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FIRE RESISTANCE OF CONCRETE-FILLED AND REINFORCED CONCRETE COLUMNS

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FIRE RESISTANCE OF CONCRETE-FILLED AND REINFORCED CONCRETE COLUMNS

By

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In The Name of God, Most Gracious, Most Merciful.

"By (the Token of) Time (through the ages), Verily Man is in Loss,
Except such as have Faith, And do righteous deeds, And (join together)
In the mutual teaching Of Truth, and of Patience and Constancy."

(The Holy Qur'an, 103)

"And My Success Can Only Come From God.
In Him I Trust, And
Unto Him I Look"

To God, The Creator
To God, The Sustainer
To God, The Most Merciful

Then, to my parents

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NOMENCLATURE

(1) Nomenclature for Chapter two

Notations

a = depth of equivalent rectangular stress block, $a = \beta C$.

Ag = gross area of section, square inch.

A_{se} = equivalent area of one end steel

A_{st} = area of the steel

C = $(k_u t)$ distance from extreme compression fibre to neutral axis.

C_c = force in concrete, kips

C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram.

E_c = modulus of elasticity of concrete, PSI

E_{cc} = excentricity of design load parallel to axis measured from the centroid of the section.

E_s = modulus of elasticity of steel, PSI

EI = flexural stiffiness of compression members.

f'_c = specified compressive strength of concrete, PSI, can be found by experimental tests for the specified concrete samples.

f_s = $E_s \varepsilon_s$ = the steel stress at any point in the ring, ksi

F_s = the net force in the steel ring, kips

$F_{s12...}F_{s4}$ = the forces acting in the steel ring, kips

fy = $E\varepsilon_y$ = specified yield strength of the steel, PSI

g = a constant

g_t = the length of the equivalent steel side, in (rectangular cross-section)

= or the mean diameter of the steel tube, in (cylindrical cross-section)

= or the outside diameter of the cross-section ,in

h = the thickness of equivalent steel ring ,in (cylindrical C-S)

h_1 = the thickness of side steel strip ,in (rectangular C-S)

h_2 = the thickness of end steel strip ,in (rectangular C-S)

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement.

I_s = moment of inertia of the equivalent steel ring about the centroidal axis of the member cross-section.

K = effective length factor for compression member

KL = effective length of column.

K_1 = $\beta_1 = 0.85 - 0.05\ (f'_c - 4)$ = a factor

K_u = C/H, a factor

$K_u t$ = the depth of the compressed area, measured from the extreme fiber on the compression side

$K_1 k_u t$ = the depth of the equivalent rectangulr stress block.

M_c = the moment of the compression forces in the concrete about the gravity axis of the section, K.lb.in

M_N = total net moment in the cross-section, K.lb.in

M_s = net moment in the equivalent steel ring, K.lb.in

$M_{s1}, \ldots M_{s4}$ = the moments acting in the steel ring, Klb.in

M_u = the ultimate moment, K.lb.in

P_o = nominal load of the cross-section, kips

P_{cr} = critical load, kips

P_N = total net load in the cross-section, kips

P_u = Ultimate load, kips

P_t = the ratio of the total area of steel to the gross area of the concrete section.

t = the length of the cross-section, in

w = the width of the cross-section, in

Greek Letters

β_1 = a factor = 0.85 for strength < 4000 PSI

= 0.85 - 0.05 (f'_c -4) for strength 4000 PSI

β = shape factor = 0.7854 for circular section.

β_d = $\frac{1}{\text{load factor}}$ = the ratio of maximum design dead load moment to maximum design total load moment, always positive.

δ = moment magnification factor for column

ε_u = ultimate concrete compressive strain = 0.003

ε_y = strain at yield in outermost steel

ε_s = strain in outermost tension steel

Θ = a variable angle

ϕ = strength reduction factor

(2) Nomenclature for Chapter four

Notations

c specific heat (J/kg°C)

f_c compressive strength of concrete at temperature T (MPa)

f'_c cylinder strength of concrete at temperature T (MPa)

f'_{co} cylinder strength of concrete at room temperature (MPa)

f_y strength of steel at temperature T (MPa)

f_{yo} yield strength of steel at room tempeature (MPa)

h coefficient of heat transfer at fire exposed surface ($W/m^2{}^\circ C$)

k thermal conductivity (W/m°C)

K effective length factor

L unsupported length of column (m)

P point

T temperature (°C)

x coordinate

y lateral deflection of column at mid-height (m)

z coordinate

<u>Greek letters</u>

α coefficient of thermal expansion

Δ increment

$\sqrt{2}\Delta h_g$ mesh width (m)

ε emissivity, strain (m)

λ heat of vaporization (J/kg)

ρ density (kg/m^3), radius of curvature (m)

σ Stefan-Boltzmann constant (W/m^2K^4)

t time (h)

ϕ concentrtion of moisture (fraction of volume)

χ curvature of column at mid-height (m^{-1})

<u>Subscripts</u>

o at room temperature

c of concrete

f of the fire

m at the points m in column

max maximum

min minimum

n at the points n in a row

L left of the x-axis

R right of the x-axis

p pertaining to proportional stress-strain relation

s of steel

T pertaining to temperature

w of water

Superscripts

j at $t = j\Delta t$

(3) *Nomenclature for Chapter five*

Notations

c specific heat ($Jkg^{-1}°C^{-1}$)

f_c compressive strength of concrete at temperature T (MPa)

f' cylinder strength of concrete at temperature T (MPa)

f'_{co} cylinder strength of concrete at room temperature (MPa)

f_y strength of steel at temperature T (MPa)

f_{yo} yield strength of steel at room temperature (MPa)

h coefficient of heat transfer at fire exposed surface ($W\ m^{-2}°C^{-1}$)

k thermal conductivity ($W\ m^{-1}°C^{-1}$)

K effective length factor

L unsupported length of column (m)

M_1 number of points P in radial direction

N_1 number of elements in tangential direction

P point

T teperature (°C)

x coordinate

V volume of water in an element (m^3)

y lateral deflection of column at mid-height (m)

z coordinate

Greek letters

α coefficient of thermal expansion

Δ increment or difference

$\Delta\xi$ mesh width (m)

ε emissivity, strain (m m^{-1})

λ heat of vaporization (J kg^{-1})

ρ density (kg m^{-3})

σ Stefan-Boltzmann constant (W m^{-2} K^{-4})

t time (h)

ϕ concentration of moisture (fraction of volume)

χ curvature of column at mid-height (m^{-1})

Subscripts

o at room temperature

c of concrete

f of the fire

m, M_1 at the points m, M_1 in radial direction

max maximum

min minimum

n, N_1 at the points n, N_1 in tangential direction

L left of the x-axis

R right of the x-axis

p pertaining to proportional stress-strain relation

s of steel

T pertaining to temperature

w of water

Superscripts

j at $t = j\Delta t$

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ABSTRACT

FIRE RESISTANCE OF CONCRETE-FILLED

AND REINFORCED CONCRETE COLUMNS

By

MOHAMED EL-SHAYEB

University of New Hampshire, May 1986

In order to calculate the fire resistance of a building component, it is necessary to know the temperature history of the component during exposure to fire. In this dissertation a numerical method is described for calculating the temperature field in fire-exposed columns. The procedure is based on a finite difference method for calculating the temperature history of concrete-filled square steel and cylindrical reinforced concrete columns. Two mathematical models and their related computer programs for these columns are presented in this dissertation.

Furthermore, the Division of Building Research of the National Research Council of Canada, is now carrying out extensive experimental studies on building columns. These studies include the testing of various columns under fire conditions. The data obtained from these tests will be used to determine the validity of the new models. In order to perform these tests the maximum allowable load must be known. Two mathematical models and their related computer programs to calculate the maximum allowable load of various column cross-sections are also presented in this dissertation.

CHAPTER 1
INTRODUCTION

1.1 Introduction

Columns are the most critical structural elements in a building in that their collapse can lead to the loss of the entire structure. Therefore, the performance of building columns in fire has long attracted considerable attention in various countries. The conventional method of obtaining information on this subject is by standard fire resistance tests. The possibility of making realistic theoretical estimates has been hampered by two factors: (1) the lack of knowledge concerning thermal properties of the commonly used protecting materials at elevated temperatures and certain rheological properties of steel, and (2) the complexity of the mechanism of heat flow, especially through physico-chemically unstable solids.

The first of these difficulties is not so serious now as it was 10-15 years ago. During the past decade information has accumulated on the thermal and rheological properties at elevated temperatures of many important building materials, among them steel and concrete. The difficulties related to the complexity of heat flow analysis have also been greatly reduced by having the calculations performed by high-speed computers. Thus many fire performance problems that not long ago had to be solved by experiment can now be solved by numerical techniques. Fire resistance of a structural member may be defined as its ability to withstand exposure to elevated temperatures without loss of its load-bearing function.

In previous publications by the Division of Building Research, some numerical techniques have already been described for the calculation of the temperature history of various one- and two-dimensional configurations typically employed in walls and floors, and of the deformation history of steel supporting elements, such as beams, joints, etc. In this thesis, the mathematical models that have been developed to calculate the maximum allowable load and fire resistance for various columns are described.

1.2 Literature Review

The fire resistance ratings for reinforced concrete columns in most North American Building Codes are mainly based on test results obtained from 1920. Since that time, design procedures have changed and the safety factor has decreased indicating a need for revision of these ratings. In view of this, studies on the fire resistance of reinforced concrete columns were started a few years ago at the Division of Building Research of the National Research Council of Canada.

The purpose of these studies was to obtain, by calculation, information on the fire resistance of columns as a function of significant parameters such as load intensity, slenderness of the column and cover thickness on the steel reinforcement. Several laboratories are now engaged in studies related to the prediction of the fire resistance of structures by calculation. In the past the fire resistance of columns was determined by empirical methods based on fire tests. Calculation of fire resistance was not possible, mainly because of insufficient knowledge of the thermal and mechanical properties of concrete at elevated temperatures and the lack of a suitable method for calculating the temperature distribution in columns.

In the last decade knowledge of material properties at the temperatures met with in fires has increased significantly. In addition, methods have been developed that enable accurate calculations of the temperature history of concrete and steel columns [1,2].

When temperature distribution in a column and the relevant material properties are known, the strength of the column can be calculated at any time during a fire by the well established methods used for columns not exposed to fire. From this information the decrease in strength because of the fire may be determined as a function of time. Under a given load, usually equal to the dead load plus the design live load, the time to failure or fire resistance can be determined.

Computer calculations of fire resistance can be obtained quickly and relatively inexpensively (less than 1% of the time and cost involved in testing). At present the Division of Building Research of the National Research Council of Canada is conducting studies to develop mathematical models for the calculations of the fire resistance of various column constructions, including reinforced concrete columns and concrete-filled steel columns. The studies are carried out jointly with the North American Concrete and Steel industries.

In these studies, twenty mathematical models for the predictions of the fire resistance of different column constructions need to be developed. So far, four mathematical models for protected steel, square reinforced concrete, rectangular reinforced concrete and concrete-filled cylindrical colums have been developed [1,2,3].

1.3 Objectives of the Present Research

This research is a part of a large joint project sponsored by the National Research Council of Canada, the Portland Cement Association and by members of the Canadian Steel Construction Council. The main objectives of this research is to develop the following models:

a. Mathematical models for determining the maximum allowable load for rectangular or square concrete-filled and cylindrical concrete-filled columns.

b. Mathematical model for determining the fire resistance of concrete-filled square steel columns.

c. Mathematical model for determining the fire resistance of reinforced concrete cylindrical steel columns.

1.4 Layout of the dissertation

The dissertation consists of seven chapters followed by a list of references and six appendices.

Chapter one provides a general introduction to the fire resistance of columns. Describes the objectives, scope and layout of this dissertation as mentioned above.

Chapter two presents a mathematical model to calculate the maximum allowable load of columns of various cross-sections.

Chapter three deals with the heat transfer theory and the finite difference method for solving Parabolic Partial Differential Equations.

Chapter four describes a mathematical model to calculate fire resistance of concrete-filled square steel columns. In this model, heat transfer equations have been solved by using a finite difference method.

The effect of moisture has been taken into consideration. Finally strains and stresses in steel and concrete have been determined.

Chapter five describes a mathematical model to calculate fire resistance of reinforced concrete cylindrical columns.

Chapter six gives output results of these computer programs and a discussion of these results.

Chapter seven presents a summary of the entire work as well as a list of the conclusions drawn.

CHAPTER 2

MAXIMUM ALLOWABLE LOAD FOR CONCRETE-FILLED COLUMNS

2.1 Introduction

A part of the joint studies between the National Research Council of Canada, the Portland Cement Association and the Canadian Steel Construction Council consists of fire tests on building columns. These tests are carried out for the determination of the effect of fire on the strength of loaded columns. The data obtained from these tests will be used to verify the validity of the mathematical models that have been developed for the prediction of the fire resistance of the columns [3,4,5].

In order to begin the test the maximum allowable load had to be known. The mathematical models for various cross-sections for concrete-filled columns have not been developed yet. Also, computer programs for calculating the test load or the maximum allowable load for reinforcing concrete columns are not available to the Division of Building Research of the National Research Council of Canada. Therefore it was necessary to develop these mathematical models and computer programs.

Since the strength of axially loaded members depends strongly on the compression strength of concrete; the steel/concrete area ratio, and the shape of the column cross-section [3,4,5,6], the maximum allowable load constantly changes. This involves long and complicated hand work calculations, which requires a lot of time and energy, full of chances of errors. Therefore it is necessary to develop a computer program for each mathematical model.

Mathematical models for various concrete-filled steel columns Figure (2.1), have been developed for calculating the maximum allowable load. A computer program for each of these mathematical models have been written.

There are two theories for designing building columns, working stress design and ultimate strength design. Recently, the latter was recommended by the American Concrete Institute. This theory is a procedure of design with a margin of safety against collapse. The basic assumptions for this theory are given.

2.2 Assumptions

The basic assumptions for the ultimate strength design theory are [7 to 15]

(1) Plane sections before bending remain plane after bending.

(2) No slip, i.e. perfect bond between steel and concrete.

(3) Tensile strength of concrete is negligible ($f'_{ct} = 0$).

(4) Strain in the concrete is proportional to the distance from the neutral axis.

(5) The maximum ultimate concrete strain at failure, $\varepsilon_u = 0.003$. Failure is not precisely a definable point, Figure (2.2).

(6) The ultimate tensile stress in the steel does not exceed f_y.

(7) The maximum compressive stress in the concrete $C_c = 0.85\ f'_c$.

(8) The stress-strain curve for the steel is bilinear, Figure (2.3).

The steps used to develop the first mathematical model will be explained in the following section.

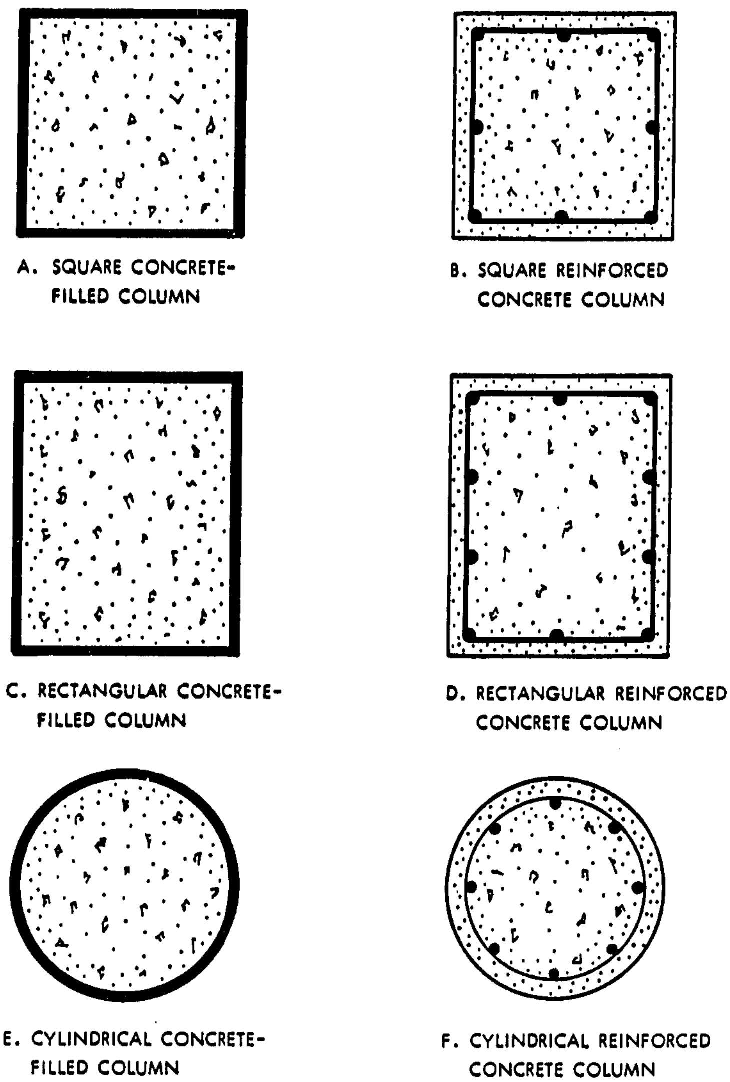

FIGURE 2.1
VARIOUS COLUMN CROSS-SECTIONS

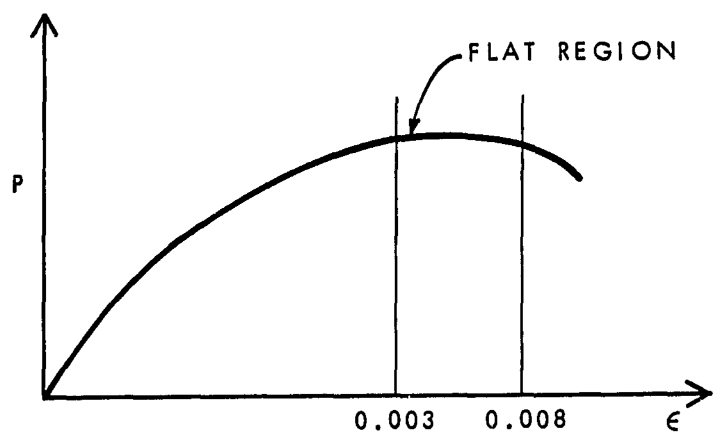

FIGURE 2.2

THE ULTIMATE CONCRETE STRAIN AT FAILURE

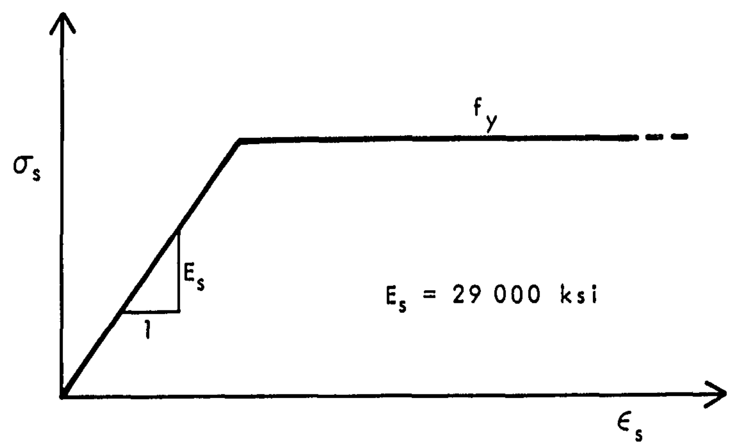

FIGURE 2.3

MODULUS OF ELASTICITY OF STEEL

FIRST MATHEMATICAL MODEL

CONCRETE-FILLED RECTANGULAR STEEL COLUMNS

The general formulation of the mathematical model for concrete-filled columns Figure (2.1a), will be explained in this section.

2.3 Load Factor

The load factor is a safety factor used in the design of building columns, taking into account the variability of the dead and live loads as expressed in the following formula [7 to 11]:

$$\text{Load Factor} = \frac{1.4\ D + 1.7\ L}{1.4D} \tag{2.1}$$

where:

D = Dead load

L = Live load = 0.4 D

1.4 = a factor of safety for dead load which is considered sufficient to take into account any miscalculation in the design due to dead load greater than the dead load ancitcpated.

1.7 = a factor of safety for live load taking into consideration unexpected earthquake or large amount of snowfall.

2.4 The Ratio β_d

The ratio β_d is the ratio of maximum factored dead load moment to maximum factored total load moment. It is always positive between zero and one. It is inverse to the load factor of Equation (2.1) as indicated below:

$$\beta_d = \frac{1.4\ D}{1.4\ D + 1.7\ L} \tag{2.2}$$

where:

$L = 0.4\ D$

2.5 Concrete Modulus (E_c)

The modulus of elasticity E_c (PSI) for normal weight concrete may be taken according to the American or the Canadian Standards [7,8] as:

$$E_c = 57000\ \sqrt{f'_c} \tag{2.3}$$

where:

f'_c = specified compressive strength of concrete, psi

2.6 Flexural Stiffness (EI)

The flexture stiffness of compression columns can be expressed as [7,8]:

$$EI = \frac{(E_c I_g/5) + E_s I_{se}}{1 + \beta_d} \tag{2.4}$$

where:

EI = flexural stiffness of compression columns

E_c = modulus of elasticity of concrete in psi = Equation (2.3).

I_g = moment of inertia of gross concrete section about the centroidal axis, including structural steel.

E_s = modulus of elasticity of steel in psi, where E_s for non-prestressed steel (may be taken as 29×10^6 psi)

I_{se} = moment of inertia of reinforcement or structural steel about the centroidal axis of column cross-section

β_d = as defined in Equation (2.2).

2.7 Critical Load

The critical load for the specific column according to the American Concrete Institute (ACI) or the Canadian Standard Association is [7 to 10]:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{2.5}$$

where:

P_{cr} = critical load

EI = as defined in Equation (2.4).

K = effective length factor for compression column

L = the length of compression column

or (KL) = effective length of compression column

2.8 Capacity Reduction Factor (ϕ)

The capacity reduction factor ϕ provides for the possibility of the concrete or reinforcing steel being of less strength than required and for the possibility of members being under strength due to inaccuracies or mistakes in construction. The values of this factor depends on the importance of the member and the mode of anticipated failure. The following values for ϕ shall be used [6,7,8]:

$$\phi = 0.70 \tag{2.6}$$

2.9 Critical Load with Reduction Factor (ϕ)

The capacity reduction factor ϕ must be considered in calculating the critical load as follows:

$$P_{(cr)_\phi} = \phi \left[\frac{\pi^2 EI}{(KL)^2}\right] \tag{2.7}$$

2.10 Concrete Stress and Strain

The relationship between concrete compressive stress distribution and concrete strain may be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests. The above requirement may be considered satisfied by an equivalent rectangular concrete stress distribution defined by the following [Figure A-1, Appendix A]:

(a) Concrete stress of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = (\beta_1)c$ from the fiber of maximum compressive strain [Appendix A].

(b) Distance c from fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis [Figure A-6, Appendix A].

(c) Factor β_1 shall be taken as the following [Appendix A]:

$$\beta_1 = 0.85 \qquad \text{for } f'_c < 4000 \text{ psi} \tag{2.8}$$

$$\beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right) \times 0.05 \qquad \text{for } f'_c > 4000 \text{ psi} \tag{2.9}$$

2.11 Minimum Eccentricity

A minimum eccentricity "e" is required. This insures that if a column is not perfectly concentrically loaded (as is normally the case) a certain moment capacity can be maintained by the column. The American Concrete Institute states that the minimum eccentricity for a concrete column is [8,9]:

$$e = 0.6 + 0.03 \text{ H} \tag{2.10}$$

while the Canadian Standard Association states:

$$e = 15 + 0.03 \text{ H} \tag{2.11}$$

where:

e = eccentricity, in or mm

H = column width, in or mm.

2.12 The Nominal Load

According to the North American Standards, for steel structure, design axial load strength at zero eccentricity (ϕP_o) will be taken as [7,8]:

$$\phi P_o = 0.85\, f'_c(A_g - A_{st}) + f_y A_{st} \qquad (2.12)$$

where:

ϕ = strength reduction factor, Equation 2.6

P_o = axial load at zero eccentricity

$0.85\, f'_c$ = the concrete compressive strength [Appendix A]

A_g = gross area of section

A_{st} = total area of steel

f_y = yield strenth of steel

2.13 Column Cross-Section

The concrete-filled column cross-section as illustrated in Figure 2.4, where:

h = thickness of constructed steel wall (in)

w = total width of column cross-section (in)

t = total length of column cross-section (in)

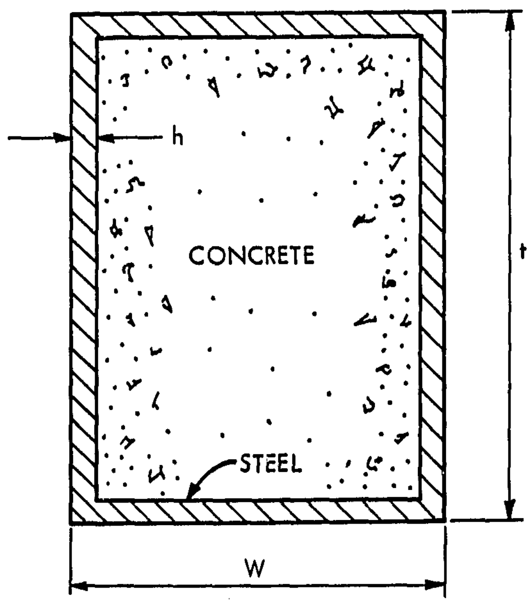

FIGURE 2.4

CROSS-SECTION OF CONCRETE-FILLED COLUMN

2.14 Forces, Moments and Neutral Axis Location

To calculate the forces and the moments in the steel and concrete, the neutral axis location with respect to the gravity axis of the cross-section must be identified. The neutral axis location has the following significance [8,9,11,12]:

a. The part of the column cross-section located above the neutral axis usually is subjected to compression.

b. The part of the column cross-section located under the neutral axis is normally subjected to tension.

In order to calculate forces and moments in the cross-section of columns, assuming various locations for the neutral axis for each of these positions, the total forces and the total moments, which column cross-section can resist must be calculated.

2.15 Case One, Figure 2.5

The strain in the steel in the compression side is less than the yield strain, i.e:

$$\varepsilon'_s < \varepsilon_y \qquad (2.15)$$

The strain in the steel in the tension side is greater than the yield strain, i.e.:

$$\varepsilon_s > \varepsilon_y \qquad (2.16)$$

where:

ε_s = strain in the tension steel

ε'_s = strain in the compression steel

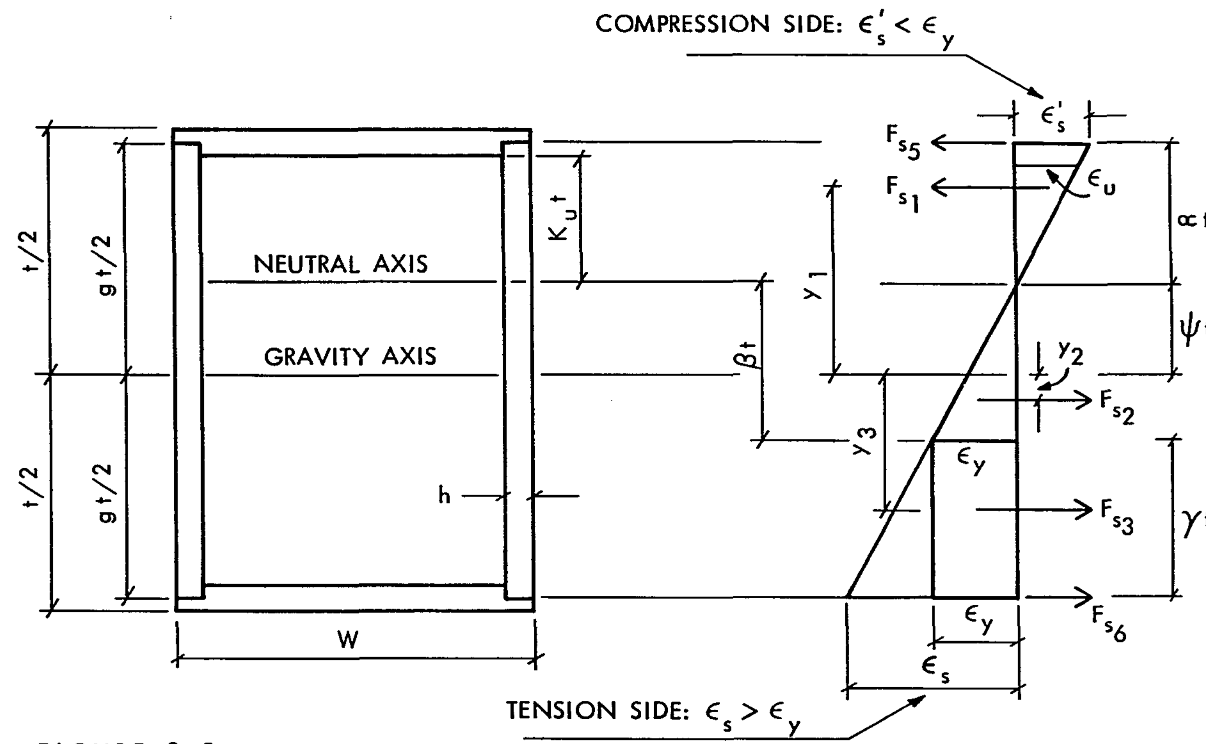

FIGURE 2.5

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE ONE)

ε_y = yield strain of the steel

From the strain diagram shown in Figure (2.5), the following relations can be obtained:

$$\alpha = K_u + \frac{R}{2} \qquad (2.17)$$

where:

$$R = \frac{h}{t}$$

$$\beta = (\alpha - \frac{R}{2}) \frac{\varepsilon_y}{\varepsilon_u} \qquad (2.18)$$

$$\phi = \frac{g}{2} - \alpha \qquad (2.19)$$

$$\gamma = g - \alpha - \beta \qquad (2.20)$$

$$\varepsilon'_s = \varepsilon_y \frac{\alpha}{\beta} \qquad (2.21)$$

Since $\varepsilon'_s = \frac{f'_s}{E}$ and $\varepsilon_y = \frac{f_y}{E}$, then Equation (2.21) becomes

$$f'_s = f_y \frac{\alpha}{\beta} \qquad (2.22)$$

2.15.1 Forces in side steel

From the strain diagram Figure (2.5), the following forces can be derived:

$$F_{s1} = \left(\frac{f'_s}{2}\right)(\alpha t)(2h) \quad (2.23)$$

$$F_{s2} = \left(\frac{f_y}{2}\right)(\beta t)(2h) \quad (2.24)$$

$$F_{s3} = (f_y)(\gamma t)(2h) \quad (2.25)$$

2.15.2 Forces in End Steel

Also, from the strain diagram Figure (2.5), the following forces can be derived:

$$F_{s5} = f'_s \cdot A_{se} = f'_s \cdot h\,(w-h) \quad (2.26)$$

$$F_{s6} = f_s \cdot A_{se} = f_s \cdot h\,(w-h) \quad (2.27)$$

Then, the total force in side and end steel, F_{st} is:

$$F_{st} = F_{s1} - F_{s2} - F_{s3} + F_{s5} - F_{s6} \quad (2.28)$$

2.15.3 Forces in Concrete

From the concrete stress block demonstrated in Figure (2.6), the net force in concrete is:

$$C_c = 0.85\, f'_c(\beta K_u t)W \quad (2.29)$$

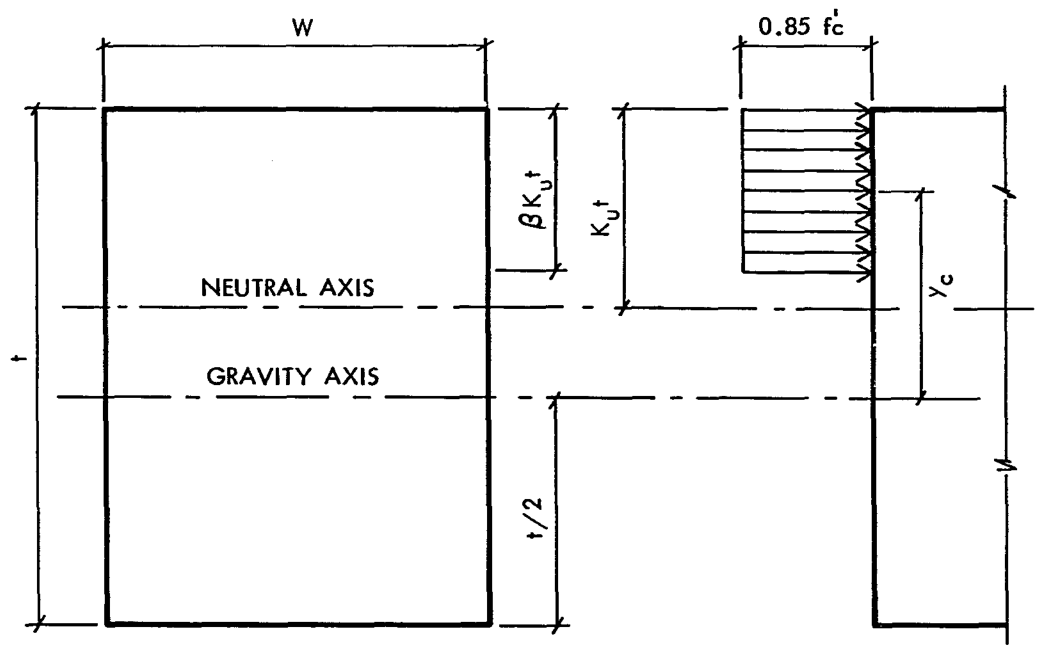

FIGURE 2.6

STRESS BLOCK OF CONCRETE FOR CONCRETE-FILLED COLUMNS

where:

C_c = the net force in concrete

$0.85\, f'_c$ = the concrete compressive strength [Appendix A]

$\beta K_u t$ = the depth of the equivalent rectangular stress block

W = the width of column cross-section

2.15.4 Total Force in Steel and Concrete

The total force in steel and concrete can be obtained from Equations (2.28) and (2.29) as given below:

$$P_N = F_{st} + C_c \tag{2.30}$$

where:

P_N = total force in steel and concrete

F_{st} = total force in steel

C_c = total force in concrete

2.15.5 Arms of Forces

The arms of forces of side and end steel are as follows:

$$y_{s1} = t(\psi + \frac{2}{3}\alpha) \tag{2.31}$$

$$y_{s2} = t(\frac{2}{3}\beta - \psi) \tag{2.32}$$

$$y_{s3} = t(\frac{\gamma}{2} + \beta - \psi) \tag{2.33}$$

$$y_{s5} = \frac{gt}{2} \tag{2.34}$$

$$y_{s6} = \frac{gt}{2} \tag{2.35}$$

$$y_c = \frac{t}{2} - \frac{\beta K_u t}{2} \tag{2.36}$$

2.15.6 Moments in Steel

The net moment in steel M_s is:

$$M_{st} = F_{s1}y_{s1} + F_{s2}y_{s2} + F_{s3}y_{s3} + F_{s5}y_{s5} + F_{s6}y_{s6} \tag{2.37}$$

2.15.7 Moments in Concrete

The net moment in concrete M_c is:

$$M_c = C_c y_c \tag{2.38}$$

2.15.8 Total Moments in Steel and Concrete

The total net moment in steel and concrete M_N can be obtained from Equations (2.37) and (2.38) as:

$$M_N = M_{st} + M_c \tag{2.39}$$

By using capacity reduction factor ϕ in Equations (2.30) and (2.39), the final net load and moment indicated below are representing one point (ϕP_N, ϕM_N) located in the interaction diagram:

$$\phi P_N = \phi(F_{st} + C_c) \tag{2.40}$$

$$\phi M_N = \phi \left(M_{st} + M_c\right) \qquad (2.41)$$

Similarly, by selecting various locations for the neutral axis and repeating the calculations as in case one, other points (ϕP_N, ϕM_N) can be obtained as indicated in [Appendix B].

2.16 Interaction Diagram

When concrete-filled columns are subjected to numerous loads and thus different moments being generated the resulting data can best be interpreted by the interaction diagram. This diagram plots the applied load (ϕP_N) verses moment (ϕM_N).

Therefore the obtained coordinates of ϕP_N and ϕM_N from the various locations of the neutral axis can be fitted together to construct the interaction diagram as indicated in Figure (2.7) [13,14,15].

In Figure (2.7), one observes that the curve generated comes to a peak at point (b). This point is known as the "balance point, Appendix A". It occurs when the strain in the extreme compression fiber reaches 0.003 and the stress in the longitudinal steel reaches its yield point [13,14,15].

Point (a) represents a concentrically loaded member (P_o, M_u = 0.0). The portion of the curve represented by (ab) pertains to that range of small eccentricity in which failure is initiated by crushing of the concrete. The portion (bc) represents that range in which failure is initiated by yielding of the tension steel.

2.17 The Ultimate Load and the Ultimate Moment

a. Construct the quadratic equation which represents the

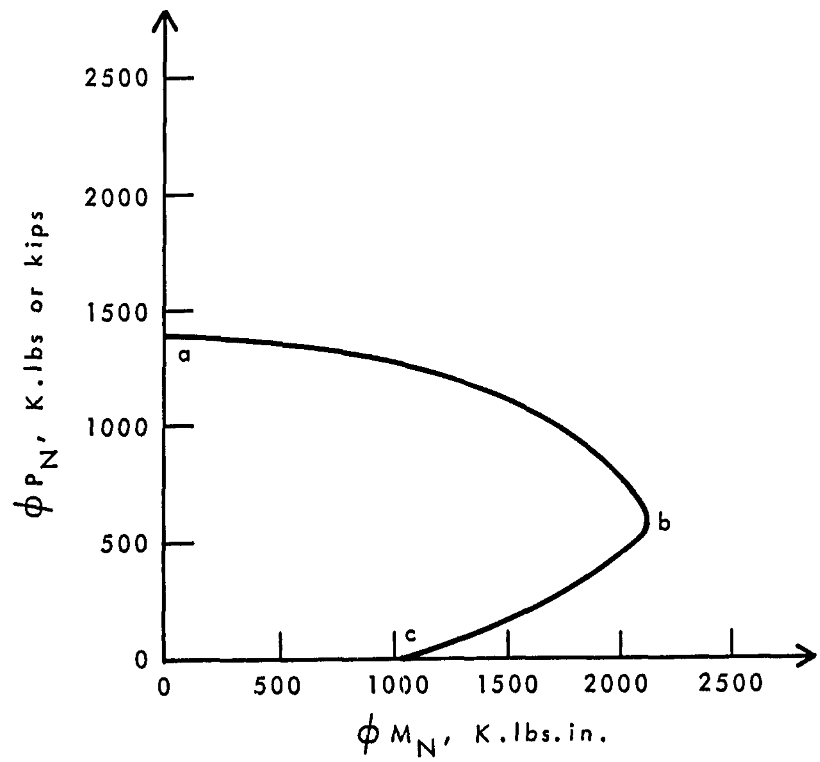

FIGURE 2.7

INTERACTION DIAGRAM

interaction diagram by using the (ϕP_N, ϕM_N) points calculated previously. The load P_N and moment M_N in this quadratic equation are unknown. From the subroutine fit polynomial (Appendix C), the quadratic equation is:

$$\phi M_N = C_1 + C_2(\phi P_N) + C_3(\phi P_N)^2 \tag{2.42}$$

where:

C_1, C_2 and C_3 are constants

b. The linear equation relating the load P_u and the moment M with the eccentricity e is:

$$M_N = P_N e \tag{2.43}$$

By solving the quadratic equation (2.42) and the linear equation (2.43), the load P_N and the moment M_N can be obtained.

2.18 Validity of P_N

a. Check the validity of the obtained value of the ultimate load P_u with comparison of the critical load (P_{cr}) as follows:

if

$$P_N < \phi P_{cr} \tag{2.44}$$

then, the value of the ultimate load (P_N) is valid, then take

$$P_u = P_N \tag{2.44`}$$

and if $$P_N > \phi P_{cr} \qquad (2.45)$$

then, the value of the ultimate load (P_N) is invalid and an iteration process must be applied until equation (2.44) is satisfied.

b. Calculate the moment magnification factor δ from the following formula [8]:

$$\delta = \frac{C_m}{1 - \frac{P_u}{\phi P_{cr}}} \qquad (2.46)$$

where:

δ = moment magnification factor for columns

C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram and equal to 1.0

P_u = the ultimate load, from Equations (2.42) and (2.43)

ϕP_{cr} = critical load, from Equation (2.7).

d. Then, the required ultimate moment M_N can be calculated:

$$M_N = P_N(e \times \delta) \qquad (2.47)$$

2.19 Test Load

A final calculation is required to determine the test load or maximum allowable load:

$$\text{Max. Allowable Load} = \frac{\text{Factored Axial Load } (\phi P_u)}{\text{Load Factor}} \qquad (2.48)$$

where:

Load Factor can be found from Equation (2.1).

The formulation of the mathematical model of concrete-filled cylindrical steel columns will be the same steps as followed for the above one, except for the forces and moments derivations. Therefore, the forces and moments formulation for the following mathematical model will be presented.

SECOND MATHEMATICAL MODEL

CONCRETE-FILLED CYLINDRICAL STEEL COLUMNS

2.20 Column Cross-Section

In order to calculate forces and moments in the cross-section of columns, assuming various locations for the neutral axis, for each of these positions obtain the total forces and the total moments in which the cross-section can resist. From Figure (2.8):

h = thickness of steel pipe

t = outside diameter of steel pipe

gt = middle diameter of steel pipe

$r = \frac{h}{t}$ = ratio of steel thickness h to the outside diameter of steel pipe

Assume any arbitrary position for the neutral axis location, draw the strain diagram as indicated in Figures (2.9) and (2.10), the following identities can be derived:

$$\cos\theta_1 = \frac{W}{gt/2} = (\frac{t}{2} - h - K_u t)/\frac{gt}{2} = (1-2r-2K_u)/g$$

$$\theta_1 = \cos^{-1}(1-2r-2K_u)/g \qquad (2.49)$$

$$\cos\theta_2 = \frac{(\frac{t}{2} - h) - \beta C}{(\frac{t}{2} - h)}$$

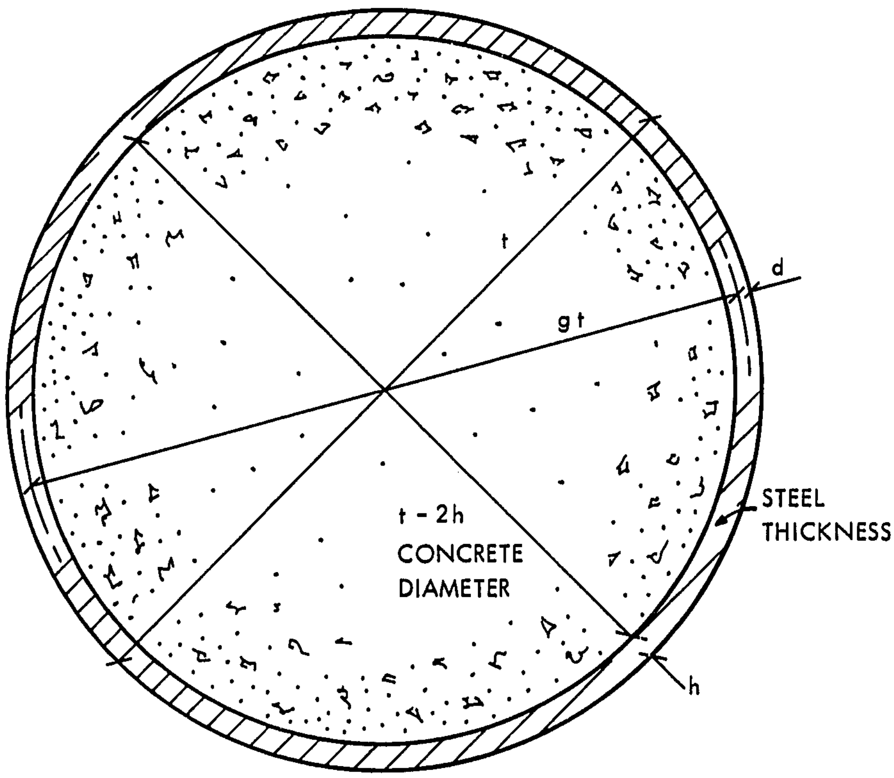

FIGURE 2.8

CROSS-SECTION OF CONCRETE-FILLED CYLINDRICAL STEEL COLUMN

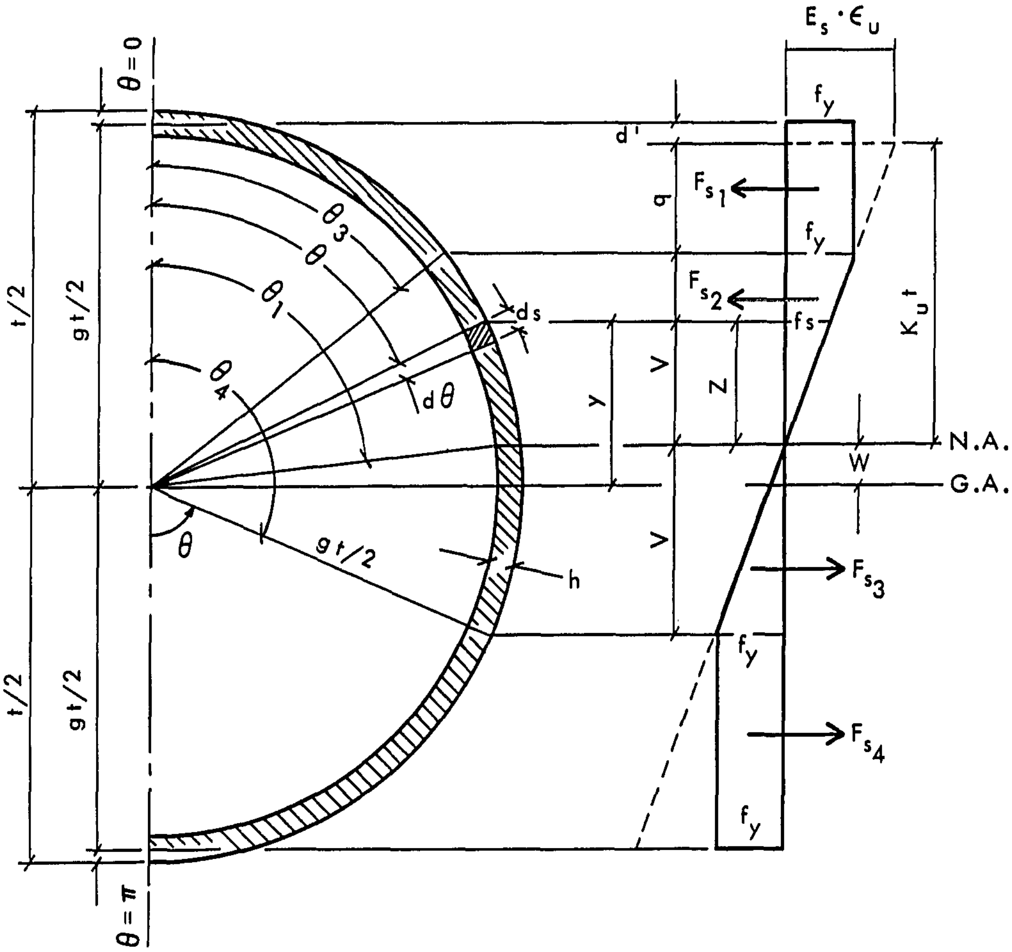

FIGURE 2.9

STRAIN DIAGRAM FOR STEEL AND CONCRETE FOR A RIGHT HALF OF CONCRETE-FILLED CYLINDRICAL STEEL COLUMN

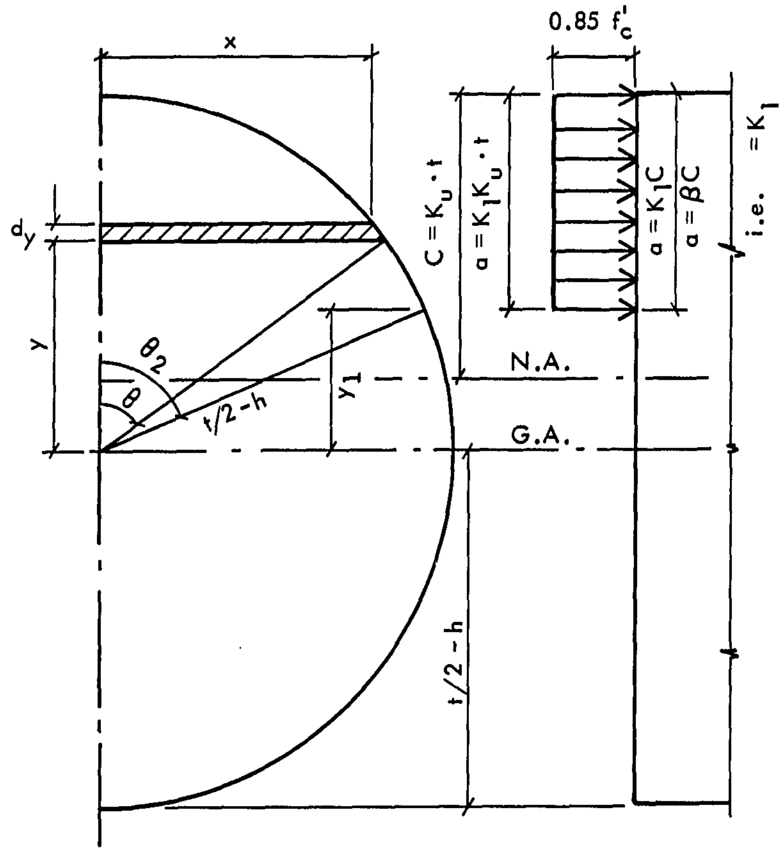

FIGURE 2.10

STRESS BLOCK OF CONCRETE FOR CONCRETE-FILLED CYLINDRICAL STEEL COLUMN

$$\theta_2 = \cos^{-1} \frac{(\frac{t}{2} - h) - \beta C}{(\frac{t}{2} - h)} \qquad (2.50)$$

$$\frac{V}{K_u t} = \frac{E_s}{E_s}\frac{\varepsilon_y}{\varepsilon_u} = \frac{\varepsilon_y}{\varepsilon_u} = \phi$$

$$V = \phi \cdot K_u t \qquad (2.51)$$

$$\cos\theta_3 = \frac{V + W}{gt/2} = \frac{(\phi \cdot K_u t) + (\frac{t}{2} - h - K_u t)}{gt/2}$$

$$\theta_3 = \cos^{-1} \left[\frac{t/2 - K_u t(1-\phi) - h}{gt/2}\right] \qquad (2.52)$$

$$\cos\theta_4 = \cos(180-\phi) = -\cos\phi = -\frac{V-W}{gt/2} = \frac{W-V}{gt/2} = \frac{(t/2 - h - K_u t) - (\phi \cdot K_u t)}{gt/2}$$

$$\theta_4 = \cos^{-1} \left(\frac{t/2 - h - K_u t - \phi K_u t}{gt/2}\right) \qquad (2.53)$$

$$\frac{gt}{2} = \frac{t}{2} - \frac{h}{2}$$

$$g = 1 - \frac{h}{t} \qquad (2.54)$$

One may also note from Figure (2.9) that the differential of arc length for the steel ring is given by the expression:

$$ds = \left(\frac{gt}{2}\right) d\theta \qquad (2.55)$$

By observing the strain diagram for the steel and concrete, and defining the steel stress at any point in the ring as f_s, the yield stress for the steel as f_y, the maximum strain in the concrete as ε_u and the yield strain in the steel as ε_y one can derive the general expression for f_s as follows:

$$\frac{f_s}{f_y} = \frac{Z}{V} = \frac{\frac{gt}{2}\left(\cos\theta - \cos\theta_1\right)}{\phi K_u t}$$

$$f_s = \left[\frac{gf_y(\cos\theta - \cos\theta_1)}{2K_u\phi}\right] < f_y \qquad (2.56)$$

2.21 Net Force in the Steel Pipe

The net force in the steel pipe can be obtained by summing the forces about the gravity axis. In the derivations compressive forces are positive and tension forces are negative. The coordinate y is measured positive upward from the gravity axis, so tension forces cause positive moment when considering the portion of the section below the gravity axis.

Considering only one half of the steel ring shown in Figure (2.9), the net force in the steel can be obtained as:

$$F_{st} = 2\left[F_{s1} + F_{s2} - F_{s3} - F_{s4}\right] \qquad (2.57)$$

$$F_{st} = 2\left[\int_{0}^{\theta_3} f_y \cdot dA_s + \int_{\theta_3}^{\theta_1} f_s dA_s - \int_{\theta_1}^{\theta_4} f_s dA_s - \int_{\theta_4}^{\pi} f_y \cdot dA_s\right] \tag{2.58}$$

From Figure (2.9) and Equation (2.55);

$$dA_s = h \cdot ds = h \frac{gt}{2} d\theta \tag{2.59}$$

By substituting Equations (2.56) and (2.59) into (2.58), we can determine the values of the integral forces as follows:

$$F_{s1} = f_y h \frac{gt}{2} \theta_3 \tag{2.60}$$

$$F_{s2} = \frac{f_y h g^2 t}{4 \phi K_u} \left[(\sin\theta_1 - \sin\theta_3) - (\theta_1 - \theta_3) \cos\theta_1\right] \tag{2.61}$$

$$F_{s3} = \frac{f_y h g^2 t}{4 \phi K_u} \left[(\sin\theta_4 - \sin\theta_1) - (\theta_4 - \theta_1) \cos\theta_1\right] \tag{2.62}$$

$$F_{s4} = f_y h \frac{gt}{2} (\pi - \theta_4) \tag{2.63}$$

Finally, by substituting Equations (2.60) to (2.63) into (2.57), we can obtain the total force in the steel pipe (F_{st}).

2.22 Net Moment in the Steel Pipe

Consequently, we will follow the same steps mentioned above for the net moment of the steel forces about the gravity axis of the cross-section. The net moment may be obtained by summing up all of the individual element of the differential moment by integration:

$$dM_s = dF_s \cdot y$$

or

$$dM_s = f_s \cdot dA_s \cdot y \qquad (2.64)$$

$$M_s = 2[F_s \cdot y] = 2[F_{s1}y + F_{s2}y + F_{s3}y + F_{s4}y]$$
$$= 2[M_{s1} + M_{s2} + M_{s3} + M_{s4}] \qquad (2.65)$$

or

$$M_s = 2[\int_0^{\theta_3} f_y \cdot y\ dA_s + \int_{\theta_3}^{\theta_1} f_s \cdot y\ dA_s + \int_{\theta_1}^{\theta_4} f_s \cdot y\ dA_s + \int_{\theta_4}^{\pi} f_y \cdot y\ dA_s] \qquad (2.66)$$

where:

$$y = \frac{gt}{2} \cos\theta \qquad (2.67)$$

Performing the integrals of Equation (2.66), yields:

$$M_{s1} = f_y\ h\ \frac{g^2t^2}{4} \sin\theta_3 \qquad (2.68)$$

$$M_{s2} = \frac{f_y\ h\ g^3t^2}{8\phi\ K_u} [\frac{1}{2} (\theta_1-\theta_3)+\frac{1}{4} (\sin2\theta_1-\sin2\theta_3)-\cos\theta_1(\sin\theta_1-\sin\theta_3)] \qquad (2.69)$$

$$M_{s3} = \frac{f_y\, h\, g^3 t^2}{8\phi\, K_u} \left[\frac{1}{2}(\theta_4-\theta_1)+\frac{1}{4}(\sin 2\theta_4-\sin 2\theta_1)-\cos\theta_1(\sin\theta_4-\sin\theta_1)\right] \quad (2.70)$$

$$M_{s4} = -f_y\, h\, \frac{g^2 t^2}{4} \sin\theta_4 \quad (2.72)$$

By substituting Equations (2.68) into Equation (2.65), the total moment in the steel pipe can be obtained.

2.23 Forces in Concrete

Figure (2.10) indicates half of the concrete cross-section with the stress block diagram. From this figure we can derive the concrete force as follows:

$$x = \left(\frac{t}{2} - h\right)\sin\theta \quad (2.73)$$

$$y = \left(\frac{t}{2} - h\right)\cos\theta \quad (2.73)$$

$$dy = -\left(\frac{t}{2} - h\right)\sin\theta\, d\theta \quad (2.74)$$

Therefore, the force in the concrete ignoring any effect of tension will be:

$$C_c = 2 \int_{y_1}^{(t/2)-h} (0.85\, f'_c)\, x\, dy \quad (2.75)$$

$$\text{At } y = y_1 \,,\quad \theta = \theta_2 \tag{2.76}$$

$$\text{At } y = t/2 - h \,,\quad \theta = 0 \tag{2.77}$$

If we substitute the values of x, dy and integral limits from Equations (2.72), (2.74), (2.76) and (2.77) into Equation (2.75) and perform the integration, then the net force in the concrete will be:

$$C_c = 0.85\, f'_c\, (\frac{t}{2} - h)^2\, (\theta_2 - \sin\theta_2 \cos\theta_2) \tag{2.78}$$

2.24 Net Moment in Concrete

From Figure (2.10), the net moment of the compression force in the concrete about the gravity axis of the section will be:

$$M_c = 2 \int_{y_1}^{(t/2)-h} (0.85\, f'_c)\, x\, y\, dy \tag{2.79}$$

By substituting the values of x, y, dy and integration limits from Equations (2.72), (2.73), (2.74), (2.76) and (2.77) into Equation (2.79), then, perform the integration yields:

$$M_c = 1.7\, f'_c\, (\frac{t}{2} - h)^3\, \frac{\sin^3\theta_2}{3} \tag{2.80}$$

From the above equations, the total net force and moment can be found, using the capacity reduction factor (ϕ), the load and moment

(ϕP_N, ϕM_N) of a point located on the interaction diagram can be obtained.

Similarly, by selecting various other locations for the neutral axis and repeating the calculations same way as for the above one, other points can be obtained for constructing the interaction diagram.

2.25 Computer Program

A comprehensive computer program for the previous two mathematical models and for various column cross-sections has been written. The computer output results and discussions will be presented in Chapter six. The input data for the six various cross-sections and the computer program list are given in Appendix C.

CHAPTER 3

HEAT TRANSFER THEORY AND FINITE DIFFERENCE FORMULATIONS

3.1 Introduction

Heat transfer to an object from gases and furnace walls may be divided into heat transfer by convection and heat transfer by radiation. The quantity of heat received per unit area, unit time, and unit temperature difference between object and surroundings, depends on many factors [16]. The most important are: temperature, composition, velocity of the gases, the thickness of the layer of gases between furnace walls and objects, the proportion between surface area of the object and inner surface of the furnace, and the emissivity of the furnace walls and object.

The exchange of heat between the gases in a furnace, the furnace walls, and an object, may be described as follows:

The gases are continuously transferring heat to walls and object, so that both attain a temperature dependent on the quantity of heat supplied to them.

The better the walls are insulated and the lower their thermal heat capacity, the higher their temperature will be. Thus through radiation more heat will be transferred from the walls to the object. Heat transfer may also be increased by enlarging the volume of the gases transferring heat to walls and object, because a thicker layer of gases gives more radiation [52]. A higher heat transfer may also be obtained by increasing the emissivity of the gases.

3.2 Heat Transfer by Convection

According to the existing information, the amount of heat transferred by convection to an object is less than 10 percent of the radiative heat [17]. It is known that above a certain level of the coefficient of heat transfer, which is easily obtained in fires and furnaces, the temperature of the surface of the exposed object (T_{ob}) will be very close to the temperature of the environment (T_f) [18,19]. hen the heat transferred by convection is:

$$Q_{conv} = h\,(T_f - T_{ob}) \tag{3.1}$$

where

Q_{conv} = heat transferred by convection

h = coefficient of heat transfer

T_f = fire temperature

T_{ob} = surface temperature of the object

In this region changes of the order of 10% will have little effect on the surface temperature and thus on the temperature in the exposed object. Therefore, to simplify the heat transfer model, the convective heat transfer will be neglected in this study.

3.3 Heat Transfer by Radiation

Furthermore, it will be assumed that the radiative heat transfer to the exposed object is approximately that of a black body. As explained subsequently, this assumption will cause only a small error.

In an actual fire, heat is received from luminous flames, which have a high emissivity. If the thickness of the flames is sufficient, the emissivity may reach values of about 0.9 or higher, and thus approaches that of a black body. For the same reason as in the case of convection, an error of the order of 10 percent in the radiative transfer will have little effect on the surface temperatures of the exposed object if the heat transfer is high. The high heat transfer from fires is simulated in furnaces by making them large, so that the flames have sufficient thickness, and by selecting furnace wall materials that produce wall temperatures close to the flame temperature.

Thermodynamic considerations show that an ideal radiator, or black body, will emit energy at a rate proportional to the fourth power of the absolute temperature of the body. When two bodies exchange heat by radiation, the net heat exchange is then proportional to the difference in T^4. Thus:

$$q_R = \sigma A(T_1^4 - T_2^4) \qquad (3.2)$$

where σ is Stefan-Boltzman constant with the value of 5.67×10^{-8} w/m^2 K^4. Equation (3.2) is called the Stefan-Boltzman law of thermal radiation, and it applies only to black bodies.

It is stated that a black body is a body which radiates energy according to the T^4 law. We call such a body black because black surfaces, such as a piece of metal covered with carbon black, approximate this type of behavior. Other types of surfaces, like a glossy painted surface or polished metal plate, do not radiate as much energy as the black body; however, the total radiation emitted by these bodies still generally follows the T^4 proportionality. To take account of the "gray" nature of such surfaces we introduce another factor into Equation (3.2), called the emissivity ε, which relates the radiation of the "gray" surface to that of an ideal black surface. In addition, we must take into account the fact that not all the radiation leaving one surface will reach the other surface since electromagnetic radiation travels in straight lines and some will be lost to the surroundings. We therefore introduce two new factors in Equation (3.2) to take into account both of these situations, so that:

$$q_R = \varepsilon_f \varepsilon_{ob} \quad \sigma A(T_1^4 - T_2^4) \qquad (3.3)$$

where ε_f is the emissivity factor for the black body which is the main source of heat and ε_{ob} is the emissivity for the gray body which is the object.

3.4 Heat Transfer by Conduction

The heat transferred by conduction in a column exposed to fire depends on the thermal properties of the concrete and steel at high tempertures.

3.4.1 Thermal Properties

Temperature rise in a column is determined by two properties of the concrete, thermal conductivity and thermal capacity. The latter is often given in the combination ρc, where ρ is the density of the concrete and c the specific heat. In addition to the above-mentioned thermal properties, another property, absorptivity, may influence temperatures. Absorptivity determines the portion of radiation incident on the exposed surface that will be absorbed by the concrete.

The thermal properties of concrete depend on the thermal properties of both the cement paste and the aggregate. Investigations [20] show that the thermal properties of cement paste are not subject to large variations; the thermal properties of aggregates, however, can vary over a wide range and have, therefore, a substantial influence.

It is common to make a distinction between siliceous and calcareous aggregates. Analysis, however, of a large number of tests on concrete blocks made with aggregates provided from regular production of five major Canadian producers showed no conclusive differences in the fire resistances of siliceous and calcareous concretes [21]. Results were similar for a series of tests on columns performed in Germany [22]. On the other hand, tests carried out in the U.S.A. on slabs [23] and columns [24] showed that the fire resistances of specimens made with calcareous aggregate are appreciably greater than those made with siliceous aggregate.

An important factor in determining the thermal properties of concrete is the molecular structure of the aggregate. For example, crystalline materials have higher conductivity than amorphous materials. It is probable that the amount of material in the aggregate that undergoes endothermic reactions (dehydration, decomposition and transformation) upon heating also has great influence on the thermal properties of the aggregate. In siliceous aggregates, for example the presence of quartz, which transforms from α-quartz into β-quartz at about 1000°F, will cause an increase in the specific heat of the aggregate. In calcareous aggregates, the presence of magnesite and dolomite, which dissociate at temperatures of 650 and 1350°F respectively, will affect the thermal properties of the aggregate. During dissociation, heat is absorbed so that the presence of magnesite and dolomite should be beneficial for the fire resistance of the column. Its effect on the thermal properties, however, is not yet known precisely [25].

The values of the material properties and physical constants for concrete and steel are given below [3]:

3.4.2 Concrete Properties

Thermal capacity of concrete (J m^{-3}°C^{-1})

for $0 < T < 200°C$, $\rho_c c_c = (0.005T + 1.7) \times 10^6$ (3.4)

for $200°C < T < 400°C$, $\rho_c c_c = 2.7 \times 10^6$ (3.5)

for $400°C < T < 500°C$, $\rho_c c_c = (0.013T - 2.5) \times 10^6$ (3.6)

for $500°C < T < 600°C$, $\rho_c c_c = (-0.013T + 10.5) \times 10^6$ (3.7)

for $T > 600°C$, $\rho_c c_c = 2.7 \times 10^6$ (3.8)

Thermal conductivity of concrete (W m^{-1}°C^{-1})

for $0 < T < 800°C$, $k_c = -0.00085T + 1.9$ (3.9)

for $T > 800°C$, $k_c = 1.22$ (3.10)

Coefficient of thermal expansion

$$\alpha_c = (0.008T + 6) \times 10^{-6} \qquad (3.11)$$

3.4.3 Steel Properties

Thermal capacity of steel (J m^{-3}°C^{-1})

for $0 < T < 650°C$, $$\rho_s c_s = (0.004T + 3.3) \times 10^6 \qquad (3.12)$$

for $650°C < T < 725°C$, $$\rho_s c_s = (0.068T - 38.3) \times 10^6 \qquad (3.13)$$

for $725°C < T < 800°C$, $$\rho_s c_s = (-0.086T + 73.35) \times 10^6 \qquad (3.14)$$

for $T > 800°C$, $$\rho_s c_s = 4.55 \times 10^6 \qquad (3.15)$$

Thermal conductivity of steel (W m^{-1}°C^{-1})

for $0 < T < 900°C$, $$k_s = -0.22T + 48 \qquad (3.16)$$

for $T > 900°C$, $$k_s = 28.2 \qquad (3.17)$$

Coefficient of thermal expansion

for $T < 1000°C$, $$\alpha_s = (0.0.004T + 12) \times 10^{-6} \qquad (3.18)$$

for $T > 1000°C$, $$\alpha_s = 16 \times 10^{-6} \qquad (3.19)$$

3.4.4 Water Properties (J m^{-3}°C^{-1})

Thermal capacity

$$\rho_w c_w = 4.2 \times 10^{-6} \quad (3.20)$$

Heat of vaporization (J kg^{-1})

$$\lambda_w = 2.3 \times 10^{6} \quad (3.21)$$

3.4.5 Physical Constants

σ = Stefan-Boltzmann constant: 5.67×10^{-8} W/m ^{2}°K^4 (3.22)

ε_f = emissivity of fire: 1 (3.23)

ε_s = emissivity of steel: 0.9, ε_c = emissivity of concrete = 0.9 (3.24)

Due to the variability of thermal properties mentioned above and the assumptions mentioned in reference [2], the equation used for heat transfer by conduction for column cross-section is [Appendix D]:

$$\frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) = \rho c \frac{\partial T}{\partial t} \quad (3.25)$$

Equation (3.25) is unsteady state partial differential equation of parabola in two variables x and z. Also in the above equation, the thermal conductivity (K) and the thermal capacity (ρc) are functions of temperature.

In order to apply Equation (3.25) to predict temperature distribution of columns cross-section, a numerical solution is required. The finite difference method will be used for this purpose.

3.5 Finite Difference Method

It is true that of numerous problems in the physical sciences and engineering that, even if the differential equations governing the physical phenomena can be formulated mathematically, the analytical solution of the resulting equations is beyond the reach of pure mathematics. A useful line of attack in such problems is to solve the equations for particular cases by numerical methods. Further, it is often found that even when a general solution to the differential equations is known it proves to be very difficult and tedious to translate the general solution into particular results for a particular problem. Therefore not only are numerical methods essential in problems which will not yield to any other method of solution but they are also often the best means of obtaining a particular solution even when a general solution can be found by analytical methods.

There exist a large number of different numerical methods for solving partial differential equations, the most important of which is the method of finite differences. Finite difference methods were discussed in the 1920's, but only in recent years, with the development of high-speed computing machines, have these methods been appllied in practical problems on a large scale. Although digital computers perform just the same operations as can be performed by hand, their speed and capacity make it possible to deal with problems whose solution is not feasible by hand calculation.

Finite difference methods seeks to replace the differential equation by algebraic equations which give relations between values of the dependent variable and proximate values of the independent variable or variables. The numerical solution then consists of solving a series of simultaneous algebraic equations to give values of the dependent variable corresponding to a number of discrete points throughout the domain of interest.

3.6 Finite Difference Approximation to Derivatives

3.6.1 Functions of a Single Variable

When a function T and its derivatives are single-valued and continuous functions of t, Figure (3.1), then by Taylor's theorem [26,27,28,29]:

$$T^{j+1} = T^j + (\Delta t)\left(\frac{dT}{dt}\right)_Q + \frac{(\Delta t)^2}{2}\left(\frac{d^2T}{dt^2}\right)_Q + \frac{(\Delta t)^3}{6}\left(\frac{d^3T}{dt^3}\right)_Q + \ldots \tag{3.26}$$

and

$$T^{j-1} = T^j - (\Delta t)\left(\frac{dt}{dt}\right)_Q + \frac{(\Delta t)^2}{2}\left(\frac{d^2T}{dT^2}\right)_Q - \frac{(\Delta T)^3}{6}\left(\frac{d^3T}{dt^3}\right)_Q + \ldots \tag{3.27}$$

Addition of these expansions gives:

$$T^{j+1} + T^{j-1} = 2T^j + (\Delta t)^2\left(\frac{d^2T}{dt^2}\right)_Q + O(\Delta t)^4_Q\,, \tag{3.28}$$

where $O(\Delta t)^4$ denotes terms containing fourth and higher powers of (Δt). Assuming these are negligible in comparison with lower powers of Δt it follows that,

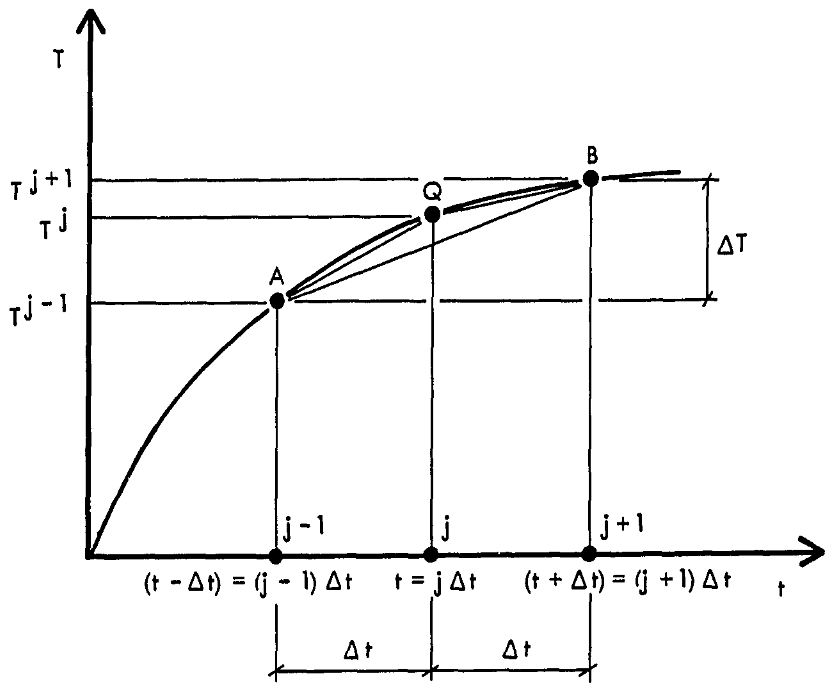

FIGURE 3.1

FINITE DIFFERENCE APPROXIMATION

$$\left(\frac{d^2T}{dt^2}\right)_Q = \frac{1}{(\Delta t)^2}\left[T^{j+1} - 2T^j + T^{j-1}\right] \quad (3.29)$$

with a leading error on the right-hand side termed the trancation error of order $(\Delta t)^2$.

The right-hand side of Equation (3.29) is the finite-difference analog to a second derivative, with a truncation error of a second order. In such a case the analog is said to be second order correct.

On subtracting Equation (3.27) from Equation (3.26) and neglecting terms of order $(\Delta t)^3$, the following can be obtained:

$$\left(\frac{dT}{dt}\right)_Q = \frac{1}{2(\Delta t)}\left[T^{j+1} - T^{j-1}\right] \quad (3.30)$$

The right-hand side of Equation (3.30) is the second order correct analog to the first derivative.

Equation (3.30) clearly approximates the slope of the tangent at point Q by the slope of the chord AB, and is called a central-difference approximation. One can also approximate the slope of the tangent at Q by either the slope of the chord OB, giving the forward-difference formula:

$$\left(\frac{dT}{dt}\right)_Q = \frac{1}{\Delta t}\left[T^{j+1} - T^j\right] \quad (3.31)$$

or the slope of the chord AQ, giving the backward-difference formula,

$$\left(\frac{dT}{dt}\right)_Q = \frac{1}{\Delta t}\left[T^j - T^{j-1}\right] \quad (3.32)$$

Both Equations (3.31) and (3.32) can be written down immediately from Equations (3.26) and (3.27) respectively assuming the second and higher powers of (Δt) to be negligible. This shows that the truncation error in these forward and backward-difference formula are both $O(\Delta t)$, and the formula are said to be first order correct analog.

3.6.2 Functions of Several Variables

A procedure similar to the one described above applies when T is a function of more than one independent variable. If the independent variables are x and y, and the x-y plane is subdivided into sets of equal rectangles of sides equal Δx and Δy as shown in Figure (3.2), then that the coordinates (x,y) of the representative mesh point Q [26 to 30] are:

$$x = m(\Delta x), \quad y = n(\Delta y)$$

where m, n are integers and the values of T at Q is denoted by:

$$T_Q = T[m(\Delta x),\ n(\Delta y)] = T(m,n)$$

Then from Equation (3.29),

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_{(m,n)} = \frac{T^j_{(m+1,n)} - 2T^j_{(m,n)} + T^j_{(m-1,n)}}{(\Delta x)^2} \qquad (3.33)$$

with a truncation error of order $(\Delta x)^2$. Similarly,

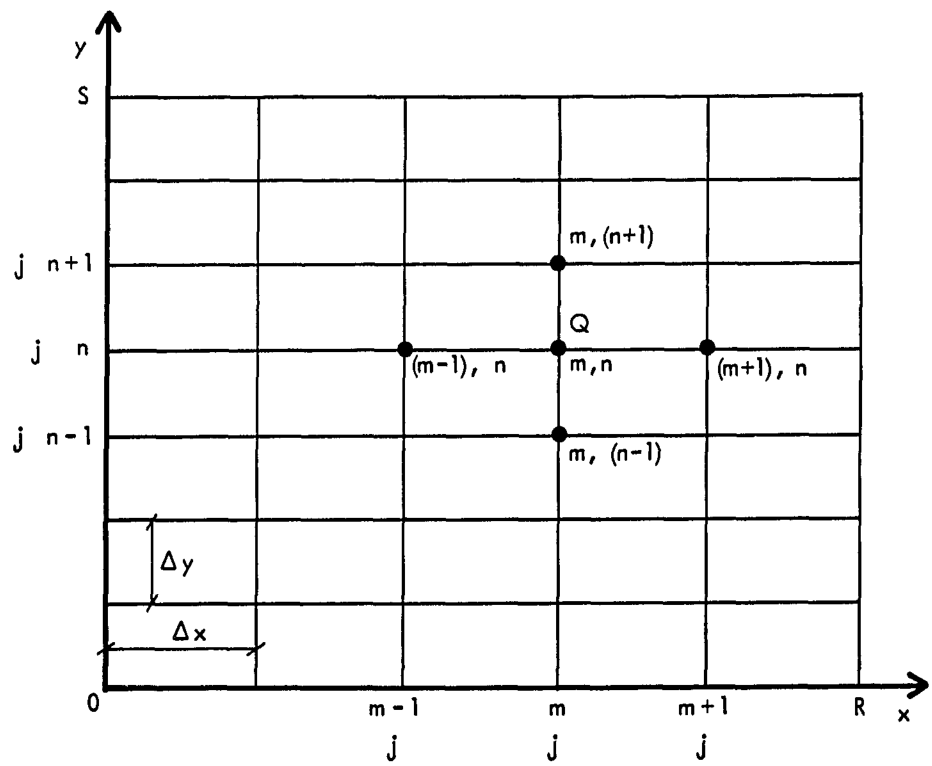

FIGURE 3.2

FINITE DIFFERENCE NOTATIONS FOR A RECTANGULAR MESH

$$\left(\frac{\partial^2 T}{\partial y^2}\right)^j_{(m,n)} = \frac{T^j_{(m,n+1)} - 2T^j_{(m,n)} + T^j_{(m,n-1)}}{(\Delta y)^2} \tag{3.34}$$

with a truncation error of $(\Delta y)^2$.

Also from Equation (3.30),

$$\left(\frac{\partial T}{\partial x}\right)^j_{(m,n)} = \frac{T^j_{(m+1),n} - T^j_{(m-1),n}}{2(\Delta x)} \tag{3.35}$$

and

$$\left(\frac{\partial T}{\partial y}\right)_{(m,n)} = \frac{T^j_{m,(n+1)} - T^j_{m,(n-1)}}{2(\Delta t)} \tag{3.36}$$

3.6.3 Finite Difference Expressions for Irregular Boundaries

When the boundary of the region is not such that a network of rectangles can be drawn so that the boundary would coincide with the nodes of the mesh, one must proceed differently at points near the boundary. In the general case of a group of five points whose spacing in nonuniform, arranged in an unequal-armed star as shown in Figure (3.3), if distances OB and OD are represented by $S(\Delta x)$ and $e(\Delta y)$ respectively than by Taylor's theorem

$$T_A = T_{(0)} - \left(\frac{\partial T}{\partial x}\right)_{(0)} (\Delta x) + \left(\frac{\partial^2 T}{\partial x^2}\right)_{(0)} \cdot \frac{(\Delta x)^2}{2} - \left(\frac{\partial^3 T}{\partial x^3}\right)_{(0)} \frac{(\Delta x)^3}{6} \tag{3.37}$$

and

$$T_B = T_{(0)} + \left(\frac{\partial T}{\partial x}\right)_{(0)} S(\Delta x) + \left(\frac{\partial^2 T}{\partial x^2}\right)_{(0)} \frac{S^2(\Delta x)^2}{2} + \left(\frac{\partial^3 T}{\partial x^3}\right)_{(0)} \frac{S^3(\Delta x)^3}{6} \tag{3.38}$$

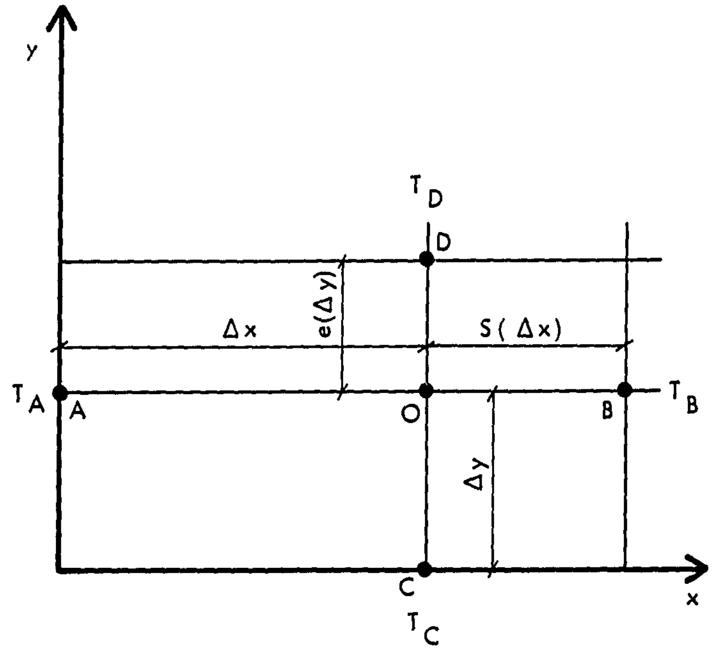

FIGURE 3.3

FINITE DIFFERENCE REPRESENTATION FOR IRREGULAR REGIONS

By multiplying Equation (3.37) by S, and adding to Equation (3.38) the following expression is obtained for the second derivative of T with respect to x.

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_{(0)} = \frac{2[ST_A - (1+S)T_0 + T_B]}{(\Delta x)^2 S(1+S)} \tag{3.39}$$

Similarly, $\left(\frac{\partial^2 T}{\partial y^2}\right)_{(0)}$ can be expressed as:

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_{(0)} = \frac{2[eT_c - (1+e)T_0 + T_D]}{(\Delta y)^2 e(1+e)} \tag{3.40}$$

The right-hand sides of Equations (3.39) and (3.40) are the first-order correct analogs to the second derivatives.

To get an expression for the first derivative, Equation (3.37) is multiplied by $(-S^2)$ and then added to Equation (3.38), so that:

$$\left(\frac{\partial T}{\partial x}\right)_{(0)} = \frac{T_B - (1-S^2)\, T_0 - S^2 T_A}{S(1+S)(\Delta x)} \tag{3.41}$$

In a similar manner

$$\left(\frac{\partial T}{\partial y}\right)_{(0)} = \frac{T_D - (1-e^2)\, T_0 - e^2\, T_C}{e(1+e)(\Delta y)} \tag{3.42}$$

It should be noted that the above analogs to the first derivatives are second order correct.

3.6.4 Numerical Solution of Parabolic Partial Differential Equations

The numerical solution by finite difference for the two-dimensions heat-transfer partial differential Equation (3.25) will be as explained below [30]. Considering the following equation:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)_{(m,n)} + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)_{(m,n)} = \rho c\left(\frac{\partial T}{\partial t}\right)_{(m,n)} \tag{3.25}$$

Then consider the two-dimension body divided into increments as shown in Figure (3.4-A). The subscript m denotes the number of raws, and the subscript n denotes the number of columns. Within the solid body the differential equation which governs the heat flow is Equation (3.25).

Consider the x-direction Figure (3.4-B), the finite difference expression representing the first term of the partial differential equation (3.25) is:

$$\frac{\partial}{\partial x}\left(K^j\frac{\partial T^j}{\partial x}\right)\Bigg|_{(m,n)} = \frac{\left\{K^j\frac{\partial T^j}{\partial x}\right\}_{(m-\frac{1}{2},n+\frac{1}{2})} - \left\{K^j\frac{\partial T^j}{\partial x}\right\}_{(m+\frac{1}{2},n-\frac{1}{2})}}{\Delta x} \tag{3.43}$$

Where the superscript j at time (t) = j(Δt)

but:

$$\left\{K^j\frac{\partial T^j}{\partial x}\right\}_{(m-\frac{1}{2},n+\frac{1}{2})} = K^j_{(m-\frac{1}{2},n+\frac{1}{2})}\frac{T^j_{(m-1,n+1)} - T^j_{(m,n)}}{\Delta x} \tag{3.44}$$

and

$$\left\{K^j\frac{\partial T^j}{\partial x}\right\}_{(m+\frac{1}{2},n-\frac{1}{2})} = K^j_{(m+\frac{1}{2},n-\frac{1}{2})}\frac{T^j_{(m,n)} - T^j_{(m+1,n-1)}}{\Delta x} \tag{3.45}$$

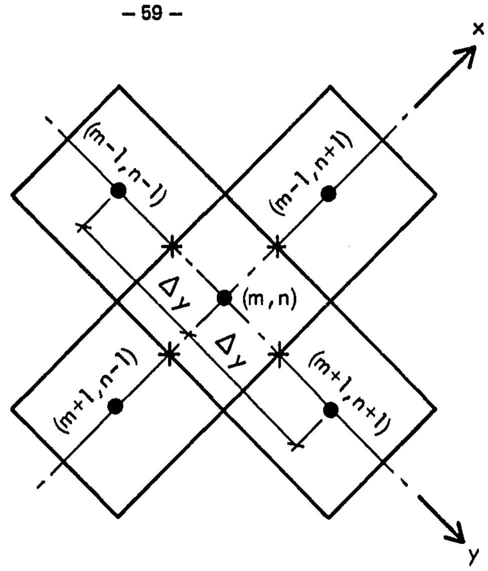

FIGURE 3.4-A

TWO-DIMENSION MESH

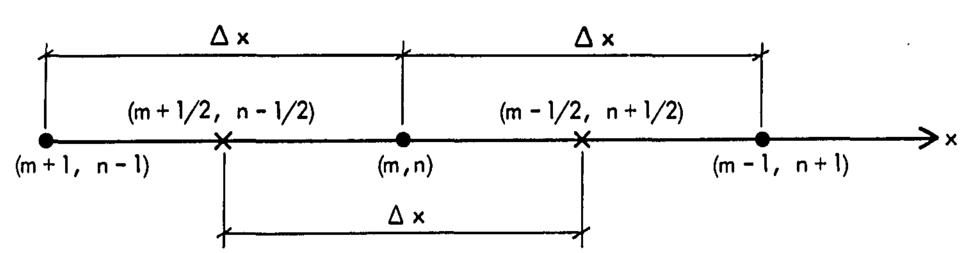

FIGURE 3.4-B

CONSIDER x-DIRECTION

Also, the variable conductivity K is:

$$K^j_{(m-\frac{1}{2},n+\frac{1}{2})} = \frac{K^j_{(m-1,n+1)} + K^j_{(m,n)}}{2} \tag{3.46}$$

and

$$K^j_{(m+\frac{1}{2},n-\frac{1}{2})} = \frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2} \tag{3.47}$$

Substituting Equations (3.46), (3.47) into Equations (3.44), (3.45), we get:

$$\left\{K^j \frac{\partial T^j}{\partial x}\right\}_{(m-\frac{1}{2},n+\frac{1}{2})} = \left(\frac{K^j_{(m-1,n+1)} + K^j_{(m,n)}}{2}\right)\left(\frac{T^j_{(m-1,n+1)} - T^j_{(m,n)}}{\Delta x}\right) \tag{3.48}$$

$$\left\{K^j \frac{\partial T^j}{\partial x}\right\}_{(m+\frac{1}{2},n-\frac{1}{2})} = \left(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2}\right)\left(\frac{T^j_{(m,n)} - T^j_{(m+1,n-1)}}{\Delta x}\right) \tag{3.49}$$

Substituting Equations (3.48), (3.49) into Equation (3.43) yields:

$$\frac{\partial}{\partial x}\left(K^j \frac{\partial T^j}{\partial x}\right)\Bigg|_{(m,n)} = \frac{1}{(\Delta x)^2}\left\{\left(\frac{K^j_{(m-1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m-1,n+1)} - T^j_{(m,n)}\right)\right.$$

$$\left. + \left(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n-1)} - T^j_{(m,n)}\right)\right\} \tag{3.50}$$

In a similar way one can get finite difference scheme for the second term of the left-hand side of Equation (3.25) as follows:

$$\frac{\partial}{\partial y}\left(K^j \frac{\partial T^j}{\partial y}\right)\Bigg|_{(m,n)} = \frac{1}{(\Delta y)^2}\Big\{\Big(\frac{K^j_{(m+1,n+1)} + K^j_{(m,n)}}{2}\Big)\Big(T^j_{(m+1,n+1)} - T^j_{(m,n)}\Big)$$

$$+ \Big(\frac{K^j_{(m-1,n-1)} + K^j_{(m,n)}}{2}\Big)\Big(T^j_{(m-1,n-1)} + T^j_{(m,n)}\Big)\Big\} \quad (3.51)$$

Also, the right-hand side of Equation (3.25) can be found from equation (3.31) as follows:

$$\rho c \frac{\partial T}{\partial t}\Bigg|_{(m,n)} = \frac{(\rho c)^j}{\Delta t}\Big[T^{j+1}_{(m,n)} - T^j_{(m,n)}\Big] \quad (3.52)$$

Substituting Equations (3.50), (3.51) and (3.52) into Equation (3.25) yields to the finite difference equation equivalent to Equation (3.25) as:

$$T^{j+1}_{(m,n)} = T^j_{(m,n)} + \frac{\Delta t}{(\rho c)^j_{(m,n)}}\Big\{\frac{1}{(\Delta x)^2}\Big[\Big(\frac{K^j_{(m-1,n+1)} + K^j_{(m,n)}}{2}\Big)$$

$$\Big(T^j_{(m-1,n+1)} - T^j_{(m,n)}\Big)$$

$$+ \Big(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2}\Big)\Big(T^j_{(m+1,n-1)} - T^j_{(m,n)}\Big)\Big] +$$

$$\frac{1}{(\Delta y)^2}\left[\left(\frac{K^j_{(m+1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n+1)} - T^j_{(m,n)}\right)\right.$$

$$\left. + \left(\frac{K^j_{(m-1,n-1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m-1,n-1)} - T^j_{(m,n)}\right)\right]\} \qquad (3.53)$$

In the special case where $\Delta x = \Delta y$, Equation (3.53) becomes:

$$T^{j+1}_{(m,n)} = T^j_{(m,n)} + \frac{\Delta t}{(\rho c)^j_{(m,n)}(\Delta x)^2}\{\left(\frac{K^j_{(m-1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m-1,n+1)} - T^j_{(m,n)}\right)$$

$$+ \left(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n-1)} - T^j_{(m,n)}\right)$$

$$+ \left(\frac{K^j_{(m+1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n+1)} - T^j_{(m,n)}\right)$$

$$+ \left(\frac{K^j_{(m-1,n-1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m-1,n-1)} - T^j_{(m,n)}\right)\} \qquad (3.54)$$

Equation (3.54) is the final form of the finite difference approximation of Equation (3.25). This obtained equation will be applied for the solutions of fire resistance of columns.

CHAPTER 4

THIRD MATHEMATICAL MODEL

FIRE RESISTANCE OF CONCRETE-FILLED SQUARE STEEL COLUMNS

4.1 Introduction

The mathematical models and the computer programs which have been developed to calculate the maximum allowable load for various columns cross-sections [31,32,33,34], in Chapter one, will be used for column's experimental tests. The purpose of these experimental tests is to validate the mathematical models of fire resistance of columns [2,3,4,5].

In this Chapter, the calculation of fire resistance of a concrete-filled square steel columns, for which no method exists at present, is developed. The various steps in the construction of the mathematical model to calculate temperatures, deformations and strength of the column are presented. A computer program for this model has been written.

4.2 Temperatures in Column

The column temperatures are calculated by using the finite difference method [35]. The method of driving the heat transfer equations and calculating the temperatures in objects exposed to heat is described in detail in Chapter three.

4.3 Division of Cross-Section Into Elements

The cross-sectional area of the column is subdivided into a number of elements, arranged in a triangular network Figure (4.1). The elements are square inside the column and triangular at the surface. For the inside elements, the temperature at the centre is taken as representative of the entire element. For the triangular surface elements, the representative points are located on the centre of each hypotenuse.

Because only columns with square cross-sections (and four axis of symmetry) will be considered, it is possible to calculate the temperature distribution in only one-eighth of the cross-sectional area of the column as illustrated in Figure (4.2). In Figure (4.1), in an x-z coordinate system, a point $P_{m,n}$ has the coordinates $x = (m-1)\Delta h_g$ and $z = (n-1)\Delta h_g$.

4.4 Temperature Calculations

It will be assumed that the columns are exposed on all sides to the heat of a fire whose tmeperature course follows that of the standard fire described in References [36, 37]. This temperature course can be approximately described by the following expressions:

$$T_f^j = 20 + 750\left[1 - \exp(-3.79553\sqrt{t})\right] + 170.41\sqrt{t} \qquad (4.1)$$

where t is the time in hours and T_f^j is the fire temperature in °C at time $t = j\Delta t$.

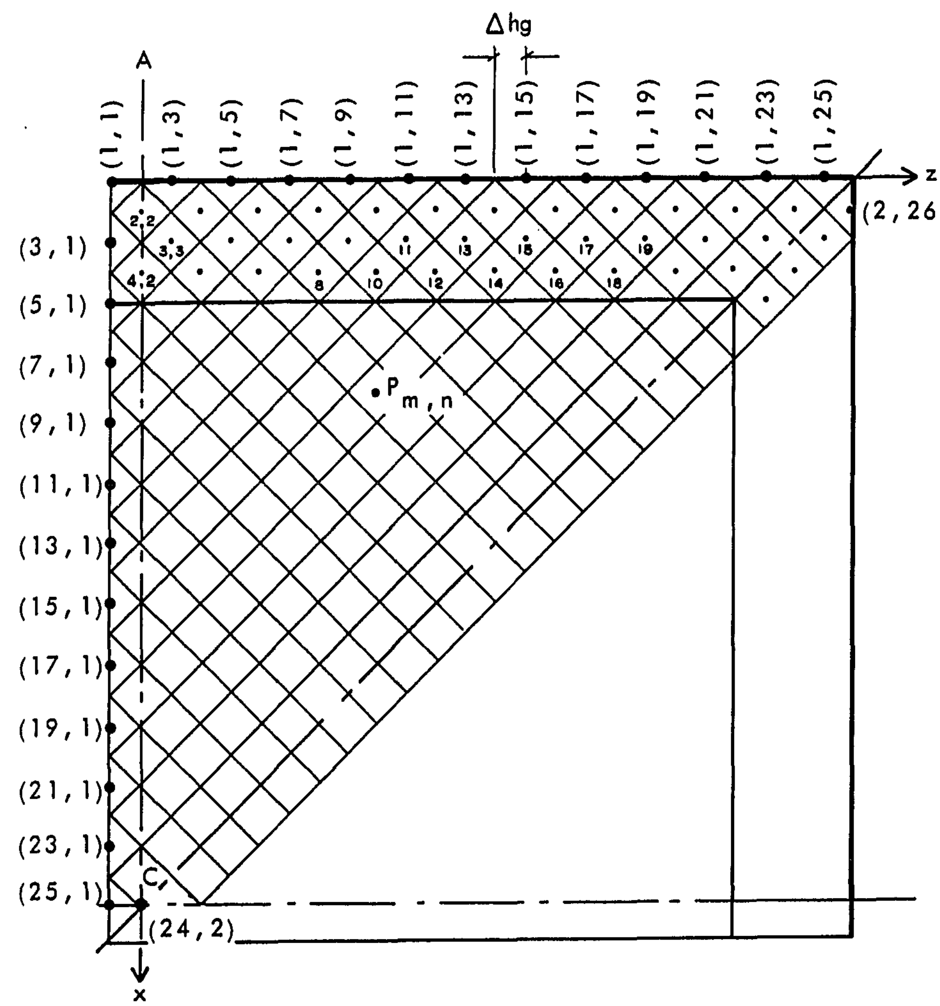

FIGURE 4.1

TRIANGULAR NETWORK OF ELEMENTS IN A ONE-EIGHTH SECTION OF COLUMN

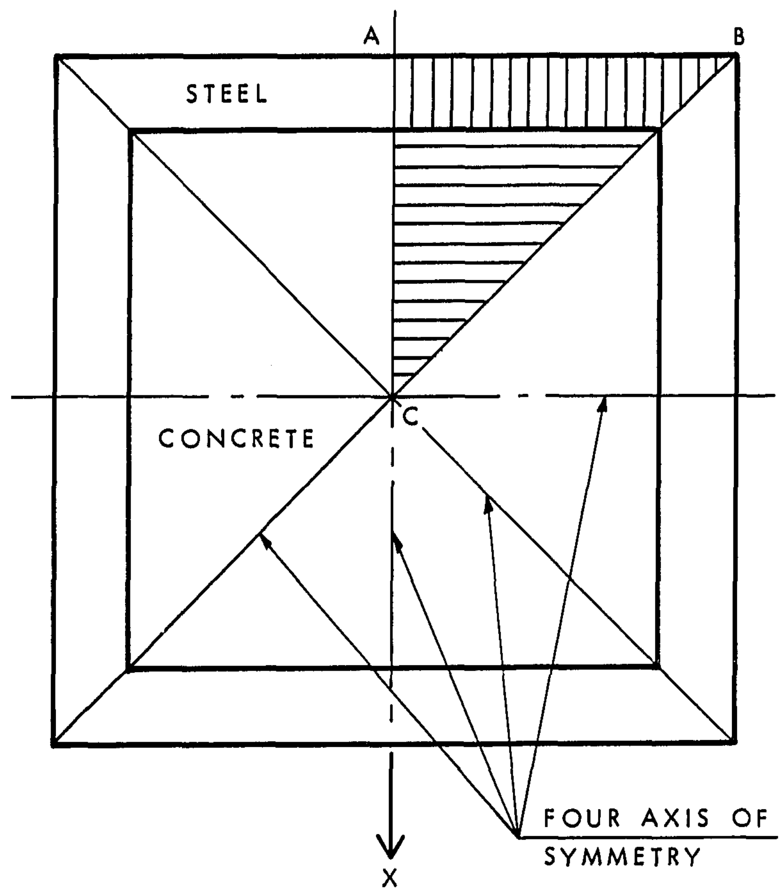

FIGURE 4.2

COLUMN CROSS-SECTION

4.5 Equations at Fire/Steel Boundary

The temperature rise in each element can be derived by making a heat balance for it, i.e. by applying the parabolic unsteady state, partial differential Equation (3.25) and its numerical solution Equation (3.54) into the region. Also, the heat transfer by radiation to the surface boundary elements must be considered as follows:

4.5.1 Heat Transfer by Radiation

This heat which has been given by Equation (4.1) will be transmitted from the fire to an elementary surface region, $R_{m,n}$ by radiation. For the fire/steel boundary, the heat transmitted by radiation along the boundary A-B (see Figure 4.3) during the period $j\Delta t < t < (j+1)\Delta t$ for a unit height of the column can be found as explained in Chapter three as:

$$q_R = A_{es} \ \sigma \ \varepsilon_f \varepsilon_s \left[(T_f^j + 273)^4 - (T_{m,n}^j + 273)^4 \right] \qquad (4.2)$$

where:

q_R = heat transfer by radiation, J/(m.hr)

A_{es} = surface area of the fire/steel boundary element

i.e. $A_{es} = 2(\Delta h_g)(1.0)$, m

σ = Stefan-Boltzman constant, $J/(hr.m^2.K^4)$

ε_f, ε_s = as defined in Equations (3.22) to (3.24), dimensionless.

T_f^j = fire temperature, K^4

4.5.2. Heat Transfer by Conduction

From the surface region $R_{(m,n)}$ along the boundary line A-B as illustrated in Figure (4.3), heat is transfer by conduction to the two neighboring regions, $R_{(m+1,n-1)}$ and $R_{(m+1,n+1)}$. This heat can be

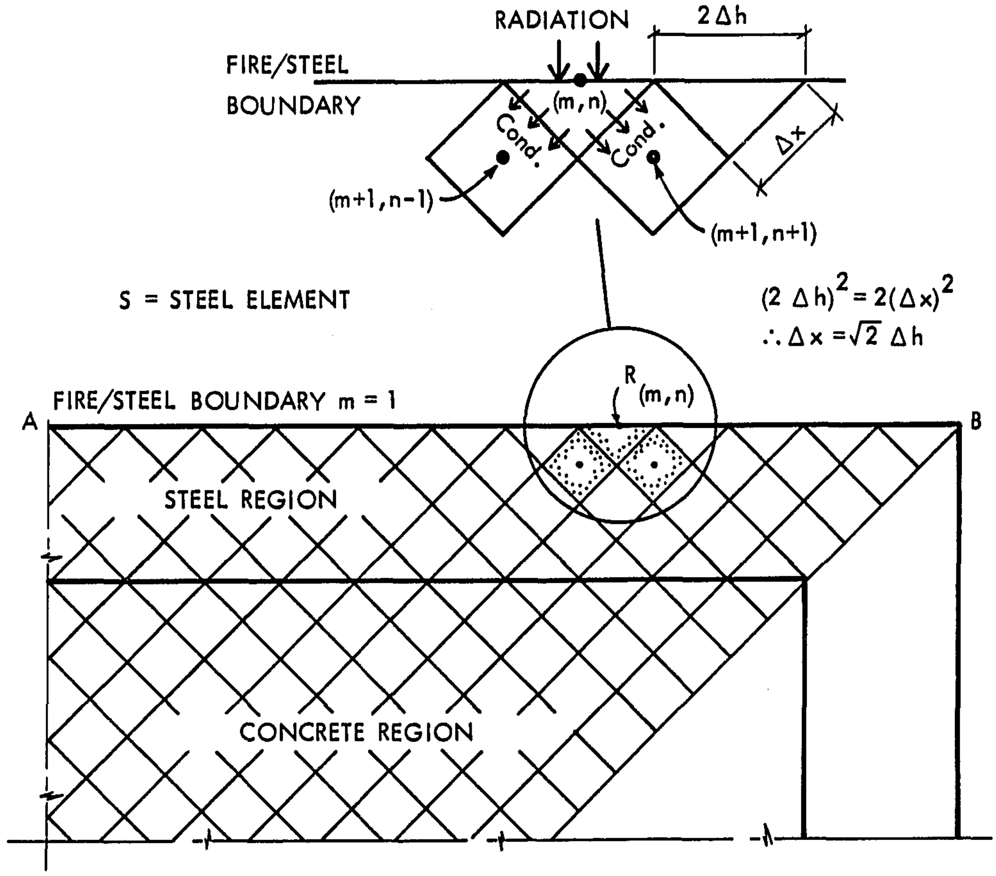

FIGURE 4.3

FIRE/STEEL BOUNDARY REGION

obtained by applying Equation (3.25) and its numerical solution as follows:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) = \rho c\,\frac{\partial T}{\partial t}$$

for fire/steel boundary:

$$\left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)\right]A_e = \left(\rho c\frac{\partial T}{\partial t}\right)A_e \qquad (3.25)$$

The two terms of the left-hand side of Equation (3.25), represent the heat transfer by conduction. From Equations (3.50), (3.51) and (3.25) it can be found:

$$\frac{\partial}{\partial x}\left(K^j\frac{\partial T^j}{\partial x}\right)\Bigg|_{(m,n)} = \frac{1}{(\Delta x)^2}\left\{\left(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n-1)} - T^j_{(m,n)}\right\}\right. \qquad (4.3)$$

$$\frac{\partial}{\partial z}\left(K^j\frac{\partial T^j}{\partial z}\right)\Bigg|_{(m,n)} = \frac{1}{(\Delta z)^2}\left\{\left(\frac{K^j_{(m+1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n+1)} - T^j_{(m,n)}\right)\right\} \qquad (4.4)$$

From Figure (4.1), it can be obtained:

$$\Delta x = \Delta z = \Delta h_g \qquad (4.5)$$

and $$(A_e)_{f/s} = \tfrac{1}{2}(2\Delta h_g)(\Delta h_g) = (\Delta h_g)^2 \qquad (4.6)$$

Using Equations (4.3) to (4.6) into the left-hand side of Equation (3.25) yields:

$$q_c = [\frac{\partial}{\partial x}(K\frac{\partial T}{\partial x}) + \frac{\partial}{\partial z}(K\frac{\partial T}{\partial z})](\Delta h_g)^2$$

$$= \frac{1}{(\Delta h_g)^2}\{(\frac{K^j_{(m+1,n+1)}+K^j_{(m,n)}}{2})(T^j_{(m+1,n-1)} - T^j_{(m,n)})$$

$$+ (\frac{K^j_{(m+1,n-1)}+K^j_{(m,n)}}{2})(T^j_{(m+1,n-1)} - T^j_{(m,n)})\}(\Delta h_g)^2 \qquad (4.7)$$

Equation (4.7) is the finite difference method representation for the parabolic unsteady state partial differential Equation (3.25).

4.5.3 <u>Sensible Heat</u>

During exposure, heat may be generated within the elements of the column cross-section, because of material decomposition. It is also possible that heat is absorbed because of dehydration or transformation processes in the material. If Q is the rate of heat generation (+) or absorption (-) per unit volume, then heat gain or loss in an elementary region $R_{(m,n)}$ for a time period Δt can be found by applying Equation (3.52). Then the sensible heat absorbed by the element in this period is [Appendix E].

$$(\frac{\partial E}{\partial t})_{AB} = Q \times \frac{\partial T}{\partial t}$$

$$= (\rho_s C_s)_{(m,n)} \times (A_e)_{(m,n)} (\frac{T^{j+1}_{(m,n)} - T^j_{(m,n)}}{\Delta t}) \qquad (4.8)$$

where:

$(\frac{\partial E}{\partial t})_{AB}$ = heat absorbed or the rate of change in energy or the sensible heat along the boundary line AB, J/(m.hr)

$(\rho_s C_s)_{(m,n)}$ = thermal capacity of steel, $J/(m^3 C°)$

$(A_e)_{(m,n)}$ = volume of an element located at the fire/steel boundary, m^2

4.5.4 Final Equation for Fire/Steel Boundary

Fire/steel boundary equation can be obtained by adding all heat gained and losses as follows:

$$q_{R_{(fire \rightarrow m,n)}} - q_{c_{\substack{(m,n \rightarrow m+1,n-1)\\ +(m,n \rightarrow m+1,n+1)}}} = (\frac{\partial E}{\partial t})_{AB_{(m,n)}} \qquad (4.9)$$

Subsituting the values of q_R, q_c and $(\frac{\partial E}{\partial t})_{AB}$ from Equations (4.2), (4.7) and (4.8) into Equation (4.9) and rearranging, the following heat balance for an elementary region $R_{(m,n)}$ is:

$$A_{es}\ \sigma\varepsilon_f\varepsilon_s\ [(T_f^j + 273)^4 - (T^j_{(m,n)} + 273)^4] -$$

$$\tfrac{1}{2}\ \{(\frac{K^j_{(m+1,n-1)} + K^j_{(m,n)}}{2})\ (T^j_{(m+1,n-1)} - T^j_{(m,n)}) +$$

$$\left(\frac{K^j_{(m+1,n+1)} + K^j_{(m,n)}}{2}\right)\left(T^j_{(m+1,n-1)} - T^j_{(m,n)}\right)\}$$

$$= (\rho_s C_s)_{(m,n)} \left(\frac{T^{j+1}_{(m,n)} - T^j_{(m,n)}}{\Delta t}\right) (\Delta h_g)^2 \qquad (4.10)$$

Because the heat must flow downhill on the temperature scale, a minus sign must be inserted into the above equation. The temperature $T^{j+1}_{(m,n)}$ at the time $(j+1)\Delta t$ for an elementary region $R_{(m,n)}$ can be obtained by rearranging equation (4.10) as follows:

$$T^{j+1}_{(m,n)} = T^j_{(m,n)} + \frac{\Delta t}{(\rho_s C_s)_{m,n} (\Delta h_g)^2}$$

$$\{\left[\frac{K^j_{s(m+1,n-1)} + K^j_{s(m,n)}}{2}\right]\left[T^j_{(m+1,n-1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{s(m+1,n+1)} + K^j_{s(m,n)}}{2}\right]\left[T^j_{(m+1,n+1)} - T^j_{(m,n)}\right]$$

$$+ (A_{es}\ \sigma\ \varepsilon_f \varepsilon_s)\left[(T^j_f + 273)^4 - (T^j_{(m,n)} + 273)^4\right]\} \qquad (4.11)$$

Equation (4.7) is the general heat balance equation for any point in the fire/steel boundary region.

4.6 Equations for Inside Steel Region

In the same way as for elementary regions at the outer boundary, the temperature inside steel region can be calculated by writing heat balance equation for the inside elementary regions. For the elements in the steel, Figure (4.4), except for the boundary elements, the temperature rise at time $t = (j+1)\Delta t$, is given by:

$$T^{j+1}_{(m,n)} = T^{j}_{(m,n)} + \frac{\Delta t}{(\rho_s C_s)^j_{(m,n)} \; (\Delta h_g)^2}$$

$$\{[\frac{K^j_{s(m-1,n-1)} + K^j_{s(m,n)}}{2}] [T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{s(m-1),n+1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n+1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{s(m+1,n-1)} + K^j_{s(m,n)}}{2}][T^j_{(m+1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{s(m+1,n+1)} + K^j_{s(m,n)}}{2}][T^j_{(m+1,n+1)} - T^j_{(m,n)}]\} \qquad (4.12)$$

4.7 Equations for Steel/Concrete Boundary

For the elements at the boundary between the steel and concrete as illustrated in Figure (4.5) the temperature rise at time $t = (j+1)\Delta t$ is:

$$T^{j+1}_{(m,n)} = T^{j}_{(m,n)} + \frac{\Delta t}{[(\rho_s C_s)^j_{(m,n)} + (\rho_c C_c)^j_{(m,n)} + (\rho_W C_W \phi^j_{(m,n)})](\Delta h_g)^2}$$

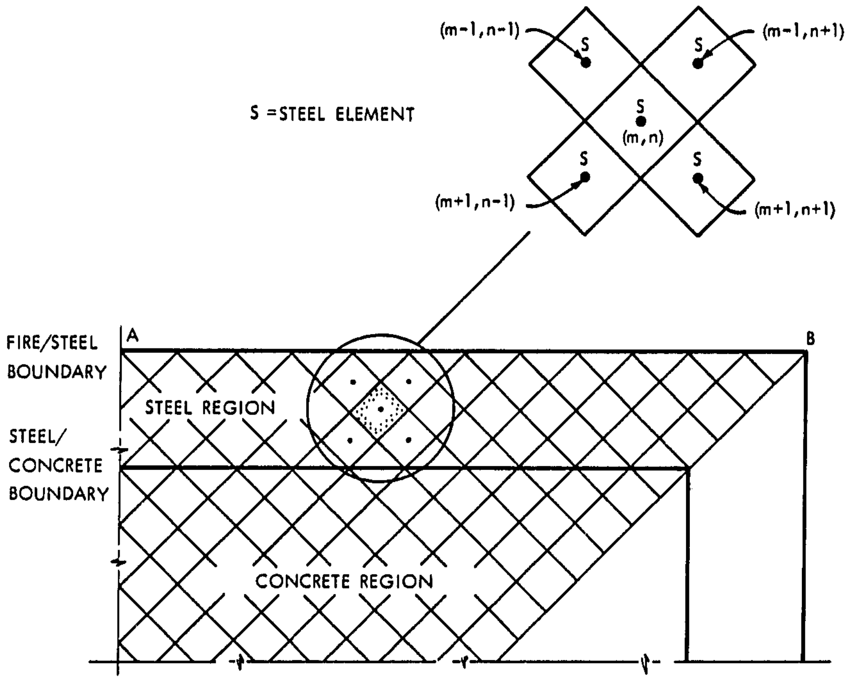

FIGURE 4.4

INSIDE STEEL REGION

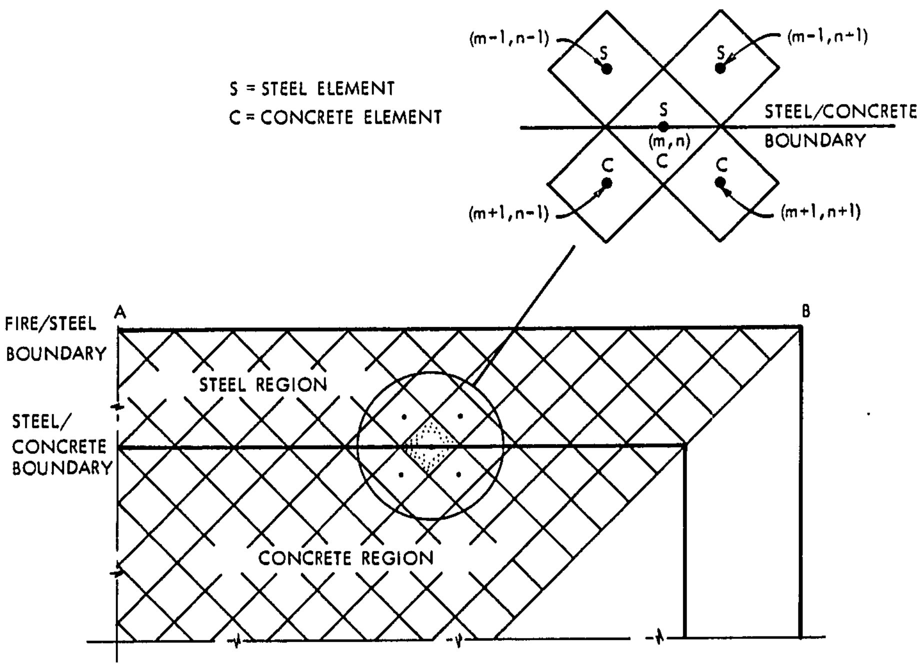

FIGURE 4.5

STEEL/CONCRETE BOUNDARY

$$\{[\frac{K^j_{s(m-1,n-1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{s(m-1,n+1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n+1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n+1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n+1)} - T^j_{(m,n)}]\} \qquad (4.13)$$

where:

$\phi^j_{(m,n)}$ = the concentration of moisture content

4.8 Equations for Inside the Concrete Region

For the elements in the concrete as illustrated in Figure (4.6), except for the elements at the boundary between the concrete and steel, the temperature rise at time $t = (j+1)\Delta t$, is given by:

$$T^{j+1}_{(m,n)} = T^j_{(m,n)} + \frac{\Delta t}{[(\rho_c c_c)^j_{(m,n)} + (\rho_W c_W \phi^j_{(m,n)})](\Delta h_g)^2}$$

$$\{[\frac{K^j_{c(m-1,n-1)} + K^j_{c(m,n)}}{2}][T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m-1,n+1)} + K^j_{c(m,n)}}{2}][T^j_{(m-1,n+1)} - T^j_{(m,n)}]$$

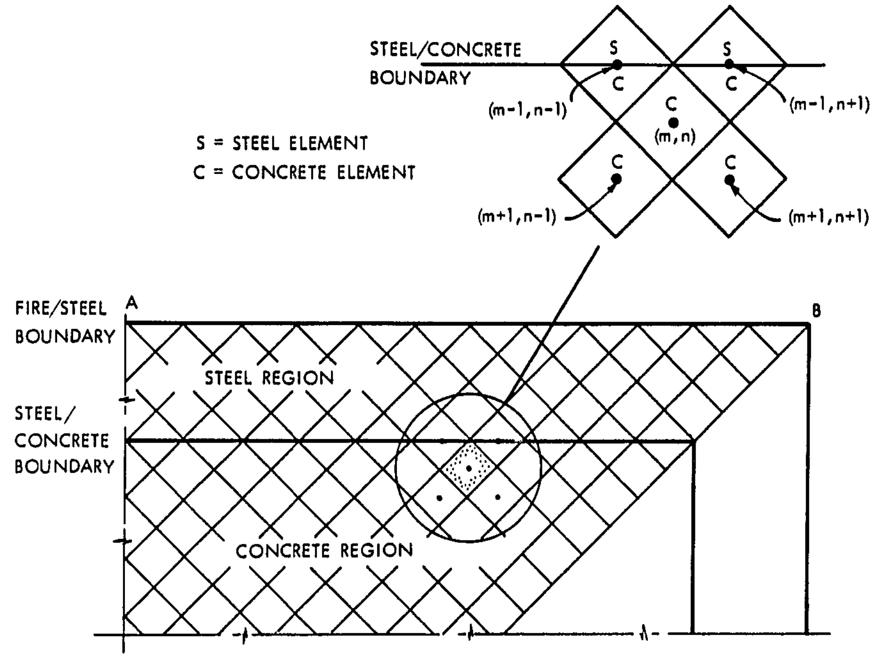

FIGURE 4.6

INSIDE CONCRETE REGION

$$+ \left[\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n-1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m+1,n+1)} - K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n+1)} - T^j_{(m,n)}\right]\} \quad (4.14)$$

4.9 Stability Criterion

In order to ensure that any error existing in the solution at some time level will not be amplified in subsequent calculations, a stability criterion has to be satisfied which, for a selected value of Δh_g, limits the maximum of the time step (Δt). Following the method described in reference [35], it can be derived that for the fire-exposed column the criterion of stability is most restrictive along the boundary between fire and steel. It is given by the condition:

$$\Delta t < \frac{2(\Delta h_g)^2(\rho_s C_s)_{min}}{4K_{s(max)} + 4(\Delta h_g)h_{max}} \quad (4.15)$$

where the maximum value of the coefficient of heat transfer during exposure to the standard fire (h_{max}) is approximately 3×10^6 J/m^{2}h°C [2].

4.10 Effect of Moisture

The effect of moisture in the concrete elements is taken into account by assuming that in each element, the moisture starts to evaporate when the temperature of the element reaches 100°C (212°F). During the period of evaporation all the heat supplied to an element is used for evaporation of the moisture, until the element is dry.

4.11 Initial Moisture at Steel/Concrete Boundary

The initial moisture content for a steel/concrete boundary element is:

$$v_{m_c} = \text{(volume of moisture content for the concrete element)}$$
$$= \text{(volume of the element) (concentration of moisture)}$$
$$= [2(\tfrac{1}{2}h_g \times \Delta h_g) \times 1.0](\phi)$$

$$v_{m_c} = (\Delta h_g)^2\phi \qquad (4.16)$$

4.12 Initial Moisture Inside Concrete Elementary Region

The initial moisture content for an element inside the concrete region is:

$$v_{m_c} = [4(\tfrac{1}{4}\Delta h_g \times \Delta h_g) \times 1.0]\phi = 2(\Delta h_g)^2\phi \qquad (4.17)$$

4.13 Change in the Volume of the Moisture Content at Steel/Concrete Boundary

From a heat balance equation, it can be derived that, per unit length of the column, the volume of the moisture content $\Delta V_{m,n}$

evaporated at the time $t = (j+1)\Delta t$ from a concrete element located at the steel/concrete boundary, Figure (4.5), as the following:

$$q_{c_{(m-1,n-1\to m,n)+(m-1,n+1\to m,n)}} = q_{c_{(m,n\to m+1,n-1)+(m,n\to m+1,n+1)}} + q_{v_{(m,n)}} \qquad (4.18)$$

where:

q_c = heat transfer by conduction

q_v = the heat used for evaporization of the moisture

the subscripts = (m-1),(m+1),(n-1),(n+1),(m,n) is for the prescribed elements.

In the above equation, the heat used for evaporization, q_v can be derived as:

$q_{v_{(m,n)}}$ = (heat used for evaporization)(change in moisture content)

$$\therefore q_{v_{(m,n)}} = (\rho_W \lambda_W) \left(\frac{\phi^{j+1}_{m,n} - \phi^{j}_{m,n}}{\Delta t}\right) \qquad (4.19)$$

By substituting the value of q_c from Equation (4.13) and the value of $q_{v_{(m,n)}}$ from Equation (4.19) into Equation (4.18) and rearranging, the moisture concentration in an element at the steel-concrete boundary, at the time $t = (j+1)\Delta t$ is:

$$\phi^{j+1}_{m,n} = \phi^{j}_{m,n} + \frac{\Delta t}{\rho_W \lambda_W (A_e \times 1.0)_{m,n}} \{[\frac{K^j_{s(m-1,n-1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{s(m-1),n+1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n+1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n+1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n+1)} - T^j_{(m,n)}])]\} \quad (4.20)$$

where:

$$A_e = 2(\tfrac{1}{2}\Delta h_g \times \Delta h_g) = (\Delta h_g)^2$$

And since,

$$V^{j+1}_{(m,n)} = [\phi^{j+1}_{(m,n)}](A_e \times 1.0)_{m,n} \quad (4.21)$$

$$V^{j}_{(m,n)} = [\phi^{j}_{(m,n)}](A_e \times 1.0)_{m,n} \quad (4.22)$$

By substituting Equations (4.21), (4.22) into Equation (4.20), the change in the volume of moisture content at the steel-concrete boundary is:

$$V^{j+1}_{m,n} = V^{j}_{m,n} + \frac{\Delta t}{\rho_W \lambda_W} \{[\frac{K^j_{s(m-1,n-1)} + K^j_{s(m,n)}}{2}][T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ \left[\frac{K^j_{s(m-1,n+1)} + K^j_{s(m,n)}}{2}\right]\left[T^j_{(m-1,n+1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n-1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m+1,n+1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n+1)} - T^j_{(m,n)}\right]\} \qquad (4.23)$$

4.14 Change in the Volume of the Moisture Content of Inside Concrete Region

In a similar manner, the moisture concentrate in an element inside the concrete region at the time $t = (j+1)\Delta t$ can be derived by using Figure (4.6) and Equations (4.14) to (4.22) which is given by:

$$\phi^{j+1}_{m,n} = \phi^j_{m,n} + \frac{\Delta t}{(\rho_W \lambda_W)(A_e \times 1.0)}$$

$$\{\left[\frac{K^j_{c(m-1,n-1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m-1,n-1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m-1,n+1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m-1,n+1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n-1)} - T^j_{(m,n)}\right]$$

$$+ \left[\frac{K^j_{c(m+1,n+1)} + K^j_{c(m,n)}}{2}\right]\left[T^j_{(m+1,n+1)} - T^j_{(m,n)}\right]\} \qquad (4.24)$$

Consequently, the change in the volume of moisture content at the inside concrete region is given by:

$$V^{j+1}_{m,n} = V^{j}_{m,n} + \frac{\Delta t}{\rho_W \lambda_W} \{[\frac{K^j_{c(m-1,n-1)} + K^j_{c(m,n)}}{2}][T^j_{(m-1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m-1,n+1)} + K^j_{c(m,n)}}{2}][T^j_{(m-1,n+1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n-1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n-1)} - T^j_{(m,n)}]$$

$$+ [\frac{K^j_{c(m+1,n+1)} + K^j_{c(m,n)}}{2}][T^j_{(m+1,n+1)} - T^j_{(m,n)}]\} \quad (4.25)$$

4.15 Auxiliary Equations

To calculate the temperatures of the elementary regions along the lines of symmetry A-C and B-C as illustrated in Figure (4.2), the temperature has to satisfy the following symmetry conditions:

along line A-C:

$$T^{j+1}_{(m,1)} = T^{j+1}_{(m,3)} \quad (4.26)$$

and along line B-C:

$$T^{j+1}_{(m+1,N-m+2)} = T^{j+1}_{(m,N-m+1)} \quad (4.27)$$

With the aid of Equations (4.1) to (4.27), and the relevant material properties given in Chapter three, the temperature distribution in the column and its surface can be calculated for any time $[t = (j+1)\Delta t]$ if the temperature distribution at the time $j\Delta t$ is known. Starting from a temperature of 20°C (68°F), the temperature history of the column can be calculated by Equations (4.1) to (4.27).

4.16 Calculation of Strength During Fire

Mechanical Properties

The most important mechanical properties that determine the strength of concrete-filled square steel columns are compressive strength (f_c'), modulus of elasticity (E_c) and ultimate strain (ε_u) of the concrete, and the yield strength (f_y) and modulus of elasticity (E_s) of the steel. A survey of the literature (18) shows that the variation of these properties with temperature is influenced by a large number of factors. The compressive strength of concrete at elevated temperatures is affected by the rate and duration of heating, the size and shape of the test specimen, and the loading during heating.

During exposure to fire the strength of the column decreases with the duration of exposure. The strength of the column can be calculated by a method based on load-deflection analysis which in turn is based on a stress-strain analysis of cross-sections [38]. In this method, the columns, which are fixed at the ends during the tests, are idealized as pin-ended columns of reduced length KL (Figure 4.8). The load on the test columns is intended to be concentric. To represent imperfections in the columns, an initial deflection y_o = 2.5 mm (0.1 in.) is assumed.

The calculation of the strains, stresses, loads and moments for each element of the column cross-section due to temperature change for a triangular network, Figure (4.2), is quite difficult. The main reason of this difficulty is the temperature representatives of the triangular elements located at the fire/steel boundary (line A-B) which can cause difficulty for stress calculations. Therefore the triangular network should be transfer to square network.

4.17 Transformation Into Square Network

To simplify the calculation of the deformations and stresses in the column, the triangular network is transformed into a square network. In Figure (4.7) a quarter section of this network, consisting of square elements arranged parallel to the x- and z-axis of the section, are shown. The width of each element of this network is Δh_g. The temperatures, deformations and stresses of each element are represented by those of the center of the element. The temperature at the center of each element is obtained by averaging the temperatures of the elements in the triangular network according to the relation:

$$(T^j_{m,n})_{square} = \left(\frac{T^j_{(m+1),(n+1)} + T^j_{m,(n+2)}}{2}\right)_{triangular} \qquad (4.28)$$

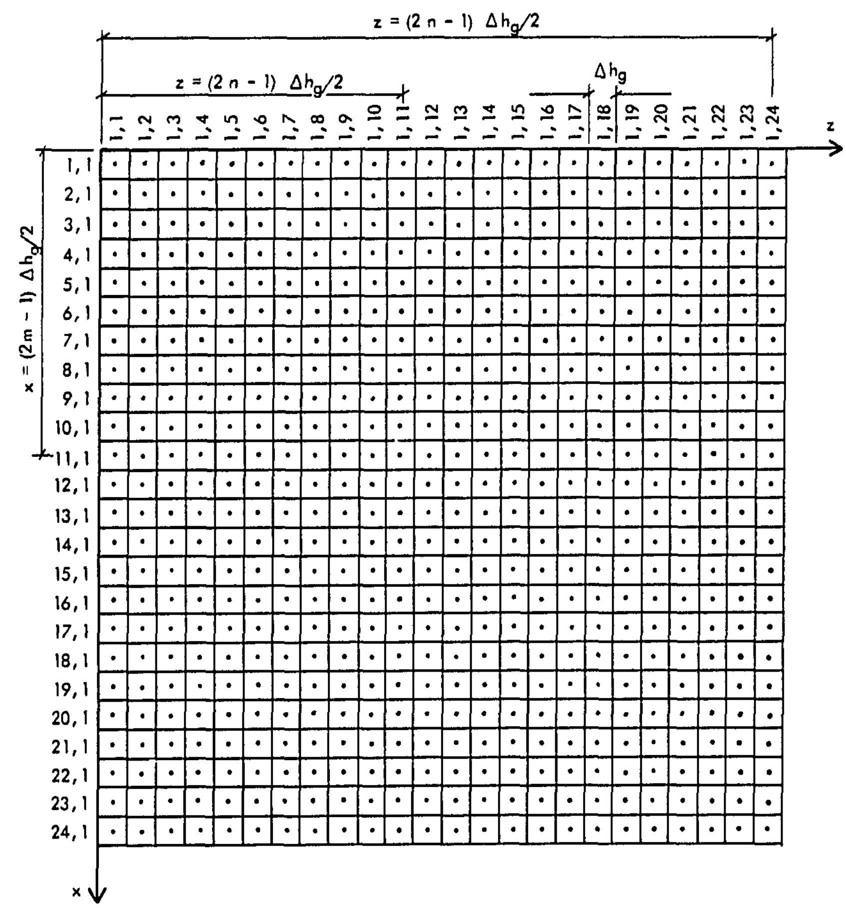

FIGURE 4.7

SQUARE NETWORK OF ELEMENTS IN A QUARTER SECTION OF COLUMN where the subscripts 'square' and 'triangular' refer to the elements of the square and triangular network.

4.18 Assumptions

The curvature of the column is assumed to vary from zero at pin-end to mid-height according to a straight line relation, as illustrated in Figure (4.8). For such a relation the deflection at mid-height (y), in terms of the curvature (χ) of the column at this height, can be given by:

$$y = \chi \frac{(KL)^2}{12} . \qquad (4.29)$$

For any given curvature (χ), and thus for any given deflection at mid-height, the axial strain is varied until the internal moment at the midsection is in equilibrium with the applied moment given by the product of load and total deflection. In this way a load deflection curve can be calculated for specific times during the exposure to fire. From these curves the strength of the column (its maximum load carrying capacity) can be determined for each time. In the calculation of column strength the following assumptions were made.

1) The properties of the concrete and steel are as given in Chapter three.
2) Concrete has no tensile strength.
3) Plane sections remain plane.
4) Initial strains in the column before the exposure to fire consists of free shrinkage of the concrete and creep. Because the shrinkage of the column during test normally compensated by

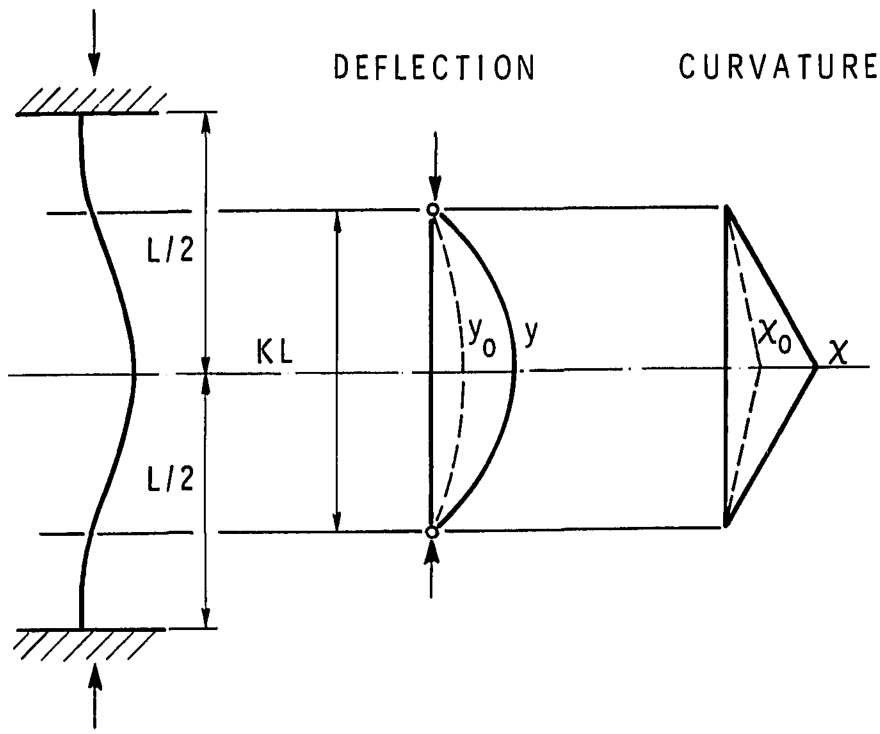

FIGURE 4.8

LOAD-DEFLECTION ANALYSIS

filling the space at both ends of the column between the concrete and steel with a plaster, the shrinkage is assumed to be negligible.

Normally, the tests of the columns start after a preloading period of about one hour. The shortening of the column due to creep in this period is assumed to be negligible.

Based on these assumptions, the change of column strength during the exposure to fire was calculated. In the calculations the square network shown in Figure (4.7) was used. Because the strains and stresses of the elements are not symmetrical with respect to the x-axis, the calculations of the strains and stresses were performed for both the network shown and an identical network at the left of the x-axis. The force and moment in the section were obtained by adding the forces carried by each element and the moments contributed by them.

The equations used in the calculations of the strength of the column during the exposure to fire are given below.

4.19 Calculations of Strains, Stresses, Loads and Moments in Steel

4.19.1 Strains in the Steel

(a) Strain due to thermal expansion

The strain in an element of the steel due to the thermal expansion is given by:

$$(\varepsilon_T)_S = \alpha_S(\Delta T) \qquad (4.30)$$

where:

$(\varepsilon_T)_S$ = free strain due to thermal expantion of steel

ΔT = temperature change of the element

= $T - 20$

α_S = coefficient of thermal expansion of steel

= $(0.004T + 12)\times10^{-6}$ for $T < 1000°C$

(b) Strain due to axial loading

For any given curvature χ, and thus for any given deflection at mid-height y, the axial strain ε is varied until the internal moment (due to temperature change) at the mid-section is in equilibrium with the applied moment given by the product

Load × (deflection + eccentricity)

where:

ε = axial strain, is varied until equilibrium (4.31)

(c) Strain due to bending

If Z_s is the horizontal distance of the steel element to the vertical plane through the x-axis of the column section as illustrated in Figure (4.7) and ρ is the radious of curvature, then the strain due to bending of the column is:

$$\varepsilon_b = \frac{Z_s}{\rho} \tag{4.32}$$

(d) Total strain

Therefore, the total strain in an element of the steel can be given as the sum of Equations (4.30), (4.31) and (4.32). For the steel at the right of the x-axis the strain $(\varepsilon_s)_R$ is given gy:

$$(\varepsilon_s)_R = -(\varepsilon_T)_s + \varepsilon + \frac{Z_s}{\rho} \qquad (4.33)$$

For the steel elements at the left of the x-axis the strain $(\varepsilon_s)_L$ is given by:

$$(\varepsilon_s)_L = -(\varepsilon_T)_S + \varepsilon - \frac{Z_s}{\rho} \qquad (4.34)$$

4.19.2 Stresses in the Steel

The stresses in the elements of the network are calculated using stress-strain relations given in references [2] and [3]. These relations can be derived from data provided by Ingberg and Sale [39], and Witteveen, Twilt and Bylaard [40]. These relations include the effect of creep at elevated temperatures and were obtained at heating rates approximately those that occur in a fire in actual practice. The relations have been generalized for other structural steels by assuming that, for a given temperature, the curves are the same for all steels, but the stress below with the stress-strain relation is linear, is proportional to the yeild strength of the steel. This is illustrated in Figure (4.9), where the stress-strain curves at 20°C (68°F) are shown for a steel with a yield strength of 250 MPa (36 psi) and for the steel, which has a yield strength of 345 MPa (64.3 psi). In Figure 4.10 the stress-strain curves of the steel are shown for various temperatures. These curves reflect that even at the very high temperature of 800°C (1472°F) the steel still possesses some strength and rigidity. The equations that describe the relation between the stress in the steel (f_y), the strain (ε_s) and the temperature of the steel (T) are as follows [2,3]:

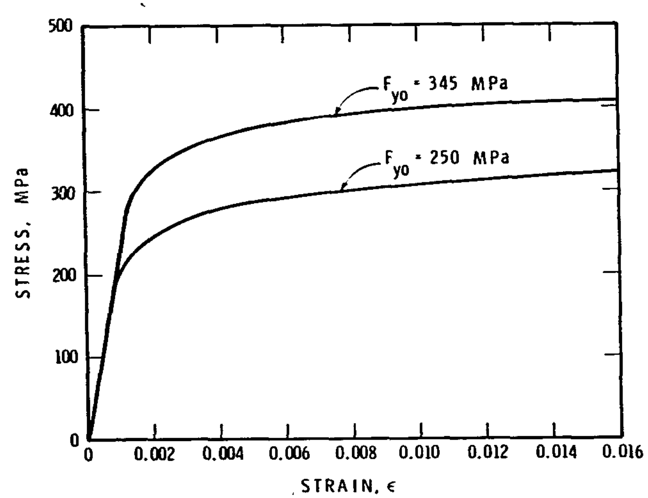

FIGURE 4.9

STRESS-STRAIN CURVES FOR TWO STEELS AT 20°C [2,3]

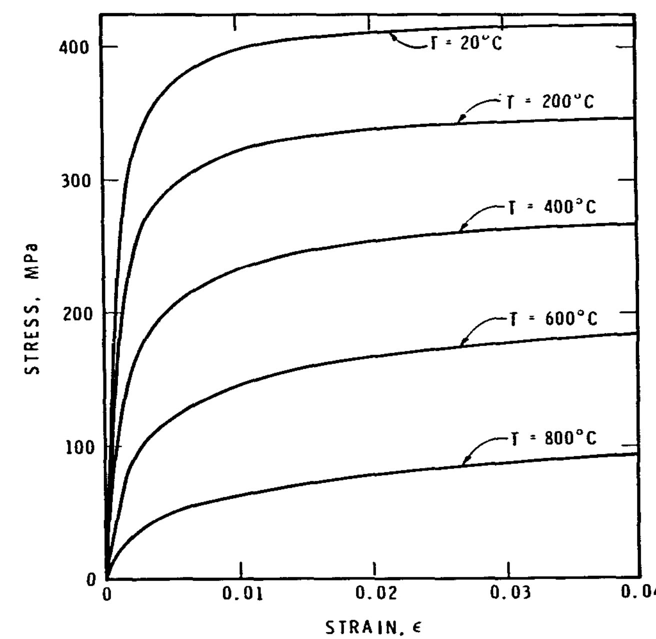

FIGURE 4.10

STRESS-STRAIN CURVES FOR 345-MPa STEEL AT VARIOUS TEMPERATURES [2,3] for $\varepsilon_s < \varepsilon_p$, $$f_y = \frac{f(T,0.001)}{0.001}\varepsilon_s \qquad (4.35)$$

where $$\varepsilon_p = 4 \times 10^{-6} f_{yo} \qquad (4.36)$$

and

$$f(T,0.001) = (50-0.04T) \times \left[1-\exp\left(-30+0.03T\right)\sqrt{0.001}\right)\right] \times 6.9 \qquad (4.37)$$

for $\varepsilon_s > \varepsilon_p$

$$f_y = \frac{f(T,0.001)}{0.001}\varepsilon_p + f\left(T,(\varepsilon_s-\varepsilon_p+0.001)\right) - f(T,0.001)\left.\right] \qquad (4.38)$$

With the aid of Equations (4.29)-(4.38) the stresses at mid-height in the steel can be calculated for any value of the axial strain (ε), curvature ($1/\rho$) and temperature (T). From these stresses the load that the steel carries and the contribution of the steel to the moments can be derived.

4.19.3 Loads in the Steel

The total load that the steel carries can be calculated by the summation of the product of stress by the area for each element located in right and left side of the x-axis of the column cross-section as the following:

The total load in steel is:

$$(P_s)_T = 2\left[\sum_{e=1}^{N} (f_{y_{SR}})_e (A_s)_e + \sum_{e=1}^{N} (f_{y_{SL}})_e (A_s)_e\right] \qquad (4.39)$$

where:

$(P_s)_T$ = the total load that the steel can carry

$(f_{y_{SR}})_e$ = the stress in the steel for an element located at right-side of x-axis (Figure 4.7).

$(f_{y_{SL}})_e$ = the stress in the steel for an element located at left-side of x-axis (Figure 4.7).

$(A_s)_e$ = the area of steel element

4.19.4 Moments in the Steel

The total moment due to the contribution of the total load carried by steel can be calculated by the summation of the product of stress by area by z coordinate of the steel for each element located in right and left side of the x-axis of the column cross-section as:

The total moment in steel is:

$$(M_s)_T = 2\left[\sum_{e=1}^{N} (f_{y_{SR}})_e \, (A_s)_e \, (Z_s)_e + \sum_{e=1}^{N} (f_{y_{SL}})_e \, (A_s)_e \, (-Z_s)_e \right] \quad (4.40)$$

where:

$(M_s)_T$ = the total moment due to the contribution of the total load carried by steel

$(Z_s)_e$ = the Z coordinate for the steel element.

4.20 Calculations of Strains, Stresses, Loads, and Moments in Concrete

4.20.1 Strains in the Concrete

In the same way as applied for steel, the strain in concrete causing stresses for elements at the right of the x-axis (Figure 4.7) can be given by:

$$(\varepsilon_c)_R = -(\varepsilon_T)_c + \varepsilon + \frac{z_c}{\rho} \tag{4.41}$$

and for elements at the left of the x-axis by:

$$(\varepsilon_c)_L = -(\varepsilon_T)_c + \varepsilon - \frac{z_c}{\rho} \tag{4.42}$$

where:

$(\varepsilon_T)_c$ = free strain due to thermal expansion of the concrete

ε = axial strain of the column

z_c = horizontal distance of the center of the element to the vertical plane through the x-axis of the column section

ρ = radius of curvature

4.20.2 Stresses in the Concrete

The stresses in the elements are calculated using the stress-strain relations described in References [2] and [3]. These relations were based on the work of Ritter [41] and Hognestad [12]. The relations have been slightly modified to take into account the creep of concrete at elevated temperatures. The modifications are based on results of work by Schneider and Haksever [42] and consist of a movement of the maxima in the stress- strain curves to higher strains with higher temperatures. These curves are shown in Figure (4.11) for a concrete with a cylinder

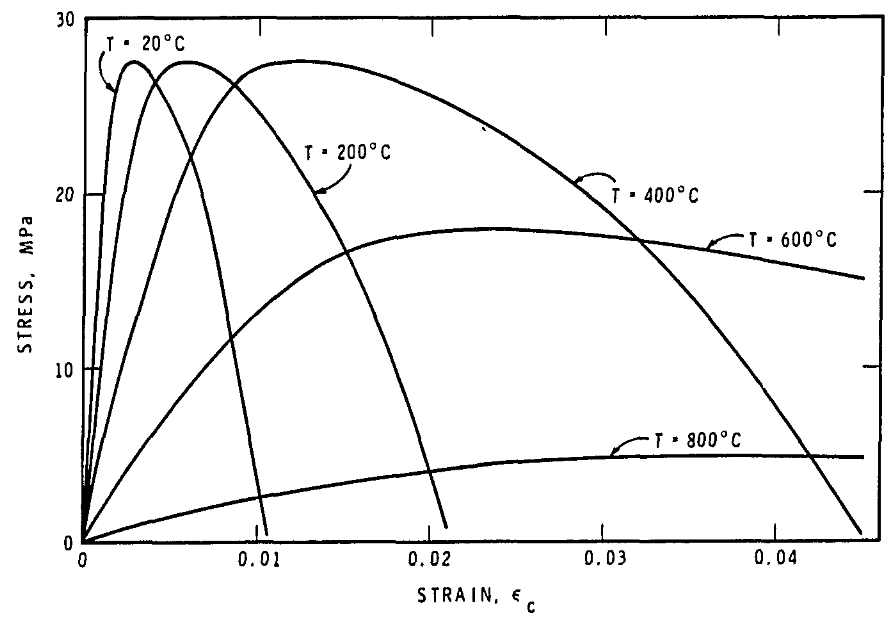

FIGURE 4.11

STRESS-STRAIN CURVES FOR 28-MPa CONCRETE AT VARIOUS TEMPERATURES [2,3] strength of 28 MPa (4 ksi). The equations that describe these curves are as follows [2,3]:

$$\text{for } \varepsilon_c < \varepsilon_{max} \quad , \quad f_c = f'_c \left[1 - \left(\frac{\varepsilon_{max} - \varepsilon_c}{\varepsilon_{max}}\right)^2\right] \tag{4.43}$$

$$\text{for } \varepsilon_c > \varepsilon_{max} \quad , \quad f_c = f'_c \left[1 - \left(\frac{\varepsilon_c - \varepsilon_{max}}{3\,\varepsilon_{max}}\right)^2\right] \tag{4.44}$$

where

$$f'_c = f'_{co} \text{ if } T < 450°C \tag{4.45}$$

$$f'_c = f'_{co} \left[2.011 - 2.353 \frac{T-20}{1000}\right] \text{ if } T > 450°C \tag{4.46}$$

$$\varepsilon_{max} = 0.0025 + (6.0T + 0.04T^2) \times 10^{-6} \tag{4.47}$$

In these equations

f_c = compressive strength of concrete at temperature T

f'_c = cylinder strength of concrete at temperature T

f'_{co} = cylinder strength of concrete at 20°C (68°F)

ε_c = strain of the concrete

ε_{max} = strain corresponding to maximum stress.

With the aid of Equations (4.41)-(4.47) the stresses in each of the concrete elements at midsection can be calculated for any value of the axial strain (ε) and curvature ($1/\rho$). From these stresses the load that the concrete carries and the contribution of the concrete to the moments can be derived.

4.20.3 Loads in the Concrete

In the same way as applied for steel, the total load in the concrete can be given:

$$(\rho_c)_T = 2[\sum_{e=1}^{N} (f_{cR})_e (A_c)_e + \sum_{e=1}^{N} (f_{cL})_e (A_c)_e] \qquad (4.48)$$

where:

$(\rho_c)_T$ = the total load which can be carried by concrete

$(f_{cR})_e$ = the compressive strength of the concrete for an element located at right-side of x-axis

$(f_{CL})_e$ = the compressive strength of the concrete for an element located at left-side of x-axis

$(A_c)_e$ = the area of concrete element.

4.20.4 Moments in the Concrete

In the same way as applied for steel, the total moment in the concrete is:

$$(M_c)_T = 2[\sum_{e=1}^{N} (f_{CR})_e (A_c)_e (Z_c)_e + \sum_{e=1}^{N} (f_{CL})_e (A_c)_e (-Z_c)_e] \qquad (4.49)$$

where:

$(M_c)_T$ = the total moment due to the total load carried by concrete,

$(Z_c)_e$ = the Z coordinate for concrete element.

4.21 Computer Program

A comprehensive computer program for this mathematical model has been written. The program output results and discussions is presented in Chapter six. The program list and the input data used are presented in Appendx E.

CHAPTER 5

FOURTH MATHEMATICAL MODEL

FIRE RESISTANCE OF CYLINDRICAL REINFORCED CONCRETE COLUMNS

5.1 Introduction

The calculation of fire resistance of a cylindrical reinforced columns, for which no mathematical model exists at present, is discussed. Details of the mathematical model to calculate temperatures, deformations and strength of the column are presented. A computer model program has been developed. Some of the output results will be discussed in Chapter six.

5.2 Temperatures of Column

The column temperatures are calculated by using the finite difference method. The method of deriving the heat transfer equations and calculating the temperatures in objects exposed to heat is described in detail in Chapter three.

5.3 Cross-Section Identities

The cross-sectional area of the column is subdivided into a number of concentric layers. From Figure (5.1), the following identities can be derived:

5.4 Identities for Fire/Concrete Boundary Layer

$$R_0 = (M - 1)\Delta\xi \qquad (5.1)$$

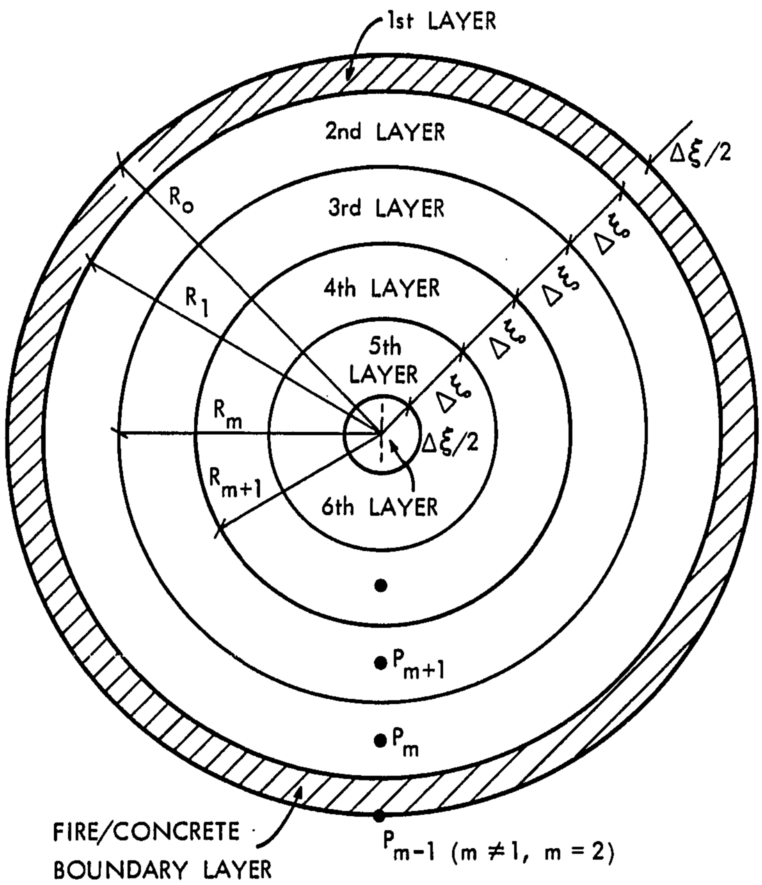

FIGURE 5.1

ARRANGEMENT OF ELEMENTARY LAYERS IN SECTION OF REINFORCED CONCRETE CYLINDRICAL COLUMN

where:

R_0 = outer radius of the column cross-section

M = total number of layers

$\frac{\Delta\xi}{2}$ = the thickness of fire/concrete boundary layer and the layer at the centre of the column

$\Delta\xi$ = the thickness of the layers except the fire/concrete boundary and the centre layers.

$$R_1 = \left(M - \frac{3}{2}\right) \Delta\xi \tag{5.2}$$

where:

R_1 = inner radius of the boundary layer

$$(A_{rs})_{m-1} = 2\pi R_o = 2\pi(M-1)\Delta\xi \tag{5.3}$$

where:

A_{rs} = surface area of the boundary layer located on the longitudinal surface of the colum

$2\pi R_0 \times 1.0 = [2\pi(M-1)\Delta\xi] \times 1.0$

$$(A_r)_{m-1} = 2\pi\left(\frac{R_1+R_0}{2}\right) \frac{\Delta\xi}{2}$$

By substituting R_1 and R_0 from Equations (5.1) and (5.2) into the above equation yields:

$$(A_r)_{m-1} = 2\pi\left(M - \frac{5}{4}\right) \frac{(\Delta\xi)^2}{2} \tag{5.4}$$

where:

A_r = cross-section area of the boundary layer

= mean perimeter $\times \frac{\Delta\xi}{2}$

5.5 Division of Cross-Section into Elementary Layers

The cross-sectional area of the column is subdivided into a number of concentric layers (M). As illustrated in Figure (5.1), the outer layer of concrete, which is exposed to fire, has a thickness of $\frac{1}{2}(\Delta\xi)$. The thickness of the last concrete layer at the centre is also $\frac{1}{2}(\Delta\xi)$. The thickness of all other layers in the cross-section is $\Delta\xi$.

5.6 Temperature Calculations

It is assumed that the entire surface of the column is exposed to the heat of a fire whose temperature course follows that of standard fire described in ASTM-E119 [36]. This temperature course can be described by the following expression:

$$T_f^j = 20 + 750\left[1-\exp(-3.79553\sqrt{t})\right] + 170.41\sqrt{t} \qquad (5.5)$$

where t is the time in hours and T_f^j is the fire temperature in °C at time $t = j\Delta t$.

5.7 Equations of Fire/Concrete Boundary

The temperature rise in each layer can be derived by making a heat balance for it, i.e. by applying the linear unsteady state partial differential equation and its numerical solution, for each layer.

Also, the heat transferred by radiation to the surface boundary layer must be taken into account. The heat balance for the fire/concrete boundary layer is as follows.

5.7.1 Heat Transfer by Radiation

Heat will be transmitted from the fire to a surface layer by radiation represented by Equation (5.5). For the fire/concrete boundary, the heat transmitted by radiation to the boundary surface layer (see Figure 5.1) during the period $j\Delta t < (j+1)\Delta t$ for a unit height of the column is as follows:

$$q_R = (A_{rs})\ \sigma\ \varepsilon_f \varepsilon_c [(T_f^j + 273)^4 - (T_{m-1}^j + 273)^4], \text{ where } m = 2$$

or

$$q_R = 2\pi(M-1)\Delta\xi\ \ \sigma\ \varepsilon_f \varepsilon_c [(T_f^j + 273)^4 - (T_{m-1}^j + 273)^4] \qquad (5.6)$$

5.7.2 Heat Transfer by Conduction

From Figure (5.1), heat is transferred from point 1 to point 2 in the radial direction of the cross-section. This heat is dependent on the radial direction (r) and temperature (T) and does not depended upon any angle. This means that the required unsteady state partial differential equation is a function of (r) and (T) as derived below:

The three-dimensional partial differential equation in cylindrical coordinates is [52]:

$$K \frac{\partial^2 T}{\partial r^2} + \frac{K}{r}\frac{\partial T}{\partial r} + \frac{K}{r^2}\frac{\partial^2 T}{\partial \phi^2} + K \frac{\partial^2 T}{\partial z^2} = \rho c \frac{\partial T}{\partial t} \qquad (5.7)$$

Equation (5.7) has three-dimensional cylindrical coordinates r, ϕ and z with constant thermal conductivity K.

As mentioned above, for this mathematical model, the heat transferred is dependent on the radial direction r. Thus, the terms include ϕ and z in Equation (5.7) must be cancelled to obtain the following Equation:

$$K \frac{\partial^2 T}{\partial r^2} + \frac{K}{r} \frac{\partial T}{\partial r} = \rho c \frac{\partial T}{\partial t} \tag{5.8}$$

Also, if the constant thermal conductivity (K) in Equation (5.8) is a variable then:

$$\left\{\frac{1}{r} \frac{\partial}{\partial r} (Kr \frac{\partial T}{\partial r})(A_r)_{f/c}\right\} = (\rho c \frac{\partial T}{\partial t})(A_r)_{f/c} \tag{5.9}$$

Equation (5.9) is a linear unsteady state partial differential equation. This equation wil be solved by difference methods at various boundary conditions for this mathematical model.

From Figures (5.1) and (5.2), the difference method solution for the left-hand side of Equation (5.9) is analogous to the Equations (3.43) to (3.51) as follows.

$$q_c = \left[\frac{1}{r} \frac{\partial}{\partial r} (K^j r \frac{\partial T^j}{\partial r})\right]_A (A_r)_{f/c}$$

$$= \left[\frac{1}{r_A} \frac{\{K^j r \frac{\partial T}{\partial r}\}_{m-1} - \{K^j r \frac{\partial T}{\partial r}\}_{m-\frac{1}{2}}}{\frac{\Delta\xi}{2}}\right] (A_r)_{f/c}$$

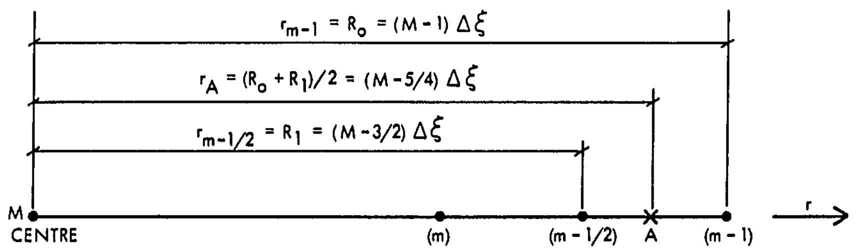

FIGURE 5.2

ENLARGED SCALE FOR POINTS $P_{(m)}$, $P_{(m-1/2)}$ AND $P_{(m-1)}$ OF FIGURE 5.1

$$\therefore q_c = \frac{1}{r_A\,(\frac{\Delta\xi}{2})}\left[\{K^j r \frac{\partial T}{\partial r}\}_{m-1} - (K^j)_{m-\frac{1}{2}}\,(r)_{m-\frac{1}{2}}\,(\frac{\partial T}{\partial r})_{m-\frac{1}{2}}\right](A_r)_{f/c} \qquad (5.10)$$

From Figures (5.1) and (5.2), the value of $\{K^j r \frac{\partial T}{\partial r}\}_{m-1}$ at the fire/concrete boundary layer is equal to zero because there is no heat conduction at the surface of the column, then:

$$\{K^j r \frac{\partial T}{\partial r}\}_{m-1} = 0 \qquad (5.11)$$

$$(K^j)_{m-\frac{1}{2}} = \frac{(K^j)_m + (K^j)_{m-1}}{2} \qquad (5.12)$$

$$(r)_{m-\frac{1}{2}} = R_1 \qquad (5.13)$$

$$(\frac{\partial T}{\partial r})_{m-\frac{1}{2}} = \frac{(T^j)_{m-1} - (T^j)_m}{\Delta\xi} \qquad (5.14)$$

Using Equations (5.4) and (5.11) to (5.14) into Equation (5.10) yields the following finite difference equation:

$$q_c = \left[\frac{1}{r}\frac{\partial}{\partial r}(K^j r \frac{\partial T^j}{\partial r})\right]_A (A_r)_{f/c}$$

$$= \frac{2\pi(M - \frac{5}{4})\frac{\Delta\xi^2}{2}}{r_A\,(\frac{\Delta\xi}{2})}\left[\frac{(K)^j_m + (K)^j_{m-1}}{2}\,(r_{m-\frac{1}{2}})(\frac{(T^j)_{m-1} - (T^j)_m}{\Delta\xi})\right]$$

(5.15)

From Equation (5.15 and Figures (5.1) and (5.2), the following can be found:

$$r_A = (M - \frac{5}{4})\ \Delta\xi \qquad (5.1)$$

$$r_{m-\frac{1}{2}} = R_1 = (M - \frac{3}{2})\ \Delta\xi \qquad (5.2)$$

Using Equations (5.1) and (5.2) into Equation (5.15) yields:

$$q_c = [\frac{1}{r}\ \frac{\partial}{\partial r}\ (K^j r\ \frac{\partial T^j}{\partial r})]$$

$$= \frac{2\pi(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}}{(M - \frac{5}{4}\ \frac{(\Delta\xi)^2}{2}}\ [\ \frac{(K^j)_m + (K^j)_{m-1}}{2}\ (M - \frac{3}{2}\ )\ \Delta\xi$$

$$(\frac{(T^j)_{m-1} - (T^j)_m}{\Delta\xi})] \qquad (5.16)$$

Rearranging Equation (5.16), the final heat conduction equation at the fire/concrete boundary is:

$$[q_c]_A = [\frac{1}{r}\frac{\partial}{\partial r}\ (K^j r\ \frac{\partial T^j}{\partial r})]_A$$

$$= 2\pi[(\frac{(K^j)_m + (K^j)_{m-1}}{2})(M - \frac{3}{2})(T^j_{m-1} - T^j_m)] \qquad (5.17)$$

5.7.3 <u>The Sensible Heat:</u>

The heat gain or loss in the fire/concrete boundary layer shown in Figure (5.1), because of heat generation or absorption is for a time period Δt can be obtained by applying the right-hand side of Equation (5.9) as follows:

$$\left(\frac{\partial E}{\partial t}\right)_A = \left(\rho c \frac{\partial T}{\partial t}\right)_A (A_r)_{f/c}$$

$$= [(\rho_c c_c)^j_{m-1} + \rho_w c_w \phi^j_{m-1}] \left(\frac{T^{j+1}_{m-1} - T^j_{m-1}}{\Delta t}\right) [2\pi(M - \frac{5}{4}) \frac{(\Delta\xi)^2}{2}] \tag{5.18}$$

where

$(\rho_c c^j_c)_{m-1}$ = thermal capacity of concrete, $J/m^3{}^\circ c$

$\rho_w c_w$ = thermal capacity of water, $J/m^3{}^\circ c$

ϕ^j_{m-1} = concentration of moisture (volume fraction)

T^{j+1}_{m-1} = the temperature at time t = $(j+1)\Delta t$, °c

T^j_{m-1} = the temperature at time t = $j\Delta t$, °c

Δt = time in hours

$\Delta\xi$ = the width of the layer, m

5.7.4 <u>The Final Equation for Fire/Concrete Boundary</u>

Using Equations (5.6), (5,17) and (5.18) to get the final equation for the boundary as follows:

$$\{q_r\} - \{\frac{1}{r}\frac{\partial}{\partial r}(Kr\frac{\partial t}{\partial r})(A_r)_{f/c}\} = \{(\rho C\frac{\partial T}{\partial t})(A_r)_{f/c}\}$$

or $$\{[2\pi(M-1)\Delta\xi]\sigma\ \varepsilon_f\ \varepsilon_f[(T_f^j + 273)^4 - (T_{m-1}^j + 273)^4]\} -$$

$$2\pi[(\frac{(k^j)_m + (K^j)_{m-1}}{2})\ (M-\frac{3}{2})(T^j_{m-1} - T^j_m)] =$$

$$[(\rho_c C_c)^j_{m-1} + \rho_w C_c \phi^j_{m-1}](\frac{T^{j+1}_{m-1} - T^j_{m-1}}{\Delta t})[2\pi(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}] \qquad (5.19)$$

Rearranging Equation (5.19), the temperature T^{j+1}_{m-1} at the time $(j+1)\Delta t$ for the fire/concrete boundary layer is:

$$T^{j+1}_{m-1} = T^j_{m-1} + \frac{\Delta t}{(M-\frac{5}{4})[\rho_c C_c)^j_{m-1} + \rho_w C_w \phi^j_{m-1}]\frac{(\Delta\xi)^2}{2}}$$

$$\{(M-1)\ \Delta\xi\sigma\varepsilon_f\varepsilon_c[(T_f^j + 273)^4 - (T^j_{m-1} + 273)^4]$$

$$- (\frac{(K^j_m) + (K)^j_{m-1}}{2})(M-\frac{3}{2})(T^j_{m-1} - T^j_m)] \qquad (5.20)$$

Equation (5.20) can be rewritten in other form which includes the volume of the moisture V^j_{m-1} instead of moisture concentration ϕ^j_{m-1} as follows:

$$T^{j+1}_{m-1} = T^{j}_{m-1} + \frac{\Delta t}{(\rho_c C_c)^j_{m-1}(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2} + \frac{\rho_w C_w}{2\pi}(2\pi)(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}\phi^j_{m-1}}$$

$$\{(M-1)\ \Delta\xi\sigma\varepsilon_f\varepsilon_c[(T^j_f + 273)^4-(T^j_{m-1}+273)^4]$$

$$-(\frac{(K^j)_m + (K)^j_{m-1}}{2})(M-\frac{3}{2})(T^j_{m-1} - T^j_m)] \qquad (5.21)$$

But, from Equation (5.4), it can be obtaines:

$$V^j_{m-1} = [(A_{r_{f/c}})_{m-1} \times 1.0] \times \phi^j_{m-1}$$

$$\therefore\ V^j_{m-1} = [2\pi(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2} \times 1.0] \times \phi^j_{m-1}$$

then Equation (5.20) in terms of moisture volume V^j_{m-1} becomes:

$$T^{j+1}_{m-1} = T^{j}_{m-1} + \frac{\Delta t}{(\rho_c C_c)^j_{m-1}(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2} + \frac{\rho_w C_w}{2\pi}V^j_{m-1}}$$

$$\{(M-1)\ \Delta\xi\sigma\varepsilon_f\varepsilon_c[(T^j_f + 273)^4-(T^j_{m-1}+273)^4]$$

$$-\ (\frac{(K^j)_m + (K)^j_{m-1}}{2})(M-\frac{3}{2})(T^j_{m-1} - T^j_m)] \qquad (5.22)$$

5.8 Equations at Inside Concrete Region

The identities and the heat balance for the layers in the concrete, except for the layer at the boundary and the centre layer will be as follows:

5.8.1 Identities at Inside Concrete Region:

By observing Figure (5.3) the following identities can be derived:

$$R_m = R_o - \frac{\Delta\xi}{2} - (m-1)\ \Delta\xi \qquad \text{where } m = 2 \tag{5.23}$$

using Equation (5.1) into Equation (5.23) yields

$$R_m = [M - m - \tfrac{1}{2}]\Delta\xi \tag{5.24}$$

where:

M = is the number of layers

m = is the layer number

$$R_{m-1} = R_m + \Delta\xi \tag{5.25}$$

using Equation (5.24) in Equation (5.25) yields

$$R_{m-1} = [M - m + \tfrac{1}{2}]\Delta\xi \tag{5.26}$$

Then, the cross-section area of the m^{th} layer is:

$$(A_r)_m = \text{mean perimeter} \times \Delta\xi$$

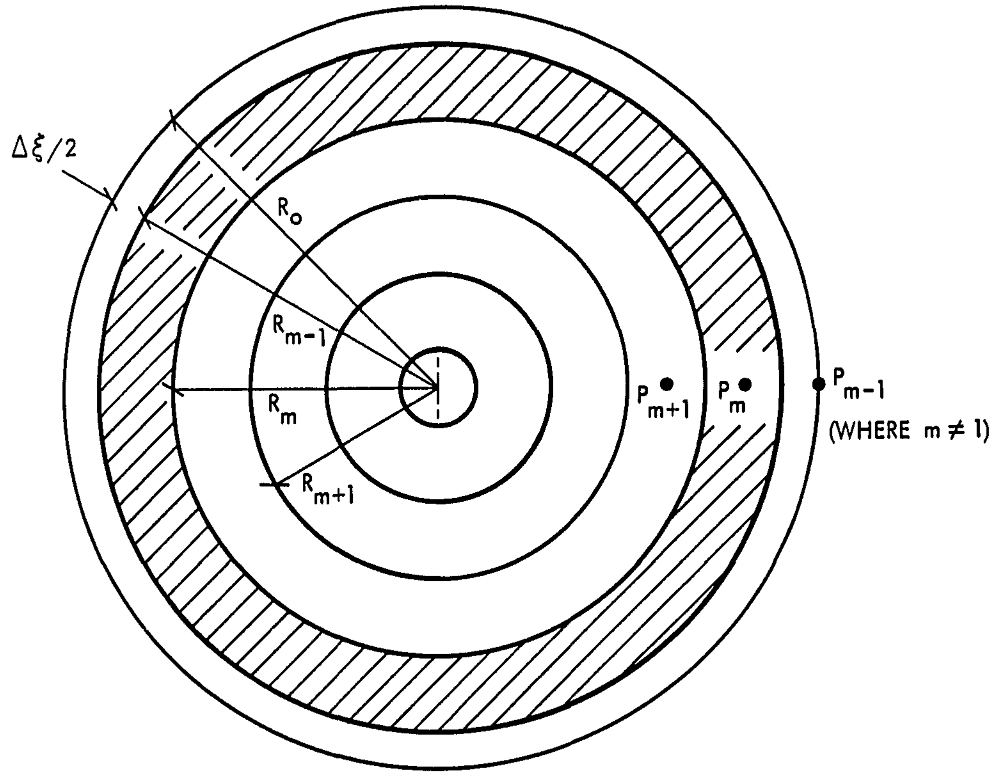

FIGURE 5.3

LAYER AT INSIDE CONCRETE REGION

$$(A_r)_m = 2\pi \left(\frac{R_m + R_{m-1}}{2}\right) \Delta\xi \tag{5.27}$$

Using Equations (5.24) and (5.26) into (5.27) yields:

$$(A_r)_m = 2\pi(M - m)\Delta\xi^2 \tag{5.28}$$

where

$(A_r)_m$ = The cross-section area for layer inside the concrete region Figure (5.2)

5.8.2 The Heat Transfer by Conduction

The heat transfer by conduction through a layer at point P_m can be found by applying Equation (5.9) and its numerical solution to the concrete layers except for the layer at the boundary and the centre layer.

From Figures (5.3) and (5.4), the difference method solution for the left-hand side of Equation (5.9) is analogus to the Equations (3.43) to (3.51) as follows:

$$q_c = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(K^j r \frac{\partial T^j}{\partial r}\right)\right]_m = \frac{1}{r_m} \frac{\{K^j r \frac{\partial T}{\partial r}\}_{m-\frac{1}{2}} - \{K^j r \frac{\partial T}{\partial r}\}_{m+\frac{1}{2}}}{\Delta\xi}$$

$$= \frac{1}{r_m(\Delta\xi)} \left[(K^j)_{m-\frac{1}{2}} (r)_{m-\frac{1}{2}} \left(\frac{\partial T^j}{\partial r}\right)_{m-\frac{1}{2}} - (K^j)_{m+\frac{1}{2}} (r)_{m+\frac{1}{2}} \left(\frac{\partial T}{\partial r}\right)_{m+\frac{1}{2}}\right] \tag{5.29}$$

From Figures (5.3) and (5.4), it can be found:

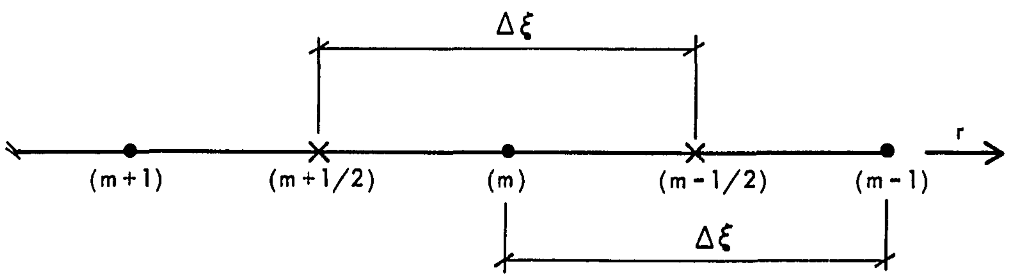

FIGURE 5.4

ENLARGED SCALE FOR POINTS $P_{(m+1)}$, $P_{(m)}$ AND $P_{(m-1)}$ OF FIGURE 5.3

$$(K^j)_{m-\frac{1}{2}} = \frac{(K^j)_m + (K^j)_{m-1}}{2} \tag{5.30}$$

$$(K^j)_{m+\frac{1}{2}} = \frac{(K^j)_m + (K^j)_{m+1}}{2} \tag{5.31}$$

$$\left(\frac{\partial T}{\partial r}\right)_{m-\frac{1}{2}} = \frac{(T^j)_{m-1} - (T^j)_m}{\Delta \xi} \tag{5.32}$$

$$\left(\frac{\partial T}{\partial r}\right)_{m+\frac{1}{2}} = \frac{(T^j)_m - (T^j)_{m+1}}{\Delta \xi} \tag{5.33}$$

$$r_{m-\frac{1}{2}} = R_{m-1} \tag{5.34}$$

$$r_{m+\frac{1}{2}} = R_m \tag{5.35}$$

Using Equations (5.30) to (5.35) into Equation (5.29) yields to the following finite difference equation:

$$q_c = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(K^j r \frac{\partial T^j}{\partial r}\right)\right]_m$$

$$= \frac{1}{r_m(\Delta \xi)} \left\{\left(\frac{(K^j)_m + (K^j)_{m-1}}{2}\right)\left(r_{m-\frac{1}{2}}\right)\left(\frac{(T^j)_{m-1} - (T^j)_m}{\Delta \xi}\right)\right.$$

$$\left. - \left(\frac{(K^j)_m + (K^j)_{m+1}}{2}\right)\left(r_{m+\frac{1}{2}}\right)\left(\frac{(T^j)_m - (T^j)_{m+1}}{\Delta \xi}\right)\right\} \tag{5.36}$$

From Equations (5.24) to (5.26) and Figures (5.3) and (5.4) the following can be determined:

$$(r_{m+\frac{1}{2}}) = R_m = [M - m - \tfrac{1}{2}]\Delta\xi \qquad (5.37)$$

$$(r_{m-\frac{1}{2}}) = R_{m-1} = [M - m - \tfrac{1}{2}]\Delta\xi \qquad (5.38)$$

From Equations (5.37) and (5.38), r_m can be obtained:

$$r_m = \frac{(r_{m+\frac{1}{2}}) + (r_{m-\frac{1}{2}})}{2} = [M - m]\Delta\xi \qquad (5.39)$$

Substitution of Equations (5.37) to (5.39) into Equation (5.36) yields:

$$q_c = [\frac{1}{r}\frac{\partial}{\partial r}(K^j r \frac{\partial T^j}{\partial r})]_m = \frac{1}{(\Delta\xi)^2}\{(\frac{K^j_{m-1} + K^j_m}{2})(\frac{M - m + \frac{1}{2}}{M - m})(T^j_{m-1} - T^j_m)$$

$$- (\frac{K^j_{m+1} + K^j_m}{2})(\frac{M - m - \frac{1}{2}}{M - m})(T^j_m - T^j_{m+1})\} \qquad (5.40)$$

5.8.3 <u>The Sensible Heat</u>

Similarly, as applied for Section 5.6.3, the sensible heat absorbed by the m^{th} layer inside the concrete region in a time period Δt is:

$$\left(\frac{\partial E}{\partial t}\right)_m = \rho C \frac{\partial T}{\partial t}$$

$$= \left[(\rho_c C_c)^j_m + \rho_w C_w \phi^j_m\right]\left(\frac{T^{j+1}_m - T^j_m}{\Delta t}\right) \tag{5.41}$$

Using Equations (5.40) and (5.41) into Equation (5.9) yields:

$$\frac{1}{(\Delta\xi)^2}\left\{\left(\frac{K^j_{m-1} + K^j_m}{2}\right)\left(\frac{M - m + \frac{1}{2}}{M - m}\right)(T^j_{m-1} - T^j_m) - \left(\frac{K^j_{m+1} + K^j_m}{2}\right)\right.$$

$$\left.\left(\frac{M - m - \frac{1}{2}}{M - m}\right)(T^j_m - T^j_{m+1})\right\} = \left\{\left[(\rho_c C_c)^j_m + \rho_w C_w \phi^j_m\right]\left(\frac{T^{j+1}_m - T^j_m}{\Delta t}\right)\right. \tag{5.42}$$

Rearranging Equation (5.42), the temperature T^{j+1}_m at the time $(j+1)\Delta t$ for an m^{th} layer inside the concrete region can be obtained:

$$T^{j+1}_m = T^j_m + \frac{\Delta t}{(M-m)\left[(\rho_c C_c)^j_m + \rho_w C_w \phi^j_m\right](\Delta\xi)^2}\left\{(M - m + \tfrac{1}{2})\left(\frac{K^j_{m-1} + K^j_m}{2}\right)\right.$$

$$\left.(T^j_{m-1} - T^j_m) - (M - m - \tfrac{1}{2})\left(\frac{K^j_{m+1} + K^j_m}{2}\right)(T^j_m - R^j_{m+1})\right\} \tag{5.43}$$

Equation (5.43) can be rewritten in another form which includes the volume of the moisture V^j_m instead of moisture concentration ϕ^j_m as follows:

$$T_1^{j+1} = T_m^j + \frac{\Delta t}{(\rho_c C_c)^j (M - m)\ (\Delta\xi)^2 + \frac{\rho_w C_w}{2\pi}\ (2\pi)(M-m)\phi_m^j\ (\Delta\xi)^2}$$

$$\{(M - m + \tfrac{1}{2})(\frac{K_{m-1}^j + K_m^j}{2})\ (T_{m-1}^j - T_m^j)$$

$$- (M - m - \tfrac{1}{2})(\frac{K_{m+1}^j + K_m^j}{2})(T_m^j - R_{m+1}^j)\} \qquad (5.44)$$

But

$$V_m^j = [(A_r)_m \times 1.0](\phi_m^j)$$

$$= [2\pi\ r_m(\Delta\xi)\ (1.0)](\phi_m^j)$$

$$= [2\pi(M-m)(\Delta\xi)^2(1.0)](\phi_m^j)$$

$$V_m^j = [2\pi(M-m)\phi_m^j\ (\Delta\xi)^2] \qquad (5.45)$$

By substituting Equation (5.45) into its Equation (5.44), then Equation (5.44) in terms of moisture volume V_m^j becomes

$$T_m^{j+1} = T_m^j + \frac{\Delta t}{(\rho_c C_c)^j (M-m)(\Delta\xi)^2 + \frac{\rho_w C_w}{2\pi}\ V_m^j}\ \{(M - m + \tfrac{1}{2})(\frac{K_{m-1}^j + K_m^j}{2})$$

$$(T^j_{m-1} - T^j_m) - (M - m - \tfrac{1}{2})(\frac{K^j_{m+1}+K^j_m}{2})(T^j_m - T^j_{m+1})\} \quad (5.45)$$

5.9 Equations For The Centre Concrete Layer

Similarly as before, by applying Equation (5.9) and its numerical solution to the centre layer as follows:

5.9.1 Heat Transfer by Conduction:

From Figures (5.5) and (5.6), the difference method solution for the left-hand side of Equation (5.10) is analogous to the Equations (3.43) to (3.51) as follows:

$$q_c = [\frac{1}{r}\frac{\partial}{\partial r}(K^j r\frac{\partial T^j}{\partial r})]_M = \frac{1}{r_{M-\frac{1}{4}}}\frac{\{K^j r\frac{\partial T^j}{\partial r}\}_{M-\frac{1}{2}} - \{K^j r\frac{\partial T^j}{\partial r}\}_M}{\frac{\Delta\xi}{2}}$$

$$= \frac{1}{(r)_{M-\frac{1}{4}}(\frac{\Delta\xi}{2})}[(K^j)_{M-\frac{1}{2}}(r)_{M-\frac{1}{2}}(\frac{\partial T^j}{\partial r})_{M-\frac{1}{2}} - (K^j)_M(r)_M(\frac{\partial T^j}{\partial r})_M] \quad (5.46)$$

From Figures (5.5) and (5.6), it can be found:

$$(r)_{M-\frac{1}{4}} = \frac{\Delta\xi}{4} \quad (5.47)$$

$$(K^j)_{M-\frac{1}{2}} = \frac{(K^j)_M + (K^j)_{M-1}}{2} \quad (5.48)$$

$$(r)_{M-\frac{1}{2}} = \frac{\Delta\xi}{2} \quad (5.49)$$

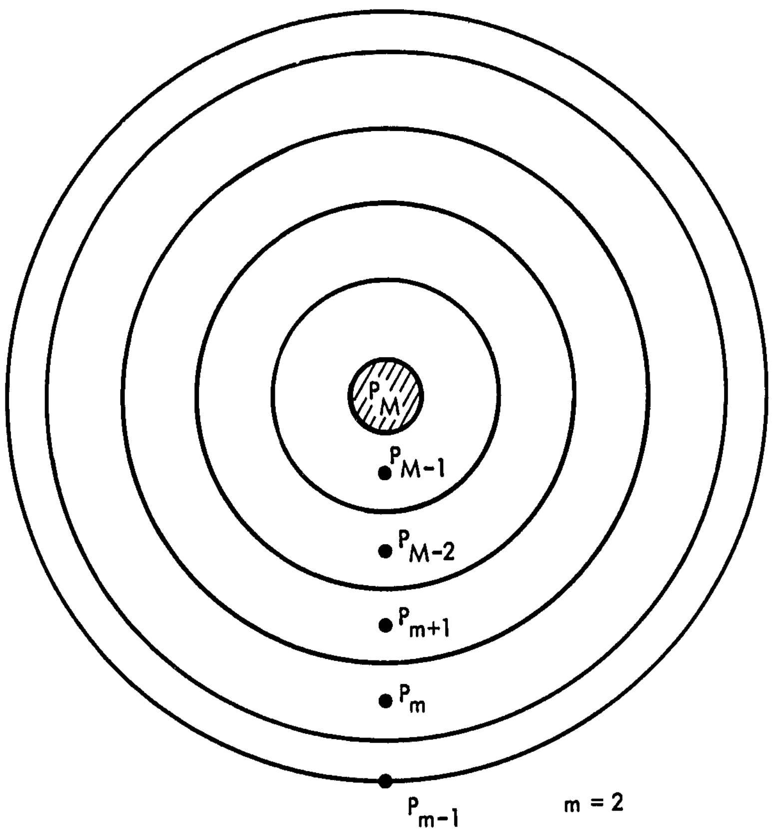

FIGURE 5.5

THE CONCRETE LAYER AT THE CENTRE OF THE CROSS-SECTION

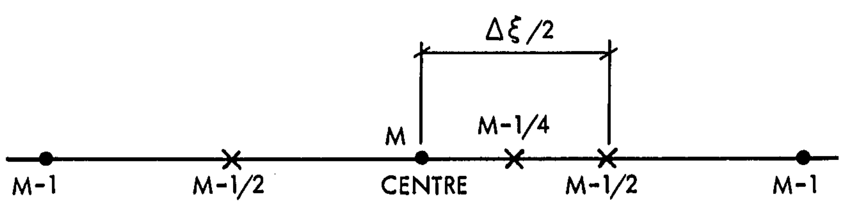

FIGURE 5.6

ENLARGED SCALE FOR POINTS P_M AND P_{M-1} OF FIGURE 5.5

$$\left(\frac{\partial T}{\partial r}\right)_{M-\frac{1}{2}} = \frac{(T^j)_{M-1} - (T^j)_M}{\Delta\xi} \tag{5.50}$$

$$(r)_M = 0 \tag{5.51}$$

Using Equations (5.47) to (5.51) into (5.46) yield:

$$q_c = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(K^j r \frac{\partial T^j}{\partial r}\right)\right]_{M-\frac{1}{2}}$$

$$= \frac{1}{\left(\frac{\Delta\xi}{4}\right)\left(\frac{\Delta\xi}{2}\right)}\left[\left(\frac{(K^j)_M + (K^j)_{M-1}}{2}\right)\left(\frac{\Delta\xi}{2}\right)\left(\frac{(T^j)_{M-1} - (T^j)_M}{\Delta\xi}\right)\right]$$

$$\therefore\ q_c = \frac{1}{\frac{(\Delta\xi)^2}{4}}\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T^j)_{M-1} - (T^j)_M\right] \tag{5.52}$$

5.9.2 The Sensible Heat:

The heat absorbed by the centre layer M in a time period Δt is:

$$\left(\frac{\partial E}{\partial t}\right)_M = \rho C \frac{\partial T^j}{\partial t}$$

$$= \left[(\rho_c C_c)^j_M + \rho_w C_w \phi^j_M\right]\left(\frac{T^{j+1}_M - T^j_M}{\Delta t}\right) \tag{5.53}$$

5.9.3 The Heat Balance Equation of the Centre Layer

$$(q_c)_M = \left(\frac{\partial E}{\partial t}\right)_M$$

or $$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(K^j r\frac{\partial T^j}{\partial r}\right)\right]_M = \left[\rho C\,\frac{\partial T^j}{\partial t}\right]_M \tag{5.54}$$

Using Equations (5.52) and (5.53) into Equation (5.54) yields

$$\frac{1}{\frac{(\Delta\xi)^2}{4}}\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T^j)_{M-1} - (T^j)_M\right]$$

$$= \left[(\rho_c C_c)^j_M + \rho_w C_w \phi^j_m\right]\left(\frac{T^{j+1}_M - T^j_M}{\Delta t}\right) \tag{5.55}$$

Rearranging Equation (5.55), the temperature T^{j+1}_M at the time $(j+1)\Delta t$ for the centre layer can be determined:

$$T^{j+1}_M = T^j_M + \frac{\Delta t}{\left[(\rho_c C_c)^j_M + \rho_w C_w \phi^j_M\right]\frac{(\Delta\xi)^2}{4}}$$

$$\left\{\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T^j)_{M-1} - (T^j)_M\right\}\right. \tag{5.56}$$

Equation (5.56) can be rewritten in another form which includes the volume of the moisture V^j_{M-1} instead of moisture concentration ϕ^j_M as follows:

$$T_M^{j+1} = T_M^j + \frac{\Delta t}{[(\rho_c c_c)_M^j \frac{(\Delta\xi)^2}{4} + \frac{\rho_w C_w}{\pi}(\pi)\frac{(\Delta\xi)^2}{4}\varphi_M^j]}$$

$$\{[\frac{(K^j)_M + (K^j)_{M-1}}{2}][(T^j)_{M-1} - (T^j)_M]\} \qquad (5.57)$$

But: $V_M^j = [(A_r)_M \times 1.0](\phi_M^j)$

$\therefore \quad V_M^j = [\pi \frac{(\Delta\xi)^2}{4} \times 1.0](\phi_M^j)$

By substituting the value of V_M^j into Equation (5.57), the final equation in terms of moisture volume is:

$$T_M^{j+1} = T_M^j + \frac{\Delta t}{(\rho_c c_c)_M^j (\frac{(\Delta\xi)^2}{4}) + \frac{\rho_w C_w}{\pi} V_M^j} \{$$

$$[\frac{(K^j)_M + (K^j)_{M-1}}{2}][(T^j)_{M-1} - (T^j)_M]\} \qquad (5.58)$$

5.10 Stability Criterion

In order to ensure that any error existing in the solution at some time level will not be amplified in the subsequent calculations, a stability criterion has to be satisfied; for a selected value of $\Delta\xi$, this limits the maximum time step Δt. Following the method described in reference [35], it can be derived that for the fire-exposed column the criterion of stability is most restrictive along the boundary between fire and concrete it is given by the condition

$$\Delta t < \frac{(\rho_c C_c)_{min} (\Delta\xi)^2}{2(K_{max} + h_{max} \Delta\xi)} \tag{5.59}$$

where $(\rho_c C_c)_{min}$ is the minimum value of the heat capacity of the steel, K_{max} the maximum value of its thermal conductivity and h_{max} the maximum value of the coefficient of heat transfer to be expected during the exposure to fire. The Units for these quantities are:

$(\rho_c C_c)_{min} = J/m^3c°$

$K_{max} = J/mhc°$

$K_{max} = J/m^2hc°$

5.11 Effect of Moisture:

The effect of moisture in the concrete is taken into account by assuming that in each layer the moisture starts to evaporate when the temperature reaches 100°C. In the period of evaporation, all the heat supplied to a layer is used for evaporation of the moisture until the layer is dry. To calculate the change in the moisture content, first the initial moisture has to be calculated.

5.12 Initial Moisture at Fire/Concrete Boundary Layer:

The total volume of moisture in the fire/concrete boundary layer is:

Volume of the moisture = (volume of the layer)(concentration of moisture)

$$V_{m-1} = [(A_r)_{f/c}\ (1.0)](\phi) \tag{5.60}$$

where:

V_{m-1} = Volume of moisture content with the first layer where m = 2

$(A_r)_{f/c}$ = the area of fire/concrete boundary layer

ϕ = concentration of moisture.

Using Equation (5.4) into Equation 5.60 yields

$$V_{m-1} = [\pi(M - \frac{5}{4})\ (\Delta\xi)^2\ (1.0)](\phi) \tag{5.61}$$

5.13 Initial Moisture at Inside Concrete Layer

The total volume of moisture content for a layer inside the concrete region is:

$$V_m = [(A_r)_m\ (1.0)]\ (\phi) \tag{5.62}$$

where:

$(A_r)_m$ = the area of the concrete layer m, where m = 2.

Using Equation (5.28) into Equation (5.62) yields

$$V_m = [2\pi(M - m)(\Delta\xi)^2\ (1.0)](\phi) \tag{5.63}$$

where:

M = is the total number of layers

m = is the layer number, m=2

5.14 Initial Moisture for the Centre Concrete Layer:

The total volume of moisture content for a concrete layer at the centre is:

$$V_M = [(A_r)_M \ (1.0)](\phi) \tag{5.64}$$

where:

$(A_r)_M$ = the area of the centre concrete layer and M is the total number of layers

From Figure (5.5),

$$(A_r)_M = \pi\frac{(\Delta\xi)^2}{4} \tag{5.65}$$

Using Equation (5.65) into Equation (5.64) yields:

$$V_M = [\pi\frac{(\Delta\xi)^2}{4} \ (1.0)]\phi \tag{5.66}$$

5.15 Change in the Volume of the Moisture Content in Fire/Concrete Layer

From a heat balance equation, it can be derived that, per unit length of the column, the volume of moisture content (ΔV_{m-1}), evaporated in time Δt from the concrete layer at the boundary between fire and concrete as follows:

$$(q_R)_{f \to (m-1)} = (q_c)_{(m-1) \to m} + (q_v)_{(m-1)} \tag{5.67}$$

where:

$(q_R)_{f \to (m-1)}$ = The heat transfer from the fire to the fire/concrete boundary layer by radiation, $\frac{J}{m.hr}$, Equation (5.6).

$(q_c)_{(m-1) \to m}$ = The heat transfer by conduction from layer (m-1) to layer m, J/(m.hr), Equation (5.18), where m = 2

$(q_v)_{m-1}$ = The heat used for evaporization of the moisture content of layer (m-1), $\frac{J}{m.hr}$. This heat of evaporization will continue until the layer becomes dry, then the heat used to raise the layer temperature is called the sensible heat.

5.15.1 Heat of Evaporization

The heat of evaporization can be calculated as follows:

$(q_v)_{m-1}$ = (water density) (heat of vaporization) (volume of the layer) (Change of moisture concentration with respect to time)

$$\therefore (q_v)_{m-1} = (\rho_w)(\lambda_w)(A_r)_{m-1} \frac{(\phi^{j+1} - \phi^j)_{m-1}}{\Delta t} \tag{5.68}$$

Using Equation (5.4) into (5.68) yields:

$$(q_v)_{m-1} = (\rho_\omega)(\lambda_w)\left[2\pi(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}\right]\frac{(\phi^{j+1}-\phi^j)_{m-1}}{\Delta t} \quad (5.69)$$

Using Equations (5.6), (5.17) and (5.69) into Equation (5.67) yields:

$$[2\pi(M-1)\Delta\xi]\sigma\varepsilon_f\varepsilon_c\left[(T_f^j + 273)^4 - (T_{m-1}^j + 273)^4\right] =$$

$$2\pi\left[\left(\frac{(K^j)_m + (K^j)_{m-1}}{2}\right)(M - \frac{3}{2})(T_{m-1}^j - T_m^j)\right]$$

$$+ (\rho_w\lambda_w)\left[2\pi(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}\right]\frac{(\phi^{j+1} - \phi^j)_{m-1}}{\Delta t} \quad (5.70)$$

Then, the moisture concentration in the layer at the fire/concrete boundary at the time t = (j+1)Δt is given by:

$$\phi_{m-1}^{j+1} = \phi_{m-1}^j + \frac{\Delta t}{\rho_w\lambda_w(M-\frac{5}{4})\frac{(\Delta\xi)^2}{2}}\{(M-1)\Delta\xi\sigma\varepsilon_f\varepsilon_c\left[(T_f^j + 273)^4 - (T_{m-1}^j + 273)^4\right]$$

$$- \left[\left(\frac{(K^j)_m + (K)_{m-1}^j}{2}\right)(M-\frac{3}{2})(T_{m-1}^j - T_m^j)\right]\} \quad (5.71)$$

Using Equations (4.23) and (4.24) into Equation (5.71), the volume of the moisture is:

$$V^{j+1}_{m-1} = V^{j}_{m-1} + \frac{2\pi\Delta t}{\rho_w\lambda_w} \{(M-1)\Delta\xi\sigma\varepsilon_f\varepsilon_c [(T^j_f + 273)^4 - (T^j_{m-1} + 273)^4]$$

$$- [(\frac{(K^j)_m + (K^j)_{m-1}}{2})(M-\frac{3}{2})(T^j_{m-1} - T^j_m]\} \qquad (5.72)$$

5.16 Change in the Volume of the Moisture Content for a Concrete Layer Inside Concrete Region

The volume of moisture content (ΔV_m), evaporated in the time Δt from a layer inside the concrete, i.e. not located at the fire/concrete boundary as follows:

$$(q_c)_{(m-1)\to(m)} = (q_c)_{(m)\to(m+1)} + (q_v)_{(m)}$$

or $$(q_v)_m = \{(q_c)_{(m-1)\to m} - (q_c)_{(m\to(m+1)}\} \qquad (5.73)$$

where:

$(q_c)_{(m-1)\to m}$, $(q_c)_{(m)\to(m+1)}$, is defined by Equation (5.40).
$(q_c)_{(m-1)\to m}$, $(q_c)_{(m)\to(m+1)}$, is defined by Equation (5.40).n as:
And the heat used for evaporization $(q_v)_m$ can be given as:

$$(q_v)_m = (\rho_w)(\lambda_w)(\frac{\phi^{j+1} - \phi^j}{\Delta t})_m \qquad (5.74)$$

Using Equations (5.40) and (5.74) into Equation (5.73) yields:

$$(\rho_w)(\lambda_w)(\frac{\phi^{j+1} - \phi^j}{\Delta t})_m = \frac{1}{(\Delta\xi)^2} \{(\frac{(K^j)_{m-1} + (K^j)_m}{2})$$

$$\left(\frac{M-m+\frac{1}{2}}{M-m}\right)(T^j_{m-1} - T^j_m) - \left(\frac{(K^j)_{m+1} + (K^j)_m}{2}\right)\left(\frac{M-m-\frac{1}{2}}{M-m}\right)(T^j_m - T^j_{m+1}) \qquad (5.75)$$

Then, the moisture content for the layer at inside concrete region at the time $t = (j+1)\Delta t$ is given by:

$$\phi^{j+1}_m = \phi^j_m + \frac{\Delta t}{(\rho_w \lambda_w)[(M-m)(\Delta\xi)^2]} \{(\frac{(K^j)_{m-1} + (K^j)_m}{2})$$

$$(M-m+\tfrac{1}{2})(T^j_{m-1} - T^j_m) - (\frac{(K^j)_{m+1} + (K^j)_m}{2})(M-m-\tfrac{1}{2})(T^j_m - T^j_{m+1})\} \qquad (5.76)$$

Using Equations (4.23) and (4.24) into Equation (5.76), the volume of the moisture is:

$$V^{j+1}_m = V^j_m + \frac{2\pi(\Delta t)}{\rho_w \lambda_w} \{(\frac{(K^j)_{m-1} + (K^j)_m}{2})(M-m+\tfrac{1}{2})(T^j_{m-1} - T^j_m)$$

$$- (\frac{(K^j)_{m+1} + (K^j)_m}{2})(M-m-\tfrac{1}{2})(T^j_m - T^j_{m+1})\} \qquad (5.77)$$

5.17 Change in the Volume of the Moisture Content for the Centre Concrete Layer:

The volume of moisture content (ΔV_M), evaporated in the time Δt from the centre layer is as follows:

$$(q_c)_{(M-1)\to(M)} = (q_v)_{(M)} \tag{5.78}$$

where:

$(q_c)_{(M-1)\to M}$ is defined by Equation (5.52)

And the heat used for evaporization $(q_v)_M$ can be given as:

$$(q_v)_M = (\rho_w)(\lambda_w)\left(\frac{\phi^{j+1} - \phi^j}{\Delta t}\right)_M \tag{5.79}$$

Using Equations (5.52) and (5.79) into Equation (5.78) yields:

$$\frac{1}{\frac{(\Delta\xi)^2}{4}}\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T)^j_{M-1} - (T)^j_M\right] = \rho_w\lambda_w\left(\frac{\phi^{j+1}-\phi^j}{\Delta t}\right)_M \tag{5.80}$$

Then, the moisture content for the center layer at the time t = (j+1)Δt is:

$$\phi^{j+1}_M = \phi^j_M + \frac{\Delta t}{\rho_w\lambda_w\left(\frac{(\Delta\xi)^2}{4}\right)}\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T^j)_{M-1} - (T^j)_M\right] \tag{5.81}$$

Using Equations (4.23) and (4.24) into Equation (5.81) yields:

$$V^{j+1}_M = V^j_M + \frac{\pi(\Delta t)}{\rho_\omega\lambda_w}\left[\frac{(K^j)_M + (K^j)_{M-1}}{2}\right]\left[(T^j)_{M-1} - (T^j)_M\right] \tag{5.82}$$

5.18 Strength of Column

5.18.1 Division of Cross-Section into Annular Elements

To calculate the deformation and stresses in the column, the cross sectional area of the column is subdivided into a number of annular elements (Figure (5.7). In Figure (5.7) the arrangement of the elements is shown in a quarter section of the column. The arrangement of elements in the three other quarter sections is identical to this. In radial direction the subdivision is the same as that shown in Figure (5.1), where the cross-section is divided into concentric layers. In tangential direction each quarter layer is divided into N_1 elements. The temperature representative of an element is assumed to be that at the centre of the element. This can be obtained by taking the average of the temperatures at the tangential boundaries of each element, previously calculated with the aid of Equations (5.1)-(5.82).

Thus for an element $P_{m,n}$ in the cross-section except for the reinforcement, the representative temperature is:

$$(T^j_{m,n})_{annular} = \left(\frac{T^j_{m-1} + T^j_m}{2}\right)_{layer} \text{, (where } m = 2)_{layer} \tag{5.83}$$

and if the location of the reinforcement at the centre of an element $P_{m,n}$, the representative temperature is:

$$(T^j_{m,n})_{annular} = (T^j_{Reinfor.})_{layer} \tag{5.84}$$

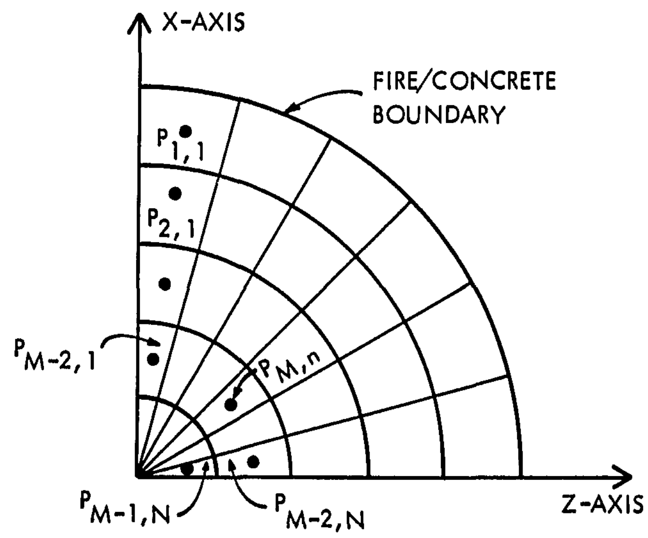

FIGURE 5.7

ARRANGEMENT OF ELEMENTS IN QUARTER SECTION OF REINFORCED CONCRETE COLUMN where the subscripts annular and layer refer to the annular elements shown in Figure (5.7) and the element layers shown in Figure (5.1).

Similarly it is assumed that the stresses and deformations at the centre of an element are representative of the whole element.

5.18.2 Assumptions

In the calculation of column strength the following assumptions were made.

1) The properties of the concrete and steel are as given in Chapter three.
2) The influence of the presence of reinforcing steel on the temperature may be neglected. Thus the column, from a thermal point of view, may be treated as consisting entirely of concrete. The temperature of the steel is assumed to be equal to the temperature in the column section at the location of the center of the steel.
3) Concrete has no tensile strength.
4) Plane sections remain plane.
5) Initial strains in the column before the exposure to fire consists of free shrinkage of the concrete and creep. Because the shrinkage of the column during test normaly is compensated by filling the spaces at both ends of the column between the concrete and steel with a plaster, the shrinkage is assumed to be negligible.

The tests of the columns are usually started after a preloading period of about one hour. The shortening of the column due to creep during this period is assumed to be negligible. The initial creep can be eliminated by selecting the length of the shortened column as the reference length from which the axial strain of the column during the test is measured.

Based on these assumptions, the change of column strength during exposure to fire was calculated. In the calculations the network of annular elements shown in Figure (5.7) was used. Because the strains and stresses of the elements are not symmetrical with respect to the x-axis, the calculations were performed for both the network shown and an identical network at the left of x-axis. The load that the column can carry and the moments in the section were obtained by adding the loads carried by each element and the moments contributed by them.

The equations used in the calculation of the strength of the column during exposure to fire are given in Chapter four.

5.19 Calculations of Strains, Stresses, Loads and Moments in Steel and Concrete:

Follow the same method explained previously in Chapter four.

5.20 Computer Program:

A comprehensive computer program for this mathematical model has been written. The computer output results and discussions will be presented in Chapter six. The program list and the input data are presented in Appendix F.

CHAPTER 6

RESULTS AND DISCUSSIONS

6.1 Introduction

In this Chapter, numerical examples representing the output of the mathematical models of Chapters two, four and five are given. In these examples, the influences of various parameters are discussed.

6.2 Maximum Allowable Load Results:

The mathematical model of cylindrical concrete-filled columns has been selected to illustrate the influence of the column cross-section area, concrete strength and the steel wall thickness on the load and moment resistance of these columns.

Figure (6.1) represents the interaction diagrams for various cross-section area with diameters of 8 in, 10 in, 12 in, 14 in and 16 in. It is clear from these interaction diagrams that, the increase of the cross-section area will increase the values of the load and moment that the column can resist.

Figure (6.2) represents the interaction diagrams for various values of concrete strength (f'_c) i.e of 3, 5, 7, 9 and 12 Kpsi. From Figure (6.2), it can be seen that the increasing of the value of concrete strength (f'_c) will result in increasing the load and moment values that the column can resist.

Figure (6.3) represents the interaction diagrams for various steel wall thickness of 0.125 in, 0.250 in, 0.750 in, 1.0 in and 1.25 in. Its clear from Figure (6.3) that the increase of the steel wall thickness

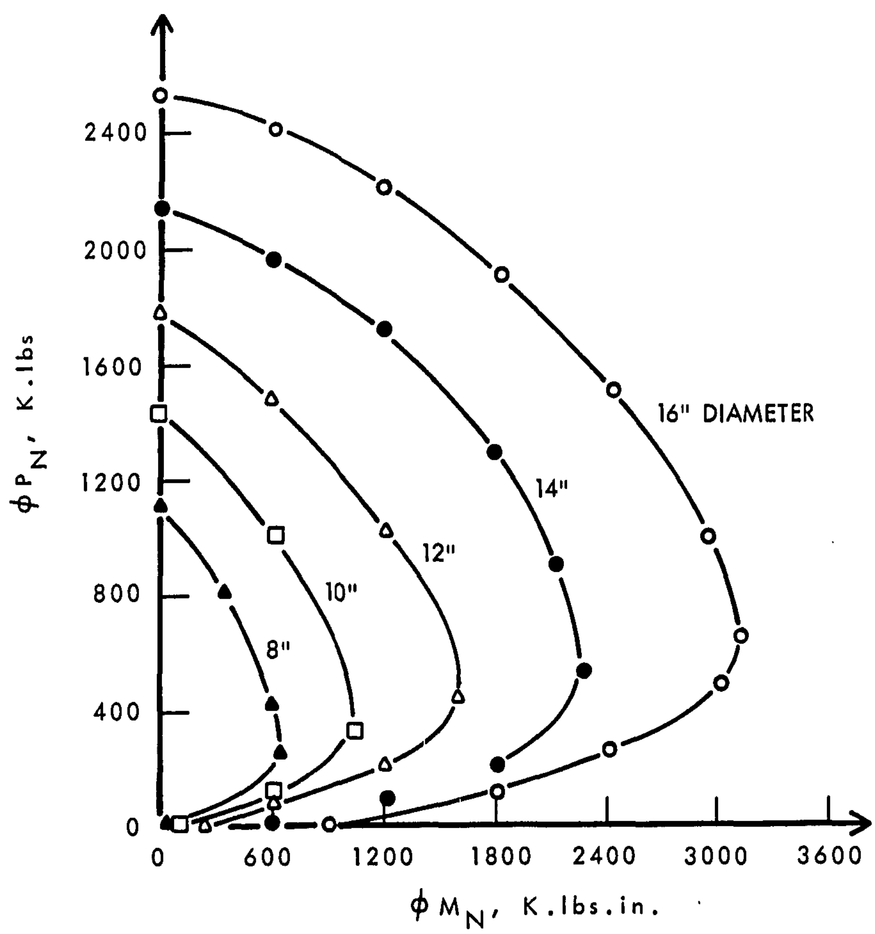

FIGURE 6.1

THE INTERACTION DIAGRAMS FOR VARIOUS CROSS-SECTION DIAMETERS OF CYLINDRICAL CONCRETE-FILLED COLUMNS ($f'_c = 5$ ksi)

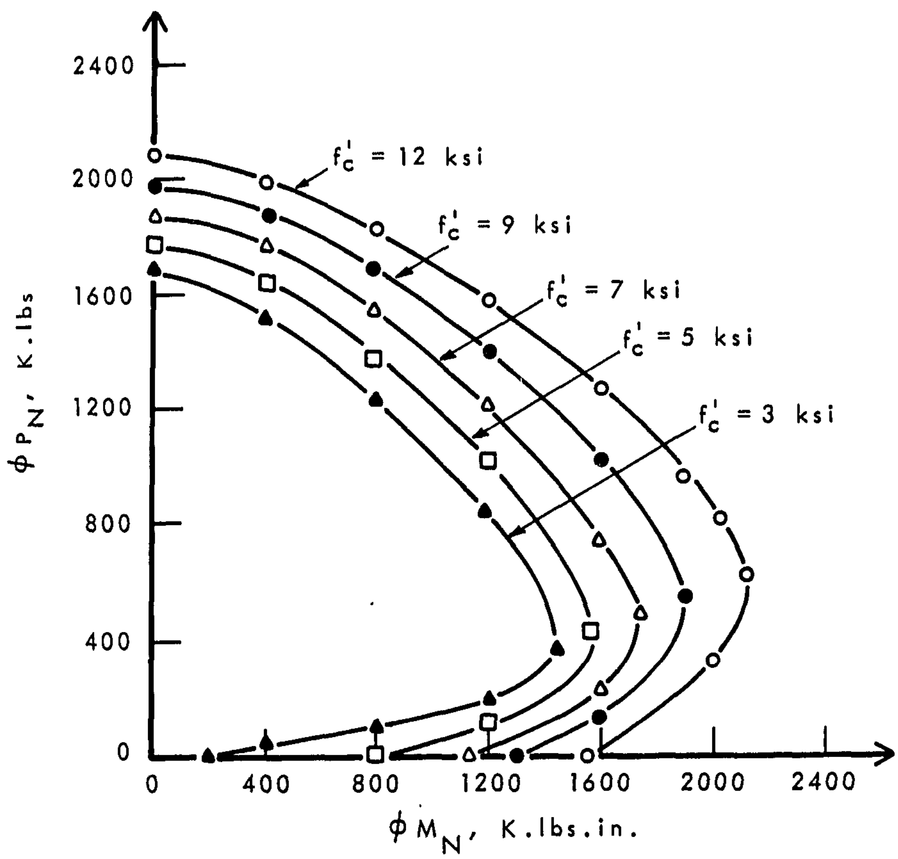

FIGURE 6.2

THE INTERACTION DIAGRAMS FOR VARIOUS VALUES OF CONCRETE STRENGTH FOR CYLINDRICAL CONCRETE-FILLED COLUMNS (12" DIAMETER)

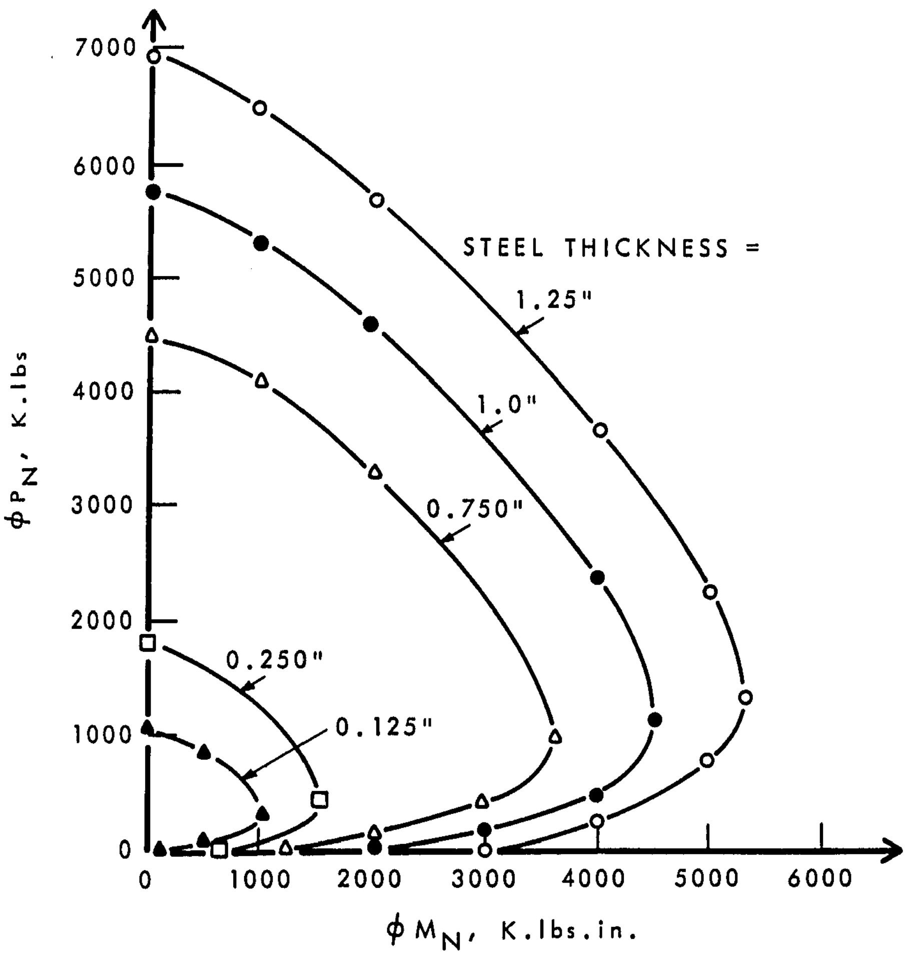

FIGURE 6.3

THE INTERACTION DIAGRAMS FOR VARIOUS VALUES OF STEEL THICKNESS FOR CYLINDRICAL CONCRETE-FILLED COLUMNS (12" DIAMETER) will result in increasing the load and moment values that the column can resist.

6.3 Fire Resistance Results

The influence of fire temperature on the temperatures history of the steel and concrete are presented for square concrete-filled steel and cylindrical reinforced concrete columns. Also, the influence of the cross-section area and the number of reinforcing bars for cylindrical reinforced concrete columns are studied.

Figure (6.4) represents the temperature/time relation for the surface of the steel and concrete for concrete-filled square steel column has (12 in x 12 in) cross-section and 1.0 in steel wall thickness.

Figure (6.5) represents the temperature/time relation for the surface of concrete and for the reinforcing bars for cylindrical reinforced concrete column (12.0 in) diameter with (8) reinforcing bars.

It is clear from Figures (6.4) and (6.5) that the heat transferred from the fire to the steel and concrete is in accordance with the classical solution of the unsteady state partial differential equation [37]. This proves the validity of the numerical solutions which have been explained in detail in Chapters four and five.

Figure (6.6) represents the total load/fire resistance relationships of cylindrical reinforced concrete columns of 12.0 in, 14.0 in and 16.0 in diameters with 8 reinforcing bars. From this Figure, it can be seen that fire resistance of a column will increase by increasing the cross-section area of the column.

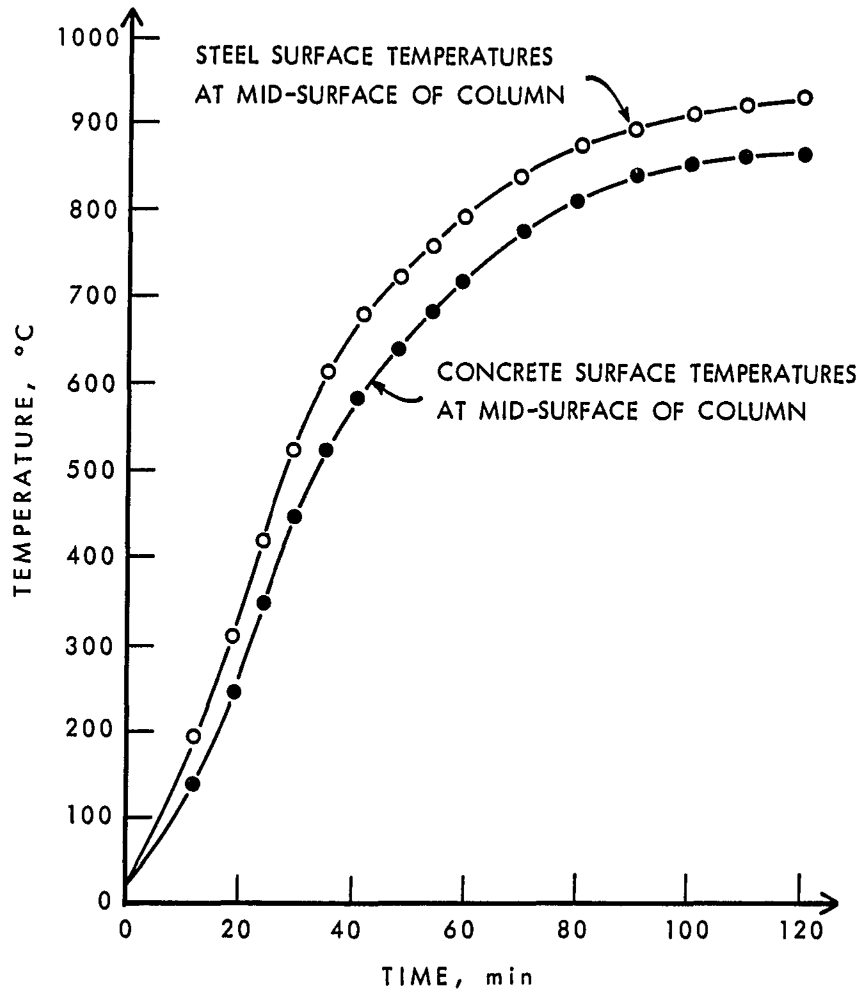

FIGURE 6.4

SQUARE HOLLOW STEEL AND CONCRETE TEMPERATURES FOR CONCRETE-FILLED SQUARE STEEL COLUMN (12"x12" AND 1" STEEL THICKNESS)

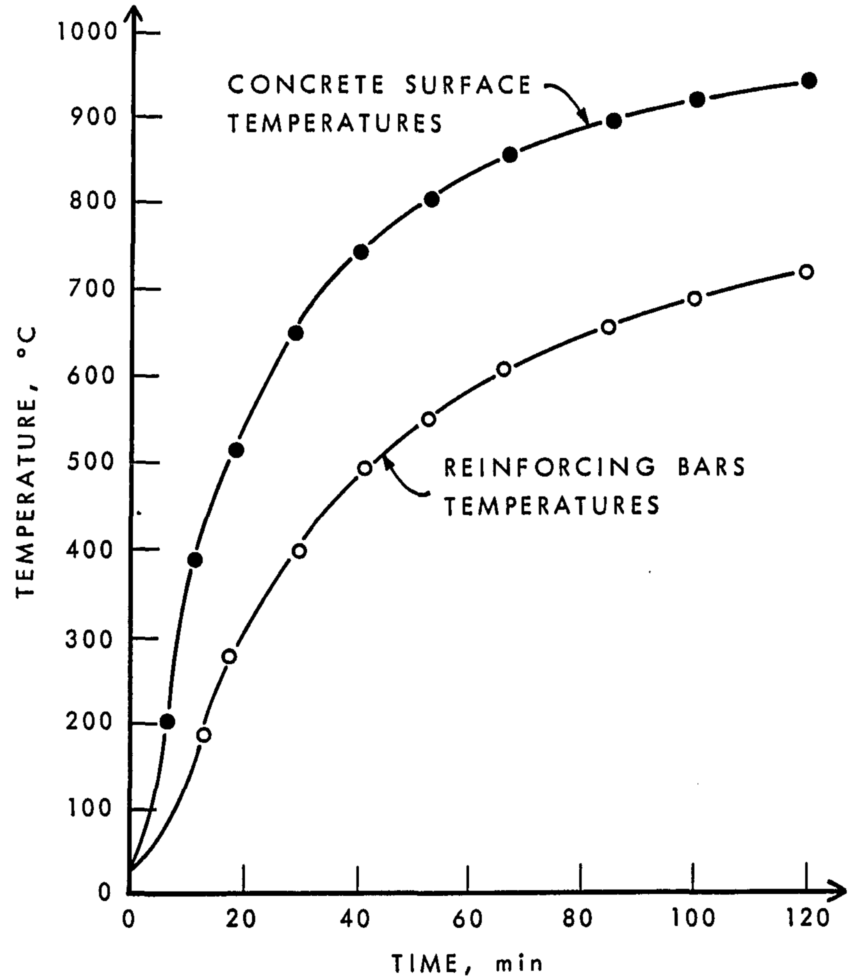

FIGURE 6.5

REINFORCING BARS AND CONCRETE SURFACE CALCULATED TEMPERATURES FOR CYLINDRICAL REINFORCED CONCRETE COLUMN (12" DIAMETER AND 8 REINFORCING BARS)

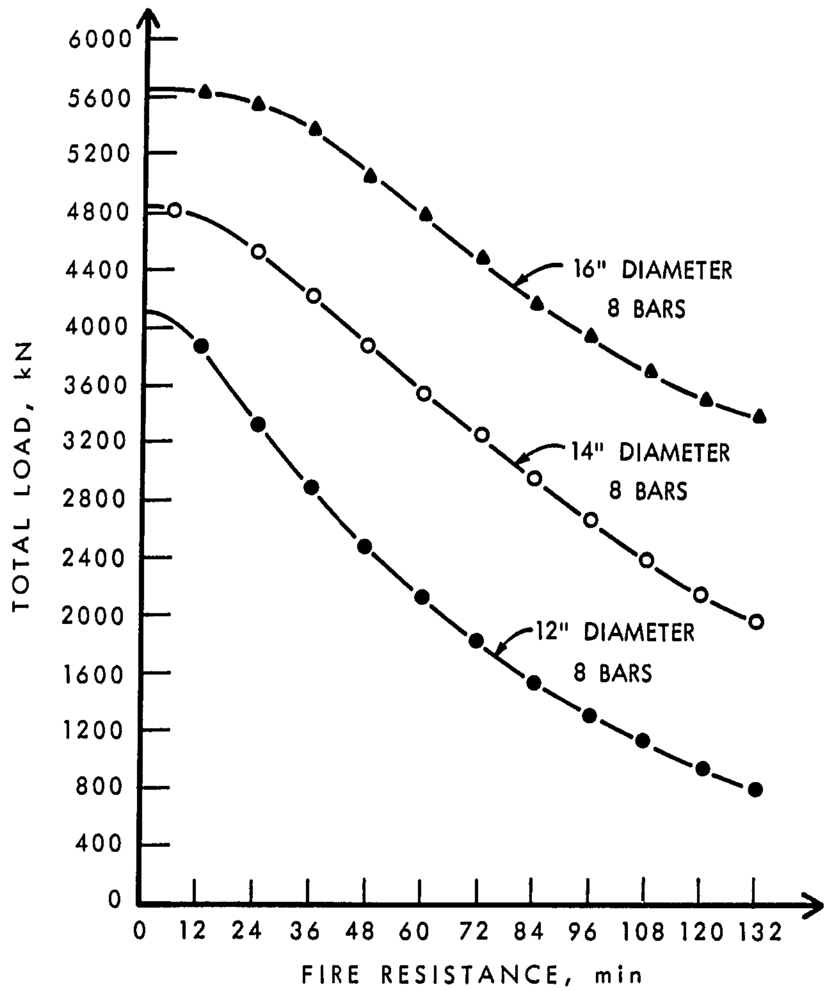

FIGURE 6.6

INFLUENCE OF LOAD ON FIRE RESISTANCE FOR VARIOUS DIAMETERS OF CYLINDRICAL REINFORCED COLUMNS (8 REINFORCING BARS)

Figure (6.7) represents the total load/fire resistance relationships of cylindrical concrete columns for various number of reinforcing bars namely for 6, 9 and 12 bars. This Figure shows that the fire resistance of a column will increase with increasing number of reinforcing bars. The influence of the number of bars on the fire resistance, however, is relatively small in comparison with the influence of the size of the column cross-section.

6.4 Discussions

6.4.1 The Maximum Allowable Load Mathematical Models

The application for the mathematical models of the maximum allowable load allows check the effects of many variables. For example if the cross-section area is varied, the effect of this on the interaction diagram could be seen. Since the computer programs of these mathematical models are written to run using either the American or the Canadian Standards, a flexibility is allowed for and once again variations can be observed in the interaction diagram calculations.

6.4.2 The Fire Resistance Mathematical Models

The variation of the cross-section area shown in Figure (6.6) has more influence on the fire resistance than the variation of the number of reinforcing bars shown in Figure (6.7). However, the predicted fire resistance of these mathematical models do appear to be of the right order of magnitude based on general experience. It is however, fully acknowledge that experimental experience is needed before the sensitivity of these models can be assessed.

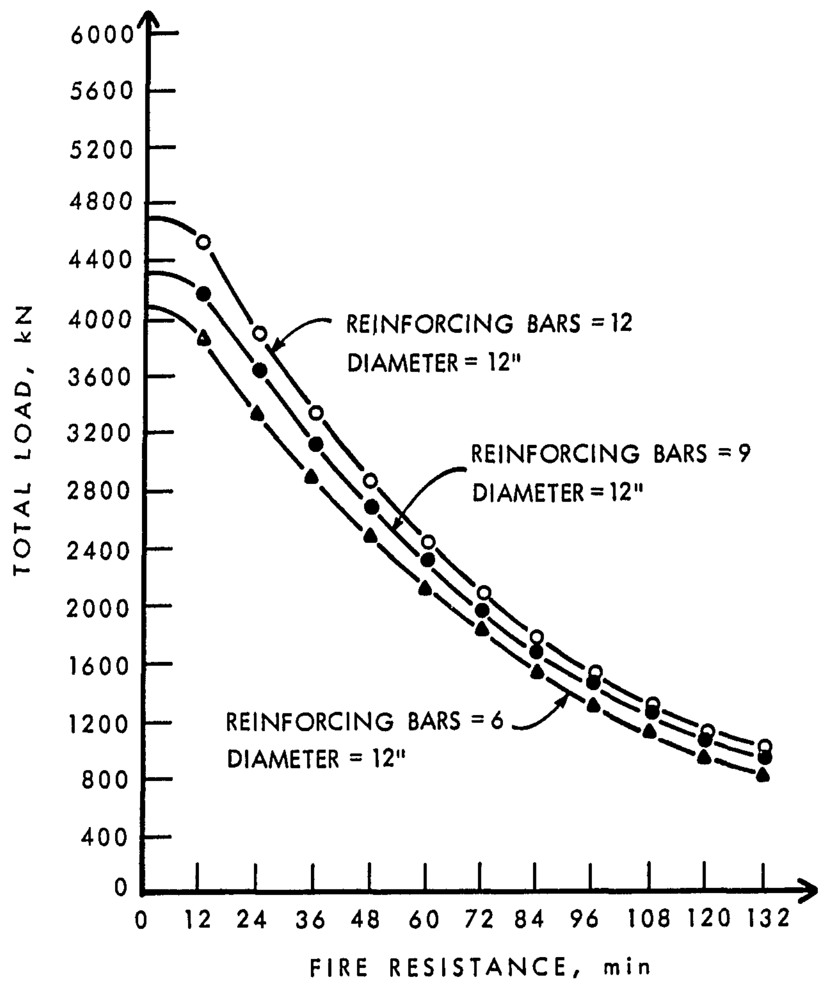

FIGURE 6.7

INFLUENCE OF LOAD ON FIRE RESISTANCE FOR VARIOUS NUMBER OF REINFORCEMENT OF CYLINDRICAL REINFORCED CONCRETE COLUMNS (12" DIAMETER)

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Original Research Contributions

The development of the four mathematical models and their related computer programs, for which no methods exist at present, are the main original contributions of this dissertation. These mathematical models can be summarized as follows:

(1) Maximum allowable load for concrete-filled rectangular steel columns.

(2) Maximum allowable load for concrete-filled cylindrical steel columns.

(3) Fire resistance of concrete-filled square steel columns.

(4) Fire resistance of cylindrical reinforced concrete columns.

In addition computer programs, have been developed for the essential need of the National Research Council of Canada. These computer programs are:

(5) Maximum allowable load for rectangular reinforced concrete columns.

(6) Maximum allowable load for cylindrical reinforced concrete columns.

7.2 Conclusions

7.2.1. Maximum Allowable Load Mathematical Models:

The computer models for the maximum allowable load of columns were run with various input data and the results of the loads were considerably affected by the following parameters:

(a) Dimension of the cross-section of the column.

(b) Compressive strength of concrete.

(c) Steel wall thickness.

7.2.2 Fire Resistance Mathematical Models:

(1) In the past the fire performance of building component could be determined only by experimentation. Recent developments, in particular development of numerical techniques and better knowledge of material properties at elevated temperatures, have made it possible to solve many fire performance problems by calculation. Calculation has the advantage that it is for less expensive and time consuming than performance tests.

(2) The two mathematical models of fire resistance, section 7.1, describes a procedure based on a finite difference method for prediction of the temperature history of concrete-filled and reinforced concrete columns.

(3) The finite difference method described in this dissertation is also suitable for the prediction of temperatures in solid concrete columns, beams and walls. It can also be used for the calculation of temperatures of any system in which a perfect conductor or well-stirred fluid is enclosed in an encasement; for example, water-filled hollow steel columns or beams exposed to a radiative heat source of varying temperature.

(4) The computer models for the fire resistance of concrete-filled and reinforced concrete columns were run with various input data and the main features of the results are as follows:

(a) fire resistance increases markedly with cross-sectional area.

(b) fire resistance increases with the increase of the number of the reinforcing bars.

7.3 Recommendations for Future Research

The following recommendations are suggested as a sequel to this study:

(1) In addition to the two mathematical models for the prediction of the temperature history of columns, which have been presented in this dissertation by using the finite difference method, the following work is further required:

(a) Experimental tests to validate the two models.

(b) Computer output results need sensitivity analysis for various input parameters.

(c) The two mathematical models can be rebuilt by using the finite element and boundary element methods. A comparison can then be made between them and the finite difference method used in this dissertation which may lead to improvement in the accuracy of theoretical results.

(2) The four Computer models for the maximum allowable loads especially for concrete-filled, require an extensive computer output analysis for various input data parameters. The results will be sufficiently abondant and valuable to publish a complete text book for mechanical and civil design engineers.

(3) The computer model for fire resistance of concrete-filled square steel columns presented in this dissertation requires a modification for rectangular cross-section columns.

(4) The mathematical models of the maximum allowable load and fire resistance for various column structures of square, rectangular and circular cross-sections need to be developed. These models could be developed for eccentric and concentric loaded columns, i.e. more than sixty mathematical models are available for further research. The various construction of these columns are as follows:

(a) Reinforced-concrete-filled steel columns.

(b) Concrete-filled steel columns with outer surface insulation.

(c) Reinforced-concrete-filled steel columns with outer surface insulation.

(d) Air-filled steel columns with outer surface insulation.

(e) Square or rectangular frame combined from four columns.

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APPENDIX A

A.1 ULTIMATE STRENGTH DESIGN THEORY

It is a procedure of design with margin of safety against collapse. It's basic assumptions are [6, 7, 8]:

(a) Plane sections before bending remain plane after bending.

(b) Strain in the concrete is proportional to the distance from the neutral axis.

(c) Tensile strength of concrete is neglected in flexural computions.

(d) No slip, i.e. perfect bond between steel and concrete.

(e) The maximum ultimate concrete strain at failure $\varepsilon_u = 0.003$.

(f) The maximum compressive stress in the concrete $C_c = 0.85f'_c$.

(g) The ultimate tensile stress in the reinforcement does not exceed f_y

(i) The modulus of elasticity of the reinforcing steel = 29×10^6 psi

(i) The compressive stress distribution in the concrete (f_c) may be well researched and decumented curve as a rectangular, trapezoidal or parabola [12]. These three cases are presented in Figure (A-1).

A.2 STRESS-STRAIN IN CONCRETE COLUMN

The shape of the stress curve is approximately the standard cylinder test curve, turned sides away as indicated in Figure (A-2).

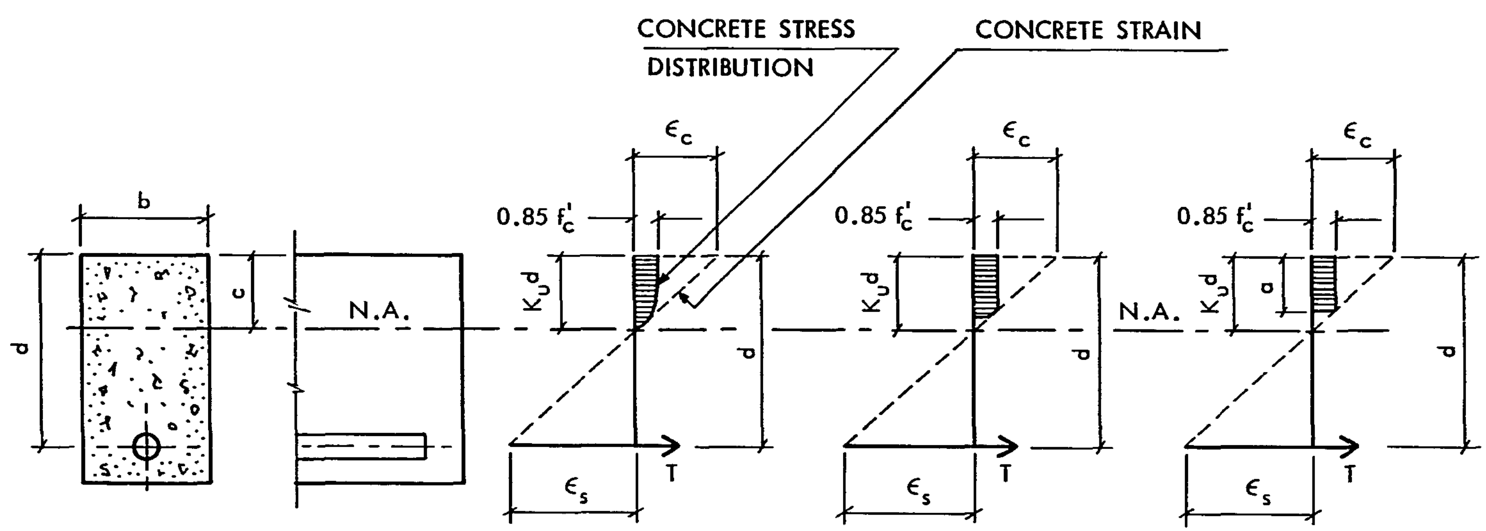


FIGURE A.1

STRESS-STRAIN IN CONCRETE COLUMN

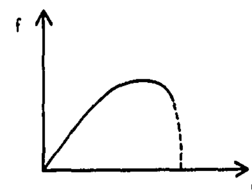


FIGURE A.2 THE STRESS CURVE

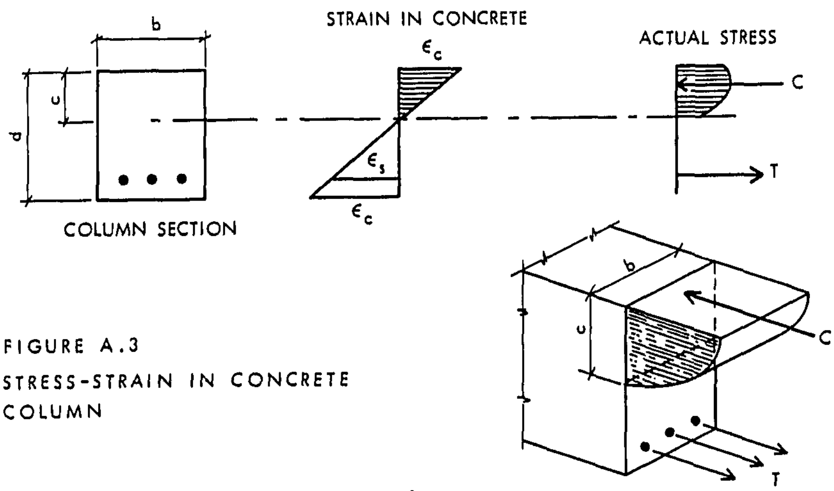


FIGURE A.3
STRESS-STRAIN IN CONCRETE COLUMN

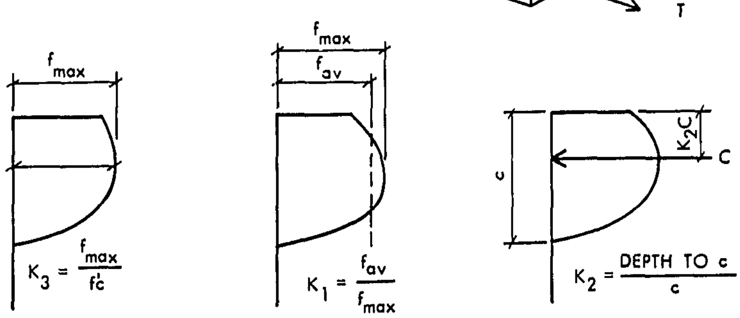


FIGURE A.4
THE CONSTANTS K_1, K_2 AND K_3

The strain and actual stresses diagrams are indicated in Figure (A-3) [43, 44].

STÜSSI (1932)

It is not necessary to know the exact shape. It is only necessary to know the following [45, 46]

(a) Total compression force C

(b) The location of the force C

- The constants K_1, K_2 and K_3:

From Figure (A-4), the total compressive force C is:

$$C = f_{av} \quad c \quad b$$

$$= K_1 \quad f_{max} \quad c \quad b$$

$$= K_1 \quad K_3 \ f_c \quad c \quad b$$

$$\therefore \quad C = K_1 K_3 f_c \quad c.b$$

where:

$$f_{av} = K_1 K_3 f'_c$$

c.b = compression zone area on the cross-section

A.3 • WHITNEY STRESS BLOCK [45]

(a) He stablished empirical values of K_1, K_2 and K_3

(b) He pushed for stress block approach

(c) He tested columns and verified that the stress block approach could be used for columns.

A.4 • WHITNEY'S RESULTS [45]

(a) He made tests on standard concrete cylinders

(b) Assuming that the average curve can be applied for compression zone in failure

(c) He checked the average curve with flexural tests, it was valid, see Figure (A-5)

(d) The values of the constants were:

(c) $K_3 = \frac{f_{max}}{f'_c}$ = Slower rate of loading in conventional structure than standard cylindrical test

$K_3 = 0.85$

(f) $K_2 = \frac{\text{depth to } C}{c} = \frac{K_2\ C}{c}$

For $f'_c < 4000$ psi

$K_2 = 0.425$

For $f'_c > 4000$ psi

$K_2 = 0.45 - \left(\frac{f'_c - 4000}{1000}\right) \quad 0.05$

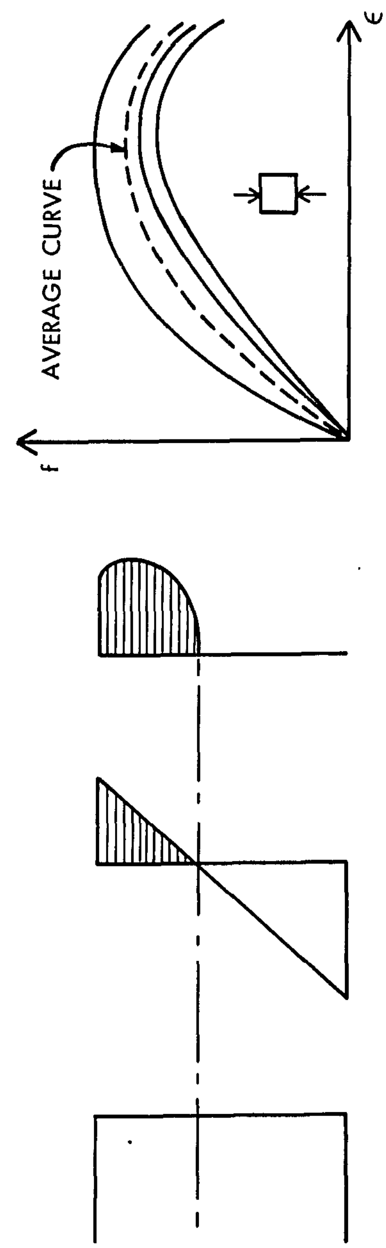


FIGURE A.5

AVERAGE CURVE

(g) $K_1 = \frac{\text{average stress}}{\text{max. stress}} = \frac{f_{an}}{f_{max}}$, see Fig. 2.9

For $f'_c \leq 4000$ psi

$$K_1 = 0.85$$

For $f'_c > 4000$ psi

$$K_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right) \quad 0.05$$

A.5 • STRESS BLOCK [12, 45]

From Figure (A-6), the total compressive force C can be obtained as:

Stress block depth = a = $2K_2c = \beta_1 c$

For $f'_c \leq 4000$ psi $\qquad \beta_1 = 0.85$

For $f'_c > 4000$ psi $\qquad \beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right) \quad 0.05$

$$\text{Force } C = K_1 \cdot K_3 \cdot f'_c \quad \frac{a}{\beta_1} \quad b$$

$$= 0.85 \, f'_c \quad a \quad b$$

$$\therefore \; C = [0.85 \, f'_c] \quad \text{stress block area (a.b)}$$

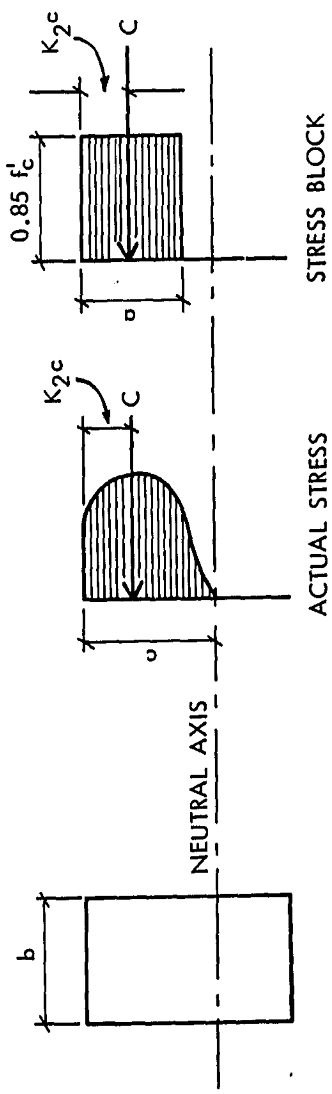


FIGURE A.6

STRESS BLOCK

A.6 BALANCE POINT AND BALANCED CONDITIONS: [11]

• Balance Point:

The balance point can be defined as the point at which the maximum moment should occur and is defined as, "under the simultaneous action of the load and the corresponding moment the concrete will reach its limiting strain (0.003) simultaneously with the tension steel reaching its yield stress f_y.

By looking into Figure (A-7) we can assume that:

ρ_b = the balanced steel ratio at the balanced conditions:

$$\varepsilon_c = \varepsilon_u = 0.003$$

$$\varepsilon_s = \varepsilon_y = \frac{f_y}{E_s}$$

$$\rho_b = \frac{A_s{}^1}{bd}$$

$$\therefore\ A_s = \rho_b \cdot b \cdot d$$

From strain diagram:

$$\frac{c}{d} = \frac{0.003}{\frac{f_y}{E_s} + 0.003}$$, by substituting the value of E_s

$$E_s = 29 \times 10^6 \text{ psi}$$

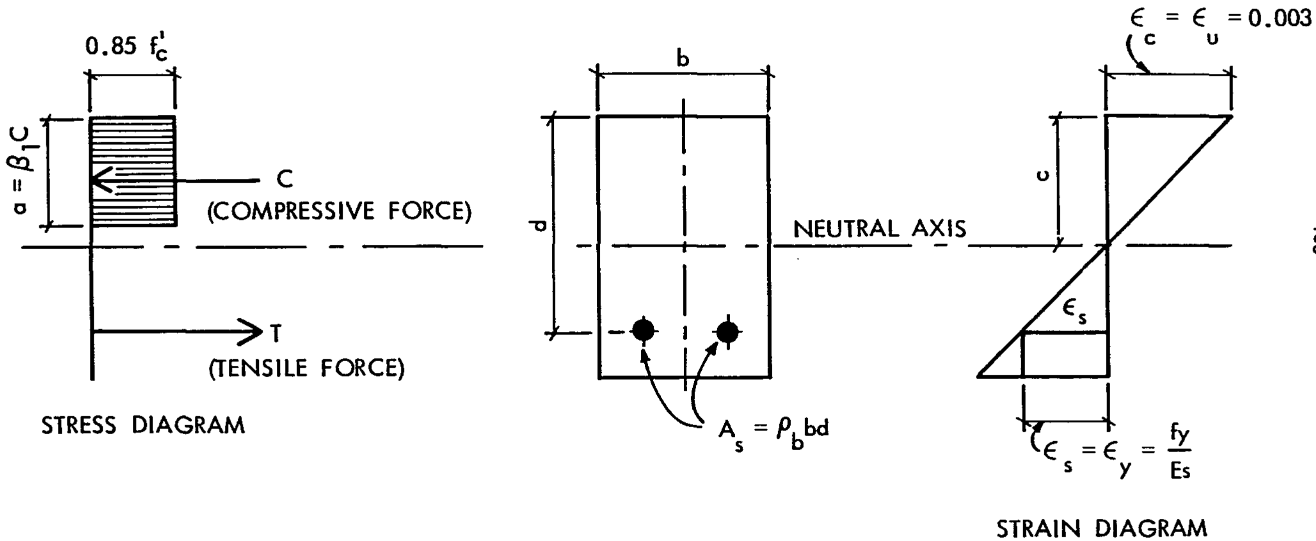


FIGURE A.7

REINFORCEMENT OF ONE SIDE OF THE CROSS-SECTION

$$\therefore \; c = d\left(\frac{87000}{f_y + 87000}\right) \qquad \text{balance point}$$

- **Balanced Conditions:**

From the stress diagram:

$$C = 0.85 \; f'_c \quad b \quad a$$

$$\therefore \; C = 0.85 \; f'_c \quad b \quad \beta_1 C$$

$$\& \; T = As \cdot f_y$$

but, we have Compressive force (C) = Tensile force (T)

$$\therefore \; 0.85 \; f'_c \quad b \quad \beta_1 C$$

$$0.85 \; \beta_1 \frac{f'_c}{fy} \quad b \left(\frac{0.003d}{\frac{f_y}{Es} + 0.003}\right) = \rho_b \cdot b \cdot d$$

$$\therefore \; \rho_b = 0.85 \; \beta_1 \; \frac{f'_c}{f_y} \left(\frac{0.003}{\frac{f_y}{29x10^6} + 0.003}\right)$$

$$\therefore \; \rho_b = 0.85 \; \beta_1 \; \frac{f'_c}{fy} \left(\frac{87000}{87000 + f_y}\right) \qquad \text{balanced condition}$$

At the balanced conditions, a sudden failure would occur with less alarming deflection, because the rotation of the cross section per unit longitudinal distance ($\frac{\varepsilon_u}{c}$ or $\frac{\varepsilon_y}{d-c}$), is relatively small. Therefore when the amount of steel is kept significantly less than that in the balanced condition, neutral axis moves upward (to satisfy C = T). In this case the rotation of the cross-section per unit distance would become larger, and failure would not occur suddenly.

A.7 *DOUBLY REINFORCED CROSS-SECTION* [48, 49, 50]:

If the cross-section is reinforced from both sides as indicated in Figure (A-8), the

$$T = A_s \quad f_y$$

$$C = C_c + C_S$$

$$= 0.85\ f'_c\ (a \cdot b - A'_s) + f'_s\ A'_s$$

$$\frac{c}{d} = \frac{0.003}{fy/E_s + 0.003} = \frac{87000}{fy + 87000}$$

From the above equations:

$$T = C$$

$$A_s \quad fy = 0.85\ f'_c\ (\beta_1 \cdot bC - A'_s) + f'_s\ A'_s$$

$$\rho_b \quad bd \quad fy = 0.85\ f'_c\ (\beta_1 \cdot bC - \rho' \cdot bd) + f'_s\ \rho' bd$$

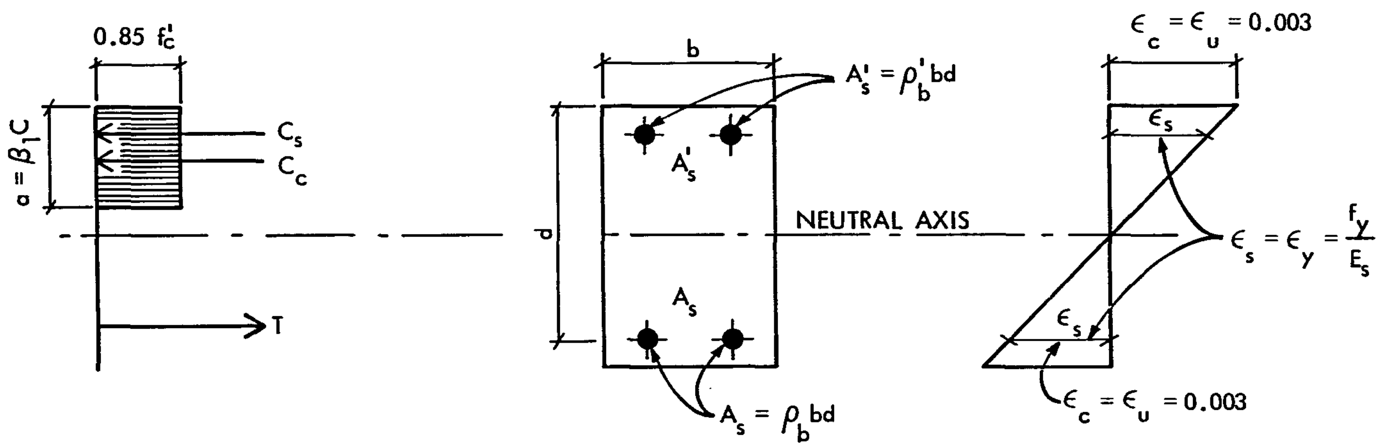


FIGURE A.8

REINFORCEMENT OF TWO SIDES OF THE CROSS-SECTION i.e. the exact balanced ratio is:

$$\rho_b = 0.85 \frac{f'_c}{fy} \{\beta_1(\frac{c}{d}) - \rho'\} + \rho' \cdot \frac{fs}{fy}$$

if $f'_s = fy$ i.e. $\frac{f'_s}{fy} = 1.0$

then, the approximate balanced ratio is:

$$\rho_b = \rho' + 0.85 \frac{f'_c}{fy} \times \beta_1(\frac{c}{d})$$

APPENDIX B

CALCULATIONS OF THE TOTAL LOAD AND MOMENTS FOR RECTANGULAR CONCRETE-FILLED

In this Appendix, the total load (ϕP_N) and the total moment (ϕM_N) have been calculated for four positions of the neutral axis. These calculation have been applied for rectangular or square concrete-filled columns [31, 32].

B.1 RECTANGULAR CONCRETE-FILLED COLUMNS:

The forces in side steel, in end steel, total load and total moments have been calculated for the following neutral axis locations:

<u>CASE 2, FIGURE B.1</u>

The strain in the steel in compression side is greater than the yield strain, i.e.

$$\varepsilon'_s > \varepsilon_y$$

The strain in the steel in tension side is greater than the yield strain, i.e.

$$\varepsilon_s > \varepsilon_y$$

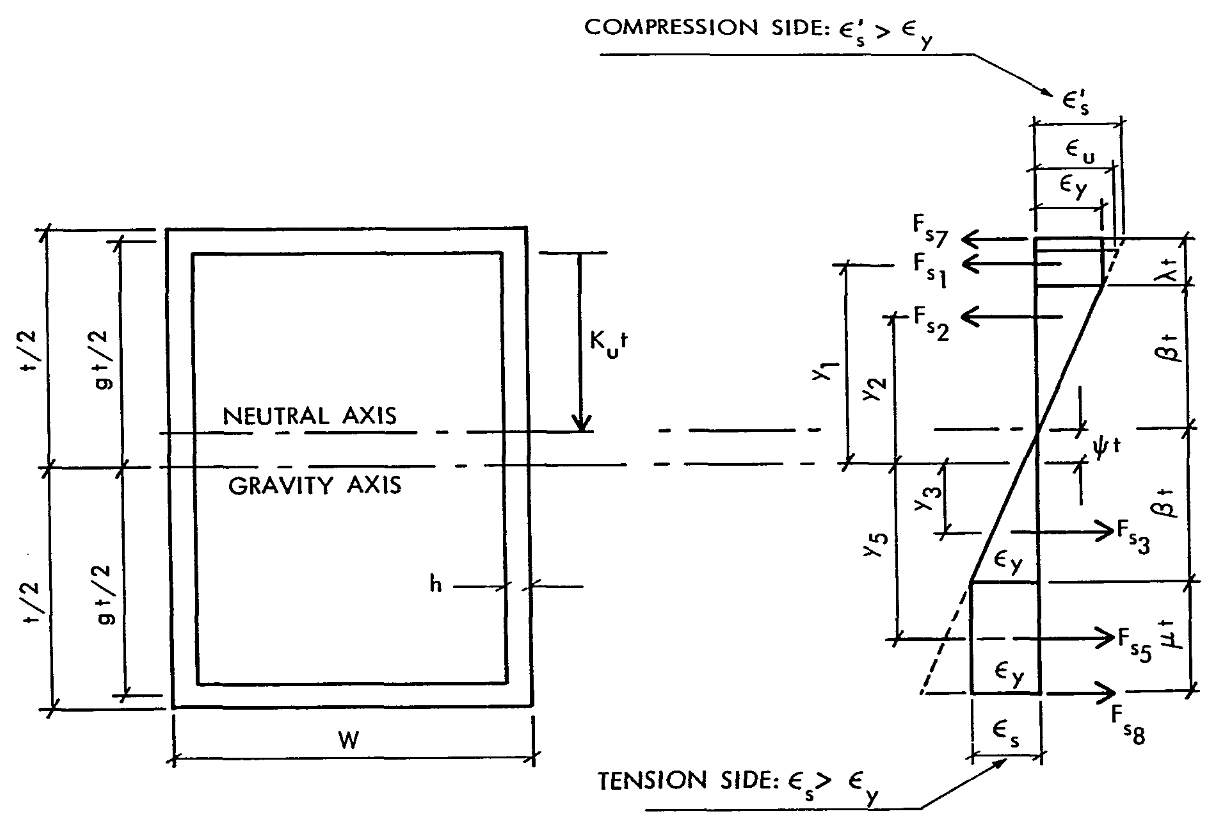


FIGURE B.1

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE TWO)

IDENTITIES

$$M = \frac{g}{2} - \beta + 4$$

$$\lambda = \frac{g}{2} - \beta - 4$$

$$4 = \frac{g}{2} - K_u - \frac{R}{2}$$

Forces In Side Steel:

$$F_{s1} = fy\ (\lambda t)(2h)$$

$$F_{s2} = (\frac{fy}{2})(\beta t)(2h)$$

$$F_{s2} = (\frac{fy}{2})(\beta t)(2h)$$

$$F_{s5} = (fy(Mt)(2h)$$

Forces In End Steel:

$$F_{s7} = fy[h(w-h)]$$

$$F_{s8} = fy[h(w-h)]$$

ARMS OF FORCES

$$y_{s1} = (4 + \beta + \tfrac{1}{2})t$$

$$y_{s2} = (\frac{2}{3}\ \beta + 4)t$$

$$y_{s3} = (\frac{2}{3}\ \beta - 4)t$$

$$y_{s5} = (\frac{M}{2} + \beta - 4)t$$

$$y_{s7} = gt/2$$

$$y_{s8} = gt/2$$

$$y_c = t/2 - \frac{\beta k_u t}{2}$$

TOTAL LOAD AND TOTAL MOMENTS

$P_N = F_{st} + C_c$

$\therefore\ P_N = F_{s1} + F_{s2} - F_{s3} - F_{s5} + F_{s7} - F_{s8} + C_c$

$M_s = F_{s1}Y_{s1} + F_{s2}Y_{s2} + F_{s3}Y_{s3} + F_{s5}Y_{s5} + F_{s7}Y_{s7} + F_{s8}Y_{s8}$

$M_c = C_c \cdot Y_c$

$M_N = M_S + M_C$

$\therefore\ \phi P_N = \phi\ (F_{st} + C_c)$

$\therefore\ \phi M_N = \phi(M_S + M_C$

CASE 3, FIGURE B.2

The strain in the steel in compression side is greater than the yield strain, i.e.:

$\varepsilon'_S > \varepsilon_y$

The strain in the steel in tension side is less than the yield strain, i.e.:

$\varepsilon_S < \varepsilon_y$

IDENTITIES

$\phi = g - ku - \frac{R}{2}$

$w = \phi + \beta - \frac{g}{2}$

$\delta = \frac{g}{2} - w$

$f_s = fy\ \frac{\phi}{\beta}$

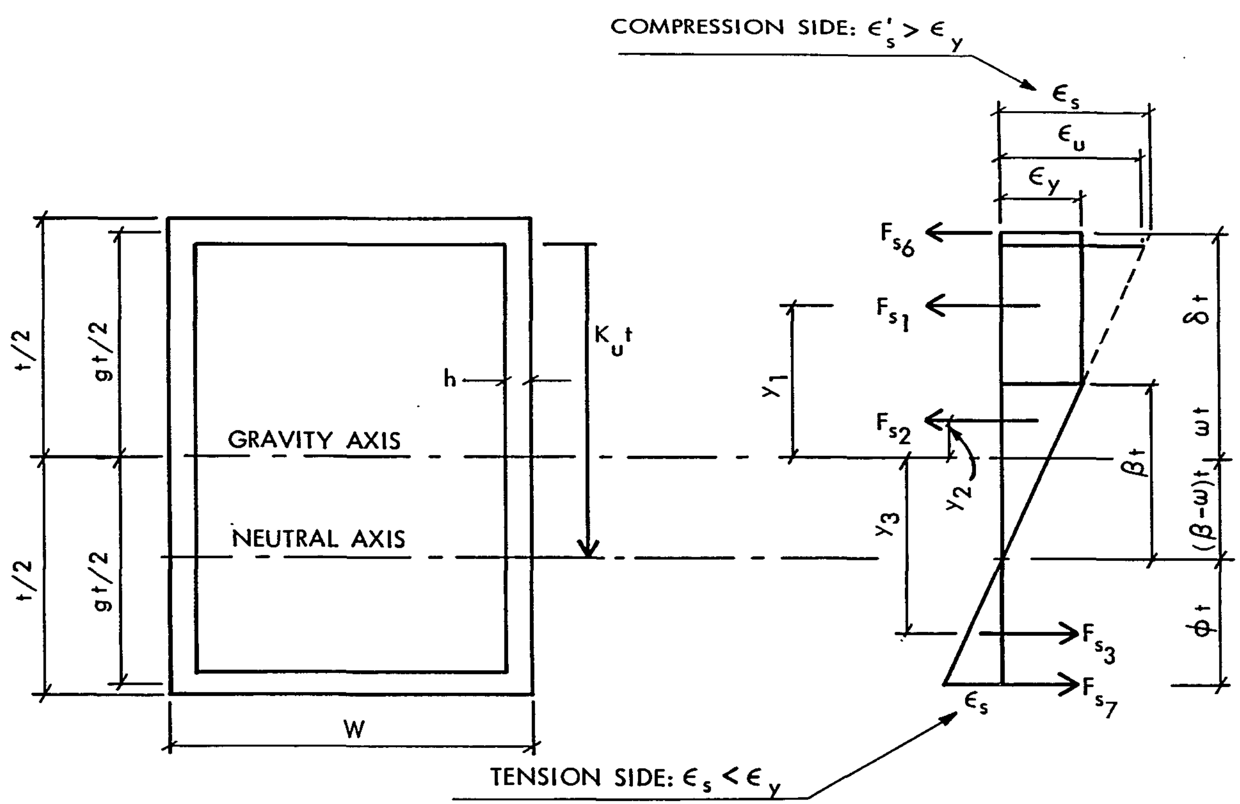


FIGURE B.2

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE THREE)

Forces In Side Steel:

$F_{s1} = fy\ (\delta t)(2h)$

$F_{s2} = (\frac{fy}{2})(\beta t)(2h)$

$F_{s3} = (fy\ \frac{\phi}{\beta})(\phi t)(2h)$

Forces in End Steel:

$F_{s6} = fy[h(w-h)]$

$F_{s7} = (fy\ \frac{\phi}{\beta})[h(w-h)]$

ARMS OF FORCES

$y_{s1} = (W + \frac{\delta}{s})t$

$y_{s2} = (W - \frac{\beta}{3})t$

$y_{s3} = (\frac{2}{3}\ \phi + \beta - W)t$

$y_{s6} = gt/2$

$y_{s7} = gt/2$

$y_c = t/2 - \frac{\beta k_u t}{2}$

TOTAL LOAD AND TOTAL MOMENTS

$P_N = F_{st} + C_c$

$\therefore\ P_N = F_{s1} + F_{s2} - F_{s3} + F_{s6} - F_{s7} + C_c$

$M_S = F_{s1}Y_{s1} + F_{s2}Y_{s2} + F_{s3}Y_{s3} + F_{s6}Y_{s6} + F_{s7}Y_{s7}$

$M_c = C_c \cdot Y_c$

$M_N = M_s + M_c$

$\therefore\ \phi P_N = \phi\ (F_{st} + C_c)$

$\therefore\ \phi M_N = \phi(M_s + M_c)$

CASE 4, FIGURE B.3

The neutral axis falling outside the cross-section causes compression only for the total cross-section.

The strain in the steel of the compressed cross-section is greater than the yield strain, i.e.:

$$\varepsilon'_s > \varepsilon y$$

IDENTITIES

$$\eta = K_u + \frac{R}{2} - g$$

$$\rho = K_u + R - \tfrac{1}{2} - \beta$$

$$\xi = \beta - \eta$$

$$\theta = \frac{g}{2} + \rho$$

$$\varepsilon_s = \varepsilon y \frac{\eta}{\beta}$$

$$f_s = fy \frac{\eta}{\beta}$$

Forces In Side Steel:

$$F_{s1} = fy\ (\theta t)(2h)$$

$$F_{s2} = (\tfrac{1}{2} fy \tfrac{\eta}{\beta})(\beta - \eta)t(2h)$$

$$F_{s3} = (fy \tfrac{\eta}{\beta})(\xi t)(2h)$$

Forces In End Steel:

$$F_{s6} = fy[h(w-h)]$$

$$f_{s7} = fy \tfrac{\eta}{\beta}\ [h(w-h)]$$

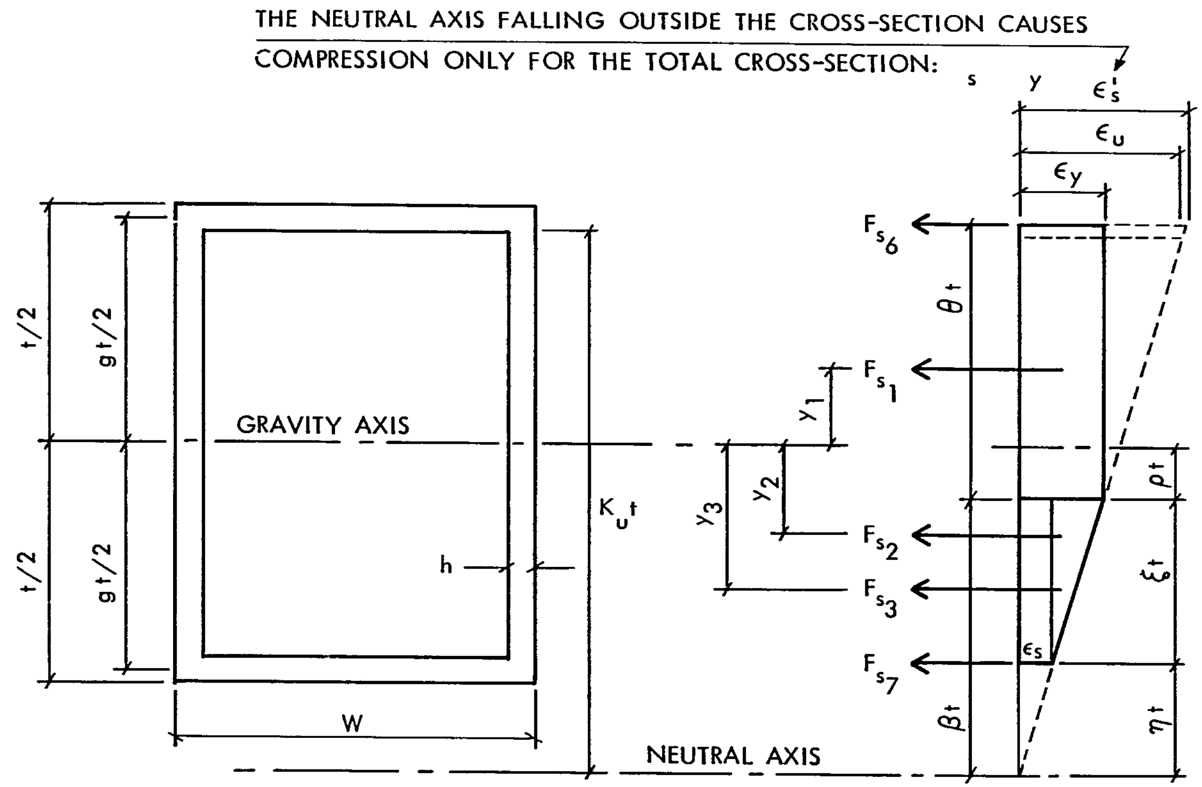


FIGURE B.3

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE FOUR)

ARMS OF FORCES

$$y_{s1} = (\frac{\theta}{2} - \rho)t$$

$$y_{s2} = (\rho + \xi/3)t$$

$$y_{s3} = (\rho + \xi/2)t$$

$$y_{s6} = gt/2$$

$$y_{s7} = gt/2$$

$$y_c = \frac{t}{2} - \frac{\beta k_u t}{2}$$

TOTAL LOAD AND TOTAL MOMENTS

$$P_N = F_{st} + C_c$$

$$\therefore P_N = F_{s1} + F_{s2} + F_{s3} + F_{s6} + F_{s7} + C_c$$

$$M_S = F_{s1}Y_{s1} - F_{s2}Y_{s2} - F_{s3}Y_{s3} + F_{s6}Y_{s6} - F_{s7}Y_{s7}$$

$$M_c = C_c \cdot Y_c$$

$$M_N = M_S + M_C$$

$$\therefore \phi P_N = \phi(F_{st} + C_c)$$

$$\therefore \phi M_N = \phi\ (M_s + M_c)$$

CASE 5, FIGURE B.4

Consider the steel is from high strength steel. The strain in the steel in compression side is less than the yield strain, i.e.

$$\varepsilon'_s < \varepsilon_y$$

The strain in the steel in tension side is less than the yield strain, i.e.

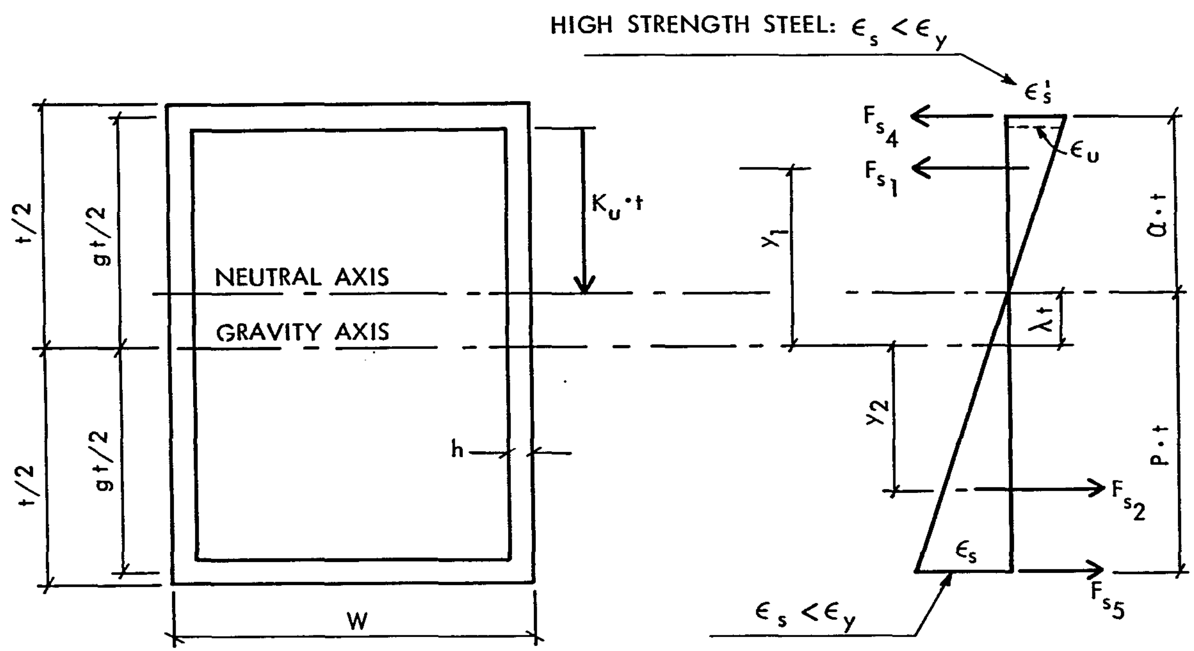


FIGURE B.4

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE FIVE)

$\varepsilon_s < \varepsilon_y$

IDENTITIES

$\alpha = K_u + \frac{R}{2}$

$\lambda = \frac{1}{2} - K_u - R$

$\rho = g - \alpha$

$f_s = fy\ Q$

$f_s' = fyQ'$

$Q' = \frac{\rho \varepsilon_u E_s}{K_u f_y}$

$Q' = \frac{(k_u + \frac{R}{2})\varepsilon_u E_s}{K_u f_y}$

Forces In Side Steel:

$F_{s1} = (\frac{1}{2}fyQ')(\alpha t)(2h)$

$F_{s2} = (\frac{1}{2}fyQ)(\rho t)(2h)$

Forces In End Steel:

$F_{s4} = (fyQ')[h(w-h)]$

$F_{s5} = (fyQ)[h(w-h)]$

ARMS OF FORCES

$y_{s1} = (\frac{2}{3}\alpha + \lambda)t$

$y_{s2} = (\frac{2}{3}\rho - \lambda)t$

$y_{s4} = gt/2$

$y_{s5} = gt/2$

$y_c = \frac{t}{2} - \frac{\beta K_u t}{2}$

TOTAL LOAD AND TOTAL MOMENTS

$P_N = F_{st} + C_c$

∴ $P_N = F_{s1} - F_{s2} + F_{s4} - F_{s5} + C_c$

$M_s = F_{s1}Y_{s1} + F_{s2}Y_{s2} + F_{s4}Y_{s4} + F_{s5}Y_{s5}$

$M_c = C_c \cdot Y_c$

$M_N = M_s + M_c$

∴ $\phi P_N = \phi(F_{st} + C_c)$

∴ $\phi M_N = \phi(M_s + M_c)$

CASE 6, FIGURE B.5

- The neutral axis falling outside the cross-section causes compression only for the total area of the cross-section. Also, the steel is from high strength steel.
- The strain in the steel of the compressed cross-section is less than yield strain, i.e.:

$$\varepsilon_s' < \varepsilon_y$$

IDENTITIES

$\alpha = K_u + \frac{R}{2} - g$

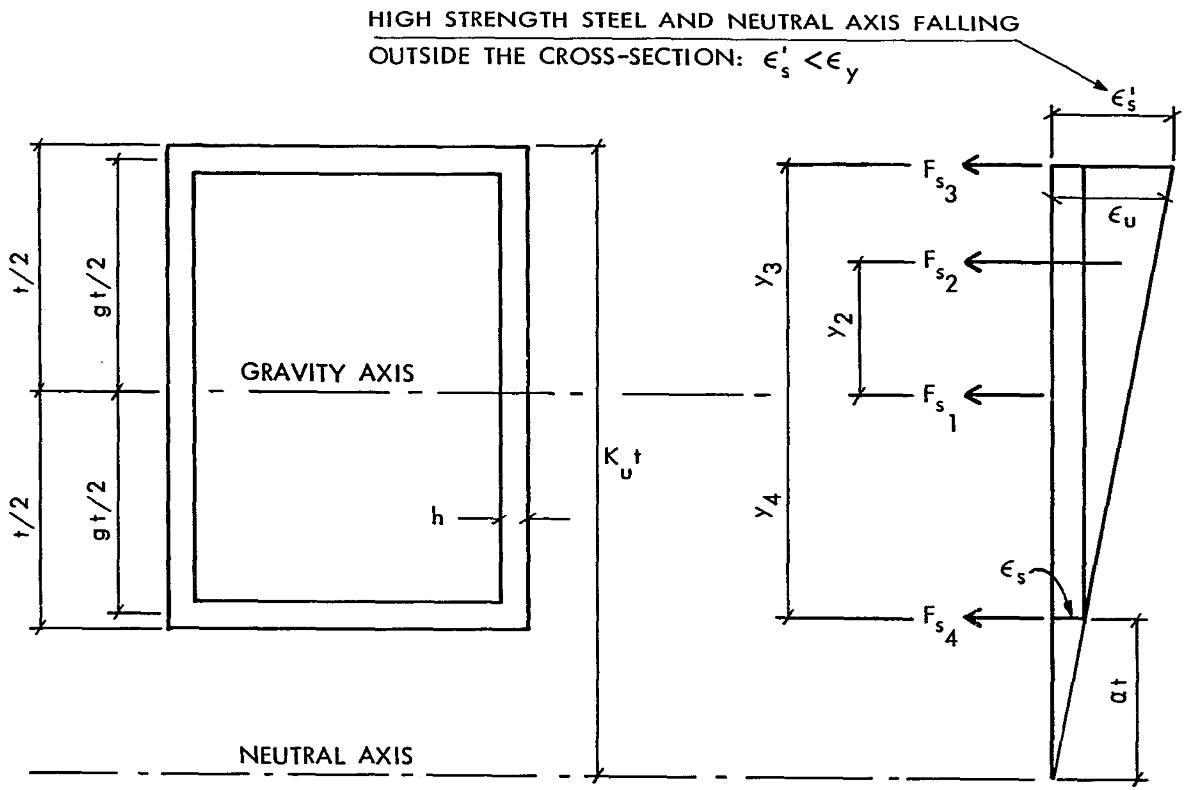


FIGURE B.5

STRAIN DIAGRAM OF THE STEEL AND CONCRETE FOR CONCRETE-FILLED RECTANGULAR STEEL COLUMN (CASE SIX)

$f_s = fyQ$

$f'_s = fyQ'$

$$Q = \frac{(K_u + \frac{R}{2} - g)}{K_u f_y}$$

$$Q' = \frac{(2K_u + R)\varepsilon_u E_s}{2K_u f_y}$$

Forces In Side Steel:

$F_{s1} = (f_s)(gt)(2h)$

$$F_{s2} = \left(\frac{f'_s - fs}{2}\right)(gt)(2h)$$

Forces In End Steel:

$F_{s3} = (fyQ')\left[h(w-h)\right]$

$F_{s4} = (fyQ)\left[h(w-h)\right]$

ARMS OF FORCES

$y_{s1} = 0.00$

$y_{s2} = gt/3$

$y_{s3} = gt/2$

$y_{s4} = gt/2$

$$y_c = \frac{t}{2} - \frac{\beta K_u t}{2}$$

TOTAL LOAD AND TOTAL MOMENTS

$P_N = F_{st} + C_c$

∴ $P_N = F_{s1} + F_{s2} + F_{s3} + F_{s4} + C_c$

$M_s = F_{s1}Y_{s1} + F + F\,Y_{sY} + F_{s4}Ys4 \quad s5\ s5$

$M_c = C_c \bullet Y_c$

$M_N = M_s + M_c$

∴ $\phi P_N = \phi(F_{st} + C_c)$

∴ $\phi M_N = \phi(M_s + M_c)$

APPENDIX C.1

C.1 MAXIMUM ALLOWABLE LOAD DATA INPUT AND SYMBOLS

Symbols Used In Input Data

IMP	A counter used to specify whether imperial or metric units are being used.
ISEC	A counter used to specify whether the cross section is square, rectangular or circular.
IOPN	A counter used to specify whether the cross section is reinforced or concrete-filled.
H	The length of the cross section
WIDTH	The width of the cross section
DG	Is a constant which if multiplied by the length H will give the distance between the two ends of the reinforcement.
THICK	The thickness of the wall of the steel concrete-filled.
DIAB	The diameter of the steel bar.
IBARS	Number of bars for two side of reinforcement.
IBARE	Number of bars for one end of reinforcement.
FC	Strength of the concrete.
FY	Yield strength of the steel.
COEFDL	Coefficient dead load.
COEFLL	Coefficient live load.
LL	Live load
K	Slenderness coefficient
AOC=0	Tells Canadian Standard
AOC=1	Tells American Standard

IEXIT A counter to stop the program or to continue.

The Counters

ITYPE	= 0	Tied column, ϕ=0.7
ITYPE	= 1	Spiral column, ϕ=0.75
IMP	= 0	Metric units
IMP	= 1	Imperial units
ISEC	= 1	Square reinforced or filled
ISEC	= 2	Circular reinforced or filled
ISEC	= 3	Rectangular reinforced or filled
IOPN	= 1	Square, rectangular or circular reinforced
IOPN	= 2	Square, rectangular or circular filled
IEXIT	= 0	Stop
IEXIT	= 1	Continue

APPENDIX C.2

Input Data

Case One, Figure C.1

Square Reinforced

READ (5,3)	IMP	ISEC	IOPN		
3 FORMAT (3I5)	1	1	1		
READ (5,5)	H	WIDTH	DG		
5 FORMAT (3D12.6)	0.16000D 02	0.16000D 02	0.70313D 00		
READ (5,6)	DIAB	IBARS	IBARE		
6 FORMAT (D12.6,2I5)	0.10000D 01	2	3		
READ (5,5)	FC	FY			
5 FORMAT (4D12.6)	0.50000D 01	0.60000D 02			
READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,I5)	0.14000D 01	0.17000D 01	0.40000D 00	0.10000D 01	1
READ (5,3)	IEXIT				
3 FORMAT (1I5)	0	(to stop)			
	1	(to continue)			

Case Two

Square Concrete-Filled, Figure C.2

READ (5,3)	IMP	ISEC	IOPN
3 FORMAT (315)	1	1	2

READ (5,5)	H	WIDTH	THICK
5 FORMAT (3D12.6)	0.16000D 02	0.16000D 02	0.25000D 00

READ (5,5)	FC	FY
5 FORMAT (2D12.6)	0.50000D 01	0.60000D 02

READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,15)	0.14000D 01	0.17000D 01	0.40000D 00	0.10000D 01	1

READ (5,3)	IEXIT	
3 FORMAT (115)	0	(to stop)
	1	(to continue)

Case Three

Rectangular Reinforced, Figure C.3

READ (5,3)	IMP	ISEC	IOPN
3 FORMAT (315)	1	3	1

READ (5,5)	H	WIDTH	DG
5 FORMAT (3D12.6)	0.18000D 02	0.16000D 02	0.61457D 00

READ (5,6)	DIAB	IBARS	IBARE
6 FORMAT (D12.6,2I5)	0.10000D 01	2	3

READ (5,5)	FC	FY			
5 FORMAT (4D12.6)	0.50000D 01	0.60000D 02			
READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,15)	0.14000D 01	0.17000D 01	0.40000D 00	0.10000D 01	1
READ (5,3)	IEXIT				
3 FORMAT (115)	0	(to stop)			
	1	(to continue)			

Case Four

Rectangular Concrete-Filled, Figure C.4

READ (5,3)	IMP	ISEC	IOPN		
3 FORMAT (315)	1	1	2		
READ (5,5)	H	WIDTH	THICK		
5 FORMAT (3D12.6)	0.18000D 02	0.16000D 02	0.25000D 00		
READ (5,5)	FC	FY			
5 FORMAT (2D12.6)	0.50000D 01	0.60000D 02			
READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,15)	0.14000D 01	0.17000D 01	0.40000D 00	0.10000D 01	1
READ (5,3)	IEXIT				
3 FORMAT (115)	0	(to stop)			
	1	(to continue)			

Case Five

Circular Reinforced, Figure C.5

READ (5,3)	IMP	ISEC	IOPN	ITYPE	
3 FORMAT (315)	1	3	1	0	
READ (5,5)	H		DG		
5 FORMAT (2D12.6)	0.18000D 02		0.70313D 00		
READ (5,6)	DIAB		NBARE		
6 FORMAT (D12.6,2I5)	0.10000D 01		8		
READ (5,5)	FC	FY			
5 FORMAT (5D12.6)	0.50000D 01	0.60000D 02			
READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,15)	0.14000D 01	0.17000D 01	0.4000D 00	0.1000D 01	1
READ (5,3)	IEXIT				
3 FORMAT (115)	0	(to stop)			
	1	(to continue)			

Case Six

Circular Concrete-Filled, Figure C.6

READ (5,3)	IMP	ISEC	IOPN
3 FORMAT (315)	1	2	2
READ (5,5)	H		THICK

5 FORMAT (5D12.6)	0.18000D 02		0.25000D 00		
READ (5,5)	FC	FY			
5 FORMAT (2D12.6)	0.50000D 01	0.60000D 02			
READ (5,7)	COEFDL	COEFLL	LL	K	AOC
7 FORMAT (4D12.6,15)	0.14000D 01	0.17000D 01	0.40000D 00	0.10000D 01	1
READ (5,3)	IEXIT				
3 FORMAT (115)	0	(to stop)			
	1	(to continue)			

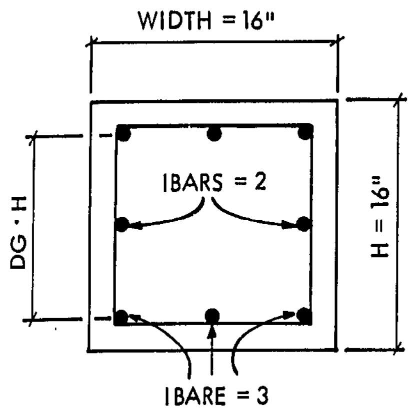


FIGURE C.1

SQUARE REINFORCED

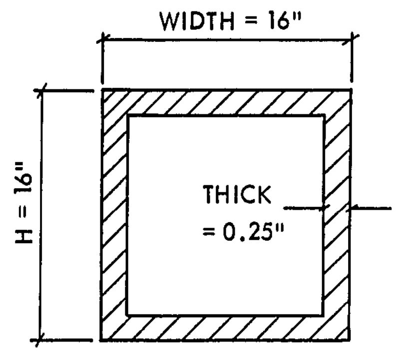
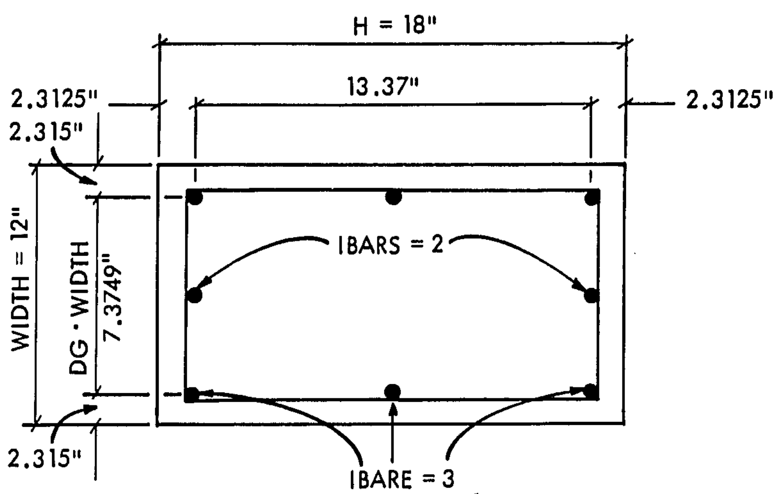


FIGURE C.2

SQUARE CONCRETE-FILLED



$$DG = \frac{\text{WIDTH} - 2(2.3125)}{\text{WIDTH}} = \frac{12 - 2\,(2.3125)}{12}$$

$$= 0.61457$$

FIGURE C.3

RECTANGULAR REINFORCED

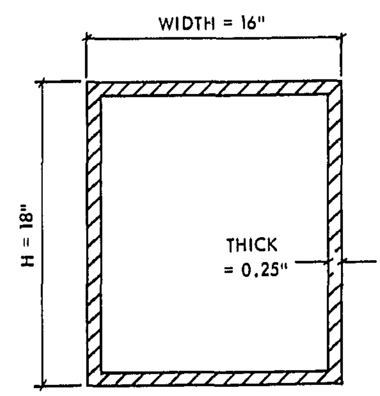


FIGURE C.4

RECTANGULAR CONCRETE-FILLED

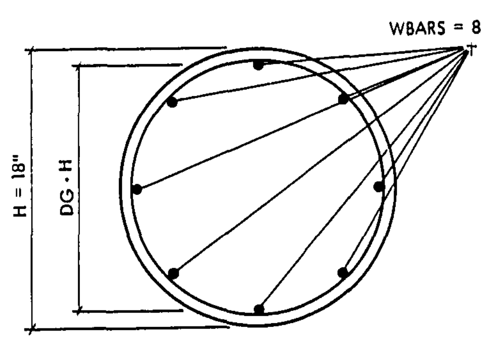


FIGURE C.5

CIRCULAR REINFORCED

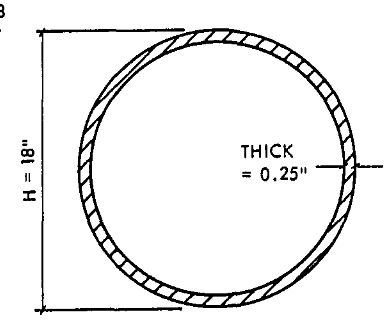


FIGURE C.6

CIRCULAR CONCRETE-FILLED

APPENDIX C.3

```
C       THIS PROGRAM HAS BEEN DEVELOPED BY:
C
C             ***************************************
C             * MOHAMED EL-SHAYEB                   *
C             * DEPARTMENT OF MECHANICAL ENGINEERING *
C             * UNIVERSITY OF NEW HAMPSHIRE         *
C             ***************************************
C
C                   ****************************
C                   ** M A I N   P R O G R A M **
C                   ****************************
C
C...THIS PROGRAM CALCULATES THE MAX. ALLOWABLE LOAD FOR
C...REINFORCED CONCRETE COLUMNS WITH SQUARE  , CIRCULAR
C...OR RECTANGULAR CROSS SECTION
C
C...DEFINITION OF SOME VARIABLES USED IN THIS PROGRAM:
C
C   AOC = IS AN OPTION TO SPECIFY WHERE AMERICAN OR
C         CANADIAN STANDARDS ARE USED.
C   AOC = 0  CANADIAN STANDARDS FOR CALCULATING THE
C            COMPRESSIVE STRENGTH OF COLUMNS ARE USED.
C   AOC = 0  CANADIAN STANDARDS FOR ECCENTRICITY ARE USED.
C   AOC = 1  AMERICAN STANDARDS FOR CALCULATING THE
C            MAXIMUM ALLOWABLE LOAD ARE USED.
C   AOC = 1  AMERICAN STANDARDS FOR ECCENTRICITY ARE USED.
C   IMP : IS AN OPTION USED TO SPECIFY WHETHER IMPERIAL
C         OR METRIC UNITS ARE BEING USED
C   IF IMP = 0 : USE METRIC UNITS
C   IF IMP = 1 : USE IMPERIAL UNITS
C
C   ISEC : IS AN OPTION TO SPECIFY SHAPE OF CONCRETE SECTION
C   IF ISEC= 1 : SQUARE SECTION IS USED
C   IF ISEC= 2 : CIRCULAR SECTION IS USED
C   IF ISEC= 3 : RECTANGULAR SECTION IS USED
C
C   IOPN : IS AN OPTION USED TO SPECIFY TYPE OF CONCRETE SECTION
C   IF IOPN = 1 : REINFORCED CONCRETE SECTION IS REQUIRED
C   IF IOPN = 2 : FILLED     CONCRETE SECTION IS REQUIRED
C
C   ITYPE : IS AN OPTION TO SPECIFY WHETHER SPIRAL OR TIED COLUMN
C   IF ITYPE = 0 : TIED COLUMN
C   IF ITYPE # 0 : SPIRAL COLUMN
C
C   NCPT = IS AN OPTION TO SELECT THE NUMBER OF C POINTS TO BE
C          FITTED IN THE INTERACTION DIAGRAM , COULD BE 28 OR ANY
C          OTHER NUMBER.
C
C   TEMP = TEMPORARY CONSTANTS
C   TEMP1= TEMPORARY CONSTANTS
C
```

```
C...............UNITS CONVERSION
C
C......FROM METRIC TO IMPERIAL      FROM IMPERIAL TO METRIC
C
C      MM  * 0.0393700——— IN        IN  / 0.0393700——— MM
C      MPA * 0.1450000——— KSI       KSI / 0.1450000——— MPA
C      KN  / 4.4457090——— KIPS      KIPS* 4.4457090——— KN
C      KN-M/ 0.1129101——— K-IN      K-IN* 0.1129101——— KN-M
C
C   H     = THE LENGTH OF THE CROSS SECTION OF THE COLUMN
C   WIDTH =THE WIDTH OF THE CROSS SECTION OF THE COLUMN
C   DG    = THE DISTANCE BETWEEN CENTRE LINES OF UPPER AND LOWER STEEL
C   PERST = PERSENTAGE OF REINFORCEMENT
C   FC    = THE COMPRESSIVE STRENGTH OF CONCRETE
C   FY      D= THE YIELD STRENGTH OF REINFORCEMENT
C   ES    = MODULUS OF ELASTICITY OF REINFORCEMENT
C   EC    = MODULUS OF ELASTICITY OF CONCORETE
C   BETA  = FACTOR USED TO CALCULATE A (WHERE A IS THE DEPTH OF
C           THE CONCRETE STRESS BLOCK)
C   EPSI  = D D D DU  = ULTIMATE STRAIN AT THE OUTER FIBERS OF CONCRETE
C   EPSY  = YIELD STRAIN OF THE REINFORCEMENT
C   EPSI  =   DRATIO BETWEEN EPSY AND EPSU ( = 0.003 )
C   C     = KU*H = DISTANCE FROM EXTREME COMPRESSION FIBER TO
C                  NEUTRAL AXIS
C   CM    = DISTANCE FROM EXTREME COMPRESSION FIBER TO NEUTRAL
C           AXIS BUT IN METRIC
C   PN    = NET LOAD WHICH LIES ON INTERACTION DIAGRAM(KIPS)
C   PNM   = SAME AS ABOVE (KN)
C   MN    = NET MOMENT ON INTERACTION DIAGRAM(K-IN)
C   MNM   = SAME AS ABOVE (KN-M)
C   PHIMN = NET MOMENT MULTIPLIED BY SAFETY FACTOR
C   PHIMNM= SAME AS ABOVE                             (KN-M)
C   PHIPN = NET LOAD MULTIPLIED BY SAFETY FACTOR      (KIPS)
C   PHIPNM= SAME AS ABOVE                             (KN  )
C   IG    = GROSS MOMENT OF INERTIA                   (M**4)
C   IS    = STEEL MOMENT OF INERTIA                   (M**4)
C   LOADF = LOAD FACTOR DEFINED AS (= (1.4D + 1.7L)/1.4D)
C           WHERE D&L ARE DEAD AND LIVE LOADS
C   KL    = EFFECTIVE LENGTH OF COLUMN                (IN )
C   KLM   = SAME AS ABOVE                             (M  )
C   MU    = MOMENT WHICH LIES ON INTERACTION DIAGRAM AND IS PART
C           OF THE MAXIMUM ALLOWABLE LOAD             (K-IN)
C   MUM   = SAME AS ABOVE                             (KN-M)
C   DIAB  = DIAMETER OF THE REINFORCEMENT             (IN  )
C   IBARS = NUMBER OF SIDE REINFORCEMENT BARS (TWO SIDES)
C   IBARE = NUMBER OF END  REINFORCEMENT BARS (ONE SIDE )
C   COEFDL= COEFFICIENT OF DEAD LOAD (=1.4)
C   COEFLL= COEFFICIENT OF LIVE LOAD (=1.7)
C   LL    = LIVE LOAD
C   K     = SLENDERNESS COEFFICIENT( FOR MOST CASES = 1.0)
C           HOWEVER IT IS VARIABLE
```

```
C   AST   = AREA OF ONE STEEL BAR
C   ASTT  = TOTAL AREA OF STEEL ( = (IBARS + 2.0*IBARE)*AST)
C   ECC   = ECCENTRICITY
C   PO    = LOAD AT WHICH MOMENT IS ZERO (LIES ON INTERACTION DIAGRAM)
C   PC    = CRITICAL LOAD (FROM EULER'S FORMULA)
C   BT    = SHAPE FACTOR ( = 1.0 - FOR SQUARE OR RECTANGULAR SECTION)
C                        ( = 0.7854 - FOR CIRCULAR SECTION)
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/MATPR/PERST,FC,FY,ES,EC
      COMMON/DIMEN/H,DG,WIDTH
      COMMON/CONST/BETA,EPSI,BT,KU
      COMMON/REINF/ASS,ASE,ASTT,DIAB,IBARS,IBARE,NBARS
      COMMON/SIKNS/THICK1,THICK2,THICK
C
      DIMENSION PN(30),C(30),PNM(30),PHIPN(30),PHIMN(30)
      DIMENSION PHIMNM(30),PHIPNM(30),CM(30)
C
      REAL IS,IG,LOADF,MN(30),MNM(30),KLM,KL,LL,MU,MUM,K
      INTEGER AOC,COEMUM,COEMUN,COEPUM,COEPUN
  110 CONTINUE
      READ(5,3) IMP,ISEC,IOPN,NCPT,ITYPE
    3 FORMAT(5I5)
      IF(ITYPE .EQ. 0)  PHI = 0.70
      IF(ITYPE .NE. 0)  PHI = 0.75
      PI    = 3.1415926540
      IF(ISEC - 2) 8,9,8
C
C...READ DATA FOR SQUARE OR RECTANGULAR SECTION
C
    8 IF(IOPN - 1) 11,11,13
C
C...CASE OF REINFORCED CONCRETE SECTION ( SQUARE OR RECTANGULAR)
C
   11 READ(5,5) H,WIDTH,DG
      READ(5,6) DIAB,IBARS,IBARE
      IF(ISEC.EQ.3) GO TO 19
      GO TO 63
   19 CONTINUE
      TEMP   = WIDTH
      WIDTH  = H
      H      = TEMP
      DG     = (H - WIDTH*(1.0 - DG))/H
   63 CONTINUE
      AST    = PI*(DIAB/2.0)**2
      ASS    = IBARS*AST
      ASE    = IBARE*AST
      ASTT   = ASS + 2.0*ASE
      NBARS  = IBARS + 2*IBARE
      THICK1 = ASS/(2.0*DG*H)
      THICK2  = ASE/(WIDTH - H*(1.0 - DG))
```

```
      AG     = H*WIDTH
      PERST  = ASTT/AG
      GO TO 17
C
C...CASE OF FILLED CONCRETE SECTION(SQUARE OR RECTANGULAR)
C
   13 READ(5,5) H,WIDTH,THICK
      IF(ISEC.EQ.1) GO TO 44
      TEMP  = WIDTH
      WIDTH = H
      H     = TEMP
   44 CONTINUE
      DG     = (H-THICK)/H
      ASE    = THICK*(WIDTH - THICK)
      ASS    = 2.0*DG*H
      ASTT   = ASS + 2.0*ASE
      AG     = H*WIDTH
      PERST  = ASTT/AG
      GO TO 17
C
C...READ DATA FOR CIRCULAR SECTION
C
    9 IF(IOPN - 1) 14,14,16
C
C...CASE OF REINFORCED SECTION( CIRCULAR)
C
   14 READ(5,5) H,DG
      READ(5,6) DIAB,NBARS
      AST    = PI*(DIAB/2.0)**2
      ASTT   = NBARS*AST
      THICK  = ASTT/(PI*DG*H)
      AG     = PI*H**2/4.0
      PERST  = ASTT/AG
      GO TO 17
C
C...CASE OF FILLED SECTION (CIRCULAR)
C
   16 READ(5,5) H,THICK
      DG     = (H - THICK)/H
      D1     = H
      D2     = H - 2.0*THICK
      ASTT   = PI*(D1**2 - D2**2)
      AG     = PI*H**2/4.0
      PERST  = ASTT/AG
   17 CONTINUE
      READ(5,5) FC,FY
    5 FORMAT(5D12.6)
    6 FORMAT(D12.6,2I5)
      READ(5,7)COEFDL,COEFLL,LL,K,AOC
    7 FORMAT(4D12.6,I5)
      IF(IMP .EQ. 1) GO TO 10
```

```
C...UNITS CONVERSION FROM IMPERIAL TO METRIC
      IF(IOPN .EQ. 1) DIAB = DIAB*0.03937
      H    = H*.03937
      DG   = DG*.03937
      FC   = FC*.145
      FY   = FY*.145
   10 ES   = 29000.0
      KL   = 150.0 * K
C...CALCULATIONS OF MINIMUM ECCENTRICITY
      IF(AOC - 0)12,12,15
C...ECCNTIRICITY ACCORDING TO CSA
   12 ECC = 0.1*H
      IF (ECC .LT. 0.984252) ECC = 0.984252
      EC = (5000*SQRT(FC/0.145))*0.145
      GO TO 18
C...ECCENTRICITY ACCORDING TO ACI 318-83
   15 ECC   = 0.6 + 0.03*H
      EC    = (57000.0*(DSQRT(1000.0*FC)))/1000.0
   18 CONTINUE
      IF(ISEC .EQ. 1 .OR. ISEC .EQ. 3) BT = 1.0
      IF(ISEC .EQ. 2 )                 BT = 0.7854
      WRITE(6,109)
  109 FORMAT(1H1)
      IF(ISEC.EQ.1.AND.IOPN.EQ.1) WRITE(6,30)
      IF(ISEC.EQ.1.AND.IOPN.EQ.2) GO TO 36
      IF(ISEC.EQ.2.AND.IOPN.EQ.1) WRITE(6,32)
      IF(ISEC.EQ.2.AND.IOPN.EQ.2) GO TO 37
      IF(ISEC.EQ.3.AND.IOPN.EQ.1) WRITE(6,34)
      IF(ISEC.EQ.3.AND.IOPN.EQ.2) GO TO 38
      GO TO 39
   36 IF(AOC .EQ. 1) WRITE(6,31)
      IF(AOC .EQ. 0) WRITE(6,41)
      GO TO 39
   37 IF(AOC .EQ. 1) WRITE(6,33)
      IF(AOC .EQ. 0) WRITE(6,43)
      GO TO 39
   38 IF(AOC .EQ. 1) WRITE(6,35)
      IF(AOC .EQ. 0) WRITE(6,45)
   39 CONTINUE
   30 FORMAT(5X,'MAX. ALLOWABLE LOAD FOR SQUARE REINFORCED COLUMN',/
     .     5X,'**** ********* **** *** ****** *********** ******',//)
   31 FORMAT(5X,'MAX. ALLOWABLE LOAD FOR SQUARE FILLED COLUMN',/
     .       5X,'**** ********* **** *** ****** ****** ******',/
     .      20X,'ACCORDING TO ACI 318-83',/
     .      20X,'********* ** *** ******',//)
   32 FORMAT(5X,'MAX ALLOWABLE LOAD FOR CYLIND. REINFORCED COLUMN',/
     .    5X,'*** ********* **** *** ******* *********** ******',//)
   33 FORMAT(5X,'MAX ALLOWABLE LOAD FOR CYLIND. FILLED COLUMN',/
     .       5X,'*** ********* **** *** ******* ****** ******',/
     .      20X,'ACCORDING TO ACI 318-83',/
     .      20X,'********* ** *** ******',//)
```

```
34 FORMAT(5X,'MAX ALLOWABLE LOAD FOR RECT. REINFORCED COLUMN',/
  .     5X,'*** *********  *** ***** *********** ******',//)
35 FORMAT(5X,'MAX ALLOWABLE LOAD FOR RECT. FILLED COLUMN',/
  .     5X,'*** ********* **** *** ***** ****** ******',/
  .        20X,'ACCORDING TO ACI 318-83',/
  .        20X,'********* ** *** ******',//)
41 FORMAT(5X,'MAX. ALLOWABLE LOAD FOR SQUARE FILLED COLUMN',/
  .         5X,'***  ********* **** *** ****** ****** ******',/
  .        10X,'ACCORDING TO THE CANADIAN STANDARDS',/
  .        10X,'********* ** *** ******** *********',//)
43 FORMAT(5X,'MAX. ALLOWABLE LOAD FOR CYLIND. FILLED COLUMN',/
  .         5X,'**** ********* **** *** ******* ****** ******',/
  .        10X,'ACCORDING TO THE CANADIAN STANDARDS',/
  .        10X,'********* ** *** ******** *********',//)
45 FORMAT(5X,'MAX. ALLOWABLE LOAD FOR RECT. FILLED COLUMN',/
  .         5X,'**** ********* **** *** ***** ****** ******',/
  .        10X,'ACCORDING TO THE CANADIAN STANDARDS',/
  .        10X,'********* ** *** ******** *********',//)
   WRITE(6,24)
24 FORMAT(20X,'SECTION DIMENSIONS',/
  .         20X,'******* **********',//)
   HCONV = H*25.40
   IF(ISEC - 2) 20,21,20
20 WIDTHC = WIDTH*25.40
   WRITE(6,22) HCONV,WIDTHC,H,WIDTH
22 FORMAT(5X,'DIMENSIONS OF THE SECTION ARE .............',
  .        D12.3,'*',D12.3,' MM','(',D12.3,'*',D12.3,' IN)')
   GO TO 29
21 WRITE(6,23) HCONV,H
23 FORMAT(5X,'DIAMETER OF THE CIRCULAR SECTION IS .......',
  .        D12.3,' MM','(',D12.3,' IN)')
29 CONTINUE
   IF(IOPN .EQ. 2) GO TO 25
   DIABM  = DIAB/0.03937
   DH     = DG*H
   DHM    = DH  /0.03937
   WRITE(6,26) DIABM,DIAB,DHM,DH,NBARS,PERST
26 FORMAT(5X,'DIAMETER OF BAR ........................ =',
  .        D12.3,' MM','(',D12.3,' IN)',/
  .        5X,'DISTANSE BET. CENTER LINES OF END STEELS. =',
  .        D12.3,' MM','(',D12.3,' IN)',/
  .        5X,'NUMBER OF BARS ......................... =',I5,/
  .        5X,'% OF STEEL.............................. =',D12.3)
   GO TO 28
25 THICKM = THICK*25.40
   DH     = DG*H
   DHM    = DH*25.40
   WRITE(6,27) THICKM,THICK,DHM,DH
27 FORMAT(5X,'THICKNESS OF THE STEEL SECTION ......... =',
  .        D12.3,' MM','(',D12.3,' IN)',/
  .        5X,'DISTANCE BET. CENTER LINES OF STEEL SEC. =',
```

```
     .       D12.3,' MM','(',D12.3,' IN)',//)
   28 CONTINUE
      KLM   = KL/0.039370
      WRITE(6,50) KLM,KL,K
   50 FORMAT(5X,'EFFECTIVE LENGTH OF COLUMN-KL ........... =',
     .       D12.3,' MM','(',D12.3,' IN)',/
     .      5X,'(NOTE K ASSUMED =',D12.3,')',//)
      WRITE(6,55)
   55 FORMAT(20X,'MATERIAL PROPERTIES',/
     .       20X,'******** **********',//)
      FCM   = FC/0.1450
      FYM   = FY/0.1450
      ECM   = EC/0.1450
      ESM   = ES/0.1450
      WRITE(6,56) ECM,EC,ESM,ES,FCM,FC,FYM,FY
   56 FORMAT(5X,'CONCRETE MODULUS-EC...................... =',
     .       D12.3,' MPA','(',D12.3,' KSI)',/
     .       5X,'STEEL MODULUS-ES......................... =',
     .       D12.3,' MPA','(',D12.3,' KSI)',/
     .       5X,'COMPRESSIVE STRENGTH OF CONCRETE-FC...... =',
     .       D12.3,' MPA','(',D12.3,' KSI)',/
     .       5X,'YIELD STRENGTH OF STEEL-FY............... =',
     .       D12.3,' MPA','(',D12.3,' KSI)')
C
      IF(IOPN .EQ. 2 .AND. AOC .EQ. 0) GO TO 71
C
      ECCM = ECC/0.03937
      IF(AOC-0) 65,65,66
   65 WRITE(6,68) ECCM,ECC
   68 FORMAT(5X,'MIN. ECC. ACCORDING TO CSA..(ECC=0.1*H)...=',
     .       D12.3,' MM ','(',D12.3,'  IN)')
      GO TO 70
   66 WRITE(6,69) ECCM,ECC
   69 FORMAT(5X,'MIN. ECC. ACCORDING TO ACI 318-83.......  =',
     .       D12.3,' MM ','(',D12.3,'  IN)')
C     NOMINAL LOAD AT ZERO ECCENTRICITY
   70 PO = 0.85*FC*(AG-ASTT) + FY*ASTT
      LOADF = (COEFDL+COEFLL*LL)/COEFDL
      WRITE(6,75) LOADF
   75 FORMAT(5X,'LOAD FACTOR.((1.4*D+1.7*L)/1.4*D)........ =',D12.3,//)
C
C...MOMENTS OF INERTIA CALCULATIONS TO GET CRITICAL LOAD
C
      IF(ISEC - 2) 51,52,51
C
C...MOMENTS OF INERTIA FOR RECTANGULAR OR SQUARE REINFORCED SECTION
C
   51 IF(IOPN - 1) 53,53,54
   53 DH  = DG*H
      IS  = (THICK1*DH**3/12.0)*2.0 + (ASE*(DH/2.0)**2.0)*2.0
      IG  = WIDTH*H**3/12.0
```

```
      GO TO 57
C
C...MOMENTS OF INERTIA FOR RECTANGULAR OR SQUARE FILLED  SECTION
C
   54 DH  = DG*H
      IS  = 2.0*(THICK*DH**3/12.0) + 2.0*(ASE*(DH/2.0)**2)
      IG  = WIDTH*H**3/12.0
      GO TO 57
C
C...MOMENTS OF INERTIA FOR CIRCULAR REINFORCED OR FILLED SECTION
C
   52 DH  = DG*H
      D1  = DH + THICK
      D2  = DH - THICK
      IS  = PI*(D1**4 - D2**4)/64.0
      IG  = PI*H**4/64.0
   57 CONTINUE
      BD  = 1.0/LOADF
      EI  = ((EC*IG)/5.0 + ES*IS)/(1.0 + BD)
C...CRITICAL LOAD CALCULATIONS
      PC  = PI**2*EI/(KL**2)
      CALL INTDG(C,PO,PHIPN,PHIMN,NCPT,PHI,ISEC,IOPN,ITYPE)
      CALL MAXLD(PHIPN,PHIMN,ECC,PC,C,LOADF,PHI,NCPT)
      GO TO 72
C
   71 CONTINUE
C
C...  CALCULATE THE COMPRESSIVE RESISTANCE OF FILLED SECTION
C     ACCORDING TO THE CANADIAN STANDARDS
C
      CALL FLDCN(ISEC,K,KL)
C
   72 CONTINUE
      READ(5,3) IEXIT
      IF(IEXIT .EQ. 0) GO TO 61
      GO TO 110
   61 CONTINUE
      STOP
      END
C
C     *********************************************
      SUBROUTINE INTDG(C,PO,PHIPN,PHIMN,NCPT,PHI,ISEC,IOPN,ITYPE)
C     *********************************************
C
C...THIS SUBROUTINE CALCULATES THE NET LOADS (PN) AND THE NET MOMENTS
C   (MN) WHICH CONSTRUCT THE INTERACTION DIAGRAM
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/MATPR/PERST,FC,FY,ES,EC
      COMMON/DIMEN/H,DG,WIDTH
```

```
      COMMON/CONST/BETA,EPSI,BT,KU
      COMMON/SIKNS/THICK1,THI' 2,THICK
C
      DIMENSION C(30),CM(30),PN(30),PHIPN(30),PHIMN(30)
      DIMENSION PHIPNM(30),PHIMNM(30),PNM(30)
C
      REAL KU,MN(30),MNM(30)
      EPSU  = 0.003
      EPSY  = FY/ES
C
C...PN AND MN VALUES AT POINT-2 (POINT-2 LIES ON PN-AXIS)
C...LET C   = INFINTY
      C(NCPT)  = 0.1E 08
      PN(NCPT) = PO
      MN(NCPT) = 0.0
C
C...CALCULATE THE VALUE OF BETA ( A = BETA*C ,WHERE A IS THE DEPTH
C   OF THE CONCRETE STRESS BLOCK, AND C IS THE N.A. LOCATION)
C
      BETA = 0.85 - 0.05*(INT(FC-4.0))
      IF(BETA .LT. 0.65) BETA = 0.65
      IF(FC .LE. 4.0) BETA = 0.85
C
C... LOCATION OF BALANCE POINT AND OTHER POINTS ON THE DIAGRAM
C
      NCPT1  = NCPT-1
      IF(IOPN - 1) 30,30,40
C
C...CASE OF REINFORCED CONCRETE COLUMN
C
   30 D = H-(H-DG*H)/2.0
      C(1) = (EPSU/(EPSU + EPSY))*D
      GO TO 50
C
C...CASE OF FILLED CONCRETE COLUMN
C
   40 D  = DG*H - THICK/2.0
      C(1) = D*(EPSU/(EPSU + EPSY)) + THICK
C
   50 CONTINUE
      DO 119 I=2,NCPT1
      C(I)    = I*H/10.0
  119 CONTINUE
      DO 120 I=1,NCPT1
      IF(IOPN .EQ. 1) KU = C(I)/H
      IF(IOPN .EQ. 2) KU = (C(I) - THICK)/H
      IF(ISEC - 2) 10,20,10
C
C...CALCULATE PN AND MN FOR SQUARE OR RECTANGULAR SECTION
C
   10 CALL FMSQ(I,C,PN,MN,IOPN)
```

```
C
      GO TO 120
C
C...CALCULATE PN AND MN FOR CIRCULAR SECTION
C
   20 CALL FMCR(I,C,PN,MN,IOPN)
  120 CONTINUE

C
C...MULTIPLY ALL INTERACTION VALUES BY PHI(=0.7,=0.75 FOR TIED AND SPI)
C
      DO 410 I=1,NCPT
      PHIPN(I) = PN(I)*PHI
      PHIMN(I) = MN(I)*PHI
  410 CONTINUE
      DO 420 I=1,NCPT
      PHIPNM(I) = PHIPN(I)*4.445709
      PHIMNM(I) = PHIMN(I)*0.1129101
      PNM(I)    = PN(I)   *4.4457090
      MNM(I)    = MN(I)   *0.1129101
      CM(I)     = C(I)    *25.40
  420 CONTINUE
      WRITE(6,430)
  430 FORMAT(1H1,35X,'*** INTERACTION DIAGRAM VALUES ***',/
     .           35X,'**********************************',//)
      WRITE(6,450)
  450 FORMAT(11X,'N.A.LOCATION',31X,'ULT.MOMENT',35X,'ULT. LOAD',/
     .       11X,'************',31X,'**********',35X,'*********',/
     .       17X,'C',41X,'MN',42X,'PN'//)
      WRITE(6,469) CM(1),C(1),MNM(1),MN(1),PNM(1),PN(1)
  469 FORMAT( D12.3,'MM(',D12.3,'IN) BAL. PT. ',D12.3,' KNM',
     .';(',D12.3,'K-IN)',10X,D12.3,' KN',';(',D12.3,'KIPS)')
      WRITE(6,468)
  468 FORMAT(/)
      DO 460 I=2,NCPT
      IF(I .EQ. NCPT ) GO TO 455
      WRITE(6,465) CM(I),C(I),MNM(I),MN(I),PNM(I),PN(I)
  465 FORMAT(D12.3,'MM(',D12.3,'IN)',10X,D12.3,' KNM'
     .,';(',D12.3,'K-IN)',10X,D12.3,' KN',';(',D12.3,'KIPS)')
      GO TO 460
  455 WRITE(6,453) MNM(I),MN(I),PNM(I),PN(I)
  453 FORMAT(10X,'***INFINITY***',16X,D12.3,' KNM',';(',D12.3,'K-IN)',
     .10X,D12.3,' KN',';(',D12.3,'KIPS)')
      GO TO 460
  460 CONTINUE
      IF(ITYPE .EQ. 0 )  WRITE(6,470)
      IF(ITYPE .NE. 0 )  WRITE(6,471)
  470 FORMAT(1H1,//,42X,'PHI = 0.70 (TIED COLUMN)')
  471 FORMAT(1H1,//,42X,'PHI = 0.75 (SPIRAL COLUMN)')
      WRITE(6,480)
  480 FORMAT(///,11X,'N.A. LOCATION',31X,'MOD. MOMENT',33X,'MOD. LOAD',
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     .         11X,'*************',31X,'***********',33X,'*********',
     .        ,17X,'C',37X,'  'IMN(KNM)',34X,'PHIPN(KN)',/)
      WRITE(6,469) CM(1),C(:  .HIMNM(1),PHIMN(1),PHIPNM(1),PHIPN(1)
      WRITE(6,468)
      DO 490 I=2,NCPT
      IF(I .EQ. NCPT) GO TO 485
      WRITE(6,465) CM(I),C(I),PHIMNM(I),PHIMN(I),PHIPNM(I),PHIPN(I)
      GO TO 490
  485 WRITE(6,453) PHIMNM(I),PHIMN(I),PHIPNM(I),PHIPN(I)
  490 CONTINUE
      RETURN
      END
C
C     ******************************
      SUBROUTINE FMSQ(I,C,PN,MN,IOPN)
C     ******************************
C
C...THIS SUBROUTINE CALCULATES THE NET LOADS AND THE NET MOMENTS
C   WHICH CONSTRUCT THE INTERACTION DIAGRAM
C
C   D      = DISTANCE FROM EXTREME COMP. FIBER TO THE CENTROID OF
C            TENSION REINFORCEMENT
C   DPRIME = H - D
C
C...ASS    = TOTAL AREA OF SIDE STEEL
C...ASE    = AREA OF ONE END STEEL
C...THICK1 = THICKNESS OF SIDE STEEL STRIP
C...THICK2 = THICKNESS OF UPPER AND LOWER STRIPS
C...RINF   = ANY NUMBER GREATER THAN H
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/MATPR/PERST,FC,FY,ES,EC
      COMMON/DIMEN/H,DG,WIDTH
      COMMON/CONST/BETA,EPSI,BT,KU
      COMMON/SIKNS/THICK1,THICK2,THICK
      COMMON/REINF/ASS,ASE,ASTT,DIAB,IBARS,IBARE,NBARS
C
      DIMENSION PN(30),C(30)
      REAL MN(30),KU,MS,MC,MR
      PI       = 3.141592654
      FR3      = 0.00
      FR4      = 0.00
      FR5      = 0.00
      FR6      = 0.00
      FR8      = 0.00
      FR9      = 0.00
      FR10     = 0.00
      R        = THICK/H
      EPSU     = 0.003
      EPSY     = FY/ES
      DPRIME   = H*(1.0 - DG)/2.0
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```
      IF(IOPN .EQ. 1) D  = H*(1.0 + DG)/2.0
      IF(IOPN .EQ. 2) D  = H - THICK/2.0
      RINF    = 10000.0
C...ALFA AND BTA ARE CONSTANTS
      BTA     = EPSY*KU/EPSU
      IF(IOPN - 1) 71,71,72
   71 ALFA    = (2.0*KU - 1.0 + DG)/2.0
      GO TO 73
   72 ALFA   = KU + R/2.0
   73 CONTINUE
      IF(IOPN .EQ. 1) GO TO 74
      IF(IOPN .EQ. 2) GO TO 75
   74 IF(C(I) .LE. RINF .AND. C(I) .GE. D) GO TO 80
      IF(C(I) .LE. H .AND. C(I) .GE. DPRIME) GO TO 70
      GO TO 120
   75 IF(C(I) .LE. RINF. AND. C(I) .GE. D) GO TO 80
      IF(C(I) .LE. H)                      GO TO 70
      GO TO 120
   70 CONTINUE
C
C...CHECK UPPER AND LOWER YIELD TO SPECIFY THE CASE NUMBER (1,2,3,OR5)
C
      ROH      = DG - ALFA
      Q        = ROH*EPSU*ES/(KU*FY)
      QP       = ALFA*EPSU*ES/(KU*FY)
      IF(Q   .LE. 1.0 .AND. QP .LE. 1.0) GO TO 50
      IF(Q   .GT. 1.0 .AND. QP .LE. 1.0) GO TO 10
      IF(Q   .LE. 1.0 .AND. QP .GT. 1.0) GO TO 30
      IF(Q   .GT. 1.0 .AND. QP .GT. 1.0) GO TO 20
   80 CONTINUE
      IF(IOPN .EQ. 1) GO TO 81
      IF(IOPN .EQ. 2) GO TO 82
   81 QP       = ALFA*EPSU*ES/(KU*FY)
      GO TO 83
   82 QP       = (ALFA + DG)*EPSU*ES/(KU*FY)
   83 CONTINUE
C
C...CHECK YIELD TO SPECIFY THE CASE NUMBER (4 OR 6)
C
      IF(QP   .GT. 1.0) GO TO 40
      IF(QP   .LE. 1.0) GO TO 60
   10 CONTINUE
C
C... THE FOLLOWING SIX CASES CALCULATE THE NET LOADS AND THE NET MOM
C    MOMENTS FOR DIFFERENT SIX ARBITRARY LOCATIONS FOR THE N.A.
C
C
C                    **************
C....                C A S E  O N E
C                    **************
C
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```
C... THE LOCATION OF THE N.A. CAUSES TENSION REINFORCEMENT REACHES
C    ITS YIELD STRENGTH WHILE COMP. REINFORCEMENT DOES NOT.
C
      IF(IOPN .EQ. 1) GO TO 11
      IF(IOPN .EQ. 2) GO TO 12
   11 ALFA     = (2.0*KU - 1.0 + DG)/2.0
      BTA      = EPSY*KU/EPSU
      PSI      = (1.0 - 2.0*KU)/2.0
      GAMA     = DG - BTA - ALFA
      GO TO 13
   12 ALFA     = KU + R/2.0
      BTA      = KU*EPSY/EPSU
      PSI      = DG/2.0 - ALFA
      GAMA     = DG - ALFA - BTA
   13 CONTINUE
C
C...FORCES IN THE SIDE STEEL
C
      FSP      = FY*ALFA/BTA
      FS1      = FSP/2.0*2.0*THICK1*ALFA*H
      FS2      = FY/2.0 *2.0*THICK1*BTA *H
      FS3      = FY *THICK1*GAMA*H*2.0
C
C...FORCES IN END STEEL
C
      FS5      = ASE*FSP
      FS6      = ASE*FY
C
C...FORCE IN CONCRETE
C
      CC        = 0.85*FC*BETA*KU*H*WIDTH
C
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
      IF(IOPN .EQ. 2) GO TO 14
      FR4      = 0.85*FC*ALFA*THICK1*H*2.0
      FR5      = 0.85*FC*ASE
      FR10     = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   14 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)    = FS1 - FS2 - FS3 + FS5 - FS6 + CC - FR4 - FR5 + FR10
C
C...ARMS OF FORCES
C
      YS1      = H*(PSI + 2.0*ALFA/3.0)
      YS2      = H*(2.0*BTA/3.0 - PSI)
      YS3      = H*(GAMA/2.0 + BTA - PSI)
      YS5      = DG*H/2.0
      YS6      = YS5
```

```
      YC       = H/2.0 - BETA*KU*H/2.0
      YR4      = H*(ALFA/2.0 + PSI)
      YR5      = YS5
      YR10     = H*(1.0 - KU*(1.0 + BETA))/2.0
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS       = FS1*YS1 + FS2*YS2 + FS3*YS3 + FS5*YS5 + FS6*YS6
      MC       = CC*YC
C
C...CORRECTION MOMENT DUE TO DISPLACED CONCRETE
C
      MR       = FR4*YR4 + FR5*YR5 - FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)  = MS + MC - MR
      GO TO 110
   20 CONTINUE
C
C
C                    ***************
C....                C A S E  T W O
C                    ***************
C
C...THE LOCATION OF THE N.A. CAUSES BOTH TENSION AND COMP. REINFORCEMEN
C   REACH THEIR YIELD STRENGTH
C
      IF(IOPN .EQ. 1) PSI = (1.0 - 2.0*KU)/2.0
      IF(IOPN .EQ. 2) PSI = (DG/2.0 - KU - R/2.0)
      RMU      = DG/2.0 - BTA + PSI
      RLMDA    = DG/2.0 - BTA - PSI
C
C...FORCES IN THE SIDE STEEL
C
      FS1      = 2.0*THICK1*RLMDA*H*FY
      FS2      = 2.0*THICK1*BTA  *H*FY/2.0
      FS3      = 2.0*THICK1*BTA  *H*FY/2.0
      FS5      = 2.0*THICK1*RMU  *H*FY
C
C...FORCES IN END STEEL
C
      FS7      = ASE*FY
      FS8      = ASE*FY
C
C...FORCE IN CONCRETE
C
      CC        = 0.85*FC*BETA*KU*H*WIDTH
C
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
```

```
      IF(IOPN .EQ. 2) GO TO 21
      FR4      = 2.0*THICK1*RLMDA*H*0.85*FC
      FR6      = 2.0*THICK1*BTA  *H*0.85*FC
      FR9      = 0.85*FC*ASE
      FR10     = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   21 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)    = FS1 + FS2 - FS3 - FS5 + FS7 - FS8 + CC
     .         - FR4 - FR6 - FR9 + FR10
C
C...ARMS OF FORCES
C
      YS1      = (PSI + BTA + RLMDA/2.0)*H
      YS2      = (2.0*BTA/3.0 + PSI)*H
      YS3      = (2.0*BTA/3.0 - PSI)*H
      YS5      = (RMU/2.0 + BTA - PSI)*H
      YS7      = DG*H/2.0
      YS8      = DG*H/2.0
      YC       = H/2.0 - BETA*KU*H/2.0
      YR4      = YS1
      YR6      = (BTA/2.0 + PSI)*H
      YR9      = YS7
      YR10     = H*(1.0 - KU*(1.0 + BETA))/2.0
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS       = FS1*YS1 + FS2*YS2 + FS3*YS3 + FS5*YS5
     .         + FS7*YS7 + FS8*YS8
      MC       = CC*YC
C
C...CORRECTION MOMENT DUE TO DISPLACED CONCRETE
C
      MR       = FR4*YR4 + FR6*YR6 + FR9*YR9 - FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)    = MS + MC - MR
      GO TO 110
C
   30 CONTINUE
C
C
C                     ******************
C....                 C A S E  T H R E E
C                     ******************
C
C... THE LOCATION OF THE N.A. CAUSES COMP. REINFORCEMENT REACHES
C    ITS YIELD STRENGTH WHILE TENSION REINFORCEMENT DOES NOT.
C
```

```
C...FORCES IN THE SIDE STEEL
C
      D        = H*(1.0 + DG)/2.0
      IF(IOPN .EQ. 1) PHI = D/H - KU
      IF(IOPN .EQ. 2) PHI = DG - KU - R/2.0
      OMEGA    = PHI + BTA - DG/2.0
      DELTA    = DG/2.0 - OMEGA
      FS       = FY*PHI/BTA
C
C...FORCES IN THE SIDE STEEL
C
      FS1      = 2.0*THICK1*DELTA*H*FY
      FS2      = 2.0*THICK1*FY*BTA*H/2.0
      FS3      = 2.0*THICK1*PHI*H*PHI/BTA*FY
C
C...FORCES IN END STEEL
C
      FS6      = ASE*FY
      FS7      = ASE*FY*PHI/BTA
C
C...FORCE IN CONCRETE
C
      CC        = 0.85*FC*BETA*KU*H*WIDTH
C
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
      IF(IOPN .EQ. 2) GO TO 31
      FR4      = 2.0*THICK1*DELTA*H*0.85*FC
      FR5      = 0.85*FC*BTA*H*2.0*THICK1
      FR8      = ASE*0.85*FC
      FR10     = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   31 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)    = FS1 + FS2 - FS3 + FS6 - FS7 + CC - FR4 - FR5 - FR8
     .         + FR10
C
C...ARMS OF FORCES
C
      YS1      = (OMEGA + DELTA/2.0)*H
      YS2      = (OMEGA - BTA/3.0)*H
      YS3      = (2.0*PHI/3.0 + BTA - OMEGA)*H
      YS5      = (OMEGA - BTA/2.0)*H
      YS6      = DG*H/2.0
      YS7      = YS6
      YC       = H/2.0 - BETA*KU*H/2.0
      YR4      = YS1
      YR5      = YS5
      YR8      = YS6
      YR10     = H*(1.0 - KU*(1.0 + BETA))/2.0
```

```
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS      = FS1*YS1 + FS2*YS2 + FS3*YS3 + FS6*YS6 + FS7*YS7
      MC      = CC*YC
      MR      = FR4*YR4 + FR5*YR5 + FR8*YR8 - FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)   = MS + MC - MR
      GO TO 110
   40 CONTINUE
C
C
C                    ****************
C....                C A S E  F O U R
C                    ****************
C
C
C...THE LOCATION OF THE N.A. CAUSES COMP. IN BOTH REINFORCEMENT
C   (COMP. REINFORCEMENT REACHES YIELD WHILE TENSION DOES NOT)
      D       = H*(1.0 + DG)/2.0
      IF(IOPN .EQ. 1) GO TO 41
      IF(IOPN .EQ. 2) GO TO 42
   41 ETA     = KU - (1.0 + DG)/2.0
      ROH     = KU - 0.50 - BTA
      GO TO 43
   42 ETA     = KU + R/2.0 - DG
      ROH     = KU + R - 0.50 - BTA
   43 CONTINUE
      ZETA    = BTA - ETA
      SETA    = DG/2.0 + ROH
      FS      = FY*ETA/BTA
C
C...FORCES IN THE SIDE STEEL
C
      FS1     = 2.0*THICK1*SETA*H*FY
      FS2     = ZETA*FY/(2.0*BTA)*2.0*THICK1*(BTA - ETA)*H
      FS3     = ETA/BTA*FY*ZETA*H*2.0*THICK1
C
C...FORCES IN END STEEL
C
      FS6     = ASE*FY
      FS7     = ASE*ETA/BTA*FY
C
C...FORCE IN CONCRETE
C
      A       = BETA*KU
      IF(A .GE. 1.0)  A=1.0
      CC        = 0.85*FC*A*H*WIDTH
C
```

```
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
      IF(IOPN .EQ. 2) GO TO 44
      FR4      = 2.0*THICK1*SETA*H*0.85*FC
      FR5      = 0.85*FC*ZETA*H*2.0*THICK1
      FR8      = ASE*0.85*FC
      FR9      = ASE*0.85*FC
      IF(BETA*C(I) .GE. D) GO TO 44
      FR10     = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   44 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)    = FS1 + FS2 + FS3 + FS6 + FS7 + CC - FR4 - FR5 - FR8
     .           - FR9 + FR10
C
C...ARMS OF FORCES
C
      YS1      = (SETA/2.0 -ROH)*H
      YS2      = (ROH + ZETA/3.0)*H
      YS3      = (ROH + ZETA/2.0)*H
      YS6      = DG*H/2.0
      YS7      = YS6
      YC       = H/2.0 - A*H/2.0
      YR4      = YS1
      YR5      = YS3
      YR8      = YS6
      YR9      = YS7
      YR10     = H*(1.0 - KU*(1.0 + BETA))/2.0
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS       = FS1*YS1 + FS6*YS6 - FS2*YS2 - FS3*YS3 - FS7*YS7
      MC       = CC *YC
      MR       = FR4*YR4 - FR5*YR5 + FR8*YR8 - FR9*YR9 + FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)  = MS + MC - MR
      IF(MN(I) .LT. 0.00)  MN(I) = 0.00
      GO TO 110
C
C
   50 CONTINUE
C
C
C                    ****************
C....                C A S E  F I V E
C                    ****************
C
C...THE LOCATION OF THE N.A. DOES NOT CAUSE YIELD IN THE REINFORCEMENT
```

```
C   IN BOTH SIDES(ONE SIDE IS UNDER TENSION AND THE OTHER UNDER COMP.)
C
      IF(IOPN .EQ. 1) GO TO 51
      IF(IOPN .EQ. 2) GO TO 52
   51 ALFA    = KU - ((1.0 - DG)/2.0)
      RLMDA   = (1.0 - 2.0*KU)/2.0
      GO TO 53
   52 ALFA    = KU + R/2.0
      RLMDA   = 0.50 - KU - R
   53 CONTINUE
      ROH     = DG - ALFA
      Q       = ROH*EPSU*ES/(KU*FY)
      QP      = ((2.0*KU - 1.0 + DG)/2.0)*(EPSU*ES/(KU*FY))
      FSP     = FY*QP
      FS      = FY*Q
C
C...FORCES IN THE SIDE STEEL
C
      FS1     = THICK1*H*QP*FY*ALFA
      FS2     = THICK1*H*Q *FY*ROH
C
C...FORCES IN END STEEL
C
      FS4     = QP*FY*ASE
      FS5     = Q *FY*ASE
C
C...FORCE IN CONCRETE
C
      CC       = 0.85*FC*BETA*KU*H*WIDTH
C
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
      IF(IOPN .EQ. 2) GO TO 54
      FR3     = 0.85*FC*2.0*THICK1*ALFA*H
      FR6     = 0.85*FC*ASE
      FR10    = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   54 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)   = FS1 - FS2 + FS4 - FS5 + CC - FR3 - FR6 + FR10
C
C...ARMS OF FORCES
C
      YS1     = (2.0*ALFA/3.0 + RLMDA)*H
      YS2     = (2.0*ROH/3.0 - RLMDA)*H
      YS4     = DG*H/2.0
      YS5     = YS4
      YC      = H/2.0 - BETA*KU*H/2.0
      YR3     = (ALFA/2.0 + RLMDA)*H
      YR6     = YS4
```

```
      YR10     = H*(1.0 - KU*(1.0 + BETA))/2.0
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS       = FS1*YS1 + FS2*YS2 + FS4*YS4 + FS5*YS5
      MC       = CC*YC
      MR       = FR3*YR3 + FR6*YR6 - FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)  = MS + MC - MR
      GO TO 110
   60 CONTINUE
C
C
C                      **************
C....                  C A S E  S I X
C                      **************
C
C
C...THE LOCATION OF THE N.A. DOES NOT CAUSE YIELD IN THE REINFORCEMENT
C   IN BOTH SIDES(BOTH SIDES UNDER COMP.)
C
      IF(IOPN .EQ. 1) GO TO 61
      IF(IOPN .EQ. 2) GO TO 62
   61 ALFA     = KU - (1.0 + DG)/2.0
      Q        = ALFA*EPSU*ES/(KU*FY)
      QP       = ((2.0*KU - 1.0 + DG)/2.0)*EPSU*ES/(KU*FY)
      GO TO 63
   62 ALFA     = KU + R/2.0 - DG
      Q        = ALFA*EPSU*ES/(KU*FY)
      QP       = (2.0*KU + R)*EPSU*ES/(2.0*KU*FY)
   63 CONTINUE
      FS       = FY*Q
      FSP      = FY*QP
C
C...FORCES IN THE SIDE STEEL
C
      FS1      = FS*DG*H*THICK1*2.0
      FS2      = (FSP - FS)/2.0*DG*H*2.0*THICK1
C
C...FORCES IN END STEEL
C
      FS3      = FSP*ASE
      FS4      = FS *ASE
C
C...FORCE IN CONCRETE
C
      A        = BETA*KU
      IF(A .GE. 1.0)  A=1.0
      CC        = 0.85*FC*A*HWIDTH
```

```
C
C...CORRECTIONS DUE TO DISPLACED CONCRETE
C
      IF(IOPN .EQ. 2) GO TO 64
      FR5     = 0.85*FC*DG*H*THICK1*2.0
      FR6     = 0.85*FC*ASE
      FR7     = 0.85*FC*ASE
      IF(BETA*C(I) .GE. D) GO TO 64
      FR10    = 2.0*THICK1*(KU*H - BETA*KU*H)*0.85*FC
   64 CONTINUE
C
C...TOTAL NET LOAD
C
      PN(I)   = FS1 + FS2 + FS3 + FS4 + CC - FR5 - FR6 - FR7 + FR10
C
C...ARMS OF FORCES
C
      YS1     = 0.00
      YS2     = DG*H/6.0
      YS3     = DG*H/2.0
      YS4     = YS3
      YC      = H/2.0 - A*H/2.0
      YR5     = 0.00
      YR6     = YS2
      YR7     = YS3
      YR10    = H*(1.0 - KU*(1.0 + BETA))/2.0
C
C...MOMENTS IN STEEL AND CONCRETE
C
      MS      = FS1*YS1 + FS2*YS2 + FS3*YS3 - FS4*YS4
      MC      = CC * YC
C
C...CORRECTION MOMENT DUE TO DISPLACED CONCRETE
C
      MR      = FR5*YR5 + FR6*YR6 - FR7*YR7 - FR10*YR10
C
C...TOTAL NET MOMENT
C
      MN(I)   = MS + MC - MR
      IF(MN(I) .LT. 0.00)  MN(I) = 0.00
C
  110 GO TO 140
  120 WRITE(6,130)
  130 FORMAT(5X,'SOMTHING WRONG YA MOHAMMAD')
  140 CONTINUE
      RETURN
      END
C
C     *******************************
      SUBROUTINE FMCR(I,C,PN,MN,IOPN)
C     *******************************
```

```
C
C...THIS SUBROUTINE CALCULATES THE TOTAL FORCE AND MOMENT
C...IN A REINFORCED CIRCULAR CONCRETE SECTION
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/MATPR/PERST,FC,FY,ES,EC
      COMMON/DIMEN/H,DG,WIDTH
      COMMON/CONST/BETA,EPSI,BT,KU
      COMMON/SIKNS/THICK1,THICK2,THICK
C
      REAL KU,MS,MC,MR,MN(30)
      REAL MS1,MS2,MS3,MS4
      DIMENSION PN(30),C(30)
      PI    = 3.1415926
      EPSY  = FY/ES
      EPSU  = 0.003
      EPSI  = EPSY/EPSU
      R     = THICK/H
      HK    = H*KU
      H2    = H/2.0
      FR    = 0.00
      MR    = 0.00
C
C...CALCULATIONS OF THE VALUES OF SETAS (S1 = SETA1,..ITC)
C
      IF(IOPN .EQ. 1) Q1  = (1.0 - 2.0*KU)/DG
      IF(IOPN .EQ. 2) Q1  = (1.0 - 2.0*R - 2.0*KU)/DG
      IF(DABS(Q1) .GT. 1.0. AND . HK .GT. H2) S1  = PI
      IF(DABS(Q1) .GT. 1.0. AND . HK .LT. H2) S1  = 0.00
      IF(DABS(Q1) .LT. 1.0) S1  = DACOS(Q1)
      IF(IOPN .EQ. 1) Q2  = (1.0 - 2.0*BETA*KU)
      IF(IOPN .EQ. 2) Q2  = (1.0 - 2.0*R - 2.0*BETA*KU)/(H - 2.0*R)
      IF(DABS(Q2) .GT. 1.0. AND . HK .GT. H2) S2  = PI
      IF(DABS(Q2) .GT. 1.0. AND . HK .LT. H2) S2  = 0.00
      IF(DABS(Q2) .LT. 1.0) S2  = DACOS(Q2)
      IF(IOPN .EQ. 1) Q3  = (1.0 - 2.0*KU*(1.0 - EPSI))/DG
      IF(IOPN .EQ. 2) Q3  = (1.0 - 2.0*KU*(1.0 - EPSI) - 2.0*R)/DG
      IF(DABS(Q3) .GT. 1.0. AND . HK .GT. H2) S3  = PI
      IF(DABS(Q3) .GT. 1.0. AND . HK .LT. H2) S3  = 0.00
      IF(DABS(Q3) .LT. 1.0) S3  = DACOS(Q3)
      IF(IOPN .EQ. 1) Q4  = (1.0 - 2.0*KU*(1.0 + EPSI))/DG
      IF(IOPN .EQ. 2) Q4  = (1.0 - 2.0*R - 2.0*KU*(1.0 + EPSI))/DG
      IF(DABS(Q4) .GT. 1.0. AND . HK .GT. H2) S4  = PI
      IF(DABS(Q4) .GT. 1.0. AND . HK .LT. H2) S4  = 0.00
      IF(DABS(Q4) .LT. 1.0) S4  = DACOS(Q4)
      Q5    = (1.0 - 2.0*BETA*KU)/DG
      IF(DABS(Q5) .GT. 1.0. AND . HK .GT. H2) S5  = PI
      IF(DABS(Q5) .GT. 1.0. AND . HK .LT. H2) S5  = 0.00
      IF(DABS(Q5) .LT. 1.0) S5  = DACOS(Q5)
C
```

```
C...CALCULATE THE NET FORCE IN THE STEEL RING(FS)
C
      CONST1 = PERST*BT*H*H/(2.0*PI)
      CONST2 = DG*FY/(2.0*KU*EPSI)
      CONST3 = FY*THICK*DG*DG*H/(4.0*EPSI*KU)
      IF(IOPN .EQ. 1) GO TO 10
      IF(IOPN .EQ .2) GO TO 20
C
C...CASE OF REINFORCED SECTION
C
   10 FS1    = CONST1*FY*S3
      FS2    = CONST2*(DSIN(S1)-DSIN(S3) - DCOS(S1)*(S1-S3))*CONST1
      FS3    = CONST2*(DSIN(S4)-DSIN(S1) - DCOS(S1)*(S4-S1))*CONST1
      FS4    = CONST1*FY*(PI-S4)
      GO TO 30
C
C...CASE OF FILLED SECTION
C
   20 FS1    = FY*THICK*DG*H*S3/2.0
      FS2    = CONST3*(DSIN(S1)-DSIN(S3) - DCOS(S1)*(S1-S3))
      FS3    = CONST3*(DSIN(S4)-DSIN(S1) - DCOS(S1)*(S4-S1))
      FS4    = FY*THICK*DG*H*(PI-S4)/2.0
C
   30 FS     = 2.0*(FS1 + FS2 - FS3 - FS4)
C
C...CALCULATE THE NET MOMENT IN THE STEEL RING(MS)
C
      CONST4 = FY*PERST*BT*DG*H**3/(4.0*PI)
      CONST5 = CONST4*DG/(2.0*KU*EPSI)
      CONST6 = FY*THICK*DG*DG*H*H/4.0
      CONST7 = CONST6*DG/(2.0*EPSI*KU)
      IF(IOPN .EQ. 1) GO TO 40
      IF(IOPN .EQ. 2) GO TO 50
C
C...CASE OF REINFORCED SECTION
C
   40 MS1    = CONST4*DSIN(S3)
      MS2    = CONST5*((S1-S3)/2.0 - (DSIN(2.0*S1)-DSIN(2.0*S3))/4.0
     .         - DCOS(S1)*(DSIN(S1)-DSIN(S3)))
      MS3    = CONST5*((S4-S1)/2.0 - (DSIN(2.0*S4)-DSIN(2.0*S1))/4.0
     .         - DCOS(S1)*(DSIN(S4)-DSIN(S1)))
      MS4    =-CONST4*DSIN(S4)
      GO TO 60
C
C...CASE OF FILLED SECTION
C
   50 MS1    = CONST6*DSIN(S3)
      MS2    = CONST7*((S1-S3)/2.0 + (DSIN(2.0*S1)-DSIN(2.0*S3))/4.0
     .         - DCOS(S1)*(DSIN(S1)-DSIN(S3)))
      MS3    = CONST7*((S4-S1)/2.0 + (DSIN(2.0*S4)-DSIN(2.0*S1))/4.0
     .         - DCOS(S1)*(DSIN(S4) - DSIN(S1)))
```

```
      MS4     =-CONST6*DSIN(S4)
   60 MS      = 2.0*(MS1 + MS2 + MS3 + MS4)
C
C...CALCULATE THE NET FORCE IN CONCRETE
C
      IF(IOPN.EQ.1) GO TO 70
      IF(IOPN.EQ.2) GO TO 80
C
C...CASE OF REINFORCED SECTION
C
   70 PC      = 0.2125*FC*H*H*(S2 - DSIN(S2)*DCOS(S2))
      GO TO 90
C
C...CASE OF FILLED SECTION
C
   80 PC      = 0.85*FC*(H/2.0 - THICK)**2*(S2 - DSIN(S2)*DCOS(S2))
C
C...CALCULATE THE NET MOMENT IN CONCRETE
C
   90 CONTINUE
      IF(IOPN .EQ. 1) GO TO 100
      IF(IOPN .EQ. 2) GO TO 110
C
C...CASE OF REINFORCED SECTION
C
  100 MC      = 0.85*FC*(H*DSIN(S2))**3/12.0
      GO TO 120
C
C...CASE OF FILLED SECTION
C
  110 MC      = 1.7*FC*(H/2.0 - THICK)**3.0*DSIN(S2)**3/3.0
  120 IF(IOPN .EQ. 2) GO TO 130
C
C...CALCULATE THE CORRECTION FORCE DUE TO DISPLACED CONCRETE(FR)
C   (R5 AND R6 ARE CONSTANTS)
      R5      = PERST*H*H*BT*S5/PI
      R6      = 0.85*FC
      FR      = R5*R6
C
C...CALCULATE THE CORRECTION MOMENT DUE TO DISPLACED CONCRETE(MR)
C   (R7 AND R8 ARE CONSTANTS)
      R7      = PERST*H**3*BT*DG*DSIN(S5)/PI
      R8      = 0.425*FC
      MR      = R7*R8
C
C...TOTAL NET LOAD AND NET MOMENT
C
  130 PN(I) = FS + PC - FR
      MN(I) = MS + MC - MR
      RETURN
      END
```

```
C
C     ******************************************
      SUBROUTINE MAXLD(PHIPN,PHIMN,ECC,PC,C,LOADF,PHI,NCPT)
C     ******************************************
C
C...SPECIAL CASE OF MINIMUM ECCENTRICITY WHICH OCCURES ON
C     INTERACTION DIAGRAM
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PHIPN(50),PHIMN(50),XP(50),YP(50),CF(10),C(50)
      REAL LOADF,MU,MC,MMN,MUM
C
C...ASSIGN COORDINATES TO ARRAYS XP AND YP.
C
      JCOUNT = 1
      YP(1)  = PHIMN(1)
      XP(1)  = PHIPN(1)
C
C...OBTAIN POINTS (TO BE FITTED) ABOVE THE BALANCE POINT
C
      DO 10 I=2,NCPT
      IF(C(I) .LT. C(1))  GO TO 10
      DIF1  = PHIPN(I) - PHIPN(I-1)
      DIF2  = PHIMN(I) - PHIMN(I-1)
      IF(DIF1 .EQ. 0.00 .AND. DIF2 .EQ. 0.00) GO TO 10
      JCOUNT = JCOUNT + 1
      XP(JCOUNT) = PHIPN(I)
      YP(JCOUNT) = PHIMN(I)
   10 CONTINUE
C
C...NPFIT = NUMBER OF POINTS TO BE FITTED
C
      NPFIT      = JCOUNT + 1
      XP(NPFIT)  = PHIMN(NCPT)
      YP(NPFIT)  = PHIPN(NCPT)
C
C     FIT DATA POINTS TO A QUADRATIC EQUATION AND DETERMINE
C     THE COEFFICIENT OF THIS EQUATION.
C
      CALL FITPOL(XP,YP,NPFIT,CF)
C
C...SOLVE THE INTERACTION DIAGRAM EQUATION WITH THE ECC. EQUATION
C
      PPN =(ECC-CF(2)-DSQRT((CF(2)-ECC)**2-4*CF(3)*CF(1)))/(2.0*CF(3))
      PHIPC  = PHI*PC
      IF(PPN .GE. PHIPC)   PPN = PHIPC-1.0
      IRANGE  = INT(PPN-XP(1))
C
C...INDEX IS AN INDICATOR TO STOP ITERATION ONCE TEST LOAD IS OBTAINED
C
      INDEX   = 0
```

```
      DO 60 I=1,IRANGE
      IF(INDEX .EQ. 1) GO TO 60
      PPN      = PPN - 1.0
      DELTA    = PHIPC/(PHIPC - PPN)
      MC       = DELTA*ECC*PPN
      MMN      = CF(1) + CF(2)*PPN + CF(3)*PPN**2
      IF(MC .GT. MMN)  GO TO 60
      PU       = PPN
      MU       = MC
      TLOAD    = PU/LOADF
      INDEX    = 1
   60 CONTINUE
      PCM      = PC*4.445709
      WRITE(6,775)
  775 FORMAT(//,20X,'*** MAXIMUM ALLOWABLE LOAD CALCULATIONS ***',/
     .       20X,'*********************************************',//)
      WRITE(6,780) PCM,PC
  780 FORMAT(5X,'CRITICAL LOAD-PC........................ =',
     .       D15.5,' KN (',D15.5,' KIPS)')
      WRITE(6,785) DELTA
  785 FORMAT(5X,'MOMENT MAGNIFICATION-DELTA.............. =',D15.5)
      PUM = PU*4.445709
      MUM = MU*0.1129101
      WRITE(6,790) PUM,PU
  790 FORMAT(5X,'FACTORED AXIAL LOAD-PU.................. =',
     .       D15.5,' KN (',D15.5,' KIPS)')
      WRITE(6,795) MUM,MU
  795 FORMAT(5X,'LOAD OCCURES AT A MOMENT OF............. =',
     .       D15.5,' KNM(',D15.5,' K-IN)')
      TLOADM= TLOAD*4.445709
      WRITE(6,805) TLOADM,TLOAD
  805 FORMAT(/,5X,'TEST LOAD(MAX. ALLOWABLE LOAD).......... =',
     .       D15.5,' KN (',D15.5,' KIPS)')
      RETURN
      END
C     **********************************
      SUBROUTINE FITPOL(XP,YP,NPFIT,CF)
C     **********************************
C
C...THIS SUBROUTINE FITS DATA POINTS TO A QUADRATIC EQUATION USING
C   LEAST SQUARE METHOD. THE FITTED EQUATION IS :
C
C      PHIMN = CF(1) + CF(2)*PHIPN + CF(3)*PHIPN**2
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XP(50),A(50,10),XY(10,10)
      DIMENSION YP(50),YA(50),CF(10)
C...NUMC = NUMBER OF COEFFICIENT OF THE FITTED EQUATION
      NUMC   = 3
      DO 15 I=1,NPFIT
      X1 = XP(I)
```

```
      X2 = X1*X1
      A(I,1) = 1.00
      A(I,2) = X1
      A(I,3) = X2
   15 CONTINUE
      DO 20 I=1,NUMC
      DO 20 J=1,NUMC
      T1 = 0.00
      DO 25 K=1,NPFIT
   25 T1 = T1 + A(K,I)*A(K,J)
   20 XY(I,J) = T1
      DO 40 I=1,NUMC
      T2 = 0.00
      DO 45 K=1,NPFIT
   45 T2 = T2 + A(K,I)*YP(K)
   40 XY(I,NUMC+1) = T2
C
      CALL SOLV(XY,CF,NUMC)
C
C...CALCULATE THE FITTED VALUES OF THE VARIABLE
      DO 50 I=1,NPFIT
      T3 = 0.00
      DO 55 K=1, NUMC
   55 T3 = T3 +A(I,K)*CF(K)
   50 YA(I) = T3
C...CALCULATE R-SQUARD VALUE
      A1 = 0.00
      A2 = 0.00
      A3 = 0.00
      DO 60 K=1,NPFIT
      A1 = A1 + (YP(K)-YA(K))**2
      A2 = A2 + YP(K)
      A3 = A3 + YP(K)**2
   60 CONTINUE
      SSE = A1
      YBS = (A2/NPFIT)**2
      SST = A3 - NPFIT*YBS
      RSQD = 1.00 - SSE/SST
      WRITE(6,200)
      WRITE(6,201)
      WRITE(6,101) CF(1),CF(2),CF(3)
  101 FORMAT(//,5X,'THE EQUATION OF THE INTERACTION DIAGRAM CURVE :',
     .       //,5X,'PHIMN =',5X,F10.5,'+',5X,F10.5,'*PHIPN +',5X,F10.5,
     .             '*PHIPN**2',//)
  200 FORMAT(1H1)
  201 FORMAT(//)
      RETURN
      END
C     ***********************
      SUBROUTINE SOLV(A,DX,N)
C     ***********************
```

```
C
C...THIS SUBROUTINE IS USED FOR SOLVING SYSTEM OF EQUATIONS :
C         ¢ A !*(X) = (B)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(10,10),DX(10)
      M = N+1
      DO 21 I=2,N
      DO 21 J=I,N
      RA = A(J,I-1)/A(I-1,I-1)
      DO 31 K=1,M
      A(J,K) = A(J,K)-RA*A(I-1,K)
   31 CONTINUE
   21 CONTINUE
      DO 41 I=2,N
      K=N-I+2
      RA = A(K,M)/A(K,K)
      DO 51 J=I,N
      L = N-J+1
   51 A(L,M) = A(L,M)-RA*A(L,K)
   41 CONTINUE
      DO 61 I=1,N
   61 DX(I) = A(I,M)/A(I,I)
      RETURN
      END
C     ***************************
      SUBROUTINE FLDCN(ISEC,K,KL)
C     ***************************
C
C     FLDCN = FILLED CONCRETE
C     THIS SUBROUTINE FILLED CONCRETE (FLDCN),CALCULATES THE COMPRESSIV
C     RESISTANCE OF CONCRETE-FILLED COLUMNS ACCORDING TO THE CANADIAN
C     STANDARDS FOR SQUARE,RECTANGULAR AND CYRCULAR COLUMNS.
C
C     RCON = RADIOUS OF CONCRETE(RC)
C     RSTL = OUTSIDE RADIOUS OF STEEL(RS)
C     ACON = AREA OF CONCRETE(AC)
C     ASTL = AREA OF STEEL(AS)
C     ICON = MOMENT OF INERTIA OF CONCRETE PART (IC)
C     ISTL = MOMENT OF INERTIA OF STEEL PART (IS)
C     RDGC = RADIOUS OF GYRATION OF CONCRETE (SMALL R SUBSCRIPT C)
C     RDGS = RADIOUS OF GYRATION OF STEEL (SMAAL R SUBSCRIPT S)
C     HCON = THICKNESS OF CONCRETE IN SQUARE OR RECTANGULAR SECTION
C     WCON = WIDTH OF CONCRETE IN SQUARE OR RECTANGULAR SECTION
C     SRTC = SLENDERNESS RATIO OF CONCRETE (KL/SMALL R SUB. C)
C     SRTS = SLENDERNESS RATIO OF STEEL (KL/SMALL R SUB. S)
C     LAMC = (LAMDA C) NONDIMENSIONAL SLENDERNESS RATIO IN COLUMN FORM.
C     LAMS = (LAMDA FOR STEEL) NONDIMENSIONAL SLENDERNESS RATIO IN
C            COLUMN FORMULA
C     FCRC = (C PRIME SUBSCRIPT SMAAL R) COMPRESSIVE RESISTANCE OF
C            CONCRETE ACTING AT THE CENTROID OF THE CONCRETE AREA
```

```
C             IN COMPRESSION
C      FCRS = (C SUBSCRIPT SMALL R) FACTORED COMPRESSIVE RESISTANCE OF
C             STEEL ACTING AT THE CENTROID OF THAT PART OF THE STEEL
C             AREA IN COMPRESSION
C       ROH = A CONSTANT = 0.02(25-L/D)
C      TUAC = TAW PRIME FOR CONCRETE , IS A CONSTANT
C      TUAS = TAW FOR STEEL , IS A CONSTANT
C       FCM = (TAW PRIME FOR CON.)(C PRIME SUBSCRIPT SMALL R)
C           = FACTORED COMPRESSIVE RESISTANCE OF CONCRETE ,MODIFIED
C       FSM = (TAW FOR CONCRETE)(C SUBSCRIPT SMALL R)
C           = FACTORED COMPRESSIVE RESISTANCE OF STEEL ,MODIFIED
C      FCMM = FCM IN KILO-NETWON
C      FSMM = FSM IN KILO-NETWON
C      FCTL = C SUBSCRIPT SMALL RC = COMPRESSIVE FORCE OF A COLUMN
C             UNDER SPECIFIED AXIAL LOAD
C       PHI = RESISTANCE FACTOR
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON/MATPR/PERST,FC,FY,ES,EC
      COMMON/DIMEN/H,DG,WIDTH
      COMMON/SIKNS/THICK1,THICK2,THICK
C
      REAL K, KL,ICON,ISTL,LAMC,LAMS
      PI    = 3.141592654
C
C...  PROPERTIES OF THE COLUMN CROSS-SECTION
C
      IF(ISEC .EQ. 2 )                  GO TO 10
      IF(ISEC .EQ. 1 .OR. ISEC .EQ. 3)  GO TO 20
   10 CONTINUE
C
C     CIRCULAR CROSS-SECTION
C
      RCON  = H/2.0 - THICK
      RSTL  = H/2.0
      ACON  = PI*RCON**2
      ASTL  = PI*(RSTL**2 - RCON**2)
      ICON  = PI*RCON**4/4.0
      ISTL  = PI*(RSTL**4 - RCON**4)/4.0
      RDGC  = DSQRT(ICON/ACON)
      RDGS  = DSQRT(ISTL/ASTL)
C
      GO TO 30
C
   20 CONTINUE
C
C     SQUARE OR RECTANGULAR CROSS-SECTION
C
      HCON  = H - 2.0*THICK
      WCON  = WIDTH - 2.0*THICK
```

```
      ACON  = HCON*WCON
      ASTL  = H*WIDTH - ACON
      ICON  = WCON*HCON**3/12
      ISTL  = WIDTH*H**3/12.0 - ICON
      RDGC  = DSQRT(ICON/ACON)
      RDGS  = DSQRT(ISTL/ASTL)
C
   30 CONTINUE
C
      SRTC  = KL/RDGC
      SRTS  = KL/RDGS
      LAMC  = SRTC*DSQRT(FC/(EC*PI**2))
      LAMS  = SRTS*DSQRT(FY/(ES*PI**2))
C
C...  FACTORED COMPRESSIVE RESISTANCE OF CONCRETE
C
      TEMP  = 1.0 + 0.25/LAMC**4.0
      TEMP1 = DSQRT(TEMP) - 0.50/LAMC**2
      FCRC  = 0.85*0.67*FC*ACON*TEMP1/LAMC**2
C
C...  FACTORED COMPRESSIVE RESISTANCE OF STEEL
C
      IF(LAMS .LE. 0.15) GO TO 40
      IF(LAMS .LE. 1.20) GO TO 50
      IF(LAMS .LE. 1.80) GO TO 60
      IF(LAMS .LE. 2.80) GO TO 70
      FCRS  = 0.90*ASTL*FY/LAMS**2
      GO TO 80
   40 FCRS  = 0.90*ASTL*FY
      GO TO 80
   50 FCRS  = 0.90*ASTL*FY*(0.99 + 0.122*LAMS - 0.367*LAMS**2)
      GO TO 80
   60 FCRS  = 0.90*ASTL*FY*(0.051 + 0.801/LAMS**2)
      GO TO 80
   70 FCRS  = 0.90*ASTL*FY*(0.008 + 0.942/LAMS**2)
C
   80 CONTINUE
C
C...  CALCULATION OF TAUC AND TAUS
C
      ROH   = 0.02*(25.0 - KL/H)
      IF((KL/H) .GE. 25.0)  ROH = 0.00
      TAUS  = 1.0/DSQRT(1.0 + ROH + ROH**2)
      TAUC  = 1.0 + 25.0*ROH**2*TAUS*FY/((H/THICK)*0.85*FC)
      FCM   = TAUC*FCRC
      FSM   = TAUS*FCRS
      FCMM  = FCM*4.445709
      FSMM  = FSM*4.445709
C
C...  TOTAL COMPRESSIVE RESISTANCE OF COLUMN
C
```

```
      FCTL  = FCM + FSM
      FCTLM = FCMM + FSMM
      WRITE(6,101)
  101 FORMAT(///)
      WRITE(6,102)
  102 FORMAT(//,20X,'*** MAXIMUM ALLOWABLE LOAD CALCULATIONS ***',/
     .         20X,'*******************************************',//)
      WRITE(6,100) FCMM,FCM,FSMM,FSM,FCTLM,FCTL
  100 FORMAT(5X,'FACTORED COMP. RESISTANCE OF CONCRETE.... =',D15.5
     .          ,'KN ( ',D15.5,' KIPS )'/
     .       5X,'FACTORED COMP. RESISTANCE OF STEEL....... =',D15.5
     .          ,'KN ( ',D15.5,' KIPS )',/
     .       5X,'FACTORED COMP. RESISTANCE OF COLUMN...... =',D15.5
     .          ,'KN ( ',D15.5,' KIPS )')
      RETURN
      END
```

APPENDIX D

DERIVATION OF PARABOLOIC UNSTEADY STATE

PARTIAL DIFFERENTIAL EQUATION FOR CONDUCTION

7.2 Heat Transferred by Conduction

When a temperature gradient exists in a column, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. We say that the energy transferred by conduction and that the heat-transfer rate per unit area is proportional to the normal temperature gradient [51, 52]

$$\frac{q}{A} \sim \frac{\partial T}{\partial x}$$

or $$q = -KA \frac{\partial T}{\partial x} \qquad \text{(D.1)}$$

where:

q = the heat transfer rate (Watt/sec)

$\frac{\partial T}{\partial x}$ = the temperature gradient in the direction of the heat flow (°c)

K = thermal conductivity of the material Watt/mc°

The minus sign is inserted so that the second principle of thermodynamics will be satisfied, i.e. heat must flow downhill on the temperature scale.

Equation (D.1) is called Fourier's law of heat conduction.

If we consider a one dimensional system shown in Figure (D.1). If the system is in a steady state, i.e. if the temperature does not change with time, then the problem is a simple one and we need only integrate

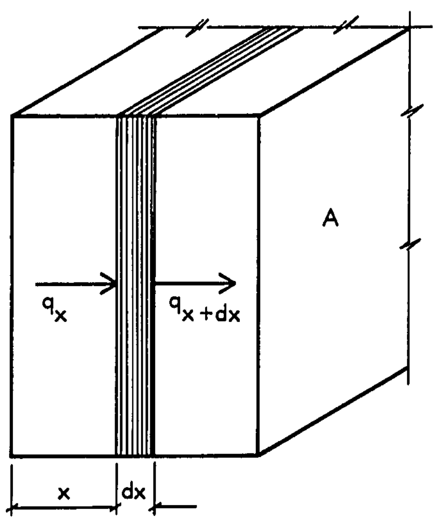

FIGURE D.1

ELEMENTAL VOLUME FOR ONE-DIMENSIONAL HEAT CONDUCTION ANALYSIS

Equation (D.1) and substitute the appropriate values to solve for the desired quantity. However if the temperature of the solid is changing with time, the situation is more complex. We consider the general case where the temperature may be changing with time and heat sources not present within the body, for element of thickness dx the following energy balance may be made (Figure D.1)

Energy in left face $= q_x = -KA \frac{\partial T}{\delta x}$ (D.2)

change in internal energy $= \frac{dE}{\delta t} \rho c(A.dx) \frac{\partial T}{\delta t}$ (D.3)

Energy out right face, q_{x+dx} can be obtained by expanding q_x in a Taylor series and retaining only the first two terms as a reasonable approximation:

$$q_{x+dx} = q_x + \frac{\partial}{\delta x} q_x \, dx \, x$$

$$= -KA \frac{\partial T}{\delta x} + \frac{\partial}{\delta x} \left(-KA \frac{\partial T}{\delta x}\right) dx$$

$$\therefore q_{x+dx} = -\left[K \frac{\partial T}{\delta x} + \frac{\partial}{\delta x} \left(K \frac{\partial T}{\delta x}\right) dx\right] A \qquad \text{(D.4)}$$

Combining Equations (D.2), (D.3) and (D.4) results:

i.e.: $$q_x - q_{x+dx} = \frac{dE}{d\tau}$$

$$-KA \frac{\partial T}{\delta x} - \left[KA \frac{\partial T}{\delta x} + \frac{\partial}{\delta x} \left(-KA \frac{\partial T}{\delta x}\right) dx\right] = \rho c(A.dx) \frac{\partial T}{\delta t}$$

$$-KA \frac{\partial T}{\delta x} + KA \frac{\partial T}{\delta x} + \frac{\partial}{\delta x} \left(KA \frac{\partial T}{\delta x}\right) dx = \rho c(A.dx) \frac{\partial T}{\delta t}$$

$$\frac{\partial}{\delta x}\left[K(A.dx)\frac{\partial T}{\delta x}\right] = \rho c(A.dx)\frac{\partial T}{\partial t}$$

$$\text{or} \quad \frac{\partial}{\delta x}\left(K\frac{\partial T}{\partial x}\right) = \rho c\frac{\partial T}{\partial t} \qquad \text{(D.5)}$$

Equation (D.5) is one-dimensional heat conduction equation. To treat more than one-dimensional heat flow we need only consider the heat conducted in and out of a unit volume in all three coordinate directions.

If we consider an infinitesimal control volume of dimensions δx, δy and δz which is oriented within a three-dimensional (x, y, z) coordinate system as in Figure (D.2). The considerations here will include the nonsteady condition of temperature variation with time t.

From Figure D.2, the energy balance yields [51,52],

$$q_x + q_y + q_z + q_{gen} = q_{x+sz} + q_{y+dy} + q_{z+dz} + \frac{\partial E}{\delta t}$$

where:

$$q_x = -K\, dydz\, \frac{\partial T}{\delta x}$$

The heat flow out of the right face of the volume element can be obtained by expanding q_x in a Taylor series and retaining only the first two terms as a reasonable approximation:

$$q_{x+dx} = q_x + \frac{\partial}{\delta x} q_x\, dx + \ldots$$

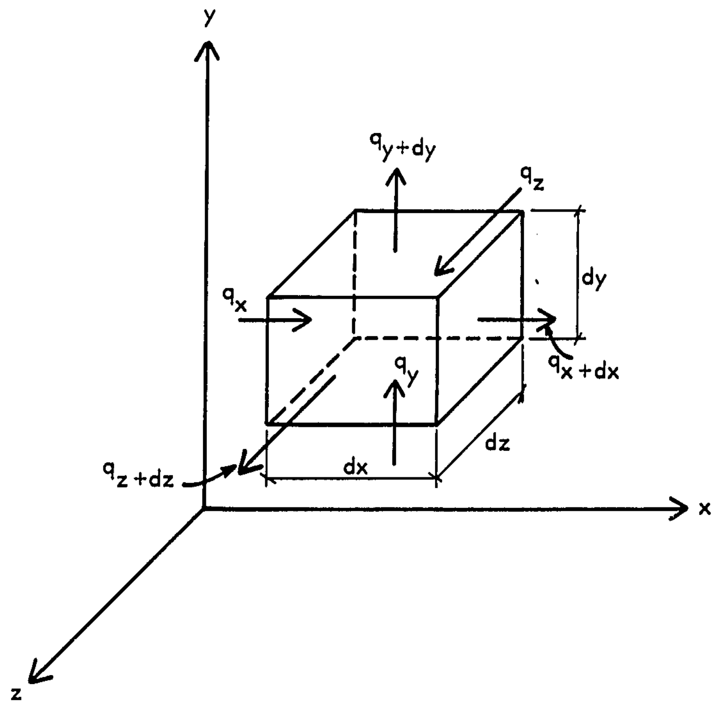

FIGURE D.2

THE VOLUME ELEMENT FOR DETERMINATION OF THE HEAT CONDUCTION EQUATION or $q_{x+dx} = \left(Kdydz \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left(-Kdydz \frac{\partial T}{\partial x}\right)dx$

$\therefore \quad q_{x+dx} = -\left[K\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dx\right]dydz$

Similarly:

$$q_y = -Kdxdz \frac{\partial T}{\partial y}$$

$$q_{y+dy} = -\left[K\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)dy\right]dxdz$$

$$q_z = -Kdxdy \frac{\partial T}{\partial z}$$

$$Q_{z+dz} = -\left[K\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right)dz\right]dxdy$$

If per unit of time and space the quantity of heat $\dot{q}\ (x,y,z,\tau)$ is generated, then the generation of heat in the volume element is:

$$q_{gen} = \dot{q}\ dxdydz$$

The net heat flow into the volume element due to conduction $[(q_x - q_{x+dx}) + (q_y - q_{y+dy}) + (q_z - q_{z+dz})]$ and the heat generated within the volume element $[q_{gen}]$ together serve to increase the internal energy of the volume element. Such an increase in the internal energy is reflected in the time rate of change in the energy storage in the volume element and can be written

$$\frac{dE}{dt} = \rho c\ dxdydz \frac{\partial T}{\partial t} \qquad (D.6)$$

Therefore an energy balance can be made on the volume element to equate the time rate of change of the energy stored to the sum of the net heat flow into the element due to conduction and the heat generated within the element, yield to the following three-dimensional heat-conduction equation [51, 52] can be obtained:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial t} \qquad (D.7)$$

where

ρc = thermal heat capacity (J/m^3-°c or Cd/cm^3-°c)

t = time

$\dot{q}$ = heat generated in the differential volume

x, y, z = Cartezion coordinates

The thermal conductivity [K] is defined as a measure of the energy transfer rate of the material, the higher the thermal conductivity, the greater the heat flow in a material.

The values of thermal conductivity [K] and thermal capacity [ρc] are known to vary with temperature in steel and concrete. Figures (D.3) and (D.4) show the temperature dependence [53] of thermal conductivity [K], and heat capacity [C].

i.e.:

$$K = K(x,y,z,t), \quad c = c(x,y,z,t)$$

So that Equation (D.6) is valid for isotropic, homogeneous media. If the heat generated internally within an element is equal to zero hence the final differential equation for three dimensional heat flow per unit volume:

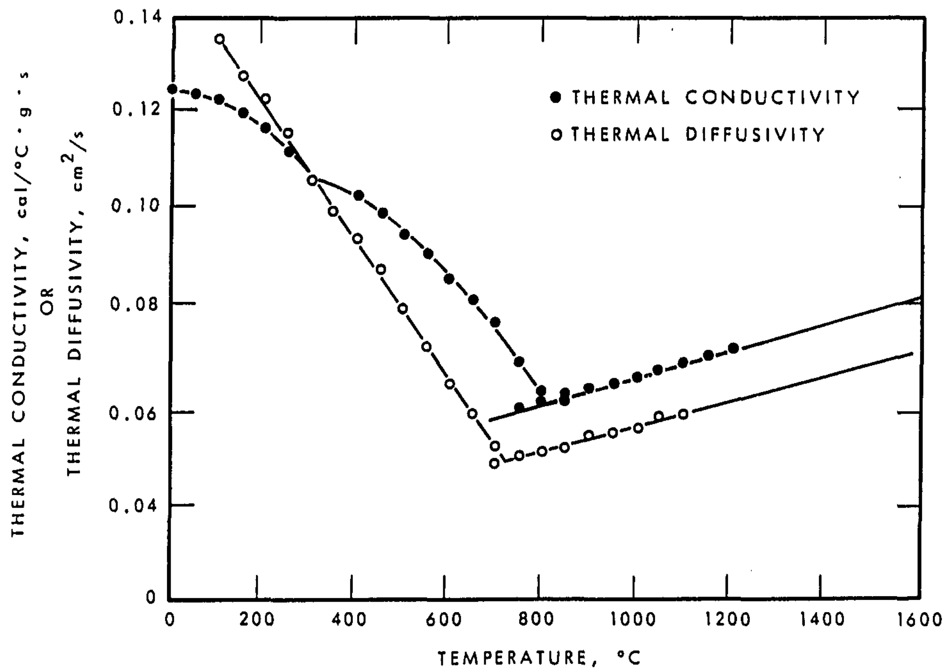

FIGURE D.3

VARIATION OF THERMAL CONDUCTIVITY AND THERMAL DIFFUSIVITY OF STEEL WITH TEMPERATURE [53]

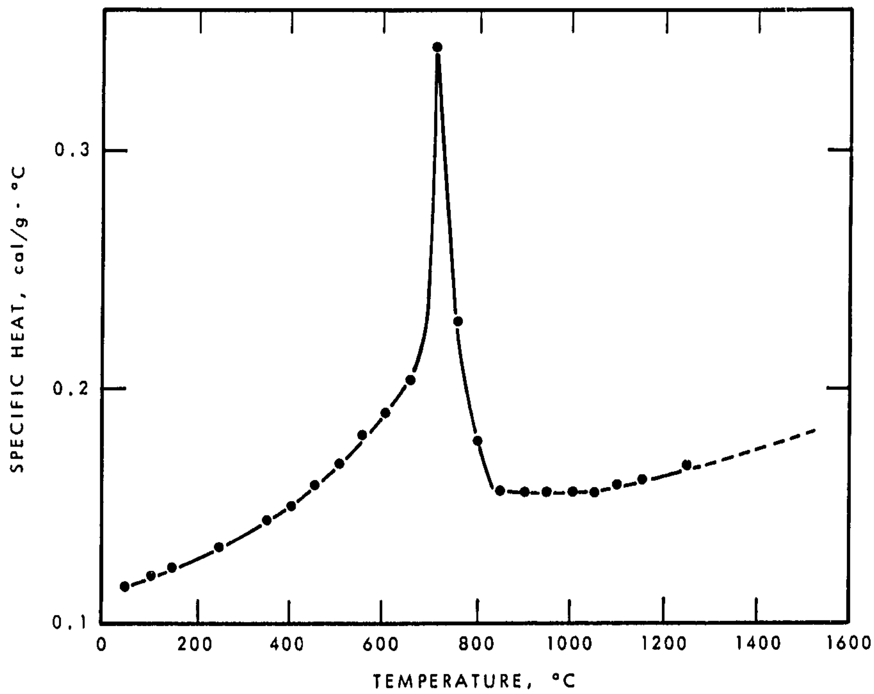

FIGURE D.4

THE VARIATION OF THE SPECIFIC HEAT OF STEEL WITH TEMPERATURE [53]

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) = \rho c\,\frac{\partial T}{\partial t} \qquad \text{(D.8)}$$

In building columns subjected to high temperatures, the thermal conductivity [K] is temperature dependent. In order to investigate the heat transfer and the stresses within a column, two-dimensional case of x-z coordinates will be considered. Therefore, the third dimension will be cancelled as indicated in Figure (D.5) and Equation (D.7) becomes [52, 53]

$$\frac{\partial}{\partial x}\left(K\,\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\,\frac{\partial T}{\partial z}\right) = \rho c\,\frac{\partial T}{\partial t} \qquad \text{(D.9)}$$

Equation (D.8) is the Parabolic Unsteady State Partial Differential Equation. This equation will be used to predict the temperature distribution for fire resistance of concrete-filled square steel columns as shown in Figure (D.5).

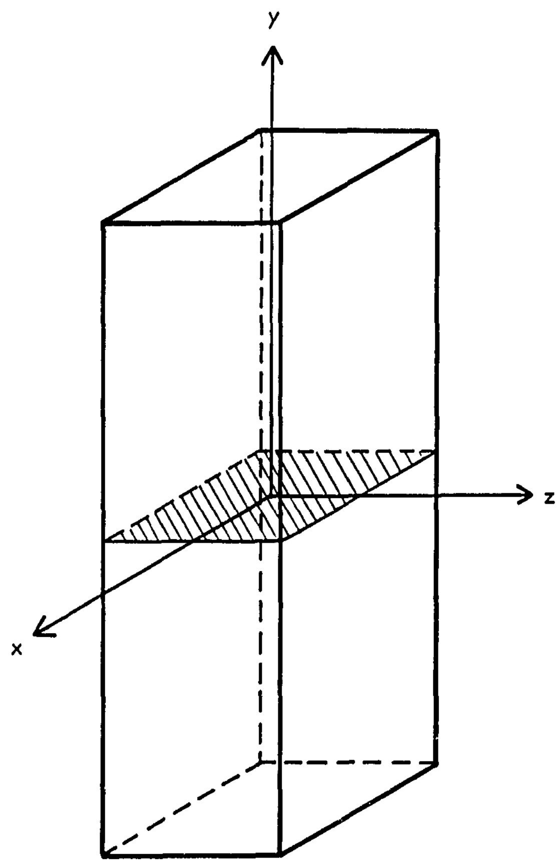

FIGURE D.5

COORDINATES OF COLUMN
CROSS-SECTION

APPENDIX E

```
C       THIS PROGRAM HAS BEEN DONE FOR THE NATIONAL RESEARCH
C       COUNCIL OF CANADA BY:
C       MOHAMED EL-SHAYEB ,PH.D. STUDENT ,UNIVERSITY OF NEWHAMPSHIRE,
C       DURHAM,NEWHAMPSHIRE,U.S.A.
C
C
C                    ________________________
C                    -**********************-
C                    -*====================*-
C                    -*=    MAIN PROGRAM   =*-
C                    -*====================*-
C                    -**********************-
C                    ________________________
C
C         FIRE RESISTANCE OF CONCRETE-FILLED SQUARE STEEL COLUMN
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB3/INDIC,NUMB,NUM,NNN,MMM,IND
      COMMON/PROPR/PHI,EMIS,EPTOT
      COMMON/AREAS/AEND1,AEND2,AEND3,AEND4
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/DIMEN/THICK,H,DHG,DHE
      COMMON/NUMB1/NI,MI,IK
      DIMENSION TJ1(135,135),TJ(135,135),TT(135,135)
      DIMENSION CAPC(135,135),CAPS(135,135),CONDC(135,135),
     .    CONDS(135,135)
      DIMENSION V(135,135),DV(135,135)
      DIMENSION EPCR(135,135),EPCL(135,135),EPSR(135,135),EPSL(135,135)
      DIMENSION FSR(135,135),FSL(135,135),FCR(135,135),FCL(135,135)
      DIMENSION Z(135,135),ASE(135,135),ACE(135,135)
      REAL MST,MCT,MT,KL
C
C
C     SYMPOLS DIFINITIONS
C
C     UNITS USED IN THIS PROGRAM ARE : JOULE,METER,HOUR,
C     DEGREE-CENTIGRADE AND NETON
C
C     THICK =THICKNESS OF STEEL WALL
C     H     =COLUMN WIDTH(WIDTH OF STEEL WALL + WIDTH OF CONCRETE)
C     FYSO  =YIELD STRENGTH OF STEEL AT ROOM TEMPERATURE
C     FDCO  =COMPRESSIVE STRENGTH OF CONCRETE AT ROOM TEMPERATURE
C     DX**2 =LENGTH OF TRIANGULAR DIVISION
C           =(SQRT(DHG+DHG))**2 = 2*DHG**2
C     ECO   = YOUNG'S MODULUS OF CONCRETE
C     MC1   =THE BENDING MOMENT IN CONCRETE
C     MS1   = THE BENDING MOMENT IN STEEL
C     EPST  =TOTAL STRAIN(EPSILON) IN STEEL
C     EPSH  =SHRINKAGE STRAIN IN CONCRETE
C     Z     =COORDINATE OF ELEMENTS
```

```
C      ASE  = AREA OF STEEL  ELEMENTS
C      ACE  = AREA OF CONCRETE  ELEMENTS
C      COMAX= MAX CONDUCTIVITY OF STEEL(KMAX)
C             UNITS ARE: (J/H.M.K  OR  J/H.M.C )
C      CAMIN= MIN THERMAL CAPACITY OF STEEL
C             UNITS ARE: (J/M3.K  OR  J/M3.C)
C      HMAX = MAX COEFFICIENT OF HEAT TRANSFERE
C             UNITS ARE: (J/M2.H.K  OR J/M2.H.C)
C
C      IWRITE = IS AN OPTION FOR WRITING THE TEMPERATURE OR NOT
C      IWRITE = 0  DO NOT WRITE TEMPETURES
C      IWRITE = 1  WRITE THE TEMPETURES
C
C      IEXIT  = IS AN OPTION FOR STOP OR CONTINUE TO EXECUTE OTHER DATA
C      IEXIT  = 0  TO STOP EXECUTION
C      IEXIT  = 1  TO CONTINUE EXECUTION OTHER DATA
C
C      EPCRC = CREEP STRAIN IN CONCRETE
C      EPTOT= THE SUMMITION OF CREEP STRAIN AND SHRINKAGE STRAIN
C              IN CONCRETE
C            = EPCRC + EPSH
C
C
  107 CONTINUE
C
C      READ STATMENTS
C
       READ(5,100) THICK,H,ECC,KL
       READ(5,100) TIMLIM,PERIOD,STTIME
       READ(5,100) EMIS,PHI,EPSH,EPCRC
       READ(5,100) COMAX,CAMIN,HMAX
       READ(5,100) FYSO,FDCO
       READ(5,106) IWRITE
C
  100 FORMAT(6D15.6)
  106 FORMAT(1I5)
C
C      SET COUNTERS EQUAL ZEROS
C
       TIME = 0.00
       TF   = 20.0
       ICO  = 0
       ICOU = 0
       EPAXL= 0.00
       Y    = 0.0001
       NUM  = 0
       IND  = 0
       INDIC= 0
       NUMB = 0
       MMM  = 0
       NNN  = 0
```

```
C
C
      EPTOT= EPSH+EPCRC
C
      CALL COORD(Z,ACE,ASE,NET)
C
    6 FORMAT(5X,'NET  = ',I5,'NI  = ',I5)
      DX2=2.0*DHG**2
      TWO = 2.0
      RT2=DSQRT(TWO)
      TSTAB=DX2*CAMIN/(4.0*COMAX+4.0*DHG*HMAX)
      NINT=1.0/(PERIOD*TSTAB)+1
      DT = 1.0/(PERIOD*NINT)
      WRITE(6,110)
  110 FORMAT(1H1,19X,'FIRE RESISTANCE',/
     .          26X,'OF',/
     .          10X,'CONCRETE-FILLED SQUARE STEEL COLUMN',/
     .          10X,'**************** ****** ***** ******',//)
      WRITE(6,120) H,THICK,ECC,KL
  120 FORMAT(5X,'COLUMN WIDTH (H).......................=',1D15.6,'(M)',
     .       5X,'THICKNESS OF STEEL FRAME (THICK)......=',1D15.6,'(M)',
     .       5X,'ECCENTRICITY (ECC)....................=',1D15.6,'(M)',
     .       5X,'EFFECTIVE LENGTH OF COLUMN (KL).......=',1D15.6,'(M)')
      WRITE(6,130) EMIS,PHI,COMAX,CAMIN,HMAX
  130 FORMAT(5X,'EMISIVITY (EMIS).......................=',1D15.6,/
     .       5X,'MOISTURE CONCENTRATION (PHI)..........=',1D15.6,/
     .       5X,'MAX. CONDUCTIVITY OF STEEL (COMAX)....=',1D15.6,/
     .       5X,'MIN. CAPACITY OF STEEL (CAMIN)........=',1D15.6,/
     .       5X,'MAX. COEFFICIENT OF HEAT TRANS.(HMAX).=',1D15.6,//)
      WRITE(6,140) FYSO,FDCO
  140 FORMAT(5X,'YIELD STREN. OF STEEL AT ROOM TEMP....=',1D15.6,/
     .       5X,'COMPR STREN. OF CONCR AT ROOM TEMP....=',1D15.6,//)
      WRITE(6,150) TIMLIM,PERIOD,DT
  150 FORMAT(5X,'TIME LIMIT (TIMLIM)..................=',1D15.6,/
     .       5X,'PERIOD ..............................=',1D15.6,/
     .       5X,'TIME INCREMENT (DT)..................=',1D15.6,//)
C     CALCULATION OF INITIAL TEMPERATURE AND INITIAL MOISTURE
C
      CALL INITL(TJ1,TJ,V,NET)
C
      IF(TIME .EQ. 0.00) GO TO 300
  190 CONTINUE
C
      ICO  = ICO+1
      ICOU = ICOU+1
      TIME = ICO*DT
C
C      CALCULATION OF THERMAL PROPERTIES OF CONCRETE
C
      MIP1 = MI+1
      DO 10  M=5,MIP1,2
```

```
      LI = NI+3-M
      N1 = 2
      IF(NET .EQ. 1  .OR.  NET .EQ. 4 ) N1=1
      DO 10  N=N1,LI,2
C
      CALL CONPR(M,N,TJ,CAPC,CONDC )
C
   10 CONTINUE
      DO 20  M=6,NI1,2
      LI = NI+3-M
      N1 = 1
      IF(NET .EQ. 1  .OR. NET .EQ. 4)  N1=2
      DO 20  N=N1,LI,2
C
      CALL CONPR(M,N,TJ,CAPC,CONDC)
C
   20 CONTINUE
C
C     CALCULATION OF THERMAL PROPERTIES OF STEEL
C
      DO 11 M=1,5,2
      LI = NI+3-M
      N1 = 2
      IF(NET .EQ. 1 .OR. NET .EQ. 4) N1 = 1
      DO 11 N=N1,LI,2
      CALL STLPR(M,N,TJ,CAPS,CONDS)
   11 CONTINUE
C
      DO 21 M=2,4,2
      LI = NI+3-M
      N1 = 1
      IF(NET .EQ. 1 .OR. NET .EQ. 4) N1 = 2
      DO 21 N=N1,LI,2
      CALL STLPR(M,N,TJ,CAPS,CONDS)
   21 CONTINUE
C
C     CALCULATION OF TEMPERATURE
C
      CALL TEMPCS(NET,TIME,DT,V,CONDC,CONDS,CAPC,CAPS,TJ,TJ1,TF)
C
C     SET TJ AT NEXT TIME STEP EQUAL TO TJ1 AT CURRENT TIME STEP
C
      IF(NET .EQ. 1 .OR. NET .EQ. 4) GO TO 30
      IF(NET .EQ. 2 .OR. NET .EQ. 3) GO TO 40
   30 CONTINUE
      DO 50 I=1,NI,2
      LI = NI-I+3
      DO 50 J=1,LI,2
      TJ (I,J) = TJ1(I,J)
   50 CONTINUE
      DO 60 I=2,NI,2
```

```
      LI = NI-I+3
      DO 60 J=2,LI,2
      TJ (I,J) = TJ1(I,J)
   60 CONTINUE
      GO TO 80
   40 CONTINUE
      DO 70 I=1,NI,2
      LI = NI-I+3
      DO 70 J=2,LI,2
      TJ (I,J) = TJ1(I,J)
   70 CONTINUE
      DO 90 I=1,NI,2
      LI = NI-I+3
      DO 90 J=1,LI,2
      TJ (I,J) = TJ1(I,J)
   90 CONTINUE
   80 CONTINUE
C
      IF(ICOU - NINT) 190,300,300
  300 ICOU  = 0
      TIMEMI  = TIME*60.0
      IF(TIMEMI .LT. STTIME) GO TO 190
C
C     CALCULATE THE AVERAGE TEMPERATURES AT THE TRANSFORMED NET
C
      CALL AVERG(TJ1,TT,NET)
C
 1500 CONTINUE
      ROH   = (KL**2/12.0)/Y
      NUM   = 0
      IND   = 0
      INDIC = 0
      NUMB  = 0
      MMM   = 0
      NNN   = 0
      EPAXL = 0.00
 1510 CONTINUE
C
C     CALCULATION OF STRAINS,STRESSES,LOADS AND MOMENTS
C
C
      CALL LOADMS(TT,EPAXL,Z,ASE,EPSR,EPSL,FSR,FSL,ROH,PST,MST,FYSO)
C
    8 FORMAT(5X,'MOMENT STEEL =',1D15.6,5X,'LOAD STEEL =',1D15.6)
C
      CALL LOADMC(TT,EPAXL,Z,ACE,EPCR,EPCL,FCR,FCL,ROH,PCT,MCT,FDCO)
C
    9 FORMAT(5X,'MOMENT COCR. =',1D15.6,5X,'LOAD COCR. =',1D15.6)
C
C     SUMMITION OF TOTAL LOADS AND MOMENTS
C
```

```
      PT = PCT + PST
      MT = MCT + MST
C
C     CHECKING THE AXIAL STRAIN AND MAKE THE BALANCE BETWEEN THE
C     EXTERNAL AND INTERNAL MOMENTS
C
      CALL CHECK(MT,PT,ECC,Y,EPAXL,TIME,TIMLIM,TIMEMI,TF,TT,
     .PST,PCT,MST,MCT,EPSR,EPSL,EPCR,EPCL,FSR,FSL,FCR,FCL,ICHEK,IWRITE)
C
      IF(ICHEK .EQ. 1) GO TO 190
      IF(ICHEK .EQ. 2) GO TO 1500
      IF(ICHEK .EQ. 3) GO TO 1510
C
      READ(5,106)IEXIT
      IF(IEXIT .EQ. 0) GO TO 108
      GO TO 107
C
  108 STOP
      END
C
C
C     ********************************
C     ================================
      SUBROUTINE COORD(Z,ACE,ASE,NET)
C     ================================
C     ********************************
C
C      THIS SUBROUTINE CALCULATES THE COORDINATES AND AREAS OF
C      THE ELEMENTS IN THE DIFFERENT NETWORKS FOR CONCRETE & STEEL
C      ALFAC=ALFA FOR CONCRETE
C     INT  = INTEGER
C     DHG  = HALF LENGTH OF THE DIAGONAL OF TRIANGULAR ELEMENT
C     DHE  = HALF LENGTH OF THE DIAGONAL OF THE TRIANGULAR ELEMENT
C            AT END FOR EVERY NET
C
C
C      ND = NUMBER OF DIVISIONS OF LENGTH ( 2.0*DHG )
C      NI = NUMBER OF HORIZONTAL DIVISIONS OF LENGTH DHG
C      MI = NUMBER OF VERTICAL DIVISIONS OF LENGTH DHG
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/AREAS/AEND1,AEND2,AEND3,AEND4
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/DIMEN/THICK,H,DHG,DHE
      COMMON/NUMB1/NI,MI,IK
      DIMENSION Z(135,135),ACE(135,135),ASE(135,135)
      DHG = THICK/4.0
      ND  = INT(H/(4.0*DHG))
      DHE = H/2.0 - 2.0*ND*DHG
C
```

```
C
      IF( DHE .EQ. 0.00 ) GO TO 10
      IF( DHE .EQ. DHG  ) GO TO 20
      IF( DHE .LT. DHG  ) GO TO 30
      IF( DHE .GT. DHG  ) GO TO 40
C
   10 CONTINUE
C
C      ***********************************************************
C      FIRST CASE (EVEN NUMBER OF TRIANGULAR ELEMENT),NETWORK NO.1
C      ***********************************************************
C
      NET = 1
      NI   = 2*ND + 1
      MI   = NI - 1
      IK   = NI - 1
      NI1  = NI - 1
      NI2  = NI - 2
      NI3  = NI - 3
      NI4  = NI - 4
      IK1  = IK - 1
      IK2  = IK - 2
C
      DO 15 I = 1,IK
      DO 15 J = 1,IK
   15 Z(I,J)  = (2*J - 1)*DHG/2.0
C
C     AEND1 = AREA OF ELEMENT (1,3) FOR NETWORK1
C     AEND2 = AREA OF ELEMENT (2,2) FOR NETWORK1
C     AEND3 = AREA OF ELEMENT (3,3) FOR NETWORK1
C     AEND4 = AREA OF ELEMENT(14,2) FOR NETWORK1
C
      AEND1 = DHG ** 2.0
      AEND2 = 2.0 * AEND1
      AEND3 = AEND2
      AEND4 = AEND2
      GO TO 50
   20 CONTINUE
C
C      ***********************************************************
C      SECOND CASE (ODD NUMBER OF TRIANGULAR ELEMENT),NETWORK NO.2
C      ***********************************************************
C
      NET = 2
      NI   = 2*ND + 2
      MI   = NI - 1
      IK   = NI - 1
      NI1  = NI - 1
      NI2  = NI - 2
      NI3  = NI - 3
      NI4  = NI - 4
```

```
      IK1  = IK - 1
      IK2  = IK - 2
C
      DO 25 I = 1,IK
      DO 25 J = 1,IK
   25 Z(I,J)  = (2*J - 1) * DHG/2.0
C
C     AEND1 = AREA OF ELEMENT (1,2) FOR NETWORK2
C     AEND2 = AREA OF ELEMENT (2,3) FOR NETWORK2
C     AEND3 = AREA OF ELEMENT (3,2) FOR NETWORK2
C     AEND4 = AREA OF ELEMENT(13,2) FOR NETWORK2
C
      AEND1 = DHG ** 2.0
      AEND2 = 2.0 * AEND1
      AEND3 = AEND2
      AEND4 = AEND2
      GO TO 50
   30 CONTINUE
C
C
C     ********************************************************
C     THIRD CASE (EVEN NUMBER OF TRIANGULAR ELEMENT + FRACTION
C                 OF AN ELEMENT
C     ********************************************************
C
      NET = 3
      NI   = 2*ND + 2
      MI   = NI - 1
      IK   = NI - 1
      NI1  = NI - 1
      NI2  = NI - 2
      NI3  = NI - 3
      NI4  = NI - 4
      IK1  = IK - 1
      IK2  = IK - 2
C
      DO 35 I = 1,IK
      Z(I,1) = 0.50 * DHE
      DO 35 J = 2,IK
C
   35 Z(I,J)  = DHE + (2.0 * J - 3.0) * DHG/2.0
C
C     AEND1 = AREA OF ELEMENT (1,2) FOR NETWORK3
C     AEND2 = AREA OF ELEMENT (2,3) FOR NETWORK3
C     AEND3 = AREA OF ELEMENT (3,2) FOR NETWORK3
C     AEND4 = AREA OF ELEMENT(15,2) FOR NETWORK3
C
      AEND1 = DHE * DHG
      AEND2 = DHG ** 2.0 + DHG * DHE
      AEND3 = 2.0 * AEND1
      AEND4 = DHG * DHE + DHE ** 2.0
```

```
      GO TO 50
   40 CONTINUE
C
C
C     ***********************************************************
C     FOURTH CASE (ODD NUMBER OF TRIANGULAR ELEMENT + FRACTION
C                   OF AN ELEMENT
C     ***********************************************************
C
      NET = 4
      NI    = 2*ND + 3
      MI    = NI - 1
      IK    = NI - 1
      NI1   = NI - 1
      NI2   = NI - 2
      NI3   = NI - 3
      NI4   = NI - 4
      IK1   = IK - 1
      IK2   = IK - 2
C
      DO 45 I = 1,IK
      Z(I,1) = 0.5 * DHE
      DO 45 J = 2,IK
   45  Z(I,J)  = DHE + ( 2.0 * J - 3.0 ) * DHG/2.0
C
C
C     AEND1 = AREA OF ELEMENT (1,3) FOR NETWORK4
C     AEND2 = AREA OF ELEMENT (2,2) FOR NETWORK4
C     AEND3 = AREA OF ELEMENT (3,3) FOR NETWORK4
C     AEND4 = AREA OF ELEMENT(14,2) FOR NETWORK4
C
      AEND1 = 0.5*DHG**2.0 + 0.5*DHE*DHG
      AEND2 = 2.0*DHG*DHE
      AEND3 = 2.0*AEND1
      AEND4 = DHG*DHE + DHE**2
   50 CONTINUE
      NI5 = NI-5
C
C      GET THE AREA OF STEEL AND CONCRETE  ELEMENTS
C
      IF ( NET .EQ. 1 .OR. NET .EQ. 2 ) GO TO 60
      IF ( NET .EQ. 3 .OR. NET .EQ. 4 ) GO TO 70
   60 CONTINUE
C
C      AREA OF STEEL ELEMENTS
C
      DO 55 I=1,4
      DO 55 J=1,IK
   55 ASE(I,J)= DHG**2.0
      DO 56 I=5,IK
      DO 56 J=NI4,IK
```

```
   56 ASE(I,J)= DHG**2.0
C
C      AREA OF CONCRETE ELEMENTS
C
      DO 57 I=5,IK
      DO 57 J=1,NI5
   57 ACE(I,J) = DHG**2.0
      GO TO 80
   70 CONTINUE
C
C      AREA OF STEEL ELEMENTS
C
      ASE(1,1) = DHE*DHG
      DO 72 I=2,4
      ASE(I,1) = ASE(1,1)
   72 CONTINUE
      DO 75 I=1,4
      DO 75 J=2,IK
   75 ASE(I,J) = DHG**2.0
C
      DO 76 I=5,IK
      DO 76 J=NI5,IK
   76 ASE(I,J) = DHG**2.0
C
C      AREA OF CONCRETE ELEMENTS
C
      DO 77 I=5,IK
   77 ACE(I,1) = DHE*DHG
      DO 78 I = 5,IK
      DO 78 J = 2,NI5
   78 ACE(I,J) = DHG**2.0
   80 CONTINUE
      RETURN
      END
C
C     *******************************
C     ===============================
      SUBROUTINE INITL(TJ1,TJ,V,NET)
C     ===============================
C     *******************************
C
C      THIS SUBROUTINE CALCULATES THE INITIAL TEMPERATURES AND
C     THE INITIAL MOISTURE CONTENT WITHIN THE CONCRETE REIGON
C     AT ROOM TEMPERATURE
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/AREAS/AEND1,AEND2,AEND3,AEND4
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/PROPR/PHI,EMIS,EPTOT
      COMMON/NUMB1/NI,MI,IK
      DIMENSION TJ1(135,135),TJ(135,135),V(135,135)
```

```
      MIP1 = MI+1
      NIP1 = NI+1
C
C      INITIAL TEMPERATURES
C
      DO 10 I = 1,NIP1
      DO 10 J = 1,NIP1
      TJ (I,J) = 20.0
      TJ1(I,J) = 20.0
   10 CONTINUE
C
C      INITIAL MOISTURE
C
      IF (NET .EQ. 1  .OR.  NET .EQ. 4) GO TO 90
      IF (NET .EQ. 2  .OR.  NET .EQ. 3) GO TO 100
C
C     N'S  = IS THE COLUMN NUMBER
C     M'S  = IS THE ROW NUMBER
C     PHI  = IS THE MOISTURE CONCENTRATION
C     V(M,N)=THE VOLUME OF THE MOISTURE FOR ELEMENT(M,N)
C           =AREA OF ELEMENT(M,N) * UNIT THICKNESS * PHI
   90 N1 = 1
      N2 = 2
      N3 = 5
      GO TO 110
  100 N1 = 2
      N2 = 3
      N3 = 4
  110 AREA = DHG**2
      V(5,N1) = AEND3/2.0*PHI
      DO 85 N = N3,NI2,2
   85 V(5,N) = AREA*PHI
      AREA = 2.0*DHG**2
      DO 86 M=6,MIP1,2
      LI = NI+2-M
      DO 86 N = N2,LI,2
      IF (N .EQ. N2) AREA = AEND2
   86 V(M,N) = AREA*PHI
      AREA = 2.0*DHG**2
      DO 87 M=7,MI,2
      LI = NI+2-M
      DO 87 N = N1,LI,2
      IF (N .EQ. N1) AREA = AEND3
   87 V(M,N) = AREA*PHI
      V(MI,2) = AEND4*PHI
      RETURN
      END
C
C      ***********************************
C      ===================================
      SUBROUTINE STLPR(M,N,TJ,CAPS,CONDS)
```

```
C      ===================================
C      ***********************************
C
C     STLPR = STEEL PROPERTIES
C
C      THIS SUBROUTINE CALCULATES THE THERMAL PROPERTIES OF THE
C      STEEL AT DIFFERENT TEMPERATURE
C
C     CAPS(M,N) = THERMAL CAPACITY OF STEEL AT ELEMENT(M,N)
C     CONDS(M,N)= THERMAL CONDUCTIVITY OF STEEL
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB1/NI,MI,IK
      DIMENSION TJ(135,135),CAPS(135,135),CONDS(135,135)
C
      IF(TJ(M,N) - 650.0) 20,20,22
   20 CAPS(M,N)=0.004D 6*TJ(M,N)+3.30D 6
      GO TO 23
   22 IF(TJ(M,N) - 725.0) 24,24,26
   24 CAPS(M,N) = 0.068D 06*TJ(M,N) - 38.30D 06
      GO TO 23
   26 IF(TJ(M,N) - 800.0) 28,28,30
   28 CAPS(M,N) = -0.086D 06*TJ(M,N) + 73.35D 06
      GO TO 23
   30 CAPS(M,N) = 4.55D 06
   23 IF(TJ(M,N) - 900.0) 32,32,36
   32 CONDS(M,N) = (-0.022*TJ(M,N) + 48.0)*3.60D 3
      GO TO 40
   36 CONDS(M,N) = 28.20D 0*3.60D 3
   40 CONTINUE
      RETURN
      END
C
C      ***********************************
C      ===================================
      SUBROUTINE CONPR(M,N,TJ,CAPC,CONDC)
C      ===================================
C      ***********************************
C
C     CONPR = CONCRETE PROPERTIES
C
C     THIS SUBROUTINE CALCULATES THE THERMAL PROPERTIES  OF CONCRETE
C     AT DIFFERENT TEMPERATURE
C
C     CAPC  = THERMAL CAPACITY OF CONCRETE
C     CONDC = THERMAL CONDUCTIVITY OF CONCRETE
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TJ(135,135),CAPC(135,135),CONDC(135,135)
      IF(TJ(M,N) - 200.0) 10,10,20
   10 CAPC(M,N) = 0.005D 06*TJ(M,N) + 1.70D 06
```

```
      GO TO 30
   20 IF(TJ(M,N) - 400.0) 40,40,50
   40 CAPC(M,N) = 2.70D 06
      GO TO 30
   50 IF(TJ(M,N) - 500.0) 60,60,70
   60 CAPC(M,N) = 0.013D 06*TJ(M,N) - 2.50D 06
      GO TO 30
   70 IF(TJ(M,N) - 600.0) 80,80,90
   80 CAPC(M,N) = -0.013D 06*TJ(M,N) + 10.50D 06
      GO TO 30
   90 CAPC(M,N) = 2.70D 06
   30 IF(TJ(M,N) - 800.0) 100,100,110
  100 CONDC(M,N) = (-0.000625D 00*TJ(M,N) + 1.50)*3.60D 03
      GO TO 120
  110 CONDC(M,N) = 1.0*3.60D 03
  120 RETURN
      END
C
C       ***********************************************************
C       ===========================================================
      SUBROUTINE TEMPCS(NET,TIME,DT,V,CONDC,CONDS,CAPC,CAPS,TJ,TJ1,TF)
C       ===========================================================
C       ***********************************************************
C
C     TEMPCS = TEMPERATURE IN CONCRETE AND STEEL
C
C     THIS SUBROUTINE CALCULATES THE TEMPERATURE CHANGES OF THE
C     ELEMENTS FOR DIFFERENT TIME INTERVAL
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/DIMEN/THICK,H,DHG,DHE
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/NUMB1/NI,MI,IK
      COMMON/PROPR/PHI,EMIS,EPTOT
      DIMENSION TJ1(135,135),TJ(135,135),V(135,135),DV(135,135)
      DIMENSION CAPC(135,135),CAPS(135,135),CONDC(135,135),
     .CONDS(135,135)
C
C     ROHCW = THERMAL CAP. OF WATER (ROHWATER*CWATER)
C     ROLAM = ROHWATER*HEAT EVAPORATION OF WATER)
C     SBC = STEFAN-BOLTIZMAN CONSTANT¢JOULE/(HOUR*M**2*(DEG. K)**4)!
C     TF  = STANDERED FIRE TEMPERATURE
      ROHCW = 4.20D 06
      ROLAM = 2.30D 09
      SBC  = 5.670D-08*3.60D 03
      U    = DSQRT(TIME)
      TF   =  20.0 + 750.0*(1.0 - DEXP(- 3.79553*U)) + 170.41*U
C
C     TEMPERATURE AT FIRE-STEEL BOUNDARY
C
      M = 1
```

```
      IF (NET .EQ. 1 .OR. NET .EQ. 4) GO TO 10
      IF (NET .EQ. 2 .OR. NET .EQ. 3) GO TO 20
   10 CONTINUE
      N1 = 3
C      AEL = AREA OF THE ELEMENT
      AEL = DHG**2.0
C      ASF = SURFACE AREA OF ELEMENT
      ASF = 2.0*DHG*1.0
      GO TO 30
   20 CONTINUE
      N1 = 2
      AEL = DHG**2.0
      ASF = 2.0*DHG * 1.0
   30 CONTINUE
      DO 40  N = N1,NI,2
      IF (N .EQ. N1 .AND. NET .EQ. 3) GO TO 50
      IF (N .EQ. N1 .AND. NET .EQ. 4) GO TO 60
      GO TO 70
   50 CONTINUE
      AEL = DHG*DHE
      ASF = 2.0*DHE*1.0
      GO TO 70
   60 CONTINUE
      AEL = DHG**2.0/2.0 + DHE*DHG/2.0
      ASF = (DHG + DHE)*1.0
C
C     EQUATION OF TEMPERATURE CHANGES AT THE FIRE-STEEL BOUNDARY
C     WHERE M = 1
C
   70 CONTINUE
      TJ1(M,N) = TJ(M,N) + DT/(CAPS(M,N)*AEL)
     .*((CONDS(M+1,N-1) + CONDS(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDS(M+1,N+1) + CONDS(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N))
     .+ ASF*SBC*EMIS*((TF+273.0)**4 - (TJ(M,N) +273.0)**4))
   40 CONTINUE
C
C      TEMPERATURE INSIDE STEEL REIGON
C
      DO 110 M=2,4
      LI = NI+1-M
      IF( NET .EQ. 1  .OR. NET .EQ. 4 ) GO TO 80
      IF( NET .EQ. 2  .OR. NET .EQ. 3 ) GO TO 90
   80 CONTINUE
      N1 = 2
      IF(M .EQ. 3) N1=3
      AEL= 2.0*DHG**2
      GO TO 100
   90 CONTINUE
      N1 = 3
      IF(M .EQ. 3) N1=2
      AEL= 2.0*DHG**2
```

```
  100 CONTINUE
      DO 110 N = N1,LI,2
      IF( N .EQ. N1 .AND. NET .EQ. 3 ) AEL = DHG*(DHG+DHE)
      IF( N .EQ. N1 .AND. NET .EQ. 4 ) AEL = 2.0*DHG*DHE
C
C      THE AVERAGE CONDUCTIVITY FOR A POINTS LOCATED ON THE BOUNDARY
C      LINE OF THE STEEL - CONCRETE REIGON
C
C
C     EQUATION OF TEMPERATURE CHANGES FOR INSIDE STEEL REIGON AT M=2
C
      TJ1(M,N) = TJ(M,N) + DT/(CAPS(M,N)*AEL)
     .*((CONDS(M-1,N-1) + CONDS(M,N))/2.0 * (TJ(M-1,N-1) - TJ(M,N))
     .+ (CONDS(M+1,N-1) + CONDS(M,N))/2.0 * (TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDS(M-1,N+1) + CONDS(M,N))/2.0 * (TJ(M-1,N+1) - TJ(M,N))
     .+ (CONDS(M+1,N+1) + CONDS(M,N))/2.0 * (TJ(M+1,N+1) - TJ(M,N)))
C
  110 CONTINUE
C
C      TEMPERATURE AT STEEL-CONCRETE BOUNDARY
C
      M = 5
      IF( NET .EQ. 1 .OR. NET .EQ. 4 ) GO TO 120
      IF( NET .EQ. 2 .OR. NET .EQ. 3 ) GO TO 130
  120 CONTINUE
      N1 = 3
      AEL= 2.0*DHG**2
      GO TO 140
  130 CONTINUE
      N1 = 2
      AEL = 2.0*DHG**2
  140 CONTINUE
      NI3 = NI-3
      DO 150  N=N1,NI3,2
      IF( N .EQ. N1 .AND. NET .EQ. 3 ) AEL = 2.0*DHE*DHG
      IF( N .EQ. N1 .AND. NET .EQ. 4 ) AEL = (DHG+DHE)*DHG
      IF( TJ(M,N) .LT. 100.0 ) GO TO 160
      IF( V(M,N) - 0.0D 00 ) 170 , 160 , 180
  170 V(M,N) = 0.0D 00
      GO TO 160
  180 CONTINUE
C
C
C     EQUATION OF MOISTURE CHANGES AT STEEL-CONCRETE BOUNDARY,M=3
C
      DV(M,N) = DT/ROLAM
     .*((CONDS(M-1,N-1) + CONDS(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
     .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDS(M-1,N+1) + CONDS(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
     .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
C
```

```
      V(M,N) = V(M,N) - DV(M,N)
      TJ1(M,N) = TJ(M,N)
      GO TO 150
  160 CONTINUE
C
C
C     EQUATION OF TEMPERATURE CHANGES AT STEEL CONCRETE BOUNDARY,M=3
C
      TJ1(M,N) = TJ(M,N) + DT/(CAPS(M,N)*AEL + ROHCW*V(M,N))
     .*((CONDS(M-1,N-1) + CONDS(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
     .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDS(M-1,N+1) + CONDS(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
     .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
C
  150 CONTINUE
C
C      CALCULATION OF TEMPERATURE INSIDE THE CONCRETE REIGON
C
C     TEMPERATURE DISTRIBUTION FOR EVEN VALUES OF M
C
      NI1 = NI-1
      DO 250  M=6,NI1,2
      LI=NI-M+2
      IF( NET .EQ. 1 .OR. NET .EQ. 4 ) GO TO 255
      IF( NET .EQ. 2 .OR. NET .EQ. 3 ) GO TO 256
  255 N1=2
      AEL=2.0*DHG**2
      GO TO 257
  256 N1=3
      AEL=2.0*DHG**2
  257 CONTINUE
      DO 250 N=N1,LI,2
      IF(NET .EQ. 3 .AND. N .EQ. N1) AEL=(DHE+DHG)*DHG
      IF(NET .EQ. 4 .AND. N .EQ. N1) AEL=2.0*DHE*DHG
      IF(TJ(M,N) .LT. 100.0) GO TO 260
      IF(V(M,N) - 0.00D 00) 270 , 260 , 280
  270 V(M,N) = 0.00
      GO TO 260
C
C     EQUATION OF MOISTURE CHANGES INSIDE CONCRETE REIGON WHEN
C     M = EVEN NUMBER = 6,8,10,..
C
  280 DV(M,N) = DT/ROLAM
     .*((CONDC(M-1,N-1) + CONDC(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
     .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDC(M-1,N+1) + CONDC(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
     .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
C
      V(M,N) = V(M,N) - DV(M,N)
      TJ1(M,N) = TJ(M,N)
      GO TO 250
```

```
  260 CONTINUE
C
C
C
C      EQUATION OF TEMPERATURE CHANGES INSIDE CONCRETE REIGON
C      AT M = EVEN NUMBER = 6,8,10,....
C
       TJ1(M,N) = TJ(M,N) + DT/(CAPC(M,N)*AEL + ROHCW*V(M,N))
      .*((CONDC(M-1,N-1) + CONDC(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
      .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
      .+ (CONDC(M-1,N+1) + CONDC(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
      .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
  250 CONTINUE
C
C       TEMPERTURE DISTRIBUTION INSIDE THE CONCRETE WHEN M IS ODD
C
       NI4=NI-4
       DO 300 M=7,NI2,2
       LI=NI-M+2
       IF(NET .EQ. 1 .OR. NET .EQ. 4) GO TO 310
       IF(NET .EQ. 2 .OR. NET .EQ. 3) GO TO 320
  310  N1=3
       AEL=2.0*DHG**2
       GO TO 330
  320  N1=2
       AEL=2.0*DHG**2
  330  CONTINUE
       DO 300 N=N1,LI,2
       IF(NET .EQ. 3 .AND. N .EQ. N1) AEL=2.0*DHE*DHG
       IF(NET .EQ. 4 .AND. N .EQ. N1) AEL=(DHE+DHG)*DHG
       IF(TJ(M,N) .LT. 100.0) GO TO 340
       IF(V(M,N) - 0.0) 350 ,340 ,360
  350  V(M,N) = 0.00
       GO TO 340
C
C      EQUATION OF MOISTURE CHANGES INSIDE CONCRETE REIGON WHEN
C      M = ODD NUMBER = 5,7,9,...
C
  360  DV(M,N) = DT/ROLAM
      .*((CONDC(M-1,N-1) + CONDC(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
      .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
      .+ (CONDC(M-1,N+1) + CONDC(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
      .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
       V(M,N) = V(M,N) - DV(M,N)
       TJ1(M,N) = TJ(M,N)
       GO TO 300
C
C      EQUATION OF TEMPERATURE CHANGES INSIDE CONCRETE REIGON WHEN
C      M = ODD NUMBER = 5,7,9,...
C
```

```
  340 TJ1(M,N) = TJ(M,N) + DT/(CAPC(M,N)*AEL+ROHCW*V(M,N))
     .*((CONDC(M-1,N-1) + CONDC(M,N))/2.0*(TJ(M-1,N-1) - TJ(M,N))
     .+ (CONDC(M+1,N-1) + CONDC(M,N))/2.0*(TJ(M+1,N-1) - TJ(M,N))
     .+ (CONDC(M-1,N+1) + CONDC(M,N))/2.0*(TJ(M-1,N+1) - TJ(M,N))
     .+ (CONDC(M+1,N+1) + CONDC(M,N))/2.0*(TJ(M+1,N+1) - TJ(M,N)))
  300 CONTINUE
C
C
C      BY USIG THE SPESIALITY OF THE SEMMETRY OF THE CROSS-SECTION
C      ALONG THE A-C,D-C LINES,THE EOLLOWING AUXILARY EQUATIONS CAN
C      BE APPLIED IN ORDER TO FIND THE TEMPERATURE DISTRIBUTION IN
C      THE TOTAL CROSS-SECTION
C
C
      N=1
      IF( NET .EQ. 1 .OR. NET .EQ. 4) M1=1
      IF( NET .EQ. 2 .OR. NET .EQ. 3) M1=2
      DO 380  M=M1,NI,2
      TJ1(M,N) = TJ1(M,N+2)
  380 CONTINUE
      DO 390  M=1,NI1
  390 TJ1(M+1,NI-M+2) = TJ1(M,NI-M+1)
      RETURN
      END
C
C      ************************************************************
C      ============================================================
      SUBROUTINE  LOADMS(TT,EPAXL,Z,ASE,EPSR,EPSL,FSR,FSL,ROH,PST,
     .                  MST,FYSO)
C      ============================================================
C      ************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/NUMB1/NI,MI,IK
      DIMENSION EPTS(135,135),EPSR(135,135),EPSL(135,135)
      DIMENSION TT(135,135),ASE(135,135),Z(135,135)
      DIMENSION FSL(135,135),FSR(135,135)
      REAL MSR,MSL,MST
C
C     UNITS OF FYSO ARE : NETON/M**2
C
      EPSP = 4.0D-12*FYSO
      ONE = 0.001
C      CALCULATION OF STRAINS IN THE STEEL BOUNDARY
C
      DO 10  M=1,4
      DO 10  N=1,NI1
      IF(TT(M,N) .GE. 1000.0) GO TO 20
      IF(TT(M,N) .LT. 1000.0) GO TO 30
   20 CONTINUE
```

```
      ALFAS = 16.0D-06
      GO TO 40
   30 ALFAS = (0.0040*TT(M,N) + 12.0)*1.0D-06
   40 CONTINUE
C
C      EPST=THE TOTAL STRAIN IN THE STEEL
C      EPSR=THE STRAIN IN THE STEEL IN RIGHT SIDE OF THE SECTION
C      EPSL=THE STRAIN IN THE STEEL IN LEFT SIDE OF THE SECTION
C      EPAXL=THE STRAIN DUE TO THE AXIAL LOAD
C      Z(M,N)/ROH = THE STRAIN DUE TO BENDING,WHERE Z IS THE COORDIN.
C
C     EPTS = STRAIN DUE TO TEMPERATURE DIFFERENCE FOR STEEL
C          = (ALFA FOR STEEL)(DELTA T)
C     EPSR = STRAIN IN STEEL IN RIGHT SIDE OF X-AXIS
C     EPSL = STRAIN IN STEEL IN LEFT SIDE OF X-AXIS
C     EPAXL= STRAIN DUE TO AXIAL LOAD
C     Z/ROH= STRAIN DUE TO BENDING
      EPTS(M,N) = -ALFAS*(TT(M,N)-20.0)
      EPSR(M,N) = EPTS(M,N) + EPAXL + Z(M,N)/ROH
      EPSL(M,N) = EPTS(M,N) + EPAXL - Z(M,N)/ROH
   10 CONTINUE
      DO 50  M=5,NI1
      DO 50  N=NI4,NI1
      IF(TT(M,N) .GE. 1000.0) GO TO 60
      IF(TT(M,N) .LT. 1000.0) GO TO 70
   60 CONTINUE
      ALFAS = 16.0D-16
      GO TO 75
   70 CONTINUE
C
C      ALFAS=IS ALFA FOR STEEL
C
      ALFAS=(0.0040*TT(M,N) + 12.0)*1.0D-06
   75 CONTINUE
      EPTS(M,N) = -ALFAS*(TT(M,N)-20.0)
      EPSR(M,N) = EPTS(M,N) + EPAXL + Z(M,N)/ROH
      EPSL(M,N) = EPTS(M,N) + EPAXL - Z(M,N)/ROH
   50 CONTINUE
C
C      CALCULATION OF STRESSES,LOAD AND MOMENTS
C
C      PSR=THE LOAD IN THE STEEL REIGON IN RIGHT SIDE
C      PSL=THE LOAD ON THE STEEL REIGON IN LEFT SIDE
C      PST=THE TOTAL LOAD ON THE STEEL REIGON
C      FSR=THE STRESS ON THE STEEL REIGON IN RIGHT-SIDE
C      FSL=THE STRESS ON THE STEEL REIGON IN LEFT-SIDE
C      MSR=THE MOMENT ON THE STEEL REIGON IN RIGHT-SIDE
C      MSL=THE MOMENT ON THE STEEL REIGON IN LEFT-SIDE
C
      PSR = 0.00
      PSL = 0.00
```

```
      MSR = 0.00
      MSL = 0.0
C
C     CALCULATION OF STRAINS & STRESSES IN FIRST TWO ROWS IN STEEL
C
      DO 80  M=1,4
      DO 80  N=1,NI1
      F001=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
     .DSQRT(ONE)))*6.90D 06
      IF(EPSR(M,N) - 0.00) 90,100,100
   90 C=1.00
      GO TO 110
  100 C=-1.0
  110 IF(DABS(EPSR(M,N))-EPSP) 120,120,130
  120 FSR(M,N)=C*F001*DABS(EPSR(M,N))/0.001
      GO TO 135
  130 F001M=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
     .DSQRT(DABS(EPSR(M,N)) - EPSP + 0.001)))*6.90D 6
      FSR(M,N)=C*(F001*EPSP/0.001 + F001M - F001)
  135 CONTINUE
      PSR = PSR + FSR(M,N)*ASE(M,N)
      MSR = MSR - FSR(M,N)*ASE(M,N)*Z(M,N)
      IF(EPSL(M,N)-0.00) 140 , 150 , 150
  140 C=1.0
      GO TO 160
  150 C=-1.0
  160 IF(DABS(EPSL(M,N))-EPSP) 170,170,180
  170 FSL(M,N) = C*F001*DABS(EPSL(M,N))/0.001
      GO TO 175
  180 F001M=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
     .DSQRT(DABS(EPSL(M,N)) - EPSP + 0.001)))*6.90D 6
      FSL(M,N)=C*(F001*EPSP/0.001 + F001M - F001)
  175 CONTINUE
      PSL = PSL + FSL(M,N)*ASE(M,N)
      MSL = MSL + FSL(M,N)*ASE(M,N)*Z(M,N)
   80 CONTINUE
C
C     CALCULATION OF STRAINS & STRESSES IN LAST TWO COLUMNS OF STEEL
C
      DO 190  M=5,NI1
      DO 190  N=NI4,NI1
      F001=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
     .DSQRT(ONE)))*6.90D 06
      IF(EPSR(M,N) - 0.00) 200,210,210
  200 C=1.00
      GO TO 220
  210 C=-1.0
  220 IF(DABS(EPSR(M,N))-EPSP) 230,230,240
  230 FSR(M,N)=C*F001*DABS(EPSR(M,N))/0.001
      GO TO 250
  240 F001M=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
```

```
     .DSQRT(DABS(EPSR(M,N)) - EPSP + 0.001)))*6.90D 6
      FSR(M,N)=C*(F001*EPSP/0.001 + F001M - F001)
      PSR = PSR + FSR(M,N)*ASE(M,N)
      MSR = MSR - FSR(M,N)*ASE(M,N)*Z(M,N)
      IF(EPSL(M,N)-0.00) 250 , 260 , 260
  250 C=1.0
      GO TO 270
  260 C=-1.0
  270 IF(DABS(EPSL(M,N))-EPSP) 280,280,290
  280 FSL(M,N) = C*F001*DABS(EPSL(M,N))/0.001
      GO TO 195
  290 F001M=(50.0-0.04*TT(M,N))*(1.0-DEXP((-30.0+0.030*TT(M,N))*
     .DSQRT(DABS(EPSL(M,N)) - EPSP + 0.001)))*6.90D 6
      FSL(M,N)=C*(F001*EPSP/0.001 + F001M - F001)
  195 CONTINUE
      PSL = PSL + FSL(M,N)*ASE(M,N)
      MSL = MSL + FSL(M,N)*ASE(M,N)*Z(M,N)
  190 CONTINUE
C      THE SUMMITION OF THE TOTAL LOADS AND MOMENTS
C
      PST = (PSR+PSL)*2.0
      MST = (MSR+MSL)*2.0
      RETURN
      END
C
C
C      **********************************************************
C      ==========================================================
      SUBROUTINE  LOADMC(TT,EPAXL,Z,ACE,EPCR,EPCL,FCR,FCL,ROH,PCT,
     .                    MCT,FDCO)
C      ==========================================================
C      **********************************************************
C
C      THIS SUBROUTINE CALCULATES THE LOADS AND THE MOMENTS IN
C      THE CONCRETE REIGON OF THE CROSS-SECTION BY STARTING
C      TO CALCULATE FIRST STRAINS,STRESSES,FORCES IN BOTH RIGHT
C      AND LEFT-SIDES OF THE CRSS-SECTION.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/PROPR/PHI,EMIS,EPTOT
      COMMON/NUMB1/NI,MI,IK
      DIMENSION EPTC(135,135),EPCR(135,135),EPCL(135,135)
      DIMENSION FCR(135,135),FCL(135,135)
      DIMENSION TT(135,135),ACE(135,135),Z(135,135)
      REAL MCT
      NI5 = NI-5
C
C      CALCULATION OF STRAINS IN THE CONCRETE REIGON
C
      DO 10  M=5,IK
```

```
      DO 10  N=1,NI5
      ALFAC = (0.0080*TT(M,N) + 6.0)*1.0D-06
      EPTC(M,N) = -ALFAC*(TT(M,N)-20.0)
      EPCR(M,N) = EPTC(M,N) + EPAXL + EPTOT + Z(M,N)/ROH
      EPCL(M,N) = EPTC(M,N) + EPAXL + EPTOT - Z(M,N)/ROH
   10 CONTINUE
C
C      CALCULATION OF STRESSES IN CONCRETE REIGON
C      FDCO = THE CYLINDER STRENGTH OF CONCRETE AT ROOM TEMP.
C             (F-PRIME-C-O)
C      FPC  = THE CYLINDER STRENGTH OF CONCRETE AT TEMP. T
C             (F-PRIME-C)
C
      PCT = 0.00
      MCT = 0.00
      DO 20  M=5,IK
      DO 20  N=1,NI5
      EPMAX=0.0025+(6.0*TT(M,N)+0.04*TT(M,N)**2.0)*1.0D-06
      FPC = FDCO*(2.011-2.353*(TT(M,N)-20)/1000.0)
      IF(FPC .GT. FDCO) FPC=FDCO
      IF(FPC .LE. 0.00) FPC=0.00
      IF(EPCR(M,N)-0.00D 00) 30 , 40 , 40
   40 FCR(M,N)=0.00D 00
      GO TO 50
   30 IF(DABS(EPCR(M,N))-EPMAX) 60,60,70
   60 FCR(M,N)=FPC*(1.0-((EPMAX+EPCR(M,N))/EPMAX)**2)
      GO TO 50
   70 FCR(M,N)=FPC*(1.0-((-EPCR(M,N)-EPMAX)/(3.0*EPMAX))**2)
      IF(FCR(M,N) .LT. 0.00) FCR(M,N)=0.00
   50 CONTINUE
      IF(EPCL(M,N)-0.00D 00) 80,90,90
   90 FCL(M,N)=0.0D 00
      GO TO 100
   80 IF(DABS(EPCL(M,N))-EPMAX) 110,110,120
  110 FCL(M,N)=FPC*(1.0-((EPMAX+EPCL(M,N))/EPMAX)**2)
      GO TO 100
  120 FCL(M,N)=FPC*(1.0-((-EPCL(M,N)-EPMAX)/(3.0*EPMAX))**2)
      IF(FCL(M,N) .LT. 0.00) FCL(M,N)=0.00
  100 CONTINUE
C
C      THE SUMMTION OF THE TOTAL LOADS AND TOTAL MOMENTS IN THE
C      CONCRETE REIGON OF THE CROSS-SECTION
C
C      PCT=TOTAL LOAD IN CONCRETE
C      MCT=TOTAL MOMENTS IN CONCRETE REIGON
C
      PCT = PCT + 2.0*(FCR(M,N)+FCL(M,N))*ACE(M,N)
      MCT = MCT + 2.0*(-FCR(M,N)+FCL(M,N))*ACE(M,N)*Z(M,N)
   20 CONTINUE
      RETURN
      END
```

```
C     ******************************
      SUBROUTINE AVERG(TJ1,TT,NET)
C     ******************************
C
C
C      IN ORDER TO CALCULATE THE STRAINS,STRESSES,LOADS AND MOMENTS
C      WE HAVE TO TRANSFER THE DISTRIBUTED TEMPERATURES FROM TRIANG.
C      ELEMENTS NETWORK TO SQUARE ELEMENTS NETWORK.THIS TRANSFORM.
C      CAN BE DONE BY AVERAGING THE OBTAINED TEMPERATURE IN THE
C      TRIANGULAR NETWORK.
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB1/NI,MI,IK
      DIMENSION TT(135,135),TJ1(135,135)
      IF( NET .EQ. 1 .OR. NET .EQ. 4 ) GO TO 400
      IF( NET .EQ. 2 .OR. NET .EQ. 3 ) GO TO 480
  400 DO 420  M=1,IK,2
      LI=IK+1-M
      DO 430  N=1,LI,2
C
C      AVERAGING THE TEMPERATURE
C
      TT(M,N) = (TJ1(M+1,N+1) + TJ1(M,N+2))/2.0
      KJ=IK+1-N
C
C      FROM SYMMETRY OF THE TRANSFORMED NET :
  430 TT(KJ,LI) = TT(M,N)
      DO 440  N=2,LI,2
      TT(M,N) = (TJ1(M,N+1) + TJ1(M+1,N+2))/2.0
      KJ=IK+1-N
  440 TT(KJ,LI) = TT(M,N)
  420 CONTINUE
      DO 450  M=2,IK,2
      LI=IK+1-M
      DO 460  N=1,LI,2
      TT(M,N) = (TJ1(M,N+1) + TJ1(M+1,N+2))/2.0
      KJ=IK+1-N
  460 TT(KJ,LI) = TT(M,N)
      IF(LI .EQ. 1) GO TO 450
      DO 470  N=2,LI,2
      TT(M,N) = (TJ1(M+1,N+1) + TJ1(M,N+2))/2.0
      KJ=IK+1-N
  470 TT(KJ,LI) = TT(M,N)
  450 CONTINUE
      GO TO 500
  480 CONTINUE
      IK=NI-1
      DO 490  M=1,IK,2
      LI=IK+1-M
      DO 510  N=1,LI,2
      TT(M,N) = (TJ1(M,N+1) + TJ1(M+1,N+2))/2.0
      KJ=IK+1-N
```

```
  510 TT(KJ,LI) = TT(M,N)
      DO 520  N=2,LI,2
      TT(M,N) = (TJ1(M,N+2) + TJ1(M+1,N+1))/2.0
      KJ=IK+1-N
  520 TT(KJ,LI) = TT(M,N)
  490 CONTINUE
      DO 530  M=2,IK,2
      LI=IK+1-M
      DO 540  N=1,LI,2
      TT(M,N) = (TJ1(M,N+2) + TJ1(M+1,N+1))/2.0
      KJ=IK+1-N
  540 TT(KJ,LI) = TT(M,N)
      IF(LI .EQ. 1) GO TO 530
      DO 550  N=2,LI,2
      TT(M,N) = (TJ1(M,N+1) + TJ1(M+1,N+2))/2.0
      KJ=IK+1-N
  550 TT(KJ,LI) = TT(M,N)
  530 CONTINUE
  500 CONTINUE
      RETURN
      END
C
C
C      ***********************************************************
C      ===========================================================
      SUBROUTINE CHECK(MT,PT,ECC,Y,EPAXL,TIME,TIMLIM,TIMEMI,TF,TT,
     .PST,PCT,MST,MCT,EPSR,EPSL,EPCR,EPCL,FSR,FSL,FCR,FCL,ICHEK,IWRITE)
C      ===========================================================
C      ***********************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB3/INDIC,NUMB,NUM,NNN,MMM,IND
      COMMON/NUMB2/NI1,NI2,NI3,NI4,IK1,IK2
      COMMON/NUMB1/NI,MI,IK
      DIMENSION EPSR(135,135),EPSL(135,135),EPCR(135,135),EPCL(135,135)
      DIMENSION FSR(135,135),FSL(135,135),FCR(135,135),FCL(135,135)
      DIMENSION TT(135,135),ITYPE(135)
      REAL MST,MCT,MT
      NI5 = NI-5
C
C      THIS SUBROUTINE IS CHECKING THE VALUE OF AXIAL STRAIN DUE
C      TO THE AXIAL LOAD AND TRY TO INCREASE IT OR DECREASE IT
C      UNTIL THE BENDING IS BALANCED
C
C
      ICHEK = 0
      PECCY=PT*(ECC+Y)
      IF(TIME-0.D0) 420,10,15
   10 FACT=2.D0
      IF(ECC .LT. 0.004)  FACT = 20.0
      GO TO 20
```

```
  15  FACT = 10.0
  20  IF(DABS(MT-PECCY) - 0.020*MT)  125,125,25
  25  IF(NUMB - 1)  30,100,100
  30  IF(NUM - 1)  40,35,35
  35  IF(DABS(MT-PECCY) - 0.020*MT)  125,125,100
  40  IF(PT - 0.00)  45,45,55
  45  IF(IND - 1)  50,55,55
  50  EPAXL = EPAXL - 0.001/FACT
      GO TO 1510
  55  IF(MT - 0.00)  115,60,60
  60  IF(DABS(MT-PECCY) - 0.020*MT)  125,125,65
  65  IF(MT - PECCY)  70,420,85
  70  IF(INDIC - 0)  420,75,80
  75  EPAXL = EPAXL + 0.001/FACT
      IND = IND+1
      GO TO 1510
  80  EPAXL = EPAXL + 0.0005/FACT
      NUM = NUM+1
      GO TO 1510
  85  IF(IND - 0)  420,90,95
  90  EPAXL = EPAXL - 0.001/FACT
      INDIC = INDIC+1
      GO TO 1510
  95  EPAXL = EPAXL - 0.0005/FACT
      NUM = NUM+1
      GO TO 1510
 100  IF(MT - PECCY)  105,125,110
 105  IF(NNN .EQ. 1) GO TO 115
      EPAXL = EPAXL + 0.0001/FACT
      NUMB = NUMB+1
      MMM = 1
      GO TO 1510
 110  IF(MMM .EQ. 1) GO TO 115
      EPAXL = EPAXL - 0.0001/FACT
      NUMB = NUMB+1
      NNN  = 1
      GO TO 1510
 115  WRITE(6,120)  NUMB
 120  FORMAT(////,5X,'MOMENTS NOT BAL. WITHIN 2%;NUMBER=',I3)
 125  WRITE(6,130)  TIMEMI
 130  FORMAT(//,5X,'TIME..................=',3X,F7.1,3X,'MIN')
      WRITE(6,135)  TF
 135  FORMAT(5X,'FIRE TEMPERATURE......=',3X,F7.1,3X,'C',//)
C
      IF(IWRITE .EQ. 0) GO TO 147
C
      WRITE(6,146)
 146  FORMAT(5X,'NOTE : FOR STEEL ELEMENTS    : ELEM. TYPE = 1',/
     .       12X,'FOR CONCRETE ELEMENTS : ELEM. TYPE = 2',//)
      WRITE(6,145)
 145  FORMAT(25X,'STRAINS & STRESSES IN STEEL',/
```

```
     .      25X,'******* * ******** ** *****',//,
     .     5X,'ELEM. NO.',3X,'ELEM. TYPE',2X,'TEMP.(C)',3X,'R. STRAIN'
     .     3X,'L. STRAIN',3X,'R. STRESS',3X,'L. STRESS',/
     .     5X,'----- ---',3X,'----- ----',3X,'--------',3X,'-- ------'
     .     3X,'-- ------',3X,'-- ------',3X,'-- ------',/
     .     43X,'MM/M',7X,'MM/M',8X,'MPA',8X,'MPA',/)
      DO 150 M=1,4
      DO 150 N=1,IK
      ITYPE(N) = 1
      A        = EPSR(M,N)*1.0D 03
      B        = EPSL(M,N)*1.0D 03
      C        =  FSR(M,N)*1.0D-06
      D        =  FSL(M,N)*1.0D-06
      WRITE(6,155) M,N,ITYPE(N),TT(M,N),A,B,C,D
  155 FORMAT(5X,'(',I3,',',I3,')',I8,6X,5D12.4)
  150 CONTINUE
      DO 160 M=5,IK
      DO 160 N=NI4,NI1
      A        = EPSR(M,N)*1.0D 03
      B        = EPSL(M,N)*1.0D 03
      C        =  FSR(M,N)*1.0D-06
      D        =  FSL(M,N)*1.0D-06
      WRITE(6,155) M,N,ITYPE(N),TT(M,N),A,B,C,D
  160 CONTINUE
      WRITE(6,165)
  165 FORMAT(///,25X,'STRAINS & STRESSES IN CONCRETE',/
     .      25X,'******* * ******** ** ********',//,
     .     5X,'ELEM. NO.',3X,'ELEM. TYPE',3X,'TEMP.(C)',3X,'R. STRAIN'
     .     3X,'L. STRAIN',3X,'R. STRESS',3X,'L. STRESS',/
     .     5X,'----- ---',3X,'----- ----',3X,'--------',3X,'-- ------'
     .     3X,'-- ------',3X,'-- ------',3X,'-- ------',/
     .     43X,'MM/M',7X,'MM/M',8X,'MPA',8X,'MPA',/)
      DO 170 M=5,IK
      DO 170 N=1,NI5
      ITYPE(N) = 2
      A        = EPCR(M,N)*1.0D 03
      B        = EPCL(M,N)*1.0D 03
      C        =  FCR(M,N)*1.0D-06
      D        =  FCL(M,N)*1.0D-06
      WRITE(6,155) M,N,ITYPE(N),TT(M,N),A,B,C,D
C
  170 CONTINUE
C
  147 CONTINUE
C
      WRITE(6,180)
  180 FORMAT(///,25X,'LOADS & MOMENTS IN STEEL & CONCRETE',/
     .      25X,'***** * ******* ** ***** * ********',//)
C
      A     = PST*1.0D-03
      B     = PCT*1.0D-03
```

```
      C     = MST*1.0D-03
      D     = MCT*1.0D-03
      E     =  PT*1.0D-03
      F     =  MT*1.0D-03
      WRITE(6,185) A,B,C,D,E,F
  185 FORMAT(5X,'LOAD   IN STEEL.................=',1D15.6,'(KN  )',/
     .       5X,'LOAD   IN CONCRETE..............=',1D15.6,'(KN  )',/
     .       5X,'MOMENT IN STEEL.................=',1D15.6,'(KN-M)',/
     .       5X,'MOMENT IN CONCRETE..............=',1D15.6,'(KN-M)',/
     .       5X,'TOTAL  LOAD.....................=',1D15.6,'(KN  )',/
     .       5X,'TOTAL  MOMENT...................=',1D15.6,'(KN-M)',//)
C
      A     = PECCY*1.00D-03
      B     = EPAXL*1.00D 03
      C     = EPAXL*0.35D 04
      D     =      Y*1.00D 03
      WRITE(6,195) A,B,C,D
  195 FORMAT(5X,'MOMENT(LOAD*(ECC + Y))..........=',1D15.6,'(KN-M)',/
     .       5X,'RELATIVE AXIAL STRAIN...........=',1D15.6,'(MM/MM)',/
     .       5X,'TOTAL AXIAL STRAIN..............=',1D15.6,'(MM   )',/
     .       5X,'LATERAL DEFLECTION AT MIDHIGHT..=',1D15.6,'(MM)',//)
      IF(Y - 0.000099)  270,420,275
  270 Y = Y + 0.00002
      GO TO 1500
  275 IF(MT - 0.D0)  280,290,290
  280 IF(TIME - TIMLIM)  285,420,420
  285 Y = 0.00010D0
      GO TO 190
  290 IF(Y - 0.0004D0)  295,300,300
  295 Y = Y + 0.00005001D0
      GO TO 1500
  300 IF(Y - 0.0010)  305,305,310
  305 Y = Y + 0.0001001D0
      GO TO 1500
  310 IF(Y - 0.0020)  315,315,320
  315 Y = Y + 0.00020010
      GO TO 1500
  320 IF(Y - 0.0100)  325,325,350
  325 Y = Y + 0.00050010
      GO TO 1500
  350 Y = 0.000100
      IF(TIME - TIMLIM)  190,190,420
  190 ICHEK = 1
      RETURN
 1500 ICHEK = 2
      RETURN
 1510 ICHEK =3
      RETURN
  420 CONTINUE
      RETURN
      END
```

APPENDIX F

```
C       THIS PROGRAM HAS BEEN DONE FOR THE NATIONAL RESEARCH
C       COUNCIL OF CANADA BY:
C       MOHAMED EL-SHAYEB ,PH.D. STUDENT ,UNIVERSITY OF NEWHAMPSHIRE,
C       DURHAM,NEWHAMPSHIRE,U.S.A.
C
C
C                     ________________________
C                     -**********************-
C                     -*====================*-
C                     -*=   MAIN PROGRAM    =*-
C                     -*====================*-
C                     -**********************-
C                     ________________________
C
C         FIRE RESISTANCE OF REINFORCED-CONCRETE CYLINDERICAL  COLUMN
C
C      SYMPOLS DIFINITIN
C
C      RO=OUTSIDE RADIOUS OF COLUMN CROSS-SECTION
C      RS=THE DISTANCE FROM THE CENTRE OF THE COLUMN TO THE REINFORCE.
C      TR  =TEMPERATURE OF THE REINFORCEMENT
C      TT  =AVERAGING TEMPERATURE
C      NR=NUMBER OF THE REINFORCEMENT BARS
C      KL=EFFECTIVE LENGTH OF COLUMN
C      ECC=ECCENTRICITY
C      ML=NUMBER OF LAYERS DIVISION
C      NI=NUMBER OF SECTORS DIVISIONS
C      EMI=EMMICITIVITY OF FIRE AND CONCRETE
C      PHI=MOISTURE CONCENTRATION
C      EPSH=SHRINKAGE STRAINS
C      EPCRC=CREEP STRAINS
C      TIMELIM=TIMELIMIT
C
C      IWRITE = IS AN OPTION FOR WRITING THE TEMPETURES RESULTS OR NOT
C      IWRITE = 0 DO NOT WRITE TEMPETURES
C      IWRITE = 1 WRITE THE TEMPETURES RESULTS
C      IEXIT  = IS AN OPTION FOR STOP OR CONTINUE TO EXECUTE
C      IEXIT  = 0  STOP EXECUTION AFTER THE CURRENT DATA
C      IEXIT  = 1  CONTINUE EXECUTION OTHER DATA PROVIDED
C
C      FYSO=YIELD STRENGTH OF CONCRETE AT ROOM TEMPERATURE
C      FOCO=COMPRESSIVE STRENGTH OF CONCRETE AT ROOM TEMPARATURE
C      RSCSM=(DENCITY(R)*SPECIFIC HEAT(C))MINIMUM FOR STEEL
C            =MINIMUM THERMAL CAPACITY OF STEEL
C      CONDSM=MAXIMUM CONDUCTIVITY FORSTEEL=(KS)MAX.
C      HMAX=THE MAXIMUM COEFFICIENT OF HEAT TRANSFER AT FIRE EXPOSED
C            SURFACE(J/M**2.HR.C)
C       TSTAB=DELTA TIME,TIME IN HOUR
C            =CAPACITY*DELTA ZETA**2/(MAX. CONDUCTIVITY+H(MAX.)*
C             DELTA ZETA
C            =(J/M**3DEGREE C)*M**2/((J/HM DEGREE C)+M(J/HM**2 DEG.C)
```

```
C     CONDCM=MAXIMUM CONDUCTIVITY OF CONCRETE=K,MAX.
C     CAPACM=MAXIMUM CAPACITY OF CONCRETE
C            =(ROH*C),FOR CONCRETE(MAX.)
C     DT=DELTA T=TIME INCREMENT
C     ICO=COUNTER FOR TIME
C     ICOU=
C     EP=(EPSILON)=AXIAL STRAIN
C     Y    =THE INITIAL DEFLECTION OF THE CROSS SECTION AT MIDHIGHT
C          OF THE COLUMN
C     EPSTL=(EPSILON TOTAL),THE TOTAL STRAIN
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION RR(40,20),THETA(40,20),ZC(40,20),ACE(40,20)
      DIMENSION EPSR(20),EPSL(20),FSR(20),FSL(20)
      DIMENSION EPCR(40,20),EPCL(40,20),FCR(40,20),FCL(40,20)
      DIMENSION CAPC(40,20),CONDC(40,20),ZS(20)
      DIMENSION TT(40),TJ1(40),TJ(40),V(40)
C
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
      COMMON/PROPR/PHI,EMIS,EPTOT
      COMMON/NUMB3/NUMB,NUM,INDIC,IND,NNN,MMM
C
      REAL MCT,MST,MT,KL
C
  107 CONTINUE
C
C     READ STATMENTS
C
      READ(5,100) RO,KL,ECC
      READ(5,100) RS,DIAMR
  100 FORMAT(5D15.6)
      READ(5,101) ML,NI,NR
  101 FORMAT(3I5)
      READ(5,100) EMIS,PHI,EPSH,EPCRC
      READ(5,100) TIMLIM,PERIOD,STTIME
      READ(5,100) COMAX,CAMIN,HMAX
      READ(5,100) FYSO,FDCO
      READ(5,106) IWRITE
  106 FORMAT(1I5)
C
C
C      SET COUNTERS EQUAL ZEROS
C
      TIME = 0.00
      TF   = 20.0
      ICO  =0
      ICOU =0
      EPAXL= 0.00
      Y    =0.00002
      NUM  =0
```

```
      IND  =0
      INDIC=0
      NUMB =0
      MMM  =0
      NNN  =0
      EPTOT = EPSH+EPCRC
      PI   =3.1415926540D 00
C
C      CALCULATION OF COORDINATES(CONCRETE AND STEEL)
C
      CALL COORD(RR,THETA,ZC,ACE,ZS)
C
C
C      TIME INCREMENT FOR STABLE SOLUTION (DELTA TAU)
C
      TSTAB = DX2*CAMIN/(2.0*(COMAX + HMAX*DX))
      NINT = 1.0/(PERIOD*TSTAB)+1
      DT = 1.0/(PERIOD*NINT)
      WRITE(6,110)
  110 FORMAT(1H1,19X,'FIRE RESISTANCE',/
     .           26X,'OF',/
     .           10X,'REINFORCED-CONCRETE CYLINDERICAL COLUMN',/
     .           10X,'******************* ************ ******',//)
      WRITE(6,120) RO,RS,DIAMR,ECC,KL
  120 FORMAT(5X,'OUTSIDE RADIUS OF COLUMN........=',1D15.6,'(M)',/
     .       5X,'DIST. FROM REINF. TO COLUMN CENTER..=',1D15.6,'(M)',/
     .       5X,'DIAMETER OF REINFORCEMENT..........=',1D15.6,'(M)'/
     .       5X,'ECCENTRICITY (ECC).................=',1D15.6,'(M)',/
     .       5X,'EFFECTIVE LENGTH OF COLUMN (KL).....=',1D15.6,'(M)')
      WRITE(6,130) EMIS,PHI,COMAX,CAMIN,HMAX
  130 FORMAT(5X,'EMISIVITY (EMIS).....................=',1D15.6,/
     .       5X,'MOISTURE CONCENTRATION (PHI)..........=',1D15.6,/
     .       5X,'MAX. CONDUCTIVITY OF CONC. (COMAX)....=',1D15.6,/
     .       5X,'MIN. CAPACITY OF CONC. (CAMIN)........=',1D15.6,/
     .       5X,'MAX. COEFFICIENT OF HEAT TRANS.(HMAX).=',1D15.6,//)
      WRITE(6,140) FYSO,FDCO
  140 FORMAT(5X,'YIELD STREN. OF STEEL AT ROOM TEMP....=',1D15.6,/
     .       5X,'COMPR STREN. OF CONCR AT ROOM TEMP....=',1D15.6,//)
      WRITE(6,150) TIMLIM,PERIOD,DT
  150 FORMAT(5X,'TIME LIMIT (TIMLIM)..................=',1D15.6,/
     .       5X,'PERIOD ..............................=',1D15.6,/
     .       5X,'TIME INCREMENT (DT)..................=',1D15.6,//)
C     CALCULATION OF INITIAL TEMPERATURE AND INITIAL MOISTURE
C
      CALL INITL(TJ1,TJ,V)
C
      IF(TIME .EQ. 0.00) GO TO 300
  190 CONTINUE
C
      ICO  = ICO+1
      ICOU = ICOU+1
```

```
      TIME = ICO*DT
C
C      CALCULATION OF THERMAL PROPERTIES OF CONCRETE
C
      CALL CONCPR(TJ,CAPC,CONDC)
C
C      CALCULATION OF TEMPERATURE INSIDE CONCRETE REIGON
C
C
      CALL TEMPC(TIME,DT,CONDC,CAPC,TF,TJ,TJ1,V)
C
C     SET TJ AT NEXT TIME STEP EQUAL TO TJ1 AT CURRENT TIME STEP
C
      DO 60 I=1,ML
      TJ(I) = TJ1(I)
   60 CONTINUE
C
      IF(ICOU - NINT) 190,300,300
  300 ICOU = 0
      TIMEMI = TIME*60.0
      IF(TIMEMI .LT. STTIME) GO TO 190
C
C     CALCULATION OF AVEREGE TEMPERATURE
C
C     IN ORDER TO CALCULATE THE STRAINS,STRESSES,LOADS AND MOMENTS
C     FOR THE C-S OF THE COLUMN,WE HAVE TO FIND THE TEMPERATURE FOR
C     THE CENTRE OF THE LAYER RING, TO DO THAT WE HAVE TO CONVERT
C     THE TEMPERATURE DISTRIBUTION FROM ORIGINAAL NETWORK TO THE
C     OTHER NETWORK BY AVERAGING THE TEMPERATURE
C
      DO 400 I=1,M1
      DO 400 J=1,NI
      TT(I)=(TJ(I)+TJ(I+1))/2.0
 400  CONTINUE
C
C     CALCULATION OF THE TEMPERATURES OF THE REINFORCEMENT
C
      I = 1
      IF(RS .LT. RO .AND. RS .GT. RR(I,1)) GO TO 43
      IF(RS .EQ. RR(I,1))                  GO TO 47
      DO 40 I=2,M1
      IF(RS .LT. RR(I-1,1) .AND. RS .GT. RR(I,1)) GO TO 43
      IF(RS .EQ. RR(I,1))                         GO TO 47
   40 CONTINUE
   43 TR = TT(I)
      GO TO 50
   47 TR = (TT(I) + TT(I+1))/2.0
   50 CONTINUE
C
C      CALCULATION OF STRAINS,STRESSES,LOADS AND MOMENTS IN STEEL
C
```

```
 1500  ROH=(KL**2/12.0)/Y
       NUM  =0
       IND  =0
       INDIC=0
       NUMB =0
       MMM  =0
       NNN  =0
       EPAXL=0.00
 1510  CONTINUE
C
       CALL LOADMS(TR,EPAXL,ZS,ROH,PST,MST,EPSL,EPSR,FSL,FSR,FYSO)
C
       CALL LOADMC(TT,EPAXL,ZC,ACE,ROH,PCT,MCT,EPCR,EPCL,FCR,FCL,FDCO)
C
C
C      SUMMITION OF TOTAL LOADS AND MOMENTS
C
       PT = PCT + PST
       MT = MCT + MST
C
C      CHECKING THE AXIAL STRAIN AND MAKE THE BALANCE BETWEEN THE
C      EXTERNAL AND INTERNAL MOMENTS
C
       CALL CHECK(MT,PT,ECC,Y,EPAXL,TIME,TIMLIM,TIMEMI,TF,TT,TR,
      .PST,PCT,MST,MCT,EPSR,EPSL,EPCR,EPCL,FSR,FSL,FCR,FCL,ICHEK,IWRITE)
C
       IF(ICHEK .EQ. 1) GO TO 190
       IF(ICHEK .EQ. 2) GO TO 1500
       IF(ICHEK .EQ. 3) GO TO 1510
C
       READ(5,106) IEXIT
       IF(IEXIT .EQ. 0) GO TO 108
       GO TO 107
C
   108 STOP
       END
C
C       **************************************
C       ======================================
       SUBROUTINE COORD(RR,THETA,ZC,ACE,ZS)
C       ======================================
C      ***************************************
C
C
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION RR(40,20),THETA(40,20),ZC(40,20),ACE(40,20),ZS(20)
C
       COMMON/NUMB1/ML,NI,NR
       COMMON/NUMB2/M1,IR
       COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
C
```

```
      PI = 3.1415926
      M1=ML-1
      DX = RO/M1
      DX2 = DX*DX
      BETA = PI/(2.0*NI)
      DO 10 I=1,M1
      DO 10 J=1,NI
      RR(I,J) = RO-(I-0.50)*DX
      THETA(I,J) = (2*J-1)*BETA/2.0
      ZC(I,J) = RR(I,J)*DSIN(THETA(I,J))
      ACE(I,J) = RR(I,J)*BETA*DX
   10 CONTINUE
C
C       CALCULATIONS OF THE COORDINATES OF STEEL
C
      GAMA = 2.0*PI/NR
      IR = NR/2+1
      DO 20 I=1,IR
      ZS(I) = RS*DSIN((I-1)*GAMA)
   20 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE INITL(TJ1,TJ,V)
C     **************************
C     ==========================
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TJ1(40),TJ(40),V(40)
C
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
      COMMON/PROPR/PHI,EMIS,EPTOT
C
      PI = 3.1415926
      DO 10 I=1,ML
      TJ(I)=20.0D 00
      TJ1(I)=20.0D 00
   10 CONTINUE
C
C       CALCULATION OF INITIAL MOISTURE CONTENT
C
C     AT FIRE-CONCRETE BOUNDARY
C
      V(1) = PI*(ML - 1.25)*DX2**1.0*PHI
      DO 30 I=2,M1
C
C     AT INSIDE CONCRETE REIGON
C
```

```
      V(I)=2.0*PI*(ML-I)*DX2*1.0*PHI
   30 CONTINUE
C
C     AT THE CENTRE OF THE COLUMN
C
      V(ML) = 0.25*PI*DX2*1.0*PHI
      RETURN
      END
C
C      ____________________________________________________________
C      ************************************************************
C      ============================================================
      SUBROUTINE CONCPR(TJ,CAPC,CONDC)
C      ============================================================
C      ************************************************************
C      ____________________________________________________________
C
C     TJ  =TEMPERATURE OF THE ELEMENT AT TIME J(DELTA)T
C     CAPC=THERMAL CAPACITY OF CONCRETE
C     ML  =NUMBER OF LAYERS DIVISION
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TJ(40),CAPC(40),CONDC(40)
C
      COMMON/NUMB1/ML,NI,NR
      DO 10 I=1,ML
      IF(TJ(I)-200.0)20,20,30
   20 CAPC(I)=0.0050D 06*TJ(I)+1.70 06
      GO TO 120
   30 IF(TJ(I)-400.00)40,40,50
   40 CAPC(I)=2.70D 06
      GO TO 120
   50 IF(TJ(I)-500.0)60,60,70
   60 CAPC(I)=0.0130D 06*TJ(I)-2.50D 06
      GO TO 120
   70 IF(TJ(I)-600.00)80,80,90
   80 CAPC(I)=-0.0130D 06*TJ(I)+10.50D 06
      GO TO 120
   90 CAPC(I)=2.70D 06
  120 IF(TJ(I)-800.0)100,100,110
  100 CONDC(I)=(-0.00085D 00*TJ(I)+1.90D 00)*3.60D 03
      GO TO 10
  110 CONDC(I)=1.220D 00*3.6D 03
   10 CONTINUE
      RETURN
      END
C
C
C      ____________________________________________________________
C      ************************************************************
C      ============================================================
```

```
      SUBROUTINE TEMPC(TIME,DT,CONDC,CAPC,TF,TJ,TJ1,V)
C      ==============================================================
C      **************************************************************
C      --------------------------------------------------------------
C
C     THIS SUBROUTINE CALCULATES THE TEMPERATURE DISTRIBUTION FOR
C     THE CONCRETE ELEMENTS
C
C
C     ROHW =(ROH)(LAMDA)=DENSITY OF WATER*HEAT OF VAPORIZATION
C     CAPW =(ROH)(C)=THERMAL CAPACITY OF WATER
C     SBC  =STEFEN-BOLTZMAN CONSTANT
C     U    =SQRT(TIME)=SQARE ROOT OF TIME
C     TF   =FIRE TEMPERATURE
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CAPC(40),CONDC(40),TJ(40),TJ1(40),V(40),DV(40)
C
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
      COMMON/PROPR/PHI,EMIS,EPTOT
C
      PI  =3.141592654
      ROHW=2.30D 09
      CAPW=4.20D 06
      SBC =5.67D-08*3.60D 03
      U   =DSQRT(TIME)
      TF  =20.0+750.0*(1.0-DEXP(-3.79553*U))+170.41*U
C
C      CALCULATION OF TEMPERATURES AT FIRE-CONCRETE BOUNDARY
C
      I=1
C
C      CHECK THE VOLUME OF THE MOISTURE
C
C     V    =VOLUME OF MOISTURE CONTENT
C     (DELTA)V=THE CHANGE IN MOISTURE DUE TO THE CHANGE IN TEMP.
C              IN THE ELEMENTS
C
C
      IF(TJ(I) .LT. 100.0) GO TO 100
      IF(V(I)-0.00) 10,100,20
   10 V(I)=0.00
      GO TO 100
C
   20 DV(I)=2.0*PI*(ML-1)*DX*SBC*EMIS/ROHW*((TF+273)**4-(TJ(I)+273)
     .**4)*DT-2.0*PI*(ML-1.50)*DT/ROHW*((CONDC(I)+CONDC(I+1))/2.0)
     .*(TJ(I)-TJ(I+1))
```

```
      V(I)=V(I)-DV(I)
      TJ1(I)=TJ(I)
      GO TO 30
  100 TJ1(I)=TJ(I)+DT/(CAPC(I)*(ML-1.25)*DX2/2.0+CAPW*V(I)/(2.0*PI))
     .*(DX*(ML-1)*SBC*EMIS*((TF+273)**4-(TJ(I)+273)**4)-((CONDC(I)+
     .CONDC(I+1))/2.0)*(ML-1.50)*(TJ(I)-TJ(I+1)))
   30 CONTINUE
C
C       CALCULATION OF TEMPERATURE INSIDE CONCRETE REIGON
C
      DO 40 I=2,M1
      IF(TJ(I) .LT. 100.0) GO TO 200
      IF(V(I)-0.00) 50,200,60
   50 V(I)=0.00
      GO TO 200
C     CALCULATE THE CHANGE IN THE MOISTURE VOLUME AT INSIDE
C     THE CONCRETE REIGON
C
   60 DV(I)=2.0*PI*DT/ROHW*((ML-I+0.50)*((CONDC(I-1)+CONDC(I))/2.0)*
     .(TJ(I-1)-TJ(I))-(ML-I-0.50)*((CONDC(I)+CONDC(I+1))/2.0)*(TJ(I)-
     .TJ(I+1)))
      V(I)=V(I)-DV(I)
      TJ1(I)=TJ(I)
      GO TO 40
C
C     CALCULATE THE TEMPERATURE DISTRIBUTION INSIDE THE CONCRETE
C     REIGON(NOT THE BOUNDARY)
C
  200 TJ1(I)=TJ(I)+DT/(CAPC(I)*(ML-I)*DX2+CAPW*V(I)/(2.0*PI))*
     .((ML-I+0.5)*0.50*(CONDC(I-1)+CONDC(I))*(TJ(I-1)-TJ(I))-
     . (ML-I-0.50)*0.50*(CONDC(I+1)+CONDC(I))*(TJ(I)-TJ(I+1)))
C
   40 CONTINUE
C
C       CALCULATION OF TEMPERATURES AT THE CENTRE OF THE COLUMN C-S
C
      I=ML
      IF(TJ(I) .LT. 100.0) GO TO 300
      IF(V(I)-0.00) 70,300,80
C
C     CALCULATE THE CHANGE IN THE VOLUME OF MOISTURE (DELTA)V
C
   70 V(I)=0.00
      GO TO 300
   80 DV(I)=PI*DT/(2.0*ROHW)*(CONDC(ML-1)+CONDC(ML))*(TJ(ML-1)-TJ(ML))
      V(I)=V(I)-DV(I)
      TJ1(I)=TJ(I)
      GO TO 90
C
C     CALCULATE THE TEMPERATURE DISTRIBUTION OF THE ELEMENTS
C
```

```
 300  TJ1(ML)=TJ(ML)+DT/(CAPC(ML)*DX2/4.0+ROHW*V(ML)/PI)*
     .0.50*(CONDC(ML-1)+CONDC(ML))*(TJ(ML-1)-TJ(ML))
  90  CONTINUE
C
C      AVERAGING TEMPERATURE
C
C
      RETURN
      END
C
C
C
C     ------------------------------------------------------------
C     ************************************************************
C     ============================================================
      SUBROUTINE LOADMS(TR,EPAXL,ZS,ROH,PST,MST,EPSL,EPSR,FSL,FSR,FYSO)
C     ============================================================
C     ************************************************************
C     ------------------------------------------------------------
C
C     IR  =NUMBER OF REINFORCEMENT BARS OF HALF SECTION
C     NR  =TOTAL NUMBER OF THE REINFORCEMENT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION EPSR(20),EPSL(20),FSR(20),FSL(20),ZS(20)
C
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
      REAL MSR,MSL,MST
C
      PI   = 3.14159260
      EPSP = 4.0D-12*FYSO
      ONE  = 0.001
      ASR  = PI*DIAMR**2/4.0
C
C      STRAINS IN THE REINFORCEMENT DUE TO TEMPERATURE
C
      IF(TR .GT. 1000.0) TR=1000.0
      ALFAS = 0.004D-06*TR+12.0D-06
      EPTS = -ALFAS*(TR-20.0)
C
C      SUMMITION OF TOTAL STRAINS
C
      DO 10 I=1,IR
      EPSR(I) = EPTS + EPAXL + ZS(I)/ROH
      EPSL(I) = EPTS + EPAXL - ZS(I)/ROH
  10  CONTINUE
C
C      CALCULATION OF STRESSES
C
```

```
C       PSR=THE LOAD IN THE STEEL REIGON IN RIGHT SIDE
C       PSL=THE LOAD ON THE STEEL REIGON IN LEFT SIDE
C       PST=THE TOTAL LOAD ON THE STEEL REIGON
C       FSR=THE STRESS ON THE STEEL REIGON IN RIGHT-SIDE
C       FSL=THE STRESS ON THE STEEL REIGON IN LEFT-SIDE
C       MSR=THE MOMENT ON THE STEEL REIGON IN RIGHT-SIDE
C       MSL=THE MOMENT ON THE STEEL REIGON IN LEFT-SIDE
C
      PSR = 0.00
      PSL = 0.00
      MSR = 0.00
      MSL = 0.00
C
C     CALCULATION OF STRAINS & STRESSES IN FIRST TWO ROWS IN STEEL
C
      DO 80  M=1,IR

      F001 =(50.0-0.04*TR)*(1.0-DEXP((-30.0+0.030*TR)*DSQRT(ONE)))*
     .      6.90D 06

      IF(EPSR(M) - 0.00) 90,100,100
   90 C=1.00
      GO TO 110
  100 C=-1.0
  110 IF(DABS(EPSR(M))-EPSP) 120,120,130
  120 FSR(M) = C*F001*DABS(EPSR(M))/0.001
      GO TO 135
  130 F001M=(50.0-0.04*TR)*(1.0-DEXP((-30.0+0.030*TR)*
     .DSQRT(DABS(EPSR(M)) - EPSP + 0.001)))*6.90D 6
      FSR(M)=C*(F001*EPSP/0.001 + F001M - F001)
  135 CONTINUE
      PSR = PSR + FSR(M)*ASR
      MSR = MSR - FSR(M)*ASR*ZS(M)
      IF(EPSL(M)-0.00) 140,150,150
  140 C=1.0
      GO TO 160
  150 C=-1.0
  160 IF(DABS(EPSL(M))-EPSP) 170,170,180
  170 FSL(M) = C*F001*DABS(EPSL(M))/0.001
      GO TO 175
  180 F001M=(50.0-0.04*TR)*(1.0-DEXP((-30.0+0.030*TR)*
     .DSQRT(DABS(EPSL(M)) - EPSP + 0.001)))*6.90D 6
      FSL(M)=C*(F001*EPSP/0.001 + F001M - F001)
  175 CONTINUE
      PSL = PSL + FSL(M)*ASR
      MSL = MSL + FSL(M)*ASR*ZS(M)
   80 CONTINUE
      MST = MSR + MSL
      PST = PSR + PSL
      RETURN
      END
```

```
C
C     ---------------------------------------------------------------
C     ***************************************************************
C     ===============================================================
      SUBROUTINE LOADMC(TT,EPAXL,ZC,ACE,ROH,PCT,MCT,EPCR,EPCL,FCR,FCL,
     .                  FDCO)
C     ===============================================================
C     ***************************************************************
C     ---------------------------------------------------------------
C
C     M1  =ML-1=NUMBER OF LAYERS DIVISIONS - 1
C     NI  =NUMBER OF SECTORS
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TT(40),ZC(40,20),ACE(40,20)
      DIMENSION EPCR(40,20),EPCL(40,20),FCR(40,20),FCL(40,20)
C
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/PROPR/PHI,EMIS,EPTOT
      REAL MCT
C
C
C      CALCULATION OF STRAINS IN THE CONCRETE REIGON
C
      DO 10  M=1,M1
      DO 10  N=1,NI
      ALFAC = (0.0080*TT(M) + 6.0)*1.0D-06
      EPTC = -ALFAC*(TT(M)-20.0)
      EPCR(M,N) = EPTC + EPAXL + EPTOT + ZC(M,N)/ROH
      EPCL(M,N) = EPTC + EPAXL + EPTOT - ZC(M,N)/ROH
   10 CONTINUE
C
C      CALCULATION OF STRESSES IN CONCRETE REIGON
C      FDCO = THE CYLINDER STRENGTH OF CONCRETE AT ROOM TEMP.
C             (F-PRIME-C-O)
C      FPC  = THE CYLINDER STRENGTH OF CONCRETE AT TEMP. T
C             (F-PRIME-C)
C
      PCT = 0.00
      MCT = 0.00
      DO 20  M=1,M1
      DO 20  N=1,NI
      EPMAX = 0.0025 + (6.0*TT(M)+0.04*TT(M)**2.0)*1.0D-06
      FPC   = FDCO*(2.011-2.353*(TT(M)-20)/1000.0)
      IF(FPC .GT. FDCO) FPC=FDCO
      IF(FPC .LE. 0.00) FPC = 0.00
      IF(EPCR(M,N)-0.00D 00) 30,40,40
   40 FCR(M,N)=0.00D 00
      GO TO 50
   30 IF(DABS(EPCR(M,N))-EPMAX) 60,60,70
```

```
   60 FCR(M,N)=FPC*(1.0-((EPMAX+EPCR(M,N))/EPMAX)**2)
      GO TO 50
   70 FCR(M,N)=FPC*(1.0-((-EPCR(M,N)-EPMAX)/(3.0*EPMAX))**2)
      IF(FCR(M,N) .LT. 0.00) FCR(M,N)=0.00
   50 CONTINUE
      IF(EPCL(M,N)-0.00D 00) 80,90,90
   90 FCL(M,N)=0.0D 00
      GO TO 100
   80 IF(DABS(EPCL(M,N))-EPMAX) 110,110,120
  110 FCL(M,N)=FPC*(1.0-((EPMAX+EPCL(M,N))/EPMAX)**2)
      GO TO 100
  120 FCL(M,N)=FPC*(1.0-((-EPCL(M,N)-EPMAX)/(3.0*EPMAX))**2)
      IF(FCL(M,N) .LT. 0.00) FCL(M,N)=0.00
  100 CONTINUE
C
C      THE SUMMTION OF THE TOTAL LOADS AND TOTAL MOMENTS IN THE
C      CONCRETE REIGON OF THE CROSS-SECTION
C
C      PCT=TOTAL LOAD IN CONCRETE
C      MCT=TOTAL MOMENTS IN CONCRETE REIGON
C
      PCT = PCT + 2.0*(FCR(M,N)+FCL(M,N))*ACE(M,N)
      MCT = MCT + 2.0*(-FCR(M,N)+FCL(M,N))*ACE(M,N)*ZC(M,N)
   20 CONTINUE
      RETURN
      END
C
C
C      ***********************************************************
C      ===========================================================
      SUBROUTINE CHECK(MT,PT,ECC,Y,EPAXL,TIME,TIMLIM,TIMEMI,TF,TT,TR,
     .PST,PCT,MST,MCT,EPSR,EPSL,EPCR,EPCL,FSR,FSL,FCR,FCL,ICHEK,IWRITE)
C      ===========================================================
C      ***********************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/NUMB1/ML,NI,NR
      COMMON/NUMB2/M1,IR
      COMMON/DIMEN/RO,RS,DX,DIAMR,DX2
      COMMON/PROPR/PHI,EMIS,EPTOT
      COMMON/NUMB3/NUMB,NUM,INDIC,IND,NNN,MMM
      DIMENSION EPSR(20),EPSL(20),EPCR(40,20),EPCL(40,20)
      DIMENSION FSR(20),FSL(20),FCR(40,20),FCL(40,20)
      DIMENSION TT(40),ITYPE(40)
      REAL MST,MCT,MT
C
C      THIS SUBROUTINE IS CHECKING THE VALUE OF AXIAL STRAIN DUE
C      TO THE AXIAL LOAD AND TRY TO INCREASE IT OR DECREASE IT
C      UNTIL THE BENDING IS BALANCED
C
C
      ICHEK = 0
```

```
      PECCY=PT*(ECC+Y)
      IF(TIME-0.DO) 420,10,15
   10 FACT=2.DO
      IF(ECC .LT. 0.004)  FACT = 20.0
      GO TO 20
   15 FACT = 10.0
   20 IF(DABS(MT-PECCY) - 0.020*MT)  125,125,25
   25 IF(NUMB - 1)  30,100,100
   30 IF(NUM - 1)  40,35,35
   35 IF(DABS(MT-PECCY) - 0.020*MT)  125,125,100
   40 IF(PT - 0.00)  45,45,55
   45 IF(IND - 1)  50,55,55
   50 EPAXL = EPAXL - 0.001/FACT
      GO TO 1510
   55 IF(MT - 0.00)  115,60,60
   60 IF(DABS(MT-PECCY) - 0.020*MT)  125,125,65
   65 IF(MT - PECCY)  70,420,85
   70 IF(INDIC - 0)  420,75,80
   75 EPAXL = EPAXL + 0.001/FACT
      IND = IND+1
      GO TO 1510
   80 EPAXL = EPAXL + 0.0005/FACT
      NUM = NUM+1
      GO TO 1510
   85 IF(IND - 0)  420,90,95
   90 EPAXL = EPAXL - 0.001/FACT
      INDIC = INDIC+1
      GO TO 1510
   95 EPAXL = EPAXL - 0.0005/FACT
      NUM = NUM+1
      GO TO 1510
  100 IF(MT - PECCY)  105,125,110
  105 IF(NNN .EQ. 1) GO TO 115
      EPAXL = EPAXL + 0.0001/FACT
      NUMB = NUMB+1
      MMM = 1
      GO TO 1510
  110 IF(MMM .EQ. 1) GO TO 115
      EPAXL = EPAXL - 0.0001/FACT
      NUMB = NUMB+1
      NNN  = 1
      GO TO 1510
  115 WRITE(6,120)  NUMB
  120 FORMAT(////,5X,'MOMENTS NOT BAL. WITHIN 2%;NUMBER=',I3)
  125 WRITE(6,130)  TIMEMI
  130 FORMAT(//,5X,'TIME..................=',3X,F7.1,3X,'MIN')
      WRITE(6,135)  TF
  135 FORMAT(5X,'FIRE TEMPERATURE......=',3X,F7.1,3X,'C',//)
C
      IF(IWRITE .EQ. 0) GO TO 147
C
```

```
      WRITE(6,146)
  146 FORMAT(5X,'NOTE : FOR STEEL ELEMENTS    : ELEM. TYPE = 1',/
     .       12X,'FOR CONCRETE ELEMENTS : ELEM. TYPE = 2',//)
      WRITE(6,145)
  145 FORMAT(25X,'STRAINS & STRESSES IN STEEL',/
     .       25X,'******* * ******** ** *****',//,
     .      5X,'BAR. NO.',3X,'ELEM. TYPE',3X,'TEMP.(C)',3X,'R. STRAIN',
     .      3X,'L. STRAIN',3X,'R. STRESS',3X,'L. STRESS',/
     .      5X,'--- ---',3X,'---- ----',3X,'--------',3X,'-- ------',
     .      3X,'-- ------',3X,'-- ------',3X,'-- ------',/
     .      43X,'MM/M',7X,'MM/M',8X,'MPA',8X,'MPA',/)
      DO 150 M=1,IR

      ITYPE(M) = 1
      A        = EPSR(M)*1.0D 03
      B        = EPSL(M)*1.0D 03
      C        =  FSR(M)*1.0D-06
      D        =  FSL(M)*1.0D-06
      WRITE(6,160) M,ITYPE(M),TR,A,B,C,D
  155 FORMAT(5X,'(',I3,',',I3,')',I8,6X,5D12.4)
  160 FORMAT(9X,I4,I8,5D12.4)
  150 CONTINUE
      WRITE(6,165)
  165 FORMAT(///,25X,'STRAINS & STRESSES IN CONCRETE',/
     .       25X,'******* * ******** ** ********',//,
     .      5X,'ELEM. NO.',3X,'ELEM. TYPE',3X,'TEMP.(C)',3X,'R. STRAIN'
     .      3X,'L. STRAIN',3X,'R. STRESS',3X,'L. STRESS',/
     .      5X,'---- ---',3X,'---- ----',3X,'--------',3X,'-- ------'
     .      3X,'-- ------',3X,'-- ------',3X,'-- ------',/
     .      43X,'MM/M',7X,'MM/M',8X,'MPA',8X,'MPA',/)
      DO 170 M = 1,M1
      DO 170 N = 1,N1
      ITYPE(N) = 2
      A        = EPCR(M,N)*1.0D 03
      B        = EPCL(M,N)*1.0D 03
      C        =  FCR(M,N)*1.0D-06
      D        =  FCL(M,N)*1.0D-06
      WRITE(6,155) M,N,ITYPE(N),TT(M),A,B,C,D
  170 CONTINUE
C
  147 CONTINUE
C
      WRITE(6,180)
  180 FORMAT(///,25X,'LOADS & MOMENTS IN STEEL & CONCRETE',/
     .       25X,'***** * ******* ** ***** * ********',//)
C
      A     = PST*1.0D-03
      B     = PCT*1.0D-03
      C     = MST*1.0D-03
      D     = MCT*1.0D-03
      E     =  PT*1.0D-03
```

```
      F     =  MT*1.0D-03
      WRITE(6,185) A,B,C,D,E,F
  185 FORMAT(5X,'LOAD    IN STEEL.................=',1D15.6,'(KN  )',/
     .         5X,'LOAD    IN CONCRETE..............=',1D15.6,'(KN  )',/
     .         5X,'MOMENT IN STEEL.................=',1D15.6,'(KN-M)',/
     .         5X,'MOMENT IN CONCRETE..............=',1D15.6,'(KN-M)',/
     .         5X,'TOTAL  LOAD.....................=',1D15.6,'(KN  )',/
     .         5X,'TOTAL  MOMENT...................=',1D15.6,'(KN-M)',//)
C
      A     = PECCY*1.00D-03
      B     = EPAXL*1.00D 03
      C     = EPAXL*0.35D 04
      D     =       Y*1.00D 03
      WRITE(6,195) A,B,C,D
  195 FORMAT(5X,'MOMENT(LOAD*(ECC + Y))..........=',1D15.6,'(KN-M)',/
     .         5X,'RELATIVE AXIAL STRAIN...........=',1D15.6,'(MM/MM)',/
     .         5X,'TOTAL AXIAL STRAIN..............=',1D15.6,'(MM   )',/
     .         5X,'LATERAL DEFLECTION AT MIDHIGHT..=',1D15.6,'(MM)',//)
      IF(Y - 0.000099)  270,420,275
 270  Y = Y + 0.00002
      GO TO 1500
 275  IF(MT - 0.D0)  280,290,290
 280  IF(TIME - TIMLIM)  285,420,420
 285  Y = 0.00010D0
      GO TO 190
 290  IF(Y - 0.0004D0)  295,300,300
 295  Y = Y + 0.00005001D0
      GO TO 1500
 300  IF(Y - 0.0010)  305,305,310
 305  Y = Y + 0.0001001D0
      GO TO 1500
 310  IF(Y - 0.0020)  315,315,320
 315  Y = Y + 0.00020010
      GO TO 1500
 320  IF(Y - 0.0100)  325,325,350
 325  Y = Y + 0.00050010
      GO TO 1500
 350  Y = 0.000100
      IF(TIME - TIMLIM)  190,190,420
  190 ICHEK = 1
      RETURN
 1500 ICHEK = 2
      RETURN
 1510 ICHEK =3
      RETURN
 420  CONTINUE
      RETURN
      END
```