# Profiles of reform in the teaching of calculus: A study of the implementation of materials developed by the Calculus Consortium Based at Harvard (CCH) Curriculum Project 

Alice Darien Lauten<br>University of New Hampshire, Durham

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# PROFILES OF REFORM IN THE TEACHING OF CALCULUS: A STUDY OF THE IMPLEMENTATION OF MATERIALS DEVELOPED BY THE CALCULUS CONSORTIUM BASED AT HARVARD (CCH) CURRICULUM PROJECT 

## BY

ALICE DARIEN LAUTEN
B.A., William Paterson College of New Jersey, 1964
M.S., South Dakota School of Mines and Technology, 1974 M.Ed., University of New Hampshire, 1992

## DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy
in
Education

September, 1996

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## A. Darien Lauten

This dissertation has been examined and approved.


Dissertation co-director, Sharon Jodie Oja Professor of Education

## Joan Versing- Mandy

Dissertation co-director, Jon Ferrini-Mundy Professor of Mathematics


Richard M. Barton, Assistant Professor of Education


Todd A. DeMitchell, Associate Professor of Education


Marie A. Gaudard, Professor of Mathematics


Karen J. Graham, Associate Professor of Mathematics


## ACKNOWLEDGMENTS

I would like to thank all the members of my research committee who each contributed their time and expertise toward the completion of this project. Nodie Oja was always there, providing a sense of direction, gentle reminders to stay focused, timely assistance, and continuous support and encouragement throughout the process. Joan Ferrini-Mundy oversaw the project design, the development of the instruments, the collection of the data, and the analysis. Her friendship, support, constructive criticism, and intimate involvement with the project made its completion possible. I also thank Dick, Joe, Beth, and Adriana Mundy who contributed more than could ever be expected of an advisor's family. Karen Graham, also integrally involved with the larger documentation and evaluation project, was a friend and regular source of critical assistance in the various phases of the project. Rick Barton carefully and insightfully assisted with the design of the instruments and the analysis of the data. Marie Gaudard read the drafts of the manuscript, attending carefully to the statistical analysis. Todd DeMitchell helped place the project within the larger perspective of undergraduate education. Each of the committee members played a unique and essential role in this effort.

Sincere appreciation is due also to Deborah Hughes Hallett, for both her role in making the CCH Curriculum Project a reality and in her insightful engagement with the evaluation and documentation effort. Special thanks also go to the site liaisons and other faculty members who gave considerable time to the thoughtful completion of the surveys. The funding for the CCH Evaluation and Documentation Project through Harvard University and the National Science Foundation is gratefully acknowledged.

I would like to extend a special note of gratitude to Ann Diller and the members of the dissertation seminar. Ann's optimistic outlook and insightful comments proved essential to all of us through the ups and downs of the dissertation experience. Others to whom I am most appreciative include Prosem 13 and Charlie Ashley. They proved to be a most supportive group of fellow travelers. I also thank all of our excellent professors who collegially engaged us as we learned from and with each other.

I am also grateful to Loren Johnson and Ellen O'Keefe and to Annette Cussan, Jen LaCorcia, Joanne Morrill, and other students at the University of New Hampshire who, through their assistance with the CCH Documentation and Evaluation project, assisted with data collection, data entry, and the progress of the project.

Most importantly, I would like to thank Gary, my husband and best friend, without whose love and support I could not have completed this project. Gary, Kathryn and Angelo, and Frederick and Heather truly "lived through" this project, and I thank them.

This investigation was supported through Harvard University and National Science Foundation award \#USE8953923. The opinions expressed are those of the author and do not necessarily reflect the position or policy of Harvard University or the National Science Foundation. No official endorsement by Harvard University or the National Science Foundation should be inferred.

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#### Abstract

PROFILES OF REFORM IN THE TEACHING OF CALCULUS: A STUDY OF THE IMPLEMENTATION OF MATERIALS DEVELOPED BY THE CALCULUS CONSORTIUM BASED AT HARVARD (CCH) CURRICULUM PROJECT


by

A. Darien Lauten<br>University of New Hampshire, September, 1996

The research question addressed in this study is: What profiles of interpretation and implementation of reform in the teaching of calculus emerge from data obtained from mathematics faculty members using Calculus Consortium Based at Harvard (CCH) Curriculum Project materials? Site liaisons from mathematics departments using CCH Curriculum Project materials in 117 academic institutions, consisting of 13 secondary schools, 30 two-year colleges, 19 doctoral and research universities, and 55 other colleges and universities, completed Initial and Site Liaison Surveys. Site liaisons and 266 other instructors from 117 academic institutions completed a Faculty Survey. Six clustering scales were developed from the survey instruments that incorporated goals for reform in calculus curriculum and instruction: CONCEPTS, an emphasis on students' conceptual understanding of the central ideas of calculus; APPROACH, visual, numeric, and analytic approach to all topics and real-world experiences; TEACHING, use of alternative classroom teaching practices; ASSESSMENT, use of alternative student assessment methods; TECHNOLOGY, use of calculators and computers in calculus courses; and ACCESS, accessibility of calculus to students traditionally underrepresented in calculus.

Cluster analysis, using data from the surveys, identified eight groups or clusters of academic institutions. The institutions within each cluster exhibited similar patterns of interpretation and implementation of reform on the six clustering scales. Thirteen validating scales, incorporating survey items not used in the cluster analysis, were used to validate cluster solution. The study provides in-depth descriptions of each cluster from the perspectives of the participants, using participant comments that relate to each of the scales.

The different patterns of reform that are revealed in the cluster descriptions demonstrate that faculty members emphasize different aspects of reform that are meaningful and important to them in their contextual situation. The study is an effort to help the reader better understand reform in calculus curriculum and instruction and recognize the complexities faced by those engaged in the reform process.

## CHAPTER I

## INTRODUCTION

Reform in the teaching of calculus at the undergraduate level has been a national issue for the past decade. Since 1988, the National Science Foundation has supported reform-based calculus curriculum and implementation projects with millions of dollars. The current study is part of a larger project that is charged with the evaluation and documentation of one of the major National Science Foundation calculus curriculum projects, the Calculus Consortium Based at Harvard (CCH) Curriculum Project (Gleason, 1989). To position the current study within the various components, this first chapter provides a brief overview of the history of reform in the teaching of calculus, the CCH Curriculum Project, and the CCH Evaluation and Documentation Project (Ferrini-Mundy, 1994). Each of these topics is addressed in greater detail in Chapter II, the Review of the Literature for the current study. We turn now to a brief history of reform in the teaching of calculus.

## Reform in the Teaching of Calculus

In the early 1980s, members of the mathematics and science community began examining the role of calculus in the undergraduate curriculum, the mathematics content of calculus courses, the teaching of calculus, and student learning of calculus. As a service course to other disciplines, calculus had, for many years, provided background needed by students in courses other than mathematics as well as in subsequent mathematics courses. However, faculty from disciplines that utilize calculus began claiming that calculus was just one of many changing mathematical tools used by scientists. These client discipline faculty suggested that
students might obtain more benefit from a calculus course that placed greater emphasis on students' conceptual understanding, the role of mathematical models, and investigations into the nature and power of calculus (Douglas, 1986d). Many mathematicians, too, had become dissatisfied with what they perceived as decreased expectations of student learning in calculus courses and an emphasis on mathematical techniques over student understanding of central calculus concepts. In addition, technology capable of performing many calculus techniques had become readily available.

In response to these calls for change, Douglas (1986b) obtained funding for a national conference, which later became known as the Tulane Conference, to address the issues. The unanticipated large number of mathematicians and scientists who attended the influential Tulane Conference divided into three workshop groups (Douglas, 1986a). The content workshop group developed a syllabus for the first two years of calculus that contained fewer topics than standard calculus and emphasized conceptual depth (see Appendix A). A second group, the methods workshop, addressed the teaching of calculus and recommended ways for making calculus teaching more interactive. The third group, the implementation workshop, addressed how to make the suggested changes for calculus instruction happen. Those at the conference recommended that funding support be sought from private and government foundations such as the Sloan Foundation and the National Science Foundation.

The CCH Curriculum Project
In 1988, the National Science Foundation announced its Calculus Initiative, calling for proposals for planning and developing reform-based calculus curriculum projects (National Science Foundation, 1988). Gleason and Hughes Hallett (1988), of Harvard University, responded to the Calculus Initiative by submitting a planning
proposal. The National Science Foundation funded the planning proposal and a subsequent curriculum development proposal (Gleason, A. \& Hughes Hallett, D., 1989). Gleason's curriculum development proposal identified faculty from six fouryear colleges and universities (including Harvard University), one two-year college, and one secondary school who, upon receipt of the funding, formed a consortium to write the CCH Curriculum Project materials. The proposal for the CCH Curriculum Project included an evaluation component. Ferrini-Mundy at the University of New Hampshire as evaluator was later named project evaluator.

## The CCH Evaluation and Documentation Project

In 1994, Ferrini-Mundy initiated the CCH Evaluation and Documentation Project, of which the current study is a component. The CCH Evaluation and Documentation Project acknowledges and respects the belief that faculty members uniquely interpret CCH Curriculum Project materials within the context of their own situation. The goals of the CCH Evaluation and Documentation Project are (a) to determine how the goals of the CCH Curriculum Project are interpreted and implemented at academic institutions using the materials; (b) to assess student learning of calculus concepts, student attitudes, and student persistence in the study of mathematics; (c) to investigate faculty perceptions of student learning and faculty attitudes and beliefs towards calculus reform; and (d) to examine and to describe the evolution of efforts to reform the teaching of calculus in the context of using CCH Curriculum Project materials.

To accomplish its goals, the CCH Evaluation and Documentation Project is divided into four phases, with each phase informing later phases. The first phase included an Initial Questionnaire (see Appendix B). The Initial Questionnaire was intended to determine which academic institutions are currently using CCH Curriculum Project materials and to identify the names of faculty members, called site
liaisons, willing to participate in the project. The second phase included the sending of a Site Liaison Survey (see Appendix C) that was intended (a) to identify factors influencing the initiation of reform efforts, (b) to identify the types of students and courses using CCH Curriculum Project materials, and (c) to determine the site's reactions to the use of the materials. The third phase included the sending of Student Surveys and Faculty Surveys to the participating sites. The Faculty Survey (see Appendix D) focused on the interpretation and use of CCH Curriculum Project materials, the pedagogical characteristics of the courses, and faculty attitudes and beliefs. The Student Survey addressed affective items. The fourth phase includes (a) a flow-through component at several sites where student participation and achievement patterns beyond calculus are studied and (b) case studies at two sites. The methods used in the four phases of the CCH Evaluation and Documentation Project adhere closely to those of the research tradition.

## The Current Study and the Research Question

The current study, a documentation project, is conducted as part of the larger CCH Evaluation and Documentation Project. As does the larger project, the current study seeks to understand and to describe how goals of the CCH Curriculum Project and goals for reform-based calculus instruction established at the Tulane Conference are interpreted and implemented at academic institutions using the CCH Curriculum Project materials. The research question addressed in this study is: What profiles of interpretation and implementation of reform in the teaching of calculus emerge from data obtained from mathematics faculty members using CCH Curriculum Project materials?

The current study develops six dimensions of reform that are based on the goals for reform in the teaching of calculus that were envisioned by those in attendance at the Tulane Conference (Douglas, 1986d) and the literature surrounding
the teaching and learning of calculus. The six dimensions of reform each incorporate a total of 10-20 items from the Initial Questionnaire, the Site Liaison Survey and the Faculty Survey. Cluster analysis is used to identify clusters or groups of academic institutions that exhibit similar patterns of implementation of the CCH Curriculum Project materials along the dimensions of reform. The dimensions of reform are described briefly below and more completely in Chapter II, the Review of the Literature. A list of the dimensions of reform (clustering scales) and a brief description of each follows.

1. Concepts. The concepts clustering scale incorporates items that reflect students' understanding of the mathematical concepts that are central to calculus. The reform-based calculus goals imply that to achieve greater depth of understanding, students should spend more time considering complex problems and less time doing routine procedures. Students should become aware of the beauty of mathematics, understand the meaning of definitions and theorems, learn to justify answers, and learn to abstract and generalize.
2. Approach. The approach clustering scale incorporates items that reflect ways instructors introduce topics and approach calculus problems. Methods include: (a) using and giving equal weight to graphic, numeric, and algebraic approaches to calculus concepts and problems, (b) using an inductive approach to calculus in which the mathematics arises from student investigations and real world problems, and (c) placing value on helping students understand the role of mathematics in modeling and understanding the real world.
3. Teaching. The teaching clustering scale incorporates items that reflect what the instructor does in the classroom and how the instructor allocates classroom time. The goals for reform-based calculus instruction emphasize the need for instructors to make classrooms more interactive; to try alternative teaching strategies;
and to engage the students in exploring, doing, writing, and speaking about mathematics.
4. Assessment. The assessment clustering scale incorporates items that reflect the various ways instructors assess student learning. According to goals for reformbased calculus instruction, instructors should change their methods of testing and use a variety of methods that correspond to the many different goals of instruction.
5. Technology. The technology clustering scale incorporates items that reflect the use of technology in CCH Curriculum Project courses. Advocates of reformbased calculus instruction agree that technology should be used to increase student understanding of calculus concepts rather than used just for the sake of using technology.
6. Access. The access clustering scale incorporates items that reflect the accessibility of calculus to a wide range of students and the success of those students in calculus and subsequent courses dependent upon calculus. According to goals for reform-based calculus instruction, calculus should be more accessible to women, nontraditional students, students in disciplines that require an understanding of calculus concepts, and ethnic minority groups traditionally underrepresented in calculus.

The six clustering scales, defined in the previous paragraphs, incorporate the major components of the goals for reform in the teaching of calculus as established by the mathematicians in attendance at the Tulane Conference. These goals have served as a foundation for the movement that seeks reform in the teaching of calculus. The six clustering scales, representing the goals for reform, serve as the framework within which the current study seeks to understand, to interpret, and to document the implementation of reform in the teaching of calculus. This understanding is sought from the perspective of the users of the materials.

The current study makes a significant contribution by providing profiles of reform, unique descriptions of how users of CCH Curriculum Project materials interpret, implement, and engage with the materials and with reform in the teaching of calculus. The descriptive profiles may inform (a) members of the mathematics community who seek to understand the influences and impact of reform in the teaching of calculus, (b) members of the mathematics community who are engaging in or contemplating reform at their own institutions, (c) members of the policymaking community who seek to understand how mathematics departments are interpreting and implementing reform-based calculus materials, and (d) members of the education community who seek to understand the processes of reform. Although the analysis in the current study is limited to one calculus reform project, others who engage in reform in the teaching of calculus can use the methodological framework to assist them in understanding their own efforts, whether they use CCH Curriculum Project or other reform-based materials.

## Summary

Further discussion about reform in the teaching of calculus, the CCH Curriculum Project, and the CCH Evaluation and Documentation Project appears in Chapter II, the Review of the Literature. The following research question guides the current study: What profiles of interpretation and implementation of reform in the teaching of calculus emerge from data obtained from faculty members using CCH Curriculum Project reform-based calculus materials? In answering the research question, the current study describes the various ways those engaged in reform in the teaching of calculus interpret and implement reform and contributes a unique methodological framework for understanding efforts to reform the teaching of calculus.

## CHAPTER II

## REVIEW OF THE LITERATURE

This background chapter addresses the many different frames of reference within which the current study is situated. The various sections of this chapter, though distinct, each illuminate and influence the current study. The movement to reform undergraduate education has occurred concurrently with reform in the teaching of calculus. The first section of the current chapter considers reform in undergraduate teaching, one dimension of the larger movement to reform undergraduate education. Although it is difficult to assess the extent to which the movement to reform undergraduate teaching and the movement to reform the teaching of calculus have influenced one another, these two reform movements share many common concerns. The second section contains an historical view of calculus reform from its inception through the funding of the CCH Curriculum Project. The history of the movement to reform the teaching of calculus provides the reader with a contextual understanding of the goals of the CCH Curriculum Project, the CCH Evaluation and Documentation Project, and the current study. The third section focuses on the CCH Evaluation and Documentation Project, its goals and four phases. The current study is a major component of that project and should be understood within the context of the larger evaluation effort.

The fourth section addresses the theoretical foundations of the clustering scales that are central to the analysis and description components of the current study. The fifth section positions the CCH Evaluation and Documentation Project within the evaluation research tradition. The sixth section considers the current reform in schoo! mathematics. Many similarities exist between this reform movement and the
movement to reform the teaching of calculus. Evaluation studies of the current movement to reform school mathematics are only now being initiated. The final section considers the evaluation of New Mathematics, an earlier reform movement in mathematics education that, like the CCH Curriculum Project, was funded by the National Science Foundation. New Mathematics and its evaluation provide the only example of a complete cycle of reform in mathematics education from curriculum development through evaluation and, therefore, is relevant to the current study. We turn now to the first of the sections, the undergraduate reform movement.

The Undergraduate Reform Movement
The current undergraduate reform effort began in the 1980s in response to a number of national reports criticizing college teaching and student learning (Association of American Colleges, 1985; Bok, 1986; Boyer, 1987; National Institute of Education, 1984). Each of these reports, in their recommendations for improvement of undergraduate education, stresses the need to place greater emphasis and reward on the teaching component of professors' responsibilities. For example, Boyer (1987) asked colleges to ask themselves questions such as "Is good teaching valued as well as research, and is it an important criteria for tenure and promotion? Is superior teaching rewarded through recognized status and salary incentives?" (p. 290). In focusing on the undergraduate reform movement, the current section attends primarily to the undergraduate reform goals that are related to teaching and learning. These teaching and learning goals are similar to many goals for reform in the teaching of calculus that are discussed later in this chapter.

With regards to teaching and learning, the reports about undergraduate education strongly suggest that faculty make greater use of active modes of teaching and increase student involvement in their learning. Stressing the importance of active student involvement in their learning, Boyer (1987) recalled and agreed with a
statement made by Adler (1982), "all genuine learning is active, not passive. It involves the use of the mind, not just the memory. It is a process of discovery in which the student is the main agent, not the teacher" (p. 23).

The reports not only advocate active modes of teaching and student involvement in their learning, but also make specific recommendations to achieve this. Research in teaching and learning conducted in the past decade at the undergraduate level supports these recommendations. We now turn to these recommendations and the literature that supports them.

Several studies (Astin, 1992; King, 1989; King, 1992) document the importance of active student participation in the learning process at the undergraduate level in all disciplines. For example, Astin (1992), after examining nearly 200 environmental and curriculum variables with students at approximately 200 colleges and universities, concluded that student-student interaction and student-teacher interaction are the best predictors of positive student cognitive and attitudinal changes in the undergraduate experience. These findings draw attention to the current emphasis not only on what is taught and learned but also how it is taught.

The report of the National Institute of Education recommends that college faculty organize small classroom discussion groups, require in-class presentations, create opportunities for student projects, and provide experiential learning situations when appropriate (National Institute of Education, 1984). The success of cooperative learning as a classroom approach at the kindergarten through twelfth grade level has been documented in many studies over the past 25 years. Slavin (1994), in a review of the research that focused on studies of at least four weeks duration, found that cooperative learning has a positive effect on achievement as long as two conditions are met: group goals and individual accountability. Johnson and Johnson (1989) surveyed 193 studies in many disciplines in which cooperative learning is compared to more traditional forms of discussion. In more than $50 \%$ of the studies the
cooperative learning approach is found to be more effective in increasing student learning than more traditional forms of instruction. The reverse is true in only $10 \%$ of the studies. Although the number of college level studies in cooperative learning is significantly less, several educational researchers (Cooper \& Mueck, 1992; Cottell. 1991; Dubinsky, 1989, October; Frierson, 1986) found that the use of small cooperative groups in undergraduate classrooms increases student involvement in their education and student learning. Faculty in undergraduate institutions anecdotally report using other instructional strategies to promote active learning, such as guided reciprocal peer questioning and guided student-generated questioning, to induce critical thinking (King, 1989; King, 1990) and instructor and studentgenerated example sequences to introduce concepts (Decyk, 1994).

Several of the reports on undergraduate education (Association of American Colleges, 1985; Boyer, 1987; National Institute of Education, 1984) suggest the use of assessment to improve teaching and increase student leaming. In response to these suggestions, many faculty in many different disciplines engage in an informal type of classroom research titled "classroom assessment" (Angelo, 1991; Cross \& Angelo, 1988; Nummedal, 1994; Wolff \& Harris, 1994). Classroom assessment differs from student assessment in that its purpose is to provide feedback to both the teacher and student about the teaching and learning process while the learning is in progress. In most situations, classroom assessment is not intended to evaluate or grade student work. Angelo (1991) defined classroom assessment as follows. "Classroom assessment consists of small-scale assessments conducted continuously in college classrooms by discipline-based teachers to determine what students are learning in that class" (p. 9). Mosteller (1989) provided an anecdotal example of a form of classroom assessment he uses that he names the "muddiest point" technique. At the end of a class he asks his students to provide him with a short written response on what was least clear in the class session. He then makes changes in the next class
session based on those responses. Others report different methods of classroom assessment and student assessment such as portfolios (Crouch \& Fontaine, 1994) and self-analysis of homework (Olmsted, 1991). This informal research to improve teaching and learning may serve as a first step to more formal action-research in the undergraduate classroom.

Alternative approaches to classroom instruction and assessment appear frequently in discussions about the need for providing access to college to an increasingly diverse population. The report of the Association of American Colleges (1985) notes that during the first two hundred years of American higher education, only those aspiring to select professions attended college, with most of the work done by people who had not gone to college. Since World War II, American colleges have experienced explosive growth of a diverse population, with more than $62 \%$ of American high school students enrolling in college (U. S. Department of Education, 1995). The National Center for Education Statistics (1995) reports that more than half of all undergraduates are women, one out of five is a member of a minority group, two out of five are over the age of 25 , and less than three out of five attend college full time. Boyer (1987) stressed that "colleges and universities must recommit themselves to the task of equality of opportunity for all . . . It is college that is crucially important to advancing prospects for black and Hispanic students" (p. 39). Several researchers (Cooper \& Mueck, 1990; Frierson, 1986; Obler, Arnold, Sigala, \& Umbdenstock, 1991), concerned with ways to increase student involvement in their learning, specifically address the needs of a diverse student body. In a particularly successful effort, Treisman (1990) found that African-American students at the University of California, Berkeley, who work cooperatively in enrichment sessions outside of class, receive calculus grades one letter higher than African-Americans not involved in the program.

In their concern for undergraduate teaching and learning. the reports (Association of American Colleges, 1985; Bok, 1986; Boyer, 1987; National Institute of Education, 1984) specifically address the first two years of undergraduate education. According to the report of the National Institute of Education (1984), "College administrators should reallocate faculty and other institutional resources toward increased service to first- and second-year undergraduate students" (p. 25) According to the report, this first goal requires a corollary recommendation that the finest instructors should work with first-year students, providing opportunities for intense intellectual interaction between instructors and students. (p. 25).

In summary, several national reports published in the 1980s criticized undergraduate teaching and learning and recommend new goals for higher education. In response, the academy began thinking more critically about teaching and student learning. Educational researchers and newly involved faculty began to focus renewed attention on how to increase the learning of an increasingly diverse body of college students. More recently, Boyer (1990) noted some promising changes in college's renewed emphasis on the scholarship of teaching, citing the University of California's recommendation that more weight be placed on teaching in faculty tenure decisions and the University of Pennsylvania's decision that the teaching of students at all levels is to be distributed among faculty members without regard to rank or seniority. The academy continues to conduct research in undergraduate student learning and address attention to improving the undergraduate experience.

In the 1980s, many mathematicians and scientists began questioning calculus teaching and learning and initiated a movement to reform the teaching of calculus. The calculus reform movement adopted many goals similar to those in undergraduate education reform. The following section addresses the problems that brought calculus reform to the national agenda and the steps taken to resolve the problems.

History of the Reform in the Teaching of Calculus
The movement to reform the teaching of calculus, also initiated in the mid1980s, parallels the recent reform efforts in undergraduate education and focuses on ways to improve the teaching and learning of calculus. Like undergraduate reform goals, goals for the reform of calculus teaching stress the need for students to become actively involved in their learning and instructors to rethink what and how they teach.

Concerned about calculus instruction, Douglas and Maurer organized a panel discussion at the Joint American Mathematical Society and Mathematical Association of America meeting held in Anaheim, California during January 1985 (Douglas, 1986a). Several hundred people attended the session, and most voiced agreement that calculus was ailing and in need of revitalizing. The participants noted that advances in technology were already changing calculus; if they wanted to influence the directions of change, the time to act was at hand. After this meeting, Douglas (1986a) received funding for a national conference at Tulane University that came to be widely recognized as the Tulane Conference and the more formal beginning of the calculus reform movement. The report of this conference (Douglas, 1986d) contains many background articles calling attention to the problems in calculus teaching and learning and a forward look at how to address the problems. At a colloquium held one year later in Washington, D. C. mathematicians and scientists continued the discussion about calculus reform and how efforts to respond to the problems were progressing (Steen, 1988a).

The brief history of calculus reform presented in this section provides background for understanding the CCH Curriculum Project, the CCH Evaluation and Documentation Project, and the current study. The next section considers the perceived problems in the teaching and learning of calculus and then turns to goals developed and steps taken to address the problems.

## The problem

"Calculus instruction, crucial but ailing" (Douglas, 1986c, p. 7) came to represent how a significant number of mathematicians talked about calculus in the mid-1980s. Questions surfaced about the position of calculus in the post-secondary education curriculum. Faculty at many academic institutions experimented with their mathematics sequence, teaching finite mathematics in addition to or in place of calculus (Douglas, 1986a; Maurer, 1986). The use of mathematics in other disciplines had changed rapidly in the past few years. Maurer (1986) examined an introductory physics book and observed that after simple examples were presented in closed form, everything else was presented with computer approximations and graphics. Engineering students used computers to solve linear systems in their first engineering course. Levin (1988) expressed the view of scientists by stating that they needed changing mathematical tools and that the calculus course must change to meet those needs. He joined others in questioning whether students might benefit more from an emphasis on conceptual understanding of calculus ideas rather than on calculus techniques. He further suggested that calculus courses address the role of mathematical models and that students investigate what calculus is, what it can do, what methods are available, and where to go to obtain deeper capabilities.

Steen (1988a), at a national colloquium conducted by the National Research Council in 1987, described calculus as (a) the culmination of the study of school mathematics, (b) a pre-requisite to the majority of programs of study in colleges and graduate schools, (c) the dominant college-level teaching responsibility of university departments of mathematics, (d) a course whose techniques were being subsumed by calculators and computers, and (e) an important component of a liberal education. He also noted that those concerned about the broad purposes of education were finding that calculus was failing students. Rather than focusing on whether the course contributed to students' ability to think clearly, communicate, and solve complex
problèms, calculus courses generally required that students only perform straightforward calculations or solve word problems for which the textbook provides a template. Maurer (1986) echoed the concern by remarking that "mathematicians talk about teaching [students] to think but describe their teaching in terms of topics and theorems" (p. 82).

Young (1988) observed that the content and the spirit of calculus had not changed between 1935 and 1986 even though the world had changed dramatically. Before 1960, although most students took mathematics in their freshman year, only students in the physical sciences, engineering, and mathematics took calculus--a sophomore course. Young attributed that change to the improvements in the secondary preparation of college-bound students brought about by the New Mathematics and based his statement on the results of the survey conducted by the Conference Board of the Mathematical Sciences of the American Mathematical Society in 1965. Young notes that by 1986, however, a greatly increased number of students enrolled in calculus ( $31 \%$ of the higher education enrollment). Although a much more heterogeneous group, this new group did not share their predecessors' enthusiasm for calculus and was often mathematically unprepared for the course (Young, 1988). Anderson and Loftsgaarden (1988) supported Young's observations and related them to their own finding that calculus had become, in many instances, an undesirable course for mathematicians to teach. According to Steen (1988b), "For far too many able students, calculus served as the end of ambitions rather than the key to success." Anderson and Loftsgaarden (1988)reported that in the 1986-7 academic year fewer than half of the students who enrolled in calculus finished the term with a passing grade.
"Guess who's coming to college?" is how Malcom and Treisman (1988, p. 130) addressed the increasing percentage of minority and returning women students and suggested that calculus increasingly serves as a barrier to many of those students.

Douglas (1986a) added that an understanding of calculus concepts can provide all students with greater access to the increasingly technological workplace. Calculus, as a study of change, sits at the center of almost all science.

As we have seen, by 1986, many mathematicians were expressing serious concern about the problems regarding what is taught in calculus courses and how it is taught. A first step, recognition of the problem on a national level, had occurred. The next section addresses steps taken in efforts to address the problem.

## Steps toward a solution

Mathematicians and scientists who attended the Tulane Conference not only discussed the problems in calculus teaching and learning but began to focus on the question, "Will this change occur thoughtfully or haphazardly?" (1986b, p. v). The following subsection looks at how the Tulane Conference was organized, how those in attendance addressed the problems in calculus, and what solutions they developed.

## The Tulane Conference

As a result of the panel discussion in Anaheim, California and concern for calculus programs, Douglas (1986c) submitted to the Sloan foundation a proposal to hold a conference at Tulane University aimed at developing alternative curricula and teaching methods for calculus at the college level. The Tulane Conference established direction for reform in the teaching of undergraduate calculus (Tucker \& Leitzel, 1995). The participants each took part in one of three workshops that addressed calculus content, calculus teaching methods, and implementation of the plans developed at the conference.

The members of the content workshop developed their outlines for three calculus syllabi, basing the outlines on previous recommendations by the Committee for Undergraduate Programs in Mathematics of the Mathematical Association of America, the Advanced Placement Calculus course descriptions, and syllabi from the home institutions of many conference
participants. The workshop participants spent the greatest amount of time developing the syllabus for a 35 hour Calculus I course (see Appendix A). Although the workshop participants agreed on the benefits of extensive computer use, they also recognized the limitation presented by the accessibility of appropriate hardware. They recommended the use of calculators until availability made computer use part of the mainstream course.

The report of the methods workshop acknowledged its vision of calculus as the language of change that includes a domain of rich and powerful ideas and listed the following goals for calculus instruction (Davis, et al., 1986). Calculus instruction should:

- develop students' understanding of concepts as well as their ability to use the relevant procedures. Instruction should be aimed at conceptual understanding and at developing in students the ability to apply the subject matter they have studied with flexibility and resourcefulness.
- expose students to a broad range of problems and problem situations and a broad range of approaches and techniques for dealing with them.
- help students develop an appreciation of what mathematics is, and how it is used.
- help students develop precision in both written and oral presentation.
- help students develop their analytical skills and the ability to reason in extended chains of argument.
- help students develop the ability to read and use text and other mathematical materials. (p. xvi)

The report of the methods workshop then suggested alternative methods for classroom teaching and assessment of student learning and ways to incorporate the use of technology in calculus courses. The alternative teaching and assessment
methods advocated by the workshop participants are further addressed in a later section of the literature review devoted to the theoretical foundations of the clustering scales for the current study. The report of the methods workshop briefly noted the need for better understanding about how students learn advanced mathematics but did not mention the existence of literature available at that time.

All three workshops operated at the same time and independently. Consequently, the implementation workshop participants lacked knowledge about the final recommendations of the other groups. More than the other two groups, this seemed to hinder the implementation group in their work; however, they did make recommendations under the headings of materials development, field testing, and public relations. Their recommendations included a recognition of the need for a new emphasis on teaching; the importance of publicizing the change in teaching style and engaging faculty in regular dialogue with other departments; and a concerted public relations effort to "bring others in the mathematics community to an understanding of and support for the proposed changes" (p. xxv).

The National Science Foundation "Calculus Initiative"
The National Science Foundation Calculus Initiative began making awards in the fall of 1988 , when it funded five multi-year projects and 20 planning grants, including one to Gleason and Hughes Hallett (1988) at Harvard University. In 1989 the National Science Foundation funded six large calculus curriculum projects, including a proposal submitted by Gleason and Hughes Hallett (1989) titled "Core Calculus Consortium: A Nationwide Project". The consortium, led by Gleason and Hughes Hallett at Harvard University, includes mathematics department faculty from the University of Arizona, Colgate University, Haverford-Bryn Mawr Colleges, the University of Southern Mississippi, Stanford University, Suffolk Community College, and Chelmsford High School. The consortium proposed to develop a core calculus curriculum that would prove attractive to a wide variety of institutions. This project
came to be known as the CCH Curriculum Project and received funding that amounted to more than two million dollars between 1989 and 1993.

In 1991, the National Science Foundation began directing program awards primarily toward dissemination and implementation of reform-based calculus curriculum materials and curriculum efforts at the pre- and post-calculus level. Starting in 1992, the National Science Foundation awarded CCH implementation grants to Evergreen State College, the University of Arizona, the University of Michigan, the State University of New York at Stony Brook, and the Peralta Community College District Office, California. Each of these awards were for more than $\$ 100,000$, and many were designed to reach out to academic institutions in neighboring regions and states. In that the present study is part of the evaluation of the CCH Curriculum Project, a later section of the current chapter addresses in greater detail the goals of the CCH Curriculum Project and the CCH Evaluation and Documentation Project. The paragraphs below consider a conference that served to increase participation in the movement to reform the teaching of calculus and publicize the National Science Foundation "Science Initiative."

## A National Colloquium

On October 28-29, 1987, the National Research Council, in collaboration with the Mathematical Association of America, conducted a colloquium, "Calculus for a New Century", attended by over six hundred mathematicians, scientists, and educators (Steen, 1988a). The conference followed the National Science Foundation proposal to congress for the calculus initiative. Purposes for the colloquium included publicizing of the new program, framing a national agenda for calculus reform, and insuring participation of the scientific and engineering communities (p. xiii). At the same time the two mathematics boards of the National Research Council, the newly formed Mathematical Sciences Education Board and the Board on Mathematical Sciences, jointly launched the Mathematical Sciences in the Year 2000 program (p.
xiii). The colloquium served to energize and expand the participation of the mathematics community in the reform of the teaching of calculus.

## Some Current Perspectives on Reform in the Teaching of Calculus

The current study is being conducted in the mid-1990s. Interest in and efforts toward reform in the teaching of calculus have continued to expand since the mid1980s. Reform in the teaching of school mathematics, also, is well established, with an increasing number of school mathematics teachers engaging with efforts to align their teaching with the frameworks established in the NCTM Curriculum and Evaluation Standards for School Mathematics (1989). Goals for reform in the teaching of calculus and goals for the teaching of school mathematics are closely aligned in their emphasis on conceptual understanding, new teaching practices, and alternative assessment methods. Some suggest that the NCTM Curriculum and Evaluation Standards for School Mathematics were influential in the initiation of the reform in the teaching of calculus. It seems reasonable to imagine that the time was at hand for the various reform movements that have paralleled each other over the past decade, including reform in the teaching of school mathematics, reform in undergraduate education, and reform in the teaching of calculus. With at least these three reform movements on the national agenda, interaction and influences between them seem a natural consequence.

Evidence of the continuing national interest in reform in the teaching of calculus is well established. Tucker and Leitzel (1995) noted that, in the study they conducted, $68 \%$ of the responding institutions reported modest or major reform efforts. However, this does not mean that all mathematicians are in agreement with the directions of reform in the teaching of calculus. Some mathematicians actively decry the movement as detrimental to the education of students. The internet provides a lively forum for sometimes vehement debates about the reform movement. Noting that teaching a reform-based calculus course requires far more time on the part
of the instructor than more standard calculus, Reed (1994, August/September) contended that a clash will come in the calculus classrooms of research universities. Teaching is a primary mission at many secondary schools, two-year colleges, and undergraduate institutions. Some are concerned that at institutions where faculty members focus primarily on their research, some change in emphasis would be essential for successful implementation of reform-based courses. A hopeful scenario might be an integration of the teaching and research cultures at research universities and similar reform efforts in other undergraduate mathematics courses.

The CCH Curriculum Project represents one example of a reform-based calculus curriculum project funded through the National Science Foundation "Calculus Initiative." The CCH Evaluation and Documentation Project and the current study investigate how some users of the CCH Curriculum Project materials respond to the changes suggested by the textbook and the movement toward reform in the teaching of calculus. The following section describes the CCH Evaluation and Documentation Project in greater detail.

The CCH Evaluation and Documentation Project
As previously discussed, the National Science Foundation funded the CCH Curriculum Project to develop a reform-based calculus textbook. Subsequent grants provided for workshops to help instructors change their calculus courses to reflect the goals of the project and the calculus content of the textbook. Several National Science Foundation implementation grants funded individual and consortiums of academic institutions in their efforts to implement calculus reform using CCH Curriculum Project materials.

The CCH Curriculum Project proposal to the National Science Foundation included a summative evaluation component listing Ferrini-Mundy at the University of New Hampshire as the independent evaluator. Ferrini-Mundy in turn, began the

CCH Evaluation and Documentation Project in spring 1994. The CCH Evaluation and Documentation Project lists the following goals (Ferrini-Mundy, 1994):

- to determine how the stated goals of CCH Curriculum Project are interpreted and implemented at sites using the CCH Curriculum Project materials. developing both broad and specific descriptions of the settings in which the CCH materials are used
- to assess student variables in CCH Curriculum Project reform settings, including learning of calculus, conceptions of mathematics, attitudes, and persistence
- to investigate faculty variables in CCH Curriculum Project reform settings, including perceptions about student learning, pedagogy, attitudes and beliefs, time commitments, and relationships with other faculty and administrators - to examine and describe the evolution of calculus reform efforts, particularly the CCH Curriculum Project, in colleges and universities and, more broadly, the process of change in undergraduate teaching and learning in the context of the CCH Curriculum Project model.

The Four Phases of the CCH Evaluation and Documentation Project
The design of the CCH Evaluation and Documentation Project reflects the viewpoint of Stake (1977) who considers description and judgment the two essential and basic acts in educational evaluation. The project design recognizes the belief that faculty members uniquely interpret and implement the CCH Curriculum Project materials within the context of their own situation. Quantitative and qualitative methods are employed to better understand how the materials are interpreted and implemented.

The CCH Evaluation and Documentation Project is being conducted in four phases, with each phase informing later phases. The first phase began in early summer 1994 with the sending of an Initial Questionnaire (see Appendix B) to the

504 academic institutions that appeared on the publisher's purchase lists. The Initial Questionnaire seeks to determine (a) which academic institutions are currently using the materials; (b) the extent of their use; (c) basic information about the institutions; (d) the names of faculty members, called site liaisons, willing to participate in the project; and (e) what information would be of interest to the sites.

The second phase, the more comprehensive Site Liaison Survey (see Appendix C), was sent in the spring of 1995 to faculty members who had previously agreed to serve as site liaisons. This survey, consists of closed and open-ended questions and is intended to: (a) identify characteristics of students and courses using CCH Curriculum Project materials that would provide an indication of how different academic institutions interpret the goals and implement the materials of the CCH Curriculum Project ; (b) determine the pedagogical approaches of faculty using CCH Curriculum Project materials; (c) determine the level and type of technology used in courses using CCH Curriculum Project materials; and (d) identify factors influencing the initiation of reform efforts. In addition, the Site Liaison Survey asks the respondents to send artifacts, such as descriptions of local evaluation efforts and blank or anonymous student examinations, that would be useful in later phases of the project. Site Liaisons are also asked about their willingness to serve as case-study sites and the nature and level of information that might be available with additional effort, such as student attitudes, scores, or comparative information.

The third phase of the project began in the late spring of 1995 when site liaisons at sixty undergraduate institutions were each sent up to fifty Student Surveys. The institutions that received Student Surveys were randomly selected, assuring representation of institution types, a wide range of geographic regions, and varying student population types and sizes. The Student Survey contains closed questions that can be analyzed using quantitative methods and open questions that can be analyzed qualitatively. The survey asks students about their (a) perceptions of faculty
interpretation and use of the materials and pedagogical practices; (b) attitudes and beliefs about calculus and mathematics; and (c) perceptions of their own learning.

In the fall of 1995, each site liaison at an institution that was currently using or had previously used CCH Curriculum Project materials was sent up to five Faculty Surveys (see Appendix D). The Faculty Survey asks the responding faculty members about their (1) background and experience; (2) impressions and use of the CCH materials; (3) pedagogical approach and assessment practices; (4) opinions about CCH student learning; and (5) beliefs about reform in the teaching of calculus. The questions on the Faculty Survey were informed by responses to the previous surveys.

The fourth phase of the project consists of a flow-through component, two case studies, and an interpretation of student examinations from approximately thirty participating academic institutions. The flow-through component is under the direction of Daniel Madden at the University of Arizona and addresses student continuation and achievement in subsequent mathematics courses. Karen Graham at the University of New Hampshire directs case studies at two CCH Curriculum Project implementation sites. Recognizing the uniqueness of each implementation site, evaluation staff visited the two sites and attended to variations in content, process, goals, implementation, politics, context, and program quality.

## The Current Study as a Component of the

## CCH Evaluation and Documentation Project

The CCH Evaluation and Documentation Project accomplishes its goals through the four phases described in the previous subsection. The current study includes the analysis of the data obtained through the survey portions of the first three phases: the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey. The Student Survey is not part of the current study. This analysis uses a statistical procedure cluster analysis. Cluster analysis groups is a statistical procedure that groups entities (academic institutions in the current study) on a measure of similarity
that permits comparisons among entities. In the current study, the measures of similarity are six clustering scales that represent dimensions of reform in the teaching of calculus, concepts, teaching, approach, assessment, technology, and access.. The six clustering scales are based on the goals for reform established by those in attendance at the Tulane Conference (Douglas, 1986). Informed by the literature, the scales were chosen to characterize the major themes of the movement to reform the teaching of calculus. Cluster analysis identifies clusters or groups of academic institutions that exhibit similar patterns in their implementation of CCH Curriculum Project materials along the clustering scales. The following section defines and discusses the theoretical foundations of the six clustering scales used in the current study.

## Theoretical Foundations of the Clustering Scales

Attention to and careful use of explicitly stated theory in the choice of clustering scales is essential to and provides the necessary foundations for a meaningful cluster analysis (Aldenderfer \& Blashfield, 1984). Recognizing the importance of the clustering scales to the success and relevance of the current study, this section addresses the theoretical foundations of the clustering scales in detail.

The six clustering scales defined and used in this study, concepts, approach, teaching, assessment, technology, and access, reflect an effort to respect the goals of reform-based calculus identified at the Tulane Conference, adopted by the authors of the CCH materials, and still discussed by the concerned community. Efforts were made to define clustering scales that include and give equal weight to the various aspects or dimensions of reform in the teaching of calculus. The following description of each clustering scale includes a listing of the specific goals of the Tulane Conference in 1986 and of the CCH Curriculum Project that define that scale as well as the literature that supports the listed goals. The careful discussion of each
clustering scale establishes the theoretical foundations of the cluster analysis in the goals of reform-based calculus and in the literature.

In reading the definitions and descriptions of the clustering scales, the reader is asked to note the following two items:
(a) The specifications for each clustering scale, of necessity, are somewhat arbitrary. For example, the teaching clustering scale could be very broad. However, the approach clustering scale incorporates several related, specific ways instructors may approach mathematics topics. Teaching issues of different types are reflected in the teaching and approach clustering scales.
(b) The discussion of each clustering scale follows a consistent pattern that first lists and cites the relevant Tulane Conference and CCH Curriculum Project goals and then references and discusses the related research literature or anecdotal evidence reported in journals. The citations included with the bulleted reform-based calculus goals indicate where the goal can be found in the Tulane Conference report or in the CCH Curriculum Project materials.

We now turn to the definitions and descriptions of the six clustering scales.

## Concepts

The following reform-based calculus goals help give definition to the concepts clustering scale. Calculus courses should:

- be leaner, contain fewer topics (Douglas, 1986, p. v).
- have greater conceptual depth (Douglas, 1986, p. v).
- cover many fewer mathematical techniques (Douglas, 1986, p. v).
- contain much less drill on routine procedures (Douglas, 1986, p. v).
- help students learn to justify or provide a rationale for answers (Douglas, 1986, p. ix).
- develop students' ability to abstract and generalize (Goldstein et al., 1986, p. ix).
- present a mathematically-sound, informal rationalization, that students can understand, for many theorems and definitions (Hughes Hallett \& Gleason, 1994).
- make clear the centrality of calculus in the study of systems that change (Douglas, 1986, p. v).
- demonstrate the beauty of mathematics (Douglas, 1986, p. v).

Foundations of the Concepts Clustering Scale in the Literature
Much of the research in student understanding of key mathematical concepts conducted in the past decade is based on the view that students' conceptual understanding evolves over time (Strike \& Posner, 1985; Tall, 1991b; Vinner, 1991). A student's ability to state the definition of a concept does not reveal whether the student has an understanding of the concept or can use the concept in solving problems. Tall and Vinner (1981) were the first to introduce the notion of a "concept image", an image that comprises the visual representations, the mental images, the experiences, and the impressions conveyed by the concept name. Tall and Vinner posited that students' concept images continually develop as they study mathematics and encounter new experiences. The concept images students hold may or may not be rich, robust, or even internally consistent. To illustrate this, Tall and Vinner described work in which they found that 14 out of 36 students claimed correctly that $\lim _{n \rightarrow \infty}\left(1+\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\ldots\right)=2$ but in the next problem these same students claimed that $.999 \ldots<1$. Although mathematicians might see the two problems as the same and immediately recognize the second as $\lim _{n \rightarrow \infty}\left(\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\ldots\right)=1$, the students did not interpret the questions as the same. Tall and Vinner interpreted the students' response to indicate that their concept image incorporates inconsistencies and is not as fully developed as their response to the first question might indicate.

Many educational researchers (Artigue \& Viennot. 1987; Cornu. 1991: Davis \& Vinner, 1986; Ervynck, 1981; Ferrini-Mundy \& Graham, 1991, January; Nemirovsky \& Rubin, 1992; Orton, 1983; Sierpinska, 1987; Tall, 1992; Vinner, 1982; Williams, 1991) have adopted the idea of a developing concept image as they seek to understand how students come to understand the central ideas of calculus such as limits, derivatives, and integrals. As in the example above, the authors of these studies found that in many instances students' concept images are unexpectedly different from those held by the mathematics community and sometimes different from that which students communicate in written work (Thompson, 1991).

A second example provides evidence of the need felt by many in attendance at the Tulane Conference and the authors of the CCH Curriculum Project textbook to develop students' "greater understanding" with "less manipulation" (Hughes Hallett \& Gleason, 1994). Graham and Ferrini-Mundy (1989) described work in which they investigate the understanding Cyndi, a calculus student, holds about the derivative concept. When asked what a student needs to understand about the derivative, Cyndi focuses on rules for taking a derivative. Presented with a graph and formula for the function $\mathrm{f}(\mathrm{x})=3 \mathrm{x}-1$, Cyndi correctly uses the formula and a derivative "rule" to determine that the derivative was 3. (See Figure 1.) When asked to relate the graph to the derivative she obtained using the formula, Cyndi responded "If you took the derivative of that function it'd also be $3 . \ldots$. So, I'm not sure if that relates to it at all, but you come up with the same thing. Like the derivative is what I think the slope is, but I don't think that always happens" (p. 7). In this example, Cyndi seemed to be able to apply procedural knowledge and, in her case, succeed in the course, receiving a B. However, in a clinical interview, she revealed that her conceptual understanding of derivative was not developed to the point an instructor might expect based on her answer to a procedural question.


Figure 1. Graph of $y=3 x-1$

As Nemirovsky (1993) noted, many traditional calculus courses do not go beyond students' acquisition of procedures and notation and students quickly forget what they have learned. As a consequence many students who have satisfactorily completed calculus are unprepared for subsequent work in mathematics, science, or engineering. The advocates of reform in the teaching of calculus believe that by placing an increased emphasis on students' conceptual understanding they will retain and be able to use the calculus they have learned. This position seems well supported in the literature.

## Summary

Survey items included in concepts pertain to an emphasis on the development of students' conceptual understanding of the mathematical concepts that are central to calculus. The reform-based calculus goals imply that students should spend less time doing routine procedures and more time developing a deep and rich understanding of concepts. Rather than memorizing definitions and statements of theorems, students should develop an understanding of their meaning.

## Approach

The following reform-based calculus goals help give definition to the approach clustering scale. Calculus course instructors should:

- provide greater conceptual depth numerically and geometrically (Douglas. 1986, p. v); present every concept graphically, numerically, and algebraically ("Rule of Three") (Hughes Hallett, 1994, p. 3).
- help students learn to analyze quantitatively and qualitatively (Goldstein et al., 1986, p. ix).
- use an inductive approach to topics; that is, develop concepts from common sense investigations and real world problems rather than from abstract definitions and theorems (Davis et al., 1986, p. xvii; Hughes Hallett, 1994, p. 2).
- use problems from the real world to serve as a context for doing mathematics and to introduce mathematical ideas (Goldstein et al., 1986, p. xvii).
- make clear the role of mathematics in understanding, explaining, and modeling the real world (Douglas, 1986, p. v).


## Foundations of the Approach Clustering Scale in the Literature

A discussion of the literature associated with giving of equal weight to graphic, numeric, and algebraic approaches and using inductive and real-world approaches follows. Substantial research documents the reluctance of students to visualize mathematics and their preference for algebraic representations (Eisenberg \& Dreyfus, 1991; Tall, 1991a; Vinner, 1989; Zimmerman \& Cunningham, 1991). Nevertheless, it has been shown that by using graphs to visualize concepts, students can obtain deeper conceptual understanding (Beckmann, 1988b; Bell \& Janvier, 1981; Eisenberg \& Dreyfus, 1986; Leinhardt, Zaslavsky, \& Stein, 1990; Tall, 1985; Tall, 1980; Zimmerman, 1991). Eisenberg and Dreyfus (1991) suggested that students' preference for algebraic representations relates directly to the reason why
diagrams are so useful in mathematics. In a graph or diagram, students must interpret a very complex and concentrated collection of information. Reformulation of all of the information in sequential form would require significantly more space. Even though "a picture is worth a thousand words," students may have difficulty interpreting all of the words or information embedded in a graphical or pictorial representation. Vinner (1989) further interpreted students' preference for analytic or algebraic representations by noting that the algebraic mode of interpretation is more common in solving routine or near-routine problems, the type of problems most common in standard calculus courses.

Several educational researchers (Confrey \& Smith, 1995; Keller \& Hirsch, 1992; Monk \& Nemirovsky, 1994; Nemirovsky, 1993; Nemirovsky \& Rubin, 1992; Thompson, 1991) have investigated the teaching of calculus from the perspective of numerical covariation. They found that students who explore familiar, real-world situations and represent them numerically develop a greater intuitive sense of the central concepts of calculus.

In a particularly compelling, but anecdotal, example, Speiser and Walter (1994) gave students a series of photos, taken at fixed intervals, of a cat running. Each picture was overlaid on a fixed grid to enable students to measure distance and rates. Although the authors' intent was to introduce tangents to a curve and derivatives, they found that the introduction of the real world context reframes the students' perspectives in ways that deepen and broaden their understanding of the calculus concepts.

In a second example, Nemirovsky and Rubin (1992) had students work with a physical model of a car that automatically generates graphs and numerical tables of the car's motion. In a clinical interview situation, Nemirovsky and Rubin asked students to predict the graph or numerical table that would result when they move the car along a track. After moving the car, the students compared the generated graphs
and tables and question or improve their model in the next trial. The researchers observed that students' understanding about the relationship between a function and its derivative increased in this situation.

In the two preceding examples, the students demonstrate deeper understanding of calculus concepts after real-world experiences that they represent numerically and graphically. At the present time a graphic, numeric, and algebraic approach to topics that incorporates investigations of familiar contextual situations seems promising. These approaches, particularly those made possible by recent technological advances. provide opportunity for additional research.

## Summary

Survey items classified under approach are defined by instructors (a) using and giving equal weight to graphic, numeric, and algebraic approaches to calculus concepts and problems and (b) using an inductive approach to calculus in which mathematics arises from student investigations and real world problems and (c) placing value on helping students understand the role of mathematics in modeling and understanding the real world.

## Teaching

The following reform-based calculus goals help give definition to the teaching clustering scale. Calculus course instructors should:

- make calculus teaching more interactive (Douglas, 1986, p. v; CCH, p. 7).
- try alternative teaching strategies (Goldstein et al., 1986, p. xvii).
- have students work in small groups solving problems in class (Goldstein et al., 1986, p. xviii).
- have students give oral presentations in class (Goldstein et al., 1986, p. xviii).
- have students engage in mathematical exploration in class (Goldstein et al., 1986, p. xvii, Hughes Hallett, 1994, p. 7).
- have students write and talk about mathematics in class (Goldstein et al., 1986, p. xviii, Hughes Hallett, 1994, p. 7).


## Foundations of the Teaching Clustering Scale in the Literature

A search of the literature in classroom teaching of calculus produced formal studies and anecdotal discussions. It appears that increasingly mathematicians are reflecting on their practice, and on student learning, and are writing about their experiences. For example, Speiser and Walter (1994) developed the catwalk example discussed earlier in this section in the context of their classroom teaching, to introduce the derivative in a situation that encourages active participation by their students.

Several studies (Brechting \& Hirsch, 1977; Davidson, 1976; Deutsch, 1960; Dubinsky, 1990; Rogers, 1988) attest to the positive effects of group work in undergraduate mathematics classrooms by documenting positive contributions to student learning and student attitudes about mathematics. Rogers (1988), in a study of teaching and learning at the State University of New York at Potsdam, a university renowned for the large number of mathematics degrees awarded, found that an emphasis on learning to think mathematically through the use of student-centered teaching methods instead of lectures contributes to student success. Urion and Davidson (1992) reported the results of five studies contrasting small-group learning and more teacher-centered instructional style. The studies, conducted in junior high, secondary, and college classrooms, supported the claim that small-group instruction prepares students as well as or better than teacher-centered methods.

## Summary

Survey items that come under teaching pertain to what the teacher does in the classroom and how the teacher allocates classroom time. The reform-based calculus goals emphasize the need for instructors to make classrooms more interactive; to try
alternative teaching strategies; and to engage the students in exploring, doing, writing, and speaking about mathematics.

## Assessment

The following reform-based calculus goals help give definition to the assessment clustering scale. Calculus course instructors should:

- change their methods of testing students (Douglas, 1986, p. v).
- use testing procedures that correspond to the goals of instruction (Goldstein et al., 1986, p. xix).
- use many different ways to assess student learning such as small group tests, lab reports (group or individual), homework exercises (group or individual), projects, portfolios, class participation, and oral presentations (Goldstein et al., 1986, pp. xvii, xviii, xix).


## Foundations of the Assessment Clustering Scale in the Literature

The CCH Curriculum Project materials include open-ended and investigative problems in the sample examination questions; however, they do not explicitly suggest the use of alternative assessment strategies such as student presentations or group tests. The National Council of Teachers of Mathematics (1993) has placed considerable emphasis on assessment to inform teaching and learning in the nation's schools. Most of the suggestions would also be appropriate for use in calculus classrooms. Pedagogical discussions frequently advocate the use of alternative methods of assessment for finding out what students do know about the material and are able to do. Some educators believe that traditional tests focus on what student's do not know.

Rogers (1988), in a study discussed previously within the description of the teaching clustering scale, found that the mathematics department at the State University of New York in Potsdam used a flexible grading and assessment scheme and attributes some student success to that approach. Deatsman (1979) studied the
effect of allowing students to retake tests on which they do poorly and found that retesting harms slower students who use the test as a rehearsal and do not prepare adequately. Anderson (1989), in an investigation about gender differences on mathematics tests, found that female mathematics majors perform as well as male mathematics majors on standard open-ended questions but are less likely than men to guess on multiple choice and true-false questions. Women average only $80 \%$ as well as men on multiple choice and true-false tests. As faculty experiment with alternative approaches to student assessment to acquire a broader picture of what students know and can do, the need for research in the area of student assessment becomes increasingly apparent.

## Summary

Survey items that come under assessment pertain to the various ways instructors assess student learning. According to reform-based calculus goals, instructors should change their methods of assessment, using a variety of methods that correspond to the many different goals of instruction.

## Technology

The following reform-based calculus goals help give definition to the technology clustering scale. Calculus instructors should:

- make use of the latest technology (Douglas, 1986, p. v).
- incorporate the use of computer programs or calculators that graph functions and do numerical computation and symbol manipulation (Goldstein et al., 1986, p. vii).


## Foundations of the Technology Clustering Scale in the Literature

Research related to the influence of technology on student learning has grown dramatically in the last few years, with several annual national conferences focusing on the use of technology in the mathematics classroom. Many studies (Ayers, Davis, Dubinsky, \& Lewin, 1988; Keller \& Hirsch, 1992; Nemirovsky \& Rubin, 1992;

Speiser \& Walker, 1994; Tall, 1987), some of which have been cited previously in this section, address visual or numeric approaches to calculus concepts in situations rich with technology. More than a few studies show that use of technology can increase students' conceptual understanding with minimal or no loss of achievement on skills-based tests (Beckmann, 1988a; Dick, 1989, March; Dunham, 1993; Heid, 1988; Ruthven, 1990; Shoaf-Grubbs, 1991; Sun, 1993; Tall, 1988). In an often cited research effort, Heid (1988) introduced the concepts of derivative and integral to students who were expected to take derivatives and find integrals on software with those capabilities. Rather than spending time on procedural skills, Heid focused on students' conceptual understanding of differentiation and integration. Only during the last weeks of the term did the students learn the "rules" for differentiation and integration. She found the students performed as well as traditionally taught students on common final exams and demonstrate greater conceptual understanding.

On the other hand, some studies point to unexpected effects on student learning introduced by the use of technology (Demana \& Waits, 1988; Dunham, 1993; Goldenberg, 1987; Lauten, Graham, \& Ferrini-Mundy, 1994, September). Goldenberg (1988), in a frequently cited study, found that students sometimes misinterpret transformations of functions when learning in a graphing utility context. Students asked to investigate the effects of the constant $b$ in the linear form $y=a x+b$ by experimenting with many graphs and abstracting a rule, sometimes misinterpret the function as "moving to the left" rather than "moving higher" as the value of $b$ increases. (See Figure 2.) He stressed the need for instructors to attend carefully to students' conceptions in a technological environment.


Figure 2. Graphs of the lines $\mathrm{y}=\mathrm{x}+1$ and $\mathrm{y}=\mathrm{x}+2$.

The increasing use of technology in calculus classrooms and the influence of technology on what is taught, how it is taught, and students' conceptual understanding contribute to the significance of this clustering scale. Douglas (1986) recognized the role of technology in a changing calculus curriculum when he stated "...the group at the Tulane Conference] began to understand that calculus instruction is going to change! Technology is not going to let calculus instruction stay the same!" (p. v). It appears that those in attendance at the Tulane Conference anticipated correctly the influence of technology on calculus courses.

## Summary

Technology pertains to the use of technology in CCH Curriculum Project courses. Advocates of calculus reform agree that technology should be used to increase student understanding of calculus concepts rather than used just for the sake of using technology.


#### Abstract

Access The following reform-based calculus goals help give definition to access. This clustering scale includes goals that have increasingly come to the fore in the years following the Tulane Conference. Calculus courses should: - incorporate the use of textbooks that are written to be read and understood by students (Hughes Hallett, 1994, p. 2). - serve as a gateway to future study in science, mathematics, and engineering by their being accessible to a wider range of students including those who tend to pursue other disciplines dependent upon calculus (Douglas, 1986, p. iv). - be accessible to students who have long been underrepresented in calculus courses, including women, minority students, and nontraditional students (Malcom \& Treisman, 1988; White, 1993). - be thought-provoking for well-prepared students while still accessible to students with weak algebra backgrounds (Hughes Hallett, Gleason, Flath, Gordon, Lomen, Lovelock, et al., 1992).

\section*{Foundations of the Access Clustering Scale in the Literature}

Calculus, conceived as a "pump not a filter" (Steen, 1988a), speaks to the need to provide greater access to calculus for an increasingly diverse population. The importance of increasing access to calculus to a broader range of students, including women, nontraditional students, students in disciplines that require an understanding of calculus concepts, and ethnic minority groups traditionally underrepresented in calculus is a timely issue. In the NCTM Curriculum and Evaluation Standards (1989), the authors speak boldly, stating "Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate" (p. 4). The word "calculus" could well be substituted for the word "mathematics."


Many studies that address gender issues (Damarin, 1994; Fenema, 1994; Frid. 1991; Jones, 1994; Kalinowski \& Buerk, in press; Leder, 1992; Shoaf-Grubbs, 1991 : Tartre, 1990) document the need for change in the culture and teaching of mathematics to increase access to women. Others address minority issues (Asera \& Treisman, 1995; Cohen, 1995) and report successful calculus programs for minorities. As mentioned previously in the current chapter, Treisman (1983) developed a successful Mathematics Workshop program for minority students at the University of California, Berkeley. Among other goals this workshop is designed to improve serious deficiencies in minority students' mathematics and study skills. Using cooperative learning methods, Treisman and his colleagues guided the students to form peer groups that were academically-oriented and a source of support for one another. Treisman found the Mathematics Workshop program increases both the participating students' mathematics grades and their persistence in mathematics classes and the general university program.

Since the initiation of reform in the teaching of calculus, academic mathematicians have formed the Humanistic Mathematics Network. This group seeks to make mathematics teaching more humanistic, opening "up the mathematical world of excitement, adventure, and satisfaction" to a broader population (White, 1993). Copes (1982) reported that he teaches from the perspective of the Perry (1970) development scheme in trying to guide students who are dualistic in their view of mathematics to move beyond thinking of mathematics as a collection of formulas to be memorized. He encourages students to think relativistically and more deeply by considering how their intuition can inform and be influenced by logic. He has found he can engage many more and a much more diverse group of students in "doing mathematics", that is, toying with ideas, looking for patterns, and balancing intuition with logical arguments (p. 145). He, like Buerk (Buerk \& Szablewski, 1993), has
found that women are attracted to mathematics when it is taught from a relativistic rather than dualistic perspective.

## Summary

Access is defined by the accessibility of calculus to a wide range of students and the success of those students in calculus and subsequent courses dependent on calculus. According to reform-based calculus goals, the types of students for whom calculus should be more accessible include women, nontraditional students, students in disciplines that require an understanding of calculus concepts, and ethnic minority groups traditionally underrepresented in calculus.

## Summary

The preceding definitions and discussions of the clustering scales establish the necessary theoretical foundations for the cluster analysis and the current study. The definitions and discussions demonstrate the strong relationship between the clustering scales and the Tulane Conference goals, the CCH Curriculum Project goals, and the literature that focuses on calculus learning and teaching.

As part of a larger evaluation project, the current study, a documentation project, may be subject to questions about the relationship between evaluation research and academic research and the place of the CCH Evaluation and Documentation Project within those traditions. The following section addresses this issue.

The Position of the CCH Evaluation and Documentation Project within the Evaluation Research Tradition

The following paragraphs address the nature of evaluation research in education and the similarities, differences, and relationships between academic research and evaluation research. This section considers the approaches used and issues encountered in the CCH Evaluation and Documentation Project and relates
them directly to the description and characterization of educational evaluation research, seeking to firmly situate the project within that tradition. The CCH Evaluation and Documentation Project uses the phrase evaluation research to describe an evaluation study that is conducted systematically and empirically through careful data collection and thoughtful analysis, adhering to qualitative or quantitative methods of research (Patton, 1990; Pitman \& Maxwell, 1992). Although issues surrounding the use of the phrase evaluation research rather than evaluation study (Popham, 1975; Worthen \& Sanders, 1987) are recognized and respected, the intent in the current section is to focus on the differences between academic or basic research and evaluation research.

Before describing the similarities and differences between evaluation research and academic research, the meaning of the term "evaluation" and of several associated terms is explored. Evaluation, as an informal term, is used to describe many of the everyday activities in which people and educators engage. For example, in choosing between two possible textbooks for a course, instructors frequently base their decision on an informal appraisal of the worth of the two alternative books to the instructional program. Decisions in informal evaluation often are based on highly subjective perceptions of which alternative is best. The current research, instead, addresses formal evaluation wherein choices are based on systematic efforts to define criteria and obtain accurate information (Stake, 1977; Worthen \& Sanders, 1987). Most professional evaluators agree with Stake (1977) when he asserts that both description and judgment about the worth or merit of a project are essential elements of a formal evaluation effort.

Scriven (1977) was the first evaluator to distinguish between the formative and summative roles of evaluation. Evaluators conduct formative evaluation during the operation of a program to provide opportunities for improvement during the development of the program. Summative evaluation, on the other hand, is conducted
at the end of a program to assist decision makers with judgments about a program's merit or worth. The CCH Evaluation and Documentation Project provides an example of a summative evaluation project, in that it is conducted after the CCH Curriculum Project materials were produced. The next section addresses similarity and differences between academic research and evaluation research.

## Academic Research and Evaluation Research

The systematic and empirical collection and analysis of data are common to both academic research and evaluation research. In both instances, researchers seek to produce knowledge not previously available (Patton, 1990; Worthen \& Sanders, 1987). Similar to academic research, the validity and credibility of an evaluation project depends in many ways upon its methodological underpinnings (Fetterman, 1988). Evaluation research, like academic research, uses the methods of research and is a type of research.

The differences between academic research and evaluation research and the various characteristics of evaluation research are described next. A common way to distinguish between the two types of research is to consider academic research as "basic research" and evaluation research as a type of "applied research" (Patton, 1990) The worth of academic research can be measured by its contribution to theory and explanations of why things occur as they do; whereas the worth of evaluation research can be judged on its contribution to making human actions and interventions more effective (Patton, 1990; Popham, 1975; Worthen \& Sanders, 1990). Evaluation research uses the same methodologies as basic research; however, the purposes and goals of evaluation research and academic research differ. The following section considers evaluation research and the position of the CCH Evaluation and Documentation Project within that tradition.

## The CCH Evaluation and Documentation Project and Evaluation Research

The CCH Evaluation and Documentation Project is an example of evaluation research. In the following discussion of evaluation research, the CCH Evaluation and Documentation Project is related directly to the characteristics of evaluation research. Those engaged in evaluation research generally conduct inquiries not just for the benefit of their own questions but for the benefit of others' questions; whereas those engaged in academic research may have greater freedom and opportunity to conduct research that seeks knowledge for its own sake (Patton, 1990; Pitman \& Maxwell, 1992; Popham, 1975; Scriven, 1977; Stake, 1977; Worthen \& Sanders, 1987). For example, those conducting the CCH Evaluation and Documentation project seek not only to answer their own questions about the implementation of reform-based calculus curriculum materials but also to answer questions posed by the Calculus Consortium Based at Harvard, the National Science Foundation, and site liaisons at academic institutions participating in the study.

Often evaluators are required to be more utilitarian than those doing basic academic research. Evaluators are generally tied to the issues, program staffing, impact, allocation of resources, and other practical questions that gave rise to the evaluation project (Patton, 1990; Stake, 1977; Worthen \& Sanders, 1987). The issues surrounding calculus reform, the goals of the CCH Curriculum Project, time constraints established by the funding agency, and the amount of funding available all serve as practical considerations in the CCH Evaluation and Documentation Project.

Several groups of individuals may affect or be affected by evaluation research, whereas in basic academic research it is common for no specific groups to be affected by the findings. The groups often affected by or holding a vested interest in the findings of an evaluation research project include sponsors, agencies or individuals who authorize the evaluation and provide necessary fiscal resources; clients, agencies or individuals who request the evaluation; participants, those with whom the
evaluators interact to obtain evaluation information; stakeholders, those who may be directly affected by the evaluation results; and audiences, individuals, groups, and agencies who have an interest in the evaluation and receive its results (Worthen \& Sanders, 1987). In the situation of the CCH Evaluation and Documentation Project. the National Science Foundation and the Calculus Consortium Based at Harvard serve as sponsors and clients. The participants are the site liaisons, faculty, and students at the participating academic institutions. The audience includes the previously mentioned sponsors and clients, the participants, mathematics faculty at academic institutions who are engaged in or considering engagement in reform in the teaching of calculus, educators interested in educational reform, administrators at academic institutions making curriculum decisions, and other policy makers and funding agencies. The listing of audiences to the CCH Evaluation and Documentation Project, provides evidence that evaluation research is an inherently political activity, with societal priorities, resource allocation, and power integrally intertwined with the evaluation project (Greene, 1994; Popham, 1975; Weiss, 1987).

Tensions between quantitative and qualitative oriented researchers need to be included in this review. Fetterman (1988) speaks of the change in direction currently underway in educational evaluation, with the shift occurring from positivist frameworks toward greater acceptance and use of qualitative frames of reference. Several evaluators (Fetterman, 1988; Goetz \& LeCompte, 1984; Lincoln \& Guba, 1985) caution others to realize that the differences between the quantitative and qualitative approaches are based on philosophical and epistemological, rather than methodological, grounds. Typically positivist evaluation researchers focus on external facts or reality apart from the subjective perceptions of individuals, whereas qualitative evaluation researchers attempt to understand human behavior through the subjective realities of human perception. Evaluators from both traditions may use quantitative or qualitative methods to collect data. Within the qualitative paradigm,
evaluators and researchers currently adhere to diverse philosophical positions and use many different approaches (Fetterman, 1988; Lincoln \& Guba, 1985; Patton. 1990; Pitman \& Maxwell, 1992).

Green (1994) attested that qualitative evaluators often describe a program being evaluated in terms of multiple evaluation standards. In this way evaluators recognize that judgment about program effectiveness depends upon the perspective of the stakeholders and potential audiences. Howson (1981) used the metaphor of journalism to describe evaluators' role of tracking down leads and using interviews, observations, and open ended questions to put together a timely story that speaks to the multiple audiences. In a similar vein, Stake (1977) argued for evaluation as portrayal rather than analysis, using various methodologies to assemble a collage that will "tell the story" of a program, or in his example, curriculum. Consistent with Stake's suggestions, the CCH Evaluation and Documentation Project uses data from surveys, interviews, and case studies to tell a story about various interpretations and implementations of calculus reform. Several evaluators (Fetterman, 1988; Patton, 1990; Pitman \& Maxwell, 1992) within the qualitative tradition advocate a situational approach by designing a study that is appropriate for specific inquiry situations and that makes ongoing recursive decisions about approaches to enhance the understanding of a project. The CCH Evaluation and Documentation Project follows this advice by basing each phase of the project on observations in previous phases and by continually seeking to be influenced by the perspective revealed by participants.

Summary
In summary, academic research and evaluation research both use the methods of research and are situated within the various philosophical and epistemological foundations of research. However, evaluation research can be thought of as applied research as opposed to basic research, often associated with academic research. The CCH Evaluation and Documentation Project, resides within the tradition of
qualitative evaluation research. seeking to understand the implementation of CCH Curriculum Project materials through the insider's or user's perspective. The use of both qualitative and quantitative data gathering techniques reflects a situational response to evaluation research, that is, using a practical design that meets the goals of the project within the time and fiscal constraints inherent in evaluation projects.

The CCH Evaluation and Documentation Project primarily addresses reform in mathematics at the undergraduate level. However, a reform movement in school mathematics, kindergarten through twelfth grade, also began in the mid 1980s and has gained in national prominence over the past decade. The many similarities between the two reform movements cause many to suggest that reform in school mathematics has had a strong influence on the movement to reform the teaching of calculus. Many members of the mathematics and mathematics education communities are involved in both movements. The following section provides a brief summary of the current movement to reform the teaching of school mathematics.

## Reform in School Mathematics

Similar to the reform movement in undergraduate education, the movement to reform school mathematics occurred in response to national reports (National Commission on Excellence in Education, 1983; National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983) calling for reform in the teaching and learning of mathematics in the nation's schools. These reports note that educators face and must meet the challenge of changing demographics and rising expectations for the nation's workforce. As the foundation for science and technology, mathematics is essential to the nation's growth and productivity. A later influential report, Everybody Counts, (National Research Council, 1989) suggests that, in the future, all students will need the level of literacy formerly required for students entering college and the facility with mathematics
formerly required for students preparing for scientific careers. Mathematics provides the "key to opportunity" in the information age. Although it is difficult to determine how any of the reform movements in the 1980s influenced the others, it does appear noteworthy that reform movements with many similar characteristics came to the fore during the 1980s.

In response to calls for reform in school mathematics, the Board of Directors of the National Council of School Mathematics established the Commission on Standards for School Mathematics in 1986 with the goal of improving of school mathematics. The commission was charged with (a) creating a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields and (b) creating a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision (NCTM, 1989, p. 1). In response to this challenge, the commission produced the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) which sets standards for school mathematics curricula and for evaluating the curriculum and student achievement.

In addition to changed mathematical content standards, the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) addresses how students learn and how teaching should change. The document recognizes findings from psychology that indicate learning does not occur through passive absorption alone. Instead, individuals approach new situations with prior knowledge, assimilate new information, and construct their own meanings. The document states that the
constructive, active view of the learning process must be reflected in the way much of mathematics is taught. Thus instruction should vary and include opportunities for--

- appropriate project work
- group and individual assignments
- discussion between teacher and students and among students
- practice on mathematical methods
- exposition by the teacher
(NCTM, 1989, p. 10)

Another strong theme in the NCTM Curriculum and Evaluation Standards for School Mathematics is that of opportunity for all. The document notes that women and most minorities study less mathematics in school and that mathematics has become a "critical filter" for employment and full participation in our society (p. 4). Providing access to mathematics for all students is necessary not only to provide equal opportunities but also for the well-being of our nation.

The Curriculum and Evaluation Standards for School Mathematics (1989) was followed by the Professional Standards for Teaching Mathematics (NCTM, 1991) and the Assessment Standards for School Mathematics (NCTM, 1995). The Professional Standards for Teaching Mathematics asks teachers to assist all students in developing mathematical power. Teachers should:
(a) select mathematics tasks to engage students' interest and intellect;
(b) provide opportunities for students to deepen their understanding of the mathematics;
(c) orchestrate classroom discourse in ways that promote the investigation and growth of mathematical ideas;
(d) use, and help students use, technology and other tools to pursue mathematical investigations;
(e) seek, and help students seek, connections to previous and developing knowledge; and
(f) guide individual, small-group, and whole-class work.
(NCTM, 1991, p. 1)

The standards for student assessment adopted in the Assessment Standards for School Mathematics (NCTM, 1995) include an understanding that student assessment should (a) be aligned with, and integral to instruction; (b) use multiple sources of assessment information; and (c) reflect the vision of learning mathematics through investigating, formulating, representing, reasoning, and applying a variety of strategies to the solution of problems. These standards incorporate a shift towards student assessment that is based on evidence from multiple sources, that is the use of many alternative assessment strategies.

The similarities between the goals for school mathematics and the goals established for reform in the teaching of calculus are noteworthy. In the current chapter, six clustering scales are defined that articulate the goals for reform in the teaching of calculus. The six scales also could provide an organizing scheme for the goals for school mathematics listed in the several preceding paragraphs. The previously listed goals for school mathematics (a) advocate increased emphasis on students' conceptual understanding with a decreased emphasis on procedural techniques (concepts), (b) suggest an inductive, investigative approach to mathematical ideas (approach), (c) suggest the use of technology (technology), (d) suggest the use of alternative classroom practices and assessment strategies (teaching and assessment), and (e) and emphasize the need to provide equal access to mathematics for all students, particularly those traditionally underrepresented in more advanced mathematics courses (access).

Anecdotal evidence suggests that the effects of the National Council of Teachers of Mathematics standards for mathematics curriculum, teaching, and student assessment are widespread. Since 1990 the National Science Foundation has funded three elementary school, four middle school, and four secondary school
comprehensive mathematics curriculum projects and many other one-year and supplementary materials projects (National Science Foundation, 1993). Informal evidence reveals that many published textbooks and curriculum materials, teacher inservice workshops, and regional and national conferences state that they are based on the frameworks suggested in the Curriculum and Evaluation Standards for School Mathematics (1989).

The similarity of the goals for reform in the teaching of school mathematics and for reform in the teaching of calculus should not be unexpected. Many of the same influential members of the mathematics and mathematics education communities have served on boards and committees and attended the conferences addressing issues surrounding mathematics education at the school and college levels. Both movements were most likely influenced by the same early reports, the increased use of technology, and research on student learning.

Documentation and evaluation efforts surrounding the current reform in school mathematics are now underway. The Recognizing and Recording Reform in Mathematics Education Project (Ferrini-Mundy \& Schram, in press) and the Quasar Project (Silver, Smith, Nelson, 1995) provide examples of projects whose goals are to document reform projects and inform those engaged in school mathematics reform.

An earlier reform movement in mathematics education, New Mathematics, also focused on kindergarten through twelfth grade ( $\mathrm{K}-12$ ) school mathematics. New Mathematics provides the only example of a complete cycle of reform in mathematics education from curriculum development through evaluation. The next section in the review of the literature provides a brief description of and then addresses the evaluation of the New Mathematics project.

The Evaluation of New Mathematics, K-12
The New Mathematics curriculum reform movement began in the 1950s in response to political pressures and national reports criticizing school mathematics. The awareness of a need for reform in school mathematics originated in a general concern over the failure of existing practices to reflect a changing culture (NLSMA, 1969, p. x). The post World-War II culture reflected a growing concern for quality, changes in the academic world, the technological revolution, and the international pressures of the Cold War. Federal agencies and private foundations invested heavily in mathematics curriculum development in grades K-12 to help meet a growing need for a more technical workforce (Conference Board of the Mathematical Sciences, 1975).

Cronbach (1977), in describing the New Mathematics K-12 materials, stated that the writers portray mathematics as part of a larger intellectual framework that emphasizes the fundamental structure of the discipline rather than as a history of ancient thought or a collection of formulas and prescriptions. The writers of these materials tried to present mathematics as a valuable part of our culture, interesting and important in itself, and worth knowing because of its ability to help people interpret their world (NLSMA, 1961, p. ix). Getting an answer became secondary to its method of derivation. In The Process of Education, Jerome Bruner (1965) provided the psychological justification for teachers to emphasize the conceptual understanding of mathematical methods, with understanding conveyed by an emphasis on the unifying structure of the discipline. Projects such as those of the SMSG (School Mathematics Study Group), UICSM (University of Illinois Committee on School Mathematics), UMMaP (University of Maryland Mathematics Project), and the Madison Project, reflected the calls for a major reconstruction of the scope, sequence, and pedagogy of school mathematics (p.ix).

In 1961, the National Science Foundation provided funding to the School Mathematics Study Group project for the National Longitudinal Study of School Mathematics (NLSMA). The NLSMA (1969) report provides an example of a curriculum evaluation effort that aims to investigate the growth of mathematical skills and abilities. This effort that compares the achievement of pupils using reform and traditional curricula was perhaps the most heavily funded evaluation of any curriculum project in this century. According to Howson, Keitel, and Kilpatrick (1981), because the New Mathematics curriculum development projects of the 1950s and 1960s were primarily concerned with updating the content of the mathematics curriculum, the evaluation of these projects focused on the mathematics the students were learning.

The National Longitudinal Study of Mathematical Abilities (NLSMA) NLSMA (1969) was organized by the School Mathematics Study Group in 1961 as a long-term evaluation study of the effects on students of various kinds of mathematics programs. The study investigated the performance of students in the several New Mathematics projects that were mentioned in the previous subsection. The following paragraphs (except where referenced otherwise) are summarized from the NLSMA reports.

The initial proposal for NLSMA to the National Science Foundation specified that a long-term study was needed to provide information for the further improvement of the school mathematics curriculum, to develop measures of mathematics achievement, to provide information for school personnel, and to gain experience in operating a large scale study in order to inform other investigators wishing to perform similar investigations. For these purposes, the National Science Foundation provided funds to the School Mathematics Study Group and Stanford University for the operation of the study. In the prefaces to each of the report volumes, the editors
emphasized that there "is not a single NLSMA report . . . There are a series of NLSMA reports" (p.ix).

The NLSMA project sought not only to assess new curriculum materials, but to identify and to investigate variables related to mathematics achievements that were independent of the curriculum. This project also obtained quantitative information on the cumulative and comparative effectiveness of mathematics curricula and identified and measured variables associated with the development of mathematical abilities. Starting in the early 1960s, large populations of students were tested in the fall and spring of each year, beginning with grades 4,7 , and 10 . Over 112,000 students from 1,500 schools in 40 states participated in the study. The more than 38 volumes of reports contain the test batteries, descriptions and statistical properties of the population scales, and large amounts of data and the related statistical analyses.

In what they themselves called an oversimplified statement, Howson, Keitel, and Kilpatrick (1981) stated that the NLSMA results showed that pupils "tended to learn what was emphasized in textbooks and not something else" (p. 193). The complexity of the study was increased by the introduction of widely varying textbooks written during the New Mathematics era and the resulting differences in their effects on what students learned. The NLSMA writers were unable to explain the differences. Howson, Keitel, and Kilpatrick also reported that NLSMA found that teachers at higher grade levels, for example eleventh or twelfth grade, tend to rely less on textbooks than teachers at lower grade levels such as first or second grade. Howson, Keitel, and Kilpatrick criticized those who conducted the evaluation for spending so much of their resources gathering and organizing the data that there was not enough energy left to analyze it (p. 194).

The NLSMA study was not the only evaluation of New Mathematics. More than a decade later, the National Advisory Committee on Mathematical Education
began a more retrospective evaluation of the effects of New Mathematics on school curriculums and student learning. This evaluation is discussed in the next section. The National Advisory Committee on Mathematical Education (NACOME) Report

In 1975, the Conference Board on the Mathematical Sciences convened a committee that attempted to present a comprehensive overview and analysis of the status of mathematics education in kindergarten through twelfth grade by considering its objectives, current and innovative practices, and attainments. The committee's account, known as the NACOME report, was financed by the National Science Foundation and provided a second major evaluation of the New Mathematics (Conference Board of the Mathematical Sciences, 1975). By 1975, battles over New Mathematics had become increasingly divisive. Continued criticism of the new curricula reflected concerns that the programs were excessively formal, deductively structured and theoretical, and ignored the intuitive interaction of mathematics with its applications. Although the writers of the NACOME report took exception to many of the arguments, it recognized that the actual impact of the reform efforts had not fulfilled the early promises. The New Mathematics projects were aimed at the high school program for college preparatory students. The NACOME report indicated that these new curricula had failed to meet the needs for basic mathematical literacy of average and low ability students (p. ix). The stage had been set again for new efforts of reform in school mathematics.

## Summary and Considerations

The evaluation of New Mathematics provides interesting and useful background to the CCH Evaluation and Documentation Project. New Mathematics was considered an effort to change only the content of mathematics that was taught. In comparison, the calculus reform effort seeks to change how calculus is taught as well as what calculus is taught. It is difficult to imagine, however, that the significant change in the mathematics content in New Mathematics did not influence how the
mathematics was taught. Consideration of the influence of New Mathematics on how the mathematics was taught may have enriched the evaluation of that project.

The evaluation of New Mathematics was conducted from a behaviorist perspective with strictly quantitative methods that precluded analysis of more subjective data that could have been obtained from interviews or from written comments from the teachers and students. The evaluators of the CCH Curriculum Project seek to obtain such additional subjective information from its more qualitative approach. However, some members of the audience for the CCH Evaluation and Documentation Project may expect a more quantitative analysis of what students know and can do. As the New Mathematics evaluators noted, implementations of curriculum projects often reflect many more influences than the project materials themselves. Faculty members at all implementation and nonimplementation sites may attend the same conferences, read the same journals, engage in the same discussions, and try similar new approaches to teaching, making comparisons between implementation and nonimplementation sites problematic.

As mentioned previously, the evaluators of New Mathematics also found that teachers at higher grade levels of mathematics were influenced less by the textbook than teachers of lower grade levels. Because the CCH Evaluation and Documentation Project is an evaluation of a higher level mathematics curriculum, it seeks to understand the extent to which the CCH Curriculum Project textbook actually influences the calculus courses in its study.

Data collection and descriptive statistics consumed most of the money and energy allotted to the evaluation of the New Mathematics project, leaving few resources for deeper analysis of the findings. Current evaluation research, such as the CCH Evaluation and Documentation Project, should remain cognizant of this danger.

## Summary

This chapter has served as background for understanding the CCH Evaluation and Documentation Project and the current study. The first two sections considered the movement to reform teaching at the undergraduate level and the movement to reform the teaching of calculus, two reform movements that occurred in response to many of the same national reports about education in the United States.

The third section looked at the theoretical foundations of the clustering scales used in the current study. These foundations reside in the goals of calculus reform and the related literature. The differences between academic research and evaluation research were then explored to position the CCH Evaluation and Documentation Project within the evaluation research tradition. The CCH Evaluation and Documentation Project is an example of evaluation research conducted from a qualitative perspective that seeks to understand a situation through the subjective realities of the participants. The current study, a component of the CCH Evaluation and Documentation Project also assumes a qualitative research perspective, using quantitative and qualitative methods to achieve that understanding. The fifth section considered the current movement to reform school mathematics. This reform movement paralles the movement to reform calculus instruction and reflects many similar goals. The final section reviewed the evaluation of New Mathematics, a previous reform movement in mathematics education. The topics addressed in this chapter are directly related to and influence the current study. The next chapter looks more closely at the methods and procedures used in this study.

## CHAPTER III

## METHODS AND PROCEDURES

The presence of reform-based calculus instruction has increased over the last decade, with support from the National Science Foundation. The mathematics community continues to question the influence and impact of the reform movement. The current study, as a component of the Calculus Consortium Based at Harvard (CCH) Evaluation and Documentation Project, is designed to address some of the questions that have arisen. The methods and procedures of the current study are integrally interwoven with those of the larger project and are discussed within this context. The first section of the current chapter considers the methodological framework for the current study.

## Methodological Framework for the Current study

The current study, as part of the CCH Evaluation and Documentation Project, reflects the methodological framework of the larger project. The position of the CCH Evaluation and Documentation Project within the evaluation research tradition is discussed at length in a section of Chapter II, the Review of the Literature for the current study. As does the larger project, the current study takes a qualitative perspective, seeking to understand and to describe the implementation of CCH Curriculum Project materials through the perspective of those who use the materials. Quantitative and qualitative methods are used in data acquisition and analysis.

Quantitative data and comments collected from surveys provide the bulk of the data in the current study. Some of the criticism of the use of mail surveys is mitigated in the current study by the particular survey population. Jaeger (1988)
suggested that "Complex issues can be examined through a mail survey only when the survey population is composed of specialists with a common background and a natural interest in the topic" (p. 313). The participants in the current study, mathematics faculty members, meet that criteria.

The surveys incorporate numerical response questions that are analyzed using quantitative statistical methods and open-ended questions that are analyzed using qualitative methods. In line with the qualitative perspectives of the study, responses to each survey have influenced subsequent surveys. For example, respondents to the Initial Questionnaire were asked, "Are there issues and questions surrounding calculus reform that are of particular interest to you or your institution? Please elaborate." Some of the questions on the subsequent Site Liaison Survey directly reflect responses site liaisons made to this question.

The CCH Evaluation and Documentation Project adheres to the tradition espoused by Stake (1977) when he described evaluation as incorporating judgment and description. The current study emphasizes the descriptive aspect as it documents patterns of implementation of CCH Curriculum Project materials, with the patterns of implementation reflecting the perspectives of the participants. The participants are described in greater detail below.

## Participants

The participants in the current study are instructors who are currently using or have previously used the CCH Curriculum Project materials as the primary course textbook. The academic institutions at which the instructors in the study teach are classified as secondary schools, two-year colleges, doctoral and research colleges, and other colleges and universities. The classification of the academic institutions is based on the 1994 "Carnegie Classification" developed by the Carnegie Foundation
for the Advancement of Teaching (1994). Of the 117 academic institutions included in the current study, 115 are in the U.S.A. and two are in Canada.

The participating instructors, or faculty members, in the current study can be further identified as site liaisons and other instructors. At each of the 117 academic institutions participating in the study, one individual who was currently using or had previously used CCH Curriculum Project materials, agreed to serve as site liaison. The 117 site liaisons completed the Site Liaison Survey and distributed the Faculty Survey to up to five other instructors who were currently using or had previously used the CCH Curriculum Project materials. Although the site liaisons were encouraged to serve as one of the five faculty members completing the Faculty Survey, six site liaisons distributed all of the five Faculty Surveys to other instructors, not completing one themselves. At some institutions, fewer than five instructors had taught calculus using the CCH Curriculum Project materials. Of necessity, site liaisons from those institutions returned fewer than five Faculty Surveys.

The number of participants in the current study, 383, includes the 117 siteliaisons and 266 other instructors. Site liaisons represent 13 secondary schools, 30 two-year colleges, 19 doctoral and research universities, and 55 other colleges and universities. The 266 other instructors represent 6 secondary schools, 77 two-year colleges, 64 doctoral and research universities, and 119 other colleges and universities. The 266 other instructors are all from institutions represented by site liaisons.

All human subjects requirements of the University of New Hampshire Institutional Review Board were met by the CCH Evaluation and Documentation Project. Because the current study is a component of the larger project, the human subjects requirements for the current study have also been met.

The distribution of the four academic institution types in the current study corresponds well with the corrected estimated population of users of CCH Curriculum

Project materials in the fall of 1995. A total of 504 academic institutions that were possibly using CCH Curriculum Project materials, based on the publisher's purchase lists, were sent Initial Questionnaires. The 504 academic institutions included 67 secondary schools, 126 two-year colleges, 92 doctoral and research university, 218 other colleges and universities, and one institution whose institution type was unknown.

Table 1
Estimated Number of Users of CCH Curriculum Project Materials in the Fall of 1995
by Institution Type and Number of Participants in the Current Study

|  | Sent Initial Survey | Responded to Initial Survey | $\begin{gathered} \text { Using } \\ \text { CCH } \\ \text { material } \\ \text { (percent of } \\ \text { total } \\ \text { responses) } \\ \hline \end{gathered}$ | Corrected estimated population of current users (percent of estimated total population) | Participating in current study (percent of participating institutions) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Secondary schools | 67 | 41 | $\begin{gathered} 31 \\ (13 \%) \end{gathered}$ | $\begin{gathered} 51 \\ (12 \%) \end{gathered}$ | $\begin{aligned} & 13 \\ & (11 \%) \end{aligned}$ |
| Two-year colleges | 126 | 70 | $\begin{gathered} 63 \\ (26 \%) \end{gathered}$ | $\begin{gathered} 113 \\ (27 \%) \end{gathered}$ | $\begin{gathered} 30 \\ (26 \%) \end{gathered}$ |
| Doctoral \& research universities | 92 | 51 | $\frac{38}{(16 \%)}$ | $\frac{68}{(16 \%)}$ | $\frac{19}{(16 \%)}$ |
| Other colleges \& universities | 218 | 127 | $\begin{gathered} 106 \\ (45 \%) \end{gathered}$ | $\begin{gathered} 181 \\ (44 \%) \end{gathered}$ | $\begin{gathered} 55 \\ (47 \%) \end{gathered}$ |
| Unknown | 1 | 0 |  |  | 0 |
| Total | 504 | 289 | 238 | 413 | 204 |

Table 1 lists the estimated population of academic institutions using CCH
Curriculum Project materials in the fall of 1995. The estimation is based on the percentage of respondents who indicated they were using the materials. The table
also lists the number of academic institutions participating in the current study by institution type. The distribution of the numbers of institution types participating in the current study is approximately the same as that of the estimated population of users. Because responses to the several surveys were voluntary and represented a year-long involvement with the project, the distribution of institution types seems especially fortuitous. The surveys themselves are described in the next section.

## Survey Instruments

The three survey instruments used in the current study consist of the Initial Questionnaire (see Appendix B), the Site Liaison Survey (see Appendix C), and the Faculty Survey (see Appendix D) that were sent to participating academic institutions as part of the CCH Evaluation and Documentation Project. As research assistant to the project, the researcher for the current study wrote the three survey instruments with the assistance of Ferrini-Mundy, Project Director, and Graham, Senior Associate to the Project.

The three surveys contain numerical-response and open-ended questions. The theoretical framework for the survey questions used in the current study is the same as that of the clustering scales. The reader is referred to Chapter II, the Review of the Literature, for the theoretical foundations of the clustering scales.

A professor from Mississippi State University, a professor from Brigham Young University, and a secondary school teacher from California, all of whom were currently using CCH Curriculum Project materials, critiqued and reviewed final drafts of the Site Liaison Survey and the Faculty Survey. Appropriate changes were made based on reviewer comments before the surveys were distributed. The professor from Mississippi State University who reviewed and critiqued the surveys is a writer of the CCH Curriculum Project materials.

Having considered the methodological frameworks, the participants, and the design of the survey instruments, we now turn to the procedures section of the current study. The procedures section looks more closely at the response patterns to the surveys, the methods of cluster analysis, and the validation of the clusters.

## Procedures

The first three sections of the procedures section address the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey. These surveys were written as part of the CCH Evaluation and Documentation Project. The goal of the current study, to develop an understanding of how mathematics departments are interpreting reform and implementing the CCH Curriculum Project materials, is also a major goal of the CCH Evaluation and Documentation Project. The many items on the three surveys that are directly related to this goal provide the data for the six clustering scales of the current study.

After discussion of the surveys, the next section addresses how the data obtained from the surveys is prepared for the cluster analysis procedure. The subsequent section addresses the methodological decisions made in the process of conducting the cluster analysis, followed by a description of the process used to validate the cluster analysis. The final section of the chapter addresses the use of the comment data from the surveys. The distribution of the Initial Questionnaire began the interaction with the participants in the current study.

Initial Ouestionnaire
An early challenge that confronted the CCH Evaluation and Documentation Project involved determining a list of academic institutions that had previously used or were currently using the CCH Curriculum Project materials as a primary calculus course textbook. When the CCH Evaluation and Documentation Project began in the spring of 1994, John Wiley \& Sons, Inc., the publishers of the CCH Curriculum

Project textbook, provided the CCH Evaluation and Documentation Project with lists of academic institutions that had purchased the CCH Curriculum Project textbook. The publisher did not have addresses or names of contact people at the institutions on the lists. The lists also contained some duplicate entries, some missing entries, and some academic institutions for which addresses were not obtainable. In the fall of 1994, the Initial Questionnaire was sent to the 504 academic institutions that appeared on the list and for which an address could be located. A faculty member from one institution that did not appear on the publisher's list asked to be included in the study as a site liaison and was sent an Initial Questionnaire.

One goal for sending the Initial Questionnaire was to determine names of faculty members who would be willing to serve as site liaison and complete the subsequent, more in-depth surveys. Individuals from 289 academic institutions responded to the Initial Questionnaire (57\%). Follow-up procedures to increase the response rate included a mailed reminder followed by a telephone or electronic-mail reminder to the individual (if known) or the mathematics department that received the questionnaire. The same follow-up procedures were repeated for each of the subsequent surveys.

Of the 289 academic institutions that responded to the Initial Questionnaire, 238 were using the CCH Curriculum Project materials in fall 1994, 27 had previously used the materials but were no longer doing so, and 24 had never used the materials. The responses to the numerical-response survey questions were entered into a statistical computer program and the responses to the open-ended questions were entered into a computer data base. The same data entry procedures were followed for each of the surveys.

Of the 238 respondents to the Initial Questionnaire who were currently using the CCH Curriculum Project materials, 190 agreed to serve as site liaisons. These

190 site liaisons were each sent a Site Liaison Survey. The following section addresses the Site Liaison Survey.

## The Site Liaison Survey

The second phase of the CCH Evaluation and Documentation Project began in April, 1995. It began with the mailing of the Site Liaison Survey to the 190 respondents to the Initial Questionnaire who indicated they were currently using the CCH Curriculum Project materials and were willing to serve as site liaisons. The Site Liaison Survey was intended to (a) identify characteristics of students and courses using CCH Curriculum Project materials, (b) determine the pedagogical approaches of faculty using the CCH Curriculum Project materials, (c) determine the level and type of technology used in CCH Curriculum Project courses, and (d) identify factors influencing reform efforts. Site liaisons who returned the Site Liaison Survey received a $\$ 25$ honorarium.

Of the 190 site liaisons who were sent the Site Liaison Survey, 131 responded for a response rate of $69 \%$. Fourteen respondents were from secondary schools, 37 from two-year colleges, 23 from doctoral and research universities, and 57 from other colleges and universities.

The third phase of the CCH Evaluation and Documentation Project included a Faculty Survey and a Student Survey. The Student Survey was sent in May, 1995 and is not included in the current study. The next section addresses the Faculty Survey.

## The Faculty Survey

The Faculty Survey was sent in October, 1995 to the 131 site liaisons who had responded to the Site Liaison Survey. The Faculty Survey was intended to determine CCH Curriculum Project instructors' (a) background and experience, (b) impressions and use of the CCH Curriculum Project materials, (c) pedagogical approach and assessment practices, (d) opinions about student learning, and (e) beliefs about student learning.

Site liaisons were each sent up to five Faculty Surveys, based on the maximum number of instructors who had taught calculus using the CCH Curriculum Project materials. Site Liaisons from 117 of the 131 academic institutions returned Faculty Surveys for a response rate of $89 \%$. The distribution of the responses among types of institutions is discussed in the participant section of the current chapter. Site liaisons received an honorarium of $\$ 25$ for coordination efforts and for completing the Faculty Survey. Other instructors who completed the Faculty Survey received an honorarium of $\$ 15$. Follow-up and data entry procedures for the Faculty Survey were the same as for the other two surveys. The next section discusses the preparation of the data for cluster analysis, the statistical method used to analyze the quantitative data.

## Preparation of Data for the Cluster Analysis

Cluster analysis is a technique used to group a data set containing information about a number of entities into relatively homogenous subgroups that are based on the similarities among the entities (Aldenderfer \& Blashfield, 1984; Lorr, 1983). A primary benefit of cluster analysis is that the procedure is especially suited to studies in which group membership and the number of groups is unknown prior to the analysis (Norusis, 1994). In the current study, the entities are the academic institutions participating in the study. The data set consists of the responses to survey items. The number of groups, the group membership, and the characteristics of implementation patterns were all unknown prior to the analysis. For these reasons, cluster analysis seems particularly suited to the current study.

The five basic steps listed below and paraphrased from Aldenderfer and Blashfield (1984) characterize all cluster analyses. The subsections after the listing of the steps describe the first three steps in terms of the current study. Sections describing the cluster analysis and the validation methods complete the chapter.
(1) selection of a sample to be clustered
(2) definition of a set of variables (scales) on which to measure the entities in the sample
(3) computation of the similarities among the entities
(4) use of a cluster analysis method to create groups of similar entities
(5) validation of the resulting cluster solution
(Aldenderfer \& Blashfield, 1984, p. 12)

## Selection of a Sample to be Clustered

The sample to be clustered consists of the academic institutions (sites) participating in the study, as described in the participants section of the current chapter. Although the site liaisons and up to five instructors at each institution were participants, for the cluster analysis all responses to each survey item were averaged (see discussion below) so that there would be one response to each survey item for each academic institution. Each academic institution is one entity or case in the cluster analysis.

## Definition of a Set of Variables or Clustering Scales

The variables used to measure the entities or cases in the study are the clustering scales. See Chapter II, the Review of the Literature for the current study, for the theoretical foundations and descriptions of the clustering scales. The tables in Appendix E list the survey items that comprise each clustering scale and provide the specifications for coding responses to each survey item as consistent, neutral, or inconsistent with reform-based calculus goals. The specifications are based on the theoretical foundations of the clustering scales and informed judgment. The next section addresses the methods used to recode the responses to the survey items.

## Generation of the Data Set Necessary for Computing the Similarities

The computer algorithm that performs the cluster analysis requires the existence of a similarity matrix or data set that contains clustering scale values for the cases. The methods used to develop the data set that is required for the computer software to compute the similarity measures are described in the paragraphs that follow.

Coding responses to the Site Liaison Survey items. The site liaisons each represent one of the 117 academic institutions and, therefore, one case in the current study. Site liaisons responses were coded into the data set exactly as they appear on the completed Site Liaison Survey.

Recoding the responses to the Faculty Survey items. The situation with Faculty Survey items is slightly different from that of the Site Liaison Survey items. As described previously, between one and five instructors from each site completed the Faculty Survey. The one to five responses from each site to each survey item were averaged and the average value became the response for that survey item. In this way each academic institution was assigned one value for each item in the Faculty Survey. After the averaging of faculty responses to each item, the data set consisted of 117 academic institutions, each with one response for all Site Liaison Survey items and one mean response across all Faculty Survey items. The responses to the Site Liaison Survey and the averaged responses to the Faculty Survey are represented in the data set in Table 2.

Table 2
Data Set Reflecting Responses to Site Liaison Survey Items and Averaged Responses to Faculty Survey Items

| Academic <br> institution <br> DH | Site Liaison <br> Survey Item <br> 1 | $\ldots$ | Site Liaison <br> Survey Item <br> n | Faculty <br> Survey Item <br> 1 | $\ldots$ | Faculty <br> Survey Item <br> p |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Site liaison <br> response | $\cdots$ | Site liaison <br> response | Average of <br> instructors' <br> responses | $\cdots$ | Average of <br> instructors' <br> responses |
| 2 | Site liaison <br> response | $\cdots$ | Site liaison <br> response | Average of <br> instructors' <br> responses | $\cdots$ | Average of <br> instructors' <br> responses |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$. | $\ldots$ | $\ldots$ | $\ldots$ |
| 117 | Site liaison <br> response |  | Site liaison <br> response | Average of <br> instructors' <br> responses |  | Average of <br> instructors' <br> responses |

Assigning 0 or 1 to survey item responses. Each Site Liaison Survey response and each averaged Faculty Survey response were next recoded to 0 or 1 . The recoding was based on the specifications for consistency with reform-based calculus listed in Appendix E. For example, consider Faculty Survey item \#43c, "Please indicate the value you place on the following aspect of calculus reform: Calculus courses should make greater use of technology" This is a Likert scale item for which the respondents are asked to select a number from 1 (value little or not at all) to 5 (value highly). A response of 3,2 , or 1 is considered inconsistent with calculus reform and a 4 or 5 is considered consistent. Suppose the computed average responses to item \#43c on the five Faculty Surveys received from the academic institution with ID \#2 is 4.2 . Since 4.2 is a consistent response (greater than 3 ), the recoded response to Faculty Survey item \#43c would be 1. Averaged responses to other types of survey items were recoded in a similar manner appropriate to the item. Table 3 represents the data set after recoding the Site

Liaison Survey and Faculty Survey items to is and 0s. The data set in Table 2 is ready for determining the average response for institutions across clustering scales.

## Table 3

## Data Set Recoded to 0s and Is

| Academic <br> Institution <br> D\# | Site Liaison <br> Survey Item <br> 1 | $\ldots$ | Site Liaison <br> Survey Item <br> n | Faculty <br> Survey <br> Item 1 | $\ldots$ | Faculty <br> Survey Item <br> p |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 or 0 | $\cdots$ | 1 or 0 | I or 0 | $\cdots$ | or 0 |
| 2 | 1 or 0 | $\cdots$ | 1 or 0 | 1 or 0 | $\cdots$ | 1 or 0 |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| 117 | 1 or 0 |  | 1 or 0 | 1 or 0 |  | 1 or 0 |

Completing the recoding. The final step was to create the cluster analysis data set. The cluster analysis data set is a matrix with rows representing academic institutions and columns representing clustering scales. The cluster analysis data set matrix was computed from the data set represented in Table 2. For each academic institution, the sum of the survey items assigned to each clustering scale was computed. The sum was then divided by the number of items assigned to that scale. This process resulted in a value between 0 and 1 , because prior to this step each academic institution had a value between 0 and 1 , inclusive, for each survey item. For example, if the site liaison from the academic institution with ID \#2 had circled responses that met "consistent" specifications to 10 of the 15 survey items assigned to the approach clustering scale, the sum of the consistent responses would be 10 . The next step would be to divide 10 by the number of survey items (15). The academic institution with ID \#2 would then have a value of $10 / 15 \approx .67$ for the approach
clustering scale. The computed value .67 , in this example, appears in the cell in the third row and third column of Table 3. At this point in the process, the data set would be a matrix with 117 rows (one for each academic institution or case) and six columns (one for each clustering scale) with each cell containing a number between 0 and I inclusive. Table 4 represents a data set that is ready for the cluster analysis procedure.

Table 4
Data Set Ready for Cluster Analysis

| Site <br> ID\# | Concepts | Approach | Teaching | Technology | Assessment | Access |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ |
| 2 | $0 \leq x \leq 1$ | $.67 a$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 117 | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ | $0 \leq x \leq 1$ |

Note. ${ }^{\text {a }}$ This cell represents the location of approach value for academic institution with ID \#2. The value for this cell has been computed.

## Summary

The preceding steps have described the methods used to prepare the quantitative data in the current study for the cluster analysis. The following subsection describes the options selected during the cluster analysis.

## Cluster Analysis

This section describes the decisions that were made in performing the agglomerative hierarchical cluster analysis in the current study. A brief description of cluster analysis appears in Appendix F. Aldenderfer and Blashfeld (1984) present a much more complete, systematic guide to the concepts, techniques, and algorithms
associated with cluster analysis. The current study used Ward's method for cluster formation, a commonly used method (Aldenderfer \& Blashfield, 1984; Lorr, 1983: Norusis, 1994). The distance measure selected for computing the distance between two items was squared Euclidean distance, one of the more frequently used distance measures (Aldenderfer \& Blashfield, 1984; Lorr, 1983; Norusis, 1994). The data values were standardized to a range of 0 to l , a method of standardization that has been shown to be optimal in most situations (Barton, 1993; Milligan \& Cooper, 1988).

The procedure for determining the number of clusters in a cluster analysis is somewhat subjective. In the current study, a method was used that attends to a "jump" in the value of the squared Euclidean distance between the two clusters being combined at some step in the cluster formation process. The jump in distance values signifies that two relatively dissimilar clusters have been merged (Lorr, 1983; Milligan \& Cooper, June, 1985). The cluster solution immediately before the jump in the value of the squared Euclidean distance is the one chosen. This method produced the eight cluster solution that is described later in Chapter IV, the Analysis and Results. The following section describes the methods used to validate the cluster analysis solution.

## Validation of the Current Study

The purpose of the cluster analysis is to identify clusters or groups of academic institutions that exhibit similar patterns (profiles) of implementation of reform-based calculus materials. The eight clusters represent eight statistically distinct patterns of implementation. The current study speaks of clusters in both contexts: clusters of academic institutions and patterns of implementation.

The current study also validates the eight clusters identified through cluster analysis through a technique known as external validation. A successful external validation of the clusters identified through cluster analysis demonstrates differences
between the clusters on external criteria relevant to the study. Aldenderfer and Blashfield (1984) contend that the value of a cluster solution that has been shown to demonstrate differences on external variables is much greater than one that has not. In the current study, the relevant criteria are validating scales. Validating scales are made up of groups of survey items that are directly related to reform in the teaching of calculus that were not used in the cluster analysis. For example, the validating scale vstatus is defined by survey items that relate to CCH Curriculum Project instructors: the percentage that are full time, their mathematics teaching experience, and their calculus teaching experience.

In the external validation in the current study, differences between clusters are demonstrated on three sets of validating scales. The first set of validating scales pertains to characteristics of the mathematics department in which the CCH Curriculum Project materials are being used, and the second set is related more directly to the use of CCH materials. A third set of validating scales is more demographic in nature. Each validating scale is composed of survey items that are not used in the cluster analysis. (See Appendix G for a listing of the survey items that comprise each validating scale and the specifications for recoding the responses as consistent or inconsistent with reform-based calculus.)

We turn now to the description of the first set of validating scales, those that are concerned with the mathematics department. The actual variable names used in the recoding of data are identified in parentheses .

## Validating Scales Related to the Mathematics Department

The mathematics department validating scales relate directly to the practices, beliefs, and values of the majority of the faculty members in the mathematics department at a participating institution. They reflect the more general departmental context in which the CCH Curriculum Project work is occurring. As do the clustering scales, the mathematics department validating scales have direction, based on the
consistency of the response with the goals of reform-based calculus instruction. The mathematics department validating scales are comprised of survey items on the Site Liaison Survey. It should be remembered throughout this section that one site liaison at each academic institution completed the Site Liaison Survey. Site liaisons were asked in the survey to base their responses on their personal understanding of the viewpoints of the majority of faculty members in their departments. It also should be recognized that many site liaisons had to make a qualitative judgment as to the most representative response to survey items. The mathematics department validating scales are described below.

Values. The values validating scale (vvalues) is defined by the apparent value placed on teaching by the institution and the mathematics department. It considers (a) the mathematics department's attitudes towards experimentation in teaching, (b) resources directed towards teaching, (c) the desirability of teaching calculus, and (d) whether the institution or department evaluates teaching (beyond using student evaluation forms).

Interest. The interest validating (vinterest) scale is defined by faculty members' apparent interest in pedagogy and reform. Survey items included in vinterest pertain to (a) the frequency mathematics department members discuss pedagogical issues, reform in the teaching of calculus, and reform in mathematics education in general and (b) the opinions department members hold about reform. Coding of the validating scale assumes that more frequent discussions are indications of increased interest in these topics.

Teaching. The teaching validating scale (vteaching) corresponds to the previously discussed teaching clustering scale. Survey items included in vteaching ask the site liaisons about the pedagogical practices of most faculty members in the department. Alternative teaching practices include the use of cooperative groups, student writing, complex problems, alternative solutions, and student exploration.

Assessment. The assessment validating scale (vassessment) corresponds to the previously discussed assessment clustering scale. It pertains to the use of alternative methods of student assessment such as group projects, class presentations, group tests, or portfolios.

Concepts. The concepts validating scale (vconcepts) corresponds to the previously discussed concepts clustering scale. Responses to vconcepts considered consistent with reform in the teaching of calculus would indicate the respondent places high emphasis on student's conceptual understanding of the major ideas of calculus and a lesser emphasis on students' practice of routine procedures.

Technology. The technology validating scale (vtechnology) corresponds to the previously discussed technology clustering scale It pertains to the use of and support for the use of technology in calculus classes.

## Validating Scales Concerned with the Use of CCH Materials

The survey items that comprise the CCH validating scales address the use of CCH materials and come from the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey. The CCH validating scales are described below.

Use-CCH. The use CCH validating scale (vuse-CCH) is based on the percentage of calculus students using CCH Curriculum Project materials.

Status. The status validating scale (vstatus) is defined by the percentage of full-time, tenured, or tenure-track faculty teaching courses using CCH Curriculum Project materials and by the length of the CCH instructors' experience teaching calculus and mathematics. Full time faculty at institutions that do not have tenure are considered tenured or tenure-track.

Interaction. The interaction validating scale (vinteraction) is defined by the percentage of instructors using CCH Curriculum Project materials who attended CCH workshops and the frequency with which the instructors meet to discuss pedagogical issues and calculus reform.

Reform. The reform validating scale (vreform) is defined by the amount of support for reform in the teaching of calculus held by instructors using CCH Curriculum Project materials.

## Demographic Validating Scales

The demographic validating scales are comprised of survey items concerned with the type of academic institution, the enrollment, the financial support provided for the use of CCH Curriculum Project materials, and whether the academic institution is public or private. These validating scales do not have direction in the sense of consistency with goals for reform in the teaching of calculus. The following paragraphs describe the demographic validating scales.

Type. The type validating scale (vtype) identifies the type of academic institution: secondary school, two-year college, doctoral or research university or other college or university.

Enrollment. The enrollment validating scale (venrollment) identifies the enrollment at institutions in the current study.

Public-private. The public-private validating scale (vpubpri) identifies whether the institution is public or private.

Support. The support validating scale (vfinsup) is defined by whether site liaisons report financial support for implementing CCH Curriculum Project materials. The financial support may have been received from outside the institution or may be special financial support from within the institution or department.


#### Abstract

Summary The external validation process serves two purposes. It first validates the cluster solution by demonstrating differences among the clusters on external validating scales. The validating scales include demographic scales, scales related to mathematics department perspectives, and scales related more directly to the use of the CCH Curriculum Project materials. The second purpose of the validation process


is to contribute to the in depth description of the individual clusters in Chapter V . Profiles of Reform in the Teaching of Calculus. The scores on the three types of validating scales provide additional contextual information that permits a richer portrayal of each cluster.

## Comment Data

The Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey contain many qualitative questions that ask the participants to comment on their beliefs, attitudes, and practices related to reform-based calculus instruction and their implementation of CCH Curriculum Project materials. The original intent in the current study was to use the comment data for additional validation of the cluster solution, that is to further differentiate between clusters. However, the coding and analysis of the comment data did not reveal quantifiable differences between clusters. On a more subjective basis, the overall tone of the comments does differ among clusters.

The second purpose of the comment data, to enrich the descriptions of the clusters, is realized in the current study. The use of comment data in the cluster descriptions gives a richer portrayal of the participants' attitudes, perspectives, and observations. The in-depth descriptions of the eight clusters comprise Chapter V, Profiles of Reform in the Teaching of Calculus.

A description of the analysis of the comment data follows. The comments on all survey items related to each clustering or validating scale were coded with the name of the appropriate scale and the direction. (See appendixes E, G, and H.) Direction was coded as consistent with goals for reform in the teaching of calculus $(+)$, inconsistent with goals for reform in the teaching of calculus $(-)$, or ambiguous or neutral ( $\pm$ ). For example, a comment such as "[CCH materials] cause me to use cooperative learning in many cases to allow for student conceptual development. I do much less lecturing," was coded as: teaching (+), concepts (+)." An example of a
neutral response is demonstrated by the response to Faculty Survey item 39. "Please make comparisons between the success and retention rates of CCH students and students in standard calculus sections." The response, "No change--maybe some flattening out in that more C's and fewer A's and B's," received a code of: access ( $\pm$ ).

The percentage of comments coded as consistent with reform-based calculus instruction was computed for each cluster on each clustering scale. Coding and counting of comment data revealed that percentages of consistent with reform comments are similar among the clusters (see Appendix H). This may reflect the way the questions were framed. Faculty Survey item 45 asks participants to "describe what aspects of reform they find most encouraging and what causes them concern." This item elicited many comments from the participants. On some clustering scales, participants from the same cluster respond with approximately equal numbers of consistent and inconsistent comments. For example, a participant from one cluster is encouraged by "the decreased role of "template problems" and symbol manipulation." Another participant feels "manipulative skills are just awful because students don't get enough drill and 'easy' problems for practice." Faculty Survey item 20 provides an second example in which the wording of questions may have contributed to the similarity in terms of consistency-with-reform comments from all clusters. "In what ways did the CCH material influence changes in your preparation and (or) teaching of calculus?" Respondents tended to either leave the question blank or describe their use of alternative teaching practices. Although the comment data does not document patterns of differences among the clusters, it provides a rich source of descriptive data for the narrative descriptions of the clusters.

## Descriptions of the Profiles of Reform

The research question addressed in the current study is: What profiles of interpretation and implementation of reform in the teaching of calculus emerge from data obtained from mathematics faculty members using CCH Curriculum Project
materials? Descriptions of the profiles of interpretation and implementation of reform in the teaching of calculus are central to the current study. The profiles or narrative descriptions are based on clusters' scores on the clustering scales, the mathematics department validating scales, the CCH validating scales, the demographic validating scales, and the comment data. Chapter V, Profiles of Reform in the Teaching of Calculus contains the descriptions of the patterns of reform.

## Summary

In order to develop profiles of implementation of CCH Curriculum Project materials, Initial Questionnaires were sent to all mathematics departments that appeared on purchaser lists provided by the publisher. Instructors who are currently using or have previously used the CCH Curriculum Project materials at 117 academic institutions are participants in the current study. Site liaison participants completed the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey, and other instructors completed the Faculty Survey only. The quantitative responses to the surveys were coded to generate the data set that was used to perform a cluster analysis on the data. Cluster analysis identified eight groups or clusters of academic institutions. The clusters also represent eight patterns of implementation of CCH Curriculum Project materials. Validation scales that incorporate survey items not used for the cluster analysis are used to validate and describe the eight clusters. Respondents' comments to survey items are used in descriptions of the clusters. The next chapter, Chapter IV, Analysis and Interpretation, first briefly describes the clusters identified in the cluster analysis, noting differences between the clusters on the clustering scales. It then proceeds with the validation process in which differences between clusters on the validating scales are demonstrated. Chapter V, Profiles of Reform in the Teaching of Calculus, presents a more detailed, narrative portrayal of each individual cluster, using scores on the clustering scales and validating scales and participants' comments to enrich the description.

## CHAPTER IV

## ANALYSIS AND INTERPRETATION

This chapter focuses on and validates the eight clusters that were identified in the current study through cluster analysis. Clusters are nonoverlapping groups of academic institutions that exhibit similar patterns of implementation of Calculus Consortium based at Harvard (CCH) Curriculum Project materials. The first section of the current chapter attends primarily to the clusters and the clustering scales. The individual clusters are described briefly and the differences between the clusters on the clustering scales are addressed. A more complete description of the eight clusters that is based on the clustering and validating scales and comment data appears in Chapter V, Profiles of Reform in the Teaching of Calculus. The second section of the current chapter validates the clusters, using an external validation process. The external validation process in the current study consists of a demonstration of differences between the eight clusters on validating scales. The external validating scales are comprised of groups of survey questions that are not used in the cluster analysis. The validating scales are defined and described in Chapter III, Methods and Procedures, of the current study.

In the following discussions about clusters' average scores and consistency rankings, it is important to remember that the clusters were identified through a process that was based solely on information reported by participants. Comparisons between clusters and cluster rankings are meant to be descriptive of the clusters' consistency with the goals of reform-based calculus instruction as the goals are interpreted in the current study (see Chapter II, Review of the Literature). For ease of reading, statements such as "the responses from the faculty at the academic
institutions in cluster $n$ result in cluster $n$ receiving a relative consistency rank of $k$ " are shortened to "cluster n's rank is $k$." The reader is cautioned not to infer any judgments about academic institutions. We turn now to the first section, introductory descriptions of the eight clusters and comparisons of the clusters on the clustering scales.

## The Eight Clusters

The cluster analysis process grouped the 117 academic institutions participating in the current study into eight nonoverlapping clusters, each containing the academic institutions most similar on the six clustering scales (concepts, approach, teaching, assessment, technology, and access). A brief review of the definitions of the six clustering scales follows below. Chapter III, Methods and Procedures, includes complete definitions and descriptions of the clustering scales. It may be helpful to remember that clustering scales have direction in terms of consistency with goals of reform-based calculus instruction.
(1) Concepts is defined by the development of students' conceptual understanding of central calculus ideas.
(2) Approach is defined by instructors (a) using and giving equal weight to graphic, numeric, and algebraic approaches to calculus topics and (b) using an inductive approach in which calculus topics arise from student investigations and real world problems.
(3) Teaching is defined by instructors' teaching practices. Reform-based calculus goals encourage the creation of classroom situations in which students are actively involved in their learning.
(4) Assessment is defined by student assessment methods, recognizing that the use of alternative assessment strategies is consistent with reform-based calculus goals.
(5) Technology is defined by the use of calculators and computers in the classroom.
(6) Access is defined by the accessibility of calculus to a wide range of students including, women, nontraditional students, students in disciplines that require an understanding of calculus concepts, and ethnic minority groups traditionally underrepresented in calculus.

The preceding clustering scales are the foundations upon which cluster analysis identified eight clusters of academic institutions most similar on the scales. The following section describes the eight clusters in terms of their average scores on each of the clustering scales.

## Mean Scores and Cluster Rankings

The following discussion is intended to help the reader interpret Table 5 and understand the rankings of the clusters on the clustering scales. A cluster's average score on each of the six scales is listed in the columns of Table 5 and determines the ranking of the clusters on each scale. The cluster with the lowest average score receives the lowest rank (1). The cluster with the highest average score receives the highest rank (8). A rank of 8 indicates the cluster is most consistent with reformbased calculus instruction relative to the other clusters on that scale. A rank of 1 indicates least consistent. For example, in Table 5, cluster 5 has an average score of .78 on concepts and ranks 8 (most consistent with goals for reform). Cluster 8, with an average score of .41 on concepts, ranks I (least consistent). The columns of Table 5 demonstrate the differences between the clusters on the six scales.

Table 5
Mean Responses and Ranking of Clusters by Clustering Scales ( 1 is low, 8 is high)

| $\begin{aligned} & \text { Cluster } \\ & \text { number } \end{aligned}$ | Clustering Scales |  |  |  |  |  | $\begin{array}{\|c\|} \hline \text { Overall } \\ \text { mean } \\ \text { score and } \\ \text { (rank) } \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | concepts | approach | teaching |  | tech | access |  |
| 1 |  |  |  |  |  |  |  |
| Mean score (s.e.) | $\begin{gathered} .50 \\ (.02) \end{gathered}$ | $\begin{array}{r} .88 \\ (.01) \end{array}$ | $\begin{array}{r} .16 \\ (.02) \end{array}$ | $\begin{array}{r} .16 \\ (.02) \end{array}$ | $\begin{gathered} .66 \\ (.02) \end{gathered}$ | $\begin{array}{r} .47 \\ (.04) \end{array}$ | . 47 |
| (Rank) | (2) | (4) | (2) | (1) | (2) | (7) | (3) |
| $\mathrm{N}=17$ |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| Mean score | . 70 | . 91 | . 21 | . 19 | . 67 | . 31 | . 49 |
| (s.e.) | (.02) | (.01) | (.02) | (.01) | (.02) | (.02) |  |
| (Rank) | (6) | (6.5) | (4) | (3) | (3) | (4) | (4) |
| $\mathrm{N}=31$ |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| Mean score | . 55 | . 75 | . 12 | . 22 | . 47 | . 21 | . 39 |
| (s.e.) | (.03) | (.03) | (.02) | (.02) | (.04) | (.04) |  |
| (Rank) | (4) | (2) | (1) | (4) | (1) | (1) | (2) |
| $\mathrm{N}=13$ |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| Mean score | . 53 | . 88 | . 25 | . 34 | . 81 | . 32 | . 52 |
| (s.e.) | (.03) | (.02) | (.02) | (.02) | (.03) | (.02) |  |
| (Rank) | (3) | (4) | (5.5) | (5) | (8) | (5) | (5) |
| $\mathrm{N}=12$ |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |
| Mean score | . 78 | . 91 | . 25 | . 37 | . 80 | . 29 | . 57 |
| (s.e.) | (.01) | (.01) | (.01) | (.02) | (.02) | (.03) |  |
| (Rank) | (8) | (6.5) | (5.5) | (6) | (7) | (3) | (6) |
| $\mathrm{N}=17$ |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |
| Mean score |  | . 88 |  |  | . 79 | . 70 | . 65 |
| (s.e.) | (.02) | (.04) | (.08) | (.04) | (.06) | (.09) |  |
| (Rank) | (5) | (4) | (8) | (7) | (6) | (8) | (8) |
| $\mathrm{N}=7$ |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |
| Mean score |  |  | . 37 | . 63 |  | . 36 | . 63 |
| (s.e.) | (.02) | (.02) | (.02) | (.02) | (.03) | (.03) |  |
| (Rank) | (7) | (8) | (7) | (8) | (5) | (6) | (7) |
| $\mathrm{N}=10$ |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |
| Mean score | . 41 | . 58 | . 17 | . 17 | . 71 | . 25 | . 38 |
| (Rank) | (.03) | (.03) | (.03) | (.02) | (.05) | (.06) |  |
| (s.e.) | (1) | (1) | (3) | (2) | (4) | (2) | (1) |
| $\mathrm{N}=10$ |  |  |  |  |  |  |  |
| Mean for all |  |  |  |  |  |  |  |
| clusters |  |  |  |  |  |  |  |
| (s.e.) | (.01) | (.01) | (.01) | (.01) | (.01) | (.02) |  |
| $\mathrm{N}=117$ |  |  |  |  |  |  |  |

A cluster's overall rank for consistency with reform-based calculus instruction is based on the cluster's mean score across all scales (see the rightmost column of Table 5). For example, cluster 6 ranks overall highest with a mean score of 65 . Cluster 6, therefore, receives a rank of 8 , indicating that the responses from institutions in cluster 6 are most consistent with the goals for reform-based calculus instruction across all six scales. By focusing on the rightmost column of Table 5, the reader can learn that Clusters 6 and 7 rank most consistent with reform while clusters 3 and 8 rank least consistent with reform, and clusters $1,2,4$, and 5 rank in the moderate range.

The average scores for each scale across all clusters, listed in the bottom row of Table 5, differ considerably. From least to greatest the average score on each clustering scale is: (a) teaching--.25, (b) assessment--.31, (c) access--.36, (d) concepts --.61 , (e) technology--.71, and (f) approach --.84. The following paragraphs describe the meanings of the different average scores on the scales.

Based on the data obtained in the current study, the average score of .84 on approach and the average score of .71 on technology indicate that instructors using CCH Curriculum Project materials are most consistent with reform-based calculus goals on approach and technology. By referring to the definition of approach, the reader can infer that most instructors in the current study report that they give equal weight to the graphic, numeric, and algebraic approach to calculus topics and use an inductive approach in which calculus topics arise from student investigations and real world problems. The relatively high average score on technology (.71) indicates that most participants in the study report that they support and use technology in the classroom.

Overall, the responses to the survey items related to concepts (.61) indicate moderate consistency with reform on the development of students' conceptual
understanding of the central ideas of calculus. The responses to items related to access (.36), teaching (.25), and assessment (.31) are in the moderately low range. The moderately low scores on teaching and assessment indicate the participants use alternative teaching practices and alternative student assessment methods less frequently than the goals for reform in the teaching of calculus might suggest.

It is also interesting to note that the overall average scores of all clusters on the clustering scales are between .38 and .65. Based on a scale from 0 to 1 , these are relatively moderate scores, suggesting that academic institutions in the current study are moderate in their consistency with reform when the scores are averaged across all clustering scales representing the various dimensions of reform.

Some of the sections in this chapter discuss the rankings of clusters on the clustering scales. A cluster is ranked "high" in consistency with reform if it ranks in the highest two clusters, "moderate" if it ranks in the middle four, and "low" if it ranks in the lowest two. When the rank of a cluster is discussed, the mean score of the cluster on the scale is written, in parentheses, to the right of the scale. It is important to attend to a cluster's mean score as well as its rank.

It is also important to consider the mean score of the clustering scale across all clusters when attending to rankings. For example, the teaching clustering scale has a low mean score across all clusters (.25). Therefore a cluster's score of .46 on teaching is relatively high compared to the other clusters, but .46 is in the mid-range on a scale from 0 to 1 ( 1 indicates the greatest consistency with reform).

The next subsection briefly describes the clusters on the clustering scales. The first several descriptions are slightly longer than the others because they include some review of the definitions of the scales in the context of the description of the cluster.

## Brief Introduction to the Clusters

This section presents brief descriptions, based on the clustering scales, of the eight clusters. This early introduction to the clusters is intended to provide
background for the validation discussion in which differences between clusters on validating scales are noted. As mentioned previously, more complete descriptions of the eight clusters appear in Chapter V, Profiles of Reform in the Teaching of Calculus.

Figure 3 presents line graphs, also called clusters, of clusters 6, 7, and 5 that illustrate the clusters' average scores on the six clustering scales. Clusters 6,7 , and 5 have the highest overall mean scores ( $.65, .63$, and .57 respectively). The reader may find it helpful to refer to the profiles while reading the descriptions of the eight clusters that follow.


Figure 3. Profiles of clusters 7,6 , and 5 on clustering scales.

## Cluster 6

Cluster 6, with overall mean score of 65 , ranks highest in consistency with the goals for reform in the teaching of calculus. Although the mean score for all
clusters on teaching is low (.25), indicating that the teaching practices of most participants in the current study are generally traditional, cluster 6 scores highest on teaching (.46). Cluster 6 ranks second high on assessment (.42). Based on the scores on teaching and assessment, instructors in cluster 6 are among the most likely to use alternative teaching practices and alternative student assessment methods.

Cluster 6 also scores highest on access (.70), indicating that, on the average, academic institutions in cluster 6 provide greater access to calculus for students traditionally underrepresented in calculus than other clusters. Cluster 6 ranks in the moderate range on concepts (.67), approach (.88), and technology (.79). It is interesting to note that cluster 6, the cluster ranked most consistent with reform, is the smallest cluster, with only seven academic institutions. One way to characterize cluster 6 is "teaching diverse students".

## Cluster 7

Cluster 7 ranks highest on four clustering scales and ranks second highest overall for consistency with reform with an overall mean score of 63 . Like cluster 6, cluster 7 ranks in the high range as compared with other clusters on teaching (.37) and assessment (.63), even though the teaching score is still quite low on a scale from 0 to 1. Cluster 7's score on assessment is considerably higher than that of any other cluster, indicating greater use of alternative methods of student assessment than other clusters. Cluster 7 also ranks second highest on concepts (.72), indicating that instructors in cluster 7 emphasize student understanding of the central ideas of calculus and place less emphasis on procedural skills. On approach (.93), cluster 7 scores highest, revealing that instructors in cluster 7 closely adhere to the approach suggested by the authors of the CCH Curriculum Project textbook. The authors suggest that instructors place emphasis on graphic, numeric, and algebraic approaches to topics and use an inductive, investigative approach to ideas that is based on real world situations.

Cluster 7 scores in the moderate range on technology (.75) and access (.36). It is important to remember that the overall mean score on technology (.71) indicates that most participants in the current study make considerable use of technology. Cluster 7 is also a small cluster with only ten academic institutions. A characterization for cluster 7 could be "teachers."

## Cluster 5

Cluster 5 ranks third highest overall on the clustering scales with an overall mean score of .57 . Technology (.80) and concepts (.78) are the two clustering scales on which cluster 5 ranks in the high range. The high score on technology indicates that many instructors in cluster 5 regularly use calculators or computers in the classroom. Cluster 5 also ranks in the high-moderate range on approach (.91). Based on the high scores on concepts and approach, it could be speculated that technology is used for investigative approaches and conceptual understanding.

Cluster 5 ranks in the moderate range on teaching (.25), assessment (.37), and access (.29). There are 17 academic institutions in cluster 5. A characterization for cluster 5 could be "technology for understanding."

Figure 4 illustrates the profiles of the average scores of clusters of clusters 4 , 2, and 1 on the clustering scales. Clusters 4,2 , and 1 have overall mean scores of .52 , .49 , and .47 respectively.


Figure 4. Profiles of clusters 4,2 , and 1 on the clustering scales.

## Cluster 4

Cluster 4, with a mean score of .52 across all clustering scales, ranks overall in the moderate range. Technology (.81) is the one clustering scale on which cluster 4 ranks in the high range, indicating a greater than average use of technology. On the other clustering scales, concepts (.53), approach (.88), teaching (.25), assessment (.34), and access (.32), cluster 4 ranks in the moderate range. Cluster 4 contains 12 academic institutions. Considering cluster 4's high rank on technology and moderate rank on the other scales, a characterization for cluster 4 could be "techies."

## Cluster 2

Cluster 2 also ranks in the moderate range with an overall mean score of .49. The largest cluster, cluster 2 contains 31 academic institutions. This is consistent with the observation that a majority ( 77 out of 117 or $66 \%$ ) of the academic institutions participating in the current study are in the four moderate ranking clusters. Cluster 2 also ranks in the moderate range on all of the clustering scales: concepts (.53), approach (.88), teaching (.21), assessment (.19), technology (.67), and access (.47). Cluster 2 could be characterized as "middle of the road."

## Cluster 1

Cluster 1 ranks in the low-moderate range with an overall mean score of . 47 . However, cluster 1 ranks second high on access (.47), indicating that academic institutions in cluster 1 provide greater than average access to calculus for students traditionally underrepresented in the course. Cluster 1 ranks in the low range on concepts (.50), teaching (.16), assessment (.16), and technology (.66). Only on approach (.88) is cluster 1 in the moderate range. Based on the average scores, it appears that academic institutions in cluster 1 provide access to calculus for a diverse group of students using relatively traditional teaching practices and student assessment methods. A characterization of cluster 1 could be "diverse students and some reform."

Figure 5 illustrates the profiles of the average scores of clusters of clusters 3 and 8 on the clustering scales. Clusters 3 and 8 have overall mean scores of .39 and .38 respectively.


Figure 5. Profiles of clusters 3 and 8 on the clustering scales.

## Cluster 3

Cluster 3 has the second lowest overall mean score (.39). Although it ranks in the moderate range on concepts (.55) and assessment (.22), cluster 3's scores are in the low range on approach (.75), teaching (.12), technology (.47) and access (.21). The scores on teaching and technology are considerably lower than those of all other clusters. There are 13 academic institutions in cluster 3. Cluster 3 could be characterized as "small steps toward reform."

## Cluster 8

Cluster 8, with an overall mean score of .38 , ranks lowest in consistency with the goals for reform in the teaching of calculus. Cluster 8 does rank in the lowmoderate range on teaching (.17) and in the moderate range on technology (.71). On
concepts (.41), approach (.58), assessment (.21), and access (.25), cluster 8 ranks in the low range. Ten academic institutions are in cluster 8 . Cluster 8 could be characterized as "technology with a little reform."

## Summary

The preceding descriptions of the clusters were intended to introduce the reader to the clusters. The next several sections compare the clusters, first on the clustering scales and then on the validating scales. The comparisons of the clusters on the validating scales are intended to demonstrate differences among clusters on variables external to the cluster analysis and thereby to validate the clusters.

## Relative Positions of Clusters on the Clustering Scales

The next section addresses differences among the clusters on the clustering scales through the use of a "box diagram" representation (see Figure 6). This visual representation draws attention to the clusters' relative consistency with reform across the scales and the spread of the clusters about the mean of each scale. The box diagrams demonstrate differences among the clusters' mean scores on clustering and validating scales through a graphical representation as the tables do through a numerical representation.


Figure 6. Ranking of clusters by clustering scales. The numbers in the boxes are cluster numbers. The dashed lines represent the average scores for the clustering scales.

Each box in Figure 6 represents a clustering scale, and the numbers in the boxes represent clusters. The dashed line in each box represents the mean score for all clusters on that particular scale. The following discussion, in addition to comparing and contrasting the clusters, is intended to assist the reader in interpreting the information presented in Figure 6. Tables similar to Table 4 and diagrams similar to Figure 6 are used throughout the validation process.

The leftmost box contains the positions of the clusters within the teaching scale. The box diagram demonstrates that cluster 7 (represented by the numeral 7 in the box labeled Teaching) has an average score between .35 and .40 on teaching, and cluster 3 has an average score between .10 and .15 on teaching. Cluster numbers positioned higher within a box indicate the cluster is more consistent with reform on that clustering scale than clusters whose cluster numbers are positioned lower in the box. Because cluster 7 is positioned higher than cluster 3 in the teaching box, the
reader can infer that responses from institutions in cluster 7 show greater consistency with reform on teaching than responses from institutions in cluster 3. Similar interpretations can be made for the other clusters and on the other clustering scales.

By tracing the position of a particular cluster across all boxes in Figure 6, the reader can observe the relative consistency with reform of a cluster across all scales. The trace of a cluster in Figure 6 corresponds to the profiles in Figures 3, 4, and 5. The use of box diagrams allows the scores of all eight clusters on the six clustering scales to be represented on the same diagram. For example, tracing the position of cluster 7 across the boxes in Figure 3 provides the reader with a visual representation of the relative ranking of cluster 7 across all six scales. The trace of cluster 7 across all scales is higher than the trace of cluster 8, indicating that institutions in cluster 7 are more consistent with the goals of reform-based calculus instruction than the institutions in cluster 8. The reader can also confirm, from Figure 3, that cluster 7 is ranked in the two clusters most consistent with reform on teaching, assessment, concepts, and approach.

The spread of the clusters about the mean of each scale is also highlighted in Figure 6. Cluster 7 is positioned a relatively large distance above the mean on assessment, whereas other clusters appear closely grouped about the mean on assessment. The position of cluster 7 indicates that cluster 7's implementation of reform-based-calculus instruction related to student assessment is more consistent with the goals for reform in the teaching of calculus than that of the other clusters. The clusters are grouped relatively close to the mean on concepts and teaching with cluster 6 slightly higher than the rest on the use of alternative teaching practices and cluster 8 slightly lower than the rest on concepts. Cluster 6 stands out as ranking considerably higher on access than the other clusters. This would indicate that academic institutions in cluster 6 provide comparatively greater access to calculus for students traditionally underrepresented in calculus. Seven clusters are grouped
relatively tightly about the mean on technology, with cluster 3 standing alone as a low outlier. A similar situation exists with cluster 8 on approach. Further analysis of the clusters based on the validating scales and the comment data will contribute to the more complete descriptions of the individual clusters in the next chapter.

## Summary

This section has briefly described each of the eight clusters in terms of the clustering scales. The section has also compared the eight clusters on the six clustering scales using two different representations, a table and a schematic box diagram. Each representation makes apparent the differences among the clusters from a different perspective. Similar representations will be used in the validation discussion in the next section. The next section begins the external validation process by demonstrating differences among the clusters on the validating scales.

## Validation of the Clusters Using External Validating Scales

Validation of the cluster solution, in the current study, refers to the process of demonstrating differences among clusters on relevant variables not used in the cluster analysis. The validating scales in the current study are of three types, those pertaining (a) to mathematics department perspectives, (b) to attitudes and practices directly concerned with the CCH Curriculum Project materials, and (c) to demographic issues. Validating scales, in any external validation process, should be relevant to the study and, when possible, should be based on the same theoretical foundations as the clustering scales. The validating scales in the current study, like the clustering scales, are based on the goals for reform in the teaching of calculus. According to Aldenderfer and Blashfield (1984), the validation of a cluster solution on external variables is the strongest validation process for a cluster solution. External validation is not possible in some studies because of the lack of data from external variables.

Because the participants were asked a large number of questions on the three surveys,
the current study has an opportunity for external validation not available to many studies. The validation sections that follow each contain (a) a brief summary of the validating scales under discussion, (b) a discussion of the differences among clusters on the validating scales, and (c) several interpretive paragraphs. The next section addresses the mathematics department validating scales.

Mathematics Department Validating Scales
A brief review of the mathematics department validating scales follows. The concepts (vconcept), teaching (vteaching), assessment (vassessment), and technology (vtechology) validating scales correspond to the concepts, teaching, assessment, and technology clustering scales respectively. The difference is that the validating scales describe the general perspective of all faculty in the mathematics department, whereas the clustering scales describe the perspective of faculty using CCH Curriculum Project materials. The vinterest mathematics department validating scale is defined by the mathematics department faculty members' interest in pedagogy and reform in the teaching of calculus and of mathematics as measured by the frequency of discussions about these topics. The vvalues mathematics department validating scale is defined by the value placed on teaching by the institution and mathematics department.

Two different representations of the data are used to demonstrate differences among the clusters on the mathematics department validating scales. The first representation is the tabular representation, and the second is the box diagram representation. Each representation highlights differences among the clusters from a different perspective. The next subsection begins the validation process through the tabular representation of average scores for clusters on the mathematics department validating scales.

## Mean Scores and Cluster Rankings

The average score and rank of each cluster over all mathematics department validating scales and the average score of each mathematics department validating scale over all clusters are shown in Table 6. Differences among the clusters on each of the mathematics department validating scales are made evident by the wide variation of each cluster's mean score on each validating scale in the columns of Table 5. For example, on vteaching the mean scores for all institutions within each cluster range from cluster 3's low mean score of 09 to cluster 7's high mean score of .61. The mean scores on vassessment vary from .15 to 70 . For each of the clustering scales there is considerable variation among the mean scores of the eight clusters. These widely varying mean scores demonstrate differences among the clusters on the mathematics department validating scales and contribute to the external validation of the clusters.

Table 6
Mean Responses and Ranking of Clusters by Mathematics Department Validating
Scales ( 1 is low, 8 is high)


The discussion now focuses on the overall relative rankings of the clusters on the mathematics department validating scales (see the rightmost column of Table 6). The clusters' mean score across all clustering scales is listed in parentheses to the right of the cluster number. Cluster 7 (.62) and cluster 5 (.51) rank as the two clusters most consistent with the goals of reform-based calculus on the mathematics department validating scales. Cluster 3 (.27) and cluster I (.39) rank as the two clusters least consistent with the goals of reform-based calculus instruction. Cluster 6 (.50), cluster 4 (.47), cluster $2(.45)$, and cluster $8(.44)$ rank in the moderate range. Note that the difference between cluster 5's score and cluster 6's score is only .01. It is interesting to note that cluster 7 ranks in the high range in consistency with the goals of reform-based instruction on both the clustering scales and on the mathematics department validating scales. Cluster 3 ranks in the low range on the clustering scales and on the mathematics department validating scales.

The box diagram in the next subsection illustrates differences among the clusters through a more visual representation. Table 6 provides exact scores for the clusters on the mathematics department validating scales, whereas the box diagram in the next section presents a pictorial representation of the differences among the clusters.

## Relative Positions of Clusters on the Mathematics Department Validating Scales

This section addresses general implementation characteristics and differences among clusters on the mathematics department validating scales through use of the box diagram representation (see Figure 7). This visual representation draws attention to the relative consistency with reform of the clusters across the mathematics department validating scales and the spread of the clusters about the mean of the mathematics department validating scales.


Figure 7. Ranking of clusters by mathematics department validating scales. The numbers in the boxes are cluster numbers. The dashed line represents the average score for the validating scale.

By tracing the clusters, the reader can note their position across the scales relative to the other clusters and relative to the mean of all clusters on the scale. For example, the traces of clusters 7 and 3 demonstrate their relatively high and low profiles respectively and their distances above and below the means.

Comparisons among the average scores on the mathematics department validating scales across all clusters provide insight into the average implementation of reform-based calculus instruction by participants in the current study. As shown in the bottom row of Table 6 and by the dashed lines in the box diagram in Figure 7, the mean scores on the mathematics department validating scales differ greatly. From least to greatest, the mean score on each mathematics department validating scale is:
(a) vteaching--.30, (b)
(b) vinterest--. 30
(c) vassessment--. 34 ,
(d) vconcept--.46, (e)
vtechnology--.63, (f) vvalues--. 63 . Overall, the mathematics departments, as reported by the site liaisons, are least consistent with reform in their teaching practices (vteaching--.30) and in their interest in pedagogy and reform (vinterest-.30). Vinterest is measured by the reported frequency of discussions about pedagogy and reform in the teaching of mathematics. The score related to mathematics department faculty's use of alternative methods of student assessment (vassessment) is relatively low (.34). The average score on vconcepts (.46) indicates relatively moderate mathematics department implementation related to this validating scale across all clusters. The mathematics departments are most consistent with reform in their use of technology (vtechnology--.63) and in their valuing of teaching (vvalues-..63).

It is important not only to consider the average scores on mathematics department validating scales, but also to consider the variation among the clusters within each scale. Cluster scores for vteaching (.30), vassessment (.34), and vvalues (.65) are relatively widely distributed about the mean, indicating relatively substantial differences in clusters' implementation of reform in the teaching of calculus along those validating scales. The two clusters most consistent with reform, clusters 6 and 7, are positioned well above the mean on vteaching (. 45 and .61 respectively), whereas cluster 3, a low ranking cluster on the clustering scales, is positioned well below the mean on vteaching (.09). The wide distance between clusters 7 and 6 and cluster 3 indicates there is a substantial difference between clusters 7 and 6 and cluster 3's implementation of reform in the teaching of calculus on vteaching. Cluster 3, a cluster low in consistency with reform on the clustering scales, stands apart as low on the valuing of teaching mathematics department clustering scale (vvalues-.43).

The scores on vinterest are relatively closely grouped about the mean (.30), indicating that many mathematics departments of institutions in the current study address pedagogy and reform with similar frequency. The scores on vconcept are also
closely grouped about the mean (.46). Cluster 3 appears to be a low outlier on vtechnology (.31). The next subsection provides some interpretation of the observations.

## Interpretation

Tables 5 and 6 and Figures 6 and 7 indicate there may be some relationship between the cluster rankings on the clustering scales and on the mathematics department validating scales. A comparison between the cluster rankings on the scales is shown in Table 7. The overall mean scores are listed in parentheses below the cluster numbers.

Table 7.
Comparison of Cluster Rankings by clustering scales and mathematics department validating scales.

|  | low |  | moderate |  |  |  | high |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Clusters ordered from lowest to highest rank on the clustering scales | $\begin{gathered} 8 \\ (.38) \\ \hline \end{gathered}$ | $\begin{gathered} 3 \\ (.39) \\ \hline \end{gathered}$ | $\begin{gathered} 1 \\ (.47) \\ \hline \end{gathered}$ | $\begin{gathered} 2 \\ (.49) \\ \hline \end{gathered}$ | $\begin{gathered} 4 \\ (.52) \\ \hline \end{gathered}$ | $\begin{gathered} 5 \\ (.57) \\ \hline \end{gathered}$ | $\begin{gathered} 7 \\ (.63) \\ \hline \end{gathered}$ | $\begin{gathered} 6 \\ (.65) \\ \hline \end{gathered}$ |
| Clusters ordered from lowest to highest rank on the mathematics department validating scales | $\begin{gathered} 3 \\ (.27) \end{gathered}$ | $\begin{gathered} 1 \\ (.29) \end{gathered}$ | $\begin{gathered} 8 \\ (.44) \end{gathered}$ | $\begin{gathered} 2 \\ (.45) \end{gathered}$ | $\begin{gathered} 4 \\ (.47) \end{gathered}$ | $\stackrel{6}{(.50)}$ | $\begin{gathered} 5 \\ (.51) \end{gathered}$ | $\begin{gathered} 7 \\ (.62) \end{gathered}$ |

Cluster 7 ranks high in consistency with reform on both the clustering scales and the mathematics department validating scales. Cluster 3 ranks low in consistency with reform on the clustering scales and on the mathematics department validating scales. Cluster 5, the third highest ranked cluster on the clustering scales, ranks second highest on the mathematics department validating scales. Cluster 1 , ranked third lowest on the clustering scales, ranks second lowest on the mathematics department validating scales. Clusters 3 and 8 rank in the three lowest and three highest ranges respectively on the two scales.

The similarity between the rankings on the clustering scales and on the mathematics department validating scales suggest some interesting questions. Is the general overall climate of a mathematics department related to the teaching of all courses in the mathematics department? In particular, does their implementation of reform-based curriculum projects such as the CCH Curriculum Project also reflect, in many instances, the characteristics of the mathematics department itself?

From a different perspective, a similar relationship exists between the average scores of clustering scales across all clusters and the average scores of the corresponding mathematics department validating scales across all clusters. Table 8 lists the overall average scores, ranked from lowest to highest, on the corresponding clustering scales and validating scales.

Table 8
Ranked Scores on Corresponding Clustering Scales and Validating Scales

| Clustering scale | Score | Validating scale | Score |
| :---: | :---: | :---: | :---: |
| Teaching | .25 | Vteaching | .30 |
| Assessment | .31 | Vassessment | .34 |
| Concepts | .61 | Vconcepts | .46 |
| Technology | .71 | Vtechnology | .71 |

The reader should note that the ranking of the clustering scale scores compare favorably to the rankings of the corresponding validating scale scores. This observation leads to additional questions. Are the teaching practices, student assessment methods, and use of technology in reform-based calculus courses similar to those in other mathematics courses within a mathematics department? The difference between the scores on concepts and vconcepts are the greatest. Is this difference related to the stated emphasis in the CCH Curriculum Project textbook on conceptual understanding over procedural practice? What is the relationship between
the emphasis in a course textbook and the approach used by the instructor? These and related questions will be explored further in the description of the individual clusters and in the final chapter.

## Summary

The first set of validating scales, the mathematics department validating scales, provide a very strong external validation of the clusters. There are considerable differences among the clusters on the scales. Similarities of cluster rankings on the corresponding clustering and mathematics department validating scales is not necessary for cluster solution validation. However, the demonstration of the similarity among rankings contributes extra strength to the validity of the cluster solution. The discussion surrounding the mathematics department validating scores also adds to the understanding of the implementation of CCH Curriculum Project materials and reform in the teaching of calculus. The second set of validating scales, described below, relate to the use of the CCH Curriculum Project materials directly.

## CCH Validating Scales

The four validating scales that are most directly related to CCH Curriculum Project faculty attitudes and practices and to CCH Curriculum Project materials are called the CCH validating scales. The CCH validating scales are comprised of survey items not used in the cluster analysis and are, therefore, external validating scales. A brief summary of the definitions of the CCH validating scales follows. The vuseCCH validating scale is defined by the percentage of calculus students that use CCH Curriculum Project textbook as the main textbook in their calculus course. Vstatus is defined by the percentage of full-time, tenured, or tenure-track faculty that are teaching or have taught CCH Curriculum Project courses. For institutions that do not have tenure, full-time status is used. Vinteraction is defined by the percentage of instructors that have attended CCH Curriculum Project workshops and the frequency with which CCH Curriculum Project instructors meet to discuss pedagogical issues or
reform in the teaching of calculus. Vreform is defined by the level of support for reform in the teaching of calculus that is held by CCH Curriculum Project instructors.

The next two subsections address differences among the eight clusters on the CCH validating scales and the overall averages of CCH clustering scale scores. The first subsection contributes to the validation of the clusters through the numerical representation of average scores for clusters on the CCH validating scales.

## Mean Scores and Cluster Rankings

Table 9 lists the clusters' average score and rank on each of the CCH validating scales. Differences between the clusters on each of the CCH validating scales are evidenced by the mean score of each cluster on each CCH validating scale in the columns of Table 9.

Table 9
Mean Responses and Ranking of Clusters by CCH Validating Scales (1 is low, 8 is
high)

| Cluster number | CCH validating scales |  |  |  | $\begin{array}{\|l} \text { Overall mean } \\ \text { and (rank) } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | vinteract | vreform | vstatus | vuse |  |
| 1 <br> Mean score <br> (s.e.) <br> (Rank) <br> $\mathrm{N}=17$ <br> 2 | $\begin{gathered} .28 \\ (.09) \\ (2) \end{gathered}$ | .94 $(.06)$ <br> (4) | $\begin{array}{r} .76 \\ (.05) \\ (4.5) \end{array}$ | $\begin{gathered} .65 \\ (.08) \\ (5) \end{gathered}$ | .66 <br> (4) |
| $\begin{gathered} \mathbf{2} \\ \text { Mean score } \\ \text { (s.e.) } \\ \text { (Rank) } \\ \mathrm{N}=31 \\ \hline \end{gathered}$ | $\begin{gathered} .29 \\ (.06) \\ (3) \end{gathered}$ | $\begin{aligned} & 1.00 \\ & (.00) \\ & (6.5) \end{aligned}$ | $\begin{gathered} .75 \\ (.04) \end{gathered}$ (3) | $\begin{gathered} .54 \\ (.07) \\ (1) \end{gathered}$ | $\begin{gathered} .65 \\ (2.5) \end{gathered}$ |
| 3 <br> Mean score <br> (s.e.) <br> (Rank) <br> $\mathrm{N}=13$ | $\begin{gathered} .31 \\ (.10) \end{gathered}$ <br> (4) | $\begin{gathered} .92 \\ (.08) \\ (2.5) \end{gathered}$ | $\begin{gathered} .69 \\ (.08) \\ (1) \end{gathered}$ | $\begin{gathered} .60 \\ (.11) \\ (2) \end{gathered}$ | .63 <br> (1) |
| Mean score (s.e.) (Rank) $\mathrm{N}=12$ | $\begin{array}{r} .36 \\ (.09) \\ (5) \end{array}$ | $\begin{gathered} .92 \\ (.08) \\ (2.5) \end{gathered}$ | .81 (.08) <br> (6) | $\begin{gathered} .63 \\ (.10) \\ (3) \end{gathered}$ | .68 <br> (5) |
| $\begin{gathered} 5 \\ \text { Mean score } \\ \text { (s.e.) } \\ \text { (Rank) } \\ \mathrm{N}=17 \\ \hline \end{gathered}$ | $\begin{gathered} .53 \\ (.09) \end{gathered}$ <br> (7) | $\begin{aligned} & 1.00 \\ & (.00) \\ & (6.5) \end{aligned}$ | $\begin{gathered} .72 \\ (.05) \\ (2) \end{gathered}$ | $\begin{gathered} .64 \\ (.07) \\ (4) \end{gathered}$ | (6) |
| 6 <br> Mean score <br> (s.e.) <br> (Rank) <br> $\mathbf{N}=7$ | $\begin{array}{r} .76 \\ (.16) \\ (8) \end{array}$ | $\begin{aligned} & 1.00 \\ & (.00) \\ & (6.5) \end{aligned}$ | $\begin{array}{r} .76 \\ (.10) \\ (4.5) \end{array}$ | $\begin{aligned} & .76 \\ & (.12) \\ & (7) \end{aligned}$ | $\begin{aligned} & .82 \\ & (8) \end{aligned}$ |
| 7 <br> Mean score <br> (s.e.) <br> (Rank) <br> $\mathrm{N}=10$ | $\begin{gathered} .50 \\ (.13) \\ (6) \end{gathered}$ | $\begin{aligned} & 1.00 \\ & (.00) \\ & (6.5) \end{aligned}$ | $\begin{gathered} .82 \\ (.06) \\ (7) \end{gathered}$ | $\begin{aligned} & .67 \\ & (.12) \\ & (6) \end{aligned}$ | $.75$ <br> (7) |
| $\mathbf{8}$ <br> Mean score <br> (s.e.) <br> (Rank) <br> $\mathrm{N}=10$ | $\begin{gathered} .18 \\ (.06) \\ (1) \end{gathered}$ | $\begin{gathered} .70 \\ (.15) \\ (1) \end{gathered}$ | $\begin{gathered} .93 \\ (.04) \\ (8) \end{gathered}$ | $\begin{gathered} .78 \\ (.09) \end{gathered}$ (8) | $\begin{gathered} .65 \\ (2.5) \end{gathered}$ |
| $\begin{aligned} & \hline \text { Mean for all } \\ & \text { clusters } \\ & \text { (s.e.) } \\ & \mathrm{N}=117 \\ & \hline \end{aligned}$ | $\begin{gathered} .37 \\ (.03) \end{gathered}$ | $\begin{gathered} .95 \\ (.02) \end{gathered}$ | $\begin{gathered} .77 \\ (.02) \end{gathered}$ | $\begin{gathered} .63 \\ (.03) \end{gathered}$ |  |

The mean scores on vinteraction vary from a low of .18 for cluster 8 to a high of .76 for cluster 6. The scores on vstatus vary from cluster 3's mean score of 69 to cluster 8's mean score of .93 . The scores on vuse-CCH vary from a low of .54 for cluster 2 to a high of .78 for cluster 8. There is less difference among the clusters' scores on vreform.

The rightmost column of Table 9 lists the overall ranking of the clusters on the CCH validating scales. Cluster 6 (.82) has the overall highest score on the CCH validating scales and ranks highest. Clusters 7 (.75) and cluster 5 (.72) are the next highest ranking clusters. The rest of the clusters, in order of rank, display relatively little difference between scores: cluster 4 (.68), cluster I (.66), cluster 2 (.65), cluster 8 (.65), and cluster 3 (.63). The relative positions of the clusters on the CCH validating scales are presented in the next subsection through the visual box diagram representation..

## Relative Positions of Clusters on the CCH Validating Scales

The visual representation of the clusters in the box diagram draws attention to the clusters' relative consistency with reform across the CCH validating scales, the spread of the clusters about the mean of the CCH validating scales, and the overall mean on each CCH validating scale.


Figure 8. Ranking of clusters by CCH validating scales. The numbers in the boxes represent cluster numbers. The dashed lines represent the average scores for the clustering scales.

The traces of the various clusters on Figure 8 demonstrate the relative consistency of the various clusters across the clustering scales. The traces of clusters 6 and 7 illustrate their high consistency ranking, whereas the trace of cluster 3 illustrates cluster 3's low rank on the CCH validating scales. The trace of cluster 8 appears the most erratic on the CCH validating scales, with relatively high consistency rankings on the percentage use of CCH Curriculum Project materials (vuse-CCH--.78) and the status of instructors using CCH Curriculum Project materials (vstatus--.93) and relatively low consistency rankings on vinteraction (.37) and vreform (.70).

Comparisons among average scores on the CCH validating scales across all clusters can provide some insight into the average implementation of reform-based calculus instruction by participants in the study. The mean scores on the CCH validating scales are given in the bottom row of Table 9 and by the dashed line in the boxes in Figure 8. From least to greatest the average mean score on each CCH validating scale is (a) vinteract--.37, (b) vuse-CCH--. 63, (c) vstatus --.77 , and (d) vreform--.95. Overall, the CCH instructions score lowest on vinteraction.

Although the mean scores on the clustering scales provide some idea of the average implementation of reform-based calculus materials on the CCH validating scales, attention must also be paid to the spread of individual clusters' scores about the mean. Figure 8 is helpful in this regard.

Although the average score for all clusters on vinteraction is lowest (.37), the spread of the vinteraction scores about the mean score is considerable. Vinteraction is measured by the frequency with which CCH Curriculum Project instructors meet to discuss pedagogical and reform issues and the percentage who have attended CCH Curriculum Project workshops. Scores on vinteraction range from cluster 8's low score of .18 to cluster 6's high score of 76 .

From the average scores of the clusters on vreform (.95), it appears that CCH Curriculum Project instructors are strongly supportive of reform in the teaching of calculus. Cluster 8's relatively low score on vreform stands apart from the other scores.

The percentage of use of CCH materials in calculus courses (vuse-CCH) averages about $63 \%$ across the clusters. The clusters' scores on vuse-CCH are comparatively closely grouped about the mean score (.63). Clusters 6 and 8 exhibit the greatest average percentages ( .76 and .78 respectively) and cluster 2 the lowest (.54). The relatively high and closely grouped mean scores on vstatus (.77) indicate
that the majority of CCH Curriculum Project instructors are full-time. tenured, or tenure-track.

The clusters are compactly grouped about the mean of .77 on vstatus except for cluster 8 with the high score of .93 . Although most of the scores on vreform are closely grouped about the high overall mean score of .95 , Cluster 8 , with a mean score of .70 , stands out as low. Cluster 8's widely varying positions on the CCH clustering scale are not readily interpretable.

The box diagram and preceding paragraphs illustrate the considerable differences among the clusters on the CCH validating scales. The next subsection presents some interpretation of the clusters' scores on the scales.

## Interpretation

The cluster scores on the CCH validating scales reveal additional insights into the implementation of CCH Curriculum Project materials. Some interpretation seems appropriate in this section. Additional interpretation is included with the more complete cluster descriptions in Chapter V, Profiles of Reform in the Teaching of Calculus and in Chapter VI, Summary, Interpretation, and Implications for Future Research.

The relatively high position of all clusters on vstatus (.77) indicates that the status and experience of instructors using CCH Curriculum Project materials overall is generally high. This may mean that some established faculty members are thinking deeply about reform and are using the materials to reflect on what it means to become involved with reform in the teaching of calculus. This could also provide evidence that reform in the teaching of calculus is being considered seriously by the community. Is the high score on vstatus indicative of a renewed emphasis on teaching at colleges and universities? Will higher status faculty continue to teach reform-based calculus after the initial enthusiasm for reform wanes? Will the dialogue surrounding reform become more reflective and less confrontational? Will
this dialogue contribute to a productive continuing evolution of the teaching of calculus and mathematics at the undergraduate level? On the other hand, as the relatively low scores on teaching seem to indicate, is it possible that high status faculty are teaching reform-based calculus using traditional teaching methods? These and other questions should be addressed as the movement to reform calculus curriculum and instruction matures.

The grouping of the clusters around the high mean on vreform (.95) is also noteworthy. A relationship between instructors choosing or agreeing to teach calculus with CCH Curriculum Project materials and their support for reform seems reasonable. It is interesting to speculate that perhaps some high status faculty are using the CCH Curriculum Project textbook to gain a better understanding of the movement to reform the teaching of calculus in order to enter the dialogue from well informed positions. However, it is somewhat perplexing that cluster 8, a small cluster, scores highest on vstatus (.93) and scores lowest on vreform (.70).

Cluster analysis, based on the clustering scales, determines a clusters' overall rank for consistency with reform-based calculus instruction. However, interesting comparisons can be made between the rankings on the clustering scales, the mathematics department validating scales, and the CCH validation scales. Such comparisons appear in Table 10. The overall mean score for each cluster appears beneath the cluster number.

Table 10
Comparison of Cluster Rankings and (Means) by Clustering Scales, Mathematics
Department Validating Scales, and CCH Validating Scales

|  | low |  |  | moderate |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Clusters ordered from <br> lowest to highest rank on <br> clustering scales | 8 <br> $(.38)$ | 3 <br> $(.39)$ | 1 <br> $(.47)$ | 2 <br> $(.49)$ | 4 <br> $(.52)$ | 5 <br> $(.57)$ | 7 <br> $(.63)$ | 6 <br> $(.65)$ |
| Clusters ordered from <br> lowest to highest rank on <br> mathematics department <br> validating scales | 3 <br> $(.27)$ | 1 <br> $(.29)$ | 8 <br> $(.44)$ | 2 <br> $(.45)$ | 4 <br> $(.47)$ | 6 <br> $(.50)$ | 5 <br> $(.51)$ | 7 <br> $(.62)$ |
| Clusters ordered from <br> lowest to highest rank on <br> CCH validating scales | 3 <br> $(.63)$ | 8 <br> $(.65)$ | 2 <br> $(.65)$ | 1 <br> $(.66)$ | 4 <br> $(.68)$ | 5 <br> $(.72)$ | 7 <br> $(.75)$ | 6 <br> $(.82)$ |

Comparisons of the rankings across the three scales indicate some consistency of the rankings across the scales. Cluster 6 has a high consistency-with-reform rank on two of the three scales and is in the high-moderate range on the third scale. Cluster 7 ranks in the high range on the three scales, whereas cluster 5 has a high moderate or high rank on the three scales. Cluster 4 ranks in the moderate range across the scales. Clusters 1,2 , and 8 rank in the low-moderate or low range on the three scales, and cluster 3 ranks in the low range on the three scales. This table, like Table 7, reinforces the possibility of a relationship between the interpretation of goals for reform-based calculus instruction in the implementation of CCH Curriculum Project materials and the mathematics departments' overall engagement with and openness to reform. Further study of this possible relationship would add to the community's understanding of reform.

## Summary

The preceding section has noted differences among the clusters on the CCH validating scales. In so doing it has contributed to the external validation of the cluster solution. The section has also provided opportunities for increased understanding of reform in the teaching of calculus by considering issues related to
the CCH validating scales. The average scores on CCH validating scales give a broad picture of reform in the teaching of calculus, whereas the clusters' individual scores on the scales document the differences between implementation at various academic institutions. These differences highlight the importance of remaining mindful of local contextual features when implementing reform. The next section considers the demographic validating scales.

## Demographic Validating Scales

The demographic validating scales provide contextual information about the clusters. These scales do not have direction in the sense of consistency with goals for reform-based calculus instruction. The first three validating scales describe (a) the types of academic institutions in the clusters, (b) the average student enrollment at institutions in the cluster, and (c) the percentages of public and private institutions in the cluster. The fourth demographic validating scale is qualitatively different from the other three scales. It addresses the percentage of institutions in each cluster for which the site liaison reports some financial support to implement the use of CCH Curriculum Project materials.

## Types of Academic Institutions in the Clusters

Table 11 presents information about the numbers and types of academic institutions in each cluster. The number in each cell indicates the number of academic institutions in each cluster by institution type. The percentage positioned highest in each cell indicates the percentage each type of institution is of the total number of institutions in the cluster. The percentage positioned lowest in each cell indicates the percentage each institution type in a cluster is of the total number of that institution type in the current study.

## Table II

## The Number and (Percent) of Each Type of Academic Institution by Cluster

| Cluster number | Secondary schools $\mathrm{N}=13$ | Two-year colleges $\mathrm{N}=30$ | Doctoral and research universities $\mathrm{N}=19$ | Other colleges and universities $\mathrm{N}=55$ | All institutions $\mathrm{N}=117$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 1 \\ \text { (row \%) } \\ \text { (column \%) } \\ \hline \end{gathered}$ |  |  |  | $\begin{gathered} 8 \\ (47 \%) \\ (15 \%) \end{gathered}$ | $\begin{gathered} 17 \\ (15 \%) \end{gathered}$ |
| $\begin{gathered} 2 \\ \text { (row \%) } \\ \text { (column \%) } \end{gathered}$ | $\begin{gathered} 2 \\ (6 \%) \\ (15 \%) \end{gathered}$ |  |  |  | $\begin{gathered} 31 \\ (26 \%) \\ \hline \end{gathered}$ |
| $\begin{gathered} 3 \\ \text { (row \%) } \\ \text { (column \%) } \end{gathered}$ |  |  | $\begin{gathered} 1 \\ (8 \%) \\ (5 \%) \end{gathered}$ |  | $\begin{array}{r} 13 \\ (11 \%) \\ \hline \end{array}$ |
| $\begin{gathered} 4 \\ \text { (row \%) } \\ \text { (column \%) } \end{gathered}$ | $\begin{gathered} 1 \\ (8 \%) \\ (8 \%) \end{gathered}$ |  |  |  | $\begin{gathered} 12 \\ (10 \%) \end{gathered}$ |
| 5 (row \%) (column \%) | $\begin{gathered} 0 \\ (0 \%) \\ (0 \%) \\ \hline \end{gathered}$ |  |  | $\begin{gathered} 10 \\ (59 \%) \\ (18 \%) \\ \hline \end{gathered}$ | $\begin{array}{r} 17 \\ (15 \%) \\ \hline \end{array}$ |
| 6 (row \%) (column \%) |  | $\begin{gathered} 1 \\ (14 \%) \\ (3 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (14 \%) \\ (5 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (14 \%) \\ (2 \%) \\ \hline \end{gathered}$ |  |
| $\begin{gathered} 7 \\ \text { (row \%) } \\ \text { (column \%) } \end{gathered}$ |  |  | $\begin{gathered} 0 \\ (0 \%) \\ (0 \%) \end{gathered}$ | $\begin{gathered} 3 \\ (30 \%) \\ (5 \%) \end{gathered}$ | $\begin{array}{r} 10 \\ (9 \%) \\ \hline \end{array}$ |
| $\mathbf{8}$ (row \%) (column \%) |  | $\begin{gathered} 1 \\ (10 \%) \\ (3 \%) \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \\ (0 \%) \end{gathered}$ |  | $\begin{gathered} 10 \\ (9 \%) \\ \hline \end{gathered}$ |

When reading Table 11, it is important to remember the distribution of the types of institutions in the current study. Of the 117 participating academic institutions, there are 13 secondary schools (11\%), 30 two-year colleges ( $26 \%$ ), 19 doctoral and research universities ( $16 \%$ ), and 55 other colleges and universities ( $47 \%$ ). The uneven distribution of institution types in the current study demands caution when discussing the percentage of an institution in a cluster. The current study seeks patterns of implementation. For this reason, Table 12 lists the actual number and percentage of institution types in each cluster and the expected number of institution types in each cluster based on the percentage representation in the current
study. The expected number is found by multiplying the percentage of a particular institution type in the current study by the number of institutions in the cluster. For example, cluster 1 contains 17 academic institutions. Because $11 \%$ of the academic institutions in the current study are secondary schools, based on an expected distribution of institution types, cluster 1 would contain two secondary schools. The number of secondary schools in cluster $1(4)$ is unexpectedly high. The following discussion about possible relationships between institution types and particular clusters is based on the data from Table 11 and Table 12. The discussion is divided into sections by institution types.

Table 12
The actual number and expected number of institution types in each cluster

|  | Secondary schools | Two-year colleges | Doctoral and research universities | Other colleges and universities | All institutions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| All clusters | $\begin{aligned} & \hline N=13 \\ & (11 \%) \end{aligned}$ | $\begin{gathered} \hline N=30 \\ (26 \%) \end{gathered}$ | $\begin{gathered} N=19 \\ (16 \%) \end{gathered}$ | $\begin{gathered} \mathrm{N}=55 \\ (47 \%) \end{gathered}$ | $\mathrm{N}=117$ |
| Cluster 1 Actual number and (percent) Expected number | $\begin{gathered} 4 \\ (24 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 2 \\ (12 \%) \\ 4 \end{gathered}$ | $\begin{gathered} 3 \\ (18 \%) \\ 3 \end{gathered}$ | $\begin{gathered} 8 \\ (47 \%) \\ 8 \end{gathered}$ | 17 |
| Cluster 2 <br> Actual number and (percent) Expected number | $\begin{gathered} 2 \\ (6 \%) \\ 3 \end{gathered}$ | $\begin{gathered} 8 \\ (26 \%) \\ 8 \end{gathered}$ | $\begin{gathered} 9 \\ (29 \%) \\ 5 \end{gathered}$ | $\begin{gathered} 12 \\ (39 \%) \\ 15 \end{gathered}$ | 31 |
| Cluster 3 <br> Actual number and (percent) Expected number | $\begin{gathered} 0 \\ (0 \%) \\ 1 \end{gathered}$ | $\begin{gathered} 3 \\ (23 \%) \\ 4 \end{gathered}$ | $\begin{gathered} 1 \\ (8 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 9 \\ (69 \%) \\ 6 \\ \hline \end{gathered}$ | 13 |
| Cluster 4 <br> Actual number and (percent) Expected number | $\begin{gathered} 1 \\ (8 \%) \\ 1 \end{gathered}$ | $\begin{gathered} 5 \\ (42 \%) \\ 3 \end{gathered}$ | $\begin{gathered} 2 \\ (17 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 4 \\ (33 \%) \\ 6 \\ \hline \end{gathered}$ | 12 |
| Cluster 5 <br> Actual number and (percent) Expected number | $\begin{gathered} 0 \\ (0 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 4 \\ (24 \%) \\ 4 \end{gathered}$ | $\begin{gathered} 3 \\ (18 \%) \\ 3 \end{gathered}$ | $\begin{gathered} 10 \\ (59 \%) \\ 8 \end{gathered}$ | 17 |
| Cluster 6 <br> Actual number and (percent) Expected number | $\begin{gathered} 4 \\ (57 \%) \\ 1 \end{gathered}$ | $\begin{gathered} 1 \\ (14 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 1 \\ (14 \%) \end{gathered}$ | $\begin{gathered} 1 \\ (14 \%) \\ 3 \end{gathered}$ | 7 |
| Cluster 7 <br> Actual number and (percent) Expected number | $\begin{gathered} 1 \\ (10 \%) \\ 1 \end{gathered}$ | $\begin{gathered} 6 \\ (60 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 3 \\ (30 \%) \\ 5 \end{gathered}$ | 10 |
| Cluster 8 Actual number and (percent) Expected number | $\begin{gathered} 1 \\ (10 \%) \\ 1 \end{gathered}$ | $\begin{gathered} 1 \\ (10 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 0 \\ (0 \%) \\ 2 \end{gathered}$ | $\begin{gathered} 8 \\ (80 \%) \\ 5 \end{gathered}$ | 10 |

Secondary schools. The representation of four secondary schools in cluster 6 is considerably higher than the expected number ( 1 expected). From a different perspective, the largest numbers of secondary schools are in cluster 6, a cluster that
ranks high in consistency with reform, and cluster 1 , a cluster that ranks moderate in consistency with reform, with four secondary schools in each cluster.

Two-year colleges. The representation of six two-year colleges in cluster 7 is considerably higher than the expected number (2 expected). Cluster 7 ranks high in consistency with reform. It is interesting to note that the largest number of two-year colleges (8) are in cluster 2, a cluster that ranks in the moderate range.

Doctoral and research universities. Cluster 2 has the highest percentage of doctoral and research universities (29\%). The representation of nine doctoral and research universities in cluster 2 is considerably higher than the expected number ( 5 expected). It should be noted that the percentage of other colleges and universities (39\%) in cluster 2 is higher than the percentage of doctoral and research universities although the representation of other colleges and universities is lower than would be expected. The largest number of doctoral and research universities are in cluster 2.

Other colleges and universities. Cluster 8 has the highest percentage of other colleges and universities (80\%). The representation of eight other colleges and universities in cluster 8 is higher than the expected number ( 5 expected). The largest number, 12, of other colleges and universities are in cluster 2. Cluster 8 ranks in the low range in consistency with reform.

There are unexpectedly large numbers of particular types of academic institutions in some clusters and some clusters have unexpectedly high representation of some institutions types. However, all institution types are represented in all clusters except for the absence of secondary schools in cluster 3 and cluster 5 . Although there are differences between percentages of institution types in each cluster, distinguishable overall patterns are not immediately evident. Possible relationships between the consistency ranking of the clusters and the types of institutions in the clusters are discussed in the interpretation subsection near the end of the current section.


#### Abstract

Average Student Enrollments Institutions in the current study vary greatly in their student enrollment, with an average enrollment of approximately 6,750 students over all. Figure 9 illustrates the second demographic validating scale, the average student enrollment at institutions in each cluster. It appears that average enrollment at the institutions in each cluster varies considerably. The academic institutions with the highest average reported enrollment are in cluster 4, a cluster with a moderate-consistency-withreform ranking. It is noteworthy that clusters 6 and 7, the clusters with the highest consistency-with-reform ranking, and clusters 3 and 8 , the clusters with the lowest consistency-with-reform ranking, are the clusters with the four lowest reported average school enrollments. A possible reason for this apparent relationship is addressed in the limitation subsection of the current section.




Figure 9. Reported average student enrollment at institutions in each cluster. Clusters are ranked left to right from least consistent to most consistent with goals for reform in the teaching of calculus.

## Public and Private Academic Institutions

A third demographic validating scale is the percentage of public institutions in each cluster (see Figure 10). Based on the reported data, cluster 4, a cluster with moderate consistency ranking, has the highest percentage of public institutions. Clusters 6 and 7, the clusters ranking highest in consistency with goals for reformbased calculus instruction have the second and third highest percentages of public institutions. Cluster 3 , ranked low on consistency with reform, has a low-moderate percentage of public institutions, whereas cluster 8 , also ranked low, has the lowest percentage of public institutions. Based on the reported data, there is a possibility of a positive relationship between the percentage of public institutions in a cluster and the clusters' consistency ranking. The evidence is not strong.


Figure 10. Reported percentage of public and private institutions in each cluster.
Clusters are ranked left to right from least consistent to most consistent with goals for reform in the teaching of calculus.

## Financial Support

The vfinsup demographic validating scale measures the percentage of site liaisons in each cluster that report some financial support for implementing the use of CCH Curriculum Project Materials. The bar graph in Figure 11 illustrates the average value for vfinsup for institutions in each cluster.

Percentage of Site Liaisons Reporting Some
Financial Support to Implement CCH Materials


Figure 11. Percentages of site liaisons reporting some financial support to implement CCH Curriculum Project materials by cluster. Clusters are ranked left to right from least consistent to most consistent with goals for reform in the teaching of calculus.

Financial support refers to either financial support from sources outside of the institution or special support from within the institution. (See Appendix B, Initial Questionnaire item \#10.) Respondents to the surveys report many different types of financial support for the implementation of CCH Curriculum Project materials. These include funding to attend CCH Curriculum Project workshops, a reduced teaching load, support for purchasing technology, and implementation grants from the National Science Foundation.

Differences among the clusters regarding the receiving of financial support appear to exist. Site liaisons from a relatively low percentage of academic institutions in clusters 3 and 8, whose consistency-with-reform rankings are lowest, report some financial support for implementation of CCH Curriculum Project materials. Site
liaisons from a relatively high percentage of academic institutions in clusters 6 and 7. whose consistency-with-reform scores are highest, report some financial support. These and other possible relationships are discussed in the next subsection, Interpretation.

## Interpretation

The demographic validating scales describe some contextual features about the academic institutions in the clusters. The data indicate that there may be a possible relationship between institution type and reform in the teaching of calculus. In cluster 6, the cluster that scores highest in consistency with reform-based calculus instruction, $57 \%$ of the academic institutions are secondary schools, a percentage considerable higher than the secondary school representation in the current study (11\%). The National Council of Teachers of Mathematics published the Curriculum and Evaluation Standards for School Mathematics in 1986. This document has had some positive influence toward reform in the teaching of mathematics at the secondary level as secondary school teachers have incorporated the suggestions for new approaches in their teaching (Ferrini-Mundy \& Schram, in press). It seems reasonable to expect that teachers who are changing their teaching approach in school mathematics would make similar changes in their teaching of calculus.

Average cluster scores on vfinsup reveal the possibility of some relationship between receiving financial support for implementation of reform-based calculus materials and the consistency rankings. The two clusters with high consistency rankings are among the top four clusters in percentage of institutions for which site liaisons report some financial support. The two clusters with lowest consistency rankings are among the bottom four clusters in the percentage of institutions reporting financial support.

A question emerges about a possible relationship between scores on vinteraction and vfinsup. Many site liaisons reported some financial assistance to
attend workshops and conferences that addressed reform in calculus instruction. Conversations initiated at conferences and workshops may have continued with other faculty at the home institution and contributed to local reform efforts.

There are two possible limitations to the current study related to two demographic variables, vfinsup and venrollment. The next section addresses these possible limitations.

## Limitations to the Current Study Related to the Demographic Variables

The first limitation relates to the vfinsup CCH validating scale. Some ambiguous responses to the survey question about financial support made coding somewhat difficult and, perhaps, problematic. Comment data revealed that some site liaisons checked "No support" if the only support they received was Wiley Publisher's sponsoring of their trip to a CCH Curriculum Project workshop, while others checked "External support" for the same situation. In addition, the questionnaire did not ask for amounts of financial support. "External support" could mean their academic institution is part of a large implementation consortium receiving considerable funding or they may have only received minimal outside support to attend a workshop.

The second limitation concerns student enrollment and the number of Faculty Surveys received from institutions. The four clusters with the highest and lowest consistency rankings, clusters 6 and 7 and 3 and 8 respectively, are the four clusters with the reported lowest average institution enrollment. A possible moderating effect may have occurred in the current study. Larger institutions may have more sections of calculus using CCH Curriculum Project materials than smaller institutions. Institutions returning five Faculty Surveys may have multiple sections of calculus that use the CCH Curriculum Project materials. It is possible that the five instructors who completed the Faculty Survey, hold diverse viewpoints about reform. Some site liaisons from institutions with many CCH sections mentioned they asked instructors
with divergent viewpoints to complete the surveys. Averaging of the diverse responses would result in moderate scores on the clustering scales. If only one or two instructors with closely aligned viewpoints have used CCH Curriculum Project materials and completed the surveys, the one response or average of two responses (if the two responses take similar perspectives) could be more extreme. This could explain the possible moderating effect on averaged responses at large institutions with many CCH sections. The next paragraph quantitatively addresses this issue.

Table 12 lists the average number of Faculty Surveys received from academic institutions in each cluster. Further inspection of the data reveals that the lowest mean number of Faculty Surveys returned represent academic institutions in clusters 7 and 6, clusters ranked most consistent with reform, and clusters 3 and 8, clusters ranked least consistent with reform. A low number of Faculty Surveys returned generally indicates that few instructors at the institutions have taught calculus using CCH Curriculum Project materials. The more moderate ranking clusters, clusters 1 , 2,4 , and 5 returned higher average numbers of Faculty Surveys.

Table 13
The Average Number of Faculty Surveys Returned from Academic Institutions by

## Cluster

$\left.\begin{array}{|c|c|c|}\hline \text { Cluster } \\ \text { number }\end{array} \quad \begin{array}{c}\text { Rank of } \\ \text { cluster }\end{array} \quad \begin{array}{c}\text { Average } \\ \text { number of Faculty } \\ \text { Ser institution }\end{array}\right]$

The current section has demonstrated differences between the clusters on the demographic validating scales. Differences between clusters, evident on demographic validating scales, support the validation of the clusters. In this section addressing demographic validating scales, including institution type, it also seems appropriate to consider how different types of institutions participating in the current study score on the clustering and validating scales. The next subsection addresses this issue.

## Clustering and Validating Scales and Institution Types

Tables 14, 15, and 16 illustrate the average scores by institution type on the clustering scales. This section provides additional descriptive data for the current study.

Table 14
Mean and (Standard Error) of Responses by Institution Types on Clustering Scales

| Institution type | concepts | approach | Clustering <br> teaching | Scales <br> assess |  | access |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline \text { Secondary } \\ \text { schools } \\ \text { mean score } \\ \text { (s. e.) } \\ \mathrm{N}=13 \\ \hline \end{gathered}$ | $\begin{gathered} .53 \\ (.04) \end{gathered}$ | $\begin{gathered} .86 \\ (.04) \end{gathered}$ | $\begin{gathered} .33 \\ (.05) \end{gathered}$ | $\begin{gathered} .29 \\ (.05) \end{gathered}$ | $\begin{gathered} .70 \\ (.03) \end{gathered}$ | $\begin{gathered} .53 \\ (.07) \end{gathered}$ |
| $\begin{gathered} \text { Two-year } \\ \text { colleges } \\ \text { mean score } \\ (\mathrm{s} . \mathrm{e} .) \\ \mathrm{N}=30 \\ \hline \end{gathered}$ | $\begin{gathered} .64 \\ (.03) \end{gathered}$ | $\begin{gathered} .88 \\ (.02) \end{gathered}$ | $\begin{gathered} .24 \\ (.02) \end{gathered}$ | $\begin{gathered} .35 \\ (.03) \end{gathered}$ | $\begin{gathered} .74 \\ (.02) \end{gathered}$ | $\begin{gathered} .34 \\ (.02) \end{gathered}$ |
| $\begin{gathered} \hline \text { Doctoral and } \\ \text { research } \\ \text { universities } \\ \text { mean score } \\ (\mathbf{s} . \mathbf{e} .) \\ \mathbf{N}=19 \\ \hline \end{gathered}$ | $\begin{array}{r} .66 \\ (.03) \end{array}$ | $\begin{gathered} .89 \\ (.02) \end{gathered}$ | $\begin{gathered} .21 \\ (.03) \end{gathered}$ | $\stackrel{.25}{(.03)}$ | $\begin{gathered} .65 \\ (.03) \end{gathered}$ | $\begin{gathered} .31 \\ (.04) \end{gathered}$ |
| $\begin{gathered} \hline \text { Other colleges } \\ \text { and universities } \\ \text { mean score } \\ \text { (s. e.) } \\ \mathrm{N}=55 \\ \hline \end{gathered}$ | $\begin{gathered} .62 \\ (.02) \end{gathered}$ | $\begin{gathered} .84 \\ (.02) \end{gathered}$ | $\begin{gathered} .20 \\ (.02) \end{gathered}$ | $\begin{gathered} .25 \\ (.02) \end{gathered}$ | $\begin{gathered} .69 \\ (.02) \end{gathered}$ | $\begin{gathered} .31 \\ (.02) \end{gathered}$ |
| Mean for all institutions (s.e.) $\mathrm{N}=117$ | $\begin{gathered} .62 \\ (.01) \end{gathered}$ | $\begin{gathered} .86 \\ (.01) \end{gathered}$ | $\begin{gathered} .23 \\ (.01) \end{gathered}$ | $\begin{gathered} .28 \\ (.01) \end{gathered}$ | $\begin{gathered} .70 \\ (.01) \end{gathered}$ | $\begin{gathered} .34 \\ (.02) \end{gathered}$ |

When interpreting the mean scores for institution types, it is again important to note that the overall average scores on teaching (.25) and assessment (.29) are low and those on approach (.87) and technology (.70) are high. This reflects the results noted with the cluster solution. Based on the data in Table 14, it appears that the scores on concepts are relatively consistent among the various types of colleges and universities, with the secondary school mean score of .53 somewhat lower than the rest. The scores on approach are uniformly close to the mean. Secondary schools scores on teaching (.33) and access (.53) stand out as higher than the mean score, while the scores for the other institution types are closer to the mean. Two-year
colleges stand out as high on assessment (.35). On technology, two-year colleges score highest (.74), and doctoral and research universities score lowest (.65).

It appears that the highest and lowest mean scores by institution type differ by more than 10 only on concepts, teaching and access. The differences on the clustering scales do not appear to be as great by institution types as they are by clusters. For example, the differences between the highest and lowest mean scores are more than .30 for all clustering scales when computed for clusters. This observation would tend to strengthen the partition by clusters of academic institutions in the current study. It seems to be the case that implementation of reform in the teaching of calculus reflects contextual situations more complex than just institution type.

Table 15

## Mean Responses and (Standard Error) by Institution Types on Mathematics

## Department Validating Scales

| Institution Type | Mathematics-department validating scales |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | vconcept | vteach | vassess | vtech | vvalues | vinterest |
| Secondary schools mean score (s.e.) $\mathrm{N}=17$ | $\begin{gathered} .38 \\ (.07) \end{gathered}$ | $\begin{gathered} .30 \\ (.08) \end{gathered}$ | $\begin{gathered} .38 \\ (.14) \end{gathered}$ | $\begin{gathered} .73 \\ (.09) \end{gathered}$ | $\begin{gathered} .79 \\ (.08) \end{gathered}$ | $\begin{array}{r} .15 \\ (.07) \end{array}$ |
| Two-year colleges mean score $\begin{aligned} & \text { (s.e.) } \\ & \mathrm{N}=31 \end{aligned}$ | $\begin{gathered} .50 \\ (.05) \end{gathered}$ | $\begin{gathered} .37 \\ (.06) \end{gathered}$ | $\begin{array}{r} .40 \\ (.09) \end{array}$ | $\begin{gathered} .72 \\ (.08) \end{gathered}$ | $\begin{gathered} .72 \\ (.04) \end{gathered}$ | $\begin{gathered} .31 \\ (.04) \end{gathered}$ |
| $\begin{gathered} \hline \text { Doctoral and } \\ \text { research } \\ \text { universities } \\ \text { mean score } \\ \text { (s.e.) } \\ \mathrm{N}=13 \\ \hline \end{gathered}$ | $\begin{array}{r} .42 \\ (.06) \end{array}$ | $\begin{array}{r} .16 \\ (.04) \end{array}$ | $\begin{array}{r} .16 \\ (.09) \end{array}$ | $\begin{gathered} .45 \\ (.09) \end{gathered}$ | $\begin{array}{r} .42 \\ (.07) \end{array}$ | $\begin{gathered} .24 \\ (.03) \end{gathered}$ |
| ```Other colleges and universities mean score (s.e.) \(\mathrm{N}=12\)``` | $\begin{gathered} .48 \\ (.03) \end{gathered}$ | $\begin{gathered} .30 \\ (.04) \end{gathered}$ | $\begin{array}{r} .36 \\ (.07) \end{array}$ | $\begin{gathered} .63 \\ (.06) \end{gathered}$ | $\begin{array}{r} .66 \\ (.04) \end{array}$ | $\begin{array}{r} .36 \\ (.04) \end{array}$ |
| Mean for all institutions $\begin{gathered} \text { (s.e.) } \\ \mathrm{N}=117 \end{gathered}$ | $\begin{array}{r} .46 \\ (.02) \end{array}$ | $\begin{gathered} .30 \\ (.03) \end{gathered}$ | $\begin{gathered} .34 \\ (.04) \end{gathered}$ | $\begin{gathered} .63 \\ (.04) \end{gathered}$ | $\stackrel{.65}{(.03)}$ | $\begin{gathered} .30 \\ (.02) \end{gathered}$ |

When reading Table 15 , it is again important to note the overall low average scores on vteaching (.28), vassessment (.33), and vinterest (.27). According to the data in Table 15, secondary schools rank highest on vassessment (.38), vtechnology (.73), and vvalues (.79); two-year colleges rank highest on vconcepts (.50) and vteaching (.37); and other colleges and universities rank highest on vinterest (.36). Secondary schools rank lowest on vinterest (.15) and vconcepts (.38); and doctoral and research universities rank lowest on vteaching (.37), vtechnology (.45), and vvalues (.42).

Table 16
Mean and (Standard Error) of Responses by Institution Types on CCH Validating
Scales
$\left.\begin{array}{|c|cccc|}\hline \begin{array}{c}\text { Institution } \\ \text { Type }\end{array} & \text { vinteract } & \text { vreform } & \text { vstatus } & \text { vuse } \\ \hline \begin{array}{c}\text { Secondary } \\ \text { schools } \\ \text { mean score } \\ \text { (s.e.) }\end{array} & .26 & 1.00 & .81 & .86 \\ \mathrm{~N}=17\end{array}\right)$

According to Table 16, the greatest differences among institution types is the percentage of use of the CCH materials. Secondary schools average an $86 \%$ use in calculus classes, whereas doctoral and research universities average $42 \%$ use. All institution types score high on vreform (.95) and relatively high on vstatus (.77). Secondary schools score the lowest on vinteract (.26) and two-year colleges score the highest (.44). Most secondary schools in the current study have only one or two
sections of calculus. This provides CCH Curriculum Project instructors with little opportunity to discuss reform in the teaching of calculus locally.

Although the purpose of the current study is to identify clusters of academic institutions that exhibit similar patterns of implementation of CCH Curriculum Project materials, many faculty at institutions are interested in the perspectives of other instructors at similar institution types. The preceding tables and brief discussion provide some such information.

## Summary

The preceding sections have validated the eight clusters by demonstrating differences among the clusters on the mathematics department validating scales, the CCH validating scales, and the demographic validating scales. Differences on the mathematics department validating scales, and the CCH validating scales were demonstrated with tables listing the mean scores and rank of the clusters on the scales and box diagrams that represent the distribution of the clusters about the mean on each scale. Differences among clusters on the demographic validating scales were demonstrated with bar graphs.

The validating scales show the ranking of clusters to be somewhat consistent across the clustering and validating scales. Although this relationship between the rankings is not necessary for validation of the clusters, it is an interesting result that strengthens the external validation of the cluster solution and warrants further study.

## Summary

The current chapter has focused on the eight clusters identified through cluster analysis. Descriptions of the clusters based on the clustering scales introduced the clusters. Then the chapter turned to the validation process. To validate the clusters, differences among the clusters were demonstrated on scales relevant to the study but not used in the cluster analysis. The large number of items on the Initial

Questionnaire, the Site Liaison Survey, and the Faculty Survey allowed for six mathematics department validating scales, four CCH validating scales, and four demographic validating scales to be defined. Because the validating scales did not use any survey items that were used in the cluster analysis, the cluster solution of eight distinct clusters was validated through a process called external validation. External validation is perhaps the most powerful process for validating a cluster solution (Aldenderfer \& Blashfield, 1984). The current chapter also presented the mean scores on the clustering and validating scales by institution type.

The next chapter describes the eight clusters individually. The descriptions of the clusters include discussions about clusters' mean scores on the clustering scales and validating scales and comment data from the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey.

## CHAPTER V

# PROFILES OF REFORM IN THE TEACHING OF CALCULUS 

Chapter IV, Analysis and Interpretation, focused on differences among the eight clusters of academic institutions identified through cluster analysis. The clustering scales were developed and defined with attention to the goals of reform in calculus curriculum and instruction established from the Tulane Conference (Douglas, 1986) and to establish a measure for describing patterns of implementation of reform. Clustering scales can be thought of as dimensions of reform in the teaching of calculus. The eight clusters represent patterns of interpretation and reform in the teaching of calculus in the context of the Calculus Consortium based at Harvard (CCH) Curriculum Project materials.

After a brief description of the clusters themselves in Chapter IV, the clusters were compared and contrasted on the clustering scales and on the mathematics department, CCH, and demographic validating scales. The validating scales, based on groups of survey questions not used in the cluster analysis, rest on the same theoretical foundations as the clustering scales.

Here the clusters are considered one at a time, presenting a verbal picture or profile of each individual cluster. Each profile is based on the cluster's scores on the clustering and validating scales and includes descriptive comments from the Initial Questionnaire, the Site Liaison Survey, and the Faculty Survey (see Appendixes B, C, and D). The eight narrative descriptions are the profiles of interpretation and implementation of reform sought in the research question guiding the current study: What profiles of interpretation and implementation of reform in the teaching of
calculus emerge from data obtained from faculty members using CCH Curriculum Project materials?

The descriptions of the clusters in this chapter can serve to help the community better understand the current movement to reform the teaching of calculus. Mathematics departments initiating or continuing to engage in reform efforts can perhaps recognize themselves in a description of a cluster, thereby better understanding the complexities and issues they are experiencing. Efforts are made in the descriptions of the clusters to portray patterns of implementation rich enough to help the various members of the mathematics community understand their own and others' efforts and experiences.

## Profiles of Reform in the Teaching of Calculus

The descriptions of the eight profiles of reform in calculus curriculum and instruction follow similar organizational structures. The clusters are described first on the demographic validating scales, second on the clustering scales, and third on the mathematics department and CCH validating scales. It is important to remember that the clusters of academic institutions were identified through cluster analysis on the basis of similarities on the clustering scales. Comment data is used throughout to enrich the descriptions of the clusters.

Several items concerning the comment data are helpful to keep in mind. The Site Liaison Survey elicited few comments. This may be due to the fact that participants were asked to write comments as the final item on primarily numerical response questions. Participants responded much more frequently to the separate questions on the Faculty Survey that requested comment type responses. For the most part, the validating scales incorporate Site Liaison Survey items and the clustering scales incorporate Faculty Survey items. Therefore, in the cluster descriptions, there are many more comments relating to clustering scales than to
validating scales. It should also be kept in mind that some participants wrote more and longer comments than other participants. The clustering scales include up to 20 survey items addressing different aspects of the scales. Therefore, the number of comments related to a particular scale often exceeds the number of participants in a particular cluster.

Comment data gives voice to the participants in the cluster. Comments were selected that represent the views stated by many participants. Use of comment data in this way is subjective. Cluster analysis identified clusters of academic institutions whose faculty members are engaging daily in reform in the teaching of calculus. Some faculty members hold strong viewpoints about reform, others are facing the dilemmas and ambiguities with an open mind, seeking to make sense of student learning in a reform-based calculus setting. All are struggling with hard issues. The descriptions portray different implementation patterns through the perspectives of those situated in the different contexts. The descriptive profiles are intended to contribute to the reader's fuller understanding of that which is "going on" in the reform of calculus instruction.

There is another issue regarding the comment data. The comments from participants in each cluster reflect the complexities of the issues embedded in reform in the teaching of calculus and in the participants' situations. Many comments reveal the respondent's struggle with the issues. For example, one participant comments, " CCH students are strong in understanding of calculus but weak in manipulative skills in differentiation and integration." Another participant is encouraged by CCH Curriculum Project emphasis on the "understanding of the central concepts of calculus" and is also concerned that "formal logical development, the sense of mathematical argument and structure is missing." On an item asking participants their views about the use of technology in first-year calculus courses, a participant
comments, "Technology can be a great help or a great distracter if students get 'confused'."

Similarly, different participants from the same institution sometimes hold widely varying views on issues. A participant from one academic institution reports being encouraged by the "emphasis on intuition, using the calculator as a tool, and relating mathematics to real-world problems." Another from the same institution is concerned that "more emphasis on abstraction and generalization should be maintained." Because the participants themselves are struggling with complex issues, the comments reveal the ambiguities.

The initial numerical process used in the cluster analysis was an averaging process. Quantitative responses from participants at each academic institution were averaged to give the institution a score on each of the clustering scales. The cluster analysis process then grouped the academic institutions according to similarities among scores on the various clustering scales. The cluster analysis process emphasized similarities among institutions based on averages within institutions.

The comment data, on the other hand, reveals similarities and differences of viewpoints within individuals and among the participants within each institution and each cluster. The intent in the current study is to portray the various ways CCH Curriculum Project materials are being implemented. The preceding chapter demonstrated differences among the clusters on the various clustering and validating scales. The current chapter presents a verbal picture of each of the clusters, respecting the diversity of viewpoints within the clusters. The descriptions highlight the predominant characteristics of the clusters as revealed by the numerical analysis and also give voice to the participants in each cluster through the comments. It is reasonable to expect the implementation of reform-based calculus materials to generate the diverse opinions held at the various institutions. For these reasons, the descriptions are not tidy, neatly packaged profiles. Instead, they convey the deep
thought and varying viewpoints of those engaged in the hard work of reform. Although each participant is in a cluster that exhibits a distinct pattern of implementation, comments help reveal the variation within that pattern.

On a different note, there are a large number of comments related to the access clustering scale. The comment data related to access communicate somewhat different information than the numerical data. The numerical items related to access in the Site Liaison Survey seek information about the percentage of students from specific population groups such as women and minorities. (See Appendix C for Site Liaison Survey items 11 and 12.) The numerical items related to access on the Faculty Survey seek information about student success rates and ask the respondent's to indicate their level of agreement with the statements: (a) mathematics department faculty should confer regularly with faculty from other disciplines and (b) first-year calculus should be accessible to a wide range of students. (See Appendix E for Faculty Survey items 38a, 43h, and 43i.)

The comment questions related to access appear primarily in the Faculty Survey and have a slightly different tone. They seek faculty opinions about the types of students who experience increased or decreased access to calculus through the use of CCH Curriculum Project materials; perceptions about the success and retention rates of CCH Curriculum Project students; and faculty opinions about student performance or attitudes in courses subsequent to and dependent upon calculus. (See Appendix E for Faculty Survey items $11,34,35$, and 39.) The large number of comments associated with access may result in part from having four distinct comment type survey items addressing this issue. Another possible explanation is that the participants have more to say on access than on other issues.

Faculty Survey items related to concepts and technology also received a large number of comments. The overall mean scores on concepts (.61) and technology (.71) are relatively high. The high mean scores may indicate that the participants are
thinking deeply about the issues surrounding concepts and technology or that these are especially salient issues in their day-to-day work. Many fewer participants commented on issues surrounding teaching and assessment. The overall mean scores for the clusters on teaching (.25) and assessment (.31) are the lowest mean scores on the clustering scales. Although participants tended to contribute the most comments to the Faculty Survey item "What do you find encouraging about the direction of calculus reform and what are your concerns?" relatively few of these comments pertain to teaching practices or assessment methods. The low scores may indicate the participants are less accustomed to talking about their day to day teaching practices.

Cluster 2, the largest cluster, is described first because it represents a typical implementation of reform. Cluster 2 ranks moderate in reform in the teaching of calculus, with a mean score across all clustering scales of .49. The overall mean score of all clusters across all clustering scales is .51 . Cluster 2 is also unique in that it scores relatively close to the mean score on each individual clustering scale. Readers interested in making comparisons between the descriptions of the clusters, may find it helpful to refer to the relatively typical implementation demonstrated by cluster 2. Some readers may be interested in speculating about the cluster in which they might fit. The final paragraph in each description assists readers in this hypothetical endeavor.

The description of cluster 2 is followed by descriptions of clusters in decreasing order of consistency with reform in the teaching of calculus. That is, cluster 2, cluster I, cluster 3, and cluster 8 are described in that order. Next the clusters ranking more consistent with reform than cluster 2 are described in increasing order of consistency with reform (in the following order: cluster 4, cluster 5, cluster 7, and cluster 6). The reader will most likely note the tone of the comments received from participants in clusters ranked higher than cluster 2 is more enthusiastic about
reform than the tone of those ranked lower than cluster 2. We turn now to a description of cluster 2.

## Cluster 2

Cluster 2 is characterized as "middle of the road." The characterization reflects cluster 2's mean score on each of the clustering scales. Cluster 2 is the largest cluster with 31 academic institutions, including two secondary schools, eight twoyear colleges, nine doctoral and research universities, and 12 other colleges and universities. Cluster 2 is the cluster with the highest number (12) and percentage (29\%) of doctoral and research universities as compared with the "expected number" ( 5 expected) and percentage ( $16 \%$ ) of doctoral and research universities. [Note: The expected number of an institution type is based on the percentage of each institution type in the entire sample. The current study includes 117 academic institutions, consisting of 13 secondary schools (11\%), 30 two-year colleges ( $26 \%$ ), 19 doctoral and research universities ( $16 \%$ ), and 55 other colleges and universities ( $47 \%$ ). The expected number of doctoral and research universities in cluster 2 is computed by multiplying the percentage of doctoral and research universities in the sample by the number of academic institutions in the cluster ( $.16 \times 31 \approx 5$ ).] The percentage of other colleges and universities (39\%) in cluster 2 is greater than the percentage of doctoral and research universities ( $29 \%$ ).

Fifty-five percent ( 17 out of 31 ) of the institutions in cluster 2 are public institutions. This is close to the $57 \%$ average for all clusters. With an average enrollment of 8,930 students, cluster 2 has the second highest enrollment. The overall average enrollment is 6,750 . Thirty-nine percent of the site liaisons in cluster 2 report some financial support for implementing CCH Curriculum Project materials (similar to the average for all institutions which is $38 \%$ ).

Overall, in terms of consistency with the goals for reform, cluster 2 scores in the moderate range on the clustering scales and moderate on the mathematics department and CCH validating scales. Table 17 lists the relative ranking of cluster 2 on the three sets of scales.

Table 17

## Rank of Cluster 2 on Clustering and Validating Scales

| Scales | High <br> consistency <br> ranking <br> (highest two) | Moderate consistency ranking <br> (middle four) | Low <br> consistency <br> ranking <br> (lowest two) |
| :---: | :---: | :--- | :--- |
| Clustering scales |  | concepts (.70), <br> approach (.91), <br> teaching (.21), access (.31), <br> assessment (.19), <br> technology (.67) |  |
| Mathematics <br> department <br> validating scales | vinterest (.36) | vconcepts(.48), vvalues (.65) <br> vteaching (.27), <br> vassessment (.26), <br> vtechnology (.66) |  |
| CCH validating <br> scales | vreform (1.00) | vinteraction (.29), <br> vstatus (.75) | vuse-CCH (.54) |

Figure 12 represents the profile of cluster 2 over all clustering scales. The solid line denotes the scores of cluster 2 and the dashed line denotes the average scores for all clusters.


Figure 12. Profile of cluster 2 on clustering scales.

Cluster 2 reveals a somewhat typical implementation of CCH Curriculum Project materials. The large number of academic institutions in cluster 2 resulted in a large number of comments. The comments reflect the complex set of issues that mathematics faculty face as they engage with reform in the teaching of calculus. Many participants express support for an emphasis on students' conceptual understanding of the central ideas of calculus (concepts--70). As one participant states, "[The] students are much better at explaining why. They are much better at trying something rather than waiting around until someone tells them how to do the problem." Another notes, "Students have a better understanding of the derivative, the integral, and functions. Students are better able to verbalize their reasoning." Some comments from cluster 2 reflect participants' concern for "the reduction in the analytical skills of students," and the "lack of theorems and proof structures," or as
one participant states, "The loss of seeing the beauty in proofs flowing simply from prior definitions and results."

Although the numeric results reveal that participants are highly supportive of the graphic, numeric, and algebraic approach to calculus topics and the emphasis on real world applications (approach--.91) few respondents commented on this issue. Some respondents indicate they are encouraged by the "open ended questions related to various applications of mathematics."

Although in the average range compared to the overall mean, cluster 2's scores on teaching (.21) and assessment (.19) are low. Although few comments address teaching and assessment, some comments mention that faculty members are giving increased attention to their teaching: "People are talking about [calculus] enthusiastically. Old ideas are being re-examined. . . The question of how students learn best is being addressed." Another participant states, "Many are actively thinking about and talking to each other about how to teach mathematics and what to teach." As these comments illustrate, the movement to reform calculus instruction has generated discussion about calculus and the teaching of calculus.

Cluster 2 scores in the average range on technology (.67). Most of cluster 2's participants' comments support the use of technology in the teaching of calculus to increase students' understanding of calculus concepts. One respondent states, "Technology should be used in ways to enhance and support the course material. . . Technology should never be used just to 'use technology'."

Cluster 2's numerical score on access (.31) is below average, indicating little diversity in the student populations. In their comments, the participants describe many groups of students who they feel benefit from increased access to calculus through the use of CCH Curriculum Project materials. These students include, "students who were poor in algebra but have good mathematical intuition," "liberal arts students who are somewhat alienated from rigorous technical discussions,"
"students with good verbal skills," and "students in soft sciences." However, some comments are consistent with the following participant's concern. "Students are not as well prepared for physics or engineering classes which require students to symbolically develop formulas. Students are not well prepared to take upper division conceptual classes such as abstract algebra and advanced calculus."

Figure 13 represents the profiles of cluster 2 on the mathematics department and CCH validating scales. The dotted line indicates the mean scores for all clusters on the validating scales. Cluster 2 scores in the moderate range on the majority of the validating scales. It is important to remember, in this and other descriptions of clusters, that statements about mathematics department faculty members represent the views of the site-liaisons. One site liaison from each institution completed the Site Liaison Survey, the survey from which the data for the mathematics department validating scales was obtained.


Figure 13. Profiles of cluster 2 on mathematics department and CCH validating scales.

Cluster 2's score on vvalues (.65), the same as the overall mean score on vvalues (.65), indicates that the institutions and mathematics departments in cluster 2 place average value on teaching. However, several participants note strong support for teaching at their institutions. One participant from cluster 2 states that "undergraduate teaching is a high priority at the institution and the department." Another respondent states that "faculty teaching is the sole basis for evaluating faculty. 'Teaching' includes activities outside the classroom but directly related to teaching, such as the development of material for class use."

Cluster 2's score on technology (.67) and score on vtechnology (.66) differ by only .01 , indicating that the CCH instructors' use of technology may be similar to that of the mathematics department faculty. The majority of CCH Curriculum Project
instructors are full time and tenured (vstatus--.75). Approximately $54 \%$ of the calculus students in cluster 2 use CCH Curriculum Project materials, a relatively low percentage compared to the overall average of $63 \%$. Overall, the participants from cluster 2 are highly supportive of reform (vreform-1.00).

Several site liaisons comment on informal discussions about reform issues among department members. The climate in some mathematics departments engaging in reform efforts emerges as an issue in cluster 2. One participant states, "[I am concerned about the] reaction from a conservative element and the bloodletting that is occurring."

It seems reasonable to assume that some readers of the foregoing and subsequent descriptions of each cluster are, on some level, trying to identify which cluster best describes their own institution. The readers who recognize themselves in cluster 2 may teach at a doctoral or research university. The mathematics department faculty, for the most part, use traditional teaching approaches and are open to the use of technology in the teaching of mathematics. The faculty meets somewhat infrequently to discuss reform and pedagogy. Overall, the academic institutions that are in cluster 2 can be classified as "middle of the road."

## Cluster 1

Cluster I can be characterized as "diverse students and some reform," reflecting cluster l's relatively high score on access (.47) and its relatively low score on four of the other clustering scales.

Cluster 1 contains 17 academic institutions: four secondary schools, two twoyear colleges, three doctoral and research universities, and eight other colleges and universities. The distribution of academic institutions is similar to that which could be expected in a cluster with 17 academic institutions.

Eight of the $17(47 \%)$ institutions are public, the second lowest percentage of public institutions. The average enrollment is 7,650 students, an enrollment in the
moderate range. Site liaisons from four of the 17 institutions in cluster I (24\%) report some financial support to implement CCH Curriculum Project materials. Twenty four percent is a low percentage compared to all institutions. Based on the preceding data cluster 1 is composed primarily of moderate sized institutions of all types (half of which are public) that have not received significant support to implement CCH Curriculum Project materials.

Overall, in terms of consistency with reform, cluster 1 ranks in the lowmoderate range on the clustering scales, in the low range on the mathematics department validating scales, and in the moderate range on the CCH validating scales. Table 18 lists cluster l's relative rank on each of the clustering and validating scales.

Table 18
Rank of Cluster 1 on Clustering and Validating Scales

| Scales | High <br> consistency <br> ranking <br> highest two) | Moderate consistency <br> ranking <br> (middle four) | Low consistency <br> ranking <br> (lowest two) |
| :---: | :--- | :--- | :--- |
| Clustering <br> scales | access (.47) | approach (.88) | concepts (.50), <br> teaching (.16), <br> assessment (.16), <br> technology (.66) |
| Mathematics <br> department <br> validating <br> scales |  | vconcepts (.46), <br> vtechnology (.65), <br> vvalues (.64), vinterest (.22) | vteaching (.21), <br> vassessment (.18) |
| CCH <br> validating <br> scales |  | vreform (.94), vstatus (.76), <br> vuse-CCH (.65) | vinteraction (.28) |

Figure 14 represents the profile of cluster 1 on the clustering scales. The profile of cluster I graphically reveals cluster l's relatively high score on access (.47) and low scores on concepts (.50), teaching (.16), assessment (.16), and technology (.66).


Figure 14. Profile of cluster 1 on clustering scales

Cluster 1 is a somewhat unique cluster in that it scores above the average range only on access (.47). Access is defined by the accessibility of calculus to a wide range of students, including women, nontraditional students, students in disciplines that require understanding of calculus concepts, and ethnic groups traditionally underrepresented in calculus and by the retention and success of these students in calculus and subsequent courses. Cluster 1 respondents are generally in agreement about students who they feel experience greater access to calculus through the use of CCH Curriculum Project materials. These students include those who are "bound for programs in business, psychology, biological science, pre-vet, and premed." Other respondents mention "students with weaker backgrounds who can rely on the calculator," "students who need to relate to mathematics in the real world in
order to stay interested," and "students with average or below average symbolic manipulation skills and students who are conceptual or visual learners."

Comments regarding CCH Curriculum Project students' success in courses subsequent to calculus are mixed in tone. Many participants state that students "are better able to analyze problems" or "understand physics problems better." Other participants report that CCH students have difficulty in other courses because "they have trouble differentiating and integrating functions" and have "trouble with proofs."

Cluster l's high score on access (.47) would also indicate the average academic institution in cluster 1 has a diverse student body. The students' diversity is described indirectly in comments regarding the types of students using CCH materials. Some respondents indicate that students whose "verbal skills are weak" and those for whom "English is a second language may have difficulty with the increased emphasis on reading the textbook and writing explanations of problem solutions."

Cluster 1 scores below average on teaching (.16), assessment (.16), and concepts (.50). In spite of the low scores on teaching and assessment, several participants who are trying new ideas and activities in their classes provide comments. These participants note "I use group work in class and do more numerical examples with the aid of technology;" and "I attempt to find and incorporate 'lab' or 'hands on' activities for class and plan to use more collaborative learning activities." Another participant speaks about asking students to write essays on open-ended topics such as, "What is a derivative?"

Although cluster l's score on technology (.66) is relatively low compared with the overall mean score of .77 , the score could be considered in the moderate range in the absolute sense. The majority of the comments addressing technology take a positive position towards calculus students' use of technology. The following
comment is representative of many. "The use [of technology] is enthusiastically welcomed by students and seems to stimulate their interest in the course."

Figure 15 represents cluster l's profiles on the mathematics department validating scales, and the CCH validating scales. Cluster 1 ranks in the average range on seven of the ten mathematics department and CCH validating scales, vconcepts (.46), vtechnology (.65), vvalues (.64), vinterest (.22), vreform (.94), vstatus (.76) and vuse-CCH (.65) and scores below average on vteaching (.21), vassessment (.18), and vinteraction (.28).


Figure 15. Profiles of cluster 1 on mathematics department and CCH validating scales

It is interesting to note that cluster l's score on vtechnology (.65) is approximately equivalent to its score on technology (.66) and score on concepts (.50) is similar to its score on vconcepts (.46). This may indicate that mathematics
department faculty members in cluster 1 and CCH Curriculum Project instructors have similar views about the use of technology and the emphasis on conceptual understanding over procedural skills. According to the relatively low score on vinterest (.22), the mathematics department appears to devote little time to discussion of pedagogy and reform. The comments from participants in cluster I indicate that few, if any, of the participants in cluster 1 are involved in cooperative efforts with other institutions that are using CCH materials or attend many conferences at which reform in the teaching of calculus might be discussed. Is it possible that participants in cluster 1 are using the CCH Curriculum Project materials in what some would call "traditional" ways? One participant comments, "My preparation for a CCH course is about the same as for any course I teach."

Readers wondering if their situation is similar to cluster 1 might consider the relatively high percentage of private institutions in cluster 1 (53\%), the wide diversity of institution types, the moderate average enrollment, and the low amount of financial support for implementing CCH Curriculum Project materials. The responses from cluster 1 indicate that the academic institutions in cluster 1 enroll a more diverse student body than the average institutions participating in the study. The mathematics departments seem to be relatively traditional in their teaching and moderate in their use of technology. The data also seems to indicate that the instructors at institutions in cluster 1 do not interact as much with one another and attend as many outside conferences and workshops as do some instructors at other participating institutions. The characterization "diverse students and some reform" seems to describe cluster 1.

## Cluster 3

Cluster 3's overall mean score across all clustering scales is second lowest (.39). The characterization, "small steps toward reform," is based on cluster 3's low overall mean score and cluster 3's lowest or second lowest score on four of the six clustering scales. Cluster 3's 13 academic institutions include no secondary schools,
three two-year colleges, one doctoral and research university, and nine other colleges and universities. The representation of other colleges and universities is higher than the expected representation (6), whereas the representations of secondary schools and doctoral and research universities are lower than expected ( 1 and 2 respectively). The average enrollment of 7,038 students at institutions in cluster 3 is in the average range. Seven or $54 \%$ of the academic institutions in cluster 3 are public, an average percent. Thirty-one percent of the site liaisons from academic institutions in cluster 3 report financial support to implement CCH Curriculum Project materials, a lowaverage percentage. The average percentage is $38 \%$. Based on the above data, it would seem that a typical institution in cluster 3 would be a public college or university of moderate size that has received a small amount of funding to implement CCH Curriculum Project materials.

Overall, cluster 3 ranks low on the clustering scales and lowest on the mathematics department and CCH validating scales. Table 19 lists the relative ranking cluster 3 on the three sets of scales.

Table 19

## Rank of Cluster 3 on Clustering and Validating Scales

| Scales | High <br> consistency <br> ranking <br> (highest two) | Moderate <br> consistency <br> ranking <br> (middle four) | Low consistency ranking <br> (lowest two) |
| :---: | :--- | :---: | :--- |
| Clustering scales | assessment (.22), <br> concepts (.55) | teaching (.12), access <br> $(.21)$, technology (.47), <br> approach (.75), |  |
| Mathematics <br> department <br> validating scales <br> (. |  | vconcept (.43) | vteaching (.09), <br> vassessment (.15), <br> vtechnology (.31), <br> vvalues (.43), <br> vinteres (.21) |
| CCH validating <br> scales |  | vinteraction (.31), <br> vreform (.92) | vstatus (.69) <br> vuse-CCH (.60) |

Figure 16 represents the profiles of cluster 3 over the clustering scales. The diagram evidences cluster 3 's relatively low scores on the various scales.


Figure 16. Profile of cluster 3 on clustering scales.

Participants in cluster 3 made relatively few comments. Cluster 3 scores lower than average on approach (.75) and technology (.47). However, both of these scores are in the moderate range in the absolute sense, reflecting the generally high scores on those scales for all clusters. The comments associated with approach generally support the use of applications to introduce topics and the use of numeric and graphical representations. "I introduce a lot of real life applications" and "[I am encouraged by] the use of technology, numerical 'rough' approximations, better emphasis on applications, good use of graphical understanding, and less emphasis on algebraic skills." Most participant comments related to technology are supportive of its use in calculus, but also indicate the respondent is thinking about the issues
involved. As one respondent states. "I wouldn't do without graphing calculators. I'm still uncertain about computer algebra systems." Another notes, "I like it [technology]. I wish I could use it in class instead of in separate labs. But it's hurting even elementary manipulation skills."

Cluster 3 scores in the average range on concepts (.55). In a comment related to concepts, one participant notes, "I have developed a better understanding of how much (and how little) students learn and have much more appreciation for the needs to revisit concepts." Comments from all clusters note that the participants are paying increased attention to issues surrounding student learning. For example, "Some students are developing a genuine understanding of calculus concepts."

Cluster 3's score on teaching (.12) is well below average and on assessment (.22) is average. A few participants comment about changed teaching practices and assessment methods, mentioning the use of "group take-home projects." Another states, "I have had student teams work on projects, write a report, and present to the class." Some comments associated with teaching relate to the use of student labs that incorporate technology. "[I] integrate 'new' material with lab study using computers." Many of cluster 3's comments that relate to access (.21) compare the success and retention rates of CCH Curriculum Project students and students in standard calculus sections and note they "seem to be the same."

Figure 17 represents the profiles of cluster 3 over the mathematics department and CCH validating scales. The diagram shows cluster 3's relatively low scores on the scales.


Figure 17. Profiles of cluster 3 on the mathematics department and CCH validating scales.

The mathematics departments at institutions in cluster 3 score below average on vteaching (.09), vassessment (.15), and vtechnology (.31). By way of speculation, it appears that the teaching practices of the CCH Curriculum Project instructors in cluster 3 are similar to the relatively traditional teaching practices of the instructors in the mathematics department on the whole. Several respondents from cluster 3 report that they are the only CCH Curriculum Project instructor at their institution and that they interact with others teaching reform-based calculus courses very infrequently. The average reported percentage of calculus students using CCH Curriculum Project materials in cluster 3 is approximately average ( $60 \%$ ).

Again it seems appropriate to speculate about readers who might recognize their situation in cluster 3. Readers from a public college or university of moderate
size that has received a small amount of funding to implement CCH Curriculum Project materials, might look further at cluster 3. Is there little use of technology in the mathematics department at their institution? Do the faculty use primarily traditional teaching practices and assessment methods? These are the characteristics of cluster 3, characterized as "small steps toward reform."

## Cluster 8

Cluster 8's overall mean score across all clustering scales is lowest (.38). The characterization, "technology with a little reform," is based on cluster 8's low overall mean score, below average scores on five of the six clustering scales and approximately average score on technology (.71).

Cluster 8, a relatively small cluster, contains ten academic institutions, including one secondary school, one two-year college, no doctoral and research universities, and eight other colleges and universities. The representation of other colleges and universities is higher than the expected number ( 5 expected). The average reported enrollment at institutions in cluster 8 is approximately 1,880 students, the lowest average enrollment. Three of the ten academic institutions (30\%) are public. Only one of the ten site liaisons at institutions in cluster 8, reports some financial support to implement CCH Curriculum Project materials. This is the lowest percentage reporting financial support for implementation. Based on the above data, it would seem that a typical institution in cluster 8 would be a small private college that has received little or no funding to implement CCH Curriculum Project materials.

Overall, cluster 8 scores in the low range on the clustering scales, in the moderate range on the mathematics department validating scales, and in the lowmoderate on the CCH validating scales. Table 20 lists the relative ranking on the three sets of scales.

Table 20

## Rank of Cluster 8 on Clustering and Validating Scales

| Scales | High consistency <br> ranking <br> (highest two) | Moderate <br> consistency <br> ranking <br> (middle four) | Low consistency <br> ranking <br> (lowest two) |
| :---: | :--- | :--- | :--- |
| Clustering scales | technology (.71) | concepts (.41), <br> teaching (.17), <br> assessment (.17), <br> approach (.17), <br> access (.25) |  |
| Mathematics <br> department <br> validating scales |  | vconcepts (.48), <br> vteaching (.34), <br> vassessment $(.34)$, <br> vtechnology (.70), <br> vinterest (.35) | vvalues (.59) |
| CCH validating <br> scales | vstatus (.93), <br> vuse-CCH (.78) |  | vinteraction (.18), <br> vreform (.95) |

Figure 18 represents the profile of cluster 8 over the clustering scales. The comparison of cluster 8's profile with an average profile demonstrates cluster 8's low ranking on all clustering scales except for technology.


Figure 18. Profile of cluster 8 on clustering scales

There are relatively few comments from participants in cluster 8. This may result from the low number of academic institutions in cluster 8 and small average institution size. Cluster 8 scores in the low range on teaching (.17) and assessment (.17). Only two participants comment on their teaching practices: "[I use] group work in labs," and "[ I use] data from magazines and newspaper articles to model real-life situations."

Cluster 8 scores in the moderate range on technology (.71) and most comments relate to the use of technology. One participant reports, "With a graphing utility and programs for roots, and Riemann Sums, we can do all that I wanted for the last 15 years. The portability of graphic calculators is wonderful." Another states, "It is a powerful tool [that] gives 'mental leverage'."

Cluster 8 scores in the low range on access (.25). Most comments related to access discuss student success in the CCH Curriculum Project course and subsequent mathematics and science courses. The comments are mixed in tone with some participants noting the success rate is "about the same," others noting "the success and retention rates are better," and a few noting "the success rate seems to be worse [for CCH students]."


Figure 19. Profiles of cluster 8 on mathematics department and CCH validating scales

Figure 19 represents the profiles of cluster 8 on the mathematics department and CCH validating scales. Cluster 8 's overall score in the moderate range on the mathematics department validating scales contrasts somewhat with its low scores on seven of the eight clustering scales. It is possible that cluster 8 participants use the CCH Curriculum Project textbook primarily because of its incorporation of
technology. The below average score on approach (.58) and relatively average score on technology (.71) might lend support to the possibility that other features of the CCH Curriculum Project textbook are compelling for this group. Cluster 8 scores highest on vstatus (.93), indicating that the large majority of the faculty teaching CCH courses are full-time or tenured. Cluster 8 also reports the highest average percentage use of CCH Curriculum Project materials in first-year calculus courses (78\%).

Again it seems interesting to speculate about readers who might recognize their institution as similar to those in cluster 8. They may represent small, private colleges or universities that receive little funding for implementing reform in the teaching of calculus. The mathematics department may be typical in its outlook towards reform. Perhaps some instructors are interested in using more technology in the teaching of calculus but are not particularly interested in other aspects of calculus reform. Speculations such as these lead to the characterization of cluster 8 as "technology with a little reform."

Of the four clusters just described (clusters 2, 1, 3, and 8), cluster 8 ranks least consistent with the goals for reform in the teaching of calculus. Cluster 2 , the first cluster described, ranks in the moderate range on all clustering scales and the other clusters were described in order of decreasing consistency with reform goals. The descriptions now turn to the four other clusters, each of which ranks above average in consistency with the goals for reform in the teaching of calculus. The next section describes cluster 4, a cluster that ranks in the moderate range, just above cluster 2 in consistency with reform. After the description of cluster 4, the remaining clusters (clusters 5, 7, and 6) are described in increasing order of consistency with goals for reform. We turn now to a description of cluster 4.


#### Abstract

Cluster 4 Cluster 4 is characterized as "techies," reflecting cluster 4's high score on technology and moderate score on the other clustering scales. Cluster 4 contains 12 academic institutions: one secondary school, five two-year colleges, two doctoral and research universities, and four other colleges and universities. The representation of two-year colleges is higher than would be expected in a cluster with twelve academic institutions ( 3 expected), whereas the representation of other colleges and universities is slightly lower (5 or 6 expected). The reported average enrollment at academic institutions in cluster 4 is approximately 9,050 students, the cluster with the highest average enrollment. Eleven of the 12 institutions in cluster 4 are public institutions ( $92 \%$ ). Cluster 4 has the highest percentage of public institutions. Seven site liaisons from institutions in cluster 4 (58\%) report some financial support to implement CCH Curriculum Project materials. Based on the preceding, a typical institution in cluster 4 would be a large public two-year college or other college or university that has financial support to implement CCH Curriculum Project materials.

Overall, cluster 4 ranks moderate on the clustering scales, moderate on the mathematics department validating scales, and moderate on the CCH validating scales. Table 21 lists the relative ranking of cluster 4 on the three sets of scales.


Table 21
Rank of Cluster 4 on Clustering and Validating Scales

| Scales | High consistency <br> ranking <br> (highest two) | Moderate <br> consistency <br> ranking <br> (middle four) | Low consistency <br> ranking (lowest two) |
| :---: | :---: | :--- | :--- |
| Clustering <br> scales | technology (.81) | concepts (.53), <br> teaching (.25), <br> assessment (.34), <br> approach (.88), <br> access (.32) |  |
| Mathematics <br> department <br> validating <br> scales | vtechnology (.71), <br> vinterest (.38) | vteaching (.29), <br> vassessment $(.42)$, <br> vvalues (.60) | vconcepts (.42) |
| CCH validating <br> scales |  | vinteraction (.36), <br> vreform (.92), <br> vstatus (.81), Vuse- <br> CCH (.63) |  |

Figure 20 represents the profile of cluster 4 over all clustering scales. The diagram illustrates cluster 4's above average score on technology and relatively average score on all other clustering scales.


Figure 20. Profile of Cluster 4 on the Clustering Scales

Cluster 4's high score on technology and average scores on the other clustering scales may indicate that technology defines reform for some of the academic institutions in cluster 4. Many respondents from cluster 4 comment enthusiastically about the use of technology: "It is here, so let's start using it. Why waste time doing derivatives of complicated expressions when we have Derive and other [computer programs];" "I used Maple in a lab for two hours per week;" "[Technology is] extremely helpful: wouldn't dream of not using it;" and "Essential--no sense teaching how to graph! We can now concentrate on what's important--the concepts and how to apply." Other participants are more cautious: "[Technology is] important after students learn mathematics concepts involved, and after students know how to solve a problem by hand."

Cluster 4's comments related to concepts (.53) are mixed. Many respondents are encouraged by the increased emphasis on conceptual understanding they note in reform-based calculus instruction. For example, "Calculus reform places more emphasis on understanding concepts, thus allowing students to solve problems they have not encountered before. It also seems to me that they enjoy learning the subject a lot more than in a standard calculus course." "More time [in reform-based calculus instruction] is available to focus on understanding the main ideas and developing a sense of intuition, or perhaps a 'feel' for the subject (as opposed to just a body of rules to be applied)." Another participant expresses concern that "We are being careless with our proofs. We are missing some of the beauty of mathematics by emphasizing its applications." Others ask, "Will we be sorry we de-emphasized algebraic manipulation?"

The comments concerning access indicate that participants feel that CCH Curriculum Project materials increase some students' access to calculus. Typical comments note increased access for "students strong in visualization but less strong in symbol manipulation;" "life science majors;" "students in economics and business;" "non-traditional older students;" and "the students who are inclined toward problem solving." Other participants express concern about "those who dislike technology;" "students with weaker language skills;" and "mathematics majors."

Although cluster 4 scores in the average range on teaching (.25) and assessment (.34), relatively few participants in cluster 4 comment on their use of alternative teaching practices and assessment methods. However, some participants mention group work and "performing experiments in labs, collecting data, and analyzing."

Figure 21 represents the profile of cluster 4 over the validating scales. Overall, cluster 4 scores in the moderate range on the mathematics department and CCH validating scales.


Figure 21. Profiles of cluster 4 on the mathematics department and CCH validating scales.

Data from the Site Liaison Survey indicates that mathematics department faculty make above average use of technology (vtechnology--.71). Cluster 4's above average score on technology and vtechnology indicate that CCH Curriculum Project faculty members may reflect the mathematics department's support for the use of technology. Cluster 4 also scores highest on the demographic validating scale, vfinsup, with $58 \%$ of the site-liaisons reporting some financial support for implementing CCH Curriculum Project materials. It seems reasonable to speculate that strong financial supports contributes to high use of technology.

Site liaisons report that an average of $62 \%$ of the calculus students in academic institutions in cluster 4 are in CCH Curriculum Project courses, a relatively
average percentage. Cluster 4 also scores above average on vinteract indicating that CCH instructors meet relatively frequently to discuss pedagogical issues and relatively many have attended CCH Curriculum Project workshops. Site liaisons' comments related to departmental perspectives on reform vary from "We are generally a pro-reform department" to "The department is split nearly evenly about the need for calculus reform."

Readers who might recognize their institution as similar to those in cluster 4 may represent a large public two-year college or other college or university that has financial support to implement CCH Curriculum Project materials. The use of technology in the mathematics department is considerably above average. Based on the high use of technology and moderate implementation of reform in the teaching of calculus, cluster 4 is characterized as "techies."

## Cluster 5

Cluster 5 is characterized as "technology for understanding," reflecting cluster 5's high score on concepts and technology. Cluster 5 contains 17 academic institutions: no secondary schools, four two-year colleges, three doctoral and research universities and ten other colleges and universities. The representation of secondary schools is less than would be expected in a cluster with 17 academic institutions (2 or 3 expected), whereas the number of other colleges and universities is greater than the expected number ( 8 expected). The reported average enrollment at academic institutions in cluster 5 is 8,450 students, placing it in the average range. Ten of the academic institutions are public and seven private, an average distribution of institution types. Eight of the site liaisons (47\%) report some financial support to implement CCH Curriculum Project materials. Based on the above data, a typical institution in cluster 5 would be a moderate-sized public college or university that may have received some financial support to implement CCH Curriculum Project materials.

Cluster 5 ranks overall in the moderate range on consistency with reformbased calculus instruction on the clustering scales and on the CCH validating scales. On the mathematics department validating scales, cluster 5 ranks in the high range.

Table 22 lists the ranking of cluster 5 on the three sets of scales.

Table 22
Rank of Cluster 5 on Clustering and Validating Scales

| Scales | High consistency <br> ranking <br> (highest two) | Moderate <br> consistency ranking <br> (middle four) | Low <br> consistency <br> ranking <br> (lowest two) |
| :---: | :--- | :--- | :--- |
| Clustering scales | concepts (.78), <br> technology (.80) | teaching (.25), <br> assessment $(.37)$, <br> approach (.91), <br> access (.29) |  |
| Mathematics <br> department <br> validating scales | vconcepts (.49), <br> vassessment (.59) | vteaching (.32), <br> vtechnology (.68), <br> vinterest (.28) |  |
| CCH validating <br> scales | vinteraction (.53), <br> vreform (1.00), <br> vuse-CCH $(.64)$ | vstatus (.72) |  |

Figure 22 represents the profile of cluster 5 over all clustering scales. Cluster 5 ranks high on concepts and technology and moderate on the other four clustering scales.


Figure 22. Profile of cluster 5 on clustering scales.

Cluster 5 is characterized by its high scores on technology (.80) and concepts (.78). Technology as a vehicle for developing students' understanding is reflected in many comments in cluster 5. A participant comments, "[Technology is] vital not just as a time saver. Its greatest role is in concept development." Others report, "It [technology] is necessary. It takes the tedium away from the calculations and graphing, allowing students to focus on the concepts and understanding rather than the mechanics;" and "[Technology is] extremely important--allows students to dig deep into concepts and really see what's going on." Others warn, "Use it where appropriate, but don't let it drive the course." Several participants suggest that technology be used to "confirm analytical work and extend the scope and interpretation of a problem."

Many comments in this cluster discuss students' positive attitudes and conceptual understanding. "Students know concepts, students are becoming better problem solvers, mathematics is fun and useful;" "At the end of the semester, my students can talk intelligently about calculus ideas;" "Many students seem to be genuinely 'turned on' by the readable text and the interesting, varied problems. A lot of fun has been restored to undergraduate mathematics." Many participants note CCH students' strong problem solving skills and mention some concern about students' algebra skills. The following comment is typical. "Their [the students'] algebraic skills are weak, but their willingness to tackle non-routine problems is stronger. They have a greater tolerance for multi-step problems whose methods of solutions are not clear."

Cluster 5 scores slightly below average on access (.29), indicating less than average diversity in the compositions of the student populations. Comments about students who experience greater or lesser access to calculus are generally consistent with comments from other clusters. One respondent notes, "Students with insight and imagination but with poor algebra skills [experience greater access to calculus]. They become enthusiastic when they see that mathematics has meaning." One comment mentions, "women, minorities, verbally-oriented" students experience greater access. Cluster 5 is one of the few clusters in which comments specifically mention women or minorities.

Cluster 5 scores in the average range on approach (.91), teaching (.25), and assessment (.37). Many respondents in cluster 5 comment on changes in their teaching practices. One participant notes, "With the introduction of CCH materials, we also added a two-hour lab each week so that students have more structured time to interact with me, the material, and each other-a lot of work but well worth it." Another participant notes, "[The CCH materials] led me away from the straight lecture method. Made me put emphasis on problem solving instead of theorem-proof
approach. I now look for ways to involve students instead of ways to polish my presentation." Another reports "much, much more time spent paying attention to students' words, their written reports of thinking processes." The respondents in cluster 5 seem to be considering the issues deeply. As one states, "My feelings come down strongly on both sides of many issues. I'll be glad when I start to feel like I once again know what I'm doing."

Figure 23 represents the profile of cluster 5 on the validating scales. Cluster 5 ranks in the high range on vconcepts, vassessment, vinteraction, and vreform and in the moderate range on the other validating scales.


Figure 23. Profiles of cluster 5 on mathematics department and CCH validating scales.

Overall, cluster 5 ranks second highest on the mathematics department validating scales, indicating that the mathematics department faculty in cluster 5 are
generally supportive of reform. Cluster 5's high scores on vconcepts (.49) and vassessment (.59) indicate that many mathematics department faculty support the emphasis on conceptual understanding over procedural skills and try alternative methods of student assessment. Some site liaisons in cluster 5 comment on discussions among mathematics department members about reform. Others mention funding to attend conferences, workshops, and meetings. Cluster 5 also scores high on vinteraction (.53), indicating that CCH Curriculum Project instructors interact regularly with others using reform-based materials. Site liaison comments indicate that some institutions are involved in regional implementation projects. An average of $64 \%$ of the calculus students in cluster 5 are reported to be in CCH Curriculum Project courses. Cluster 5 also scores high on vreform (1.00), indicating CCH instructors are highly supportive of reform in the teaching of calculus.

Readers who might recognize their institution as similar to those in cluster 5 may represent a moderate-sized two-year college or other college or university that has some financial support to implement CCH Curriculum Project materials. The mathematics department is open to new ideas and holds informal and formal discussions about reform and pedagogy. Faculty members support the use of technology to enhance student understanding. "Technology for understanding" characterizes academic institutions in cluster 5.

## Cluster 7

Cluster 7's characterization as "the teachers" is based on cluster 7's high scores on concepts, approach, teaching, and assessment. Cluster 7 ranks second high in consistency with reform in the teaching of calculus.

Cluster 7 is a relatively small cluster consisting of ten academic institutions: one secondary school, six two-year colleges, no doctoral and research universities, and three other colleges and universities. The representation of two year colleges in cluster 7 is considerably higher than the expected number ( 2 expected). Cluster 7 has
twice as many public institutions as private, placing it in the average range for the ratio of public to private institutions. The average enrollment at institutions in cluster 7 is approximately 6,900 students. Fifty percent of the site liaisons in cluster 7 report some financial support to implement CCH Curriculum materials, a relatively high percentage. Based on the above data, a typical institution in cluster 7 is a moderatesized, public two-year college with some financial support for the implementation of CCH Curriculum Project materials.

Table 23 lists the ranking of cluster 7 on the clustering scales, the mathematics department validating scales, and the CCH validating scales. Overall, cluster 7 ranks second highest on the clustering scales, highest on the mathematics department validating scales, and second highest on the CCH validating scales.

Table 23

## Rank of Cluster 7 on Clustering and Validating Scales

| Scale | High consistency <br> ranking <br> (highest two) | Moderate <br> consistency ranking <br> (middle four) | Low consistency <br> ranking <br> (lowest two) |
| :---: | :--- | :--- | :--- |
| clustering <br> scales | concepts (.72), <br> teaching (.37), <br> assessment (.63), <br> approach (.93) | technology (.75), <br> access (.36) |  |
| mathematics <br> department <br> validating <br> scales | vconcepts (.57), <br> vteaching (.61), <br> vassessment (.70), <br> vvalues (.86) | vtechnology (.65), <br> vinterest (.35) |  |
| CCH validating <br> scales | vreform (1.00), <br> vstatus (.82) | vinteraction (.50), <br> vuse-CCH (.63) |  |

Figure 24 represents the profile of cluster 7 on the clustering scales. The diagram illustrates cluster 7's relatively high scores on concepts (.72), approach (.93). teaching (.37), and assessment (.63).


Figure 24. Profile of cluster 7 on the clustering scales.

Cluster 7's high scores on the clustering scales are evident in the positive and enthusiastic tone of the comments. One participant reports, "I completely changed my approach. This was wonderful. It made me have the excitement and enthusiasm of a first-year teacher. The class has group projects, labs, discourse [among students], probing of students' ideas." Many comments describe the use of group work in class: "[I use] cooperative learning in many cases to allow for student conceptual development. I do much less lecturing;" "[I spend] more time thinking through alternative solution methods, more use of team activities during class to bring a variety of viewpoints to the solution of problems, more connection of calculus
interpretation of real world relationships;" and "[I spend more time] selecting good activities for students to do without a lecture longer than ten minutes."

Many participants from cluster 7 comment on their placement of equal emphasis on graphic, numeric, and algebraic approaches to topics and relate this approach to increased access to calculus for more students: "Students can solve problems using multiple approaches, thereby increasing their chances of solving new problems. Students can make connections to mathematical models--algebraic and numeric--which increase their understanding. Students can check or verify results themselves--empowering the student, not the back of the book;" and "Multiple perspectives for presenting material allow an entry point into the material for more students."

Participants from cluster 7 also comment on their use of alternative methods for student assessment: "Grades are based on a variety of things: in class tests, take home tests, group quizzes, and group and individual projects;" and "My students pick from a menu of . . . units of assessment. . . . They choose from exams, homework, project or paper, concept map, oral presentation, labs, corrections file, and 'make your own unit.'."

Cluster 7 scores in the average range on technology (.75). The participants comment on their use of technology, but the comments almost seem to consider the use of technology accepted and not necessary to discuss. For example, "Students have been using technology at lower level courses. It is only natural to continue and expand its usage in the calculus;" and "Technology needs to be assumed available--it should not become a teaching emphasis, but a tool to help understanding."

Many participants in cluster 7 note CCH students' positive attitude towards mathematics: "They [CCH Curriculum Project students] seem to be more involved in discussions and present their opinion more readily;" and "These students are quicker to 'pull out' their array of tools and to view problems from different perspectives.

Their attitudes are better." One participant's observation is consistent with many. "Attitude and excitement in learning is excellent, but skills are poor;"

Cluster 7's score on access (.36) indicates that CCH Curriculum Project courses in cluster 7 most likely have average diversity in their student populations. Many respondents in cluster 7 comment about students who they believe have greater access to calculus through a reform-based course. These participants suggest that "business and life science students and students with weak algebra skills have greater access." Others comment that, "Women have greater access [to calculus]. Women students, I find, like the writing/explaining. Also, men and women seem to be on equal ground--fewer know-it-all guys."

Respondents in cluster 7 also comment on their own struggles with the issues involved in reform. Some ask, "How much paper and pencil symbolic manipulation is needed?;" "Does it [the material] challenge the very bright math student who intuited this all before reform?;" and "Is it enough for mathematics majors?"


Figure 25. Profiles of cluster 7 on mathematics department and CCH validating scales.

Figure 25 represents the profile of cluster 7 on the validating scales. It is interesting to note that cluster 7 scores above average on vconcepts (.57) and concepts (.72), vteaching (.61) and teaching (.37), and vassessment (.70) and assessment (.63). Cluster 7 scores in the average range on vtechnology (.65) and technology (.75). The similarities between clustering scale scores and mathematics department validating scale scores may support the conjecture that implementation of reform-based calculus instruction may reflect departmental perspectives. Cluster 7 also scores highest on vvalues (.86), indicating that academic institutions in cluster 7 place high value on teaching. Institutions that value teaching may be more likely to support CCH instructors in their efforts to try alternative pedagogical approaches.

According to site liaisons' reports, an average of $67 \%$ of the calculus students in cluster 7 use CCH Curriculum Project materials. Cluster 7's high score on vstatus
(.82) indicates the CCH Curriculum Project faculty members in cluster 7 are generally full-time or tenured. This is especially interesting because of the higher than expected representation of two-year colleges in cluster 7. Table 15 in Chapter II, Analysis and Interpretation, reveals that, overall, two-year colleges participating in the current study have the lowest average score on vstatus (.73).

Readers who recognize their institution as similar to those in cluster 7 may represent a moderate-sized, public two-year college with some financial support to implement CCH Curriculum Project materials. The mathematics department and institution value teaching and support faculty members who try new ideas.

Technology use is accepted, but not overly emphasized. "Teachers" characterizes academic institutions in cluster 7 .

## Cluster 6

Cluster 6 is characterized as "teaching diverse students" because of its high scores on teaching (.46) and access (.70). The smallest cluster with only seven academic institutions, cluster 6 includes four secondary schools, one two-year college, one doctoral and research university, and one other college and university. The representation of secondary schools is considerably higher than the expected number ( 1 expected). Cluster 6 has the highest percentage of secondary schools of any cluster (40\%). Cluster 6 has five public and two private institutions, a relatively high percentage of public institutions (71\%). Academic institutions in cluster 6 have relatively low average student enrollment, 4,030. Forty-three percent of the site liaisons in cluster 6 report some financial support for implementing CCH Curriculum Project materials. Based on the preceding data, a typical institution in cluster 6 would be a public secondary school with some financial support for implementing CCH Curriculum Project materials.

Overall, cluster 6 ranks highest on the clustering scales and the CCH validating scales and moderate on the mathematics department validating scales. Table 24 lists the ranking of cluster 6 on the three sets of scales.

Table 24
Rank of Cluster 6 on Clustering and Validating scales

| Scales | High consistency <br> ranking <br> (highest two) | Moderate <br> consistency <br> ranking <br> (middle four) | Low consistency <br> ranking <br> (lowest two) |
| :---: | :--- | :--- | :--- |
| Clustering scales | teaching (.46), <br> assessment $(.42)$, <br> access (.70) | concepts (.67), <br> technology (.79), <br> approach (.88) |  |
| Mathematics <br> department <br> validating scales | vteaching (.45), <br> vtechnology (.71), <br> vvalues (.85) | vassessment (.43), <br> vinterest (.21) | vconcepts (.32) |
| CCH validating <br> scales | vinteraction (.76), <br> vuse-CCH $(.76)$, <br> vreform (1.00) | vstatus (.76) |  |

Figure 26 represents the profile of cluster 6 on the clustering scales. The profile illustrates cluster 6's high rank on teaching, assessment and access.


Figure 26. Profile of cluster 6 on clustering scales.

Cluster 6 scores high on access (.70) both in a relative sense and in an absolute sense. The overall mean score on access is .36 . The high score indicates that CCH Curriculum Project courses in cluster 6 have a diverse student population. A reader might conjecture that the high representation of secondary schools contributes to this score. This may be a contributing factor, but, overall, secondary schools score . 53 on access.

The many comments from participants in cluster 6 share an enthusiasm for reform in the teaching of calculus. The following are representative of comments concerning access. "This book [the CCH Curriculum Project textbook] is very accessible to majors from the humanities and sciences;" "Calculus is no longer being used to filter out students from further study of certain disciplines." Another participant notes, "[Student] understanding seems to be much better. Students who
used to flounder in the algebra now have access to this course;" "Women and poor algebra students experience greater access, but all students benefit;" "Students who rely on verbal communication (as opposed to symbolic) [have greater access to calculus]." Some participants express concern about students whose second language is English, "Foreign students have trouble with all the words, as do many Americans."

Also an access issue, many participants in cluster 6 contribute comments expressed by faculty members who have CCH Curriculum Project students in courses that rely on calculus. Representative comments include, "[CCH students in subsequent courses] are no longer intimidated by applications and problem solving;" and "Favorable. Students are willing to try a variety of techniques. [CCH students] are conceptually strong, but algebraically weak."

Although cluster 6's score on teaching (.46) and assessment (.42) are considerably higher than the overall mean scores, relatively few comments relate directly to teaching and assessment. One participant observes, "I think [the course] makes students ask me different kinds of questions than those of traditional calculus. And it makes the class more of a lab class than a passive traditional lecture." Another speaks of "more class time [used\} for student activity and group work." Others comment on their use of projects and student presentations.

Cluster 6's score on technology (.79) is in the average range. The comments indicate that the use of technology in cluster 6 is accepted and widespread. One respondent notes that the classroom has "a computer for each pair of students that is used regularly." Another comments that students use "calculators all the time and use Maple and other software also."

The profiles of cluster 6 on the mathematics department and CCH validating scales are represented in Figure 27. Cluster 6 scores in the average range on the mathematics department validating scales and highest on the CCH validating scales.


Figure 27. Profiles of cluster 6 on mathematics department and CCH validating scales.

Cluster 6's high scores on vteaching (.45), vtechnology (.71), vvalues (.85), vinteraction (.76) indicate that the mathematics department faculty in cluster 6 interact with each other regularly and value and attend to their teaching. Several of the site liaisons in cluster 6 comment that their sites are in collaborative projects with other academic institutions using CCH Curriculum Project materials. One participant comments that "Our weekly calculus meeting is popular and the single most important factor in our success." Another participant reports, "Instructors and other faculty meet one hour each week to discusses courses and their teaching." Comments suggest that interaction with others engaged in reform contributes to a successful implementation of reform-based materials.

CCH Curriculum Project faculty members are highly supportive of reform as evidenced by cluster 6's high score on vreform (1.00). An average of $75 \%$, a relatively high percentage, of the calculus students at institutions in cluster 6 use CCH Curriculum Project materials.

Readers who recognize their institution as similar to those in cluster 6 might represent a public secondary school with a diverse student body. Their institution is in a consortium with other academic institutions using CCH Curriculum Project materials. The mathematics department faculty members hold frequent discussions with each other and with others from other institutions. Enthusiasm for teaching and trying new ideas is characteristic of the mathematics faculty. The use of technology in the mathematics department is commonplace. "Teaching diverse students" characterizes academic institutions in cluster 7.

## Summary

This chapter has looked individually at the eight clusters identified through cluster analysis. The patterns of implementation of reform in the teaching of calculus differ considerably among the clusters. Some clusters emphasize the use of technology, others appear to use technology in a matter-of-fact manner and focus on the use of alternative teaching practices or assessment methods. Few participants comment directly about ethnic diversity, although many participants speak of diversity issues concerning nontraditional students, students with weak mathematics or verbal backgrounds, and students for whom English is a second language.

The comments reveal that many instructors using CCH Curriculum Project materials are not blindly accepting all of the tenets of reform in the teaching of calculus. They are struggling with the issues that comprise the dimensions of reform and question which aspects of reform best meet the needs of their students in their situations. The comments also seem to indicate that the instructors using the CCH

Curriculum Project materials are attending to how their students learn and interact with calculus curriculum materials. The comment data contributes to the understanding of reform in many ways. The comments reveal common as well as unique implementation features and topics of concern. After focusing on characteristics of individual clusters it seems appropriate to consider the common issues that emerge through the comments. The issues listed below each represent sets of comments that appear in every cluster.

It appears that instructors note and appreciate CCH Curriculum Project students' increased understanding of concepts central to calculus and students' willingness to tackle problems. CCH instructors see benefits to the use of graphic, numeric, and algebraic approaches to all topics and appreciate the emphasis on connections between calculus and real world situations. The majority of CCH Curriculum Project instructors use technology and appreciate its usefulness in increasing student understanding of topics. Some instructors are trying alternative classroom teaching practices such as cooperative group learning and student assessment methods such as student projects, or class presentations.

At the same time, CCH Curriculum Project instructors report that they are concerned about the complex issues surrounding reform. They note that students are leaving calculus with different skills and understandings than students in standard courses. Many wonder if their students have the symbol manipulation skills and understandings about formal definitions and proofs they might need for subsequent work in mathematics and science. Others are concerned about students' possible over-reliance on technology.

Reform in the teaching of calculus is not clearly defined. Different dimensions of reform are important, meaningful, and helpful to faculty members at the various institutions. The current study contributes to the community's understanding of reform in calculus instruction by providing in-depth descriptions
that portray, through the words and perspectives of those engaged with reform. the various ways reform in calculus instruction is being implemented. The next chapter reflects on the current study and considers its limitations and the opportunities for further study.

## CHAPTER VI

## SUMMARY, $\operatorname{INTERPRETATION,~AND~IMPLICATIONS~FOR~FUTURE~WORK~}$

Reform in calculus curriculum and instruction is now entering its second decade. Chapter II, Review of the Literature, described the origins of the reform movement. The mathematicians at the Tulane Conference in 1986 recognized problems in calculus courses and collaboratively tried to establish curricular and pedagogical goals for reform. They had hopes for acceptance and implementation of these goals but could not have foreseen the influence the goals would have on calculus courses nationwide.

Reform in undergraduate teaching and in school mathematics have also been "in the air" for the last decade. The directions of these reform efforts compare favorably with the goals for reform in the teaching of calculus. Research studies, scholarly and informal discussions in each domain, and an overlap of key people have most likely contributed to tendencies of the movements to inform one another. The current study recognizes the influences and seeks to increase the understanding of reform in the teaching of calculus through the perspectives of those teaching calculus. The next section provides a brief discussion of the current study.

The Current Study
The current study is situated within the Calculus Consortium Based at Harvard (CCH) Evaluation and Documentation Project (Ferrini-Mundy, 1994). This larger project uses the methods of research, from a qualitative perspective, (a) to determine how the goals of the CCH Curriculum Project (Gleason, 1988) are interpreted and implemented at academic institutions using project materials and (b) to examine and
describe the evolution of efforts to reform the teaching of calculus in the context of using the CCH Curriculum Project materials. The current study, although a documentation project and not an evaluation project, shares these goals.

The research question in the current study is somewhat qualitative in nature, seeking understandings and descriptions of patterns of interpretation and implementation of the goals for reform in the teaching and learning of calculus and of the goals of the CCH Curriculum Project. The research question asks: What profiles of interpretation and implementation of reform in the teaching of calculus emerge from data obtained from faculty members using CCH Curriculum Project materials?

Cluster analysis methodology was used to develop this understanding. Cluster analysis was chosen because it is structure seeking rather than structure imposing, descriptive and taxonomic in nature. A successful cluster analysis requires firm theoretical foundations. The goals for reform in the teaching of calculus and the literature about how mathematics students come to understand calculus concepts provided the necessary theoretical foundations. For the purposes of this research study, cluster analysis identified clusters or groups of academic institutions that are similar in their interpretation and implementation of reform-based calculus instruction on six clustering scales or dimensions of reform in the teaching of calculus. External validating scales were used to validate the clusters. The validating scales were defined by the larger contextual framework in which the participants engage in, interpret, and make sense of reform in the teaching of calculus.

The cluster analysis identified eight distinct clusters or profiles of similar implementation. Participants in the current study are mathematics department faculty at the 117 academic institutions that comprise the clusters. The portrayals of the clusters in Chapter V, Profiles of Reform in the Teaching of Calculus, describe how faculty at the participating academic institutions interpret and engage with reform in
the teaching of calculus. The descriptions are presented from the perspectives of the participants, using participant comments that relate to each of the clustering scales.

Implementation of reform in the teaching of calculus is complex. Each contextual situation is different; students at various institutions hold different goals, expectations, and backgrounds. The issues surrounding goals for reform and the complexity of local situations contributes to the many difficult decisions mathematics departments face as they implement reform. The descriptions of the clusters in Chapter V graphically illustrate the multi-dimensional aspect of reform. Although all those who engage in reform may struggle with similar issues, the relative importance of each issue and the interplay of the issues is unique in each situation. The descriptions of the clusters in Chapter V, Profiles of Reform in the Teaching of Calculus form the core of the results of the current study. Rather than summarize the descriptions of the patterns of reform described in the previous chapter, the current chapter considers commonalties between clusters, observations that seem significant, reflections on the processes of the current study, and implications for future research. The next section considers perspectives that are common to the clusters.

## Perspectives on Reform in the Teaching of Calculus

The emergence of comments expressing common perspectives, struggles, and dilemmas across all clusters was somewhat unexpected. The experiences and interpretation of reform by the various participants throughout the current study seemed continually to reveal differences among participants, institutions, and clusters. However, study of the comments from each cluster, revealed that participants from all clusters face common issues. Although the portrayals of unique clusters presented in Chapter V address the main research issue in the current study, the common perspectives that emerge in the comments across clusters also contribute to the understanding of calculus reform. The common perspectives include those for which
the instructors are encouraged by reform and those about which they feel concern. Many of the issues that emerge in the current study are subjects of disagreements among mathematicians who strongly support reform in the teaching of calculus and mathematicians who are less supportive. The strong language surrounding reform within the mathematics community has been mentioned previously in the current study and continues in 1996. At a time of change and experimentation it is reasonable to expect that people will hold near contradictory views. As noted previously, individual comments by the same participant often reveal the participant's struggle with the issues and dilemmas. We now turn to the common perspectives that emerge from the comment data. The common issues that appear in the comment data should serve as a stimulus for further investigation and research. It should be understood that this section is interpretive and based on the researcher's own understanding of reform.

The concepts clustering scale relates to several goals for reform in the teaching of calculus. According to the Tulane Conference goals, calculus should develop students' conceptual understanding of the major ideas of calculus, should cover fewer topics, should cover many fewer mathematical techniques, and should contain much less drill on routine procedures. The authors of the CCH Curriculum Project materials (Hughes Hallett \& Gleason, 1994) state that the course presents a mathematically-sound, informal rationalization, that students can understand, for many theorems and proofs.

Participants' from all clusters express an appreciation of CCH Curriculum Project students' increased understanding of concepts central to calculus. Many participants describe students' ability to "talk about calculus concepts such as the meanings of the derivative and the integral" and assert that students "have a much better understanding of the central concepts of calculus." At the same time, CCH instructors express concern that "students are not exposed to the concept of proof, or
the beauty of a formal proof." CCH instructors also ask, "To what degree is facility with symbol manipulation necessary for student success in future courses?"

The approach clustering scale incorporates Tulane Conference goals suggesting that calculus courses use an inductive approach to topics, develop concepts from common sense investigations and real world applications and provide greater depth numerically and geometrically. The CCH Curriculum Project materials state that all topics are presented from three perspectives; graphic, numeric, and algebraic. Participants in all clusters appreciate the benefits to student learning resulting from increased emphasis on graphic and numeric representations. Participants in each cluster also comment that the graphic, numeric, and algebraic approach to topics provides increased access to calculus for students who may be more graphically or numerically oriented than algebraic. Instructors appreciate the connections made between calculus and the world outside the mathematics classroom.

The Tulane Conference goals suggest that calculus course instructors should make calculus teaching more interactive by using alternative teaching practices. The goals also suggest that calculus course instructors should change their methods of testing students, using assessments that correspond to the goals of instruction. According to the comment data, instructors in all clusters are trying new methods of classroom teaching and student assessment, such as the use of cooperative groups, oral presentations, and small-group projects. Analysis of the quantitative data reveals that, overall, the participants are least consistent with reform on the teaching and assessment clustering scales. The quantitative results indicate that most participants generally use relatively traditional teaching practices and student assessment methods. Taken together, the comment data and quantitative data may indicate that the participants are trying new ideas and are moving towards incorporating those that work best in their own situations into their regular practice.

The technology clustering scale reflects the Tulane Conference goal suggesting that calculus instructors make use of the latest technology in their courses. Comments from participants in all clusters indicate appreciation for the benefits of using technology. In the participants' views, these benefits include an opportunity to focus on conceptual understanding and more complex problems, with technology handling the tedious numerical calculations. At the same time, participants express concern about students' possible over reliance on technology. They worry that some students may use the technology as a "black box" and solve problems without an understanding of the underlying concepts.

The final clustering scale, access, relates to Tulane Conference goals suggesting that calculus should serve as a gateway, for a wider range of students, to future study in science, mathematics, and engineering. More recent goals for reform in the teaching of mathematics suggest that calculus be accessible to students who traditionally have been underrepresented in calculus courses, including women, minority students, and nontraditional students (Solow, 1994). Comments from participants in all clusters support the view that CCH Curriculum Project courses are more accessible to returning adults, students in the life sciences, liberal arts, and economics, and students who have weak algebra backgrounds. Questions in comments arise about whether CCH Curriculum Project courses are less accessible to students with verbal difficulties or whose English language skills are weak. Few comments mention ethnic minorities or women directly. Mixed views in comments emerge about whether the CCH Curriculum Project course is the best calculus course for mathematics majors. Some participants from all clusters question whether a course that emphasizes informal definitions and justifications of theorems may discourage future mathematicians from continuing in mathematics programs. Other participants claim in comments that the CCH Curriculum Project course is an appropriate course for mathematics majors.

The common perspectives in the comments from individuals in the eight clusters relate directly to the goals for reform in the teaching of calculus established at the Tulane Conference. Considering that the perspectives relate to the widely accepted goals for reform as well as specific goals developed by the authors of the CCH Curriculum Project materials, there is reason to believe that these perspectives may be representative of reform-based calculus courses using materials other than CCH Curriculum Project materials. The perspectives identified in the current study may increase the understanding about reform, inform the dialogue surrounding the issues, and provide a stimulus for further research. The next section considers limitations of the current study.

## Limitations of the Current Study

The current study seeks to understand and describe patterns of implementation of reform-based calculus instruction in the context of the CCH Curriculum Project materials through the perspectives of instructors using the project materials. One of the weaknesses of cluster analysis is that with a different set of cases (or academic institutions as in the current study), the clusters identified may not have the same characteristics. However, this limitation does not necessarily detract from the current study. This study has attempted to provide the community with in-depth descriptions of how reform-based calculus curriculum materials are being interpreted and implemented. Although other academic institutions using CCH Curriculum Project materials may interpret and engage with reform in a manner similar to one of the described patterns, they may exhibit a pattern that combines features of more than one cluster or adds features to a particular cluster. Mathematics faculty from academic institutions can learn from the patterns that have been described and can contribute descriptions of their unique patterns of implementation.

The current study also demonstrates that reform in calculus curriculum and instruction looks different, and has different emphases, in different groups of institutions. Mathematics faculty engaged in reform are struggling with hard issues. For the most part, they are facing the dilemmas and ambiguities with an open mind to the possibilities reform presents. There is no "one way" to implement or look at calculus reform, nor is there a single interpretation of CCH Curriculum Project materials. Instructors and institutions place priority on goals that have meaning for them in their own situation. Within the various patterns or profiles of reform, different features become salient, reflecting the distinctive contextual features of the situation.

Similarly, it seems reasonable to speculate that the patterns of reform identified in the current study may represent patterns exhibited by users of other reform-based calculus materials. The clustering scales represent goals for reformbased calculus instruction that have been adopted by many calculus curriculum projects. The implementation patterns described in the current study may be useful as a base for descriptions of implementations of other materials.

A second limitation of the current study lies in the data collection method. Site liaisons were asked to provide responses to items on the Site Liaison survey that request information about the perspectives of the faculty in their mathematics department. For example, Question 35 on the Site Liaison Survey states, "How well do the following statements characterize the viewpoints of most faculty members in your department? Please base responses on your personal understanding. We recognize many site liaisons will have to make a qualitative judgment as to the most representative response." The site liaisons' opinions about the mathematics department perspectives are, at best, an indirect measure.

The Site Liaison Survey also invited comments as the final item on a primarily numerical response question. Few site liaisons chose to include comments under this
format. Based on the limited amount of comment data obtained from the Site Liaison Survey, changes were made on the Faculty Survey. On the Faculty Survey, separate items were used to invite the respondents to make comments For example, Question 20 on the Faculty Survey states, "In what ways did the CCH material influence changes in your preparation and (or) teaching of calculus?" Separate comment type items such as the one just quoted elicited the many comments received from the participants that responded to the Faculty Survey. These Faculty Survey comments were used in the cluster descriptions. Descriptions of mathematics department perspectives, however, may be limited because of the low number of comments on the Site Liaison Survey.

The current study revealed a possible relationship between mathematics department perspectives and the implementation of reform-based calculus instruction. It seems reasonable to speculate about whether the overall degree to which calculus instructors engage with reform ideas and activities is indicative of the departments' perspectives on reform. More comment data from the Site Liaison Survey could have contributed toward an understanding of this possible relationship. Further work to investigate this possible relationship would contribute to the community's understanding of reform.

Another limitation concerns the uneven number of Faculty Surveys returned from academic institutions. At many academic institutions, fewer than five instructors have taught calculus using CCH Curriculum Project materials. At others, although five or more instructors have taught using the materials, fewer than five Faculty Surveys were returned. One ramification of this situation is that the values of variables used in the cluster analysis may not accurately reflect the viewpoints of the instructors at the academic institution. Another limitation lies in a possible moderating effect, discussed in Chapter IV, Analysis and Results. Briefly, if five instructors with diverse opinions return surveys, the method of computing scores,
averaging, might tend to "average out" the diverse viewpoints as compared with the scores of institutions with only one or a few respondents. Future studies could explore the effects of various viewpoints within a department on the patterns of implementation of reform.

Many of the limitations of the current study suggest areas for future research that might increase the community's understanding of reform in the teaching of calculus. Interpretive observations and questions have been noted as they seemed appropriate in Chapters IV, Analysis and Interpretation, and Chapter V, Reform in the Teaching of Calculus. The section below provides additional interpretative observations.

## Observations and Interpretation

The current study has included interpretative discussions as they arose during descriptions of the validation process and descriptions of the patterns of reform in the teaching of calculus. However, it seems important to review the interpretations in light of the entire study and to add to the interpretations as appropriate.

The comparison of the ranking of clusters on the clustering and validating scales in Chapter IV, Analysis and Interpretation, revealed that, for most clusters, clusters' consistency-with-reform rankings are consistent across the scales . (See Table 25.) Cluster 7 ranks second highest on the clustering and CCH validating scales and highest on the mathematics department scales. Cluster 6 ranks highest on the clustering and CCH validating scales and third highest on the mathematics department validating scales. Cluster 3 ranks second lowest on the clustering scales and lowest on the mathematics department and CCH validating scales.

Table 25
Comparison of Cluster Rankings and (Means) by Clustering Scales. Mathematics
Department Validating Scales, and CCH Validating Scales

|  | low |  |  | moderate |  |  |  | high |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Clusters ordered from <br> lowest to highest rank on <br> clustering scales | 8 <br> $(.38)$ | 3 <br> $(.39)$ | 1 <br> $(.47)$ | 2 <br> $(.49)$ | 4 <br> $(.52)$ | 5 <br> $(.57)$ | 7 <br> $(.63)$ | 6 <br> $(.65)$ |  |
| Clusters ordered from <br> lowest to highest rank on <br> mathematics department <br> validating scales | 3 <br> $(.27)$ | 1 <br> $(.29)$ | 8 <br> $(.44)$ | 2 <br> $(.45)$ | 4 <br> $(.47)$ | 6 <br> $(.50)$ | 5 <br> $(.51)$ | 7 <br> $(.62)$ |  |
| Clusters ordered from <br> lowest to highest rank on <br> CCH validating scales | 3 <br> $(.63)$ | 8 <br> $(.65)$ | 2 <br> $(.65)$ | 1 <br> $(.66)$ | 4 <br> $(.68)$ | 5 <br> $(.72)$ | 7 <br> $(.75)$ | 6 <br> $(.82)$ |  |

Although further study is needed, the relationship noted concerning clusters' consistency-with-reform rankings across the clustering and validating scales may extend to individual institutions that comprise the cluster. The unexpected consistency-with-reform rankings across the scales strengthens the validation of the cluster solution and contributes an interesting possible characteristic of reform in the teaching of calculus. An issue raised in the previous section again emerges. Is the overall degree to which calculus instructors engage with reform ideas and activities indicative of the departments' perspectives on reform? It seems reasonable to expect that this possible relationship is not limited to the use of the CCH Curriculum Project materials and may extend to the implementation of other reform-based calculus curriculum materials.

A more specific possible relationship appears to exist between the way CCH instructors at an institution interpret reform on individual clustering scales and the way site liaisons perceive their colleagues in the mathematics department interpret reform on the corresponding validating scales. For example, the way CCH instructors interpret reform on technology may relate to the way mathematics department faculty members interpret reform on vtechnology, the corresponding validating scale.

Table 26.

## Ranking of Clusters on Corresponding Clustering and Validating Scales

| Clustering scale | low |  |  |  |  |  |  | high |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| concepts | 8 | 1 | 4 | 3 | 6 | 2 | 7 | 5 |  |
| vconcepts | 6 | 4 | 3 | 1 | 8 | 2 | 5 | 7 |  |
|  |  |  |  |  |  |  |  |  |  |
| teaching | 3 | 1 | 8 | 2 | 4 | 5 | 7 | 6 |  |
| vteaching | 3 | 1 | 2 | 4 | 5 | 8 | 6 | 7 |  |
|  |  |  |  |  |  |  |  |  |  |
| assessment | 1 | 8 | 2 | 3 | 7 | 6 | 5 | 4 |  |
| vassessment | 3 | 1 | 8 | 2 | 4 | 6 | 5 | 7 |  |
|  |  |  |  |  |  |  |  |  |  |
| technology | 3 | 1 | 2 | 8 | 7 | 6 | 5 | 4 |  |
| vtechnology | 3 | 1 | 7 | 2 | 5 | 8 | 6 | 4 |  |

Note. The numbers in the table cells are cluster numbers. The clusters are ranked from low to high (left to right) in consistency with the goals for reform in the teaching of calculus.

Table 26 lists the cluster numbers ranked from low to high on the corresponding clustering and mathematics department validating scales: concepts and vconcepts, teaching and vteaching, assessment and vassessment, technology and vtechnology. Many clusters exhibit consistent rankings on the corresponding clustering and validating scale. For example, cluster 7 scores in the high range on teaching and vteaching and on concepts and vconcepts. Cluster 5 scores in the high range on concepts and vconcepts, in the moderate range on teaching and vteaching, and in the high range on assessment and vassessment. Cluster 4 scores in the lowmoderate range on concepts and vconcepts, in the moderate range on teaching and vteaching, and highest on technology and vtechnology. Cluster 3 scores lowest on teaching and vteaching and technology and vtechnoiogy, and in the moderate range on
concepts and vconcepts. These and other consistencies of rankings on corresponding scales support the conjecture that CCH instructors may interpret reform in ways similar to other mathematics department faculty at their institution.

Comparison of the clusters' overall consistency-with-reform ranking and vteaching validating scale ranking also appears to demonstrate that many clusters exhibit consistency between the rankings on the two scales. Vteaching addresses the pedagogical practices of most faculty in the mathematics department. The pedagogical practices include use of cooperative groups, student writing, use of complex problem solving situations, encouragement of alternative solutions to problems, and encouragement of student exploration in mathematics. Table 27 lists the cluster rankings on the scales.

Table 27

## Ranking of Clusters on Overall Consistency with Reform and on Vteaching

| Clustering scale | low |  |  |  |  |  |  | high |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ranking of clusters' <br> overall consistency <br> with reform on <br> clustering scales | 8 <br> $(.38)$ | 3 <br> $(.39)$ | 1 <br> $(.47)$ | 2 <br> $(.49)$ | 4 <br> $(.52)$ | 5 <br> $(.57)$ | 7 <br> $(.63)$ | 6 <br> $(.65)$ |
| Ranking of clusters on <br> vteaching mathematics <br> department validating <br> scale | 3 <br> $(.09)$ | $(.21)$ | $(.27)$ | 4 <br> $(.29)$ | 5 <br> $(.32)$ | 8 <br> $(.34)$ | 6 <br> $(.45)$ | 7 <br> $(.61)$ |

Cluster 7 and cluster 6 both rank in the high range on both scales, whereas cluster 3 ranks in the low range on both scales. Clusters $1,2,4$, and 5 also exhibit consistent rankings on the scales. Only cluster 8's rank differs considerably on the two scales. The consistencies between cluster rankings on the two scales would tend to support the conjecture that a relationship may exist between a mathematics department's openness to alternative teaching practices and their implementation of reform-based calculus materials. Faculty in mathematics departments perceived to
support the use of alternative pedagogical practices may be more willing to engage with calculus reform and be more consistent with the goals for reform in the teaching of calculus in their implementation of reform-based calculus materials.

Vvalues measures the institutional and departmental support for teaching.
Table 28 compares each cluster's overall mean score for consistency with the goals for reform in the teaching of calculus and the score on wvalues.

Table 28
Ranking of Clusters on Overall Consistency with Reform and on Vvalues

| Clustering scale | low |  |  |  |  |  |  | high |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ranking of clusters' <br> overall consistency <br> with reform on <br> clustering scales | 8 <br> $(.38)$ | 3 <br> $(.39)$ | 1 <br> $(.47)$ | 2 <br> $(.49)$ | 4 <br> $(.52)$ | 5 <br> $(.57)$ | 7 <br> $(.63)$ | 6 <br> $(.65)$ |
| Ranking of clusters on <br> vvalues mathematics <br> department validating <br> scale | 3 <br> $(.43)$ | 8 <br> $(.59)$ | 4 <br> $(.60)$ | 1 <br> $(.64)$ | 2 <br> $(.65)$ | 5 <br> $(.68)$ | 6 <br> $(.85)$ | 7 <br> $(.86)$ |

Note. The numbers in the table cells are cluster numbers. The clusters are ranked from low to high (left to right) in consistency with the goals for reform in the teaching of calculus.

Cluster 7 and cluster 6 rank high on overall consistency with reform and on the vvalues scale. Cluster 3 and cluster 8 rank low on both scales. Cluster 5 ranks third high on both scales. These comparisons raise questions about a possible relationship between the valuing of and support for teaching at an institution and the institution's consistency with reform in its implementation of reform-based calculus materials. Does the valuing of teaching at an institution contribute to a climate that encourages faculty to try new ideas and reflect upon student learning in the context of their teaching practices? To what extent is an institution's support for teaching conducive to creating a situation that provides greater access to a wide range of
students to calculus? In such a situation are more students successful in calculus and able to continue in courses dependent upon calculus?

It is interesting to speculate on a possible relationship between a cluster's overall rank on consistency with reform in the teaching of calculus and the percentage of site liaisons in a cluster reporting financial support for the implementation of CCH Curriculum Project materials. As mentioned previously in the validation of the clusters in Chapter IV, most site liaisons did not describe the type or amount of financial support received. However, the data indicate the site liaisons' perceptions about whether financial support was received. The first two rows of Table 29 rank the clusters on overall consistency with reform and percentage of site liaisons reporting some financial support for implementation. Clusters 7 and 5 rank second and third highest on both scales respectively, cluster 8 ranks lowest on both scales, clusters $I$ and 3 both rank in the low to low-moderate range on both scales, and cluster 2 ranks in the moderate range on both scales. Clusters 4 and 6 exhibit inconsistent rankings. The consistency between most cluster rankings on the two scales supports the conjecture that there may be a relationship between the financial support to implement reform at an institution and the consistency of the implementation of CCH Curriculum Project materials with the goals for reform.

Table 29
Ranking of Clusters on Overall Consistency with Reform and on Vfinsup

| Clustering scale | low |  |  |  |  |  |  | high |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ranking of clusters on <br> overall consistency <br> with reform on <br> clustering scales and <br> (consistency score) | 8 <br> $(.38)$ | 3 <br> $(.39)$ | 1 <br> $(.47)$ | 2 <br> $(.49)$ | 4 | 5 | 7 | 6 |
| Ranking of clusters on <br> percentage of site | 8 | 1 | 3 | 2 | 6 | 5 | 7 | 4 |
| liaisons reporting some <br> financial support to | $10 \%$ | $24 \%$ | $31 \%$ | $39 \%$ | $43 \%$ | $47 \%$ | $50 \%$ | $58 \%$ |
| implement CCH <br> Curriculum Project <br> materials (vfinsup) |  |  |  |  |  |  |  |  |
| Ranking of clusters <br> and (mean score) on <br> the technology <br> clustering scale. | 3 <br> (.47) | 1 <br> $(.66)$ | 2 <br> $(.67)$ | 8 <br> $(.71)$ | 7 <br> $(.75)$ | 6 <br> $(.79)$ | 5 <br> $(.80)$ | 4 <br> $(.81)$ |

The bottom row of Table 29 ranks the clusters on technology. Because of the frequently heard concern about the high cost of technology, it seems appropriate to compare cluster rankings on technology and vfinsup. Cluster 4 scores highest on technology and vfinsup; clusters 7,6 , and 5 rank in the high moderate range on both scales, cluster 2 ranks in the moderate range on both scales, and clusters 1 and 3 rank in the low to low-moderate range on both scales. Cluster 8 's rankings are inconsistent. The data indicate some consistency between the cluster rankings, raising the possibility that there is a relationship between the use of technology and financial support to implement CCH Curriculum Project materials.

The preceding discussions have addressed a possible relationship between a cluster's scores on the mathematics department validating scales and on the clustering scales. The observations stand in contrast to a commonly discussed issue regarding reform. Researchers and practitioners, when considering how mathematics departments initiate and sustain reform, sometimes suggest the presence of one key faculty member, who initiates reform and is influential in sustaining reform within the
department. Questions arise concerning what happens to reform if the key person leaves. Few comments in the current study refer to a key person at an institution who is responsible for the continuation of reform. More often participants refer to group or departmental decisions about use of the CCH Curriculum Project textbook. Did the respondents neglect to mention the key person in their comments? What is the role of a key reformer in the development of a well-defined curricular innovation? Is reform maturing to the point where more members of departments are considering engagement with reform in the teaching of calculus?

As has been noted previously, reform in the teaching of calculus involves addressing complex issues. Influences toward reform come from many different sources and reform takes many different directions. Some members of the mathematics community complete the statement, "Reform in the teaching of calculus is $\qquad$ ," with one or two short phrases. The completion phrases may include "technology," "use of cooperative group learning," "a watered down calculus for all," "a less rigorous course," "class projects," "mathematical modeling," or "applications."

The cluster descriptions in Chapter V, Profiles of Reform in the Teaching of Calculus, illustrate that institutions implement reform in many different ways. For some academic institutions, technology drives reform; for others technology is just another tool for understanding concepts and the course does not emphasize its use. The two clusters scoring highest overall on consistency with reform score in the moderate range on technology. Some respondents actively engaged in reform speak of lecturing less and having small groups work on problems; others continue to use a traditional lecture format in their teaching. Some reform-based calculus courses are conducted exclusively in a lab situation with computers available for all classes. Others describe a somewhat traditional classroom situation with the instructor presenting the material. Some participants describe the centrality of student projects that engage the students in complex problems and situations. Others do not use
projects. Many participants respond to those who claim reform-based calculus is a watered-down calculus by mentioning the complex problems in the textbook and the many hours they spend making sure they can do all problems they have asked the students to do. Very few respondents suggest the course is easy. Although some claim the course is less rigorous, definitions of rigor may vary widely. Rigor is not a well defined term among the mathematics community. For some participants, their course emphasis is on applying the calculus to real world situations. Others prefer less emphasis on applications. As the definitions of the clustering scales indicate, the goals for reform in the teaching of calculus are multi-dimensional. Instructors and institutions place priority on goals that have meaning for them and their own situation. The danger is to define reform too narrowly, not recognizing the richness provided by the diverse viewpoints. There remains much to be learned about reformbased calculus teaching and learning. The preceding paragraphs have indicated many areas for further work. The next section considers the dimensions of reform individually and raises questions for additional future study.

## Implications for Future Study

Implications for future study have been addressed as they arose in the current study and in the preceding interpretation section. However, it seems useful to relate many of the implications to the clustering scales and, thereby, to the goals for reform in the teaching of calculus established from the Tulane Conference on which the clustering scales are based. In this brief section, questions related to each clustering scale are raised.

The average score on concepts (.61) is moderately high. Concepts pertains to an emphasis on the development of students' conceptual understanding of the mathematical concepts that are central to calculus. Students should spend more time developing a deep and rich understanding of concepts and less time doing routine
procedures. Questions arise such as: What do mathematicians and calculus instructors mean by conceptual understanding? How can students' conceptual understanding best be measured? How do students come to understand concepts in calculus? How is that understanding influenced by or related to students' understanding of similar terms used in everyday contexts?

Anecdotal evidence, as reported by the participants in the current study, suggests that students in CCH Curriculum Project courses have better understanding of central calculus concepts than students in standard courses. In 1990, the Mathematical Association of America's Committee on the Teaching of Undergraduate Mathematics issued a source book for college mathematics teaching that began with a statement of goals for instruction (Schoenfeld, 1992). One of the six goals speaks directly to students' conceptual understanding.

Mathematics instruction should develop students' understanding of important concepts in the appropriate core content. . . . Instruction should be aimed at conceptual understanding rather than at mere mechanical skills, and at developing in students the ability to apply the subject matter they have studied with flexibility and resourcefulness. (Schoenfeld, 1992, p. 345)

Overall, the participants in the current study indicate they are pleased with students' increased understanding of concepts. However, some participants question the balance between conceptual understanding and procedural skills. Participants note: "We have de-emphasized algebraic manipulation. I am uneasy about the unforeseen consequences of that" and "[I am concerned about] the lack of foundation skills such as routine manipulations. I am very concerned about the increasing number of situations where students can't do the mathematics." Leitzel and Dossey (1994)
present a different but related question, "How much change can be accomplished in either calculus or pre-calculus courses and still meet the current expectations of client disciplines? Are client disciplines changing curriculum and teaching at the same rate as mathematics?" (p. 44). These issues are complex. Considerable thought and research are needed in these areas.

The overall mean score on approach (.84) is the highest of all clustering scales. Approach pertains to instructors using graphic, numeric, and algebraic approaches to calculus concepts and problems and an inductive approach to calculus in which mathematics arises from student investigations and real world problems. In the current study there appears to be a relationship between the textbook and the implemented curriculum. What are the influences of the textbook on an instructors' curricular decisions and on student learning? How do these influences differ among different types of faculty? That students experience difficulty moving among representations of mathematics concepts is documented in the literature (Janvier, 1987; Lesh, Post, \& Behr, 1987). In what ways does the use of graphic, numeric, and algebraic approaches to all topics influence students' understanding and ability to move between graphic, numeric, and algebraic representations?

Many instructors comment enthusiastically on their changed teaching practices and changed methods of student assessment. These new practices and methods include the use of cooperative learning, class presentations, projects, or calculus labs. On the other hand, the teaching and assessment clustering scales have the overall lowest mean scores ( .25 and .31 , respectively). What is the relationship between self perceptions about teaching practices and actual changes in practice? What are the influences of changes in undergraduate teaching practices and student assessment methods on student learning of calculus concepts? How best can students' conceptual understanding be assessed? What is the role of technology in student assessment?

Many comments provide anecdotal evidence about students' improved attitudes towards mathematics and calculus when alternative methods of teaching and student assessment are used. What characterizes these changed attitudes? In what ways are students' beliefs about mathematics changed as a result of alternative teaching and student assessment methods? What is the relationship between improved student attitudes toward mathematics and students' problem-solving skills? Research conducted in the undergraduate mathematics classroom is somewhat limited, although accounts of some formal studies and anecdotal discussions are available (for examples, see Brechting \& Hirsch, 1977; Dubinsky, 1990; Rogers, 1988; Treisman, P. U., 1983; Urion \& Davidson, 1992). What can be learned about student understanding through continued research conducted in undergraduate classrooms?

Technology is defined by the use of calculators or computers in the calculus course. Graphing calculators or computers are used on a regular basis in a majority of the courses instructed by participants in the current study. The overall high mean score (.71) on technology also indicates strong support for its use. What characterizes the relationship between reform in calculus instruction and technology? Some completed work and some work in progress address how students come to understand the central concepts of calculus in the presence of technology (Heid, M. K., 1988; Keller, B. A., \& Hirsch, C. R., 1992; Lauten, A. D., Graham, K., \& Ferrini-Mundy, J., 1994; Monk, S., \& Nemirovsky, R., 1994; Tall, D. O., 1988). Technology continues to change and more work is needed. Symbol manipulation software is increasingly available on hand-held technology. What is an appropriate balance between students' need to be able to compute derivatives and integrals by hand and students' use of technology to differentiate and integrate? In what ways do the characteristics of graphing utilities influence students' understanding of the function concept and other central ideas of calculus, such as pointwise versus across-time understanding, end-
behavior, continuity, differentiability, and limits? Technology is more than a tool. How can technology be used more effectively by students to explore calculus notions, to make and investigate conjectures, to understand concepts, to pose better problems, and to apply their understanding in new situations ?

The relationship between increased access to calculus for traditionally underrepresented students and current efforts toward reform in the teaching of calculus is unclear on the basis of this study. Many participants suggest that students with weak algebra backgrounds are more successful in CCH Curriculum Project courses than in traditional courses. Does success in calculus give these same students greater access to subsequent mathematics courses and courses in client disciplines? What curriculum changes influence access? What changes in teaching practice influence access? What is the role of technology in increasing access?

On a broader note, what does it mean to be "doing calculus reform" as reform in the teaching of calculus continues to evolve? Several list-serves on the internet are devoted to discussions about reform-based calculus instruction. Analysis of the discussions could provide additional documentation and insight into the reform movement. How have the issues changed over time? What influence is held by those deeply involved in (or critical of) the reform movement?

Efforts are now underway to extend reform to the teaching of precalculus, linear algebra, and other undergraduate mathematics courses. What will be the influences of those changes on the calculus curriculum? What will be the influences of the debates surrounding calculus reform on those changes?

The current study has tried to document and increase the community's understanding of reform in the teaching of calculus in the context of the CCH Curriculum Project. Descriptions of profiles of reform efforts and reflection on reform has led to the many questions listed above that are in need of further investigation.

## Conclusion

The current study has presented eight patterns of interpretation and implementation of reform in the teaching of calculus in the context of CCH Curriculum Project materials. The patterns of implementation can help others understand reform efforts and can serve to guide institutions as they make decisions about reform-based calculus instruction.

The current study suggests that CCH Curriculum Project instructors, in general, support the directions set by the authors including the "Rule of Three" approach (graphic, numeric, and algebraic) and an "inductive" and investigative approach that includes more real-world problems. The participants indicate that they believe the three-fold approach to topics allows multiple entry points to calculus and makes calculus more accessible to a wider range of students. Instructors appreciate the use of an investigative approach to engage students in the learning of calculus and to help students understand the role of mathematics in modeling the real world. The instructors report that they generally support the decreased emphasis on manipulative skills, but many question to what level students should be able to perform some procedural skills by hand.

Most courses using CCH Curriculum Project materials are taught by full-time, tenured, or tenure-track faculty members. The participants indicate they are thinking deeply about the issues surrounding reform and generally report that they believe their students are gaining deeper understanding of the central concepts of calculus and are better able to see the use of calculus in other courses. Some express concern about whether the "formal logical development, the sense of mathematical argument and structure is missing." The majority of participants express enthusiasm for the use of technology in calculus although some express concern that students may rely too heavily on technology and "trust graphing calculators over their own judgment.".

Most participants indicate they are pleased with CCH Curriculum Project materials and plan to continue using them. Very few participants speak of returning to traditional calculus materials.

The current study demonstrates that there are many patterns for implementation of reform-based calculus. Furthermore, participant comments reveal significant variations within the profiles of reform. Within the contextual situations of the academic institutions and the unique student populations, faculty members place emphasis on those aspects of reform that are important in their situation. The current study has sought to describe the dilemmas, ambiguities, and struggles with hard issues from the perspective of those who are engaging with reform on a daily basis. The current study is an effort to reveal what is "going on" in calculus reform, helping the reader better understand calculus reform while recognizing the complexities faced by those participating in reform efforts. As one of the CCH Curriculum Project authors stated in a personal interview, "We are all in the same stream, we're just in different currents" (A. Pasquale, personal communication, March 29, 1996). Perhaps some of the currents are the profiles of reform that initially seemed to be neat and tidy ways of describing reform implementations. However, closer attention to the participants' comments revealed that the currents themselves are turbulent and complex. Each holds its own subcurrents or variations based on the participants' unique situations and emphases.

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## APPENDIXES

## APPENDIX A

## APPENDIX A

## EXCERPTS FROM REPORTS OF THE CONTENT WORKSHOP AT THE TULANE CONFERENCE

The content workshop group spent the majority of its time establishing student competencies and topics for Calculus I, a course in the core concepts of the calculus for a general audience. The student competencies and list of topics appear below (Douglas, 1986d).

## Student Competencies

The first group below lists items students should be able to do; while the second lists items students should see and know.

## Group 1

- Students should be able to give a coherent mathematical argument.
- Students must be able not only to give answers but also to justify them.
- Calculus should teach students how to apply mathematics in different contexts, to abstract and generalize, and to analyze quantitatively and qualitatively.
- Students should learn to read mathematics on their own.
- Calculus must also teach mechanical skills, both by hand and by machine.


## Group 2

- Students must understand the fundamental concepts of calculus: change and stasis, behavior at an instant and behavior in the average, and approximation and error.
- Students must also know the vocabulary of calculus used to describe these concepts, and they should feel comfortable with that vocabulary when it is used in other disciplines.

A Listing of Topics with suggested time allotments

The derivative
The suggested course hour to attend to each topic is listed in the left column.
1 Honesty day (show why we are interested in the concept and where it arises) Present a problem asking for instantaneous rate of change or local magnification constant (marginal cost, velocity)

Interpret graphically as the slope of the tangent line and conclude the derivative is $\lim (\Delta x->0)=[f(x+\Delta x)-f(x)] / \Delta x$.

## 2-4 Dictionary of functions

 Introduce:a. $\quad x^{r}, b^{x}, \log _{b}(b>1), \sin (x), \cos (x)$, and $\tan (x) \backslash$
b. graphically and numerically presented functions
c. new functions from old (addition, multiplication, dividsion, composition)

Notes: Treat $\log _{b} \mathrm{x}$ as the inverse of $\mathrm{b}^{\mathrm{x}}$ without getting bogged down in the properties of the log function; use the "black box" keys on a calculator as a source of numerically presented functions.

Limits and continuity
Use the precise english definition of "lim $f(x)=L$ " rather than the $\varepsilon-\delta$ definition. Examples: " $\mathrm{f}(\mathrm{x})$ can be made arbitrarily close to L by making x sufficiently close, but not equal to, a." or "small changes in x produce small changes in $\mathrm{f}(\mathrm{x})$ ". Numerically define a contuous function as one whose value at any $x$ can be computed to any given number of digits by putting into $f$ any number near enough $x$ and graphically as one whose graph can be drawn "without lifting pencil from paper".

6 Definition of $f(x)$, notation, linearity, polynomials.
Only compute $f^{\prime}(x)$ "the long way" for a few easy functions such as $x^{3}$ or $x^{2}+$ 5 x and derive the power rule and do addition and scalar multiplication so students differentiate the general polynomial functions the same hour. Graphically obtain $f^{\prime}(x)$ from $f(x)$ and obtain the relationship between the qualitative behavior of $f(x)$ and the sign of $f^{\prime}(x)$.

8 Graphically find $d(\sin (x)) / d x$ and quickly derive that $f^{\prime}(x)=k b^{x}$ where $f(x)=$ $b x$, where $k$ is the slope at $x=0$. Define $e$ to be the value of $f(x)$ such that the slope at $\mathrm{x}=0$ is 1 .

9 New-from-old differentiation.
Derivative of sums, products, reciprocals, and quotients.
10. The chain rule, inverse functions.
11. Implicit differentiation. Treat as a formal, manipulative process motivated by the notation. $d\left(y^{3}\right) / d x=3 y^{2} d y / d x$.

## Uses of the Derivative

12-15 Qualitative analysis of functions (curve sketching) Include (with f not just a polynomial function) sign of $f$, increasing/decreasing, critical points, $\mathrm{f}^{\prime \prime}$, concavity, points of inflection,
$\max / \mathrm{min}$ on a closed interval. graphical representation of $f$ and $f$ (given the graph of $f$, find the graph of $f$ ).

16-17 Root-finding, Newton's Method, Rolle's Theorem
Use of "solve key" on caculator.
18-19 Linear approximation and error
Derive $f(x)-f(a)=f(a)(x-a)+f^{\prime \prime}(c)(x-a)^{2} / 2$ and interpret big Oh notation: $\Delta y=f(a) \Delta x O\left((\Delta x)^{2}\right)$

## 20-21 Extrema ("word") problems

## The Integral

22 Honesty-day
Summing examples include work, present value of money, total distance, any averaging problem (temperature, depth of river), and eare. Contrast with the derivative: total versus instantaneous, global versus local information, (formally) functional versus operator. "Finish with the definition of $\int_{a}^{b}$ as the limit of Riemann sums.

23-24 Numberical integration--rectangle, midpoint, trapezoidal, and Simpson's rule. Students should be able to observe the order of error and use a calculator for the value of a definite integral, but be able to give rough estimates to assure reasonable estimates.
25 Properties of the definite integral: linearity, $\int_{a}^{b}=\int_{a}^{c}+\int_{c}^{b}, \mathrm{~m}(\mathrm{~b}-\mathrm{a}) \leq$ $\int_{a}^{b} \mathrm{M}(\mathrm{b}-\mathrm{a})$
Justify change of variagble in integrals $\int_{a}^{b} f(g(x)) g^{\prime}(x) d x=\int_{c}^{d} f(y) d y$ by
Riemann sums and the Mean Value Theorem.

26-27 The area function, $\mathrm{A}(\mathrm{x})=\int_{u}^{r} f$.
28 The Fundamental Theorem of Calculus $A^{\prime}(\mathrm{x})=\mathrm{f}(\mathrm{x})$. The FTC in the form $\int_{a}^{\mathrm{x}} f=\mathrm{F}(\mathrm{b})-\mathrm{F}(\mathrm{a})$ needs to use $\mathrm{F}^{\prime}(\mathrm{x})=\mathrm{A}^{\prime}(\mathrm{x})$ $=>F(x)=A(x)+C$. Either treat this as an "obvious" fact intuitively or prove using the mean value theorem.

29-30Antidifferentiation
Do $\sin (x), \cos (x), e^{x}, 1 / x, \ln (x)$ (given as oracle as $x \ln x-x+C$ but easily checked). Antidifferentiate $x \sin \left(x^{2}\right)$ by substitution or guess-and-check. Uses of Integral

31-32 Areas between curves and average value
33-35 Easy differential equations: Include (a) acceleration-velocity-position problems, including non-constant acceleration; (b) exponential growth, $\mathrm{y}^{\prime}= \pm$ ky ; (c) cyclic behavior, $\mathrm{y}^{\prime \prime}=-\mathrm{y}$.
(pp. vix-xi)

## General Comments

Topics that are missing include Related rates, L'Hopital's rule, $\varepsilon-\delta$ definition of limit, and precalculus material on lines, circles, and domain and range of functions. Material that is reduced includes limits and continuity, computing derivatives from the quotient definition, and computing integrals from the Riemann sum definition. Treatments that are changed from standard calculus include graphical and numerical treatment of functions, big Oh notation, Newton's method, numberical integration, and the use of hand calculators. (p. xiv)

## APPENDIX B

## Initial Questionnaire: CCH Evaluation and Documentation Project

Please return as soon as possible in the enclosed envelope.
(CCH Evaluation and Documentation Project: Mathematics Department, Kingsbury Hall;
University of New Hampshire: Durham. NH 03824.)
Name:
School name $\qquad$
Mailing address: $\qquad$
office phone $\qquad$ e-mail $\qquad$ fax $\qquad$
Please describe your school by checking the appropriate response.

1. Type of school: secondary school $\qquad$ 2-yr. college $\qquad$ 4 yr. college $\qquad$ university $\qquad$
2. Public or private institution: public $\qquad$ private $\qquad$
3. If college or university, number of undergraduate students:

If secondary school, number graduating this year: $\qquad$
4. Number of mathematics faculty: full time $\qquad$ part time $\qquad$
5. If 4-yr. college or university, highest degree awarded in mathematics: BA/BS _ MA/MS $\qquad$ Ph.D. $\qquad$ Other $\qquad$ (indicate degree)

## Please describe your calculus program and current use of CCH materials by responding to the following questions.

6. Is your department using CCH materials this fall? yes $\qquad$ no $\qquad$
7. Has your department used the CCH materials in the past? yes $\qquad$ no $\qquad$
If you responded yes, what academic term(s) or semester(s) did you use the material? (ex. Spring 93) PLEASE indicate all.
8. If your department is not using CCH materials this fall, but has used them in the past, please explain.
9. Do graduate student teaching assistants participate in the calculus sequence(s) that use CCH materials? (Check all that apply.) no participation __ grade for course section(s) lead discussion or lab sections $\qquad$ teach course section(s) other (please describe briefly) $\qquad$

Please respond to the following questions regarding calculus reform and evaluation.
10. Have you or your mathematics department received any special financial support to implement calculus reform or to introduce the CCH material? (CHECK all that apply, indicating amount if known.)
No support $\qquad$ Departmental support $\qquad$
Institutional support $\qquad$ External support $\qquad$
Comments $\qquad$
11. Have you or your institution conducted any informal or formal evaluation of the use of CCH material or calculus reform efforts? (Check all that apply.)
No evaluation $\qquad$ Comparison of grades in CCH and non-CCH settings Attitude questionnaires $\qquad$ Tracking of students $\qquad$ $\overline{\text { Other (please describe) }}$
12. Are there issues and questions surrounding calculus reform that are of particular interest to you or your institution? Please elaborate.

We would like to understand the types of calculus courses your department offers and in which of those settings you use CCH material.
13. The boxes in the fourth through eighth columns are to be completed only if you are using CCH materials in some or all sections of that particular calculus sequence.
COLLEGE OR UNIVERSITY:

| First term calculus: name or type of sequence. | Offered at your institution? | If yes, are you Using CCH materials? | Course number | Total \# of calculus sections | average <br> \# of <br> students <br> per <br> section | \# of sections using $\mathbf{C C H}$ material | average <br> \# of <br> students <br> per <br> section |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| offer just one type of calculus (If so, complete this row only) | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ |  |  |  |  |  |
| honors calculus | $\begin{array}{\|c} \hline \text { yes } \\ \text { no } \\ \hline \end{array}$ | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ |  |  |  |  |  |
| specialty calculus (e.g. business, life science, etc.) | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ |  |  |  |  |  |
| mainstream calculus | $\begin{gathered} \text { yes } \\ \text { no } \\ \hline \end{gathered}$ | $\begin{gathered} \text { yes } \\ \text { no } \\ \hline \end{gathered}$ |  |  |  |  |  |
| other (describe) | $\begin{aligned} & \text { yes } \\ & \text { no } \end{aligned}$ | $\begin{gathered} \text { yes } \\ \text { no } \end{gathered}$ |  |  |  |  |  |



## SECONDARY SCHOOL:

| adv. placement <br> calculus | yes <br> no | yes <br> no |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| non-adv. placement <br> calculus | yes <br> no | yes <br> no |  |  |  |  |  |

If you feel any of your answers in question \#13 need further clarification, please explain.

Please indicate your interest in further participation in the CCH evaluation project.
14. As we indicated in the cover letter to this initial questionnaire, you will receive a second survey this spring. In addition, we invite you and your institution to become more deeply involved in this project. Please indicate your further willingness to participate by checking all that apply.
$\qquad$ a. I am willing to serve as site liaison and coordinate the administration of the faculty/student survey at my institution this spring.
$\qquad$ b. Our institution is willing to be considered as a case study site.
$\qquad$ c. Although we completed the questionnaire, no one at our institution is able to serve as site liaison.

## APPENDIX C

# SURVEY B <br> CCH Evaluation and Documentation Project 

## SITE LIAISON QUESTIONNAIRE

## Spring 1995

## CCH EVALUATION AND DOCUMENTATION PROJECT SURVEY B: TO BE COMPLETED BY SITE LIAISON

Thank you for agreeing to serve as a site liaison for this project. Survey B is part of the evaluation and documentation of the Calculus Consortium Based at Harvard (CCH) single variable (or tirst-year) calculus project. This current evaluation and documentation is being done at the request of CCH . as an independent activity. The CCH project is sponsored by the National Science Foundation as a Calculus Curriculum Reform Project.

The purposes of this effort are to learn how the stated goals of CCH are interpreted and implemented. to investigate student and faculty variables. to examine and describe the evolution of calculus reform efforts, and to promote collaboration and sharing among those involved in the implementation and evaluation of calculus reform. Plans include conducting the evaluation and documentation in four major phases as outlined in the back of this booklet. When reading these descriptions it might be helpful to keep in mind that this survey is part of phase B .

## ABOUT THIS SURVEY

This survey includes nine parts.
L Structural characteristics
II. Characteristics of student participation
III. Assessment of students
IV. Use of Technology
V. Supplementary instructional materials
VI. Instructor support
VII. Departmental perspectives
VIII. Reform issues
IX. Availability of data and/or artifacts

Time needed to complete the survey is approximately one hour. We welcome written comments in the spaces provided or in any section of the survey. It is important that all site liaisons participate in the survey so the resuits will fairly represent present and previous users of CCH materials.

Your answers will be kept strictly confidential. Note that this is not an evaluation of the particular implementation at any individual institution. This research is conducted under stringent university and government regulations designed to safeguard study participants. Identification codes are used only for follow-up purposes, such as linking Questionnaire A and Survey B responses and determining case study and tracking sites. Results of Questionnaire A and Survey B will be reported only in summary or statistical form so that neither individuals nor their institutions can be identified.

Thank you for contributing your time and thoughtful responses to this evaluation and documentation effort. You will receive an honorarium of $\$ 25$ for completing this survey. Please complete and return the enclosed postcard separately so we may process your check. We hope that you will find the questions professionally meaningful and interesting.

[^0]
## General Information and instructions:

- Please complete this survey as soon as possible and return it in the enclosed envelope. (CCH Evaluation and Documentation Project. Department of Mathematics, Kingsbury Hall, University of New Hampshire, Durham, NH 03824)
- When we use the phrases "CCH textbooks" or "CCH materials" in this survey, we are referring to the first year calculus course (single variable calculus) developed by the Calculus Consortium Based at Harvard. Unless specifically mentioned in a particular question, this survey does not address the multi-variable or precalculus materials.
- This survey will be completed by site liaisons at universities. 4-year colleges. 2-year colleges and secondary schools using CCH textbooks. Please answer the questions as best as you can for your institution. If you find a question particularly difficult to answer because of your situation. please try to answer it and write a brief note explaining your answer or your difficulty.
- Unless specifically suggested otherwise for a particular set of questions, please answer all questions in terms of the most typical "first term" calculus class using the CCH textbook. By "first term" we mean the initial calculus course at your institution rather than a time of year.
- The word "term" indicates semester, quarter, trimester, or a comparable word or phrase at your institution.
- If your institution has several types of CCH calculus courses, please answer in terms of the type with the greatest total number of CCH students.
- Examples of "lab" type classes: students work the entire period (or almost the entire period) in small groups. They may use technology, use manipulatives (concrete objects), or solve problems.
- Examples of "regular" or "large lecture" classes: classes in which the teacher and/or students present material either for the entire class period or for some part of the class period. There also may be some group work with or without the use of technology during the class period.
- Reminder: The faculty (and student) questionnaires will comprise Survey C. In those questionnaires, faculty members will be asked to base answers on their own courses.
- Please note that in question 55 we ask if you have copies of any CCH final examinations you are willing to send us. We sincerely thank you if you are able to do so.
- Several site liaisons mentioned in Questionnaire A that they had conducted evaluation efforts or had collected some comparison data. Again, if you have conducted such investigations, we would greatly appreciate (and thank you for) your sending us copies of the data or the findings of those efforts. See question 57.


## I. STRUCTURAL CHARACTERISTICS

## CIRCLE ONE RESPONSE.

1. Which of the following best describes the CCH classes at your institution?
All "lab" type classes ..... 1
All "regular" classes with 45 or fewer students per class ..... 2
All "large lecture" classes (more than 45 students per class) ..... 3
A mixture of "large lecture" and small (less than 45 student) "lab" classes. ..... 4
A mixture of large lecture and small (less than 45 student) "regular" classes. ..... 5
Other. ..... 6
2. Approximately how many hours per week do CCH classes meet at your institution?Include any lab hours for which credit is given.
Less than 3 hours .....  1
Three hours .....  2
Four hours ..... 3
Five hours ..... 4
More than 5 hours .....  5
3. How many classes per week meet for two or more hours at a time?
None. .....  .1
One. ..... 2
Two. ..... 3
Three ..... 4
Four or more ..... 5
4. Is there an entering-student advisory process in mathematics?
yes .......... 1 no ..... 2
CIRCLE ALL APPROPRIATE RESPONSES.
5. What are the prerequisites for enrolling in "first term" calculus at your institution?
Three years of college preparatory mathematics required. .....  1
Four years of college preparatory mathematics required ..... 2
A placement test required (including ETS Advanced Placement tests) ..... 3
A college pre-calculus course required ..... 4
Prerequisites exist but they are advisory only ..... 5
No prerequisites ..... 6
Other ..... 7

## CIRCLE ALL APPROPRIATE RESPONSES.

6. How are students placed into the "first term" CCH sections?
All first term calculus sections use CCH materials ..... 1
Students choose to be in CCH sections ..... 2
Assignment to CCH sections is random ..... : 3
Assignment to CCH sections is based on major ..... 4
Assignment to CCH sections is based on some form of pretest results ..... 5
Other ..... 6
7. At your institution, what percentage of the mathematics faculty are adjunct (or part - time)?
0-25\% ..... I
26-50\% ..... 2
51-75\% ..... 3
76-100\% ..... 4
8. What is the percentage of each type of CCH instructors at your institution?
Tenured senior level (college) or full-time faculty (secondary school) ..... 1
Tenure track junior faculty (college) ..... 2
Non-tenure track instructors ..... 3
Graduate students ..... 4
Adjunct or part-time faculty ..... 5
Other ..... 6
9. If "lab" sections are included as part or all of the CCH course credit, who isresponsible for conducting the "lab" sections at your institution?
All classes are regular calculus course sections. ..... I
Tenure track faculty (secondary school or college) ..... 2
Non-tenure track instructors ..... 3
Adjunct faculty ..... 4
Graduate students ..... 5
10. Are there any types of scheduled non-credit classes, "labs", or other sessions offered to help students in CCH classes?

$$
\text { yes .......... } 1 \text { no .......... } 2
$$

If so, please describe the type of help, frequency, and who is responsible for providing this help.

## CIRCLE ONE NUMBER IN EACH LINE.

11. Of the students in "first term" CCH sections. please estimate (if possible) the percent of students that are:

| $0-25 \%$ | $26-50 \%$ | $51-75 \%$ | $76-100 \%$ | Not <br> available |
| :--- | :--- | :--- | :--- | :--- |


| a | Women................... | 1 | 2 | 3 | 4 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b. | Non-traditional students. | 1 | 2 | 3 | 4 | 9 |
| c. | Math majors............ | 1 | 2 | 3 | 4 | 9 |
| d. | Physical science majors. | 1 | 2 | 3 | 4 | 9 |
| e | Engineering majors.. | I | 2 | 3 | 4 | 9 |
| f. | Life science majors... | 1 | 2 | 3 | 4 | 9 |
| g. | Business majors...... | 1 | 2 | 3 | 4 | 9 |
| h. | Other majors............ | 1 | 2 | 3 | 4 | 9 |
| i. | Have previously taken calculus. | I | 2 | 3 | 4 | 9 |

12. Of the students in "first term" CCH sections, please estimate (if possible) the percent of students that are:

|  |  |  | Not <br> $0-25 \%$$\quad 26-50 \%$ | $51-75 \%$ |
| :--- | :--- | :--- | :--- | :--- |


| a | African-American..... | 1 | 2 | 3 | 4 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b. | American Indian or Alaskan Native. $\qquad$ | 1 | 2 | 3 | 4 | 9 |
| c. | Asian-American. | 1 | 2 | 3 | 4 | 9 |
| d. | Hispanic-American... | 1 | 2 | 3 | 4 | 9 |
| e | White (nonHispanic) American.. | 1 | 2 | 3 | 4 | 9 |
| f. | International ............. | 1 | 2 | 3 | 4 | 9 |

f. Other (Please explain Briefly)

## II. CHARACTERISTICS OF STUDENT PARTICIPATION

## CIRCLE ONE NUMBER IN EACH LINE.

13. Approximately how often do students in CCH SECTIONS take part in the following activities during time scheduled for calculus? We realize many site liaisons will have to make a qualitative judgment as to the most representative response.

h. Work in small groups on mathematics problems.............................................................
i. Practice procedural skills. $\qquad$ 1
j. Make conjectures and explore more than one possible method to solve a calculus problem...... 1 2
k. Write in journals. $\qquad$
14. Work on small group or individual projects that take several class meetings to complete $\qquad$ 1 2 3 4
m. Write about how to solve a problem on an assignment or test. $\qquad$ 1
n. Engage in mathematical exploration $\qquad$ 1 2 34

3
4
3
4
o. If you wish. please use this space to make additional comments about characteristics of student participation during CCH classes.

If all calculus students in your institution use CCH textbooks. omit question 14 and proceed to question 15. If you have more than two types of first year calculus course. answer in terms of the type of calculus course (other than CCH ) with the greatest number of students.
14. Approximately how often do students in these courses take part in the following activities during time scheduled for calculus? We recognize many site liaisons will have to make a qualitative judgment as to the most representative response.

CIRCLE ONE NUMBER IN EACH LINE.

|  |  | Rarely <br> or <br> never | Once or <br> twice a <br> month | Approx. <br> once a <br> week | Almost <br> every <br> class <br> meting |
| :--- | :--- | :--- | :---: | :---: | :---: |
| a | Participate in a calculus lab............................... | 1 | 2 | 3 | 4 |

o. Please provide the name of the text or briefly describe the course for which you completed \#14.
III. ASSESSMENT OF STUDENTS
CIRCLE ALL APPROPRIATE RESPONSES.
15. Which of the following are used to assign grades in CCH calculus sections? We recognize many site liaisons will have to make a qualitative judgment as to the most representative response.
Individual tests of mastery of content material ..... 1
Small group tests of mastery of content material. ..... 2
Lab reports (individual grades) ..... 3
Lab reports (group grades) ..... 4
Humework exercises (individual grades based on number correct) .....  .5
Homework exercises (group grades based on number correct) ..... 6
Homework exercises (individual grades based on number attempted) ..... 7
Homework exercises (group grades based on number attempted) ..... 8
Frequent in-class quizzes .....  .9
Projects (individual grades) ..... 10
Projects (group grades) ..... 11
Common final exams for all sections ..... 12
Journals ..... 13
Class participation ..... 14
Portfolio ..... 15
16. If you wish, please use this space to make additional comments about studentassessment.

## rv. USE OF TECHNOLOGY

17. Circle ALL computer programs that are commonly used in CCH classes or labs.

None ................................................................................................................. 1
MAPLE ................................................................................................................................... 2
MATHEMATICA ............................................................................................ 3
DERIVE ........................................................................................................... 4
MATLAB .......................................................................................................... 5
[SETL ............................................................................................................... 6
THEORIST ........................................................................................................ 7
University of Arizona Software ......................................................................... 8
Other software program(s). (Please list)

## CIRCLE ONE NUMBER IN EACH LINE:

Again, we recognize many site liaisons will have to make a qualitative judgment as to the most representative response.
18. May students in CCH classes at your institution use computers on most (or all) tests?
yes $\qquad$ no $\qquad$
19. Are students in CCH courses encouraged to use one particular graphing calculator?
yes .......... 1 no .......... 2

If yes, list brand and model:
20. May students in CCH classes at your institution use graphing calculators on most (or all) tests?
yes .......... 1 no .......... 2
21. Which phrase best describes the use of technology in CCH classes at your institution?
none or minimal ................ 1 moderate ................. 2 heavy ..................... 3
22. If you are using computer software and are a member of a group of institutions or part of a program that provides instructional material or support for the use of that software, please describe your situation and the impact of the program briefly. If not, proceed to question 23.
V. SUPPLEMENTARY INSTRUCTIONAL MATERIALS.
CIRCLE ALL APPROPRIATE RESPONSES23. What type(s) of non-CCH supplementary materials do students in CCH classes use?
Students use no non-CCH supplementary materials
..........................................................1
Materials for occasional use with technology ..... 2
Lab activities ..... 3
Materials for activities using manipulatives
Material intended to supplement or cover topics omitted in text. ..... 5
List topics
Supplementary exercises to reinforce basic algebra skills

$\qquad$6
Supplementary exercises to reinforce basic calculus skills
$\qquad$List skills
24. What CCH materials do students use in CCH courses other than the textbook?
Students use no other CCH materials
Test questions (on tests) from sample test items in the instructors' manual
.......................2
Student solution manual
.................................................................................................... ..... 3
25. If you wish, please use this space to make additional comments about the use of supplementary materials at your institution.

## VI. INSTRUCTOR SUPPORT

## CIRCLE ONE NUMBER IN EACH LINE

26. Does your institution provide release time and/or funding for instructors using CCH materials to attend staff development activities, conferences or workshops?

| a release time: | yes............. 1 | no............ 2 |
| :--- | :--- | :--- | :--- |
| b. funding: | yes.......... 1 | no.......... 2 |

If you wish. please provide comments. You may choose to indicate whether funding or release time is for local, regional, or national staff development activities. conferences or workshops.
27. Does your institution provide release time or funding for other preparation time related to teaching CCH calculus?

| a release time: | yes.............. 1 | no............. 2 |  |
| :--- | :--- | :--- | :--- |
| b. | funding: | yes.......... 1 | no......... 2 |

If you wish. please provide comments.

## CIRCLE ONE RESPONSE

28. What percentage of instructors teaching CCH courses attended one or more CCH sponsored workshops (or other workshops that used CCH materials as a platform)?

No one
less than $25 \%$....................................................................................................... 2
26-50\% ............................................................................................................. 3
51-75\% .............................................................................................................. 4
76 - $100 \%$............................................................................................................. 5
29. If you are involved in collaborative efforts with other institutions using CCH materials, please describe your situation briefly. If not, proceed to question 30.
30. If you are involved in interdisciplinary efforts with other departments at your institution that involve the use of CCH materials, please describe your situation briefly. If not, proceed to question 31.
31. If you wish. please use this space to provide additional comments about support for department members involved with students in CCH courses.

## CIRCLE ONE NUMBER IN EACH LINE

32. Based on your personal understanding of the priorities of your department and institution, indicate your agreement with the following statements.

| Least | Most |
| :--- | ---: |
| descriptive | descriptive |

a Calculus is considered a desirable course to teach..................................... 1 2 3 4
b. Your institution values and/or rewards teaching. $\qquad$ $\begin{array}{ll}1 & 2\end{array}$
45
c. Your mathematics department values and/or rewards teaching...............

12
d. Your mathematics department values and/or rewards experimentation in teaching............................................... 1 2 3
e. Your mathematics department or institution evaluates faculty teaching (beyond student evaluations)................ 123
f. Department resources are directed toward teaching.......................... 1 2 2 3 4
33. If you wish. please use this space to make additional comments about instructor support at your institution.

## VII. Departmental Perspectives

## CIRCLE ONE NUMBER IN EACH LINE

34. How well do the following statements characterize the viewpoints of most faculty members in your department? Base responses on your personal understanding. We recognize many site liaisons will have to make a qualitative judgment as to the most representative response.

|  | Least descri |  |  | Most <br> descriptive |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a Formal theorems should be emphasized. | I | 2 | 3 | 4 | 5 |
| b. Axiomatic structure should be emphasized. | 1 | 2 | 3 | 4 | 5 |
| c. Mathematical concepts should be emphasized. | I | 2 | 3 | 4 | 5 |
| d. Mathematical techniques should be emphasized. | 1 | 2 | 3 | 4 | 5 |
| e. Applications of mathematics should be emphasized. | 1 | 2 | 3 | 4 | 5 |
| f. Use of technology in mathematics courses should be increased. | I | 2 | 3 | 4 | 5 |
| g. Alternative pedagogical strategies should be used. | 1 | 2 | 3 | 4 | 5 |

h. Comments

## CIRCLE ONE NUMBER IN EACH LINE

35. How well do the following statements describe the pedagogical approaches of most faculty members in your department? Base responses on your personal understanding. We recognize many site liaisons will have to make a qualitative judgment as to the most representative response.

|  | Least <br> descriptive |  |  |  | Most <br> descriptive |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| a $\quad$ Use the lecture method...................... | 1 | 2 | 3 | 4 | 5 |

## CIRCLE ONE NUMBER IN EACH LINE

36. How often do CCH course instructors do the following?

| Rarely | Once or | Once or | Once or |  |
| :--- | :--- | :--- | :--- | :--- |
| or | twice a | twice a | twice a | Not |
| Never | term | month | week | applicable |

a Meet as a group to coordinate instructional

b. Meet as a group to discuss pedagogical issues.. $\qquad$ $1 \quad 2$

3
4
9
c. How often do CCH course instructors meet as a group to discuss other issues related to calculus reform. $\qquad$ l

3
4
9
d. Please comment, particularly if you responded "not applicable."
37. How often does the mathematics department at your institution meet?............ I
Rarely
or

Never $\quad$\begin{tabular}{l}
Once or <br>
twice a <br>
term

 

Once or <br>
twice a <br>
month

$\quad$

Once or <br>
twice a <br>
week
\end{tabular}

38. How often does the mathematics department meet to discuss the following?

|  |  | Rarely or Never | Once or twice a term | Once or twice a month | Once or twice a week |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | Pedagogical issues............................. | 1 | 2 | 3 | 4 |
| b. | Reform in calculus............................ | 1 | 2 | 3 | 4 |
| c. | Reform in mathematics education in general. | 1 | 2 | 3 | 4 |

39. If you wish, please use this space to make any further comments concerning departmental perspectives.

## VIII. REFORM ISSUES

40. CIRCLE THE ONE RESPONSE that is most descriptive of the initiation of the use of a reformed calculus text at your institution.

The mathematics department as a whole discussed calculus reform and agreed to try to initiate reform in calculus courses.

Most (or all) of the calculus instructors alone discussed calculus reform and agreed to try to initiate reform in calculus courses2

One, or a small group of, calculus instructors decided to try to initiate reform in
calculus course(s).
41. CIRCLE THE ONE RESPONSE that is most descriptive of the initial use of CCH materials at your institution.

The entire mathematics department agreed to the use of CCH materials. 1

The calculus instructors as a group, rather than the entire mathematics department, originally decided to use CCH materials

2

One, or a small group of, the mathematics department faculty decided to use
the CCH textbook in their own calculus classes................................................ 3
One, or a small group of, the mathematics department faculty decided to use the CCH textbook in all (or most all) calculus classes, including classes they did not teach.

## CIRCLE ONE NUMBER IN EACH LINE

42. Please answer all questions based on your personal understanding of the situation when reformed base calculus materials such as CCH materials were first used at your institution.

|  | yes | 0 |
| :---: | :---: | :---: |
| a Students were given an explanation about reformed calculus prior to their enrollment in a CCH class. | 1 | 2 |
| b. Students were given an explanation about reformed calculus after they had enrolled in a CCH class.. | 1 | 2 |
| c The department chair was consulted about the change............ | 1 | 2 |
| d. The dean or principal was consulted/informed about the change $\qquad$ | 1 | 2 |
| d. The board of trustees, superintendent or school board was consulted/informed about the change. | 1 | 2 |
| e. The change was publicized to the broader community in your institution beyond the mathematics department. | 1 | 2 |
| f. The change was publicized to the broader community beyond your institution. | 1 | 2 |
| g. Other or comments |  |  |

## CIRCLE ONE NUMBER IN EACH LINE

43. How well do the following describe the likelihood of each statement occurring at your institution? Base your response on your personal understanding.

|  |  | Least <br> likely |  |  | Most likely |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a | Use CCH materials in one or a few sections in the future. |  | 2 | 3 | 4 | 5 |
| b. | Use the CCH textbook in all (or almost all) sections in future years. | 1 | 2 | 3 | 4 | 5 |
| c. | Try non-CCH reform based calculus texts in future terms or years. | 1 | 2 | 3 | 4 | 5 |
| d. | Use the CCH precalculus materials within in the next year or two. | 1 | 2 | 3 | 4 | 5 |
| e | Use the CCH multivariable materials within the next year or two $\qquad$ | 1 | 2 | 3 | 4 | 5 |
| f. | Return to all traditional calculus texts .. | I | 2 | 3 | 4 | 5 |
|  | Other or comments |  |  |  |  |  |

4. Is the CCH textbook being used as a pilot program at your institution. the results of which will determine future use?

$$
\text { yes .......... } 1 \text { no .......... } 2
$$

45. Is the continued use of CCH materials dependent upon the sustained interest of a small percent of the calculus teachers at your institution?
yes .......... 1 no .......... 2
46. List all "reform based" calculus text books, other than CCH textbooks. your institution has previously used as the primary textbook in a calculus course.
47. List all "reform based" calculus textbooks, other than CCH textbooks, your institution is currently using as the primary textbook in a calculus course..
48. List all "reform based" calculus textbooks, other than CCH texts, your institution is considering as the primary textbook in a calculus course.

## CIRCLE ONE NUMBER IN EACH LINE

49. The following two questions refer to the reaction of others to the introduction of reformed calculus materials at your institution. Indicate your impression of the description of the accuracy of following statements.

| Least <br> descriptive |  | Most <br> descriptive |  |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |

b. You have used CCH materials for two or more years and student resistance is not an issue. $\qquad$ I
2
3
4
5
c. Parent resistance to CCH materials has been an issue.

1
2
3
4
5
d. Publicity about the introduction of CCH materials to the broader community, beyond the mathematics department, occurred without your attention or initial awareness.............. I 2

3
4
5
e. Other or comments
50.
Least Most Not
a The physical sciences departments have been supportive of the change to CCH materials..................... 1
b. The engineering department has been supportive of the change to CCH materials........ $\begin{array}{llllllll} & 1 & 2 & 3 & 4 & 5 & 9\end{array}$
c. The life sciences departments have been supportive of the change to CCH materials........ $\begin{array}{lllllll}1 & 2 & 3 & 4 & 5 & 9\end{array}$
d. The social sciences departments have been supportive of the change to CCH materials.................... $1 . \begin{array}{lllllll} & 2 & 3 & 4 & 5 & 9\end{array}$
e. Comments
51. CIRCLE THE ONE RESPONSE you consider most descriptive of faculty interest in calculus reform at your institution.
More than half of the mathematics department faculty members express
disinterest in calculus reform........................................................ I
More than half of the mathematics department faculty members hold strong opinions (positive or negative) about calculus reform. 2
52. If you wish. please use the space below to make any further comments concerning reform issues at your institution.

## IX. AVAILABILITY OF DATA AND/OR ARTIFACTS

53. Based on your present understanding of your department and institution, please CIRCLE ALL ITEMS that would be available, after obtaining appropriate permissions and clearances, for our use. Please remember that this is not an evaluation of the particular implementation at any individual institution.
Syllabi I
Assessment models: quizzes and tests.................................... 2
Other assessment models such as group problems, portfolios
or projects.......................................................................... 3
Assignments............................................................................ 4
Student lab manuals................................................................ 5
Teaching notes........................................................................ 6
Material used with English as a second language students
(ESL)........................................................................................ 7
Examples of common exam questions .................................... 8
Anonymous samples of student work in some or all of the
above categories................................................................. 9
54. If you wish, please use the space below to make any further comments about the availability of data and/or artifacts.
55. We would appreciate copies of any or all final examinations you have given to any classes using the CCH calculus materials. If you are willing. please include them when you return this survey or mail them in a separate envelope. We thank you for assisting us in this way.
56. This question refers to the availability of comparison data or evaluation effort findings. Based on your present understanding of your department and institution. please CIRCLE ALL ITEMS that would be available, after obtaining appropriate permissions and clearances, for our use. We realize not every institution has collected this data.

Data from common exam questions used across CCH and non-CCH sections. If yes, explain briefly I

Student comments on course evaluations. 2

Background data on students. If yes, explain briefly 3

Data on first to second semester, term, or year transition to or from CCH courses. If yes, explain briefly. $\qquad$ 4

Data related to students' majors. If yes, explain briefly. $\qquad$ 5

Other evaluation data or reports of CCH implementation.. If yes,
explain briefly
$\qquad$

Additional available data. Explain briefly. 7
57. Some of you have collected comparison data or have conducted evaluation efforts of your own and may be willing to share the data or the findings. If so, you may wish to include copies when you return this survey or mail them separately. We thank those of you who are able to send us this information.
58. If you wish. please use the space below to make any further comments about the availability of comparison data or evaluation effort findings.

## X OPPORTUNITIES FOR COMMUNICATION

59. Some site liaisons have asked for names of faculty and/or institutions using computer software or graphing calculators in conjunction with the CCH materials. We plan to make available a list that includes the name of the site liaison. institution. phone number, address, and e-mail address (if available). Would you like to be included on such a list?

$$
\text { yes .......... } 1 \text { no .......... } 2
$$

60. Some secondary site liaisons have asked for names of faculty and/or institutions using CCH materials in secondary school classes. We plan to make available a list that includes the name of the site liaison, institution, phone number, address. and email address (if available). If you are a secondary school site, would you like to be included on such a list?
yes .......... 1 no .......... 2
61. If there is interest, we are willing to make available a list of post-secondary as well as secondary school sites that are using CCH materials. This list would include the name of the site liaison, institution, phone number, address, and e-mail address (if available). Would you like to be included on such a list?

$$
\text { yes .......... } 1 \text { no .......... } 2
$$

THANK YOU FOR PARTICIPATING IN THIS SURVEY!
Please add any additional comments you wish.

## CCH Evaluation and Documentation Project

Phase A-Questionnaire: An initial contact was made with all sites who are currently using or have used the CCH materials. This questionnaire was intended to identify appropriate contact names at each site. gather basic information about the nature of the site and the nature of the implementation of the materials. determine which sites were willing to participate in faculty and student data-gathering, and determine what information would be of interest to the sites.

Phase B-Extensive Site Survey: Mailed in February to all site liaisons, this survey examines the contextual features of the CCH implementation. Site liaisons who complete this survey are compensated for their time. This survey is intended to determine the nature and level of information that might be available with additional effort (student attitude, scores, comparative information, tracking data, other evaluation data. etc.). identify factors influencing the initiation of reform efforts, identify the types of students/courses using CCH materials, and determine department and university reaction to the use of the materials.

Phase C-Students and Faculty: On the basis of the results of Phase A, some sites will be selected to receive the Phase $C$ packet of materials sufficient for surveying five faculty members, and two sections of students. The faculty survey will focus on the interpretation and use of the CCH materials, the pedagogical characteristics of the courses, and faculty attitudes and beliefs. The student survey will include affective as well as conceptual items. Site liaisons will obtain whatever local permissions are necessary and facilitate the distribution and return of the materials. They will be compensated for their work.

Phase D-Case Studies: Current plans call for a "tracking" emphasis at a few sites, in addition to a "case study/student learning" emphasis at other sites. Certain basic data will be gathered by telephone interviews with those sites, so that some cross site analysis will be possible. Evaluation staff visits will also be conducted at each of these sites, facilitated by on-site documenters. We will collect artifacts such as: exams and assessment tools, syllabi and assignments, student work, attitude surveys of students and faculty, student responses to "standardized" final exam items, and additional information.

## APPENDIX D

## FACULTY SURVEY

## CCH Evaluation and Documentation Project

## Fall 1995

$\qquad$

## CCH EVALUATION AND DOCUMENTATION PROJECT FACULTY QUESTIONNAIRE

Thank you for agreeing to complete this questionnaire. The Faculty questionnaire is part of the evaluation and documentation of the Calculus Consortium Based at Harvard (CCH) single-variable (or first-year) calculus project. This current evaluation and documentation is being done at the request of CCH, as an independent activity. The CCH project is sponsored by the National Science Foundation as a Calculus Curriculum Reform Project.

The purposes of this effort are to learn how the stated goals are interpreted and implemented, to investigate student and faculty variables, to examine and describe the evolution of calculus reform efforts, and to promote collaboration and sharing among those involved in the implementation and evaluation of calculus reform. Plans include conducting the evaluation and documentation in four major phases as outlined in the back of this booklet. When reading these descriptions it might be helpful to keep in mind that this questionnaire is part of phase $\mathbf{C}$.

## About This Questionnaire

This questionnaire includes eight parts.
i. Introductory Questions
II. CCH Materials
III. CCH Workshops
IV. Instruction
V. Your CCH Course
VI. Student Learning
VII. Student Assessment
VIII. Calculus Reform

Time needed to complete this faculty questionnaire is approximately one hour. We welcome written comments in the spaces provided or in any section of the questionnaire. Your participation is voluntary and will not affect any professional relationships. Your answers will be kept strictly confidential. Note that this is not an evaluation of the particular implementation at any individual institution. This research is conducted under university and government regulations designed to safeguard study participants. Identification codes are used only for follow-up purposes, such as linking Questionnaire A, Survey B, Student Survey, and Faculty Questionnaire responses and determining case study and flow-through study sites. Results of the questionnaires and surveys will be reported only in summary or statistical form so that neither individuals nor their institutions can be identified.

Thank you for contributing your time and thoughtful responses to this evaluation and documentation effort. After completing the questionnaire, please seal it in the enclosed envelope and give it to your site liaison who will return all questionnaires from your institution. Your site liaison will record your social security number and indicate you completed the questionnaire on a form we have provided. You will receive an honorarium of $\$ 15$ for completing the questionnaire. We hope that you will find the questions professionally meaningful and interesting.

## FOR FURTHER INFORMATION

If you have any questions about the questionnaire, please feel free to call us at the University of New Hampshire.
Darien Lauten, Project Manager. $603862-2320$ or $603868-7133(\mathrm{~h})$;
e-mail: dlauten@christa.unh.edu
Joan Ferrini-Mundy, UNH, Project Director: 603 862-2320
Karen Graham, Project Co-Director. 603 862-2320

## general information and instructions

- This questionnaire is intended for current or previous instructors of CCH single variable calculus. If you no longer instruct first year calculus or use the CCH textbook, please answer in terms of your most recent experience using the CCH materials.
- When we use the phrases "CCH textbooks" or "CCH materials" in this questionnaire, we are referring to the first-year calculus course (single-variable calculus) developed by the Calculus Consortium Based at Harvard. Unless specifically mentioned in a particular question, this questionnaire does not address the multi-variable or precalculus materials.
- This questionnaire will be completed by CCH instructors at universities, 4-year colleges, 2 -year colleges and secondary schools using CCH textbooks. Please answer the questions as best as you can for your situation. We find comments explaining your situation helpful. You may choose to omit any question.
- The word "term" indicates semester, quarter, trimester, or a comparable word or phrase at your institution.
- If this is your first experience teaching with CCH materials, you may feel you would like to complete this questionnaire again after you have had more time to reflect upon your experience. Please complete this questionnaire now and ask your site liaison to note on the return form that you would like an additional questionnaire sent for you to complete at a later date. In order to include information from your second questionnaire in our analysis, we should receive it by February, 1996.


## I. INTRODUCTORY QUESTIONS

1. Please circle the one response that best describes your teaching situation.
a. Full-time faculty member ....................................................... 1

If you are a full-time faculty member, please select one of the following responses
i. tenure track .................................................................. i
ii. non-tenure track...............................................................ii
iii. these categories do not apply at my institution....................iii
b. Part-time or adjunct faculty member........................................ 2
c. Graduate student.................................................................. 3
d. Other (please describe) ....................................................... 4
2. Please circle the response(s) that best describe(s) the highest academic degree you have received.
a. Ph.D. ..... 1
major field of study:
b. Ed.D. ..... 2
major field of study:
c. LL.D. or M.D. ..... 3
d. M.S. or M.A. ..... 4
major field of study:
e. B.S. or B.A ..... 5
major field of study:f. Other (please describe)6
3. Including the current academic year, please indicate the number of years you have taught mathematics, rounding when necessary. If during some years you taught at both the secondary and collegiate levels, please select the teaching level that best reflects your primary teaching responsibilities.
a. Collegiate level $\qquad$

| 1 | $2-3$ | $4-5$ | $6 \cdot 10$ | $11-15$ | more than 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

b. Secondary level $\qquad$ 1
c. Other (please describe). $\qquad$ 1 23 4 6
4. Including the current academic year, during how many years have you taught at least one term of first year calculus?

| PLEASE CIPCLE ONE NUMBER |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $2-3$ | $4-5$ | $6-10$ | $11 \cdot 15$ | more than 15 |
| 1 | 2 | 3 | 4 | 5 | 6 |

5. a. Is your institution currently using CCH materials as the primary textbook in at least one first year calculus section or course?
yes ...... 1 no ...... 2
If you answered "no", please answer the following questions, otherwise proceed to \#6.
b. When did your institution most recently use the CCH textbook as the primary textbook in any first-year calculus section or course?
c. Does your institution plan to use the CCH textbook as the primary textbook in any first-year calculus section or course in the foreseeable future?
yes ...... 1
no ...... 2
not sure ...... 3
6. In the chart below, please indicate the types of course sections you are currently teaching. If you are between terms, refer to the previous term or the most recent term you taught mathematics.

PLEASE ANSWER ALL STATEMENTS THAT PERTAIN TO YOUR SITUATION.

|  | yes | по | * of sections | $\begin{array}{\|l\|} \text { total } \# \text { of } \\ \text { students } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| a. Single-variable CCH calculus ................................ | 1 | 2 | - | - |
| b. Other CCH courses (please specify) ........................ | 1 | 2 | $\square$ |  |
| c. Non-CCH first-year calculus (Please specify textbook used) | 1 | 2 | - | - |
| d. Interdisciplinary courses involving CCH materials. <br> (Please describe briefly.) | 1 | 2 | - | - |
| e. Interdisciplinary courses involving calculus but not using CCH materials (Please describe briefly.). | 1 | 2 | - | - |
| f. Other mathematics courses (Please list).................. | 1 | 2 | - | - |
| g. Other non-mathematics courses (Please list)............. | 1 | 2 | - | - |

## II. CCH MATERIALS

7. This question addresses your opinion about the treatment and amount of emphasis placed on topics in the CCH materials. Please select one response for presentation and one response for the amount of attention. We are asking you to select two responses for each line. If you are unfamiliar with the CCH presentation of a particular topic, please leave that line blank. (Presentation refers to the way the authors chose to introduce, treat, and develop each topic.)

|  | PRESENTATION |  |  | amount of emphasis |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { below } \\ \text { average } \end{gathered}$ | average | $\begin{gathered} \text { above } \\ \text { average } \\ \hline \end{gathered}$ | $\begin{gathered} \text { Not } \\ \text { enough } \end{gathered}$ | About right | $\begin{aligned} & \text { Too } \\ & \text { much } \\ & \hline \end{aligned}$ |
| a. Functions ............................. | 1 | 2 | 3 | 1 | 2 | 3 |
| b. Limit concept........................ | 1 | 2 | 3 | 1 | 2 | 3 |
| c. Taking limits ........................ | 1 | 2 | 3 | 1 | 2 | 3 |
| d. Concept of the derivative .......... | 1 | 2 | 3 | 1 | 2 | 3 |
| e. Techniques of differentiation ...... | 1 | 2 | 3 | 1 | 2 | 3 |
| f. Applications of the derivative...... | 1 | 2 | 3 | 1 | 2 | 3 |
| g Antidifferentiation................... | 1 | 2 | 3 | 1 | 2 | 3 |
| h. Fundamental Theorem of Calculus. | 1 | 2 | 3 | 1 | 2 | 3 |
| i. Concept of the definite integral.... | 1 | 2 | 3 | 1 | 2 | 3 |
| j. Techniques of integration.......... | 1 | 2 | 3 | 1 | 2 | 3 |
| k. Applications of integration | 1 | 2 | 3 | 1 | 2 | 3 |
| I. Concept of differential equation .. | 1 | 2 | 3 | 1 | 2 | 3 |
| m. Solving differential equations ..... | 1 | 2 | 3 | 1 | 2 | 3 |
| n. Applications of differential equations | 1 | 2 | 3 | 1 | 2 | 3 |
| o. Series ............................... | 1 | 2 | 3 | 1 | 2 | 3 |
| Please add any other topics that you feel we should have included. |  |  |  |  |  |  |
| $i$. | 1 | 2 | 3 | 1 | 2 | 3 |
| $i{ }^{\text {i }}$ | 1 | 2 | 3 | 1 | 2 | 3 |

Comments:
8. Many reform calculus projects make the following claims. Please indicate below your feelings about whether the CCH textbook addresses each of these claims.

|  | PLEASE CIRCLE ONE NUMBER IN EACH LINE  <br> Strongly <br> disagree Strongly <br> agree |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a. Requires deeper understanding of calculus concepts than standard calculus textbooks..... | 1 | 2 | 3 | 4 | 5 |
| b. Requires less routine manipulation than more conventional calculus textbooks. | 1 | 2 | 3 | 4 | 5 |
| c. Covers less material in greater depth than conventional calculus textbooks. | 1 | 2 | 3 | 4 | 5 |
| d. Introduces topics graphically, numerically, and analytically, giving equal time to each component $\qquad$ | 1 | 2 | 3 | 4 | 5 |
| e. Develops concepts from common sense investigations rather than from abstract definitions $\qquad$ | 1 | 2 | 3 | 4 | 5 |
| f. Supports meaningful use of computers and (or) calculators | 1 | 2 | 3 | 4 | 5 |
| g. Is written for students to read...................... | 1 | 2 | 3 | 4 | 5 |
| h. Develops a sense of how mathematics is used in today's world. | 1 | 2 | 3 | 4 | 5 |

9. Among the above claims, the CCH Instructor's Manual indicates that the CCH textbook is written with the following goals in mind. Please further discuss your observations and opinions about how well the CCH materials accomplish these goals.
a. "Let formal definitions and proofs evolve from a long process of common sense investigations, rather than to start with abstract definitions."
b. Emphasize "The Rule of Three: Every concept should be introduced graphically, numerically, and analytically."
10. a. Please state your own definition of the phrase "mathematical rigor".
b. Based on your own definition, please describe the general level of mathematical rigor in the CCH materials.

c. Please explain your response briefly.
11. a. Some instructors have observed that CCH materials provide access to a wider range of calculus students than standard or traditional calculus materials.
Please indicate your agreement with that observation.

| Strongly <br> disagree |  | Strongly <br> agree |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

b. Please describe the types of students (if any) you believe experience greater access to calculus through the use of CCH materials.
c. Please describe the types of students (if any) you believe experience decreased access to calculus through the use of CCH materiais.
12. Some respondents to previous CCH surveys have indicated that the way they introduced the CCH textbooks affected students' attitudes about the course. The word "introduce" refers to spoken or written words that inform students of possible differences between CCH and standard calculus textbooks.
a. How did you introduce the CCH textbook to your students?
b. Are you satisfied with the introduction you used?

```
yes ..... 1 no ...... }1\mathrm{ not sure ...... }
```

c. If you answered "no", please explain why not and describe how would you would change that introduction.
13. Please indicate how helpful you find the following items in the CCH Instructor's Manual.

14. In your opinion, what are the strengths of the CCH materials?.
15. What changes would you suggest if a revision of the CCH first year single-variable calculus textbook is published?

## III. CCH WORKSHOPS

16. a. Have you attended one or more workshops about the CCH single-variable calculus materials?
yes ...... 1 no ...... 2
If you answered "yes" to part "a", please answer the following questions that refer to these workshops. Otherwise, proceed to \#17.
b. What benefits did you receive from your workshop experience?
c. In what ways could the workshop have been improved to better meet your needs?
d. In what ways did the workshop(s) change (or reinforce) your perceptions of calculus reform and (or) the CCH materials?
e. Please complete the following by circling one number.


## IV. INSTRUCTION

17. Please indicate your agreement with the following statements about first-year calculus (not limited to CCH). Please give us your opinion about the level of emphasis for each item. You may find it helpful to read all of the items before responding.

PLEASE CIRCLE ONE NUMBER IN EACH LINE Little or
no Heavy
emphasis emp___________

18. The following question refers to the amount of time you spend outside of class each week on the following activities as they relate to teaching a CCH course. Please round to the nearest number of hours.

| PLEASE CIRCLE ONE NUMBER IN EACH LINE |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 hour | 2 | 3 | 4 | 5 | 6 hours |  |
| or less | hours | hours | hours | hours | or more |  |


19. Please compare the instructor time requirements for teaching the CCH course with teaching standard calculus courses. If you have not used CCH materials more than once, please omit part "b".

| once, please omit part b. | please circle one number in each line |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Takes considerably less time |  | Takes a similar amount of time |  | Takes considerably more time |
| a. First experience using CCH materials | 1 | 2 | 3 | 4 | 5 |
| b. Subsequent experiences using CCH materials. | 1 | 2 | 3 | 4 | 5 |

20. In what ways did the CCH material influence changes in your preparation and (or) teaching of calculus?
21. In what ways did the CCH materials influence how you think about calculus concepts or ideas?

## V. YOUR CCH COURSE

22. This question addresses your teaching of topics in the CCH textbook. Please add any comments you feel would help us interpret your responses in the space provided.

|  |  | Follow the textbook presentation | E CIRCLE ON <br> Use a more standard presentation | UMBER IN EACH LINE <br> Use a presentation different from both the CCH and a standard calculus approach | Omit this topic |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a. | Functions ............................. | 1 | 2 | 3 | 4 |
| b. | The limit concept.................... | 1 | 2 | 3 | 4 |
| c. | Taking limits ........................ | 1 | 2 | 3 | 4 |
| d. | Concept of derivative ............... | 1 | 2 | 3 | 4 |
|  | Techniques of differentiation ...... | 1 | 2 | 3 | 4 |
|  | Applications of the derivative...... | 1 | 2 | 3 | 4 |
|  | Antidifferentiation.. | 1 | 2 | 3 | 4 |
| h. | Fundamental Theorem of Calculus | 1 | 2 | 3 | 4 |
|  | Concept of the definite integral.... | 1 | 2 | 3 | 4 |
|  | Techniques of integration.......... | 1 | 2 | 3 | 4 |
|  | Applications of integration ........ | 1 | 2 | 3 | 4 |
| 1. | Concept of the differential equation $\qquad$ | 1 | 2 | 3 | 4 |
|  | Solving differential equations ..... | 1 | 2 | 3 | 4 |
|  | Applications of differential equations | 1 | 2 | 3 | 4 |
| o. | Series .............................. | 1 | 2 | 3 | 4 |
|  | Other (Please list and circle ap $i$. | opriate numb 1 | 2 | 3 | 4 |
|  | ii. | 1 | 2 | 3 | 4 |

## Comments:

23. Do you change the order of the presentation of any topics in the CCH textbook?
yes $\qquad$ 1
no $\qquad$ 2

If you answered "yes", please describe any changes that come to mind.
24. Do you add topics to your CCH course that are not included in the CCH textbook?
yes .1
no 2

If you answered "yes", please list the topics you add.
25. Please select the response that best represents the expected student use of technology in your CCH course.
rarely or never frequently class meeting.
a. Calculations using calculators $\qquad$ 1

2 3
b. Graphing on graphics calculators $\qquad$ 1 2 3
c. Producing or manipulating graphs on a computer.

1
2
3
d. Using symbol manipulation software on a computer or calculator with that capability ...... 1
e. Using or writing programs on a programmable calculator .................................................. 1 12

3
f. Using or writing programs on a computer ......... 1 2 3
g. Using spreadsheets or other numerical tableproducing programs on a computer or calcuiator with that capability. $\qquad$ 1
2
3
h. Other uses of calculators or computers (piease describe)

1
2
3
26. Please discuss your views about the use of technology in first-year calculus courses.
27. In a typical week of CCH calculus teaching, please estimate the amount of class time spent on each of the following activities.
please circle one number in each line $\begin{array}{llllll}0-10 \% & 11-20 \% & 21-50 \% & 51-70 \% & 71 \cdot 90 \% & 91-100 \%\end{array}$
a. Lectures or mini-lectures $\qquad$
1 五
2

| 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- |
| 3 | 4 | 5 | 6 |
| 3 | 4 | 5 | 6 |
| 3 | 4 | 5 | 6 |
| 3 | 4 | 5 | 6 |
| 3 | 4 | 5 | 6 |

f. Writing in joumals............................
g. Writing about a mathematics concept or how to solve a problem (in everyday language)
h. Engaging in student dominated large group discussions .........................
12
i. Engaging in exploratory activities that do not have "right answers" or prescribed procedures ....................
j. Lab activities.

| 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 2 | 3 | 4 | 5 | 6 |


| . | Individual quizzes or tests .............. | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Small group quizzes or tests............ | 1 | 2 | 3 | 4 | 5 | 6 |
| m. | Other (Please describe)................. | 1 | 2 | 3 | 4 | 5 | 6 |

28. Please describe any conditions at your institution that prevent you from using teaching approaches that you would like to use in your CCH class.
29. Do you consider calculus a desirable course to teach at your institution?
yes .1
no ...... 2
30. Please comment briefly on any innovations you have included in your course.

## VI. STUDENT LEARNING

31. In some student surveys, the responding students indicated concern about examples in CCH materials not serving as templates or scripts for exercise problems. Please estimate how many of your students share this concern by circling one number.

| Few or <br> none | 2 | Many <br> or all |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |

32. Please discuss briefly your observations about your students' ability to interpret information from a qualitative graph for which they are not given an equation.
33. The following items ask you to compare students' understanding of calculus topics in CCH courses versus standard courses.
a. Please list the calculus topics, processes, or procedures that, in your opinion, CCH students understand better than students in standard calculus courses.
b. Please list the calculus topics, processes, or procedures that, in your opinion, CCH students do not understand as well as students in standard calculus courses.
c. Did any of the differences, that you noted in parts "a" or "b" above surprise you? Please explain.
34. There is considerable interest in what happens when CCH students move on to other mathematics courses. What are your impressions, and those of your colleagues who receive CCH students, about student performance or attitudes in other mathematics courses?
35. Please discuss briefly any observations made by faculty from other departments regarding CCH student performance related to calculus in courses they teach and their impressions of CCH student understanding of calculus. If you can, please include examples.
36. Some institutions use "gateway tests," tests that measure student mastery of skills and procedures taught in previous courses and needed for current or subsequent courses. Students often are given a specified length of time to pass these tests in order to receive credit for the current course. Do you require any type of "gateway test" in your CCH course?
yes ...... 1 no ...... 2
If you answered "yes", please briefly describe the test, its method and time-frame of administration, and the ramifications for a student not passing.

## VII. STUDENT ASSESSMENT

## 37. Beside each item that you might use to assign grades in CCH calculus sections, please indicate the percent of the final term grade assigned to that item.

a. Quizzes and tests that measure individual mastery of content
material.........................................................................................__ \%
b. A final examination that measures individual mastery of content material........__\%
c. Small group tests of mastery of content material ........................................__ \%
d. Lab reports (individual grades)................................................................___\%
e. Lab reports (group grades) ....................................................................__\%
f. Quizzes, tests, or examinations of material leamed in labs...........................___ \%
g. Homework exercises (individual grades) ..................................................___\%
h. Homework exercises (group grades) .......................................................__ \%
i Individual projects ...............................................................................___\%
j. Projects .........................................................................................__\%
i. individual grades ....................................................................___
ii. group grades ........................................................................___
k. Journals ........................................................................................... ___ \%
I. Class participation ..............................................................................___\%
m. Portfolios ..........................................................................................___\%
n. Other (Please describe below) ..............................................................____\%
38. Many who question the success of calculus reform ask about student success rates and the retention of students in reform based calculus courses. This question addresses those issues. Please provide your best estimate concerning average student retention and success rates in your CCH course. If you are teaching a CCH course for the first time this term and it is too early to answer this question, please proceed to \#39.

|  | Please CIRCLE ONE Number in each line |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0-10\% | 11-20\% | 21.50\% | 51-70\% | 71-90\% | 91-100\% |
| a. Complete the course with a C or better | 1 | 2 | 3 | 4 | 5 | 6 |
| b. Withdraw from the course before the end of the term | 1 | 2 | 3 | 4 | 5 | 6 |
| c. Complete the course with a D or worse | 1 | 2 | 3 | 4 | 5 | 6 |

39. Please make comparisons between the success and retention rates of CCH students and students in standard calculus students.

## VIII. CALCULUS REFORM

40. Please indicate your support of calculus reform by circling one number.

41. Please indicate how well the CCH materials are aligned with your interpretation of calculus reform by circling one number.

42. Please indicate your opinion about the extent to which each of the following has influenced calculus reform.

| PLEASE CIRCLE ONE NUMBER IN EACH LINE |
| :---: |
| strong <br> little or no <br> influence |

a. Students' pre-college preparation in mathematics

1

| 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 |

43. Please indicate the value you place on each of the following aspects of calculus reform.
a. The calculus syllabus should contain fewer topics
...................................................
b. The calculus syliabus should place greater emphasis on numeric and geometric interpretations ns...
Calculus courses should make greater use of technology
d. Decreased emphasis should be placed on the practice of algorithmic procedures.

1
e. Time should be set aside for students to explore complex problems
. Calculus teaching should become more interactive (students interacting with students and students interacting with the instructor).
g. Time should be set aside in calculus courses for students to develop an understanding of how mathematics is used in today's world.
h. Faculty teaching calculus should confer regularly with other disciplines that expect their students to take calculus
....................................
First-year calculus should be accessible to a wide range of students
Comments:
please circle one number in each line Value

| litte or <br> not at all |
| :---: |


| 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |

44. Please indicate the value you place on each of the following items that many proponents of calculus reform have felt students should learn or understand.

|  | please circle one number in each une Value <br> litte or <br> Value <br> not at all <br> highly |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a. The concept of change............................... | 1 | 2 | 3 | 4 | 5 |
| b. The concepts of local and global behavior .......... | 1 | 2 | 3 | 4 | 5 |
| c. The concepts of approximation and error........... | 1 | 2 | 3 | 4 | 5 |
| d. The role of mathematics in modeling and understanding the real world. | 1 | 2 | 3 | 4 | 5 |
| e. The beauty of mathematics .......................... | 1 | 2 | 3 | 4 | 5 |
| f. That functions are not just given by formulas but can be represented graphically or by tables of data. | 1 | 2 | 3 | 4 | 5 |
| g. The need to give a coherent mathematical argument to justify answers. | 1 | 2 | 3 | 4 | 5 |
| h. To apply mathematics in different contexts......... | 1 | 2 | 3 | 4 | 5 |
| i. To generalize results.................................. | 1 | 2 | 3 | 4 | 5 |
| j. To analyze non-routine problem situations both quantitatively and qualitatively. $\qquad$ | 1 | 2 | 3 | 4 | 5 |
| k. To read mathematics | 1 | 2 | 3 | 4 | 5 |
| I. To develop a deeper understanding of calculus concepts through the use of technology | 1 | 2 | 3 | 4 | 5 |
| m. To remain engaged with one problem (perhaps as a project) for an extended period of time............. Comments: | 1 | 2 | 3 | 4 | 5 |

45. What do you find encouraging about the direction of calculus reform and what are your concerns?

| Aspects of calculus reform you find encouraging | Aspects of calculus reform that concern you |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

THANK YOU FOR COMPLETING THIS QUESTIONNAIRE!
Please add any additional comments you wish.

## CCH Evaluation and Documentation Project

Phase A-Questionnaire: An initial contact was made with all sites who are currently using or have used the CCH materials. This questionnaire was intended to identify appropriate contact names at each site, gather basic information about the nature of the site and the nature of the implementation of the materials, determine which sites were willing to participate in faculty and student data-gathering, and determine what information would be of interest to the sites.

Phase B-Extensive Site Survey: Mailed in March, 1995 to all site liaisons, this survey examined the contextual features of the CCH implementation. Site liaisons who completed this survey were compensated for their time. This survey was intended to determine the nature and level of information that might be available with additional effort (student attitude, scores, comparative information, flow-through data, other evaluation data, etc.), identify factors influencing the initiation of reform efforts, identify the types of students/courses using CCH materials, determine department and university reaction to the use of the materials, and obtain blank copies of student assessment materials.

Phase C-Students and Faculty: On the basis of the results of Phase A, some sites have been selected to receive the Phase C Student Surveys or Faculty Questionnaires. Phase C includes surveying five faculty members and up to 50 students. The faculty questionnaire focuses on the interpretation and use of the CCH materials, the pedagogical characteristics of the courses, and faculty attitudes and beliefs. The student survey includes affective items. Site liaisons facilitate the distribution and return of the materials. Site liaisons and faculty members completing the faculty questionnaires will receive honoraria.

Phase D-Case Studies: Current plans call for a "flow-through" emphasis at a few sites where student participation and achievement patterns beyond calculus will be studied. In addition, case studies will be conducted at two other sites. Certain basic data will be gathered by telephone interviews with those sites, so that some cross site analysis will be possible. Evaluation staff visits will also be conducted at each of these sites, facilitated by on-site documenters who will collect artifacts such as: exams and assessment tools, syllabi and assignments, student work, attitude surveys of students and faculty, and additional information.

## APPENDLX E

## APPENDIX E

## CLUSTERING SCALES

Appendix E contains two sets of tables. The first set of tables lists the survey items assigned to each clustering scale for the cluster analysis. The second set of tables lists the survey items that were used for comment data in the description of the profiles of reform.

Tables E1 through E6 list the survey items assigned to each clustering scale for the cluster analysis, indicating the responses considered consistent with the goals of reform-based calculus and those considered neutral or inconsistent. For each survey item, the academic institution (case) received a value of 1 for responses considered consistent with calculus reform and a 0 for inconsistent responses.

The "balance point" for responses to each item was assigned a value of 0 . For example, on Likert type items, cases with responses less than or equal to 3 received a value of 0 , and cases with responses greater than 3 received a value of 1 . See the table for the assignment of values to survey items with categorical responses.

Note also that participating academic institutions returned between one and five Faculty Surveys. If an institution returned more than one Faculty Survey, the responses to each item were averaged. The averaged response determined whether the institution received a value of 0 or 1 on the item. Each participating academic institution returned only one Site Liaison Survey; therefore, responses to the Site Liaison Survey were not averaged and a 0 or 1 was assigned directly.

## Table EI

## Concepts

| Scale <br> variable <br> name | Survey questions <br> B: refers to Site-Liaison Survey <br> F: refers to Faculy Survey | Response(s) <br> consisent <br> with reform- <br> based calculus <br> (neg. | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1) |
| :---: | :--- | :---: | :---: | :---: |
| C01 | B13i (neg. <br> students practice procedural <br> skills | 1,2 | 3,4 | $\geq 3$ |
| C02 | B13j Frequency students make <br> conjectures and explore more <br> than one possible method to <br> solve a calculus problem | 4,5 | 1,2, | $\leq 2$ |
| C03 | F17a (neg.) Values emphasis <br> on formal definitions in calculus | 1,2 | $3,4,5$ | $\geq 3$ |
| I |  |  |  |  |


| C16 | F44e Values students should <br> understand the beauty of <br> mathematics | 4,5 | $1,2,3$ | $\leq 3$ |
| :---: | :--- | :---: | :---: | :---: |
| C17 | F44g Values the need to give a <br> coherent mathematical argument <br> to justify answers | 4,5 | $1,2,3$ | $\leq 3$ |
| C18 | F44i Values students' ability <br> to generalize results | 4,5 | $1,2,3$ | $\leq 3$ |
| C19 | F44m Values that students <br> should learn to remain engaged <br> with one problem (perhaps as a <br> project) for an extended period <br> of time | 4,5 | $1,2,3$ | $\leq 3$ |

Table E2
Approach

| Variable name for scale | Survey questions <br> B: refers to Site-Liaison Survey F: refers to Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> $(0)$ | Response(s) assigned a value of 0 (Others assigned a value of 1 ) |
| :---: | :---: | :---: | :---: | :---: |
| AP01 | F17g Values an emphasis on applications of real world problems | 4,5 | 1,2,3 | $\leq 3$ |
| AP02 | F25 b, c Frequency students use calculators or computers for graphs | 2, 3 | 1 | $\leq 1.5$ |
| AP03 | F25g Frequency students use calculator or computer for spreadsheets or numerical tables | 2,3 | 1 | $\leq 1.5$ |
| AP04 | F43b Values greater emphasis on numeric and geometric interpretations | 4, 5 | 1,2,3 | $\leq 3$ |
| AP05 | F43g Values setting aside more time to develop an understanding of how mathematics is used in today's world | 4,5 | 1,2,3 | $\leq 3$ |
| AP06 | F44b Values students understanding the concepts of global and local behavior | 4,5 | 1,2,3 | $\leq 3$ |
| AP07 | F44C Values concepts of approximation and error | 4,5 | 1, 2, 3 | $\leq 3$ |
| AP08 | F44d Values the role of mathematics in modeling and understanding the real world | 4,5 | 1,2,3 | $\leq 3$ |
| AP09 | F44f Values that students should understand functions are not just given by a rule or a formula | 4, 5 | 1, 2, 3 | $\leq 3$ |
| AP10 | F44h Values the importance of students learning to apply mathematics in different contexts | 4,5 | 1,2,3 | $\leq 3$ |
| AP11 | F44j Values students' ability to analyze problem situations both quantitatively and qualitatively | 4,5 | 1,2,3 | $\leq 3$ |

Table E3

## Teaching

| Scale variable name | Survey questions <br> B: refers to Site-Liaison Survey <br> F: refers to Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) inconsistent with reformbased calculus (or neutral) (0) | Response(s) assigned a value of 0 (Others assigned a value of 1 ) |
| :---: | :---: | :---: | :---: | :---: |
| TA01 | B13a Frequency students participate in a lab | 3,4 | 1,2 | $\leq 2$ |
| TA02 | BI3f Frequency students take lecture notes | 1,2 | 3,4 | $\geq 3$ |
| TA03 | B13g Frequency students use concrete materials to explore calculus ideas | 3,4 | 1,2 | $\leq 2.5$ |
| TA04 | B13h Frequency students do small group work on mathematics problems during class | 3,4 | 1,2 | $\leq 2.5$ |
| TA05 | B13k Frequency students write in journals during class | 3,4 | 1,2 | $\leq 2.5$ |
| TA06 | BI31 Frequency students work on projects during class | 3,4 | 1, 2 | $\leq 2.5$ |
| TA07 | BI3m Frequency students write about how to solve a problem on assign or test | 3,4 | 1,2 | $\leq 2.5$ |
| TA08 | BI3n Frequency students engage in mathematical exploration | 3,4 | 1,2 | $\leq 2.5$ |
| TA09 | B23c Use supplementary materials for lab activities | 1 | 0 | 0 |
| TA10 | B23d Use supplementary materials for activities using manipulatives | 1 | 0 | 0 |
| TAII | F17j Values the use of alternative teaching strategies | 4, 5 | 1, 2, 3 | $\leq 3$ |
| TA12 | F27a Amount of class time spent on lectures or minilectures | 1,2 | 3, 4, 5, 6 | $\geq 2.5$ |
| TA13 | F27b Amount of class time spent on using manipulatives | 2, 3, 4, 5, 6 | 1 | $\leq 1.5$ |
| TA14 | F27d Amount of class time spent on group work on problems in class | 3, 4, 5, 6 | 1,2 | $\leq 2.5$ |
| TA15 | F27f Amount of class time spent on writing in journals | 2,3,4, 5, 6 | 1 | $\leq 1.5$ |
| TA16 | F27g Amount of class time students spend writing about a mathematics concept or how to solve a problem | 2, 3, 4, 5, 6 | 1 | $\leq 1.5$ |


| TA17 | F27h Amount of class time <br> spent on student dominated <br> large group discussions in class | $3,4,5,6$ | 1,2 | $\leq 2.5$ |
| :--- | :--- | :---: | :---: | :---: |
| TA18 | F27i Amount of class time <br> spent engaging in exploratery <br> activities that do not have "right <br> answers" or prescribed <br> procedures | $3,4,5,6$ | 1,2 | $\leq 2.5$ |
| TA19 | F27j Amount of class time <br> spent on lab activities | $3,4,5,6$ | 1,2 | $\leq 2.5$ |
| TA20 | F43f Agrees calculus <br> teaching should become more <br> interactive | 4,5 | $1,2,3$ | $\leq 3$ |

Table E4

## Assessment (Assess)

| Scale variable name | Survey questions <br> B: refers to Site-Liaison Survey <br> F: refers to Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) <br> ( | Response(s) assigned a value of 0 (Others assigned a value of 1) |
| :---: | :---: | :---: | :---: | :---: |
| ASO1 | B15b Grades include small group tests of mastery of content | 1 | 0 | 0 |
| AS02 | B15 c Grades include lab reports (individual grades) | 1 | 0 | 0 |
| AS03 | B15 d Grades include lab reports-(group grades) | 1 | 0 | 0 |
| AS04 | BISf Grades include homework exercises (group grades-\# correct) | 1 | 0 | 0 |
| AS05 | B15h Grades include homework exercises (group grades-\# attempted) | I | 0 | 0 |
| AS06 | BI5j Grades include projects (individual grades) | 1 | 0 | 0 |
| AS07 | BI5k Grades include projects (group grades) | 1 | 0 | 0 |
| AS08 | B15m Grades include journals | 1 | 0 | 0 |
| AS09 | B15n Grades include class participation | 1 | 0 | 0 |
| AS 10 | B150 Grades include portfolios | 1 | 0 | 0 |
| AS 11 | B18 Students may use computers on tests | 1 | 0 | 0 |
| AS13 | B20 Students may use graphing calculators on tests | 1 | 0 | 0 |
| ASI4 | F17k Values the use of alternative assessment strategies | 4, 5 | 1,2,3 | $\leq 3$ |
| AS15 | F271 Amount of class time spent on small group quizzes or tests | 2, 3, 4, 5, 6 | 1 | $\leq 1.5$ |
| AS 16 | F37 This is \% of emphasis on course grade. | $\begin{gathered} \text { Sum of } c, d, \\ e, i, j, k, l, \\ m>50 \% \end{gathered}$ | $\begin{gathered} \text { Sum of c, d, } \\ e, i, j, k, l, \\ m \leq 30 \% \end{gathered}$ | $\leq 30$ |

Table E5
Technology

| Scale |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: |
| Sariable <br> name | Survey questions <br> B: refers to Site-Liaison Survey <br> F: refers to Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based calculus <br> (1) | Response(s) <br> inconsisten <br> with reform- <br> based calculus <br> (or neutral) <br> (0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1$)$ |
| TN01 | B13b, Frequency Students <br> work at a computer in lab or <br> class situation | 3,4 | 1,2 | 1,2 |
| TN02 | B13d, e Frequency Students <br> work with a graphing <br> calculator in lab or class <br> situation | 3,4 | 1,2 | 1,2 |
| TN03 | B17 Use of computers | $2-8$ or <br> other | 1 | 1 |
| TN04 | B21 Self-reported amount <br> of use of technology | 2,3 | 1 | 1 |
| TN05 | B23b Students use <br> supplementary materials for <br> occasional use with <br> technology | 1 | 0 | 0 |
| TN06 | F17h Agrees with emphasis <br> on the use of technology in <br> Calculus I | 4,5 | $1,2,3$ | $\leq 3$ |
| TN07 | F25a Frequency students use <br> calculators for computation | 2,3 | 1 | $\leq 1.5$ |
| TN08 | F25 Frequency students use <br> calculator or computer for <br> symbol manipulation | 2,3 | 1 | $\leq 1.5$ |
| TN09 | F25 e, frequency students <br> use calculator or computer to <br> write programs | 2,3 | 1 | $\leq 1.5$ |
| TN010 | F43c Values greater use of <br> technology | 4,5 | $1,2,3$ | $\leq 3$ |
| TN011 | F44l Values that students <br> learn to develop a deeper <br> understanding of calculus <br> concepts through the use of <br> technology | 4,5 | $1,2,3$ | $\leq 3$ |

Table E6

## Access

|  | Scale <br> variable <br> name | Survey questions <br> B: refers to Site-Liaison Survey <br> F: refers to Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based calculus <br> (I) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) |
| :--- | :--- | :---: | :---: | :---: |
| AC01 | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1) |  |  |  |
| AC02 | BIIb \% Nontraditional students | $2,3,4$ | 1,2 | 1,2 |
| AC03 | BIIg \% Life Science Majors | $2,3,4$ | 1 | 1 |
| AC04 | BIIh \% Business Majors | $1,3,4$ | 1 | 1 |
| AC05 | B12a, b, d > 25 \% Non-Asian <br> minority (Black, American <br> Indian or Alaskan Native, or <br> Hispanic) <br> AC06F38a Student success rates <br> AC07F43h Faculty should confer <br> regularly with other disciplines <br> that expect their students to take <br> calculus | 4,5 | 1 | 1 |
| AC08 | F43i First-year calculus <br> should be accessible to wide <br> range of students | 4,5 | $1,2,3$ | $\leq 3$ |

Tables E7 through E13 list the survey items used for comment data in the descriptions of the profiles of reform. Please note that the question numbers associated with Site Liaison Survey items are preceded with an "S" and those associated with Faculty Survey items are preceded with an " F ".

Table E7
Survey Items Used for Comment Data Associated with Concepts

| Item Number |  |
| :--- | :--- |
| F44 | Please indicate the value you place on each of the <br> following items that many proponents of calculus reform <br> have felt students should learn or understand. <br> Comments: |

## Table E8

## Survey Items Used for Comment Data Associated with Approach

| Item Number |  |
| :--- | :--- |
| F9 a and b | Among the above claims, the CCH Instructor's Manual indicates <br> that the CCH textbook is written with the following goals in <br> mind. Please further discuss your observations and opinions <br> about how well the CCH materials accomplish these goals. |
| a. "Let formal definitions and proofs evolve from a long <br> process of common sense investigations, rather than to start with <br> abstract definitions." |  |
| b. Emphasize "The Rule of Three: Every concept should <br> be introduced graphically, numerically, and analytically." |  |

Table E9
Survey Items Used for Comment Data Associated with Teaching

| Item Number |  |
| :--- | :--- |
| F 20 | In what ways did the CCH material influence changes in <br> your preparation and (or) teaching of calculus? |
| F30 | Please comment briefly on any innovations you have <br> included in your course. |
| F21 | Comments listed at the end of the items. See Appendix D <br> for the list of items. <br> This question addresses you teaching of topics in the CCH <br> textbook. Please add any comments you feel would help <br> us interpret your responses in the space provided. |

Table E10
Survey Items Used for Comment Data Associated with Assessment

| Item Number |  |
| :--- | :--- |
| F30 | Please comment briefly on any innovations you have <br> included in your course. |

Table Ell
Survey Items Used for Comment Data Associated with Technology

| Item Number |  |
| :--- | :--- |
| F26 | Please discuss your views about the use of technology in <br> first-year calculus courses. |

Table El2
Survey Items Used for Comment Data Associated with Access

| Item Number |  |
| :---: | :---: |
| $F 11 \mathrm{~b}$ and c | b. Please describe the types of students (if any) you believe experience greater access to calculus through the use of CCH materials. <br> c. Please describe the types of students (if any) you believe experience decreased access to calculus through the use of CCH materials. |
| F 34 | There is considerable interest in what happens when CCH students move on to other mathematics courses. What are your impressions, and those of your colleagues who receive CCH students, about student performance or attitudes in other mathematics courses? |
| F 35 | Please discuss briefly any observations made by faculty from other departments regarding CCH student performance related to calculus in courses they teach and their impressions of CCH student understanding of calculus. If you can, please include examples. |
| F39 | Please make comparisons between the success and retention rates of CCH students and students in standard calculus students. |

## Table E13

## Survey Items Used for Comment Data Associated with all clustering scales

| Item Number |  |
| :--- | :--- |
| F 17 | Comments listed at the end of the item: <br> Please indicate your agreement with the folowing <br> statements about first-year calculus (not limited to CCH). <br> Please give us your opinion about the level of emphasis for <br> each item. |
| F45 | What do you find encouraging about the direction of <br> calculus reform and what are your concerns? |

## APPENDLX F

## APPENDIX F

## CLUSTER ANALYSIS

The primary statistical technique used in the current study is cluster analysis. Cluster analysis is frequently used in the social and biological sciences. Because some readers may be unfamiliar with the process, a description follows.

The current study employs a common clustering method called agglomerative hierarchical cluster analysis. In this method clusters are formed by grouping cases into larger and larger clusters until all cases are members of a single cluster. The complete clustering process requires an exact number of steps that is equivalent to one less than the number of cases. On the first stép, all cases are treated as individual clusters. At each subsequent step, either two cases, one case and one previously formed cluster, or two previously formed clusters are combined, according to their similarity measure. At the final step, all cases are merged into one large group. Clustering methods, by definition, produce nonoverlapping clusters at each step of the process. These nonoverlapping clusters are nested into larger, more inclusive disjoint clusters at each subsequent step. It is common to stop the clustering procedure immediately before a clustering step that combines clusters that have a relatively large distance measure compared to the previous distance measures.

Table F1 and Figure Fl (modified from (Norusis, 1994, pp. 91 \& 93), with the accompanying discussion, illustrate the forming of a hierarchical structure using the statistics computer program SPSS (1995). SPSS will also be used in the cluster analysis portion of the current study. The clustering schedule in Table Fl identifies the cases or clusters being combined at each stage (or step) of the clustering process.

Table F1
Clustering Schedule

| Clusters combined |  |  |  |  |  |  |  | Stage cluster first appears |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stage | Cluster A | Cluster B | Coefficient | Cluster A | Cluster B | Next Stage |  |  |
| 1 | 11 | 17 | .114695 | 0 | 0 | 10 |  |  |
| 2 | 9 | 20 | .306903 | 0 | 0 | 8 |  |  |
| 3 | 8 | 18 | .309227 | 0 | 0 | 9 |  |  |
| 4 | 1 | 3 | .374859 | 0 | 0 | 5 |  |  |
| 5 | 1 | 2 | .529696 | 4 | 0 | 10 |  |  |
| 6 | 5 | 15 | .606378 | 0 | 0 | 7 |  |  |
| 7 | 4 | 5 | .870016 | 0 | 6 | 15 |  |  |
| 8 | 9 | 10 | .934909 | 2 | 0 | 11 |  |  |
| 9 | 6 | 8 | 1.352618 | 0 | 3 | 14 |  |  |
| 10 | 1 | 11 | 1.405148 | 5 | 1 | 16 |  |  |
| 11 | 9 | 12 | 1.559987 | 8 | 0 | 12 |  |  |
| 12 | 9 | 13 | 1.990205 | 11 | 0 | 17 |  |  |
| 13 | 16 | 19 | 2.820897 | 0 | 0 | 19 |  |  |
| 14 | 6 | 7 | 3.106108 | 9 | 0 | 16 |  |  |
| 15 | 4 | 14 | 4.238165 | 7 | 0 | 17 |  |  |
| 16 | 1 | 6 | 4.378198 | 10 | 14 | 18 |  |  |
| 17 | 4 | 9 | 12.151937 | 15 | 12 | 18 |  |  |
| 18 | 1 | 4 | 19.552841 | 16 | 17 | 19 |  |  |
| 19 | 1 | 16 | 33.338039 | 18 | 13 | 0 |  |  |

This example consists of an original set of 20 cases, numbered 1 through 20.
The numbers listed vertically on the left side of the table, or first column, represent stages in the clustering process. The numbers in the second, third, fifth, and sixth columns represent cases (or academic institutions). The first row of the clustering schedule represents stage 1 in which cases 11 and 17 are combined. Stage 1 represents the first solution in which 19 clusters have been formed from the 20 cases. Stage 2 represents the 18 cluster solution, and so on. Stage 19 represents the solution in which all cases have been combined into one cluster. In practice the clustering process stops, based on stopping criteria, prior to the forming of one large cluster that incorporates all cases.

The squared Euclidean distance between case 11 and case 17 in row 1 is listed in the Coefficient column as .114695. Cases 11 and 17 are the two closest cases based on the computed squared Euclidean distance measure. Under the heading Clusters combined, the term cluster can refer to either an individual case or a previously formed multicase cluster, depending on whether two cases, a case and a previously formed cluster, or two previously formed cluster exhibit the greatest similarity. The cluster number is always the same as the number of its earliest case. The first cluster, formed by combining cases 11 and 17, is known throughout the clustering process as Cluster 11. In subsequent stages, other cases are included in Cluster 11. In stage 4, cases I and 3 are combined to form cluster 1. Stage 5 identifies the merging of cluster 1 and case 2. The column labeled Stage cluster Ist appears indicates at which stage a multicase cluster is first formed. In the row for stage 5, cluster 1 and case 2 are combined. The number, 4, in the column labeled Cluster $\mathbf{A}$ indicates that cluster 1 was previously involved in a merge in stage 4 . The number, 0 , appears in the column labeled Cluster B , indicating that case 2 has not appeared in a previous stage. The column labeled Next stage identifies the next stage at which another case or cluster is combined with the current case or cluster. In the row for stage 4 , the number, 5 , in the column labeled Next stage indicates that Cluster 1 is next involved in a merge in stage 5.

Examination of the coefficient values provides an idea of how unlike the cases or clusters being combined are. Small coefficients indicate that clusters containing fairly homogeneous cases are being merged. Large coefficients indicate that clusters containing relatively dissimilar cases are being combined. Generally agglomeration, or clustering, should be stopped as soon as the increase between two adjacent steps becomes relatively large. In this example there is a relatively large increase in the value of the distance measure from a four-cluster to a three-cluster solution (stages 16 and 17) and the four cluster solution seems reasonable.

Figure Fl. Dendrogram.


Dendogram diagrams, also used to represent the clustering process, illustrate the rescaled Euclidean distance between combined cases or clusters. For computergraphics purposes, the computer program SPSS automatically rescales the actual squared Euclidean distances between cases or previously-formed clusters to numbers between 0 and 25. The rescaled distance preserves the ratio between distances. By looking at Table F1 and Figure F1, one can see that cases 11 and 17, 9 and 20, 8 and 18, and 1 and 3 were all separated by relatively small Euclidean distances and combined in the first 4 stages. Case 2 was merged with cluster 1 , composed of cases 1 and 3, very early in the clustering process as were cases 5 and 15 , case 4 and cluster 5 , cluster 9 and case 10, case 6 and cluster 8, and cluster 1 and cluster 11. The
differences between the rescaled Euclidean distances are not discernible on the dendogram shown above. The next discernible distance on the dendogram represents the merging of cluster 9 and case 12 in stage 11 .

Looking at the dendogram from right to left, one can see the one-cluster solution at rescaled distance 25 , the two-cluster solution at rescaled distance 15 , the three cluster solution at rescaled distance 10 and the four-cluster solution at rescaled distance 5 . As previously discussed, the four-cluster solution appears reasonable in this example.

## APPENDIX G

## APPENDIX G

## VALIDATING SCALES

The following validating scales are used for validation, comparison, and description of the clusters of academic institutions identified through cluster analysis. The validating scales are divided into three categories, (a) demographic validating scales, (b) mathematics department validating scales, and (c) CCH validating scales. Each type of validating scale contains two sets of tables. The first set of tables lists the survey items used for quantitative validation of the cluster solution. The second set of tables lists the survey items that were used for comment data in the descriptions of the profiles of reform. Questions preceded with an " A " indicate the question appears on the Initial Questionnaire; questions preceded with a "B" indicate Site Liaison Survey items, and questions preceded with an "F" indicate Faculty Survey items.

The tables associated with the quantitative validation of the cluster solution indicate the responses considered consistent with the goals of reform-based calculus and those considered neutral or inconsistent. For each survey item, the academic institution (case) received a value of 1 for responses considered consistent with calculus reform and a 0 for inconsistent responses.

The "balance point" for responses to each item was assigned a value of 0 . For example, on Likert type items, cases with responses less than or equal to 3 received a value of 0 , and cases with responses greater than 3 received a value of 1 . See the tables for the assignment of values to survey items with categorical responses.

## Demographic Validating Scales

The demographic validating scales pertain to the institution type, whether the institution is private or public, and the financial support received to implement the CCH Curriculum Project materials.

Table Gl.

## Four Demographic Validating Scales

|  | Survey Item <br> A indicates Initial Questionnaire <br> Balidating <br> Bidicates Site Liaison Survey <br> Scale | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> 0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assinged a <br> value of 1) |
| :--- | :--- | :---: | :---: | :---: |
| vtype | A1 Institution Survey <br> (seconpe <br> college, doctoral or research <br> university, other college or <br> university | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ |
| vpubpri | A2 Public or private <br> institution | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ |
| vfinsup | A10 Financial support | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ |
| vstuenro | A3 Student enrollment | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ |

Table G2 list the survey items used for comment data in the descriptions of the profiles of reform. Please note that the question numbers associated with Site Liaison Survey items are preceded with an " S " and those associated with Faculty Survey items are preceded with an " $F$ ".

Table G2
Survey Items Used for Comment Data Associated the Demographic Validating Scales

| Item Number |  |
| :--- | :--- |
| A 10 | Have you or your mathematics department received any <br> special financial support to implement calculus reform or <br> to introduce the CCH material? (See Appendix B for <br> survey and list of items.). Comments. |
| B 26 | Does your institution provide release time and/or funding <br> for instructors using CCH materials to attend staff <br> development activities, conferences, or workshops? <br> Comments. |

## Mathematics Department Validating Scales <br> Mathematics Department: Value of Teaching

The value mathematics department validating scale pertains to institution and mathematics department values and attitudes about teaching.

Table G3
The Values (Vvalues) Validating Scale

| Scale <br> Variable <br> Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) inconsistent with reformbased calculus (or neutral) (0) | Response(s) assigned a value of 0 (Others assigned a value of L ) |
| :---: | :---: | :---: | :---: | :---: |
| VVT01 | B32b Institution values or rewards teaching | 4, 5 | 1,2,3 | $\leq 3$ |
| VVT02 | B32d Mathematics department values and/or rewards experimentation in teaching | 4,5 | 1,2,3 | $\leq 3$ |
| VVT03 | B32e Mathematics department or institution evaluates faculty teaching (beyond student evaluation) | 4,5 | 1,2,3 | $\leq 3$ |
| VVT04 | B32f Department resources directed towards teaching | 4, 5 | 1,2,3 | $\leq 3$ |
| VVT05 | B32a Calculus is considered a desirable course to teach | 4,5 | 1,2,3 | $\leq 3$ |

## Mathematics Department: Interest in pedagogy and reform

The interest validating scale pertains to mathematics department faculty
members' interest in reform and pedagogy

## Table G4

## The Interest (Vinterest) Validating Scale

|  | Scale |  |  |  |
| :---: | :--- | :---: | :---: | :---: |
| Variable <br> Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1) |
| VTT01 | B38a Frequency department <br> discusses pedagogical issues | 3,4 | 1,2 | $\leq 2$ |
| VTT02 | B38b Frequency department <br> discusses calculus reform | 3,4 | 1,2 | $\leq 2$ |
| VTT03 | B38c Frequency department <br> discusses reform in general | 3,4 | 1,2 | $\leq 2$ |
| VTT04 | B51 Descriptive of <br> department opinions about <br> reform | 2 | 1 | $\leq 1.5$ |

## Mathematics Department: Teaching

The teaching validating scale corresponds to the teaching clustering scale and pertains to mathematics department faculty.

## Table G5

The Teaching (Vteaching) Validating Scale

| Scale <br> Variable <br> Name | Survey Item <br> Bindicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1) |
| :---: | :--- | :---: | :---: | :---: |
| VT01 | B34g Values use of <br> alternative pedagogical <br> strategies | 4,5 | $1,2,3$ | $\leq 3$ |
| VT02 | B35a Uses lecture method <br> (neg) | 1,2 | $3,4,5$ | $\geq 3$ |
| VT03 | B35b Uses cooperative <br> groups | 4,5 | $1,2,3$ | $\leq 3$ |
| VT04 | B35c Ask students to write <br> about mathematics | 4,5 | $1,2,3$ | $\leq 3$ |
| VT05 | B35d Ask students do <br> mathematics projects | 4,5 | $1,2,3$ | $\leq 3$ |
| VT06 | B35g Encourages discussion <br> of alternative solutions | 4,5 | $1,2,3$ | $\leq 3$ |
| VT07 | B35h Encourages student <br> exploration during class period | 4,5 | $1,2,3$ | $\leq 3$ |

## Mathematics Department: Student Assessment

The assessment validating scale corresponds to the assessment clustering scale and pertains to mathematics department faculty.

## Table G6

The Assessment (Vassessment) Validating Scale

| Scale | Survey Item <br> Sariable <br> Name indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> $(1)$ | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutral) <br> (0) | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1 ) |
| :---: | :--- | :---: | :---: | :---: |
| VASOI | B35i Use variety of <br> assessment methods | 4,5 | $1,2,3$ | $\leq 3$ |

## Mathematics Department: Concepts

The concepts validating scale corresponds to the concepts clustering scale and pertains to mathematics department faculty.

Table G7

## The Concepts (Vconcepts) Validating Scale

| Scale <br> Variable <br> Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calaulus <br> (or neutral) <br> $(0)$ | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1 ) |
| :---: | :--- | :---: | :---: | :---: |
| VC01 | B34a Values emphasizing <br> formal theorems (neg) | 1,2 | $3,4,5$ | $\geq 3$ |
| VC02 | B34b Values emphasizing <br> axiomatic structure (neg) | 1,2 | $3,4,5$ | $\geq 3$ |
| VC03 | B34d Values emphasizing <br> mathematical techniques (neg) | 1,2 | $3,4,5$ | $\geq 3$ |
| VC04 | B34e Values emphasizing <br> applications of mathematics | 4,5 | $1,2,3$ | $\leq 3$ |

## Mathematics Department: Technology

The technology validating scale corresponds to the technology clustering scale and pertains to mathematics department faculty.

## Table G8

The Technology (Vtechnology) Validating Scale

| Scale Variable Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) inconsistent with reformbased calculus (or neutral) (0) | Response(s) assigned a value of 0 (Others assigned a value of 1) |
| :---: | :---: | :---: | :---: | :---: |
| VTNOI | B35e, f Uses computers or graphing calculators in teaching | 4,5 | 1, 2, 3 | $\leq 3$ |
| VTN02 | B34f Supports increased use of technology | 4,5 | 1,2,3 | $\leq 3$ |

Mathematics Department: Comment Data
Table G9
Survey Items Used for Comment Data Associated with the Mathematics Department
Validating Scales

| Item Number |  |
| :--- | :--- |
| B 11 | Does your institution provide release time or funding for other <br> preparation time related to teaching CCH calculus? |
| B 13 | Approximately how often do students in CCH sections take part <br> in the following activities during time scheduled for calculus. <br> (See Appendix C for survey and list of items.) Comments. |
| B 15 | Which of the following are used to assign grades in CCH <br> sections. (See Appendix C for survey and list of items.) <br> Comments. |
| B 16 | If you wish, please use this space to make additional comments <br> about student assessment. |
| B 22 | If you are using computer software and are a member of a group <br> of institutions or part of a program that provides instuctional <br> material or support for the use of that software, please describe <br> you situation and the impact of the program briefly. |
| B 52 | If you wish, please use the space below to make any further <br> comments concerning reform issues at your institution. |

## Validating Scales Related Directly to CCH Curriculum Project

The validating scales related directly to the CCH Curriculum Project are defined by survey items that pertain directly to CCH Curriculum Project instructors or to the use of CCH Curriculum Project materials.

## CCH Curriculum Project : Percentage of Use of Materials

The use- CCH validating scale pertains to the percentage of use of CCH materials for calculus classes.

Table G10
The Use- CCH (Vuse-CCH) Validating Scale

| Scale Variable Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) consistent with reformbased calculus (1) | Response(s) inconsistent with reformbased calculus (or neutral) <br> (0) | Response(s) assigned a value of 0 (Others assigned a value of 1) |
| :---: | :---: | :---: | :---: | :---: |
| VU01 | Al3 Percent of use of CCH materials (percentage computed for each institution and compared directly) | n/a | n/a | n/a |

## CCH Curriculum Project: Faculty Status

The status validating scale pertains to the status of CCH instructors.

Table Gll
The Status (Vstatus) Validating Scale

| Scale <br> Variable <br> Name | Survey Item <br> Bindicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calaulus <br> (or neutral) <br> (0) | Response(s) <br> aspinged a <br> value of 0 <br> (Ohers <br> assigned a <br> value of 1) |
| :---: | :--- | :---: | :---: | :---: |
| VS01 | B8 \% CCH faculty are <br> tenured or tenure track and <br> full time. | $>99 \%$ | $\leq 99 \%$ | $\leq 99 \%$ |
| VS02 | F3 CCH instructors' <br> mathematics teaching <br> experience (compared <br> directly) | $4,5,6$ | $1,2,3$ | $\leq 3.5$ |
| VS03 | F4 CCH instructors' <br> calculus teaching experience <br> (compared directly) | $4,5,6$ | $1,2,2$ | $\leq 3.5$ |

## Interaction

The interaction validating scale pertains to CCH instructors' interactions.

Table G12

## The Interaction (Vinteract) Validating Scale

| Scale <br> Variable <br> Name | Survey Item <br> B indicates Site Liaison Survey <br> Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) <br> inconsistent <br> with reform- <br> based calculus <br> (or neutra) <br> $(0)$ | Response(s) <br> assigned a <br> value of 0 <br> (Others <br> assigned a <br> value of 1$)$ |
| :---: | :--- | :---: | :---: | :---: |
| VI01 | B28 Percent of CCH <br> instructors that attended CCH <br> workshops | 4,5 | $1,2,3$ | $\leq 3$ |
| V102 | B36b Frequency CCH <br> instructors meet to discuss <br> pedagogical issues | 3,4 | 1,2 | $\leq 2.5$ |
| VI03 | B36c Frequency CCH <br> instructors meet to discuss <br> calculus reform | 3,4 | 1,2 | $\leq 2.5$ |

CCH Curriculum Project: Attitude towards reform and calculus teaching
The reform validating scale pertains to CCH instructors' attitudes towards reform. The attitude validating scale pertains to CCH instructors' attitude towards teaching calculus.

Table G13
The Reform (Vreform) Validation Scales

| Scale Variable Name | Survey Item <br> B indicates Site Liaison Survey Findicates Faculty Survey | Response(s) <br> consistent <br> with reform- <br> based <br> calculus <br> (1) | Response(s) inconsistent with reform- based calculus (or neurral) (0) | Response(s) assigned a value of 0 (Others assigned a value of 1) |
| :---: | :---: | :---: | :---: | :---: |
| Vreform | F40 Support for reform | 4,5 | 1,2,3 | $\leq 3$ |

## CCH Curriculum Project: Comment Data

Table G14 list the survey items used for comment data in the descriptions of the profiles of reform. Please note that the question numbers associated with Site Liaison Survey items are preceded with an " S " and those associated with Faculty Survey items are preceded with an " $F$ ".

## Table G14

Survey Items Used for Comment Data Associated with the CCH Validating Scales

| Item Number | B 29 If you are involved in collaborative efforts with other <br> institutions using CCH materials, please describe you <br> situation briefly. <br> B 30 If you are involved in interdisciplinary efforts with other <br> departments at your institution that involve the use of CCH <br> materials, please describe your situation briefly. <br> B 36 How often do CCH course instructors do the following? <br> (See Appendix C for survey and list of items.) Comments. <br> B 39 If you wish, please use this space to make any further <br> comments concerning departmental perspectives. <br> B 40 Circle the one response that is most descriptive of the <br> initiation of the use of a reformed calculus text at your <br> institution. (See Appendix C for survey and list of items.) <br> Comments. <br> B 41 Circle the one response that is most descriptive of the <br> initial use of CCH materials at your institution. (See <br> Appendix C for survey and list of items.) Comments. <br> B 43 How well do the following describe the likelihood of each <br> statement occurring at your institution? Please base your <br> response on your personal understanding. (See Appendix <br> C for survey and list of items. These items concern future <br> use of CCH textbook.) Comments. |
| :--- | :--- |

## APPENDLX H

## APPENDIX H

COMMENT DATA

This table, concerning the use of comment data, is described in Chapter III, Methods and Procedures, in the section titled Comment Data.

Table HI. Comment Data for Cluster 1 (17 institutions, 53 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 23 | 16 | 0 | $59 \%$ |
| Approach | 26 | 1 | 0 | $96 \%$ |
| Teaching | 17 | 0 | 0 | $100 \%$ |
| Assessment | 4 | 0 | 0 | $100 \%$ |
| Technology | 58 | 5 | 6 | $84 \%$ |
| Access | 68 | 57 | 39 | $41 \%$ |

Table H2. Comment Data for Cluster $2(31$ institutions, 113 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 55 | 52 | 0 | $51 \%$ |
| Approach | 42 | 0 | 0 | $100 \%$ |
| Teaching | 21 | 1 | 0 | $97 \%$ |
| Assessment | 5 | 1 | 0 | $83 \%$ |
| Technology | 98 | 8 | 14 | $82 \%$ |
| Access | 144 | 71 | 50 | $54 \%$ |

Table H3. Comment Data for Cluster 3 ( 13 institutions. 26 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 12 | 5 | 0 | $70 \%$ |
| Approach | 7 | 0 | 0 | $100 \%$ |
| Teaching | 9 | 0 | 0 | $100 \%$ |
| Assessment | 2 | 0 | 0 | $100 \%$ |
| Technology | 29 | 6 | 5 | $73 \%$ |
| Access | 23 | 16 | 17 | $41 \%$ |

Table H4. Comment Data for Cluster 4 ( 12 institutions, 46 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 32 | 21 | 0 | $61 \%$ |
| Approach | 11 | 1 | 0 | $92 \%$ |
| Teaching | 18 | 7 | 0 | $72 \%$ |
| Assessment | 0 | 0 | 0 | 0 |
| Technology | 56 | 18 | 2 | $74 \%$ |
| Access | 61 | 40 | 25 | $49 \%$ |

Table H5. Comment Data for Cluster $5(17$ institutions. 65 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 41 | 32 | 0 | $56 \%$ |
| Approach | 24 | 0 | 0 | $100 \%$ |
| Teaching | 52 | 2 | 0 | $96 \%$ |
| Assessment | 10 | 1 | 0 | $91 \%$ |
| Technology | 79 | 10 | 0 | $89 \%$ |
| Access | 86 | 55 | 35 | $49 \%$ |

Table H6. Comment Data for Cluster 6 ( 7 institutions. 20 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 15 | 9 | 1 | $76 \%$ |
| Approach | 11 | 1 | 0 | $92 \%$ |
| Teaching | 7 | 0 | 0 | $100 \%$ |
| Assessment | 6 | 1 | 0 | $86 \%$ |
| Technology | 17 | 1 | 2 | $85 \%$ |
| Access | 30 | 14 | 4 | $63 \%$ |

Table H7. Comment Data for Cluster 7 ( 10 institutions. 31 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 20 | 12 | 1 | $61 \%$ |
| Approach | 15 | 0 | 0 | $100 \%$ |
| Teaching | 21 | 0 | 0 | $100 \%$ |
| Assessment | 13 | 1 | 0 | $93 \%$ |
| Technology | 29 | 4 | 3 | $81 \%$ |
| Access | 65 | 23 | 13 | $71 \%$ |

Table H8. Comment Data for Cluster 8 ( 10 institutions. 19 Faculty Surveys received)

| Clustering scale | Number of <br> comments <br> consistent <br> with reform | Number of <br> comments <br> inconsistent <br> with reform | Number of <br> neutral <br> comments | Percentage <br> of consistent <br> comments |
| :---: | :---: | :---: | :---: | :---: |
| Concepts | 4 | 4 | 0 | $50 \%$ |
| Approach | 12 | 0 | 0 | $100 \%$ |
| Teaching | 2 | 1 | 0 | $67 \%$ |
| Assessment | 3 | 0 | 0 | $100 \%$ |
| Technology | 32 | 4 | 3 | $82 \%$ |
| Access | 18 | 29 | 9 | $32 \%$ |


[^0]:    FOR FURTHER INFORMATION
    If you have any questions about the survey, please feel free to call us at the University of New Hampshire.
    Darien Lauten. Project Manager: 603 862-3620 or 603 868-7133; e-mail:
    dlauten@christa.unh.edu
    Joan Ferrini-Mundy , Project Director: 603 862-2684

