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Aspects of String Compactification

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ASPECTS OF STRING COMPACTIFICATION

by

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ABSTRACT

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This dissertation addresses some interesting problems in string compactification relevant to phenomenology, especially cosmological models derived from string theory. Most attention is drawn to stabilizing the moduli and discussing the cosmology solutions given the effective potential obtained from string theory. We first discuss compactification of type IIB string theory in the presence of flux. With the moduli stabilized, we obtain the effective potential in the large volume limit. We then focus on getting the vacua of the potential and solving the cosmology equations. Finally we compare the solutions with observations in cosmology.

Chapter 1

Introduction

1.1 Why String Compactification

The Standard Model of particle physics together with general relativity in $3+1$ dimensions have been proved to be a very successful low energy effective field theory of nature. The predictions of the Standard Model and general relativity up to energy scales of 100 GeV have been tested by numerous high energy physics experiments as well as some of the cosmological observations which showed good agreement between the data and the predictions. However, the Standard Model Lagrangian consists of free parameters (the coupling constants, mixing angles, etc) which cannot not be defined/constrained by the theory. To be compatible with the observation results, those free parameters need to be fine tuned extensively which makes the theory less satisfactory from the phenomenological point of view. Moreover, the Standard Model is not a complete fundamental theory in the sense that it does not include quantum gravity into its framework. The quantization of general relativity perturbatively at high energies is a nontrivial task due to the non-renormalizable nature of the infinities (divergences) encountered in the procedure.

There are many extensions to the Standard Model such as Minimal Supersymmetric Standard Model (MSSM) and the Grand Unified Theories (GUT). However, those extensions usually introduce new structures and parameters which make the theory even more complicated This is not satisfactory since any truly fundamental theory should be able to explain the source of the parameters and include all the fundamental interactions (gravity, etc) in its framework

String theory, on the other hand, is a unified and consistent theory which can solve the divergence problem at high energy scales and unifies the elements in quantum field theory, including gravity In string theory, the fundamental objects in nature are not point particles but one-dimensional strings (perturbative) and higher dimensional objects like the branes which characterize the nonperturbative regime String theory also lives in more than 3+1 dimensions The critical dimension of (supersymmetric) string theory is 10 (or 11 for M-theory) If string theory is correct, the low energy effective perturbative descriptions of string theory should be equivalent to general relativity and quantum field theory in $3 + 1$ dimensions

To derive the 3+1 dimensional physics from the ten-dimensional string theory, we need to do dimensional reduction, ie, via string compactifications, over the extra six dimensions Those extra dimensions are often called the internal space The parameters which characterize the internal space, like the length and the volume, aka the moduli, provide an intrinsic explanation for the origin of the parameters in the Standard Model physics The procedure of dimensional reduction involves stabilizing the moduli and quantizing the background flux (nonvanishing tensor fields, quantization of $F_{\mu\nu}$ of electromagnetism, in the internal space) In this dissertation, we will focus on string compactifications on six-dimentional Calabi-Yau manifolds

The study of string compactification can lead to better understanding of both practical problems, such as the driving force behind the formation of cosmological structure, and theoretical problems, such as how to constrain/define string theory beyond perturbation theory Phenomenologically, for our purpose in this dissertation, the effective action contains a Calabi-Yau sector and a Standard Model sector

1.2 Cosmology From String Theory

The major goal of this dissertation is to use the result from string compactification to study perhaps the most fundamental subject in modern cosmology the early universe Most theories/models used in studying the early universe are based on assumptions and fine tunnings due to the extreme aspects (the energy scale, etc) of the universe String theory can provide us with a powerful tool for exploring the origin and evolution of the universe Due to the large number of choices for the fluxes in string theory, the result is that there are a large number of vacua The spectrum of these vacua is called the string landscape Because the number of the vacua is so huge, it is almost unrealistic to find the right vacuum for our universe by studying each individual vacuum which is known as the string landscape problem As a result, we can say that string theory is not really a single theory but rather a large set of theories describing different universes Without a sampling mechanism for the string landscape, it is hard to figure out the correct theory for our universe To work around this problem, we often use moduli stabilization with flux to get some realistic models by requiring the resulting low energy effective theory being consistent with the four-dimensional Standard Model physics To guide for model building, the statistical approach, the study of the overall distribution of the vacua, are often used (1) (2) (3)

The advantage of a string inspired or derived inflationary model is that the low energy effective theory comes directly from a unified theory (string theory) Thus it enables us to connect string theory with observations in cosmology, from which one can test or constrain parameters in string theory Since the string energy scale is usually much higher than the energy scale in particle physics, it is almost impossible to find the fingerprint of string theory in current high energy physics experiment The early universe, on the other hand, involves processesat or near the string scale, and thus becomes an ideal test ground for theories beyond the Standard Model physics such as string theory

1.3 Dissertation Structure

The dissertation consists of two parts: string compactifications and string cosmology. Starting from an introduction to complex geometry in Chaper 2, we present some basic concepts and results for Calabi-Yau manifolds as well as compactifications of type IIB theory on Calabi-Yau orientifolds. In Chapter 3, we briefly discuss flux compactifications and demonstrate the procedure of moduli stabilization in type IIB thoery. Then in Chapter 4, we show the standard scalar field theory and the solutions in cosmology. We derive the effective action from string compactifcation based on the previous chapters in Chapter 5 and use it to build an inflationary model. In Chaper 6, we calculate the non-gaussianity for our inflationary model using both analytical method and numerical simulation. We summarize the dissertation in Chapter 7. Finally, the appendix contains both the Mathematica codes and the analytic detail that we used to solve the equations of motion in Chapter 4. The diagram below gives the main structure of this dissertation:

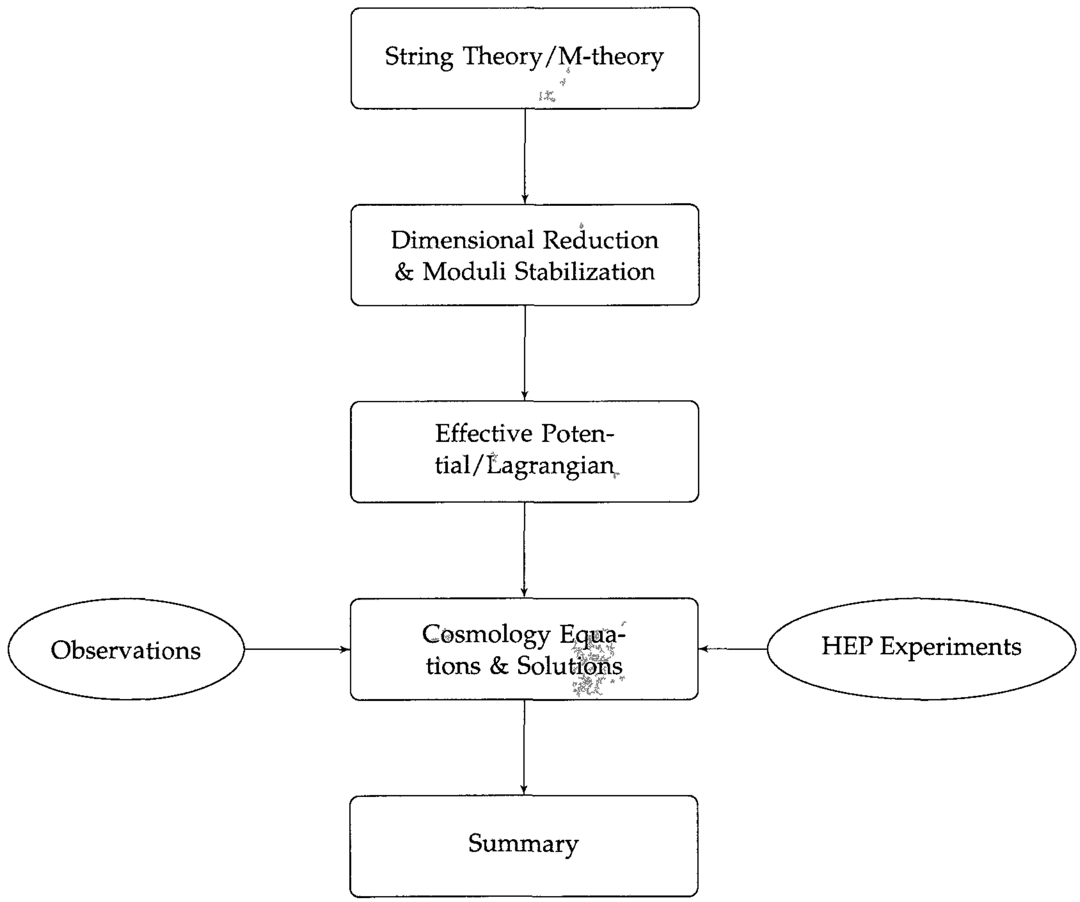


Figure 1.1: The main structure of the thesis.

Chapter 2

String Compactification - Mathematical Preliminary

2.1 A First View on Extra Dimensions

2.1.1 The Mathematical Nature

It looks obvious that the world we live in is a (3+1) dimensional spacetime. However, it is possible that there might be much more information which are hidden from our sight in some invisible extra dimensions. In fact, if we are relocated to a three-dimensional hypersphere of some higher-dimension space we may not notice the difference from our own space.

Mathematics tell us that the geometry in higher dimensions are startlingly different from that in lower dimensions. For example, as shown, the two linked rings in Figure 2.1 are not separable without breaking one or the other. However, in a four-dimensional space, these seemingly inseparable rings in three dimensions can be naturally separated via continuous deformation. It is those mathematical properties that give unique physical characteristics can be used to explain important ideas in physics.

Since we do not have the ability to visualize higher than three-dimensions, it might be hard for us accept the possibility that we might indeed live in such a world. The

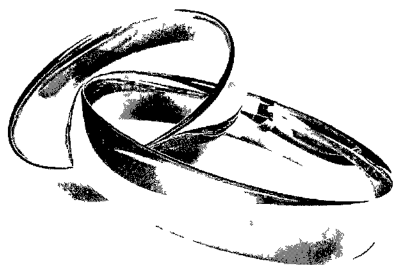


Figure 2.1: Two linked rings in three dimensions. Computer generated.

spatial intuition people have is based on the three-dimensional image of the world that is projected onto their brain. On the other hand, a blind person's spatial intuition is primarily the result of tactile experience which may turn out to be deeper. Studies show that people who were born blind but gained sight afterward usually have a hard time distinguishing a square object from a circle during their first few days after gaining vision (via medical treatment). In contrast, they can immediately tell that a torus is different from a sphere (4). In mathematics, we know that topological structures like a square and a circle are topologically equivalent, while a torus and a sphere are not. It seems that our belief that what we see are the absolute truth sometimes limit our perception of the world. In this sense, mathematics provides us with the ultimate tool for exploring the nature beyond the reaches of our intuition. In string theory, we try to build a single, consistent mathematical frame work that can describe the fundamental interactions in nature.

2.1.2 Physics Beyond Three-Dimensions

People have been considering the possible existence of extra spatial dimensions beyond three for many years. In the 1920's, Kaluza (5) and Klein (6) first studied the case that electromagnetism and gravity in a five-dimensional spacetime where the fifth dimension is curled up in a tiny circle Despite of their failure to derive any realistic four-dimensional theory from this consideration, the Kaluza-Klein theory is very inspiring Since then, people have been developing and using the dimensional reduction techniques to study problems involving extra-dimensions which later on became one of the fundamental ingredients for string theory (7)

In Kaluza-Klein (KK) theory, the massless free particle, graviton, lives in a flat $R^{3+1} \times S^1$ spacetime The circle S^1 is the compact fifth dimension which is curled up, such that the coordinate is periodic

$$x_4 \sim x_4 + 2\pi l \tag{2 1}$$

The metric for this configuration is

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + {dx_4}^2 \tag{2 2}$$

where the index μ, ν label the coordinates of the four dimensional spacetime and l denote the radius of S^1 Under the assumption, the Poincare invariance $ISO(1,4)$ is broken down to $ISO(1,3) \times U(1)$

A generalization of the KK theory is the case where the n-dimensional spacetime is decomposed to a $(n-1)$-dimensional spacetime and a circle S^1 via the dimensional reduction techniques developed by Scherk and Schwarz (8)

In string theory, the fundamental objects of nature are not zero-dimensional point particles but are one-dimensional strings as well as higher dimensional branes The idea of extra dimensions or compactified spaces plays an important/essential role in the theory It is often assumed that the sizes of the compactified spaces are small enough (usually considered as the size of the order of the Planck length) that makes them remain hidden from to observations However, the large extra dimensions in string theory are also being actively studied (9) One of the most recent development is the accessibility of extra dimensions at the LHC(the Large Hadron Collider) regarding which studies show that a rather low energy scale (string tension) seems to be particularly compatible with the existence of large extra dimensions which might leave signatures in LHC (10)

By studying the string dynamics, the goal is to derive the low energy effective theory quantum field theory in four dimensions, i e , the Standard Model physics

2.2 Compactification in String Theory

Unlike many non-string phenomenological studies that have been done, which are often based on standard field theory models that are built upon a bottom-up approach, we will derive the effective field theory from string theory via compactification To make contact between ten-dimensional string theory and the four-dimensional physics, which is usually called dimension reduction in string theory, we need to study what happens to the extra six dimensions

Throughout this thesis, we will consider the six internal dimensions to be a Calabi-Yau space of complex dimension three The reason behind this choice is that we want the resulting 3+1 dimensional theory to possess $\mathcal{N} = 1$ supersymmetry For this to happen, it was shown that the internal space must be Ricci-flat and Kahler (11) One can also show that if the internal dimensions possess $SU(3)$ holonomy, the D=3+1 theory which results from $\mathcal{N} = 1$, $D = 10$ string theory will have precisely $\mathcal{N} = 1$ supersymmetry[1] Since Calabi-Yau manifold is Ricci-flat and Kahler and Calabi-Yau three-folds have $SU(3)$ holonomy, it satisfies the above consistency requirements

A full discussion of the mathematics regarding the Calabi-Yau manifolds is far beyond the scope of this thesis The following section introduces some basic concepts of the Calabi-Yau manifold which is useful for later discussion For a comprehensive discussion on Calabi-Yau manifolds, we refer to (12) (13)

[1] $\mathcal{N} = 1$ supersymmetry arises when the ten-dimensional string theory is the heterotic string compactified on a Calabi-Yau manifold or type II compactifications on Calabi-Yau orientifolds

2.2.1 Some Geometry

This section we will briefly review the basic concepts and results in complex manifold, which can be found in various textbooks of complex manifold, such as (14) (15).

Differential Manifold An m-dimensional *differential manifold* M satisfies

- M is a topological space;
- M is endowed with a topology which consists of open subsets $\{U_i\}$ which covers M;
- Let the homeomorphism from U_i to an open subset U_i' of R^m be ϕ_i. The map $\psi_{ij} = \phi_i \circ \phi_j^{-1}$ is infinitely differentiable.

From the definition, we can assign a coordinate representation to the function $f \quad M \to R$ via the map $f \circ \phi_i^{-1} : \phi_i(U_i) \to R$ which is a map from $R^m \to R$. The differentiability of f can be analyzed using multi-variable calculus. For the overlap of two subsets: $U_{ij} = U_i \cap U_j$, the differentiability of f is the same for both of the coordinate representation $f \circ \phi_i^{-1}$ and $f \circ \phi_j^{-1}$. And we have the relation:

$$f \circ \phi_i^{-1} = f \circ \phi_j^{-1} \circ \psi_{ij} \tag{2.3}$$

where $\psi_{ij} = \phi_j \circ \phi_i^{-1}$ is called the *transition function.*

Analysis on Differential Manifold On a manifold M, a vector is defined to be a **tangent vector** to a curve in M. Let $0 \in (a, b)$, a curve $c : (a, b) \to M$, and a function $f : M \to \mathbb{R}$. A *tangent vector* on a manifold M at $c(0) \in M$ is a directional derivative of the function $f(c(t))$ *along the curve* $c(t)$ at $t = 0$. In terms of local coordinate:

$$X = \frac{dx^\mu(c(t))}{dt}|_{t=0} \frac{\partial}{\partial x^\mu} = X^\mu \frac{\partial}{\partial x^\mu}$$

$$\frac{df(c(t))}{dt}|_{t=0} = X^\mu \cdot \frac{\partial f}{\partial x^\mu}$$

where

$$\frac{\partial f}{\partial x^\mu} = \frac{\partial (f \circ \phi^{-1}(x))}{\partial x^\mu},$$

and

$$f(p) = f \circ \phi^{-1}(x), \quad \phi(p) = x, \quad \text{for } p \in M$$

Complex Manifold The next concept is the notion of a *complex manifold* M is a complex manifold if

- M is a differential manifold,
- the transition function ψ_{ij} is holomorphic

A holomorphic function f $C^m \to C$ is a complex valued function $f(z) = u + iv$ which satisfies the Cauchy-Riemann relations for each $z^\mu = x^\mu + iy^\mu$

$$\frac{\partial u}{\partial x^\mu} = \frac{\partial v}{\partial y^\mu}, \qquad \frac{\partial v}{\partial x^\mu} = -\frac{\partial u}{\partial y^\mu} \tag{2 4}$$

Complex Structure On a complex manifold M, the complex structure is defined as a linear map J_p between two complex vector space, J_p $T_pM \to T_pM$, by

$$J_p\left(\frac{\partial}{\partial x^\mu}\right) = \frac{\partial}{\partial y^\mu}, \quad J_p\left(\frac{\partial}{\partial y^\mu}\right) = -\frac{\partial}{\partial x^\mu} \tag{2 5}$$

or equivalently,

$$J_p\left(\frac{\partial}{\partial z^\mu}\right) = i\frac{\partial}{\partial z^\mu}, \quad J_p\left(\frac{\partial}{\partial \bar{z}^\mu}\right) = -i\frac{\partial}{\partial \bar{z}^\mu} \tag{2 6}$$

Note that J_p is a type $(1,1)$ tensor and satisfies $J_p^2 = -1$ It takes the form

$$J_p = \begin{pmatrix} 0 & -I_m \\ I_m & 0 \end{pmatrix} \tag{2 7}$$

with respect to the basis

$$\{\frac{\partial}{\partial x^1}, \quad , \frac{\partial}{\partial x^m}, \frac{\partial}{\partial y^1}, \quad , \frac{\partial}{\partial y^m}\} \tag{2 8}$$

Equavalently, on the (anti)holomorphic bases

$$J_p = \begin{pmatrix} iI_m & 0 \\ 0 & -iI_m \end{pmatrix} \tag{2 9}$$

which when written in tensor form is

$$J_p = idz^\mu \otimes \frac{\partial}{\partial z^\mu} - id\bar{z}^\mu \otimes \frac{\partial}{\partial \bar{z}^\mu} \tag{2 10}$$

Kähler manifolds and Holonomy group A Riemannian metric g of a complex manifold M is called a Hermitian metric if

$$g(J_p X, J_p Y) = g(X, Y) \tag{2 11}$$

for any $X, Y \in T_p M$

We further define a *Kahler form* Ω of a Hermitian metric g if Ω satisfies

$$\Omega(X, Y) = g(J_p X, Y), \quad X, Y \in T_p M \tag{2 12}$$

We apply the metric compatibility condition on the Hermitian manifold One get the covariant derivatives of the complex structure.

$$\nabla_\kappa J = \nabla_\kappa J = 0 \tag{2 13}$$

From the metric compatibility requirement, one can derive the connection coefficients

$$\Gamma^\lambda_{\kappa\mu} = g^\nu \lambda \partial_\kappa g_{\mu\bar{\nu}}, \quad \Gamma^{\bar{\lambda}}_{\kappa\bar{\nu}} = g^{\lambda\mu} \partial_\kappa g_{\mu\bar{\nu}} \tag{2.14}$$

The Riemann curvature tensor R is

$$\mathcal{R}(X, Y)Z = (\nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X\,Y]})Z \tag{2 15}$$

where $X, Y, Z \in T_pM$ and ∇ is the covariant derivative

One can show that the only Riemann tensor is

$$\mathcal{R}_{\mu\nu} = -\partial_\nu\partial_\mu \log G \tag{2 16}$$

where $G = \det(g_{\mu\nu})$ And hence the Ricci form

$$\mathcal{R} = i\mathcal{R}_{\mu\nu} dz^\mu \wedge dz^\nu = -i\partial\bar{\partial}\log G \tag{2 17}$$

A Hermitian manifold M is a Kahler manifold if its Kahler form is closed

$$d\Omega = 0, \tag{2 18}$$

where the Kahler form is a real two-form, defined by

$$\Omega = i g_{\mu\nu} dz^\mu \wedge dz^\nu \tag{2 19}$$

A little algebra reveals that, to satisfy (2 18), the Kahler metric $g_{\mu\nu}$ must have the following properties

$$\frac{\partial g_{\mu\nu}}{\partial z^\lambda} = \frac{\partial g_{\lambda\nu}}{\partial z^\mu}, \quad \frac{\partial g_{\mu\nu}}{\partial \bar{z}^\lambda} = \frac{\partial g_{\mu\lambda}}{\partial \bar{z}^\nu} \tag{2 20}$$

Clearly, any $g_{\mu\nu}$ that given by

$$g_{\mu\nu} = \partial_\mu\partial_\nu K(z, \bar{z}) \tag{2 21}$$

satisfies (2 20), where K is a scalar function on the Kahler manifold It can be shown that any Kahler metric can be written as (2 21) The function K is called the Kahler potential of the Kahler manifold The Kahler potential is locally a real function in any coordinate chart on the manifold The Kahler metric can be globally defined by imposing the *Kahler* *tranformations* on the Kahler potential

$$K(z,\bar{z}) \sim K(z,\bar{z}) + h(z) + \tilde{h}(\bar{z}) \tag{2 22}$$

The holonomy groups of Kähler manifold We now take a brief look at the holonomy groups of Kahler manifold which is the last element we need to introduce before discussing the Calabi-Yau manifold Consider a loop C which begins and ends at a point p on a Hermitian manifold (M, g) with complex dimension m M is equiped with a metric connection Γ Parallel transport a vector $X \in T_pM$ along the loop C The orientation of the new vector X' after the parallel transportation will generally end up being different from X's Let

$$X' = hX \tag{2 23}$$

where h turns out to be an $U(m)$ transformation We find that $U(m)$ is decomposed into $SU(m)\times U(1)$ For the Ricci-flat metric, this means that the parallel transportation group is contained in $SU(m)$ Repeating the procedure by parallel transporting the vector along all possible loops at p, we get a collection of $h \in SU(m)$ The group regarding the change of the orientations after these parallel transportations is called the holonomy of M, which is a subgroup of $SU(m)$

2.2.2 Calabi-Yau Manifold

A Calabi-Yau manifold is a compact Kahler manifold with a vanishing first Chern class (16)

$$c_1 = \left[\frac{\mathcal{R}}{2\pi}\right] = 0 \tag{2 24}$$

where g is the metric for the manifold which is thus flat It was proven by Yau that a compact Kahler manifold with vanishing first Chern class admits a Ricci-flat metric The r-th Chern class of a manifold M is defined as of $c_r(M) = H^{2r}(M), r = 1, \ ,m$ It can be obtained from the expansion

$$c(M) = 1 + \sum_{j} c_j(M) = 1 + tr(\mathcal{R}) + tr(\mathcal{R} \wedge \mathcal{R} - 2(tr(\mathcal{R}))^2) + \tag{2 25}$$

where $\mathcal{R}$ is the curvature two-form Thus the first Chern class is equal to the trace of the curvature two-form

For $n = 1$ (complex dimension) the only Calabi-Yau manifold (denoted as CY_1) is the torus T, for $n = 2$ we have the K_3 manifold For $n \leq 3$ there are more than one Calabi-Yau manifold The $n = 3$ case is the main focus of this chapter

The Calabi-Yau 3-fold We are interested in Calabi-Yau manifolds of 3 (complex) dimensions One such example is the quintic hypersurface on $\mathbb{P}^4$, the 4 dimensional complex projective plane, with homogeneous coordinate $(z_1, \quad , z_5)$ It satisfies the follow polynomial equation

$$P(z_1, \quad , z_5) = 0 \tag{2 26}$$

where P is some homogeneous polynomial of degree five

One can show that the expansion of (2 25) takes the form (12)

$$c(M) = \frac{(1+J)^{n+1}}{1+qJ} = 1 + \sum_{k=0}^{r} \binom{n+1}{k} (-q)^{r-k} J^r \tag{2 27}$$

where n is the dimension of $\mathbb{P}^4$, q is the degree of the quitic polynomial and J is Kahler form on $\mathbb{P}^n$ As expected, c_1 vanishes for $n = 4$, $q = 5$ here

Hodge numbers of the Calabi-Yau 3-fold The Betti number b_p of the de Rham Cohomology group $H^p(M)$ of the Kahler manifold M is a sum of the Hodge numbers

$$b_j = \sum_{p=0}^{j} h_{p\, j-p} \tag{2 28}$$

Any Calabi-Yau orbifold can be characterized by its Hodge numbers Because the cohomology group $H^p(M)$ is isomorphic to $H^{m-p}(M)$, where m is the dimension of M, the

Hodge numbers satisfy the following relation

$$h_{p,0} = h_{m-p,0} \tag{2.29}$$

There are two additional duality relations

$$h_{p,q} = h_{q,p}, \quad h_{p,q} = h_{m-q,m-p} \tag{2.30}$$

In the case of a Calabi-Yau 3-fold, we have

$$h_{0,0} = 1, \quad h_{1,0} = h_{0,1} = 0$$

Thus we have the following Hodge diamond

$$\begin{array}{c|ccccccc|}
 & & & & 1 & & & \\
 & & & 0 & & 0 & & \\
 & & 0 & & h_{1,1} & & 0 & \\
\text{d=3} & 1 & & h_{2,1} & & h_{1,2} & & 1 \\
 & & 0 & & h_{1,1} & & 0 & \\
 & & & 0 & & 0 & & \\
 & & & & 1 & & &
\end{array}$$

The Eular number of the Calabi-Yau 3-fold is

$$\chi = \sum_{p=0}^{2m} (-1)^p b_p = \sum_{p=0}^{2\,3} (-1)^p b_p = 2(h^{1,1} - h^{2,1}) \tag{2.31}$$

which can also be obtained by the *index theorem* (17) (18):

$$\chi = \sum_{r,s} (-1)^{r+s} h_{r,s} \tag{2.32}$$

Example of Calabi-Yau Manifold Let us consider the most popular and simplest CY manifold the quintic hypersurface in $\mathbb{P}_4$ given by the solutions which satisfy the homogeneous polynomial equation

$$Q(z) = \sum_{\sum n_i = 5} c(n_0, n_1, n_2, n_3, n_4) z_0^{n_0} z_1^{n_1} z_2^{n_2} z_3^{n_3} z_4^{n_4} = 0 \qquad (2\ 33)$$

This equation contains $\frac{(5+5-1)!}{5!((5+5-1)-5)!} = 126$ (complex) coefficients $c(n_0, n_1, n_2, n_3, n_4)$ (19) Restraining the coordinates in $GL(5)$, the number is reduced to $126 - 25 = 101$ The number of independent coefficients of the quintic polynomial then gives the number of complex structure deformations, i e , $h_{2\,1}$ As a Kahler manifold, it has one Kahler deformations, ie $h_{1\,1} = 1$, which is inherited from the fact that CP^4 is a Kahler deformation

2.2.3 Type IIB Compactifications On Calabi-Yau Threefold

There are many ways to obtain an $\mathcal{N} = 1$ supersymmetric theory in 4-dimensions from 10-dimensional string theory type II A/B on Calabi-Yau orientifolds, orbifold compactifications of the heterotic string, F-theory on Calabi-Yau four-fold, as well as M-theory on manifolds of G2 holonomy (20) (21) In this thesis, we choose to focus on type IIB theory compactification with branes and fluxes There is a reason why we choose type IIB instead of type IIA Firstly, type IIB compactifications on Calabi-Yau three-fold, in appropriate limit, is equivalent to F-theory compactified on Calabi-Yau four-folds (22) [2] Secondly, the nontrivial warp factor (see Chapter 3) indicates that the manifold is not exactly a Calabi-Yau manifold, which will make the problem more complicated However, in type IIB compactification the underlying manifold can be approximately treated as a Calabi-Yau manifold when the volume of which is large and the warp factor only introduces a small perturbation This is not the case in type IIA compactification (23) On the other hand, due to mirror symmetry between type IIA and type IIB orien-

[2] More generally, there is a limit, in which type IIB compactifications on Calabi-Yau n-fold after orientifolding is equivalent to F-theory compactified on Calabi-Yau $(n+1)$-folds

tifolds, both compactifications on Calabi-Yau three-fold can be shown to be equivalent (23) Therefor, for the sake of simplicity we focus the discussion in this thesis to type IIB orientifolds

Compactifications of type II string theory on Calabi-Yau three-fold gives an $\mathcal{N} = 2$ theory in four dimensions The massless bosonic spectrum[3] of type IIB string theory in D =10 dimensions consists of the metric g, the dilaton ϕ and the axion l, a two-form B_2 in the NS-NS sector, a two form C_2 and a four form C_4 in the R-R sector (24)

The textbook D=10 low energy effective action from type IIB theory is given by

$$\begin{aligned} S^{10}_{IIB} = &- \int \left(\frac{1}{2}R + \frac{1}{4}d\phi \wedge *d\phi + \frac{1}{4}e^{-\phi}H_3 \wedge *H_3\right) \\ &- \frac{1}{4}\int \left(e^{2\phi}dl \wedge *dl + e^{\phi}F_3 \wedge *F_3 + frac12F_5 \wedge *F_5\right) \\ &- \frac{1}{4}\int C_4 \wedge H_3 \wedge F_3, \end{aligned} \tag{2 34}$$

where the field strengths are

$$\begin{aligned} H_3 &= dB_2, \quad F_3 = dC_2 - ldB_2, \\ F_5 &= dC_4 - \frac{1}{2}dB_2 \wedge C_2 + \frac{1}{2}B_2 \wedge dC_2 \end{aligned} \tag{2 35}$$

Choose the coordinates such that the metric after compactification is block diagonal, i e ,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{I\bar{J}}dy^I d\bar{y}^J \tag{2 36}$$

where x^μ are the Minkowski spacetime coordinates and y^I are the coordinates of the Calabi-Yau manifold

The Kahler form after deformation is given by (24)

$$J = ig_{I\bar{J}}dy^I d\bar{y}^J = v(x)^A\omega_A, \quad A = 1, 2, \quad , h^{(1\,1)},$$

[3]We will mainly focus on the non-frermionic degrees of freedom

where the $v(x)^A$ are $h^{(1,1)}$ scalar fields and the (1,1)-forms ω_A form a basis of the cohomology group $H^{(1,1)}$ on the Calabi-Yau threefold.

The deformations corresponding to the (2,1)-forms are (25)

$$\delta g_{IJ} = \frac{i}{||\Omega||^2} \bar{z}^B (\bar{\chi}_B)_{I\bar{I}J} \Omega^{\bar{I}J}_J, \quad B = 1, \quad , h^{1,2} \tag{2.37}$$

where the product of the the holomorphic (3,0)-form

$$||\Omega||^2 = \frac{1}{3!} \Omega_{IJK} \Omega^{IJK} \tag{2.38}$$

and $\bar{\chi}_B$ form a basis of $H^{(1,2)}$.

Similarly, B_2, C_2 and C_4 can be expanded in terms of the harmonic forms

$$\begin{aligned} B_2 &= B_2(x) + b^A(x)\omega_A, \quad A = 1, 2, \quad , h^{(1,1)}, \\ C_2 &= C_2(x) + c^A(x)\omega_A, \quad A = 1, 2, \quad , h^{(1,1)}, \\ C_4 &= D_2^A(x) \wedge \omega_A + V^B(x) \wedge \alpha_B - U_B(x) \wedge \beta^B + \rho_A(x)\tilde{\omega}^A, \quad B = 1, \quad , h^{1,2}. \end{aligned} \tag{2.39}$$

where $\tilde{\omega}^A$ are harmonic (2,2)-forms and α_B , β^B are harmonic three-forms.

In the end, the $N = 2$ massless spectrum consists of the gravity multiplet $(g_{\mu\nu}, V^0)$, the vector multiplets (V^B, z^B), the hypermultiplets (v^A, b^A, c^A, ρ_A) and a double-tensor multiplet (B_2, C_2, ϕ, l) (24):

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h^{(2,1)}$	(V^B, z^B)
hypermultiplets	$h^{(1,1)}$	(v^A, b^A, c^A, ρ_A)
double-tensor multiplet	1	(B_2, C_2, ϕ, l)

Figure 2.2: $\mathcal{N} = 2$ multiplets for type IIB compactification on Calabi-Yau manifolds.

One can insert the harmonic forms defined above into (2.34) and obtain the $N = 2, D = 4$ low energy effective action (for details of the calculation, see (26)):

$$S^{(4)} = \int -\frac{1}{2}R + \frac{1}{4}\mathrm{Re}\mathcal{M}_{KL}F^K \wedge F^L + \frac{1}{4}\mathrm{Im}\mathcal{M}_{kl}F^K \wedge *F^L - G_{KL}dz^K \wedge *d\bar{z}^L - h_{ab}dq^a \wedge *dq^b, \tag{2.40}$$

where $F^K = dV^k$ and

$$F_K = \int_{CY} \Omega \wedge \alpha_K, X^K = \int_{CY} \Omega \wedge \beta^K, \quad F_{KL} = \partial F_K / \partial X^L \tag{2.41}$$

Here $\mathcal{M}_{KL}$ is the gauge kinetic metric which can be expressed in terms of F_{KL} and X^K. q^a denotes all $h^(1,1) + 1$ hypermultiplets and h_{ab} is a quaternionic metric.

Now we have briefly reviewed the $N = 2$ low effective action from type IIB compactification. One thing to note is that here the potential is flat, i.e., the supergravity potential vanishes, and none of the moduli are stabilized. In the next chapter we will focus on deriving the $N = 1$ effective action by imposing the orientifold projections.

Chapter 3

Flux Compactifications and Moduli Stabilization

In this chapter, we will discuss the $D = 4$ dimensional, $N = 1$ supersymmetric low energy effective action for string compactifications on Calabi-Yau orientifolds in the presence of background fluxes Type IIB compactifications on Calabi-Yau threefold result in an $N = 2$ theory in four dimensions However, by adding D-branes/orientifold planes, the amount of supersymmetry is broken and we get an $N = 1$ theory which is phenomenologically of the most interest

In what follows, we will consider type IIB string theory compactification on Calabi-Yau three-fold in the presence of O3/O7 orientifold planes in the manifold This, in an appropriate limit, is equivalent to F-theory compactification on an elliptically fibered Calabi-Yau 4-fold (22) (27) The O3/O7 planes are considered in order to further ensure the stabilization of the D-brane configuration[1] For comprehensive reviews, see (28) (29) (30)

[1]We do not consider the O5/O9 planes, not only because no 5-form flux is included in our configuration but because there is no equivalent F-theory counterpart for the O5/O9 case

3.1 Flux Compactifications

A non-trivial scalar potential will lead to the stabilization of moduli The scalar potential arises due to the non-trivial fluxes or branes Consider a warped geometry where the associated fluxes threading cycles of the internal manifold are non-vanishing An $(n+1)$-form field strength $F = DA$, where A is an n-form potential, generates a magnetic flux

$$\int_{\Sigma_{n+1}} F \tag{3 1}$$

where Σ_{n+1} is a nontrivial cycle of the Calabi-Yau manifold

The same field also generates an electric flux

$$\int_{\Sigma_{D_{(n+1)}}} \star F \tag{3 2}$$

in D-dimensions, where $\star$ is the Hodge dual operator

By Bianchi identity (28), any flux (NS or RR) should satisfy

$$\frac{1}{(2\pi\sqrt{\alpha'})^n}\int_{\Sigma_{n+1}} F \in \mathbb{Z} \tag{3 3}$$

for any $n+1$-cycle , which only depends on the homological properties of the cycle

3.2 Type IIB Flux Compactifications On Calabi-Yau O3/O7 Orientifold Planes

We start from type IIB theory compactified on a Calabi-Yau threefold The string worldsheet parity operator Ω_p only acts on the internal manifold while keeping the four dimensional Minkowski spacetime untouched By definition Ω_p swaps the left- and rigth-moving sectors of the closed string and flips the two end points of the open string via

(31)

$$\text{Closed:} \quad \Omega_p : (\sigma_1, \sigma_2) \to (2\pi - \sigma_1, \sigma_2);$$
$$\text{Open:} \quad \Omega_p : (\tau, \sigma) \to (\tau, \pi - \sigma). \tag{3.4}$$

where σ is a discrete holomorphic isometry which leaves both the metric and complex structure (and hence the Kähler form) invariant. The combination of Ω_p and σ forms the orientation projection.

Note that Ω_p and σ are both of order two and commute

$$\Omega_p^2 = \sigma^2 = 1 \tag{3.5}$$

Introduce the the spacetime fermion number in the left-moving sector, F_L. We are free to choose two different orientation projections

$$\sigma^*\Omega = -\Omega, \quad \mathcal{O} = (-1)^{F_L}\Omega_p\sigma^*$$
$$\text{or}$$
$$\sigma^*\Omega = \Omega, \quad \mathcal{O} = \Omega_p\sigma^* \tag{3.6}$$

where Ω is the holomorphic three-from and σ^* is the pull-back of σ. Here $\mathcal{O}$ is a symmetry operator. The first choice leads to O3/O7 orientifold planes while the second choice gives O5/O9 planes. We will focus on the former in what follows.

As discussed in the previous section, if we choose the orientation projections

$$\sigma^*\Omega = -\Omega \tag{3.7}$$

it will lead to Calabi-Yau orientifolds with O3/O7 planes.

There holomorphic groups $H^{(p,q)}$ split into two (even and odd) eigenspaces under the action of σ^*

$$H^{(p\,q)} = H_+^{(p\,q)} \oplus H_-^{(p\,q)} \tag{3 8}$$

each having $h_+^{(p\,q)}$ and $h_-^{(p\,q)}$ dimensions, respectively

Omitting the details (32), we give the $N = 1$ spectrum of the O3/O7 orientifold compactifications

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h_+^{(2\,1)}$	V^k
chiral multiplet (1)	$h_-^{(2\,1)}$	z^k
chiral multiplet (2)	1	(ϕ, l)
chiral multiplet (3)	$h_-^{(1\,1)}$	(b^a, c^a)
chiral/linear multiplets	$h_+^{(1\,1)}$	(v^a, ρ_a)

Figure 3 1 $\mathcal{N} = 1$ multiplets of O3/O7 orientifold compactification

Compared to Figure 2 2 we notice that the $h^{(2\,1)}$ vector multiplets in the $\mathcal{N} = 2$ theory are decomposed into $h_+^{(2\,1)}$ vector multiplets and $h_-^{(2\,1)}$ chiral multiplet in the $\mathcal{N} = 1$ theory

Before deriving the effective action in type IIB compactifications, we need to consider the background three-form fluxes H_3 and F_3 on Calabi-Yau manifold Define the complex flux

$$G_3 = F_3 - \tau H_3, \quad \tau = l + ie^{-\phi} \tag{3 9}$$

Due to the fluxes, the metric in (2 36) is rendered to

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g^{IJ} dy^I d\bar{y}^J \tag{3 10}$$

where $A(y)$ is called the warp factor and x and y are the coordinates of the Minkowski and internal manifold (Calabi-Yau) respectively

If no brane sources are included, the fluxes and the warp factor $A(y)$ will turn out to be trivial (otherwise the equation of motion cannot be satisfied) in the supergravity approximation due to the no-go theorem (33) This will rule out the warped compactifications of string theory to a Minkowski or de Sitter spacetime To circumvent the no-go theorem, it is necessary to include brane sources to cancel the undesired contribution in the equation of motion from the warp factor and fluxes (34) In general, Dn-branes are used to cancel the contributions from n-form fluxes F_n This leads to the so called tadpole-cancellation condition which determines the charge of the branes In the case of the D3-brane and the three-form flux F_3, the tadpole-cancellation condition reads (35)

$$\frac{1}{2\kappa^2 T_3}\int_M H_3 \wedge F_3 + Q_3 = 0 \tag{3 11}$$

where Q_3 is the total charge carried by the D3-brane[2]

Without discussing the details, we give the effective action of the O3/O7 flux compactification (24)

$$\begin{aligned} S^{(4)}_{O3/O7} = \int_{M_{3\,1}} & \frac{1}{2}R - V \\ & + \frac{1}{4}\mathrm{Re}\mathcal{M}_{kl}F^k \wedge F^k + \frac{1}{4}\mathrm{Im}\mathcal{M}_{kl}F^k \wedge *F^l + \text{terms of multiplets} \end{aligned} \tag{3 12}$$

where the rest are the terms generated by the various multiplets which we do not write down explicitly here

Our goal is to find the $\mathcal{N} = 1$ $D = 4$ low energy effective action for the compactification In $\mathcal{N} = 1$ supergravity, the standard F-term scalar potential can be expressed in

[2]Actually, there are other object which can also give contributions to Q_3

terms of a Kähler potential $\mathcal{K}$ and a holomorphic superpotential W by (39)

$$V = e^{\mathcal{K}}\left(\mathcal{G}^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3W\bar{W}\right) \quad (3.13)$$

$$D_iW = \partial_iW + W\partial_i\mathcal{K} \quad (3.14)$$

$$\mathcal{G}_{i\bar{j}} = \partial_i\partial_{\bar{j}}\mathcal{K} \quad (3.15)$$

where the superpotential and the scalar potential have a natural interpretation in the context of complex manifold. In order to generate the scalar potential in (3.12), it can be shown (40) that the classical superpotential is given by the following expression

$$W = \int_{CY} \Omega \wedge G_3. \quad (3.16)$$

3.3 Moduli Stabilization In Type IIB String Theory

The key in string phenomenology is to obtain realistic vacua from string theory. Moduli stabilization is a necessary step to obtain stabilized vacua from string compactifications. These moduli usually include the complex structure moduli and Kähler moduli. We will discuss two types of model in moduli stabilization: the KKLT (36)[3] method and the Large Volume Scenario (LVS) (41), with the latter as the main focus of this chapter.

KKLT

Let's briefly review the KKLT method. In the KKLT set-up, all moduli are stabilized on a Calabi-Yau O3/O7 orientifold to obtain metastable de Sitter vacua in type IIB theory, when anti-D branes have been added, by considering nontrivial NS and RR three-form fluxes (36).

Start from F-theory compactification on an elliptic Calabi-Yau fourfold. The tadpole

[3]Stands for the four authors: Kachru, Kallosh, Linde and Trivedi.

condition is

$$\frac{\chi(X)}{24} = N_{D_3} + \frac{1}{2\kappa^2 T_3}\int_M H_3 \wedge F_3 \tag{3 17}$$

where N_{D_3} is the number difference between the D3 branes and anti-D3 branes and $\chi(X)$ is Euler characteristic of the underlying fourfold X

The three-form fluxes generates a Gukov-Vafa-Witten (42) superpotential for the complex structure moduli

$$W = \int_M \Omega \wedge G_3 \tag{3 18}$$

where $G_3 = F_3 - \tau H_3$ and Ω is the holomorphic three-form of the Calabi-Yau manifold τ is the type IIB axio-dilaton

The Kahler potential at tree level reads (43)[4]

$$\mathcal{K} = -3\ln[-\imath(T - \bar{T})] - \ln[-\imath(\tau - \bar{\tau})] - \ln\left(-\imath\int_M \Omega \wedge \bar{\Omega}\right) \tag{3 19}$$

The first term depends only on the radius modulus, and the second and the third term depend on the dilaton and compplex structure moduli, respectively

The scalar potential is given by the standard supergravity fomula (3 13)

$$V = e^{\mathcal{K}}\left(\mathcal{G}^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3W\bar{W}\right) = e^{\mathcal{K}}\left(\mathcal{G}^{\imath\bar{\jmath}} D_\imath W D_{\jmath} \bar{W}\right) \tag{3 20}$$

where I, J run over all moduli, while $\imath, \jmath$ only run over the complex structure and the dilaton The potential is thus independent of the volume modulus T, due to the special form of (3 19)[5]

The presence of Euclidean D3-brane instantons leads to non-perturbative correction to the superpotentail and hence the scalar potential Considering the leading order

[4]For simplicity, we consider the case where $h_{1\,1} = 1$, and hence there is a single Kahler modulus that determines the volume of the Calabi-Yau manifold

[5]When $h_{1\,1} > 1$ the potential is still independent of all the Kahler moduli

corrections from instantons, the superpotentail (3.19) becomes

$$W = W_0 + \sum_i A_i e^{-a_i T_i} \tag{3.21}$$

where the Kähler moduli $T_i = \tau_i + i b_i$ with τ_i and b_i being the four cycle volume around which the D3 brane wraps and the axion, respectively. The parameter A_i's depend only on the complex structure moduli. The non-perturbative corrections thus make the scalar potential depend on the Kähler moduli.

The condition

$$D_i W = 0, \tag{3.22}$$

where the derivatives are along the complex structure moduli and dilaton, defines the supersymmetric vacua which turns out to be anti de Sitter.

Large Volume Scenario

The KKLT scenario discussed in the previous section has some limitations such as narrowly allowed parameter space which are often not desirable in building realistic models. The LVS improves the idea of KKLT and eliminates many restrictions of the KKLT scenario.

In the LVS, the Kähler potential is given by

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\xi\, g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}}\right) - \ln(-i(\tau - \bar{\tau})) - \ln\left(-i\int_M \Omega \wedge \bar{\Omega}\right) \tag{3.23}$$

Here g_s is the string coupling, l_s is the string length, Ω is the holomorphic three-form on the Calabi-Yau manifold M, G_3 is the background field (flux) that is chosen to thread 3-cycles in M. $\mathcal{V}$ is the Calabi-Yau volume and

$$\xi = -\frac{\zeta(3)\,\chi(M)}{2(2\pi)^3} \tag{3.24}$$

where χ is the Euler number of M.

The axion-dilaton field is $\tau = C_0 + i\, e^{-\phi}$, and the integrals involving Ω are implicitly functions of the complex structure moduli. The fields $T_i = \tau_i + ib_i$ are the complexified Kähler moduli where τ_i is a 4-cycle volume (of the divisor $D_i \in H_4(M,\mathbb{Z})$) and b_i is its axionic partner arising ultimately from the 4-form field. Here $a_i = 2\pi/N_i$ for some integer N_i, for each field, that is determined by the dynamical origin of the exponentials in the superpotential ($N_i = 1$ for brane instanton contributions, $N_i > 1$ for gaugino condensates). Finally, $\mathcal{V}$ is the dimensionless classical volume of the compactification manifold M (in Einstein frame, but measured in units of the string length). In terms of the Kähler class $J = \sum_i t^i D_i$ (by Poincaré duality $D_i \in H^2(M,\mathbb{Z})$), with the t^i measuring the areas of 2-cycles, C_i,

$$\mathcal{V} = \int_M J^3 = \frac{1}{6}\kappa_{ijk} t^i t^j t^k \, , \tag{3.25}$$

where κ_{ijk} are the intersection numbers of the manifold. $\mathcal{V}$ should be understood as an implicit function of the complexified 4-cycle moduli T_k via the relation

$$\tau_i = \partial_{t_i}\mathcal{V} = \frac{1}{2}\kappa_{ijk} t^j t^k \tag{3.26}$$

The first term in the Kähler potential comes from the α' correction (35). It can be expanded in inverse powers of the volume $\mathcal{V}$.

Similar to the KKLT scenario (3.21), the superpotential takes the form

$$W = \int_M G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i} \tag{3.27}$$

Again, the second term comes from the non-perturbative correction

$$W_{\rm np} = \sum_i A_i e^{-a_i T_i} \tag{3.28}$$

We have assumed that all of the Kähler moduli T_i appear in the superpotential (see (44) for examples) and that we use a basis of 4-cycles such that the exponential terms in

W take the form $e^{-a_i T_i}$. As these exponentials arise from an instanton expansion, in order to only keep the first term as we have done, the 4-cycle volumes must be sufficiently large to ensure that $Re(a_i T_i) \gg 1$.

The scalar potential has the following form (41)

$$V = V_{\text{np1}} + V_{\text{np2}} + V_{\alpha'} \tag{3.29}$$

where

$$V_{\text{np1}} = e^{\mathcal{K}}(G^{i\bar{j}}\partial_i W_{\text{np}} \partial_{\bar{j}} \bar{W}_{\text{np}}) \tag{3.30}$$

$$V_{\text{np2}} = e^{\mathcal{K}}\{G^{i\bar{j}}\partial_i W_{\text{np}} \partial_{\bar{j}} K(\bar{W}_0 + \bar{W}_{\text{np}}) + \partial_i K(W_0 + W_{\text{np}})\partial_{\bar{j}} \bar{W}_{\text{np}}\} \tag{3.31}$$

and

$$V_{\alpha'} = (G^{i\bar{j}}\partial_i K \partial_{\bar{j}} K - 3)|W|^2 \tag{3.32}$$

To write down the explicit expression of the potential, we need to obtain the metric $G^{i\bar{j}}$ of which the full analytic form has been calculated in (27). In the large volume limit, $G^{i\bar{j}}$ can be expanded in terms of the inverse of the volume. Keeping only the first few terms in the expansion will result in a much simpler expression. The same approximation can be applied to the potential. Unlike the KKLT scenario where the resulting vacua are supersymmetric AdS, in the LVS the vacua obtained are non-supersymmetric AdS.

Finally, to obtain dS vacua, we need some uplifting mechanism to uplift the minima of potential to positive values. To do this, a uplifting term V_{uplift} is added to the scalar potential. The form of the term V_{uplift} in (3.13) depends on the kind of supersymmetry breaking effects that arise from other sectors of the theory. We take

$$V_{\text{uplift}} = \frac{\gamma}{\mathcal{V}^2} \tag{3.33}$$

which will describe the energy of a space-filling antibrane (36), fluxes of gauge fields living on D7-branes (45), or the F-term due to a non-supersymmetric solution for the complex structure/axion-dilaton moduli (72)

The assumption is that the complex structure moduli and the axion-dilaton acquire heavy masses without the uplift contribution and they are then decoupled from the low-energy theory Thus their contributions to $\mathcal{K}$ and W are constants for our purposes[6]

$$\begin{aligned} \mathcal{K} &= -2\ln\left(\mathcal{V}+\frac{\xi}{2}\right)-\ln\left(\frac{2}{g_s}\right)+\mathcal{K}_0, \\ W &= \frac{g_s^{\frac{3}{2}}}{\sqrt{4\pi}}\left(W_0+\sum_i A_i e^{-a_i T_i}\right), \end{aligned} \tag{3 34}$$

where $\mathcal{K}_0$ (W_0) is the complex structure Kahler potential (superpotential), evaluated at the locations where the complex structure moduli have been fixed It was shown in (41) that, when the Euler number, $\chi < 0$, for generic values of W_0 (and hence of the background fluxes G_3), the scalar potential for the Kahler moduli has a minimum where the volume $\mathcal{V}$ of the Calabi-Yau manifold M is very large – the associated energy scale is a few orders of magnitude lower than the GUT scale Furthermore, in these Large Volume Scenarios there is a natural hierarchy – one of the Kahler moduli is much larger than the others and dominates the volume of the manifold For our purposes they are also attractive because the scalar potential admits an expansion in inverse powers of the large volume $\mathcal{V}$ This will allow us to carry out analytical calculations of inflation arising from Kahler moduli rolling towards the large volume minimum of the potential, see chapter 5 & 6

The condition that the volume is large enough translates into a special choice of the Calabi-Yau manifold since the volume is explicitly determined by the geometry of the four-cycle (3 25) To further simplify the problem, we usually choose a particular type of manifold where there is one big four-cycle and all others are relatively small This

[6] In the case of the F-term breaking due to the complex structure/axion-dilaton moduli (72), the contribution of the complex structure and axion-dilaton moduli to the scalar potential does depend on the volume (3 33)

will give rise to a volume of the form

$$\mathcal{V} \propto \lambda_1 \tau_1^{3/2} - \sum \lambda_j \tau_j^{3/2} \sim \lambda_1 \tau_1^{3/2} \tag{3.35}$$

where τ_1 denotes the big four-cycle.

In the next chapter, we will use the result from the LVS discussed above to obtain our effective action for string cosmology.

Chapter 4

String Cosmology

4.1 Probems in The Big Bang Model

In the standard Big Bang model the universe is described by different stages with either radiation or matter domination. The evolution of the universe in the theory is a process of decelerated expansion. However, this theory runs into several problems which cannot be solved unless we assume that the universe undergoes an epoch of accelerated expansion, i.e., inflation, in the early universe (46).

The backbone of the conventional Big Bang model is Einstein's theory of general relativity

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{4.1}$$

where $G_{\mu\nu}, T_{\mu\nu}, \Lambda$ and G are the Einstein tensor, the energy-momentum tensor, the cosmological constant and the gravitational constant. Hereafter we set the units $\hbar = c = 1$.

The observations tell us that the universe is extremely isotropic and homogeneous on large scales. The natural choice for the metric is the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{4.2}$$

With a positive cosmological constant (which has been confirmed by observations) the Einstein equation (4 1) yields

$$H^2 = \frac{8\pi}{3M_p^2}\rho - \frac{k}{a^2} \tag{4 3}$$

where ρ is the energy density of the universe This is the so called Friedmann equation in cosmology

Define the critical density

$$\rho_c = \frac{3H^2M_p^2}{8\pi}, \tag{4 4}$$

and (4 3) can be written as

$$\Omega - 1 = \frac{k}{a^2H^2} \tag{4 5}$$

where $\Omega = \frac{\rho}{\rho_c}$ Thus, for t (present stage) and t_0 (initial stage)

$$\frac{\Omega(t_0) - 1}{\Omega(t) - 1} = \frac{a^2(t)}{a^2(t_0)} \tag{4 6}$$

In the standard Big Bang model, a is decreasing and this ratio is extremely small ($\sim 10^{-56}$) Since we know from observations that the present $\Omega(t)$ is very close to unity, the initial condition $\Omega(t_0)$ must be fine tuned extremely close to unity in order to satisfy the Friedmann equation This fine tunning problem is usually called the *flatness problem*

There is another initial condition problem, called the *horizon problem* In the conventional Big Bang model, the very early universe consists of many causally-disconnected bubbles(regions) During the expansion those bubbles remain and become more causally-disconnected(acausal) On the other hand, we know that the universe today is very homogeneous (meaning that the properties in different regions are extremely similar) It is difficult to explain why regions that are not in causal contact (i e , outside each other's *horizon*, hence the name "horizon problem") can have almost identical properties

To solve these problems, one may either claim that the initial universe is unnatural, i e , born with extremely fine tuned initial conditions, or modify the theory to make it be compatible with the most generic (natural) initial conditions In physics we tend to favor the theory from a natural choice of initial conditions. The most successful such theory is the inflation paradigm which is the main subject that will be discussed throughout this chapter.

4.2 The Idea of Inflation

The observation tells us that the universe on large scales is extremely homogeneous and isotropic. We are interested in studying the physics laws and initial conditions in the very early universe which can lead to such a homogeneous and isotropic universe. In the last section, we show that there are problems in Big Bang theory which prevent it from becoming a consistent theory of the universe. Inflation provides an robotic mechanism which solves many of the problems appears in the standard Big Bang universe. And the predictions of inflation are strikingly accurate when compared to the observation data, such as the Cosmic Microwave Background (Figure 4.2).

In the standard Big Bang model, the Hubble radius $(aH)^{-1}$ is increasing. We find that the initial problems discussed in the previous section can be easily solved by assuming that the Hubble radius is decreasing rapidly in the early stage of the universe. Consequently, the scale factor a is increasing exponentially, causing a dramatic inflation of the universe. The condition which defines inflation is

$$\frac{d}{dt}(aH)^{-1} = \frac{-\ddot{a}}{(aH)^2} < 0, \tag{4.7}$$

i.e.,

$$\ddot{a} > 0. \tag{4.8}$$

Equivalently, the following important parameter

$$\epsilon = -\frac{\dot{H}}{H^2} = 1 - \frac{\ddot{a}}{aH^2} < 1. \tag{4.9}$$

which is the condition for inflation. In the so called *slow-roll* paradigm, $\epsilon \ll 1$ which

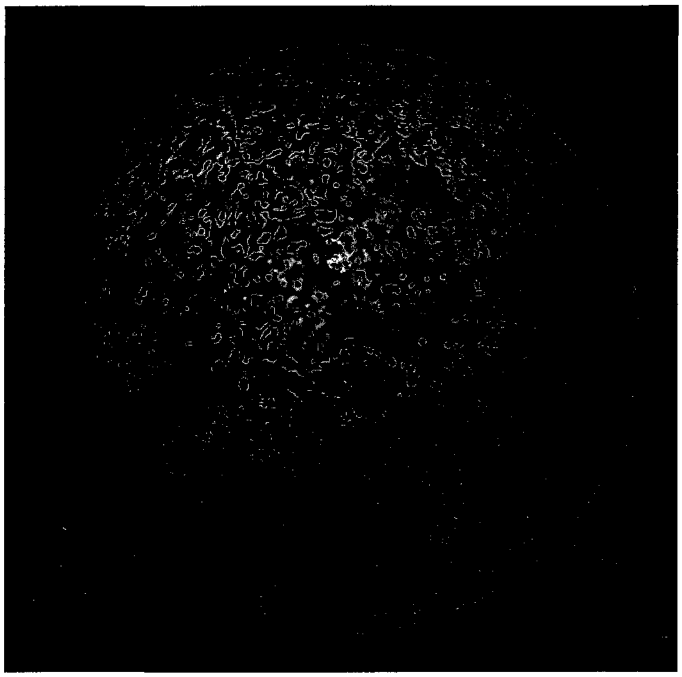


Figure 4.1: The Cosmic Microwave Background (CMB) measured by the Wilkinson Microwave Anisotropy Probe (WMAP). Inflation is believed to be a key part of the evolution of the universe when comes to study the fundamental microphysical nature of the CMB origin. Credit: NASA/WMAP team.

usually indicates that the associated potential in the model is very flat. ϵ can be expressed in terms of the kinetic energy and the potential using the Friedmann equation which will be discussed in the next section.

4.3 The Scalar Field Theory

As we have discussed in the previous chapter, there are many scalar fields arising from flux compactifications. The general study of scalar field in cosmology theory will be performed within the current section. The content of this section can also be found in (83). In the next chapter, we will focus on a specific class of models coming from flux compactifications.

4.3.1 The Background Equations of Motion

Assuming that we get an effective Lagrangian from string theory, the next step is to derive the equations of motion in cosmology and obtain the solutions. It is natural to split the spacetime metric into two parts: the exact homogeneous background metric and the small inhomogeneous deviations from the background metric. The line element reads

$$ds^2 = (g^{(0)}_{\mu\nu} + \delta g_{\mu\nu})dx^\mu dx^\nu \tag{4.10}$$

where we can choose the background metric to be the Friedmann-Robertson-Walker (FRW) metric

$$g^{(0)}_{\mu\nu} = \delta_{ij}[1 + 4kr^2]^{-2} . \tag{4.11}$$

We will work in the flat universe in which $k = 0$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix} . \tag{4.12}$$

The line element for the background becomes

$$ds^2 = -dt^2 + a^2(t) g^{(0)}_{ij} dx^i dx^j = a^2(\eta)(-d\eta^2 + g^{(0)}_{ij} dx^i dx^j) \tag{4.13}$$

where η is the conformal time: $d\eta = \frac{dt}{a}$.

We start with the scalar field theory for a general form of Lagrangian. The action in which the scalar fields ϕ are minimally coupled to gravity:

$$S = \int d^4x \sqrt{-g}[\frac{R}{2\kappa^2} - \frac{1}{2} h_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi)] \tag{4.14}$$

where h_{ab} is the metric on the space of fields, ϕ^a-space. We work in units where $\kappa^2 = 8\pi G_N = {M_p}^{-2} = 1$. M_p is the Planck mass, $\sim 2.4 \times 10^{18}$ Gev.

The background metric and fields, $g_{\mu\nu}$ and ϕ^a, satisfy the usual equations of motion obtained by varying the action S (4.14). This gives rise to the following field equations for the background fields ϕ^a,

$$[\frac{D}{dx^\mu} + \frac{1}{\sqrt{-g}}(\partial_\mu \sqrt{-g})] h_{ab} g^{\mu\nu} \partial_\nu \phi^b + V_{,a} = 0\,, \tag{4.15}$$

where $\frac{D}{dx^\mu}$ is the covariant derivative. For flat space, $g = -a^6$, and assuming that the fields are homogeneous, $\phi^a = \phi^a(t)$, Eq. (4.15) becomes

$$\frac{d^2\phi^a}{dt^2} + \gamma^a_{bc} \frac{d\phi^b}{dt} \frac{d\phi^c}{dt} + 3H \frac{d\phi^a}{dt} + h^{ab} \frac{\partial V}{\partial \phi^b} = 0\,, \tag{4.16}$$

where γ^a_{bc} is the Christoffel connection on the space of fields.

Introduce the notation

$$\dot{\phi^a} = \frac{d\phi^b}{dt}, \ddot{\phi^a} = \frac{D}{dt}\dot{\phi^a} = \dot{\phi^b} \nabla_b \phi^a, \tag{4.17}$$

where $\frac{D}{dt}$ denotes the covariant derivative on the coordinate space, while ∇_b is the covariant derivative on the field space (labeled by b).

The field equations for ϕ^a can be simplified

$$\phi^a + 3H\phi^a + V^{\ a} = 0 \tag{4 18}$$

Varying S with respect to the spacetime metric gives the Einstein equations

$$3H^2 = \frac{1}{2}\phi^a\phi_a + V\,, \tag{4 19}$$

$$H = -\frac{1}{2}\phi^a\phi_a \tag{4 20}$$

The background equations determine the evolution of the inflaton and how long inflation will last, the e-folding time, N In most cases, the light fields and the heavy fields[1] are decoupled, i e , the heavy fields remain constant most of the time when the light fields is rolling Thus, the inflaton is always one, or a subset, of the light fields which controls the dynamics of inflation However, in what follows, we will not make any assumptions about which fields are frozen and which are dynamic, but rather establish this fact as part of the calculation

4.3.2 Perturbations

In general, the metric perturbation $\delta g_{\mu\nu}$ has three different types of components scalar, vector and tensor, classified by the way in which each component transforms under coordinate transformations In what follows, we will focus on the scalar perturbation which is responsible for the existence of inhomogeneities in the universe

The most general scalar perturbations on the background spacetime metric is (53) (54)

$$ds^2 = -(1+2A)dt^2 + 2aB_{\ i}dx^i dt + a^2[(1-2\psi)\delta_{ij} + 2E_{\ ij}]dx^i dx^j \tag{4 21}$$

where A, B, E are scalar functions of spacetime coordinates

Now we have the freedom to choose a gauge such that two of the metric perturba-

[1]The "light" fields have effective masses less than the Hubble parameter, $m_h^2 = V_{II} < V \sim H^2$, while the "heavy" fields are heavier than the Hubble parameter

tions in (2.10), (A, B, ψ, E), vanish. Throughout this paper, we will apply the following gauge conditions

$$B = \psi = 0 \tag{4.22}$$

There are also other gauge choices, such as the *longitudinal gauge* defined by $B = E = 0$ and the *synchronous gauge* in which $A = B = 0$. There is a nice discussion of different gauge choices given by Hu (117). For the sake of simplifying the equations of motion, we find it most convenient to use the gauge (4.22).

The perturbative Einstein equations are

$$\delta G^0_0 = \delta T^0_0, \quad \delta G^0_i = \delta T^0_i \tag{4.23}$$

where G^μ_ν is the Einstein tensor and T^μ_ν is the stress tensor:

$$T^\mu_\nu = h_{ab} g^\mu_\rho \partial_\rho \phi^a \partial_\nu \phi^b - g^\mu_\nu (\frac{1}{2} h_{ab} g^{\rho\gamma} \partial_\rho \phi^a \partial_\gamma \phi^b - V) \tag{4.24}$$

Under the gauge (4.22), the metric perturbations are (55)

$$\delta g_{00} = -\delta g^{00} = 2A, \quad \delta g_{0i} = \delta g^{0i} = 0. \tag{4.25}$$

Let ϕ^a and their perturbations, $\delta\phi^a$, be evaluated at a particular (comoving) wavenumber k, i.e.,

$$\phi^a = \phi^a_k(t) = \int d^3x \phi^a(\boldsymbol{x}, t) e^{ikx} \tag{4.26}$$

$$\phi^a(\boldsymbol{x}, t) = \frac{1}{(2\pi)^3} \int d^3k \phi^a_k(t) e^{-ikx}, \tag{4.27}$$

and similar for the perturbations. In what follows, unless otherwise stated, we will omit k in ϕ^a_k for simplification.

The perturbative Einstein equations become

$$-6H^2A - 2k^2HE = -A\phi^a\phi_a + \phi^a\frac{D}{dt}\delta\phi_a \quad (4\ 28)$$

$$2HA = \phi_a\delta\phi^a \quad (4\ 29)$$

Combine these equations with (4 18) to get

$$A + k^2E = \delta\phi^a\frac{D}{dt}\left(\frac{\phi_a}{H}\right) \quad (4\ 30)$$

Consider the metric perturbation in (4 15), we get the following equation

$$\frac{D^2}{dt^2}\delta\phi^a + 3H\frac{D}{dt}\delta\phi^a + R^a_{cbd}\phi^c\phi^d\phi^b + \frac{k^2}{a^2}\delta\phi^a + \delta\phi_b V^{\ ab} = (A+k^2E)\phi_a + 2A(\phi_a + 3H\phi_a) \quad (4\ 31)$$

Note that the scalar potential A and the distortion E on the right hand side can be eliminated using utilizing(4 29) and (4 30)

$$\begin{aligned}(A + k^2E)\phi_a + 2A(\phi_a + 3H\phi_a) &= \delta\phi^a\frac{D}{dt}\left(\frac{\phi_a}{H}\right)\phi_a + \frac{1}{H}\phi_a\phi_b\delta\phi^b + 3\phi_a\phi_b\delta\phi^b \\ &= \frac{\delta\phi_b}{a^3}\frac{D}{dt}[\frac{a^3}{H}\phi^a\phi^b]\end{aligned}$$

Finally, the equation of motion for the perturbed fields are (55)

$$\frac{D^2}{dt^2}\delta\phi^a + 3H\frac{D}{dt}\delta\phi^a + R^a_{cbd}\phi^c\phi^d\phi^b + \frac{k^2}{a^2}\delta\phi^a + \delta\phi_b V^{\ ab} = \frac{\delta\phi_b}{a^3}\frac{D}{dt}[\frac{a^3}{H}\phi^a\phi^b] \quad (4\ 32)$$

where the covariant derivatives and the Riemann curvature tensor are all evaluated in the field (ϕ) space

To simplify the problem of solving the equations of motion for the perturbations, we work with the canonical field-space metric[2] in the spatially flat gauge Eq (4 32) then

[2]In general, the moduli space metric, h_{ab} is neither canonically normalized nor field independent However, we show that in the class of models we are considering it is possible to make a field-dependent field redefinition such that the metric remains (approximately) flat throughout and after the inflationary period

becomes (53)

$$\delta\phi^I + 3H\delta\phi^I + \frac{k^2}{a^2}\delta\phi^I + \sum_J [V^I_J - \frac{8\pi G}{a^3}\frac{D}{dt}(\frac{a^3}{H}\phi^I\phi_J)]\delta\phi^J = 0 \tag{4 33}$$

To solve this equation, we use the conformal time $\tau = \int a^{-1}dt$ instead of the cosmic time t

$$\begin{aligned} \frac{D}{dt} &= \frac{1}{a}\frac{D}{d\tau} \\ \frac{D^2}{dt^2} &= \frac{1}{a}\frac{D}{d\tau}(\frac{1}{a}\frac{D}{d\tau}) = -\frac{a}{a^2}\frac{D}{d\tau} + \frac{1}{a^2}\frac{D^2}{d\tau^2}\,, \end{aligned} \tag{4 34}$$

where $'$ denotes differentiation with respect to τ and denotes differentiation with respect to t Then, by making the change of variables, $\delta\phi^I = \frac{1}{a}u_I$, where the superscripts get changed to subscripts for later convenience, we have

$$\begin{aligned} \delta\phi^I &= \frac{u_I}{a} - \frac{u_I a}{a^2} = \frac{u_I{}'}{a^2} - \frac{u_I a}{a^2} \\ \delta\phi^I &= \frac{u_I}{a} - 2\frac{u_I a}{a^2} + 2\frac{u_I a^2}{a^3} - \frac{u_I a}{a^2} = \frac{u_I{}''}{a^3} - 3\frac{a}{a^3}u_I{}' + (2\frac{a^2}{a^3} - \frac{a}{a^2})u_I \end{aligned} \tag{4 35}$$

We also use the slow-roll approximation during inflation,

$$\epsilon = \frac{\frac{1}{2}\phi^I\phi_I}{H^2} << 1 \tag{4 36}$$

and integrate the conformal time by parts (56) (57),

$$\tau \simeq -\frac{1+\epsilon}{aH} + O(\epsilon^2) \tag{4 37}$$

Thus, in terms of the conformal time (4 37), Eq (4 33) becomes

$$u_I'' + (k^2 - \frac{2}{\tau^2})u_I = \frac{3}{\tau^2}\sum_J M_{IJ}u_J \tag{4 38}$$

where the mass matrix M_{IJ} is given componentwisely by

$$M_{IJ} = \epsilon\delta_{IJ} + 2\epsilon_{IJ} - \eta_{IJ} - \frac{2}{3}\epsilon_{IJ}(\frac{\phi^I}{\phi^I} + \frac{\phi^J}{\phi^J}) + \frac{2}{3}\epsilon\epsilon_{IJ} + O(\epsilon^2) \tag{4 39}$$

and the multi-field slow-roll parameters are defined as follows

$$\epsilon_{IJ} = \frac{1}{2}\frac{\phi^I\phi^J}{H^2} \tag{4 40}$$
$$\eta_{IJ} = \frac{1}{V}\frac{\partial^2 V}{\partial\phi^I\partial\phi^J} = \frac{V_{IJ}}{V}$$

Note that the last term, $\frac{2}{3}\epsilon\epsilon_{IJ}$, is second order in slow-roll parameters and may be ignored The forth term in (4 39) can also be treated as a second order term for the light fields For the heavy fields, this term may be a first order term, $\sim \mathcal{O}(1)\epsilon_{IJ}$ For completeness sake, we will keep all the terms in our analysis throughout this paper

To solve (4 38), we have to decouple the equations by making a rotation U such that

$$U^{-1}MU = diag\{\lambda_I\}\,, \tag{4 41}$$

where λ_I are the eigenvalues of M And U is given by the similarity transformation

$$U = \left(\vec{g_1}\; \vec{g_2} \quad \vec{g_n} \right)\,, \tag{4 42}$$

where $\vec{g_i}$ are the eigenvectors of of M

Thus, by introducing the new fields, v_I,

$$u_I = U_{IJ}v_J\,, \quad \text{or} \quad v_I = U_{IJ}^{-1}u_J\,, \tag{4 43}$$

we get that Eq (4 38) is decoupled

$$v_I'' + (k^2 - \frac{\mu_I^2 - \frac{1}{4}}{\tau^2})v_I = 0\,, \tag{4 44}$$

where

$$\mu_I{}^2 = \frac{9}{4} + 3\lambda_I \tag{4 45}$$

Now we want a solution satisfying the Minkowski-like vacuum initial conditions (47) when $k\tau_i \gg 1$ $(k \gg aH)$

$$v_I \sim \frac{e^{-ik\tau_i}}{\sqrt{2k}} \tag{4 46}$$

The solution is

$$v_I = \frac{\sqrt{\pi}}{2} e^{i\frac{(\mu_I + \frac{1}{2})\pi}{2}} (-\tau)^{\frac{1}{2}} H^{(1)}_{\mu_I}(-k\tau) e_I(k)\,, \tag{4 47}$$

where $\hat{e}_I$ are the normalized Gaussian variables[3], satisfying (53) (58) (59)

$$\langle \hat{e}_I(k) \rangle = 0 \tag{4 48}$$

$$\langle \hat{e}_I(k) \hat{e}^*_J(k') \rangle = \delta_{IJ} \delta^3(k - k') \tag{4 49}$$

4.3.3 The Asymptotic Solution

We are mainly interested in the solution after Hubble exit when $k < aH$ or $k\tau \to 0$

For small λ_I, [4]

$$\mu_I = \frac{3}{2} + \lambda_I + O(\lambda_I{}^2), \text{ for } \lambda_I \ll 1$$

Eq (4 47) becomes

$$v_I \simeq i e^{i\frac{\pi}{2}\lambda_I} (1 + C\lambda_I) \frac{1}{\sqrt{2k}} (-k\tau)^{-1-\lambda_I} \hat{e}_I(k), \text{ for } k\tau \to 0\,, \tag{4 50}$$

where $C = 2 - log2 - \gamma$ (γ is the Euler-Mascheroni constant)

[3]The fluctuation can be treated as a *random field* which is a *Gaussian process* The homogeneous universe can be divided into a set of sample space with different values of random fields mapped on it

[4]These solutions correspond to the light fields Recalling (4 39), all the components in M_{IJ} related to the light fields are first order in slow-roll parameter Thus the corresponding eigenvalues λ_I are small, too

For large and negative λ_I,[5] the order μ_I is complex (with a large imaginary part) and $|\mu_I| >> 1$ We need to expand the Hankel function of large complex order (For more details, see Appendix A)

Recall (4 45), when λ_I is large and negative,

$$\mu_I = \sqrt{\frac{9}{4} + 3\lambda_I} = \pm \imath\rho_I ,\ \rho_I > 0,\ |\rho_I| >> 1 , \quad (4\ 51)$$

which is purely imaginary. Using (A.6), the solution (4 47) becomes

$$v_I \simeq \frac{1}{\sqrt{2}}(-\tau)^{\frac{1}{2}}(1+\rho_I^2)^{-\frac{1}{4}} e^{1+(\alpha-\frac{\pi}{2})\rho_I} e^{\imath\frac{\pi}{4}} \omega_I \hat{e}_I(k) \quad (4\ 52)$$

where [6]

$$\omega_I = e^{-\rho_I\pi}(\frac{z}{2})^{-\imath\rho_I} e^{\imath(\frac{\alpha}{2}-\rho_I+\rho_I log\sqrt{1+\rho_I^2})} - (\frac{z}{2})^{\imath\rho_I} e^{-\imath(\frac{\alpha}{2}-\rho_I+\rho_I log\sqrt{1+\rho_I^2})} \quad (4\ 53)$$

and $z = -k\tau$.

Note that for large ρ_I,

$$|\omega_I|^2 = 1 + 2\cos\beta(z) e^{-\rho_I\pi} + e^{-2\rho_I\pi} \approx 1 \quad (4\ 54)$$

where $\beta(z)$ is some function of $z = -k\tau$ according to (4 53) As we can see, the dependence on k for $|\omega_I|^2$ (and hence the power spectrum), mainly given by the second term in (4 54), is exponentially suppressed In the limit $\rho_I \to \infty$

$$v_I \propto (-\tau)^{\frac{1}{2}}(1+\rho_I^2)^{-\frac{1}{4}} e^{1+(\alpha-\frac{\pi}{2})\rho_I} \overset{\rho_I\to\infty}{\longrightarrow} (-\tau)^{\frac{1}{2}} \frac{e}{\sqrt{\rho_I}} , \quad (4.55)$$

and the solution for large ρ_I is suppressed by a factor of $\frac{1}{\sqrt{\rho_I}} \sim \frac{1}{\sqrt{m}}$

[5] These solutions correspond to the heavy fields For the heavy fields, the relevant components in M_{IJ}(and thus λ_I) are dominated by the diagonal elements of the matrix (M_{II}) which are the curvature of the potential, $\sim -\frac{V_{II}}{V} \propto -\frac{m_I^2}{H^2}$

[6] The dimension of (4 52) is $|\tau|^{\frac{1}{2}}$, while the dimension of (4 50) is $|k|^{-\frac{1}{2}}$ They are the same since $k\tau$ is dimensionless

This asymptotic solution can also be partially obtained from the following consideration. Consider the perturbation equation (4.44) when μ_I^2 is large and negative and $k\tau$ is small. It is approximate to the equation

$$v_I'' + (-\frac{\mu_I^2}{z^2})v_I = 0\,,$$

where we change the variable $\tau \to z = -k\tau$. The solution of this equation is

$$v_I(z) \propto z^{\frac{1}{2}} z^{\pm i\sqrt{-\frac{1}{4}-\mu_I^2}}\,,$$

which behaves similarly as (4.52).

In summary, the perturbation solutions are

$$v_J(-k\tau) \overset{k\tau\to 0}{\simeq} \begin{cases} ie^{i\frac{\pi}{2}\lambda_J}(1+C\lambda_J)\frac{1}{\sqrt{2k}}(-k\tau)^{-1-\lambda_J}e_J(k), & |\lambda_J| << 1\,, \\ \frac{1}{\sqrt{2}}(-\tau)^{\frac{1}{2}}(1+\rho_J^2)^{-\frac{1}{4}}e^{1+(\alpha-\frac{\pi}{2})\rho_J}e^{i\frac{\pi}{4}}\omega_J e_J(k), & -\lambda_J > 1\,, \end{cases}$$

where $\rho_J = \sqrt{-(\frac{9}{4}+3\lambda_J)}$.

4.4 Perturbation Spectrum

4.4.1 The n-point function

The statistical properties of the primordial perturbation from inflation is characterized by the spectrum. The power spectrum of the perturbation is defined as the two-point function of the perturbed field $\phi(x)$(where we omit the δ):

$$\left\langle \phi_k^i \phi_{k'}^j \right\rangle = \delta^{ij} P(k)(2\pi)^3\delta^3(k+k') \tag{4.56}$$

where ϕ_k is the Fourier coefficient of $\phi(x)$:

$$\phi_k = \int \phi(x)\frac{d^3k}{(2\pi)^{\frac{3}{2}}} \tag{4.57}$$

Thus the two-point function is just the expectation value of the product of Fourier coefficients with respect to some distribution function, which are assumed to be mostly Gaussian in inflation. The three-point and four-point (and higher) functions of the fields characterize the non-Gaussian effect. For purely Gaussian distribution, the odd n-point functions vanish and only the even n-point functions remain. Because we are also interested in non-Gaussian effects, we will not make the assumption that the distribution is purely Gaussian. In multi-field theory, the *primordial curvature perturbation* reads (86) (87) (88)

$$\zeta(\mathbf{x}) = \delta N = N_i \delta\phi^i + \frac{1}{2} N_{ij} \delta\phi^i \delta\phi^j + \frac{1}{6} N_{ijk} \delta\phi^i \delta\phi^j \delta\phi^k + \ldots \tag{4.58}$$

where $N_i, N_{ij}, \ldots$ are derivatives of the e-folds with respect to the fields ϕ^i. $\delta\phi^i$ are evaluated on the initial (flat) slice while the derivatives of N are evaluated on the unperturbed trajectory with respect to the unperturbed fields at Hubble crossing (89). In the later sections, we will use $_*$ to denote Hubble crossing.

We are interested in the n-point functions of the primordial curvature perturbation ζ, which can be connected to observations. They can be calculated using the Feymann diagrams, following the usual Feymann rules.

At the tree level, the two-point function, three-point function and four-point function of the primordial curvature perturbation read (86)

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = N_i N^j \left\langle \phi^i_{k_1} \phi^j_{k_2} \right\rangle \tag{4.59}$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = N_i N_j N^{kl} \left(\left\langle \phi^i_{k_1} \phi^k_{k_2} \right\rangle \left\langle \phi^j_{k_1} \phi^l_{k_3} \right\rangle + 2 \text{ permutations} \right) \tag{4.60}$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = N_i N_j N_k N^{lmn} \left(\left\langle \phi^i_{k_1} \phi^l_{k_2} \right\rangle \left\langle \phi^j_{k_1} \phi^m_{k_3} \right\rangle \left\langle \phi^k_{k_1} \phi^n_{k_4} \right\rangle + 3 \text{ permutations} \right) \tag{4.61}$$

$$+ N_i N_j N^k_l N^{mn} \left(\left\langle \phi^i_{k_1} \phi^k_{k_2} \right\rangle \left\langle \phi^j_{k_2} \phi^m_{k_3} \right\rangle \left\langle \phi^l_{k_3} \phi^n_{k_4} \right\rangle + 11 \text{ permutations} \right) \tag{4.62}$$

where again we go to the Fourier space. Note that we have used the assumption

$$\langle \zeta \rangle = \langle \phi^i \rangle = 0 \tag{4.63}$$

We will continue the discussion on n-point functions in the section of non-Gaussianity. At the moment we focus on calculating the power spectrum, i.e., $\left\langle \phi^i_{k_1}\phi^j_{k_2}\right\rangle$ or $P(k)$.

4.4.2 The Curvature and Isocurvature Perturbations

It is convenient to decompose the scalar field perturbations into *adiabatic perturbation*[7] (parallel to the background trajectory) and *entropy perturbation*[8] (orthogonal to the background trajectory). We then define the adiabatic component

$$\delta\sigma = \sum_I \delta\phi_I \cos\theta^I \tag{4.64}$$

and the non-adiabatic component

$$\delta s^2 = \sum_I \delta{\phi_I}^2 - \delta\sigma^2 \,, \tag{4.65}$$

with

$$\cos\,\theta^I = \frac{\dot{\phi}_I}{\sqrt{\sum_I \dot{\phi_I}^2}} = \frac{\phi_I}{\dot{\sigma}} \tag{4.66}$$

and

$$\delta\phi^I = \frac{1}{a} U_{IJ} v_J \tag{4.67}$$

Note that in (4.67), the sum over the v_J is for the light solutions only, since the perturbations of the heavy fields are strongly suppressed due to the expand of the universe (see the discussion in Section 4.4.3 for more details).

By definition, the two-point correlation functions (i.e., power spectra) are given by

$$C_{xy}\delta^3(\vec{k}-\vec{k}') = \frac{4\pi k^3}{(2\pi)^3}\langle x(\vec{k})y^*(\vec{k}')\rangle \tag{4.68}$$

[7] Also called curvature perturbations

[8] Also called non-adiabatic perturbations or isocurvature perturbations

where $x, y = \delta\sigma, \delta s$ For example,

$$C_{\sigma\sigma}(k)\delta^3(k-k') = \frac{k^3}{2\pi^2}\langle\delta\sigma^*\delta\sigma\rangle = \frac{k^3}{2\pi^2}\sum_{IJ}\langle\delta\phi_I^*\delta\phi_J\rangle\cos\theta^I\cos\theta^J = \frac{k^3}{2\pi^2 a^2}\sum_{IJp}\cos\theta^I\cos\theta^J U_{Ip}U_{Jp}\langle v_p^* v_p\rangle \quad (4\ 69)$$

We next turn to the calculation of the power spectra using these correlation functions The comoving[9] curvature perturbation is defined by

$$\mathcal{R} = \psi + \frac{H}{\sigma}\delta\sigma \quad (4\ 70)$$

In spatially flat gauge ($\psi = 0$), the curvature perturbations become (47) (53)

$$\mathcal{R} = \frac{H}{\sigma}\delta\sigma \quad (4\ 71)$$

In multi-field inflation, in addition to the curvature perturbation, the isocurvature perturbations arise from the fluctuations orthogonal to the background trajectory

$$\mathcal{S} = \frac{H}{\sigma}\delta s \quad (4\ 72)$$

The power spectrum of $\mathcal{R}$ is defined as the expectation value of the Fourier components, which is just the ensemble average of the perturbations

$$\langle\mathcal{R}_k\mathcal{R}_{k'}\rangle_* = \frac{2\pi^2}{k^3}P_{\mathcal{R}}(k)\delta(\mathbf{k}-\mathbf{k}')|_* \quad (4\ 73)$$

$$P_{\mathcal{R}}(k)_* = \frac{H^2}{\sigma^2}C_{\sigma\sigma}(k)|_* \quad (4\ 74)$$

Because of slow-roll approximation, the spectrum is usually calculated at Hubble cross-

[9]Comoving means absent of peculiar motion Comoving observers, such as large galaxies and galaxy clusters, measure zero momentum density at their own positions (38) Their position, $\vec{x}$, is time-independent in the unperturbed universe Their physical coordinate is $a(t)\vec{x}$

ing, denoted by $_*$. In practice, Hubble crossing is often taken to be 50 or 60 e-foldings before the end of inflation (60) (61). Due to the presence of isocurvature perturbation, the spectrum can change after Hubble crossing, which will be discussed in the following section.

The power spectrum can be expanded around some k_0 (62) (63)

$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}(k_0)(\frac{k}{k_0})^{n_s(k_0)-1+\frac{1}{2}\alpha \ln \frac{k}{k_0}}, \tag{4.75}$$

where

$$n_s(k) - 1 = \frac{d\ln P_{\mathcal{R}}^2(k)}{d\ln k}, \ \tilde{\alpha} = \frac{dn_s}{d\ln k}. \tag{4.76}$$

We have assumed that the momentum dependence of the running, $\tilde{\alpha}$, can be neglected. In addition, $\tilde{\alpha}$ itself is of second order in slow-roll and should be small. We next turn to the power spectrum of the isocurvature fluctuation, $P_{\mathcal{S}}$, and the correlation power spectrum, $P_{\mathcal{RS}}$. The power spectrum of the isocurvature fluctuation, $P_{\mathcal{S}}$, and the correlation power spectrum, $P_{\mathcal{RS}}$, can be obtained in a similar way to the curvature perturbations. The non-adiabatic components have the general form

$$\delta s_{(i)} = \beta^I_{(i)} \delta\phi_I, \qquad i = 1, 2, ..., n-1 \tag{4.77}$$

where n is the number of the fields. And

$$\begin{pmatrix} \delta\sigma \\ \delta s_{(1)} \\ \vdots \\ \delta s_{(n-1)} \end{pmatrix} = Q \begin{pmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \vdots \\ \delta\phi_n \end{pmatrix} \tag{4.78}$$

where the rotation matrix $Q \in SO(n)$. For example, in a four-field model containing two heavy fields (ϕ_1, ϕ_2) and two light fields (ϕ_3, ϕ_4),

$$\beta^I_{(1)} = (1,0,0,0), \beta^I_{(2)} = (0,1,0,0), \beta^I_{(3)} = (0,0,-\cos\theta^4, \cos\theta^3)$$

where

$$\cos\theta^I = \frac{\dot{\phi}_I}{\dot{\sigma}}$$

In a three field model, where there are two heavy fields (ϕ_1, ϕ_2) and one light fields ϕ_3,

$$\beta^I_{(1)} = (1,0,0), \beta^I_{(2)} = (0,1,0)$$

For the decoupled case, as will be discussed in section 4.4.3, we can totally ignore the heavy fields, and the coefficients reduce to the simpler forms

$$\beta^I_{(i)} \simeq \begin{cases} (0,0,0,0), (0,0,0,0), (0,0,-\cos\theta^4, \cos\theta^3); \\ (0,0,0), (0,0,0). \end{cases}$$

for the four- and three-field models respectively.

The perturbations are then given by

$$\delta\phi_I = \frac{1}{a} U_{IJ} v_J, \qquad \text{summed over the light } v_J\text{'s}\,, \tag{4.79}$$

from which the correlation functions follow

$$\langle \delta s^* \delta s \rangle = \sum_i \left\langle \delta s^*_{(i)} \delta s_{(i)} \right\rangle = \frac{1}{a^2} \sum_i \beta^I_{(i)} \beta^J_{(i)} U_{Ip} U_{Jp} \left\langle v^*_p v_p \right\rangle \tag{4.80}$$

$$\langle \delta\sigma^* \delta s \rangle = \sum_i \left\langle \delta\sigma^* \delta s_{(i)} \right\rangle = \frac{1}{a^2} \sum_i \cos\theta^I \beta^J_{(i)} U_{Ip} U_{Jp} \left\langle v^*_p v_p \right\rangle \tag{4.81}$$

As before, the heavy v_p's are ignored in the calculation.

Thus, the power spectrum of the isocurvature fluctuation, P_S, and the correlation power spectrum, $P_{\mathcal{RS}}$, at Hubble crossing are given by

$$P_{\mathcal{S}}(k)_* = \frac{H^2}{\sigma^2} C_{ss}(k)|_* \quad (4.82)$$

$$C_{\mathcal{RS}}(k)_* = \frac{H^2}{\sigma^2} C_{\sigma s}(k)|_* \quad (4.83)$$

where the two-point functions are given by

$$C_{ss}(k)\delta^3(k-k') = \frac{k^3}{2\pi^2}\langle \delta s^* \delta s \rangle \quad (4.84)$$

$$C_{\sigma s}(k)\delta^3(k-k') = \frac{k^3}{2\pi^2}\langle \delta \sigma^* \delta s \rangle \quad (4.85)$$

For future reference, it is convenient to define a dimensionless measure of the correlation angle between the power spectra (53),

$$\cos\Delta = \frac{C_{\mathcal{RS}}}{P_{\mathcal{R}}^{\frac{1}{2}} P_{\mathcal{S}}^{\frac{1}{2}}} \quad (4.86)$$

4.4.3 The Evolution of Perturbations After Hubble Exit

For purely adiabatic perturbations, the curvature perturbation is a constant on super-horizon scales during the primordial era[10] (38) (47) In this case, the observable perturbations are calculated at horizon crossing However, as Wands *et al* have pointed out (53) (64) (65), the presence of entropy perturbations can change the curvature perturbation In general, the time dependence of the curvature and isocurvature perturbation has the following form (65) (66)

$$\dot{\mathcal{R}} = \alpha H \mathcal{S} \quad (4.87)$$

$$\dot{\mathcal{S}} = \beta H \mathcal{S} \quad (4.88)$$

[10]The primordial era is defined as the period between Hubble exit and Hubble entry when the comoving scale, equals the Hubble scale, $\frac{a}{k} = \frac{1}{H}$

or in terms of the transfer functions

$$\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} 1 & T_{RS} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} R_* \\ S_* \end{pmatrix} \tag{4.89}$$

The curvature perturbation on super-horizon scales is conserved if the perturbations are purely adiabatic or if the non-adiabatic perturbation is negligible. This general conclusion does not even depend on the slow-roll approximation or the form of the gravitational field equations (the specific theory of gravity) (64).

As we can see from the solutions of the perturbation equations (4.50) and (4.52), for each scale $(1/k)$, the spectrum of the perturbations with $-\lambda_J > 1$ decay rapidly as the universe expands, $\left\langle \frac{1}{a^2}|v|^2 \right\rangle \sim \frac{1}{a^3}$. The spectrum of the perturbations with $|\lambda_J| << 1$, on the other hand, changes slowly, $\left\langle \frac{1}{a^2}|v|^2 \right\rangle \sim \left\langle \frac{1}{a^2}|v|^2 \right\rangle_* [1 + O(\epsilon) + O(\frac{m_l}{H})]$, to leading order in the slow-roll parameters and the masses of the light fields over Hubble parameter. Thus we can ignore the contributions from the former and simplify the calculation.

Recall (4.43) or

$$\delta\phi_I = \frac{1}{a} U_{IJ} v_J$$

where U_{IJ} is the transfer matrix determined by the mass matrix M of (4.39). If we assume that the heavy fields and the light fields are decoupled in such a way that the cross components M_{IJ} (or M_{JI}), with I and J identified as light fields and heavy fields respectively, are subdominant compare to the non-cross components, then

$$\delta\phi_{I_l} \approx \frac{1}{a} U_{I_l J_l} v_{J_l}, \quad I_l \& J_l \text{ denote the light fields,} \tag{4.90}$$

and

$$\delta\phi_{I_h} \approx \frac{1}{a} U_{I_h J_h} v_{J_h}, \quad I_h \& J_h \text{ denote the heavy fields.} \tag{4.91}$$

This is true for most inflationary models encountered so far. For counterexamples, one has to use the full transfer matrix as in (4.43). Under the above assumption, the perturbations of the light fields, $\left\langle|\delta\phi_{I_l}|^2\right\rangle \sim \left\langle|\delta\phi_{I_l}|^2\right\rangle_* [1 + O(\epsilon) + O(\frac{m_l}{H})]$, decay much slower than the perturbations of the heavy fields, $\left\langle|\delta\phi_{I_h}|^2\right\rangle \sim \frac{1}{a^3}$ Therefore, in this case, one can neglect the contributions from the heavy fields when we calculate the curvature and isocurvature perturbations since the fluctuations in the heavy fields are strongly suppressed[11]

If there is a single light field (with all other fields being heavy), the perturbations are purely adiabatic and the comoving curvature perturbation remains constant during inflation If there is more than one light field, the cosmological inflation is driven by all the light fields In addition to the adiabatic perturbation, they also produce entropy perturbation orthogonal to the background trajectory In this case, the curvature perturbation is no longer a constant on super-horizon scales during inflation The coupling between the entropy perturbation and the adiabatic perturbation, given by the (4 87) and (4 88), determines the evolution of the perturbations during and after inflation

In a typical two light field inflationary model, for example, with arbitrary potential and arbitrary background trajectory, it was shown (53) (65) that the scale-dependence of the observable spectra is determined by the slow-roll parameters at Hubble exit and the current observable cross-correlation The amplitude of the power spectra are determined by the power spectra calculated at Hubble exit and the transfer functions which parameterize the detailed physics after Hubble exit until the end of reheating, given by (4 89) (65)

$$\begin{aligned} P_{\mathcal{R}} &= (1 + T_{\mathcal{RS}}^2) P_{\mathcal{R}*} + 2T_{\mathcal{RS}} C_{\mathcal{RS}*} \\ P_{\mathcal{S}} &= T_{\mathcal{SS}}^2 P_{\mathcal{S}*} \\ C_{\mathcal{RS}} &= T_{\mathcal{SS}} C_{\mathcal{RS}*} + T_{\mathcal{RS}} T_{\mathcal{SS}} P_{\mathcal{S}*} \end{aligned} \tag{4 92}$$

[11]We can always do this unless the amplitude of the non-adiabatic fluctuation is greatly amplified at the end of inflation in the preheating stage (64) (68) (69)

Chapter 5

Inflation in Flux Compactifications

5.1 String Theory in Cosmological Inflation: Motivation

There are several motivations to incorporate string theory into the study of cosmology The field theory method in the frame work of quantum field theory and general relativity becomes invalid in the regime in the very early universe where the energy scale is extremely high Secondly, string theory may provide an explanation for the cosmological singularity which cannot be reached using the standard field theory method Also, cosmology can be an ideal place for concrete tests of string theory Not only the low energy effective theory derived from string theory can be tested by cosmological observations, some topological effects such as the cosmic strings may also be tested in cosmology

The key step in string cosmology is to identify the low energy effective theory from string theory via dimension reduction No total success has been achieved so far in deriving the four-dimensional effective Lagrangian from string compactification However, a lot of progress has been made in the direction To accomplish the task, one would like to specify the local sources (such as the D-branes, orientifold planes), turn on the flux, invoke moduli stabilization and include the quantum corrections (such as α' and g_s corrections) There are usually many moduli, the internal degrees of freedom, involved in string compactification, which in principle are all necessary for determining the four-dimensional Lagrangian. For example, in Calabi-Yau compactifications, there are Kähler moduli, complex structure moduli and the dilaton. We will briefly discuss the method we used in this dissertation, which was originally proposed by Conlon and Quevedo (49).

5.2 Kähler Moduli Inflation Model

In what follows we focus on a particularly inflationary model derived from string theory consisting of multiple Kähler moduli, in the large volume limit (also known as the Large Volume Scenario) (41) (67). We adopt the model originally proposed by Conlon and Quevedo in (49) and subsequently studied in (50) (51). For more details, and in particular the conventions, see Chapter 2 and Chapter 4 as well as (67).

In Chapter 3, we demonstrated how to stabilize the moduli in the LVS case. In addition, in Chapter 4, we introduced the Kähler moduli inflationary model and studied the solutions from scalar field theory cosmology. We now explicitly construct a practical model which shows interesting results compared with observations.

Focusing on the dynamics of the scalar fields relevant for inflation, the supergravity action is (we will work in the Einstein frame, and in units where $M_P^2 = 1$)

$$S_{\mathcal{N}=1} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \mathcal{G}_{i\bar{j}}D_\mu\phi^i D^\mu\bar{\phi}^j - V(\phi_i, \bar{\phi}_i)\right] . \tag{5.1}$$

In the Conlon-Quevedo model, one aims to construct a scalar potential of the following form

$$V = V_0\left(1 - Ae^{-\tau} + O(e^{-2\tau})\right) \tag{5.2}$$

where τ is some filed. This simple type of potential has a flat direction along τ which only appears exponentially in the potential. It can be shown in field theory cosmology that this type of potential naturally generates the desired slow-roll inflation.

To get such a potential, we start from the superpotential W by considering nonperturbative effects, i.e., (3.27) in Chapter 3. These nonperturbative effects may come from, for example, instantons arising from D3-branes wrapping four-cycles. At tree-level, we can approximate the superpotential as (see (3.21))

$$W \approx W_0 + \sum_i A_i e^{-a_i T_i} \tag{5.3}$$

The scalar potential becomes

$$V = e^K K^{i\bar{j}} \left[a_i A_i a_j \bar{A}_j e^{-a_i T_i - a_j \bar{T}_j} - \left(\partial_i K \cdot W a_j \bar{A}_j e^{-a_j \bar{T}_j} + c.c. \right) \right] \tag{5.4}$$

Note that T_i only appear in the exponent and are our candidates for inflaton which is defined in the next section.

An uplifting term may arise from, for example, the $\bar{D}3$ brane (36)

$$V_{\text{uplift}} \sim \frac{1}{T^3} \sim \frac{1}{\mathcal{V}^2} \tag{5.5}$$

There may exist other sources which give contributions to the uplifting term, resulting in

$$V_{\text{uplift}} \sim \frac{1}{\mathcal{V}^\alpha}, \quad \frac{4}{3} < \alpha < 2. \tag{5.6}$$

where $\mathcal{V}$ is the Calabi-Yau volume. Here we only consider the case where $\alpha = 2$.

After uplifting, the effective potential should look like (σ denotes the moduli field)

5.3 Explicit Setups

Several previous works have considered inflation in the large volume setting, e.g., (49; 73; 74; 75). Here we include all Kähler moduli, and not just the light modes. Although we find that the heavy modes, corresponding to Kähler moduli that are stabilized before inflation takes, do not affect the dynamics during inflation in the models that we have

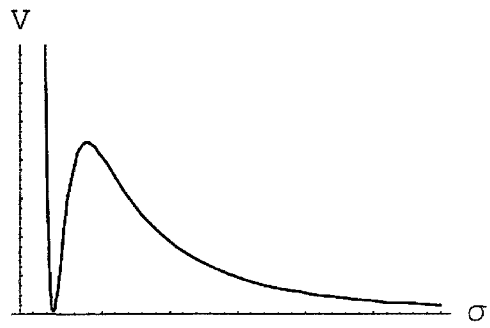


Figure 5.1: The effective potential with positive vacuum energy.

studied, these modes do change after inflation has ended.

Slow roll inflation can occur in a region of the field space where the potential is positive and very flat. We will look for this in the Large Volume Scenarios described above, where, at the minimum of the scalar potential, there is a hierarchy amongst the Kähler moduli

$$\tau_1 \gg \tau_2, \tau_3, \tau_4 \cdots \tag{5.7}$$

which we will use to simplify the effective potential.

For transparency of the equations, we will assume that the intersection numbers k_{ijk} are such that in the basis of 4-cycles, τ_i, the volume takes the diagonal form (67)

$$\mathcal{V} = \alpha(\tau_1{}^{\frac{3}{2}} - \sum_{i=2} \lambda_i \tau_i{}^{\frac{3}{2}}) = -\alpha \sum_{i=1} \lambda_i \tau_i{}^{\frac{3}{2}} \tag{5.8}$$

where $\lambda_1 = -1$, and $\lambda_i, i \geq 2$ are usually positive.

With the volume taking the above form we can explicitly compute the metric on the moduli space, $\mathcal{G}_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}$, which is needed both for the metric, h_{ij}, and for the scalar potential, V, appearing in the four dimensional action (4.14). By expanding in inverse powers of $\mathcal{V}$, keeping terms to $O(\mathcal{V}^{-2})$, we obtain

$$\mathcal{G}_{i\bar{j}} = \frac{3\alpha\lambda_i}{8(\mathcal{V}+\frac{\xi}{2})\tau_i{}^{\frac{1}{2}}}\delta^{ij} + \frac{9\alpha^2\lambda_i\lambda_j\sqrt{\tau_i\tau_j}}{8(\mathcal{V}+\frac{\xi}{2})^2}\,. \tag{5.9}$$

With the axions minimized in the potential, the effective potential then becomes (49)

$$V = \sum_{i=2}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i \alpha} e^{-2a_i\tau_i} - \sum_{i=2}^{n} \frac{4a_i A_i W_0 \tau_i}{\mathcal{V}^2} e^{-a_i\tau_i} + \frac{3\xi {W_0}^2}{4\mathcal{V}^3} + \frac{\gamma {W_0}^2}{\mathcal{V}^2}, \quad (5\ 10)$$

where we have assumed that $\mathcal{K}_0$ can be chosen such that the overall scale of the potential is simplified, i e , overall factors of g_s and 2π are not present, see (3 34) Here we have expanded V to $O(\mathcal{V}^{-3})$ to include the leading α'-corrections, $\frac{3\xi {W_0}^2}{4\mathcal{V}^3}$, as well as the uplift term, $\frac{\gamma}{\mathcal{V}^2}$ The parameters in the potential can be chosen and tuned under certain constraints (73) (74) (76)

To determine the local minimum (vacuum) of the potential we need to solve the equations

$$\frac{\partial V}{\partial \tau_i} = 0 \quad (5\ 11)$$

While it is difficult to get the analytical results[1], these equations can always be solved numerically

It is more convenient to work in the canonical frame, rather than the form taken by the supergravity metric in Eq (5 9), since we have already solved the perturbation equations in the canonical frame[2] Although it is difficult to find the exact transformations which can diagonalize the metric, we do find a canonical frame which is a good approximation as long as $\tau_1 \gg \tau_i$, of which the field space transformations are

$$\phi^1 = \sqrt{\frac{3\lambda_1(1+3\lambda_1)}{4}}\, log(\tau_1) \quad (5\ 12)$$

$$\phi^i = \sqrt{\frac{4\lambda_i}{3{\tau_1}^{\frac{3}{2}}}}\, {\tau_i}^{\frac{3}{4}},\ i \geq 2 \quad (5\ 13)$$

During, as well as after inflation, the metric, in terms of the above redefined fields, ϕ^i, remains canonically normalized, to leading order in inverse powers of the volume

[1] Although one can make approximations to solve the minimum equations analytically as in (49) and (50), it is desirable to solve them numerically As we can show by numerical analysis, the analytical solutions after approximation will likely spoil the results

[2] Note that comparing the kinetic energy terms in the actions (4 14) and (5 1), respectively, we find that $h_{ij} = 2\mathcal{G}_{ij}$, with $\mathcal{G}_{ij}$ given in (5 9)

Considering that the original metric, $\mathcal{G}_{ij}$ is a Kahler metric which neither is field independent nor diagonal, this result is somewhat surprising

5.4 Model Study

In general, a multi-field inflationary model should contain both the heavy fields and the light fields[3] To obtain inflation we choose the initial conditions such that the light fields are displaced away from the local minimum and the heavy fields are at the corresponding local minimum once the initial values of the light fields are chosen We expect that the heavy fields will be frozen as the light fields approach the minimum As we will see later in the numerical analysis, the light fields carry all the kinetic energy and are responsible for the creation of inflation The number of e-foldings or the duration of inflation is determined by how far away the light fields are displaced from the minimum The heavy fields will only begin to move and oscillate together with the light fields around the local minimum shortly after the end of inflation

In what follows, we will discuss two example models based on the discussion in the previous section In both cases there are two heavy fields/moduli The former has a single light field (inflaton) and the latter has two By assigning appropriate values to the parameters in the effective potential, we solve the background equations of motion numerically Next, we perform the field transformation (5 12), (5 13) to get the the kinetic energy in its canonical form Then we use the perturbation solutions(light) to compute the curvature and isocurvature perturbations Finally, we calculate the spectra and tilts at Hubble exit Our models can be easily reduced or generalized

Let us construct an inflationary model with two heavy moduli, τ_1 and τ_2, and two light modulus, τ_3 and $\tau 4$ This is essentially the Conlon-Quevedo model (49) However, we do not assume that the initial values of the heavy moduli are the same as the final values after inflation

[3]As has been shown in (51), the fields that are heavy (light) during inflation may become light (heavy) after inflation ends So the heaviness (or the lightness) of a field is determined not only by the corresponding parameters, but to a large extent also by its position/value in the field space

The parameters in the effective potential (5.10) are set to be

$$\alpha = \frac{1}{9\sqrt{2}}, a_2 = \frac{2\pi}{300}, a_3 = \frac{2\pi}{100}, A_2 = 0.2, A_3 = 0.001, A_4 = 0\ 001$$

$$\lambda_1 = -1, \lambda_2 = 0.1, \lambda_3 = 0.005, \lambda_4 = 0.005, W = 500, \xi = 40, \gamma = 9.75 \times 10^{-6}$$

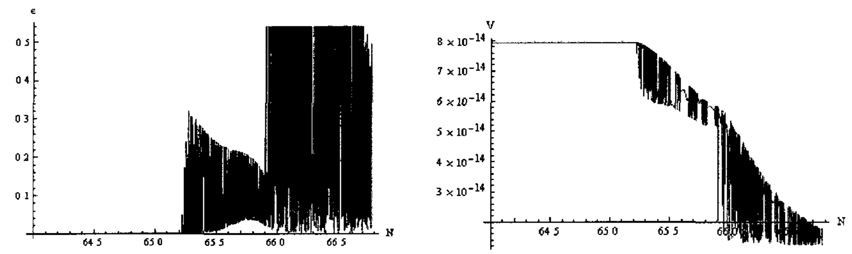
$$\tau_{1min} = 62100.7, \tau_{2min} = 234.1, \tau_{3min} = 69\ 0202$$

The initial conditions are

$$\tau_1(0) = 76212.1, \tau_2(0) = 246.99, \tau_3(0) = 472, \tau_3(0) = 492,$$

$$\dot{\tau_1}(0) = \dot{\tau_2}(0) = 0, \dot{\tau_3}(0) = -1.72 \times 10^{-19}, \dot{\tau_4}(0) = -1.5 \times 10^{-19}$$

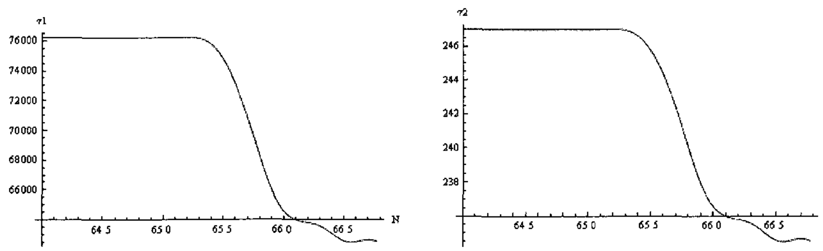
In the Appendix, we show the numerical codes used to solve the equations of motion. The results can be summarized as follows:



(a) (b)

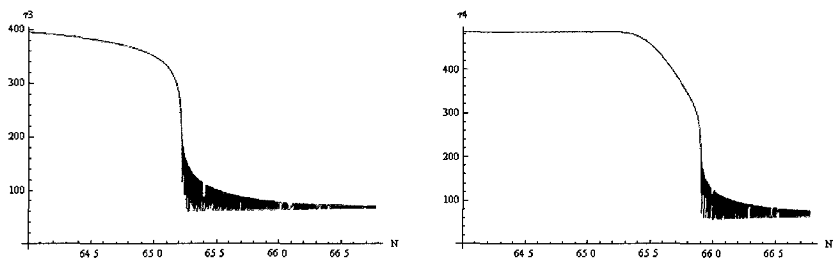
Figure 5.2: a) The slow-roll parameter ϵ. b) The potential V.

The generated inflation lasts about $N_{tot} \approx 66$. In this example, strictly speaking, inflation does not end initially when the inflaton(τ_3) begins to oscillate. It ends when other fields also begin to oscillate.



(a) (b)

Figure 5.3: Evolution of the heavy fields in the last few e-foldings. a) τ_1. b) τ_2.



(a) (b)

Figure 5.4: Evolution of the light fields in the last few e-foldings. a) τ_3. b) τ_4.

The spectral indices at $N_* = 60$ e-foldings before the end of inflation are found to be

$$n_{\mathcal{R}*} = 0.9639\,(\tilde{\alpha}_{\mathcal{R}}{=}4.69\times 10^{-5}), \quad (5.14)$$

$$n_{\mathcal{S}*} = 0.9879\,(\tilde{\alpha}_{\mathcal{S}}{=}4.85\times 10^{-5}), \quad (5.15)$$

$$\cos\Delta_* = -0.00501, \epsilon{=}1.44\times 10^{-13}\,. \quad (5.16)$$

The correlation angle $\cos\Delta_*$ is very small, consistent with (53) where

$$\cos\Delta_* \simeq -2C\eta_{\sigma s*} = -2C\cos\theta^I\beta^J\frac{V_{IJ}}{V}|_* = -0.00507$$

We can study more cases with different number of fields involved. As expected, we find that there is successful inflation if the parameters in the potential are chosen appropriately. For example, the following plot shows the evolution of the light field in

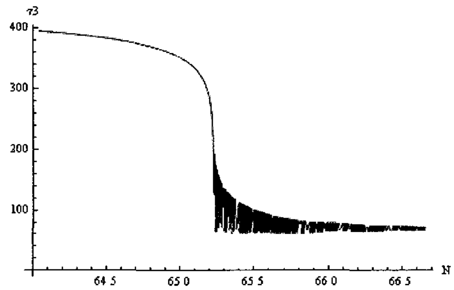


Figure 5.5: Evolution of the light field τ_3 in the last few e-foldings.

a three-field case containing two heavy fields:

5.5 Discussion

The specific class of models from string compactification we used shows some promising results. The potential has a nice shape which is ideal for generating slow-roll inflation. Along the path of the light fields (those with $\lambda_I < 1$) the potential s very flat, and the light fields will slowly roll to their local potential minimum during inflation. Our calculation shows that their perturbations are almost invariant and only decay slowly as the universe expands. The resulting power spectrum of the curvature perturbation is almost scale invariant due to the small λ_I. The heavy fields (those with $-\lambda_I > 1$), on the other hand, are frozen during inflation, until shortly after the end of inflation when they start to move from the initial location to the final minimum. Their perturbations decay rapidly as the universe expands, which can be neglected.

It is also quite easy to get desired number of e-folds before the end of inflation by adjusting the parameters in the effective potential. Calculations show that the spectral indexes, n_s, corresponding to $N_* = 60$ e-folds duration of inflation are 0.956 (single field) and 0.962-0.964 (multi-field), which agrees very well with current observations: $n_s = 0.968 \pm 0.012$ (68% CL)(62).

Chapter 6

Non-gaussianities

Gaussian fluctuations are described by the two-point function and the corresponding power spectrum. Non-gaussianity is a measure of the deviation from Gaussianity. It is usually represented by the nonlinear parameter $f_{\rm NL}$ which is roughly the ratio between the bispectrum and the square of the spectrum (see the Section 6.1). The current observation (WMAP) shows that $-10 < f_{\rm NL} < 74$ (95 % CL)(62). Future experiments such as the Planck Satellite should be able to give more accurate measurements (~ 5) (61). The amount of Non-gaussianity theory has a profound impact on the investigation of the early universe. For example, any observation of $f_{\rm NL} \sim O(1)$ or higher will rule out the single field inflation. It is therefore important to study the non-linear effects in inflationary models that can can give rise to non-gaussian fluctuations.

In single field inflationary case, the non-linear parameters f_{NL} and g_{NL}, which characterize the size of non-gaussianity, can be calculated in terms of the slow-roll parameters (84) (85). The result is generally small, of the order of slow-roll parameters. In multi-field models, there are usually both heavy fields and light fields. The light fields are believed to drive inflation and heavy fields are frozen during inflation. It is often assumed that the dominate contribution to non-gaussianity comes from the inflaton. However, for an arbitrary scalar potential, it is not clear that whether the other (non-inflaton) fields have any sizable contribution to non-gaussinity. We would like to address this issue, at least for the model studied in this paper.

In the inflaton scenario in which the primordial curvature perturbation originates from the inflaton, other light fields only play a role in assisting with stabilizing the potential. In the curvaton scenario, however, the curvaton (light, non-inflaton field) may have significant contribution to the primordial perturbation if its energy density grows large enough at a later time after inflation but before it decays into radiation. We explore the possibility of a curvaton scenario and compute the amount of non-gaussianity generated by the curvaton.

We focus on a string inspired model based on the large volume scenario (41) (67) (49) (similar to what have been discussed in the previous chapter). We study different configurations of the model with different numbers of scalar fields(moduli) and with various values of the volume of the Calabi-Yau to provide hints of the microscopic physics by connecting non-gaussianity (if observable) with the model parameters (moduli, the volume, etc).

The outline of the chapter, which mostly follows the work done in (79), is as follows. We first review the non-gaussian perturbations and the δN formalism in Section 6.1. In Section 6.2, we introduce the scalar potential arising in the large volume scenario of type IIB string compactifications. We then apply the separable potential method to the above multi-field inflationary model in Section 6.3. In Section 6.4, a numerical analysis is carried out that extends the previous analytical study beyond slow-roll. Comparing the two methods, we find a good agreement in the regions where they overlap. Finally, in Section 6.5 we study under what conditions a curvaton may exist after the end of inflation in this type of model derived from string theory, and calculate the contributions to f_{NL}.

6.1 Non-gaussian perturbations and the δN formalism

In the following, we briefly review some general facts about the non-gaussian perturbation and δN formalism which is a powerful tool to calculate non-gaussian effects. (For more detailed discussions, see eg (84) (86).)

Let us define the e-folds time

$$N = \int_{t_*}^{t_c} H dt \tag{6.1}$$

where t_* is usually chosen to be some time during inflation (the initial flat slice) and t_c is some epoch later with constant curvature perturbation (the final slice of uniform density). In multi-field theory, the primordial curvature perturbation reads (86) (87) (88)

$$\zeta(\mathbf{x}) = \delta N = N_i \delta\phi^i + \frac{1}{2} N_{ij} \delta\phi^i \delta\phi^j + \frac{1}{6} N_{ijk} \delta\phi^i \delta\phi^j \delta\phi^k + ... \tag{6.2}$$

where $N_i, N_{ij}, ...$ are derivatives of the e-folds with respect to the fields ϕ^i. $\delta\phi^i$ are evaluated on the initial (flat) slice while the derivatives of N are evaluated on the unperturbed trajectory with respect to the unperturbed fields at Hubble crossing (89). In the later sections, we will use $_*$ to denote Hubble crossing.

All the higher n-point functions can be evaluated using Feynman diagram under the usual diagram rules (86). For example, the Feynman diagram for the connected 3-point function and 4-point function at tree level look like[1]:

The result for three-point correlation function of non-gaussianity is

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_\zeta(k_1, k_2, k_3) \tag{6.3}$$

where $\zeta_{\mathbf{k_i}}$ are the Fourier coefficients of $\zeta(\mathbf{x})$. If slow-roll is satisfied, the bispectrum is

[1] These figures were drawn using JaxoDraw.

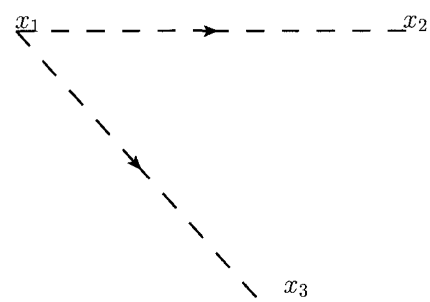


Figure 6.1: The 3-point function at tree level.

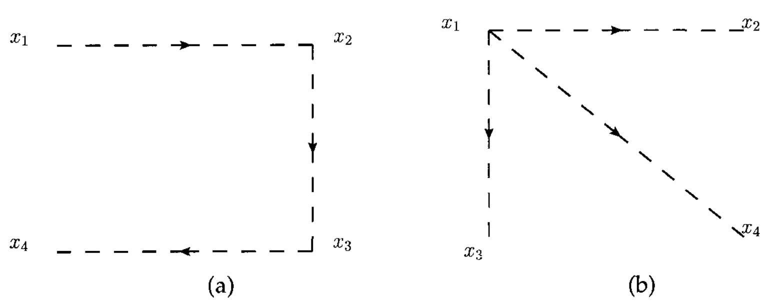


Figure 6.2: The 4-point function at tree level.

completely specified by the non-linear parameter $f_{NL}(k_1, k_2, k_3)$ (90) (92):

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{NL}(k_1, k_2, k_3)[P_\zeta(k_1)P_\zeta(k_2) + \text{cyclic permutations}] \tag{6.4}$$

where $P_\zeta(k_i)$ is the power spectrum.

Using the δN formula, the non-linear parameter f_{NL} is given by (93) (94) (95)

$$\frac{6}{5} f_{NL}(k_1, k_2, k_3) = \frac{k_1^3 k_2^3 k_3^3}{k_1^3 + k_2^3 + k_3^3} \frac{B_\zeta(k_1, k_2, k_3)}{4\pi^4 P_\zeta^2} = \frac{\sum_{i,j} N_i N_j N_{ji}}{\sum_l N_l^2} + O(r) \tag{6.5}$$

where r is the tensor to scalar ratio. The correction $O(r)$ is a k_i-dependent geometric term (95) (96). In standard slow-roll inflation,

$$r \simeq 16\epsilon, \quad \epsilon \sim \frac{1}{2} \frac{V_i V^i}{V^2} \ll 1 \tag{6.6}$$

Thus $O(r)$ is much less than unity due to slow-roll condition and observation constraints on r (97) (98). From now on we focus on the first term in (6.5) and redefine the momentum-independent non-linear parameter

$$f_{NL} = \frac{5}{6} \frac{\sum_{i,j} N_i N_j N_{ji}}{(\sum_l N_l^2)^2} \tag{6.7}$$

The four-point function has the form

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \zeta_{\mathbf{k_4}} \rangle = (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) T_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \mathbf{k_4}) \tag{6.8}$$

Neglecting corrections of the order of the slow-roll parameters, the trispectrum T_ζ reads (84) (90) (91)

$$\begin{aligned} T_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \mathbf{k_4}) =& \tau_{NL}[P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_{14}) + 11 \text{ permutations}] \\ &+ \frac{54}{25} g_{NL}[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + 3 \text{ permutations}] \end{aligned}$$

where $k_{14} = |\mathbf{k}_1 - \mathbf{k}_4|$. Note that, unlike the bispectrum, the trispectrum depends on the directions of k_i's. The parameters, when corrections of slow-roll order are neglected, are given by (88)

$$\tau_{NL} = \frac{N_{ij}N^{ik}N^jN_k}{(N_lN^l)^3} \tag{6.9}$$

$$g_{NL} = \frac{25}{54}\frac{N_{ijk}N^iN^jN^k}{(N_lN^l)^3} \tag{6.10}$$

Applying the Cauchy-Schwarz inequality to (6.7) and (6.9), we get the following relation (99)

$$\tau_{NL} \geq \frac{36}{25}f_{NL}^2 \tag{6.11}$$

Again, we adopt the potential (5.10), which we for convenience repeat below,

$$V = \sum_{i=2}^{n}\frac{8(a_iA_i)^2\sqrt{\tau_i}}{3\mathcal{V}\lambda_i\alpha}e^{-2a_i\tau_i} - \sum_{i=2}^{n}\frac{4a_iA_iW_0\tau_i}{\mathcal{V}^2}e^{-a_i\tau_i} + \frac{3\xi W_0{}^2}{4\mathcal{V}^3} + \frac{\gamma W_0{}^2}{\mathcal{V}^2},$$

Recall that that the axions are minimized, and we have assumed that $\mathcal{K}_0$ can be chosen such that the overall scale of the potential is simplified, i.e., overall factors of g_s and 2π are not present, see (3.34). Here we have expanded V to $O(\mathcal{V}^{-3})$ to include the leading α'-corrections, $\frac{3\xi W_0{}^2}{4\mathcal{V}^3}$, as well as the uplift term, $\frac{\gamma}{\mathcal{V}^2}$. The parameters in the potential can be chosen and tuned under certain constraints (73; 74; 75; 76).

To canonically normalize the metric, we can apply the field transformations discussed in chapter 5,

$$\phi^1 = \sqrt{\frac{3\lambda_1(1+3\lambda_1)}{4}}\log(\tau_1) \tag{6.12}$$

$$\phi^i = \sqrt{\frac{4\lambda_i}{3\tau_1{}^{\frac{3}{2}}}}\tau_i{}^{\frac{3}{4}},\ i \geq 2 \tag{6.13}$$

by keeping terms to leading order in the expansion of inverse powers of the volume, and in the large volume scenario

$$\mathcal{V} \approx \alpha\tau_1{}^{\frac{3}{2}}\,. \tag{6.14}$$

This field redefinition results in a good approximation to the canonical metric. After canonical normalization, the equations of motion read

$$\ddot{\phi}^i + 3H\dot{\phi}^i + \frac{\partial V}{\partial \phi^i} = 0, \quad i = 1, ..., n. \tag{6.15}$$

To get successful inflation, we take the following steps (for more detail, see (83)). First, find the global minimum of the potential by

$$\frac{\partial V}{\partial \tau_i} = 0, \quad i = 1, ..., n. \tag{6.16}$$

Written explicitly,

$$\frac{\partial V}{\partial \mathcal{V}} = 0 \tag{6.17}$$

$$\mathcal{V} = \frac{3\alpha\lambda_i W}{4a_i A_i} \frac{1 - a_i\tau_i}{\frac{1}{4} - a_i\tau_i} \sqrt{\tau_i} e^{a_i\tau_i}, \quad i = 2, ..., n. \tag{6.18}$$

where (6.17) is obtained by

$$0 = \frac{\partial V}{\partial \tau_1} = \frac{\partial V}{\partial \mathcal{V}} \frac{\partial \mathcal{V}}{\partial \tau_1}, \tag{6.19}$$

and we apply (6.14)

$$\frac{\partial \mathcal{V}}{\partial \tau_i} = 0, \quad i \geq 2 \tag{6.20}$$

to get (6.18). To have a small but positive cosmological constant, we also require that

$$V_{\min} > 0 \tag{6.21}$$

In practice we want $a_i\tau_i \gg 1$ so that all higher order non-perturbative corrections of the form $e^{-ma_i\tau_i}$, with integer $m > 1$, in the scalar potential are negligible and the effective potential (5.10) becomes a valid approximation. We find that the global minimum of the potential only exist with the parameters lying in certain regions of the parameter space.

Let the global minimum be

$$\mathcal{V}_{\min}, \tau_{i\min}. \tag{6.22}$$

We have

$$V_{\min} = P\frac{W_0^2}{\mathcal{V}^3} \tag{6.23}$$

where

$$P = \frac{-3}{2}\sum_{i=2}^{n}\frac{\alpha\lambda_i}{a_i^{\frac{3}{2}}}(\ln\mathcal{V} - \ln C_i)^{\frac{3}{2}} + \frac{3\xi}{4} + \gamma\mathcal{V} \tag{6.24}$$

and

$$C_i = \frac{3\alpha\lambda_i W_0}{4a_i^{\frac{3}{2}}A_i} \tag{6.25}$$

Then, for inflation to start, we displace the fields away from the global minimum along the flat direction of the potential, and find the corresponding local minimum. Denote the values of the fields at the local minimum by

$$\mathcal{V}_{\text{ini}}, \tau_{i\text{ini}}. \tag{6.26}$$

which are the initial conditions of the model. By solving the equation of motion (6.15), we should get successful inflation in which the fields evolve slowly toward the global minimum ($\tau_{i\min}$).

6.2 The Inflaton Scenario

6.2.1 Separable Potential Method

In order to use the δN-formalism, we need to calculate the derivatives of the number of e-folds, N, with respect to the fields. For an arbitrarily shaped potential, this can be done using numerical method which will be discussed in the next section, while the analytic treatment of non-gaussianity is known for being difficult. If the potential satisfies certain criteria ((95), (100), for example), the e-folds can be obtained by analytical integration In the model introduced in Section 5 3, it has been shown (51) (83) that the non-inflaton (or heavy) fields are frozen before the end of inflation and only the light fields evolve during inflation Thus, before the end of inflation, we can ignore the dependence of the potential on the heavy fields and treat the potential as a function of the light fields only Assume that we have a volume modulus, τ_1, a heavy modulus, τ_2, and two light moduli, τ_3 and τ_4 (as in (83)), corresponding to the canonically normalized fields (5 12)-(5 13), ϕ_3 and ϕ_4 Suppose τ_1 and τ_2 are frozen during inflation, the potential (5 10) can be seperated as two functions each depending only on one of the light fields (ϕ_3, ϕ_4)

$$V = U(\phi_3) + W(\phi_4) \tag{6 27}$$

The terms which contain the frozen fields have been absorbed into $U(\phi_3)$ and $W(\phi_4)$ Next we will follow the *separable potential method* developed by Vernizzi and Wands (95) to calculate the derivatives of the e-folds In the canonical frame, assuming the background fields only have time dependence, the background equations of motion read

$$\phi^a + 3H\phi^a + V^{\ a} = 0, \quad a = 3, 4 \tag{6 28}$$

where

$$V^{\ a} = \frac{\partial V}{\partial \phi^a} = \begin{cases} U', & a = 3, \\ W', & a = 4 \end{cases} \tag{6 29}$$

And the Einstein field equations are[2]

$$3H^2 = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi_a + V \tag{6 30}$$

$$H = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi_a \tag{6 31}$$

[2]Again we set the planck mass $M_{pl}^2 = 1$

Using the slow-roll approximation, we have

$$H^2 \simeq \frac{V}{3} \tag{6.32}$$

$$\frac{1}{H} \simeq -\frac{3\dot{\phi}_3}{U'} \simeq -\frac{3\dot{\phi}_4}{W'} \tag{6.33}$$

By integrating (6.33) we then get

$$\int \frac{d\phi_3}{U'} = \int \frac{d\phi_4}{W'} + C \tag{6.34}$$

The number of e-folds becomes

$$\begin{aligned} N &= \int_*^c \frac{H^2}{H} dt \\ &= \int_*^c \frac{U}{3H} dt + \int_*^c \frac{W}{3H} dt \\ &= -\int_*^c \frac{U}{U'} d\phi_3 - \int_*^c \frac{W}{W'} d\phi_4 \end{aligned} \tag{6.35}$$

Here $_*$ and $_c$ denote the time at Hubble exit and some time after Hubble exit, respectively. We usually choose the latter as the time, t_c, for some constant Hubble parameter (uniform energy density hypersuface), H_c. Then we let t_* vary and compute the derivative of N with respect to the initial fields at t_*. The results, derived by Vernizizi and David (95), are, with $N_{i*} = \partial N/\partial \phi^i|_{t=t_*}$,

$$N_{3*} = \frac{1}{\sqrt{2\epsilon_{3*}}} \frac{U_* + Z_c}{V_*}, \tag{6.36}$$

$$N_{4*} = \frac{1}{\sqrt{2\epsilon_{4*}}} \frac{W_* - Z_c}{V_*}, \tag{6.37}$$

$$N_{33*} = 1 - \frac{\eta_{3*}}{2\epsilon_{3*}} \frac{U_* + Z_c}{V_*} + \frac{1}{V_* \sqrt{2\epsilon_{3*}}} \frac{\partial Z_c}{\partial \phi_{3*}}, \tag{6.38}$$

$$N_{44*} = 1 - \frac{\eta_{4*}}{2\epsilon_{4*}} \frac{W_* - Z_c}{V_*} - \frac{1}{V_* \sqrt{2\epsilon_{4*}}} \frac{\partial Z_c}{\partial \phi_{4*}}, \tag{6.39}$$

$$N_{34*} = \frac{1}{V_* \sqrt{2\epsilon_{3*}}} \frac{\partial Z_c}{\partial \phi_{4*}} = -\frac{1}{V_* \sqrt{2\epsilon_{4*}}} \frac{\partial Z_c}{\partial \phi_{3*}}. \tag{6.40}$$

where

$$\epsilon_a = \frac{1}{2}\left(\frac{V_{,a}}{V}\right)^2, \quad \eta_a = \frac{V_{,aa}}{V}, \quad Z = \frac{W\epsilon_3 - U\epsilon_4}{\epsilon_3 + \epsilon_4} \tag{6.41}$$

and

$$\sqrt{\epsilon_{3*}}\frac{\partial Z_c}{\partial\phi_{3*}} = -\sqrt{\epsilon_{4*}}\frac{\partial Z_c}{\partial\phi_{4*}} = -\sqrt{2}V_*\frac{V_c{}^2}{V_*{}^2}\frac{\epsilon_{3c}\epsilon_{4c}}{\epsilon_{3c}+\epsilon_{4c}}\left(1 - \frac{\epsilon_{4c}\eta_{3c}+\epsilon_{3c}\eta_{4c}}{(\epsilon_{3c}+\epsilon_{4c})^2}\right) \tag{6.42}$$

Note that this method relies on the slow-roll approximation (6.32). It requires that the final slice at t_c must lie within the slow-roll regime. To calculate the amount of non-gaussianity generated near and after the end of inflation, one needs to find an alternative method valid beyond slow-roll. For example, the authors of (100) proposed an analytic method, valid for certain classes of inflation models with separable Hubble functions, which can be used to study non-gaussianity after inflation ends. Although their analysis applies to certain types of potentials with exponential terms, the detailed conditions are not satisfied for the scalar potential (5.10). In section 5, we present a numerical analysis valid beyond the slow-roll regime.

6.2.2 Estimate of f_{NL}

Two light fields case: In the model discussed in Section 5.3, the two light fields (ϕ_3, ϕ_4) serve as candidates for inflaton. We know make a rough estimate of f_{NL} in this case. Let us assume that ϕ^4 is the assisting field and thus lighter than the inflaton ϕ^3. During slow-roll we would then expect that

$$\epsilon_4 < \epsilon_3 \ll 1, \eta_4 < \eta_3 \ll 1, \eta_i \gg \epsilon_i, \quad i = 3, 4 \tag{6.43}$$

since it follows that $V_{,ii} \gg V_i$ from the flatness of the potential (5.10) and the fact that both fields are light, $\eta_i = \frac{V_{,ii}}{V_i} \ll 1$. Let

$$|W(\phi^4)| = p|U(\phi^3)|, \quad p > 0, \tag{6.44}$$

$$\epsilon_4 = s\epsilon_3, \quad 0 < s < 1, \tag{6.45}$$

and

$$\eta_4 = q\eta_3, \quad 0 < q < 1. \tag{6.46}$$

Although, in general, p, s, q are functions of time, they are not expected to change dramatically in the slow-roll regime. In particular, we can treat p as a constant due to the flatness of the potential. Using the separable potential method introduced in the previous section, we get

$$N_{3*} = \frac{1}{\sqrt{2\epsilon_{3*}}}\frac{p}{1+s_c}, \quad N_{4*} = \frac{s_c}{\sqrt{s_*}}N_{3*} \tag{6.47}$$

$$N_{33*} \approx 1 - \frac{p}{1+s_c}\frac{\eta_{3*}}{2\epsilon_{3*}} + \frac{s_c(s_c+q_c)}{(1+s_c)^3}\frac{\eta_{3c}}{\epsilon_{3*}} \tag{6.48}$$

$$N_{44*} \approx 1 - \frac{s_c q_*}{s_*}\frac{p}{1+s_c}\frac{\eta_{3*}}{2\epsilon_{3*}} + \frac{s_c}{s_*}\frac{s_c(s_c+q_c)}{(1+s_c)^3}\frac{\eta_{3c}}{\epsilon_{3*}} \tag{6.49}$$

$$N_{34*} \approx -\frac{1}{\sqrt{s_*}}\frac{s_c(s_c+q_c)}{(1+s_c)^3}\frac{\eta_{3c}}{\epsilon_{3*}} \tag{6.50}$$

The non-linear parameter f_{NL}, defined by (6.7) and evaluated at t_* for a fixed t_c, then becomes

$$f_{NL} \approx x\eta_{3*} + y\epsilon_{3*} \tag{6.51}$$

where the coefficients are

$$x = \frac{1}{2}(1+\frac{s_c^2}{s_*})^{-2}\left[\frac{s_c(s_c+q_c)}{p^2(1+s_c)}(1-\frac{s_c}{s_*})^2\frac{\eta_{3c}}{\eta_{3*}} - \frac{1+s_c}{p}(1+q_*\frac{s_c^3}{s_*^2})\right],$$
$$y = \frac{2}{p^2}\frac{(1+s_c)^2}{1+\frac{s_c^2}{s_*}}$$

both of which are of $O(\frac{1}{p^2})$ because of the slowly changing s and q.

Mixed case: Consider a model in which there are two heavy fields, ϕ_1 and ϕ_2, and one light field ϕ_3 (inflaton). As usual, ϕ_1 corresponds to the volume modulus and is frozen during inflation. But we drop the constrain that the heavy field ϕ_2 is frozen. We want to see how much the heavy field contributes to the non-gaussianity. The potential (5.10) can be separated as

$$V = \tilde{U}(\phi_2) + \tilde{W}(\phi_3) \tag{6.52}$$

where

$$\tilde{U}(\phi_2) = \frac{8(a_2A_2)^2\sqrt{\tau_2}}{3\mathcal{V}\lambda_2\alpha}e^{-2a_2\tau_2} - \frac{4a_2A_2W_0\tau_2}{\mathcal{V}^2}e^{-a_2\tau_2} \tag{6.53}$$

$$\tilde{W}(\phi_3) = \frac{8(a_3A_3)^2\sqrt{\tau_3}}{3\mathcal{V}\lambda_3\alpha}e^{-2a_3\tau_3} - \frac{4a_3A_3W_0\tau_3}{\mathcal{V}^2}e^{-a_3\tau_3} \tag{6.54}$$

First, since ϕ_2 is heavy and ϕ_3 is light,

$$\eta_2 = \frac{V_{,22}}{V} \sim O(1), \eta_3 = \frac{V_{,33}}{V} \ll 1; \quad \epsilon_2 \ll \epsilon_3 \tag{6.55}$$

Because the inflaton ϕ_3 is usually displaced far away along the flat direction of the potential, we would expect that

$$\tilde{W}(\phi_3) \ll \tilde{U}(\phi_2) \tag{6.56}$$

In addition, we assume that during slow-roll

$$\epsilon_c \sim \epsilon_*, \quad \eta_c \sim \eta_* \tag{6.57}$$

We then have from (6.36) and (6.37)

$$N_{2*} \sim \frac{\sqrt{\epsilon_{2*}}}{\epsilon_{3*}}, \quad N_{3*} \sim \frac{1}{\sqrt{\epsilon_{3*}}} \tag{6.58}$$

Since ϕ_2 is heavy and ϕ_3 is light it follows from (6.55) that

$$N_{3*} \gg N_{2*} \tag{6.59}$$

and (6.42) can be written

$$\frac{\partial Z_c}{\partial \phi_{2*}} \sim V\frac{\epsilon_{2c}}{\epsilon_{3c}}\frac{\eta_{2c}}{\sqrt{\epsilon_{2*}}}, \quad \frac{\partial Z_c}{\partial \phi_{3*}} \sim V\frac{\epsilon_{2c}}{\epsilon_{3c}}\frac{\eta_{2c}}{\sqrt{\epsilon_{3*}}} \tag{6.60}$$

Using these results we can estimate the expressions for N_{ij}, and (6.38-6.40) becomes

$$N_{22*} \sim O(1), \quad N_{33} \sim -\frac{\eta_{3*}}{\epsilon_{3*}}, \quad N_{23*} \sim \frac{\epsilon_{2c}}{\epsilon_{3c}}\frac{\eta_{2c}}{\sqrt{\epsilon_{2*}\epsilon_{3*}}} \tag{6.61}$$

from which it follows that

$$N_{33*} \sim \frac{\eta_{3*}}{\eta_{2c}}\sqrt{\frac{\epsilon_{3*}}{\epsilon_{2c}}}N_{23*} \tag{6.62}$$

By assumption, (6.55), the slow-roll factors satisfy

$$\frac{\epsilon_{3c}}{\epsilon_{2c}} \gg 1\,, \quad \sqrt{\frac{\epsilon_{3*}}{\epsilon_{2*}}} \gg 1\,, \quad \frac{\eta_{3c}}{\epsilon_{2c}} \gg 1, \quad \frac{1}{\eta_{2c}} \sim O(1), \tag{6.63}$$

Using (6.63) in (6.62) we then arrive at the following hierarchy among the N_{ij*}

$$N_{33*} \gg N_{23*} \gg N_{22*} \tag{6.64}$$

Therefore,

$$f_{NL} = \frac{5}{6}\frac{\sum_{i,j=2}^{3} N_i N_j N_{ji}}{(\sum_l N_l^2)^2} \approx \frac{5}{6}\frac{N_{33}}{N_3^2} \sim O(\eta_3) \ll 1 \tag{6.65}$$

with the dominant contribution coming from the light field ϕ_3.

In summary, both examples discussed in this section yield $f_{NL} \sim O(\eta) + O(\epsilon) \ll 1$ (where η, ϵ are the slow-roll parameters for the inflaton). The result is similar to that of the standard slow-roll inflation, see for example, (84).

6.2.3 Explicit Setups

Example 1 Let's choose the parameters in the potential(5.10) as

$$\alpha = \frac{1}{9\sqrt{2}}, a_2 = \frac{2\pi}{300}, a_3 = \frac{2\pi}{100}, a_4 = \frac{2\pi}{100}A_2 = 0.2, A_3 = 0.001, A_4 = 0.001.$$

$$\lambda_1 = -1, \lambda_2 = 0\ 1, \lambda_3 = 0\ 005, \lambda_4 = 0\ 005, W = 500, \xi = 40, \gamma = 9\ 75 \times 10^{-6}$$

To calculate ϵ_a, η_a, Z, etc, we need to solve the equation of motion for the background fields which can usually be done numerically Choose the initial conditions to be

$$\tau_1(0) = 76212\ 1, \tau_2(0) = 246\ 99, \tau_3(0) = 472\ 42, \tau_3(0) = 491\ 54,$$

$$\dot{\tau}_1(0) = \dot{\tau}_2(0) = 0, \dot{\tau}_3(0) = -1\ 72 \times 10^{-19}, \dot{\tau}_4(0) = -1\ 5 \times 10^{-19}$$

The volume in this setup is $\mathcal{V} \sim 10^6$ which is within a reasonable range $10^3 - 10^8$ (101) This will give 60 e-folds before the end of inflation The nonlinear coefficients are [3]

$N(H_c)$	N=20	N=30	N=40	N=50	N=55	N=59
f_{NL}	0 0146	0 0147	0 0147	0 0147	0 0147	0 0147
τ_{NL}	0 000308	0 000312	0 000312	0 000311	0 000311	0 000311

Example 2 As a second example, we choose the parameters such that the volume is relatively small, $\mathcal{V} \sim 10^3$

$$\alpha = \frac{1}{9\sqrt{2}}, a_2 = \frac{2\pi}{80}, a_3 = \frac{2\pi}{80}, A_2 = 0\ 04, A_3 = 1\ 2 \times 10^{-4}, A_4 = 1\ 2 \times 10^{-4}$$

$$\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 0\ 01, \lambda_4 = 0\ 01, W = 1, \xi = 35, \gamma = 2\ 65 \times 10^{-3}$$

$$\tau_1(0) = 1781\ 356, \tau_2(0) = 51\ 039, \tau_3(0) = 282, \tau_3(0) = 285,$$

$$\dot{\tau}_1(0) = \dot{\tau}_2(0) = 0, \dot{\tau}_3(0) = -1\ 40948 \times 10^{-9}, \dot{\tau}_4(0) = -1\ 21344 \times 10^{-9}$$

The inflation lasts for $N = 62\ 5$ e-folds

[3]The different values of N are computed at correspondingly different, constant, values of the Hubble parameter, H_c

$N(H_c)$	N=20	N=30	N=40	N=50	N=55	N=59
f_{NL}	0.0171	0.0167	0.0148	0.0108	0.0121	0.0170
τ_{NL}	0.000423	0.000429	0.000465	0.000464	0.000333	0.000423

One can check that the above explicit results are consistent with the conclusion in section (4.2) and (4.3) that $f_{NL} \sim O(\eta) + O(\epsilon)$.

6.3 Numerical Methods

6.3.1 The Finite Difference Method

Numerically it is straightforward to solve the equations of motion for the background fields without applying slow-roll approximation (83). The advantage of the numerical method is that we do not need to rely on slow-roll approximation (although we still need to assume slow-roll at Hubble exit (95)) and no assumption about the shape of the potential is needed.

We will use the *finite difference method* (102) to calculate the derivatives of $N = N(\phi_1, ..., \phi_n; H_c)$[4] up to the second order beyond the slow-roll regime.

First Order derivative The finite difference method gives

$$N_i = \frac{1}{2h_i}[N(\phi_1, ..., \phi_i + h_i, ...\phi_n) - N(\phi_1, ..., \phi_i - h_i, ...\phi_n)] + O(h^2) \tag{6.66}$$

Second Order derivative When $i = j$,

$$N_{ii} = \frac{1}{h_i^2}[N(\phi_1, ..., \phi_i + h_i, ...\phi_n) - 2N(\phi_1, ..., \phi_n) + N(\phi_1, ..., \phi_i - h_i, ...\phi_n)] + O(h^2) \tag{6.67}$$

[4] Here ϕ_i are understood to be the field values at the Hubble exit.

and when $i \neq j$,

$$N_{ij} = \frac{1}{4h_i h_j}[N(\phi_1, \quad , \phi_i + h_i, \quad , \phi_j + h_j, \quad \phi_n) - N(\phi_1, \quad , \phi_i + h_i, \quad , \phi_j - h_j, \quad \phi_n)$$
$$- N(\phi_1, \quad , \phi_i - h_i, \quad , \phi_j + h_j, \quad \phi_n) + N(\phi_1, \quad , \phi_i - h_i, \quad , \phi_j - h_j, \quad \phi_n)] + O(h^2)$$
(6 68)

Once we get N_i's and N_{ij}'s, we are ready to calculate the non-gaussianity using the δN-formalism discussed previously[5]

6.3.2 Example

We numerically solve the background equations of motion for the model introduced in Section 5 3 Then we use the δN formalism to calculate the non-linear parameters f_{NL} and τ_{NL} The parameters in the potential(5 10) and initial conditions are chosen the same as in Section 6 2 3, Example 1

$N(H_c)$	N=20	N=30	N=40	N=50	N=55	N=59	N=60 4
f_{NL}	0 00874	0 0125	0 0142	0 0143	0 0143	0 0143	0 0143
τ_{NL}	0 000300	0 000274	0 000292	0 000295	0 000296	0 000362	0 000346

These are very close to the results obtained by the analytical method in Section 6 2 3 Remarkably, notice that the values of the nonlinear parameters does not change much near ($N = 59$) and after ($N = 60$ 4) the end of inflation when slow-roll condition breaks down It is reasonable to suspect that the non-gaussianity evolves very slowly through inflation and even preheating era In practice, we may just use the separable potential method to compute non-gaussianity under slow-roll condition and use the result as an approximation to those in regimes beyond slow-roll

[5]We do not compute the third order derivative of N, and thus g_{NL}, since the term containing it is proportional to $O(h^3)$ which is very small and the error bars can be relatively huge

6.4 Curvaton Scenario

So far we have only considered the inflationary scenario in which we assume that the non-gaussianity is generated by the inflaton However, it is necessary to investigate the possibility of a curvaton scenario which does not effect the dynamics during inflation but may play a major role in the oscillation stage

6.4.1 Curvaton evolution

In a multi-field inflationary model, there will in general be several light fields, with one of them, called inflaton, dominating the dynamics of inflation Other light fields, on the other hand, have very little effect during inflation and are usually neglected However, under certain circumstances, a light field other than the inflaton may be identified as the curvaton (103) (104), σ, which sometimes generates significant non-gaussianity after the end of inflation

After the end of inflation, the inflaton quickly starts to oscillate about its potential minimum It then decays into radiation (photon) when its decay rate $\Gamma_{inf} \simeq H$, where the decay rate Γ_{inf} can be calculated once the coupled Lagrangian is given During the oscillation process, if $\Gamma_{\rm inf} > \Gamma_\sigma$, the inflaton will decay first, leaving the curvaton as the only light field[6] Right after the inflaton decays into radiation, the curvaton energy density is still subdominant However, the massless radiation decreases faster, $\sim \frac{1}{a^4}$, than the massive particles associated with the curvaton, $\sim \frac{1}{a^3}$, as the universe expands Thus the relative energy density of the non-relativistic curvaton may increase until it decays into radiation, at which point it may even dominate the total energy density

6.4.2 The existence of the curvaton

For simplicity, we assume that during inflation all the fields, except ϕ_1 and ϕ_n, stay close to their VEVs and are thus heavy We can write the potential as

[6]We assume that there are no other fields, such as those associated with cold dark matter, etc

$$V \sim V_0 + V_1 + V_n \tag{6.69}$$

where V_1 and V_n are potentials for ϕ_1 and ϕ_n, and V_0 is the (almost) constant part of the potential due to the (almost) frozen ϕ_i, with $1 < i < n$.

If the curvaton exists, its mass must be less than the Hubble parameter. Thus the quadratic potential V_1 should be small compare to V_0. The inflaton ϕ_n is displaced far away from its VEV, and its potential is suppressed by orders of $\frac{1}{\mathcal{V}}$

$$V_n = \frac{8(a_n A_n)^2 \sqrt{\tau_n}}{3\mathcal{V}\lambda_n \alpha} e^{-2a_n\tau_n} - \frac{4a_n A_n W_0 \tau_n}{\mathcal{V}^2} e^{-a_n\tau_n} \sim \frac{1}{\mathcal{V}^{3+\beta}}, \quad \beta > 0, \tag{6.70}$$

negligible if compared to V_0.

As a result, (6.69) is dominated by V_0

$$V \simeq V_0 = P_0 \frac{W_0^2}{\mathcal{V}^3} \tag{6.71}$$

where

$$P_0 = -\frac{3}{2}\sum_{i=2}^{n-1} \alpha\lambda_i \langle\tau_i\rangle^{\frac{3}{2}} + \frac{3}{4}\xi + \gamma\mathcal{V} \tag{6.72}$$

$$\langle\tau_i\rangle \simeq a_i^{-1}\left(\ln\mathcal{V} - \ln C_i\right) \tag{6.73}$$

where $\langle\tau_i\rangle$ is the value of the ith moduli at its minimum and the uplifting parameter $\gamma\mathcal{V} \sim O(1)$(see Section 5.3).

Near the potential minimum, the masses of the canonicalized fields, ϕ_1 and $\phi_i, i \geq 2$, given by (5.12) and (5.13) are

$$m_1^2 = Q_1 \frac{W_0^2}{\mathcal{V}^3}, \tag{6.74}$$

$$m_i^2 = Q_i \frac{W_0^2}{\mathcal{V}^2}, \quad i \geq 2. \tag{6.75}$$

The coefficients

$$Q_1 = -\frac{63}{4}\sum_{\imath=2}^{n}\alpha\lambda_\imath \langle\tau_\imath\rangle^{\frac{3}{2}} + \frac{81}{8}\xi + 6\gamma\mathcal{V} \tag{6.76}$$

$$Q_\imath = -\frac{5}{4} + 4a_\imath^{-1}\langle\tau_\imath\rangle + 4a_\imath^{-2}\langle\tau_\imath\rangle^2 \tag{6.77}$$

and

$$C_\imath = \frac{3\alpha\lambda_\imath W_0}{4a_\imath^{\frac{3}{2}}A_\imath} \tag{6.78}$$

As expected, the fields $\phi_\imath, 2 \le \imath \le n-1$ are heavier than the Hubble parameter in the large volume limit

$$m_\imath^2 \sim \frac{1}{\mathcal{V}^2} > H^2 \simeq \frac{1}{3}V_0 \sim \frac{1}{\mathcal{V}^3} \tag{6.79}$$

by (6.23).

Since the field ϕ_1 is our candidate for the curvaton field, it should be lighter than the Hubble parameter, i.e.,

$$0 < Q_1 < \frac{1}{3}P_0, \tag{6.80}$$

More explicitly,

$$\frac{61}{4}\sum_{\imath=2}^{n}\alpha\lambda_\imath\langle\tau_\imath\rangle^{\frac{3}{2}} + \frac{1}{2}\alpha\lambda_n\langle\tau_n\rangle^{\frac{3}{2}} > \frac{79}{8}\xi + \frac{17}{3}\gamma\mathcal{V} \tag{6.81}$$

$$\frac{63}{4}\sum_{\imath=2}^{n}\alpha\lambda_\imath\langle\tau_\imath\rangle^{\frac{3}{2}} < \frac{81}{8}\xi + 6\gamma\mathcal{V} \tag{6.82}$$

From (6.81) and (6.82) it then follows that

$$\alpha\lambda_n\langle\tau_n\rangle^{\frac{3}{2}} > \frac{1}{7}\xi - \frac{20}{9}\gamma\mathcal{V} \tag{6.83}$$

In the simplest setup where all the fields $\phi_\imath, \imath \ge 2$ are identical in parameters($\lambda_\imath, a_\imath$, etc), we get from (6.81) and (6.82)

$$n < 1\ 55 \tag{6.84}$$

which is not possible since the integer n has to be at least 2. Hence a different setup than the simplest one is needed to satisfy the curvaton condition.

Equation (6.81) and (6.82) are the *necessary* conditions for the existence of the curvaton scenario. They are very restrictive, however, as shown by the simple example above, and will require some fine tuning to make them be satisfied.

6.4.3 The decay rate

Consider the Lagrangian of the scalars coupled to the gauge field (photon) of the form

$$\mathcal{L}_g = -\frac{\lambda_\tau}{4M_{Pl}}\tau F_{\mu\nu}F^{\mu\nu} \tag{6.85}$$

where λ_τ is the coupling for the modulus field τ which can only be the small four-cycle (the inflaton) (105).

The other parts of the Lagrangian can obtained by quadratic expansion around the potential minimum. By canonically normalizing the kinetic terms and diagonalizing the mass matrix terms

$$\mathcal{L}_0 = -V_{\min} + \frac{1}{2}\partial_\mu\psi_i\partial^\mu\psi^i - \frac{1}{2}m_i^2\psi_i\psi^i \tag{6.86}$$

where ψ_i are the canonically normalized fields and also eigenfunctions of the mass matrix.

In what follows, we consider a model consisting of multiple moduli: $\tau_1, \tau_2, ..., \tau_n$, where τ_1 is the large four-cycle and all other moduli are small. Typically, most of the small cycles are close to their vevs and thus are heavy during inflation. Only the inflaton is displaced far from its vev. Let the inflaton be τ_n. So the relevent moduli here will be τ_1 and τ_n and other moduli play the role of stabilizing the potential. Starting from the Lagrangian in (4.14), it is possible to simultaneously diagonalize the kinetic terms and the mass matrix terms under the assumption that the mass matrix is independent of the fields, which is a good approximation close to the minimum of the potential. For simplicity, let us first diagonalize the kinetic terms using the field transformations (5.12) and (5.13). The Lagrangian reads

$$\mathcal{L}_0 = -V_{\min} + \frac{1}{2}\partial_\mu\hat{\phi}_1\partial^\mu\hat{\phi}_1 + \frac{1}{2}\partial_\mu\hat{\phi}_n\partial^\mu\hat{\phi}_n - \frac{1}{2}m_1^2\hat{\phi}_1^{\ 2} - \frac{1}{2}m_n^2\hat{\phi}_n^{\ 2} - m_{1n}^2\hat{\phi}_1\hat{\phi}_n$$

where

$$m_{1n}^2 = \frac{\partial^2 V}{\partial\phi_1\partial\phi_n} = Q_{1,n}\frac{W_0^2}{\mathcal{V}^{\frac{5}{2}}} \tag{6.87}$$

$$Q_{1,n} \simeq -3\sqrt{2\alpha\lambda_n}a_n\left\langle\tau_n\right\rangle^{\frac{7}{4}}, \tag{6.88}$$

and the effective fields $\hat{\phi}_i = \phi_i - \langle\phi_i\rangle$ represent the oscillation amplitude of the field about its potential minimum.

From now on we omit the hat over ϕ. Following (106), we calculate the eigenvalues and eigenvectors of the mass metrix

$$M^2 = \begin{pmatrix} m_1^2 & m_{1n}^2 \\ m_{1n}^2 & m_n^2 \end{pmatrix} \tag{6.89}$$

$$M^2 v_i = e_i v_i, \quad i = 1, 2. \tag{6.90}$$

where the eigenvectors v_i are normalized such that $v_i^T v_j = \delta_{ij}$.

The transformation takes the form

$$\begin{pmatrix} \phi_1 \\ \phi_n \end{pmatrix} = \begin{pmatrix} v_1 \end{pmatrix}\psi_1 + \begin{pmatrix} v_2 \end{pmatrix}\psi_2 \tag{6.91}$$

In the large volume limit, we find that

$$\phi_1 \simeq \psi_1 + O\left(\mathcal{V}^{-\frac{1}{2}}\right)\psi_2 \simeq \psi_1 \tag{6.92}$$

$$\phi_2 \simeq O\left(\mathcal{V}^{-\frac{1}{2}}\right)\psi_1 + \psi_2 \simeq \psi_2 \tag{6.93}$$

which means that we can approximate the Lagrangian by

$$\mathcal{L}_0 \simeq -V_{\rm min} + \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_n\partial^\mu\phi_n - \frac{1}{2}m_1^2\phi_1^2 - \frac{1}{2}m_n^2\phi_n^2 \tag{6.94}$$

Rewrite the Lagrangian for the gauge sector (105)

$$\mathcal{L}_g = -\frac{k}{4M_{Pl}}\tau F_{\mu\nu}F^{\mu\nu} = -\frac{k}{4M_{Pl}}(\hat{\tau_n} + \langle\tau_n\rangle)F_{\mu\nu}F^{\mu\nu} \tag{6.95}$$

where k is a normalization factor. We can set $\tau = \tau_n$ since the D7 branes only wrap the small four-cycle τ_n (101).

In terms of the (approximately) canonicalized fields $\phi_\imath$, the gauge field (radiation) Lagrangian takes the form

$$\mathcal{L}_g = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \sum_i \frac{\lambda_{\phi_\imath}}{4M_{Pl}}\phi_\imath F_{\mu\nu}F^{\mu\nu} \tag{6.96}$$

which corresponds to

$$k = \langle\tau_n\rangle^{-1}, \tag{6.97}$$

$$\lambda_{\phi_1} = \frac{\sqrt{6}}{2}, \quad \lambda_{\phi_n} = \left(\frac{3\mathcal{V}}{4\alpha\lambda_n}\right)^{\frac{1}{2}}\langle\tau_n\rangle^{-\frac{3}{4}}. \tag{6.98}$$

The complete Lagrangian reads

$$\begin{aligned}\mathcal{L} = &- V_{\rm min} + \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_n\partial^\mu\phi_n - \frac{1}{2}m_1^2\phi_1^2 - \frac{1}{2}m_n^2\phi_n^2 \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda_{\phi_1}}{4M_{Pl}}\phi_1 F_{\mu\nu}F^{\mu\nu} - \frac{\lambda_{\phi_n}}{4M_{Pl}}\phi_n F_{\mu\nu}F^{\mu\nu}\end{aligned} \tag{6.99}$$

From the Lagrangian (6.99), it is straightforward to get the decay rates

$$\Gamma_{\phi\to\gamma\gamma} = \frac{\lambda_\phi^2 m_\phi^3}{64\pi M_{Pl}^2} \tag{6.100}$$

i e,

$$\Gamma_{\phi_1 \to \gamma\gamma} = \frac{3W_0^3}{128\pi M_{Pl}^2} Q_1^{\frac{3}{2}} \frac{1}{\mathcal{V}^{\frac{9}{2}}} \tag{6 101}$$

$$\Gamma_{\phi_n \to \gamma\gamma} = \frac{3W_0^3}{256\pi M_p^2 \alpha \lambda_n} \left(\frac{Q_n}{\langle \tau_n \rangle} \right)^{\frac{3}{2}} \frac{1}{\mathcal{V}^2} \tag{6 102}$$

Thus,

$$\Gamma_1 \sim \frac{1}{\mathcal{V}^{\frac{9}{2}}} \ll \Gamma_n \sim \frac{1}{\mathcal{V}^2} \tag{6 103}$$

This indicates that the curvaton ϕ^1 decay indeed occurs some time after decay of the inflaton The amount of non-gaussianity generated by the curvaton is determined by its relative energy at the time of decay, which will be shown in the next section

6.4.4 The nonlinear parameter

The curvaton starts to decay when the Hubble parameter drops below the decay rate of the curvaton [7]

$$H \sim \Gamma_{\sigma \to \gamma\gamma} \tag{6 104}$$

Using the sudden decay approximation (assuming the decay happens instantaneously), the nonlinear parameter of the curvaton perturbation can be shown to be (94) (107)

$$f_{NL} = \frac{5}{4r_{\text{dec}}} \left(1 + \frac{gg''}{g'^2} \right) - \frac{5}{3} - \frac{5r_{\text{dec}}}{6} \tag{6 105}$$

where the dimensionless ratio

$$r_{\text{dec}} = \frac{3\rho_{\sigma\text{dec}}}{3\rho_{\sigma\text{dec}} + 4\rho_{\gamma\text{dec}}} \tag{6 106}$$

and $\rho_{\sigma\text{dec}}$ and $\rho_{\gamma\text{dec}}$ are the energy density for the curvaton and radiation at the time when the curvaton decays, respectively

The function g characterizes the dependence of the curvaton, $\sigma(=\phi_1)$, at the begin-

[7] In what follows, the curvaton in our scenario, ϕ_1, is relabeled σ to conform with previous work on curvatons in the literature

ning of its oscillation, on its value at Hubble crossing, σ_*, i.e., $\sigma = g(\sigma_*)$. Assuming the absence of the nonlinear evolution of the curvaton, we have $g'' = 0$ and

$$f_{NL} = \frac{5}{4r_{\text{dec}}} - \frac{5}{3} - \frac{5r_{\text{dec}}}{6} \tag{6.107}$$

If the curvaton energy density dominates the total energy density when it decays, the corresponding nonlinear parameter is

$$f_{NL} \sim -\frac{5}{4} \tag{6.108}$$

On the other hand, if $r_{\text{dec}} \ll 1$, then $f_{NL} \gg 1$. Note that if $r_{\text{dec}} \sim 0.58$, $f_{NL} \sim 0$.

The initial energy density of the curvaton σ, when it begins to oscillate, is

$$\rho_\sigma \simeq \frac{1}{2} m_\sigma^2 \sigma^2 \tag{6.109}$$

where σ is the oscillation amplitude of the curvaton. To estimate its value, we use the arguments similar to (101) (108). Analogous to the Hawking radiation in black holes, the quantum fluctuation $\delta\sigma$ of the light field σ during inflation in de Sitter space has the power spectrum (109)

$$P_{\delta\sigma} = \left\langle |\delta\sigma|^2 \right\rangle = \left(\frac{H_\star}{2\pi} \right)^2 = T_H^2 \tag{6.110}$$

where T_H is the Hawking temperature and the label $\star$ denotes the Hubble exit; and

$$H_\star^2 \simeq \frac{1}{3} P_0 \frac{W_0^2}{\mathcal{V}^3} \tag{6.111}$$

by (6.71). It indicates that the amplitude of quantum fluctuation

$$\delta\sigma \sim T_H = \frac{H_\star}{2\pi} \tag{6.112}$$

The amount of quantum fluctuation is comparable to the classical (slow roll) motion when

$$\delta\sigma \sim \dot{\sigma}\delta t_\star \sim \frac{V_{,\sigma}}{H_\star} H_\star^{-1} \tag{6.113}$$

where the slow-roll condition of the light field σ has been used, and $\delta t_\star = H_\star^{-1}$ is the change in time during one e-fold. We view the onset of the quantum regime as the time when the oscillation takes place. The typical (initial) value of σ constraint by the quantum fluctuations thus satisfies the conditon

$$\frac{\partial V}{\partial \sigma} \simeq H_\star^3, \tag{6.114}$$

Since the potential is quadratic under assumption, the value of σ reads

$$\sigma \simeq \frac{V_\star^{\frac{3}{2}}}{m_\sigma^2} \tag{6.115}$$

where $V_\star = V_0 \simeq 3H_\star^2$.

The initial ratio between the curvaton energy density and the total energy density is

$$\Omega_{\rm in} = \frac{\rho_\sigma}{\rho_{\rm tot}} \sim \frac{\frac{1}{2}m_\sigma^2\sigma^2}{3H_{\rm in}^2 M_{pl}^2} = \frac{V_\star^3}{6m_\sigma^4} \tag{6.116}$$

where

$$H_{\rm in} = m_\sigma \tag{6.117}$$

and we set $M_{pl} = 1$ as usual.

Since

$$m_\sigma^2 = m_1^2 = Q_1 \frac{W_0^2}{\mathcal{V}^3}, \tag{6.118}$$

we have

$$\Omega_{\rm in} = \frac{P_0^3 W_0^2}{6Q_1^2} \frac{1}{\mathcal{V}^3} \tag{6.119}$$

Now assume that the oscillation stage only lasts for a few e-folds ($\Delta N = \int_{\rm in}^{\rm dec} H dt$). We should have $\Omega_{\rm in} \ll e^{-\Delta N} < 1$. Then, under the sudden decay approximation, the ratio r_{dec} of Eq. (6.106) can be related to Ω_{in} by (110)

$$r_{\text{dec}} \simeq \frac{3}{4}\Omega_{\text{in}}(1-\Omega_{\text{in}})^{-\frac{3}{4}}\left(\frac{H_{\text{in}}}{\Gamma}\right)^{\frac{1}{2}}$$
$$\simeq \frac{3}{4}\Omega_{\text{in}}\left(\frac{m_\sigma}{\Gamma}\right)^{\frac{1}{2}}$$
$$\simeq \frac{\sqrt{6\pi}}{3}\frac{P_0^3 W_0^{\frac{5}{2}}}{Q_1^{\frac{5}{2}}}\frac{1}{\mathcal{V}^{\frac{3}{2}}} \tag{6.120}$$

In terms of $H_\star$,

$$r_{\text{dec}} \sim W_0^{\frac{3}{2}}\frac{H_\star}{M_{pl}} \tag{6.121}$$

where we temporarily restore the Planck mass. Notice that the Hubble parameter during inflation is generally much smaller than the Planck mass. Given the fact that W_0 should not be very large, the ratio r_{dec} can be quite small. By (6.107), this will give rise to a large positive $f_{\text{NL}} > 1$, see also (101).

6.5 Discussion

In the inflaton scenario, the analytical method (i.e., the separable potential method) and the numerical method give very similar result for the specific class of string models introduced in the previous chapter, even though the analytical method is only valid in the slow-roll regime. The nonlinear parameter we get is very small which is typical for slow-roll inflation models. One thing to note is that the result does not vary much for different CY volume, as long as the volume is reasonably large. As has been shown, the nonlinear parameter is mostly determined by the slow-roll parameters which themselves are usually very small. The amount of non-gaussianity are unlikely to be detected by the current cosmology experiments. However, it has been argued that certain types of experiments which can detect such small amount of non-gaussianity (113).

The curvaton scenario, on the other hand, if exists, can give rise to sufficiently large amount of non-gaussianity. As shown by calculation, the condition for existence of the curvaton scenario is very restrictive. To satisfy those very restrictive conditions, the parameters need to be fine tuned appropriately. The resulting non-gaussianity is determined by the potential and the CY volume. These large non-gaussianity effects in curvaton scenarios have also been computed in closely related type IIB flux compactifications (101).

Chapter 7

Summary

7.1 Conclusions

Throughout this thesis we discussed several aspects of string compactifications, including the extra dimensions, type IIB theory on Calabi-Yau manifolds and O3/O7 planes, moduli stabilizations and particular models including the KKLT and LVS. We also studied the cosmological equations and solutions and computed the observable results for inflation scenario.

As an important part of the thesis, the main focus of the moduli stabilization section is on KKLT type model and its extension, LVS. In KKLT model, the no-scale structure is broken by the non-perturbative effect (D-brane instantons) which helps to stabilize the moduli. The uplifting mechanism, by breaking the supersymmetry via adding anti-D3 branes or D7-brane flux, turns the already stabilized supersymmetric AdS minimum into a positive dS minimum. The LVS emphasizes on the perturbative α' correction to the Kähler potential which leads to the existence of non-supersymmetric Ads vacua in the large volume limit. There are, however, as has been pointed out by (114), some open problems which need to be addressed in the future: certain fluxes may alter the corrections, open string moduli are largely ignored, the uncertainty of direct supersymmetry breaking in string theory, etc. At present, the assumption is that these problems will not affect the main results we get from LVS

The effective potential we get is a function of scalar moduli which is desirable from a phenomenological point of view as the equations of motion can be solved analytically following the standard treatment of the scalar field theory in cosmology There is an important result from the solution the distinction between the light field and the heavy fields The perturbations of the the light fields change very slowly, while the perturbations of the heavy fields decay rapidly due to the expansion of the universe As a result, the power spectrum is completely determined by the light fields in the Lagrangian The heavy fields has no contribution to the power spectrum but are necessary in stabilizing the minimum of the potential The fact that there often exist many moduli fields after moduli stabilization gives rise to many possibilities to the cosmological inflation scenario There could be more than one light fields and the task of finding the inflaton path is not straightforward as in the single light field case One often needs numerical computations to identify along which path inflation evolves Luckily there is a lot of freedom in tuning the parameters in our potential, which will allow us to get the desired vacua And these parameters are mostly calculable or at least can be constrained Moreover, many necessary approximations are made in order to simplify the calculation

An interesting fact worth mentioning here, which has been discussed in (51), is that the Calabi-Yau volume (as long as it is large enough) almost does not affect the spectral index n_s In fact, we find n_s is always close to 0 96 which is well within the observed value 0 963 $\pm$ 0 012 (115) An earlier estimate (49) shows that for this type of models $n_s \sim 1 - 2/N$ where N, usually taken as 50-60, is the e-folds inflation lasts

The power spectrum contains the information regarding the evolution of the energy density of inflation Non-gaussianity, on the other hand, is a good measure for the interaction between the fields which is not sensitive to the power spectrum The correlations of the three-point and four-point functions tells the departure from a Gaussian distribution So any large non-gaussianity if detected will rule out many inflation models and put constrains on the field composition Non-gaussianity can even be a powerful probe of other scenarios than inflation since a small non-gaussianity ($F_{NL} \leq 1$) will in principle rule out all the alternative scenarios proposed so far other than inflation (116) So far the detection of non-gaussianity is not very satisfatory due to the huge error bar The up and comming experiments like the Planck satellite (61) can measure non-gaussianity at a more accurate level and will put tighter constrains on the value of non-gaussianity

We made an important step toward string phenomenology by utilizing the results from string compactifications to solve cosmological problems Deriving the low energy effective theory from string theory is a nontrivial task Many parts of string compactifications are still far from being full understood This is also the reason why we focus on some specific ideas (KKLT, LVS) instead of deriving the result from a more theoretical perspective Nevertheless, the present work sheds some light on constructing realistic models from string compactification via moduli stabilization

The mechanisms in string compactification such as moduli stabilization lead to an effective theory which otherwise is impossible to obtain using non-string methods The observations, such as the measure of the spectrum index and the amount of the non-gaussianity, can then pin down the values of the string related parameters, such as the shape and the size of the internal space, and provide a way to somewhat reveal the exact form of string theory Looking into the future, the steady development in cosmological observations and string theory will surely yield more interesting connections between the two and will most likely sharpen our understanding of the nature of string theory

7.2 Future Research

In the cosmological model we constructed, the volume of the Calabi-Yau has always be assumed to be large This allows us to ignore higher order terms in the expansion in terms of the inverse power of the volume This not only simplifies the scalar potentail, but also makes it flat enough which is ideal for generating slow-roll inflation One may wonder, instead of imposing the "uplifting" mechanism, if the LVS assumption can be used to construct models with different mechanisms of obtaining dS vacua At the same time, the future experiments such as the Planck satellite will generates new and more accurate data There will be a new wave of interaction between the phenomenological models from string theory and the observations

In Chapter 2, we explained the reason behind our choice of type IIB theory instead of other types of string compactification type IIA, heterotic, M-theory on G2 manifold, etc Different types of stirng theories are connected by dualities At the level of effective action in certain limits[1], these different compactifications are equivalent So in principle, other types of string compactification may also lead to interesting models that can be connected with observations In fact, there are many works on these models from different string theories It would be more interesting to compare the results between these models

Again, as has been emphasized earlier, we have only looked in a small region of the allowed configuration space The models have been shown so far are only specific examples which are relatively easy to access with our current knowledge of string compactification And we are far from the goal of fully solving the moduli problem and computing all the parameters Even more, without a practical screening mechanism for the string landscape, finding the physically relevant string vacua is a formidable task We hope, by further studying the connections between the phenomenology (such as string cosmology) and string compactification, we could gain a better understanding of the structure and solutions of string theory

[1]For example, in the large wolume/complex structure limit for type IIA and type IIB

Appendix A

Appendix to Chapter 4

A.1 The Hankel Function of Large Complex Order

Eq. (4.47) are the solutions for the equation of motion

$$v_I = \frac{\sqrt{\pi}}{2} e^{\imath \frac{(\mu_I + \frac{1}{2})\pi}{2}} (-\tau)^{\frac{1}{2}} H^{(1)}_{\mu_I}(-k\tau) \hat{e}_I(k) \,.$$

To get an approximation (and simplified) expression for the case where λ_I is large and negative, we first use the Frobenius expansion of the Bessel function around the origin

$$\begin{aligned} J_\mu(z) &= \left(\frac{z}{2}\right)^\mu \left[\frac{1}{\Gamma(\mu+1)} - \frac{1}{\Gamma(\mu+2)}\left(\frac{z}{2}\right)^2 + \frac{1}{2!\Gamma(\mu+3)}\left(\frac{z}{2}\right)^4 - \cdots\right] \\ &= \left(\frac{z}{2}\right)^\mu \sum_{k=0}^{\infty} (-1)^k \frac{(z/2)^{2k}}{\Gamma(1+\mu+k)k!}, \; z \ll 1 \end{aligned} \tag{A.1}$$

To the lowest order,

$$\left\{ J_\mu \simeq \frac{(z/2)^\mu}{\Gamma(1+\mu)}, J_{-\mu} \simeq \frac{(z/2)^{-\mu}}{\Gamma(1-\mu)} \right\}, \; z \ll 1 \tag{A.2}$$

So

$$H^{(1)}_\mu(z) = \frac{e^{-\imath\mu\pi} J_\mu(z) - J_{-\mu}(z)}{-i \sin \pi\mu} \simeq \frac{e^{-\imath\mu\pi} \frac{(z/2)^\mu}{\Gamma(1+\mu)} - \frac{(z/2)^{-\mu}}{\Gamma(1-\mu)}}{-i \sin \pi\mu} \tag{A.3}$$

Stirling's formula for large ν approximation of the Gamma function gives

$$\Gamma(\nu) = \nu^{\nu} e^{-\nu} \sqrt{\frac{2\pi}{\nu}} [1 + \frac{1}{12\nu} + \frac{1}{288\nu^2} + O(\nu^{-3})], \; |\nu| > 1, \; |\arg(\nu)| < \pi - \epsilon \tag{A.4}$$

where ϵ is any small positive number.

Let $\nu = 1 \pm i\rho$ where $\rho > 1$, to lowest order,

$$\begin{aligned} \Gamma(1 \pm \imath\rho) &\approx (1 \pm i\rho)^{1\pm\imath\rho} e^{-1\mp\imath\rho} \sqrt{\frac{2\pi}{1 \pm i\rho}} \\ &= (1 \pm i\rho)(1 \pm i\rho)^{\pm\imath\rho} e^{-1} e^{\mp\imath\rho} \sqrt{2\pi} \frac{1}{(1+\rho^2)^{\frac{1}{4}}} e^{\mp\imath\frac{\alpha}{2}} \\ &= \sqrt{2\pi}(1+\rho^2)^{\frac{1}{4}} e^{-1-\alpha\rho} e^{\pm\imath(\frac{\alpha}{2} - \rho + \rho \log\sqrt{1+\rho^2})} \end{aligned} \tag{A.5}$$

where $1 \pm i\rho = \sqrt{1+\rho^2} e^{\pm\imath\alpha}$.

So for $\rho > 1$, the Hankel function is

$$H_{\mu}^{(1)}(z) \approx \begin{cases} \sqrt{\frac{2}{\pi}}(1+\rho^2)^{-\frac{1}{4}} e^{1+\alpha\rho}\omega, & \mu = i\rho \\ \sqrt{\frac{2}{\pi}}(1+\rho^2)^{-\frac{1}{4}} e^{1+(\alpha-\pi)\rho}\omega, & \mu = -\imath\rho \end{cases} \tag{A.6}$$

where

$$\omega = e^{-\rho\pi}(\frac{z}{2})^{-\imath\rho} e^{\imath(\frac{\alpha}{2} - \rho + \rho \log\sqrt{1+\rho^2})} - (\frac{z}{2})^{\imath\rho} e^{-\imath(\frac{\alpha}{2} - \rho + \rho\imath \log\sqrt{1+\rho^2})} \tag{A.7}$$

A.2 Solving the equation of motion using Mathematica

Here is the Mathematica codes used in Chapter 5 for solving the equations of motion (4.16). The example contains two heavy fields, τ_1 and τ_2, and two light fields, τ_4 and τ_5. The last few plots shows the evolution of these fields and the slow-roll parameters ϵ.

$$a2 = \frac{2\pi}{300};\ a3 = \frac{2\pi}{100};\ a4 = \frac{2\pi}{100};\ A2 = 0.2;\ A3 = 1*10^{\wedge}(-3);\ A4 = 1*10^{\wedge}(-3);\ \alpha = \frac{1}{9\sqrt{2}};$$

$$\lambda1 = -1;\ \lambda2 = 0.1;\ \lambda3 = 0.005;\ \lambda4 = 0.005;\ w = 500;\ \xi = 40;\ \mu = 1;\ \gamma = 9.75*10^{-6};$$

$$V = \frac{8\,(a2\,A2)^2\sqrt{\tau2[t]}}{3\,v\,\lambda2\,\alpha}\,e^{-2\,a2\,\tau2[t]} - \frac{4\,a2\,A2\,w\,\tau2[t]}{v^2}\,e^{-a2\,\tau2[t]} + \frac{8\,(a3\,A3)^2\sqrt{\tau3[t]}}{3\,v\,\lambda3\,\alpha}\,e^{-2\,a3\,\tau3[t]} - \frac{4\,a3\,A3\,w\,\tau3[t]}{v^2}\,e^{-a3\,\tau3[t]} + \frac{8\,(a4\,A4)^2\sqrt{\tau4[t]}}{3\,v\,\lambda4\,\alpha}\,e^{-2\,a4\,\tau4[t]} - \frac{4\,a4\,A4\,w\,\tau4[t]}{v^2}\,e^{-a4\,\tau4[t]} + \frac{3\,\xi\,w^2}{4\,v^3} + \frac{\gamma\,w^2}{v^2};$$

$$v = \alpha\,\tau1[t]^{\frac{3}{2}};$$

$$G[1,\ 1] = \frac{3\,\alpha\,\lambda1\left(v + 3\,\alpha\,\lambda1\,\tau1[t]^{3/2}\right)}{8\,v^2\sqrt{\tau1[t]}};$$

$$G[1,\ 2] = \frac{9\,\alpha^2\,\lambda1\,\lambda2\sqrt{\tau1[t]\ \tau2[t]}}{8\,v^2};$$

$$G[1,\ 3] = \frac{9\,\alpha^2\,\lambda1\,\lambda3\sqrt{\tau1[t]\ \tau3[t]}}{8\,v^2};$$

$$G[1,\ 4] = \frac{9\,\alpha^2\,\lambda1\,\lambda4\sqrt{\tau1[t]\ \tau4[t]}}{8\,v^2};$$

$$G[2,\ 1] = \frac{9\,\alpha^2\,\lambda1\,\lambda2\sqrt{\tau1[t]\ \tau2[t]}}{8\,v^2};$$

$$G[2,\ 2] = \frac{3\,\alpha\,\lambda2\left(v + 3\,\alpha\,\lambda2\,\tau2[t]^{3/2}\right)}{8\,v^2\sqrt{\tau2[t]}};$$

$$G[2,\ 3] = \frac{9\,\alpha^2\,\lambda2\,\lambda3\sqrt{\tau2[t]\ \tau3[t]}}{8\,v^2};$$

$$G[2,\ 4] = \frac{9\,\alpha^2\,\lambda2\,\lambda4\sqrt{\tau2[t]\ \tau4[t]}}{8\,v^2};$$

$$G[3,\ 1] = \frac{9\,\alpha^2\,\lambda1\,\lambda3\sqrt{\tau1[t]\ \tau3[t]}}{8\,v^2};$$

$$G[3,\ 2] = \frac{9\,\alpha^2\,\lambda2\,\lambda3\sqrt{\tau2[t]\ \tau3[t]}}{8\,v^2};$$

$$G[3,\ 3] = \frac{3\,\alpha\,\lambda 3\,\left(v + 3\,\alpha\,\lambda 3\,\tau 3[t]^{3/2}\right)}{8\,v^2\,\sqrt{\tau 3[t]}};$$

$$G[3,\ 4] = \frac{9\,\alpha^2\,\lambda 3\,\lambda 4\,\sqrt{\tau 3[t]\,\tau 4[t]}}{8\,v^2};$$

$$G[4,\ 1] = \frac{9\,\alpha^2\,\lambda 1\,\lambda 4\,\sqrt{\tau 1[t]\,\tau 4[t]}}{8\,v^2};$$

$$G[4,\ 2] = \frac{9\,\alpha^2\,\lambda 2\,\lambda 4\,\sqrt{\tau 2[t]\,\tau 4[t]}}{8\,v^2};$$

$$G[4,\ 3] = \frac{9\,\alpha^2\,\lambda 3\,\lambda 4\,\sqrt{\tau 3[t]\,\tau 4[t]}}{8\,v^2};$$

$$G[4,\ 4] = \frac{3\,\alpha\,\lambda 4\,\left(v + 3\,\alpha\,\lambda 4\,\tau 4[t]^{3/2}\right)}{8\,v^2\,\sqrt{\tau 4[t]}};$$

```
V1 = inG[1, 1] D[V, τ1[t]] + inG[1, 2] D[V, τ2[t]] + inG[1, 3] D[V, τ3[t]] + inG[1, 4] D[V, τ4[t]];
V2 = inG[2, 1] D[V, τ1[t]] + inG[2, 2] D[V, τ2[t]] + inG[2, 3] D[V, τ3[t]] + inG[2, 4] D[V, τ4[t]];
V3 = inG[3, 1] D[V, τ1[t]] + inG[3, 2] D[V, τ2[t]] + inG[3, 3] D[V, τ3[t]] + inG[3, 4] D[V, τ4[t]];
V4 = inG[4, 1] D[V, τ1[t]] + inG[4, 2] D[V, τ2[t]] + ınG[4, 3] D[V, τ3[t]] + inG[4, 4] D[V, τ4[t]];
gττ1 = gamma[1, 1, 1] τ1'[t] τ1'[t] + gamma[1, 2, 2] τ2'[t] τ2'[t] +
  gamma[1, 3, 3] τ3'[t] τ3'[t] + gamma[1, 4, 4] τ4'[t] τ4'[t] + 2 gamma[1, 1, 2] τ1'[t] τ2'[t] +
  2 gamma[1, 1, 3] τ1'[t] τ3'[t] + 2 gamma[1, 1, 4] τ1'[t] τ4'[t] +
  2 gamma[1, 2, 3] τ2'[t] τ3'[t] + 2 gamma[1, 2, 4] τ2'[t] τ4'[t] + 2 gamma[1, 3, 4] τ3'[t] τ4'[t];
gττ2 = gamma[2, 1, 1] τ1'[t] τ1'[t] + gamma[2, 2, 2] τ2'[t] τ2'[t] +
  gamma[2, 3, 3] τ3'[t] τ3'[t] + gamma[2, 4, 4] τ4'[t] τ4'[t] + 2 gamma[2, 1, 2] τ1'[t] τ2'[t] +
  2 gamma[2, 1, 3] τ1'[t] τ3'[t] + 2 gamma[2, 1, 4] τ1'[t] τ4'[t] +
  2 gamma[2, 2, 3] τ2'[t] τ3'[t] + 2 gamma[2, 2, 4] τ2'[t] τ4'[t] + 2 gamma[2, 3, 4] τ3'[t] τ4'[t];
gττ3 = gamma[3, 1, 1] τ1'[t] τ1'[t] + gamma[3, 2, 2] τ2'[t] τ2'[t] +
  gamma[3, 3, 3] τ3'[t] τ3'[t] + gamma[3, 4, 4] τ4'[t] τ4'[t] + 2 gamma[3, 1, 2] τ1'[t] τ2'[t] +
  2 gamma[3, 1, 3] τ1'[t] τ3'[t] + 2 gamma[3, 1, 4] τ1'[t] τ4'[t] +
  2 gamma[3, 2, 3] τ2'[t] τ3'[t] + 2 gamma[3, 2, 4] τ2'[t] τ4'[t] + 2 gamma[3, 3, 4] τ3'[t] τ4'[t];
gττ4 = gamma[4, 1, 1] τ1'[t] τ1'[t] + gamma[4, 2, 2] τ2'[t] τ2'[t] +
  gamma[4, 3, 3] τ3'[t] τ3'[t] + gamma[4, 4, 4] τ4'[t] τ4'[t] + 2 gamma[4, 1, 2] τ1'[t] τ2'[t] +
  2 gamma[4, 1, 3] τ1'[t] τ3'[t] + 2 gamma[4, 1, 4] τ1'[t] τ4'[t] + 2 gamma[4, 2, 3] τ2'[t] τ3'[t] +
  2 gamma[4, 2, 4] τ2'[t] τ4'[t] + 2 gamma[4, 3, 4] τ3'[t] τ4'[t];
dotσ = Sqrt[G[1, 1] τ1'[t]^2 + G[2, 2] τ2'[t]^2 + G[3, 3] τ3'[t]^2 + G[4, 4] τ4'[t]^2 +
    2 G[1, 2] τ1'[t] τ2'[t] + 2 G[2, 3] τ2'[t] τ3'[t] + 2 G[2, 4] τ2'[t] τ4'[t] +
    2 G[3, 4] τ3'[t] τ4'[t] + 2 G[1, 4] τ1'[t] τ4'[t] + 2 G[1, 3] τ3'[t] τ1'[t]];
```

```
H = Sqrt[V/(3 - 1/2 dotσ^2)]; ϵ1[t] = 1/2 dotσ^2;

τ1min = 76212.0966316556` ; τ2min = 246.98550953829155` ; τ30 = 471; τ40 = 473;
dτ30 = -D[Log[V], τ3[t]] /. {τ1[t] → τ1min, τ2[t] → τ2min, τ3[t] → τ30, τ4[t] → τ40};
dτ40 = -D[Log[V], τ4[t]] /. {τ1[t] → τ1min, τ2[t] → τ2min, τ3[t] → τ30, τ4[t] → τ40};
s4 = NDSolve[{(τ1''[t] + gττ1)/(3 - 1/2 dotσ^2) + τ1'[t] == -V1/V, (τ2''[t] + gττ2)/(3 - 1/2 dotσ^2) + τ2'[t] == -V2/V,
   (τ3''[t] + gττ3)/(3 - 1/2 dotσ^2) + τ3'[t] == -V3/V, (τ4''[t] + gττ4)/(3 - 1/2 dotσ^2) + τ4'[t] == -V4/V, τ1[0] == τ1min, τ2[0] == τ2min,
   τ3[0] == τ30, τ4[0] == τ40, τ1'[0] == 0, τ2'[0] == 0, τ3'[0] == dτ30, τ4'[0] == dτ40},
  {τ1[t], τ2[t], τ3[t], τ4[t]}, {t, 0, 200}, MaxSteps → 500000, AccuracyGoal → 4]
```

NDSolve::mxst: Maximum number of 500000 steps reached at the point t == 62.6763959712055`. >>

```
{{τ1[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ2[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ3[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ4[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t]}}
```

```
s4prime = NDSolve[{(τ1''[t] + gττ1)/(3 - 1/2 dotσ^2) + τ1'[t] == -V1/V, (τ2''[t] + gττ2)/(3 - 1/2 dotσ^2) + τ2'[t] == -V2/V,
   (τ3''[t] + gττ3)/(3 - 1/2 dotσ^2) + τ3'[t] == -V3/V, (τ4''[t] + gττ4)/(3 - 1/2 dotσ^2) + τ4'[t] == -V4/V, τ1[0] == τ1min, τ2[0] == τ2min,
   τ3[0] == τ30, τ4[0] == τ40, τ1'[0] == 0, τ2'[0] == 0, τ3'[0] == dτ30, τ4'[0] == dτ40},
  {τ1'[t], τ2'[t], τ3'[t], τ4'[t]}, {t, 0, 200}, MaxSteps → 500000, AccuracyGoal → 4]
```

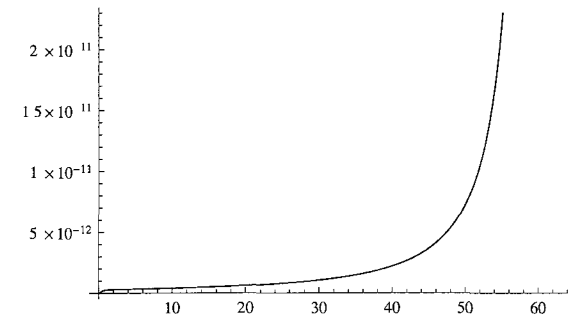
NDSolve::mxst: Maximum number of 500000 steps reached at the point t == 62.6763959712055`. >>

```
{{τ1'[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ2'[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ3'[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t],
  τ4'[t] → InterpolatingFunction[{{0., 62.6764}}, <>][t]}}
```

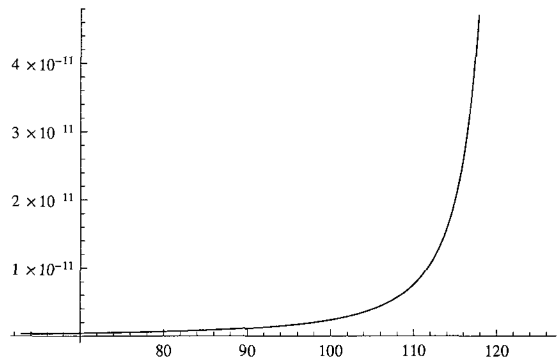
```
tf = Extract[Head[Extract[τ1[t] /. s4, {1}]], {1, 1, 2}]
```

```
62.6764
```

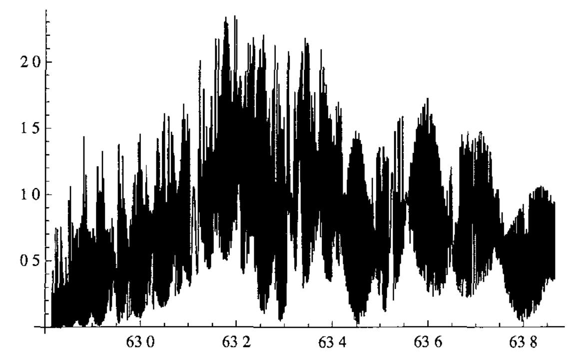
```
Plot[Evaluate[(ϵ1[t] /. s4) /. s4prime], {t, 0, tf}]
```



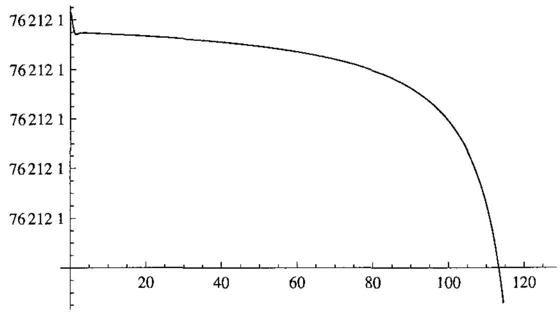

```
Plot[Evaluate[(ϵ1[t] /. s4) /. s4prime], {t, 62.89, tf}]
```



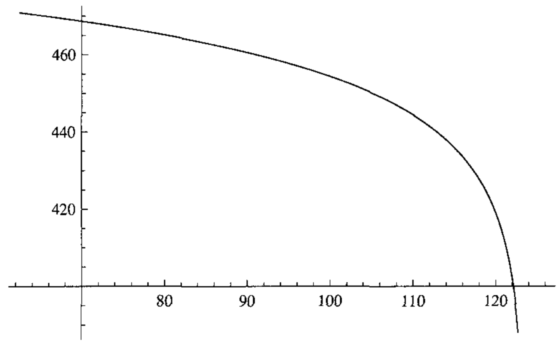

```
Plot[Evaluate[(ϵ1[t] /. s4) /. s4prime], {t, 62.8, tf}]
```



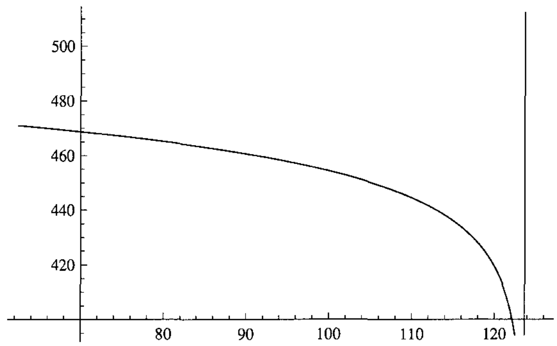
```
Plot[Evaluate[(τ1[t] /. s4) /. s4prime], {t, 0, tf}]
```



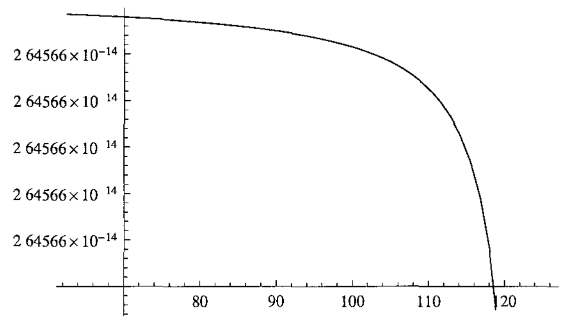

```
Plot[Evaluate[(τ2[t] /. s4) /. s4prime], {t, 62.78, tf}]

Plot[Evaluate[(τ3[t] /. s4) /. s4prime], {t, 62.5, tf}]
```



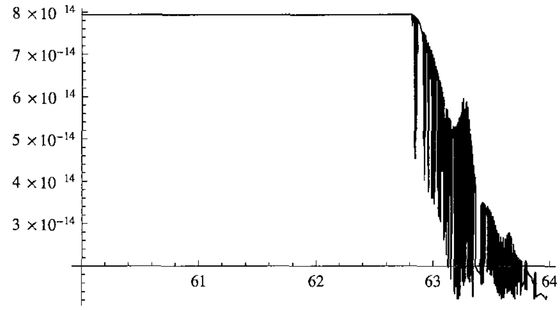

```
Plot[Evaluate[(τ4[t] /. s4) /. s4prime], {t, 62.5, tf}]
```



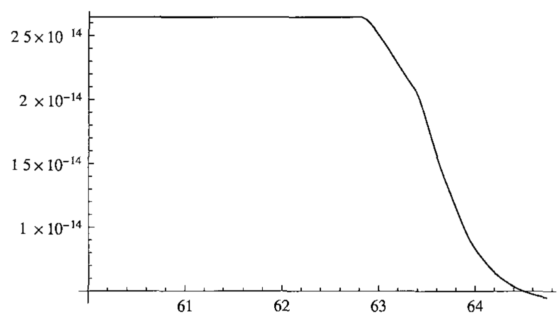
```
Plot[Evaluate[(H^2 /. s4) /  s4prime], {t, 62.5, tf}]
```




```
Plot[Evaluate[(V /. s4) /. s4prime], {t, 60, tf}]
```

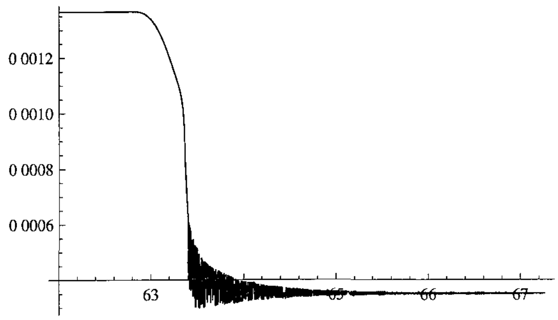



```
Plot[Evaluate[(H^2 /. s4) /. s4prime], {t, 60, tf}]
```



The normalized field ϕ

```
Plot[Evaluate[(Sqrt[(2 α λ4)/(3 α τ1[t]^(3/2))] τ4[t]^(3/4) /. s4) /. s4prime], {t, 62, tf}]
```



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