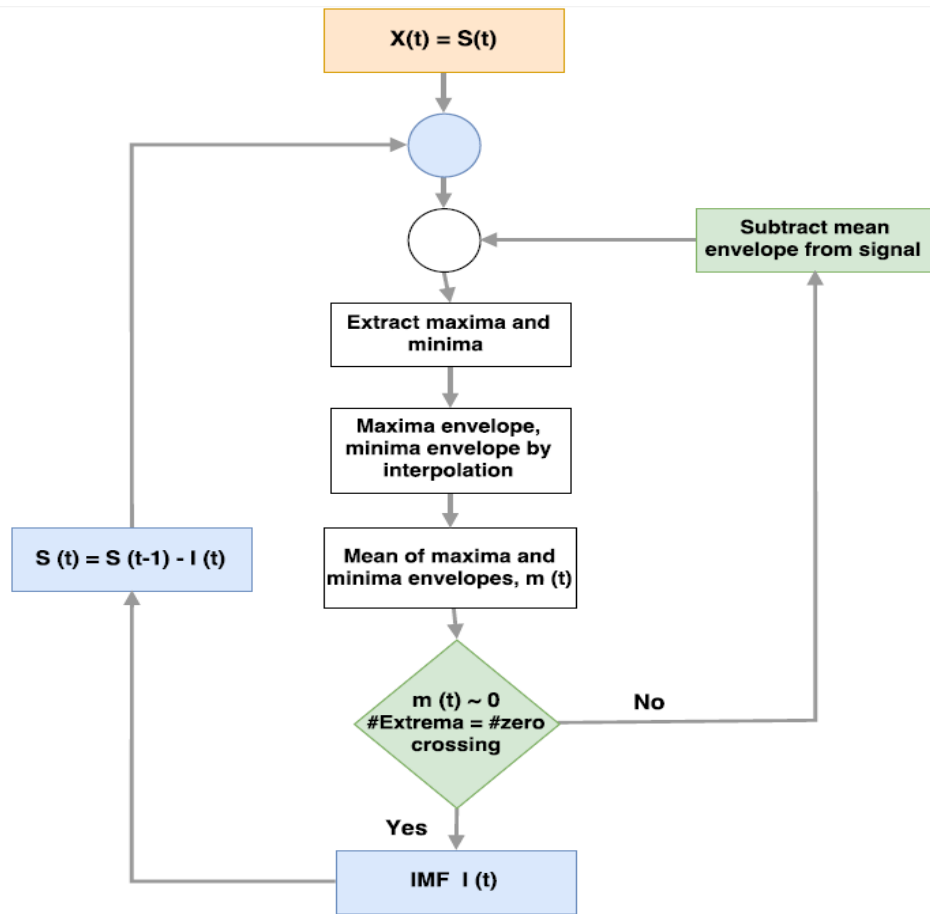
$$x_j^r = x(t) - \sum_{n=1}^{n=j} x_n(t)$$

The functions  $x_j(t), j = 1, 2, \dots, N$ , resolve the frequencies from the received signal  $x(t)$  such that they are nearly orthogonal to one another.



**Figure 2.8** Example of sifting process, showing steps of getting IMF1 and residual signal [46].

From Figure 2.9 the flow chart of the EMD algorithm is introduced. As explained before, at each IMF there is only one extrema point between any two successive zero crossings. The frequency of the signal could be directly deduced/presumed by measuring the distribution of the zero crossings of the signal. Further, the IMFs have symmetric envelopes and a zero-mean value. Because of these characteristics, the IMF's are referred to as being mono-component.



**Figure 2.9** Flowchart of the EMD.

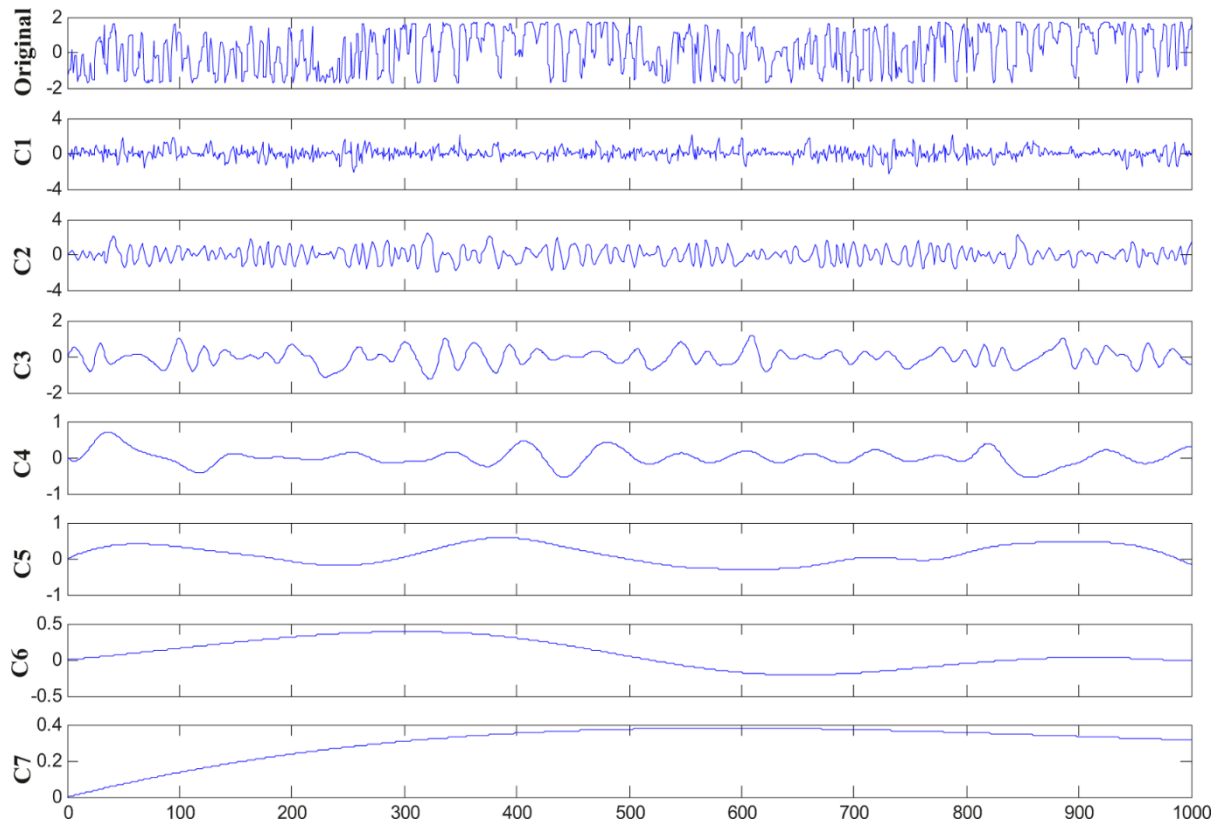
Practically, the EMD has been shown to be very efficient in extracting relevant components in many different applications including non-stationary signals. There are many examples to demonstrate the effectiveness of EMD, for example, processing audio signals [41], GPS (Global Position System) signals [42], gravitational waves [43], seismic signals [44], etc.

### 2.6.3 Generation of The IMFs for Noise Signal

One of the important features of EMD is that it behaves like a dyadic filter for a white noise input signal. The decomposition process of a white noise signal gives the frequency of the IMFs which in turn follows an exponential trend. The first IMF represents the fastest modes of oscillation (highest frequency) in the received signal, and with following IMFs, the frequency, as measured by the number of zero crossings, decay exponentially as the index of the IMF.

To explain this point more clearly, the first  $IMF_n$ , where  $n=1$ , represents the highest frequency in the received signal. The second  $IMF_2$  will represent the second highest frequency and as the number  $n$  increases, the following IMFs will have lower frequency than preceding ones.

The final IMF, always has just one zero crossing. Based on the simulation results, a relationship between the IMF and the zero crossing is found [40]. In this relation, the number of zero crossings in an intrinsic mode function is proportional to  $e^{-0.6n}$ , where  $n$  is the index of the IMF. In the same way, the energy of the IMFs also reduces according to an exponential rule. Figure 2.10 represents the simulation results of the first 7 IMFs of the EMD decomposition of a white noise waveform [40].



**Figure 2.10** Energy distribution across IMFs. (C stands for component).

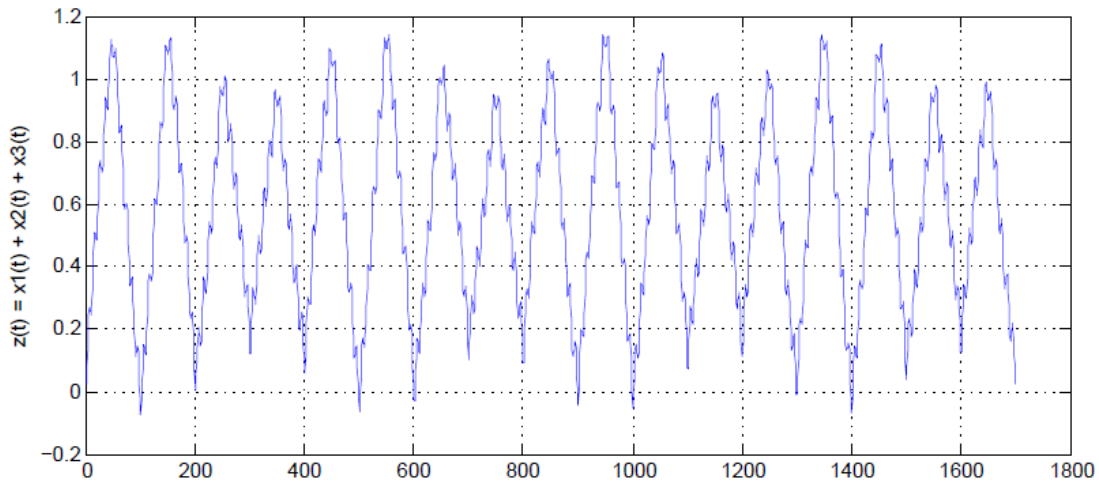
### 2.6.4 EMD in Comparison to Wavelets

In this section, a comparison between EMD and its closest competitor (Wavelet method) is discussed.

A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale.

To clarify the difference [40]:

At the beginning, the signal considered in this example is formed from the summation of three signals. The first signal  $x_1(t)$  is a triangular wave with a maximum amplitude of 1 and its minimum is 0. The second signal is given as  $x_2(t) = 0.3 x_1(10t)$ , and finally the third signal is given as  $x_3(t) = 0.1 \sin(2 \pi ft)$ , where  $f$  is roughly in the same range as the fundamental frequency of the first signal  $x_1(t)$ . The three signals are plotted in Figure 2.11 Where  $z(t)$  represents the summation of the three signals as  $z(t) = x_1(t) + x_2(t) + x_3(t)$ .



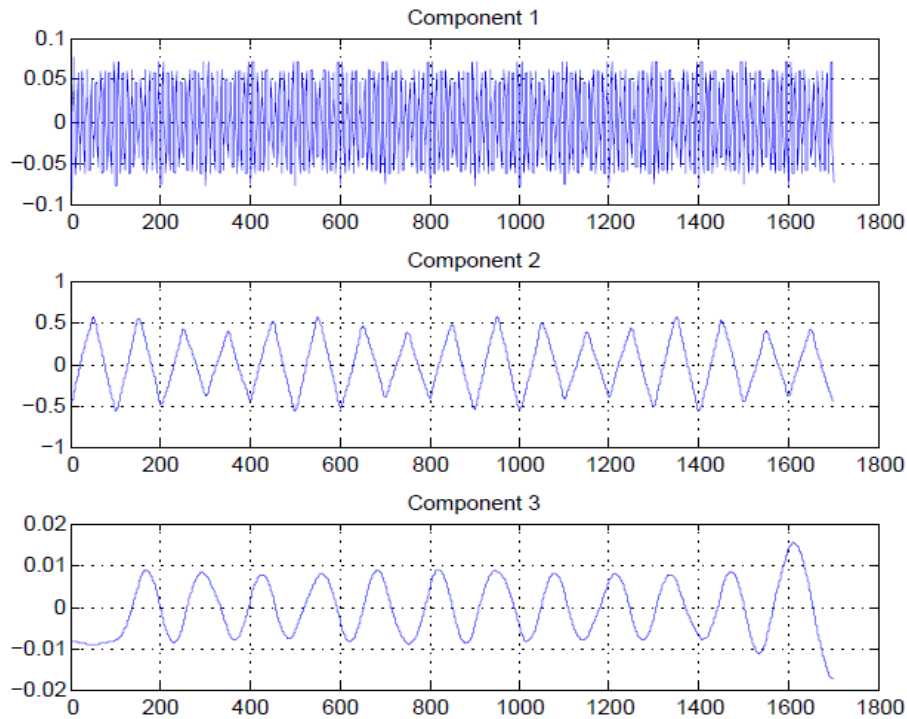
**Figure 2.11** Plotting signal  $z(t) = x_1(t) + x_2(t) + x_3(t)$ .

This example can easily be applied in any physical system, simply  $z(t)$  could be considered as two triangular waves which represents  $x_1(t)$  and  $x_2(t)$  and a sinusoidal wave  $x_3(t)$ , where the chosen signal waves represent the physically meaningful components of  $z(t)$ . Harmonic decomposition techniques such as wavelets fail to give

such a physically meaningful decomposition because of the implicit assumption that a signal is formulated of harmonic oscillatory components.

A wavelet decomposition is primarily dependent on successively passing any received signal through low-pass and high-pass time-invariant filters. The scales that the signal is divided into are determined iteratively by pre-determined high-pass and low-pass filters. One main advantage of the sifting process over wavelets is the adaptive decomposition which implies that EMD is not affected by the local features of a given signal, and does not depend on predefined filters or windows. Figure 2.12 represents the three most significant IMFs that result from applying EMD to  $z(t)$ .

To sum it up, because of the ability of EMD to adapt to the local features of the surrounding/sensed environment that makes the EMD algorithm more practical in decomposition of  $z(t)$ .



**Figure 2.12** The IMFs components resulting from applying EMD to  $z(t)$ .



## Chapter 3

# A Blind Energy Detection Scheme Using EMD for Spectrum Sensing

Noise estimation plays an essential role in enhancing the performance of non-coherent spectrum sensors such as energy detectors. If the noise power is misestimated, detector performance may deteriorate. In this chapter, we present an energy detector based on the behavior that Empirical Mode Decomposition (EMD) has towards vacant channels (noise-dominant).

### 3.1 Introduction

Based on the definition of the EMD in chapter 2 (see Sec (2.6)), the EMD algorithm can be outlined as follows [13]:

- 1) Identify all extrema points (local maxima and minima) of input signal  $y(n)$  and interpolate them (cubic spline interpolation) to find the upper and lower envelopes  $e_{\max}(n)$  and  $e_{\min}(n)$  respectively.
- 2) Find the local mean:  $m(n) = (e_{\min}(n) + e_{\max}(n))/2$
- 3) Extract the detailed signal:  $h(n) = y(n) - m(n)$

- 4) If  $h(n)$  does not satisfy the stoppage criteria then the process is repeated and  $h(n)$  is the input to step (1). Otherwise,  $h(n)$  is the  $i^{\text{th}}$  IMF and the residue is  $r(n) = y(n) - h(n)$  will be processed as input signal (steps 1-4).

The original input signal can be reconstructed as follows:

$$\hat{y}(n) = \sum_{i=1}^k M_i(n) + R(n) \quad (3.1)$$

where  $\hat{y}(n)$  is the reconstructed signal,  $n$  is the sample index,  $K$  is the total number of IMFs,  $M_i(n)$  is the  $i^{\text{th}}$  mode (IMF), and  $R(n)$  is the trend of  $\hat{y}(n)$ .

One characteristic of EMD is that the sum of all IMFs (see equation (3.1)) results in the original signal, which shows the additive reconstructive nature of IMFs in  $l_1$ -norm sense. The IMFs are also additive in the  $l_2$ -norm sense. The sum of IMFs powers approximates the power of the processed signal,  $y(n)$  and resemble the outputs of a dyadic filter bank [47]. Also, the sample size plays an essential role in the behavior of the EMD sifting performance as EMD was designed originally to process continuous signals.

Therefore, oversampling is required to capture all possible extrema, whereas, missing an extrema during the sifting process might result in losing an oscillation (or produce false envelopes) and hence the reconstructed signal,  $\hat{y}(n)$  will not represent the original signal [49].

Conversely, uncorrelated noise samples will not be influenced by missing extrema as the resulting envelopes will reflect the highest frequency content and thus get sifted largely by the first few modes.

## 3.2 Proposed Scheme

In this section, an EMD-based spectrum sensing approach is presented. The basic idea is that if the IMF energies deviate from the monotonically decreasing trend, this will result in the decision of occupancy.

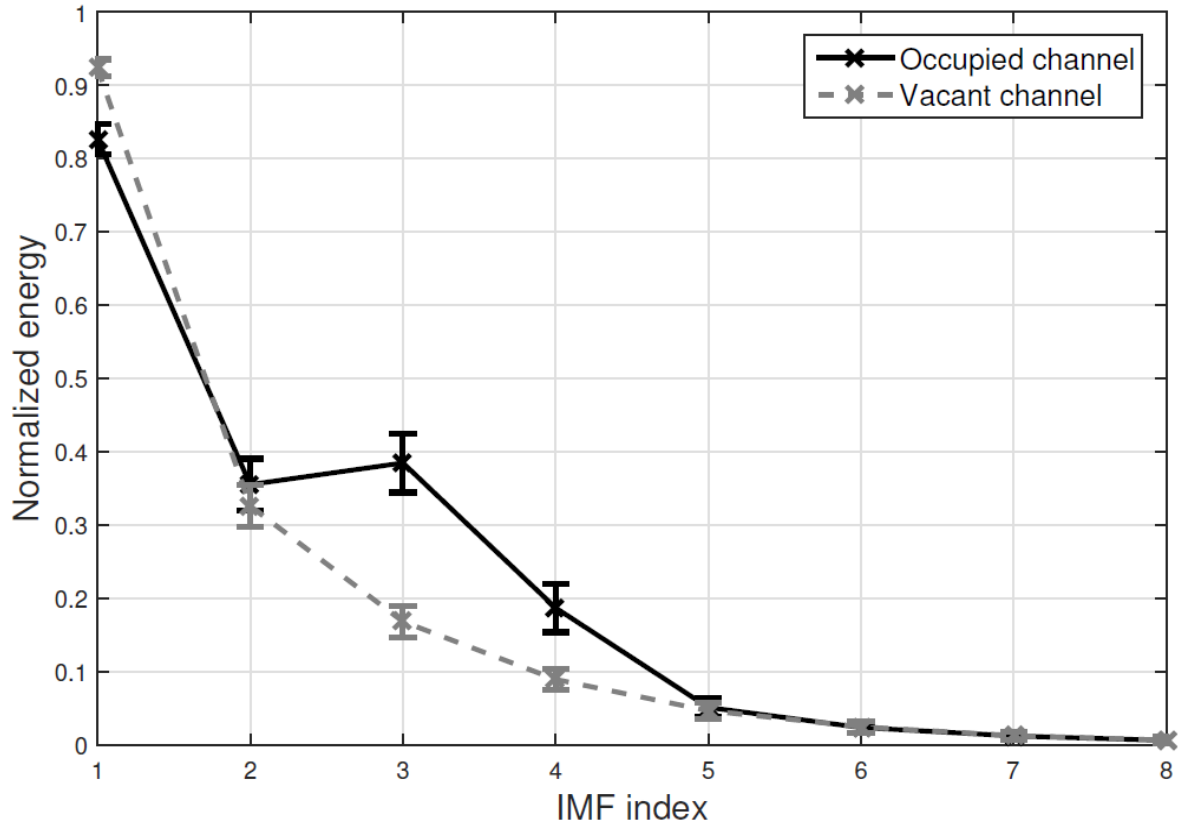
### 3.2.1 EMD Characteristics of Noise-Dominant Channels

In Section 3.1, it was shown that the IMFs resemble the output characteristics of dyadic filter. For a time-series signals contaminated with uncorrelated noise, the EMD sifting process extracts the noise components (fast oscillations) in the lower IMF indices and the signal components (slow oscillations) in the higher ones (see Sec (2.6.3)).

The energy of these IMFs reflects the contributions of both noise and signal components to the total energy of the input noisy signal. Herein, if the noise contribution of the input noisy signal was much higher than the signal, or the noise-dominant signal is presented, then the IMF energies will follow a decreasing trend (exponential decay).

To understand this characteristic more clearly, we present figure 3.1 which shows how the IMF energies behave when a received signal is noise-dominant (vacant channel) or noisy signal (occupied channel) at SNR = -5 dB.

From this figure, it is shown that the energy of IMF3 (in black solid line) deviates from the vacant channel model (in grey dashed line). The justification of such deviation is due to the presence of PU, where the contribution of signal is added to the noise leading to non-decreasing trend of the IMF energies and hence provide an indication of occupancy. The monotonic decreasing trend of IMF energies is exploited in this thesis to set a null hypothesis test for spectrum sensing purposes.



**Figure 3.1** A comparison of IMF energies for an occupied and vacant channel.

### 3.2.2 EMD-Based Detection Scheme

The received signal  $y(n)$  is the time series representation of a given band under test. Applying EMD approach on  $y(n)$  will result in a set of IMFs (see Sec 3.1). The energy of the  $i^{\text{th}}$  IMF is given as follows:

$$W_i = \sum_{n=1}^N M_i^2(n) \quad (3.2)$$

The detection decision is made upon a simple test of the IMF energies given in (3.2), in which the channel vacancy is decided if the following condition is met:

$$\log_2(W_i) > \log_2(W_{i+1}) \quad , i = 1 \dots K - 1 \quad (3.3)$$

Consequently, if the condition in (3.3) is not satisfied, a possible channel occupancy is declared. However, and due to the non-linear filtering nature of EMD, relying on the condition (3.3) might result in false detection especially in low SNR regimes.

To cope with the problem of false detection, an energy model with confidence interval limit can be utilized based on the work in [50]. Ideally, the energy model with confidence interval is based on the noise-dominant model, which requires the knowledge of the received signal's noise. The noise dominant energy model of the  $i^{\text{th}}$  IMF can be given as follows:

$$\widehat{W}_i = W * 2^{-i} \quad , i \geq 2 \quad (3.4)$$

where  $\widehat{W}_i$  is the  $i^{\text{th}}$  estimated noise-dominant IMF energy,  $W$  is the energy of the total actual noise in the received signal, and the based 2 exponents refers to the energy decreasing rate of IMFs conducting the findings that EMD behaves like a dyadic filter [47]. Practically, the model in (3.4) requires a knowledge of noise energy ( $W$ ), which is unknown, hence estimating the noise energy plays an essential role in this respect.

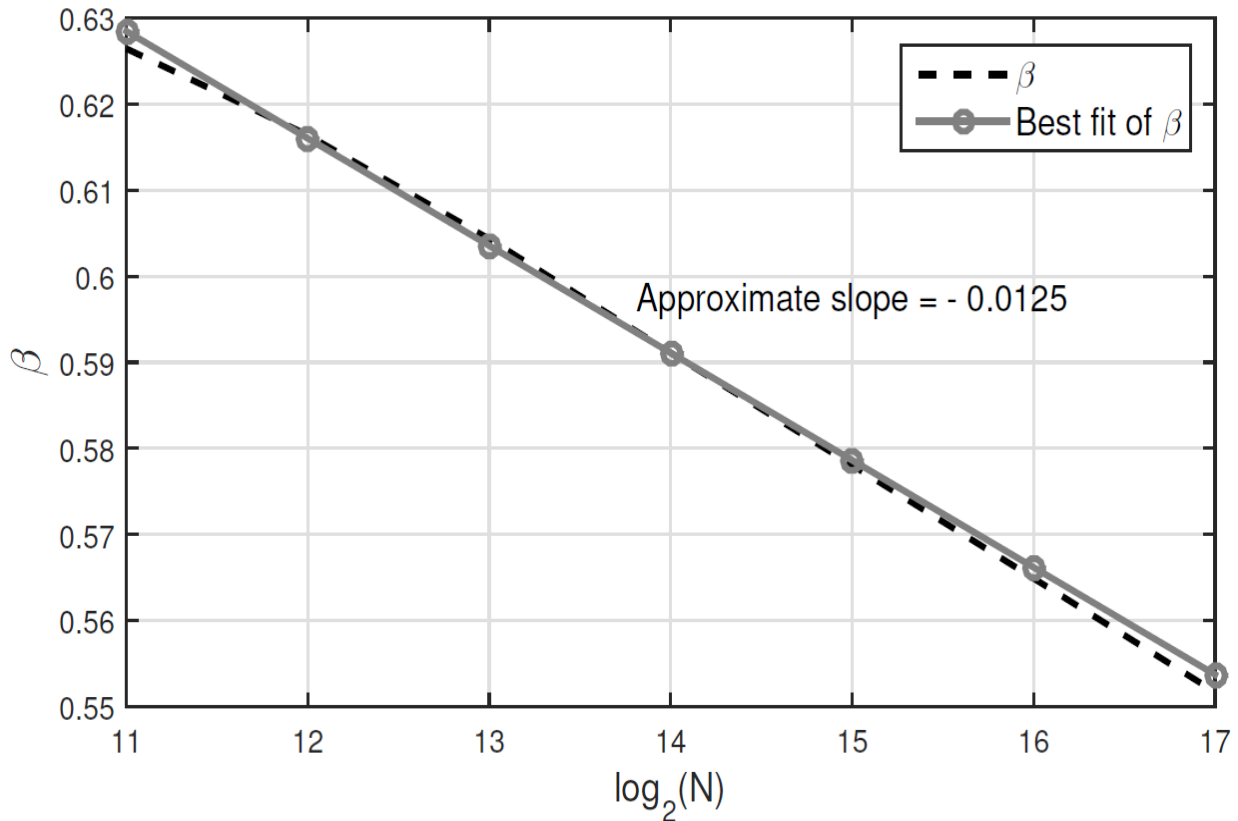
In [50], for a noisy signal, the first IMF,  $M_1(n)$ , is assumed to be dominated mostly by noise, hence a noise-dominant energy model based on the use of  $M_1(n)$  can be utilized to set a statistical boundary limit. The noise-dominant energy model is given as follows:

$$\widehat{W}_i = (\sqrt{\rho} * W_1) * 2^{-i} \quad , i \geq 2 \quad (3.5)$$

where  $W_1$  is the energy of the first IMF, and  $\sqrt{\rho}$  is a scaling factor. In (3.5), the term  $(\sqrt{\rho} * W_1)$  is the estimation of the received noise energy. However, in [50], the scaling factor  $(\sqrt{\rho})$  is a fixed scale thus, the noise estimation will be a function of the first IMF energy only.

In this thesis, we propose an adaptive scaling factor that can result in better estimation of the received signal's noise energy. Figure 3.2 illustrates the relationship between the number of samples (base 2) and  $\beta$ , where  $\beta$  is the ratio of the energy of the first IMF to the total energy of the received signal's noise and is given as follows:

$$\beta = W_1/W \tag{3.6}$$



**Figure 3.2** The relationship between the number of samples  $\log_2(N)$  of the first IMF and  $\beta$  (dashed line).

From Figure 3.2, it is shown that  $\beta$  decreases as  $N$  increases with an approximate slope denoted by  $S$ , where  $S = -0.0125$ . Compared to the best fit model (solid line) using least-squares fit polynomial coefficients, the figure is generated by averaging 5000 trials. The empirical relationship between  $N$  and  $\beta$  can be modeled by a linear fit:

$$\beta(N) = S \log_2(N) + \beta(0) \quad (3.7)$$

where  $\beta(0)$  is the y-intercept of the linear fit. Based on (3.7), eq. (3.4) can be re-written as follows:

$$\widehat{W}_1 = ((S \log_2(N) + \beta(0)) * W_1) * 2^{-i}, i \geq 2 \quad (3.8)$$

From (3.8), the estimated noise model will be a function of  $N$  and the first IMF energy. Subsequently, the energy model with a confidence interval that is based on the noise-dominant model ( $\widehat{W}_1$ ) can be written as follows:

$$\log_2(T_i) = 2^{ai+b} + \log_2(\widehat{W}_1), i \geq 2 \quad (3.9)$$

where  $T_i$  is the  $i^{\text{th}}$  IMF energy at a given confidence interval ( $\alpha$ ), and  $\mathbf{a}$  and  $\mathbf{b}$  are the fitting polynomial parameters for the noise-dominant model (3.4) and these parameters are given in [51], Table 3.1] for both  $\alpha = 95\%$  and  $99\%$ .

The energy model, (3.9), is used as an adaptive threshold to discriminate the IMFs that have a high signal contribution from the ones of noise-dominant contributions. In that sense, and for spectrum sensing purposes, the designed probability of false alarm of the proposed energy detector is denoted by  $P_{fa} = 1 - \alpha$ .

On the other hand, the probability of detection (denoted by  $P_d$ ) is the probability that at least one of the IMF energies in (3.2) exceeds the energy model in (3.9) such as:

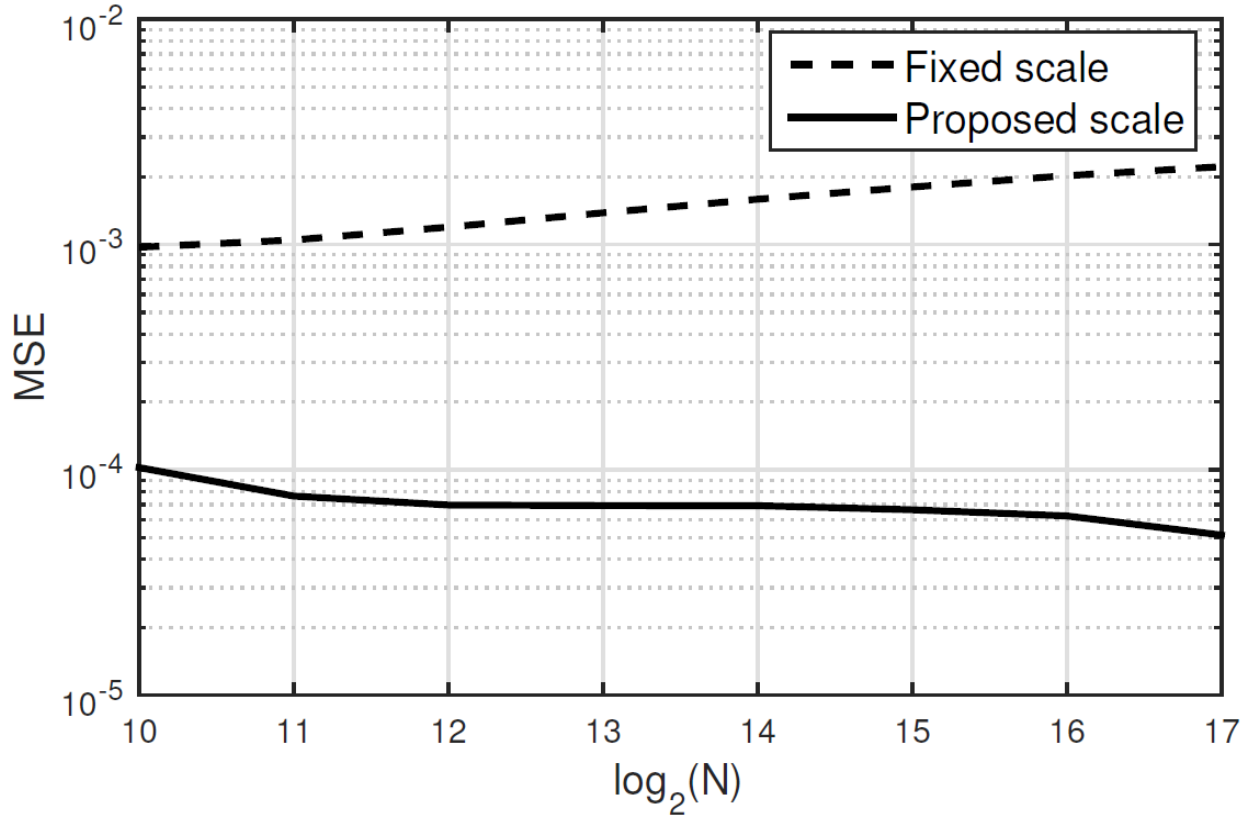
$$P_d = \text{Prob} (\exists (\log_2 W_i > \log_2 T_i)), i \geq 2 \quad (3.11)$$

### 3.3 Simulation Results

In this section, single channel detection is performed to determine occupancy for different SNR and sample size scenarios. A baseband OFDM modulated signal with an observation period ( $T$ ) is used with different sampling sizes;  $N = 1000, 2000, \text{ and } 4000$ . The channel is assumed to be flat and additive white Gaussian noise and the noise power is varied base on a range of SNR values. In this section, Monte Carlo simulations are carried out, where all results are the average of 5000 trials.

Three different energy models, known noise model (3.4), fixed scale model (3.7), and proposed adaptive scale model (3.8), are used to evaluate the model in (3.9) in which these models have significant effect on the overall performance of the proposed detector. The known noise model refers to the assumption that the noise is known a-priori, and that is not valid practically, but used here for demonstration purposes. The fixed scale model refers to the fact that the energy of the first IMF is scaled by a fixed quantity ( $\sqrt{\rho}$ ).





**Figure 3.3** shows the mean squared error of both fixed scale and proposed scale with respect to known noise model (the noise power is unity).

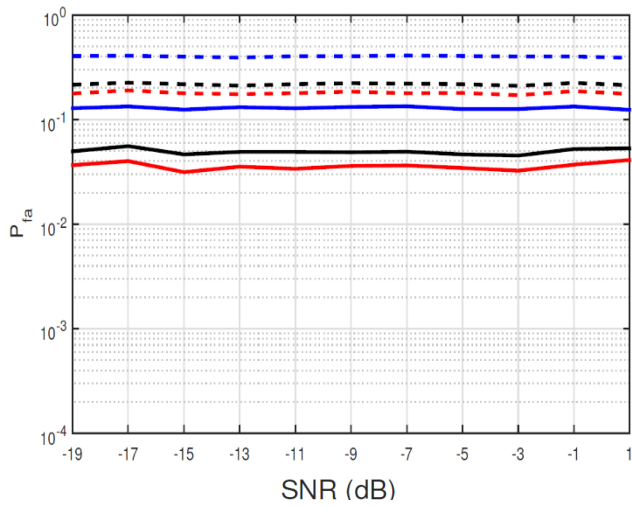
To compare the proposed scale model versus the fixed scale model, the Mean Squared Error (MSE) between the models and the known noise is compared over different sample sizes (N). Figure 3.3 illustrates the MSE of both fixed and proposed scale models about the known noise model. From that figure, it is obvious that the proposed scale is more comparable to the known noise than the fixed scale model.

The justification of that is due to the adaptive scaling of the first IMF energy (function of sample size) that results in better modelling for the total noise energy estimation of the

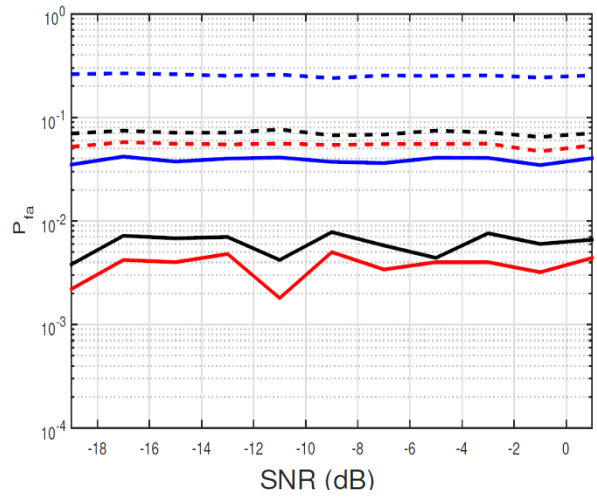
received signal. However, the use of a fixed scale model ( $\sqrt{\rho}$ ) yields in a misestimation of the noise energy and hence more deviation from the known noise model.

Next, the performance of the EMD-based energy detector is examined for each of the models (known noise, fixed scale, proposed scale) for different sample size and SNR values. The EMD-based detector is designed for two different probability of false alarms,  $P_{fa}(\text{designed}) = 0.05$  for  $\alpha = 0.95$ , and  $P_{fa}(\text{designed}) = 0.01$  for  $\alpha = 0.99$ . But remember: EMD is designed for continuous signal, which means in our case (discrete signal) EMD needs to have enough samples to give the desired performance. So, the best noise model is the one that can achieve the desired  $P_{fa}$  using a fewer number of samples than the other noise models.

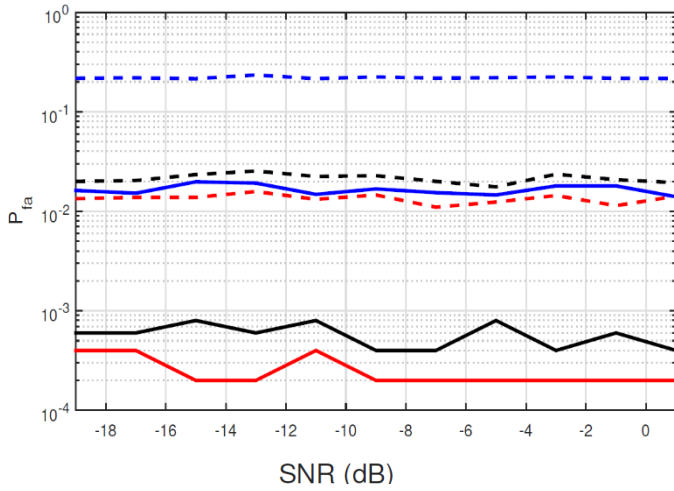
In figure 3.4 (a-c), increasing the number of samples from ( $N = 1000$  to  $2000$ ) has a significant effect on making the proposed EMD based detector achieve the designed / desired probability of false alarm. Also it is clear that the proposed scale model performs closely to the known noise model. But on the other hand, the fixed scales model cannot achieve the designed  $P_{fa}$ , it requires higher sample size at least double the number of samples ( $N = 4000$ ) to satisfy the designed false alarm rate for  $P_{fa}(\text{designed}) = 0.01$ .



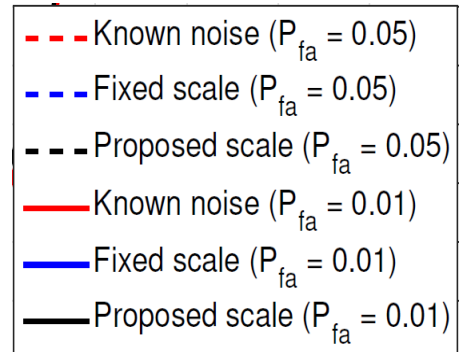
(a)  $N = 1000$



(b)  $N = 2000$



(c)  $N = 4000$

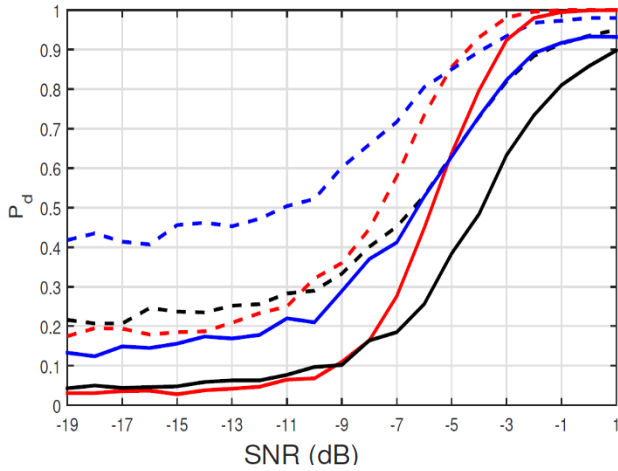


**Figure 3.4** illustration of the probability of false alarm for different SNR values and sample sizes for the known noise, fixed, and proposed scale energy models.

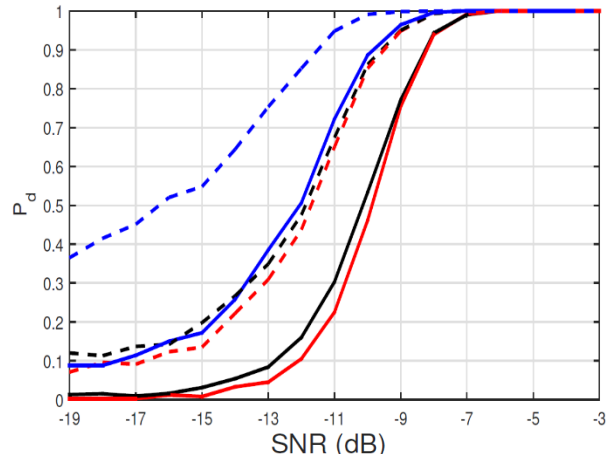
On the other hand, figure 3.5 illustrates the detector performance (for 0.01 and 0.05 designed  $P_{fa}$ ) in terms of probability of detection ( $P_d$ ) for different scenarios. The  $P_d$  of the proposed scale model performs tightly to the known noise model and that performance is enhanced as the sample size increases. However, it is shown that the fixed scale model performs better than the other two models but that is jeopardized to its high false alarm rate exhibited in Fig 3.4.

For further explanation, there is a relation between  $P_d$  &  $P_{fa}$  ( $P_d = 1 - P_{fa}$ ), that means to check any detector performance it is important to check simultaneously both its  $P_d$  &  $P_{fa}$ . As an example, in figure 3.5 (b) at SNR = -12 the fixed scale model achieves 0.9 of detection and in the same case our proposed model achieves same detection percent at SNR=-11. But, if we look at the figure 3.4(b) at the same SNR value the fixed scale model will have 0.03 of false alarm, but our proposed model will have only 0.01 false alarm.

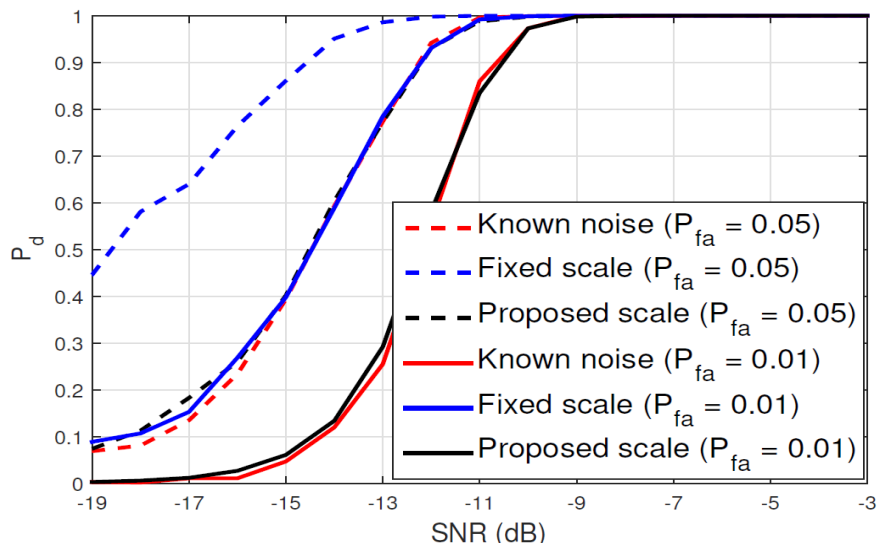
To conclude, the proposed scale model shows higher ability to estimate the noise of the received signal than the fixed scale model, and hence reveals better tradeoff between the  $P_d$  and  $P_{fa}$ .



(a)  $N = 1000$



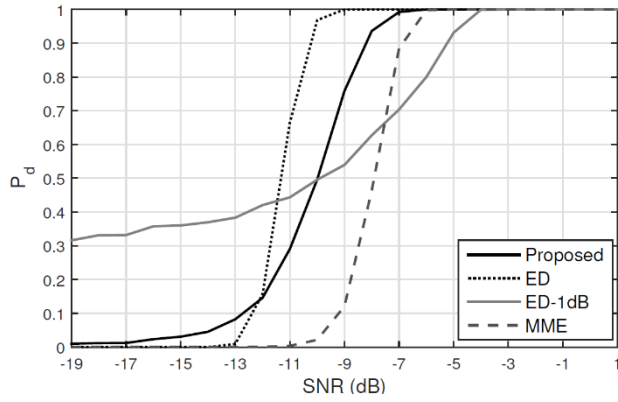
(b)  $N = 2000$



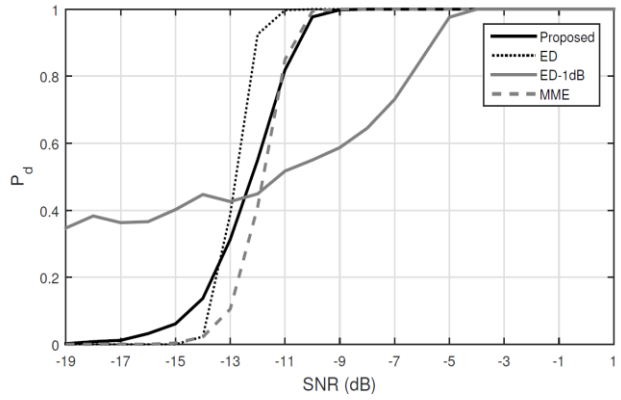
**Figure 3.5** illustration of the probability of detection for different SNR values and sample sizes for the known noise, fixed, and proposed scale energy models.

Finally, the proposed scale model is selected to build an EMD-based energy detector and compared to other known techniques such as the Energy detector model (ED) in [31], and Maximum-Minimum Eigenvalue detector (MME) [36]. In this comparison, the designed  $P_{fa}$  for all techniques is set to 0.01. Both MME and the proposed method are blind and adaptive in terms of setting the detection threshold, unlike the ED which is sensitive to noise uncertainty [32]. Furthermore, both MME and the proposed method requires no prior knowledge about the noise power, where MME is independent in its decision of noise power, while the proposed method estimates it adaptively. In figure 3.6, the comparison between the techniques show that ED performs better than all other techniques. However, in the presence of noise uncertainty (1dB), the performance of ED deteriorates significantly. The proposed method outperforms both MME (with a smoothing factor = 8) and the ED with noise uncertainty by a gain of 2 and 3 dB respectively at  $P_d = 1$ .

(Note: In figure 3.6 the relative performance maintained for different number of samples (N=2000, 4000)).



(a)  $N = 2000$



(b)  $N = 4000$

**Figure 3.6** A comparison of the probability of detection of the proposed method versus ED and MME for  $N = 2000, 4000$ .

## Chapter 4

# Conclusion and Future Work

### 4.1 Conclusion

The major focus of this work was to present a detection scheme that can allow for estimation of noise in the presence of the signal which is essential for energy detection based schemes.

In chapter 2, we focused on the operation principle of the cognitive radio and the theoretical side of spectrum sensing. An overview of various detection methods and standalone spectrum sensing techniques along with their classifications was given, and their suitability to different use cases was discussed. Two detection algorithms, Energy Detector (ED) and Eigenvalue based detection were introduced in detail in order to give theoretical background for the implementations presented in chapter 3.

Clearly, each of the detection algorithms is fit for a purpose and has their pros and cons. For example, ED is fast, computationally simple and energy efficient compared to other alternatives. However, ED threshold is a function of noise power which is in turn assumed at a prior time or estimated through measurements. Consequently, misestimating the noise power might result in a severe degradation in the detector performance.



In chapter 3, we presented our method which is an energy detector based on the behavior that Empirical Mode Decomposition (EMD) has towards vacant channels. This method exploits the ability of EMD to decompose a signal into IMFs in which the energy of these modes can be utilized for channel detection. The energy of the first IMF is scaled to evaluate the noise-dominant model which is used to obtain a detection threshold at some confidence interval ( $\alpha$ ).

Next, the performance of the EMD-based energy detector was examined for each of the models (known noise, fixed scale, proposed scale) at different sample sizes and SNR values. The proposed scaling model exhibited better performance than the fixed scale model in terms of  $P_d$  and  $P_{fa}$ . Further, the proposed scale model was selected to build an EMD-based energy detector and compare it to other known techniques such as the Energy detector model (ED) and Maximum-Minimum Eigenvalue detector (MME). Simulation results indicated the EMD-based energy detector (built upon the use of the proposed scale model) outperforms both ED (with a noise uncertainty) and MME over a range of SNR values.

Hence, if we employ our proposed technique to sense the signal in a cognitive environment, better results could be achieved, thereby making a way towards efficient spectrum utilization.

## **4.2 Future Work**

As an empirical approach, EMD can be investigated for the future work as follows:

- EMD is difficult to model mathematically, because its algorithm procedure consists of ad-hoc steps and because of its non-linear sifting process. Thus, investigating

a mathematical framework will make the algorithm more solid and easier to analyze.

- The EMD is based on an iterative use of spline interpolation, therefore it is computationally expensive. To be applied effectively as an online processor, more efficient sifting approaches must be researched.

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