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Money, output and real wages in a New Keynesian framework with heterogeneous labor and monopsonistic firms

Robert J. Martel

University of New Hampshire, Durham

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**MONEY, OUTPUT AND REAL WAGES IN A NEW KEYNESIAN FRAMEWORK
WITH HETEROGENEOUS LABOR AND MONOPSONISTIC FIRMS**

BY

**ROBERT J. MARTEL
S.M., Massachusetts Institute of Technology, 1966
B.S., Boston University, 1962**

DISSERTATION

**Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of**

Doctor of Philosophy

in

Economics

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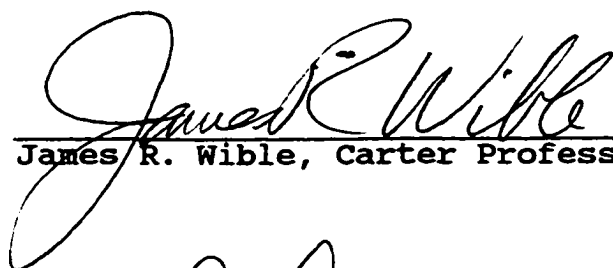
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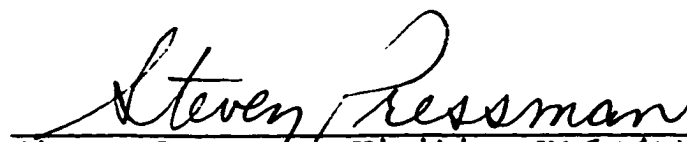
Dissertation Director, Michael D. Goldberg
Associate Professor of Economics



James R. Wible, Carter Professor of Economics



W. David Bradford III, Associate Professor of Economics
Medical University of South Carolina



Steven Pressman, Visiting ~~Associate~~ Professor of
Economics, Trinity College



David Colander, Christian A. Johnson Distinguished
Professor of Economics, Middlebury College

Date

11/20/98

I dedicate this volume to my wife Tina, without whom I never could have begun, let alone finish. They also love deeply, who sacrifice and wait.

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ABSTRACT

MONEY, OUTPUT AND REAL WAGES IN A NEW KEYNESIAN FRAMEWORK WITH HETEROGENEOUS LABOR AND MONOPSONISTIC FIRMS

by

Robert J. Martel
University of New Hampshire, December, 1998

Representative agent models do not match up well with three stylized facts of the business cycle: a money-output connection, countercyclical markups, and acyclical real wages. This thesis investigates whether a New Keynesian model which departs from the representative agent assumptions and models heterogeneity and imperfect competition in the labor market is more consistent with these stylized facts.

One possible explanation of countercyclical markups and acyclical real wages is that labor markets are monopsonistic and monopsony power is weaker during expansions than in recessions. This would require that the elasticity of labor supply be procyclical. This is not possible if worker preferences are homothetic.

An aggregate labor supply function for heterogeneous labor is constructed. Labor is indivisible, and workers are heterogeneous with respect to their nonlabor income endowments and preferences for risk. Nonlabor income is assumed to be distributed Lognormally. Workers' optimizing choices in a take-it-or-leave-it job market are determined by a reservation

wage function which relates reservation wages to nonlabor income. Aggregate labor supply is a composite function of the Lognormal distribution of nonlabor income and the reservation wage function.

The parameters of the aggregate labor supply function are affected by changes in the aggregate price level and the interest rate. An increase in the money supply increases aggregate labor supply. If workers have increasing relative risk aversion, an increase in money also increases the elasticity of labor supply. The magnitude of this increase depends upon the magnitudes of the interest- and wealth-elasticities of the aggregate money demand function.

If firms are monopsonistic, the elasticity of the aggregate labor supply function will be procyclical with respect to monetary policy, and markups will be countercyclical. A calibrated version of the model indicates that the real wage would be weakly countercyclical, acyclical, or weakly procyclical, depending on the short-term elasticities of the price level and the interest rate with respect to changes in M2. The model implies a wealth-effects transmission channel from monetary policy to aggregate labor supply, employment and output, restoring a traditional Keynesian theme of a monetary theory of production.

CHAPTER I

REAL WAGES, MARKUPS, AND THE MONEY-OUTPUT CONNECTION

1.1 Introduction

Macroeconomic models can be classified according to how well their predictions match up with two well-established stylized facts of the business cycle:[Fischer, (1988)]

- (1) changes in the nominal money stock are positively correlated with changes in real output;
- (2) the aggregate real wage is acyclical or weakly procyclical.

Each stylized fact has had its own history of debate and has been the subject of an extensive body of empirical and theoretical research. While most economists would probably agree that these two statements have been descriptive of most U. S. business cycles, when it comes to providing theoretical explanations the contemporary literature breaks down into two competing schools of thought: the New Keynesian (NK) and the Real Business Cycle (RBC) theories¹.

¹In the U. S. the published debates have been largely confined to these two mainstream schools of thought. This is not to discount, by their exclusion here, the theoretical contributions of other schools of thought such as the Post-Keynesian, Non-Walrasian and Post-Walrasian, which have had greater acceptance abroad.

In this chapter I examine representative models of both schools and show that neither school has had much success in explaining both stylized facts in a single model. The principal conclusion of this chapter is that, within the conventional paradigms of macroeconomic theory, it has been difficult to construct a model in which changes in money and output are positively correlated and the aggregate real wage is acyclical or weakly procyclical.

These two stylized facts represent nontrivial characteristics of national economies. The first one offers a rationalization of the Keynesian Phillips curve and leads to the inference that cyclical fluctuations are influenced by monetary policy. However, it is also consistent with the RBC view that money is neutral but responds to changes in real output. The second stylized fact implies that labor productivity and labor's share of national income are procyclical, which is hard to reconcile with the Keynesian assumption of a stable short-run aggregate production function with fixed capital and decreasing returns to labor. (Sargent and Wallace, 1974; Canzoneri, 1977; Hall, 1991)) The RBC explanation is that the aggregate production function is unstable and subject to cyclical shocks to labor productivity. Thus, the two main schools of thought in macroeconomics offer contrasting explanations of how cyclical shocks are propagated in the

aggregate labor market. It will be demonstrated in this chapter that neither school gets it quite right, i. e., neither explanation is consistent with both stylized facts.

This inconsistency appears to involve the core arguments of the New Keynesian-RBC debate, and begs an explanation. The source of the difficulty may reside in a common premise of the two schools rather than their differences. What is common to both RBC and New Keynesian models, and to macroeconomics generally, is the method by which the atomistic choice-theoretic behavior specified for individual agents is attributed one-for-one to their corresponding aggregates -- the assumption of a representative agent. One of the main arguments of this chapter, and of this entire thesis, is that the discrepancy between macro models and the stylized facts about real wage behavior and the non-neutrality of money is due to the restrictive assumptions inherent in the representative agent method of aggregation, which preclude certain relationships at the macro level.

It is well known that representative agent aggregation is valid for consumers if and only if their preferences are quasi-homothetic [Lewbel 1989, Kirman 1992, Martel 1996], and for firms if and only if their production functions are identical and linearly homogeneous [Sargent (1987)]. These assumptions restrict the marginal responses of all agents on

the same side of a market to be identical and independent of scale, i. e., to be homogeneous. Heterogeneity of marginal responses is excluded by assumption. This exclusion may be problematic when applied to the aggregate labor market, since there is ample evidence in the literature that labor is heterogeneous and that heterogeneity often matters for labor market outcomes². [Ehrenberg (1971), Coleman (1984), Kydland (1984), Heckman and Sedlacek (1985), Keane, Moffitt and Runkle (1986).] The evidence is more than circumstantial. The review of the extant literature in this chapter suggests that heterogeneity in the labor market may be an important factor in the real-wage anomaly.

It is a contention of this chapter that part of the difficulty in constructing macro models that match up with these two stylized facts is that the behavior that is observed empirically is precluded theoretically by the representative method of aggregation. This proposition is the focal point for the overall research agenda of this thesis, which is to demonstrate that, by moving away from a representative agent framework and employing a more general method of aggregation, it is possible to fit both stylized facts concerning money and real wages within an otherwise " Keynesian" setup.

²There also was the Cantabrigian debate over the homogeneity of capital and the existence of an aggregate production function, which will not be resurrected here. Solow (1957), Ackley (1961), Kuh (1966) and Fisher (1969) also expressed doubts about the existence of a meaningful aggregate production function.

In this chapter I present evidence in favor of this main proposition, discuss some of the methodological issues and problems that need to be addressed in the research, and pose some questions and working hypotheses which will be explored in the thesis. The chapters which follow will explore ways of modeling aggregate behavior in the labor market when those restrictive assumptions are relaxed and heterogeneity is modeled explicitly. A concluding section provides a guide to the contents of Chapters II through V.

1.2 The Real Wage Anomaly

One of the more intriguing puzzles in macroeconomics is why movements in the aggregate real wage are small relative to fluctuations in employment and output over the business cycle. Figure 1-1 shows the historical relationship between the aggregate real wage and output (both variables detrended and in logs). The real wage appears to have been procyclical during the 1970's, but outside of that time period it is not possible to discern a persistent relationship from the graph. Table 1-1 shows estimated elasticities of the aggregate real wage with respect to output and employment for the period 1949-1993. Only two of the elasticities are significantly different from zero, and those have low values³.

³The elasticities in Table 1-1 are the results of replications by Abraham and Haltiwanger (1995) of several previous studies.

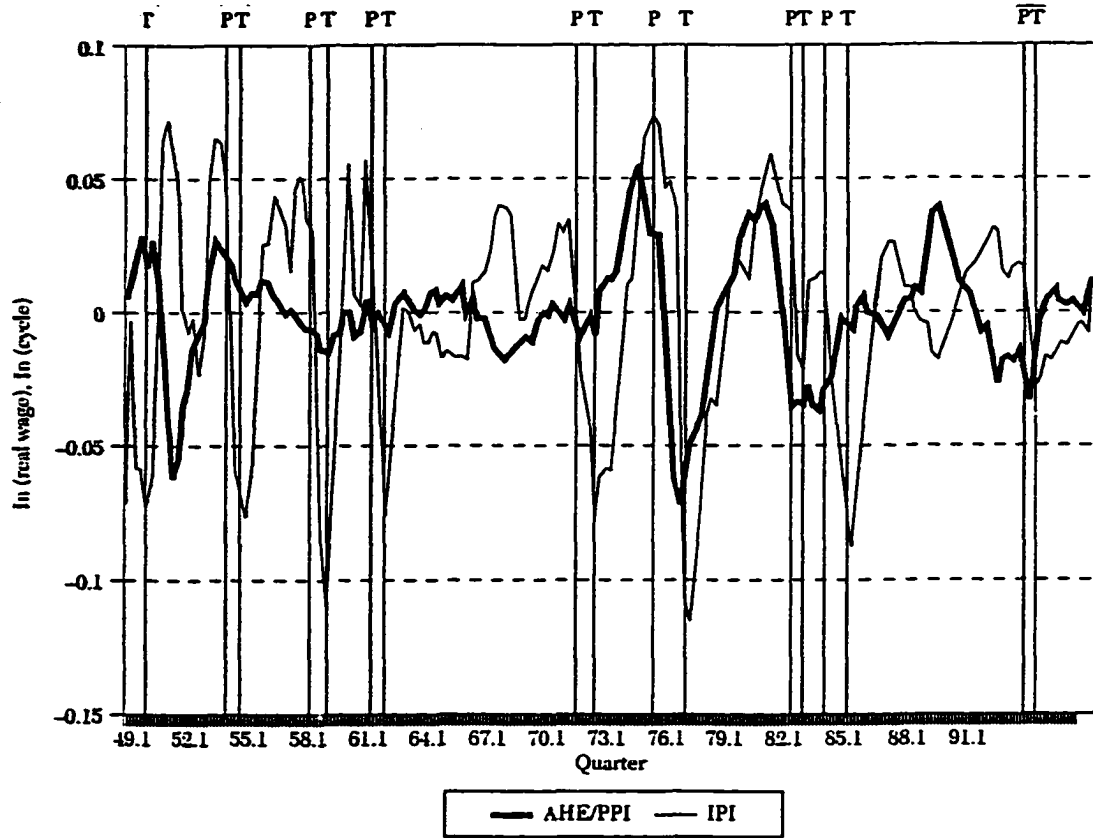


Figure 1-1. Aggregate Real Wage Vs. Industrial Output, 1949-1994
(From Abraham and Haltiwanger (1995))

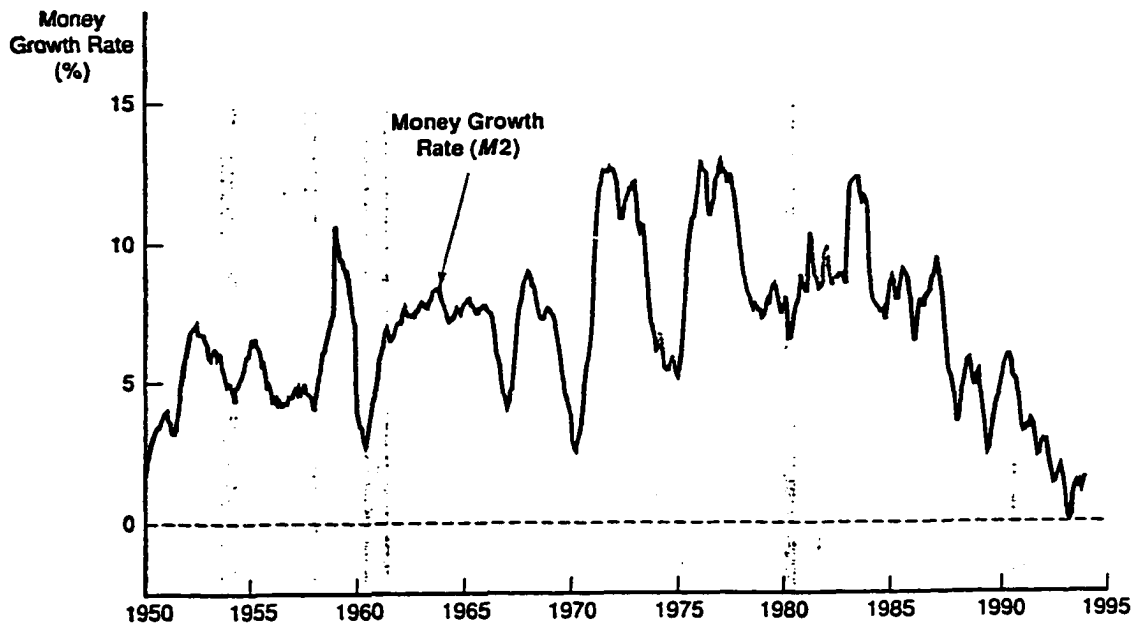


Figure 1-2. M2 Annual Growth Rate Vs. the Business Cycle
(Federal Reserve Board)

TABLE 1-1

CYCLICAL ELASTICITY OF THE AGGREGATE REAL WAGE
Quarterly BLS Data, Hodrick-Prescott Detrended

<u>Real Wage Measure</u>	<u>Cyclical Indicator</u>	<u>Elasticity</u>		
		<u>1949-69</u>	<u>1970-93</u>	<u>1949-93</u>
AHE/PPI	IPI	-.141 (.091)	+.186* (.090)	.007 (.076)
AHE/PPI	Employment	-.222 (.154)	.020 (.152)	-.098 (.110)

HEI/PPI	IPI	-.185* (.087)	.080 (.043)	-.024 (.082)
HEI/PPI	Employment	-.296 (.152)	.001 (.070)	-.147 (.115)

From Abraham and Haltiwanger (1995), Table 3. Estimation by OLS on logarithmic data. Numbers in parentheses are standard errors.

*Statistically significant at 5%; others are not significant.

AHE: Average Hourly Earnings. PPI: Producers' Price Index.

IPI: Industrial Production Index. HEI: Hourly Earnings Index.

Data for HEI are 1949-1988.

TABLE 1-2

CYCLICAL ELASTICITIES OF DISAGGREGATED REAL WAGES
Based on PSID Annual Data

<u>Investigators</u>	<u>Data Period</u>	<u>Individual Elasticities</u>	<u>Aggregate Elasticity</u>	<u>Bias</u>
Stockman (1983)	1967-80	-1.31	-.96(ns)	---
Coleman (1984)	1968-79	-1.53		
Mather (1987)		-1.07		
Solon & Barsky (1989)	1967-84	-1.26	-.78	+.48
Solon, Barsky & Parker (1994)	1967-87	-1.16	-.57	+.59

From Abraham and Haltiwanger (1995), Table 5. Cyclical variable is the unemployment rate. GDP deflator used by all investigators.

(ns) = not significant; all others are significant at 5% or lower.

One can infer from Figure 1-1 and Table 1-1 that there is no set pattern to the behavior of the aggregate real wage; its elasticity was weakly procyclical in the 1970's, weakly countercyclical at other times, seldom significantly different from zero, and when estimated over two or more decades the elasticities essentially cancel out. From the data in Table 1-1, one cannot reject the hypothesis that the aggregate real wage is acyclical with respect to employment⁴.

There is some evidence that real wages in the U. S. are more procyclical at lower levels of aggregation (Bils, 1985). Several investigators have found real wages estimated from individual wage data to be procyclical. These findings are summarized in Table 1-2. The BLS measures of aggregate wages are not adjusted for changes in composition of the workforce, and thus have a composition bias. The bias is countercyclical because low-wage workers are disproportionately represented in layoffs during recessions and in hires during expansions; thus their wages and hours are weighted accordingly in computing the economy-wide average wage. Solon, Barsky and Parker (1994) estimated a composition bias of +0.59 in the elasticity of the real wage with respect to unemployment during the

⁴The choice of a cyclical indicator depends on what hypothesis one is interested in testing. Since the real product wage is determined in the labor market, the natural indicator would seem to be employment or the unemployment rate. In Table 1-1 the difference between the elasticities with respect to the IPI and employment during 1970-93 can be attributed to the fact that industrial output was more cyclical than total employment during that period. Other problems of measurement are discussed in Abraham and Haltiwanger (1995).

1967-87 period⁵. (Table 1-2, row 5).

This finding suggests that real wages were more procyclical during that period (and presumably the entire 1949-1993 period) than would be inferred from the data in Table 1-1. It should be noted that, however measured, real wages were unusually procyclical during the 1970's, which was a period of stagflation with historically high inflation and supply-side productivity shocks. The direction and magnitude of composition bias in other time periods is unknown.

The preponderance of the evidence is that the aggregate real wage is neither countercyclical nor strongly procyclical (i.e. elasticity approaching +1). Depending on the time period examined, the source of fluctuations, and the statistical methods employed, observed aggregate real wages are either acyclical or weakly procyclical⁶. The issue of composition bias serves as a reminder that the national labor "market" is quite heterogeneous, with distributions of human capital, skills, productivity and tastes for work which may engender

⁵Using Okun's 3:1 rule to convert changes in the unemployment rate to changes in GDP, the estimated adjustment for composition bias in Table 1-1 would be $3(.59) = +.177$ for the IPI elasticity in the period 1970-93, resulting in an adjusted procyclical elasticity of $(.186 + .177) = +.363$, which is moderately procyclical. This adjustment is not applicable to other time periods or cyclical measures in Table 1-1 due to lack of comparability.

⁶Abraham and Haltiwanger (1995) provide a contemporary review and analysis of the extensive empirical literature on real wage behavior. They conclude that after adjusting for composition bias the aggregate real wage is procyclical, but they do not commit to magnitudes or time periods.

non-homogeneous marginal responses leading to fallacies of composition. One of the major arguments of this chapter and thesis is that modeling such heterogeneous responses of the labor market is an important avenue to a theoretical explanation of the money-output connection and acyclical real wages.

1.2 Countercyclical Markups

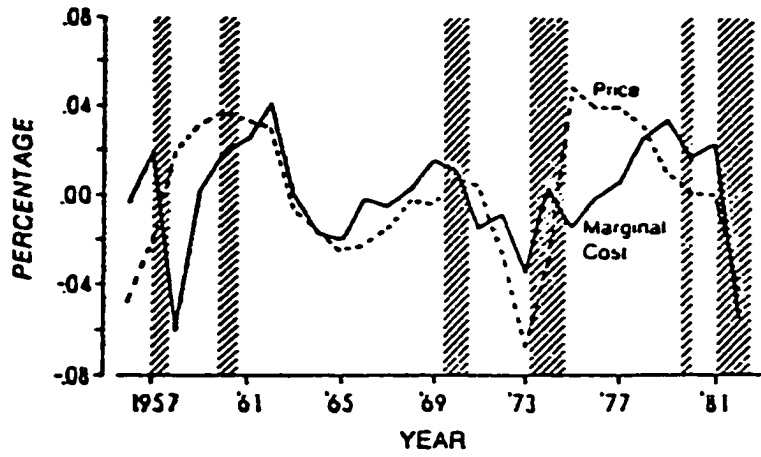
A profit-maximizing monopolistic firm will set its product price above marginal cost, which creates a markup:

$$\mu(P^*) = \frac{P^* - C'[q(P^*)]}{P^*} = -\frac{1}{\eta(P^*)} \quad (1.1)$$

Here P^* is the monopolist's optimum price, $q(P^*)$ the corresponding optimum quantity to produce and sell, $C'(q(P^*))$ is marginal cost, and $\eta(P^*)$ is the elasticity of product demand, the inverse of which is Abba Lerner's index of monopoly power. There is ample evidence that many U. S. industries are imperfectly competitive and, more significantly, that price markups in those industries are countercyclical (Bils, 1987, 1989; Hall, 1988; Rotemberg and Woodford, 1991). Equation 1.1 shows that if a firm's markup is countercyclical then its monopoly power is also countercyclical (i.e. is weaker in expansions and stronger in recessions). Equivalently, the elasticity of industry demand, $\eta(P^*)$, in an imperfectly competitive industry, must be procyclical.

Figure 1-3 (a) and (b) show the cyclical behavior of markups over aggregate marginal cost in U. S. manufacturing industries, as calculated by Bils (Panel (a)) and Rotemberg and Woodford (Panel (b)). The divergence of markups from the level of employment is evident in the 1973-75 and 1981-83 recessions and in the expansion of the mid-1980's. Bils estimated the elasticity of markups with respect to aggregate production employment to be -0.333 , highly significant, and persistent in sign across two-digit industries.

As with the real-wage anomaly, the hypothesis that demand elasticities are procyclical is problematic for the representative agent method of aggregation, which is employed almost universally in macroeconomics. An industry demand schedule is an aggregation of individual consumer demand schedules. The representative agent method of aggregation requires the underlying assumption of identical, homothetic consumer preferences, which in turn implies that the elasticity of consumer demand is invariant with respect to changes in scale (income or wealth). In this case, the representative consumer's demand curve shifts iso-elastically with a constant markup in response to changes in aggregate income. The evidence of strongly countercyclical markups contradicts a representative agent model, and suggests that it may be productive to depart somewhat from the representative agent assumptions and model heterogeneity in the economy.



The behavior of aggregate marginal cost and aggregate price

Figure 1-3(a). Cyclical Behavior of Markups
(From Bils (1988))

DEVIATIONS FROM TREND OF HOURS AND THE MARKUP

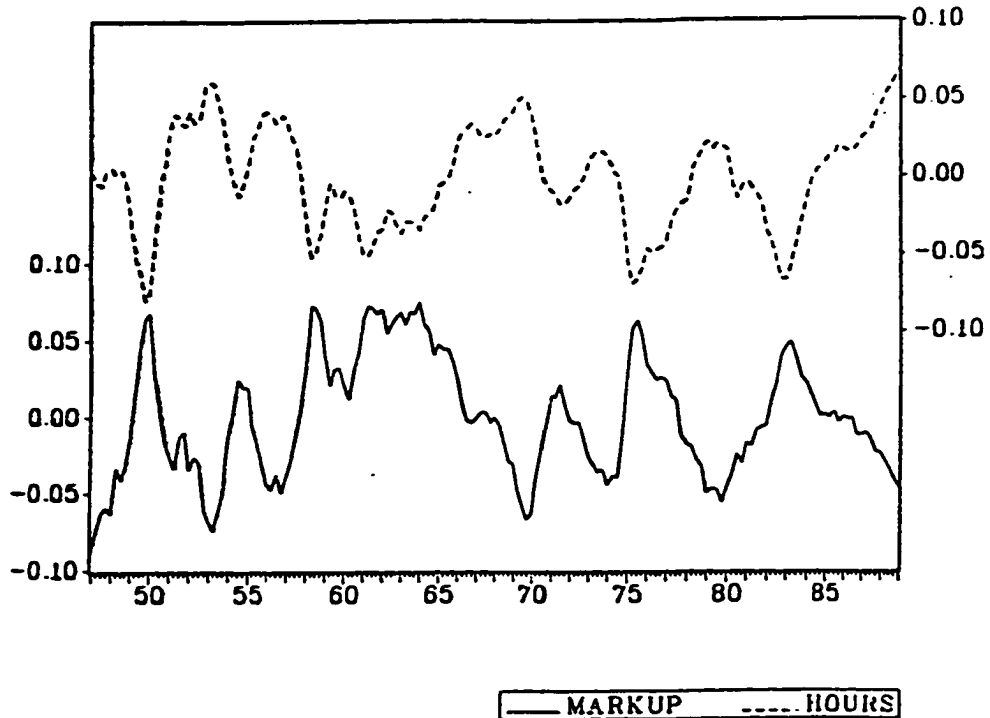


Figure 1-3(b). Cyclical Behavior of Markups
(From Rotemberg and Woodford (1991))

The real wage and markup anomalies are closely related. Keynes (1939), in responding to evidence offered by Dunlop (1938) that real wages in England were not countercyclical as implied by the General Theory, offered an explanation in terms of offsetting countercyclical markups in imperfectly competitive industries, an idea which he attributed to Kalecki (1938). However, it is evident from the previous discussion that Keynes' explanation is also inconsistent with representative agent aggregation. An explanation of acyclical or procyclical real wage behavior in terms of cyclical elasticities of supply or demand is problematic within a representative consumer framework.

New Keynesian models rationalize sticky prices by assuming that imperfectly competitive firms have weak incentives to change prices when there is a change in demand. The weak incentive may be attributed to menu costs (Mankiw, 1985) or non-optimizing behavior (Akerlof and Yellen, 1985). Most of these models, including the two just cited, assume constant marginal cost. The effect of a change in demand on the markup will depend on the firm's price-setting behavior in relation to its marginal cost. If marginal cost is constant and menu costs are small, a profit-maximizing firm will maintain its price and the markup in Equation 1.1 will be constant, contrary to what is generally observed.

On the other hand, if marginal cost is increasing and the firm maintains its price, the markup will be countercyclical, which is observed but is also inconsistent with representative agent aggregation over consumers. It appears that the implications of the New Keynesian imperfect competition models need to be reconciled with the empirical evidence on markups. Again, it may be fruitful to move away from the representative agent framework in order to accomplish this.

1.4 The Money-Output Connection

One of the more important stylized facts of macroeconomics is that lagged changes in real GDP over the business cycle are positively correlated with changes in the nominal money supply. The empirical evidence in support of this proposition is quite strong⁷. Figure 1-2 shows that every post-1950 recession in the U. S. has been preceded by a significant decline in the rate of growth of M2, followed by a sharp reversal of that trend during recessions and an increase in the growth rate during subsequent expansions. (The 1990-91 recession was an exception.) Although visually impressive, such a graph is hardly proof that a positive correlation exists over time.

⁷The empirical evidence on the money-output connection is discussed greater detail in Chapter II.

The landmark study of money by Friedman and Schwarz (1963) concluded that changes in money cause changes in economic activity. More recent econometric studies employing bivariate and multivariate causality tests (Sims, 1972; Mishkin, 1983) also found that changes in money Granger-cause output⁸. The positive correlation is not in question; however, the direction of causality is an issue which separates the two schools of business-cycle theory -- Keynesians and Real Business Cycle (RBC) advocates -- who have agreed to disagree on the issue of the neutrality of money⁹.

The non-neutrality of money implies the existence of significant non-homogenous demand or supply responses somewhere in the economy. In a Keynesian setup these are generally assumed to be caused by nominal rigidities in prices or wages, and much of the New Keynesian research agenda has been directed at establishing choice-theoretic microfoundations for such rigidities. These have generally taken the form of

⁸Recent surveys of the empirical evidence on the money-output connection are Bernanke (1986), Romer and Romer (1989), Blanchard (1990) and B. Friedman (1995). Although some studies have not found a causal relationship, most conclude that the rate of money growth is a causal factor in real output fluctuations at business-cycle frequencies. Causation is implied when the innovation in money is exogenous to changes in output.

⁹A third interpretation of the evidence, associated with the Post-Keynesian school, is that money is endogenous to the real economy, and that the positive correlation between money and output is due to the procyclical demand for and supply of credit. This interpretation places the central bank in a more or less accommodative rather than a proactive posture. Brunner and Meltzer (1993, pp. 55-58) critique this interpretation and reject it on empirical grounds. Nevertheless, the credit channel hypothesis is very much alive. (See Bernanke and Gertler (1995) for a recent review.)

frictions or "menu" adjustment costs which cause prices or wages to be sticky. But the incorporation of sticky prices or wages in a model of imperfect competition also affects the cyclical behavior of markups and the real wage in response to external shocks. This becomes evident when the markup is expressed as:

$$\mu(\ell) = \frac{P}{\frac{W}{F_\ell}} = \frac{PF_\ell}{W} = \frac{1}{1 + \frac{1}{\eta}} \quad (1.2)$$

where ℓ is labor, F_ℓ is its marginal product, W is the money wage, P is the product price, and η is the elasticity of product demand. Equation 1.2 can be rearranged thusly:

$$\frac{W}{P} = \left[1 + \frac{1}{\eta}\right] F_\ell = \frac{F_\ell}{\mu(\ell)} \quad (1.3)$$

With fixed technology and diminishing returns to labor in the conventional short-run production function, F_ℓ is countercyclical. If nominal wages are sticky and prices are flexible, the real wage will be countercyclical if markups are constant (including the special case of zero, i.e., perfect competition¹⁰). This is the Traditional Keynesian result which nevertheless is counterfactual. Keynes's conjecture to Dunlop was that $\mu(\ell)$ might be countercyclical enough to offset the influence of F_ℓ and cause W/P to be constant or even procyclical. Thus, if the goods market is imperfectly competi-

¹⁰This assumes decreasing returns to labor, i.e. $F_{\ell\ell} < 0$.

tive the cyclical behavior of the real wage will depend on the cyclical behavior of markups and the elasticity of F_L .

Conversely, if prices are sticky and wages are flexible, then, (a) markups will be constant and the real wage will be countercyclical if marginal cost is constant, or (b) markups will be countercyclical and the real wage will be strongly procyclical if marginal cost is increasing and the elasticity of labor supply is low. (Romer 1996, pp. 218-219). This outcome is closer to the stylized facts, but the real wage may be too strongly procyclical. Also, the assumption that prices are stickier than wages may not be realistic.

If both wages and prices are sticky, we have the case of generalized disequilibrium in which the out-of-equilibrium adjustment path of the real wage in response to demand shocks is generally ambiguous, although under certain conditions it could be procyclical (Barro and Grossman, 1976, pp. 95-98.)

Thus, the responses of markups and the aggregate real wage to an external shock also depend critically on how nominal and real rigidities are specified in a model. The behavior of markups and the real wage implied by the New Keynesian models of imperfect competition are virtually predetermined by the stickiness assumptions, and are not entirely consistent with the empirical record.

1.5 Survey of Representative Macroeconomic Models

This section presents some evidence in support of the proposition that most macroeconomic models find it difficult to explain the actual behavior of real wages and the non-neutrality of money. The various schools of macroeconomic theory are classified along these two dimensions, and then the implications of representative models of each class are summarized¹¹.

Figure 1-4 classifies the main schools of macroeconomic theory according to the neutrality of money and the predicted cyclical behavior of the real wage. The Classical and Real Business Cycle models preserve the classical dichotomy between nominal and real variables, and therefore make no causal connection between money and output. Traditional Keynesian and Post-Keynesian models can explain involuntary unemployment but predict a countercyclical real wage. Christiano and Eichenbaum (1992) point out that Keynesian models understate, and RBC models overstate, the degree of positive correlation between real wages and employment.

¹¹It would be impossible to be all-inclusive in this survey, and therefore I have selected models which have been judged to be canonically representative of their class based on their citations and inclusion in compendia. It is possible that counter-examples in the literature have been overlooked.

**BEHAVIOR OF
AGGREGATE REAL WAGE $\frac{W}{P}$**

		COUNTERCYCLICAL	ACYCLICAL OR PROCYCLICAL
YES	TRADITIONAL KEYNESIAN	DISEQUILIBRIUM	
	POST-KEYNESIAN	NEW KEYNESIAN	
MONEY-OUTPUT CONNECTION?			
NO	CLASSICAL	REAL BUSINESS CYCLES	

Figure 1-3. Classification of Macroeconomic Models

Both the Disequilibrium and New Keynesian classes of models are capable, under certain conditions, of exhibiting rigid or procyclical real wages. However, their implications are not quite in accord with the acyclical real wage behavior shown in Table 1-1. The specifics are discussed below.

1.5.1 Traditional Keynesian and Post-Keynesian Models

Traditional Keynesian models based on *The General Theory* are summarized in Table 1-3. Keynes' model in *The General Theory* (1936) had a causal role for money but implied a countercyclical real wage. To clarify ideas, it may be useful to describe in modern terms why this was so.

Keynes accepted the classical theory of a competitive labor market in which the real wage is equal to the marginal product of labor. With capital and technology fixed in the short run, the marginal product of labor declines with increasing employment, so that:

... with a given organization, equipment and technique, real wages and the volume of output (and hence of employment) are uniquely correlated, so that, in general, an increase in employment can only occur to the accompaniment of a decline in the rate of real wages.

[Keynes, 1936, Chap. 2, p. 17]

In the short-period framework of *The General Theory* Keynes implicitly ruled out shocks to technology or the productivity

TABLE 1-3

TRADITIONAL KEYNESIAN MODELS

<u>MODEL</u>	<u>MONEY-OUTPUT</u>	<u>BEHAVIOR OF W/P</u>
Keynes G. T. (1936)	Non-neutral	Countercyclical
Hicks IS/LM (1937)	Non-neutral	Countercyclical (with Phillips curve supply side)
Keynes (1939) Reply to Dunlop-Tarshis	Non-neutral	Acyclical or Pro- cyclical if markups are countercyclical

TABLE 1-4

REPRESENTATIVE NEW KEYNESIAN MODELS

<u>MODEL</u>	<u>MONEY-OUTPUT</u>	<u>BEHAVIOR OF W/P</u>
<u>STICKY W OR P</u>		
Baily (1974) W/P	None (Real Model)	Rigid (Optimal Path)
Fischer (1977) Taylor (1980) W	Non-neutral (Staggered Wage-Setting)	Rigid (Constant Markup Assumed)
Mankiw (1985) P	Non-neutral (Menu costs in goods market)	Procyclical
Akerlof & Yellen (1985) W	Non-neutral (Near-rational Wage-setting)	Rigid (maximizers) Countercyclical for non-maximizers
Blanchard & Kiyotaki (1987) W, P	Non-neutral (Menu costs in both goods & labor markets)	Rigid (W and P are fixed)
Ball & Romer (1990) W, P	Non-neutral (Real and nominal rigidities; heterogeneous labor, efficiency wages)	Acyclical (assumed)

of labor, implying that firms would always be on their (stationary) labor demand curves.

This situation is shown in Panel (a) of Figure 1-5. Keynes posited that nominal wages would be rigid downward, so that if prices fell in a recession the real wage would be above the market-clearing level. Employment and output would be constrained by effective demand, and there would be involuntary unemployment. The real wage would move countercyclically along the labor demand curve in response to changes in effective demand, e.g., as between point A and point B in Figure 1-5(a)¹².

Dunlop (1938) and Tarshis (1938) confronted Keynes with data purporting to show that real and money wages in England were positively correlated; under the assumption that money wages are procyclical, one could infer that the real wage is also procyclical, not countercyclical as Keynes had implied¹³. Interestingly, Keynes' reply included, among other considerations, the conjecture that the degree of imperfect

¹²Not all interpreters of Keynes would agree with this exegesis, e. g. Davidson (1994), Chap. 11. A more extreme Post-Keynesian view is that neither the demand for or supply of labor depend upon the real wage, e. g., Applebaum (1979), Eichner (1985). The diversity of Post-Keynesian visions of the labor market makes it difficult to include them in the scope of this thesis.

¹³Tarshis (1939) subsequently recanted his conclusions from the data in his 1938 paper, concluding in the end that the data implied a countercyclical real wage. Coleman (1984) pointed out that Dunlop and Tarshis have often been misquoted, and sets the record straight.

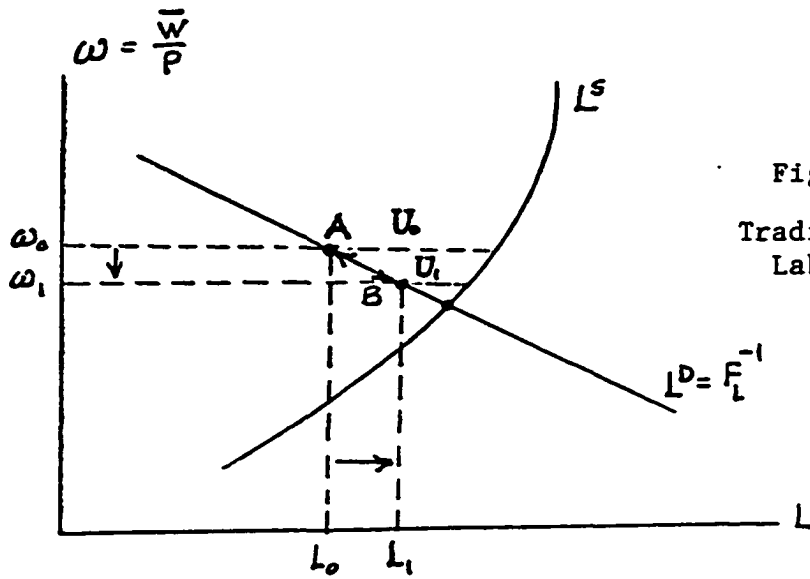


Fig. 1-4 (a)
Traditional Keynesian
Labor Market

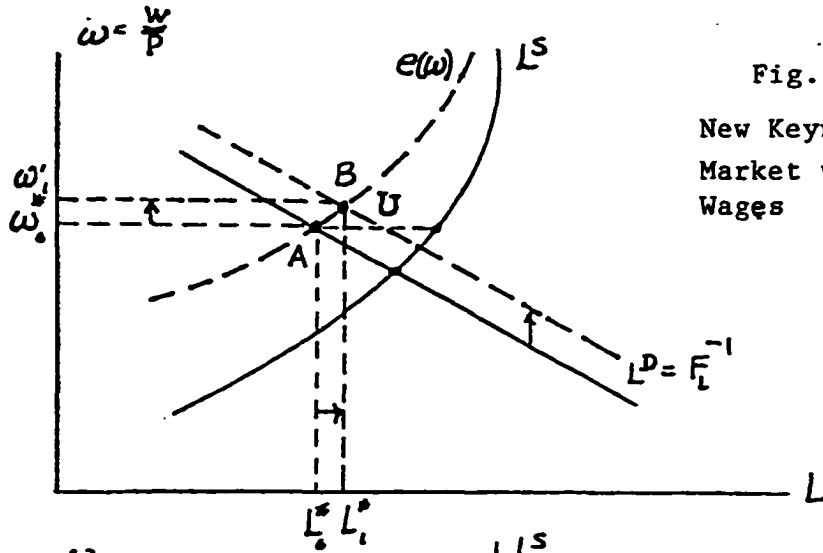


Fig. 1-4 (b)
New Keynesian Labor
Market with Efficiency
Wages

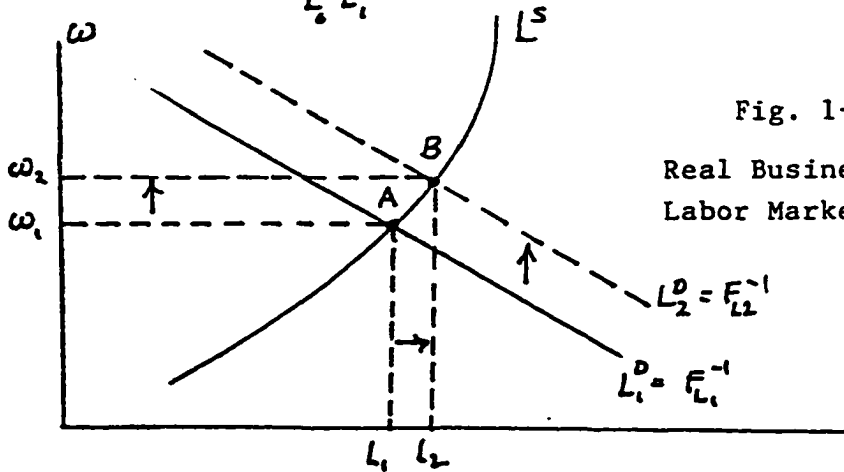


Fig. 1-4 (c)
Real Business Cycle
Labor Market

competition, reflected in markups of price over marginal cost, varied countercyclically so as to offset the downward influence of the labor demand curve, a *deus ex machina* he attributed to Kalecki. This idea was never incorporated into the Traditional Keynesian legacy. The countercyclical real wage of the General Theory was carried over in the Hicks-Hansen interpretation of Keynesian economics. Thus, Traditional Keynesian models became subject to the criticism that they were counterfactual with respect to the behavior of the aggregate real wage, and ignored the supply side of the economy.

Post-Keynesian models are a different genotype of the General Theory and to a great extent are not comparable with any of the other classes. There are many variations, but most have in common the non-neutrality of money, imperfect competition in the goods market, and markup pricing that is determined by the anticipated internal financing needs of firms rather than short-term profit maximization. (Eichner (1973, 1985, 1991), Chick (1983), Post-Keynesians appear to lack a consensus on how the labor market functions, or whether a conventional market model is even relevant (Applebaum 1979; Eichner 1985, Chap. 5; 1991, Chap. 1) They are classified here with the Traditional Keynesians, but are considered beyond the scope of this thesis and will not be discussed further.

1.5.2 Keynesian Disequilibrium Models

Disequilibrium interpretations of Keynes [e. g., Patinkin (1965. Chap. XIII)], Leijonhufvud (1968, 1981), Clower (1965) Barro and Grossman (1971, 1976)] assumed wage-price rigidities which prevent one or more markets from clearing, rationalizing both involuntary unemployment and a money-output connection. The behavior of the real wage in response to aggregate demand shocks depends on specific assumptions regarding its out-of-equilibrium adjustment path, and in general is ambiguous. Barro and Grossman (1976, p. 95-99) argued that the out-of-equilibrium adjustment behavior of the real wage is likely to be less countercyclical than in a market-clearing model, and can be procyclical. Although the Barro-Grossman disequilibrium model is *pas de rigueur* in America, much of the New Keynesian research program is devoted to developing choice-theoretic foundations for the wage and price rigidities that were merely assumed in that model.

1.5.3 New Keynesian Models

Some representative New Keynesian models are listed in Table 1-4. These are partial equilibrium micro models of the labor or goods market which were developed to provide choice-theoretic microfoundations for nominal and real rigidities that previously had been assumed. The implications of these

models are extrapolated to the macro level by invoking the representative agent assumption.

The earlier sticky-wage models [Fischer (1977); Taylor (1980)] also implied and predicted a countercyclical real wage. Baily (1974) presented a highly stylized model of risk-sharing in an industry in which the optimum strategy for all firms was to maintain a rigid real wage¹⁴. More recent models have attempted to rationalize sticky prices based on imperfect competition and menu costs or other externalities in the goods market. A money-output connection follows from the presence of nominal rigidities, but most of these models assume rather than predict a rigid real wage (e.g. Akerlof and Yellen (1985)). Thus, New Keynesian models tend to treat the real wage as a free parameter, to be specified to fit the circumstances. It appears that the assumption of rigid nominal wages, which engendered criticism of Traditional Keynesian and Disequilibrium models, has been supplanted in many New Keynesian models by the assumption of a rigid real wage.

¹⁴Baily's model and a related 1975 paper are discussed in Appendix C, where it is shown that the model provides a rationale for the assumption of monopsony power in the aggregate labor market. Also, implicit contract models are not discussed here. According to Rosen (1985):

Contract theory neither resolves nor illuminates questions of Keynesian unemployment based on nominal wage and price rigidities, money illusion and non-market clearing. Explanations for 'sticky' wages and prices that impede efficient labor utilization must be sought in other quarters.

The 1990 paper by Ball and Romer appears to have all of the necessary ingredients, but instead of deriving implications for the real wage, the authors calibrated their model so that it would be acyclical, in keeping with the empirical record¹⁵. Many New Keynesian models arrive at a rigid real wage by fixing either W or P and assuming a constant ratio W/P , which implies that markups are exactly as countercyclical as the marginal product of labor, a rather special case. (See Equation 1.3.) Thus, for the New Keynesian class of models, money is non-neutral but the theoretical underpinnings of acyclical real wages are, for the most part, not fully worked out.

1.5.4 Efficiency Wage Models

A similar pattern is evident in the Efficiency Wage models listed in Table 1-5, a subcategory of the New Keynesian literature. The efficiency wage hypothesis specifies that the productivity of workers depends positively on their real wages, and is embodied in an effort function $e(w)$ which supplants the labor supply curve. Given their product prices, firms set the nominal wage to minimize labor cost per efficiency unit of labor, which occurs where the real-wage elasticity of $e(w)$ is unity. Firms are on their labor demand

¹⁵Ball and Romer were using their model to argue that both nominal and real rigidities are important for the non-neutrality of money, and in that exercise the real wage was a free parameter that had to be pinned down.

TABLE 1-5
EFFICIENCY WAGE MODELS

<u>MODEL</u>	<u>MONEY-OUTPUT</u>	<u>BEHAVIOR OF W/P</u>
Solow (1979)	Neutral	Rigid*
Weiss (1980)	Neutral (Adverse selection in hiring)	Rigid*
Akerlof (1982)	Neutral (higher effort and wages are gift)	Rigid*
Shapiro & Stiglitz (1984)	Neutral (Shirking constraint; labor supply not relevant) exchanges)	Procyclical with technology shocks; otherwise ambiguous
Akerlof & Yellen (1985)	Non-neutral (Imperfect competition, menu costs in goods & labor markets)	Rigid* (Constant Markup assumed)
Chatterji & Sparks (1991)	Neutral (Continuous effort function and endogenous perform- ance standard)	Procyclical with productivity shocks

*Real wage rigidity is either assumed or implied only in a partial equilibrium or representative agent framework.

curves at a real wage that is higher than that which will clear the labor market and thus there is involuntary unemployment. This is shown in Panel (b) of Figure 1-5, where the equilibrium real wage is determined initially at point A, the intersection of the efficiency wage locus $e(w)$ with the labor demand curve. The efficiency real wage w_0 is higher than the market-clearing wage w_c , and involuntary unemployment is represented by the distance AD.

It turns out that the implications of efficiency wages for the cyclical behavior of the real wage are model-dependent. One of the problems in this literature is that these are partial equilibrium micro models, and the assumed sources of cyclical fluctuations in the labor market are not always clearly specified.

In the simple efficiency wage model in which the efficiency wage locus is a function of only the real wage, the labor supply curve plays no role in determining employment or the real wage; it merely determines the amount of involuntary employment. In this case, factors which shift the labor supply curve will not affect the efficiency wage or employment. Because of this, technology shocks are the only source of cyclical fluctuations in simple efficiency wage models, and these will produce a procyclical real wage. (For a positive shock this is shown as movement from point A to point B in

Figure 1-5, Panel (b).) According to this theory, the cyclical elasticity of the real wage would be close to the elasticity of the effort function $e(w)$, which is +1 at the optimum efficiency wage. This is comparable to what RBC models imply, and based on the evidence in Table 1-1, is counterfactual.

In the Shapiro-Stiglitz shirking model, the unemployment rate is a shift variable for the effort function, $e(w, U)$, which is interpreted as a *shirking constraint* on workers. An outward shift of the labor supply curve due to, for example, a monetary shock will increase involuntary unemployment. The higher unemployment rate decreases the chances that a worker who is fired for shirking will be rehired quickly, and therefore decreases the expected payoff from shirking at the current wage. Thus, an increase in the unemployment rate allows firms to lower their wages optimally without increasing shirking. This results in a downward shift of the efficiency wage locus (i. e., the shirking constraint), which follows the labor supply curve. In the absence of a technology shock the result will be a lower optimum efficiency wage and increased employment, i. e., a countercyclical real wage.

Shocks to technology in the Shapiro-Stiglitz model will shift both the labor demand curve and the shirking constraint but in opposite directions because the change in unemployment

will have a negative feedback effect on the propensity to shirk. In this case, the effect of unemployment on the shirking constraint will reinforce the direction of movement of the real wage and partially offset the effect on employment and output¹⁶. Thus, with unemployment as a shift variable in the efficiency wage locus, shocks to technology will produce a strongly procyclical real wage, as Shapiro and Stiglitz claimed. However, if cyclical shocks shift both the labor demand and labor supply curves in the same direction, the movement of the real wage is ambiguous.

In efficiency wage models, firms set wages unilaterally according to an uncontested profit-maximizing rule which is independent of labor supply. Workers are willing wage-takers because the efficiency wage offer is higher than their reservation wage and there is involuntary unemployment. It would seem that in order to be able to set the wage unilaterally and thereby create involuntary unemployment, firms must have some degree of monopsony power in the labor market¹⁷. This point is reinforced by the fact that the labor market does not clear at the efficiency wage (there is excess

¹⁶The possible outcomes are the same as when supply and demand curves move in opposite directions.

¹⁷Except for a brief discussion by Weiss (1990) in the context of a nutrition model, the relationship between efficiency wages and monopsony power has not been explored in the literature. Also, the capacity of firms to "set" the real wage requires that they have significant market power over both product prices and wages. This is seldom made explicit in the efficiency wage literature.

notional supply) and there are unemployed workers who would be willing to work for less but, like the Outsiders of Lindbeck and Snower (1988), lack the bargaining power to bid down the wage and gain employment. This suggests that a monopsony model of the aggregate labor market in which the degree of monopsony power to set the wage varies over the business cycle might be a useful way to describe the labor market¹⁸. This Kalecki-Keynes idea redux is developed more completely in Chapter IV of this thesis.

While most efficiency wage models assume that workers are identical and have the same effort function, the adverse selection models of Guasch and Weiss (1980) and Weiss (1990) assume that workers are heterogeneous in productivity, but firms can only imperfectly screen for and monitor the productivity of individuals. If workers' reservation wages are highly correlated with their productivity, then offering higher wages is one way of attracting and retaining more productive workers¹⁹. This is a fairly common practice for employers in primary, high-skill labor markets (Reynolds (1970), Rees (1973), Lang and Leonard (1987)). Thus, paying

¹⁸The monopsony wage clears the market below the competitive equilibrium wage, whereas the efficiency wage is a non-market-clearing wage above the competitive equilibrium wage. Monopsonistic firms might pay more than the monopsony wage to retain productive workers, but possibly less than the efficiency wage, the difference being determined by the cost of monitoring.

¹⁹A critical screening parameter in hiring is an applicant's wage history, or "salary requirements" i. e, their reservation wage. Reservation wages that are too high or too low may be equally valid reasons for rejection. Thus, a revealed reservation wage sends a signal about worker quality.

higher wage differentials may be a simple and direct way for firms to deal with the problems of adverse selection and moral hazard when workers vary in quality (Weiss, 1990)²⁰. One implication of the adverse selection models is that worker heterogeneity can be a cause of unemployment and layoffs in primary markets. Heterogeneity in the workforce may also be a factor contributing to anomalous real wage behavior.

Efficiency wage models have gained acceptance in the Keynesian camp because they offer choice-theoretic microfoundations for a rigid non-market-clearing wage, and thus for involuntary unemployment. But therein lies their limitation, for these are partial equilibrium models of a firm's choices in its markets, not general equilibrium macro models. Micro behavior is imputed to the economy as a whole by assuming a representative agent on each side of every market. As stated previously, representative agent aggregation is valid only when the marginal responses of individual agents are homogeneous. If efficiency wages are a means of sorting out heterogeneous workers in local labor markets that do not clear, it is not obvious that representative agent aggregation is appropriate. An open question then, for the New Keynesian literature, is the question of whether the real wage and markup

²⁰There are other ways, such as Okun's Toll (1981), probation periods, piecework, tenure systems, and requiring workers to post performance bonds. All of these are problematical, as discussed in Weiss (1990).

anomalies are due in part to inconsistent aggregation of heterogeneous labor.

Efficiency wage models appear to be an implicit acknowledgment that labor markets tend to be imperfectly competitive, that workers are heterogeneous, and that problems of asymmetric information and adverse selection abound in the employment relationship. These kinds of market imperfections are dealt with more explicitly in the implicit contract and job search literature, which has largely been supplanted by the efficiency wage construct. Because efficiency wages impart only real rigidity, money is neutral in the absence of nominal rigidities. This is evident in Table 1-3, where money is neutral in all of the models except for Akerlof and Yellen (1985), which also incorporated imperfect competition and menu costs.

It is understood that for changes in money to have a persistent effect on employment and output, there must also be a source of persistent real rigidity in the system. (Blanchard and Fischer, 1989). Efficiency wage constructions provide microfoundations for real wage rigidity in a partial equilibrium setting. However, the behavior of the efficiency real wage in a general equilibrium setting depends on the particular efficiency wage model employed, and in general is ambigu-

ous²¹ [(Blanchard (1986), Ball and Romer (1990)]. Efficiency wage constructions provide a rationale for involuntary unemployment, but as yet do not provide much insight into the behavior of the aggregate real wage in a general equilibrium framework. Thus, in their present state of development, they do not have clear-cut implications for the behavior of the aggregate real wage (Romer, 1996, p.458).

1.5.5 Real Business Cycle Models

Table 1-6 lists some of the RBC models that have been cited frequently in the literature. None of these standard RBC models contain money. They are general equilibrium models with continuous market clearing, in the Classical tradition. For the most part, business cycle fluctuations are assumed to be caused by random exogenous shocks to productivity -- real as opposed to monetary driving forces. This is equivalent to shifting the labor demand curve against a stationary labor supply curve in equilibrium, as shown in Panel (c) of Figure 1-5. Since the elasticity of the aggregate labor supply curve is assumed to be small, these models predict a strongly procyclical real wage, with an elasticity of the real wage with respect to employment of nearly +1, which is

²¹A search of the literature did not yield a general equilibrium macro model which incorporates efficiency wages in the labor market.

TABLE 1-6
REPRESENTATIVE REAL BUSINESS CYCLE MODELS

<u>MODEL</u>	<u>SOURCE OF SHOCKS</u>	<u>BEHAVIOR OF W/P</u>
Kydland & Prescott (1982)	Total factor productivity	Strongly Procyclical $\mathcal{E}_y[\omega] = +1.4$ $\rho(\omega, n) = +.90$
Long & Plosser (1983)	Stochastic productivity	Strongly Procyclical $\rho(\omega, n) = +.97$
Prescott (1986)	Labor Productivity	Strongly Procyclical $\rho(\omega, n) = +.95$
Hansen (1985)	Labor Productivity (Indivisible labor)	Strongly Procyclical $\rho(\omega, n) = +.87$
King, Plosser & Rebelo (1988)	Labor Productivity	Strongly Procyclical $\rho(\omega, n) = +.90$
Eichenbaum, Hansen & Singleton (1988)	Labor Productivity	Strongly Procyclical (no estimate given)
----- MODIFIED RBC MODELS -----		
Rogerson & Wright (1988)	Monetary Shocks (Indivisible labor, Sticky W, Wealth effects)	Countercyclical $-1 < \mathcal{E}_y[\omega] < 0$
Hansen & Wright (1992)	Productivity, plus government spending	Procyclical $\rho(\omega, n) = +.49$ to $+.76$
Christiano & Eichenbaum (1992)	Labor Productivity & government spending (Indivisible Labor)	Procyclical $\rho(\omega, n) = +.575$ to $+.81$
Baxter & King (1993)	Government purchases temporary, permanent	Countercyclical $\mathcal{E}_y[\omega] = -.70$ on impact*

*Temporary and permanent changes in government purchases have different long-term real wage elasticities; for temporary purchases the elasticity converges to $-.25$, and for permanent purchases the long-run elasticity is $+1.0$.

counterfactual. (Compare the elasticities in Table 1-6 with those in Table 1-1). Since RBC models provide no role for money in the business cycle, they deny a causal (but not necessarily a statistical) relationship between money and output. The observed money-output correlation is explained as central bank accommodation to real output fluctuations, an interpretation of the data which is controversial. Thus, the standard RBC models do not offer a comfortable fit to the two stylized facts²².

Some investigators have modified the standard Kydland-Prescott RBC model by incorporating government spending, wealth effects, and even sticky wages and monetary shocks (Rogerson and Wright, 1988). The addition of government spending shocks reduces the procyclicality of the real wage. The implications of the Rogerson and Wright model are almost Keynesian, except there is no involuntary employment. However, none of these modified RBC models fit our two criteria as well as the New Keynesian models of Table 1-4 (Particularly Ball and Romer (1990)). They are equilibrium models in which there is no involuntary unemployment. Gali (1996) has criticized these multi-shock RBC models and presented evidence that a New Keynesian model with imperfect competition, sticky prices and efficiency wages (e.g., as in Ball and Romer, 1990) is more

²²RBC models have been criticized on other technical grounds which are not relevant to the present discussion. (See Mankiw, Rotemberg and Summers (1985), McCallum (1988), Mankiw (1989).

consistent with the observed low correlation between labor productivity and employment.

Although the RBC class of models is interesting, they will not be considered further in this thesis. Instead, the thesis will focus on the Keynesian view of the business cycle, with the objective of discovering what is necessary in order to reconcile this class of models with the actual behavior of real wages, markups, and the money-output connection.

1.5.6 Summary and Conclusions

Traditional Keynesian, New Keynesian and RBC models have difficulty explaining the empirical behavior of the aggregate real wage. The findings and some of their implications are summarized in Table 1-7. It is evident that, in the cases of imperfect competition, counterfactuality extends to the markup and the implied elasticity of aggregate demand. No class of models considered here is problem-free. The various assumptions about flexible or sticky nominal wages, prices, real wages, imperfect competition, markups and the neutrality of money have produced a rich and informative body of literature, which nevertheless falls short of explaining both stylized facts introduced at the beginning of this chapter. Since the macroeconomic implications of these models also depend on the

TABLE 1-7
 MODEL PREDICTIONS OF REAL WAGE AND MARKUP BEHAVIOR

<u>Model Class</u>	<u>Source of Rigidity</u>	<u>Real Wage</u> ω	<u>Markup</u> μ	<u>Elasticity of Demand</u> η
Traditional Keynesian	Nominal Wages	Counter-Cyclical*	None*	∞
Keynesian	Goods Prices	Strongly Procyclical*	Counter-Cyclical	Pro-Cyclical**
Efficiency Wage	Real Wage and Goods Prices	Rigid(?)* Strongly Procyclical* Counter-Cyclical*	Counter-Cyclical	Pro-Cyclical**
Dis-Equilibrium	Nominal Wages and Goods Prices	Rigid* or Procyclical*	None* Constant*	∞ ∞
Real Business Cycle	None	Strongly Procyclical*	None*	∞

*Empirically inconsistent
 **Theoretically inconsistent

representative agent method of aggregation, it may be necessary to depart from that framework in order to explain the behavior of markups and real wages²³.

We have arrived at the principal working hypothesis of this thesis, which is that both the real wage anomaly and the markup anomaly are caused by sources of agent *heterogeneity* in the economy which are not being captured by representative agent models. The principal research objective of this thesis is to determine if, by moving away from the representative agent framework, modeling heterogeneity explicitly and employing a more general method of aggregation, it is possible to explain both the real wage anomaly and the markup anomaly within an otherwise standard Keynesian setup where money causes output.

1.6 What Lies Ahead

The approach that will evolve over the remaining chapters will be to develop an alternative labor-market model based on imperfect competition and heterogeneous workers in the labor market. This investigation is in the tradition of the New Keynesian literature, in that it will explore the implications of imperfect competition for the money-output connection and the behavior of the real wage. It differs in that it will

²³This was advocated previously by Coleman (1984) and Heckman (1984).

examine the implications of monopsony power of firms in the labor market, and it departs from the representative agent method of aggregating individual labor supply. Neither of these approaches have been prominent in the macroeconomic literature.

In Chapter II, I develop and analyze a static equilibrium model in which prices and wages are perfectly flexible but a causal relationship between money and output exists due to a wealth effect in aggregate labor supply. Thus, I show that if wealth is a significant determinant of aggregate labor supply, neither wage nor price rigidity is necessary to have a money-output connection. The model is a useful tool for analyzing the behavior of the real wage over a monetary-induced business cycle without the confounding effects of wage and price rigidities. If such rigidities are present, they will tend to reinforce the effects on output and the real wage implied by the model.

In Chapter III of this thesis I show that the combination of monopsony power with homogeneous workers in the labor market does not, in of itself, improve upon the situation described in this chapter. I also prove that countercyclical markups and acyclical real wages cannot be explained in terms of cyclical elasticities of product demand or labor supply within a representative agent framework. This poses a poten-

tial problem for macroeconomics, because without the representative agent assumption, Walrasian microfoundations do not carry over to the behavior of aggregates in a straightforward or systematic way.

Chapter IV presents the development of an aggregate labor supply function for heterogeneous workers who are employed by monopsonistic firms. If workers have nonhomothetic preferences with increasing relative risk aversion, then the elasticity of this aggregate labor supply function will be strongly procyclical in response to monetary policy actions. This a sufficient condition for markups to be countercyclical and a necessary condition for the real wage to be acyclical or procyclical. The analysis in Chapter IV is confined to the aggregate labor market and leads to partial equilibrium conclusions.

Chapter V reports the comparative static results of imbedding the aggregate labor supply function of Chapter IV in the general equilibrium model of Chapter II. This final step is important because, as in the case of efficiency wage theory, partial equilibrium arguments can be misleading for macroeconomics. In this extended general equilibrium model, the implied behavior of the real wage is ambiguous, because it depends on magnitudes of certain key elasticities in the model. A partial equilibrium analysis indicates that the real

wage will be weakly countercyclical, acyclical, or weakly procyclical depending on the magnitudes of certain monetary elasticities. Chapter V concludes with some observations regarding the methodology employed and areas warranting further research.

CHAPTER II

A CONNECTION BETWEEN MONEY AND OUTPUT VIA WEALTH EFFECTS IN A FLEXIBLE-PRICE MODEL

2.1 Introduction and Overview

This chapter presents a flexible-price macroeconomic model in which employment and real output are affected by changes in outside money when real wealth is included as an argument of the excess demand functions for goods, money and labor. A one-time exchange of bonds for money reduces the real value of government bond holdings, producing a reverse wealth effect in all three markets. The money multipliers for employment and output are unambiguously positive in the absence of Ricardian equivalence. The important conclusion of this chapter is that, when wealth effects are consistently specified, neither Keynesian rigidities nor reverse causation arguments are necessary for explaining the well-documented positive correlation between lagged changes in the monetary base and output. This conclusion holds as long as there is not perfect Ricardian equivalence.

Like Traditional Keynesian models (defined in Chapter I), the real wage in the model of this chapter is unambiguously countercyclical, and thus the model is also subject to criticism for that reason. However, unlike Traditional

Keynesian models, wages and prices in this model are perfectly flexible, all markets clear, and yet money is not neutral with respect to employment and output. Thus, the model demonstrates that it is not necessary to have rigid or sticky wages or prices in order for changes in money to produce changes in employment and output. The model is Keynesian in the sense that changes in the quantity of real money affect employment and output through a wealth effect on labor supply. As such, the model provides a "Keynesian" laboratory in which the cyclical behavior of the real wage and markups can be explored without the restrictive assumptions of sticky wages and/or prices. This exploration is carried out in Chapters III through V of this thesis.

2.2 The Correlation Between Money and Output

One of the more important stylized facts of macroeconomics is that changes in real GDP over the business cycle are positively correlated with lagged changes in the nominal money supply. The statistical evidence in support of this fact is overwhelming. Except for the brief 1990 recession, every other official recession since 1945 has been preceded by a decline in the rate of growth in the money stock, and cyclical expansions have been either preceded by or roughly coincident with increases in the rate of money growth. Almost all econometric studies of this relationship since 1970, employing

a variety of models and estimation techniques, have confirmed to some degree the original findings and conclusions of Friedman and Schwarz (1963) that there is a significant positive correlation between money and real output in the short run.

Friedman and Schwarz identified a number of "turning points" at which a change in the rate of growth of M1 was followed by a statistically and economically significant change in the growth rate of GDP. They concluded from this that changes in money can "cause" changes in output¹. Anderson and Jordan (1968) estimated significantly positive coefficients for a regression of quarterly changes in lagged GDP on money. Sims (1972) found that money Granger-caused output in a simple bivariate autoregression. Sims (1980) later added interest rates and other control variables to the estimating equation and found that the interest rate was significant but the money aggregate was not.

Using annual data from 1954-1976, Barro (1977) found that only "unanticipated" innovations in money affected output. Mishkin replicated Barro's study with quarterly data and longer lags, and concluded that both anticipated and unantici-

¹All of these empirical studies, including the more recent vector autoregressions, are subject to Tobin's critique (1970), *post hoc, ergo proper hoc*. Statistical correlation with a lagged independent variable does not prove physical causality one way or the other.

pated changes in money have long-lasting effects on output. Bernanke (1986) and Bernanke and Blinder (1988) found evidence that the availability of credit in the banking system significantly affected lagged real output; however, Meltzer (1995) has questioned the quantitative importance of the credit channel. Romer and Romer (1989) updated the Friedman and Schwarz study using content analysis of contemporary Federal Reserve Board records, and concluded that six of eight U. S. postwar recessions from 1945 to 1980 were preceded by deliberate contractionary Federal Reserve policy actions intended to fight inflation.

Other investigators have come to somewhat different conclusions from the data. Litterman and Weiss (1987) concluded that the nominal interest rate leads changes in output (inversely), and that the role of money is insignificant. King and Plosser (1984) found a weak and brief effect of changes in the monetary base on output, with both M1 and the credit channel being insignificant; Eichenbaum and Singleton (1986) found that even the monetary base was insignificant. Kydland and Prescott (1982) in the context of a real business cycle model, argued that the correlation between money and output represents a reverse-causation: the central bank's monetary interventions are reactions to current and forecasted economic activity, and are partly endogenous. Shapiro (1994) critiqued Romer and Romer, and concluded that the Federal

Reserve's policy actions during the period were endogenous to current and forecasted conditions in the economy. Thus, part of the money-output connection may be circular, although the central bank can exert an influence that is either restraining, neutral or stimulative. Ultimately, it is the actions of independent agents -- firms, households and financial institutions, which determine the relationship between the monetary base and economic activity².

While a short-run positive correlation between money and output is well established, the economic mechanisms through which money affects output is not well understood. Attempts to quantify the importance of transmission channels such as interest rates, the exchange rate, asset prices, Tobin's q , wealth, bank credit and firm balance sheets have had meager success (Mankiw, 1994; Mishkin, 1995). To quote Sims (1992):

Monetary aggregates tend to move in the same direction as aggregate economic activity, as has been repeatedly documented. ...the profession as a whole has no clear answer to the question of the size and nature of the effects of monetary policy on aggregate activity.

Thus, the nature of the money-output connection itself is somewhat of a puzzle that tends to Balkanize the profession, and the debate is a continuing one.

²For contemporaneous reviews of this literature, see Cagan (1989), Blanchard (1990), Mankiw (1994) and Mishkin (1995).

No attempt will be made here add to the body of empirical evidence regarding the money-output connection, or to reconcile the different interpretations. The issue is brought up only to point out that the several monetary transmission channels that have been heretofore proposed have weak empirical validity, and that this is an area of ongoing research.

In this chapter I show that, in a flexible-price version of the static IS/LM model, a one-time change in the monetary base affects the interest rate and nominal wages and prices. These in turn affect the real value of household financial wealth, and if wealth is an explanatory variable in the excess demand functions for goods, money and labor, employment and output will also be affected. This represents an extension of the wealth monetary transmission channel to aggregate labor supply. If firms are also assumed to have monopsony power in the labor market, as in Chapter IV and V, then the wealth transmission channel will extend to labor demand also, which under monopsony is not independent of the elasticity of labor supply.

Much of the modern analysis of monetary economics has been undertaken within an optimal monetary growth framework. The seminal paper by Sidrauski (1967) showed that in a Walrasian setup, money is both neutral and superneutral with respect to real interest rates and output, thereby overturning

the finding of Tobin (1965) that increases in money growth rates lead to reductions in real interest rates and increases in the level of output³. Subsequent authors, however, have shown that the strong super-neutrality result of Sidrauski does not hold up in less restrictive specifications, so that in general changes in money growth rates do lead to changes in real interest rates and output levels in the long run. But, these departures from super-neutrality have not gone consistently in one direction. Some studies found that higher monetary growth rates lead to a higher equilibrium capital stock and output (e. g., Fischer [1979] and Cohen [1985]), while others found the converse (e. g., Stockman [1986] and Wang and Yip [1992]⁴. Perhaps partly due to this ambiguity, the sentiment in the literature is that although departures from super-neutrality are supported by theory, the magnitude of these departures are small⁵. Thus, the lesson from the optimal monetary growth literature seems to be that money

³Mundell [1963] showed in a traditional non-growth framework that changes in anticipated inflation led to less than one-for-one changes in nominal interest rates when wealth was an argument in the consumption function. As such, the negative relationship between monetary growth rates and real interest rates is often called the Mundell-Tobin effect.

⁴Wang and Yip [1992] show that in the popular money-in-utility-function setup, the effect of money on output depends crucially on the cross partials of the utility function and that the comparative statics are inconclusive overall. Turnovsky [1987] shows that the matter also depends on whether the existing tax structure induces firms to finance their investment by issuing bonds or by issuing equities.

⁵Danthine et al. (1987) provide calculations suggesting that although violated, the super-neutrality proposition is not an unreasonable approximation. The studies of Lucas (1980), Geweke (1986) and Stockman (1986) provide empirical evidence supporting this view.

plays no role for real outcomes in a Walrasian setting with perfect markets.

This view that money plays no "real" role in a Walrasian framework is the premise separating two currently dominant "schools" of thought that have evolved on the subject of business cycle behavior -- the Real Business Cycle (RBC) and New Keynesian schools⁶. The RBC models, which maintain the standard assumptions of the Walrasian setup, attribute short-term fluctuations in output to real shocks rather than monetary shocks. They explain the observed correlation between money and output as a consequence of a reaction by the monetary authority to changes in output rather than the other way around (Kydland and Prescott, 1982; King and Plosser, 1984).

The New Keynesian view is that a causal mechanism from money to output does exist, and this has led to exploring departures from the frictionless Walrasian setup in one or more markets. These departures include money illusion; nominal wage rigidity due to implicit long-term employment contracts; staggered wage-setting; nominal price rigidity due to menu costs and second-order gains from changing prices under

⁶These "schools" of research have been surveyed by Rotemberg (1987), Fischer (1988), Blanchard and Fischer (1989), Phelps (1990), and Mankiw and Romer (1991, Vol.1).

conditions of imperfect competition; and "efficiency wage" constructions. Although none of these explanations have dominated the field of play, it would not be inaccurate to say that the non-neutrality of money at business-cycle frequencies has come to be identified with the New Keynesian school, which relies upon inhomogeneities and non-tatonnement market imperfections⁷.

The objective of this chapter is to show that when Ricardian equivalence does not hold in a Walrasian framework, any monetary policy action which increases the ratio of outside money to outside bonds (e. g., an open market purchase by the central bank) will lead to a decrease in real interest rates and an increase in employment and output, i. e., I show that money is not neutral even with perfectly flexible prices and market-clearing. The channel through which an open market operation affects real interest rates and output is through its effect on real wealth; a parity exchange of money for bonds takes place in nominal terms but not in real terms, and real economic activity is affected.

In order to show this in the simplest way, the static, general equilibrium framework of Patinkin (1965) will be employed. But, unlike Patinkin's model, which incorporated

⁷The Monetarist and Post-Keynesian schools also weigh in on the non-neutrality of money without necessarily subscribing to New Keynesian mechanisms.

real wealth as an argument only of the consumption and money demand functions, in this model real wealth is incorporated as an argument in *all* household excess demand functions, i. e., wealth is also assumed to affect aggregate labor supply. With real wealth connecting all three excess demand equations, (goods, money and labor), the model is not block recursive with respect to the labor market. An open-market purchase of bonds increases nominal prices and wages and lowers the interest rate, the net effects of which are to reduce both the real wage and the real value of household financial wealth. The demand for labor increases due to the reduction in the real wage, and the supply of labor increases because the wealth effect more than offsets the reduction in the price of leisure. Employment and output increase, along with investment, income, and the demand for real money balances.

The main implication of the model is that, when wealth is consistently incorporated into a Walrasian framework, changes in money affect real outcomes and the direction of this effect is unambiguously positive. As long as there is at least some departure from perfect Ricardian equivalence, so that households consider some fraction of their government bond holdings as net wealth, a positive money-output connection will result. If agents consider none of their bond holdings as real wealth, then money neutrality reappears.

It is important to note that the wealth channel through which money affects real outcomes in this model is absent in most optimal monetary growth models. This is because Ricardian equivalence is a common assumption in this class of models, implying that government bonds play no role in agents' intertemporal budget constraints. In these models, a one-time swap of bonds for money leads only to a proportionate increase in goods prices, with no effect on real outcomes. That is also the case in the model presented here, if agents are Ricardian.

The results derived from the model in this chapter also match up well with Aiyagari and Gertler (1985), which show in an overlapping generations framework that open market operations are non-neutral with respect to real interest rates in fiscal regimes that are "non-Ricardian", (i. e., where government debt is not fully backed by future direct taxes). They conclude that government bonds may matter in a manner as described in the traditional literature [e. g., Patinkin (1965), Mundell (1971)]. Aiyagari and Gertler assumed, however, that output is exogenous, so that no money-output connection exists in their model. The results of this chapter show in a simple static framework that the wealth effects uncovered by Patinkin (1965) in a non-optimizing framework and by Aiyagari and Gertler (1985) in an optimizing framework also

work to influence the levels of employment and real output⁸.

The model presented in this chapter has a causal money-output connection in the absence of Keynesian wage-price rigidities. The introduction of such non-Walrasian features would augment the non-neutrality properties derived here, but are not necessary for the basic results and play no role in the model presented here.

The results obtained in this chapter have some obvious implications for the money neutrality debate. First, it is not necessary to leave the Walrasian market-clearing paradigm, as the New Keynesians do, in order to have a theoretical link between money and output. This is not to say that the New Keynesian research program is on the wrong track, but merely that the money-output relation need not depend on a Keynesian rigidity or inhomogeneity. Second, advocates of the New Classical/RBC view are no longer required to argue that the only causal relationship is from output to money, or to rationalize the existence of a monetary reaction function. Under plausible assumptions which do not violate the Walrasian paradigm, a causal money-output relationship has been revealed

⁸Leeper (1991) and Woodford (1994) also examine non-Ricardian fiscal regimes in an optimizing framework. As with Aiyagari and Gertler (1985), output is assumed to be exogenous in these papers. See also Marini and van der Ploeg (1988).

which could serve as a legitimate explanation of the observed positive correlation.

The significance of this model for the broader investigation of this thesis is that real wealth is a shift variable for the aggregate labor supply function. A decline in real wealth due to an open-market operation will shift the labor supply curve out, increasing aggregate labor supply, equilibrium employment and output. If the wealth effect on labor supply is strong enough, employment and output could increase even if the real wage does not decline; this opens up the possibility that the real wage could be acyclical or even procyclical. This also avoids the situation described in Chapter I where the assumption of nominal and real rigidities in New Keynesian models pre-determines the behavior of the real wage. Thus, the model provides a framework for analyzing the unrestricted behavior of the real wage in a monetary-induced business cycle.

The rest of the chapter is organized as follows: the complete model is specified in Section 2. The comparative static analysis of an open market purchase are summarized in Section 3, and a detailed mathematical analysis of the model is contained in Appendix B. Section 4 analyzes the implications of degrees of Ricardian equivalence. Section 5 presents some conclusions.

2.3 The Standard Flexible-Price Macro Model

Consider a closed monetary economy in short-run static equilibrium with no net growth in population or labor supply. The economic agents of interest are large numbers of households and firms and a single government entity which includes a central bank. These three classes of agents trade in perfectly competitive markets for goods, labor services, money and financial assets. Firms trade with households in the goods, labor and financial asset markets, demanding labor services and funds to finance real investment, and supplying goods for consumption and investment.

As is customary in models of business cycle phenomena, the capital stock is assumed to be fixed. Households receive wages and profits from firms with which they purchase goods for consumption and equity shares to add to their financial wealth. Since the model is static, agents are assumed to hold static expectations concerning nominal and real values. The government purchases goods for public consumption and finances its operations by levying taxes and issuing fiat money and debt in the form of perpetuities paying \$1 per year. The government is the monopolist issuer of fiat money. In this chapter, the economic behavior of agents within each sector is assumed to be sufficiently homogeneous so as to justify its representation by a representative agent. Except for having

real wealth as an argument in the labor supply function the model is a fairly standard one, and in the interest of brevity further discussion of microfoundations will be omitted except where it is pertinent.

2.3.1 Market Equilibrium Conditions

General equilibrium conditions on the aggregate excess demand functions in the four markets are specified as follows:⁹

$$\text{Goods Market: } c^d(z, \Omega) + i^d(r, y) + g - y^s(w) = 0 \quad (2-1a)$$

$$\text{Money Market: } m^d(y, r, \Omega) - m = 0 \quad (2-1b)$$

$$\text{Labor Market: } \ell^d(w) - \ell^s(w, \Omega) = 0 \quad (2-1c)$$

$$\text{Asset Market: } f^d(z, y, r, m, \Omega) - f^s(r, y) = 0 \quad (2-1d)$$

$$\text{Government Budget Constraint: } g + \frac{B}{P} - \tau_o = \frac{dM}{P} + \frac{dB}{rP} \quad (2-1e)$$

Where z is disposable income, Ω is real household wealth, $w = W/P$, $m = M/P$, and signs under arguments denote the signs of partial derivatives which are the basic behavioral assumptions of the model. $m^d = m^s$ and $f^d = f^s$ are stock equilibrium relations, since the demand and supply of money and earning assets are in terms of balances.

⁹A glossary of mathematical symbols with definitions is contained in Appendix A. Note that the balance sheet constraint on assets allows the market for financial assets to be ignored.

Household real wealth is defined as real money balances plus the present value of claims to perpetual cash flow streams from government bonds and private equities:

$$\Omega = \Omega (P, W, r; M, \kappa B) = \left[\frac{M}{P} + \frac{\kappa B}{rP} + \pi \left(\frac{W}{P} \right) / r \right], \quad 0 \leq \kappa \leq 1 \quad (2)$$

where κ is a parameter representing the extent to which the representative household regards its holdings of government bonds to be nominal wealth, with $\kappa = 0$ representing perfect Ricardian equivalence. Note that Ω is interpreted as net of private sector liabilities and inside money, which cancel out in the aggregate.

In order to remain faithful to the static framework, it is assumed that all endogenous changes in B/P are financed with lump sum taxes and that $\frac{dM}{P} = \frac{dB}{rP} = 0$. Since it is the

effects of one-time changes in the level of money balances through open market operations that are of interest, these assumptions are relatively unimportant¹⁰. The endogenous variables of this system, therefore, are P , W , r , τ and τ_0 , and the exogenous variables are the government policy variables g , M and B . Given the equilibrium values for the

¹⁰It is assumed, therefore, that τ adjusts endogenously, i. e., $\tau = \tau_0 + B/P$. Assuming that interest payments are bond financed would not affect the non-neutrality of money in the model, but it would give rise to intrinsic dynamics which would be inconsistent with the static nature of the setup.

endogenous variables, the equilibrium values of Ω , ℓ and y can be determined. The representative agent in each sector is subject to a budget constraint: household desired aggregate consumption and saving is constrained by disposable income and wealth; firms pay out all profits to households and thus must finance new investment by selling new equities; finally, as shown in (2-1e), together with the assumption that $\frac{dM}{P} = \frac{dB}{rP} = 0$,

the government must finance continuing purchases with lump sum taxes.

2.3.2 Important Features of the Model

Equations (2-1) describe a standard textbook flexible-price model in which the classical dichotomy has been bridged by the addition of a labor market equilibrium condition which depends on real wealth. Since Ω appears in all three excess demand functions the three markets are linked by wealth effects. The model is not dichotomous and must be solved out simultaneously. Thus employment and output are not determined independently of money market equilibria. In general, the classical dichotomy between real and nominal variables breaks down in the presence of real wealth effects. All of the endogenous variables are free to move instantaneously to clear

the four markets. The model contains no Keynesian rigidities, stickiness or inhomogeneities.

As indicated previously, this model follows Patinkin (1965) except that wealth is incorporated into the labor supply function. It does not appear from the literature that the implications of open market operations (involving a swap of bonds for money) have been examined in such a setting. Phelps (1972) does add wealth to the labor supply function, but the comparative static exercise conducted (which is mainly graphical) is a swap of money for capital. There are no government bonds in Phelps' model, and so the issue of Ricardian equivalence does not arise.

Another important difference is that Phelps constrains the interest rate to be equal to the marginal product of capital. With this assumption, the real interest rate increases with output due to a monetary expansion, contrary to the finding here that it must decline in order to equilibrate the goods market at a higher level of output. This difference highlights the fact that Phelps interpreted his model as applying to the long-run steady-state, whereas the model presented here is more applicable to the short-run business cycle where the capital stock is assumed to be fixed and monetary intervention is more relevant.

Like Patinkin, and in contrast to Phelps' graphical analysis, standard comparative static analysis will be employed to determine stability conditions and sign the money multipliers. This approach also reveals that the money-output connection holds up except in the extreme case of perfect Ricardian equivalence for government bonds.

2.4 Analysis of the Model

The comparative static analysis of this non-recursive 3x3 system involves a fair amount of algebra, and the details are consigned to Appendix B. Only the principal results will be summarized and interpreted in this chapter. However, it may be helpful to introduce here some notation and definitions from Appendix B.

Differentiation of the system of equations (2-1) and rearrangement produces the matrix differential equation

$A \cdot dv = Gdu$, where:

$$A \cdot dv = \begin{bmatrix} \begin{matrix} (-) \\ EDG_p \end{matrix} & \begin{matrix} (-) \\ EDG_x \end{matrix} & \begin{matrix} (+) \\ EDG_w \end{matrix} \\ \begin{matrix} (+) \\ EDM_p \end{matrix} & \begin{matrix} (-) \\ EDM_x \end{matrix} & \begin{matrix} (-) \\ EDM_w \end{matrix} \\ \begin{matrix} (+) \\ EDL_p \end{matrix} & \begin{matrix} (-) \\ EDL_x \end{matrix} & \begin{matrix} (-) \\ EDL_w \end{matrix} \end{bmatrix} \begin{bmatrix} dP \\ dx \\ dW \end{bmatrix} \quad (2-4)$$

$$G \cdot du = \begin{bmatrix} c_x^d & -1 & -\frac{1}{P} c_Q^d & \frac{-\kappa}{rP} c_Q^d \\ 0 & 0 & \frac{1}{P} (1 - m_Q^d) & \frac{-\kappa}{rP} m_Q^d \\ 0 & 0 & \frac{1}{P} \ell_Q^s & \frac{\kappa}{rP} \ell_Q^s \end{bmatrix} \begin{bmatrix} d\tau_o \\ dg \\ dM \\ dB \end{bmatrix} \quad (2-5)$$

In (2-4), "EDX" denotes excess demand in the market for X, subscripts denote partial differentiation, and the signs are signs of the first partial derivatives. (The partial derivative expansions of each term in A are derived in the Appendix, but will not be needed in most of what follows.) The signs of the first partials depend on the original behavioral assumptions contained in (2-1) and on the stability conditions derived in Appendix B.

2.4.1 Analysis of an Open Market Operation

The primary question of interest is the effect of pure monetary policy on real economic variables. I will examine the implications of a one-time increase in M with a proportional decrease in B/r, i. e., of setting $dB = -rDM$ in (2-5), with $dg = d\tau_o = 0$. (Details are in Appendix B). It may seem that the value of κ is important for the results of this model; however it turns out that as long as $\kappa > 0$, the value of κ affects the magnitudes of the comparative static derivatives but not their algebraic signs. The results for the case

where $\kappa = 1$ are summarized here. The implications for Ricardian equivalence ($0 \leq \kappa < 1$) are discussed in Section 2.5.

COMPARATIVE STATIC DERIVATIVES FOR $\kappa = 1$

$$\frac{dP}{dM} = -|A|^{-1} \{ P^{-1} [\overset{(-)}{EDG}_r \cdot \overset{(-)}{EDL}_w - \overset{(-)}{EDL}_r \cdot \overset{(+)}{EDG}_w] \} > 0 \quad (2-6a)$$

$$\frac{dW}{dM} = |A|^{-1} \{ P^{-1} [\overset{(-)}{EDG}_p \cdot \overset{(-)}{EDL}_s - \overset{(+)}{EDL}_p \cdot \overset{(-)}{EDG}_r] \} > 0 \quad (2-6b)$$

$$\frac{d\Omega}{dM} = \overset{(+)}{\Omega}_p \left(\frac{dP}{dM} \right) + \overset{(-)}{\Omega}_r \left(\frac{dR}{dM} \right) + \overset{(-)}{\Omega}_w \left(\frac{dW}{dM} \right) < 0 \quad (2-6c)$$

$$\frac{dw}{dM} = -|A|^{-1} \{ P^{-2} [-\overset{(-)}{\ell}_\Omega (\overset{(-)}{\Omega}_p + w\overset{(-)}{\Omega}_w)] (-\overset{(-)}{i}_r^d) \} < 0, \quad w = \frac{W}{P} \quad (2-6d)$$

$$\frac{dR}{dM} = |A|^{-1} \{ P^{-1} [\overset{(-)}{EDG}_p \cdot \overset{(-)}{EDL}_w - \overset{(+)}{EDL}_p \cdot \overset{(+)}{EDG}_w] \} < 0 \quad (2-6e)$$

The results in (2-6a) and (2-6b) are not surprising, since P and W are nominal and would increase with M in any event. The result in (2-6c) also seems unsurprising, since Patinkin found that an open market purchase decreased real wealth. But the result is far from obvious in the full 3x3 system. In Appendix B it is shown that the sum of the first three terms is negative. One way to see this is to note that that (2-6d) holds unambiguously, which means that Ω must decline in order to restore equilibrium in the labor market.

The sign of (2-6e) is unambiguously negative due to stability condition (i) [See Appendix section B.2] and here these results contradict Phelps (1972), but agree with Patinkin and economic reasoning: an increase in money balances and a corresponding decline in the supply of outstanding bonds will cause bond prices to increase; the interest rate must decline in order to equilibrate the goods market following the wealth shock to consumption.

The partial derivatives of Ω are:

$$\Omega_P = - \frac{\left[\frac{M}{P} + \frac{\kappa B}{rP} - \frac{Wl^d}{rP} \right]}{P} > 0 \quad (2-7a)$$

$$\Omega_r = - \frac{\left[\frac{\kappa B}{rP} + \frac{\pi}{r} \right]}{r} < 0 \quad (2-7b)$$

$$\Omega_w = - \frac{[l^d]}{rP} < 0 \quad (2-7c)$$

The sign of (2-7a) is strictly ambiguous, but the fact that the nominal wage bill is on the order of 10 times greater than

nominal interest payments on bonds and money implies that it is positive¹¹.

The conclusion is that an even exchange of bonds for money between the public and private sectors increases both the price level P and nominal wages W . The net effect of dP and dW reduces real wealth Ω , which creates disequilibrium in all three markets. The decline in real wealth stimulates an increase in labor supply which is accommodated along a stationary and downward sloping labor demand curve by a reduction in the real wage. This produces a higher level of employment and output. In the goods market the effect of the reduction in the real wage and wealth on consumption demand is more than offset by an increase in income and the increased investment demand induced by a decline in the interest rate. Finally, an open-market operation causes money supply to increase in real terms, i. e. the increase in the price level is less than proportional to the increase in M , and money is not neutral.

It is shown in Appendix B that dP/dM has its largest positive value when $\kappa = 0$, i. e., when $dP/dM = P/M$. For

¹¹Economic Report of the President, 1992. As of 1991:IV, total employee compensation [Table B-22] was \$3,425.1 Billion; Federal net interest paid [Table B-79] was \$190.5 Billion; M2 [Table B-65] was \$3,425.4 Billion and the monetary base [Table B-67] was \$324.78 Billion. (All data in annualized 1991 dollars.) Imputing a 5% annual interest rate to the monetary base, Wl^d exceeded $(.05M_b + B)$ by \$3,2178.4 Billion, or 16 times. Using M2 the ratio is 8 times. Therefore Ω_p is unambiguously positive.

$\kappa > 0$, $dP/dM < P/M$, which means that $d(M/P)/dM > 0$ and the real money supply increases. Thus, the demand for real balances must also increase, implying that the income and substitution effects on money demand are greater than the wealth effect.

2.4.2 Summary of the Results.

Under the behavioral assumptions of the model (which are standard) the reduction in the value of real wealth induced by an open market purchase stimulates private agents to increase employment and output, their rates of saving and investment, and their holdings of real money balances. A portion of the original private wealth, in the form of bonds, will have vanished and the motivation of the private sector is to regain lost utility by increasing income and the rate of wealth accumulation. An open-market sale of bonds would have just the opposite effects all around.

Although this static short-period model is somewhat antiquated, its predictions are unambiguous and match up well with the stylized facts of active monetary intervention; Expansionary monetary policy stimulates investment, employment and output, with a corresponding rise in prices and wages and a decline in the real interest rate. The implications of the

model that depart from the stylized facts of the business cycle are (a) any unemployment is voluntary, since the labor market always clears; (b) with decreasing returns to labor, the aggregate real wage is countercyclical. These properties derive from the Walrasian heritage of the model.

The terms "involuntary unemployment" is another way of saying the labor market does not clear. Such a non-Walrasian feature is characteristic of many New Keynesian models (See the related discussion in Chapter I.) The results of this section indicate that although non-clearing markets may be important for explaining involuntary employment, they are not necessary for money to be non-neutral.

The fact that the observed aggregate real wage appears to be anything but countercyclical has prompted Kuh (1966) and more recently Hall (1991) to conjecture that the aggregate marginal product of labor is constant, or nearly so. In the model presented here, the more elastic the labor demand curve, the larger will be the money multiplier on output, i . e., the wealth effect will be more pronounced. In the extreme case of constant returns to labor, $y_\ell = a = w$, $y_{\ell\ell} = 0$, and the labor demand function $\ell^d(w)$ is undefined. Conditions in the labor market will then be defined by $\ell = \ell^s(a, \Omega)$; the equilibrium values of ℓ and y are determined by labor supply which, with a constant real wage, is a function of only Ω . The real

effect of an open-market transaction is transmitted to output via the multiplier:

$$\frac{dy}{dM} = a l_{\Omega}^s \frac{d\Omega}{dM} > 0$$

Thus, with a constant-returns technology the real wage is fixed and the wealth effects of an open-market transaction on employment and output in this model would be stronger.

2.5 Ricardian Equivalence

A case has been made for a positive causal connection between money and output in the absence of Ricardian equivalence ($\kappa=1$). As one would expect, for the case of perfect Ricardian equivalence ($\kappa = 0$) it is possible to show after some algebraic manipulation, that $dP/P = dW/W = dM/M$, and therefore $d(W/P)/dM = 0$, $d(M/P)/dM = 0$ and $d\Omega/dM = 0$. Bonds exchange for money, but bonds are not counted in real wealth so wealth doesn't change and money is neutral. It is clear that it is the presence of government bonds as a component of wealth and not money or equities that causes the non-neutrality of open-market operations in this model.

The question of whether government bonds represent net wealth to the private sector remains a controversial one. (see Barro (1974, 1976), Buchanan (1976), Feldstein (1976), and more recently Bernheim (1987), and Buiter (1991)). The issue

is whether it is reasonable to assume that government bonds are perceived by private agents as a temporary postponement of increased tax liabilities which are equivalent in present value, i.e., whether government bonds should be counted as net wealth by the private sector¹². In the final analysis this is an empirical question, but it becomes a theoretical issue because of two sets of canonical assumptions that are normally made in macroeconomic modeling, namely, those of rational expectations and the representative agent.

The Ricardian equivalence hypothesis is closely allied with the perfect foresight version of rational expectations. For an individual bondholder to equate his claim to a definite stream of cash receipts to another stream of uncertain tax liability payments and cancel them out in his mind requires not only something close to perfect foresight, but some additional assumptions. One of those assumptions is that the bondholder will be the taxpayer who is liable for those tax bills when they are levied. There are several ways that an individual bondholder can avoid the tax levy even under the assumption of infinitely lived agents or intergenerational altruism, but the main point is that the distribution of bond holdings and the distribution of their implied tax burden among households are not necessarily the same unless one is

¹²It appears that Ricardo recognized the prevalence of "fiscal illusion", and did not actually subscribe to the concept which has been associated with his name. See O'Driscoll (1977) and Bernheim (1987).

talking about a representative agent. In that event, the representative agent consumer pays all of the taxes, either directly or indirectly through ownership of all the firms, and government bond payments simply go from one pocket of the representative agent to the other. If bonds are perpetuities and government debt service is financed out of current taxes, then not much foresight is needed if tax collections are coincident with bond receipts. The absence of a store of value will be obvious to the representative agent.

The model described in this chapter is a representative agent model, but it is also a timeless static equilibrium model in which agent expectations are static. As such it is open to criticism on those grounds¹³. The representative agents of this model are naive and myopic, and plausibly could have fiscal illusion with respect to the incidence and inevitability of future taxes. If so, then there still may be a case for wealth effects. The real issue here has to do with the level of aggregation. Bonds and other debt contracts of the private sector do cancel out when aggregating over the private sector, even on a balance sheet basis. Cancelling out all debt claims between the government and private sectors on the basis of equivalent present value of assets and liabilities would be equivalent to aggregating the government and

¹³In the concluding section of this chapter I argue that the implications of this static model should hold up in a dynamic framework with rational expectations.

private sectors together and eliminating the distinction¹⁴. That would be moving in the direction of a Robinson Crusoe economy, and although there are uses for such a model, it does not make for very interesting macroeconomics.

Thus, although the Ricardian equivalence hypothesis follows as a logical consequence of rational expectations and representative agent aggregation, it potentially involves a fallacy of composition. What can be perceived as net wealth by a subcategory of consumers who are bondholders, cannot be net wealth for the collective representative agent¹⁵. On the other hand, if some fraction of bondholders have fiscal illusion, or bondholders as a group do not completely discount their claims for implied taxes and regard some fraction of their bond holdings as net wealth, then Ricardian equivalence will be imperfect for a representative agent who is truly representative. and there may be a wealth effect from government bonds. This possibility will now be investigated.

In this model $\kappa = 1$ (non-equivalence) and $\kappa = 0$ (perfect equivalence) represent the polar extremes of the Ricardian

¹⁴To be consistent, the same equivalence proposition should be applied to transfer payments and other pecuniary government services. Taking this to its logical conclusion would also eliminate financial markets, firms and money, institutions which enable consumers to deal with time and uncertainty.

¹⁵The fact that foreign entities who are not subject to direct U. S. taxation hold approximately 15% of Treasury securities means that the implied tax burden is somewhat larger than the domestic bond receipts.

equivalence controversy. An interesting question is what happens in the intermediate case when bondholders regard some positive fraction of their bond holdings as wealth, i. e. when $0 < \kappa < 1$. Given the structure of the model, deriving the comparative static results for this case is a rather tedious exercise. (See Appendix B, Section B.4). The argument will be summarized here.

2.5.1 Comparative Static Results for $0 < \kappa < 1$

Suppose that the model has a money-neutral equilibrium solution for an open-market operation when $0 < \kappa < \epsilon < 1$, where ϵ can be arbitrarily small. Then it follows that $dP/P = dW/W = dM/M$, $d(W/P) = 0$ and $dr/dM = 0$. Now examine the implications for $d\Omega$:

$$\Omega = \left[\frac{M}{P} + \frac{\kappa B}{rP} \right] + \frac{\pi}{r} = \frac{M}{P} \left[1 + \frac{\kappa \theta}{r} \right] + \frac{\pi}{r}$$

where $\theta = B/M$, and under the neutrality conditions assumed,

$$\frac{d\Omega}{dM} = \left(\frac{\kappa M}{rP} \right) \frac{d\theta}{dM} + \text{terms which cancel to 0}$$

Thus, in order for money to be neutral we must have either

(a) $\kappa = 0$, or (b) $\frac{d\theta}{dM} = 0$. The first condition in (a) has

been ruled out by assumption. As for (b), we have:

$$\frac{d\theta}{dM} = \frac{1}{M} \left[\frac{dB}{dM} - \frac{B}{M} \right]$$

so that $d\theta/dM = 0$ if and only if $dB/B = dM/M$, i. e., if $\theta = B/M$ remains constant. But this implies that an open-market operation, for which θ is most definitely not constant, will be non-neutral when $0 < \kappa < 1$. Any positive amount of government bonds that is regarded behaviorally as private wealth will produce a non-neutral response to an open-market operation in this model¹⁶. Money neutrality requires perfect Ricardian equivalence on government bonds. This conclusion can be stated as follows:

With real wealth effects in all three markets, an open-market purchase is non-neutral and increases employment and output as long as $0 < \kappa \leq 1$.

The consequences of imperfect Ricardian equivalence for other real variables can be similarly derived. In Appendix B, Section B.4, it is shown that the signs of dP/dM and dW/dM are both positive and independent of the value of κ . It is also true that:

$$\frac{d\left(\frac{W}{P}\right)}{dM} = \frac{1}{P} \left[\frac{dW}{dM} - \frac{W}{P} \frac{dP}{dM} \right] \quad (2.8)$$

¹⁶the magnitude of the non-neutral response will depend directly on the value of κ in the interval $[0,1]$.

Substituting the expressions for dP/dM and dW/dM for the case where $0 < \kappa \leq 1$ in Appendix B, Section B.4 into Equation 2.8, expanding and collecting terms, I derive the result that the real wage declines unconditionally. Thus:

$$\frac{dw}{dM} \begin{cases} = 0, \kappa = 0 \\ < 0, 0 < \kappa \leq 1 \end{cases} \quad (2.9)$$

Applying the same reasoning as before, it follows that real wealth Ω must also decline in order to achieve equilibrium in the labor market, and the implications of the model will hold except for the extreme case of $\kappa = 0$, i. e., perfect Ricardian equivalence. Of course, the closer κ is to 1, the larger in magnitude those implications will be.

The foregoing analysis also makes it clear that any change in the ratio of bonds to money will be nonneutral. This would include the fabled "helicopter drop" of currency, or more realistically, a one-time devaluation of the currency. The fact that neutrality arises in the constant θ case is uninteresting because there is no mechanism for maintaining a constant ratio of bonds to money while the monetary base is being changed. Open-market operations are the principal vehicle for fine-tuning the rate of growth of the money supply and are undertaken precisely because they are not neutral. They are, in Woodford's (1995) terminology, delicate instruments of non-Ricardian monetary policy.

2.6 Conclusions

This chapter has presented a model in which money is non-neutral with respect to the real interest rate, employment and output in a Walrasian general equilibrium framework when wealth is incorporated consistently into all household excess demand functions and Ricardian equivalence does not hold perfectly. The predictions of the model are found to be unambiguous and surprisingly "Keynesian," An increase in money causes a decrease in the real interest rate, an increase in employment and output, and increases in nominal wages and prices.

The source of non-neutrality is the fact that the central bank can make a parity exchange of money for bonds only in nominal terms; it cannot bring about a parity exchange in terms of the components of real wealth. The central bank is in an analogous position to Keynes' workers who attempted to set their real wage by bargaining for a nominal wage; it behaves as if it had "money illusion" in the bond market, except of course there is no illusion. The intention of nominal open-market operations is to affect real economic activity, and the monetary policy transmission mechanism is a change in the real value of private wealth. The swap of bonds for money reduces the real value of wealth held in the private sector, and correspondingly reduces the real value of govern-

ment obligations. Monetization of the government's debt entails a corresponding reduction in real private wealth, to which agents respond predictably: they strive harder to regain lost utility.

The model presented here is a static, non-optimizing general equilibrium model in which capital, money and bonds are exogenous and expectations are static, i. e., a traditional static macroeconomic model with naive agents. Nevertheless, the implications of this simple framework match up well with those of the optimal monetary growth literature. As in that literature, when Ricardian equivalence holds perfectly, open market operations imply no wealth effects and money is neutral. But, if Ricardian equivalence holds less than perfectly, then open market operations do imply wealth effects, which turn give rise to an unambiguous causal money-output connection.

It is fairly standard practice to assume that capital stocks are fixed when analyzing macroeconomic fluctuations at business-cycle frequencies. An important question, however, is whether the non-neutrality results obtained here would hold up in a dynamic framework in which money and bonds have non-zero growth rates and agents form rational expectations. The analysis of Mundell and Tobin, which assumed output to be exogenous, showed that with real wealth as an argument of

consumption and money demand functions, changes in the growth rate of money cause changes in real wealth and real interest rates. It should be evident from the results derived in this chapter that if wealth is consistently incorporated into a dynamic framework, so that wealth is also an argument for labor supply, then monetary policy actions would be both non-neutral and non-superneutral.

The question naturally arises as to whether the qualitative effects of money on output via real wealth represent significant quantitative effects. No attempt will be made here to answer this empirical question, other than to point out that the higher the elasticity of aggregate labor demand, the greater will be the magnitude of the money multipliers on real interest rates and output. It is important to note that the monetary transmission mechanism in the present model, which operates through an effect on wealth, differs from the mechanism present in most of the optimal growth literature, the significance of which is typically assumed to be small. In the optimal growth literature, perfect Ricardian equivalence is typically assumed at the outset, which precludes any wealth effect from monetary policy actions.

It appears that the wealth effects examined in this chapter, which arise in the case of less-than-perfect Ricardian equivalence, are becoming of more interest in the liter-

ature. This is partly due to the fact that wealth effects seem to be important for solving the price-indeterminacy problem which arises when the central bank targets interest rates instead of a monetary aggregate (Leeper, 1991; Woodford, 1994.) It is also partly due to the significance that wealth effects have for monetary and fiscal policy, as argued by Aiyagari and Gertler (1985) and Marini and van der Ploeg (1988).

It appears that wealth fell out of favor as an explanatory variable at about the same time the neo-classical synthesis unraveled. This may have been unjustified. It does not seem unreasonable to assume that households take their current real financial wealth into account when making important choices about consumption, saving and leisure; or that subsequent changes in the market value of financial wealth as a consequence of inflation and interest-rate fluctuations compel those decisions to be revised. But reasonableness is not always a reliable guide. Empirical evidence would be more persuasive.

CHAPTER III

MONOPSONISTIC LABOR MARKETS AND AGGREGATE REAL WAGES

3.1 Introduction and Overview

An important conclusion in Chapter I was that it appears to be difficult to construct an aggregate model in which money and output are positively correlated and real wages are acyclical or mildly procyclical. This conclusion appears to hold up over a broad range of macro-models including those ordinarily classified as Traditional Keynesian, New Keynesian, and Real Business Cycle models¹.

The Traditional Keynesian approach to rationalizing the non-neutrality of money was to posit some form of nominal wage-price rigidity or stickiness which worked by influencing aggregate labor supply. A major criticism of this class of models was their prediction of a countercyclical real wage, as well as failure to explain why nominal rigidities would persist under conditions of perfect competition.

¹Chapter I contains a review of the relevant macroeconomic literature and arguments which lead to this conclusion. The macro-model classifications "Traditional Keynesian, New Keynesian", etc., used in this Chapter are also defined in Chapter I.

New Keynesian models have provided a number of competing explanations for wage-price stickiness, including models of imperfect competition in which the economic incentives for price-setting agents to change prices is weak. Efficiency-wage models of the labor market imply rigid real wages and neutral money under profit maximization (Akerlof and Yellen). Models of monopolistic competition require the additional assumption of second-order menu costs in order for money to be non-neutral, and some of these models predict strongly procyclical real wages which are counterfactual². Christiano and Eichenbaum (1992) point out that, in general, Keynesian models understate and Real Business Cycle models overstate the degree of positive correlation observed between real wages and employment.

Chapter II presented a flexible-price general equilibrium macro model with wealth effects in labor supply. In that model, as with Traditional Keynesian models, open-market operations resulted in changes in employment and output through their effect on aggregate labor supply. However, the model in Chapter II makes a "Keynesian" connection between changes in money and output under the assumptions of perfect competition and perfectly flexible wages and prices, thereby

²Real Business Cycle models also exhibit strongly procyclical real wages, but deny a role for money. These models will not be considered further in this thesis.

avoiding one of the major criticisms of Traditional Keynesian models. This result suggests that, while nominal wage-price rigidities may be necessary for disequilibrium explanations of unemployment, they are not necessary for establishing a connection between money and output. However, as with the Traditional Keynesian models which rely on aggregate labor supply to produce a money-output connection, the real wage in the model of Chapter II is unambiguously countercyclical. Thus, the model described in Chapter II is unsatisfactory to that extent.

The objective of the next three chapters of this thesis is to show that it is possible to reconcile, within a Keynesian framework, both sets of stylized facts: the positive correlation between money and output, and acyclical or mildly procyclical real wages.

This chapter explores the implications of imperfect competition in the labor market for the behavior of real wages and markups. The question explored in this chapter is: Does the assumption of monopsony power in the labor market help to reconcile Keynesian-type models with the empirical record on real wages? In a Keynesian model with a perfectly competitive labor market, the profit-maximizing equilibrium real wage will be countercyclical if the marginal product of labor declines with employment when the capital stock is fixed,

which is generally assumed. However, with monopsony power in the labor market the profit-maximizing equilibrium real wage is less than the marginal product of labor by a markdown factor which depends directly on the degree of monopsony power. If monopsony power in the labor market is weaker in expansions than it is in recessions, then the markdown will vary accordingly and the real wage may be procyclical, acyclical, or at least less countercyclical than it would be under perfect competition. This chapter explores in some detail the implications of the standard monopsony model for this question, and in particular, determines the conditions that are necessary for a monopsonistic labor market to exhibit acyclical or procyclical real wages.

In addition to providing a possible explanation of the absence of countercyclical real wages, there are three reasons for investigating the implications of monopsony power in the aggregate labor market. First, it complements the more recent New Keynesian literature on imperfect competition in the goods market; in fact, monopsonistic labor markets have been largely overlooked in the macroeconomic literature. Second, the behavior of the labor market is critical to a Keynesian money-output connection, and unlike New Keynesian models of imperfect competition in which behavior in the labor market is implicit, with monopsony power the aggregate labor market takes center stage and the elasticity of aggregate labor

supply plays an important role in determining equilibrium outcomes. Finally, the monopsony assumption provides a different interpretation of the empirical evidence on markups of price over marginal cost, i. e., as markdowns of real wages from the marginal product of labor.

Econometric studies of U. S. industry data have consistently found evidence that markups of prices over marginal factor cost are countercyclical [Bils (1987), Rotemberg and Woodford (1991)]. The reasons for this are not well understood. The assumption of monopsonistic labor markets allows for an interpretation that the observed markups are actually markdowns which vary countercyclically with monopsony power. With monopsony power there is a close relationship between the behavior of markdowns and real wages, which will be examined in this Chapter.

The main contribution of this chapter is to show that in the Traditional Keynesian setup, incorporating monopsony power into the labor market in a representative agent framework, does not lead to either acyclical or procyclical aggregate real wages. In fact, with monopsony power the real wage is more countercyclical than if the labor market were perfectly competitive. Correspondingly, markups in the standard monopsony model are procyclical. Both of these predictions are counterfactual, and move us further away from explaining

the stylized facts. Thus, in the Traditional Keynesian setup, monopsony power alone is of no help in explaining the real wage and markup anomalies.

Further analysis in this chapter shows that the stumbling block here is the representative agent method of aggregation, the logical requirements of which preclude the elasticities of either either aggregate goods demand or aggregate labor supply from being procyclical. Thus, the major conclusion of this chapter is that macro models which utilize the representative agent method of aggregation cannot explain the cyclical behavior of real wages or markups in terms of cyclical changes in demand or supply elasticities.

Thus, one way to to reconcile the Keynesian money-output connection with real wage behavior may be to relax some of the restrictive assumptions of the representative agent method. That is the objective of Chapter IV, which extends the analysis of monopsony power in the labor market to the case where workers are heterogeneous and representative agent aggregation is not applicable to labor supply.

The chapter proceeds as follows: Section 3.2 briefly describes the real wage and markup anomalies. Section 3.3 relates these anomalies to standard models of imperfect competition, and presents an interpretation in terms of

monopsonistic labor markets. Section 3.4 discusses the role of monopsony power in the labor market search literature, which provides some justification for assuming monopsony power in a static equilibrium framework. The principal results of the chapter are derived in Section 3.5, where the standard monopsony model is analyzed in some detail. Section 3.6 restates the conclusions of the chapter. Some supporting arguments and mathematical derivations are contained in Appendix C.

3.2 The Real Wage and Markup Anomalies

Macroeconomic theory has found it difficult to provide a satisfactory explanation for two persistent stylized facts of the business cycle: (1) The aggregate real wage is acyclical, and at times slightly procyclical [Abraham and Haltiwanger (1995)]; (2) Markups of price over marginal cost are countercyclical in concentrated industries. [Bils (1987), Hall (1988), Rotemberg & Woodford (1991)]³. (A summary of this evidence is contained in Chapter I.)

The observed behavior of the aggregate real wage has been an anomaly in macroeconomics ever since Keynes grappled with

³A procyclical variable increases with output and employment, a countercyclical variable moves just the opposite. References to cyclicity imply the existence of a business-cycle generating mechanism in the economy which causes output and employment to move in irregular cycles.

it in *The General Theory*. The absence of a strong correlation between aggregate real wages and employment contradicts the theory of competitive labor markets, in which the real wage can be either procyclical or countercyclical depending on whether the labor demand curve shifts or the labor supply curve shifts when employment and output change⁴. The absence of any consistent statistical relationship between the aggregate real wage and employment is an anomaly which has motivated a number of reappraisals of the applicability of the standard competitive market model to the aggregate labor market. Any of the standard assumptions -- market clearing, perfect competition, declining marginal labor productivity, utility and profit maximization, perfect information, leisure as a normal good, and the representative agent (RA) method of aggregation -- are suspect for the labor market. It appears that for a model of the aggregate labor market to be consistent with the stylized facts, at least one of these canonical assumptions may have to be abandoned.

The cyclical pattern of price markups is also not well accounted for by standard price theory. Econometric studies of U. S. industry data have consistently found evidence that markups in concentrated industries are countercyclical. [Bils

⁴Obviously, one curve must shift more than the other for the real wage to change. Real Business Cycle models assume that the sources of cyclical fluctuations are shocks to technology or productivity which shift only the labor demand curve. Many of the Older Keynesian models assume fixed technology and associate shifts of the labor supply curve with changes in aggregate demand.

(1987, 1989); Hall (1988); Rotemberg and Woodford (1991)]. Bils found that markups in two-digit level industries decreased by 3.3% for each 10% expansion in output; Rotemberg and Woodford found elasticities of markups relative to hours worked on the order of -1. This evidence is consistent with the view that many goods markets are imperfectly competitive, and that the degree of market power possessed by firms varies over the cycle. However, theoretical mechanisms which could explain cyclical market power at the aggregate level are not well developed.

The real wage and markup anomalies are closely related and may have a common source. The idea that monopoly power in goods markets (measured as the percentage markup of price over marginal cost) might vary countercyclically originated with Pigou (1937) and Kalecki (1938), and was mentioned by Keynes (1939) as a possible explanation for the early Dunlop-Tarshis finding of procyclical real wages. Keynes' conjecture was that if monopolistic firms use markup pricing and tend to lower their markups in an expansion, the ratio of nominal prices to wages will decline, resulting in a procyclical real wage. Although the stylized facts on markups and real wages are closely related, it will be shown later in this chapter that the necessary conditions for real wages to be procyclical are stronger than for markups to be countercyclical; the former imply the latter, but not conversely, i.e., Keynes'

conjecture is not enough to obtain acyclical or procyclical real wages.

3.3 The Interpretation of Countercyclical Markups

3.3.1 Countercyclical Monopoly Power

In this subsection I discuss countercyclical markups in terms of monopoly power in the goods market. The case of monopsony power in the labor market is treated in the next subsection.

The studies of markups by Bils and Rotemberg and Woodford shared a common premise that goods markets were monopolistically competitive and labor markets were perfectly competitive, and therefore attributed the cyclicity of markups to changes in the degree of monopoly power over the business cycle. With these assumptions, the relationship between the markup and monopoly power is given by the first order condition for profit maximization for a monopolist.

Assume that a monopolist has a short-term production function $y = F(\ell, k)$ where labor ℓ is variable and capital k is fixed, with $F_\ell > 0$, $F_{\ell\ell} < 0$; the monopoly price is given by the industry inverse demand curve $P(y)$. With a perfectly

competitive labor market, the money wage W will be the market wage. The monopolist's profit is then:

$$\pi(y, W, r) = P(y)y - Wl - rk \quad (3.1)$$

Since capital is assumed to be fixed in the short run, y is a function of only the level of employment. The first-order condition for profit maximization is, in markup form:

$$\frac{PF_l}{W} = \left[\frac{1}{1 + \frac{1}{\eta}} \right] = \frac{1}{[1 - L]}, \quad \eta < 0 \quad (3.2)$$

Here F_l is the marginal product of labor, η is the elasticity of total market demand faced by the firm, and $L = |1/\eta|$ is Lerner's index of monopoly power. ($0 \leq L < 1$). With perfect competition assumed in the labor market and capital fixed in the short run, marginal cost is W/F_l . With $L > 0$, price exceeds marginal cost and the difference is called a markup.

Equation 3.2 gives the optimum markup for a profit-maximizing monopolist, and it is evident that a countercyclical markup at the firm or industry level implies countercyclical monopoly power L and procyclical elasticity of demand, η . Thus, a profit-maximizing monopolist will lower its markup in response to an increase in demand if and only if the elasticity of demand also increases in the vicinity of the equilibrium price as the demand curve shifts out. However, if the market demand curve shifts iso-elastically, the markup will remain constant.

The problem is that there appears to be very little basis in conventional price theory for demand elasticities to be procyclical. In the most common setup of the consumer's problem, namely, a homothetic utility function and linear budget constraint, all demand elasticities are constant with respect to the scale variable, income. A market model in which demand elasticities vary with income requires the assumption of non-homothetic preferences. This is not a particularly acute problem for microeconomic analysis, since it has been known for some time that the assumption of homothetic preferences, which implies linear Engel curves, has been repeatedly rejected by household expenditure studies. (Prais and Houthakker (1971 Chap. 2; Deaton and Muellbauer (1980), pp.142-145; Deaton (1992) p.9). Microeconomists have no basis for assuming that consumption preferences of individuals or households are homothetic.

When it comes to aggregation and macroeconomics, however, nonhomothetic preferences are problematic, because they invalidate the representative agent method of aggregation and require some form of nonlinear aggregation⁵. When aggregation is nonlinear there is no simple correspondence between the functional forms which are used to describe the behavior

⁵Aggregation issues are not confined to macroeconomics. The first level of aggregation over consumers is a household demand curve.

of individual agents and firms, and those which would consistently describe the corresponding behavior of aggregates. The whole idea of micro-foundations for macroeconomics is challenged. This may be why nonhomothetic preferences are not prevalent in the macroeconomic literature.

A theoretical basis for countercyclical market power can be found in game-theoretic analyses of collusive oligopolistic behavior (Friedman (1977, 1983), Rotemberg and Saloner (1986), Bagwell and Staiger (1995)). One conclusion of this line of research is that tacitly colluding oligopolies are likely to behave more competitively (i. e., engage in deviating price wars) during booms than in busts. Of the models tested by Rotemberg and Woodford [1991], a model of implicit oligopolistic collusion gave the best fit to industry markup data. These models were reduced form markup equations at the firm or industry level, not structural general equilibrium models. The extension of this approach to a general equilibrium framework is an active area of research by Rotemberg, Saloner and others. Although it is potentially an alternative explanation of countercyclical markups, it is not within the scope of this thesis, which is to explore the implications of imperfect competition in the labor market.

3.3.2 Countercyclical Monopsony Power

One of the major messages of this thesis is that there is an alternative interpretation of the evidence in the literature on markups, which is that *firms engage in strategic wage-setting behavior over the business cycle, and have stronger monopsony power over wages in recessions than in expansions*. If, in contrast to the studies cited in the previous section, if firms are assumed to be price-takers and wage-setters, then what appears to be a markup of price over marginal cost can be interpreted as a markdown of the wage from the firm's marginal revenue product. With perfect competition assumed in the goods market and monopsony power assumed in the labor market, the profit-maximizing markdown relationship becomes:

$$\frac{PF_{\ell}}{W} = \left[1 + \frac{1}{\epsilon}\right], \quad \epsilon > 0 \quad (3.3)$$

where ϵ is the wage-elasticity of labor supply. If both the goods and labor market were perfectly competitive, the profit-maximizing price P would be equal to marginal cost W/F_{ℓ} and there would be no markup or markdown. However, with monopsony power in the labor market, the marginal cost of labor is $\frac{W}{F_{\ell}}\left[1 + \frac{1}{\epsilon}\right]$ and is greater than average cost W by the amount of

the markdown⁶. The real wage W/P is less than the marginal product of labor by the proportionality factor $[1 + 1/\epsilon]^{-1}$. The ratio PF_L/W which was interpreted as a price markup in Equation 3.2 can now be interpreted as a wage markdown in Equation 3.3. If labor markets are monopsonistic, and if wage markdowns are countercyclical, the implication would be that the elasticity of labor supply ϵ in Equation 3.3 must be procyclical and monopsony power $1/\epsilon$ must be countercyclical. The inference would be that labor supply in equilibrium is less elastic (less sensitive to wage differentials) in recessions than in expansions, and conversely⁷. As in the case of monopoly, the challenge is to explain why ϵ might be procyclical, and to determine what assumptions about the labor market are consistent with that explanation.

Of course, a monopoly-monopsony combination is possible⁸. The corresponding first-order condition would then be:

⁶The marginal cost of adding one worker is the incrementally higher wage W paid to that worker, plus $(\partial W/\partial L)L$, the wage increase paid to all infra-marginal workers in the absence of wage discrimination. The latter expression is equal to W/ϵ , the amount of the wage markdown from the marginal revenue product. See also Figure 3-1.

⁷This is just an inference from the empirical record. It does not say that ϵ is procyclical in the standard monopsony model. In fact, in Section 3.5.3, I prove that just the opposite is the case. An interpretation in terms of collusive wage-setting behavior, analogous to Rotemberg and Woodford [1991], is possible but will not be pursued here.

⁸This general case may be more realistic, since monopsony power in the labor market may be associated with strong monopoly power in the goods market. According to Joan Robinson (1932, p.227), "A monopolist must necessarily be a monopsonist of the factors which he employs". This statement would be especially applicable to a workforce with firm-specific or industry-specific skills.

$$\frac{PF_{\ell}}{W} = \frac{\left[1 + \frac{1}{\epsilon}\right]}{\left[1 + \frac{1}{\eta}\right]} = M(\eta, \epsilon) \quad (3.3)$$

Here the ratio PF_{ℓ}/W reflects both price markups over marginal cost as in Equation 3.2 and wage markdowns from marginal revenue product as in Equation 3.3. This equation shows that hypotheses about the cyclicity of markups $M(\eta, \epsilon)$ are necessarily joint hypotheses about the cyclical behavior of η and ϵ . Since η and ϵ are not observable, it appears that it would be difficult to identify empirically their separate contributions. This identification problem, which tends to confound statistical estimates of labor supply and demand elasticities from aggregate data, is discussed in Killingsworth (1983) and Pencavel (1986).

The interpretation of the markup evidence depends on what assumptions one wishes to make about the competitiveness of goods and factor markets. The studies cited previously focused on monopoly power $1/\eta$ and assumed that labor markets were perfectly competitive ($1/\epsilon = 0$). I will take the opposite approach and assume that labor markets are monopsonistic and explore the implications of monopsony power and heterogeneity in the aggregate labor market for macroeconomic outcomes, especially the the predicted pattern of real wages and markdowns. In order to focus on the implications of

monopsony power in the labor market I will henceforth assume in this thesis that the goods market is perfectly competitive, i. e., $1/\eta = 0$.

3.4 Monopsonistic Labor Markets in Search Equilibrium

Although the case of a pure monopsony employer has often been regarded as somewhat of a textbook *curiosum*, applicable only to isolated company towns, professional sports leagues and university faculties, the construction of models involving monopsony power has been an important line of research in labor market theory. (Boal and Ransom [1997] provide a recent comprehensive survey.) This is because a competitive market-clearing model has been unable to account for some important and commonly observed phenomena, (e.g., involuntary unemployment, sticky money wages, acyclical real wages, wage differentials, etc.) As a consequence, many theorists have abandoned the frictionless Walrasian *tatonnement* paradigm in favor of an explicit analysis of out-of-equilibrium wage-setting and market adjustment processes (Lilien and Hall, [1986])⁹. This has produced a voluminous literature on the economics of search and disequilibrium dynamics in labor markets.

⁹Much of the impetus for this research was the search for microfoundations of Keynesian macroeconomics, which began in the 1970's, e. g., Phelps (1970).

Two dominant characteristics of labor market search theory are: (a) an emphasis on matching workers to job vacancies, implying that the relevant labor supply to a firm is the number of workers rather than hours, with individual labor supply assumed to be fixed; and (b) the assumption that firms set wages (or wage offers) which workers either take or reject. There is no presumption in this literature that the labor market operates like a competitive auction market. Monopsony power is implicit in most search theory models as a consequence of the assumption that firms make wage offers and workers are wage-takers. Arrow [1958] pointed out that, in the absence of an auctioneer, price-setting necessarily defaults to agents and price-setting by agents is the *de facto* exercise of market power¹⁰. Thus, it should not be surprising that monopsony power is implicit in most search models of the labor market. (E.g., Phelps [1968], Mortensen [1970], Lippman and McCall [1976], Pissarides [1976] and Baily [1975], where monopsony power is explicitly recognized). However, the monopsony power of search theory occurs in a dynamic, out-of-equilibrium market-adjustment framework. An important question is whether the dynamic monopsony power in

¹⁰Arrow's observation applies strictly where agents' offer prices are uncontested and become the transaction price. This is seldom, if ever the case in oligopoly. In Bertrand duopoly, for example, there is a dynamic pricing sequence which leads to the competitive price as an equilibrium. Union wage bargaining involves bilateral monopoly, the outcome of which is generally indeterminate. These exceptions are not considered to be relevant to the aggregate U. S. labor market.

search disequilibrium persists or vanishes when a state of equilibrium is attained.

Most macro models, including the model described in Chapters II and V, are highly aggregated general equilibrium models. There are several advantages to utilizing a static equilibrium framework in this investigation: (a) It is simpler to formulate and explain; this helps to highlight the essential differences between the monopsony model and other approaches; (b) Within the scope of this thesis, we are interested in examining the implications of monopsony power in the labor market for macroeconomic *equilibrium* outcomes; the models in Chapter II and V are static equilibrium models. Thus, the specification of monopsony power in those models needs to be an equilibrium one, since dynamics are not specified. The question is, then, does the assumption of monopsony power in an underlying disequilibrium search process imply the existence of monopsony power in equilibrium?

Appendix C contains a brief review of the relevant literature on this topic, specifically, the works of Mortensen (1970), Diamond (1971, 1982), Rothschild (1973), Baily (1975) and Pissarides (1976, 1988). The important conclusion from this review is that *if wage-setting firms are governed by profit-maximizing behavior in disequilibrium trading, then in the absence of any countervailing market power the optimum*

wage path will converge to the monopsony wage in equilibrium. In general the long-run equilibrium or stationary state resulting from a dynamic monopsony search process retains a markdown and is not a competitive equilibrium (Baily (1975), Diamond, (1982)). Thus, for profit-maximizing firms, monopsony power in disequilibrium implies monopsony power in equilibrium -- the auctioneer is not rehired.

This conclusion provides some justification for the methodology employed in this and subsequent chapters, which assumes monopsony power in the labor market and utilizes a static equilibrium framework. There are also precedents for this approach in the literature; Okun (1981) and Chick (1983), for example, feature the static monopsony labor market model in their respective interpretations of the economics of Keynes, while asserting that the underlying market adjustment process is a search process.

3.5 A Neoclassical Static Monopsony Model

This section explores the implications of a standard neoclassical model of the labor market for the elasticity of labor supply and the cyclicity of markdowns and the real

wage. Here the analysis is conducted in a static partial equilibrium framework¹¹. It is shown more formally that, while procyclical elasticity of labor supply is a necessary and sufficient condition for markdowns to be countercyclical, a stronger sufficiency condition is required for the real wage to be acyclical or procyclical. The most important finding in this section is that labor supply elasticity in the standard choice-theoretic monopsony model will always be countercyclical when preferences are assumed to be convex and homothetic. This parallels the situation for monopoly, where homothetic preferences imply constant demand elasticities.

These results lead to the conclusion that representative agent models, which preserve homotheticity in aggregation, are incapable of explaining countercyclical markups or acyclical real wages in terms of procyclical elasticities of either goods demand or labor supply. Thus, these anomalies cannot be explained by standard market models of imperfect competition that employ the representative agent method.

3.5.1 A Standard Model of Monopsony

The essential features of nondiscriminating monopsony are: (a) the firm faces an upward-sloping labor supply curve;

¹¹In Chapter V the analysis will be conducted in a macroeconomic general equilibrium framework.

(b) all workers are paid the same wage, and (c) profit maximizing by the firm results in a monopsony wage that is less than labor's marginal revenue product. The real wage and the equilibrium levels of employment and output are all less than would prevail under perfect competition. Labor receives less than its marginal revenue product, and the difference is appropriated by the firm as monopsony rents.

A firm maximizes profits when it hires labor services up to the point where the marginal product of labor equals its marginal cost. A monopsonist firm faces an upward sloping labor supply curve and must raise its wage offers in order to attract additional job applicants. It is assumed that the firm knows the labor supply curve $\ell^s(w)$, and sets the wage to maximize the following profit function:

$$\pi(w) = F(\ell, k) - w\ell^s(w) - rk \quad (3.4)$$

where $F(\ell, k)$ is assumed to be a linearly homogeneous quasi-concave production function; ℓ represents a flow of labor services; k represents the capital stock, which is assumed to be fixed in the short run over the business cycle; $w = W/P$ is the real wage; and r is the rental cost of capital. The

first-order condition for profit maximization equates the marginal revenue product of labor with its marginal cost¹²:

$$PF_{\ell} = W[1 + 1/\epsilon] \quad (3.5)$$

Here, ϵ is the real wage elasticity of a static labor supply curve, defined as

$$\epsilon = (\partial \ell^S / \ell^S) / (\partial w / w) \quad (3.6)$$

If the monopsonist firm does not discriminate when it raises its wage offer, it must offer the higher wage level to all of its employees and the marginal cost of hiring an additional worker is the nominal wage W plus the monopsony markdown W/ϵ which represents the infra-marginal cost¹³. (Note: Equation 3.6 is a rearrangement of the markdown equation 3.3.) It will be convenient to work with equation 3.6 in real terms:

$$F_{\ell} = w[1 + 1/\epsilon] \quad (3.7)$$

¹²The firm chooses W (for a given P) and accepts the supply of labor at that wage. Since the locus of profit-maximizing employment is on the labor supply curve, this is equivalent to choosing the profit-maximizing level of employment. The quasi-concavity of $F(\ell, k)$ guarantees the second-order conditions for a maximum.

¹³The possibility of discriminating monopsony is excluded here. One way to justify this is to assume that discrimination among identical workers would lower productivity and induce quits. The monopsonist incurs an opportunity cost of foregone producer surplus for choosing not to discriminate.

where $w = W/P$. This can also be written:

$$w = ([1 + 1/\epsilon]^{-1})F_{\ell} = EF_{\ell} \quad (3.8)$$

Equation 3.7 shows that with monopsony the equilibrium real wage is less than the marginal product of labor by the amount of the markdown, $w/\epsilon_{\ell w}$. The markdown factor is:

$$E(\epsilon) = [1 + 1/\epsilon]^{-1}, \quad 0 < E \leq 1 \quad (3.9)$$

Following Lerner [1934], $1/\epsilon$ is a measure of the firm's monopsony power over the real wage; higher elasticity of labor supply implies lower monopsony power (i. e., a smaller markdown), and conversely. The limiting case of a perfectly elastic labor supply curve corresponds to perfect competition where both firms and workers are wage-takers. The markdown factor E can be interpreted as an index of labor market competitiveness, since $E \rightarrow 1$ as $1/\epsilon \rightarrow 0$, and $E \rightarrow 0$ as $1/\epsilon \rightarrow \infty$. Under pure monopsony, employment, output and the real wage are less than if the firm were a perfectly discriminating monopsonist or if the labor market were perfectly competitive. Figure 3-1 shows these relationships, along with the amount of the monopsony markdown w/ϵ .

It is evident from Equation 3.9 that countercyclical markdowns under profit maximizing monopsony imply procyclical

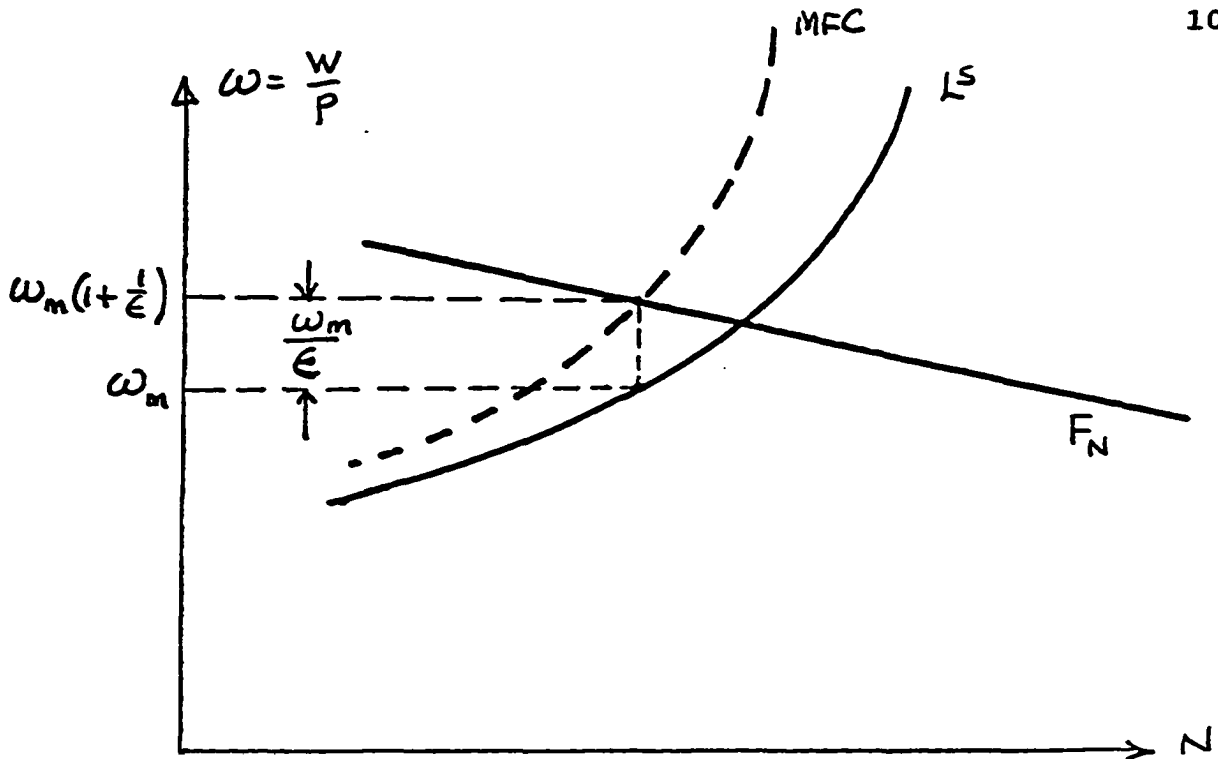


Figure 3-1. Monopsony Power in the Labor Market

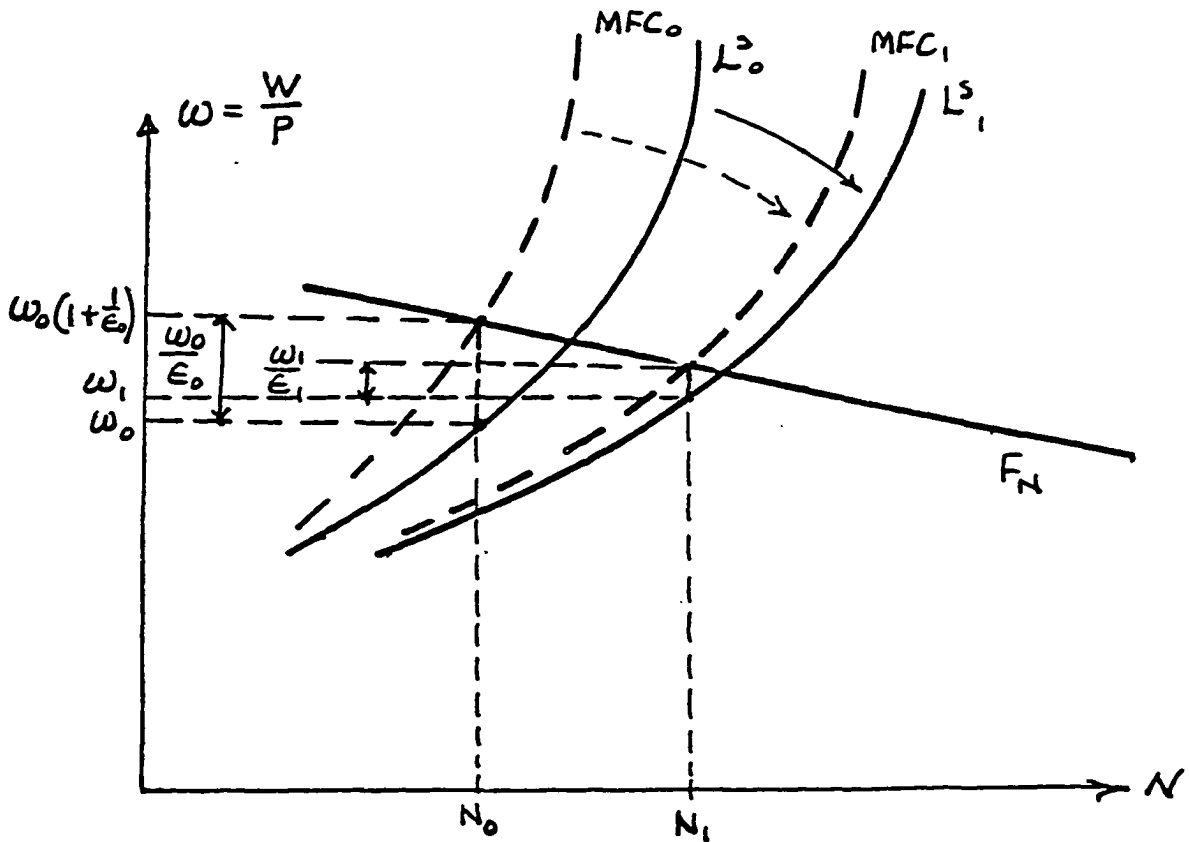


Figure 3-2. Countercyclical Monopsony Power

elasticity of labor supply ϵ ; in the monopsony model they are logically equivalent. However, I show in the next section that the procyclical requirement on ϵ is stronger for acyclical or procyclical real wages than for countercyclical markdowns.

3.5.2 Endogenous Elasticity of Labor Supply

In order to have cyclical changes in output and employment, there must be some mechanism which shifts either the aggregate marginal product curve or the aggregate marginal labor cost curve (or both) to a new equilibrium point in real terms¹⁴. Two possible mechanisms have been treated extensively in the business cycle literature: (1) the marginal product of labor curve shifts against a stationary labor supply curve, because of exogenous productivity shocks to technology $F(\ell, k)$ (the Real Business Cycle (RBC) mechanism); or (2) the labor supply curve shifts against a stationary marginal product curve, due (for example) to workers' monetary misperceptions of the real wage or perceived changes in real wealth (the Traditional Keynesian mechanism). Here, I will make use of the framework established in Chapter II, and assume a Keynesian shift of the labor supply curve due a wealth effect. (The specific source of labor supply shifts is

¹⁴Unless otherwise stated, only equilibrium trading will be assumed in this thesis.

not important here; the immediate purpose is to focus on the cyclical behavior of ϵ . However, throughout this chapter and subsequently the analysis is confined to models in which only the labor supply curve shifts).

For the purpose of exposition I will assume a labor supply curve that is monotonically increasing with w , so that $1/\epsilon_{\ell w} = \epsilon_{w\ell}$, the elasticity of the real wage with respect to employment which is a function of ℓ , and $E = [1 + \epsilon_{w\ell}]^{-1}$. Now assume a non-neutral one-time increase in the nominal money stock (as described in Chapter II) which decreases real nonlabor income through a negative effect on wealth, which in turn causes the labor supply curve to shift out so that employment and output increase¹⁵. (A more complete analysis of this mechanism in macroeconomic general equilibrium is presented in Chapter II). With employment increasing, the elasticity of the profit-maximizing real wage with respect to employment is given by applying the elasticity operator \mathcal{E} to Equation 3.8.

$$\mathcal{E}_{\ell}[w^*] = \mathcal{E}_{\ell}[E] + \mathcal{E}_{\ell}[F_{\ell}] \quad (3.10)$$

(?) (-)

¹⁵ In the wealth effects macro-model of Chapter II, an open-market purchase of government bonds increases W and P nonproportionally and decreases the value of real wealth Ω from which nonlabor income is derived. The net effect in the model is an increase in employment and output. To quote Hall [1980], "Changes in the money stock unambiguously shift the labor supply function."

Equation (3.10) follows from the elasticity of the product EF_2 in equation (3.8). For the elasticity of the equilibrium real wage with respect to employment to be non-negative, the sum of the elasticities of E and F_2 must be non-negative. A necessary condition for the monopsony real wage to be procyclical is for $\mathcal{E}_2[E] > 0$. Since $E = [1 + 1/\epsilon_{lw}]^{-1}$, the wage elasticity of labor supply ϵ_{lw} must increase as employment and output increase. However, this is only a necessary condition; A sufficient condition for the optimum real wage to be either acyclical or procyclical is for (3.10) to be non-negative, i. e.:

$$\mathcal{E}_2[w^*] = \mathcal{E}_2[E] + \mathcal{E}_2[F_2] \geq 0 \quad (3.11)$$

It can be shown that:

$$\mathcal{E}_2[E] = \frac{\mathcal{E}_2[\epsilon_{lw}]}{1 + \epsilon_{lw}} \quad (3.11)$$

so that the elasticity of ϵ_{lw} must be procyclical and large enough to satisfy the inequality in Equation 3.11. Thus, procyclical elasticity alone is sufficient for markdowns to be countercyclical, as indicated by Equations 3.3 and 3.9, but for the real wage to be either acyclical or procyclical the stronger inequality condition in (3.11) must be satisfied, due to the assumed negative elasticity of the marginal product of labor. Thus, even if ϵ_{lw} is procyclical, the real wage could be countercyclical, acyclical or procyclical depending on the magnitude of the elasticity of ϵ_{lw} . Of course, as long as ϵ_{lw}

is procyclical, w will be less countercyclical than if $\epsilon_{\ell w}$ were constant or decreasing. Real wages are more likely to be procyclical if the marginal product of labor F_{ℓ} is inelastic and $\epsilon_{\ell w}$ is procyclical and highly elastic with respect to employment¹⁶.

The conclusions from this analysis, for the Traditional Keynesian setup, are:

1. *Under static equilibrium monopsony, a necessary condition for real wages to be procyclical and for markdowns to be countercyclical is for the elasticity of labor supply to depend on factors that shift the labor supply locus, i. e., it must be endogenous to the driving forces of the business cycle.*
2. *Procyclical elasticity of labor supply is a sufficient condition for markdowns to be countercyclical, but only a necessary condition for real wages to be acyclical or procyclical.*

¹⁶For a monopsonist the inverse of the marginal product of labor curve is not a labor demand curve. The firm's demand for labor is determined by the equality of marginal product and marginal labor cost, and depends on $\epsilon_{\ell w}$. Also, it should be clear from the preceding discussion that the reduced-form elasticity of w in (3.11) represents the response of ϵ to a shift of the labor supply curve, and is not the same as the change in $\epsilon_{\ell w}$ along a stationary labor supply curve.

3. A sufficient condition for real wages to be either acyclical or procyclical is that $\mathcal{E}_\ell[\epsilon]$, the elasticity of the elasticity of labor supply, be positive and large enough in magnitude so that

$$\frac{\mathcal{E}_\ell[\epsilon]}{(1+\epsilon)} \geq |\mathcal{E}_\ell[F_\ell]| \quad (3.12)$$

This is a necessary and sufficient condition for the real wage to overcome the "tyranny of the labor demand curve."

3.5.3 Elasticity of Labor Supply in the Standard Model

In Appendix C, Section C2, I show that in the standard monopsony model with homothetic preferences, the elasticity of labor supply ϵ is always countercyclical, implying that the standard monopsony model presented here is inconsistent with countercyclical markdowns or acyclical real wages. The cause of this inconsistency is the assumption of homotheticity.

In the standard labor-market model, labor supply is a function of the real wage and nonlabor income: $\ell^s = \ell^s(w, v)$. This means that the elasticity of labor supply is also a function of w and v , with $\partial\epsilon/\partial w < 0$, $\partial\epsilon/\partial v > 0$. When homothetic preferences are specified in the standard monopsony model, ϵ increases with real nonlabor income v . What does this imply for the cyclicity of ϵ ? The only shift variable

for the labor supply function in this model is real nonlabor income v , which may be regarded as being derived from property or wealth. Since $\partial \ell^s / \partial v < 0$, a decline in v will cause the labor supply curve to shift out, increasing the level of employment and output in monopsony equilibrium. But since $\partial \epsilon / \partial v > 0$, ϵ will decrease and thus be countercyclical. As the labor supply curve shifts out and the economy expands, monopsony power $1/\epsilon$ will increase, the markdown will be procyclical and the real wage will be more countercyclical than it would be in a perfectly competitive labor market. Thus, the implications of the standard monopsony model with homothetic preferences are inconsistent with the stylized facts of countercyclical markdowns and acyclical real wages. In this respect the standard monopsony model is even more counterfactual than a perfectly competitive model of the labor market.

The reason why this standard setup is counterfactual, with or without monopsony power, is the assumption of homothetic preferences $U(c, \ell)$ which determines the algebraic sign of the change in the elasticity of the demand for leisure, η_ℓ in Equation C2.8, reproduced below:

$$\frac{\partial \epsilon}{\partial v} = \left[\frac{-\ell^d}{(1-\ell^d)} \right] \frac{\partial \eta_\ell}{\partial v} - \frac{\eta_\ell}{(1-\ell^d)} \left[\frac{\partial \ell^d}{\partial v} \right] > 0 \quad (3.13)$$

(In Equation 3.13, ℓ represents leisure and ℓ^d represents the demand for leisure, in keeping with the notation in Appendix C and Equation C2.8) With homothetic preferences, $\partial \eta_\ell / \partial v = 0$, and from Equation 3.13, $\partial \epsilon / \partial v > 0$. In order for $\partial \epsilon / \partial v$ to be negative, $\partial \eta_\ell / \partial v$ would have to be positive and large enough to offset the positive influence of the second term (an elasticity condition corresponding to Equation (3.11)); but if this were the case, individual preferences would not be homothetic¹⁷.

This result by itself does not constitute an adequate basis for rejecting the neoclassical model of individual labor supply or the standard monopsony model at the level of the firm. The evidence on markups and real wages refers in most instances to highly aggregated data, and may be irrelevant for many purposes of microeconomic analysis¹⁸. The implications for aggregate models of the labor market are more serious, and are addressed in the next subsection.

¹⁷Rotemberg and Woodford (1991) also make this point for the elasticity of product demand. Their econometric tests rejected homothetic models of demand.

¹⁸There is some evidence that real wages are more procyclical at lower levels of aggregation, and that aggregation introduces a countercyclical bias in measurements of the real wage. (See Chapter I). Such evidence is not favorable to models which employ the representative agent.

3.5.4 Implications For Aggregate Labor Supply

In macroeconomic modeling it is common practice to assume a "representative agent" whose behavior is described by demand and supply functions which are implicitly assumed to be an exact linear aggregation of corresponding individual demands and supplies. i. e., an aggregate labor supply curve is posited as:

$$L^s(w, v) = \sum_i \ell^s(w, v_i) \quad (3.14)$$

where L^s and ℓ^s are assumed to have the same homogeneous functional form. When exact linear aggregation is applicable, it preserves a one-to-one correspondence between the functional forms at the macro and micro levels, thereby rationalizing the idea of "microfoundations" for macroeconomics.

Notwithstanding the intuitive appeal of (3.13) as an "adding up" method, it is mathematically consistent only for the quasi-homothetic class of indirect utility functions which have the Gorman polar form (Deaton and Muellbauer, [1980, Sec. 6.3]):

$$U(W, P, Y_i) = a_i(W, P) + b(W, P)Y_i \quad (3.14)$$

Where Y_i is total income or expenditure. For exact linear aggregation of (3.14), a_i and Y_i can vary among individuals, but $b(W,P)$ cannot. When $a_i = 0$, preferences are homothetic. The corresponding demand for leisure, which can be derived using Roy's identity, can be written in the following form:

$$l^d(W, P, Y_i) = \alpha_i(W,P) + \beta(W,P)Y_i \quad (3.15)$$

In Equation 3.15, $\alpha_i(W,P)$ is a minimum leisure requirement that can vary among workers, $\beta(W,P)$ is the marginal propensity to consume leisure out of total income, which must be the same for all workers, and $Y_i = WT + V_i$ is the total income endowment. Leisure is a normal good and therefore $\beta > 0$. Since leisure and consumption are the only two commodities they must be gross substitutes, and therefore $[\partial\alpha_i/\partial W + \partial\beta/\partial W] < 0$.

It is apparent that demands must be linear in total income Y_i with the same coefficient $\beta(W,P)$, i. e., all workers must have linear Engel curves for leisure with the same slope β , but possibly different intercepts α_i . When preferences are homothetic, $\alpha_i = 0$ for all workers, Engel curves are identical rays through the origin, the income-elasticity of leisure demand is unity, and the wage-elasticity of leisure demand is the elasticity of $\beta(W,P)$ which is independent of the distribu-

tion of worker endowments¹⁹. When preferences are quasi-homothetic, only aggregate or per capita endowments of nonlabor income and time matter for aggregate demand.

The labor supply function corresponding to equation (3.15) is:

$$\ell^s(W, P, Y_i) = T - \alpha_i(W, P) - \beta(W, P)Y_i \quad (3.16)$$

The individual labor supply function derived from the solution to the consumer's problem will have this form if and only if the consumer preferences represented by $U(c, \ell)$ are quasi-homothetic²⁰. In other words, for exact linear aggregation (and thus the representative agent) to be valid, the functional forms in Equations (3.15) and (3.16) are required at the micro-level; that functional form is a solution to the consumers' problem if and only if the individual's budget constraint is linear and continuous and preferences are quasi-homothetic, as in Equation (3.14).

¹⁹Deaton (1992) and Deaton and Muellbauer (1987) emphasize that linear Engel curves, which are implied by the homotheticity requirement of the representative agent method, have been consistently rejected in empirical studies.

²⁰The class of quasi-homothetic utility functions include Cobb-Douglas, with $a_i = 0$, Constant Elasticity of Substitution (CES) with a_i equal to a positive constant, and the Stone-Geary Linear Expenditure System (LES) with a_i constant and minimum demand quantities c_0, ℓ_0 that are positive.

If the time endowment T is the same for all workers, linear aggregation of the labor supply function in (3.16) will reproduce the same form with average endowments Y and $\alpha(W,P)$ in place of Y_i and $\alpha_i(P)$, respectively, and the same value for β . The elasticity of individual labor supply for this functional form is:

$$\epsilon_{LW} = \frac{W \left[\frac{\partial \alpha_i}{\partial W} + \frac{\partial \beta}{\partial W} Y_i \right]}{[\alpha_i + \beta Y_i - T]} \quad (3.17)$$

where $Y_i = WT + V_i$. At the micro level, individual elasticities at a given wage W depend on individual endowments Y_i and $\alpha_i(P)$, and therefore may vary among individuals. Under exact linear aggregation the elasticity of aggregate labor supply will have the same functional form as Equation (3.17), with Y and $\alpha(W,P)$ replacing Y_i and α_i . (Because elasticity is a logarithmic measure, there is no linear aggregative relationship between individual and aggregate elasticities).

The cyclicity of ϵ for the quasi-homothetic class of labor supply functions can be determined by differentiating Equation (3.17) with respect to Y , the income or scale variable:

$$\frac{\partial \epsilon}{\partial Y} = -W \left[\frac{\epsilon^S \left(\frac{\partial \beta}{\partial W} \right) + \left(\frac{\partial \alpha}{\partial W} + \frac{\partial \beta}{\partial W} \right) \beta}{(\epsilon^S)^2} \right] \quad (3.18)$$

Since $\partial\beta/\partial W < 0$ and $(\partial\alpha/\partial W + \partial\beta/\partial W) < 0$, it follows that $\partial\epsilon/\partial Y > 0$. Since $Y = WT + V$ where V is nonlabor income, it also follows that $\partial\epsilon/\partial V$ will be positive in both the aggregate and disaggregated versions of Equation (3.17), and from the derivation and accompanying discussion in Appendix C, the elasticity of labor supply in the aggregate model will be countercyclical, not procyclical, whenever representative agent aggregation is valid. The assumption of monopsony power in a Keynesian model with representative agent aggregation of the labor market will result in countercyclical real wages and procyclical markdowns, both of which are counterfactual.

3.6 Conclusions

The main conclusion of this chapter is that the assumption of monopsony power in a Keynesian labor market does not, by itself, lead to an explanation of countercyclical markdowns or acyclical aggregate real wages when the representative agent method of aggregation is employed²¹. The representative agent method is valid only if preferences are quasi-homothetic. If preferences are assumed to be quasi-homothetic, then the elasticity of labor supply of the representative worker will be countercyclical (as it is for each individual worker), and this implies procyclical markdowns and an

²¹This conclusion holds in the absence of positive exogenous shocks to the marginal product of labor, and applies also to the possibility of explaining these two anomalies in terms of procyclical elasticity of goods demand.

aggregate real wage that is more countercyclical than it would be under perfect competition.

Alternatively, if preferences are assumed to be not quasi-homothetic, then nonlinear aggregation does not preserve functional forms derived at the micro level, and the logical basis for microfoundations is compromised. This would be counter to the macroeconomic research program of the past two decades²².

This chapter has revealed that the assumptions which underlie the representative method of aggregation preclude an explanation of the real wage and markup anomalies in terms of cyclical elasticities of labor supply or goods demand. If the elasticities of aggregate goods demand and aggregate labor supply have any economic significance, the empirical record on markups and real wage behavior suggests that perhaps the real aggregate economy is not that homothetic. Therefore, the next chapter of this thesis departs somewhat from representative agent aggregation in the labor market, and explores the implications of monopsony power when labor is heterogeneous and the distribution of agent characteristics matters.

²²It should be noted that the representative agent (homotheticity) assumption is ubiquitous in macroeconomic theory, and is implicit in all attempts to extrapolate theories of the individual consumer and the firm to higher levels of aggregation.

CHAPTER IV

AN AGGREGATE LABOR SUPPLY FUNCTION FOR HETEROGENEOUS LABOR

4.1 Introduction and Overview

In the flexible-price macro model of Chapter II, a one-time increase in the money stock raised the price level, lowered the interest rate, and reduced the real value of aggregate wealth. Since the aggregate labor supply function in the model of Chapter II was based on a representative agent, only the mean or per capita real wealth effect mattered for labor supply¹. In Chapter III it was shown that when a representative agent is assumed for aggregate labor supply, an increase in labor supply due to an aggregate wealth effect will always be accompanied by a decreasing or countercyclical wage-elasticity of labor supply. The implication for monopsonistic labor markets is that the real wage would be more countercyclical (and therefore more counterfactual) than under perfect competition. The principal conclusion of Chapter III was that the representative agent assumption precludes an explanation of countercyclical markups and procyclical real wages in terms of procyclical elasticity of aggregate labor supply.

¹In the representative agent model of Chapter II the aggregate or mean-level wealth effect is sufficient to shift the aggregate labor supply curve and make the connection between a change in the money stock and real output. However, the real wage in that model is countercyclical.

In this Chapter I show that, by moving away from the representative agent framework for labor supply and allowing for worker heterogeneity and distribution effects, it is possible to construct an aggregate labor supply function that exhibits procyclical elasticity, which was shown in Chapter III to be a necessary condition for real wages to be acyclical and a sufficient condition for markups to be countercyclical when labor markets are monopsonistic. This is accomplished in a standard decision-theoretic framework with the additional requirement that preferences are nonhomothetic. The resulting aggregate labor supply function has the potential, when imbedded in a general equilibrium framework, of predicting cyclical behavior of markups and the real wage that is more consistent with the stylized facts.

The spirit of this investigation is squarely within the New Keynesian literature, in that it explores the implications of imperfect competition in the labor market for the money-output connection and the behavior of real wages. It differs in that it does not utilize the representative agent method of aggregation for labor supply, a feature that is common to both the New Keynesian and Real Business Cycle literatures. Consequently, the marginal responses of individual workers are not restricted by the homotheticity assumption, and the distribution of workers' marginal responses matters.

Individual labor supply is assumed to be constrained by a fixed work week and workers are assumed to be heterogeneous with respect to their nonlabor income endowments and their preferences for risk. This heterogeneity gives rise to a distribution of reservation wages which provides the link between individual and aggregate labor supply.

A key assumption is that nonlabor income is derived primarily from holdings of financial assets, augmented on the low end by transfer payments. Therefore, the distribution of nonlabor income, and consequently of reservation wages, is proportional to the size distribution of real wealth holdings in the workforce, which is assumed to be Lognormal². The aggregate labor supply function takes on the properties of the size distribution of financial wealth in the workforce.

I show that if workers have nonhomothetic preferences with increasing relative risk aversion then the elasticity of the reservation wage function will be an increasing function of real nonlabor income; if real nonlabor income declines for all workers, the elasticity of the reservation wage function will decrease, implying (i) aggregate labor supply will increase at all wages, and (ii) the elasticity of labor supply will increase. Thus, the real wage-elasticity of labor supply

²There is theoretical and empirical support for the lognormal assumption in the literature, e. g., Sargan (1957), Atkinson (1975), Pestieau and Possen (1979) and Vaughn (1988).

will be procyclical. This result requires only the assumption of increasing relative risk aversion, and is independent of the functional form assumed for the distribution of wealth. I also show that if nonlabor income is assumed to be distributed Lognormal, the point elasticity of the distribution is inversely related to the *inequality of the distribution of nonlabor income* in the labor force, measured by the variance of the logarithm. A decrease in the inequality of the distribution of nonlabor income will cause the elasticity of the distribution to increase over a central range of reservation wages which includes the mean. This result is specific to the Lognormal distribution.

I also show that if workers have increasing relative risk aversion for wealth, they will rebalance their financial-asset portfolios in response to an open-market purchase of bonds by the central bank in a heterogeneous manner. Wealthier workers with higher relative risk aversion will reduce their bond holdings and income disproportionately, and will absorb a disproportionate amount of the additional money created by the banking system. This will reduce the dispersion of the distribution of income from bonds in the workforce, which will cause the elasticity of the labor supply function to increase as it shifts out. An open market sale of bonds would produce the opposite result.

Computations with a calibrated version of the model produce large positive (i. e., procyclical) labor supply elasticities for aggregate wages corresponding to labor-force participation rates of 40% to 78%. This range includes the current U. S. labor-force participation rate of 66% at the calibrated mean reservation wage. Sensitivity tests indicate that the model is fairly robust against variations in its key parameters. The model is critically dependent on its two principal assumptions: nonhomothetic preferences with increasing relative risk aversion; and Lognormal distributions of wealth and nonlabor income in the work force.

This chapter makes a theoretical connection between monetary policy actions and aggregate labor supply by modeling the effects of open market operations on the distribution of wealth and nonlabor income. Open market operations shift the aggregate labor supply curve and change its elasticity procyclically over a relevant range of reservation wages. This "monetary theory" of aggregate labor supply, together with the assumption of monopsony power in labor markets, provides a theoretical basis for countercyclical markups and acyclical or procyclical real wages within an otherwise Traditional Keynesian setup, and may serve to restore some respectability to that class of models.

The Chapter is organized as follows: The static monopsony model with heterogeneous labor supply is formally specified and developed in Section 4.2. In Section 4.3 the theoretical and empirical relevance of the Lognormal distribution to this problem is discussed, and the mathematical properties of the distribution are described. Section 4.4, supplemented by Appendix D, contains the principal analytical result of the chapter, which is the construction of an aggregate labor supply function, the elasticity of which depends on monetary policy actions. Section 4.5 describes the calibration of the model to the U. S. economy, and the computational procedures. Computational results are presented in Table 4-4 and Exhibits 4-1 through 4-15 at the end of Section 4.5, and are the basis of the claims made for the model. Section 4.6 presents some conclusions that may be drawn from the research described in this chapter.

4.2 Heterogeneous Labor Supply

In this section I develop an aggregate labor supply function for a workforce that is heterogeneous with respect to nonlabor income. The approach taken preserves the principle of individual utility maximization, while allowing aggregate labor supply to be determined by the distribution of wealth in the workforce. The aggregate supply function thus derived is

a monotonic transformation of the distribution of wealth in the workforce.

4.2.1 Heterogeneous Workers

I will investigate the implications of two possible sources of heterogeneity in individual labor supply: (1) increasing relative risk aversion with respect to nonlabor income, and (2) a distribution of reservation wages based on heterogeneous nonlabor income endowments.

Increasing relative risk aversion is a departure from homotheticity that has some theoretical and empirical support in the literature (Arrow, 1970). However, it has the distinct disadvantage of excluding most of the standard utility functions used in economic theory, and requires some form of non-linear aggregation for both leisure and consumption. In an attempt to preserve a role for standard assumptions about preferences, I will also examine the case where preferences are identical and homothetic, but workers differ in their nonlabor income endowments and therefore in their reservation wages. The advantage of focusing on nonlabor income as the heterogeneous parameter is that it is potentially measurable, it has a plausible connection to macroeconomic variables, and it already plays a role in the standard labor-market model.

4.2.2 Indivisible Labor

The first assumption will be to equate hours with workers by assuming that labor services can be traded only in fixed quantities of h hours per period. Thus, the labor supply schedule to a firm, an industry and the economy will be measured by the number of workers who choose to work h hours at the prevailing wage.

There is considerable support in the macroeconomic literature for this approach to modeling aggregate labor supply. First, macroeconomic theory and policy have been more concerned with changes in the number of persons employed and unemployed than in total hours or changes in the length of the work week. Many specifications of the aggregate labor market in theoretical models make no important distinction between the number of workers and hours per worker. Second, although there are small adjustments in the length of the work week in the manufacturing sector, it is well established that most of the quarterly and annual variation in aggregate man-hours comes through fluctuations in the number of workers employed rather than in hours per worker. Some evidence for this claim is presented in Tables 4-1 and 4-2 below, which are taken from Heckman (1984) and based on Coleman (1984). To quote Heckman from the original:

Table 4-1
Fluctuations in Total Hours, Hours per Worker, and
Number of Employees 1970-1979
Deviation from Trend, in Percentage Points
Annual Data

<u>Year</u>	<u>Total Hours</u>	<u>Hours Per Worker</u>	<u>No. of Employees</u>
1970	-3.12	-0.92	-2.20
1971	-2.17	0.03	-2.20
1972	1.44	0.51	0.93
1973	2.30	0.27	2.04
1974	-0.95	-0.59	-0.34
1975	-5.85	-0.47	-5.41
1976	1.25	0.48	0.77
1977	2.28	0.29	2.00
1978	3.11	0.28	2.84
1979	1.71	0.16	1.57
(70+71+74+75)/2	-6.05	-0.98	-5.08

Source: Table 103, BLS Handbook of Labor Statistics, Bulletin 2070.
 Taken from Heckman (1984) and Coleman (1984)

Table 4-1
By Industry
Average Percentage Point Deviation from Trend for the Two
Contractions of the 1970's, 1970-1971 and 1974-1975

	<u>Total Hours</u>	<u>Hours Per Worker</u>	<u>No. of Employees</u>
Private Business	-6.1	-1.0	-5.1
Total Non-Agricultural	-5.2	-1.0	-4.2
Mining	-2.9	-1.2	-1.6
Construction	-11.2	-0.3	-11.0
Manufacturing	11.4	-2.0	-9.5
Transportation and Public Utilities	-5.1	-1.3	-3.9
Wholesale and Retail Trade		-2.9	-0.04
Finance, Insurance and Real Estate	-2.5	0.2	-2.7
Services		-2.5	-0.6
			-1.9

Source: Tables 103 and 72, BLS Handbook of Labor Statistics, Bulletin 2070
 Business Statistics, 1979. Taken from Heckman (1984) and Coleman (1984).

...any serious empirical model of business-cycle labor market fluctuations must account for manhour variations at the extensive margin (employment or labor-force entry decisions) as well as manhour variations at the intensive margin The 'representative consumer' always works, and so an interior solution labor supply theory is invoked to account for the facts. Tables 1 and 2 reveal how poorly a representative consumer model describes the facts. As Coleman (1984) stresses, the data do not support a representative consumer model... As long as representative consumer models are used, micro evidence cannot be used to "calibrate" macro models.

The analysis of Chapter III is consistent with Heckman's comments, in that it was demonstrated that the representative agent construction precludes a model from exhibiting acyclical real wages.

Other labor economists seem to concur with Heckman. Knieser and Goldsmith (1987), in their survey article on models of the aggregate labor market, report aggregate elasticities based on annual data for the entire postwar period 1948-1985, and conclude that:

...the elasticity of aggregate employment with respect to real GNP [.44] is over twice the elasticity of the average workweek with respect to real GNP [.20]. Thus, cyclic movements in labor utilization appear to be dominated by changes in the number of workers with jobs rather than characterized by short hours or worksharing arrangements... Any satisfactory model of the aggregate labor market must be consistent with these empirical regularities.*In order to shed light on aggregate movements in employment, micro-economic research must account for changes in labor force participation rather than variations in hours of work of the continuously employed individual.*
[Italics added]

Pencavel (1986, p. 83), also citing Coleman, concludes that:

...the larger part of the movement in aggregate manhours over the business cycle is attributable to movements in the number of workers employed and not to movements in the hours worked of those continuously employed.

Lilien and Hall (1986) arrive at the same conclusion based on their analysis of annual U.S. data on per capita hours from 1956 - 1983, which updates and confirms Heckman's conclusion from BLS data. Finally, Deaton and Muellbauer (1980, Ch. 11, pp. 283-290) discuss the implications of quantity rationing in the labor market, and conclude that:

...for short-run analysis there are many types of jobs in which it is more realistic to take hours as given or at any rate set by employers. To some extent this is true even in jobs where overtime at a higher wage rate is common.

There also is precedent in the macroeconomic literature for treating labor supply as discrete and indivisible. (e.g., Sherman and Willett [1972]; Branson [1989], p. 123; Hall and Taylor [1983] pp. 452-455). Hansen (1985), Rogerson (1988) and Rogerson and Wright (1988) have developed models with "indivisible labor", where it is assumed that each worker is endowed with the same fixed unit of labor per time period.

Treating aggregate labor supply as a flow of workers providing fixed quantities of labor is also consistent with the search models of labor market adjustment which are assumed to underlie the static equilibrium framework. Thus, the assumption that individual labor hours are traded only in fixed quantities has considerable precedent in the business cycle literature. With fixed individual labor supply the operative labor supply decision of the consumer is whether to work at the extensive margin; utility maximization still

applies, but with a highly constrained opportunity set. Aggregation of labor supply will be over workers at their extensive margins of labor-force participation³

4.2.3 Behavioral Assumptions About the Labor Market

I will assume that the labor services of individual workers are purchased only in discrete units of h hours per worker per time period, due to technological or institutional quantity constraints, and will abstract from variations in hours per worker and perturbations in h . Thus, this assumption is taken merely as an institutional datum⁴.

I also assume a large population of potential labor-force participants who have identical tastes for non-negative

³The approach here differs somewhat from that of Rogerson and Hanson (1988), who assumed that individual preferences for leisure are discrete and therefore nonconvex. Instead, I assume that preferences are convex but are quantity-constrained, resulting in a discrete, nonconvex opportunity set for the individual worker. The nonconvexity will be overcome in aggregation by assuming a continuous distribution of reservation wages over workers.

⁴There is some variation in h by industry and occupation, and one could extend the model by assuming a finite set $\{h_i\}$ and working with an aggregate average h . This complication will be avoided here. Also, no presumption is made here that the institutional datum h is Pareto optimal, or anything other than an accepted norm of implicit labor contracts given by history and the institutional setting. The origins, history and political economy of the 8-hour workday and the 40-hour workweek in the U. S. and England have been researched by Dankert, Mann and Northrup [1965]; Langenfelt [1974], Cross [1988, 1989], Hinrichs, Roche and Sirianni [1991], all of whom emphasize the important roles of religious and cultural norms and the political influence of organized labor. The standard 40-hour workweek in U. S. manufacturing has not changed significantly since 1946, and there appears to be no strong movement underway to reduce it. (Owen [1989]). It appears to be in long-run equilibrium.

quantities of a composite consumption good c and nonmarket (leisure) time ℓ which can be completely represented by a concave subutility function $U(c, \ell)$ with the standard properties that $U_c > 0$, $U_\ell > 0$, $U_{cc} < 0$ and $U_{\ell\ell} < 0$.

In addition, I assume that each worker has tastes and preferences for holding a portfolio of endowed wealth Ω as a store of value, disaggregated in the form of real money balances $m = M/P$ and earning assets $f = F/P$ (where $m + f = \Omega$), and that those preferences can be represented by a concave subutility function $H(m, f)$ which possesses the same properties as U . Earning assets provide utility by earning a return in the form of income, and appreciation which is not certain. Thus, earning assets are risky. Money balances provide utility by facilitating transactions and providing a risk-free store of value compared to earning assets.

Each worker has choices to make regarding c , ℓ , m and f . I will assume that workers have identical preferences over c, ℓ, m and f which are weakly separable, i. e.,

$$\Psi(c, \ell, m, f) = \Psi[U(c, \ell), H(m, f)]$$

This will enable the analysis of optimum choices to be carried out in separate stages, wherein U and H are optimized independently subject to their own constraint sets. That being the

case, I will defer discussion of the workers' portfolio balancing problem until later in the chapter (Section 4.4 and Appendix D, Section 7) where it will play a major role. In order to concentrate on the labor market at this stage, I will assume that each worker's portfolio balance problem has been pre-solved, and each worker holds an amount of endowed wealth $\Omega_i = a_i\Omega_i + (1-a_i)\Omega_i$, where a_i is the fraction of wealth held in earning assets. This results in each worker receiving portfolio income from the yield on earning assets: $y_i = r f_i = a_i r \Omega_i$. This, together with transfer payments t_i from the government, constitutes a worker's source of nonlabor income $v_i = Y_i + t_i$.

In the most general case, Ω_i , a_i and t_i , and consequently v_i , are all different for each worker. Thus I assume that workers have identical convex indifference maps but differ in their wealth endowments and nonlabor income. I also assume that the number of workers is sufficiently large and compact that the distribution of nonlabor income among the worker population can be well described by a cumulative distribution function $\Phi(v)$ that is continuous and twice differentiable.

4.2.4 Individual Labor Supply Subject to Quantity Constraints

This section contains a more formal specification of individual labor supply subject to quantity constraints.

The approach to specifying the labor market follows Killingsworth (1983), Rogerson (1988) and Branson (1989).

With the imposed quantity constraint, ℓ can be either 0 or $1-h$, but not both. The consumer's problem becomes, with the additional quantity constraints:

$$\begin{aligned} & \text{Max } U(c, \ell), \text{ subject to the constraints} \\ & c, \ell \\ & c \leq v_i + w(1-\ell) \\ \text{and } & \ell = 1 \\ & \text{or } \ell = 1 - h \end{aligned} \tag{4.1}$$

where $w = W/P$ is the real wage, and 1 is the maximum time available in the time period. Because of the discrete labor hours constraint, the constraint set of (4.1) is not convex and therefore the Kuhn-Tucker conditions for a constrained maximum are not applicable⁵. In general the marginal rate of substitution between c and ℓ will be different from the real wage w at both corner points, and will not be a guide to which point has the higher level of utility. (See Figure 4-1).

For a worker with nonlabor income v_i who is offered a real wage w to work exactly h hours, the choice is between only two points on his budget line $c_i = v_i + w(1-\ell)$: choose between work and leisure based on the highest total (direct)

⁵The sets $\{c, 1-h\}$ and $\{c, 1\}$ are disjoint, i. e. $\{c, 1-h\} \cap \{c, 1\} = \emptyset$, and therefore $\{c, 1-h\} \cup \{c, 1\}$ cannot be a convex set.

utility between $U(v_i, 1)$ and $U(v_i + wh, 1-h)$. For a given h , each worker's choice will depend on the values of w and v_i . The offered wage at which a worker is just indifferent between working h hours or not at all is called his reservation wage, ω_i , and is formally defined by the identity :

$$V_i(v, \omega, h) = U_0(v_i, 1) - U_1(v_i + \omega_i h, 1-h) \equiv 0 \quad (4.2)$$

Equation (4.2) expresses the worker's indifference at the reservation wage. U_0 is the section of the utility surface along $\ell = 1$, and U_1 is the section along $\ell = 1-h$. (See Figure 4-1).

It is assumed that monopsony firms set the wage w that is offered to all workers, including the marginally unemployed workers. (See Chapter III for a discussion of the monopsony assumption and its motivation.) The worker's binary choice between work and nonwork can be reduced to a reservation wage rule:⁶

$$h_i = \begin{cases} 0, & w < \omega \\ h, & w \geq \omega \end{cases} \quad (4.3)$$

The reservation wage ω will be a monotonic increasing function of v , $\psi(v)$, the properties of which can be derived from (4.2)

⁶This follows the development in Killingsworth [1983]

via the implicit function theorem⁷. (The derivation is in Appendix D). Thus, values of v , $\omega = \psi(v)$, w and h completely parameterize the individual worker's constrained decision.

Figure 4-1 illustrates the binary choice confronting an individual worker whose labor supply is constrained to a fixed number of hours h . Each worker will be on an indifference curve corresponding to their nonlabor income. v_0, v_1, v_2 , etc., and each worker has a reservation wage $\omega_i = \psi(v_i)$ at which they are just indifferent between working h hours at that wage or not working and receiving only their nonlabor income v_i . The opportunity set of a worker with nonlabor income endowment v_i is constrained to the two sets of points $\{(v_i, 1); (v_i + wh, 1-h)\}$, $w \geq 0$, and thus the only relevant portions of the indifference map are along the separate vertical lines defined by $\ell = 1$ and $\ell = 1-h$. In general, a worker's indifference curve will not be tangent to the offer-wage budget line at points such as A_0, A_1 , etc. along the hours constraint line in Figure 4.1, and the worker's marginal rate of substitution at these points will not equal the reservation wage.

⁷ The theorem is applicable here due to the assumptions about $U(c, \ell)$, which also guarantee that $\psi(v)$ will be monotonic. See Appendix D, Section D1.

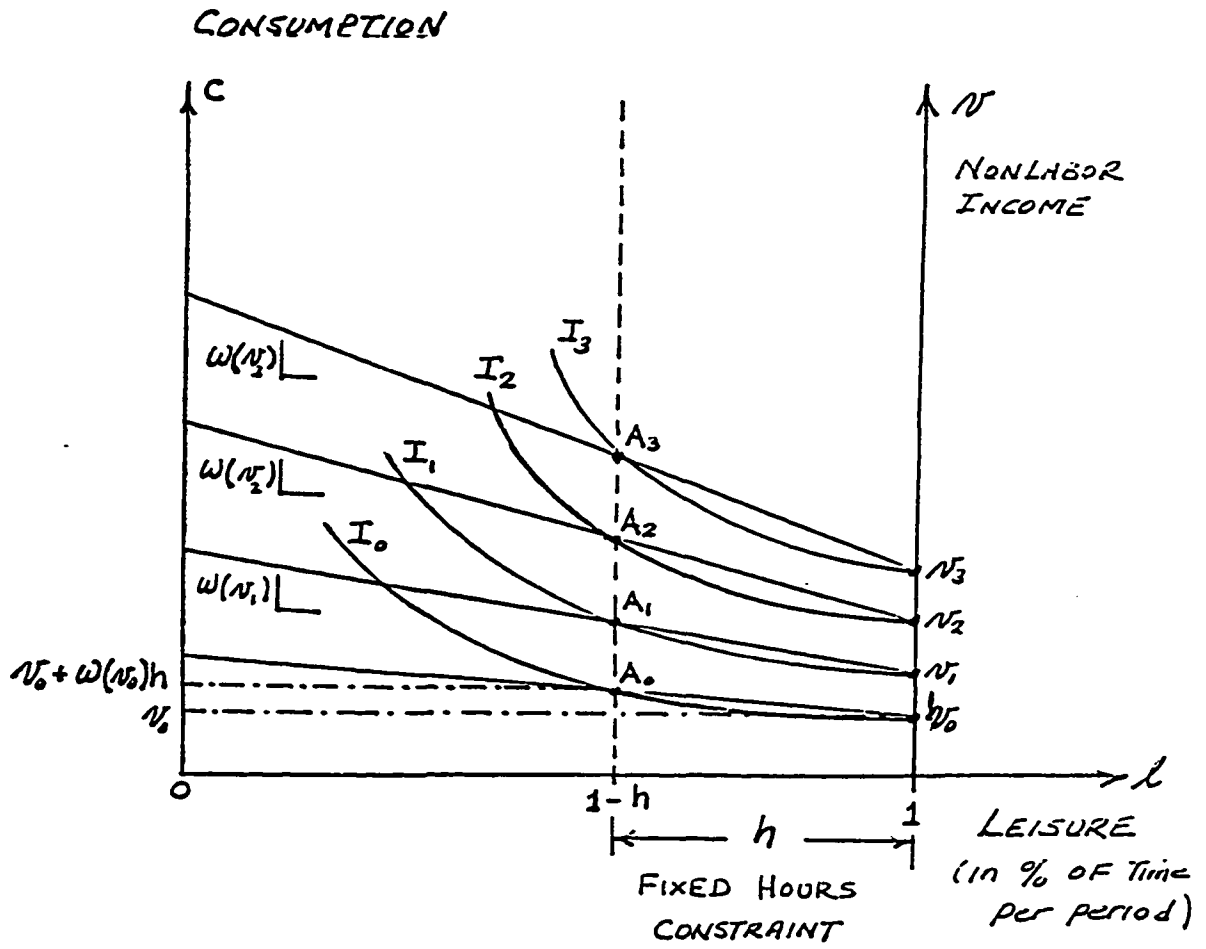


Figure 4-1. Constrained Labor-Leisure Choice

Although a worker's choice to accept work at or above his reservation wage may be the best he can do under the quantity constraint, in general it will not be globally optimal absent that constraint. Some workers will be *over-employed* in the sense that they would prefer to work fewer than h hours at the prevailing wage, and others with lower reservation wages may be *underemployed* in the sense that they would be willing to work more hours at the prevailing wage and are receiving rents above their reservation wage⁸.

4.2.4 Aggregate Labor Supply

The main implication of quantity constraints in this model is that, to a first approximation, changes in aggregate labor supply over the business cycle will be determined by the participation decisions of potential workers at the extensive margin in response to changes in the wage offer or their endowments, rather than adjustments in hours of employed workers. The supply of 'h-hour' blocks of labor services to a firm (and ultimately the economy) will be determined by the number of individuals who choose to participate in the labor force at various wage levels. The aggregate labor supply curve, then, represents a supply of workers at the margin of

⁸The behavioral pressures against the fixed hours constraint may give rise to shirking and absenteeism by over-employed workers, and moonlighting by underemployed workers. The model presented here abstracts from those possibilities.

labor-force participation, and will be determined by the cumulative distribution of reservation wages in the workforce, (designated by $\Phi(w) = \Pr(\omega \leq w)$, an approach suggested by Ben-Porath (1973).

The general functional form of the aggregate labor supply function can be expressed as follows:

$$\begin{aligned}
 L^s(w, \alpha) &= Nh\Phi(w, \alpha), \\
 0 &\leq \Phi(w, \alpha) \leq 1, \\
 \lim_{w \rightarrow \infty} \Phi(w, \alpha) &= 1 \\
 \partial\Phi/\partial w &= \phi(w, \alpha) \geq 0
 \end{aligned}
 \tag{4.4}$$

The cumulative distribution function $\Phi(w, \alpha)$, with parameter vector α , represents the proportion of the workforce population N having a reservation wage ω less than or equal to w , and therefore who are willing to work h hours at the wage w . The marginally employed worker has a reservation wage $\omega = w$. The parameter vector α contains shift variables which capture exogenous changes in the distribution of nonlabor income.

The concavity of U ensures that it has decreasing absolute risk aversion, which in turn ensures that the reservation function $\omega = \psi(v)$ is monotonic. (See Appendix D, Section D1.) It follows that:

$$\Phi(w, \alpha) = \Pr(\omega \leq w) = \Pr(\psi \leq w) = \Pr(v \leq g(w)),$$

where $g = \psi^{-1}$ (See Figure 4-2). Then (4.4) can be written in a composite form as:

$$L^s(w, \alpha) = Nh\Phi(g(w), \alpha), \quad g(w) = \psi^{-1}(v) \quad (4.5)$$

The aggregate labor supply function is a composite function of (a) $\Phi(v, \alpha)$ the cumulative distribution of nonlabor income (nonhuman wealth) in the workforce, and (b) the inverse of the reservation wage function, $g(w) = \psi^{-1}(v)$, which relates the reservation wage, and thereby the labor force participation rate, to nonlabor income.

Figure 4-2 illustrates three possible reservation wage functions, with constant (ψ_0), increasing (ψ_1) and decreasing (ψ_2) elasticities, and their respective inverses $g_i(\omega)$. The wage-elasticity of this aggregate labor supply function is equal to the product of two elasticities:

$$\epsilon_{Lw} = \mathcal{E}_g[\Phi(g)] \cdot \mathcal{E}_w[g(w)] \quad (4.6a)$$

or
$$\epsilon = \gamma \cdot \theta \quad (4.6b)$$

where $\mathcal{E}_x[y] = d(\log y)/d(\log x)$ is the elasticity operator.

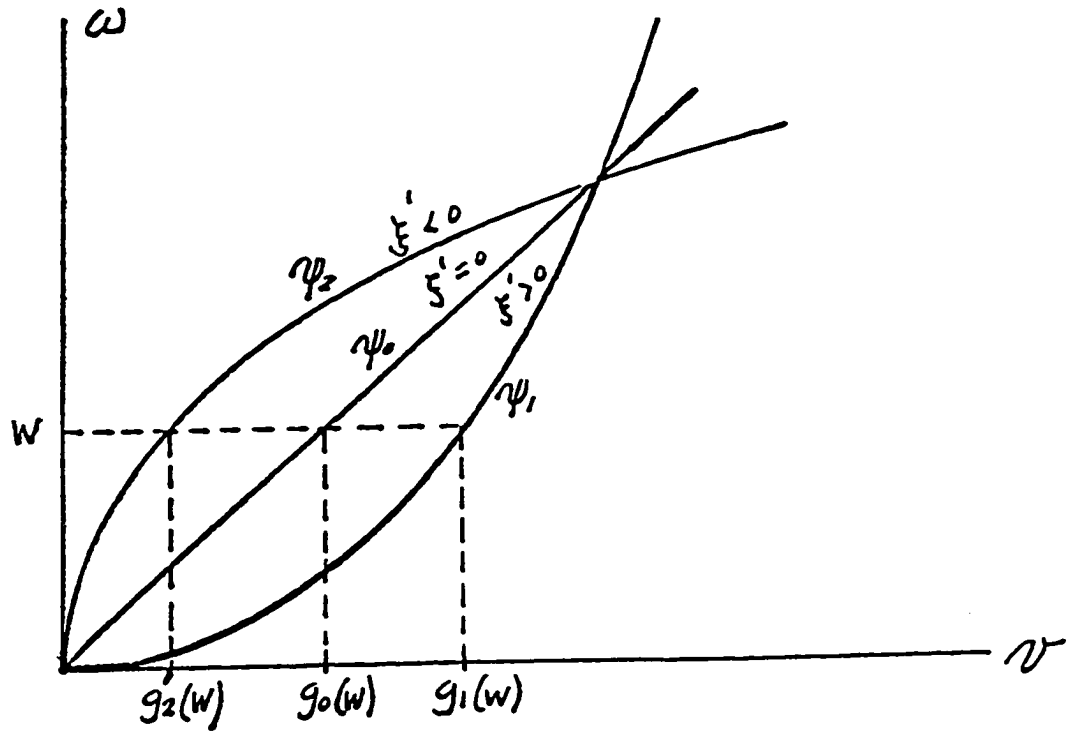


Figure 4-2. Reservation Wage Functions ψ and Their Elasticities, ξ

Equation (4.6) follows directly from (4.5) and the elasticity of a composite function; the elasticity of aggregate labor supply is the product of the elasticities of the two functions $\phi(g)$ and $g(w)$. Note that since ψ is monotonic, $\mathcal{E}_w[g(w)] = 1/\xi = \theta$.

Changes in the elasticities of either ϕ or g will result in corresponding changes in $\epsilon_{\ell w}$. Expressed in terms of elasticities of elasticities, $\mathcal{E}_x[\epsilon] = \mathcal{E}_x[\gamma] + \mathcal{E}_x[\theta]$. If x represents a procyclical variable such as employment, output, money supply or the price level, A positive value for $\mathcal{E}_x[\epsilon]$ means that ϵ will also be procyclical.

It remains to consider the determinants of these functions in this model, derive expressions for their elasticities and subject them to comparative static analysis.

4.2.5 Elasticity of The Reservation Wage Function

Individual preferences determine aggregate labor supply through the reservation wage function $\psi = \psi(v)$, and will also affect the elasticity of aggregate labor supply through the inverse function $g(w)$. The reservation wage function also establishes a link between aggregate labor supply and the distribution of nonlabor income (or wealth).

The properties of ψ are derived in Appendix D, where it is shown that ξ , the elasticity of ψ is constant for homothetic preferences (ψ_0 in Figure 4-2), and increasing with v for preferences that have increasing relative risk aversion R_R (e. g., ψ_1 in Figure 4-2)⁹. In the homothetic case $\xi' = \theta' = \mathcal{E}_v[\theta] = 0$, and the elasticity of aggregate labor supply will be unaffected by the reservation wage function ψ as v changes. However, in the case of increasing relative risk aversion, $\xi' > 0$, $\theta' < 0$, $\mathcal{E}_v[\theta] < 0$ and therefore $\mathcal{E}_v[\theta]$ will make a negative contribution to $\mathcal{E}_v[\epsilon]$ i. e.:

$$\mathcal{E}_v[\epsilon] = \mathcal{E}_v[\gamma] + \mathcal{E}_v[\theta] \quad (4.7)$$

(?) (-)

This means that ϵ will increase whenever nonlabor income declines. Since labor supply increases when v declines, the change in ϵ will be procyclical.

Thus, a reservation wage function based on homothetic preferences will not change the elasticity of labor supply as nonlabor income declines. However, a function based on preferences with increasing relative risk aversion will increase the elasticity of labor supply as nonlabor income declines. Since labor supply also increases when nonlabor income declines, IRRA will cause ϵ to be procyclical.

⁹Since uncertainty has not been explicitly modeled, R_R can be interpreted as a measure of the relative concavity of $U(c, \ell)$. Decreasing relative risk aversion can be ruled out theoretically; see Appendix D.

Thus, heterogeneity of preferences, in the specific form of increasing relative risk aversion for nonlabor income (wealth) is one possible source of procyclical elasticity of aggregate labor supply.

Arrow (1970) shows that increasing R_R is the only non-homothetic specification that is consistent with the Expected Utility Theorem, and also argues that it is the only specification that is consistent with a wealth elasticity of the demand for cash balances greater than unity, which has been found in several empirical studies. Cash balances appear to be luxury goods, or at least not necessities, and that stylized fact is consistent with increasing R_R .

It remains to evaluate the possible effects of changes in the elasticity of $\Phi(g)$ on ϵ . It is clear that if preferences are homothetic, θ is constant and the only way that ϵ can change is through changes in γ . A necessary and sufficient condition for ϵ to be procyclical is that $\mathcal{E}_v[\gamma] < 0$ in the vicinity of the equilibrium real wage. For a general cumulative distribution function Φ there is no assurance that this will be the case, or that $\mathcal{E}_v[\gamma]$ will be monotonic over the entire domain of Φ . To evaluate point elasticities, it will be necessary to assume a specific

functional form for Φ . That issue will be taken up in the next section¹⁰.

4.3 The Distribution of Nonlabor Income and Wealth

4.3.1 Nonlabor Income and Wealth

The real nonlabor income of a household is defined as the sum of real financial and transfer income. More specifically:

$$v = V/P = (B/P + T/P + \pi) \geq 0 \quad (4.8)$$

where B is nominal interest receipts on government bonds, π is total returns from private equity claims, T represents nominal transfer receipts from the government sector, and P is a price index. Excluding the equalizing effect of transfer payments, which accrue mostly to lower income households, the distribution of income from bonds and equities is likely to be similar to the distribution of the holdings of those assets in the workforce. For example, the flow of nonlabor income can be related to nonhuman wealth Ω as follows:

$$v = r[(B+T)/rP + \pi/r] = r[\Omega - M/P] \quad (4.9)$$

¹⁰Once the homotheticity assumption is abandoned, distributions and the functional forms used to represent them matter.

Here M is the aggregate money supply (e. g., $M2$), Ω is the stock of real nonhuman wealth including "transfer" wealth, and r is the real market rate of interest on long-term claims. (Note that in (4.9), v is not a function of r .) Thus, the distribution of v would be proportional to the distribution of $f = (\Omega - M/P)$, which consists of bonds, equity and transfer payments in the workforce.

I have assumed a very large number of households which can be ordered on R^+ according to their nonlabor income v at a point in time. I also assume that their measure on that interval is such that the distribution of nonlabor income over households can be represented by a continuous distribution function $\phi(v, \alpha)$ that is twice differentiable in v and each element of the parameter vector α . Two critical questions are: How is nonlabor income distributed in the population, and what would be a reasonable functional form to describe it?

4.3.2 Stylized Empirical Facts on Nonlabor Income

There is an extensive body of empirical literature on the distribution of income in the U. S. and Britain (e. g., Lydall (1973), Champernowne (1973), Blinder (1974), Smith (1975, 1980), Atkinson (1983), Slottje (1989), and Bergstrand et al (1994); also, the Review of Income and Wealth). Nearly

all of these studies have focused on the distribution of earned income, and even when property income is included there is very little data on the separate distribution of nonlabor income as defined in (4.8) above. Most income distribution data include retired, disabled, the very wealthy and other individuals who are not part of the labor force. Available data appears to be insufficient to directly estimate a distribution of nonlabor income only among potential workforce participants. Nevertheless, it is possible to infer some of the essential characteristics of such a distribution from the income distribution data that is available.

There is a broad consensus among the several studies of the distribution of wages and salaries and total income. This was best summarized by Lydall (1973, pp. 66-67), whom I will paraphrase in the interest of brevity:

(1) the distribution is "hump-shaped", and if it is confined to adult males working full time the left-hand tail is asymptotic to the income axis.

(2) the central part of the distribution, between the 10th and 80th percentile, is close to Lognormal. However, the tails of the distribution contain an excess of frequencies compared with the Lognormal (i. e., are leptokurtic in the log of income).

(3) The upper tail often approximately follows the Pareto law for at least the top 20 per cent of the distribution.

Thus the distribution of earned income is unimodal, positively skewed, leptokurtotic, and asymptotic to the income axis.

Neither the Lognormal nor the Pareto distributions give a satisfactory fit over the entire range of incomes.

If the distribution of nonlabor income is considered to be roughly proportional to the distribution of wealth, as suggested by Equation (4.8), then there is some theoretical justification for assuming that the distribution of nonlabor income is Lognormal, based on Gibrat's "Law of proportional effect." (Gibrat, 1957). If z_t is a stochastic variable distributed according to an arbitrary distribution $F_t(z_t)$ at time t , and subsequent values of z are generated by a process $z_{t+1} = u_t \cdot z_t$ where the u_t are i.i.d random variables, then $z_t = z_0 \cdot \prod u_k$, where z_0 follows some arbitrary initial distribution. This is a first-order Markov process; z_t depends only on z_{t-1} and the random element u_t . The evolution of z can also be written as $\log z_t = \log z_0 + \sum (\log u_k)$. Under the assumptions which are necessary for the application of the Central Limit Theorem, z_t will be described by a self-reproducing Lognormal distribution. [Aitchison and Brown (1954, 1963)].

This has some intuitive appeal because the evolution of the distribution of financial wealth can be viewed as such a process, where $\Omega_{t+1} - \Omega_t = \rho \Omega_t$ and ρ is a random return that is i.i.d across households. This random variable ρ is not necessarily a pure interest rate, but a periodic effective

return that reflects the combined effects of saving, dis-saving, asset management, market returns and risk. This is similar to Friedman's view (1953) that the distribution of income is the result of different choices by individuals with different tastes and preferences, tempered by chance¹¹.

Gibrat's law of proportional effect was used by Champernowne (1936; 1973), Kalecki (1938), and Aitchison and Brown (1963) to derive the size distribution of total income. Champernowne's limiting distribution was Pareto, but Aitchison and Brown (1954) showed that, with a small alteration of his assumptions, Champernowne's model produces a Lognormal distribution. Wold and Whittle (1957) and Steindl (1972) used methods similar to Champernowne's to derive an asymptotically Pareto distribution of wealth, and Vaughn (1988) extended their results using a life-cycle model of saving. Their results are also subject to the Aitchison and Brown critique. Sargan (1957) developed the most comprehensive model of wealth accumulation, for which the only tractable solution was a Lognormal distribution. Using Gibrat's method, Pestieau and Possen (1979) derived a Lognormal distribution of wealth directly, and studied the effects of risk aversion and government tax parameters on the inequality of the distrib-

¹¹Blinder (1974) has criticized such stochastic models of income and wealth distribution for their lack of decision-theoretic content. His criticism does not apply, however, to the models of Sargan (1957) and Vaughn (1979, 1988), which are based on intertemporal optimization.

ution. Thus, the Lognormal distribution has been established both empirically and theoretically as a useful description of the distribution of total income for populations which include but are not limited to the workforce.

Another reason for choosing the Lognormal distribution in this investigation is that it more likely to accurately represent the relevant range of nonlabor income than the Pareto distribution. The concentration of transfer payments near the lower end of the income range is likely to create a mode, or Lydall's "hump", which the Pareto cannot produce. Also, most households in the highest quintile of the income and wealth distribution, where the Pareto fits best, are not likely to be included in any useful definition of the labor force. Financial wealth in the U. S. is highly concentrated with a Gini coefficient of .90; In 1989 the top 1% of wealth-holders owned 48% of the financial wealth in the U. S. (Wolff, 1994). Approximately 30% of these were over age 65, but it is reasonable to assume that all were financially independent¹². Thus, the thinner upper tail of the Lognormal distribution should be a better approximation to the distribution of wealth in the labor force. The Pareto distribution

¹²Persons over age 65 made up about 20% of both the lowest and highest deciles of the wealth distribution in 1979 (Radner and Vaughn, 1987). Excluding them would thin out both tails. Excluding the independently wealthy would thin out the upper tail significantly, due to their large share of total wealth.

also has the disadvantage of having an infinite variance for some very reasonable values of its single parameter.

For these reasons, the Lognormal distribution will be used to characterize the distribution of financial wealth and nonlabor income in the remainder of this chapter. This is posited as a reasonable choice in order to proceed. We do not really know what the distributions of wealth and nonlabor income in the workforce are. More research in this area is needed, especially on the extent to which the distributions of income and wealth change over the business cycle.

4.3.3 The Lognormal Distribution¹³

The Lognormal distribution is:

$$\Lambda(x | \mu, \sigma^2) = \int_0^x \frac{1}{\sqrt{2\pi} \sigma x} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right] dx = N(\ln x | \mu, \sigma^2), x > 0 \quad (4-10)$$

where $N(\cdot | \mu, \sigma^2)$ represents the Normal distribution with mean μ and variance σ^2 . The j th moment of Λ about the origin is:

$$l_j' = \exp\{j\mu + \frac{1}{2}j^2\sigma^2\} \quad (4.11)$$

from which the mean α and variance β^2 of Λ are given by

¹³The classical reference on the Lognormal distribution and its applications is Aitchison and Brown (1963).

$$\alpha = \exp\{\mu + \frac{1}{2}\sigma^2\} \quad (4.12)$$

$$\beta^2 = [\exp\{\sigma^2\} - 1]\exp\{2\mu + \sigma^2\}$$

Other useful characteristics of the Lognormal are:

$$\text{mode:} \quad \exp\{\mu - \sigma^2\} \quad (4.13)$$

$$\text{median:} \quad \exp\{\mu\} \quad (4.14)$$

$$\text{coefficient of variation } \beta/\alpha = \gamma = [\exp\{\sigma^2\} - 1]^{\frac{1}{2}} \quad (4.15)$$

Note that μ is the arithmetic mean of ln x and also the geometric mean and the log median of x. The parameter σ^2 is the variance of ln x, and although it is a measure of relative dispersion from (4.15), it is not the variance of x, which is given in (4.12). The Lorenz measure of income inequality for the Lognormal is can be defined in terms of the standardized Normal distribution (Aitchison and Brown, *op. cit.*):

$$L = 2N(\sigma/\sqrt{2} | 0, 1) - 1 \quad (4.16)$$

Values of L can be obtained from tables of the standardized Normal distribution for values of σ . Note that $\partial L/\partial \sigma > 0$, so a decrease in σ^2 corresponds to a decrease in income or wealth inequality.

Figure 4-3(a) shows a family of Lognormal density functions for $\mu = 0$ and several values of σ^2 . Figure 4-3(b) shows density functions for $\sigma^2 = 0.5$ and several values of μ .

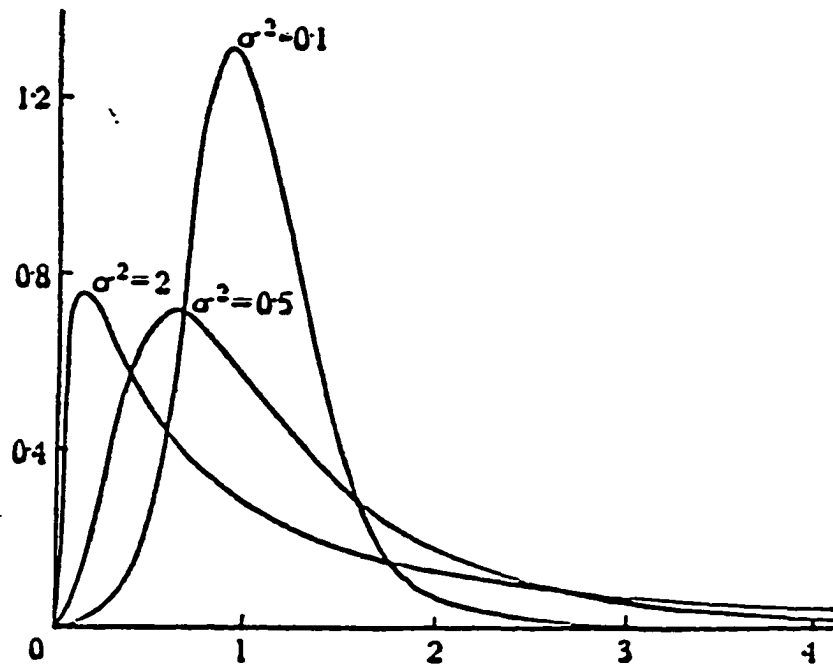


Figure 4-3(a) Lognormal density functions for $\mu = 0$ and various values of σ^2 .

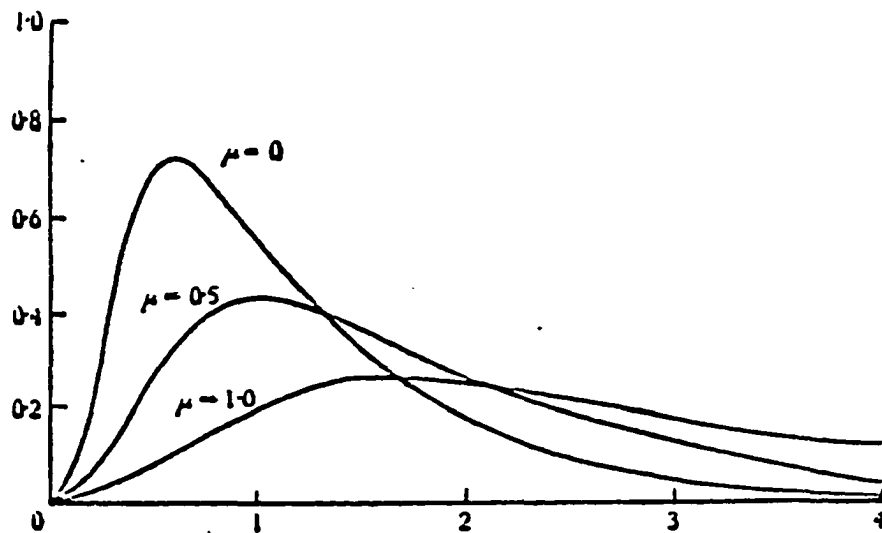


Figure 4-3(b) Lognormal density functions for $\sigma^2 = .5$ and various values of μ

the relationship between the Normal and Lognormal distributions allows the following useful transformation:

$$\Lambda(w) = \int_0^w \lambda(w | \mu, \sigma^2) dw = \int_{-\infty}^z \phi(z) dz = N(z | 0, 1) \quad (4.17)$$

$$z = \frac{\ln w - \mu}{\sigma}$$

If nonlabor income and reservation wages are measured in log units, then all the properties of the Normal distribution apply. This fact is utilized to advantage in Appendix D and the computations of Section 4.5.

Another useful property of the Lognormal distribution is that it has a closed form under a log-linear transformation of the random variable. Aitchison and Brown (op. cit.) prove the following theorem:

If X is distributed Lognormal $\Lambda(\mu, \sigma^2)$ and $Y = aX^b$, then Y is distributed Lognormal $\Lambda(\ln a + b\mu, b^2\sigma^2)$.

It is evident that a multiplicative shift of magnitude a will change μ by $\ln(a)$ but will not affect σ^2 . However, an exponential transformation X^b will affect both μ and σ^2 as shown. This property of the Lognormal will turn out to be useful in the analysis to follow.

4.4 The Lognormal Aggregate Labor Supply Function

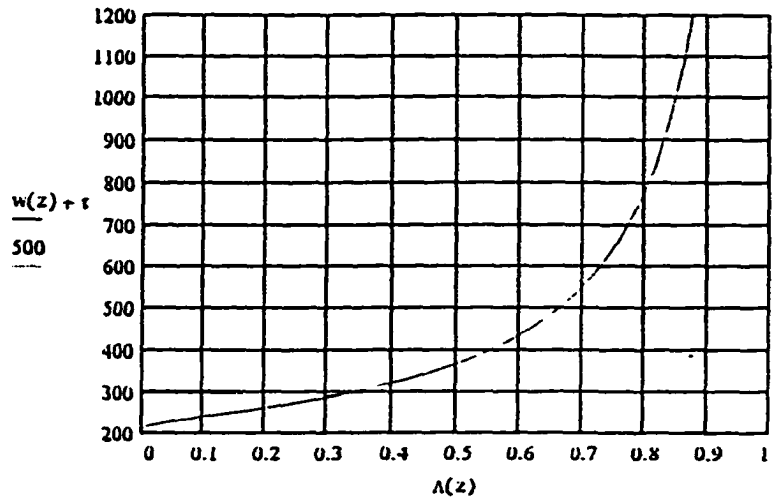
Specifying $\Lambda(w)$ for the function $\Phi(g(w))$ in Equation (4.5) yields the following expression for the aggregate labor supply function:

$$L^s(w; \mu, \sigma^2) = Nh\Lambda(w|\mu, \sigma^2), \quad w > 0 \quad (4.18)$$

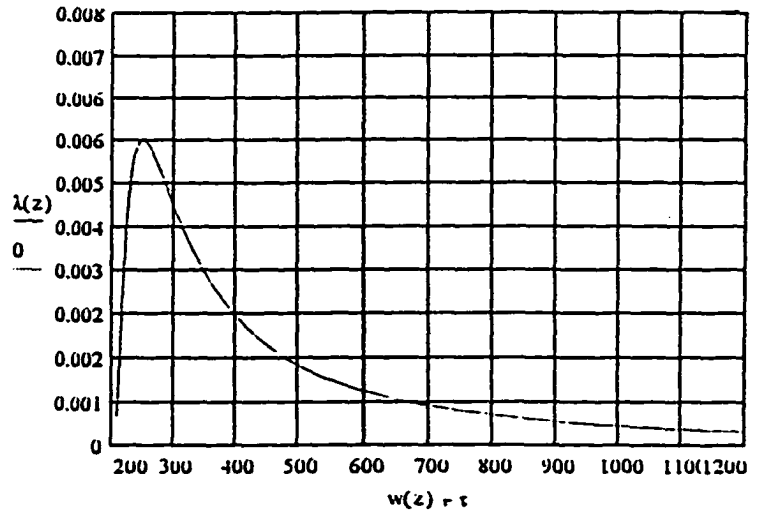
Here N represents the total size of the available workforce, h is the fixed number of hours per worker, and $\Lambda(w)$ is the cumulative distribution of reservation wages, which in this instance is a linear transformation of the distribution of nonlabor income v , which itself is a linear transformation of the distribution of wealth. The simplest form of a reservation wage function derived from homothetic preferences is $\psi(v) = v$, whereupon $g(w) = \psi^{-1}(v) = w$. In general U will be non-homothetic, the argument of Λ will be $g(w)$ and a composite function will be involved, as indicated in Equation (4.6). This aggregate labor supply function has two shift variables μ and σ^2 , both of which enter into the moments of Λ .

Figure 4-4 shows graphs of the labor supply function $\Lambda(w)$ and its elasticity $\epsilon(w)$ and marginal factor cost functions, for a mean reservation wage of \$500 per week, a minimum wage of \$200 per week, and a coefficient of variation of approximately 1. This is a benchmark case that will be used in Section 4.5.

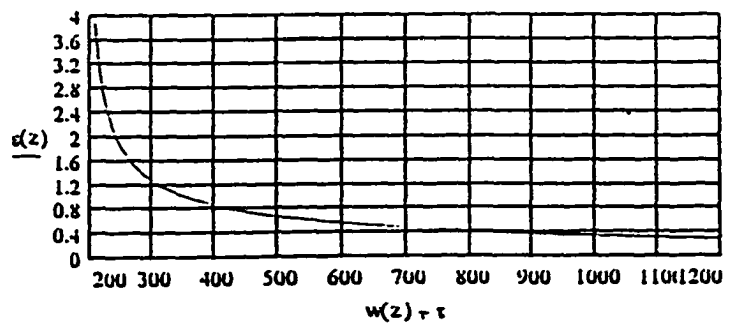
FIGURE 4-4 NONHOMOTHETIC CASE VIII



LOG-NORMAL LABOR SUPPLY FUNCTION



DENSITY FUNCTION OF RESERVATION WAGES



ELASTICITY OF LABOR SUPPLY

4.4.1 Procyclical Aggregate Labor Supply

In the Traditional Keynesian setup, with technology and labor productivity assumed to be fixed in the short run, increases in aggregate output and employment are associated with shifts of the aggregate labor supply curve. That is exactly the mechanism that is assumed to be at work in the present model. The shift variables of the lognormal aggregate labor supply function are μ and σ^2 , which are also the parameters of the distribution of nonlabor income. Thus, changes in the distribution of nonlabor income, or wealth from which it is derived, will cause the aggregate labor supply curve to shift and change its shape.

Expressions for the partial derivatives of Λ with respect to μ and σ^2 are derived in Appendix D and are summarized below. (Subscripts denote partial derivatives.)

$$\Lambda_{\mu} = \frac{1}{\sigma} \int_{-\infty}^z z \phi(z) dz < 0 \quad (4.19)$$

$$\Lambda_{\sigma^2} = \frac{1}{2\sigma^2} \int_{-\infty}^z (z^2 - 1) \phi(z) dz \quad (4.20)$$

$$\Lambda_{\sigma^2} \begin{cases} > 0, & \ln w < \mu \\ < 0, & \ln w > \mu \end{cases} \quad (4.21)$$

These partial derivatives show that (a) a decline in geometric mean nonlabor income v will increase labor supply at all wages; (b) an increase in the inequality of nonlabor income, measured by an increase in σ^2 , will increase the labor supply of lower wage workers ($\ln w < \mu$) and decrease the labor supply of higher-wage workers ($\ln w > \mu$). *This is the response of aggregate labor supply to an increase in the inequality of workers' nonlabor income*¹⁴.

Conversely, a decline in σ^2 decreases the inequality of v and increases the labor supply of high-wage workers and reduces the labor supply of lower-wage workers. The density of reservation wages will increase in the vicinity of the log median¹⁵. Since the elasticity of the distribution function $\Lambda(w)$ at a point w_0 is a measure of its relative density there, an increase in the concentration or density of reservation wages corresponds to an increase in the elasticity of labor supply. As σ^2 declines, nonlabor income becomes more homogeneous and consequently labor supply becomes more elastic in the vicinity of the median reservation wage. In the limit as

¹⁴The symmetry of the dispersion effect is due to the symmetry of σ^2 , the variance of $\ln w$, about μ in the Lognormal distribution.

¹⁵It follows from Equations (4.11), (4.12) and (4.13) and Figure 4-3 that a decline in σ^2 alone will shift the mean reservation wage to the left and the mode to the right, leaving the median unchanged. A decline in μ alone will shift all three parameters to the left. A decline in both μ and σ^2 will increase the concentration of reservation wages around the new lower median e^μ . Aggregate labor supply will increase at most wages, and most importantly, the elasticity of labor supply will increase in the vicinity of the original log median reservation wage.

$\sigma_2 \rightarrow 0$, all workers have the same nonlabor income and reservation wage, and labor supply is perfectly elastic. This indicates an inverse relationship between the dispersion of wealth (i. e. inequality) the elasticity of labor supply at the log median reservation wage. The more compact the distribution of wealth, and therefore the distribution of reservation wages, the greater the response of labor supply to a change in the wage, i. e., the greater is the elasticity of labor supply.

4.4.2 The Money-Elasticity of Aggregate Labor Supply

The shift variables μ and σ^2 are metrics of the nonlabor income distribution. Nonlabor income payments are in nominal values, with real values determined by the price level. The geometric mean or log median of v can also be expressed in terms of nominal values V and the price level P :

$$\mu = E[\ln v] = E[\ln (V/P)] = E[\ln V] - \ln P \quad (4.22)$$

Thus, an increase in the price level P without a proportional increase in nominal nonlabor income V , reduces μ and consequently, from Equations (4-12) through (4-14), reduces the mean α , variance β^2 , mode and median of the distribution.

An increase in the price level alone will shift the density function $\phi(v)$ to the left, increasing its skewness and increasing $\phi(w)$ at every w .

To the extent that contractual payments of nonlabor income in Equation (4.7), particularly bond interest and transfer payments, are fixed in nominal terms and are not indexed, nonlabor income will not move proportionally with changes in the price level and the aggregate labor supply function will shift due to a change in the price level. The function will be static with respect to changes in P only if all nonlabor income payments are perfectly indexed. In the analysis that follows, I assume that the aggregate price level is a shift variable for this labor supply function through its effects on μ .

In flexible-price macro models, an increase in money is usually associated with an increase in aggregate demand and the price level. For example, in the model of Chapter II the response to a one-time open-market purchase is a less-than-proportional increase in the price level and a reduction in the interest rate. For the aggregate labor supply function developed in this chapter, an increase in the price level P unaccompanied by a proportional increase in nominal median nonlabor income will reduce all values of v proportionately,

shifting the entire aggregate labor supply curve toward increased supply at all wages¹⁶.

The outward shift in the labor supply curve will due to an increase in P will reduce ϵ at every wage, because the mean of the distribution will be declining while the marginal values will increase at each wage. Thus, monetary policy, through its affect on the price level, will shift this aggregate labor supply function, but the price-level effect on ϵ will be countercyclical.

It turns out that an increase in P alone will have no effect on the dispersion parameter σ^2 , because:

$$\begin{aligned}\sigma^2 &= E[(\ln v)^2] - (E[\ln v])^2 \\ &= E[(\ln V)^2 - 2(\ln V)(\ln P) + (\ln P)^2] - (E[\ln V] - \ln P)^2 \\ &= E[(\ln V)^2] - (E[\ln V])^2\end{aligned}$$

Thus σ^2 , the measure of dispersion or inequality of the Lognormal distribution is unaffected by changes in a scale factor relating v and V , which is what the price level is¹⁷. A change in P will shift Λ procyclically, but it will not

¹⁶In Chapter V this shift mechanism for labor supply is imbedded in a full-fledged general equilibrium macro model.

¹⁷This result is specific to the Lognormal distribution, because of the log-linear property described in Section 4.3.3. Note that an increase in the price level will have an effect on β^2 , the variance of the Lognormal distribution, through its effect on μ . (See Equation 4.12).

change the dispersion of Λ and therefore will not have a procyclical effect on ϵ , the elasticity of the distribution.

4.4.3 Portfolio Rebalancing with Increasing R_R

In the macroeconomic model of Chapter II, an open-market purchase of bonds for high-powered money reduces the total amount of bonds outstanding and also reduces the interest rate and the value of real wealth. If workers are holding optimum portfolio allocations of bonds and money prior to an open-market operation, then those allocations will be changed to reflect the new asset quantities and relative prices. In giving up bonds for money, workers will reduce their holdings of risky assets but also forego interest income. How the asset reallocation is accomplished over the population of workers will determine the effect on the distribution of nonlabor income. In this section I will show that if workers rebalance their portfolios in a way that is nonhomogeneous, open market operations will affect σ^2 , and consequently the elasticity ϵ .

The portfolio balancing problem is well known in the literature, and only the points essential to the analysis at hand will be emphasized here. A more rigorous argument based Arrow (1970), and Diamond and Stiglitz (1974) is presented in

Appendix D, Section 7, which will be incorporated by reference.

In terms of the notation of Equation (4.8), I define two classes of real financial assets in the aggregate economy:

Earning assets: $f = (B/rP + \pi/r)$

Real money balances: $m = (M/P)$

Earning assets provide a return r which is not certain, while money offers no return but provides transaction services in consumption.

Aggregate portfolio shares are: $A = f/\Omega$ and $(1-A) = m/\Omega$. The value of A in the aggregate is determined by the ratio of the value of earning assets to total financial wealth, which is influenced by monetary policy. In particular, for the non-neutral open-market purchase of bonds described in Chapter II:

$$\frac{dA}{dM} = - \frac{\left[\frac{\Omega}{P} \left[1 - \frac{M}{P} \frac{dP}{dM} \right] - \frac{M}{P} \Omega_M \right]}{\Omega^2} < 0 \quad (4.24)$$

The sign of Ω_M is negative from the results of Chapter II. Thus, a non-neutral open-market purchase necessarily reduces the proportion of aggregate wealth that will be held in risky income-producing assets such as bonds.

For an individual wealth-holder, I define:

$$a_i = f_i/\Omega_i, \quad (1-a_i) = m_i/\Omega_i$$

Individual wealth-holders allocate their wealth between f and m to maximize the utility of services obtained by holding them. A standard assumption is that the utility function for wealth $H(\Omega) = H(f + m)$ is concave and at least twice differentiable, so that wealth-holders are risk averse and prefer to diversify between f and m . The standard setup, with indifference curves tangent to a budget constraint at an interior solution, is illustrated in Figure 4-5, (a) and (b).

It is well known that, in the two-asset case, if preferences are assumed to be identical and homothetic, then wealth-holders have constant relative risk aversion (R_R) and the marginal rate of substitution between risk-free and risky assets will be independent of the size of total wealth holdings. (See Appendix D, Section D7.) Therefore, identical homothetic preferences imply that the relative demands, or optimum portfolio shares, for the two asset classes, will be the same for all wealth-holders and equal to the aggregate portfolio. *It also implies that the fractional portfolio balance adjustments to an open-market operation will be identical for all workers, and therefore homogeneous across*

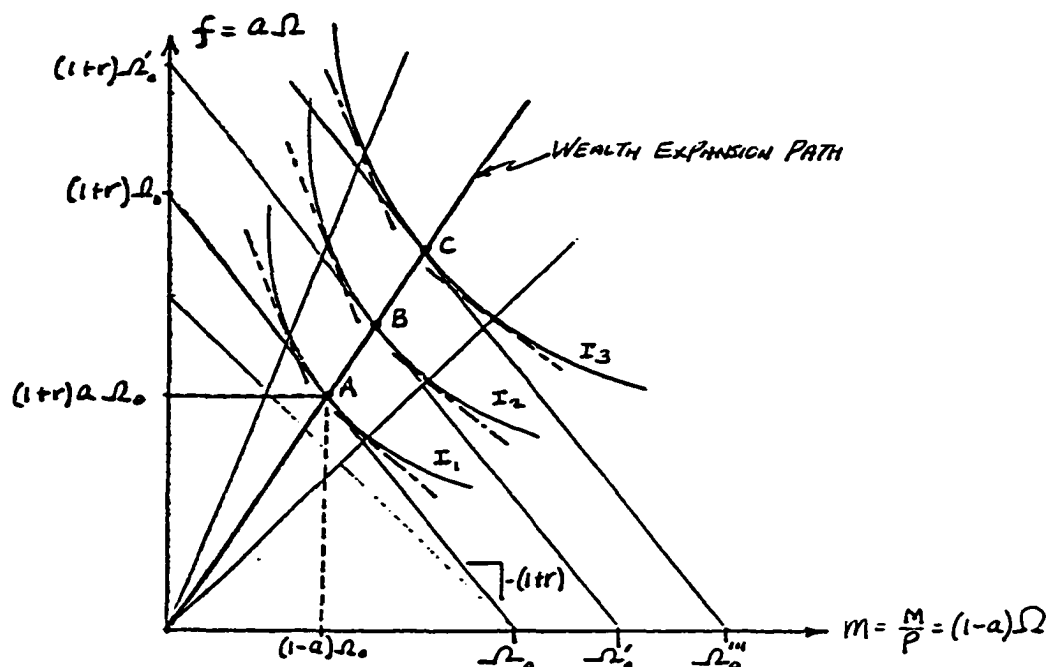


Figure 4-5(a). Portfolio balance decision with Homothetic Preferences (Constant Relative Risk Aversion)

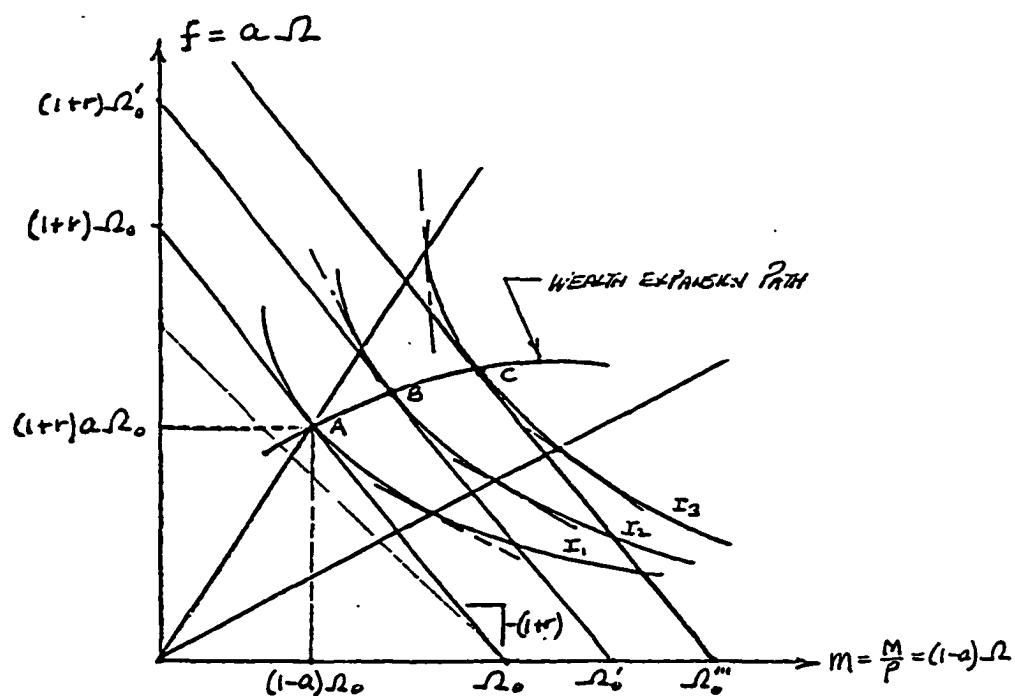


Figure 4-5(b). Portfolio balance decision with Increasing Relative Risk Aversion

workers. Homogeneous rebalancing adjustments will not change the dispersion of the distribution of v .

The case of homothetic preferences is illustrated in Figure 4-5(a). The assumption that H is homothetic implies the following: (a) a_i is independent of Ω_i and is equal to A for all wealth-holders; (b) the wealth-expansion path of optimum portfolios will be a ray from the origin, along which the marginal rate of substitution will be constant; [This is shown as points A-B-C in Figure 4-5(a)]; (c) f and m are normal goods but neither is a luxury good; (d) the Engel curves for both f and m are rays with 45° slopes.

Thus, whatever adjustments wealth-holders make to their portfolio shares $[a_i, 1-a_i]$, in response to an open market operation, it will be the same for all wealth-holders, and the change in income yield v_i will be proportional to Ω_i and therefore proportional to $r\Omega_i = v_i$ ¹⁸. It has already been shown above, in the case of changes in P , that a proportional change in v has no effect on the dispersion parameter σ^2 of the Lognormal distribution of v . Therefore, homothetic preferences rule out any distribution effects on v from an open-market operation.

¹⁸The interest rate r is also changing, but the substitution and wealth effects of this are independent of the size of wealth holdings and consequently are the same for all wealth-holders. That is the main implication of the homotheticity assumption.

However, if H has increasing relative risk aversion, a_i will decrease with Ω_i . (See Appendix D, Section D7.) Increasing relative risk aversion implies: (a) the marginal rate of substitution between f and m will increase with wealth; (b) wealthier investors will require a higher expected rate of return to hold the same portfolio mix, and will hold a smaller proportion of the risky asset for the same return; (c) the wealth-expansion path of optimum portfolios will have a decreasing slope, favoring m as shown in Figure 4-5(b); (d) money balances are a luxury good¹⁹.

The functional properties of increasing R_R can be demonstrated by applying the elasticity operator \mathcal{E}_Ω to the marginal rate of substitution in the first-order condition for the constrained maximization of $H(f, m)$, given by Equation D7.2 in Appendix D:

$$\mathcal{E}_\Omega \left[-\frac{df}{dm} \right] = \mathcal{E}_\Omega \left[\frac{H_m}{H_f} \right] = \mathcal{E}_\Omega [H_m] - \mathcal{E}_\Omega [H_f] = [-R_R(m) + R_R(f)] = \mathcal{E}_\Omega [(1+r)]$$

This expression gives the relationship between the elasticities of marginal utility of f and m as Ω increases along the wealth expansion path, which is a locus of optimum points (f^*, m^*) for a constant r . If $1+r$, $f/m = a/(1-a)$ and df/dm

¹⁹With decreasing absolute risk aversion, the amount of risky assets held will increase with wealth, but their share of wealth in the portfolio will decrease. Formally, $0 < \mathcal{E}_\Omega[f_i] < 1$ and $\mathcal{E}_\Omega[a_i] < 0$

remain constant as Ω increases, the path will be a ray from the origin as shown in Figure 4-5(a). But then the above expression is zero and relative risk aversion must be the same for all points (f,m) along the path; since a different r will map out a different path, that must be true for the entire indifference map. Constant relative risk aversion implies that the utility function H is homothetic; the wealth expansion path will be a ray from the origin, along which the marginal rate of substitution on each intersecting indifference curve will be the same. Along that path, the own-price elasticity of the demand to hold absolute amounts of f and m will also be constant, as will be their portfolio shares a and $(1-a)$.

Arrow (1970) showed that if H has increasing R_R , the above expression is positive. [Arrow, 1970, Appendix [5]]; see also Appendix D, Section D7.) In that case, along a ray (where $f/m = a/(1-a)$), $-df/dm$, $(1+r)$ and H_m/H_f will have positive wealth elasticity and therefore will be increasing. The slope of the wealth expansion path (with $(1+r)$ constant) will be $df/dm = a/(1-a)$, and from Arrow's proof:

$$\mathcal{E}_\Omega[df/dm] = \mathcal{E}_\Omega[a/(1-a)] = \mathcal{E}_\Omega[a] - \mathcal{E}_\Omega[(1-a)] < 0$$

Since, necessarily, $\mathcal{E}_\Omega[a] + \mathcal{E}_\Omega[(1-a)] = 0$, the conclusion is that increasing R_R implies that $\mathcal{E}_\Omega[a] < 0$. Portfolio shares

in the risky asset class will decline with wealth and the wealth expansion path will have a declining slope, as shown in Figure 4-5(b)²⁰. Since the proof of this holds without restriction on values of f and m , the indifference contours in f - m space must be asymmetrical, as shown in figure 4-5(b). This is consistent with the assumption that m is a risk-free asset.

Thus, with an open-market operation wealth holders must rebalance their portfolios, but with increasing R_R this rebalancing occurs in a nonhomogeneous way. Workers with higher amounts of income from bonds will reduce their bond income more than proportionally to those receiving lower bond income, and will absorb a disproportionate amount of the additional money created by the open market operation. The result of this portfolio balance trading in the financial markets will be a reduction in the dispersion of nonlabor income in the workforce, i. e.

$$\frac{\partial \sigma^2}{\partial M} = \frac{\partial \sigma^2}{\partial A} \frac{\partial A}{\partial M} < 0 \quad (4.26)$$

Assuming perfect information and efficient, frictionless financial markets this will happen instantaneously.

²⁰It is assumed that H has decreasing absolute risk aversion, so that the amount invested in earning assets f increases with Ω even though the portfolio share declines.

With increasing relative risk aversion, a_i is a function of Ω_i and r , and for a continuous distribution of Ω over workers we may write $a(r, \Omega)$. Note that $a(r, \Omega)$ is a relative demand function which depends on preferences, but since preferences depend on wealth and wealth is distributed over the workforce, the distribution of $a(r, \Omega)$ will depend on the distribution of Ω , subject to the aggregate constraint:

$$A = \int_0^{\infty} a(r, \Omega) \lambda(\Omega) d\Omega \quad (4.27)$$

where $\lambda(\Omega)$ is the density function for wealth in the workforce²¹. Thus, for a given r , A is the weighted average of $a(r, \Omega)$, and will be a function of the parameters of the density function $\lambda(\Omega)$.

To use a concrete example, let $H(\Omega)$ be Pratt's utility function with decreasing R_A and increasing R_R as introduced in Appendix D, Section D4, Equation D4.3.:

$$H(\Omega) = -\exp(-\Omega^\rho/\rho) \quad 0 < \rho < 1$$

²¹The relative demand for money may also be a function of an individual's income, in which case $a(\cdot) = a(r, y, \Omega)$, where $y = v$ if the individual is not working and $y = v + wh$ if working. Whether both income and wealth need to be included as scale variables in the money demand function is not a settled issue in the literature. To simplify the notation, the income variable will be suppressed in the material immediately following.

Relative risk aversion for this function is:

$$-H''\Omega/H' = \Omega^\rho + (1 - \rho) \approx \Omega^\rho \text{ for large } \Omega.$$

Thus the wealth-elasticity of relative risk aversion is approximately ρ . Since relative risk aversion increases proportionally with Ω at the rate ρ , a reasonable assumption would be that to a first approximation $a(r, \Omega)$ declines proportionally with increasing wealth at the same rate, i. e., $\mathcal{E}_\Omega[a] = -\rho$. This corresponds to the function $a(r, \Omega) = \Omega^{-\rho}$. If we also assume that Ω is distributed in the workforce according to a Lognormal distribution $\Lambda(\Omega | \mu_\omega, \sigma_\omega^2)$, then we have:

$$A = \int_0^\infty \Omega^{-\rho} \left[\frac{1}{\sqrt{2\pi} \sigma_\omega \Omega} \exp\left[-\frac{(\ln \Omega - \mu_\omega)^2}{2\sigma_\omega^2}\right] \right] d\Omega \quad (4.28)$$

$$= \exp(-\rho\mu_\omega + \frac{1}{2}\rho^2\sigma_\omega^2) \quad (4.29)$$

Although we do not know the values of the parameters μ_ω and σ_ω^2 for the distribution of wealth in the workforce, the relationship between A and σ^2 , the dispersion parameter of the nonlabor income distribution, can be found by utilizing Theorem 2.1 of Aitchison and Brown, previously cited in Section 4.3, in the following argument.

If v is distributed $\Lambda(\mu, \sigma^2)$ and $f = v/r$, then f is distributed $\Lambda(-\ln r + \mu, \sigma^2)$. With increasing relative risk aversion, $f = a(r, \Omega)\Omega$. Using the Pratt utility function, $f = (\Omega^{1-\rho})\Omega$, where ρ is the elasticity of relative risk aversion. But if Ω is distributed $\Lambda(\mu_\omega, \sigma_\omega^2)$, then f must also be distributed $\Lambda((1-\rho)\mu_\omega, (1-\rho)^2\sigma_\omega^2)$. Equating the dispersion parameters in the two expressions for the distribution of f gives $\sigma_\omega^2 = \sigma^2/(1-\rho)^2$. Substituting this in Equation 4.29 and deriving the elasticity of σ^2 with respect to A , yields:

$$\mathcal{E}_A[\sigma^2] = \frac{\partial \sigma^2}{\partial A} \frac{A}{\sigma^2} = \frac{2(1-\rho)^2}{\rho^2 \sigma^2} > 0 \quad (4.30)$$

Thus, for this nonhomothetic utility function, the elasticity of the change in σ^2 associated with a change in A depends directly on $[(1-\rho)/\rho]^2$ and inversely on the value of σ^2 . The effect of portfolio rebalancing on the dispersion of nonlabor income, represented by σ^2 , will be greater for small values of σ^2 (low Gini coefficients) and for values of ρ less than $\frac{1}{2}$.

These results lead to the conclusion that, with increasing relative risk aversion and a Lognormal distribution of wealth, an open-market purchase of bonds will reduce the dispersion (inequality) of the distribution of financial wealth and nonlabor income. This provides a second channel through which changes in monetary policy can not only shift the aggregate labor supply function Λ , but most importantly,

change its elasticity²². In this formulation, both μ and σ^2 are shift variables for aggregate labor supply $\Lambda(w)$ and its elasticity ϵ ; μ is affected by P and σ^2 is affected by r and Ω indirectly through A .

The implications of the flexible-price macro model in Chapter II, if not significantly altered by the additional monopsony assumption, lead to the conclusion that an open-market operation will shift this aggregate labor supply function and change its elasticity through its short-run effects on P , r , and Ω . This constitutes, in effect, a monetary theory of aggregate labor supply.

The comparative static derivatives of Λ and ϵ with respect to P , r and A are derived in Appendix D, section D5 (Equation D5.11) and section D6 (Equation D6.10). The results are summarized below:

$$\mathcal{E}_M[\epsilon] = -\mathcal{E}_M[\Lambda] - \frac{z}{\sigma} \mathcal{E}_M[P] + \frac{(z^2 - 1)(1 - \rho)^2}{\rho^2 \sigma^2} \mathcal{E}_r[A] \mathcal{E}_M[r] \quad (4.32)$$

In these two equations, $\mathcal{E}_x[y]$ is the elasticity of y with respect to x , $z = (\ln w - \mu)/\sigma$, and $\phi(z)$ and $N(z|0,1)$ are the standard Normal density and cumulative distribution functions.

²²Note that this argument implies that monetary policy will have a countercyclical effect on the dispersion (or inequality) of nonlabor income. With increasing R_R , relative demand functions for financial assets will be non-homogeneous.

$$\begin{aligned}
\mathcal{E}_M[\Lambda] = & -\frac{1}{\sigma} \left[\frac{\int_{-\infty}^z z \phi(z) dz}{N(z|0,1)} \right] \mathcal{E}_M[P] \\
& + \frac{(1-\rho)^2}{\rho^2 \sigma^2} \left[(z^2-1) - \frac{\int_{-\infty}^z (z^2-1) \phi(z) dz}{N(z|0,1)} \right] \mathcal{E}_r[A] \mathcal{E}_M[r]
\end{aligned} \tag{4.31}$$

The elasticity $\mathcal{E}_A[\sigma^2]$ from Equation 4.30 above has also been incorporated in the model. In the next section, these equations are evaluated numerically for calibrated values of μ , σ^2 and ρ for the Lognormal distribution and plausible values of the monetary elasticities on the right hand side.

4.5 Numerical Analysis of $\mathcal{E}_M[\Lambda]$ and $\mathcal{E}_M[\epsilon]$

The story that emerges from the foregoing analysis is the following:

The aggregate labor supply function based on the heterogeneous preferences of workers and the distributions of wealth and nonlabor income forms part of the wealth effects transmission channel from monetary policy actions to real output. The aggregate labor supply curve is shifted procyclically by changes in the aggregate price level, and the elasticity of labor supply is affected procyclically by changes in the economy-wide interest rate. With monopsony power in the

labor market, this will cause the markdown to be countercyclical and the real wage to be less countercyclical than under perfect competition. *A money-output connection has been established in a Keynesian model that is capable of exhibiting countercyclical markdowns and acyclical real wages. Q. E. D.*

The objective in this section is to determine if the model equations 4.31 and 4.32 produce positive money-elasticities of Λ and ϵ for plausible values of their parameters. The equations will be calibrated to representative benchmark values of their parameters for the U S. economy, the elasticities of Λ and ϵ will be computed, and various metrics will be examined.

4.5.1 Calibration of the Labor Supply Function

Values of the parameters μ and σ^2 will be chosen to approximate a realistic distribution of reservation wages in the U. S. economy. I will utilize the three-parameter Lognormal distribution with minimum value $\tau = \$200$, representing a fixed 40-hour week at a minimum wage of \$5.00 per hour. The mean of the Lognormal is given in Equation (4.12) as $\alpha = \exp(\mu + \frac{1}{2}\sigma^2)$. I will choose μ and σ^2 to set $\alpha = \$300$, so that $\tau + \alpha = \$500$, which was approximately the average weekly wage in the U. S. economy in 1997. The range of the real wage variable will then be $w \geq \$200$. Using a weekly wage

based on a fixed 40-hour week is consistent with the previous specification of indivisible labor in Section 4.2.4, with $h = 40$ hours per week.

The dispersion parameter σ^2 determines the Lorenz measure of inequality of nonlabor income in Equation (4.16)²³:

$$L = 2N(\sigma/\sqrt{2}|0,1) - 1.$$

Calibrating σ^2 to an empirical Lorenz index for wealth or nonlabor income will complete the calibration of Λ . Table 4-2 shows Gini (Lorenz) coefficients for labor and nonlabor income from IRS data for the years 1952-1981 [Slottje, (1989)]. The IRS definition of nonlabor income includes interest, dividends, rents and other nonlabor income reported on annual income tax returns²⁴. The Gini coefficients in Table 4-2 were calculated by Slottje after fitting a Beta distribution of the second kind to the data. (The Beta-2 distribution is a generalization of the Pareto and Lognormal distributions). Slottje's Gini coefficients for nonlabor income are about .44, and nonlabor income is about

²³The Lorenz measure L is also called the Gini coefficient in the literature, cf. Slottje (1989) discussed below.

²⁴The IRS data used by Slottje includes rental income which is not included in the definition of v , and excludes nontaxable transfer payments which are included in v . Most likely these are poor substitutes in terms of their place in the distribution, so it is not clear what effect this difference would have on the Gini coefficients in Table 4.3.

TABLE 4-2

FROM SLOTTJE (1989)

**GINI COEFFICIENTS OF INEQUALITY FOR THE
MARGINAL DISTRIBUTIONS OF LABOR EARNINGS,
NON-LABOR INCOME AND TOTAL INCOME**

Year	Labor Earnings	Non-Labor Income	Total Income
-----	-----	-----	-----
1952	0.315984	0.417585	0.308228
1953	0.308689	0.415573	0.301865
1954	0.314139	0.414786	0.306825
1955	0.292814	0.383159	0.286143
1956	0.309093	0.400836	0.302263
1957	0.304369	0.399035	0.297942
1958	0.328202	0.426538	0.321043
1959	0.328636	0.425430	0.321852
1960	0.324847	0.424995	0.318466
1961	0.327761	0.422926	0.321165
1962	0.327357	0.421785	0.320973
1963	0.331319	0.422219	0.324817
1964	0.334521	0.422931	0.328128
1965	0.336319	0.420617	0.329817
1966	0.354057	0.442451	0.347503
1967	0.357665	0.445950	0.351236
1968	0.359180	0.445534	0.352791
1969	0.350569	0.447051	0.345054
1970	0.352415	0.447873	0.347007
1971	0.349765	0.444327	0.344480
1972	0.347857	0.437317	0.342563
1973	0.346091	0.433483	0.340735
1974	0.345076	0.429350	0.339255
1975	0.343290	0.437048	0.337841
1976	0.339111	0.429775	0.333742
1977	0.355421	0.446047	0.349723
1978	0.319399	0.402100	0.314278
1979	0.351257	0.435840	0.345278
1980	0.341018	0.438637	0.336195
1981	0.338273	0.443703	0.333119

one-third more concentrated than labor earnings ($L = .34$) and total income ($L = .33$). Although Slottje's Gini coefficients were derived from annual data, they should be equally valid for weekly income because the Gini coefficients of both the Beta-2 distribution and the Lognormal distribution are invariant with respect to a constant multiple of the income variable²⁵

One problem in utilizing Slottje's data here is that it includes all taxpayers, including the retired and other nonworkers, whereas the wealth and nonlabor income of interest here is confined to the workforce²⁶. Wolff and Marley (1989) report Gini coefficients of .82 for financial wealth (excluding real estate), but this also includes the retired and very wealthy who are not in the workforce. In 1989 the wealthiest 1% of the U. S. population held 48% of financial wealth, and the top 20% held 96% [Wolff (1994)]. Financial wealth is highly concentrated in the general population, and most of it is owned directly or indirectly by individuals who are not in the workforce. It is likely that both financial wealth and nonlabor income derived from it are less concentrated in the workforce than in the general population, but we really don't

²⁵Weekly time series data on nonlabor income would probably have a larger coefficient of variation than annual data. However, the relevant distribution here is cross-sectional, over individuals. There is no apparent reason why the inequality of the distribution of nonlabor income over individuals would differ significantly if measured weekly instead of annually.

²⁶Cross-sectional data on nonlabor income for workforce participants was not readily available, but might be reconstructed from the Survey of Consumer Finance, a project which might be useful for future research.

know. The scant statistical evidence indicates that a Gini coefficient in the range of 0.4 to 0.5 would be reasonable for the distribution of nonlabor income in the workforce. I will use the mid-range value of 0.45.

A minimum weekly reservation wage of $\tau = \$200$, a mean $\alpha = \$500$ and a Lorenz coefficient of .45 correspond to calibrated values of $\mu = 5.347$ and $\sigma = .8444$ ($\sigma^2 = .713$) for the Lognormal distribution. For this distribution, the mean of \$500 corresponds to a labor-force participation rate of 66.0%, which is close to the official estimate for the U. S. economy. (*Economic Report of the President, 1992, Table B-34.*)

4.5.2 Calibration of Monetary Elasticities

What remains is to calibrate the elasticities in Equations (4.31) and (4.32). I will test a range of values for each, but a plausible benchmark case should have some empirical support. $\mathcal{E}_M[P]$ is positive and less than 1 for a non-neutral open-market operation. Initially I will set it equal to 0.8 and test for sensitivity to higher and lower values.

Note that $\mathcal{E}_M[r]$ is the reciprocal of the interest-elasticity of money demand in the Keynesian liquidity preference relationship. Although M represents the nominal money supply, a change in M causes a change in the equilibrium value

of r along the money demand schedule, and the elasticity of this change in r is the reciprocal of the interest-elasticity of money demand.

The relevant interest rate depends on the definition of money and its assumed substitutes. An open market operation is a swap of government bonds for high-powered base money, but most bonds traded in open-market operations have short maturities and the change in the monetary base acts through fractional reserve multipliers to change the monetary aggregates. The relevant money aggregate to use here should include those forms of money stocks which are utilized by investors as risk-free alternatives to bonds and equities, i. e., money which is held for the speculative motive. I will use M2 as the relevant measure of money, because checkable savings accounts and money-market mutual funds are superior to demand deposits as a store of value and are commonly used as cash accounts in portfolio management [Laidler (1977,1980)]. The institutional money instruments in M3 have little relevance for workforce participants. Dornbusch and Fischer (1977) and McCallum (1989) advocate M2 as the relevant definition of money in its speculative function. From 1987 to 1992 The Federal Reserve used the growth rate of M2 as an intermediate policy target²⁷.

²⁷Since 1992, Federal Reserve policy has been to target inflation and the federal funds rate, letting M2 be endogenous to the economy within broad target bands.

Most empirical studies of the money demand function have estimated cross-elasticities of M1 with respect to short-term rates of near-money substitutes such as savings accounts and time deposits²⁸. These estimates of the interest-elasticity of M1 have varied widely (See Table 4-3), and there is no consensus on whether long rates or short rates, real or nominal rates are more important determinants of M1 demand. Precision is not possible in these circumstances, but all that is required for the present purpose is a representative order-of-magnitude value for the interest-elasticity of M2.

Laidler (1980) employed several different structural models to estimate M2 demand elasticities from U. S. data for the period 1953-1978. His estimates for the three-month T-bill interest elasticity of M2 ranged from $-.121$ to $-.176$ with a clustering in the $-.147$ to $-.176$ range. (Coincidentally, these are close to the averages for time deposit and long-term bond interest elasticities for M1 in Table 4-3, including Laidler's own estimate of $-.150$ for the long-term bond.). Initially, I will set the value of $\mathcal{E}_r[M^d]$ to $-.15$, and test the sensitivity of the model to higher and lower values. This value appears to be representative of the short-run interest

²⁸See, for example, the surveys by Feige and Pearce (1976), Laidler (1977), and Judd and Scadding (1982).

TABLE 4-3. Interest Elasticity of Money Demand

<u>M1 Interest Elasticities</u>			
<u>Investigator</u>	<u>Time Deposits</u>	<u>90-Day T-Bills</u>	<u>Long-Term T-Bonds</u>
Feige (1964)	-.093		
Hamburger (1966)	-.185	-.014	-.160 -.246
Lee (1967)	-.094	-.019	----
Tiegen (1969)	---	-.104	-.123
Gramlich & Kalchbrenner (1970)	-.349	-.190	----
Goldfeld (1973)	-.279	-.049	----
Goldfeld (1976)	-.039	-.038	----
Hafer & Hein (1979)	-.040	-.02	-.200
Laidler (1980)	---	-.04	-.160
Average	<u>-.154</u>	<u>-.06</u>	<u>-.178</u>

elasticity of M2 relative to short-term government securities, the principal trading vehicle in open-market operations.

$\mathcal{E}_r[A]$, the interest-elasticity of the proportional demand for risky assets, depends on the utility function assumed to represent preferences. According to Tobin's (1958) version of the portfolio balance model, $A(r)$ should be inelastic at high interest rates and highly elastic at low rates, approaching perfect elasticity as r approaches zero. Tobin calculated bounds for this elasticity for an assumed quadratic utility function. Quadratic utility, however, has neither decreasing

R_A nor increasing R_R , and is inappropriate in the present context. Tobin's use of indifference curves in mean-variance space requires special assumptions which are inconsistent with both decreasing absolute risk aversion and increasing relative risk aversion [Borch (1969), Feldstein (1969), Hart (1975)].

However, given values for $\mathcal{E}_r[M^d]$ and $\mathcal{E}_\Omega[M^d]$, $\mathcal{E}_r[A]$ can be determined from the following relationship (Equation (D5.8) in Appendix D):

$$\mathcal{E}_r[A] = \left(\frac{(1-A)}{A} \right) \left[-\mathcal{E}_r[M^d] + A \left[\mathcal{E}_\Omega[M^d] - 1 \right] \right] \quad (4.33)$$

Representative values for the U.S. economy at the end of 1991 are: $f = \$5.035$ Trillion, $m = M2 = \$3.439$ Trillion, which give a value of $A = f/(f + m) = .594$. (*Economic Report of the President, 1992*). A benchmark value of $-.15$ will be used for $\mathcal{E}_r[M]$, as indicated above. Note that for homothetic preferences (constant relative risk aversion) $\mathcal{E}_\Omega[M^d] = 1$, and for nonhomothetic preferences with increasing relative risk aversion $\mathcal{E}_\Omega[M^d] > 1$. This implies that $\mathcal{E}_r[A]$, and thus $\mathcal{E}_M[\epsilon]$, will be larger for preferences with increasing relative risk aversion²⁹. I will compute results for both kinds of preferences and test the sensitivity of the results to the assumed values of these elasticities.

²⁹ Preferences enter into the determination of the elasticity of ϵ via $\mathcal{E}_r[A]$, $\mathcal{E}_r[M^d]$, $\mathcal{E}_\Omega[M^d]$ and the portfolio balance model.

The remaining parameter is ρ , the elasticity of relative risk aversion. From the argument leading to Equation 4.30 we have:

$$(1 - \rho)^2 = \sigma^2 / \sigma_\omega^2$$

The value of $\sigma^2 = 0.713$ has already been specified. With the assumption that Ω is distributed $\Lambda(\mu_\omega, \sigma_\omega^2)$, a value for σ_ω^2 can be derived from the Lorenz (Gini) coefficient for Λ from the relation $L_\omega = 2N(\sigma_\omega/\sqrt{2}|0,1) - 1$. It appears that Gini coefficients for the distribution of financial wealth in the workforce have not been reported in the literature; however, an approximate value can be inferred from Gini coefficients of total household wealth for the employed.

Table 4-4 lists Gini coefficients derived from several cross-sectional studies of households. Gini coefficients for measures of the household wealth of workers are shown in Wolff (1980) and Diaz-Gimenez et al (1997). The Gini coefficients in Table 4-4 correspond to broader definitions of wealth than the one used in this thesis³⁰. What is needed here is an empirically derived Gini coefficient for the size distrib-

³⁰The data in Table 4-3 exemplifies some of the methodological problems in the measurement of wealth. Different investigators have utilized different data bases, made different adjustments to the data, and employed different definitions of wealth. Consequently, Gini coefficients from different studies are not strictly comparable.

Table 4-4. Gini Coefficients for Household Wealth

<u>YEAR</u>	<u>SOURCE DATA¹</u>	<u>WEALTH DEFINITION²</u>	<u>GINI</u>	<u>REFERENCE</u>	
1969	<u>MESP (NBER)</u> Professional, Managerial, Clerical and Sales, Craft and Operative Workers (80%)	Total Assets	0.66	Wolff (1980)	
		Service and Unskilled Workers (20%)	Total Assets	0.74	
	Total Sample	Disposable Wealth	0.73	Wolff (1987)	
1983	<u>SCF</u>	Total Sample	Disposable Wealth	0.79	Wolff (1992)
		Total Sample	Net Worth	0.74	Weicher (1997)
1989	<u>SCF</u>	Total Sample	Marketable Wealth	0.83	
			Financial Wealth	0.93	
	<u>By Income</u>	Top quintile		0.78	Wolff (1994)
		2nd quintile		0.69	
		3rd quintile	Marketable	0.72	
		4th quintile	Wealth	0.77	
	1984 SIPP		0.69		
1988 SIPP		0.69			
1989	<u>SCF</u>	Total Sample	Net Worth	0.74	Weicher (1997)
1992	<u>SCF</u>	Total Sample	Net Worth	0.73	Weicher (1997)
				0.83	Wolff (1998)
	Total Sample	Net Worth	0.78	Diaz-Gimeniz, Quadrini, & Rios-Rull (1997)	
	All Workers		0.74		

¹MESP: a synthetic data base constructed by Richard Ruggles in the 1970's for the NBER.
SCF: Survey of Consumer Finances, Federal Reserve Board.

SIPP: Survey of Income and Program Participation, US Bureau of the Census.

²Total Assets = financial assets + home equity + consumer durables + business equity.

Disposable Wealth = Total Assets + other real estate + cash value of pensions - debt.

Marketable Wealth = Disposable Wealth - consumer durables.

Net Worth = Total Assets plus autos and other real estate - debt.

Financial Assets = Marketable Wealth - home equity.

ution of Treasury securities, equities and money in the workforce population; in lieu of that, a rough estimate will be conjured from the data in Table 4-4.

It is well known that financial wealth is more concentrated than household net worth in total population. (A Gini coefficient of 0.90 vs. 0.80.) That may not be true of the labor force, however, because financial assets are held disproportionately by the very rich who are not labor force participants in the sense being used here. Wolff (1995) shows that the richest 10% of households own approximately 90% of stocks and bonds and 60% of deposits. Excluding this top tier would leave 10% of stocks and bonds and 40% of deposits to be held by the workforce. The degree of concentration of these remaining financial assets in the workforce population is unknown, but it is most certainly less than their concentration in the total population. In order to proceed, I will make an instrumental assumption (based on Table 4-4) that the Gini coefficient for the size distribution of financial assets in the workforce lies within the range of 0.66 to 0.74. I will choose the midpoint of this range, 0.70, and test for sensitivity to the upper limit of 0.74. The value of 0.70 is close to the Gini coefficients of wealth for the middle

quintiles of household income in the 1989 SCF, and also for the SIPP data³¹.

Setting $2N(\sigma_\omega/\sqrt{2}|0,1) = 0.70$ yields $\sigma_\omega^2 = 2.15281$, and this in turn gives:

$$\rho = 1 - \sigma/\sigma_\omega = 1 - (.8444)/(1.4672) = .424$$

The corresponding value for $L_\omega = 0.74$ is $\rho = .471$.

This completes the initial calibration of the model. Computational results are presented in the next section.

4.5.3 Computational Results

It may be helpful to restate the purpose of the investigation at this stage. The key question is: Does this calibrated labor supply function exhibit positive elasticities of both Λ and $\epsilon_{l\omega}$ over a relevant range of labor-force participation rates in response to a one-time increase in money? If so, then the function exhibits procyclical wage-elasticity, suggesting that its underlying economic mechanism could serve as an explanation of countercyclical markups and procyclical real wages. If not, then we have another

³¹Wolff (1998) points out that, unlike the SCF, the upper tail of the wealth distribution is missing from both the SIPP and PSID data bases, making them more useful for studying the wealth accumulation behavior of the middle class.

counterfactual model and the problem introduced in Chapter I remains.

A positive money-elasticity of $\Lambda(w)$ at a real wage w_0 implies that an open-market purchase will increase the supply of labor at that wage. A positive money-elasticity of ϵ at the same real wage w_0 means that ϵ is procyclical at that wage; this occurs when the relative density of reservation wages increases at that wage. If both elasticities are positive over a range of wages, it implies that labor supply increases with procyclical elasticity over that range. A relevant range of the aggregate real wage for the U. S. economy would correspond to a range of labor-force participation rates of 65% to 80%. which encompasses the observed rates for the aggregate labor force. [(Ehrenberg and Smith (1991); *Economic Report of the President, 1992*), Table B-34]. Since this aggregate labor supply function is a cumulative Lognormal distribution, labor-force participation rates are the values of the function, which can be obtained from tables of the standard Normal distribution $N(z|0,1)$. For example, the mean wage of \$500 for this three-parameter Lognormal distribution corresponds to an LFPR of $N(.41646|0,1) = 66.0\%$. Thus, elasticities at and above the mean reservation wage will be relevant.

Finally, where the elasticities are both positive, their magnitudes should be realistic in terms of the money-employment relationship. At the end of 1991 the civilian workforce was 125.7 million, representing a labor-force participation rate of 66% of the total population. The unemployment rate was 7.1%, and total employment was 116.7 million workers. At the same time, nominal M2 was \$3.439 Trillion, and had grown at a rate of 3.0% over the previous 12 months. Given these numbers, an M2 labor supply elasticity of +1 would imply that a 1% (\$34.4 Billion) one-time increase in M2 would induce a 1% increase in aggregate labor supply (1.25 million workers). With an M2 multiplier of approximately 8, this would correspond to a 4.3 Billion (1.4%) increase in the monetary base of \$317 Billion. The monetary base actually grew by 6.4% during 1991, so the implied expansionary policy would constitute 22% of the total monetary base expansion for the year. A monetary elasticity of labor supply of +2 would require only one-half as much increase in the monetary base.

Fifteen cases were computed, each representing a different parameterization of the model. The details are shown in Exhibits 4-1 through 4-16 located at the end of this section. The results are summarized in Table 4-5.

The computational setup (in Mathcad 4.0) for the homo-

thetic Case H is shown in Exhibit 4-1(a); computed values and graphs are shown in Exhibit 4-1(b). The M2-elasticity of labor supply is positive over the entire range of wages, but the M2-elasticity of ϵ is positive only up to the weekly wage of \$395, corresponding to a labor-force participation rate (LFPR) of $N(-.1|0,1) = 46.0\%$. Above that wage the elasticity is negative and ϵ is countercyclical. At the mean reservation wage of \$500 the elasticity of ϵ is -0.5 and ϵ is countercyclical³². In fact, ϵ is countercyclical over the entire relevant range of reservation wages. This result is consistent with the main conclusion of Chapter III, i. e., *it is not possible for ϵ to be procyclical at the average aggregate wage if preferences are homothetic.*

I will now turn to the results of the nonhomothetic cases NH-1 through NH-7.

To represent preferences with increasing relative risk aversion requires that the elasticity of the reservation wage function $\psi(v)$ be an increasing function of nonlabor income v , i. e., $\xi' > 0$. (See Section 4.2.5 and the proof in Appendix

³²Aitchison and Brown (1963) argue that the median (geometric mean e^{μ}) is a more logical measure of central tendency for the Lognormal distribution. In the homothetic case of Exhibit 4-1, ϵ is slightly countercyclical at the median of \$410 with an elasticity there of $-.083$. Nevertheless, the empirical evidence on acyclical real wages relates to the equilibrium real wage, which appears to be in the vicinity of the mean of \$500, at a labor-force participation rate of 66%.

Table 4-5. Computational Results of the Labor Supply Model

Exhibit/Case	PARAMETERS				COMPUTED QUANTITIES			
	L=.45 L_w	EMP	M2 ERM	EQM	$\mathcal{E}_M[\epsilon]_\alpha$	$+W^-$	$+LFPR^-$	
4-1 H	.70	.8	-.15	1.0	-0.5	\$ 395	46.0%	
4-2 NH-1	.70	.8	-.15	1.1	+0.5	\$ 835	71.2%	
4-3 NH-2				1.3	+1.3	1,300	74.4	
4-4 NH-3				1.5	+2.1	1,599	75.9	
4-5 NH-4			-.25	1.1	+0.34	730	70.1	
4-6 NH-5			-.05	1.1	+1.3	1,300	74.4	
4-7 NH-6		.95	-.15	1.1	+0.33	682	69.5	
4-8 NH-7	.74	.8	-.15	1.1	+0.06	539	66.9	
4-9 NH-8	.70	.8	-.15	***	+3.0	\$ 694	77.1%	
4-10 NH-9			-.25	***	+1.9	664	76.8	
4-11 NH-10		.5	-.15	***	+3.5	720	78.1	
4-12 NH-11		.8	-.10	***	+4.5	714	77.9	
4-13 NH-12	.74	.8	-.15	***	+1.8	663	76.0	
4-14 NH-13	.70	.95	-.25	1.05	+0.05	575	66.5%	
4-15 NH-14		.95	-.25	***	+1.7	647	75.2	
4-16 NH-15	.74	.10	-.15	***	+2.6	752	77.9	

EMP = $\mathcal{E}_M[P]$; ERM = $\mathcal{E}_r[M^d]$; EQM = $\mathcal{E}_\Omega[M^d]$; L = Gini Coefficient of nonlabor income. L_w = Gini coefficient of wealth.

$\mathcal{E}_M[\epsilon]_\alpha$ is the elasticity of ϵ at the mean reservation wage of \$500 per week, where the labor-force participation rate is 66.0%.

$+W^-$ and $+LFPR^-$ are the maximum reservation wage and labor-force participation rate for which the elasticity of ϵ is positive.

*** EQM = $\xi = \delta \ln(w + 1)$, which increases with w .

section D3.). The simplest specification is that $\xi' = \delta > 0$, a constant, from which it follows that:³³

$$\begin{aligned}\xi &= \delta v \\ \omega &= \psi(v) = \exp(\delta v) - 1 \\ v &= \psi^{-1}(\omega) = g(\omega) = (\ln(\omega + 1))/\delta \\ \theta &= \mathcal{E}_w[g] = \omega / [(\omega + 1) \ln(\omega + 1)]\end{aligned}\tag{4.34}$$

This specification was implemented by substituting the function $g(w)$ in place of w as the argument of $\Lambda(w|\mu, \sigma^2)$ in the elasticity equations 4.25 and 4.26 of Section 4.4.

This is facilitated by setting:³⁴

$$\begin{aligned}z &= (\ln[g(w)] - \mu) / \sigma, \\ \text{whereupon} \quad g(w) &= \exp(\mu + \sigma z) = \ln(w(z) + 1) \\ \text{and} \quad w(z) &= \exp(\delta \exp(\mu + \sigma z)) - 1\end{aligned}\tag{4.35}$$

³³Setting the constants of integration to 1 in the derivation of the reservation wage function ψ implies that $\xi(0) = 0$ and $\psi(0) = 0$, i. e., workers without nonlabor income have a reservation wage equal to the minimum weekly wage $\tau = \$200$.

³⁴In principle the chain rule for the elasticity of a composite function, which is the basis for Equations 4.6 and 4.7, can also be used, but $\mathcal{E}[\gamma]$ and $\mathcal{E}[\theta]$ are functions of $g(w)$ and the integrals in Equations 4.25 and 4.26 still have to be evaluated. The transformation between the Normal and Lognormal makes this straightforward.

Equations 4.31 and 4.32 were evaluated using the standard Normal distributions $\phi(z)$ and $N(z|0,1)$ as before, and the results were transformed back in terms of $w(z)$ using Equation 4.35. The constant parameter δ was set to calibrate the distribution at the mean reservation wage of \$500 per week, and incorporates the scale factor $h = 40$ hours in a work week. All of this is shown in the computational setup for the non-homothetic case NH-1 in Exhibit 4-2(a).

Table 4-5 shows the results for this representation of nonhomothetic preferences, for assumed values of 1.1, 1.3 and 1.5 for $E\Omega M$, representing the range of values estimated by Meltzer (1963), [1.15]; Laidler (1971), [1.4]; and Friedman (1959), [1.8]. The elasticities of both Λ and ϵ are positive and larger than in the homothetic case, and increase with $E\Omega M$. (See Table 4-5 and Exhibits 4-2(b) through 4-4(b). In Case NH-1 the M2-elasticity of ϵ at the mean reservation wage of \$500 is +0.5 and increases significantly with the higher values of $E\Omega M$ in Cases NH-2 and NH-3. More importantly, it is positive up to a reservation wage of \$1,599 per week, corresponding to a labor-force participation rate (LFPR) of 75.9%. For this particular specification of nonhomothetic preferences and monetary elasticities, ϵ is procyclical and elastic with respect to a one-time change in M2.

Using the parameters of case NH-1 as a benchmark, Cases NH-4, 5 and 6 test the partial sensitivity of the results to changes in the assumed values of the monetary elasticities ERM and EMP. Increasing ERM to $-.25$ (which is about the largest estimate found for T-bond interest elasticity and is five times the average estimate for T-bills) reduced the magnitude of $\mathcal{E}_M[\epsilon]$ slightly, from $+0.5$ to $+0.34$, and lowered the upper bound of its positive range slightly, to 70.1% . Reducing ERM to $-.05$ had precisely the same effect on the benchmark case as increasing ERM from 1.1 to 1.3 . Increasing EMP to 0.95 in Exhibit 4-7 had about the same effect on the benchmark case as increasing ERM from $-.15$ to $-.25$, and decreasing EMP to 0.5 (not shown) increased the elasticities and positive range by small amounts. Thus, it appears that for this specification of nonhomothetic preferences, implications of the model are somewhat sensitive to the assumed values of the monetary elasticities, although there is a vector of empirically supported values for which the elasticity of ϵ at the mean wage is positive.

The computed results of the benchmark Case NH-1 were not sensitive to small changes in the Gini coefficient for nonlabor income. However, Case NH-7 (Exhibit 4-8) indicates that there are limits to the degree of wealth inequality for which this model will exhibit procyclical elasticities. Increasing the Gini coefficient of the wealth distribution

from .70 to .74 (which corresponds to $\rho = .471$) reduced the elasticity of ϵ at the mean to +0.06 and made ϵ counter-cyclical above it³⁵. Increasing the Gini to 0.8 (not shown) produced results very close to the nonhomothetic case (Exhibit 4-1(b)), with $\mathcal{E}_M[\Lambda] < 1$. This result implies that monetary policy would have a weaker influence on aggregate labor supply if the concentration of financial wealth in the workforce were greater than a Gini coefficient of 0.74.

Cases NH-8 through NH-13 are based on an alternative representation of preferences in the model. In Cases NH-1, -2 and -3, $\mathcal{E}_Q[M^d]$ was treated as exogenous and results were obtained for arbitrary values of 1.1, 1.5, and 1.5. For each case, $\mathcal{E}_Q[M^d]$ was assumed to be constant over the full range of v and w . This appears to be inconsistent with the nonhomothetic specification that $\xi' > 0$, because both $\mathcal{E}_Q[M^d]$ and ξ measure the proportional response of workers to a change in a scale variable. The reservation wage function $\psi(v)$ can be interpreted as the Engel curve of the demand for leisure in the constrained work/leisure choice problem of Section 4.2.3, with ξ as its elasticity. If leisure and money balances are complements and are luxury goods and if preferences have

³⁵Note that elasticity of ϵ of 0 at the mean reservation wage is still more procyclical than in the homothetic case, where the elasticity of ϵ at the mean is -0.5. This can also be seen by comparing the graphs of Exhibits 4-1 and 4-8 at the mean reservation wage of \$500.

increasing relative risk aversion, then the wealth elasticity of the demand for money balances should change proportionally with the income elasticity of the demand for leisure³⁶. This feature can be incorporated in the model by setting:

$$\mathcal{E}_Q[M^d] = \xi \quad (4.36)$$

so that $\mathcal{E}_Q[M^d]$ is endogenous and will increase at the same rate as ξ . For the present reservation wage function $\xi = \delta v = \ln(w + 1)$, which is greater than one for $w > e - 1 = 1.718$. However, this function causes $\mathcal{E}_Q[M^d]$ to increase too rapidly, producing values in excess of 2.0 around the mean, which are not supported by empirical evidence. The alternative specification utilized in Cases NH-7 through NH-11 is:

$$\begin{aligned} \xi &= \delta v^\delta, \quad 0 < \delta < 1 \\ \psi(v) &= \exp(v^\delta) - 1 \\ g(w) &= [\ln(w + 1)]^{1/\delta} = v \\ w(z) &= \exp[\exp[\delta(\mu + z\sigma)]] - 1 \\ \delta v^\delta &= \delta \ln(w(z) + 1) \end{aligned} \quad (4.37)$$

³⁶This assumes that relative risk aversion increases at approximately the same rate in both subutility functions $U(c, l)$ and $H(f, m)$. There is no a priori reason for this to be true, but neither is there any empirical evidence to the contrary. One could argue that risk is risk, and aversion to it should be similar in different contexts. The subutility functions could be parameterized to allow for differences in R_p .

Exhibit 4-9(a) shows the computational setup for this specification. A value of 0.305 for δ calibrates the mean reservation wage at $r + \$300 = \500 . From Equation 4.37, $\xi' = \delta^2 v^{\delta-1} = \delta^2 [\ln(w + 1)]^{(\delta-1)/\delta}$, and since $\delta < 1$, this specification produces a lower growth rate in $\mathcal{E}_\Omega[M^d]$ that declines with v .

The computed results for this nonhomothetic specification are shown in Table 4-5 and Exhibits 4-9 through 4-13. Case NH-8 is the basic elasticity computation with the same parameter values as in NH-1. The elasticity of ϵ at the mean is +3.0 and ϵ is procyclical up to a LFPR of 77.1%. The value of the endogenous elasticity $\mathcal{E}_\Omega[M^d]$ in Equation 4.36 is +1.56 at the median wage and +1.74 at the mean. These values are close to that for Case NH-3 and represent strong relative risk aversion and procyclicality of Λ and ϵ . Because the reservation wage functions ψ in Exhibits 4-2(b) and 4-9(b) are different, the respective density functions of reservation wages are different even though they are based on the same underlying distribution of nonlabor income $\Lambda(v|\mu, \sigma^2)$. The essential difference between these two nonhomothetic representations is that the first reservation wage function is a strong downward compression of the distribution of nonlabor income, and the second one is much less so.

Cases NH-9 through NH-12 perform the same sensitivity tests as in NH-4 through NH-6. This specification has about

the same degree of sensitivity to the value of $\mathcal{E}_r[M^d]$ (compare Cases NH-9 and NH-4), and to the value of L_ω (Case NH-7 and NH-12). This version of the model can tolerate a higher Gini coefficient for wealth (0.74), undoubtedly due to its higher elasticity of relative risk aversion.

Cases NH-13 and NH-14 test each of the two model specifications for a "worst-case" situation where money is almost neutral ($EMP = .95$), the interest-elasticity of M2 is high ($-.25$), and the wealth-elasticity of money demand is relatively low (1.05). The result in each case is a small but positive elasticity of ϵ at the mean, and a reduction of 3-4% on the upper bound LFPR's. The parameters most critical to these results are ρ , the wealth-elasticity of R_R , and $\mathcal{E}_\Omega[M_d]$, the wealth-elasticity of money demand, which has to be greater than 1 for ϵ to be procyclical at the mean reservation wage.

Finally, in Exhibit 4-16(b), NH-15, I show a "New Keynesian" case, with sticky prices ($\mathcal{E}_M[P] = .10$), low elasticity of money demand ($\mathcal{E}_r[M^d] = -.15$), a wealth Gini of 0.74, and endogenous wealth elasticity of money demand³⁷. The result is a procyclical elasticity of + 2.6 for ϵ at the mean wage, which remains procyclical up to the LFPR of 77.9%.

³⁷To be rigorous here, a low value of $\mathcal{E}_M[P]$ implies sticky prices only in a dynamic framework. In the static framework of Chapters II and V prices jump instantaneously, but with a low value of $\mathcal{E}_M[P]$, not very far.

The elasticity of Λ in the vicinity of the mean is about +1.25. Thus, New Keynesian price rigidities would enhance the procyclical results of this model.

In all of these cases, ϵ is procyclical at the mean reservation wage of \$500 and LFPR of 66%. In general, the upper bound LFPR's are in the range of 70 -78%. Killingsworth (1983, p. 103) claims that labor-force participation rates for women are in the 50% - 60% range, and for men are 80% - 90%. Ehrenberg and Smith (1991) quote participation rates of 77% for men, 58% for women, and 67% total, from BLS data for 1979. This agrees with the figure of 66% for 1991 in the 1992 *Economic Report of the President*. The upper-bound LFPR's shown in Table 4-5 indicate that the range of procyclical ϵ includes all of these figures, and therefore the results are relevant to the U. S. labor market. Of course, the elasticity of ϵ will be small if the equilibrium real wage in the economy is close to the upper-bound LFPR of 78%.

Thus, it appears that the aggregate labor supply function developed in this chapter is capable of exhibiting strongly procyclical elasticity of Λ and ϵ over a relevant range of labor-force participation rates, for plausible values of its parameters. This is conditional on the assumption that consumer preferences have increasing relative risk aversion.

It was established in Chapters I and III that procyclical elasticity of labor supply at the equilibrium wage is a sufficient condition for the markdown of the real wage from labor's marginal product to be countercyclical. This follows directly from the markdown relationship:

$$PF_{\ell}/W = [1 + 1/\epsilon] = e$$

Thus, the model can serve as one explanation of countercyclical markups, i. e., as countercyclical markdowns in a monopsonistic labor markets. It remains to be determined whether the magnitudes of the elasticities shown here are large enough to offset the negative elasticity of an aggregate labor demand function and cause the equilibrium real wage to be acyclical or procyclical. This analysis is carried out in Chapter V.

In the first set of computations (Cases NH-1 through NH-7). both the magnitude of the elasticity of ϵ at the mean wage and the range of LFPR's for which the elasticity was positive were sensitive to changes in the monetary elasticities $\mathcal{E}_M[P]$, $\mathcal{E}_r[M^d]$ and $\mathcal{E}_R[M^d]$, and the assumed Gini coefficient for wealth. In the second set (Cases NH-8 through NH-12) the model was more stable with respect to changes in these parameters. A sensitivity test indicated that, if the Gini coefficient of the distribution of wealth in the workforce

were higher than 0.74, it would be difficult to obtain a positive $\mathcal{E}_M[\epsilon]$ above the mean wage without assuming extreme values for the monetary elasticities.

The benchmark Cases NH-1 (Exhibit 4-2) and NH-8 (Exhibit 4-9) were parameterized conservatively with respect to the price-elasticity of money supply and the interest-elasticity of money demand. If prices are sticky, in the sense that the short-run elasticity of the price level with respect to money is less than 0.8, and/or the interest-elasticity of M2 demand is less than $-.15$, then this model will predict even stronger procyclicality of Λ and ϵ .

The interest-elasticity of the demand for money balances $\mathcal{E}_r[M^d]$, plays a key role in the model, as it does in the monetary policy effectiveness debate. Low elasticities imply that exogenous changes in the money supply will have a strong influence on interest rates and aggregate demand in the short run. (The monetarist view is that the influence is too strong and destabilizing.) Similarly, in the model of this chapter, with nonhomothetic preferences the procyclical behavior of labor supply and its elasticity is driven by the reciprocal of $\mathcal{E}_r[M^d]$; low values therefore imply that monetary policy has a strong influence on aggregate labor supply. This is consistent with its influence on aggregate demand. Also, to the extent that the money demand function is unstable due to

disintermediation or changes in the velocity of money, the money-elasticity of aggregate labor supply will also be unstable.

4.6 Conclusions

I have shown in this chapter that, by abandoning the representative agent framework and incorporating heterogeneity of labor as an endogenous characteristic, it is possible to construct a model of the aggregate labor market that appears to be more consistent with some persistent stylized facts of macroeconomics -- facts which the representative agent is powerless to explain.

The analysis in this chapter has revealed a potential source of countercyclical markdowns and procyclical elasticity of aggregate labor supply: (1) nonhomothetic preferences with increasing relative risk aversion, and (2) an aggregate labor supply function based on the distribution of financial wealth and nonlabor income in the workforce, the shift characteristics of which are determined by the effects of central bank open-market operations.

Increasing relative risk aversion implies that the elasticities of the demand for leisure and the demand for real money balances increase with income or wealth. The method of

aggregation employed in this chapter was distributional: the entire distribution of heterogeneous characteristics (in this instance, nonlabor income and risk aversion) was modeled and formed the structure of an aggregate labor supply function. That function is a composite function of (a) a Lognormal distribution of nonlabor income and (b) a reservation wage function relating individual reservation wages to nonlabor income. The cyclical properties of this aggregate labor supply function were then investigated.

The Lognormal aggregate labor supply function developed in this chapter exhibited procyclical elasticity at labor-force participation rates of up to 78% for plausible values of its parameters. This result holds only for nonhomothetic preferences with increasing relative risk aversion. This property, together with the assumption of monopsony power in the labor market, makes this aggregate labor supply model a viable candidate for explaining countercyclical markdowns, which requires only that ϵ be procyclical.

Whether the elasticity properties of this labor supply function can also serve to explain acyclical or procyclical real wages is a more complicated question. First, there is the question of whether the magnitudes of $\mathcal{E}_M[\epsilon]$ are large enough to offset the negative elasticity of the aggregate marginal product of labor curve in the standard monopsony

labor market model (See Equation (3.15) in section 3.5.2 and the conclusions of that section, in Chapter III). This raises the empirical question of what the elasticity of the aggregate marginal product of labor is, and how to characterize aggregate labor demand in a monopsony model³⁸. Finally, the behavior of the labor supply function derived in this chapter depends on nonhomogeneous relative demands for financial assets and money, which may have other macroeconomic implications. Therefore, this question needs to be investigated in a general equilibrium framework similar to the model of Chapter II, That is the objective of Chapter V.

What can be claimed for this model is that it implies that, with increasing relative risk aversion in monopsonistic labor markets, real wages will be less countercyclical than they would be in a representative agent model of labor supply. As discussed in Chapter I, aggregate real wages have been at most mildly procyclical, and then only at certain times and under certain conditions. An acyclical real wage is also consistent with the empirical record. What is encouraging is that the computed magnitudes of $\mathcal{E}_M[\epsilon]$ at the mean in the calibrated version of this model are in the range of +3 to

³⁸Recall, from Chapter III, that a monopsonist does not have a demand schedule that is independent of the elasticity of supply.

+5 for reasonable ranges of the parameters, and ϵ is strongly procyclical at the observed labor-force participation rate of 66%.

In contrast with the representative agent labor market of Chapter II, the wealth effects in labor supply in this chapter are distributed, and depend on the particular functional forms used to represent nonhomothetic preferences and the distribution of wealth in the workforce. The results obtained here are specific to the Lognormal distribution; they cannot be obtained, for example, with the Pareto distribution. However, the research literature indicates that the Lognormal may be the best choice in this context. If the representative agent framework is abandoned, then distributions of heterogeneous agent characteristics, such as wealth, tastes for work and leisure, and aversion to risk will matter for macroeconomic outcomes. It is important to represent these distributions by functional forms that have theoretical and empirical justification.

The model developed in this chapter articulates a monetary theory of aggregate labor supply for heterogeneous workers. The aggregate labor supply function is directly related to the size distribution of financial wealth in the workforce. With real wealth as a shift variable in the labor supply function, it is not surprising that non-neutral

monetary policy actions will have an affect on labor supply. That was established back in Chapter II. What is new in this chapter is that the distributive wealth effects of a Lognormal distribution cause the elasticity of labor supply to be strongly procyclical over a relevant range of labor-force participation rates. In addition to reducing per capita real wealth, an increase in money reduces the inequality of the size distribution of wealth, which increases the elasticity of aggregate labor supply in the vicinity of the mean reservation wage. In this model the inequality of the distribution of financial wealth matters for labor market outcomes, and incidentally, for monetary policy effectiveness.

The willingness of risk-averse agents to adjust their portfolio balances in response to changes in expected returns on financial assets is an important question in monetary theory, since it helps to determine the interest-elasticity of the money demand function. And so it is also in this aggregate labor supply function, where the non-neutrality of open-market operations, the portfolio balance decision, and the elasticity of the Keynesian liquidity preference function play major roles in determining the cyclical properties of the aggregate labor supply function. The critical assumption here, however, is that consumers have increasing relative risk aversion with respect to the leisure-labor decision and the portfolio balance decision, from which it follows that the

income-elasticity of leisure demand and the wealth-elasticity of money demand are both significantly greater than unity. Without that assumption, the model has no important implications for the cyclicalities of markdowns and real wages.

These monetary ideas are not without controversy, and the model presented here is subject to some of the same disagreements and criticisms that have been associated with the monetary policy effectiveness debate. Nevertheless, from an historical perspective the aggregate labor supply model presented in this chapter may be viewed as a somewhat belated followup of Keynes' attempt to construct a monetary theory of production.

znorm0A.mcd

EXHIBIT 4-1(a) HOMOTHETIC CASE
ELASTICITY OF ELASTICITY COMPUTATIONS
LOGNORMAL LABOR SUPPLY FUNCTION

LOGNORMAL DISTRIBUTION CALIBRATION

$$z := -3, -2.9 \dots -3 \quad \alpha := 300 \quad \beta := 510 \quad \tau = 200 \quad \gamma = \frac{\beta}{\alpha + \tau} \quad \sigma = \sqrt{\ln(\gamma^2 + 1)}$$

$$v = 1 \quad \beta_2 := \alpha^2 \cdot \gamma^2 \quad \beta_2 = 9.364 \cdot 10^4 \quad \gamma = 1.02 \quad \sigma = 0.844 \quad \sigma^2 = 0.713$$

$$\Omega(z) := z \quad \Omega(z) := z^2 - 1 \quad \mu := \ln(\alpha) - .5 \cdot \sigma^2 \quad L := 2 \cdot \text{cnorm}\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \quad \mu = 5.347$$

$$L = 0.45 \quad \rho = 0$$

MONETARY CALIBRATION

$$M2 = 3.439 \quad F = 5.035 \quad A = \frac{F}{M2 + F} \quad A = 0.594$$

$$\text{EMP} = .8$$

$$\text{ERM} = -.15 \quad \text{EQM} = 1$$

$$\text{ERA} = \frac{1 - A}{A} (-\text{ERM} + A \cdot (\text{EQM} - 1)) \quad \text{Equation (D5.8)}$$

$$\text{ERA} = 0.102$$

$$\text{EMA} = \frac{\text{ERA}}{\text{ERM}} \quad \text{EMA} = -0.683$$

Homothetic Reservation Wage Function

$$\xi = 1 \quad \psi = \frac{v}{40} \quad g(w) = w \cdot 40$$

$$w(z) = \exp(\mu + \sigma \cdot z)$$

$$\text{Mean} = \exp(\mu + .5 \cdot \sigma^2) + \tau$$

$$\text{Mean} = 500$$

$$\text{Median} = \exp(\mu) + \tau$$

$$\text{Median} = 410.021$$

$$K1(z) = \left[\frac{1}{\text{cnorm}(z)} \int_{-3}^z \frac{\Omega(z)}{\sigma} \frac{1}{\sqrt{2 \cdot \pi}} \exp\left(-\frac{z^2}{2}\right) dz \right] \cdot \text{EMP}$$

$$K2(z) = \left[\int_{-3}^z \Omega(z) \frac{1}{\sqrt{2 \cdot \pi}} \exp\left(-\frac{z^2}{2}\right) dz \cdot \left(\frac{1}{\text{cnorm}(z)}\right) \right] \cdot \text{EMA}$$

$$E\Lambda(z) = K1(z) + K2(z) \quad \text{(Equation D5.11)}$$

M - ELASTICITY OF AGGREGATE LABOR SUPPLY

$$E\epsilon(z) := -E\Lambda(z) - \frac{\Omega(z)}{\sigma} \cdot \text{EMP} + \Omega(z) \cdot \text{EMA} \quad \text{(Equation D6.11)} \quad \text{M - ELASTICITY OF AGGREGATE LABOR SUPPLY ELASTICITY}$$

$$\lambda(z) := \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma \cdot w(z)} \exp\left[-\frac{1}{2} \cdot (z)^2\right]$$

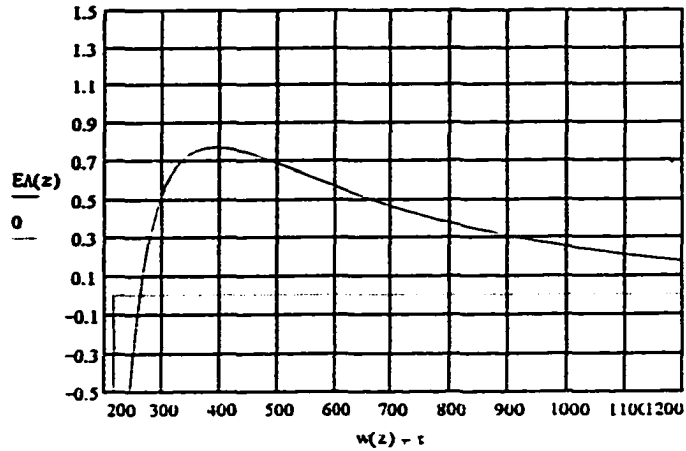
LOGNORMAL DENSITY FUNCTION
OF NONLABOR INCOME

znorm0A.mcd

LFPR W
cnorm(z)w(z) + τ EA(z) Ez(z)

0.00135	216.673	0	-2.622
0.00187	218.142	-0.68	-1.634
0.00256	219.74	-1.079	-0.94
0.00347	221.48	-1.287	-0.451
0.00466	223.373	-1.367	-0.104
0.00621	225.432	-1.36	0.142
0.0082	227.673	-1.295	0.317
0.01072	230.111	-1.192	0.441
0.0139	232.765	-1.066	0.527
0.01786	235.652	-0.926	0.586
0.02275	238.793	-0.779	0.625
0.02872	242.212	-0.631	0.648
0.03593	245.931	-0.484	0.659
0.04457	249.979	-0.342	0.661
0.0548	254.382	-0.205	0.656
0.06681	259.174	-0.077	0.644
0.08076	264.389	0.044	0.627
0.0968	270.062	0.156	0.605
0.11507	276.236	0.258	0.578
0.13567	282.954	0.351	0.548
0.15866	290.263	0.434	0.513
0.18406	298.217	0.507	0.475
0.21186	306.871	0.571	0.432
0.24196	316.288	0.626	0.386
0.27425	326.535	0.671	0.335
0.30854	337.685	0.707	0.279
0.34458	349.817	0.735	0.218
0.38209	363.019	0.754	0.152
0.42074	377.383	0.765	0.08
0.46017	393.014	0.769	0.002
0.5	410.021	0.766	-0.083
0.53983	428.528	0.756	-0.174
0.57926	448.665	0.74	-0.274
0.61791	470.576	0.719	-0.382
0.65542	494.418	0.693	-0.498
0.69146	520.361	0.663	-0.625
0.72575	548.59	0.63	-0.761
0.75804	579.307	0.594	-0.908
0.78814	612.73	0.555	-1.067
0.81594	649.098	0.515	-1.238
0.84134	688.671	0.475	-1.422
0.86433	731.731	0.434	-1.619
0.88493	778.585	0.393	-1.831
0.9032	829.568	0.354	-2.056
0.91924	885.043	0.315	-2.297
0.93319	945.406	0.279	-2.554
0.9452	1011.088	0.245	-2.826
0.95543	1082.558	0.213	-3.114
0.96407	1160.326	0.183	-3.418
0.97128	1244.946	0.157	-3.739

EXHIBIT 4-1(b) HOMOTHETIC CASE



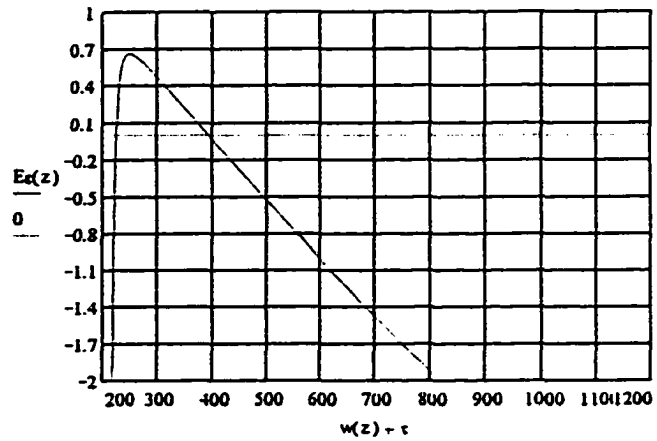
M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 410.021 $\alpha = 300$ $L = 0.45$

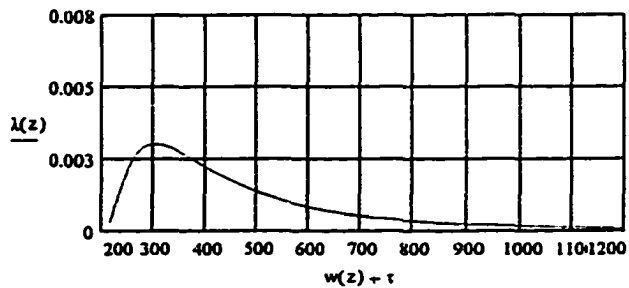
$\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^{-5}$ $\gamma = 1.02$

$\rho = 0$

EMP = 0.8 ERM = -0.15 EQM = 1 ERA = 0.102



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



DENSITY OF RESERVATION WAGES

znorm0A.mcd

**ELASTICITY OF ELASTICITY COMPUTATIONS
LOGNORMAL LABOR SUPPLY FUNCTION**

LOGNORMAL DISTRIBUTION CALIBRATION

$$z := -3, -2.9, \dots, 1.5 \quad \alpha := 300 \quad \beta := 510 \quad \tau := 200 \quad \gamma := \frac{\beta}{\alpha + \tau} \quad \sigma := \sqrt{\ln(\gamma^2 + 1)} \quad \sigma = 0.844$$

$$v := 1 \quad \beta_2 := \alpha^2 \cdot \gamma^2 \quad \beta_2 = 9.364 \cdot 10^4 \quad \gamma = 1.02 \quad \sigma^2 = 0.713$$

$$\mu := \ln(\alpha) - .5 \cdot \sigma^2 \quad L := 2 \cdot \text{cnorm}\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \quad L = 0.45 \quad \mu = 5.347$$

$$\rho = .424$$

MONETARY CALIBRATION

$$M2 := 3.439 \quad F := 5.035 \quad A = \frac{F}{M2 + F} \quad A = 0.594$$

EMP := .8

ERM := -.15 EQM := 1.2

ERA := $\frac{1 - A}{A} \cdot (-ERM + A \cdot (EQM - 1))$ Equation (D5.8)

ERA = 0.184

EMA := $\frac{ERA}{ERM}$ EMA = -1.224

**Calibrate Mean Reservation Wage
For Nonhomothetic Function**

$$\delta := \frac{\ln(\alpha + 1)}{\alpha} \quad \delta = 0.019$$

**Nonhomothetic
Reservation Wage Function**

$$\xi := \delta \cdot v$$

$$\psi = \exp(\delta \cdot v) - 1 \quad g(w) = \frac{\ln(w + 1)}{\delta}$$

$$w(z) = \exp(\delta \cdot \exp(\mu + z \cdot \sigma)) - 1$$

$$\text{Mean} := \exp(\delta \cdot \exp(\mu + .5 \cdot \sigma^2)) - 1 + \tau$$

Mean = 500

$$\text{Median} = \exp(\delta \cdot \exp(\mu)) - 1 + \tau$$

Median = 253.347

$$K1(z) = \left[\frac{1}{\text{cnorm}(z)} \int_{-3}^z \frac{f1(z)}{\sigma} \cdot \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{z^2}{2}\right) dz \right] \cdot \text{EMP}$$

$$K2(z) := \left[\int_{-3}^z \Omega(z) \cdot \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{z^2}{2}\right) dz \cdot \left(\frac{1}{\text{cnorm}(z)}\right) \right] \cdot \text{EMA}$$

$$\lambda(z) = \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma \cdot w(z)} \cdot \exp\left[-\frac{1}{2} \cdot (z)^2\right]$$

**LOGNORMAL DENSITY FUNCTION
OF NONLABOR INCOME**

EΛ(z) := K1(z) + K2(z) (Equation D5.11) **M - ELASTICITY OF AGGREGATE
LABOR SUPPLY**

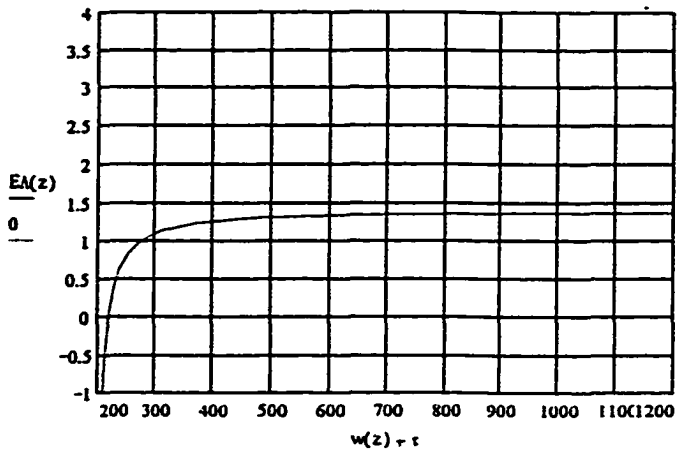
Eε(z) := -EΛ(z) - $\frac{f1(z)}{\sigma} \cdot \text{EMP} + \Omega(z) \cdot \text{EMA}$ (Equation D6.11)

M - ELASTICITY OF AGGREGATE LABOR SUPPLY ELASTICITY

LFPF W
 $cnorm(z)w(z) + \tau$ $E\lambda(z)$ $E\varepsilon(z)$

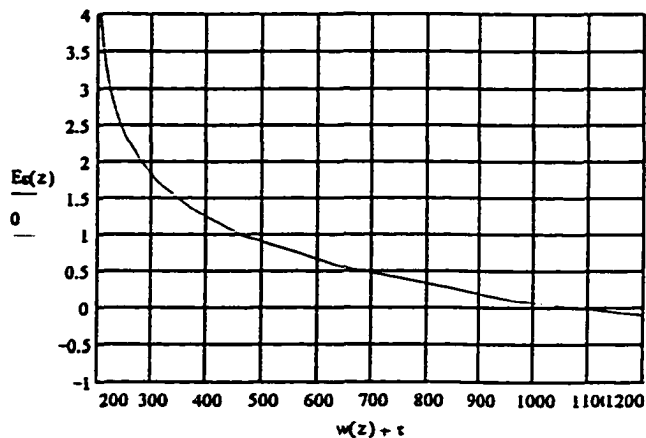
0.00135	200.373	0	-22.501
0.00187	200.412	-5.963	-14.763
0.00256	200.456	-9.703	-9.313
0.00347	200.505	-11.924	-5.444
0.00466	200.56	-13.105	-2.678
0.00621	200.622	-13.574	-0.688
0.0082	200.693	-13.556	0.751
0.01072	200.773	-13.207	1.796
0.0139	200.865	-12.637	2.557
0.01786	200.97	-11.924	3.111
0.02275	201.092	-11.121	3.512
0.02872	201.232	-10.268	3.8
0.03593	201.396	-9.392	4.002
0.04457	201.588	-8.515	4.138
0.0548	201.814	-7.65	4.224
0.06681	202.082	-6.808	4.269
0.08076	202.404	-5.997	4.282
0.0968	202.792	-5.222	4.267
0.11507	203.264	-4.487	4.23
0.13567	203.846	-3.795	4.172
0.15866	204.569	-3.148	4.095
0.18406	205.478	-2.546	4.001
0.21186	206.638	-1.991	3.889
0.24196	208.136	-1.482	3.761
0.27425	210.103	-1.02	3.616
0.30854	212.726	-0.603	3.453
0.34458	216.289	-0.232	3.272
0.38209	221.225	0.096	3.071
0.42074	228.209	0.382	2.849
0.46017	238.324	0.626	2.605
0.5	253.347	0.832	2.336
0.53983	276.281	1	2.042
0.57926	312.355	1.133	1.719
0.61791	370.977	1.233	1.366
0.65542	469.677	1.302	0.98
0.69146	642.393	1.344	0.559
0.72575	957.602	1.36	0.099
0.75804	1559.787	1.354	-0.401
0.78814	2768.969	1.328	-0.945
0.81594	5332.314	1.285	-1.536
0.84134	11096.932	1.229	-2.176
0.86433	24921.912	1.161	-2.868
0.88493	60483.124	1.085	-3.616
0.9032	$1.592 \cdot 10^5$	1.003	-4.42
0.91924	$4.57 \cdot 10^5$	0.918	-5.285
0.93319	$1.441 \cdot 10^6$	0.832	-6.212

EXHIBIT 4-2(b) NONHOMOTHETIC CASE I

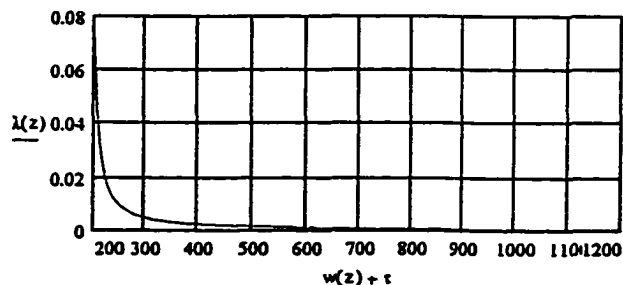


M-ELASTICITY OF LABOR SUPPLY*

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.019$
 EMP = 0.8 ERM = -0.15 EQM = 1.2 ERA = 0.184



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

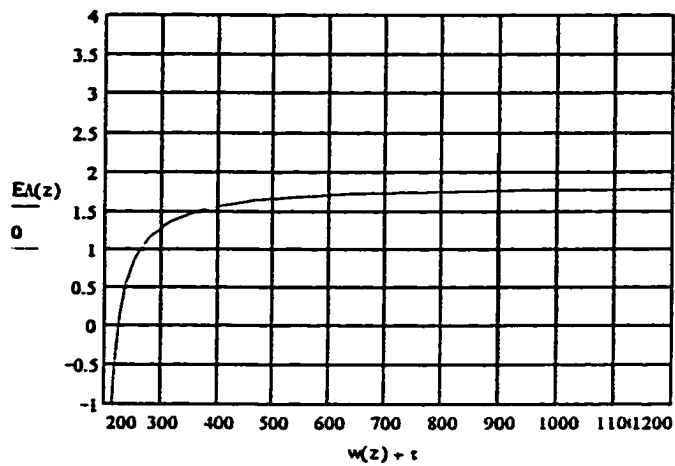


DENSITY OF RESERVATION WAGES

znorm1A.mcd

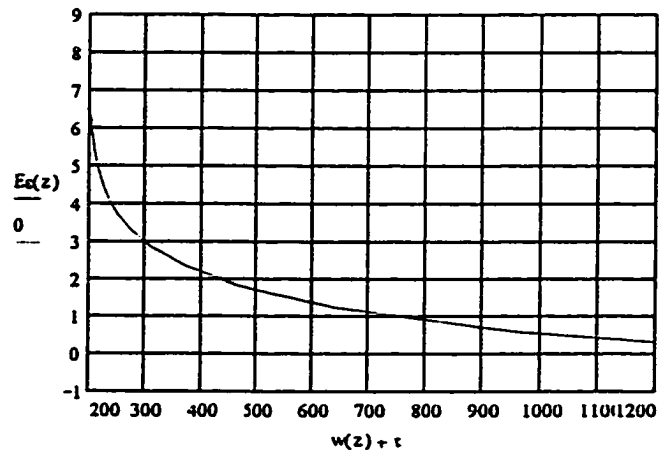
LFPR	W	$E\lambda(z)$	$E\epsilon(z)$
$cnorm(z)w(z) + \tau$			
0.00135	200.373	0	-33.703
0.00187	200.412	-8.94	-22.163
0.00256	200.456	-14.562	-14.031
0.00347	200.505	-17.918	-8.258
0.00466	200.56	-19.721	-4.129
0.00621	200.622	-20.458	-1.156
0.0082	200.693	-20.466	0.995
0.01072	200.773	-19.978	2.56
0.0139	200.865	-19.159	3.701
0.01786	200.97	-18.122	4.534
0.02275	201.092	-16.949	5.139
0.02872	201.232	-15.699	5.576
0.03593	201.396	-14.413	5.885
0.04457	201.588	-13.121	6.097
0.0548	201.814	-11.845	6.235
0.06681	202.082	-10.601	6.312
0.08076	202.404	-9.401	6.342
0.0968	202.792	-8.252	6.332
0.11507	203.264	-7.161	6.288
0.13567	203.846	-6.131	6.214
0.15866	204.569	-5.166	6.113
0.18406	205.478	-4.267	5.987
0.21186	206.638	-3.435	5.837
0.24196	208.136	-2.67	5.663
0.27425	210.103	-1.973	5.465
0.30854	212.726	-1.342	5.242
0.34458	216.289	-0.776	4.992
0.38209	221.225	-0.274	4.716
0.42074	228.209	0.166	4.409
0.46017	238.324	0.546	4.071
0.5	253.347	0.869	3.699
0.53983	276.281	1.137	3.29
0.57926	312.355	1.354	2.842
0.61791	370.977	1.522	2.351
0.65542	469.677	1.645	1.813
0.69146	642.393	1.727	1.225
0.72575	957.602	1.771	0.584
0.75804	1559.787	1.782	-0.115
0.78814	2768.969	1.763	-0.877
0.81594	5332.314	1.719	-1.704
0.84134	11096.932	1.653	-2.601
0.86433	24921.912	1.571	-3.572
0.88493	60483.124	1.475	-4.621
0.9032	159206.05	1.369	-5.753
0.91924	457019.452	1.257	-6.969
0.93319	$1.441 \cdot 10^6$	1.143	-8.274

EXHIBIT 4-3 (b) NONHOMOTHETIC CASE II

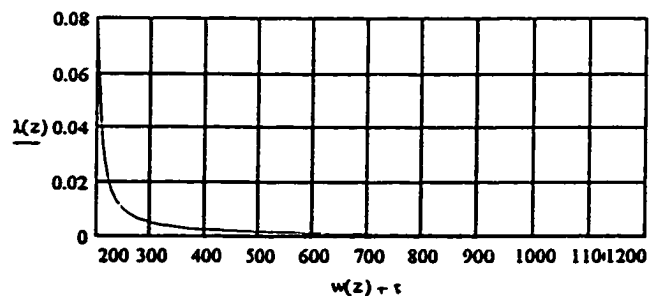


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.019$
 EMP = 0.8 ERM = -0.15 EQM = 1.4 ERA = 0.265



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

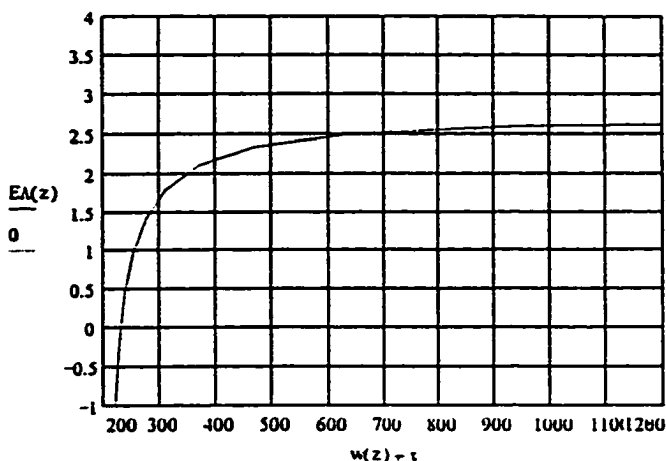


DENSITY OF RESERVATION WAGES

znorm2A.mcd

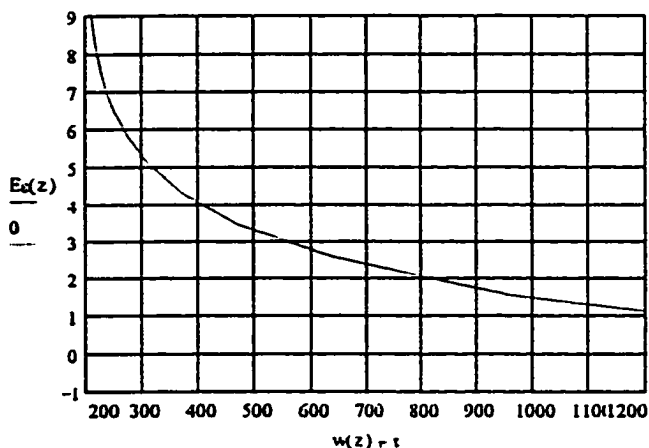
LPR	W	$\epsilon(z)$	$\epsilon(z)$
$\text{cnorm}(z)w(z) + \tau$		$EA(z)$	$E\epsilon(z)$
0.00135	200.373	0	-56.107
0.00187	200.412	-14.894	-36.961
0.00256	200.456	-24.282	-23.467
0.00347	200.505	-29.906	-13.885
0.00466	200.56	-32.951	-7.03
0.00621	200.622	-34.225	-2.093
0.0082	200.693	-34.285	1.484
0.01072	200.773	-33.521	4.088
0.0139	200.865	-32.201	5.989
0.01786	200.97	-30.517	7.379
0.02275	201.092	-28.605	8.394
0.02872	201.232	-26.56	9.128
0.03593	201.396	-24.453	9.652
0.04457	201.588	-22.333	10.016
0.0548	201.814	-20.236	10.256
0.06681	202.082	-18.188	10.398
0.08076	202.404	-16.209	10.461
0.0968	202.792	-14.313	10.46
0.11507	203.264	-12.508	10.403
0.13567	203.846	-10.803	10.298
0.15866	204.569	-9.202	10.15
0.18406	205.478	-7.708	9.961
0.21186	206.638	-6.323	9.733
0.24196	208.136	-5.046	9.468
0.27425	210.103	-3.879	9.163
0.30854	212.726	-2.819	8.819
0.34458	216.289	-1.866	8.434
0.38209	221.225	-1.016	8.005
0.42074	228.209	-0.266	7.53
0.46017	238.324	0.385	7.004
0.5	253.347	0.943	6.425
0.53983	276.281	1.412	5.788
0.57926	312.355	1.796	5.088
0.61791	370.977	2.101	4.32
0.65542	469.677	2.331	3.479
0.69146	642.393	2.494	2.559
0.72575	957.602	2.594	1.553
0.75804	1559.787	2.639	0.456
0.78814	2768.969	2.634	-0.739
0.81594	5332.314	2.587	-2.039
0.84134	11096.932	2.503	-3.45
0.86433	24921.912	2.39	-4.98
0.88493	60483.124	2.254	-6.633
0.9032	$1.592 \cdot 10^5$	2.101	-8.417
0.91924	$4.57 \cdot 10^5$	1.937	-10.337
0.93319	$1.441 \cdot 10^6$	1.766	-12.398

EXHIBIT 4 - 4 (b) NONHOMOTHETIC CASE III

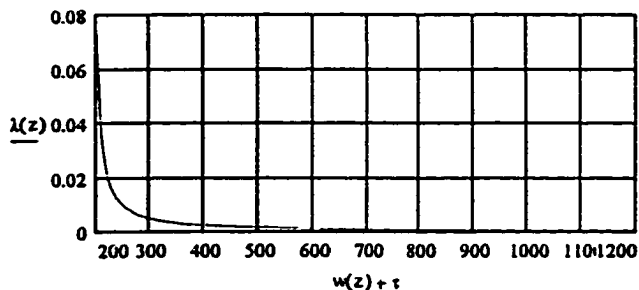


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\rho = 0.424$ $\delta = 0.019$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 EMP = 0.8 ERM = -0.15 EQM = 1.8 ERA = 0.427



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

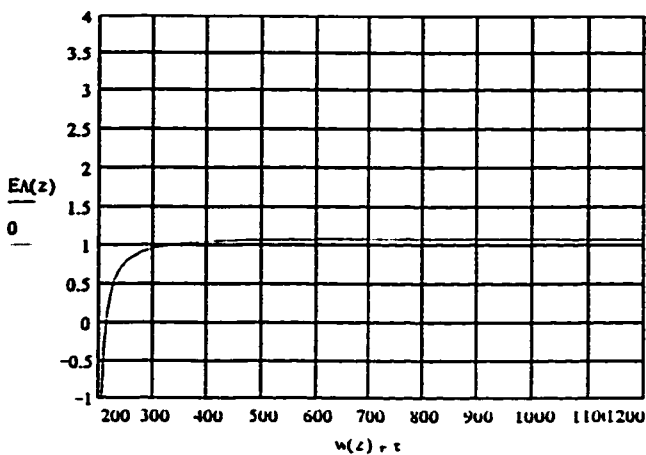


DENSITY OF RESERVATION WAGES

znorm3A.mcd

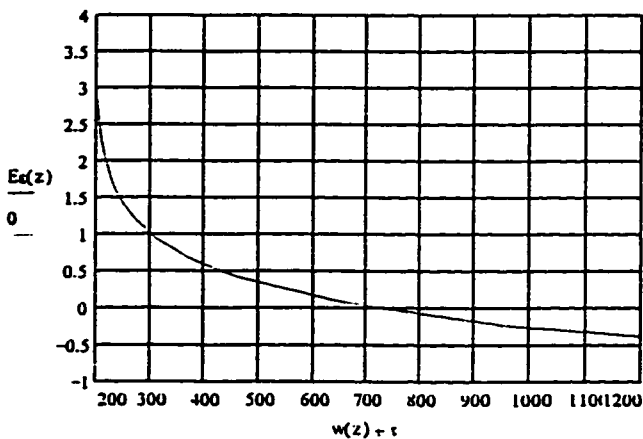
LFPR	W	$EA(z)$	$Ez(z)$
0.00135	200.373	0	14.65897
0.00187	200.412	-3.87888	-9.58413
0.00256	200.456	-6.30069	-6.01011
0.00347	200.505	-7.728	-3.47434
0.00466	200.56	-8.4748	-1.66283
0.00621	200.622	-8.75585	-0.36083
0.0082	200.693	-8.71915	0.57967
0.01072	200.773	-8.46746	1.26143
0.0139	200.865	-8.07264	1.7563
0.01786	200.97	-7.58528	2.11489
0.02275	201.092	-7.04123	2.37303
0.02872	201.232	-6.46608	2.55632
0.03593	201.396	-5.87816	2.68309
0.04457	201.588	-5.29068	2.76654
0.0548	201.814	-4.71317	2.81621
0.06681	202.082	-4.15254	2.83901
0.08076	202.404	-3.61378	2.83993
0.0968	202.792	-3.10046	2.82253
0.11507	203.264	-2.61512	2.78936
0.13567	203.846	-2.1595	2.74216
0.15866	204.569	-1.73475	2.68208
0.18406	205.478	-1.34155	2.6098
0.21186	206.638	-0.98023	2.52564
0.24196	208.136	-0.6508	2.42962
0.27425	210.103	-0.35303	2.32151
0.30854	212.726	-0.08648	2.20086
0.34458	216.289	0.14948	2.06706
0.38209	221.225	0.35564	1.91929
0.42074	228.209	0.53297	1.75661
0.46017	238.324	0.68256	1.57792
0.5	253.347	0.80563	1.38199
0.53983	276.281	0.90357	1.16744
0.57926	312.355	0.97784	0.93281
0.61791	370.977	1.03006	0.67647
0.65542	469.677	1.06193	0.39674
0.69146	642.393	1.07526	0.09179
0.72575	957.602	1.07192	-0.24024
0.75804	1559.787	1.05385	-0.6013
0.78814	2768.969	1.02305	-0.99337
0.81594	5332.314	0.9815	-1.41844
0.84134	11096.932	0.93119	-1.87852
0.86433	24921.912	0.87408	-2.37555
0.88493	60483.124	0.81205	-2.9114
0.9032	159206.05	0.74688	-3.48787
0.91924	457019.452	0.68023	-4.1066
0.93319	1.441·10 ⁶	0.61358	-4.7691

EXHIBIT 4-5 (b) NONHOMOTHETIC CASE IV

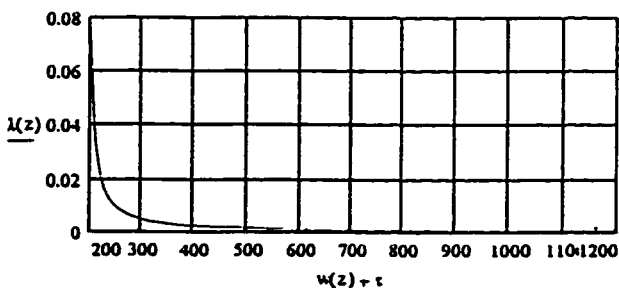


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.34721$ Median = 253.34675 $\alpha = 300$ $L = 0.44958$
 $\sigma^2 = 0.71315$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.01902$
 EMP = 0.8 ERM = -0.25 EQM = 1.1 ERA = 0.21134



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

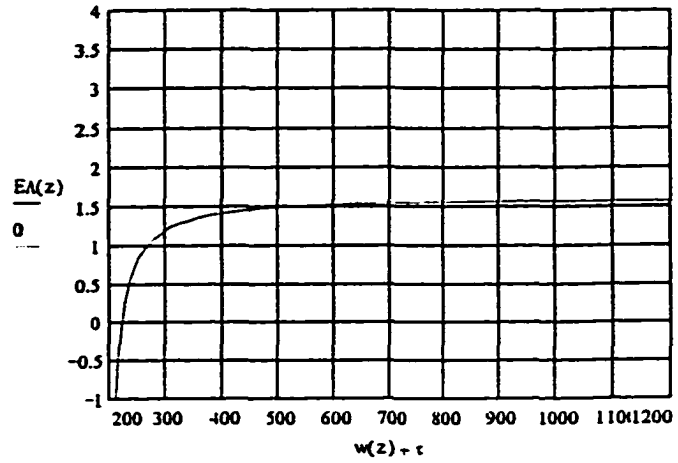


DENSITY OF RESERVATION WAGES

znorm4A.mcd

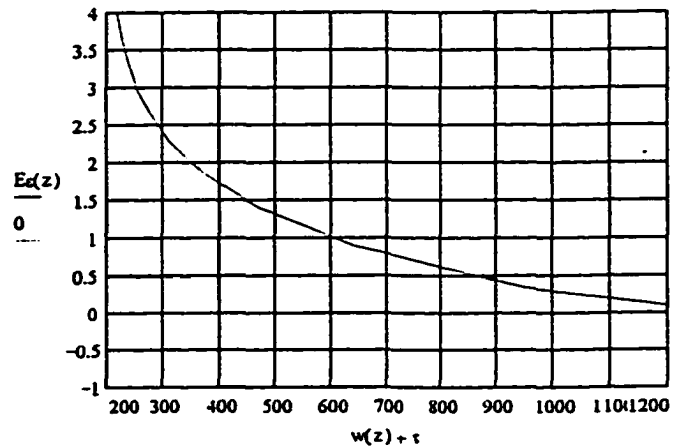
LPR	W	EA(z)	Ez(z)
cnorm(z)w(z) + τ			
0.00135	200.373	0	-28.102
0.00187	200.412	-7.451	-18.463
0.00256	200.456	-12.132	-11.672
0.00347	200.505	-14.921	-6.851
0.00466	200.56	-16.413	-3.403
0.00621	200.622	-17.016	-0.922
0.0082	200.693	-17.011	0.873
0.01072	200.773	-16.593	2.178
0.0139	200.865	-15.898	3.129
0.01786	200.97	-15.023	3.822
0.02275	201.092	-14.035	4.326
0.02872	201.232	-12.983	4.688
0.03593	201.396	-11.902	4.943
0.04457	201.588	-10.818	5.118
0.0548	201.814	-9.747	5.229
0.06681	202.082	-8.705	5.291
0.08076	202.404	-7.699	5.312
0.0968	202.792	-6.737	5.299
0.11507	203.264	-5.824	5.259
0.13567	203.846	-4.963	5.193
0.15866	204.569	-4.157	5.104
0.18406	205.478	-3.406	4.994
0.21186	206.638	-2.713	4.863
0.24196	208.136	-2.076	4.712
0.27425	210.103	-1.497	4.54
0.30854	212.726	-0.973	4.347
0.34458	216.289	-0.504	4.132
0.38209	221.225	-0.089	3.893
0.42074	228.209	0.274	3.629
0.46017	238.324	0.586	3.338
0.5	253.347	0.85	3.018
0.53983	276.281	1.069	2.666
0.57926	312.355	1.243	2.281
0.61791	370.977	1.377	1.858
0.65542	469.677	1.474	1.396
0.69146	642.393	1.535	0.892
0.72575	957.602	1.566	0.341
0.75804	1559.787	1.568	-0.258
0.78814	2768.969	1.545	-0.911
0.81594	5332.314	1.502	-1.62
0.84134	11096.932	1.441	-2.388
0.86433	24921.912	1.366	-3.22
0.88493	60483.124	1.28	-4.118
0.9032	$1.592 \cdot 10^5$	1.186	-5.087
0.91924	$4.57 \cdot 10^5$	1.088	-6.127
0.93319	$1.441 \cdot 10^6$	0.987	-7.243

EXHIBIT 4 - 6 (b) NONHOMOTHETIC CASE V

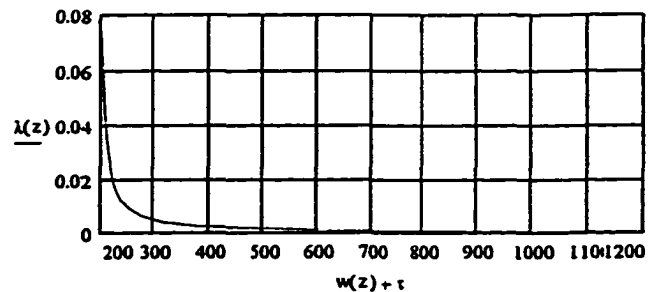


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.019$
 EMP = 0.8 ERM = -0.05 EQM = 1.1 ERA = 0.075



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

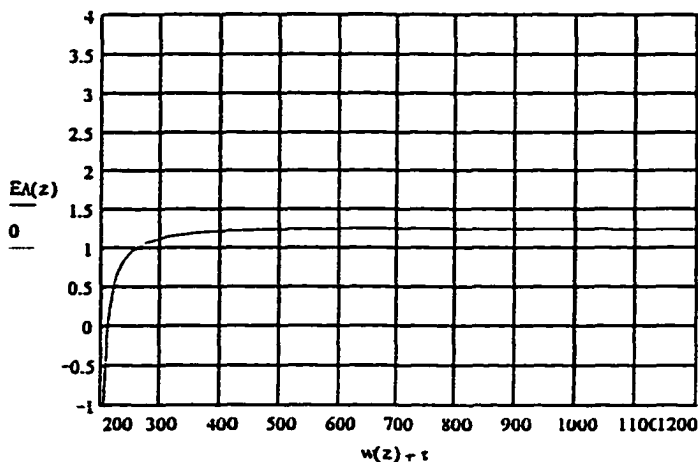


DENSITY OF RESERVATION WAGES

znorm5A.mcd

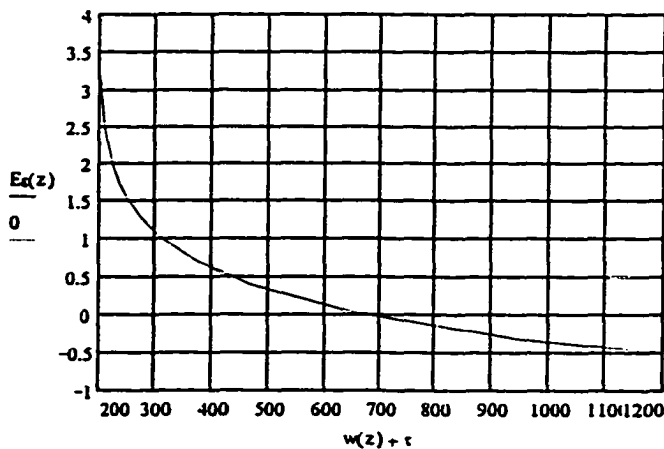
LFP	W	$\epsilon_{norm}(z)w(z) + \tau$	$E\Lambda(z)$	$Ez(z)$
0.00135	200.373	0	-16.367	
0.00187	200.412	-4.33	-10.694	
0.00256	200.456	-7.03	-6.699	
0.00347	200.505	-8.62	-3.864	
0.00466	200.56	-9.449	-1.84	
0.00621	200.622	-9.758	-0.385	
0.0082	200.693	-9.712	0.666	
0.01072	200.773	-9.426	1.427	
0.0139	200.865	-8.98	1.979	
0.01786	200.97	-8.432	2.379	
0.02275	201.092	-7.82	2.667	
0.02872	201.232	-7.174	2.871	
0.03593	201.396	-6.514	3.011	
0.04457	201.588	-5.855	3.103	
0.0548	201.814	-5.207	3.157	
0.06681	202.082	-4.579	3.181	
0.08076	202.404	-3.975	3.181	
0.0968	202.792	-3.4	3.16	
0.11507	203.264	-2.857	3.121	
0.13567	203.846	-2.347	3.067	
0.15866	204.569	-1.872	2.997	
0.18406	205.478	-1.433	2.915	
0.21186	206.638	-1.03	2.818	
0.24196	208.136	-0.662	2.708	
0.27425	210.103	-0.331	2.585	
0.30854	212.726	-0.034	2.447	
0.34458	216.289	0.228	2.295	
0.38209	221.225	0.457	2.126	
0.42074	228.209	0.653	1.941	
0.46017	238.324	0.818	1.737	
0.5	253.347	0.953	1.514	
0.53983	276.281	1.06	1.27	
0.57926	312.355	1.141	1.003	
0.61791	370.977	1.196	0.712	
0.65542	469.677	1.229	0.394	
0.69146	642.393	1.241	0.047	
0.72575	957.602	1.235	-0.33	
0.75804	1559.787	1.212	-0.741	
0.78814	2768.969	1.174	-1.186	
0.81594	5332.314	1.125	-1.669	
0.84134	11096.932	1.066	-2.191	
0.86433	24921.912	1	-2.756	
0.88493	60483.124	0.928	-3.364	
0.9032	159206.05	0.853	-4.018	
0.91924	457019.452	0.776	-4.72	
0.93319	$1.441 \cdot 10^6$	0.7	-5.472	

EXHIBIT 4 - 7 (b) NONHOMOTHETIC CASE VI

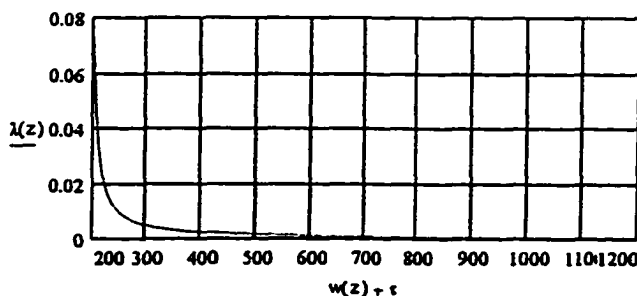


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.019$
 EMP = 0.95 ERM = -0.15 EQM = 1.1 ERA = 0.143



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

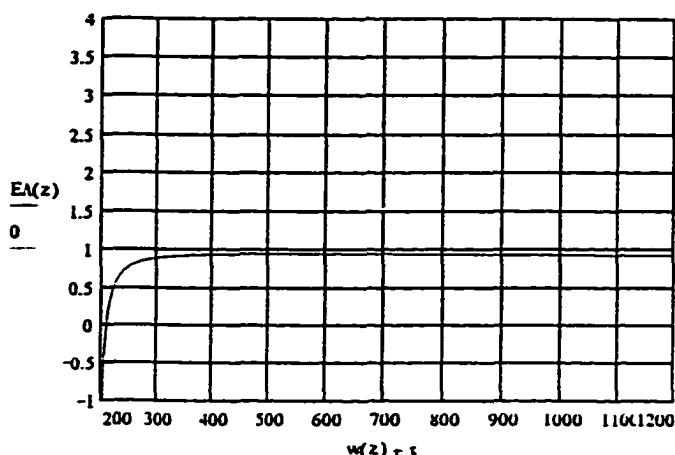


DENSITY OF RESERVATION WAGES

znorm6A.mcd

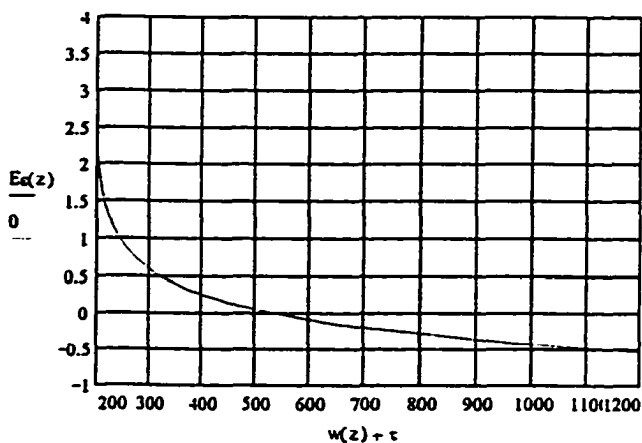
LFPF	W	$\ln \text{norm}(z)w(z) + \tau$	$E\lambda(z)$	$E\varepsilon(z)$
0.00135	200.373	0	-10.652	
0.00187	200.412	-2.814	-6.937	
0.00256	200.456	-4.562	-4.322	
0.00347	200.505	-5.584	-2.468	
0.00466	200.56	-6.109	-1.144	
0.00621	200.622	-6.294	-0.193	
0.0082	200.693	-6.247	0.492	
0.01072	200.773	-6.045	0.988	
0.0139	200.865	-5.74	1.347	
0.01786	200.97	-5.368	1.606	
0.02275	201.092	-4.957	1.791	
0.02872	201.232	-4.523	1.921	
0.03593	201.396	-4.082	2.009	
0.04457	201.588	-3.643	2.066	
0.0548	201.814	-3.213	2.097	
0.06681	202.082	-2.796	2.108	
0.08076	202.404	-2.396	2.103	
0.0968	202.792	-2.016	2.084	
0.11507	203.264	-1.659	2.053	
0.13567	203.846	-1.324	2.012	
0.15866	204.569	-1.013	1.96	
0.18406	205.478	-0.726	1.899	
0.21186	206.638	-0.464	1.829	
0.24196	208.136	-0.226	1.749	
0.27425	210.103	-0.012	1.66	
0.30854	212.726	0.178	1.561	
0.34458	216.289	0.344	1.451	
0.38209	221.225	0.488	1.331	
0.42074	228.209	0.61	1.198	
0.46017	238.324	0.711	1.053	
0.5	253.347	0.792	0.894	
0.53983	276.281	0.854	0.721	
0.57926	312.355	0.899	0.531	
0.61791	370.977	0.927	0.324	
0.65542	469.677	0.939	0.099	
0.69146	642.393	0.938	-0.147	
0.72575	957.602	0.925	-0.414	
0.75804	1559.787	0.901	-0.704	
0.78814	2768.969	0.867	-1.018	
0.81594	5332.314	0.826	-1.358	
0.84134	11096.932	0.779	-1.727	
0.86433	24921.912	0.728	-2.124	
0.88493	60483.124	0.673	-2.552	
0.9032	159206.05	0.616	-3.011	
0.91924	457019.452	0.559	-3.504	
0.93319	$1.441 \cdot 10^6$	0.502	-4.032	

EXHIBIT 4-8 (b) NONHOMOTHETIC CASE VII

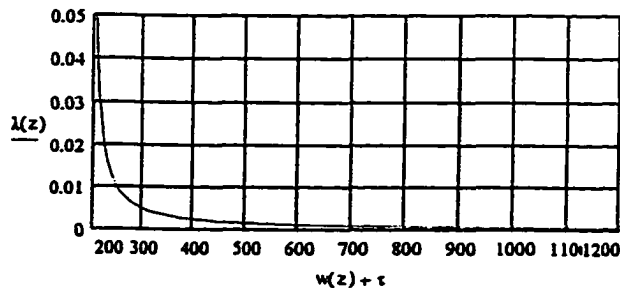


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.471$ $\delta = 0.019$ $L_D = .74$
 EMP = 0.8 ERM = -0.15 EQM = 1.1 ERA = 0.143



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



"DENSITY OF RESERVATION WAGES

znorm7AB.mcd

EXHIBIT 4-9 (a)

znorm7A.mcd

ELASTICITY OF ELASTICITY COMPUTATIONS
LOGNORMAL LABOR SUPPLY FUNCTION

LOGNORMAL DISTRIBUTION CALIBRATION

$$z := -3, -2.9, -1.5 \quad \alpha = 300 \quad \beta = 510 \quad \tau = 200 \quad \gamma = \frac{\beta}{\alpha + \tau} \quad \sigma = \sqrt{\ln(\gamma^2 + 1)}$$

$$v := 1 \quad \beta_2 = \alpha^2 \cdot \gamma^2 \quad \beta_2 = 9.364 \cdot 10^4 \quad \gamma = 1.02 \quad \sigma = 0.844 \quad \sigma^2 = 0.713$$

$$\mu = \ln(\alpha) - .5\sigma^2 \quad L = 2 \cdot \text{cnorm}\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \quad \mu = 5.347$$

$$L = 0.45 \quad \rho = .424$$

Calibrate Mean Reservation Wage

For Nonhomothetic Function

$$\Omega(z) = z \quad \Omega(z) = \frac{(z^2 - 1) \cdot (1 - \rho)^2}{\rho^2 \cdot \sigma^2}$$

$$\delta = \frac{\ln(\ln(\alpha + 1))}{\ln(\alpha)} \quad \delta = 0.305$$

Nonhomothetic Reservation Wage Function

$$\xi = \delta \cdot v^\delta \quad \psi = \exp(v^\delta) - 1 \quad g(w) = \ln(w + 1)^{\frac{1}{\delta}}$$

$$w(z) = \exp(\exp(\delta \cdot (\mu + z \cdot \sigma))) - 1$$

$$\text{Mean} = \exp\left[\exp\left[\delta \cdot (\mu + .5\sigma^2)\right]\right] - 1 + \tau$$

$$\text{Mean} = 500$$

$$\text{Median} = \exp(\exp(\delta \cdot \mu)) - 1 + \tau$$

$$\text{Median} = 366.057$$

MONETARY CALIBRATION

$$M2 = 3.439 \quad F = 5.035 \quad A = \frac{F}{M2 + F}$$

$$\text{EMP} = .8 \quad A = 0.594$$

$$\text{ERM} = -.15 \quad \text{EQM}(z) = \delta \cdot \ln(w(z) + 1)$$

$$\text{ERA}(z) = \frac{1 - A}{A} (-\text{ERM} + A \cdot (\text{EQM}(z) - 1)) \quad \text{Equation (D5.8)}$$

$$\text{EMA}(z) = \frac{\text{ERA}(z)}{\text{ERM}}$$

$$K1(z) = \left[\frac{1}{\text{cnorm}(z)} \int_{-3}^z \frac{\Omega(z)}{\sigma} \frac{1}{\sqrt{2 \cdot \pi}} \exp\left(-\frac{z^2}{2}\right) dz \right] \cdot \text{EMP}$$

$$\lambda(z) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma \cdot w(z)}} \exp\left[-\frac{1}{2} \cdot (z)^2\right]$$

LOGNORMAL DENSITY FUNCTION
OF NONLABOR INCOME

$$K2(z) = \left[\int_{-3}^z \int_{-3}^z \Omega(z) \frac{1}{\sqrt{2 \cdot \pi}} \exp\left(-\frac{z^2}{2}\right) dz \cdot \left(\frac{1}{\text{cnorm}(z)}\right) \right] \cdot \text{EMA}(z)$$

$$\text{EA}(z) := K1(z) + K2(z) \quad \text{(Equation D5.11)} \quad \text{M-ELASTICITY OF AGGREGATE LABOR SUPPLY}$$

$$\text{Ee}(z) := -\text{EA}(z) - \frac{\Omega(z)}{\sigma} \cdot \text{EMP} + \Omega(z) \cdot \text{EMA}(z) \quad \text{(Equation D6.11)}$$

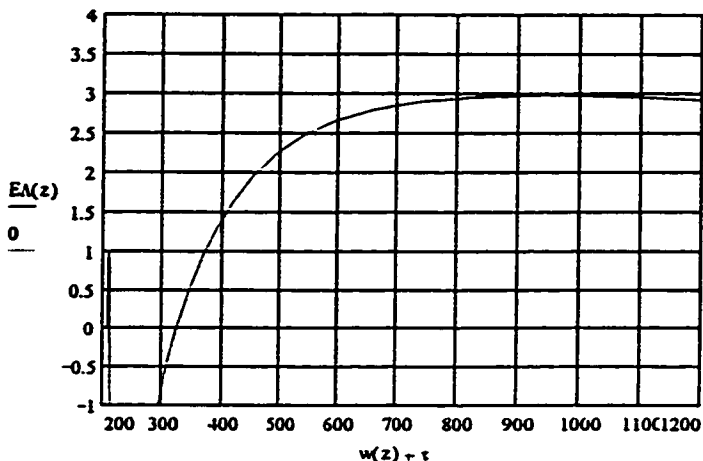
M-ELASTICITY OF AGGREGATE LABOR SUPPLY ELASTICITY

znorm7A.mcd

LFPR W
 $cnorm(z)w(z) + \tau$ $EA(z)$ $E\epsilon(z)$

0.00135	209.605	0	4.326
0.00187	210.28	0.886	2.259
0.00256	211.017	1.008	1.086
0.00347	211.823	0.692	0.478
0.00466	212.707	0.145	0.227
0.00621	213.677	-0.505	0.2
0.0082	214.744	-1.177	0.313
0.01072	215.919	-1.821	0.512
0.0139	217.216	-2.408	0.762
0.01786	218.651	-2.919	1.044
0.02275	220.241	-3.345	1.341
0.02872	222.006	-3.684	1.646
0.03593	223.97	-3.934	1.954
0.04457	226.16	-4.098	2.259
0.0548	228.606	-4.18	2.56
0.06681	231.346	-4.183	2.855
0.08076	234.421	-4.113	3.142
0.0968	237.88	-3.976	3.419
0.11507	241.782	-3.778	3.684
0.13567	246.192	-3.525	3.936
0.15866	251.191	-3.225	4.172
0.18406	256.871	-2.884	4.39
0.21186	263.343	-2.508	4.587
0.24196	270.738	-2.105	4.758
0.27425	279.209	-1.682	4.9
0.30854	288.943	-1.246	5.008
0.34458	300.16	-0.803	5.076
0.38209	313.126	-0.361	5.099
0.42074	328.16	0.075	5.067
0.46017	345.648	0.497	4.974
0.5	366.057	0.899	4.81
0.53983	389.955	1.277	4.563
0.57926	418.036	1.626	4.222
0.61791	451.148	1.94	3.772
0.65542	490.335	2.216	3.199
0.69146	536.885	2.452	2.486
0.72575	592.393	2.645	1.613
0.75804	658.843	2.793	0.559
0.78814	738.714	2.897	-0.697
0.81594	835.113	2.958	-2.183
0.84134	951.956	2.976	-3.924
0.86433	1094.196	2.955	-5.949
0.88493	1268.128	2.897	-8.292
0.9032	1481.789	2.806	-10.984
0.91924	1745.493	2.688	-14.062
0.93319	2072.535	2.546	-17.563

EXHIBIT 4-9 (b) NONHOMOTHETIC CASE VIII



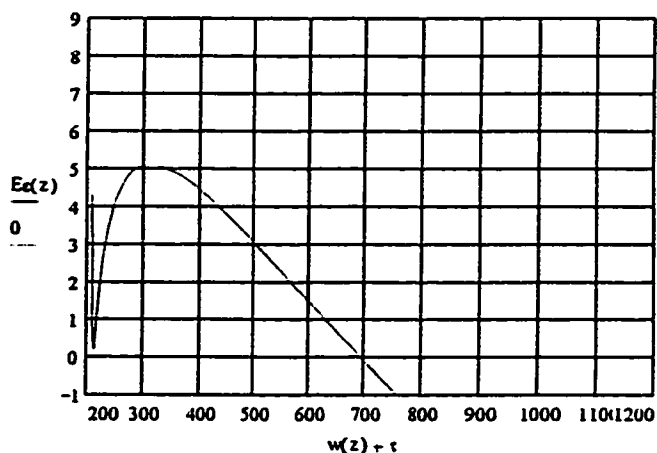
M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$

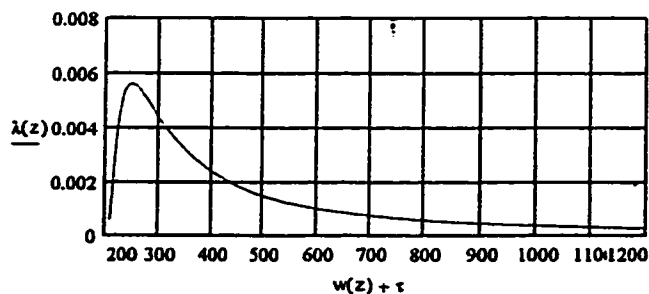
$\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$

$\rho = 0.424$ $\delta = 0.305$

EMP = 0.8 ERM = -0.15 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



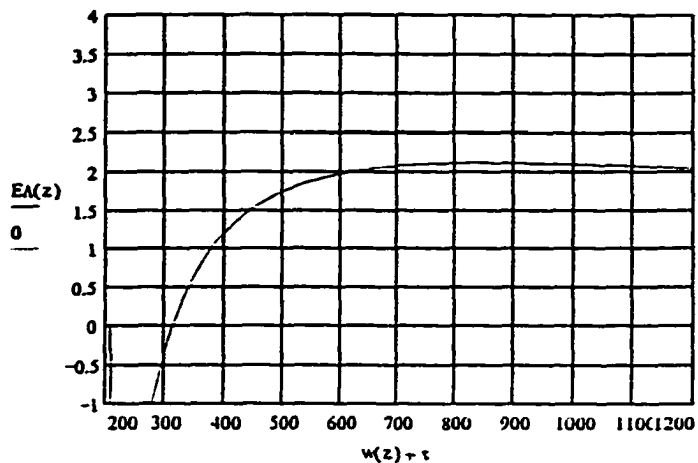
DENSITY OF RESERVATION WAGES

znorm7A.mcd

LFPR W
 $cnorm(z)w(z) + \tau$ $E\lambda(z)$ $E\epsilon(z)$

0.00135	209.605	0	-1.924
0.00187	210.28	-0.663	-1.591
0.00256	211.017	-1.332	-1.186
0.00347	211.823	-1.956	-0.765
0.00466	212.707	-2.509	-0.355
0.00621	213.677	-2.98	0.032
0.0082	214.744	-3.365	0.39
0.01072	215.919	-3.667	0.72
0.0139	217.216	-3.891	1.023
0.01786	218.651	-4.042	1.301
0.02275	220.241	-4.124	1.559
0.02872	222.006	-4.145	1.797
0.03593	223.97	-4.109	2.019
0.04457	226.16	-4.023	2.227
0.0548	228.606	-3.89	2.421
0.06681	231.346	-3.715	2.604
0.08076	234.421	-3.505	2.774
0.0968	237.88	-3.262	2.933
0.11507	241.782	-2.992	3.079
0.13567	246.192	-2.699	3.213
0.15866	251.191	-2.387	3.334
0.18406	256.871	-2.06	3.44
0.21186	263.343	-1.724	3.528
0.24196	270.738	-1.381	3.598
0.27425	279.209	-1.036	3.647
0.30854	288.943	-0.693	3.671
0.34458	300.16	-0.357	3.666
0.38209	313.126	-0.03	3.63
0.42074	328.16	0.284	3.556
0.46017	345.648	0.581	3.44
0.5	366.057	0.857	3.275
0.53983	389.955	1.111	3.055
0.57926	418.036	1.34	2.771
0.61791	451.148	1.541	2.416
0.65542	490.335	1.713	1.978
0.69146	536.885	1.855	1.448
0.72575	592.393	1.966	0.813
0.75804	658.843	2.046	0.061
0.78814	738.714	2.095	-0.824
0.81594	835.113	2.115	-1.857
0.84134	951.956	2.107	-3.055
0.86433	1094.196	2.073	-4.435
0.88493	1268.128	2.016	-6.019
0.9032	1481.789	1.939	-7.826
0.91924	1745.493	1.844	-9.878
0.93319	2072.535	1.736	-12.198

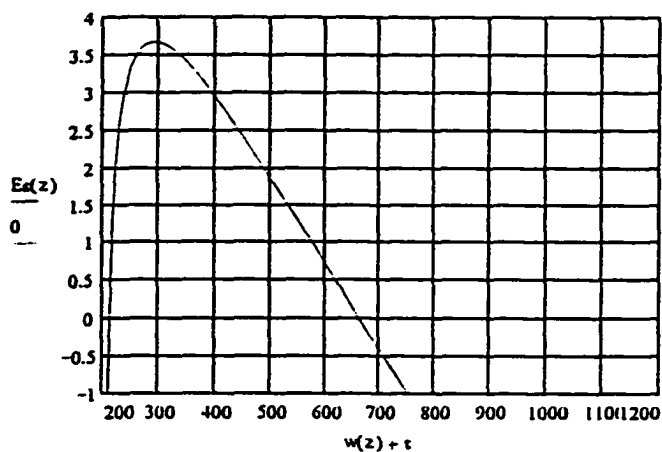
EXHIBIT 4 - 10 (b) NONHOMOTHETIC CASE IX



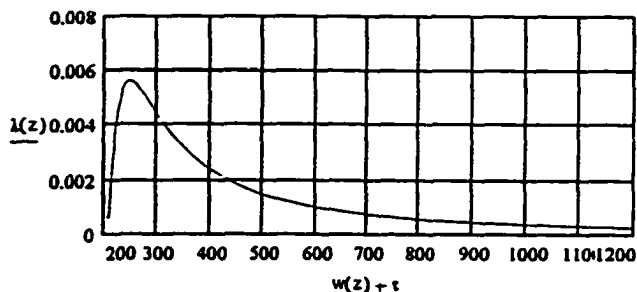
M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$

EMP = 0.8 ERM = -0.25 $E\Omega(z) = \delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



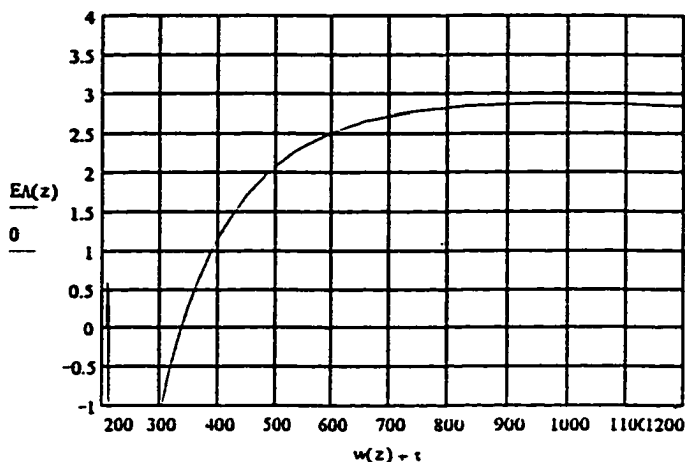
DENSITY OF RESERVATION WAGES

znorm8A.mcd

LFPR W
 $c_{norm}(z)w(z) + \tau$ $E\lambda(z)$ $E\epsilon(z)$

0.00135	209.605	0	3.26
0.00187	210.28	0.597	1.518
0.00256	211.017	0.524	0.575
0.00347	211.823	0.079	0.133
0.00466	212.707	-0.553	0.001
0.00621	213.677	-1.255	0.061
0.0082	214.744	-1.956	0.239
0.01072	215.919	-2.613	0.486
0.0139	217.216	-3.201	0.774
0.01786	218.651	-3.705	1.084
0.02275	220.241	-4.119	1.405
0.02872	222.006	-4.441	1.728
0.03593	223.97	-4.671	2.051
0.04457	226.16	-4.813	2.37
0.0548	228.606	-4.87	2.682
0.06681	231.346	-4.848	2.987
0.08076	234.421	-4.752	3.284
0.0968	237.88	-4.588	3.57
0.11507	241.782	-4.364	3.844
0.13567	246.192	-4.084	4.104
0.15866	251.191	-3.757	4.349
0.18406	256.871	-3.389	4.575
0.21186	263.343	-2.986	4.781
0.24196	270.738	-2.557	4.961
0.27425	279.209	-2.108	5.113
0.30854	288.943	-1.646	5.231
0.34458	300.16	-1.178	5.309
0.38209	313.126	-0.711	5.343
0.42074	328.16	-0.252	5.323
0.46017	345.648	0.194	5.242
0.5	366.057	0.619	5.09
0.53983	389.955	1.019	4.857
0.57926	418.036	1.388	4.53
0.61791	451.148	1.723	4.095
0.65542	490.335	2.019	3.538
0.69146	536.885	2.273	2.842
0.72575	592.393	2.484	1.987
0.75804	658.843	2.649	0.952
0.78814	738.714	2.769	-0.285
0.81594	835.113	2.844	-1.749
0.84134	951.956	2.876	-3.468
0.86433	1094.196	2.867	-5.471
0.88493	1268.128	2.821	-7.789
0.9032	1481.789	2.741	-10.457
0.91924	1745.493	2.632	-13.509
0.93319	2072.535	2.499	-16.983

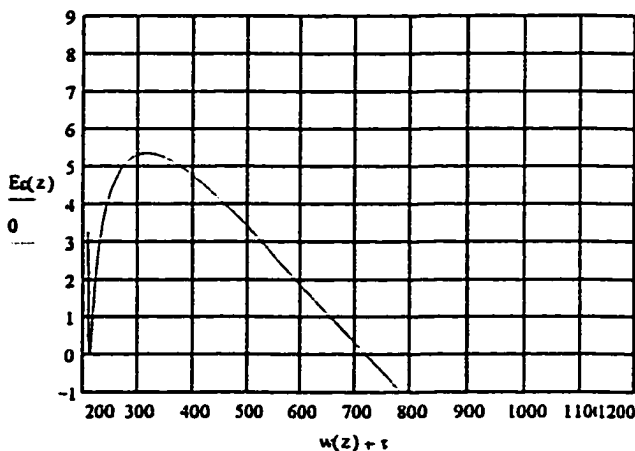
EXHIBIT 4 - 11 (b) NONHOMOTHETIC CASE X



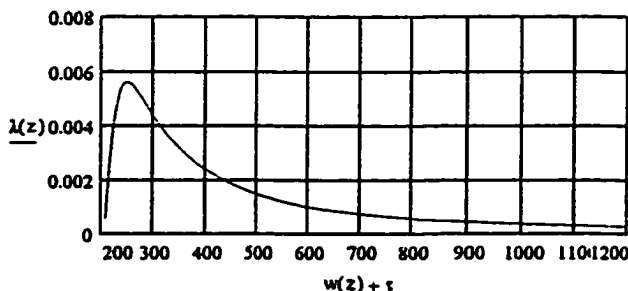
M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$

EMP = 0.5 ERM = -0.15 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



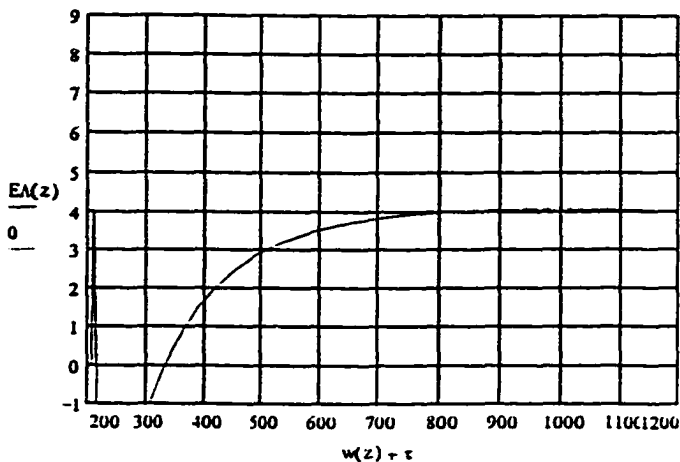
DENSITY OF RESERVATION WAGES

znorm9A.mcd

LFPF W
 $znorm(w(z) + \tau EA(z) Ee(z) Ew(z)$

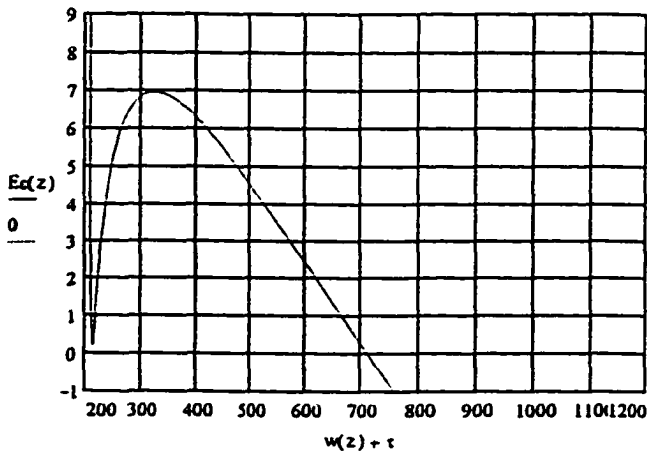
0.00135209.605	0	12.138	2.483	
0.00187210.28	2.822	7.07	-1.342	
0.00256211.017	3.933	3.926	-3.093	
0.00347211.823	4.003	2.032	-3.558	
0.00466212.707	3.462	0.955	-3.248	
0.00621213.677	2.587	0.41	-2.493	
0.0082214.744	1.557	0.216	-1.506	
0.01072215.919	0.486	0.251	-0.425	
0.0139217.216	0.553	0.437	0.662	
0.01786218.651	1.515	0.721	1.699	
0.02275220.241	2.371	1.069	2.652	
0.02872222.006	3.107	1.458	3.501	
0.03593223.97	3.715	1.872	4.235	
0.04457226.16	4.193	2.3	4.85	
0.0548228.606	4.542	2.734	5.347	
0.0668231.346	4.767	3.17	5.728	
0.08076234.421	4.873	3.602	6	
0.0968237.88	4.868	4.027	6.168	
0.11507241.782	-4.76	4.44	6.241	
0.13567246.192	4.559	4.839	6.227	
0.15866251.191	4.273	5.22	6.133	
0.18406256.871	3.913	5.578	5.97	
0.21186263.343	3.488	5.909	5.744	
0.24196270.738	-3.01	6.207	5.466	
0.27425279.209	2.489	6.467	5.141	
0.30854288.943	1.936	6.68	4.777	
0.34458300.16	1.361	6.839	4.38	
0.38205313.126	0.774	6.935	3.953	
0.42074328.16	0.187	6.957	3.499	
0.46017345.648	0.391	6.892	3.018	
0.5	366.057	0.952	6.728	2.508
0.53983389.955	1.485	6.448	1.962	
0.57926418.036	1.983	6.034	1.371	
0.61791451.148	2.438	5.468	0.721	
0.65542490.335	2.845	4.725	-0.007	
0.69146536.885	3.198	3.783	-0.838	
0.72575592.393	3.493	2.612	-1.801	
0.75804658.843	3.727	1.182	-2.932	
0.78814738.714	3.9	-0.539	-4.276	
0.81594835.113	4.011	-2.59	-5.88	
0.84134951.956	4.062	-5.01	-7.8	
0.864331094.196	4.057	-7.842	-10.096	
0.884931268.128	3.998	-11.133	-12.834	
0.90321481.789	3.891	-14.932	-16.084	
0.919241745.493	3.743	-19.293	-19.916	
0.933152072.535	3.559	-24.27	-24.404	

EXHIBIT 4-12 (b) NONHOMOTHETIC CASE XI

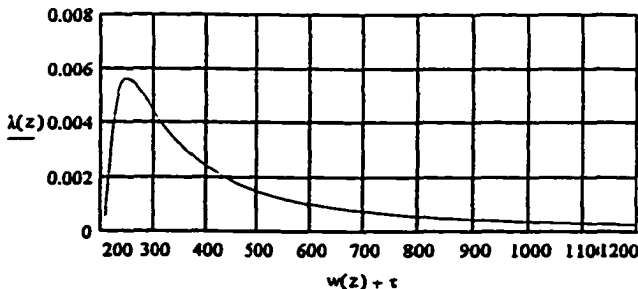


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$
 EMP = 0.8 ERM = -0.1 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



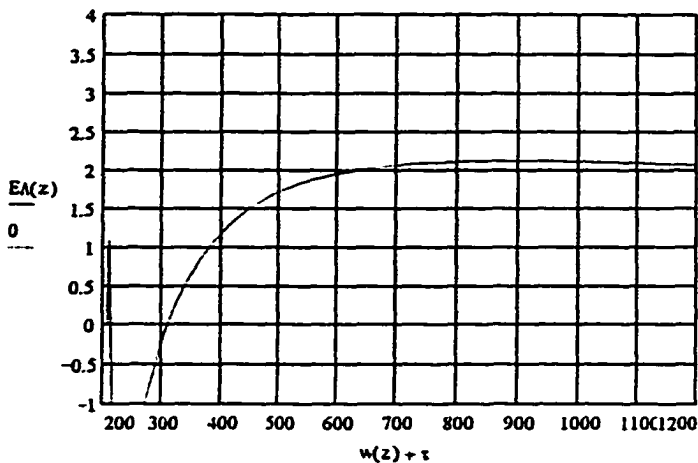
DENSITY OF RESERVATION WAGES

znorm10A.mcd

LFPR W
 $cnorm(w(z) + \tau) EA(z) Ee(z) Ew(z)$

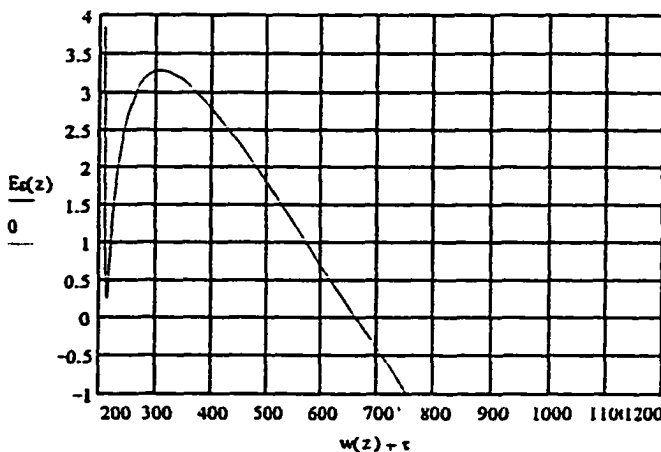
0.00135209.605	0	3.856	0.789	
0.00187210.28	0.85	2.169	-0.396	
0.00256211.017	1.098	1.173	-0.847	
0.00347211.823	0.991	0.618	-0.856	
0.00466212.707	0.688	0.346	-0.61	
0.00621213.677	0.287	0.254	0.228	
0.0082214.744	0.148	0.276	0.213	
0.01072215.919	0.577	0.371	0.667	
0.0139217.216	0.976	0.511	1.103	
0.0178218.651	1.331	0.679	1.505	
0.0227220.241	1.633	0.863	1.86	
0.0287222.006	1.879	1.056	2.164	
0.03593223.97	2.067	1.253	2.415	
0.04457226.16	2.198	1.451	2.613	
0.0548228.606	2.274	1.647	2.759	
0.0668231.346	2.298	1.84	2.856	
0.0807234.421	2.272	2.028	2.906	
0.0968237.88	2.201	2.21	2.914	
0.11507241.782	2.088	2.384	2.883	
0.13567246.192	1.938	2.549	2.816	
0.1586251.191	1.756	2.703	2.719	
0.1840256.871	1.545	2.844	2.594	
0.2118263.343	1.311	2.971	2.445	
0.2419270.738	1.057	3.081	2.276	
0.2742279.209	-0.79	3.17	2.09	
0.3085288.943	0.514	3.235	1.89	
0.34458300.16	0.232	3.273	1.677	
0.38205313.126	0.049	3.279	1.454	
0.42074328.16	0.326	3.248	1.22	
0.46017345.648	0.595	3.174	0.975	
0.5	366.057	0.851	3.051	0.718
0.53983389.955	1.091	2.871	0.444	
0.57926418.036	1.311	2.625	0.148	
0.61791451.148	1.509	2.305	-0.177	
0.65542490.335	1.681	1.9	-0.54	
0.69146536.885	1.827	1.398	-0.954	
0.72575592.393	1.943	0.787	-1.434	
0.75804658.843	2.031	0.051	-1.997	
0.78814738.714	2.089	-0.825	-2.664	
0.81594835.113	2.118	-1.858	-3.458	
0.84134951.956	2.119	-3.066	-4.406	
0.864331094.196	2.094	-4.47	-5.537	
0.884931268.128	2.044	-6.092	-6.88	
0.90321481.789	1.974	-7.953	-8.468	
0.919241745.493	1.885	-10.079	-10.334	
0.933152072.535	1.781	-12.495	-12.512	

EXHIBIT 4-13 (b) NONHOMOTHETIC CASE XII

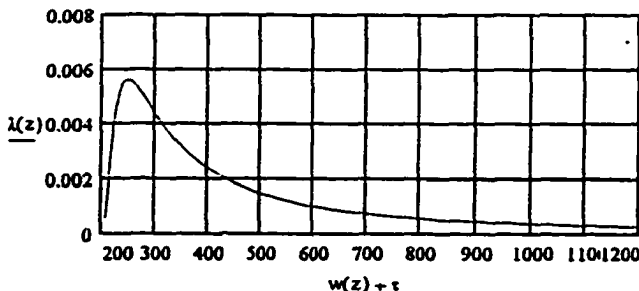


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.471$ $\delta = 0.305$ $L_\Omega = .74$
 EMP = 0.8 ERM = -0.15 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY

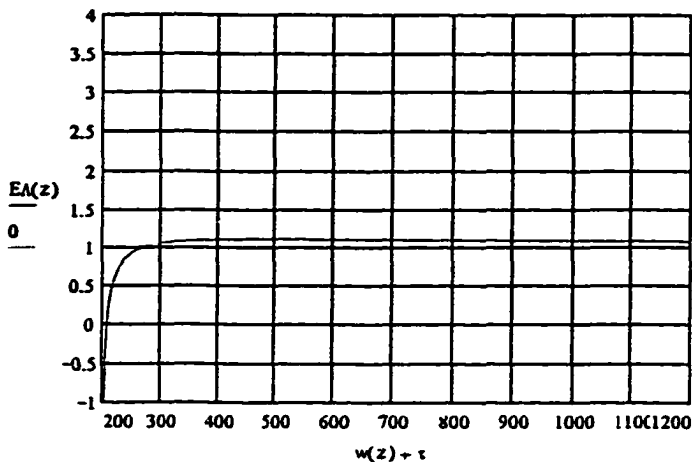


DENSITY OF RESERVATION WAGES

znorm11A.mcd

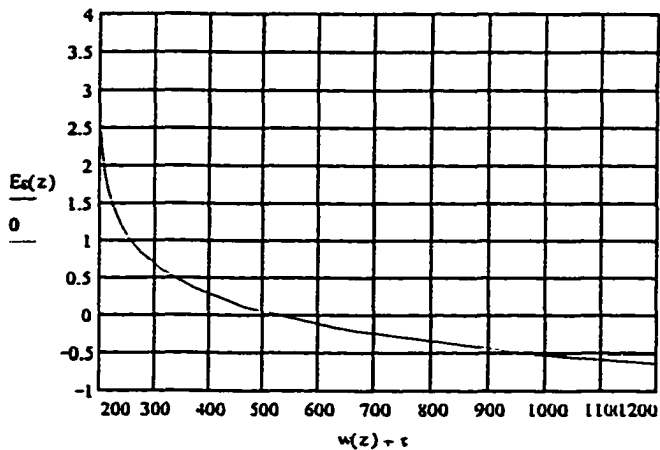
LFPR	W	$EA(z)$	$E\epsilon(z)$
$cnorm(z)w(z) + \tau$			
0.00135	200.373	0	-12.446
0.00187	200.412	-3.288	-8.104
0.00256	200.456	-5.33	-5.047
0.00347	200.505	-6.522	-2.88
0.00466	200.56	-7.134	-1.332
0.00621	200.622	-7.349	-0.221
0.0082	200.693	-7.293	0.58
0.01072	200.773	-7.056	1.16
0.0139	200.865	-6.698	1.579
0.01786	200.97	-6.262	1.881
0.02275	201.092	-5.78	2.097
0.02872	201.232	-5.273	2.249
0.03593	201.396	-4.757	2.352
0.04457	201.588	-4.243	2.417
0.0548	201.814	-3.739	2.454
0.06681	202.082	-3.251	2.466
0.08076	202.404	-2.784	2.46
0.0968	202.792	-2.34	2.438
0.11507	203.264	-1.921	2.401
0.13567	203.846	-1.53	2.352
0.15866	204.569	-1.166	2.291
0.18406	205.478	-0.831	2.219
0.21186	206.638	-0.524	2.136
0.24196	208.136	-0.247	2.043
0.27425	210.103	0.003	1.938
0.30854	212.726	0.224	1.821
0.34458	216.289	0.419	1.692
0.38209	221.225	0.586	1.551
0.42074	228.209	0.729	1.395
0.46017	238.324	0.846	1.224
0.5	253.347	0.94	1.037
0.53983	276.281	1.012	0.833
0.57926	312.355	1.063	0.61
0.61791	370.977	1.095	0.367
0.65542	469.677	1.109	0.102
0.69146	642.393	1.107	-0.186
0.72575	957.602	1.091	-0.5
0.75804	1559.787	1.062	-0.841
0.78814	2768.969	1.022	-1.21
0.81594	5332.314	0.973	-1.61
0.84134	11096.932	0.918	-2.043
0.86433	24921.912	0.856	-2.509
0.88493	60483.124	0.792	-3.012
0.9032	$1.592 \cdot 10^5$	0.725	-3.552
0.91924	$4.57 \cdot 10^5$	0.657	-4.131
0.93319	$1.441 \cdot 10^6$	0.591	-4.75

EXHIBIT 4-14 (b) NONHOMOTHETIC CASE XIII

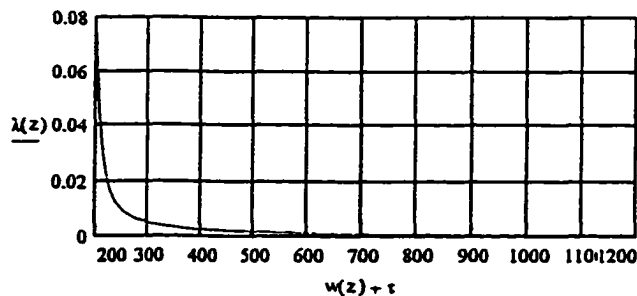


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 253.347 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.019$
 EMP = 0.95 ERM = -0.25 EQM = 1.05 ERA = 0.191



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



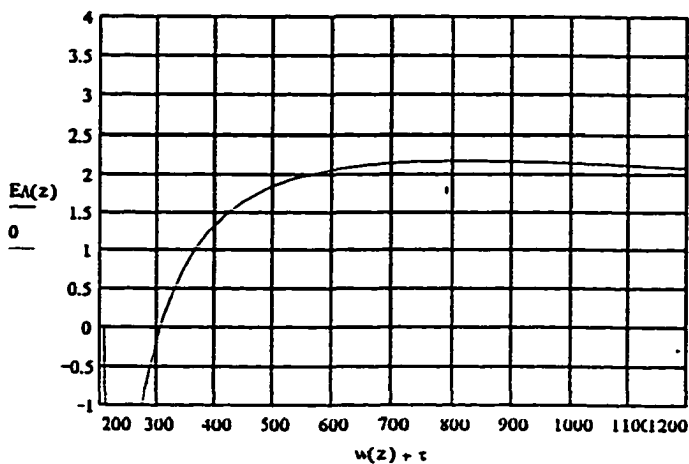
DENSITY OF RESERVATION WAGES

znorm12A.mcd

LFPR W
 $cnorm(z)w(z) + \tau$ $E\lambda(z)$ $Ez(z)$

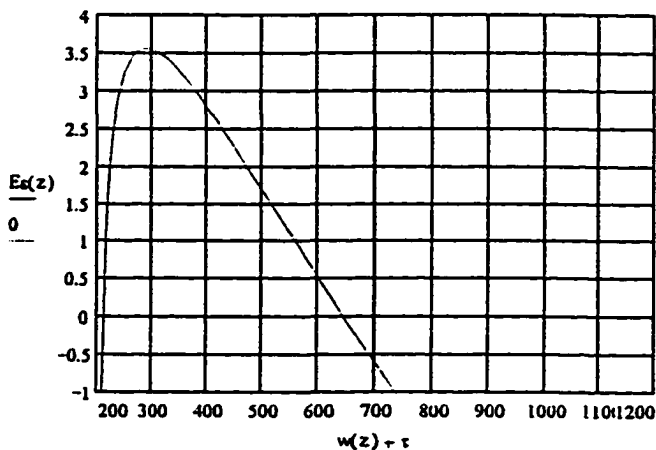
0.00135	209.605	0	-1.391
0.00187	210.28	-0.518	-1.22
0.00256	211.017	-1.09	-0.931
0.00347	211.823	-1.65	-0.592
0.00466	212.707	-2.16	-0.242
0.00621	213.677	-2.605	0.101
0.0082	214.744	-2.976	0.427
0.01072	215.919	-3.272	0.733
0.0139	217.216	-3.495	1.017
0.01786	218.651	-3.648	1.281
0.02275	220.241	-3.737	1.527
0.02872	222.006	-3.767	1.756
0.03593	223.97	-3.741	1.971
0.04457	226.16	-3.665	2.172
0.0548	228.606	-3.544	2.36
0.06681	231.346	-3.383	2.537
0.08076	234.421	-3.185	2.703
0.0968	237.88	-2.956	2.857
0.11507	241.782	-2.699	3
0.13567	246.192	-2.419	3.129
0.15866	251.191	-2.121	3.246
0.18406	256.871	-1.808	3.347
0.21186	263.343	-1.484	3.431
0.24196	270.738	-1.155	3.497
0.27425	279.209	-0.823	3.54
0.30854	288.943	-0.493	3.559
0.34458	300.16	-0.169	3.55
0.38209	313.126	0.146	3.508
0.42074	328.16	0.447	3.428
0.46017	345.648	0.732	3.306
0.5	366.057	0.997	3.135
0.53983	389.955	1.24	2.908
0.57926	418.036	1.458	2.617
0.61791	451.148	1.65	2.254
0.65542	490.335	1.812	1.809
0.69146	536.885	1.944	1.27
0.72575	592.393	2.047	0.626
0.75804	658.843	2.118	-0.136
0.78814	738.714	2.16	-1.03
0.81594	835.113	2.172	-2.074
0.84134	951.956	2.157	-3.282
0.86433	1094.196	2.117	-4.675
0.88493	1268.128	2.054	-6.27
0.9032	1481.789	1.971	-8.09
0.91924	1745.493	1.872	-10.155
0.93319	2072.535	1.76	-12.489

EXHIBIT 4-15 (b) NONHOMOTHETIC CASE XIV

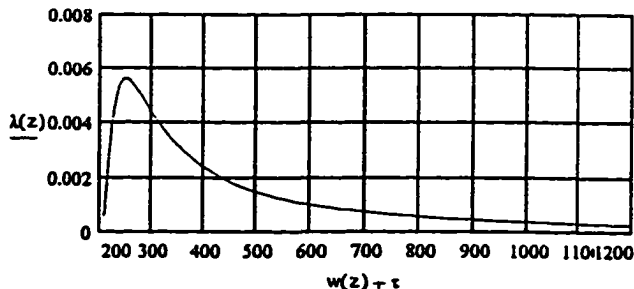


M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$
 EMP = 0.95 ERM = -0.25 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



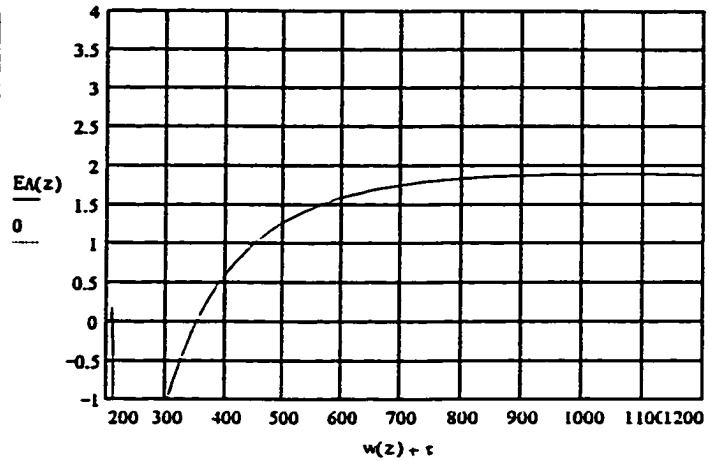
DENSITY OF RESERVATION WAGES

znorm13A.mcd

LPR W
 $cnorm(z)w(z) + \tau$ $E\lambda(z)$ $E\epsilon(z)$

0.00135	209.605	0	1.37
0.00187	210.28	0.174	0.441
0.00256	211.017	-0.032	-0.018
0.00347	211.823	-0.441	-0.188
0.00466	212.707	-0.94	-0.182
0.00621	213.677	-1.461	-0.07
0.0082	214.744	-1.964	0.103
0.01072	215.919	-2.424	0.312
0.0139	217.216	-2.827	0.538
0.01786	218.651	-3.166	0.774
0.02275	220.241	-3.439	1.011
0.02872	222.006	-3.645	1.247
0.03593	223.97	-3.786	1.48
0.04457	226.16	-3.865	1.709
0.0548	228.606	-3.885	1.932
0.06681	231.346	-3.85	2.149
0.08076	234.421	-3.763	2.359
0.0968	237.88	-3.63	2.561
0.11507	241.782	-3.455	2.756
0.13567	246.192	-3.242	2.941
0.15866	251.191	-2.997	3.115
0.18406	256.871	-2.723	3.277
0.21186	263.343	-2.427	3.424
0.24196	270.738	-2.112	3.555
0.27425	279.209	-1.784	3.666
0.30854	288.943	-1.448	3.755
0.34458	300.16	-1.108	3.817
0.38209	313.126	-0.769	3.848
0.42074	328.16	-0.435	3.844
0.46017	345.648	-0.112	3.798
0.5	366.057	0.197	3.705
0.53983	389.955	0.488	3.556
0.57926	418.036	0.758	3.344
0.61791	451.148	1.003	3.06
0.65542	490.335	1.221	2.692
0.69146	536.885	1.41	2.23
0.72575	592.393	1.568	1.66
0.75804	658.843	1.694	0.968
0.78814	738.714	1.789	0.138
0.81594	835.113	1.852	-0.846
0.84134	951.956	1.885	-2.003
0.86433	1094.196	1.889	-3.354
0.88493	1268.128	1.867	-4.919
0.9032	1481.789	1.82	-6.722
0.91924	1745.493	1.754	-8.788
0.93319	2072.535	1.67	-11.141

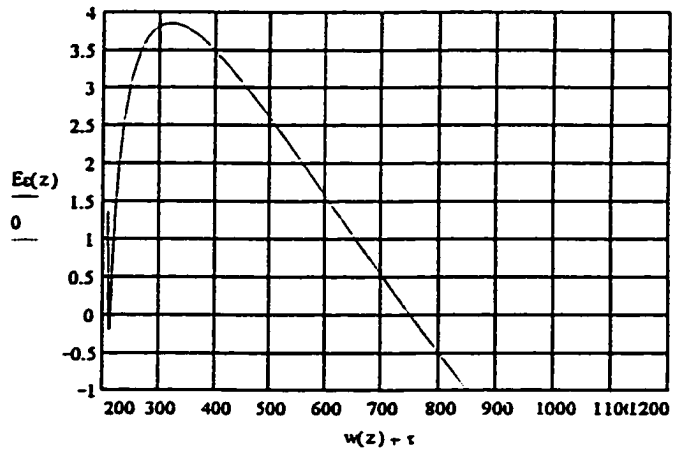
EXHIBIT 4 - 16 (b) NONHOMOTHETIC CASE XIV



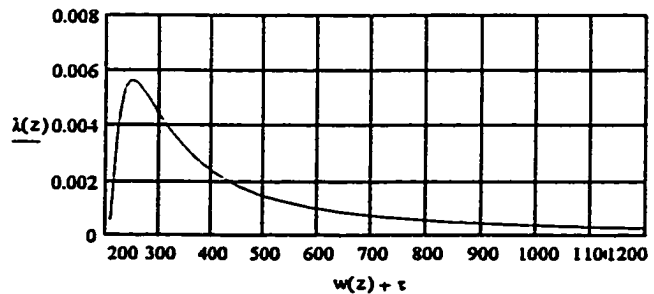
M-ELASTICITY OF LABOR SUPPLY

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.471$ $\delta = 0.305$

EMP = 0.1 ERM = -0.15 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



DENSITY OF RESERVATION WAGES

znorm13b.mcd

CHAPTER V

A FLEXIBLE-PRICE MODEL WITH NON-NEUTRAL MONEY, COUNTERCYCLICAL MARKDOWNS AND ACYCLICAL REAL WAGES.

5.1 Introduction

In this chapter I derive the implications of the labor supply model of Chapter IV for the behavior of markups and the real wage. This is done in two stages; (1) a partial equilibrium analysis, which extends the analysis in Chapter IV to the real wage, and (2) a general equilibrium analysis, utilizing the flexible-price model of Chapter II.

Neither approach leads to a satisfactory conclusion. The partial equilibrium method excludes feedback from other markets in the determination of the real wage, and the results are somewhat sensitive to the magnitudes of parameters. The comparative static employed in Chapter II captures the inter-market relationships, albeit in a static framework, but yields ambiguous results for the real wage because it abstracts from magnitudes. These limitations suggest that a different modeling technique, e. g., computer simulation, which can combine the best features of the two methods, might be a more useful methodology for analyzing this problem.

5.2 A Partial Equilibrium Analysis

With the model of Chapter IV in hand, it is time to return to the question originally posed in Chapter I and elaborated in Chapter III: Is monopsony power in this model sufficiently countercyclical to cause the real wage to be acyclical or weakly procyclical? Equivalently, is the markdown of the real wage in the labor market sufficiently countercyclical to offset the assumed negative elasticity of the marginal productivity of labor?

The fundamental relationship is the first-order condition for profit maximization by a monopsonistic firm::

$$w[1 + 1/\epsilon] = we = F_{\ell} \quad (5.1)$$

where $(e - 1)$ is the degree of monopsony power and $1/e$ is the markdown factor applied to F_{ℓ} . The amount of the markdown is w/ϵ . I will denote the real wage which satisfies this condition as w^* , and write as an identity $w^*e \equiv F_{\ell}$. This may be expressed in terms of elasticities with respect to money as:

$$\mathcal{E}_M[w^*] + \mathcal{E}_M[e] = \mathcal{E}_M[F_{\ell}] \quad (5.2)$$

from which $\mathcal{E}_M[w^*]$ can be derived.

It is straightforward to show that:

$$\mathcal{E}_M[e] = -\mathcal{E}_M[\epsilon]/(1 + \epsilon) \quad (5.3)$$

The expression for $\mathcal{E}_M[\epsilon]$ as a function of the equilibrium real wage was derived in Chapter IV, and the function was plotted for various cases. Also:

$$\mathcal{E}_M[F_\ell] = \left[\frac{\partial F_\ell^*}{\partial \ell^*} \frac{\ell^*}{F_\ell^*} \right] \left[\frac{\partial \ell^*}{\partial M} \frac{M}{\ell^*} \right] \quad (5.4)$$

The first term in brackets is the elasticity of the marginal product of labor with respect to employment, which is the reciprocal of η , the total elasticity of labor demand in a perfectly competitive labor market when output is variable. This elasticity has been estimated at the industry level by several investigators, as reported in Hammermesh (1993, Table 3.2):

	<u>η</u>
Meese (1980),	-1.73
Symons and Layard (1984)	-1.54
Layard and Nickell (1986)	-1.19
Burgess (1988)	-1.85
Nickell and Symons (1990)	-1.92
Average	<u>-1.65</u>

These estimates are based on quarterly data for manufacturing industries in the U. S. and Britain. Estimates based on annual data fell in the range of -0.50 to -0.95, as reported by Hammermesh. I will assume the value of -1.0, which is the theoretical value for a Cobb-Douglas output technology, and is conservative here. Thus, in Equation 5.2 the first term in brackets will be -1.

The second term in brackets is the elasticity of profit-maximizing employment ℓ^* with respect to a change in money. For a given optimum real wage w^* , monopsony employment is determined on the labor supply curve $\Lambda(w^*, \mu, \sigma^2)$, so this elasticity is $\mathcal{E}_M[\Lambda]$, which was also derived and computed in Chapter IV. Thus, in terms of functions for which numerical values can be computed, Equation 5.2 becomes:

$$\mathcal{E}_M[w^*] = -\mathcal{E}_M[\Lambda] - \mathcal{E}_M[e] \tag{5.5}$$

(+)(-)

The intuitive interpretation of Equation 5.5 is that a one-time change in money shifts the labor supply curve and also changes its elasticity at the equilibrium real wage, in the same direction as the shift (i. e., procyclically.) In the monopsony case, this changes both monopsony power and the size of the markdown in the opposite direction (countercyclically). To the extent that the money-elasticity of monopsony power

exceeds the absolute value of the money-elasticity of the marginal product curve, the real wage will be procyclical. If the elasticities on the right-hand side of Equation 5.5 are exactly offsetting, the real wage will be acyclical. For a perfectly competitive labor market, $\mathcal{E}_M[e] = 0$, and $\mathcal{E}_M[w^*]$ has the negative elasticity of the marginal product curve where it intersects the labor supply curve.

Equation 5.5 was computed for several of the cases in chapter IV. The computations are shown in Exhibits 5-1 through 5-7 at the end of this chapter. The results are summarized in Table 5-1.

Computed values of the money-elasticity of the equilibrium real wage are shown in the last column of Table 5-1. In the homothetic Case H-1, the negative elasticity of ϵ adds to the negative elasticity of F_L to make the elasticity of the real wage -1.0, which is more countercyclical than it would be under perfect competition (-0.7). This is consistent with the conclusion in Chapter III that that monopsony with homothetic preferences would make the real wage more countercyclical. In Case NH-8, corresponding to the benchmark case of Figure 4-9 in Chapter IV, the real wage is weakly countercyclical with an elasticity of -0.44. This is less countercyclical than it would be for a perfectly competitive labor market.

Table 5-1. Partial Equilibrium Analysis of Real Wage Behavior

EXHIBIT/CASE	PARAMETERS		COMPUTED ELASTICITIES			
	EMP	ERM	$\mathcal{E}_M[\epsilon]_\alpha$	$-\mathcal{E}_M[e]_\alpha$	$-\mathcal{E}_M[\Lambda]_\alpha$	$\mathcal{E}_M[w^*]_\alpha$
5-1 H-1	.8	-.15	- 0.5	- 0.30	- 0.7	- 1.0
5-2 NH-8	.8	-.15	+ 3.0	+ 1.82	- 2.26	- 0.44
5-3 NH-10	.5	-.15	+ 3.4	+ 2.06	- 2.1	- 0.04
5-4 NH-11	.8	-.10	+ 4.5	+ 2.7	- 3.0	- 0.27
5-5 NH-15	.1	-.15	+ 3.9	+ 2.36	- 1.92	+ 0.44
5-6 $L_\Omega = .70$.3	-.10	+ 5.5	+ 3.2	- 2.7	+ 0.50
5-7 $L_\Omega = .74$.3	-.10	+ 3.3	+ 2.0	- 1.8	+ 0.20

EMP = $\mathcal{E}_M[P]$, ERM = $\mathcal{E}_r[M]$. L_Ω is the Gini coefficient of the distribution of wealth.

All computed elasticities are valued at the mean reservation wage of \$500, where $\epsilon = 0.65$ and the labor-force participation rate is 66%.

The total elasticity of aggregate labor demand is assumed to be -1.

Although the benchmark case of Chapter IV had large positive elasticities for ϵ and Λ , the magnitude of $\mathcal{E}_M[\epsilon]$ was not quite large enough to offset an assumed value of -1 for η . If the average of -1.65 for estimated values of η were used, the elasticity of the real wage would be + 0.45. Thus, the elasticity of w^* in the model is sensitive to the value assumed for η .

The implied elasticity of the real wage is also sensitive to the values assumed for the monetary elasticities. In Case NH-10, reducing EMP from 0.8 in the benchmark case to 0.5

makes the real wage acyclical at an elasticity of $-.04$. This corresponds to a weaker short-run effect of money on the price level (i. e., sticky prices¹.) and a stronger effect on real labor supply, employment and output. Case NH-11 shows the effect of assuming a lower interest-elasticity of money demand ($-.10$). The real wage is less countercyclical than the benchmark case.

Case NH-15, with $EMP = 0.1$, corresponds to a very sticky price level in the short-run, which makes the real wage moderately procyclical at an elasticity of $+0.44$. This is not surprising, since sticky prices with flexible nominal wages can produce a strongly procyclical real wage in a perfectly competitive labor market (Romer, 1996). However, that case is a model of imperfect competition in the goods market, in which the effective labor demand curve is vertical and shifts against an inelastic labor supply curve. In the present model, sticky prices would augment the procyclical elasticity of a shifting labor supply curve. As pointed out in Chapter I, one of the criticisms of the New Keynesian imperfect competition models is that they tend to predict strongly procyclical real wages. There is an indication here that incorporating sticky prices in this model would result in a real wage that is only

¹Again, sticky prices imply sluggish adjustment in a dynamic setting. Some degree of price or wage stickiness may be necessary to justify a strong and persistent money-output connection. The static flexible-price model of Chapter II may be deficient in this respect.

weakly procyclical, which is more consistent with the stylized facts.

Exhibits 5-6 and 5-7 show that combinations of moderately lower values for EMP and ERM than the benchmark case can generate a weakly procyclical real wage, for Gini coefficients of wealth of 0.70 and 0.74.

The conclusion from the foregoing analysis is this model is capable of exhibiting a real wage that is weakly counter-cyclical, acyclical or weakly procyclical, with the outcome being parameter-dependent. In the particular calibration of the model shown here, the cyclicity of the real wage was somewhat sensitive to the assumed values of the monetary elasticities $\mathcal{E}_M[P]$ and $\mathcal{E}_r[M]$, and the elasticity of aggregate labor demand, η . These elasticities are not precisely known, and may vary over time.

This conclusion is qualified by the fact that it is based on a partial equilibrium framework, where any possible influences on the real wage from the goods and money markets have been excluded. That issue is addressed in the next section.

5.3 A Static General Equilibrium Framework

In this section the aggregate labor supply function of Chapter IV is incorporated into the flexible-price static general equilibrium model of Chapter II, and the implications for the behavior of the real wage under monopsony are investigated. The method of comparative statics is employed; this involves re-specifying and signing the excess demand functions of the model, and deriving comparative static derivatives, as was done in Chapter II. Appendix B to Chapter II will be referenced in order to minimize duplication here.

5.3.1 Excess Demand Functions

The revised excess demand functions of the model are:

$$\begin{aligned} \text{EDG} &= c^d(z, \Omega) + i^d(r, y) + g - y^s(w) = 0 \\ \text{EDM} &= y^\beta r^\gamma \Omega^{1+\rho} - M/P = 0, \quad \beta > 0, \gamma < 0, 0 < \rho < 1 \quad (5.6) \\ \text{EDL} &= \ell^d(ew) - \Lambda(w, \mu, \sigma^2) = 0 \end{aligned}$$

where $y^s(w)$ is output, z is disposable income, Ω is wealth, y is total income, r is the interest rate, $w = W/P$, M is the money stock and P is the price level, all as defined in Chapter II. The definition of household wealth in this chapter includes the present value of transfer payments T/rP , (e. g., unemployment compensation, Social Security, unearned income

credits and welfare assistance) which establishes a minimum floor on nonlabor income and corresponding wealth. Thus:

$$\Omega = \frac{M}{P} + \frac{\kappa B + T}{rP} + \frac{\pi}{r}, \quad 0 < \kappa \leq 0 \quad (5.7)$$

Monopsony power is incorporated in the labor market by specifying the labor demand function as $\ell^d(ew)$, where $e = (1 + 1/\epsilon)$. This follows directly from the inverse of the first-order condition for profit maximization, Equation 5.1. Aggregate labor supply is the Lognormal distribution form developed in Chapter IV. The dimensionality of flows in the model is per capita per week, so that Λ is the proportion of the labor force that is willing to work a 40-hour week at the real wage w . I have also defined a more specific aggregate money demand function with wealth elasticity $1+\rho$, which is consistent with the specification of increasing relative risk aversion in Chapter IV².

5.3.2 Partial Derivatives

Equilibrium profits in the monopsony case are:

$$\pi^* = y^s(\ell^d(ew), k) - w\ell^d(ew) = \pi^*(ew).$$

²Increasing relative risk aversion could also be specified in the goods market. This would effect the results obtained here only if imperfect competition were also assumed in the goods market, in which case the elasticity of demand would be a function of the level of income or wealth. Since I have assumed perfect competition in the goods market in order to focus on the labor market, I employ a standard Keynesian consumption function here. Imperfect competition with IRRA in both goods and labor markets would be an interesting extension.

Differentiating π^* with respect to P, W and r, with subscripts denoting partial derivatives, I obtain:

$$\begin{aligned}\pi_p^* &= [y_\ell^s - w] \ell_{ew}^d \left[-\frac{W}{p^2} e + we_p \right] + \frac{W}{p^2} \ell^d(ew) \\ &= -\frac{W}{p^2} \frac{W}{\epsilon} \ell_{ew}^d [e - Pe_p] + \frac{W \ell^d}{p^2} > 0\end{aligned}\quad (5.8)$$

$$\pi_w^* = \frac{1}{p} \frac{W}{\epsilon} \ell_{ew}^d [e + We_w] - \frac{\ell^d}{p} < 0\quad (5.9)$$

(-) (+)

$$\pi_r^* = \frac{W}{\epsilon} \ell_{ew}^d [we_r] < 0\quad (5.10)$$

(-) (+)

Here I have used the fact that in monopsony, $[y_\ell^s - w] = w/\epsilon$, the amount of the markdown. The signs of these three partial derivatives follow from the fact that $\ell_{ew}^d < 0$ and e_p , e_w and e_r have the opposite signs of $\epsilon_p < 0$, $\epsilon_w < 0$ and $\epsilon_r < 0$, which are known from the analysis in Chapter IV³. The sign of $e - Pe_p$ is not obvious, but this expression was evaluated numerically for Case NH-8 and found to be unconditionally positive, with a value of +5 at the mean reservation wage. Thus, the effect of an increase in the price level is to increase the conditional demand for labor. The only diff-

³These signs hold over a relevant range of reservation wages, which for the benchmark Case NH-8 of Chapter IV, includes wages up to \$694 per week at a labor-force participation rate of 77%.

erence from Chapter II thus far is that for the monopsonistic labor market $\pi_r^* < 0$, whereas in the perfectly competitive labor market of Chapter II, $\pi_r^* = 0$.

These signs enable the partial derivatives of Ω , which contains π/r , to be determined.

$$\Omega_P = -\frac{1}{P} \left[\frac{M}{P} + \frac{kB+T}{rP} - \frac{W\ell^d}{rP} \right] - \frac{W}{rP^2} \frac{W}{\epsilon} \ell_{ew}^d [e - Pe_P] > 0 \quad (5.11)$$

The sign here follows from the sign of π_p^* and the reasoning in Chapter II that the real wage bill $w\ell^d$ is much greater than nonlabor income from equities, bonds and transfer payments.

$$\Omega_W = \frac{\pi_W}{r} < 0 \quad (5.12)$$

$$\Omega_r = -\frac{1}{r} \left[\frac{\kappa B+T}{rP} + \frac{\pi}{r} - \pi_r \right] < 0 \quad (5.13)$$

All of the partial derivatives of Ω have the same sign as in Chapter II.

With these facts established, it is possible to sign the partial derivatives of the excess demand functions and examine the 3 x 3 matrix of partial derivatives, \mathbf{A} , of Chapter II and Appendix B. Since the excess demand functions for the goods market are the same as in Chapter II, there will no change there, and:

$$EDG_p < 0, \quad EDG_r < 0, \quad \text{and} \quad EDG_w > 0.$$

Inspection of the money demand function reveals that

$$m_y^d = \beta y^{\beta-1} B > 0, \quad \beta > 0, \quad B > 0.$$

$$m_\Omega^d = (1+\rho)\Omega^\rho C > 0, \quad 0 < \rho < 1, \quad C > 0$$

$$m_r^d = \gamma r^{\gamma-1} D < 0, \quad \gamma < 0, \quad D > 0.$$

These are the same signs as in Chapter II, so the signs of the partial derivatives of EDM are also unchanged.

$$EDM_p > 0, \quad EDM_r < 0, \quad EDM_w < 0.$$

However, conditions in the labor market are different.

$$EDL = \ell^d(ew) - \Lambda(w, \mu, \sigma^2)$$

$$EDL_p = -\frac{1}{P} \left[\frac{W}{P} \left[\ell_{ew}^d \left[e - Pe_p \right] - \lambda \right] - \Lambda_\mu \right] > 0, \quad w + \tau \leq 600 \quad (5.14)$$

The sign here appears to be ambiguous, but evaluating the expression numerically for the benchmark case NH-8 (Exhibit 4-9 in Chapter IV) revealed that EDL_p is positive up to an equilibrium real wage of \$600, or a labor-force participation rate of 73%.

$$EDL_r = \ell_{ew}^d \left[we_r \right] - \Lambda_r < 0, \quad w + \tau > 300 \quad (5.15)$$

Here again, the sign appears to be ambiguous, but a numerical evaluation for case NH-8 of Chapter IV revealed that EDL_r is negative for a real wage above \$300.

$$EDL_W = \overset{(-)}{\ell} \overset{(+)}{e} \left[w e_w + \frac{1}{P} e \right] - \frac{\overset{(+)}{\lambda}}{P} < 0 \quad (5.16)$$

Thus, for the benchmark case NH-8, the partial derivatives of EDL have the same sign as in Chapter II for an equilibrium real wage in the range of \$300 to \$600 per week (Labor-force participation rates of 34% to 73% in the model.) Since none of the elements of the A matrix change sign over that range, most of the comparative static results of Chapter II also hold over that range, including the stability conditions and, more importantly, the signs of

$$\frac{dP}{dM} > 0, \quad \frac{dW}{dM} > 0, \quad \frac{dr}{dM} < 0, \quad 300 \leq w + \tau \leq 600 \quad (5.17)$$

5.3.3 Comparative Static Derivatives

The derivative of primary interest here is dw/dM ,

$$\frac{d\left[\frac{W}{P}\right]}{dM} = \frac{1}{P} \left[\frac{dW}{dM} - \frac{W}{P} \frac{dP}{dM} \right] \quad (5.18)$$

To derive the sign of this it is necessary to expand dP/dM and dW/dM and recombine them in the above expression, as was done in Chapter II. I will do this for the benchmark case NH-8 in Chapter IV, and the case of no Ricardian equivalence, i. e. $\kappa = 1$.

For $\kappa = 1$, we have from Appendix B to Chapter II:

$$(5.19) \quad \frac{dW}{dM} = \frac{-1}{P|A|} \left\{ \overset{(-)}{EDG}_P \cdot \overset{(-)}{EDL}_r - \overset{(+)}{EDL}_P \cdot \overset{(-)}{EDG}_r \right\}$$

Substituting the partial derivatives for each component:

$$\begin{aligned} \frac{dW}{dM} = \frac{-1}{P|A|} \left\{ \overset{(-)}{\left[-(1-c_z^d - i_y^d) Y_w^s \left[-\frac{W}{P^2} \right] + c_{\Omega P}^d \left[\overset{(-)}{\ell_{ew}^d [we_r] - \Lambda_r} \right] \right]} \right. \\ \left. - \overset{(+)}{\left[\frac{W}{P^2} \left[\overset{(+)}{\ell_{ew}^d [e - Pe_P] - \lambda} \right] + \frac{1}{P} \Lambda_{\mu} \left[\overset{(-)}{c_{\Omega r}^d \Omega_r + i_r^d} \right] \right]} \right\} \quad (5.20) \end{aligned}$$

Similarly, for dP/dM we have:

$$\frac{dP}{dM} = \frac{-1}{P|A|} \left[\overset{(-)}{EDG}_r \cdot \overset{(-)}{EDL}_w - \overset{(-)}{EDL}_r \cdot \overset{(+)}{EDG}_w \right] \quad (5.21)$$

which upon substitution becomes:

$$\begin{aligned} \frac{dP}{dM} = \frac{-1}{P|A|} \left\{ \overset{(-)}{\left[c_{\Omega r}^d \Omega_r + i_r^d \right] \left[\overset{(-)}{\ell_{ew}^d \left[e_w + \frac{1}{P} e \right] - \frac{\lambda}{P}} \right]} \right. \\ \left. - \overset{(-)}{\left[\ell_{ew}^d [we_r] - \Lambda_r \right] \left[\overset{(+)}{-(1-c_z^d - i_y^d) Y_w^s \frac{1}{P} + c_{\Omega w}^d} \right]} \right\} \quad (5.22) \end{aligned}$$

Upon subtracting W/P times Equation 5-22 from Equation 5-20, several terms cancel, and after some reduction I obtain⁴:

⁴The trick here is to collect terms on EDL_r and EDG_r and recognize $\Omega_r + W\Omega_w$ as a common factor.

$$\frac{dw}{dM} = \frac{-1}{P^2 |A|} \left\{ \begin{aligned} & \overset{(-)}{EDG}_r \left[\frac{W}{P} \left[\overset{(-)}{\ell}_{ew}^d [-we_w - e_p] \right] - \frac{1}{P} \overset{(-)}{\Lambda}_\mu \right] \\ & + \overset{(+)}{c}_\Omega^d \left[\overset{(-)}{\Omega}_p + \frac{\overset{(-)}{W}}{P} \overset{(-)}{\Omega}_w \right] \overset{(-)}{EDL}_r \end{aligned} \right\} \quad (5.23)$$

where:

$$\overset{(-)}{EDG}_r = \overset{(+)}{c}_\Omega^d \overset{(+)}{\Omega}_r + \overset{(-)}{i}_r^d < 0$$

$$\overset{(-)}{EDL}_r = \overset{(-)}{\ell}_{ew}^d \left[\overset{(+)}{we}_r \right] - \overset{(+)}{\Lambda}_r < 0$$

The sign of dw/dM is ambiguous. This is disappointing, but not surprising because even in the partial equilibrium setup the sign of the elasticity of the real wage depends on magnitudes. In Chapter II, the real wage was unambiguously countercyclical; here, it is ambiguous, which allows for the possibility that it could be acyclical or procyclical, depending on magnitudes.

In Equation 5.23, the effect on dw/dM of e_r in $\overset{(-)}{EDL}_r$ is positive, and is channeled through a wealth effect on consumption, $\overset{(+)}{c}_\Omega^d [\overset{(-)}{\Omega}_p + w\overset{(-)}{\Omega}_w]$, which is diminished by the increase in P and W . The effects of e_w and e_p on dw/dM are negative, and are channeled through the interest-rate effect on demand, $(\overset{(+)}{c}_\Omega^d \overset{(+)}{\Omega}_r + \overset{(-)}{i}_r^d)$. The real wage is more likely to be acyclical or weakly procyclical if the interest-elasticity of monopsony power is large relative to the interest-elasticity of demand.

The general equilibrium approach reveals interactions between the goods market and the labor market in the determination of the real wage, which could not be taken into account in the partial equilibrium analysis. The connection is through the interest rate. A one-time increase in money raises the price level and the nominal wage, and lowers the interest rate. This causes a decline in wealth, which has a negative effect on consumption and a positive effect on labor supply. With monopsony, effective labor demand and labor supply are linked via the elasticity of labor supply in marginal cost, ϵ_w . The new equilibrium in the labor market is determined by the M-elasticity of ϵ_w as the labor supply curve shifts out. With homothetic preferences $e_r = 0$, and w and $e = [1 + 1/\epsilon]$ are functions of W and P only. Equation 5.23 shows that in this case $dw/dM < 0$, and in fact, due to the presence of the term $[-we_w - e_p]$ it is more negative than under perfect competition. With increasing relative risk aversion, however, e_r is positive over a relevant range of w , and this brings the positive term $c_\Omega^d[\Omega_p + w\Omega_w] \cdot EDL_r$ into play. The net effect on dw/dM will depend on relative magnitudes of the two terms in Equation 5.23.

The method of comparative statics is less revealing when the signs of total derivatives depend on magnitudes, as is the case here.

5.4 Conclusions

A Keynesian model with monopsony power in the labor market, wealth effects in all markets, and increasing relative risk aversion in consumer preferences, will have a causal connection between money and output and exhibit countercyclical markdowns. The behavior of the real wage will be weakly countercyclical, acyclical, or weakly procyclical, depending on the strength of the wealth effect in the labor market, and the values of monetary elasticities which are parameters of the model. These results are broadly consistent with the stylized facts introduced in Chapter I.

The results are somewhat sensitive to values assumed for the interest-elasticity and wealth-elasticity of the money demand function, and the short-run elasticity of the price level with respect to innovations in money. They are also conditional upon assumed Gini coefficients of 0.45 for the distribution of nonlabor income in the workforce, and 0.70 for the distribution of financial wealth in the workforce, values which have been crudely estimated here.

The model presented in this thesis has not been tested empirically. Rejection of any of the following hypotheses could be considered a rejection of the model as constructed and described here:

(a) In the aggregate, firms have significant market power to set wages in their industries.

(b) The wealth-elasticity of the demand for real money balances among members of the labor force is greater than unity. Alternatively, the wealth-elasticity of the demand for earning assets (stocks and bonds) is less than unity.

(c) The Gini coefficient for the distribution of financial wealth in the workforce is less than 0.80.

(d) The Gini coefficient for the distribution of nonlabor income in the workforce is greater than 0.4 and less than 0.60

(e) The distribution of nonlabor income in the workforce follows a Lognormal distribution.

(d) The distribution of financial wealth in the workforce follows a Lognormal distribution.

(e) The distribution of nonlabor income (or financial wealth) in the workforce becomes more unequal during contractions and less unequal during expansions of the business cycle.

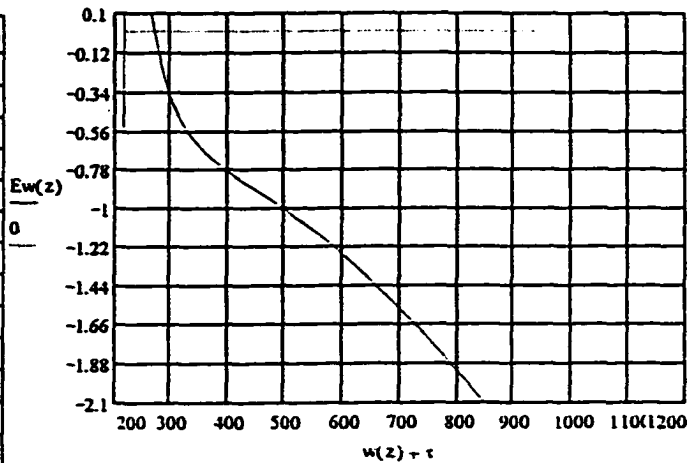
A final note on the methodology employed in this thesis: When representative agent aggregation is not valid, the distributions of heterogeneous agent characteristics matter for the behavior of aggregates. The method of aggregation must then incorporate information about those distributions in the form of sufficient statistics. In the model presented here, the sufficient statistics were: The parameters of the Lognormal distribution of nonlabor income; the wealth elasticity of the demand for money; and the wealth-elasticity of the demand for leisure. Statistical estimates of these parameters have seldom, if ever, been made. One impediment to more widespread modeling of heterogeneity in economics may be that we lack the empirical knowledge to do it well. On the other hand, econometricians usually try to measure things that their colleagues regard as important. The research program on heterogeneous agents is in the early stage of its life-cycle, and promises to reveal more about what we really need to know about heterogeneity and the distributions of agent characteristics.

The method of comparative statics which served well in Chapter II was not very revealing in this chapter. This is not a very powerful way to evaluate a model in which the magnitudes of elasticities of second order are important. A computer simulation or Computable General Equilibrium model might be more useful in future research of this nature.

LFPF W
 cnorm(z)w(z) + τEA(z) Eε(z) Ew(z)

0.00135	216.673	0	-2.622	-0.536
0.00187	218.142	-0.68	-1.634	0.338
0.00256	219.74	-1.079	-0.94	0.877
0.00347	221.48	-1.287	-0.451	1.188
0.00466	223.373	-1.367	-0.104	1.343
0.00621	225.432	-1.36	0.142	1.392
0.0082	227.673	-1.295	0.317	1.369
0.01072	230.111	-1.192	0.441	1.299
0.0139	232.765	-1.066	0.527	1.197
0.01786	235.652	-0.926	0.586	1.075
0.02275	238.793	-0.779	0.625	0.943
0.02872	242.212	-0.631	0.648	0.805
0.03593	245.931	-0.484	0.659	0.667
0.04457	249.979	-0.342	0.661	0.531
0.0548	254.382	-0.205	0.656	0.398
0.06681	259.174	-0.077	0.644	0.272
0.08076	264.389	0.044	0.627	0.152
0.0968	270.062	0.156	0.605	0.04
0.11507	276.236	0.258	0.578	-0.065
0.13567	282.954	0.351	0.548	-0.162
0.15866	290.263	0.434	0.513	-0.251
0.18406	298.217	0.507	0.475	-0.332
0.21186	306.871	0.571	0.432	-0.406
0.24196	316.288	0.626	0.386	-0.473
0.27425	326.535	0.671	0.335	-0.534
0.30854	337.685	0.707	0.279	-0.589
0.34458	349.817	0.735	0.218	-0.638
0.38209	363.019	0.754	0.152	-0.684
0.42074	377.383	0.765	0.08	-0.727
0.46017	393.014	0.769	0.002	-0.768
0.5	410.021	0.766	-0.083	-0.808
0.53983	428.528	0.756	-0.174	-0.849
0.57926	448.665	0.74	-0.274	-0.892
0.61791	470.576	0.719	-0.382	-0.94
0.65542	494.418	0.693	-0.498	-0.993
0.69146	520.361	0.663	-0.625	-1.053
0.72575	548.59	0.63	-0.761	-1.123
0.75804	579.307	0.594	-0.908	-1.204
0.78814	612.73	0.555	-1.067	-1.299
0.81594	649.098	0.515	-1.238	-1.409
0.84134	688.671	0.475	-1.422	-1.535
0.86433	731.731	0.434	-1.619	-1.681
0.88493	778.585	0.393	-1.831	-1.846
0.9032	829.568	0.354	-2.056	-2.033
0.91924	885.043	0.315	-2.297	-2.241
0.93319	945.406	0.279	-2.554	-2.472

EXHIBIT 5-1 HOMOTHETIC CASE



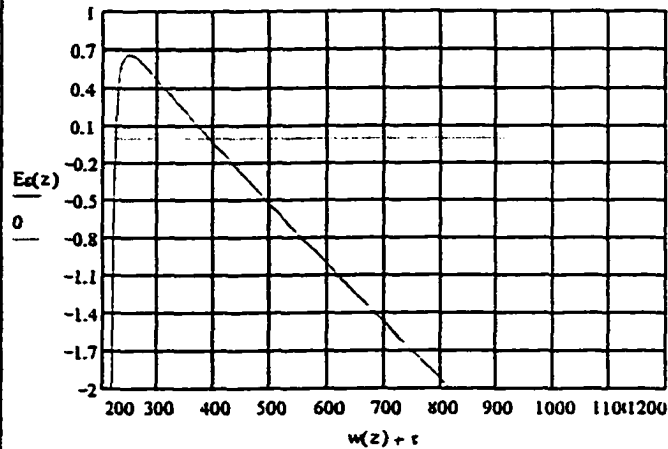
M-ELASTICITY OF THE REAL WAGE

$\mu = 5.347$ Median = 410.021 $\alpha = 300$ $L = 0.45$

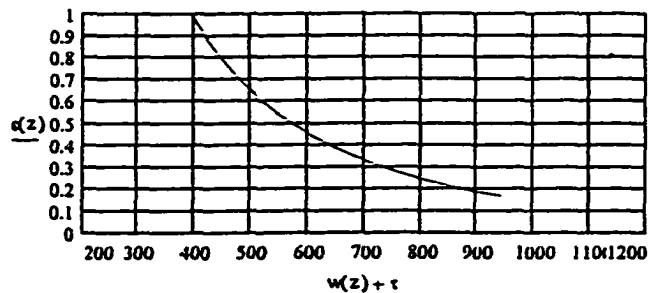
$\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$

$\rho = 0$

EMP = 0.8 ERM = -0.15 EQM = 1 ERA = 0.102



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



ELASTICITY OF LABOR SUPPLY

znorm0b.mcd

EXHIBIT 5-2 (a)

znorm7AA.mcd

ELASTICITY OF ELASTICITY COMPUTATIONS
LOGNORMAL LABOR SUPPLY FUNCTION

LOGNORMAL DISTRIBUTION CALIBRATION

$$z := -3, -2.9, -1.5 \quad \alpha := 300 \quad \beta := 510 \quad \tau := 200 \quad \gamma = \frac{\beta}{\alpha + \tau} \quad \sigma = \sqrt{\ln(\gamma^2 + 1)}$$

$$v := 1 \quad \beta_2 := \alpha^2 \cdot \gamma^2 \quad \beta_2 = 9.364 \cdot 10^4 \quad \gamma = 1.02 \quad \sigma = 0.844 \quad \sigma^2 = 0.713$$

$$\mu := \ln(\alpha) - .5 \cdot \sigma^2 \quad L = 2 \cdot \text{cnorm}\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \quad L = 0.45 \quad \rho = .424$$

Calibrate Mean Reservation Wage
For Nonhomothetic Function

$$\Pi(z) = z \quad \Omega(z) = \frac{(z^2 - 1) \cdot (1 - \rho)^2}{\rho^2 \cdot \sigma^2}$$

$$\delta = \frac{\ln(\ln(\alpha + 1))}{\ln(\alpha)} \quad \delta = 0.305$$

Nonhomothetic Reservation Wage Function

$$\xi = \delta \cdot v^\delta \quad \psi = \exp(v^\delta) - 1 \quad g(w) = \ln(w + 1)^{\frac{1}{\delta}}$$

$$w(z) = \exp(\exp(\delta \cdot (\mu + z \cdot \sigma))) - 1$$

$$\text{Mean} = \exp\left[\exp\left[\delta \cdot (\mu + .5 \cdot \sigma^2)\right]\right] - 1 + \tau$$

$$\text{Mean} = 500$$

$$\text{Median} = \exp(\exp(\delta \cdot \mu)) - 1 + \tau$$

$$\text{Median} = 366.057$$

MONETARY CALIBRATION

$$M2 = 3.439 \quad F = 5.035 \quad A = \frac{F}{M2 + F}$$

$$\text{EMP} = .8 \quad A = 0.594$$

$$\text{ERM} = -.15 \quad \text{EQM}(z) = \delta \cdot \ln(w(z) + 1)$$

$$\text{ERA}(z) = \frac{1 - A}{A} \cdot (-\text{ERM} + A \cdot (\text{EQM}(z) - 1)) \quad \text{Equation (D5.8)}$$

$$\text{EMA}(z) = \frac{\text{ERA}(z)}{\text{ERM}}$$

$$\lambda(z) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma \cdot w(z)}} \cdot \exp\left[-\frac{1}{2} \cdot (z)^2\right]$$

$$K1(z) = \left[-\frac{1}{\text{cnorm}(z)} \int_{-3}^z \frac{\Pi(z)}{\sigma} \cdot \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{z^2}{2}\right) dz \right] \cdot \text{EMP}$$

LOGNORMAL DENSITY FUNCTION
OF NONLABOR INCOME

$$\varepsilon(z) = \lambda(z) \cdot \frac{w(z)}{\text{cnorm}(z)}$$

$$K2(z) = \left[\int_{-3}^z \Omega(z) \cdot \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{z^2}{2}\right) dz \cdot \left(\frac{1}{\text{cnorm}(z)}\right) \right] \cdot \text{EMA}(z)$$

w - ELASTICITY OF
AGGREGATE LABOR SUPPLY

$$E\Lambda(z) = K1(z) + K2(z) \quad \text{(Equation D5.11)} \quad \text{M - ELASTICITY OF AGGREGATE LABOR SUPPLY}$$

$$E\varepsilon(z) = -E\Lambda(z) - \frac{\Pi(z)}{\sigma} \cdot \text{EMP} + \Omega(z) \cdot \text{EMA}(z) \quad \text{(Equation D6.11)}$$

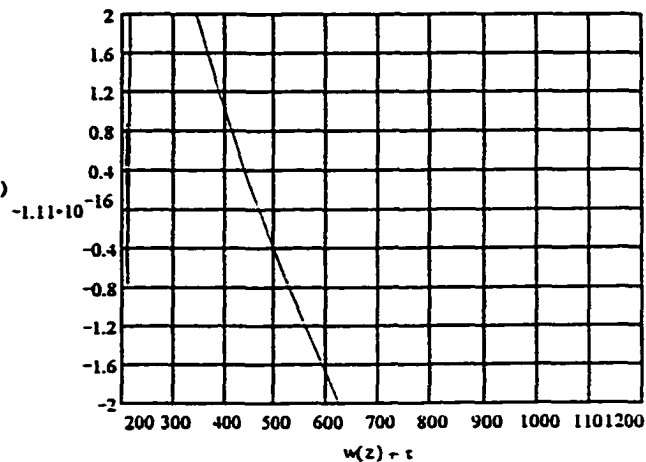
$$Ew(z) = \frac{E\varepsilon(z)}{\varepsilon(z) + 1} - E\Lambda(z)$$

M - ELASTICITY OF THE ELASTICITY OF
AGGREGATE LABOR SUPPLYMONEY - ELASTICITY OF
THE REAL WAGE

znorm7AA.mcd

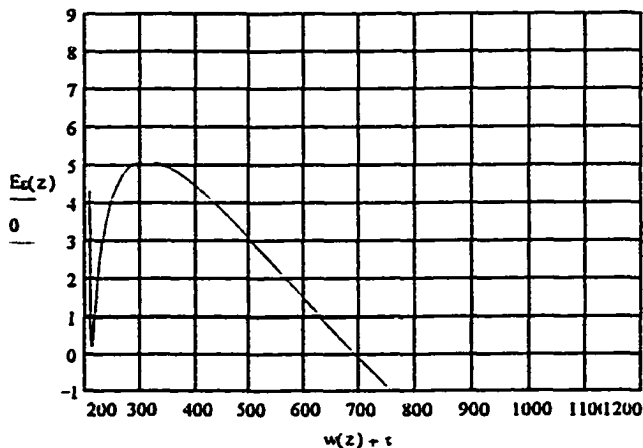
LFPR	W	$\ln(w(z) + \tau)EA(z)$	$Ez(z)$	$Ew(z)$
0.00135	209.605	0	4.326	0.885
0.00187	210.28	0.886	2.259	-0.413
0.00256	211.017	1.008	1.086	-0.775
0.00347	211.823	0.692	0.478	-0.588
0.00466	212.707	0.145	0.227	-0.094
0.00621	213.677	-0.505	0.2	0.551
0.0082	214.744	1.177	0.313	1.251
0.01072	215.919	1.821	0.512	1.945
0.0139	217.216	2.408	0.762	2.597
0.01786	218.651	2.919	1.044	3.185
0.02275	220.241	3.345	1.341	3.697
0.02872	222.006	3.684	1.646	4.128
0.03593	223.97	3.934	1.954	4.477
0.04457	226.16	4.098	2.259	4.744
0.0548	228.606	-4.18	2.56	4.933
0.06681	231.346	-4.183	2.855	5.049
0.08076	234.421	4.113	3.142	5.096
0.0968	237.88	3.976	3.419	5.08
0.11507	241.782	3.778	3.684	5.007
0.13567	246.192	3.525	3.936	4.882
0.15866	251.191	3.225	4.172	4.712
0.18406	256.871	2.884	4.39	4.502
0.21186	263.343	2.508	4.587	4.259
0.24196	270.738	2.105	4.758	3.987
0.27425	279.209	1.682	4.9	3.691
0.30854	288.943	1.246	5.008	3.376
0.34458	300.16	0.803	5.076	3.044
0.38209	313.126	0.361	5.099	2.697
0.42074	328.16	0.075	5.067	2.338
0.46017	345.648	0.497	4.974	1.964
0.5	366.057	0.899	4.81	1.574
0.53983	389.955	1.277	4.563	1.162
0.57926	418.036	1.626	4.222	0.72
0.61791	451.148	1.94	3.772	0.24
0.65542	490.335	2.216	3.199	-0.295
0.69146	536.885	2.452	2.486	-0.901
0.72575	592.393	2.645	1.613	-1.6
0.75804	658.843	2.793	0.559	-2.417
0.78814	738.714	2.897	-0.697	-3.383
0.81594	835.113	2.958	-2.183	-4.533
0.84134	951.956	2.976	-3.924	-5.903
0.86433	1094.196	2.955	-5.949	-7.537
0.88493	1268.128	2.897	-8.292	-9.478
0.9032	1481.789	2.806	-10.984	-11.775
0.91924	1745.493	2.688	-14.062	-14.476
0.93319	2072.535	2.546	-17.563	-17.631

EXHIBIT 5-2(b) NONHOMOTHETIC CASE VIII

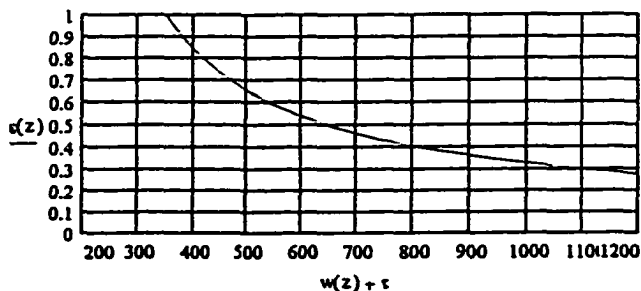


M-ELASTICITY OF THE REAL WAGE

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$
 EMP = 0.8 ERM = -0.15 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



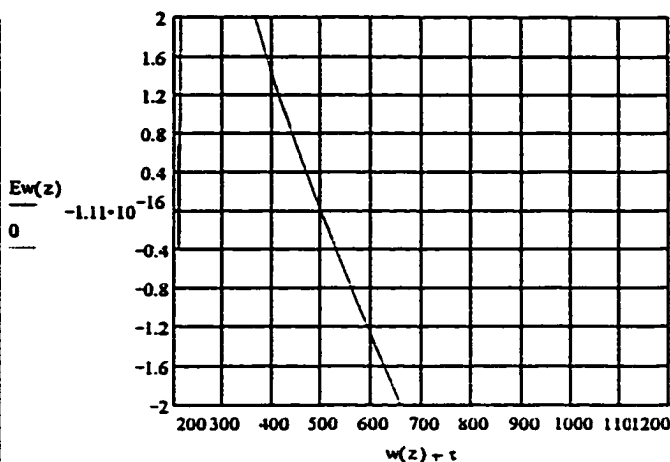
ELASTICITY OF LABOR SUPPLY

znorm7AA.mcd

LFPR W
 $cnorm(z)w(z) + \tau$ $E\Lambda(z)$ $E\varepsilon(z)$ $Ew(z)$

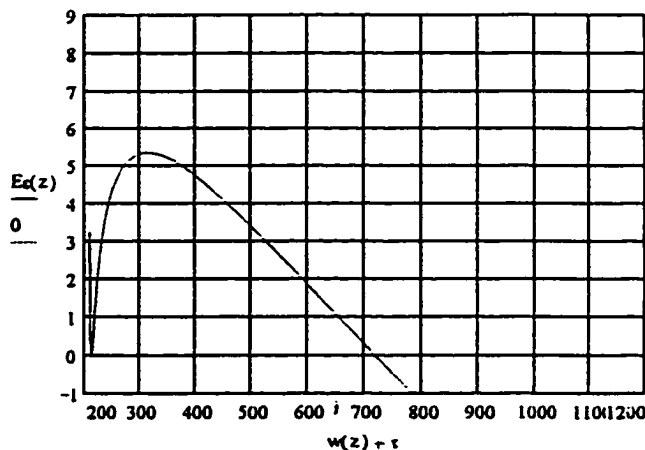
0.00135	209.605	0	3.26	0.667
0.00187	210.28	0.597	1.518	0.279
0.00256	211.017	0.524	0.575	-0.4
0.00347	211.823	0.079	0.133	-0.05
0.00466	212.707	-0.553	0.001	0.553
0.00621	213.677	-1.255	0.061	1.269
0.0082	214.744	-1.956	0.239	2.012
0.01072	215.919	-2.613	0.486	2.731
0.0139	217.216	-3.201	0.774	3.393
0.01786	218.651	-3.705	1.084	3.982
0.02275	220.241	-4.119	1.405	4.488
0.02872	222.006	-4.441	1.728	4.907
0.03593	223.97	-4.671	2.051	5.24
0.04457	226.16	-4.813	2.37	5.49
0.0548	228.606	-4.87	2.682	5.659
0.06681	231.346	-4.848	2.987	5.754
0.08076	234.421	-4.752	3.284	5.779
0.0968	237.88	-4.588	3.57	5.741
0.11507	241.782	-4.364	3.844	5.646
0.13567	246.192	-4.084	4.104	5.499
0.15866	251.191	-3.757	4.349	5.307
0.18406	256.871	-3.389	4.575	5.076
0.21186	263.343	-2.986	4.781	4.811
0.24196	270.738	-2.557	4.961	4.519
0.27425	279.209	-2.108	5.113	4.204
0.30854	288.943	-1.646	5.231	3.871
0.34458	300.16	-1.178	5.309	3.522
0.38209	313.126	-0.711	5.343	3.16
0.42074	328.16	-0.252	5.323	2.786
0.46017	345.648	0.194	5.242	2.4
0.5	366.057	0.619	5.09	1.998
0.53983	389.955	1.019	4.857	1.577
0.57926	418.036	1.388	4.53	1.129
0.61791	451.148	1.723	4.095	0.643
0.65542	490.335	2.019	3.538	0.106
0.69146	536.885	2.273	2.842	-0.5
0.72575	592.393	2.484	1.987	-1.197
0.75804	658.843	2.649	0.952	-2.009
0.78814	738.714	2.769	-0.285	-2.967
0.81594	835.113	2.844	-1.749	-4.106
0.84134	951.956	2.876	-3.468	-5.463
0.86433	1094.196	2.867	-5.471	-7.08
0.88493	1268.128	2.821	-7.789	-9.003
0.9032	1481.789	2.741	-10.457	-11.279
0.91924	1745.493	2.632	-13.509	-13.956
0.93319	2072.535	2.499	-16.983	-17.085

EXHIBIT 5-3 NONHOMOTHETIC CASE X

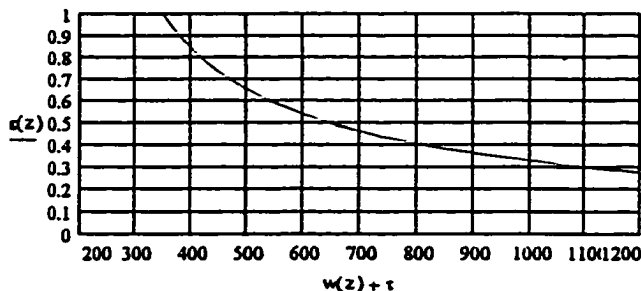


M-ELASTICITY OF THE REAL WAGE

$\mu = 5.347$ Median = 366.057 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.713$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.424$ $\delta = 0.305$
 EMP = 0.5 ERM = -0.15 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



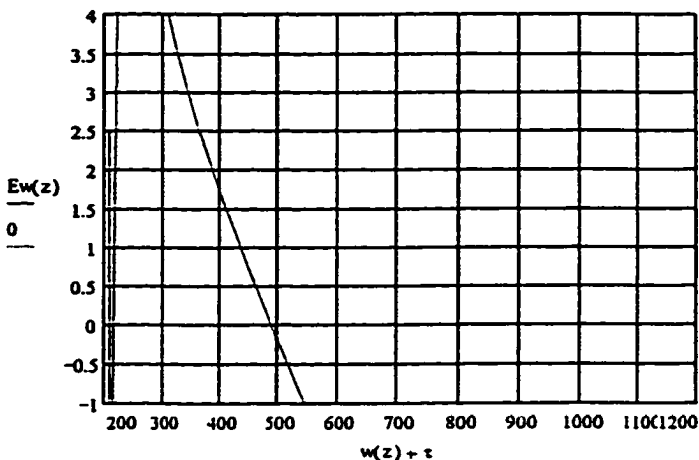
ELASTICITY OF LABOR SUPPLY

znorm9AA.mcd

LPR W
 $cnorm(z)w(z) + \tau E\Lambda(z)E\epsilon(z) Ew(z)$

0.00135	209.6	0	12.14	2.48
0.00187	210.28	2.82	7.07	-1.34
0.00256	211.02	3.93	3.93	-3.09
0.00347	211.82	4	2.03	-3.56
0.00466	212.71	3.46	0.95	-3.25
0.00621	213.68	2.59	0.41	-2.49
0.0082	214.74	1.56	0.22	-1.51
0.01072	215.92	0.49	0.25	-0.42
0.0139	217.22	-0.55	0.44	0.66
0.01786	218.65	-1.51	0.72	1.7
0.02275	220.24	-2.37	1.07	2.65
0.02872	222.01	-3.11	1.46	3.5
0.03593	223.97	-3.72	1.87	4.23
0.04457	226.16	-4.19	2.3	4.85
0.0548	228.61	-4.54	2.73	5.35
0.06681	231.35	-4.77	3.17	5.73
0.08076	234.42	-4.87	3.6	6
0.0968	237.88	-4.87	4.03	6.17
0.11507	241.78	-4.76	4.44	6.24
0.13567	246.19	-4.56	4.84	6.23
0.15866	251.19	-4.27	5.22	6.13
0.18406	256.87	-3.91	5.58	5.97
0.21186	263.34	-3.49	5.91	5.74
0.24196	270.74	-3.01	6.21	5.47
0.27425	279.21	-2.49	6.47	5.14
0.30854	288.94	-1.94	6.68	4.78
0.34458	300.16	-1.36	6.84	4.38
0.38209	313.13	-0.77	6.94	3.95
0.42074	328.16	-0.19	6.96	3.5
0.46017	345.65	0.39	6.89	3.02
0.5	366.06	0.95	6.73	2.51
0.53983	389.96	1.48	6.45	1.96
0.57926	418.04	1.98	6.03	1.37
0.61791	451.15	2.44	5.47	0.72
0.65542	490.33	2.84	4.73	-0.01
0.69146	536.88	3.2	3.78	-0.84
0.72575	592.39	3.49	2.61	-1.8
0.75804	658.84	3.73	1.18	-2.93
0.78814	738.71	3.9	-0.54	-4.28
0.81594	835.11	4.01	-2.59	-5.88
0.84134	951.96	4.06	-5.01	-7.8
0.86433	1094.2	4.06	-7.84	-10.1
0.88493	1268.13	4	-11.13	-12.83
0.9032	1481.79	3.89	-14.93	-16.08
0.91924	1745.49	3.74	-19.29	-19.92
0.93319	2072.53	3.56	-24.27	-24.4

EXHIBIT 5-4 NONHOMOTHETIC CASE XI



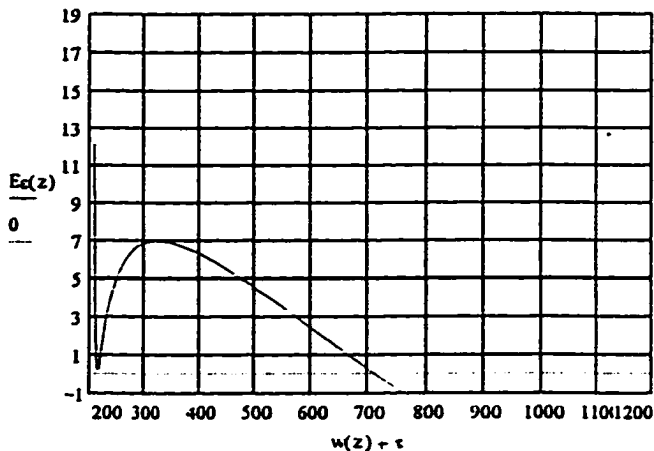
M-ELASTICITY OF THE REAL WAGE

$\mu = 5.35$ Median = 366.06 $\alpha = 300$ $L = 0.45$

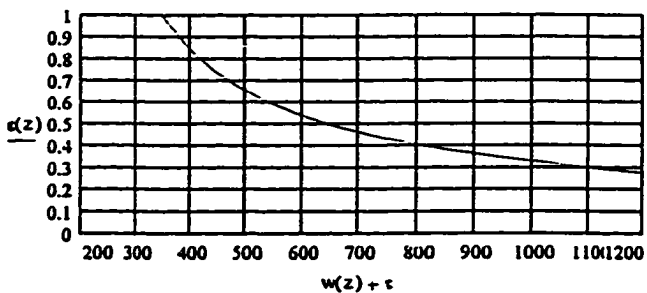
$\sigma^2 = 0.71$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$

$\rho = 0.42$ $\delta = 0.31$

EMP = 0.8 ERM = -0.1 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



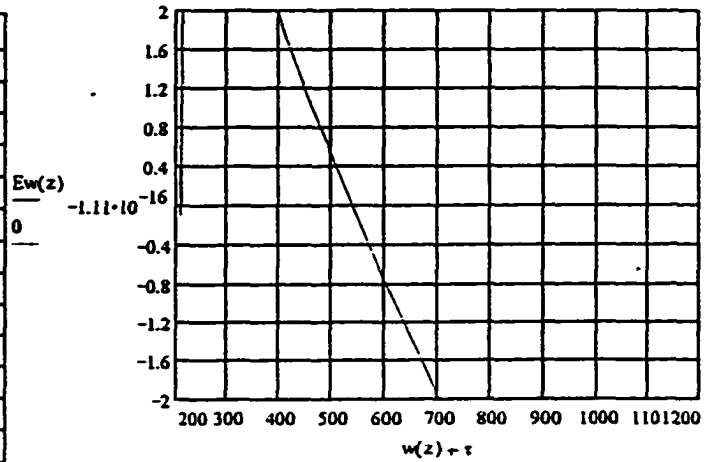
ELASTICITY OF LABOR SUPPLY

znorm10B.mcd

LFPR W
 $\text{cnorm}(z)w(z) + \tau E\Lambda(z)E\epsilon(z) Ew(z)$

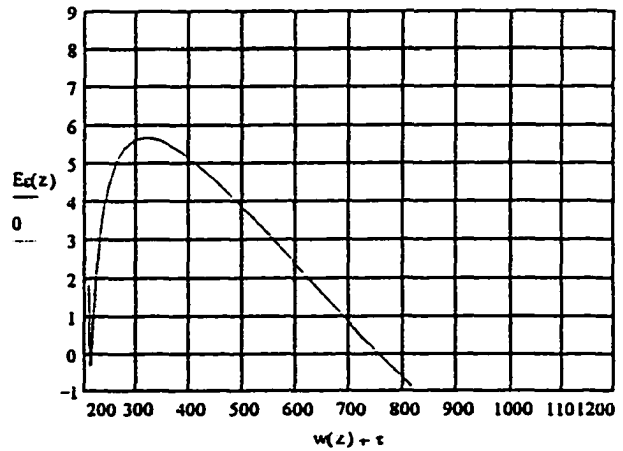
0.00135	209.6	0	1.84	0.38
0.00187	210.28	0.21	0.53	-0.1
0.00256	211.02	-0.12	-0.11	0.1
0.00347	211.82	-0.74	-0.33	0.67
0.00466	212.71	-1.48	-0.3	1.41
0.00621	213.68	-2.25	-0.12	2.23
0.0082	214.74	-2.99	0.14	3.03
0.01072	215.92	-3.67	0.45	3.78
0.0139	217.22	-4.26	0.79	4.45
0.01786	218.65	-4.75	1.14	5.04
0.02275	220.24	-5.15	1.49	5.54
0.02872	222.01	-5.45	1.84	5.95
0.03593	223.97	-5.65	2.18	6.26
0.04457	226.16	-5.77	2.52	6.48
0.0548	228.61	-5.79	2.84	6.63
0.06681	231.35	-5.73	3.16	6.69
0.08076	234.42	-5.6	3.47	6.69
0.0968	237.88	-5.41	3.77	6.62
0.11507	241.78	-5.14	4.06	6.5
0.13567	246.19	-4.83	4.33	6.32
0.15866	251.19	-4.47	4.58	6.1
0.18406	256.87	-4.06	4.82	5.84
0.21186	263.34	-3.62	5.04	5.55
0.24196	270.74	-3.16	5.23	5.23
0.27425	279.21	-2.68	5.4	4.89
0.30854	288.94	-2.18	5.53	4.53
0.34458	300.16	-1.68	5.62	4.16
0.38209	313.13	-1.18	5.67	3.78
0.42074	328.16	-0.69	5.66	3.38
0.46017	345.65	-0.21	5.6	2.98
0.5	366.06	0.25	5.46	2.56
0.53983	389.96	0.67	5.25	2.13
0.57926	418.04	1.07	4.94	1.67
0.61751	451.15	1.43	4.53	1.18
0.65542	490.33	1.76	3.99	0.64
0.69146	536.88	2.04	3.32	0.03
0.72575	592.39	2.27	2.49	-0.66
0.75804	658.84	2.46	1.48	-1.46
0.78814	738.71	2.6	0.27	-2.41
0.81594	835.11	2.69	-1.17	-3.54
0.84134	951.96	2.74	-2.86	-4.88
0.86433	1094.2	2.75	-4.83	-6.47
0.88493	1268.13	2.72	-7.12	-8.37
0.9032	1481.79	2.65	-9.75	-10.62
0.91924	1745.49	2.56	-12.77	-13.26
0.93319	2072.53	2.44	-16.21	-16.36

EXHIBIT 5-5 NONHOMOTHEIC CASE XV

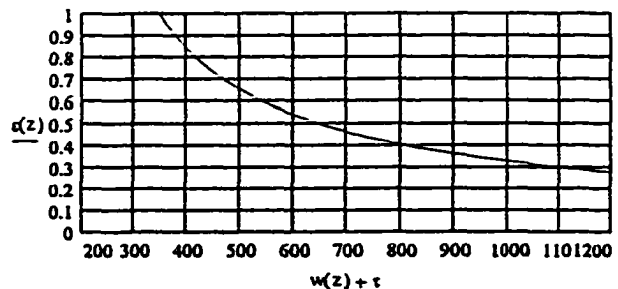


M-ELASTICITY OF THE REAL WAGE

$\mu = 5.35$ Median = 366.06 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.71$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.42$ $\delta = 0.31$ $L_\Omega = .70$
 EMP = 0.1 ERM = -0.15 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY

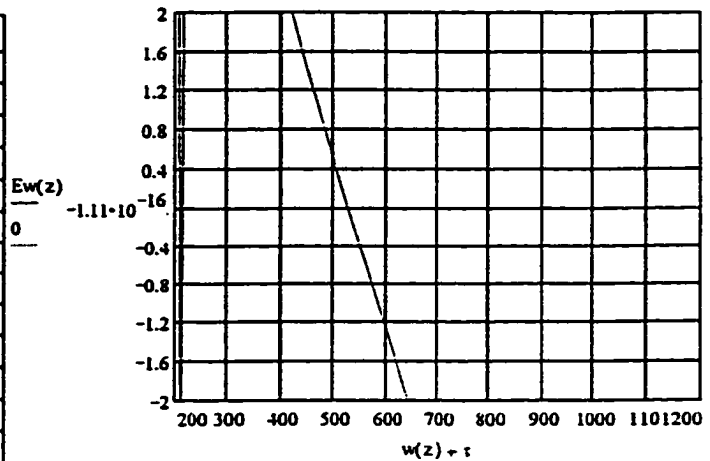


ELASTICITY OF LABOR SUPPLY

znorm11B.mcd

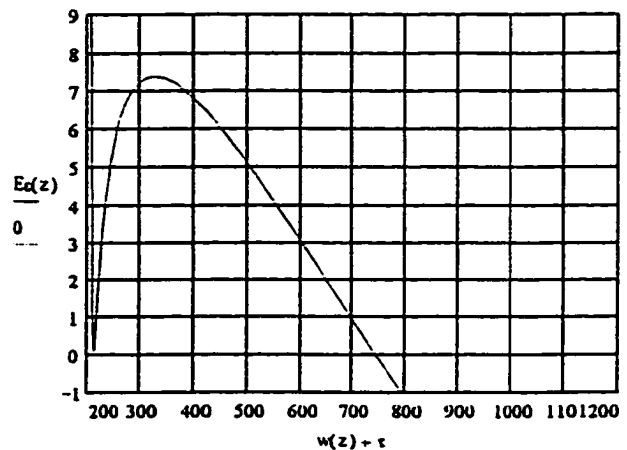
LFPR	W	$cnorm(z)w(z) + \tau EA(z)$	$Ez(z)$	$Ew(z)$
0.00135	209.6	0	10.36	2.12
0.00187	210.28	2.34	5.84	-1.12
0.00256	211.02	3.13	3.08	-2.47
0.00347	211.82	2.98	1.46	-2.66
0.00466	212.71	2.3	0.58	-2.17
0.00621	213.68	1.34	0.18	-1.3
0.0082	214.74	0.26	0.09	-0.24
0.01072	215.92	-0.83	0.21	0.88
0.0139	217.22	-1.88	0.46	1.99
0.01786	218.65	-2.83	0.79	3.03
0.02275	220.24	-3.66	1.17	3.97
0.02872	222.01	-4.37	1.59	4.8
0.03593	223.97	-4.94	2.03	5.51
0.04457	226.16	-5.38	2.48	6.09
0.0548	228.61	-5.69	2.94	6.56
0.06681	231.35	-5.88	3.39	6.9
0.08076	234.42	-5.94	3.84	7.14
0.0968	237.88	-5.89	4.28	7.27
0.11507	241.78	-5.74	4.71	7.31
0.13567	246.19	-5.49	5.12	7.25
0.15866	251.19	-5.16	5.51	7.12
0.18406	256.87	-4.75	5.89	6.93
0.21186	263.34	-4.29	6.23	6.67
0.24196	270.74	-3.76	6.55	6.35
0.27425	279.21	-3.2	6.82	6
0.30854	288.94	-2.6	7.05	5.6
0.34458	300.16	-1.99	7.23	5.18
0.38209	313.13	-1.36	7.34	4.72
0.42074	328.16	-0.73	7.38	4.25
0.46017	345.65	-0.11	7.34	3.74
0.5	366.06	0.48	7.2	3.22
0.53983	389.96	1.05	6.94	2.65
0.57926	418.04	1.59	6.55	2.05
0.61791	451.15	2.08	6.01	1.39
0.65542	490.33	2.52	5.29	0.66
0.69146	536.88	2.9	4.38	-0.17
0.72575	592.39	3.22	3.24	-1.13
0.75804	658.84	3.49	1.84	-2.25
0.78814	738.71	3.69	0.15	-3.58
0.81594	835.11	3.82	-1.87	-5.17
0.84134	951.96	3.9	-4.25	-7.07
0.86433	1094.2	3.91	-7.04	-9.34
0.88493	1268.13	3.87	-10.3	-12.04
0.9032	1481.79	3.78	-14.05	-15.26
0.91924	1745.49	3.65	-18.37	-19.05
0.93319	2072.53	3.48	-23.3	-23.49

EXHIBIT 5-6 NONHOMOTHETIC CASE XVI

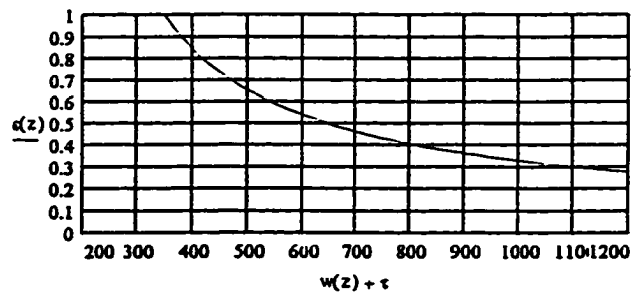


M-ELASTICITY OF THE REAL WAGE

$\mu = 5.35$ Median = 366.06 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.71$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.42$ $\delta = 0.31$ $L_{\Omega} = .70$
 EMP = 0.3 ERM = -0.1 $E\Omega M(z) = \delta \cdot \ln(w(z) + 1)$



M-ELASTICITY OF LABOR SUPPLY ELASTICITY



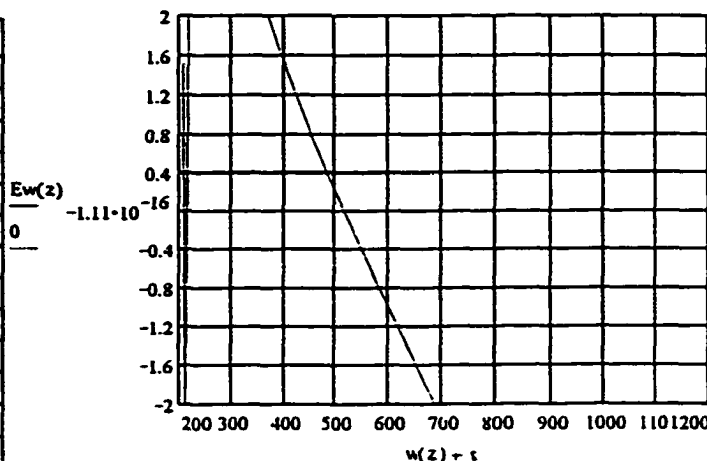
ELASTICITY OF LABOR SUPPLY

znorm11D.mcd

LFPR W
 $cnorm(z)w(z) + \tau EA(z)Ee(z) Ew(z)$

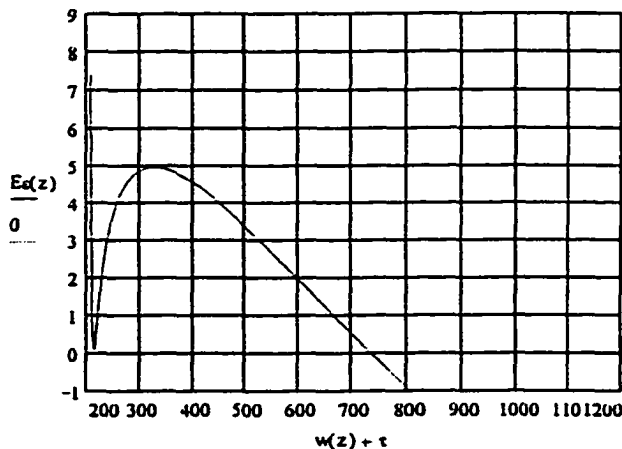
0.00135	209.6	0	7.42	1.52
0.00187	210.28	1.69	4.22	-0.81
0.00256	211.02	2.29	2.26	-1.81
0.00347	211.82	2.23	1.1	-1.99
0.00466	212.71	1.79	0.47	-1.69
0.00621	213.68	1.15	0.17	-1.11
0.0082	214.74	0.42	0.09	-0.4
0.01072	215.92	-0.32	0.15	0.36
0.0139	217.22	-1.03	0.31	1.11
0.01786	218.65	-1.68	0.53	1.82
0.02275	220.24	-2.26	0.78	2.46
0.02872	222.01	-2.75	1.06	3.03
0.03593	223.97	-3.15	1.36	3.52
0.04457	226.16	-3.45	1.66	3.93
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0.0968	237.88	-3.83	2.88	4.76
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0.27425	279.21	-2.05	4.6	3.94
0.30854	288.94	-1.65	4.75	3.67
0.34458	300.16	-1.24	4.87	3.39
0.38209	313.13	-0.82	4.94	3.08
0.42074	328.16	-0.4	4.97	2.76
0.46017	345.65	0.02	4.93	2.42
0.5	366.06	0.42	4.83	2.06
0.53983	389.96	0.8	4.65	1.68
0.57926	418.04	1.16	4.38	1.27
0.61791	451.15	1.49	4	0.82
0.65542	490.33	1.78	3.51	0.32
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0.72575	592.39	2.25	2.09	-0.9
0.75804	658.84	2.43	1.13	-1.67
0.78814	738.71	2.56	-0.03	-2.58
0.81594	835.11	2.65	-1.41	-3.67
0.84134	951.96	2.69	-3.05	-4.97
0.86433	1094.2	2.7	-4.97	-6.53
0.88493	1268.13	2.67	-7.2	-8.38
0.9032	1481.79	2.61	-9.77	-10.59
0.91924	1745.49	2.51	-12.73	-13.19
0.93319	2072.53	2.39	-16.11	-16.23

EXHIBIT 5-7 NONHOMOTHETIC CASE XVII

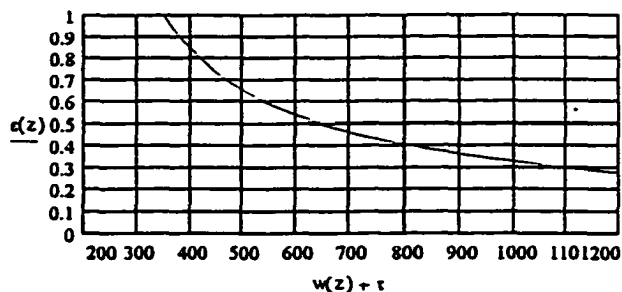


M-ELASTICITY OF THE REAL WAGE

$\mu = 5.35$ Median = 366.06 $\alpha = 300$ $L = 0.45$
 $\sigma^2 = 0.71$ Mean = 500 $\beta^2 = 2.601 \cdot 10^5$ $\gamma = 1.02$
 $\rho = 0.47$ $\delta = 0.31$ $L_D = .74$
 EMP = 0.3 ERM = -0.1 EQM(z) = $\delta \cdot \ln(w(z) + 1)$



M - ELASTICITY OF LABOR SUPPLY ELASTICITY



ELASTICITY OF LABOR SUPPLY

znorm11E.mcd

APPENDICES

APPENDIX A

GLOSSARY OF MATHEMATICAL SYMBOLS

NOTE: Partial derivatives are generally denoted by subscripts

- A** Jacobian matrix of excess demand functions (Chapters II and V)
- [A]** determinant of **A** (Chapters II and V; Appendix B)
- A** proportion of aggregate wealth held in bonds and equities (Chapter IV and Appendix D)
- a_i** proportion of wealth that an individual worker prefers to hold in bonds and equities.
- α** Mean reservation wage; mean of the Lognormal distribution
- B** quantity of government bonds held by the private sector; each paying annual interest of \$1 in perpetuity.
- β** variance of the Lognormal distribution
- c^d** consumption flow demand
- δ** calibration parameter; elasticity of ξ
- EDX** excess demand function for market X; X = G, M, L
- ε, ε_{lw}** real wage-elasticity of aggregate labor supply
- E** = $[1 + 1/\epsilon]^{-1} = 1/e$
- e** = $[1 + 1/\epsilon]$
- E_x[y]** elasticity operator; elasticity of y with respect to x.
- f^d** stock demand for real asset holdings
- f^s** stock supply of real assets
- Φ(w)** cumulative distribution function for reservation wages
- φ(z)** standardized Normal density function
- g(w)** inverse reservation wage function = $\psi^{-1}(v) = v$
- γ** coefficient of variation of the Lognormal distribution

- h** fixed hours in a work week (40)
H(f, m) subutility function for stock holdings of real money balances and risky financial assets
 η elasticity of demand (labor demand in Chapters IV & V)
 η_L elasticity of demand for leisure (Chapter III and IV)
 i^d investment flow demand = \dot{K}^d
 κ fraction of government bond holdings perceived as net financial wealth by households.
 \bar{K} fixed stock of physical capital
 ℓ^d demand for labor services
 ℓ^s supply of labor services
 $\Lambda(\cdot)$ Lognormal cumulative distribution function
 $\lambda(\cdot)$ Lognormal probability density function
L Lorenz (Gini) coefficient of concentration.
M nominal outside money stock
 $m = \frac{M}{P}$
 $m^d = \left(\frac{M}{P}\right)^d$ stock demand for real money balances
 μ log median of Λ
 ω reservation wage
 Ω Total wealth; financial and transfer wealth.
P aggregate price level for goods
 π flow of real profits on capital

- $\psi(v)$ reservation wage function
- r interest rate on earning assets
- R_A Measure of absolute risk aversion = U''/U'
- R_R Measure of relative risk aversion = $\ell U''/U'$
- ρ wealth-elasticity of relative risk aversion, R_R
- σ^2 dispersion parameter for Λ ; variance of $\log w$
- T transfer payments to households
- τ_0 base tax rate on income (exogenous)
- τ $\tau_0 + \frac{B}{P}$, endogenous tax rate on income (Chapter II)
minimum reservation wage (Chapter IV & V)
- $U(c, \ell)$ subutility function for consumption and leisure
- v real nonlabor income = V/P
- W nominal aggregate wage rate
- $w = \frac{W}{P}$
- ξ elasticity of $\psi(v)$
- y aggregate real income
- y^s aggregate real output (supply)
- $z = y - \tau$, disposable real income (Chapter II)
 $= (\ln w - \mu)/\sigma$ (Chapter IV and V)
- $\theta = \frac{B}{M}$ ratio of bonds to outside money

APPENDIX B TO CHAPTER II

B.1 Differentiation of Excess Demand Functions

(All symbols are defined in Appendix A)

The excess demand functions of the model are:

$$\begin{aligned} EDG &= c^d(z, \Omega) + i^d(r, y) + g - y^s(w) = 0 \\ EDM &= m^d(y^s(w), r, \Omega) - m^s = 0 \\ EDL &= \ell^d(w) - \ell^s(w, \Omega) = 0 \end{aligned}$$

where household real disposable income is $z = y^s(w) + \frac{B}{P} - \tau$
and $\tau = \tau_o + \frac{B}{P}$, and $w = \frac{W}{P}$.

Household real wealth is $\Omega = \frac{M}{P} + \frac{\kappa B}{rP} + \frac{\pi}{r}$, $0 \leq \kappa \leq 1$

Firm Profits are $\pi^* = y^s(\ell^d(w), \bar{k}) - w\ell^d(w) = \pi^*(w)$.
With subscripts denoting partial derivatives:

$$\begin{aligned} \pi_p^* &= y_l^s \ell_w^d \left(-\frac{W}{P^2} \right) - \left[w \ell_w^d \left(-\frac{W}{P^2} \right) + \ell^d \left(-\frac{W}{P^2} \right) \right] \\ &= -\frac{W}{P^2} [\ell_w^d [y_l^s - w] - \ell^d] = \frac{W}{P^2} \ell^d > 0 \\ \pi_w^* &= \frac{1}{P} [\ell_w^d (y_l^s - w) - \ell^d] = \frac{-\ell^d}{P} < 0 \\ \pi_r^* &= 0 \end{aligned}$$

$$\begin{aligned} \text{Then } \Omega_p &= -\frac{1}{P} \left[\frac{M}{P} + \frac{\kappa B}{rP} - \frac{W \ell^d}{rP} \right] > 0 \quad (W \ell^d \gg B + r \cdot M) \\ \Omega_w &= -\frac{\ell^d}{rP} < 0 \\ \Omega_r &= -\frac{1}{r} \left[\frac{\kappa B}{rP} + \frac{\pi}{r} \right] < 0 \end{aligned}$$

I also make the standard behavioral assumptions that

$$0 < c_z^d + i_y^d < 1, c_\Omega^d > 0; 0 < m_y^d < 1, m_r^d < 0, m_\Omega^d > 0; \ell_v^d < 0, \ell_w^s > 0, \ell_\Omega^s < 0$$

Differentiating the model:

$$EDG_P = - \left(1 - c_z^d - i_y^d \right) y_w^s \left(-\frac{W}{P^2} \right) + c_\Omega^d \Omega_P < 0$$

$$EDG_r = c_\Omega^d \Omega_r + i_r^d < 0$$

$$EDG_W = - \left(1 - c_z^d - i_y^d \right) y_w^s \frac{1}{P} + c_\Omega^d \Omega_w > 0$$

To sign EDG_P and EDG_W , assume $(1 - c_z^d - i_y^d) > 0$ and that direct effects of P and W are greater than the indirect effects via Ω . Then $EDG_P < 0$, $EDG_W > 0$.

$$EDM_P = m_y^d y_w^s \left(-\frac{W}{P^2} \right) + m_\Omega^d \Omega_P + \frac{M}{P^2} > 0$$

$$EDM_r = m_r^d + m_\Omega^d \Omega_r < 0$$

$$EDM_W = m_y^d y_w^s \frac{1}{P} + m_\Omega^d \Omega_w < 0$$

$$EDL_P = (\ell_v^d - \ell_w^s) \left(-\frac{W}{P^2} \right) - \ell_\Omega^s \Omega_P > 0$$

$$EDL_r = -\ell_\Omega^s \Omega_r < 0$$

$$EDL_W = \frac{1}{P} (\ell_v^d - \ell_w^s) - \ell_\Omega^s \Omega_w < 0$$

$$EDG_{\tau_o} = -c_z^d, \quad EDM_{\tau_o} = EDL_{\tau_o} = 0$$

$$EDG_g = +1, \quad EDM_g = EDL_g = 0$$

$$EDG_M = \frac{1}{P} c_\Omega^d, \quad EDM_M = (m_\Omega^d - 1) \frac{1}{P}, \quad EDL_M = -\frac{1}{P} \ell_\Omega^s$$

$$EDG_B = \frac{\kappa}{IP} c_\Omega^d; \quad EDM_B = \frac{\kappa}{IP} m_\Omega^d; \quad EDL_B = -\frac{\kappa}{IP} \ell_\Omega^s$$

The model in total differential form is $\text{Adv} = \text{Gdu}$, where:

$$\mathbf{A} \cdot d\mathbf{v} = \begin{bmatrix} \overset{(-)}{\text{EDG}}_P & \overset{(-)}{\text{EDG}}_r & \overset{(+)}{\text{EDG}}_W \\ \overset{(+)}{\text{EDM}}_P & \overset{(-)}{\text{EDM}}_r & \overset{(-)}{\text{EDM}}_W \\ \overset{(+)}{\text{EDL}}_P & \overset{(-)}{\text{EDL}}_r & \overset{(-)}{\text{EDL}}_W \end{bmatrix} \begin{bmatrix} dP \\ dr \\ dW \end{bmatrix}$$

(B1)

$$\mathbf{G} \cdot d\mathbf{u} = \begin{bmatrix} C_z^d & -1 & -\frac{1}{P} C_Q^d & -\frac{\kappa}{rP} C_Q^d \\ 0 & 0 & \frac{1}{P} (1 - m_Q^d) & -\frac{\kappa}{rP} m_Q^d \\ 0 & 0 & \frac{1}{P} l_Q^s & \frac{\kappa}{rP} l_Q^s \end{bmatrix} \begin{bmatrix} dt_0 \\ dg \\ dM \\ dB \end{bmatrix}$$

B.2 Stability Analysis

Assume that the trajectories of the endogenous variables P , r , W in the neighborhood of an equilibrium point P^* , r^* , W^* are governed by:

$$\dot{P} = k_1 \text{EDG}(P, r, W)$$

$$\dot{r} = k_2 \text{EDM}(P, r, W)$$

$$\dot{W} = k_3 \text{EDL}(P, r, W)$$

where $k_1, k_2, k_3 > 0$ are arbitrary constants. Linearizing the system (A1) in the neighborhood, I obtain:

$$\boldsymbol{\beta} = \begin{bmatrix} \overset{(-)}{k_1 \text{EDG}}_P & \overset{(-)}{k_1 \text{EDG}}_r & \overset{(+)}{k_1 \text{EDG}}_W \\ \overset{(+)}{k_2 \text{EDM}}_P & \overset{(-)}{k_2 \text{EDM}}_r & \overset{(-)}{k_2 \text{EDM}}_W \\ \overset{(+)}{k_3 \text{EDL}}_P & \overset{(-)}{k_3 \text{EDL}}_r & \overset{(-)}{k_3 \text{EDL}}_W \end{bmatrix} = \begin{bmatrix} \overset{(-)}{k_1 a_{11}} & \overset{(-)}{k_1 a_{12}} & \overset{(+)}{k_1 a_{13}} \\ \overset{(+)}{k_2 a_{21}} & \overset{(-)}{k_2 a_{22}} & \overset{(-)}{k_2 a_{23}} \\ \overset{(+)}{k_3 a_{31}} & \overset{(-)}{k_3 a_{32}} & \overset{(-)}{k_3 a_{33}} \end{bmatrix}$$

The necessary and sufficient conditions for the system to be stable in the neighborhood of (P^*, r^*, W^*) are: (Routh-Hurwitz)

1. $\text{tr } \boldsymbol{\beta} < 0$ which is satisfied

$$2. \quad k_1 k_2 \begin{vmatrix} \overset{(-)}{a_{11}} & \overset{(-)}{a_{12}} \\ \overset{(+)}{a_{21}} & \overset{(-)}{a_{22}} \end{vmatrix} + k_1 k_3 \begin{vmatrix} \overset{(-)}{a_{11}} & \overset{(+)}{a_{13}} \\ \overset{(+)}{a_{31}} & \overset{(-)}{a_{33}} \end{vmatrix} + k_2 k_3 \begin{vmatrix} \overset{(-)}{a_{22}} & \overset{(-)}{a_{23}} \\ \overset{(-)}{a_{32}} & \overset{(-)}{a_{33}} \end{vmatrix} > 0 \text{ for all } k_1, k_2, k_3$$

$\quad \quad \quad |\boldsymbol{\beta}|_{12} \quad \quad \quad |\boldsymbol{\beta}|_{13} \quad \quad \quad |\boldsymbol{\beta}|_{23}$

Alternatively, it follows from $|\beta_{23}| > 0$ that $A-F < 0$, and if $B-E < 0$ it must also hold for stability that $C-D < 0$, or:

$$\begin{array}{c} (+) \quad (+) \quad (-) \quad (+) \quad (-) \\ EDG_N [EDM_P \cdot EDL_r - EDL_P \cdot EDM_r] < 0 \\ \frac{EDM_P}{EDM_r} < \frac{EDL_P}{EDL_r} \end{array}$$

$$\text{or} \quad \frac{dr}{dP} \Big|_{M^d=M^s} > \frac{dr}{dP} \Big|_{l^d=l^s}$$

In addition, it easy to show that $\frac{dr}{dP} \Big|_{y^d=y^s} < 0$ unconditionally. With these sign conditions, $|\beta| < 0$ and the third Routh-Hurwicz condition is met.

To summarize the stability conditions:

$$(i) \quad 0 < \frac{dW}{dP} \Big|_{l^d=l^s} < \frac{dW}{dP} \Big|_{y^d=y^s} < \frac{dW}{dP} \Big|_{M^d=M^s}$$

In W - P space ($dr=0$) the slope of the money market equilibrium locus is greater than the slope of the goods market locus, which in turn is greater than the slope of the labor market locus. All three slopes are positive.

(See Figure B-1(a))

$$(ii) \quad \frac{dW}{dr} \Big|_{M^d=M^s} < \frac{dW}{dr} \Big|_{l^d=l^s}$$

In W - r space ($dP=0$) the slope of the labor market equilibrium locus is greater than the slope of the money market equilibrium locus. Both slopes are negative.

(See Figure B-1(b)). It is also true that:

$$\frac{dW}{dr} \Big|_{y^d=y^s} > \frac{dW}{dr} \Big|_{l^d=l^s}$$

but it is not an imposed stability condition.

$$(iii) \quad \frac{dr}{dP} \Big|_{l^d=l^s} < \frac{dr}{dP} \Big|_{M^d=M^s}$$

In r - P space ($dW=0$) the slope of the labor market equilibrium locus is less than the slope of the money market locus. Both slopes are positive. (See Figure B-1(c)). The following also holds but is not an imposed stability condition:

$$\frac{dr}{dP} \Big|_{y^d=y^s} < \frac{dr}{dP} \Big|_{M^d=M^s}$$

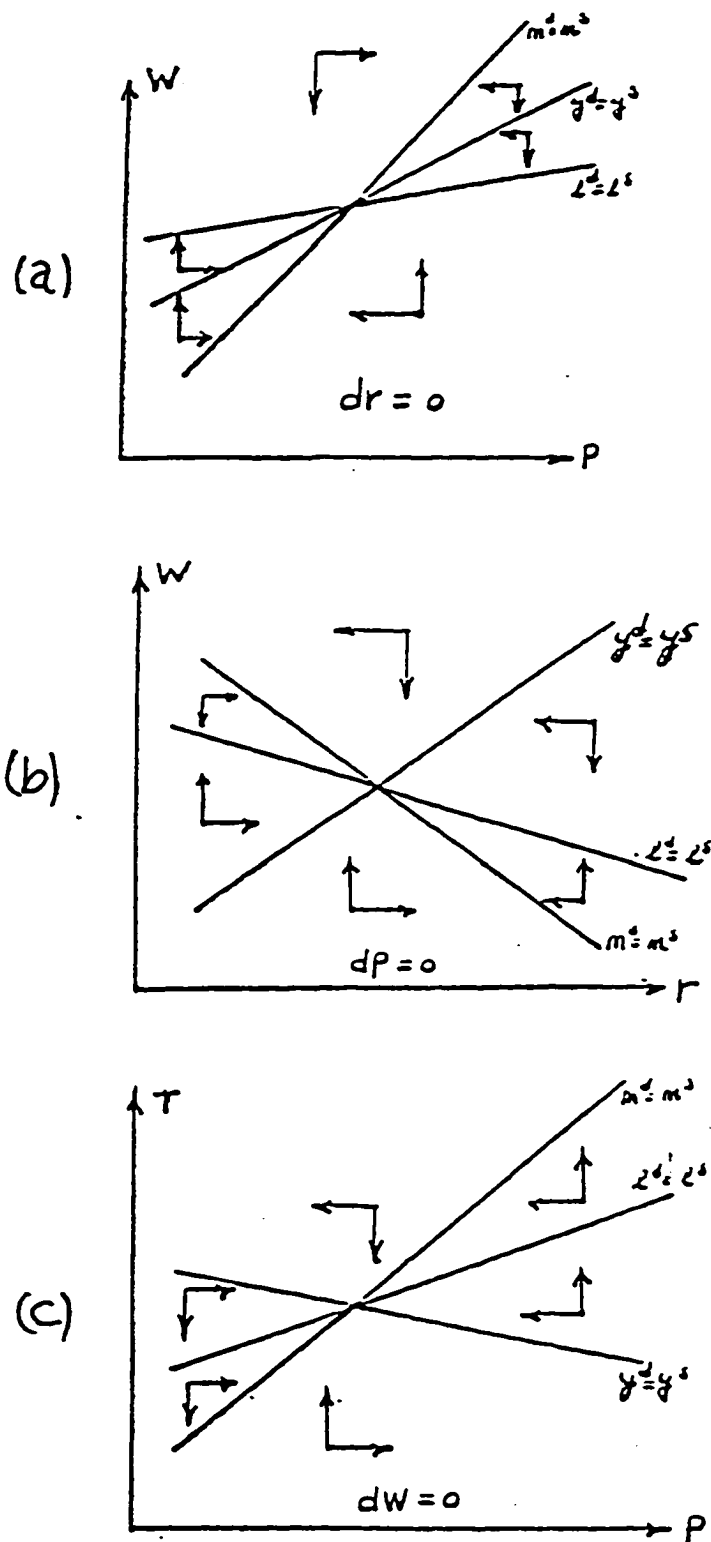


Figure B-1. Equilibrium Loci for the three markets: Goods, Money and Labor

These three conditions, along with the conventional assumption that $0 < (c_z^d + i_y^d) < 1$ and $EDG_p < 0$, $EDG_w > 0$, are sufficient for stability of the linearized system (A1).

B.3 Comparative Static Analysis of Open-Market Operations

A one-time purchase of bonds for outside money is specified by: $dM = -\frac{dB}{r}$, $dr_o = dg = 0$. Applying Cramer's Rule to (A1) and using the co-factor expansion of the determinant along the substituted column vector, I obtain:

$$\begin{aligned} \frac{dP}{dM} = \frac{1}{|A|} & \left\{ \left[-\frac{1}{P} c_{\Omega}^d (1-\kappa) \right] \left[\overset{(-)}{EDM}_r \cdot \overset{(a)}{\overset{(-)}{EDL}}_w - \overset{(-)}{EDL}_r \cdot \overset{(-)}{EDM}_w \right] + \right. \\ & - \left[\frac{1}{P} (1-m_{\Omega}^d (1-\kappa)) \right] \left[\overset{(-)}{EDG}_r \cdot \overset{(-)}{EDL}_w - \overset{(-)}{EDL}_r \cdot \overset{(+)}{EDG}_w \right] + \\ & \left. + \left[\frac{1}{P} \ell_{\Omega}^s (1-\kappa) \right] \left[\overset{(-)}{EDG}_r \cdot \overset{(-)}{EDM}_w - \overset{(-)}{EDM}_r \cdot \overset{(+)}{EDG}_w \right] \right\} \end{aligned} \quad (A2)$$

Stability condition (i) guarantees that (a) > 0 , and so $\frac{dP}{dM} > 0$ unconditionally. It is evident that $\frac{dP}{dM}$ is a minimum when $\kappa = 1$, a maximum when $\kappa = 0$.

$$\begin{aligned} \frac{dr}{dM} = \frac{1}{|A|} & \left\{ \left[\frac{1}{P} c_{\Omega}^d (1-\kappa) \right] \left[\overset{(+)}{EDM}_p \cdot \overset{(b)}{\overset{(-)}{EDL}}_w - \overset{(+)}{EDL}_p \cdot \overset{(-)}{EDM}_w \right] + \right. \\ & + \left[\frac{1}{P} (1-m_{\Omega}^d (1-\kappa)) \right] \left[\overset{(-)}{EDG}_p \cdot \overset{(c)}{\overset{(-)}{EDL}}_w - \overset{(+)}{EDL}_p \cdot \overset{(+)}{EDG}_w \right] + \\ & \left. - \left[\frac{1}{P} \ell_{\Omega}^s (1-\kappa) \right] \left[\overset{(-)}{EDG}_p \cdot \overset{(d)}{\overset{(-)}{EDM}}_w - \overset{(+)}{EDM}_p \cdot \overset{(+)}{EDG}_w \right] \right\} \end{aligned} \quad (A3)$$

Stability condition (i) guarantees that (b) > 0, (c) > 0, and also implies (d) < 0, so it appears that the sign of $\frac{dr}{dM}$ might depend on the value of κ . For $\kappa=1$, $\frac{dr}{dM} < 0$.

Expansion of (A3) and cancellation of terms produces the result that for $0 \leq \kappa \leq 1$:

$$\frac{dr}{dM} = \frac{-1}{P|A|} \left[\kappa \frac{1}{P} \left[\frac{M}{P} + \frac{B}{rP} \right] \left[c_{\Omega}^d (\ell_W^d - \ell_W^s) \frac{1}{P} + \ell_{\Omega}^s \left[-(1 - c_z^d - i_y^d) y_W^s \frac{1}{P} \right] \right] \right] < 0$$

$$\begin{aligned} \frac{dW}{dM} = \frac{1}{|A|} \left\{ \left[-\frac{1}{P} c_{\Omega}^d (1-\kappa) \right] \left[\overset{(e)}{EDM_P} \cdot \overset{(e)}{EDL_r} - \overset{(e)}{EDL_P} \cdot \overset{(e)}{EDM_r} \right] + \right. \\ \left. - \left[\frac{1}{P} (1 - m_{\Omega}^d (1-\kappa)) \right] \left[\overset{(f)}{EDG_P} \cdot \overset{(f)}{EDL_r} - \overset{(f)}{EDL_P} \cdot \overset{(f)}{EDG_r} \right] + \right. \\ \left. + \left[\frac{1}{P} \ell_{\Omega}^s (1-\kappa) \right] \left[\overset{(g)}{EDG_P} \cdot \overset{(g)}{EDM_r} - \overset{(g)}{EDM_P} \cdot \overset{(g)}{EDG_r} \right] \right\} \end{aligned} \quad (A4)$$

(e) < 0 from stability condition (iii) and this results in a negative contribution to $\frac{dW}{dM}$. Terms (f) and (g) are unambiguously > 0 and make a positive contribution to $\frac{dW}{dM}$.

If $\kappa = 1$, only (f) has an effect on $\frac{dW}{dM}$ and $\frac{dW}{dM} > 0$.

For $\kappa=1$, $\frac{dW}{dM} > 0$.

Expanding (A4) and cancelling terms produces the result:

$$\frac{dW}{dM} = \frac{-1}{P|A|} \left\{ \left[-c_{\Omega}^d (\Omega_r + m_r^d (1-\kappa)) - i_r^d [1 - m_{\Omega}^d (1-\kappa)] \right] \overset{(h)}{EDL_P} \right. \\ \left. - \ell_{\Omega}^s [\Omega_r + m_r^d (1-\kappa)] \overset{(h)}{EDG_P} \right. \\ \left. + \ell_{\Omega}^s i_r^d (1-\kappa) \overset{(h)}{EDM_P} \right\} > 0, \quad 0 \leq \kappa \leq 1$$

B.4 Non-Neutrality Results

Real Wage

For $0 \leq \kappa \leq 1$, $\frac{dP}{dM} > 0$, $\frac{dr}{dM} < 0$, $\frac{dW}{dM} > 0$, and $\frac{d\left(\frac{W}{P}\right)}{dM} = \frac{1}{P} \left[\frac{dW}{dM} - \frac{W}{P} \frac{dP}{dM} \right]$

Expanding $\frac{dW}{dM}$ and $\frac{dP}{dM}$, it can be shown that, if $\kappa = 1$,

$$\frac{d\left(\frac{W}{P}\right)}{dM} = -\frac{1}{P^2 |A|} \left[-\overset{(-)}{\ell}_\Omega^g \left[\overset{(-)}{\Omega}_P + \frac{\overset{(-)}{W}}{P} \overset{(-)}{\Omega}_N \right] \left[-\overset{(-)}{i}_r^d \right] \right] < 0 \quad \kappa=1$$

because $\overset{(-)}{\Omega}_P + \frac{\overset{(-)}{W}}{P} \overset{(-)}{\Omega}_N = -\frac{1}{P} \left[\frac{M}{P} + \frac{\kappa B}{rP} \right] < 0$.

The real wage declines, i.e., $\frac{dW}{W} < \frac{dP}{P}$. By subtracting $\frac{dW}{dM}$ from $\frac{W}{P} \frac{dP}{dM}$ within each corresponding co-factor expansion,

it can be proven that $\frac{d\left(\frac{W}{P}\right)}{dM} < 0$ for $0 < \kappa \leq 1$ unambiguously,

provided that $\frac{d\Omega}{dM} \Big|_{y^d=y^s} \leq \frac{d\Omega}{dM} \Big|_{m^d=m^s}$ i. e., the wealth effect in the money market is not less than the wealth effect in the goods market.

Real Wealth

For $\kappa = 1$, $\frac{d\Omega}{dM} = \overset{(+)}{\Omega}_P \frac{dP}{dM} + \overset{(-)}{\Omega}_r \frac{dr}{dM} + \overset{(-)}{\Omega}_W \frac{dW}{dM} < 0$

The sign of $\frac{d\Omega}{dM}$ is not obvious even for $\kappa = 1$, but by expanding the derivatives and collecting terms one obtains the result that $\overset{(-)}{\Omega}_P \frac{dP}{dM} + \overset{(-)}{\Omega}_r \frac{dr}{dM} < 0$, which makes $\frac{d\Omega}{dM} < 0$ and, for the case $\kappa = 1$:

$$\frac{d\Omega}{dM} = \frac{-1}{P^2 |A|} \left(\overset{(-)}{\Omega}_P + \frac{\overset{(-)}{W}}{P} \overset{(-)}{\Omega}_N \right) \left[\overset{(-)}{i}_r^d (\overset{(-)}{\ell}_N^d - \overset{(-)}{\ell}_N^s) \right] < 0 \quad \kappa=1$$

Since $\overset{(-)}{\Omega}_P + \frac{\overset{(-)}{W}}{P} \overset{(-)}{\Omega}_N = -\frac{1}{P} \left[\frac{M}{P} + \frac{B}{rP} \right]$. The sign of $\frac{d\Omega}{dM}$ depends only on the basic behavioral assumptions $\overset{(-)}{i}_r^d < 0$, $\overset{(-)}{\ell}_W^d < 0$, $\overset{(-)}{\ell}_W^s > 0$ and the stability conditions which guarantee that $|A| < 0$.

$$\text{Finally, } \frac{dl^s}{dM} = l_w^s \frac{d\left(\frac{W}{P}\right)}{dM} + l_Q^s \frac{dQ}{dM}.$$

Expanding this expression, collecting and cancelling terms, I find that

$$\frac{dl^s}{dM} = - \frac{1}{P^2 |A|} \left[\left(\Omega_P + \frac{W}{P} \Omega_w \right) (l_Q^s \cdot i_r^d) (l_w^d) \right] > 0$$

labor supply increases due to the wealth effect, even though $\frac{W}{P}$ has declined. Since the labor market always clears in this model, $l^d = l^s$ and the new equilibrium is at a higher level of employment and output. A causal connection between money and output is established via a wealth effect on labor supply.

APPENDIX C TO CHAPTER III

C1. Monopsonistic Labor Markets in Search Equilibrium

The search theory of labor markets differs from the static supply-demand model in two fundamental ways: (1) an emphasis on matching workers to job vacancies, implying that the relevant labor supply to a firm is the number of workers rather than worker-hours, with individual labor supply assumed to be fixed; (2) the assumption that firms set wages (or make wage offers) which workers either take or reject¹. This literature also deals with markets which are in a state of adjustment in which disequilibrium trading takes place. Arrow (1958) pointed out that, in the absence of an auctioneer, price-setting necessarily defaults to agents and price-setting is the *de facto* exercise of market power. Thus, dynamic monopsony power is implicit in most disequilibrium search models of the labor market.

The sources of monopsony power in these models are typically asymmetric information and unequal trading costs. Participants in the labor force have imperfect information about the existence and location of job vacancies and/or the

¹Howitt and McAfee [1987] and Howitt [1988] describe search models in which wages are determined by bargaining between workers and firms. The equilibrium solution of the bilateral monopoly is indeterminate unless an arbitrary surplus-sharing rule is introduced (McDonald and Solow [1981]). The monopsony model investigated here could be generalized by introducing a sharing rule that is endogenous and procyclical with respect to labor's share.

distribution of wage offers by firms. Trading opportunities can be improved through search, at a cost of household resources, including time and foregone wages.

The existence of a marginal cost-benefit tradeoff in search models implies an optimum search or stopping rule. Prospective hires are imperfectly informed and will turn down wage offers that are too low relative to their expectations. Individual firms seeking to expand employment find that they can increase the rate of new hires by raising their wage offers, i. e., in these models, firms have dynamic monopsony power over the flow of net hires during the employment adjustment process. Thus, firms are able to regulate the flow of new hires and quits, and consequently the level of equilibrium employment, by raising or lowering their nominal wage offers. The supply-side response is determined by the wage-elasticity of labor supply, in or out of equilibrium.

In Mortensen's (1970) seminal model, the long-run labor supply curve is perfectly elastic, but the equilibrium real wage is less than the marginal product of labor by the cost to the firm of financing the marginal cost of hiring one more worker. Thus Mortensen's model retains a markdown in equilibrium, but one that is smaller than in the static monopsony case².

²In a timeless static model, financing the marginal labor cost is out of the question so it must be included in current expenditure flow, i. e., the incremental higher wage is paid to all employed workers in the absence of wage discrimination.

Diamond [1971] showed in a consumer random search model that if profit-maximizing sellers fully exploit their informational advantage against consumers, the optimum price for all sellers in equilibrium is the static monopoly price. Although this model deals with monopoly power, the equilibrium result can be carried over to the case of pure monopsony by a symmetrical construction in which firms as buyers of labor services have more information or lower trading costs than workers.

Baily [1975] showed that the stationary solution to the intertemporal optimization problem of the representative firm with dynamic monopsony power is similar to the static monopsony equilibrium, with a markdown W/ϵ_d where W is the stationary wage and ϵ_d is the intertemporal elasticity of labor supply to the firm³. In fact, countercyclical monopsony power is implicit in Baily's model, although Baily did not point out this feature. The operative assumptions of Baily's model are: (a) the firm is a price-taker in both the goods and capital markets ; (b) the firm can increase its flow supply of labor services by raising its relative wage offer according to a recruiting function $(dL^s/dt)/L^s = g(W)$, $g' > 0$; (c) there are decreasing returns to successive wage increases in recruiting labor, i. e. $g'' < 0$; and (d) firms maximize profits.

³This result was confirmed by Pissarides [1976, Chap. 4, p. 56].

Baily derives the optimum dynamic wage-setting path as:

$$PF_L = W[1 + 1/\epsilon_d] \quad (C1.1)$$

Here PF_L is the marginal revenue product of labor, W is the money wage, $\epsilon_d = Wg'/(r-g)$ is the dynamic elasticity of labor supply, g is the recruiting function and r is the intertemporal discount rate. Along the adjustment path the markdown is W/ϵ_d and the corresponding measure of dynamic monopsony power is $1/\epsilon_d$.

In Baily's model, monopsony power varies with employment due to the properties of the recruiting function $g(W)$. Since the value of g is positive in expansions, negative in contractions and zero in steady-state employment, countercyclical monopsony power is possible in Baily's model. With steady-state output and employment, $g = 0$, and the elasticity of labor supply in steady-state equilibrium is:⁴

$$\epsilon_{ss} = W^*g'(W^*)/r \quad (C1.2)$$

It is straightforward to show in Baily's model that $\mathcal{E}(\epsilon_d)$, the elasticity of ϵ_d with respect to the wage, will be positive along the optimum adjustment path from one steady-state equilibrium to another, if and only if:

$$\mathcal{E}_w[g'] = |Wg''/g'| < (1 + \epsilon_d) \quad (C1.3)$$

This result holds also at the equilibrium wage W^* where $g(W^*) = 0$. Thus, a second-order constraint on the recruiting function $g(W)$, restricting the rate of diminishing returns, is

⁴Baily's steady-state solution to (C1.1) is identical to that of Mortenson, with the implicit steady-state elasticity of labor supply equal to $W^*g'(W^*)/r$, where W^* is the steady-state wage.

required to ensure procyclical elasticity of labor supply in equilibrium⁵. The important conclusion from Baily's paper is that monopsony power, or a wage markdown, is retained in long-run equilibrium and monopsony power in equilibrium can be procyclical.

Another approach to making market power endogenous exploits externalities from trading in thin and thick markets. Diamond [1982a], Pissarides [1976, 1988] and Andolfatto [1996] developed search equilibrium models in which the returns from trade depend on the volume of trading, e. g, the number of job vacancies or the number of workers searching. The market coordinating mechanism is search and trading by agents in lieu of an auctioneer. The steady-state equilibrium level of employment is supported by an underlying search process.

These models can have multiple equilibria as a consequence of increasing returns in the search technology, and typically have Keynesian features even though they are equilibrium models. The labor market is modeled as a bilateral monopoly, with bargaining power over wages shared between firms and workers in inverse proportion to their relative trading costs. Monopsony would be the case where workers have significantly higher trading costs than firms; monopsony power would be cyclical if the cost ratio depended on the labor

⁵These results were derived by the author from Baily's steady-state equations. They are mentioned here because similar relationships will be found in the heterogeneous reservation wage model presented in Chapter IV.

force participation rate. This potential source of cyclical monopsony power is not utilized in this thesis.

C2. Elasticity of Labor Supply in the Standard Monopsony Model

The individual labor supply function is derived from the solution to the consumers' consumption-leisure choice problem⁶:

$$\begin{aligned} \text{Max } U(c, \ell) \quad \text{subject to } Pc + W\ell \leq Y = V + WT \quad (\text{C2.1}) \\ c \geq 0, \quad 0 \leq \ell \leq T, \quad V \geq 0 \end{aligned}$$

where U is a quasi-concave utility function, c is the quantity of a composite consumption good with price index P , ℓ is leisure time with nominal wage price W , and Y is the sum of the individual's endowments of nominal nonlabor income V and the value of total time T in the period. The interior solution to (C2.1) yields Marshallian demand functions for consumption and leisure as functions of prices and income:

$$\begin{aligned} c^d &= c^d(W, P, V) \quad (\text{C2.2}) \\ \ell^d &= \ell^d(W, P, V) \end{aligned}$$

Because the budget constraint in (C2.1) is linear and continuous, these demand functions are homogeneous of degree zero in W , P and V . Euler's Theorem applied to leisure demand gives:

⁶This basic exposition follows Killingsworth (1983)

$$W \frac{\partial \ell^d}{\partial W} + P \frac{\partial \ell^d}{\partial P} + V \frac{\partial \ell^d}{\partial V} = 0 \quad (\text{C2.3})$$

The homogeneity property also implies that only relative prices matter, and using the product price P as numeraire we may express the demand for leisure in real terms as:

$$\ell^d = \ell^d(w, 1, v) = \ell^d(w, v) \quad (\text{C2.4})$$

where $w = W/P$ and $v = V/P$. Without loss of generality, I will set $T = 1$ and interpret ℓ^d as the fraction of time spent in nonmarket activities each period. Individual labor supply is then the mathematical complement to leisure demand:

$$\ell^s(w, v) = 1 - \ell^d(w, v) \quad (\text{C2.5})$$

With the behavioral assumptions made thus far and the functional form of $U(c, \ell)$ unspecified, the signs of all partial derivatives of the demand equations in (C2.2) are ambiguous. If we assume, as is commonly done, that both consumption and leisure are normal goods and are gross substitutes, then

$$\partial c^d / \partial v > 0, \quad \partial \ell^d / \partial v > 0, \quad \partial c^d / \partial w > 0, \quad \text{and} \quad \partial \ell^d / \partial p > 0.$$

If we make the additional assumption, also commonly done, that substitution effects dominate income effects in both consumption and leisure, then it follows from the Slutsky equation that

$$\partial \ell^d / \partial w < 0, \quad \partial \ell^d / \partial p > 0, \quad \partial c^d / \partial w > 0, \quad \text{and} \quad \partial c^d / \partial p < 0.$$

These additional assumptions imply a negative sign for the wage-elasticity of leisure demand:

$$\eta_{\ell} = \frac{\partial \ell^d}{\partial W} \frac{W}{\ell^d} = \frac{\partial \ell^d}{\partial W} \frac{W}{\ell^d} < 0 \quad (\text{C2.6})$$

Since labor supply is the mathematical complement of leisure demand, its corresponding first partial derivatives have opposite signs from leisure demand and its wage-elasticity is unambiguously positive: $\epsilon_{\ell w} > 0$.

Without making additional specifications, the second partial derivatives of the demand functions are ambiguous. Ordinarily this is of little concern in the analysis of consumer demand and labor supply, but in the present instance we are interested in the direction of endogenous change in $\epsilon_{\ell w}$, which is a second order effect. If, however, we make the additional (and crucial) assumption of homothetic preferences, then the signs of all second partial derivatives can be determined.

First, a homothetic utility function $U(c, \ell)$ implies a linear income expansion path and Engel curves that are rays through the origin, i. e., the income elasticities of leisure demand and consumption are unity. Second, the marginal rate of substitution $U_{\ell}/U_c = W/P$ is constant along any income expansion path; this in turn implies that the point elasticities of the demand functions of (C2.2) are invariant with respect to changes in income alone. Thus, shifts in consumption and leisure demand due to changes in real income alone, with relative price W/P constant, will be iso-elastic, i. e.,

$$\partial(\eta_w)/\partial v = \partial(\eta_\ell)/\partial v = \partial(\eta_c)/\partial v = 0$$

This, however, will not be the case for labor supply, the mathematical complement of leisure demand. From (C2.5), the elasticity of labor supply is related to the elasticity of leisure demand as follows:

$$\epsilon_{\ell w} = \left[\frac{-\ell^d}{(1-\ell^d)} \right] \eta_\ell > 0 \quad (\text{C2.7})$$

Partial differentiation of (C2.7) with respect to v gives the result:

$$\frac{\partial \epsilon}{\partial v} = \left[\frac{-\ell^d}{(1-\ell^d)} \right] \frac{\partial \eta_\ell}{\partial v} - \frac{\eta_\ell}{(1-\ell^d)} \left[\frac{\frac{\partial \ell^d}{\partial v}}{(1-\ell^d)} \right] > 0 \quad (\text{C2.8})$$

The positive sign follows from the homotheticity property which makes $\partial \eta / \partial v = 0$, the assumption that $\eta_\ell < 0$, and that leisure is a normal good. This result can also be obtained by differentiating $\epsilon_{\ell w}$ directly⁷:

⁷The homotheticity assumption enables all second partial derivatives of the demand equations to be signed. Euler's and Young's theorems and the property of unit income elasticity can be used to derive for labor supply:

$$\partial^2 \ell^s / \partial w^2 < 0, \quad \partial^2 \ell^s / \partial v^2 \geq 0, \quad \partial^2 \ell^s / \partial v \partial w > 0$$

These signs, along with those of the first partials, can be used to obtain the result in (C2.9). The corresponding first partials of ℓ^d have the exact opposite signs.

$$\frac{\partial \epsilon}{\partial v} = w \left[\frac{\ell^s \frac{\partial^2 \ell^s}{\partial w \partial v} - \frac{\partial \ell^s}{\partial w} \frac{\partial \ell^s}{\partial v}}{(\ell^s)^2} \right] > 0 \quad (\text{C2.9})$$

The positive sign is determined by the fact that $\partial \ell^s / \partial v < 0$ and $\partial^2 \ell^s / \partial v \partial w > 0$.

The economic interpretation of $\partial \epsilon / \partial v > 0$ is that an individual with high nonlabor income occupies a higher indifference curve and therefore has higher absolute demands for both consumption and leisure. With homothetic preferences, the wage-elasticity of leisure demand is constant along any income expansion path, but the wage-elasticity of labor supply increases with income because the total available time T in the period is fixed and the desired ratio of leisure time to work time increases with income.

In Equation (C2.7), η_ℓ is constant but the ratio $\ell^d / (1 - \ell^d)$ increases with v , resulting in $\epsilon_{\ell w}$ increasing with v . Since v is the only shift variable in $\ell_s(w, v)$ and $\partial \ell^s / \partial v < 0$, a decrease in v is associated with an outward shift of the labor supply function and an increase in employment and output. Thus, in the standard setup with homothetic preferences, $\epsilon_{\ell w}$ is always countercyclical, implying that monopsony power will be procyclical and the real wage will be countercyclical in the standard monopsony model.

APPENDIX D TO CHAPTER IV

D1. The Reservation Wage Function

The properties of the reservation wage function $\psi(v)$ are inherent in the utility function and the budget and hours constraints of the consumer's problem. Since consumers are assumed to differ only in their endowments, they share the same reservation wage function. The function V in Equation (4.2) is continuous and twice differentiable in (v, ω) due to the standard assumptions about $U_0 = U(v, 1)$ and $U_1 = U(v + \omega h, 1-h)$. Also, note that $\partial V / \partial \omega = [\partial U_1 / \partial c] h \neq 0$. Thus the implicit function theorem applies, and the implicit function for the reservation wage $\omega = \psi(v)$ can be investigated. Differentiating Equation (4.2) (and dropping the terms with arguments h and $1-h$ which are constants) yields the result:

$$dV = \left(\frac{\partial U_0}{\partial c} - \frac{\partial U_1}{\partial c} \right) dv - \frac{\partial U_1}{\partial c} h d\omega = 0 \quad (\text{D1.1})$$

and solving (D1.1) for $d\omega/dv$:

$$\frac{d\omega}{dv} = \frac{1}{h} \left[\frac{\frac{\partial U_0}{\partial c}}{\frac{\partial U_1}{\partial c}} - 1 \right] > 0 \quad (\text{D1.2})$$

The inequality in (D1.2) follows from the fact that U_0 and U_1 are strictly concave and $(v + \omega h) > v$ for $\omega > 0$.

Equation (D1.2) also follows from the fact that U_0 and U_1 are concave and therefore have decreasing absolute risk aversion (Hall, Lippman & McCall [1979], Ch. 7). It has been long established in the labor market search literature that if the individual utility function has decreasing absolute risk aversion (DARA), the reservation wage increases monotonically with nonlabor income v or wealth Ω (Danforth, 1979)¹.

The derivable properties of ψ are functions of the first and second partial derivatives of U_0 and U_1 . The elasticity of ψ is:

$$\xi = \frac{\psi}{v} v = \frac{1}{h} \left[\frac{\frac{\partial U_0}{\partial c}}{\frac{\partial U_1}{\partial c}} - 1 \right] \frac{v}{\psi} > 0 \quad (\text{D1.3})$$

Since ψ is monotonic, it has an inverse function $\psi^{-1} = g(\omega)$ with elasticity $\mathcal{E}_\omega[g(\omega)] = 1/\xi$. It is evident from Equation (4.6) in Chapter IV that ϵ_{ω} and ξ will be inversely related. If $d\xi/dv > 0$, then $\partial\epsilon/\partial v < 0$ and a decline in v , which is necessary for the labor supply curve to shift out, will increase ϵ_{ω} , which will then be procyclical.

¹In a context of job search with uncertainty, a utility function with DARA implies a declining absolute risk premium for the choice of an uncertain income stream from job search as nonlabor income or wealth increases. Although uncertainty and search dynamics are not being modeled here, the relative risk aversion, or concavity, of $U(c, l)$ plays a role in determining $\partial\epsilon_{\omega}/\partial v$; see below.

The first derivative of the elasticity ξ is:

$$\frac{d\xi}{dv} = \xi' = \frac{\psi'}{\psi} \left[\frac{\psi'}{\psi} v + 1 - \xi \right] \quad (\text{D1.4})$$

from which it can be seen that the sign of ψ'' is pivotal to the sign of ξ' . Differentiating Equation (D1.2) yields:

$$\psi'' = \frac{\frac{\partial U_1}{\partial c} \left[\frac{\partial^2 U_0}{\partial v \partial c} \right] - \frac{\partial U_0}{\partial c} \left[\frac{\partial^2 U_1}{\partial v \partial c} \right]}{\left[\frac{\partial U_1}{\partial c} \right]^2} \quad (\text{D1.5})$$

and since

$$\frac{\partial^2 U_0}{\partial v \partial c} = \frac{\partial^2 U_0}{\partial c^2} = U_0'' < 0$$

and

$$\frac{\partial^2 U_1}{\partial v \partial c} = \frac{\partial^2 U_1}{\partial c^2} \left[1 + \frac{d\omega}{dv} h \right] = U_1'' \left[\frac{U_0'}{U_1'} \right] < 0$$

then (D1.5) can be written:

$$\psi'' = \frac{1}{h} \left[\frac{U_0''}{U_1'} - \left(\frac{U_1''}{U_1'} \right) \left(\frac{U_0'}{U_1'} \right)^2 \right] \quad (\text{D1.6})$$

Equation (D1.6) can be rearranged to yield the following relationship:

$$\psi' = \frac{\left(\frac{U'_0}{U'_1}\right)R_A(U_1) - R_A(U_0)}{h\left(\frac{U'_1}{U'_0}\right)} \leq \geq 0 \quad (\text{D1.7})$$

where $R_A(U) = -U''/U'$ is the Arrow-Pratt measure of absolute risk aversion. The sign of Equation (D1.7) depends on cardinal utility measure and is ambiguous without specification of a functional form for $U(c, \ell)$.

For a general concave utility function with decreasing relative risk aversion (DARA), the most that can be said about $\psi(v)$ is that it is positive and monotonically increasing with v . For the class of homothetic utility functions it is easy to show that $\psi' = \psi/v$, $\xi = 1$, $\psi(v) = \kappa v$ where κ is an arbitrary constant, and $\psi'' = 0$; in that case it can be verified from Equation (D1.4) that $\xi' = 0$. Thus, the assumption of homothetic preferences implies a linear reservation wage function ψ with constant elasticity ξ everywhere.

Inspection of Equations (D1.4) and (D1.7) reveals that utility functions that are more concave than homothetic functions will have $\psi'' > 0$ and $\xi' > 0$, with opposite signs prevailing for utility functions that are less concave. If $\xi' \neq 0$, then the elasticity of $\psi(v)$ will be changing with v , and will have an effect on ϵ_{LW} via $\mathcal{E}_W[g]$ that will be due only to nonhomothetic preferences.

D2. Non-homothetic Preferences and Relative Risk Aversion

A closely related measure of concavity is the relative risk aversion $R_r(U) = U''v/U' = vR_A$, which is also the elasticity of the marginal utility of income. In terms of relative risk aversion, Equation D1.7 becomes:

$$\psi' = \frac{\left(\frac{U'_0}{U'_1}\right) \frac{R_r(U_1)}{(v+\omega h)} - \frac{R_r(U_0)}{v}}{h\left(\frac{U'_1}{U'_0}\right)} \quad (\text{D2.1})$$

For a homothetic utility function R_r is constant for all values of c and ℓ , i.e., the elasticity of marginal utility is constant everywhere. In the special case of log utility, $R_r = 1$ and it can be shown by substitution in Equation (D2.1) that $\psi'' = 0$, and also from Equations (D1.3) and (D1.4) that $\xi = 1$ and therefore $\xi' = 0$. This corresponds to the reservation wage function ψ_0 in Figure 4-2, which is representative of all homothetic utility functions. Thus, the assumption of homothetic preferences rules out any dynamic effect of ξ , the elasticity of the reservation wage function, on ϵ , the elasticity of labor supply. This result is consistent with the conclusions of Chapter III.

If the utility function is not homothetic, then R_r will be either increasing or decreasing with v , and from Equations (D2.1) and (D1.4), ψ'' and ξ' will be correspondingly positive or negative. In either case, the change in $\xi(v)$ will have a

corresponding effect on ϵ_{lw} . Thus, the operative implications of nonhomothetic preferences for the elasticity of the reservation wage function can be reduced to the assumption of whether R_R is increasing or decreasing with income in the worker population².

D3. Relative Risk Aversion and Procyclical ϵ_{lw}

Arrow (1970) showed that a utility function must be bounded from above and from below in order for lotteries to be consistently ranked according to their Expected Utility, and for a bounded utility function:³

$$\lim_{x \rightarrow 0} R_R(U(x)) \leq 1 \quad \text{and} \quad \lim_{x \rightarrow \infty} R_R(U(x)) \geq 1 \quad (\text{D3.1})$$

Equation (D3.1) implies that only two possibilities are consistent with the Expected Utility Theorem: constant or increasing R_R . The constant case is homotheticity with $\psi'' = 0$ which has already been considered. If utility is assumed to be nonhomothetic with increasing R_R , then it is evident from Equations D2.1 and D1.4 that $\psi'' > 0$ and $\xi' > 0$.

²Preferences could be heterogeneous across workers, or all workers could have identical nonhomothetic preferences. The relevant heterogeneous property which determines the sign of ξ' is relative, or proportional, risk aversion.

³A bounded utility function precludes an infinite utility measure for arbitrarily large lottery payoffs, which would invalidate the Expected Utility Theorem. See Arrow (1970).

Since $\mathcal{E}_\omega[g(\omega)] = 1/\xi$, it then follows from Equation (4.6) in Chapter IV that $\partial\epsilon/\partial v < 0$, and a decline in v which causes the labor supply curve to shift out will also be accompanied by an increase in the elasticity of labor supply. This case corresponds to reservation wage function ψ_1 with $\xi' > 0$ in Figure 4-2. Arrow points out that increasing R_R is consistent with empirical findings that the wealth elasticity of the demand for money balances is at least unity.

Thus, one possible source of procyclical elasticity of labor supply would be nonhomothetic preferences with increasing relative risk aversion (IRRA). With increasing R_R , the risk premium component of the reservation wage function decreases more than proportionally with nonlabor income, causing the elasticity of aggregate labor supply to increase as the labor supply curve shifts out.

The remaining possibility, decreasing relative risk aversion, corresponding to reservation wage function ψ_2 in Figure 4-2, can be eliminated on the basis of (D3.1), and the empirical estimates that the wealth-elasticity of the demand for money balances is not less than unity.

D4. Conditions on Preferences for Procyclical ϵ_{LM}

By differentiating the expressions for R_A and R_R , it is straightforward to show that a utility function $U(x)$ which

has decreasing R_A and increasing R_R must satisfy the double inequality:

$$0 < \left[\frac{U'''}{U''} - \frac{U''}{U'} \right] < \frac{1}{x} \quad (\text{D4.1})$$

or, in elasticity terms:

$$0 < \left[\mathcal{E}_x[U'''] - \mathcal{E}_x[U''] \right] < 1 \quad (\text{D4.2})$$

The set of utility functions $\{U(x)\}$ which satisfy (D4.1) and (D4.2) will have monotonically increasing reservation wage functions with elasticities that increase with nonlabor income v , creating a procyclical influence on the elasticity of labor supply in Equation (D4.6)⁴. An example of such a utility function, due to Pratt (1964), is:

$$U(x) = -\exp[-\rho^{-1}(x + \beta)^\rho], \quad 0 < \rho < 1, \quad \beta \geq 0 \quad (\text{D4.3})$$

For the case $\beta = 0$, $R_A = [x^\rho + (1-\rho)]x^{-1}$ which is decreasing in x ; $R_R = [x^\rho + (1-\rho)]$ which is increasing in x , with nearly constant elasticity ρ when x is large.

⁴With increasing R_R , the income-elasticity of the demand for leisure (and consumption) increases with income; thus the income-elasticity of labor supply, the complement of leisure demand, decreases with income, and conversely. Thus with increasing R_R , a decline in nonlabor income increases the elasticity of labor supply.

5. Shift of the Aggregate Labor Supply Function

The Lognormal labor supply function is:

$$\Lambda(w | \mu, \sigma^2) = \int_0^w \frac{1}{\sqrt{2\pi\sigma w}} \exp \left\{ -\frac{1}{2} \left[\frac{\ln w - \mu}{\sigma} \right]^2 \right\} dw \quad (\text{D5.1})$$

Partial differentiation under the integral sign:

$$\Lambda_\mu = \frac{1}{\sigma} \int_0^w z \lambda(w) dw < 0 \quad \text{because} \quad \int_{-\infty}^{\infty} z \varphi(z) dz = 0 \quad (\text{D5.2})$$

$$\Lambda_{\sigma^2} = \frac{1}{2\sigma^2} \int_0^w [z^2 - 1] \lambda(w) dw \quad (\text{D5.3})$$

$$= \frac{1}{2\sigma^2} \int_{-\infty}^z [z^2 - 1] \varphi(z) dz \quad \begin{cases} > 0, & z < 0 \\ < 0, & z > 0 \end{cases} \quad (\text{D5.4})$$

Here I employ the transformation $z = (\ln w - \mu)/\sigma$ which enables the substitution of the standardized normal density $\varphi(z)$ for $\lambda(w)$. This convention will be maintained throughout.

An increase in P due to an open-market purchase reduces μ but not σ^2 . (See Section 4.4.2) However, an open-market operation necessarily reduces A , the proportion of total wealth held in the risky asset (bonds) and this reduces σ^2 through the nonhomogeneous portfolio adjustment process described in Section 4.4.3 and Appendix Section D7 below.

It also follows from the portfolio balance relationship that $\mathcal{E}_A[\sigma^2] > 0$, so a decline in r reduces A and thereby σ^2 .

Thus P , directly through $d\mu/dP$, and r , indirectly through $(\partial\sigma^2/\partial A)(\partial A/\partial r)$, will shift and compress the aggregate labor supply curve.

The partial elasticities $\mathcal{E}_P[\Lambda]$ and $\mathcal{E}_r[\Lambda]$ of Λ are derived below.

$$\frac{\partial \Lambda}{\partial P} \Big|_w = \Lambda_\mu \frac{d\mu}{dP} = -\frac{1}{P} \Lambda_\mu = -\frac{1}{\sigma P} \int_{-\infty}^z \varphi(z) dz > 0 \quad (\text{D5.5})$$

The price-elasticity of Λ is:

$$\mathcal{E}_P[\Lambda] = \frac{-1}{N(z|0,1)} \int_{-\infty}^z \frac{z}{\sigma} \varphi(z) dz \geq 0 \quad (\text{D5.6})$$

An increase in P increases aggregate labor supply at all reservation wages.

The interest-elasticity works through A and σ^2 :

$$\frac{\partial \Lambda}{\partial r} = \Lambda_{\sigma^2} \frac{\partial \sigma^2}{\partial r}$$

$$\mathcal{E}_r[\Lambda] = \frac{1}{2} \left[\frac{\int_{-\infty}^z [z^2 - 1] \varphi(z) dz}{N(z|0,1)} \right] \left[\frac{\partial \sigma^2}{\partial A} \frac{A}{\sigma^2} \right] \left[\frac{\partial A}{\partial r} \frac{r}{A} \right] \quad (\text{D5.7})$$

For the Pratt IRRA utility function in Equation D4.3, the elasticity $\mathcal{E}_A[\sigma^2]$ was found previously to be $+2(1-\rho)^2/\rho^2\sigma^2$.

$\mathcal{E}_r[A]$ is the interest-elasticity of A in the portfolio balance relationship, and is related to the interest-elasticity and wealth-elasticity of the money demand function, as will be shown in the following development.

$$\frac{\partial A}{\partial r} \frac{r}{A} = \frac{r}{A\Omega} \left[F_r^d + \Omega_r (F_\Omega^d - A) \right] \quad (\text{D5.8})$$

where F^d and M^d represent the demand for risky financial assets and money balances, respectively. Here I maintain consistency with the specifications of money and asset demand in Chapter II, Section 2.1, i. e., $M^d = M^d(y, r, \Omega)$ and $F^d = F^d(y, r, \Omega, \dots)$, so that there are wealth effects in both demand functions. Note that changes in r can affect A in two ways: a substitution effect F_r^d , and a wealth effect $\Omega_r (F_\Omega^d - A)$. The following identities also apply:

$$A\Omega = F^d, \quad F_r^d = -M_r^d, \quad F_\Omega^d = 1 - M_\Omega^d, \quad \frac{M^d}{F^d} = \frac{(1-A)}{A}, \quad \frac{\Omega_r}{\Omega} r = -A$$

Substituting these identities and rearranging terms results in the following expression for $\mathcal{E}_r[A]$:

$$\mathcal{E}_r[A] = \left(\frac{1-A}{A} \right) \left[-\mathcal{E}_r[M^d] + A \left[\mathcal{E}_\Omega[M^d] - 1 \right] \right] > 0 \quad (\text{D5.9})$$

Thus $\mathcal{E}_r[A]$ can be expressed exclusively in terms of the interest- and wealth-elasticities of money demand. $\mathcal{E}_r[A]$ can be evaluated numerically for given values of A and these two elasticities. Note, however, that if preferences are homothetic, $\mathcal{E}_\Omega[M^d] = \mathcal{E}_\Omega[F^d] = 1$, i. e., asset demand elasticities

are constant with respect to scale. In that case, there is no wealth effect on A , and:

$$\mathcal{E}_r[A] = \left(\frac{(1-A)}{A} \right) \mathcal{E}_r[M^d] \quad (\text{D5.10})$$

the magnitude of which is considerably less than one.

The macro variables P and r are affected by changes in the stock of money. In the model of Chapter II, it was found that $dP/dM > 0$ and $dr/dM < 0$ unambiguously, and that an open-market purchase of bonds was non-neutral. If the aggregate labor supply function Λ were to be imbedded in that model, it would have a total money-elasticity as follows:

$$\begin{aligned} \mathcal{E}_M[\Lambda] = & -\frac{1}{\sigma} \left[\frac{\int_0^z z \varphi(z) dz}{N(z|0,1)} \right] \left[\frac{dP}{dM} \frac{M}{P} \right] \\ & + \frac{(1-\rho)^2}{\rho^2 \sigma^2} \left[\frac{\int_0^z (z^2-1) \varphi(z) dz}{N(z|0,1)} \right] \left[\frac{\partial A}{\partial r} \frac{r}{A} \right] \left[\frac{dr}{dM} \frac{M}{r} \right] \end{aligned} \quad (\text{D5.11})$$

This expression can be evaluated numerically for given values of ρ , σ , and the elasticities on the right-hand side.

D6. Endogenous Change in Labor Supply Elasticity

The real-wage elasticity of aggregate labor supply is:

$$\epsilon = \frac{\lambda w}{\Lambda} = \epsilon(w, \mu, \sigma^2) \quad (\text{D6.1})$$

which shows that the elasticity of Λ is equivalent to its relative density. The partial derivative of ϵ with respect to one of its parameters x is:

$$\frac{\partial \epsilon}{\partial x} = \frac{\frac{\partial(\lambda w)}{\partial x}}{\Lambda} - \epsilon \frac{\frac{\partial \Lambda}{\partial x}}{\Lambda} \quad (\text{D6.2})$$

For $x = \mu$:

$$\frac{\partial \epsilon}{\partial \mu} = \frac{\epsilon}{\sigma} \left[z - \frac{\int_0^z z \varphi(z) dz}{N(z|0,1)} \right] \quad (\text{D6.3})$$

For $x = \sigma^2$:

$$\begin{aligned} \frac{\frac{\partial(\lambda w)}{\partial \sigma^2}}{\Lambda} &= \frac{\epsilon}{2\sigma^2} [z^2 - 1] \\ \frac{\frac{\partial \Lambda}{\partial \sigma^2}}{\Lambda} &= \frac{\frac{1}{2\sigma^2} \int_0^z [z^2 - 1] \varphi(z) dz}{N(z|0,1)} \end{aligned} \quad (\text{D6.4})$$

$$\frac{\partial \epsilon}{\partial \sigma^2} = \frac{\epsilon}{2\sigma^2} \left[(z^2 - 1) - \frac{\int_{-\infty}^z [z^2 - 1] \varphi(z) dz}{N(z|0,1)} \right] \quad (\text{D6.5})$$

The sign of the right-hand side will depend on the value of z , and the expression will have to be evaluated numerically.

The change in elasticity due to an increase in P at a given wage is through μ only:

$$\begin{aligned} \frac{\partial \epsilon}{\partial P} &= \frac{\partial \epsilon}{\partial \mu} \left(\frac{-1}{P} \right) \\ &= \frac{\epsilon}{\sigma P} \left[-z + \frac{\int_{-\infty}^z z \varphi(z) dz}{N(z|0,1)} \right] < 0 \end{aligned} \quad (\text{D6.6})$$

A reduction in μ due to an increase in P will reduce ϵ unambiguously. There is no effect on the dispersion parameter σ^2 from P .

The price-elasticity of ϵ is, from (D6.6)

$$\mathcal{E}_P[\epsilon] = \frac{1}{\sigma} \left\{ -z + \frac{\int_{-\infty}^z z \varphi(z) dz}{N(z|0,1)} \right\} < 0 \quad (\text{D6.7})$$

Upon comparing Equations (D6.7) and (D5.6), it is evident that:

$$\mathcal{E}_p[\epsilon] = -\mathcal{E}_p[\Lambda] - \frac{z}{\sigma} \quad (\text{D6.8})$$

Since $\mathcal{E}_p[\Lambda] > 0$, an increase in the price level alone will increase ϵ only at large negative values of z , well below the median μ . There is, however, another channel through σ^2 . The elasticity of the distribution will be increased in a region about the log median if the kurtosis of the density function $\lambda(w)$ is decreased by a change in the shift parameters. The kurtosis of the Lognormal density function is an increasing function of σ^2 only⁵. Thus, a decline in the log variance σ^2 will increase the relative density of $\lambda(w)$ in the vicinity of μ , which will increase elasticity there. (The variance and skewness of $\lambda(w)$ will also decrease with a decline in σ^2 .)

From Equation (D6.5):

$$\mathcal{E}_r[\epsilon] = \frac{1}{2} \left[(z^2 - 1) - \frac{\int_{-\infty}^z [z^2 - 1] \varphi(z) dz}{N(z|0,1)} \right] \left[\frac{2(1-\rho)^2}{\rho^2 \sigma^2} \left[\frac{\partial A}{\partial r} \frac{r}{A} \right] \right] \quad (\text{D6.9})$$

and comparing this with Equation D5.7, it is evident that:

⁵The kurtosis $\gamma_2 = a^2(a^6 + 6a^4 + 15a^2 + 16)$ where $a^2 = \exp(\sigma^2) - 1$. A decline in σ^2 reduces the kurtosis of the Lognormal density function.

$$\mathcal{E}_r[\epsilon] = -\mathcal{E}_r[\Lambda] + \frac{(1-\rho)^2}{\rho^2\sigma^2} (z^2-1)\mathcal{E}_r[A]$$

Finally, the total money-elasticity of ϵ , with respect to a one-time open-market purchase, is derived from Equations (D6.7) and (D6.9) as:

$$\begin{aligned} \mathcal{E}_M[\epsilon] = \frac{\partial \epsilon}{\partial M} \frac{M}{\epsilon} = \frac{1}{\sigma} \left[-z + \frac{\int_0^z z\varphi(z) dz}{N(z|0,1)} \right] \left[\mathcal{E}_M[P] \right] \\ + \frac{(1-\rho)^2}{\rho^2\sigma^2} \left[(z^2-1) - \frac{\int_0^z (z^2-1)\varphi(z) dz}{N(z|0,1)} \right] \left[\mathcal{E}_r[A] \right] \left[\mathcal{E}_M[r] \right] \end{aligned} \quad (\text{D6.10})$$

Like Equation (D5.11), this expression can be evaluated for given values of the elasticities on the right-hand side.

Comparing Equation (D6.10) with Equation (D5.11) reveals that:

$$\begin{aligned} \mathcal{E}_M[\epsilon] = -\mathcal{E}_M[\Lambda] - f_1 \left[\mathcal{E}_M[P] \right] + f_2 \left[\mathcal{E}_r[A] \right] \left[\mathcal{E}_M[r] \right] \\ f_1 = \frac{z}{\sigma}; \quad f_2 = \frac{(1-\rho)^2(z^2-1)}{(\rho^2\sigma^2)} \end{aligned} \quad (\text{D6.11})$$

For a non-neutral open-market operation the elasticity $\mathcal{E}_M[P]$ is positive and less than one, most likely greater than 0.5. $\mathcal{E}_r[A]$ is the elasticity of proportional demand for risky assets in Tobin's portfolio balance model of Keynesian liquidity preference (Tobin, 1958). (See Chapter IV, Section

liquidity preference (Tobin, 1958). (See Chapter IV, Section 4.4 and Figure 4-5). This elasticity will be determined by the particular utility function assumed to represent preferences, but can be calculated from Equation (D5.9) given empirical estimates of the interest-elasticity and wealth-elasticity of money demand, and A , the share of risky assets in financial wealth. The third elasticity, $(dr/dM)(M/r)$ is just the reciprocal of $\mathcal{E}_r[M^d]$.

Given plausible values for these three elasticities, Equations (D5.11) and (D6.10) or (D6.11) can be evaluated numerically for calibrated parameters of $\Lambda(w|\mu, \sigma^2)$, the standard Normal density function $\phi(z)$ and distribution function $N(z|0,1)$, and ρ , the elasticity of relative risk aversion. This is done in Chapter IV, Section 5.

D7. Optimum Portfolio Balancing with Increasing R_r

This section follows the analysis and relies upon proofs in Arrow (1970) and Diamond and Stiglitz (1974). It is included to provide a more rigorous support of the discussion in Section 4.4.3.

An individual with an endowment of wealth Ω_0 at the beginning of a time period can hold it in either earning assets f which provide an uncertain periodic return R , or in riskless money balances m which provide no return. The return

$$R = r + r/r_e - 1$$

Here, $1/r$ is the current price of the risky asset and $1/r^e$ is the expected price at the end of the period. Individual preferences for wealth and its components m and f are described by a concave utility function $H(\Omega)$. It will be assumed here that $H(\Omega)$ has decreasing absolute risk aversion [$\partial/\partial\Omega(H''/H') < 0$] and increasing relative risk aversion [$\partial/\partial\Omega(H''\Omega/H') > 0$]. Thus, individuals are risk-averse and prefer to diversify between f and m .

Since wealth at the end of the period is uncertain, the individual desires to choose a portfolio allocation between f and m that will maximize his/her expected utility from wealth at the end of the period. The consumer's problem in the asset market is:

$$\text{Max}_a E[H(\Omega_1)] = E[H(1-a)\Omega_0 + a(1+R)\Omega_0] = E[H(\Omega_0(1 + aR))]$$

$$0 \leq a \leq 1$$

Where the expectation E is with respect to the uncertain return R . The only decision variable is a , and the constraint $f + m = \Omega_0$ is incorporated in the substitution for Ω_1 . Differentiating $E[H]$ twice with respect to a gives the first- and second-order conditions for a maximum:

$$\text{F.O.C.} \quad E[H'(\Omega_0(1 + aR)R\Omega_0)] = 0$$

$$\text{S.O.C.} \quad E[H''(\Omega_0(1 + aR)R^2\Omega_0^2)] \leq 0$$

The second order condition is satisfied by the concavity of H . The first order condition equates marginal utilities of f and m at the optimum; this occurs when the marginal rate of substitution between f and m equals their price ratio, which is $(1 + R)$. The standard setup with indifference curves tangent to budget lines is shown in Figure 4-5 (a) and (b) in Section 4.4.3.

The first-order condition gives the optimum a^* as an implicit function of Ω_0 and R , and we may write $a^* = a(\Omega, r)$. (With static, regressive expectations, $r^e = r = R$). Implicit differentiation of a^* with respect to r in the first-order condition gives:

$$\frac{da^*}{dr} = - \frac{E[H'' [a^* r \Omega^2] + H' \Omega_0]}{E[H'' [r^2 \Omega^2]]} > 0 \quad (D7.1)$$

The denominator of D7.1 is the negative second-order condition. Arrow (1970; Appendix [4]) proved that the numerator is positive if H has decreasing absolute risk aversion (R_A), which has been assumed here. It follows that $\mathcal{E}_r[a^*] > 0$, and a reduction in r reduces a^* , the optimum portfolio share of the risky asset.

Implicit differentiation of a^* with respect to Ω_0 gives:

$$\frac{da^*}{d\Omega_0} = - \frac{E[H'' [r\Omega_0 + a^* r^2 \Omega^2]]}{E[H'' r^2 \Omega_0^2]} \leq 0 \quad (D7.2)$$

The denominator of D7.2 is the negative second-order condition, and Arrow (op. cit., Appendix [5]) proved that the

numerator is negative if H has increasing relative risk aversion (R_R) (which has also been assumed), and zero if H has constant relative risk aversion, i. e., if H is homothetic. It follows that if H is homothetic, a^* is independent of wealth and is the same for all wealth-holders, and since $\partial^2 a^* / \partial r \partial \Omega = 0$, portfolio share adjustments to changes in r will be homogeneous over wealth-holders. But with increasing R_R , $\mathcal{E}_\Omega[a^*] < 0$, and the optimum portfolio share a^* will decline with an increase in Ω at a constant interest rate r . It can be shown that increasing R_R implies that $\partial^2 a^* / \partial \Omega \partial r > 0$. so that the marginal rate of substitution between f and m increases with wealth. Thus, portfolio adjustments in response to changes in r will not be homogeneous over wealth-holders.

In response to an open market operation, which reduces Ω and r , bond-holders with increasing R_R will reduce their portfolio shares allocated to bonds proportionally to their wealth. Since wealthier bond-holders hold a lower percentage of their wealth in bonds to begin with, their portfolio rebalancing actions will reduce their bond income v more than proportionally. Relative nonlabor incomes will decline more than proportionally, compressing the dispersion of the distribution.

Applying the elasticity operator \mathcal{E}_Ω to $v = ra^*\Omega$, remembering that $a^* = a(\Omega, r)$, yields the relationship:

$$\mathcal{E}_M[v] = \left\{ \mathcal{E}_M[r] [1 + \mathcal{E}_r[a^*]] \right\} + \left\{ \mathcal{E}_M[\Omega] [1 + \mathcal{E}_\Omega[a^*]] \right\} < 0 \quad (\text{D7.3})$$

The first term on the right is the substitution effect of a change in r due to a change in aggregate M , and is always negative. The second term is the wealth effect of a change in aggregate M . If preferences are homothetic, $\mathcal{E}_\Omega[a^*] = 0$ and the wealth effect is only the effect of a proportional reduction in earning assets, which is also negative on v , but homogeneous over bond-holders. With increasing relative risk aversion, $\mathcal{E}_\Omega[a^*]$ will be negative but greater than -1 ; the net wealth effect on v will still be negative, but not as large, because a reduction in Ω will increase a^* somewhat. What is not apparent from this equation is that, with increasing relative risk aversion, the substitution effect will be much greater at higher values of Ω and v due to the increasing concavity of the utility function, i. e. $\mathcal{E}_r[a^*(\Omega, r)]$ is an increasing function of Ω , and a^* and v will decline more than proportionally, i. e., $\partial^2 a^* / \partial \Omega \partial r > 0$. This causes the wealth expansion path to have a decreasing slope, as shown in Figure 4-5(b), Section 4.4.3

Diamond and Stiglitz (1974), following Arrow (1970) and Pratt (1964), proved the following:

(1) *The absolute value of risky asset holdings increases (decreases) with wealth if absolute risk aversion decreases (increases) with wealth.*

i. e., $\mathcal{E}_\Omega[f] > 0$ if and only if H has decreasing R_A

(2) *The fraction of risky asset holdings increases (decreases) with wealth if relative risk aversion decreases (increases) with wealth.*

i. e., $\mathcal{E}_\Omega[a^*] < 0$ if and only if H has increasing R_R .

As pointed out earlier in Sections D2 through D4 of this Appendix, a utility function with increasing relative risk aversion is nonhomothetic in a systematic way that has important implications for both the consumption-leisure and portfolio balance choices.

With increasing R_R , $\mathcal{E}_r[a^*] > 0$ and $\mathcal{E}_\Omega[a^*] < 0$, A reduction in r , the (expected) rate of return on bonds, induced by an open-market operation will lower the reward/risk ratio for all bond-holders, but wealthier ones with higher relative risk aversion will reduce their bond allocations proportionally to their wealth. Since these wealthier investors hold a lower percentage of bonds in their portfolios, their actions will result in a reduction in $v_i = r a_i \Omega_i$ that is more than proportional to the value of v_i prior to the open-market operation. This will compress the distribution of v , reducing the variance of its logarithm, σ^2 . The loss of bond income, which will be proportionally larger for wealthier workers, can be interpreted as a risk premium paid to reduce risk, one which increases with wealth.

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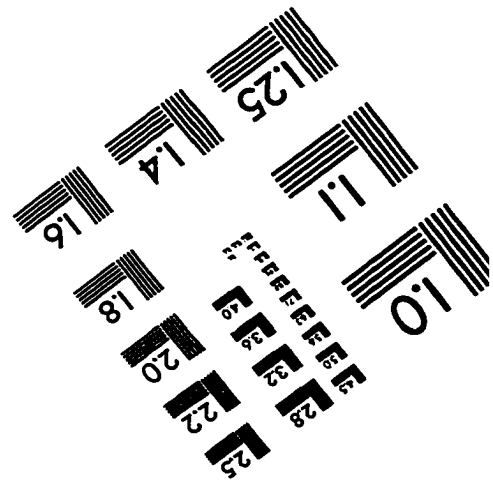
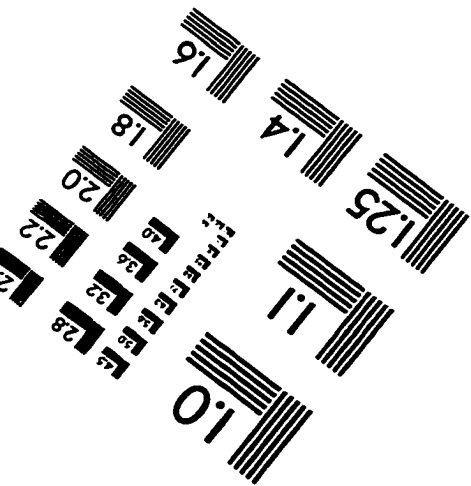
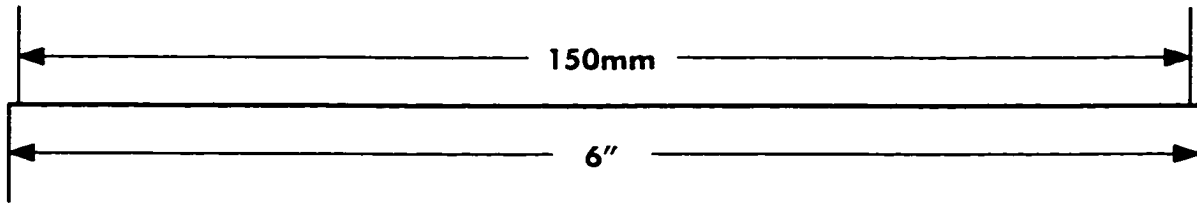
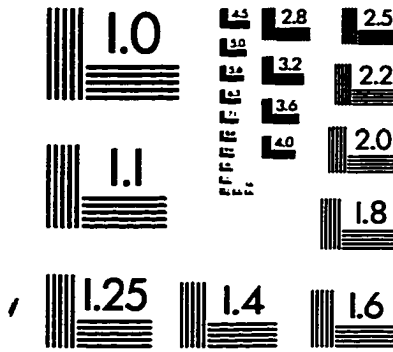
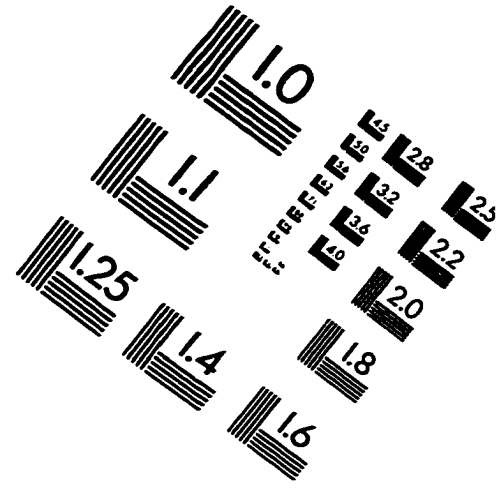
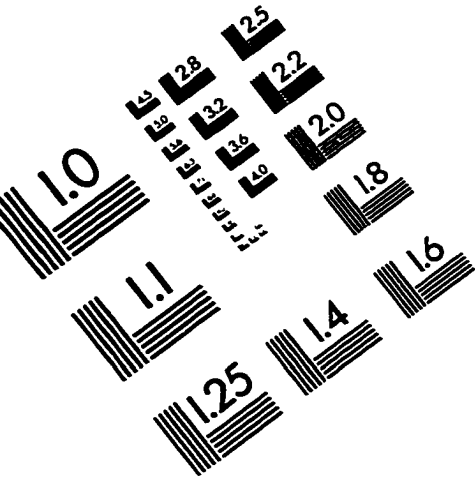
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IMAGE EVALUATION TEST TARGET (QA-3)



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