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Exploring lesson study as a form of professional development for enriching teacher knowledge and classroom practices

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Exploring Lesson Study as a Form of Professional Development

for Enriching Teacher Knowledge and Classroom Practices

BY

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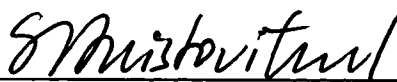
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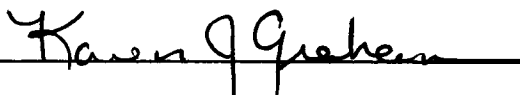
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DEDICATION

This work is dedicated with all my love to my daughter Katherine Rose

Mitcheltree. May she use this as an inspiration to always persevere to achieve

her lifetime goals!

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ABSTRACT

Exploring Lesson Study as a Form of Professional Development for Enriching Teacher Knowledge and Classroom Practices

by

Melissa K. Mitcheltree
University of New Hampshire, December, 2006

This study tracked the development of teacher knowledge through a professional development experience called lesson study. Lesson study is a Japanese professional development process in which a group of teachers develop a series of lessons using the following stages: planning lessons, teaching/observing the lessons, reflecting on lessons taught as well as sharing and discussing the lessons with their colleagues (Lewis, 2002). The aim of this study was to explore how the lesson study process influenced teacher knowledge of mathematics content and pedagogical content.

Study participants were four secondary mathematics teachers from a rural high school in the Northeast. All participants were certified in teaching secondary mathematics and varied in their educational background and teaching experience. These four teachers and the researcher created a Mathematics Lesson Study Group at the high school level. Throughout the 2004-2005 school year, this group met to plan three different mathematics lessons. After planning

each lesson, one teacher from the group taught the lesson to his/her class of students while the other teachers observed. Following the teaching/observing stage of the lesson development, the teachers had a debriefing meeting to reflect on how the lesson went and to propose possible revisions.

Each stage of the lesson study process- planning, teaching/observing, and debriefing was examined carefully in order to determine how the various aspects of that stage contributed to the development of the teachers' mathematics content and pedagogical content knowledge. This information was gathered from videotapes and teachers' notes taken at all meetings, journal reflections following the meetings, initial and final interviews, and classroom observations. All data was analyzed qualitatively. Results indicate that the teachers' mathematics content knowledge in the form of substantive and syntactic knowledge was influenced throughout all stages of the lesson study process. In addition, during each of the stages of lesson study the teachers' pedagogical content knowledge evolved in the areas of prior knowledge connections, anticipating student misconceptions, questioning, choosing example problems, assessing student understanding during the lesson, and curriculum knowledge. Lastly, the results support how important the components of planning and reflection, within the lesson study model, are to the enrichment of teachers' knowledge and classroom practices.

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CHAPTER 1

INTRODUCTION

Purpose

This research is a study of teacher knowledge development in the context of in-service professional development experience for mathematics teachers. The purpose of this research was to understand how mathematics teachers' knowledge progresses as they participate in lesson study as a professional development experience. Lesson study is a Japanese professional development method in which teachers systematically examine their classroom practice in order to improve instruction (Fernandez & Chokshi, 2002). Lewis (2002) reports the following four stages of the Japanese model of the Lesson Study Cycle: Goal-Setting and Planning, Research Lesson, Lesson Discussion, and Consolidation of Learning. In the first stage of lesson study, a group of teachers work together to establish a goal or a set of goals that they want to accomplish with their students and teachers meet regularly to plan a lesson. Once the group of teachers plans the lesson in the second stage of lesson study,

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at least one of the teachers conducts the lesson while the other teachers observe how the lesson is carried out in the classroom. In the third stage of lesson study after the lesson is taught, the group of teachers meet s again to debrief or reflect on the success of the group's lesson. In the fourth stage, if desired the teachers re-teach a refined lesson and study it again. In this study, stage one is referred to as the planning stage, stage two is referred to as the teaching/observing stage, stage three is referred to as the debriefing stage, and the fourth stage was not carried out. As the teachers work together to plan, teach/observe, and debrief the lesson, there are many opportunities for the teachers to share instructional strategies and learn from one another. During this type of professional development experience teachers are given the opportunity to enhance their knowledge of mathematics content and pedagogy. The purpose of this study was to examine and clearly document the elements of the lesson study experience that aid in the development of teacher knowledge. The results of this study add to the limited amount of research on lesson study in the United States and provide an examination of lesson study and its effect on teacher knowledge.

Why Lesson Study and Teacher Knowledge?

As we focus on teacher knowledge, we must consider the complex nature of this knowledge. Teacher knowledge will be examined in two forms – mathematics content knowledge and pedagogical content knowledge. Mathematics content knowledge also referred to by others as subject matter

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knowledge (Ball & Bass, 2000; Ball 1990; Mosenthal & Ball, 1992) is the organization of knowledge of mathematics in the mind of the teacher (Shulman, 1986). Pedagogical knowledge consists of the components of the teaching process that could be applied to any content area such as lesson planning, classroom management, and assessment. Kauchak and Eggen (1993) state "pedagogical knowledge is the information we gather about the process of teaching itself from research and the experience of expert teachers that helps us understand connections between teaching and learning" (p.11). Pedagogical content knowledge is the content knowledge that is necessary for teaching (Shulman, 1986). Ball and Bass (2000) use the term pedagogical content knowledge to describe a "unique subject-specific body of pedagogical knowledge that highlights the close interweaving of subject matter and pedagogy in teaching" (p.87). Pedagogical content knowledge is a subset of content knowledge that is necessary for planning and executing lessons. In teaching mathematics, pedagogical content knowledge may include useful representations; unifying concepts; clarifying examples and counter examples; helpful analogies; and important relationships and connections among concepts (Grouws & Schultz, 1996).

My experience as a secondary mathematics teacher has sparked my interest in the professional development of in-service teachers. The professional development activities initiated by the school system, where I was once an employee, consisted of small amounts of information about diverse topics, rather than setting out to reach long-term goals for the school or individual teachers.

Many professional development workshops or activities have teachers consider how the instructional strategy or "best practice" can be used in the classroom, but may not have the teacher actively plan an activity or lesson that can be directly applied to their own classroom. This type of professional development activity does not contribute as well to my knowledge growth. This lack of quality experiences have prompted my research and the desire to add to mathematics education research in the area of professional development.

Professional development experiences should serve as a bridge between knowledge teachers possess and the new demands of an ever changing society (U.S. Department of Education, 1995). After their undergraduate course work and preparation exercises, teachers arrive at their schools possessing the mathematics content and pedagogical knowledge they acquired, but in need of continuing appropriate activities to enhance their knowledge base. One of the characteristics of lesson study, as a form of professional development, is that lesson study values teachers. According to Lewis (2002), lesson study "is a system of research and development in which teachers advance theory and practice through the careful study of their own classrooms, constantly testing and improving on 'best practices'" (p.12).

Lynn Liptak, one of the first United States principals to implement lesson study contrasts traditional professional development and lesson study in the following chart.

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TRADITIONAL	LESSON STUDY
• Begins with answer • Driven by outside "expert"	• Begins with question • Driven by participants
• Communication flow: Trainer → teachers	• Communication flow: among teachers
• Hierarchical relations between trainer and learners	• Reciprocal relations among learners
• Research informs practice	• Practice *is* research

Table 1.1 Contrasting Views of Professional Development (Lewis, 2002,p.12)

Lesson study values teachers in professional development in several ways. First, teachers are in control of the discussions that take place during lesson study. Second, there is communication between teachers as they conduct the professional development experience which can be directly applied to their classrooms. Third, lesson study places teachers in an active role as researchers and implementers.

The nature of lesson study, as a form of professional development, lends itself well to the complex nature of teacher knowledge. An important part of lesson study is group planning of lessons that will actually be taught by at least one member of the group (Fernandez, Chokshi, Cannon, & Yoshida, 2001). Another major component of lesson study is reflection (Fernandez et al, 2001). The teachers take the time to reflect on the general everyday activities that go into planning and implementing the lessons. Reflection among the lesson study group members also takes place within debriefing sessions after the lessons are taught (Fernandez et al, 2001). The complex nature of teacher knowledge is the

focus of this study.   The nature of lesson study, as a professional development experience, will help illuminate the effect of lesson study on teacher knowledge.

Research Questions

The research question guiding this study follows directly from my purposes presented above and the conceptual argument that will be presented in Chapter Three and is central to my inquiry: How does lesson study influence teacher knowledge and classroom practices?   In this study, I set out to discover how the mathematics teachers further their teacher knowledge in relation to the lesson study experience. I claim that an effective professional development activity should cause teachers to think about their mathematics content and/or pedagogical content knowledge and how they can use it to improve student learning. In Chapter Three, I will further develop the conceptual argument that lesson study is a professional development experience that has the power to do this. In this study, I explored how the teachers participating in lesson study develop their classroom practices particularly their teacher knowledge.

While there is one main question governing this inquiry, there are other topical questions that contribute to the central focus:

1. What elements of the lesson planning stage contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

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2. What aspects of the teachers' observations of the taught lessons contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

3. How does reflecting on the lesson study process contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

In order to examine how the lesson study experience contributes to teacher knowledge, I needed to look at how each component of lesson study impacted the teachers' knowledge base. Therefore, each stage of the lesson study process- planning, teaching/observing the lesson, and debriefing were examined carefully in order to determine what aspects of teacher knowledge were influenced by the lesson study experience as well as to examine the interaction between mathematics content and pedagogy.

Lesson planning is a major part of the lesson study process. Grouws and Shultz (1996) stress that planning for instruction is where mathematics content knowledge, pedagogical knowledge, and pedagogical content knowledge converge. Heibert, Gallimore, and Stigler (2002) view lesson plans as the unit of analysis for converting practitioner knowledge into professional knowledge. Fernandez and Yoshida (2001) add that lesson study is fueled by lesson planning – an act that comes naturally to teachers. In addition, Byrum, Jarrel, and Munoz (2002) found that one of the benefits for Kentucky teachers in their study is that lesson study changed their thinking process when planning lessons.

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As one teacher taught each lesson and the other teachers observed the learning that took place in the classroom, I analyzed videotapes and observer notes to examine the elements of this stage that contributed to the development of the teachers' mathematics content and pedagogical content knowledge. It was also important to examine the teachers' reflections on the lesson study experience. Another major component of lesson study is the reflection that takes place in the debriefing sessions. Lewis (2002) and Byrum, Jarrel, and Monzo (2002) point out that Japanese teachers feel that they learn from observing others. Itzel (2002) reports of Delaware teacher's interest in lesson study because of the opportunity to improve instruction through discussions and reflections with other colleagues.

By collecting data related to each of these topical questions, I strived to illustrate the main objective of this inquiry – examining how lesson study influences teacher knowledge and classroom practices.

Organization of Dissertation

Chapter Two is an in-depth literature review of the two forms of teacher knowledge – mathematics content knowledge and pedagogical content knowledge; professional development in general; and lesson study. Chapter Three focuses on the theoretical perspective in which I approached this project. It provides a conceptual argument that ties together the forms of teacher knowledge, lesson planning and reflection, and how lesson study has the potential to improve teacher knowledge. Chapter Four provides a detailed

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description of the research methodology utilized in this study. It includes the research design, data collection procedures, as well as, descriptions of data coding and analysis. Chapter Five reports results of the data analysis for lesson #1, lesson #2, and lesson #3 across the stages of lesson study. Chapter Six includes the results of data analysis by stages across lesson #1, lesson #2, and lesson #3. Chapter Seven contains the teacher by teacher results from the data analysis of interviews, journals, and classroom observations. Lastly, Chapter Eight concludes with a discussion of the results reported in Chapters Five, Six, and Seven, implications of the results, and possible directions for future research.

CHAPTER 2

LITERATURE REVIEW

Literature was reviewed in the following three broad subject areas that helped shape this study: literature related to the forms of teacher knowledge-mathematics content knowledge and pedagogical content knowledge; literature related to professional development; and literature related to lesson study.

Mathematics Content Knowledge

Mathematics content knowledge provides a base of material that is necessary in order for teachers to consider what pedagogical or pedagogical content knowledge they will need to teach a specific mathematical concept. The NCTM *Professional Standards for Teaching Mathematics* (1991), supports this claim as it states,

> Knowledge of both the content and discourse of mathematics is an
>
> essential component of teacher's preparation for the profession. Teachers'
>
> comfort with, and confidence in, their own knowledge of mathematics
>
> affects what they teach and how they teach it. Their conceptions of
>
> mathematics shape their choice of worthwhile mathematical tasks, the

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kinds of learning environments they create, and the discourse in their classrooms. (p.132)

Shulman and Grossman (1992) identify two aspects of content knowledge, also referred to as subject matter knowledge. First, content knowledge consists of the understanding of key facts, concepts, principles, and explanatory frameworks within a discipline, known as substantive knowledge. In mathematics substantive knowledge includes mathematical facts, concepts, and computational algorithms (Brown & Borko, 1992). Second, content knowledge consists of the rules of evidence and proof within the discipline, known as syntactic knowledge (Shulman & Grossman). Syntactic knowledge includes an understanding of methods of mathematical proof and forms of argument mathematicians' use (Brown & Borko).

Ball (1990) states

Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways – with story problems, pictures, situations, and concrete materials. They need to understand the subject flexibly enough so that they can interpret and appraise students' ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions. (p.458)

Ball offers three criteria that characterize the kind of substantive knowledge teachers need. First, teachers' knowledge of concepts and procedures must be correct. Second, teachers must also understand the underlying principles and meanings of the mathematical concepts. Third, Ball

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stresses the need for teachers to understand and appreciate the connections between mathematical concepts. As a conclusion of her study of American and Chinese elementary teachers, Ma (1999) states "to understand the key to reform: whatever the form of classroom interactions might be, they must focus on substantive mathematics" (p.151). Ma explains further that we cannot expect a classroom to have a tradition of inquiry mathematics when the teacher's knowledge of the mathematics taught in elementary school is limited to procedures.

Ball (1990) refers to syntactic knowledge as "knowledge about mathematics". Ball has studied the ways in which prospective teachers' ideas about mathematics influence their representations of mathematics. It should not be assumed that people understand the meanings of the mathematical processes that they have learned to perform. Interviews in Ball's study indicate that prospective teachers lack explicit understanding of concepts and procedures even when they can perform the calculations involved. Ball argues that in order to teach mathematics effectively, individuals must have knowledge of mathematics characterized by an explicit conceptual understanding of the principles and meaning underlying mathematical procedures. Shulman (1986) offers a similar claim when he writes, "The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied" (p.9).

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In the early 1990's two influential mathematics organizations published documents recommended the mathematics content knowledge necessary for mathematics teachers. According to the *Professional Standards for Teaching Mathematics* (1991) there are appropriate mathematical concepts and procedures to be studied at any level of mathematical study. In the elaboration section of Standard 2: Knowing Mathematics and School Mathematics, this document lists the mathematical content knowledge essential for teachers in grades K-4, 5-8, and 9-12. In 1991, the Mathematical Association of America also published – *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics* (Leitzel). This document describes the collegiate mathematical experiences that a teacher needs in order to be an "ideal" mathematics teacher in classrooms of the 1990's and beyond. Similar to the elaboration section of Standard 2, the sections of *A Call for Change* address Standards for the mathematical preparation of teachers common to all grade levels and then specifically for K-4, 5-8, and 9-12. The specific Standards describe broad knowledge and understanding of mathematics needed by mathematics teachers.

The Mathematical Education of Teachers (MET) is a report prepared by the Conference Board of Mathematical Sciences (CBMS) and was published by the American Mathematical Society (AMS) in cooperation with the Mathematics Association of America (MAA) in 2001. It was designed as a resource for departments of mathematics at the post-secondary level. The document lists recommendations for mathematics departments on the number of semester

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hours teachers should take at each of the elementary, middle, and high school levels, and details and explanations of the mathematics that should be addressed in content courses for teachers at each of the levels. One particular statement that this document proclaims is that the mathematics that teachers need to know in order to teach mathematics is substantively different from the usual mathematics offered by mathematics departments and that this mathematics is worthy of study. This mathematics is what Lee Shulman refers to as *pedagogical content knowledge,* Zalman Usiskin calls this *teachers' mathematics,* and MET describes it as *mathematical knowledge for teaching.*

Pedagogical Content Knowledge

Researchers (Ball & Bass, 2000; Steinbring, 1998; Grouws & Schultz, 1998) are looking at the content and nature of teachers' special subject matter understanding. Shulman (1986) describes pedagogical content knowledge as content knowledge which is pedagogical knowledge that goes beyond simply subject matter to the dimension of subject matter for teaching. According to Shulman, ". . . to think properly about content knowledge requires going beyond knowledge of facts or concepts of a domain. It requires understanding of the structure of the subject matter" (p.9). In addition, he states that pedagogical content knowledge includes" . . . the ways of representing and formulating the subject that makes it comprehensible to others" (p.9).

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Ball and Bass (2000) point out teachers build up bundles of such knowledge over time as they teach the same mathematical topics.

These bundles of pedagogical content knowledge contain mathematical knowledge along with knowledge of learners, learning, and pedagogy. Ball and Bass (2000) indicate that these bundles can be beneficial to mathematics teachers in the course of a lesson because they can help the teacher anticipate areas where a student may have difficulty. Within his pedagogical content knowledge the teacher can have alternative methods and explanations ready for those students having trouble learning the concepts. Ball and Bass point out that a body of such bundled knowledge may not always equip teachers with the flexibility needed to manage the complexity of the teaching process. They argue, "Teachers also need to puzzle about the mathematics in a student's idea, analyze a textbook presentation, and consider the relative value of two different representations in the face of a particular mathematical issue" (p.88). They add that in order to do this, teachers' need a kind of mathematical understanding that is "pedagogically useful and ready, not bundled in advance with other considerations of students or learning or pedagogy" (p.88).

Ball and Bass (2000) remind us that no body of pedagogical content knowledge can be extensive enough to adequately anticipate what every student may think or how the instruction of some mathematical topic may evolve in a class. When teachers are involved in novel situations, they must coordinate all areas of their knowledge- that of content, students, learning, and pedagogy. They may not be able to pull out one of their strategies or an answer on the spot.

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Another part of pedagogical content knowledge can be categorized as curricular knowledge. The curriculum and the materials associated with it are the core of a teacher's pedagogy (Shulman, 1986). Shulman presents the knowledge of alternative curriculum materials for a given subject or topic within a grade as one aspect of curricular knowledge. A second aspect of curricular knowledge is the teachers' understanding of curriculum materials under study by students in classes they are taking simultaneously. Shulman states "this lateral curriculum knowledge . . . underlies the teacher's ability to relate the content of a given course or lesson to topics or issues being discussed simultaneously in other classes" (p.10). A third aspect of curricular knowledge is for the teacher to be familiar with the curriculum materials that precede and follow the classes he is currently teaching.

Hill and Ball (2004) are currently using the phrase "knowing mathematics for teaching" to further describe pedagogical content knowledge. They examine knowledge for teaching mathematics in two ways: specialized knowledge of content and common knowledge of content. Specialized knowledge of content is unique to mathematics teachers where as common knowledge of content is readily known by non-teachers. Hill and Ball believe that "teachers of mathematics need *both* types of content knowledge to teach this subject matter competently" (p.333).

There are several studies that are worth reporting here to display the importance of including curricular knowledge as a part of pedagogical content knowledge. These studies show a connection between professional

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development experiences and teacher knowledge. Cohen and Hill (2000) use data from 1994 survey of California elementary teachers and 1994 student California Learning Assessment System (CLAS) to examine the influence of assessment, curriculum, and professional development on teacher practice and student achievement. They found that teachers' opportunities to learn about reform affect their knowledge and practices. When these opportunities were situated in curriculum that was designed to be consistent with the reforms, and in curriculum in which their students studied, teachers reported practice that was consistently closer to the aims of the policy. Furthermore, Cohen and Hill state "Since the assessment of students' performances was consistent with the student and teacher curriculum, teachers' opportunities to learn paid off for students' math performance" (p.329). Wiley and Yoon (1995) investigated the impact of teachers' learning opportunities on student performance on the 1993 CLAS. They found higher student achievement when teachers had extended opportunities to learn about mathematics curriculum and instruction. Brown, Smith, and Stein (1996) analyzed teacher learning, practice, and student achievement data collected from four QUASAR project schools. They found that students had higher scores when teachers had more opportunities to study a coherent curriculum designed to enhance both teacher and student learning. As a result of her study of American and Chinese elementary teachers, Ma (1999) stresses the importance the role that curricular materials, including textbooks, might play in reform. "Teachers need not have an antagonistic relationship with

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textbooks. My data illustrate how teachers can both use and go beyond the textbook." (Ma, 1999, p.150)

There has been much research done on teacher knowledge in connection with pre-service teachers. Research on pre-service mathematics teachers has focused on the connection between subject matter knowledge and pedagogical content knowledge (Ball, 1988; Hutchinson, 1997). Ball & Wilson (1990) examine the mathematics content that prospective mathematics teachers bring to the classroom. Fuller (1996) reports on a study done to compare novice and experienced elementary teachers' pedagogical knowledge and pedagogical content knowledge of three major topics in mathematics: whole number operations, fractions, and geometry. Lowery (2002) reports on a study done in a methods course with content specific instruction in elementary mathematics and science that involved the students working with in-service elementary teachers. The findings confirmed the acquisition of pedagogical content knowledge and the extent of knowledge construction by pre-service teachers. Frykholm and Glasson (2005) report on a study done to examine the content knowledge, pedagogical content knowledge, attitudes and beliefs with respect to the integration of mathematics and science, that prospective secondary mathematics and science teachers bring to their teacher preparation programs. This study went further and explored a collaborative model that would foster pre-service teachers' desire and ability to connect mathematics and science instruction.

Hill and Ball (2004) describe an effort to evaluate California's Mathematics Professional Development Institutes (MPDIs) using novel measures of

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knowledge for teaching mathematics. No previous measures have been employed to understand how useful and useable knowledge of mathematics develops in teachers. Their analyses of these measures showed that teachers participating in MPDIs improved their performance on these measures during the extended summer workshop portion of their experience. Hill and Ball state, "Our results show that teachers *can* learn mathematics for elementary school teaching in the context of a single professional development program. This alone is news: policymakers, mathematics educators, and others can successfully design programs that improve teacher's content knowledge for teaching, a goal named prominently in many of today's published reports, policy recommendations, and research programs." (p.345)

Two other studies that look at mathematics content knowledge in in-service teacher education are centered on summer mathematics institutes. Mosenthal and Ball (1992) analyze how the staff of SummerMath for Teachers helps elementary school teachers develop constructivist teaching practices. The analysis showed that the program is based on a principled conception of the subject matter, but developing teachers' subject matter was not an explicit objective of the program. Jones and Holder (2001) report on a ten-day summer mathematics content institute held in Alaska. They set out to determine what content teachers have the opportunity to learn in a summer content institute and how is the content decided. They report "the opportunity to learn for teachers in this institute was founded on a deliberate responsiveness to their interests, social

as well as academic needs, and specific teaching contexts" and add that "the content was decided by the teacher-presenters" (p.13).

Professional Development

As the nation searches for ways to increase students' learning, improving classroom teaching is receiving renewed attention. Researchers are currently focusing on providing teachers with opportunities for high quality professional development (Heibert, Gallimore, & Stigler, 2002; Stigler & Heibert, 1999; Lee, 2001). In the early 1970's, the goal of in-service education was to bring outside expertise to teachers to increase their knowledge base. In the 1980's an extremely technical and simplistic view of teaching was dominant (Lee, 2001). According to Lee, the current focus of professional development has widened to include not only teachers but also the professional organizations to which teacher's belong.

Lee (2001) suggests several items that teacher educators and teachers should keep in mind as they strive to enrich professional development in their schools. First, professional development should be lifelong and relevant to student learning. Second, schools need to spend less time counting hours and programs that a teacher participates in and spend more time examining what happens as a result of their participation. Third, teachers need to become active decision makers in the process of designing and implementing professional development opportunities. Fourth, planning professional development should begin with the end goal in mind and should encourage teacher involvement in the

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planning process. Fifth, follow-up professional development should be provided – such as opportunities for practice in the classroom.

"Professional development experiences serve as the bridge between where prospective and experienced teachers are now and where they will need to be to meet the new challenges of guiding all students in achieving to higher standards of learning and development" (U.S. Department of Education, 1995, p.2). The image of a bridge is a useful metaphor for those who provide professional development opportunities for mathematics teachers. Viewed as a bridge, professional development is a link between where the teacher is and where they want to be. Each professional development program requires a careful and unique design to best meet the needs of the teachers and students. According to Susan Loucks-Horsley (1998), the scene in mathematics teacher professional development does not resemble the ideal of a sturdy bridge to the future. Loucks-Horsley states, "Instead, the professional development experience is typically weak, limited, and fragmented, incapable of supporting them as they carry the weight of adequately preparing future citizens. Programs fall short of helping teachers develop the depth of understanding they must have of mathematics content, as well as how best to help their students learn" (p.1).

Designing Professional Development for Teachers of Science and Mathematics, (Loucks-Horsley, Hewson, Love, Dyasi, Friel, Mumme, Sneider, & Worth, 1998), a publication of the National Science Foundation, includes five principles of effective professional development. These principles were

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developed as a result of research, theory, and the "wisdom" of experienced practicing professional developers.

1. Professional development experiences must have students and their learning at the core- and that means all students.
2. Excellent mathematics teachers have a very special and unique kind of knowledge that must be developed through their professional development learning experiences.
3. Principles that guide the improvement of student learning should also guide professional learning for teachers and other educators.
4. The content of professional learning must come from both research and practice.
5. Professional development must align with and support system-based changes that promote student learning. (p.3)

The current study's focus on teacher knowledge is embedded within the second principle listed above. Here, the special and unique kind of knowledge Loucks-Horsley et al (1998) are referring to is pedagogical content knowledge. Loucks-Horsley et al indicate that although knowledge of general pedagogy and mathematics content are critical, they are not enough. Thus, they state "the goal of developing pedagogical content knowledge must be the focus of professional development opportunities for teachers" (p.4).

Garet, Porter, Desimone, Birman, & Yoon (2001) report on a study using a national probability sample of 1,027 mathematics and science teachers. The study provides the first large scale empirical comparison of the effects of different characteristics of professional development on teachers' learning. Their results indicate three core features of professional development activities that have significant positive effects on teachers' self-reported increases in knowledge and skills as well as changes in classroom practice. First, the professional

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development activity must focus on content knowledge. A second core feature of professional development involves the activities within the experience for teachers to become actively engaged in meaningful discussion, planning, and practice. Such activities may provide teachers with opportunities to observe expert teachers or to be observed teaching in their own classroom and obtain feedback. Active learning may include the opportunity for teachers to link ideas introduced during professional development experiences to what they do currently in their own classrooms. Two other elements of active learning are examining and reviewing student work and provide teachers with opportunities to give presentations, lead discussions, or produce written work. The third core feature of professional development concerns the extent to which the activities are perceived by teachers to be a part of a coherent program of teacher learning. Elements of coherence may include connections with school goals and activities, alignment with state and district standards and assessments, and communication among teachers who are engaged in efforts to reform their teaching in similar ways. Garet et al (2001) state that it is primarily through these core features of professional development experiences that the following structural features significantly affect teacher learning: the type of activity; collective participation of teachers from the same, school, grade or subject; and the duration of the activity.

Stigler and Heibert (1999) state, "Improvement [in teaching] will not happen by itself. It will require designing and building a research-and-development system that explicitly targets steady, gradual improvement of teaching and learning." (p.131) They advocate for a professional development

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program like lesson study in the United States. They include the following as one of the aspects of success of lesson study: "teachers who participate in lesson study see themselves as contributing to the development of knowledge about teaching as well as to their own professional development" (p.125). They point out that teachers in Japan feel they are contributing to the knowledge base of the teaching profession as a result of participating in lesson study. In their study of this Japanese professional development activity, Stigler and Heibert find that Japan has succeeded in developing a system, which not only develops teachers, but also develops knowledge about teaching. This knowledge is relevant to classrooms and can be shared among members of the teaching profession

In 2002, Heibert, Gallimore, and Stigler propose connections between teachers' knowledge and lesson study. Heibert et al were looking for a way for teacher's practitioner knowledge to emerge from the teacher's classroom to a shared, professional knowledge base for teaching. Heibert et al state, "To improve classroom teaching in a steady, lasting way, the teaching profession needs a knowledge base that grows and improves" (p.3). They define practitioner knowledge as the kind of knowledge teachers generate through active participation and reflection of their own classroom practice. Features of practitioner knowledge include: being linked with practice and integrated and organized around problems of practice. Heibert et al point out that within practitioner knowledge, content knowledge, pedagogical knowledge, and pedagogical content knowledge are intertwined, not according to type, but according to the problem the knowledge is intended to address. In order for the

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practitioner knowledge to become professional knowledge, it must be made public and storable and shareable. Heibert et al state "Collaboration, then, becomes essential for the development of professional knowledge, not because collaborations provide teachers with social support groups, but because collaborations force their participants to make their knowledge public and understood by colleagues" (p.7). Heibert et al further explain that teachers need to have a means of storing knowledge in a form that can be assessed and used by others.

Heibert, Gallimore, and Stigler (2002) also address how teachers can represent the knowledge they construct in a more principled and abstract form than in the past, while retaining its practical character. One possibility presented by Heibert et al is for daily lessons to be the unit of analysis. Analyzing lessons requires the teachers to focus on the many elements that make up their teaching. Lessons are small enough units that the complexity of teaching can be reduced to a manageable size. Analysis of lessons provides an organized way to move what was learned in one context or classroom into another.

Lesson Study

Heibert, Gallimore, and Stigler (2002), propose that Japanese lesson study is a system that could support the transformation from practitioner knowledge into professional knowledge. More will be elaborated on this proposition once we discuss the lesson study process.

Lesson study is a professional development process teachers in Japan engage in to continually improve the quality of the experiences they provide their students. It involves a group of teachers working together to accomplish three main activities. First, the teachers spend a great deal of time to identify a lesson study goal (Fernandez, Choshi, Cannon, and Yoshida, in press). For example, the teachers may decide that the students need to develop better problem solving skills or critical thinking skills. Or, they may come to a consensus within the group that a certain topic or concept is holding the students back from learning; therefore, they may decide to focus on subtraction or dividing fractions. The objective of this first step is for the teachers to select a goal to work on that will help them to move closer to their aspirations for students.

Next, the teachers select several target lessons to work on as a group. These lessons are called "study lessons" because they will be used to study how to meet the lesson study goal that has been chosen by the teachers (Fernandez et al, in press). The content of the "study lessons" comes from research as well as from practical experience of the teachers. The teachers begin by writing a detailed lesson plan. In the process of creating this plan, the teachers discuss topics such as: determining the content that will be taught, organizing the lesson and allocating time to different parts, anticipating students' responses to the lesson and the specific problems they are asked to work on, and deciding how student performance will be evaluated during the lesson. These study lessons are then taught in real classrooms. Anyone at the school can observe the teaching of these lessons and analyze them.

After the lesson has been taught, the teachers who worked on the lesson as well as those who observed the lesson come together for a debriefing meeting. The purpose of this reflection component is for the teachers to discuss what the lesson taught them about their students and the goal they set out to explore. Also, it is not uncommon for teachers who planned the lesson to decide to revise their lesson plan and re-teach the lesson to another group of students (Fernandez et al, in press).

Now, let's look at Heibert, Gallimore, and Stigler's (2002) proposition that lesson study can be the vehicle which transforms practitioner knowledge into professional knowledge. Lesson study groups generate knowledge that shares key features with practitioner's knowledge. While engaged in lesson study, the teachers work on a problem that is directly linked to their practice. Also, the lesson study groups focus on how the knowledge can be made most comprehensible by the students. Heibert et al state "the lesson provides a unit of practice in which the knowledge of teachers gets integrated into a useful form" (p.10). Heibert et al propose that lesson study generates practitioner knowledge but within a system containing features identified earlier as essential for transforming such knowledge into a professional knowledge base. Chokshi and Fernandez (2004) argue that lesson study can "serve as the vehicle by which practitioners can deepen their understanding of content" (p.521). The collaborative nature of lesson study allows teachers to learn basic content from one another as they plan lessons together. More important though is that the content knowledge developed during lesson study is in an embedded context.

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The task of learning the content is closely intertwined with the pedagogy and can be immediately applied in the classroom. Chokshi and Fernandez caution that learning content through lesson study is not an automatic process. The teachers involved must recognize these learning opportunities and develop productive strategies for capitalizing on them.

Catherine Lewis's (2000) work with science teachers in Japan exposed her to lesson study. She suggests several ways in which lesson study contributes to the improvement of instruction in Japanese classrooms based on interviews and observations. First, Japanese teachers feel that they learn from the feedback they get on their own teaching and the new ideas gained from watching others teach. Second, lesson study has helped Japanese teachers implement new topics added to the curriculum. The teachers have the opportunity to think through problems and questions with other teachers who have already worked with the new material in the classroom (Lewis & Tsuchida, 1998). Third, deciding on a lesson study goal encourages teachers to connect individual teachers' practices to the school goals. Fourth, lesson study gives teachers a chance to bring up, discuss, and perhaps reconcile competing goals or visions of education.

Fernandez and Chokshi (2002) state "we do not believe that there can be a 'one-size fits all' approach for integrating lesson study into the U.S. educational landscape. Instead, we encourage creative experimentation with lesson study that allows teachers to engage in high-quality learning experiences" (p.129). Chokshi and Fernandez (2004) add that lesson study has the following

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characteristics: lesson study is teacher directed, a concrete process, collaborative, and helps to build professional knowledge. The teachers involved in lesson study decide how to explore their chosen goals and student needs as they reflect on their current practice. Wilms (2003) supports this claim when he states that lesson study is a "continuous cycle of classroom problem-solving – a Plan, Do, Check, Act process – that is carried out by teachers themselves" (p.606). This examination of practice is concrete because the main activities of lesson study are embedded in what goes on each day in the classroom. Lesson study is also collaborative because teachers spend time together with a common purpose, sharing experiences. Lastly, continuous lesson study work can help teachers build a shared body of professional knowledge.

From his experience observing lesson study in Japan, Watanabe (2002) gives recommendations of what teachers in the U.S. and in other countries can learn from Japanese lesson study. First, Watanabe claims, "a successful lesson study group requires the development of a shared culture through collective participation" (p.38). Second, Watanabe proposes that teachers need to develop the habit of writing a detailed instruction plan that will be understandable by all teachers and observers involved in the lesson study process. Third, teachers need to develop a unit perspective. A typical Japanese instruction plan is not focused on just one day's lesson. Fourth, teachers need to anticipate students' thinking. Watanabe states "thinking about how students might respond to a given task is one of the main activities of a research lesson" (p.38). Fifth, Watanabe encourages teachers to learn to observe well. Observations recorded

during the teaching of the lesson are very important in the reflective component of lesson study. Sixth, teachers must play a central role in developing lesson study. As Lewis (2000) notes, lesson study should honor the central role of teachers. Lastly, Watanabe (2002) claims that knowledgeable others should be involved in every step of lesson study. Knowledgeable others, professionals from outside the school, are needed to help teachers transcend the limits of their own content knowledge (Harper, 2002)

Teachers at Patterson School 2, a public K-8 school in New Jersey, began lesson study in their school in September 1999. A group of Japanese teachers from Greenwich Japanese School in Connecticut also participated in this lesson study project. The teachers in Patterson 2 broke up into 4 lesson-planning sub-groups. The Japanese teachers rotated attendance at the planning meetings of each sub-group working on one lesson. Each group taught its lesson for the first time in January and a second time in February. In the spring, the groups each worked on a second lesson with less help from the Japanese teachers (Fernandez & Yoshida, 2001)

Fernandez and Yoshida (2001) report their observations at Patterson School 2. In particular, they identify features of lesson study that can inform our understanding about how to structure teacher learning. First, they point out that lesson study is based on a school-wide vision of improving teaching. At Patterson School 2, they observed this when the lesson study group asked every teacher in the school for suggestions as they strived to narrow down a goal for the group. Next, Fernandez and Yoshida, observed that lesson study asks

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teachers to plan, implement, and refine lessons with the premise that this exercise leads to reflection rather than asking teachers to examine their practice with the premise that reflection leads to teacher growth. Fernandez and Yoshida state, "This distinction is crucial because it means that lesson study is fueled by lesson planning, something that comes naturally to teachers, rather than by critical examination of practice, an activity that is harder to sustain" (p.35). Lastly, Fernandez and Yoshida found that lesson study places the focus on students rather than on teaching. Lesson study is about teachers working together to determine how to best serve their students.

Lynn Liptak (2002), principal of Patterson 2 School, gives her support of lesson study by stating,

> For too long, professional development time has been allocated to outside experts to 'train' teachers rather than given to teachers to reflect collaboratively on their practice. We need to tap outside expertise; we need to improve our content and pedagogical knowledge. But the professional development process needs to occur in the context of our classrooms and be driven as an on-going activity by professional practitioners. (p.7)

Janice Itzel (2002), teacher-on-loan in the Delaware Department of Education, assisted five Delaware school districts that implemented lesson study. In her explanation of why lesson study took hold in Delaware, Itzel states,

> The process of teachers observing lessons, conducting research, and revising lessons encourages not only the sharing of pedagogical and

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content knowledge, but also reflection. When teachers, through discussion and reflection, can improve the 'what' and the 'how' of teaching, and when these improvements are based on students' needs, this is professional development at its best. (p.10)

Lewis, Perry, & Hurd (2004) state "Lesson study is not just about improving a single lesson. It's about building pathways for ongoing improvement of instruction." (p.18) They elaborate with the following seven key pathways to instructional improvement that underlie successful lesson study: increased knowledge of subject matter, increased knowledge of instruction, increased ability to observe students, stronger collegial networks, stronger connection of daily practice to long-term goals, stronger motivation and sense of efficacy, and improved quality of available lesson plans.

Byrum , Jarrell, & Munoz (2002) report of a lesson study initiative implemented in 25 high schools/learning centers in the Jefferson County, Kentucky School District. Their action research involved five high schools in the district grouped together based on similar characteristics. Byrum et al found that the teachers cited many benefits of the lesson study initiative in terms of instructional practices, planning, and assessment. The teachers agreed that the opportunity to observe other colleagues teaching the same lesson in their own classroom was invaluable. Most of the teachers agreed the lesson study initiative changed their thinking process when planning lessons. In terms of assessment, the teachers experienced benefits in student assessment, and perhaps more importantly, self-assessment. The teachers were required to

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reflect and assess their own teaching, and they began to realize the importance of revising the lesson immediately after teaching it. Lastly, Byrum et al report that the biggest threats to the lesson study initiative were the cost and time involved in the process.

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CHAPTER 3

THEORETICAL FRAMEWORK

This study provided the participating teachers the opportunity to collaboratively plan, teach/observe, and debrief mathematics lessons that could be directly applied to their own classrooms. In the different stages of lesson study, the teachers have many opportunities to share ideas and reflect on their current teaching practices. In this chapter I take the definitions and related research of the forms of teacher knowledge, teacher professional development in general, and lesson study that were described in the literature review and develop a theoretical argument for why lesson study is a professional development tool that can be used to improve the participating teachers' knowledge in these areas.

Forms of Teacher Knowledge

This study is framed around the two forms of teacher knowledge: mathematics content knowledge and pedagogical content knowledge. The perspective on pedagogical content knowledge is based on the work of Shulman (1986) and Ball and Bass (2000). This knowledge may take on the form of

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guiding questions, example problems, various representations of a concept, and the ability to answer questions and anticipate possible misconceptions. Teachers need this type of knowledge in order to explain the mathematics concepts to their students in a coherent and systematic way. One particular subset of pedagogical content knowledge that is closely examined is curricular knowledge. Shulman (1986) defines this as familiarity with the current curriculum materials a teacher is working with, but also the curriculum from the previous and future mathematics course. The teachers need a broad picture of the mathematics concepts in order to explain to the students how one concept is connected to another and for the students to understand why it is important to learn the various mathematics concepts.

Shulman(1986) and Ball and Bass (2000) use the term pedagogical content knowledge to describe a special kind of knowledge needed by mathematics teachers. As Ma (1999) indicates, for teachers to develop this form of knowledge they need a "profound understanding of mathematics." Ma describes "profound understanding of mathematics" in terms of depth, breadth, and thoroughness. According to her, "depth" refers to the teacher's ability to connect ideas to the larger and more powerful ideas of mathematics. "Breadth" refers to the teacher's ability to connect ideas of similar conceptual power. "Thoroughness" refers to the teacher's ability to weave ideas into a coherent whole. I claim that this description given by Ma is a characteristic of pedagogical content knowledge.

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In this study, mathematics content knowledge is examined in terms of substantive knowledge and syntactic knowledge (Ball, 1990; Brown and Borko, 1992; and Shulman and Grossman, 1992). As students prepare to become mathematics teachers, it only makes sense that they have the knowledge of their subject matter. Knowing mathematics involves understanding specific concepts and procedures as well as simply the process of doing mathematics. Brown and Borko, (1992) and Shulman and Grossman (1992) refer to this knowledge as substantive knowledge. Teachers' mathematics knowledge must go beyond this in order to understand the essence of mathematics. They must also have the syntactic knowledge that allows them to understand the development of mathematics in terms of formal proof and argument.

NCTM (1991) states that mathematics teachers must have a deep understanding of the mathematics of the school curriculum. The teachers need opportunities to revisit school mathematics topics in ways that will allow them to develop connections among concepts. Ball and Bass (2000) point out that mathematics content knowledge is essential in order to listen to students and hear what ideas they are expressing and where they might be heading. In addition, Ball and Bass state that knowing the mathematics content is necessary in order for teachers to be inventive in creating worthwhile learning opportunities while keeping all students' needs in mind.

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Lesson Planning and Reflection

Lewis (2002) reports the following four stages of the Japanese model of the Lesson Study Cycle: Goal-Setting and Planning, Research Lesson, Lesson Discussion, and Consolidation of Learning. In this study, stage one is referred to as the planning stage, stage two is referred to as the teaching/observing stage, stage three is referred to as the debriefing stage, and the fourth stage was not carried out.

I propose that reflection and lesson planning are important teacher practices that affect teacher knowledge. It is often the case that teachers begin their teaching careers with the knowledge they acquired in their preparation programs, but do not strive to enhance that knowledge as they continue to teach. One possibility is that the professional development opportunities for such teachers have placed less emphasis on the reflection process. Teachers may not have taken the time to take what they learn in the professional development activity and examine how it has affected their classroom practices. Teacher's knowledge, skills, and dispositions are, in varying degrees, the product of what they have experienced as pupils, the professional training they have received, and their experience as teachers. According to Ball and Mosenthal (1990), researchers who are concerned with helping teachers change and develop their practices must consider how to influence most effectively the complex web of ideas, understanding, and habits that will shape what teachers actually do in their classrooms.

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A major component of lesson study is the reflection that takes place not only in the debriefing sessions, but throughout the entire lesson study process. For example, the teachers' reflections on the planning process may include the awareness of or the improvement of the knowledge they used to plan the lesson or the need to look for resources to plan future lessons. Lewis (2000) and Byrum, Jarrel, & Munoz (2002) point out that Japanese teachers feel that they learn a lot from the feedback they get on their own teaching and the new ideas gained from observing others. Itzel (2002) reports that Delaware teachers are interested in lesson study because of the opportunity to improve instruction through discussions and reflections with other colleagues.

Lesson planning is one of the essential components of teaching that determines what teachers will actually do in their classrooms. "Planning is the process by which teachers make decisions about how they intend to use their instructional time to enhance their students' mathematics learning." (Brown & Smith, 1997, p.140) By planning together, teachers in Brown & Smith's summer staff development seminars learned to transform their pedagogical content knowledge into plans for instruction that embodied the new knowledge. When these teachers planned together, they carefully selected problems for students, discussed what difficulties students might have, and considered how they would determine if students were really learning what was intended of them. Grouws and Shultz (1996) suggest that planning for instruction is where mathematics content and pedagogical content knowledge converge as one focuses on student understanding of the mathematics concepts. Heibert, Gallimore, and Stigler

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(2002) view lesson plans as the unit of analysis for converting practitioner knowledge into professional knowledge.

Planning study lessons is one of the major components of lesson study as a form of professional development (Fernandez, Chokshi, Cannon, & Yoshida, 2001). The teachers in the lesson study group can share their knowledge of mathematics content and pedagogical content. In the planning stage of lesson study is where the teachers are challenged to extend their own knowledge based on findings from research, other teacher's practical experience, or information from knowledgeable others. Fernandez and Yoshida (2001) add that lesson study is fueled by lesson planning. In addition, Byrum, Jarrel, & Munoz (2002) found one of the benefits for the Kentucky teachers is that lesson study has changed their thinking process when planning lessons.

Lesson Study for Improving Teacher Knowledge

I propose that lesson study is a professional development experience that can improve teacher knowledge. Lesson study measures up against Loucks-Horsley et al's (1998) principles of effective professional development (see pg. 22). In particular, lesson study is a process that has all students and learning as the central concern. In fact, lesson study groups spend much thoughtful time deciding on a lesson study goal – one that will promote student learning and meet the needs of the individual students. The content of professional learning that takes place in lesson study comes from the practical experience of the teachers. In addition, as these teachers devise study lessons, they have the

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opportunity to share their mathematics content knowledge, and pedagogical content knowledge with other teachers.

Lewis, Perry, & Hurd (2004) claim lesson study provides key pathways to instructional improvement. Two of these pathways are teachers' increased knowledge of subject matter and increased knowledge of instruction. As teachers in a lesson study group collaboratively plan lessons, they discuss the essential concepts and skills that their students need to learn, look at how the lesson fits into the curriculum and consider students prior knowledge and how they will respond to the planned lesson. While the teachers engage in these activities, they generate many questions about the subject matter. The teachers within the group can often answer such questions themselves and if not they may need to locate outside resources to assist them. In addition to enriching the teachers' mathematics content knowledge, the sharing of teaching experiences and the pedagogical decisions the group makes throughout the planning process provides excellent opportunities for the teachers to enhance their knowledge of instructional practices.

Three core features of professional development activities that have positive effects on teachers' knowledge growth reported in the study from Garet et al (2001) are embedded in the lesson study process (see pg.22). The teachers focus on content knowledge as they discuss the specific content and skills within the lesson they are planning. The entire lesson study process is an active learning process which involves teachers planning lessons for their own classrooms, observing other teachers teaching the lesson, and providing

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feedback. In addition, lesson study can be a part of a coherent program of teacher learning. The lesson study group starts out with goals and through the entire planning, observing, teaching, and debriefing process, the group produces a final lesson as a product. Then, this cycle is repeated and the teachers produce more lessons and have more opportunities for knowledge growth.

The works of Heibert and Stigler (1999) and Heibert, Gallimore, & Stigler (2002) have begun to look at teacher knowledge in connection with lesson study. They claim practitioner knowledge of teachers, which includes mathematics content knowledge, and pedagogical content knowledge needs to be transformed into a professional knowledge base for teaching. The means by which this occurs successfully in Japan is lesson study; thus they propose that such an effective system like the one in Japan is needed in the United States.

Within all stages of the lesson study process – there is the potential for interaction between mathematics content and pedagogy which will contribute to the development of teachers' pedagogical content knowledge. The second principle of effective professional development presented by Louckes-Horsley et al (1998), "Excellent mathematics teachers have a very special and unique kind of knowledge that must be developed through their professional development learning experiences" (p. 3), is emphasizing the need for professional development experiences that focus on pedagogical content knowledge. I claim lesson planning and reflection, two important components of lesson study as a professional development experience, have the potential to improve all forms of teacher knowledge, in particular, pedagogical content knowledge.

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CHAPTER 4

METHODOLOGY

Research Design

The method of inquiry for this study is a qualitative research approach. The following two main characteristics of this study: working with a small group of teachers and collecting a large amount of rich detailed data has led me to a qualitative approach. A lesson study group consisting of high school mathematics teachers was formed in a rural high school in the Northeast. My role was one of participant -observer, as one of the teachers who participated in all aspects of the lesson study group.

The participants in the study were five high school mathematics teachers- Alex, Craig, Mike, Lisa, and Melissa. All names used in the study are pseudonyms. At the time of the study, I was in my second year as a member of the mathematics department at the high school; i.e. I worked with the other four teachers for only one year prior to the study. All of the participants are full-time, secondary mathematics teachers and participated in all aspects of the study.

Mike has business experience and received his Master's in Secondary Education from a university in 1997. He taught middle school for nine years in a

different state. His teaching load was algebra and geometry honors level. He has also done private tutoring and taught summer school for a few years. This is Mike's first year at this high school.

Craig has a teaching degree in secondary education. He has taught at this school for 8 years. The following is a list of classes he has taught: Integrated I,II,V,VI, transitional math, college geometry, functions, honors geometry, and honors advanced algebra.

Alex started out as a chemical engineer. Then, as he worked with boy scouts, he became interested in teaching. He received his teaching degree, substitute taught for 6 months, and then taught at a private high school for 20 years all in the same state. While at the private school, he taught chemistry, pre-calculus, calculus, physics, and algebra. He taught science and math at a charter middle school for a couple of years while getting his Master's Degree in Professional Development. When he came to the Northeast, he taught one year at a different high school. The classes he taught were geometry, algebra, introduction to pre-algebra, and consumer math. This is his first year at this high school.

This is Lisa's first year of teaching. She has a B.S. in Mathematics Education and Masters of Arts in Teaching. She did her student teaching at a high school in the Northeast. She taught integrated algebra and geometry to freshmen and sophomores.

The high school has approximately 1000 students with approximately 120 faculty, administration and support staff. The typical class size for level 2 mathematics courses is 20-25 students; for honors and level 1 mathematics courses class size is 25-30 students. The level 2 mathematics curriculum consists of the following courses: transitional mathematics, Integrated Mathematics I through VI which includes topics in algebra and geometry. The level 1 curriculum includes: Algebra I, Algebra II, College Geometry, Algebra III, Functions, Trigonometry, Calculus, and Statistics. The honors curriculum includes Algebra I, Algebra II, Advanced Geometry and Algebra, Functions, Trigonometry, and AP Calculus AB. The high school uses block scheduling which includes four, eighty-six minute classes and one, fifteen minute homeroom period. The teachers are assigned to three of the four blocks, monitor one homeroom period per day, and have a twenty minute duty during three weeks of their planning block. Professional development activities that have been done at this high school include: district-wide workshop days, departmental work days, reading in your discipline followed by discussion, and seminars held by teachers within the district or experts from outside the district. The teachers are encouraged to attend conferences and professional meetings outside the school and are reimbursed up to $300 per year for expenses.

The lesson study group went through the stages of lesson study - plan, teach/observe, and debrief - three times throughout the 2004-2005 school year. Therefore, they planned three different study lessons. A timeline of data collection is located in the appendix. Prior to data collection, the participants

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were given information about lesson study in the form of a video (Curcio, 2002) a journal article (Fernandez and Chokshi, 2002), and a portion of my literature review on lesson study. The mathematics content for each of the three lessons was decided upon by the group members.

At the end of the lesson study group's first meeting- the goal setting meeting, Craig volunteered to teach the first lesson. He was teaching geometry classes this semester so the lesson was a geometry lesson. The goal was to teach and debrief the lesson by Thanksgiving break; Craig looked through his curriculum and planned what topic he would be teaching at this time and the group came up with the topic of proving that quadrilaterals with certain conditions are parallelograms for the first lesson.

At the end of the debriefing meeting for lesson #1, the group decided that Lisa would teach lesson #2. She was teaching two classes of Honors Algebra II and a transitional math class. The group members decided that they would like to plan a lesson for the honors level students; therefore, they chose the Honors Algebra II class. Lisa figured out what she would be teaching in the curriculum when lesson #2 was to be taught. This led the group to plan an introduction to functions lesson for lesson #2.

At the end of the debriefing meeting for lesson #2, the group decided that Alex would teach lesson #3. He was teaching two classes of Algebra II and an Integrated Math IV class. The group members decided that they would like to plan a lesson for the integrated level students; therefore, they chose the Integrated Math IV class. The integrated (level 2) curriculum has the students

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work with the algebra and geometry concepts at a slower pace. If a student takes all six of these courses, they will be prepared for a pre-calculus course at the college level. Alex figured out what he would be teaching in the curriculum when lesson #3 was to be taught. This led the group to a problem solving lesson on linear equations for lesson #3.

Data Collection

The data consisted of interviews, observations, videotapes, meeting notes, and journal reflections. Each will be described in detail below.

Interviews

To collect background information on the teachers prior to the lesson study experience, I conducted initial interviews with each teacher. Each teacher was asked the same series of questions as shown in table 4.1

1. Give a detailed description of your educational background and teaching experiences.
2. What do you like to teach the most? Why?
3. What do you like to teach the least? Why?
4. What do you do to plan your lessons?
5. How do you assess your students?
6. What do you think of the curriculum materials and textbook that you are   currently using?
7. What qualities do you think a mathematics teacher should have?
8. What information do you think a mathematics teacher needs to know or to be able to do in order to teach mathematics?
9. For you, what is the best part about teaching mathematics or teaching in general?

Table 4.1 Initial Interview Questions

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Each initial interview was videotaped and took approximately 30 minutes. After the lesson study group had gone through the stages of lesson study three times, the researcher conducted a final interview with each teacher. The teachers were asked to reflect on the following two questions prior to the interview and to turn in written responses:

1. Describe how the lesson study experience changed your understanding of mathematics. Please include how your knowledge of particular concepts and methods or procedures changed. Also, include how your understanding of the connections from one concept to another changed.

2. Describe how the lesson study experience affected the teaching of mathematics in your own classroom. Please include how you incorporated ideas acquired through the lesson study group in your own teaching. Be specific in terms of any useful representations, unifying concepts, clarifying examples and counter examples, helpful analogies, or information that was helpful to prepare for student misconceptions.

Then each teacher was asked five common questions (listed in table 4.2) and several additional questions based on their written responses to the ones above.

1. What do you do to plan your daily lessons?
2. Has lesson study affected the way you plan your lessons?
3. What qualities do you think a math teacher should have?
4. How did you see this through the lesson study experience?
5. What types of knowledge do you think a mathematics teacher needs? Did any of this come out of the lesson study experience?

Figure 4.2 Common Final Interview Questions

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Each final interview was videotaped and took approximately 30 minutes. All interview data was transcribed. One of the participants interviewed me as well.

Observations

In order to collect information on each individual teacher, I conducted classroom observations. This provided data on what was happening in each teacher's classroom outside of the lesson study experience. I took detailed notes on how the teacher presented the mathematics content and the students' reactions and questions. Also, I took notes on the problems and assignments that were given to the students. Classroom observations were conducted at three different times throughout the study. First, each teacher was observed during three blocks of classes prior to or while the lesson study group was planning lesson #1. Second, each teacher was observed during one to two blocks of classes after the lesson study group completely finished lesson #2. Lastly, each teacher was observed during two blocks of classes after the lesson study group completely finished lesson #3 and after all final interviews were completed. The exact dates of these observations can be found in the Timeline of Data Collection in the appendix. I opened up my classroom to observations by any of the lesson study participants, but none of them observed my classes.

Videotapes

A major component of data collection for this study was the videotapes of each meeting of the lesson study group. The videotapes were transcribed. The

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initial meeting was the goal-setting meeting in which the lesson study group established the goals they wanted to accomplish for each of the lessons that they planned collaboratively. Once the group started planning their three different lessons, the meetings of the group consisted of several planning meetings, the teaching/observing of the lesson, and the debriefing meeting. For lesson #1, there were four planning meetings that ranged in length from forty-five minutes to one hour and fifteen minutes. In addition to the eighty-six minute teaching of lesson #1, there was an hour long debriefing meeting. For lesson #2, there were two planning meetings that totaled 5 hours and fifteen minutes in length. In addition to the eighty-six minute teaching of lesson #2, there was an hour long debriefing meeting. For lesson #3, there were four planning meetings that varied in length from one hour to an hour and twenty minutes. In addition to the eighty-six minute teaching of lesson #3, there was a forty minute long debriefing meeting. The exact dates of the meetings of the lesson study group can be found in the TimeLine of Data Collection in the appendix.

During the debriefing meetings, the teacher, who taught the lesson, had the first opportunity to comment on their reaction to the lesson. Then, the other teachers gave their feedback and discussed questions/issues that were raised during the planning sessions. They described how these concerns were addressed by the instructional decisions they made for the study lesson. Each observer commented on a specific aspect of the lesson, and then gave other observers the opportunity to comment on this point or related aspects of the lesson (Choskin, Ertle, Fernandez, & Yoshida, 2001).

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Meeting Notes

The teachers' notes of all meetings were collected and photocopied. This includes all the meetings described above that were videotaped. This also includes the teachers' observation notes, taken while one of the teachers in the group was teaching lesson #1, #2, and #3. The other teacher participants who were observing had a specific observation task that varied from one lesson to the next. Some examples of tasks include: watch assigned groups of students, keep track of the time for each part of the lesson, or keep track of questions students ask. The observers were not to interfere with the natural process of the lesson. The observers were permitted to circulate around the classroom during seatwork and to communicate with students for clarifying purposes only. The person teaching the lesson distributed seating charts, so that the observers could refer to students by name in the debriefing sessions (Choskin, Ertle, Fernandez, & Yoshida, 2001).

Journal Reflections

After each stage of the lesson study process - planning, teaching/observing, and debriefing, each teacher wrote private reflections in a journal. This included reflections after every meeting of the lesson study group. The teachers were asked to respond to the following prompts in each of their journal entries:

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1. Write your reaction to each stage of the lesson study process. In particular how each stage of the lesson study process affected you personally.

2. Explain how you may use something that we discussed as a lesson study group in your own day-to-day lessons. It doesn't matter how big or small this idea may be.

3. Reflect on the quality of this professional development experience and compare to other professional development activities that you have participated in this school year or in the past.

The goal was to keep their reflections more open-ended and to encourage the teachers to write as much as they would like about this professional development experience. This data provided me with each teacher's specific perception of the impact of lesson study. These journal entries were collected and the teachers received no feedback. I also wrote journal articles following the same prompts.

Data Coding and Analysis

To analyze the data in this qualitative research project, I used grounded theory analysis. According to Strauss (1987), "The focus of the data is *not* merely on collecting or ordering a mass of data, but on *organizing many ideas* which have emerged from the analysis of the data" (p.23). During the first phase of analysis, coding and the development of core categories began. During this first phase of analysis, I examined the videotapes of all meetings of the lesson

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study group. While watching the videos, detailed notes were taken and then theoretical memos were written. These memos included short descriptions of portions of the videos that caught my attention. I used the memos as an initial recording of learning opportunities that were taking place between the teachers. In addition to watching the videos for each planning meeting, teaching/observing of the lesson, and debriefing session, I read through the meeting notes taken by each teacher and journal reflections submitted by each teacher for each different meeting. To my initial memos, I added comments from these data sources or wrote additional memos.

From the theoretical memos of the videos, meeting notes, and journal reflections, I began coding the data. The coding began using key words such as lesson planning, choosing example problems, prior knowledge connections, motivation, and curricular knowledge – all related to forms of teacher knowledge or professional development. More theoretical memos were written as a method of keeping track of coding results and stimulating further coding. Each of the stages of lesson study for lesson #1, #2, and #3 – planning, teaching/observing, and debriefing were analyzed further and more detailed coding was written out in the form of memos. These memos were in the form of spreadsheets that aligned the subcategories from the initial coding of the stages of the lesson study process under four core categories – mathematics content knowledge, pedagogical knowledge, pedagogical content knowledge and professional development. It was critical to the analysis that the theory emerge out of the data through an on-going process of data collection, coding, and writing memos.

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According to Strauss, the goal of the open coding process described above is to verify and saturate individual codes. Strauss states, ". . . Eventually the code gets saturated and is placed in relationship to other codes, including its relation to the core category or categories – if, indeed, they or it are not actually the core" (p.32).

In addition to open coding, I proceeded toward axial coding. Axial coding involves intense analysis done around one category at a time. The result of axial coding is cumulative knowledge about relationships between the category and other categories and subcategories. Strauss explains that axial coding is unlikely to take place during the earliest days or weeks of data analysis, but is used more during the lengthy period of open coding before the researcher becomes committed to core categories and so moves into selective coding. Selective coding is the process of coding systematically for the core category. During this process I specifically looked for conditions or consequences that relate to the core category. Memos during this process became more focused and were important in achieving theory integration. During the selective coding process, I watched the videotapes of each meeting of the lesson study group again and selected different episodes that involved the categories from the initial coding. Descriptions of these episodes were written and then analyzed according to the core categories. Other categories such as making connections to prior knowledge, questioning, choosing example problems, and assessing student understanding during the lesson emerged. As I analyzed each of the episodes, I looked specifically for conditions and consequences that related to the core

categories. In addition, I used the core categories to lead me to conclusions regarding the teacher's knowledge growth throughout the different stages of the lesson study process. The results of this lesson by lesson analysis are written out in detail in Chapter Five.

In addition to examining the videotapes, meeting notes, and journal entries, I analyzed the stages of the lesson study process and then used the initial and final interview and classroom observation data to enhance or help to explain the stage by stage analysis. For example, I examined the episodes from the planning stage for all three lessons and looked for similar situations in which teacher learning was taking place. Similarly I did this for the teaching/observing stage and debriefing stage. In each of the stages, I organized the teacher learning experiences according to the main categories mathematics content knowledge and pedagogical content knowledge. These are further divided into and analyzed according to the smaller categories that emerged from the data. The results of the stages across lessons analysis are reported in Chapter Six.

The teacher by teacher analysis required me to watch once again all the interview videos and write memos on this data. Next, I read through all classroom observations and wrote memos on this data for each teacher. Lastly, I read through all journal entries written by each teacher and wrote memos on this data. The results of this data were first included individually for each type of data – initial and final interviews, classroom observations, and journal entries. Then, it was examined collectively and complied into a teacher by teacher summary of progress. As I analyzed each of these rich pieces of data, I looked specifically

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for conditions that related to the core categories and then the smaller categories.

I used the categories to make more focused memos to help lead to conclusions

regarding the teacher's individual knowledge growth throughout the lesson study

process. The results of this analysis are discussed in detail in Chapter Seven.

CHAPTER 5

LESSON BY LESSON RESULTS

In this chapter, specific episodes from the meetings of the lesson study group are analyzed. The episodes include a brief description of the event and/or specific dialogue that transpired between the teachers. A short description of each lesson is given before the analysis of the episodes for that lesson. Detailed lesson plans for each lesson can be found in the appendix.

The analysis begins with the first meeting of the lesson study group- the goal setting meeting. Recall that this is when the teachers formulate goal(s) that they want to accomplish in each of the lessons. Analysis of all episodes from the planning, teaching, and debriefing meetings for all three lessons is organized by lessons and categories. The categories emerged from the data and are descriptions of pedagogical content knowledge. They encompass patterns in the teachers' behavior in the process of learning as the teachers participate in lesson study. All of the episodes for lesson #1 follow the goal setting meeting and fall under the following categories: mathematics content knowledge, meaning and connections to prior knowledge, choosing example problems, anticipating possible student misconceptions, questioning, and assessing student

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understanding during the lesson, curricular knowledge All of the episodes for

lesson #2 are next and fall under the following categories: mathematics content

knowledge, meaning and connections to prior knowledge, choosing example

problems, anticipating possible student misconceptions, questioning, and

assessing student understanding during the lesson. All of the episodes for

lesson #3 are given last and fall under the following categories: meaning and

connections to prior knowledge, choosing example problems, assessing student

understanding during the lesson, motivation, critiquing video, vocabulary, and

curricular knowledge.

Goal Setting Meeting

Episode 1

The purpose of the meeting was to set up goals the group would try to

accomplish in their study lessons. In the dialogue below the teachers reflected

on what they would like to see in students at their school.

> Melissa: Today, we need an overall goal for all the lessons that we
> plan.
> Mike: Is this one goal for one day?
> Melissa: No, it is an overall goal. For example, if we were in middle
> school we may really want to concentrate on fractions so we would
> make sure we would pick lessons that deal with fractions. Another
> example would be problem solving skills so in our lesson we could
> include some kind of problem solving activity. We could target the
> freshman and sophomores because of the sophomore test. Alex in
> your initial interview you talked about a central idea or golden
> nugget that you called it that the students would take with them
> each day.
> Alex: …you really want to do the golden nugget but all these

other things are overwhelming and you miss the golden moment....
Melissa: We need to be able to relate it to all the lessons that we will be teaching. An overall goal.
Mike: I would like to be able to teach lessons and have the kids retain it. Because today for example, we were working in algebra on systems of equations – substitution, now doing linear combination. And then I went back today and gave them a sheet which required them to find a solution by graphing which we did weeks ago. So many kids couldn't remember how to plot on the y-axis, how to count slope, which way to go. Ideally I would want them to remember these things not really having to ask me or with a quick word or two. But some of them I had to teach it over again to them. Same thing with the integrated class....They seem to go from day to day. Get through today and forget what happened yesterday.
Craig: I don't want to be rude, but I don't know if this is what we are trying to do. Don't get me wrong this is good conversation.
Melissa: We are trying to come up with a goal here, maybe we can incorporate these things into one goal.

Next, Craig suggests that the teachers share what they are each teaching this semester so that they can consider if there is similar content.

Mike: I like that. [motivation factor for a goal] because what I mentioned before if they are motivated they are probably going to retain it.
Lisa: Motivation is always something that is important.
Melissa: So I have motivation and creating meaningful lessons that will help the students retain the information. These are things that we can apply to anyone's curriculum. Does anyone have anything else?
Lisa: Understanding concepts instead of memorizing procedures.

Choosing goals for the group was not an easy task. Each of the teachers

brought to the group their own perspectives and expectations. The teachers

negotiated and sorted through each other's ideas in order to decide on goals that

would work best for the group.

Craig was initially thinking that the lesson study goal had to be specifically

mathematical in nature. He wrote in his journal that he was frustrated with the

group for discussing classroom issues rather than mathematical concerns. Craig

centers his lesson planning on the mathematics content but as he shared in his

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initial interview, instead of writing detailed lesson plans for classes that he has taught before, he just writes bullets for the main topics. It seems that while Craig emphasizes the mathematics content, in the actual presentation of his lessons a lot is left to improvisation. On the other hand, it appears that he is concerned with the pedagogical issue of motivation. He mentioned in the same interview that he does not enjoy teaching the introductory integrated math classes because the students are not motivated, and he has trouble motivating and involving these students with his lessons.

Lisa was also expecting the goals to be more content based. She wrote in her journal that she was excited about the goals concerning motivation and conceptual learning because she feels these are important aspects of teaching. She added, "I believe retaining information is important to some extent, but I feel it can be overemphasized. I realize that mathematics is very cumulative, but I think that if a student understands something conceptually, then he or she can derive or reach an answer without having memorized every step or procedure". Apparently, Lisa is concerned that in some teaching practices retention can be over emphasized to the extent of memorization of procedures. In Lisa's view, students' ability to derive mathematics concepts from previous ones leads to conceptual understanding.

In Alex's initial interview he talked about looking for the "golden nugget" in the lesson and then choosing examples for the students to complete based on this key idea. It seems that Alex thought this "golden nugget" was the key idea that would make the lesson meaningful. Also in this interview he discussed his

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least favorite math class to teach as a pre-algebra class because it was challenging for him beyond the mathematics. He described how he had to establish relationships with the students and to try to increase their motivation. Thus it seems Alex would like to improve on motivating his students.

Mike started the group off by discussing how he would like his students to retain the information. "Retain" was the term he chose to use here to describe how he wants his students to remember the mathematics concepts from one day to the next. It appears Mike was looking for ways to aid the students in their ability to learn the concepts. His suggestion that motivation may be an important factor in this was reinforced in his journal entry when he stated that he was excited about the motivation goal.

From this initial meeting of the lesson study group, it looks like the teachers can be divided into two groups. Craig and Lisa seem to be more focused on the mathematics content while Alex and Mike are more focused on pedagogy. This brings up a broader issue of a balance between the mathematics content knowledge and pedagogical content knowledge. As the teachers continue to participate in lesson study, more evidence unfolds to help us see if the teachers develop a balance between these two forms of knowledge.

Lesson #1

Lesson #1 was a geometry lesson planned by the lesson study group and taught by Craig. The lesson consisted of five conditions for proving a quadrilateral is a parallelogram. This lesson was centered on four theorems, but

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Craig asked the students various guiding questions that led them from one theorem to the next theorem. Several episodes from the planning, teaching/observing, and debriefing meetings are analyzed below. A detailed lesson plan for lesson #1 can be found in the appendix.

Mathematics Content Knowledge

Episode 2

During planning meeting #3, Craig went to the board to go through each proof that he would teach the students in the lesson. When he got to the 2nd proof (if opposite angles in a quadrilateral are congruent, then the quadrilateral is a parallelogram), he could not complete the poof the way he initially started it. He started the proof by drawing an auxiliary line and set out to use congruent triangles, but had trouble proving the two triangles congruent. All the teachers attempted to come up with a different approach in order to complete the proof. After they worked on the proof and consulted the textbook, they decided to begin the proof with the sum of the angles in a quadrilateral are 360 degrees. Craig then completed the proof at the board. Lisa suggested that the proof could still be proven the way Craig initially started it. However, Craig was not completely convinced, therefore, it was not pursued further. They decided that the students will probably start the proof in a different direction as well (because the 1st proof involves drawing a line and congruent triangles). Craig's approach would be to let them start it this way and then tell them to concentrate on angles.

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Since Craig didn't normally plan out his lessons, instead of him starting off the proof on a different path in front of the teachers during a planning meeting this could have happened during the teaching of the lesson. After trying several different approaches at the board, Craig said, "I can't believe I can't think of this . . . I'm embarrassed". But, Craig received support from the other teachers at this time. It is evident that the teachers needed this time to discuss the mathematics content involved in the proof – Craig's first approach would work but would require more steps. In Craig's view the shorter proof was the correct one. Since one of his students may complete the proof differently than Craig, it is important for him to be familiar with other ways to prove the theorem. It would be beneficial for Craig to let the students start on their own the proof that he thinks they will begin differently. Then, they can discuss how the different approaches led the students on different paths, but to the same conclusion. It looks like Lisa would have determined this if the group would have spent more time investigating this approach. Her perspective that derivation leads to conceptual understanding leads me to believe that she would encourage her students to investigate different approaches to prove a theorem.

Meaning and Connections to Prior Knowledge

Episode 3

During planning meeting #1, the teachers began their discussion with the first part of A *Tool for Planning and Describing Study Lessons* which includes

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background information on the lesson. Recall that the three lesson study goals for the group are: for each lesson to motivate the students, to develop meaningful lessons that will help the students retain the information, and to emphasize conceptual knowledge as well as procedural knowledge. The teachers' discussion on students memorizing procedures led them to a possible answer to one of the questions from the planning tool: Why does this gap between our aspirations and reality exist and how can we close this gap? They decided that in order to develop their lesson study goals, they must display to the students the meaning behind each concept. One way that they suggested this can be done is to make connections to prior knowledge. They question whether or not their motivation goal can be accomplished in a similar way. This led the teachers to discuss rewards systems and looking at mathematics as a puzzle or game as possible methods for motivation.

Mike: I think some of the kids try to memorize things rather than concepts and that is why they are not retaining like we would like them to.
Craig: They are memorizing procedures?
Melissa: Do you think they are just memorizing procedures or do they memorize the concepts and then forget them?
Mike: I think they want to know step-by-step exactly how to do something. They try to memorize that step-by-step rather than think something through.
Craig: I agree.
Lisa: That is how they go about transferring that incorrectly into something different. They haven't recognized that that doesn't work there too and just memorize that.
Alex: If I was to... as a quick analogy if I was to go to a pond and was very thirsty and did not have a cup or a bucket with which to scoop out water I'd go home pretty much empty handed. The analogy is simple in the classroom if the kids are lacking a place in their brain to deposit the new knowledge there can be an oasis, ocean of knowledge where there is really not a place to put it, but it seems to me that you can give them a purpose. That purpose provides new avenue in the brain to store that

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information and that gives meaning and purpose.... I don't think I honestly do a good enough job I really don't in pursuing the sense of purpose, why we are doing this.

Craig: I think that helps us answer the third question on this list. Why does this gap between our aspirations and reality exist and how can we close the gap? We are not necessarily giving meaning to it. To close the gap we need to provide meaning to the concept.... Even if it is simple as that you need this concept to learn the next skill or to learn next concept.

Alex: To state that and to show them the hook.... I have seen it done successfully where you hook and bait, you hook and bait throughout the lesson. You want to know that step because you have provided that meaning.

The teachers take a moment to restate and write down what Craig and Alex have just discussed.

Mike: I keep thinking to myself that you can say all you want that you are going to need this for the next concept, but I don't see this motivating a whole lot of kids.

Craig: So that [motivation] wouldn't be one of the ways to close this gap.

Alex: I did a little reward system. I had two young ladies that would normally have been shall we say less than nice etiquette in a classroom. But they had won one of the competitions. Then we did it a second time. ....Suddenly I saw it, it was right there as an instructor I saw in her brain it click -I am going to get this- she mentally rose to the occasion where she said I am going to win this, me and my partner are going to win this. Low and behold they won, they beat clean sweep because of a little motivational connector – candy.... My point is two young ladies who could have easily been side tracked or distracted or kind of blown this off took it graciously.

Craig: In a less than physical motivating way also to kind of go by what you are saying you are going to need this for the next thing. I often times will explain you are seeing this map that currently doesn't make any sense because it is all part of the grand picture. You do not see the grand picture and you probably won't see the grand picture maybe for another couple of years. But the problem is this is still a piece of that puzzle. When you put together a couple pieces of the puzzle great you may have a little thing here but you still may not know where it goes in the whole thing. So that is how I try to relate to them something that they grasp – puzzles, games. Things that they understand the little pieces of and how they are important to the game. So, I often try to make that kind of a connection. . . .

Alex: So you kind of empathize with them.

Craig: So I am kind of saying no you don't see where this is going but you will because it is all part of a larger game. And if you don't know the rules of the game you can't play. So, that is how I'll try to run with that in a less than physical more of an academic motivational way. A hybrid of the two

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could certainly work. That is one thing that I have tried and generally kids kind of respond to that...

Melissa: a lot of it is trying to explain the why. Sometimes it is a stretch because it might just be a small skill that we are learning that will evolve into something.

Craig: Like proving something is a parallelogram. Whoopee! It is great that you can prove that it's got two pairs of parallel sides or whatever. Why is that important? So that is going to become a crucial part of the lesson. This is something that I try to do on a daily basis. It is great that we have learned this. But why?

The teachers have different perspectives on what leads students to retention or conceptual understanding. It appears that Mike, Craig, and Lisa believe that students' memorization of procedures doesn't lead them to retention. Each of them pointed out that students try to remember step-by-step processes. Mike and Craig emphasized that their students want the procedure so that they don't have to think about the problems. Lisa added that since the students are just memorizing procedures, they try to transfer the same procedure to problems in which it doesn't apply. Evidently, Mike, Craig, and Lisa wanted their students to understand mathematics conceptually, but struggled with how to get them to achieve this. In Alex's water analogy he assumed that there exist places in the brain to deposit information. His view on cognition is not consistent with the other teachers.

In this episode the teachers started to see how they can promote conceptual understanding. They considered how they could attach meaning to the concepts. Alex suggested that to attach meaning to a concept the teachers need to explain why the concept is important and how this concept ties in with other concepts. This is what Alex was referring to when he said that the teacher

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needs to "hook and bait" throughout the lesson. It appears Alex was thinking about connections among concepts and how this helps to create meaning, but connections is not expanded upon at this time.

Mike questioned how creating meaning for the concept is going to motivate the students. Mike believes that telling the students that they will need to know the concept for future mathematics classes is not going to motivate most students to want to learn it. The teachers' other suggestions for motivation such as candy reward systems and thinking of mathematics as a puzzle or game will not work for all students either. The teachers learned from Craig that he tries to motivate his students on a daily basis by explaining why it is important for them to learn the concepts. It seems that as Craig focuses on the mathematics concepts, he looks at how the concepts are connected. This is part of Craig's "why" for the students.

This planning meeting gave the teachers the opportunity to start to see how their goals can be achieved. The teachers' discussions also revealed their different perspectives on students focus on procedures versus conceptual understanding. As the teachers came up with the idea of attaching meaning to the concept, the issue of connections surfaced, but will require more attention at future discussions.

Episode 4

In planning meeting #1, the teachers discussed another question from A Tool for Planning and Describing Study Lessons: Why is this mathematics

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important? In the dialogue below the teachers discussed in general why proofs are important. Craig shared information from two articles about proofs that he had read. The teachers concluded that proofs are important for students to learn because the proof process gives the students a way to explain the concepts.

> Lisa: Well, I was thinking along the line at first of proofs and why proofs are important. But since you are doing that all along that is not the driving force of this lesson.
>
> Craig: They will know proof really well by this point.
>
> Melissa: That may not be the underlying objective of this lesson, but why? We are going to be doing proofs. Why are proofs important? I still think we should think about why knowing something about parallelograms is important.
>
> Alex: I recall a year or so back when I was interviewing for a position, one of the schools one of the questions they asked what is your position on proofs? ...I found out later that their school does not do proofs. I was not offered a position. The point is that there are different attitudes so the question is legitimate.
>
> Craig: There are two things that I have read recently that help me drive home the point of proof. One thing that I have read recently is that at the higher level of mathematics anything that has been proven is considered trivial.
>
> Alex: Yes, exactly!
>
> Craig: So it is not the past knowledge. It's where do I go. How I can go further. And that is what I try to teach the kids. This is one of the things that we have mentioned here. Making those connections on their own, once you have learned how to prove something you learned how to make connections to advance yourself further and further. So you are looking at all this other stuff you already know going I already know this is true why can I show these next few things are true. Once you've shown it that immediately goes into the used pile.... Even just what we did two weeks ago is trivial now. We already did it; we know it let's use it to do something else. The second thing that I read which kind of contradicts that is that again at the higher level of mathematics the question has become is rigorous proof worthwhile. Is it become less about knowing something is true with absolute certainty and become more about can I convince you that I am right?... That was a very interesting article that a student brought in too. And I will talk about that in classes too. I'll say look some people don't believe in proof. They believe it has become more can I convince you that I am right. I spin it so that they understand why we do proofs – if someone asks you why you hate INSYNC, tell me why, convince me why they suck. Tell me why. And they can respond to that. I say good take that convincing and apply it to math.

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Alex: Craig I could see you writing a book in a couple years and it is called Mathematics by Convincing. O.K. class, please write a convincer. Change the terminology, and you can change the whole thing.
Craig: That is a lot of it they are scared by the word proof....It is a vocabulary issue as well. Why are proofs important? They help students explain the concept and that is the driving question to understand the concepts.
Alex: It is a way (one of perhaps several ways) to help students learn how to explain something. So really it is a way to get to higher level thinking.

It is interesting how Lisa said that proofs are not the driving force of this lesson. In fact, Lisa appreciates proof because she sees it as the driving force for conceptual understanding as we noticed in her journal entry when she stated, "....I think that if a student understands something conceptually, then he or she can derive or reach an answer without having memorized every step or procedure". In her view, proofs and derivations are important methods of inquiry in mathematics which in the theory we call syntactical knowledge. In the above remark it appears that Lisa just realized that since Craig is employing proofs all throughout the semester the focus for this lesson is not on the process of writing proofs per se but rather on using proofs to conceptualize various relations among the elements of a parallelogram.

Out of all the teachers in the group, Craig had the most experience teaching geometry. As I observed his geometry classes several times throughout the school year, I saw how he gets his students to use the theorems that they have already proven to advance further into the geometry concepts. Craig included the following aspects of proof: ownership, explanation, connections, validity, and level of rigor in his discussion. Craig claimed that proofs are needed in order for the students to further advance their geometry

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knowledge. It appears that Craig also would like students' experience proving theorems to help them make conjectures on their own. During this portion of planning meeting #1, Craig turned the discussion toward students making connections. It seems that Craig believes that in order for students to understand the geometry concepts they must be able to make the connections from one proof to the next proof and to offer conjectures.

Craig contradicted himself when he jumped to the issue of looking at proofs as ways to convince that the statement is true. It seems that if students do not accept a proved statement with absolute certainty, then they would not have the theorem to use to advance further in their geometry knowledge. If Craig wants to establish the importance of how one proof connects to another, then simply establishing convincing arguments doesn't validate the theorem to be used at a later time. It is evident that Craig used this time during the planning meeting to sort out his own thoughts about why proofs are important. His conclusion that proofs help students explain the concepts has much more embedded within it – connections and conjecturing. Lisa and Craig would both agree that these are results of the proof process that will help students establish conceptual understanding.

Episode 5

In planning meeting #2 for lesson #1, the teachers began to discuss the third main part of A *Tool for Planning and Describing Study Lessons*: Lesson Information. In the following dialogue, Alex talked about how teachers view

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mathematics as a continuum and that students may not learn one entire concept during one class period. Craig described mathematics as a long narrative in which concepts are added and expanded upon each day with no clear ending.

> Alex ...You come in and actually walk away knowing an entire block of information. Another words it wasn't like a continuum, it wasn't like we've been here, and we're now here and now we're going to go here it was independent. You could walk out of that, and if I didn't have another math class that was worth it. That kind of approach. I'm not sure how that happens. I think we do that but often we teach so much in a continuum and that is why when kids are absent it throws us off so much....
> Craig: Mathematics is like a story. It is a long narrative and you certainly can't skip sentences and paragraphs. You have to read the whole thing. And if you happen to miss a day you have to go back and reread what was taught or what was learned or instructed.
> Lisa: I think it can be both at the same time. You were talking about being completely done after one lesson. In some sense that can happen like you can really get into something, but I don't think it should ever be completely done. You know like you should still be able to see it.
> Craig: Agreeing with what you're saying, mathematics is never done. There is always more to be done especially in geometry. We focus O.K. we learned something now considered trivial. What can we do with it?

In this dialogue, the teachers discussed their personal views of the nature of mathematics. Alex's statement about walking away from a class knowing an entire block of information that may not connect to anything else comes from his science background. It appears that he would like to see this with mathematics. But as Craig pointed out with his story analogy there is always more mathematics that can be studied and in order to see the complete picture concepts shouldn't be omitted or disconnected. It is evident that Alex was concerned with pedagogy and how the lesson looked while Craig remained focused on the mathematics and students' development of concepts.

It is evident that Lisa was also focused on the development of concepts. Lisa pointed out how the teachers can get deeply involved in a concept, but this same concept should become prior knowledge for a later topic. In Craig's last statement he was referring back to one of the articles he read on proofs. He considered each learned concept as trivial and wants his students to figure out how this prior knowledge connects to new concepts. In this episode it is clear that Craig and Lisa continued to discuss the connections of mathematics concepts.

Episode 6

The debriefing meeting for lesson #1 began with Craig commenting on how he felt the lesson played out in the classroom. The dialogue below includes some of the other teachers' reactions to the teaching of lesson #1.

> Mike: The key thing that I noticed was that it flowed smoothly. You could tell that...as you were taking them through and walking them step-by-step through this process that they were recalling information that they had learned prior to that day.... Some kids who were very vocal and they probably raise their hands a lot... and everybody has those kinds of kids. But even looking around at some of the other kids, they understood it too for the most part, and then they were looking at each other's stuff. I saw a little interplay between the kids during the lesson.

> Later Mike adds:

> Mike: The kids that aren't vocal are they getting it?
> Craig: No quiz on that yet. But I can tell you just as we move forward with other figures they understand it's got these things so it's a parallelogram and this so it's a rectangle, so they are immediately applying their knowledge.
> Melissa: Students remembered the prior knowledge that they needed. It was clear throughout the lesson that they were making connections to the material they needed to know in order to do the lesson.

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Mike and Melissa pointed out that the students used their prior knowledge to connect the old geometric concepts with the new concepts. Craig added that the students were then able to use the knowledge they learned in this lesson on parallelograms when they were studying squares and rectangles. In Craig's journal entry, he wrote, "during class, they did well drawing on prior knowledge – the ones I called on that didn't give answers were probably to shy to venture a guess, right or wrong. That is more of a confidence issue than a knowledge issue." From the evidence presented above, it is clear that Craig did not just talk about the importance of students making connections. It is clear from the observers' comments that he acted upon this when teaching the lesson. Mike made reference to Craig's guiding questions which helped to make the lesson flow smoothly. Mike called this the "key thing that he noticed". Since Craig led the students with thoughtful questions and the students recalled the necessary prior knowledge, the flow of the lesson went as well as any teacher would want it to.

Mike stated that Craig was "walking them step-by-step through the process." Since the lesson contained four proofs that were stated and completely proved, the observers could see that the students understood the process involved with the proofs. However, evidently there was much sharing of conceptual knowledge as well, for the observers stated that the students displayed their understanding of the prior geometric concepts that they needed in order to come up with the correct steps for the proofs. Not only was this done

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verbally between the students and Craig, but Mike observed students discussing the proofs among themselves during the lesson.

Choosing Example Problems

<u>Episode 7</u>

From planning meeting # 2, Mike, Lisa, Melissa, and Alex saw the need for an opening activity or problem, but did not have time to plan it. Craig was reluctant to try anything new in his classroom. The dialogue below from planning meeting #3 begins with Melissa pointing out various application type problems that she found in the textbook.

> <u>Melissa</u>: Did anyone think of anything that we could do as an opener to kind of motivate the lesson even if we do it after the homework is checked? I was looking through the book that Craig actually uses.
> Melissa points out some application problems from the book.
> <u>Melissa</u>: Would these motivate them to want to talk about these theorems?
> <u>Alex</u>: Or on reverse side would it frustrate some, say I can't do that.
> <u>Craig</u>: That's what is going to happen with this class. They'll look at it and go I have no idea. And then I will try to get them to think about it more and to come up with their own conjectures, or whatever.
> <u>Alex</u>: When Craig mentioned that the kids like the algebra of geometry I wonder if there would be an algebra question that they could literally put a parallelogram on the board and they could be successful based on their intuitive instinct as to how to solve for x and y.
> <u>Craig</u>: Put the problem up, solve for x and y without telling them this is a parallelogram?
> <u>Melissa</u>: Maybe we don't need something like this....

Melissa points out one other thing that she found in the textbook as a possible opener [Ask students to draw segments AB and CD so they both share a midpoint M. Instruct them to use a ruler to draw quadrilateral ACBD. Ask what conjecture they might make about ACBD].

> <u>Craig</u>: That is something I could do.
> <u>Lisa</u>: I think that is the kind of thing I was thinking about.

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Alex: That is kind of neat. You are guiding them. Plus it is step-by-step. You are guiding them through a procedure and pretty hard not to be successful after that.

Craig: I would definitely do something like that. Don't give them the answer though. We want them to try it. Then can we later say by the way remember that conjecture that you tried to make.

Lisa: Can you do this for each of the sufficient conditions for a parallelogram?

Craig: Well, when I teach this, I start with opposite sides congruent (I do the theorems in order). I'll walk through opposite sides congruent. I'll walk through the proof then I'll show it. Then I'll say what about the other properties we know. We know that if it is a parallelogram opposite angles are congruent. Well, what if opposite angles are congruent does that make it a parallelogram? I'll have them draw that situation and attempt the proof. They'll have just done one way can they do it on their own. Then I will do the same thing. The diagonals bisect each other what if that makes a parallelogram. That answers this (the opener). That is what we were talking about a couple of weeks ago what is the converse of the theorem that you've started with? Do you think they work? That is kind of what we are doing.

Melissa: Do we do this before going over the homework or after?

Craig: We don't have to go over the homework first. That is what I do, but we don't have to do that. I can say hold onto your homework for a minute and let's do this.

Alex: If they didn't do the homework or the homework wasn't done or they were absent is that a critical part of the lesson.

Craig: Not really because I am going to remind them if it is a parallelogram, then blu blu blu.

Alex: So that is not a necessary lead in.

Craig: They will want to go over the homework.... But, the questions are applications of the theorems not the theorems themselves. We will have proved them in class the day before. So, they'll know the properties. We can do it without going over the homework.

It appears that although Craig doesn't write out formal lesson plans, he had a definite plan for this lesson in his mind. Alex's statement that the proposed activities might frustrate the students gave Craig an excuse for not accepting one of the earlier suggestions for the lesson. The teachers needed to find an activity that was within Craig's comfort zone in order for him to agree to it. As Alex's pointed out in the dialogue, the activity needed to be a procedure that the

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students could follow. However, once the students do the steps within the procedure, then they must use their conceptual knowledge of geometry to venture a conjecture about the quadrilateral. Apparently, once Craig heard an example that would require the students to apply prior geometric concepts and to conjecture, he was willing to use it. It also appears that Craig wanted the students to think about the correctness of their own conjecture as well as other students' conjectures who share theirs because he didn't want to give the students the answer right away.

Lisa wanted Craig to do this type of activity to introduce each of the theorems. This is too much for Craig, for he wanted to present the theorems as he usually did. As Craig explained to Lisa how he presents the theorems in order like in the textbook, he offered a few of the guiding questions that he would use in the lesson. This displays that Craig had a definite plan for the lesson in his mind and was not willing to deviate from it. Craig did agree to discuss the homework problems another day. These minor adjustments to Craig's own plan for the lesson made it the group's lesson and allowed Craig to try new things without being overwhelmed by the changes.

Anticipating Possible Student Misconceptions

Episode 8

In the dialogue that follows from planning meeting #1, the teachers discussed this question from A *Tool for Planning and Describing Study Lessons*: What misconceptions might students have?

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Melissa: Besides students having trouble understanding proofs, is there anything else that you can think of since you've taught this before that might come up?

Craig: A lot of times they want to jump to rectangles. They hear quadrilaterals and four sided figures and immediately think of rectangles and squares and don't think of anything in between mostly because they don't know what a parallelogram is.... there are two things that throw them – the first thing is showing one pair of sides congruent and parallel and they always get concerned about that other side. What about the other sides? Why don't we need to say anything about them?

Melissa: Because it only takes that one pair.

Craig: All you need is that one pair.... The other thing that throws them is diagonals. They haven't seen diagonals thoroughly up until this point. If we are talking about the proof part of it they will know what a diagonal is and they will know that the diagonals bisect each other. But, they will have a hard time connecting the diagonals bisecting each other to making a parallelogram because they will see the four triangles but will prove the wrong two congruent. They will try to prove the top two or bottom two instead of the opposites. They will try so hard to get those two congruent not seeing that it gives them nothing to work with but half of a parallelogram.... Of the five ways to prove, they do all right with the others because it is mostly direct application of parallel lines and direct application of congruent triangles. So that could be another misconception: non-retention of congruent triangles.

Melissa: Using all of those postulates to prove triangles congruent.

Craig: Right forgetting SSS, SAS, . . .

Here, Craig was very specific as to where the students will make their

mistakes and very quick to share this knowledge with the teachers. Craig's ability

to provide the teachers with this list comes from his mathematics content

knowledge- syntactic knowledge of geometric proofs. Since he had several

years experience teaching geometry, it seems like he had pedagogical content

knowledge of how the students do in completing the proofs on their own.

Episode 9

In the dialogue that follows from planning meeting #2, the teachers discussed

what they do if students do not understand the homework from the previous

night.

Melissa: So after we go over the homework do you want to do another problem before going into the lesson or do you want to go right into the lesson?

Craig: I would go right in unless I see that they just don't get it. Then I would do one problem.

Alex: Kind of in the spur of the moment.

Craig: What I would do usually in that case is pull one of these ones from the corners of the book or grab a homework problem that they didn't do. Then maybe I would give them the critical thinking [problem]... Maybe I would grab a couple of these.

Melissa: So you don't want to decide beforehand. It is usually a spur of the moment decision. Is that how you all do it?

Craig: That is how I do it. It is pretty random.

Lisa: I guess it's spur of the moment.

Melissa: Sometimes you can't always anticipate.

Mike: Sometimes I know. I know as we all know a lot of times they don't ask questions they just wait. So I will have a question or two ready. O.K. let's see what you know.... It comes off the top [of my head] many times, but I try to have some things prepared.

Alex: I think that is where mastery of the content comes in, you know teacher qualifications.

Craig: I am such a terrible planner. For me that it is always off of the top of my head. O.K. they didn't get it, need a problem. I will either dig back in my head and find one or quickly grab the book. Hey what about this one.

Melissa: You are familiar with the material.

Alex: You can almost feel where the class is.

Melissa: You may recognize this the day before. Sometimes if I rushed to get the lesson in, or I can just tell when I am going over examples that they might have it but are really not sure so I can anticipate that they are going to have questions on this and then thinking what other problem can I give them in case they have questions. If something happened overnight that they did O.K. than I don't need that question.

Lisa: I don't usually think of a question ahead of time, but I'll try to anticipate what they might have trouble with. I will do examples and solve for and explore in my planning. I 'm not sure that I would pull out an extra problem. Sometimes I would try to pull out an extra problem. But otherwise I would try to answer their questions.

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It is impossible to anticipate every possible question the students will ask or mistake the students will make. The teachers agreed here that if the students are having trouble with the concepts then they will give them more problems to solve. The difference lies in how they come up with these problems. Craig admitted that his inability to plan forces him to find a problem in the book or make up one at the particular moment. This requires the teacher to be familiar with the textbook or curriculum materials in order to find a problem easily. Mike will often have a problem or two ready. Lisa explained how she takes time while she is planning to solve example problems herself and consider where the students may have trouble.

Like Craig, Alex another non-planner comes up with problems at the particular moment. Alex stated that a teacher's mathematics content knowledge is what allows them to be able to come up with a problem at the particular moment. This ability to choose or make-up an appropriate problem is part of a teachers' pedagogical content knowledge because not only must they know the mathematics content but also they must be able to explain the content in more than one way. Lisa stated "I would try to answer their questions". This is one of the most important things that mathematics teachers need to be able to do. They need to use their knowledge of the content to explain the concepts in a variety of ways to reach different learners.

Episode 10

In the teaching of lesson #1, Craig has finished the 1st proof. He asked for any other conjectures. The students looked at the list on the board and

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suggested proving opposite angles congruent. They started this proof on the board the way they started the first proof. When this didn't work, a student suggested to draw another diagonal and to use vertical angles. They saw that this doesn't get them anywhere, so they started anew with a clean drawing. Craig then suggested a non-triangle approach.

> Craig: tell me about <A, <B, <C, <D.
> Student: They all add up, there measures all add up to 360 degrees.

As a class they proceeded to finish the proof.

Craig chose to let the students begin the proof in several different directions. As a lesson study group, in the planning sessions, the teachers anticipated the students making the mistake of trying to do this proof like the first one. Craig told the group that he would let them try the different possibilities first before leading them in the right direction. This choice helped Craig to see that the students knew to try their prior knowledge of congruent triangles and parallel lines. With one hint about the 4 angles, he was able to get a student to recall prior knowledge about the sum of their measures. Also, these students were working as mathematicians. They came up with a conjecture to prove and then they tried certain geometry concepts until they found ones that would get them on track for completing this proof.

Craig did a great job of observing student misconceptions on where to go with the proof. He could have just told them the correct way to do the proof. The way he preceded allowed the students to see that there are many different directions that can be taken in proofs, but they must always keep in mind the

conjecture that they are proving and their prior knowledge that is going to get them to that conjecture. He asked a guiding question to redirect their thinking in order to prove the conjecture. Since the lesson study group had experienced this misdirection themselves, Craig was prepared for student misunderstanding. Evidently, the preparation by the lesson study group helped equip him with the necessary pedagogical content knowledge to work through this classroom situation in a successful way.

Questioning

Episode 11

During planning meeting #2, Alex wanted to come up with an essential question for this lesson. In the dialogue that follows the teachers offered their opinions about essential questions in general and developed one for this lesson.

> Alex: Is there an essential question? I'm not sure if we created an essential question.
> Melissa: We didn't. No! You are right.
> Alex: If a visitor were to come into the classroom and observe could they feel as though they could answer successfully a key question. This is one thing that I mentioned in my reflection. What is the essential question that we are trying to answer?
> Melissa: If we don't have one in mind right now maybe as we are developing the lesson more specifically we can do that.
> Craig: Not being a fan of essential questions or that whole concept, I honestly can do without one. Understanding where our school is headed and how every lesson should have some kind of an essential question, if one happens to comes up great and if not I'm not worried about it.
> Melissa: So as we go though [with the lesson] we will keep that in mind.

> Next the teachers began to discuss the actual process of the lesson and approximately 15 minutes later:

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Alex: Craig, you are really stating the essential question. Can you restate that?

Craig: What information do we need to determine a quadrilateral is a parallelogram?

Alex: Isn't that the essential question? Kind of ? Sort of? It is general enough. Or maybe it's not the essential question? If I walk out of that lesson with five pieces of information that allowed me to determine... I could walk out of that lesson with hey I have this, this, and this and it's this.

Craig: In essence you will do what you were talking about. You will walk away with one thing. But what I try to do is here is that one thing but here is how it relates to where we've been and where we're going.... I don't always make that connection every day.

Approximately 35 minutes later as the teachers discussed how to begin the lesson Lisa suggested starting the lesson with a question. Melissa offered this question: How can we show that a quadrilateral is a parallelogram? This ends up the essential question that Alex requested.

The administration at the high school would like the teachers to use essential questions in each of their lessons. There has been very little training at the school on how to write essential questions. In addition, the administration has not explained clearly why it has become such an initiative at this school.

Alex is the teacher of the group that insists on developing an essential question for lesson #1. The essential question is supposed to be open-ended enough to spark student discussion. The one that Craig offered- What information do we need to determine a quadrilateral is a parallelogram- is not an essential question because the students could simply list information to answer the question. The essential questions for each lesson should lead students into the discussion of the concepts within the lesson. Lisa's suggestion to start the lesson with a question is a common way to incorporate essential questions into the lesson. Since teachers at this school are being mandated to use them in their classrooms, the questions are often forced and end up non-essential.

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Once again, in Alex's view the students must walk away from the class with one main idea. Evidently, his science background was showing through again. Craig emphasized that in a mathematics class the students should learn one main concept, but it is important for them to see the connection of this concept to what they've already learned and to what they will learn in the future. It appears that Craig continued to focus on connections while Alex was more focused on pedagogy.

Episode 12

In teaching lesson #1, Craig asked the students how we show that certain quadrilaterals are also parallelograms. This led him into the first theorem to prove: if opposite sides are congruent, then a quadrilateral is a parallelogram. This was the information (prior knowledge) that the students gave Craig and that he referred to on the board throughout the dialogue that follows:

> If ABCD is a parallelogram then, (there is a drawing of the parallelogram too)
>
> 1) AB ∥ CD, AD ∥ CB
> 2) AB ≈ CD, AD ≈ BC
> 3) <A ≈ < C, <B ≈ < D
> 4) AC and BD bisect each other
> 5) Consecutive angles are supplementary.

Craig writes on the board: How do you show that ABCD is a parallelogram? (and draws a diagram with it).

> Student 1: Show each angle adds up to 360 degrees.
> Craig: If each angle adds up to 360?
> Student 1: No all together.
> Craig: Oh! If the sum of 4 angles is 360 degrees. Possibility? (he writes this on the board.

Craig: But, can I give you a counter example. (He draws a kite on the board then says) That doesn't really look like a parallelogram does it, but it is a quadrilateral and in any quadrilateral we already know the sum of the angles is 360 degrees. The sum of the angles 360 degrees doesn't necessarily mean it is a parallelogram. What else? How could we show for sure it is a parallelogram?

Students 2: Maybe using SSS Theorems.

Craig: Those are for triangles, although when we proved these theorems (pointing to the ones on board and written above) we went back to congruent triangles then so it might have something to do with it. Well, what is the easiest way to determine whether or not this is a parallelogram (as he points to the quadrilateral drawn on the board)?

Student 3: Opposite sides parallel.

Craig: Opposite sides parallel, definition. That would be the only way you know right now, right? Find out if opposite sides are parallel so that would be one way. But that is basically just restating the definition, isn't it? What is a converse?

Student 4 (Derek): The opposite of something. In a statement, you change it around.

Craig: Any ideas how we could use this?

Student 4 (Derek): Maybe you could do the converse of one of our other theorems.

Craig: Maybe if we tried the converse of one of these (and points to the ones on the board). (While writing this on the board) Let's try this: If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. This is Derek's conjecture.

Then there is a question from a student.

Student 5: Is there a counter example to that because doesn't a square have opposite angles congruent and a rectangle.

Craig: We haven't talked about squares. Don't bring in squares and rectangles yet. Have patience young man.

Next, Derek gives him the given and the prove statements and he wrote them on the board.

Craig: How do we go from congruency to parallel lines? I don't know, it sounds wicked hard. What do we know?

Student 6: Segment AB is congruent to segment DC and segment AD is congruent to segment BC (Craig shows this in the diagram on the board).

Craig: What is the only way we can show this is a parallelogram? If we could show what about opposite sides?

Several students in unison: That they are parallel.

Craig: Didn't we just say if we have a quadrilateral with opposite sides parallel, then the quadrilateral is a parallelogram? That is the definition.

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That is the only thing we have right now to show that a quadrilateral is a parallelogram. So I have to go from congruent sides to parallel sides. How are we going to do this? Any ideas?

Student 7: Construct a line.

Craig: What line?

Student 7: Construct line AC or BD.

Craig: What does this do for us?

Student 7: That line is congruent to itself so the triangles are congruent.

Craig: Oh congruent triangles! Who said congruent triangles before? Derek did. We are going to use congruent triangles. Again prior knowledge applied to a new situation.

Craig talked through the rest of the proof with more students' help.

Craig: Again we have prior knowledge bringing forth to a new situation. We've used congruent triangles and parallel lines.

Then as he began to write the proof on the board,

Craig: O.K. I forgot what we did.

Then different students told him what to write for the proof.

Throughout this dialogue there was an open discussion between Craig and his students. Craig didn't just give the students all the information he wanted them to know about parallelograms. He guided them through it by asking questions that led him from one theorem to the next. It appears that Craig was familiar with the mathematics content in order to ask good guiding questions. The ease with which he connected all of the prior knowledge of the theorems on parallelograms and definitions with the new knowledge of proving the converses displays how comfortable Craig was with the mathematics. Also, it appears that Craig was comfortable with this instructional approach.

Several times throughout the portion of the lesson Craig reminded the students how they were applying their prior knowledge to new concepts. As the

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teachers were planning the lesson, they considered the students' prior knowledge necessary in order to learn the new concepts. As he was teaching the lesson, Craig clearly identified to them that this is what they were doing. Craig asks "What do we know" when he wanted the students to examine the geometric concepts that can be applied in the specific situation and after the students correctly applied them he stated, "Prior knowledge applied to a new situation". Evidently, Craig did not just talk to his colleagues about the importance of connections; he made this importance explicit to his students. The last statement in this dialogue about him forgetting what they did was an excellent way to have different students go though and explain how the proof should go. This was one way that Craig assessed student understanding of this proof.

Assessing Student Understanding During the Lesson

Episode 13

While teaching lesson #1, Craig wrote on the board the given and proof statements for the 3rd converse (the diagonals bisect each other). He explained to the students that they now have three things they can use to prove that the quadrilateral is a parallelogram: the definition, opposite sides congruent, and opposite angles congruent. He gave the students nine minutes to do the proof on their own. While they worked he walked around to see how they were doing and answered their questions. It ends up that the students did a good job with the proof, but most of them left out the same step. After stating that the diagonals

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bisected each other (the given). The second step should be that the smaller segments formed are congruent. The students used this to prove triangles congruent, but forgot to state that the segments were congruent in this second step. As Craig went over this proof on the board, he told the students about this mistake.

This lesson consisted of a lot of notes. Taking the time to have the students try this proof on their own and have the opportunity to assess their progress was a good pedagogical decision. The students had confidence from proving the previous two theorems together as a class. They were ready to try one on their own. This allowed Craig to assess the students understanding during the lesson. Craig was not just walking aimlessly around the room while the students were working. He checked on the students' progress and answered their questions. It is clear that Craig had a clear understanding of the proof in his mind in order to catch this mistake and to answer all of their different questions. His pedagogical choice of pointing out this mistake at the board made it clear to the students that they need to make sure they include this step, but also that they were not alone in missing the step. It appears that Craig used this as an opportunity to reinforce the steps of reasoning involved in a proof. He used this example to remind the students that each step in the proof must be supported by the previous step.

Episode 14

In the debriefing meeting, Alex commented on how Craig answered a student's question as he walked around the room.

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> Alex: I was impressed...a student might have had three or four steps written and how you were able to almost like a game of chess instantly go to a step (I wouldn't have a clue) now this is reversed. I thought it was very technical the comment that you made. Wow, for the student to be there so quickly and then for you to be able to diagnosis so quickly.
> Melissa: Do you think that comes with experience?
> Craig: (Agreeing with a nod) Knowing where you are headed, knowing what they should have on paper.

Later in the discussion, Lisa tells Craig that she was impressed that he didn't use any notes during the lesson. Craig said that he never does, for he just has it in his head. She pointed out that he did exactly as we had planned leaving nothing out even without notes. Craig says that he will sometimes forget things, but in some way it always comes up at a later time.

Alex was impressed with Craig's ability to answer students' questions and to find their mistakes. Since Craig went through all of the theorems with the teachers during one of the planning meetings, the proofs where fresh in his mind. It appears that he had the pedagogical content knowledge he needed in order to help the students find their mistake. Craig agreed that the ability to do this comes with experience.

It wasn't surprising that Craig did not use any notes during the lesson. I saw Craig look at the lesson plan for the lesson a few minutes prior to teaching, but he did not refer to it at all during the class. The group was very pleased with Craig's teaching of the lesson. The group members appeared surprised the lesson went so well because of the way Craig described his planning method or lack of it and his teaching method "chalk and talk." Craig views himself as an average teacher. In his journals, he mentioned how it is strange how these teachers look up to him. He wasn't used to having his teaching be the center of

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other people's attention. It looks like this was a good experience for Craig

because he got some validation of his teaching (self-confidence that he might not

have had before) and was able to learn new things from the other teachers as

well.

Curriculum Knowledge

<u>Episode 15</u>

In planning meeting #1, the teachers discussed where the current

geometry lesson fits into the curriculum and what concepts will follow the lesson.

> Melissa: Do we have a specific unit in our curriculum for parallelograms? I
> don't think we do.... If we had a unit, it would come from quadrilaterals? Is
> that right?
> Craig: This would probably fall under a unit parallelograms. From
> parallelograms contained in that unit would be proving quadrilaterals
> parallelograms, rectangles, rhombuses, squares. Or maybe it would be
> the whole quadrilateral thing. I don't know.
> Melissa: This was developed before I came. I'm not sure where they got
> the list of topics listed here (pointing to the curriculum).
> Craig: To be honest with you everything came from right here.
> Melissa: The book.
> Craig: Each of these is sections in the book.
> Melissa: What prior knowledge is necessary to learn the content?
> (Reading from a *Tool for Planning and Describing Study Lessons*)
> Craig: We talked about that- parallel lines, congruent triangles, basic
> geometry knowledge.
> Melissa: What new knowledge can be developed from what the students
> learn in this unit? (Reading from A *Tool for Planning and Describing Study
> Lessons*).
> Craig: Special situations. This is a quadrilateral. This is a special
> quadrilateral called a parallelogram. Why is it special? Properties and
> applications. How can we prove it is a parallelogram? And that is where
> we be at. Then it becomes this is a parallelogram but it is called a
> rectangle. What makes it special? How is this special? How can we
> prove that it's special. Then the same thing with a square. The same
> thing with a rhombus.

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Mike: They are also building skill for doing future proofs. This is just more practice and practice.

While looking at the geometry topics listed in the curriculum that would follow this lesson, the teachers begin the following discussion:

Mike: Did you say a rhombus is a kite?
Craig: In a special way right, it has two pairs of congruent sides.
Mike: But a rhombus has to have opposite sides congruent, no a rhombus has to have 4 congruent sides.
Craig: Kite by definition is a quadrilateral with two pairs of congruent sides. By that vague of a definition, a rhombus is a quadrilateral with two pairs of congruent sides. It also happens to have two pairs of parallel sides and in a kite they are not parallel.
Mike: You are saying a rhombus is a kite but a kite is not a rhombus. I thought you meant it the other way around.
Craig: It may specify in the definition of a kite... that a kite is a non-parallelogram. If that is the case a rhombus is definitely not a kite because a rhombus is a parallelogram. The one that throws them is when is a rectangle a rhombus? As he beats his head imitating the students he says, when it is a square.

Each teacher in the mathematics department at this high school had a binder that contained the curriculum for each of the mathematics courses taught. The teachers referred to the geometry curriculum during this dialogue. Craig should be the most familiar with it since he was the only one currently teaching the course. But, it seems like he has spent very little time referring to it. The curriculum lists topics to be covered during each major section of content. It should also contain a detailed unit plan for each major section, but the department was still completing these units at the time of the study. There wasn't a unit plan for the section on quadrilaterals.

Craig easily stated to the teachers the prior knowledge the students need for the lesson and how the students will apply the concepts on parallelograms to future lessons on squares, rectangles, and rhombuses. Evidently, this

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examination of the curriculum allowed the teachers who are not currently teaching geometry to see where the lesson on parallelograms fits into the geometry course. It is important for the teachers to see what has already been taught and what other topics will be taught after the lesson. Although it initially appeared that Craig was not familiar with the curriculum document for the geometry course, it appeared that he was familiar with the concepts within the curriculum. As Craig described the way the students will use the new knowledge on parallelograms he stated several good guiding questions. He included the following questions: what makes a square, rectangle, or rhombus special? How is this special? How can we prove that it is special? These questions will lead students to the characteristics of rectangles, squares, and rhombuses that differentiate them from one another. Evidently, Craig not only was clear about the geometry content for this lesson, but he also had planned out in his mind how he would present the next topic on special parallelograms.

When discussing the prior knowledge that the students will need Mike added the ability to prove. He stated, "They are also building skill for doing future proofs. This is just more practice and practice". It seems like Mike was pointing out students' procedural knowledge here. This was practice of the proof process, but as the students construct meaning of the new theorems they must connect many old and new concepts. Tin Lisa and Craig's view, the reasoning and conceptual understanding involved in writing the proofs is more important than looking at developing new theorems as practicing the procedure of proving.

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In the interplay between Mike and Craig about the definitions of a rhombus and a kite the two teachers examined their own knowledge. The teachers entered into this discussion as they looked at the topics in the curriculum that would follow the lesson on parallelograms. It appears that taking the time to look at the curriculum led the teachers to a discussion to verify specific mathematics content.

Lesson #2

Lesson #2 was an Honors Algebra II lesson planned by the lesson study group and taught by Lisa. The content of the lesson was an introduction to functions. For the beginning of the lesson, the teachers decided to have the students do an experiment to generate data. The students displayed the data on the board in graphical form. Then, Lisa gave the students notes on functions using the student data. A detailed lesson plan for Lesson #2 can be found in the appendix. Various episodes from the planning, teaching/observing, and debriefing meeting are analyzed below.

Mathematics Content Knowledge

Episode 16

During planning meeting #1, the teachers answered questions from A *Tool for Planning and Describing Study Lessons* Section IB: Narrative Overview of

Background Information. In this dialogue they discussed why functions are important for the students to learn.

> Craig: To understand how numbers cooperate. [After he laughs and a short pause]
> Well, what's the end result of functions? We are going to talk about a linear function. What is the end result of it? Why would you need it? Why do you want to use it? Where are they useful? What is their application value?
> Melissa: [As she looks at Lisa], Do you want to talk about specifically linear functions or functions in general?
> Lisa: I'm not sure how far we will get, but I think probably just functions, functions and graphs, evaluating, vertical line test, and then the next section is linear functions and slope.
> Melissa: Introduction to functions, so say functions in general.
> Craig: Functions are used to model real-world situations. That is the driving force behind the functions course.
> There is a discussion about whether this material is review for the students or not. Then they get back into why it is important.
> Craig: So functions are used to model real-world situations. Why else are functions important? How can we tie that into one number goes in one number comes out? ... What makes f(x), independent variable, dependent variable? Or are they just going to have to realize that here is the basic procedure and we will move into the conceptual as we address linear functions, quadratic functions, and cubic functions?
> Craig asks Lisa: Can you give me an example since I haven't taught functions in forever exactly, best guess what would be taught that day just so I think a little bit before or after?
> At this time then the group discusses the material that the book includes in the section that will be taught. Then, they go back to why functions are important.
> Craig: So that they can understand the relationship between numbers. I mean yes functions are used to model real-world situations, but they are not going to get that from one number goes in one number comes out. How are these two things related? How do you get from one thing to another?
> Melissa: It doesn't have to be a relationship between numbers. It can be applied to something else in real life.

Next, Alex discussed some experiment ideas that he used in the past. There was much more discussion of this later, but he added that you "create a function to make a prediction of something that cannot be measured". Later in the

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meeting, Craig found a statement in a textbook that said students learn functions because they are used "to make reasonable predictions of future trends".

Craig was thinking out loud in this dialogue to try to come to grips with why functions are important. He presented to the group a series of questions to get them thinking on the right track. These are similar to the types of guiding questions that we saw Craig use in lesson #1. It appears that Craig continues to develop his ability to pose questions to guide the discussion. The group could have stopped with the fact that functions are used to model real world situations. But, Craig wanted to be able to explain further. He went back to the definition of a function to try to offer a better explanation. Then he stopped to see if the teachers just want the students to understand functions as a procedure or from more of a conceptual perspective. It seems like Craig was looking for curricular knowledge when he asked Lisa, "Can you give me an example since I haven't taught functions in forever exactly, best guess what would be taught that day just so I think a little bit before or after?" As Craig focused on the mathematics, it appears that he needed to consider how the function concepts were connected.

Through all of this Alex sat quietly and took it all in. In Alex's initial interview, he shared that he had taught just as many science classes as he had math classes. When he does share something, the group didn't realize how great it was until later. He briefly discussed some experiments that he had used with his students to show how they do something to get some result. Then, he explained that the students must create a function in order to make a prediction about something that they cannot measure. It appears that this explanation of

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why functions are important goes back to the definition of a function. Evidently,

this time as Alex described pedagogical techniques, he also relied on the

mathematics to aid him in his explanation.

Episode 17

As the teachers begin to plan the lesson in planning meeting #1, Alex asks

for more specific details of the function concepts.

> Alex: Are there maybe 5, 3 , or start with even 2 questions, you know specific questions that if a child has had this topic or has had this day of lecture they should know?
> Lisa: Well, what is a function, maybe the difference between a function and a relation?
> Craig: Refresh my memory; it really has been a while since I taught this.
> Lisa: For every x there is at most one y.
> Melissa: In a function.
> Alex: Another words, they would need to know a precise definition is what you would be looking for?
> Lisa: Yea, maybe not be able to just spit that back out at me, but to understand what that means.
> Melissa: Being able to give an example of something that would be considered a function or not.
> Craig: So the absolute value function is or is not a relation?
> Melissa: Yes, all relations are functions. A circle is not a function or a parabola on its side is not a function.
> Later,
> Craig: Pattern recognition is also prior knowledge that the students need.
> Alex: A function is really a relation with a pattern. Right?
> Craig: Sure! I'm still sketchy on the word relation.
> Alex: A relation is just a set of ordered pairs. All it is.
> Craig goes onto say how it has been 3 or 4 years since he has taught functions.
> The teachers wanted the students to know the definition of a function and

more importantly what the definition means. A student could easily memorize the

definition and not really understand what a function is. Lisa wanted to avoid this.

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It appears that Lisa wanted the students to have a conceptual understanding of functions rather than memorization of the definition of a function.

Craig didn't recall the term relation. The teachers gave Craig examples to try to help him fill in the gap in his substantive knowledge. As Craig gave the group pattern recognition as another topic for prior knowledge, Alex used this to help Craig understand what a relation is. Alex says "a function is really a relation with a pattern". This is a very general way of describing a function, but Alex used something that Craig was familiar with to start with. It looks like this was exactly what the teachers wanted to do with their students, for they wanted to connect the concepts that the students were familiar with to the new concepts. But, Craig needed more explanation of what a relation is. Alex added, "A relation is just a set of ordered pairs". Here again Alex related this definition to something Craig was familiar with - ordered pairs.

Episode 18

In planning meeting #1, the teachers discussed what concepts about functions are important for the lesson. Alex continued to lead the group through recalling the important function concepts.

> Alex: ...Maybe you want to stress there are three ways to express the difference.
> Melissa: Different representations of a function?
> Alex: One way is verbal and one way is numerical in set notation, that kind of way. And the third way is typically graphing. What you want to do is express verbally the connections, you want to express numerically the connections, or you want to express graphically the connections so they can physically see the connection of the x and y.
> Melissa: You could actually have four. Usually the equation is called algebraically and numerically could just be in a table.

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Mike: I remember teaching this there was actually five. I think an arrow diagram.
Melissa: Yea, the actual diagram of the mapping.
Mike: I remember we had to teach the kids five ways.
Alex: If they could remember four of them or three of them when they graduate, hey more power to them. But if we only taught them two ways .
. . . .
Lisa: This might be good for some of them.
Melissa: ...showing them the different representations maybe not requiring them to know everyone of them but being able to represent it in different ways helps us to reach different learners.

As Alex listed three different representations of a function that he recalled, he offered a very good reason for why these different representations are important. He stressed how the students must be able to see the connection between the x and y values. Evidently, Alex was concentrating on the mathematics and in particular connections and representations.

Mike added that there are actually five different representations. Alex offered that showing them all five is good, but they don't need to know all of them. The teachers decided that the number of ways there are is not as important as finding the best way for each student to understand the concept of a function. It appears that the teachers wanted to use these different representations to help reinforce the definition of a function. Although here they are specifically talking about functions, it seems like the teachers display how important it is to be able to explain a concept in more than one way.

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Prior Knowledge Connections

Episode 19

During planning meeting #1, the teachers answered some of the questions from A *Tool for Planning and Describing Study Lessons*. Here, they were under Section III Unit Information: How this unit is related to the curriculum. In the following dialogue they were discussing the question: What prior knowledge is necessary (to learn the content that this unit focuses on)?

> Melissa: What prior knowledge do they need?
> Craig: They need to know how to evaluate expressions. They need to know how to substitute a value... Ordered pairs and graphing.
> Melissa: You are assuming they know how to do that?
> Lisa: I'm not sure if they are going to need a reminder or not. They've seen that.
> Craig: Is this where we would see if they know abscissa and ordinate or do we want to leave them out?
> Mike: I don't think those words are that important to be honest with you.
> Craig: I don't either.
> Mike: Unless you just going to say hey by the way . . .
> Craig: ...Let them know that at higher levels things are often referred to with different vocabulary. The x-axis is sometimes called the abscissa – and very well could be named that on the sophomore test, could be named that on the SAT, or even on the AP. They might use that vocabulary so you should know it....
> Melissa: More prior knowledge is the idea of sets. Are we assuming that they know domain and range?
> Craig: I wouldn't assume it even for honors kids. They may have heard but they don't truly know it....
> Melissa: Have they done any graphing before? Whenever they actually will be given the function and they will be setting up the table to graph?
> Lisa: Some of them have done graphing. I think a lot of them have graphed y=mx+b.

Lisa did teach these same students Honors Algebra I, but she was not confident in stating the concepts they should know for a lesson on functions.

Craig stated a vocabulary issue – whether to use the words abscissa and

ordinate. Since the students might see these vocabulary words on standardized tests, Craig stated that the students need to know what they mean. The way Mike and Craig talked about these terms, it appears that they wouldn't use them on a regular basis in the classroom. Instead they just want the students to remember them if they happen to be on one of these tests.

Two other vocabulary words domain and range must also be included in this lesson on functions. However, the teachers decided that this is not something that they should assume the students completely understand. The students have been introduced to these terms in the past, but will need more work with them in this lesson. This was a good call by the teachers. If they assumed too many concepts were prior knowledge, then the students would have trouble making the connections to the new concepts.

Choosing Example Problems

Episode 20

As the teachers continued to decide what information about functions is important to include in the lesson, the teachers discussed possible example problems. In this dialogue from planning meeting #1, they come up with real-life examples of functions and non-functions.

> Mike: When you where saying about giving an example do you mean giving a mathematical example or give an example of like each person theoretically has its own social security number? That kind of thing that would be like a real-life that would be a function.
> Lisa: Yea, that would be good to, but also an example when it wouldn't be a function.
> Mike: People in town all have the same zip code. People are inputs; the zip code is the output.

Lisa: But that wouldn't be a function, they all have the same zip code.
Mike: Yea, that's right what is the other way I am thinking?
Alex: That would be a function because each person is only assigned one value. You could have one thousand x-values and as long as that x is paired with one value. So in other words, if I live in a town I have a zip code, I am paired with one specific y-value, only one y-value.
Mike: Yes, that's a function.
Melissa: Yes, it would be a horizontal line.
Alex: Well, how about if one person has two residences. If they live in two towns, O.K. they can be assigned...
Craig: So you get one x-value -one person having two different y- values.
Alex: One person with two zip codes would be an example of a non-function with that analogy. You have a relation with the two ordered pairs, but you wouldn't have a function.

The teachers took time during the planning meeting to develop specific example problems for the lesson. Several aspects of these examples are important to note. First, the teachers wanted to come up with a real-life example problem. They attempted to make the problem more meaningful for the students. If there weren't any students in the class with different zip codes, then this would be a constant function, which may not be the best example to use in order to emphasize the definition of a function. Also, the students may question whether zip codes are really assigned to people. In addition, Lisa not only wanted examples of a function, but also a relation that is not a function.

As the teachers talked through the zip code example, it appears that they clarified their own understanding of the definition of a function. While thinking aloud, Lisa and Mike revised their thinking about whether the zip code example is a function or not. Alex also offered a non function example of a person holding two different residences. Craig translated the real-life example into more mathematical terms when he states, "So you get one x-value- one person having two different y-values". Another aspect of this example to think about would be if

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there is a student in the class that lives in two towns. If the teachers wanted to include the students in the class as the domain of these functions, then they must consider this. Evidently, talking through such examples was beneficial for these teachers as they reviewed the definition of function.

Episode 21

During planning meeting #1, the group spent a lot of time discussing an experiment which involved marbles attached to pieces of spaghetti. The idea was to see how many marbles it takes to break a certain number of spaghetti. They first decided that they wanted it to be the opener for the lesson. Alex suggested that the guiding question be for the students to make a prediction. This dialogue started out with Lisa trying to figure out what is the independent and what is the dependent variable for the marble breaking spaghetti experiment.

> Lisa: Say two pieces of spaghetti; say its six marbles break it...
> Mike: It is always going to be spaghetti depends on number of marbles.
> Lisa: Yes, but the number of spaghetti is the domain and the marbles is the range.
> Melissa: The number of spaghetti is changing. And the x is usually what you change. The y depends on x. So the spaghetti breaking depends on the number of marbles. So the marbles is the x.
> Mike: That is the input, yes.
> Lisa: The spaghetti breaking or the number of spaghetti down?
> Melissa: The number of original pieces that you have that break. Isn't that what you are saying?
> Alex: Correct
> Lisa: That is the dependent one? No, I am still backwards.
> Melissa: Y depends on x. The spaghetti depends on the number of marbles...
> Craig: Spaghetti just hangs out. O.K. You attach paper clips and start dropping jelly beans or whatever in the bucket. So that is the number that keeps changing. This number (pointing at imaginary spaghetti in his hand) has not changed.... How many numbers get you to this y-value?...

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So, two spaghettis you have now changed your y-value. What x-value will get you to that y-value?

Lisa: Yea, you're changing your y-value. Is that what you're doing? So we are going backwards from what I am normally thinking.

Mike: You are not totally wrong...

Lisa: This is what I am thinking. We are starting with five pieces of spaghetti, how many does that take to break? Twelve marbles? And then you change it. You put two spaghettis. It only takes three marbles. That is what I am thinking because you don't know how many marbles it is going to take so you are not plugging that in first.

Mike: That's right; usually you plug in an x and get a resulting y. She is saying is how many marbles will it take to break starting with six spaghettis.

Lisa: Spaghetti should be x and figure out how many y, how many marbles.

Craig: The problem I have with that is in terms of thinking in terms of input and output. You know you are putting in marbles. That I see confusing.

At this time Lisa suggested using marbles and graduated cylinders filled with water. Melissa asked how this would be different than the spaghetti experiment.

Craig: The difference with the water is that for each one you add you see a difference. The problem with the spaghetti is that you don't have an immediate effect. You put one in and nothing happens. So, you have not evaluated your function. Alright, f(1) currently has no value if we are doing marbles counting as x. It is only when you actually achieve that value that causes it to break that you have an input value.

Then, Alex and Craig talk more about the spaghetti experiment.

Craig: Instead of a linear function is that more of a step function? Which certainly we wouldn't want to get into if this is an introductory lesson.

The group members were thinking hard about this experiment. Lisa appears to be unclear on how to set up the dependent and independent variables at first. Then, once she had time to explain what she was thinking, the rest of the group understood what she was saying. It appears that the teachers considered the causal relationship of how many marbles it takes to break a certain amount of spaghetti as an interpretation in science rather than simply thinking of the number of spaghetti and number of marbles as elements of an ordered pair in

mathematics. They use the statement "y depends on x" to try to determine which is the independent and which is the dependent variable.

Throughout this discussion, it appears that the teachers relied on the definition of a function to sort out if this is a good experiment to use or not. They wanted the experiment to motivate the students as they generate their own data to be used to develop the rest of the concepts on functions. In addition, they wanted the students to be able to easily establish which part is the independent and which part is the dependent variable.   The teachers used different mathematical terms to express the x and y variables in the experiment as they used the terms independent and dependent, domain and range, and input and output. They decided that the spaghetti experiment is too confusing; therefore they abandon this idea.

Episode 22

During planning meeting #2, the teachers look through the section in the book to pick out homework problems. Since they have not followed the section exactly as it is presented in the book, they must filter through the problems. The dialogue that took place here involved the teachers recognizing what concept of functions was necessary in order for the students to complete the problems.

> Melissa: I like these two where they have to look at the graph to determine if it is a function or not. And then this one they have to actually get the domain, range, graph it, and determine if it is a function or not and explain why.
> Craig: Yea I like those.
> Melissa: We don't really go over graphing a function. Have they ever plotted points before, you know set up a t-chart?

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Lisa: You know, I think they have done it. They've done y=mx+b, and I know they've done graphing.

Craig: So they could look at number 45 f(x) = absolute value of (x+5). Now that they know f(x) and y is the same thing. No they are not graphing, never mind they are evaluating.

Melissa: They are actually asking them to determine if the function is linear and then evaluate the function. We haven't really stressed linear either. That is a lot to put in this section. Then they talk about slope and they just assume that you've gone over linear functions.

Craig: But, this section covers linear functions. They actually define linear functions.

Melissa: Are we going to do that?

Lisa: It seems like a lot.

Craig: We're just defining functions. You can do the second half of the section the next day. Say alright, O.K. the functions you created are called linear and connected by a line.

Melissa: If you want to do more with determining if a relation is a function there are more on the first page.

Lisa: Yea I would like that.

Craig: Right, because...25-27 reinforces the vertical line test while 22-24 just has you look at the values.

Melissa: This one is using a mapping diagram. Do we want them to use a mapping diagram? I don't know? Just one of them?

Craig: That is the circle thing right?

Melissa: You usually use that when you are introducing functions, but do we ever use the mapping to determine if it is a function.

Craig: You could just change the directions for them to use the vertical line test.

Craig: I think 33 is a good question.

Melissa: They could have a bunch of different ways that they could explain that.

Craig: And then 51-54.

Melissa: Do we want them to evaluate any functions?

Alex: How about 49 and 50?

Melissa: Yes, that would be good they have them evaluate. There are some of those quantitative comparison problems.

Alex: My recommendation would be to do 59-62. By the way 63 is awesome.

Craig: They have tossed those from the SAT. I totally ignore those now.

Alex: What about from which is bigger?

Melissa: This gives them a chance to evaluate

Craig: I'm sorry I didn't even look at the problems. In terms of quantitative comparison I'd throw them out. Those are good problems.

Melissa: If we include 53, it does say to graph the relation. We could always tell them not to graph.

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Craig: But it is set up in an ordered pair format so they should be able to figure that out.
Melissa: I think these are good problems.
Lisa: I think so too. I like this book a lot better.

This process of choosing problems required the teachers to determine what concepts the students will need to know about functions in order to complete each type of problem. One of the main ideas of the lesson was the definition of a function. Craig didn't refer to this when he was confirming the following sets of problems, "25-27 reinforces the vertical line test while 22-24 just has you look at the values". In order for the students to determine if the relation is a function, they may use these tools, but the tools come from the definition of a function. The teachers questioned whether or not to have the students use a mapping diagram to determine if a relation is a function. Evidently, the teachers were brought back to their discussion on the different representations of a function.

Craig got hung up on the format of the quantitative comparison problems. Rather than look at the mathematics content involved, he eliminated them at first because of the format. After his colleagues pointed out the content involved he decided they were good problems. It is clear that the time that the teachers spent discussing the function concepts in previous planning meetings prepared them for sorting though the possible problems and choosing the ones appropriate for the lesson.

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Anticipating Possible Student Misconceptions

<u>Episode 23</u>

As the group was deciding what order they wanted to discuss the function concepts during planning meeting #2, Lisa and Mike mentioned two questions that the students might ask.

> <u>Lisa</u>: Why is it only the vertical line test?
> Melissa said that she would explain and show them how it is the x-values that cannot be repeated based on the definition of a function.
> <u>Mike</u>: What is so important about something being a function?
> <u>Alex</u>: Look, if you are trying to predict something you want to know if it is a function or not because what if you have a launching machine and put in 2. I want to know that it is only going to go to one place and not two. So, if it is a function it is only going to go to one place. If it is not a function, then it might go to two places which probably isn't good.
> One other question or possible misconception that Mike mentioned is that students have trouble understanding functional notation. The students may look at f(x) as f times x.

Lisa and Mike came up with good questions that the students might ask. It appears that the teachers examined the definition of a function as they attempted to answer these questions. Alex used an excellent real-life example to bring home why it is important for a relation to be a function. This example displays the importance of a function having a unique y-value for each x-value. Alex points this out when he stated, "So, if it is a function it is only going to go to one place. If it is not a function, then it might go to two places which probably isn't good".

Notation is an issue that students often have trouble with especially functional notation. It was a good idea for Mike to bring this to Lisa's attention since this was her first time teaching functional notation. It is clear that the

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planning meetings provided the teachers with time to repeatedly review the function concepts.

Episode 24

When teaching lesson #2, Lisa discussed functional notation with the students. She had already written out that f(x) means "f of x" and that you are evaluating the function at a value for x. She then wrote the following on the board:

$f(x) = \frac{1}{2} x + 50$
$f(2) = \frac{1}{2} (2) + 50$
$f(2) = 1 + 50$
$f(2) = 51.$

Here is the dialogue that followed:

> Brian: Yea, wouldn't you solve it from there and you would divide by 2. You have 51 divided by 2 equals f.
> Lisa: We are not actually solving. What we are doing is substituting two in for our function. So it is not really an equation you are going to go through to solve at the end. Does this answer your question?
> Brian: Not really because it is still 2f. Isn't 2 times f equals 51.
> Lisa: It is not 2f. This is a certain notation. So, it is always going to be written as f of some number and you input it into whatever your function is. I know it looks like multiplication there and it looks like you can write it as 2f.
> Brian: So f of x is equal to y.
> Lisa: Yes, there you go.
> Tom: So basically the two is kind of invisible.
> Lisa: Well, whatever is inside the parentheses here is what you are going to substitute into your equation.
> Tom: It is invisible because you wouldn't divide the two out.
> Lisa: Yes, think of the whole thing (puts a box around f(2)) as its own side.
> Brian: It is equal to y.
> Lisa: Yes, and that is what we are headed to.

She then wrote on the board:
Compare: $y = \frac{1}{2} x + 50$
$\qquad f(x) = \frac{1}{2} x + 50$
$\qquad y = f(x)$

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In a planning meeting Mike brought up the possible misconception that the students often think of f of x as f times x. It appears that this helped to prepare Lisa for the questions from the students. Lisa did a good job of answering Brian and Tom's questions here. When Brian sees f(2) = 51, he assumed the parentheses mean multiplication and that they were solving for the variable f. Lisa explained "This is a certain notation. So, it is always going to be written as f of some number and you input it into whatever your function is. I know it looks like multiplication there and it looks like you can write it as 2f."
She had planned to tell them that f(x) is the same as y, but they asked there questions before she got a chance to. But, it was nice to hear Brian say "so f of x equals y". After Brian stated this, Tom explained how it makes sense to him. He needed to think of the 2 as invisible so that he didn't try to divide by 2. To reply to Tom's comment, Lisa didn't just repeat what she had said to Brian. Instead she visually puts a box around f(2) to show how it was all one term and not two terms separated by parentheses. It seems like Lisa used her substantive knowledge of functions to help her frame her responses to their questions. Her pedagogical choice of choosing to state that y= f(x) last was interesting one. However, this went with Lisa's style of teaching. She let her honors students come to conclusions on their own or with some guidance from her.

Mike and Craig commented during the debriefing session on Lisa's pedagogical ability to clear up this misconception among these two students as well as the others that may have been thinking the same thing. Mike said that he tells the students write away that f of x does not mean f times x. This may

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prevent the misconception, but it may not provide the opportunity for a student to tell the class that f of x equals y.

Questioning

<u>Episode 25</u>

During the first planning meeting the group has talked about whether or not to include the vertical line test or to make sure that the students understand the definition of function before presenting it. As they went through the outline of what Lisa was going to teach in planning meeting #2, they found a place in it where the vertical line test would fit in nicely. So, they decided they would teach this. Then, this dialogue follows.

> <u>Melissa</u>: It would be nice if we could get them to say it. They are not going to get that by one graph unless we draw...
> <u>Lisa</u>: I could definitely ask them. Can you think of any method? Since this is not a function can you come up with a method that you might be able to use to test a graph that is given if it is a function or not?
> <u>Melissa</u>: Yea, then maybe you could draw a few functions and non-function on the board. Like if you draw a circle on the board and maybe ask if this is a function or not based on their definition for every x there is one y and then establish this and label beside it is not a function. Then give them a parabola.
> <u>Lisa</u>: Yes, I can do this.
> <u>Melissa</u>: And they may not exactly state vertical line. But, you can give them a chance to lead into it.

Lisa and Melissa wanted the students to come up with the vertical line test on their own. Lisa posed a good guiding question when she stated, "... can you come up with a method that you might be able to use to test a graph that is given if it is a function or not?" Melissa suggested that Lisa give them more examples in the form of graphs that the students can generalize from. Evidently, Lisa

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thinks that in order to pull this off in the classroom she must use her knowledge of functions (examples that are and are not functions) and ask guiding questions that could lead the students to the test. The students' ability to understand the definition of a function and how it relates to a graph will help them put together the idea of the vertical line test. It can be seen on the teaching of the lesson tape that Lisa was able to get a student to point out that a vertical line could be used as a method to test if a relation is a function.

Episode 26

When Melissa is giving her reaction to the lesson at the debriefing meeting, she stated that she would have liked to see the students talk more during the presentation of the functions notes. She felt that Lisa could have gotten more from the students without telling them everything. Craig pointed out that Lisa would have needed to use more guiding questions.

> Melissa: I liked how you were able to use everything with the experiment. I was hoping for more discussion. I didn't think there would be quite as much notes with them writing everything down. I was thinking we would get more from them. Maybe they don't know enough. Unless we just have to try to get it from them.
> Craig: More leading type questions.
> Melissa: Yeah, maybe.... You stated that if the x-values never repeat, you have a function. I think that could be telling them too much. They could have figured that out on their own by giving them more examples or something.
> Lisa: I could have gotten more from them.
> Melissa: That would have slowed you down though too. It is matter of how much time you want to spend. Standing back and waiting a little longer for the responses and then giving them another question or two to lead them into the answer....I liked how at the end you had problems for them to work on. And at that point then I could watch and see and observe how much understanding they had based on the questions they were asking. But during the notes I wasn't sure. Maybe if you would have put more

examples or just more leading questions you could have got more from them.

Lisa: Yeah!

Lisa's style of teaching is more of a discovery approach. She usually doesn't tell her students much without them giving her a lot first. It appears that due to Lisa's inexperience teaching functions or her nervousness, she chose not to pose guiding questions on her own during the lesson. Craig was quick to offer a solution on how Lisa could get more from the students. It is clear that Craig's ability to ask guiding questions in the geometry lesson came from his experience teaching the course. But it is important to keep in mind that Craig reviewed all of the geometry concepts with the lesson study group. Thus, it seems that this attention to the details of the lesson aided in his ability to lead the students along throughout the lesson.

By giving the students time at the end of class to work on problems, the teacher observers were able to assess the students' understanding. Before the students left class, they were able to get a feel for how the students were doing with the new concepts. These problems were the problems that the group members had spent time deciding upon and were not exactly like the problems that they had done during the lesson. Thus, they were a good indication of what the students had learned during class. However, with more discussion during the lesson, Lisa could have been assessing student understanding throughout the lesson.

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Assessing Student Understanding During the Lesson

Episode 27

The following dialogue from the debriefing meeting is more comments about the teaching of lesson #2:

> Lisa: That kid Brian, the one that kept raising his hand who made the class move along. He was putting together things that surprised me.... I didn't expect that necessarily from the class because I didn't know what background they had and I also didn't expect that from him.
> Melissa: So he doesn't normally do that?
> Lisa: Well we have been doing more factoring and things like that. He gets bogged down with things that look difficult and with procedures, but he can apparently put together and apply concepts really nicely.
> Alex: Was he the one that said you mean y?
> Melissa: He said you mean y equals f of x.
> Alex: It takes a student like that. For me it varies between maybe four students who can help drive the lesson. One student will drive the class more than another one. It kind of oscillates. If you can have those four, life is good with kids that can help drive the lesson.
> Lisa: And actually one of them who also does that. He was one of the ones that weren't on camera today. He was out. And the few others who I think generally do that were nervous because of the camera; but also, I think they are more procedural students. They like to be told what to do. They want to think independently, but it is hard for them. They are just very good at being good students.

Lisa was surprised by Brian's connections among the function concepts. Lisa learned something about this student as she taught the lesson today. It appears that she was able to assess his understanding as he asked questions and made connections among the concepts. According to Lisa, Brian has trouble with problems that look difficult or that require a procedure to solve. Alex pointed out that it is great to have students like this in your classes to help you move the lesson along. Lisa had other students that normally do this. Her comment, "They are just very good at being good students" is interesting. In

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Lisa's view, students that can follow procedures and not necessarily think independently are considered good students. By good students, it appears that she meant students that are capable of getting good grades, but may not truly understand the concepts.

Lesson #3

Lesson #3 was planned by the lesson study group for Integrated Mathematics IV students and taught by Alex. Alex wanted to incorporate technology into this lesson on application problems involving linear equations. The teachers spent time during their planning meeting watching possible DVD's that Allen could incorporate into the lesson. Instead of the DVD's, he used music and a storyline to motivate the students. A detailed lesson plan for Lesson #3 can be found in the appendix. Various episodes from the planning, teaching/observing, and debriefing meeting are analyzed below.

Creating Meaning and Prior Knowledge Connections

Episode 28

Craig and Mike missed planning meeting #3. In the dialogue that follows from planning meeting #4, Craig asked the other teachers a question about one of the example problems.

> Craig: I have a question about the end of this. After you come up with your value of 160 cm, will they then need to convert that to feet and inches to give it a little more meaning because quite honestly I don't know if they are going to know what 160 cm is?

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Alex: I know. We did a whole section on ratios and of course you do shadows. The ability to go from a meter height and then convert that to feet in the same problem was really challenging for most of the kids. Healthy, good to do, real yes, but wow.

Craig: It would be good way to bring back that skill, Say hey remember we talked about ratios.

Alex: We talk about real life. In real life you don't talk in cm. In science we measure in cm, but you talk in inches. What do you think? Would it be worthy to have a calculation?

Craig: If they've done the conversions before...

Alex: ...O.K.

Craig: Say - remember we talked about conversions. We converted meters to feet. Here we are going to convert cm to inches. It makes that one more connection.

Alex: It makes it more real too. The whole point is a real life situation.

Craig: To be quite honest with you I can't tell you how tall 160 cm is.

Alex: People who ski there skis are measured in cm. Still in your mind your saying how high is it really?

Craig: That would be a very good example to mention. Tell me if you ski. How tall are you skis? Then they realize when they take their skis off and stand them up that is their height 5ft, 6ft.

Alex: That is awesome, use that prior knowledge.

Mike: Are they going to remember that?

Alex: I can show them on Monday or Tuesday (before this lesson).

Craig: Just walk them through it.

Alex: ...How would you do that conversion? Would you do as a ratio? Would you solve an equation for a ratio? Or would you take the number of cm and divide by 2.54? What would be your strategy?

Lisa: How did you do it before?

Alex: I wanted to get in as much theory. We did it as a ratio. Basically, we know that 169cm is to x inches as 2.54. We actually solved the equation as opposed to saying just divide by 2.54.

They all agreed to do it this same way.

Craig: Then you will have to change the inches to feet. If you get 56 inches, how many feet is that?

Alex asked if it is "worthy to have a calculation". He consulted with the other teachers on taking the time to do this conversion. They assured him that it would allow the students to connect this lesson to something they already learned. Alex assumed that the students would need assistance setting up the correct proportion. It appears that Alex was not confident that his students would

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recall the necessary prior knowledge to do the conversion. Mike also questioned if they would remember it. It seems like these teachers were considering the level of student and were concerned with how these students often memorize procedures. Craig suggested that he "just walk them through it" if they don't remember how to do the conversion. Once again, it appears that Craig was focused on the students seeing a connection between concepts.

Alex offered a good way for students to be able to picture how high 170 cm is with ski lengths. If the students can picture the height of the femur in inches and then feet, they may find the problem more meaningful. Evidently, the teachers also wanted to include this conversion in order to help fulfill their goal to make the lesson meaningful. Another thing that they considered about the conversion was that by having the answer in units that the students understood they will be able to see if their answer makes sense.

Choosing Example Problems

Episode 29

Alex, Lisa, and Melissa were the only group members present for planning meeting #3 to plan the details of this lesson on application problems involving writing linear equations. Alex was called away from the meeting for a few minutes, so Melissa and Lisa looked through some textbooks for example problems. When Alex returned, they shared with him what they had found. There was a femur problem in which the students must pick out two ordered

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pairs and write the equation of the line in order to answer the question. Alex got excited about this example and wanted to actually use a model of a femur for the students to measure. He said he would work out the details of this. Melissa suggested the following format for the lesson: introduction, students do five problems together in groups, go over these problems on the board, then give two problems at the end of class for the students to solve on their own. Then, Lisa made a suggestion.

> Lisa: Is there any way we could work into having them create a problem? And then maybe trade with someone else and solve each other's? But it may have to be really guided.
> Melissa: So, that could take the place of five problems. Maybe give them a couple maybe two or three to do and then have them make up one that they can give to another group.
> Alex: That is an interesting thought…If they create a problem and they solve it….They exchange with each other that would be awesome. Now they are thinking about what the problem is made up of and they are not going to make it sound weird because they have to solve it and give it to another team.
> Melissa: One thing we could do is. Let's say they had an example at the beginning whether it was on the video clip or something that you did Alex and then they had two or three that they had to do in their groups.
> Alex: Would that example be written out extremely thoroughly?
> Melissa and Lisa: Yes!
> Lisa's suggestion was for the students to create their own problem. After

solving several linear equation application problems, the students would pose their own problems and model the process for solving them. Lisa emphasized that this would have to be a guided practice exercise for the students. The students may have never done this before, for they have not done it in Alex's class. Lisa ended up developing a template for the creation of linear application problems that she shared with the group at the next planning meeting. Evidently, Lisa believed that if students posed their own problems and solved each other's

problems, they would display their understanding of this type of linear application problem. It appears that Alex believed this activity would require the students to think about the real-life context of a problem and not just the procedure for solving it.

At the end of this dialogue Alex asked if the first example problem he gives the students should be "written out extremely thoroughly." Melissa and Lisa agreed that it should be. They consider the learning styles of the students and how the students will need such an example to do similar types of problems. It appears that in Melissa and Lisa's view, this level of student needs to see at least one example very clearly in order to model the solution process to do similar types of problems.

Episode 30

Alex really liked the problem in one of the DVD's that involved finding the patterns of tiles and predicting how many tiles will be necessary to create that pattern. The way the woman in the DVD solves the problem is by looking for a pattern and just writing out an equation based on the pattern. In the dialogue below, they discussed which types of problems they wanted to include and Melissa pointed out to Alex that in the tile problem the students are not using two points to derive the equation of the line.

> Alex: We may only have time to show them one kind of problem.
> Melissa: I'm not sure what you mean by kind of problem.
> Alex: For example, one would involve two points. Another one would involve....They come up with slope. What if they are given a point and the y-intercept?

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Lisa: So you are talking about given different information. Two points, a point and the slope, what else?
Melissa: Slope and y-intercept. Let's look at the application problems [in the book] and see how they are. I think most of them are two points.

Several minutes later they went back to looking for example problems. Alex suggested that the students do the tile problem from the DVD, and then model another one.

Melissa: The only thing that I am concerned about is that they are not given two points in that one. That is one where they are just writing the equation based on what she is talking about- how they are adding one, taking one away or whatever. They are just writing an equation, not given two points and finding the slope.
Alex: But that is an application. In other words, that could be one- give them an m, give them a b, give them two points
Lisa: What are they given?
Melissa: The pictures of the tiles and that's it.
Lisa: There is more problem solving involved. I think.
Alex: I'll take another look.
Melissa: I think this is a good problem, but I don't think it is going to model the application problems that we are finding in these books. Just by them seeing that example and doing another one like it, I'm not sure they will be able to do these ones (points to the ones in the book).
Alex: What if our goal is that we want them to see and practice two application problems. One of them is the tile where they can see how they can take a real life example and write the equation right away.... And then I give them another sheet of paper that has a very similar tile problem. And based on the template they filled out watching her do it, now I give them a slightly different tile problem but in such a way that they can do that one.
Melissa: And that is going to be a similar thing where they just write the equation down?
Alex: Exactly, the goal of that is to write an equation.

During the rest of this meeting the teachers tried to find problems from different textbooks to use in the lesson. They rule out problems based on different reasons. If the problems were not set up as two points, then they reworked the problem to be set up that way. They also changed problems to get rid of decimals or to make it a friendlier slope. There was one example that they ruled out because it had a time factor in which the students had to relate t=0 to the

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year 1980. They found a model for a template that they can use. They decided on a temperature problem that relates Celsius and Fahrenheit temperature scales.

In this episode the teachers were carefully deciding what problems they wanted to use in this lesson. First, they considered the mathematics content involved in linear application problems and discussed which information – slope, two points, y-intercept - should be given in the problems. As the teachers were looking for example problems in the textbook, they negotiated which problems they wanted to include based on the type of given information, the numbers involved, and student interest. The teachers wanted the students to understand the process of solving linear application problems when given enough · information to set up two points. They didn't want the students to be confused by the numbers involved, time factors, or any other parts of the problem that can be unnecessarily confusing for the students. It is evident that during this planning time the teachers used their content knowledge and the nature of Alex's students to assist them in choosing example problems.

Episode 31

At planning meeting #4, Lisa distributed the problems that she wrote based on the discussion at the last meeting and possible templates that could be used. The dialogue below is the discussion over which problems the teachers preferred:

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Craig: I really like the cell phone problem. I think that is an excellent real world, very meaningful to them application. And I think that should be the second problem.

Note: The original cell phone problem created by Lisa is in the appendix at the end of Lesson Plan #3.

Melissa: Don't do the temperature (Celcius/Farenheit) problem?
Craig: Just because this is far more meaningful...The other thing about the cell phone problem is that there is a lot more follow through. You don't just get one answer and your done there are several other things she's got in here. What does the y-intercept mean, what does the slope mean... taking those numbers from the equation and giving them more meaning, even more beyond the archeology which gives more meaning to the numbers and values but the y-intercept of that equation doesn't mean much. Here you are taking it one step further with that cell phone problem. I think it is an excellent problem.
Melissa: Would we have them answer all of these questions all at one time?
Alex: If one of the intentions was to model the problem would it be wise in the femur problem we've not modeled the discussion of the y-intercept. We've found the y-intercept strictly as an intermediary to find the height of the person. But, we have not discussed it.
Craig: Right, you've done that twice.
Melissa: You are kind of going a little bit further.
Craig: You are certainly pushing them beyond what I think would be expected in Integrated IV. You are now pushing them to a higher standard. You are saying there are other parts of this problem that relate back to the algebra.... I hate the word rigor, but you are making it a more academic problem in disguise.
Alex: What do you think Mike?
Mike: I think they are going to have trouble with it.
Alex: That is my gut feeling.
Craig: In a class of nine, you can really get into it.
Alex: What we can do is stress the issue that is comes down to just the ordered pairs. If you just treat them as numbers you don't have to get all caught up in that it is a femur. There is a purity of mathematics which is strictly numbers and it's surrounded by this application thing. You and I know if we were to give them two ordered pairs they would solve it readily.
Melissa: This will show if they really understand.

Note: The original hiking problem is in the appendix at the end of Lesson Plan #3.

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Alex: The idea of the hiking one- we want them to see that 2 hours yields 6 miles and 7 hours yields 14 miles. Was that the idea? The ordered pairs?

Craig: Although, I don't know that I would use the vocabulary that describes time as a function. You are not talking about functions, you are talking about equations. Write the equation that describes the hiking distance in terms of time spent walking. But you are at that point defining x and y for them.

Alex: Write an equation that describes your hiking distance and time.

Craig: Write an equation that describes your time spent hiking and the distance traveled.

The teachers used this time to revise the problems that they developed at the previous planning meeting. It appears that the teachers like the cell phone problem because it is meaningful, but they were concerned that it is too challenging. It looks like this problem would call upon the students' conceptual understanding of slope and y-intercept rather than simply the memorization of a procedure. As Craig embraced this, Mike and Alex acted cautiously. It seems like Mike and Alex are concerned again about the level of the student.

It appears that Craig's suggestions for the hiking problem focused more on the students' ability to apply the mathematics concepts. For example in rephrasing the problem, Craig suggested to try not to define which is x and which is y. Craig's rewording of the problem, "Write an equation that describes your time spent hiking and the distance traveled" accomplished this request. It seems like Craig wanted to challenge the students more. It can clearly be seen that the planning meeting provided the teachers with time to share and to negotiate ideas in order to develop problems that would work best for Alex's students.

Assessing Student Understanding During the Lesson

Episode 32

During the teaching of lesson #3 as the class was working out the first example problem, a student named Cindy said "we do this like we did in the homework problems."

> Cindy: I don't think I did it correctly on the homework.
> Alex: Well, do you see it now? The homework kind of warmed you up to it. Now, the fact that you can say that is great.
> Cindy: You know how you have to pick the x and the y? Where y was, where y(1) would stand for, I think I put any number.
> Alex: O.K. Is it a little clearer now? That was the idea for you to practice a little bit. Notice I didn't even go over them. Just the fact that you looked at them, you tried to punch out some numbers, you are a lot better equipped to do it today, then if you did nothing.
> Cindy: Yes!
> Cindy pointed out at the beginning of the example problem that she sees

how this is connected to what they did in their homework. A student saw a connection. This was great for Alex to hear especially since the group members made the decision to not go over the previous night's homework. But, later on Alex got even more unsolicited information from the student that she did it incorrectly last night, but understands what she was doing wrong. A student's self assessment is always good to hear. It may not be vocalized. The teacher may have to discover this as he/she walks around the room and checks homework or has the students try a few more of the same type of problems at the beginning of the class. The teacher must make pedagogical decisions to determine how he/she will assess students' understanding of the previous day's lesson. It appears that this was not only a way to assess students

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understanding, but also a way to see if students made connections with their work from the previous day.

Motivation

Episode 33

Throughout the teaching of lesson #3, Alex used music. He played such songs as *Chariots of Fire, Under the Sea, Star Wars*, and *Pink Panther*. The pedagogical decision to use music was Alex's. He started out with it to motivate the students, one of the group's lesson study goals. He explained how he would listen to the particular type of music to psyche him up for something or to listen while trying to solve a problem.

The use of music surprised the students and really kept them interested in the lesson. They would wait for the next choice of music to come on. This may not work in every teacher's classroom. It seems like it would depend on the frequency of use, the number of students, and the type of music one chooses.

Episode 34

After 20 minutes into the teaching of lesson #3, Alex began the storyline behind the femur problem. He had a reporter from *Time Magazine* call and ask for a group of students that could solve the missing bone problem for a reward. The class had one hour to solve the problem before a reporter from *U.S. News and World Report* stole the story. Alex had the real-life example problem on the

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overhead. They did one example first in order to solve the problem presented by *Time.*

The storyline that Alex created was very interesting and fun for the students. They were interested from the beginning to the end. This was another one of Alex's pedagogical decisions. This was a very big motivator for the students, one of the lesson study group's goals. Often, the students need something like this to make the mathematics concepts meaningful to them, another one of the lesson study group's goals. If the students were trying to remember how to write the equation of a line from a set of ordered pairs several days later, all they would need to be reminded of is the bone or femur problem and they would be able to visualize the procedure they used. Apparently, the use of the storyline helped Alex to achieve two of the group's lesson study goals.

Episode 35

During the debriefing meeting, Alex told the group how he felt the lesson went. The dialogue below includes the other teachers' comments.

> Craig: The adaptation to time was good.
> Mike: I thought you did an excellent job. I was very entertained. You got the point across....Like Lisa said earlier today when I commented to her I said wasn't that great. She said "I was so motivated by that. I want to do something like it."
> Lisa: I went to my next class and I was like I got to do something fun. It was great.
> Melissa: Will you consider using music more in your class? They seem to like it.
> Alex: They did. I could have theme music.... You play the music when you want them to solve a problem....
> Lisa: I would be so embarrassed. My face would be so red. I wouldn't be able to do it.

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Melissa: That is another thing that is interesting in working with other people. I would not have thought to do that in my classroom. It worked wonderful in yours. You pulled it off great. The students were motivated. It was a wonderful lesson. I would not have seen that if I had not been working with you guys because that is not something that I would have thought of. So it's great to get these ideas. We had different ideas for problems and things like that like creating problems on their own. That is one thing my honors trig students are writing out three application problems – this is one of their journal entries. They are solving them on another sheet of paper and on Friday they will exchange and solve each others. I wouldn't have thought about that if I hadn't been working with this group. I had heard other teachers using music before. Now I can see how you used it and think about how it might work in my classroom.

Alex: After everyone left...I was thinking to myself.... We had one lesson that anyone of us five teachers would have carried it out completely differently....The idea is communication as oppose to I have to do it exactly Craig's way or he has to do it my way. This group is so mature to see that. It is more a question of the ideas, how to communicate, and how to get the spirit in us so the kids can sense it and see it. What we do help us do that. Each of us is a professional. We care. We are dedicated to helping students...

Alex not only entertained the students with the music and storyline, but he also entertained the teachers observing the lesson. The group members were not upset that Alex had to deviate from the lesson plan due to time. They all liked the storyline and music that kept the students motivated throughout the lesson and attached meaning to the lesson (two of their goals). These pedagogical decisions made by Alex had not been used in the other teacher's classrooms before. It looks like the teachers have more things to consider using in their own classrooms. The teachers realize that they may not carry out the lesson in the same way due to their own personality. Alex's final comments in this dialogue display how much he appreciated the support of the group members. It appears that the communication among teachers to develop a motivating and meaningful lesson really inspired him to teach the group's lesson,

Critiquing Video

Episode 36

Alex wanted to use a video as part of the instruction time in his class. Melissa helped Alex locate three DVD's that the group watched during this planning meeting. They watched the first DVD and give their reaction to it. Craig said the DVD is thorough, but boring. He pointed out that the women in the DVD uses y=ax+b instead of what is in our textbook y = mx+b. Melissa added that she doesn't use the word slope. Craig also said that she doesn't use a consistent process to solve one problem to the next problem. The group agreed that the problems are good problems, but they may be too easy for Alex's students. Alex said that his students would have trouble creating the equations in the DVD, but they could solve them. The group asked Alex again why he wants to use a video in the first place. He wanted to be free to see what student are doing and thinking in the classroom. In terms of what he has seen in the first DVD, Craig didn't see how this DVD would help him teach these concepts.

As the teachers watch the first DVD, they examined the mathematics content and how the mathematics is taught. It appears that the teachers were thinking about how they teach the topic of linear equation application problems and were comparing it to how it is presented in the video. They picked up on some notation and vocabulary concerns that are not consistent with the way they teach. They also have concerns on whether the problems were appropriate for Alex's students. Evidently, the teacher saw discrepancies between their own pedagogical content knowledge on the topic with what and how it is presented in

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the DVD. The other teachers respected why Alex wanted to use a DVD in his classroom, but at this time they are not seeing how the first DVD would work.

Vocabulary

Episode 37

In planning meeting #4 as the teachers discussed the example problems and templates that they planned to use in the lesson, Lisa asked the group the following question:

> Lisa: Have they seen independent and dependent variables?
> Alex: I'm going to say no, not in any strong capacity.
> Craig: Not in that vocabulary.
> Lisa: So will you always have to tell them when they see a word problem like this which one is x and which one is y?
> Craig: Well, what I do when I am trying to indicate you substitute x to find y is that which one would you need to know first? And generally in the integrated when I taught it...we didn't use independent and dependent variable. Literally it was which one do you need to know first. Kind of like when you are defining a word problem you know one number is twice the other which one do you need to know first?
> Lisa: O.K.
> Alex: The way I would handle that would be what is it that we are trying to find? We want to know a person's height. That is unknown, that's what we are trying to find and that is going to be the y. That is going to be the dependent.

Lisa's question about independent and dependent variables was one of vocabulary. In some math classes, these would be the terms used any time the teacher is discussing lines. In Lisa's honors' classes, she would use these terms. Since this is a lower level math class, Alex and the other teachers decided that this terminology was not necessary. For the time being, Alex had

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decided to not give this information to the students. At a later time, he may feel that they could come up with this on their own. The teachers discussed strategies that they would present to the students to help them determine the x and y-coordinates of the ordered pair. Craig suggested for the students to find x first while Alex suggested for the students to find y first. It is clear that the time spent together as a lesson study group has made the teachers comfortable sharing different ideas.

Curricular Knowledge

<u>Episode 38</u>

During planning meeting #1, Alex discussed the material that he would have already covered in his Integrated Math IV class prior to the lesson that the group plans for him to teach. This led the teachers into the dialogue below about various curriculum issues.

> <u>Melissa</u>: Unfortunately, this is so disjointed. It is not your fault it is the curriculum. You did similar triangles, transformations in the coordinate plane that is where the matrices are, now polynomials, and will come back to slope and systems of equations.
> <u>Alex</u>: ...to focus in on linear equations. Our rational was that would cut across all classes Algebra II, Algebra I, Integrated III, Integrated IV.
> <u>Melissa</u>: So it could be something you all could use later on.
> A few minutes later:
> <u>Melissa</u>: Just so you know where this fits into the whole scheme of things in Integrated V I started with systems of equations. We graphed a few lines and reviewed slope. We did all the different methods for solving a system of equations including determinants. Then we did systems of inequalities. So that is the first chapter in Integrated V
> <u>Alex</u>: And that is part of the curriculum for IMV . . .
> <u>Melissa</u>: So pretty much what is at the end of your Chapter 6, they do again in Integrated V.
> <u>Alex</u>: But you have a different book.

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Melissa: Yes, an algebra book, not an integrated math book.
Lisa: A student in integrated series, how many times will they exposed to slope and their equations? Every single year?
Melissa: At least Integrated III, IV, V
Mike: Integrated II, we didn't get into that in I.
Lisa: By the time they get to that are they really sick of it? Or do they just forget?
Melissa: They don't remember it.
Alex: ...There is less mental exertion into the topic... If IMV goes in the same manner that I discovered with IMIV. It was night and day between IMIII and IMIV. I am guessing you are going to have the same even jump again between IMIV and IMV.
Melissa: You are going to have more of the students weeded out and are more serious because they don't have to take it. Even in IMIV they don't have to take it.
Lisa: They need four math classes.
Alex: Three, isn't it three.
Lisa: They only need three. Has the school thought about requiring eight? That they take math all the time?

Before this group even decided on a definite topic for the lesson that they would plan, they found it important to talk about what the students would already have learned in this class and what material would follow this lesson. The main curriculum issue was that the topics are disjoint. As a teacher presenting one chapter after another in this course, it is difficult to demonstrate to the students how one chapter connects with another or how the concepts they just learned will or will not be used in the next chapter. These are students where math is most likely not their favorite subject or something that comes easily to them. It seems like without this lesson study opportunity to sit down and talk about curriculum with their colleagues, the teachers that weren't teaching in the integrated curriculum would not have learned what is taught in some of these classes. This may lead the teachers to discuss this farther at a department meeting or it may be something that just stays in the back of their mind for now.

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Lisa brought up a good point when she asked how many times the students in the integrated curriculum study lines. This surfaced another curriculum issue repetition versus depth. She also asked if the students become bored with the topic, or if they really need to review it that many times. Alex made a good point when he said that it is nice to see the students' progression on this topic.

The group even goes broader in their discussion of curriculum when Lisa learned for the first time that the students only need three semesters of math for graduation. As they began to develop a lesson for this class, it was good for these teachers to know that the nine students in this class were not required to take it. They have fulfilled their math requirement, but are going beyond it because they are considering going to college and will need the math in order to pass the SAT's. These students who normally struggle with math or do not particularly like it are taking a math class beyond their high school requirement. It is clear that these were all important things for the teachers to consider as they begin to develop this lesson for these particular students.

In this chapter, the analysis of various episodes from each of the three lessons was organized according to categories. These categories emerged from memos written after I watched the videos of all meetings and read through all meeting notes and journal entries. The categories: mathematics content knowledge, meaning and connections to prior knowledge, choosing example problems, anticipating possible student misconceptions, questioning, assessing student understanding during the lesson, motivation, critiquing video, vocabulary,

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and curricular knowledge all fall under the broader heading pedagogical content knowledge. These categories help to display the patterns in the teachers' behavior in the process of learning. In the next chapter, I present the analysis of the three lessons across the planning, teaching/observing, and debriefing stages. Once again the categories which include mathematics content knowledge and various aspects of pedagogical content knowledge emerge from the data.

CHAPTER 6

RESULTS BY STAGES ACROSS LESSONS

After examining the individual episodes from Lesson #1, #2, and #3 described and analyzed in Chapter 5, the next step was to analyze the changes across the lessons within each stage – planning, teaching, and debriefing. The results recorded in this chapter refer back to the specific dialogue from the episodes in Chapter 5. Parts of the dialogue will be included here, but for the entire dialogue please go back to the specific episode. The results are separated in each stage according to mathematics content knowledge and pedagogical content knowledge.

Planning Stage

Mathematics Content Knowledge

In the planning meetings for lesson #1 and #2, the teachers refreshed their mathematics content knowledge on geometry proofs and function concepts. As the teachers plan the details of lesson #1, they write out the steps to each of the four proofs that Craig will do in the lesson. It appears that the teachers used

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their substantive knowledge to figure out the steps for proving when a quadrilateral is a parallelogram. As written in Episode 2, when Craig runs through each proof, he ran into difficulty when he started the 2nd proof. This required all of the teachers to work together to find a different approach to the proof. They worked on their own and consulted textbooks to refresh their syntactic knowledge for that specific proof. Evidently, their discussion of the geometric concepts involved in the proof also brought to light the substantive knowledge they needed to use. Also, it appears that the teachers' concentration on the details of the content helped them to see how they wanted to present the concepts to the students. In the first lesson it was not completely clear which teacher besides Craig felt comfortable with the geometry topic. All the teachers wanted to go through each step of every proof in detail before the lesson was taught.

In lesson #2, from the discussions in the planning stages, it was clear who felt comfortable and who did not feel comfortable with the topic of functions. By lesson #2, it appears that these teachers had developed a relationship within the group that they felt comfortable sharing this information with each other. Lisa clearly let the group know that she needed their help clarifying the function concepts before she taught the lesson. When Craig admitted not remembering concepts, the group knew he needed to be reminded of the substantive knowledge of the concept of functions. It appears that the teachers spent a lot of time during the planning meetings organizing their substantive knowledge on the

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function concepts. One such thing discussed in Episode 17 was the difference between a function and a relation.

> Lisa: Well, what is a function, maybe the difference between a function and a relation?
> Craig: Refresh my memory; it really has been a while since I taught this.
> Lisa: For every x there is at most one y.
> Melissa: In a function.
> Alex: Another words, they would need to know a precise definition is what you would be looking for?
> Lisa: Yea, maybe not be able to just spit that back out at me, but to understand what that means.
> Melissa: Being able to give an example of something that would be considered a function or not.
> Craig: So the absolute value function is or is not a relation?
> Melissa: Yes, all relations are functions. A circle is not a function or a parabola on its side is not a function.
> Later,
> Craig: Pattern recognition is also prior knowledge that the students need.
> Alex: A function is really a relation with a pattern. Right?
> Craig: Sure! I'm still sketchy on the word relation.
> Alex: A relation is just a set of ordered pairs. All it is.

Here the teachers review the definitions of a function and a relation. It appears that as they considered what they wanted the students to know from the lesson, they discussed their own knowledge of the concepts. The teachers also review the different ways of representing a function in the dialogue that follows from Episode 18.

> Alex: ...Maybe you want to stress there are three ways to express the difference.
> Melissa: Different representations of a function?
> Alex: One way is verbal and one way is numerical in set notation, that kind of way. And the third way is typically graphing. What you want to do is express verbally the connections, you want to express numerically the connections, or you want to express graphically the connections so they can physically see the connection of the x and y.
> Melissa: You could actually have four. Usually the equation is called algebraically and numerically could just be in a table.

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> Mike: I remember teaching that there was actually five. I think an arrow diagram.
> Melissa: Yea, the actual diagram of the mapping.
> Mike: I remember we had to teach the kids five ways.

Evidently, as the teachers thought individually about the function concepts, they shared this substantive knowledge with each other. Once they shared the details of the mathematics content, then they decided what was best to include in the lesson.

During the planning meetings in lesson #3, it seems like the teachers looked more at the mathematics content when they were examining the videos. As written out in Episode 36, the teachers were trying to see if the examples presented in the videos contained the appropriate mathematics content for the lesson to be taught. Craig pointed out that the women in the video uses y=ax+b instead of what is in our textbook y = mx+b and that she doesn't use a consistent process to solve each problem. Melissa added that she doesn't use the word slope. Apparently, they were comparing the substantive knowledge on linear equations that they had in their minds with the way it was presented in the video. The teachers pointed out the differences and discussed how this would have an effect on the way they would have to present the material to the students.

Pedagogical Content Knowledge

In the planning meetings for lesson #1, the teachers were examining their pedagogical content knowledge as they strive to have students make connections to prior knowledge, to provide meaning for each concept, and to avoid student memorization of procedures. As they look at the mathematics

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content involved in the lesson, they must also use their pedagogical content knowledge to help them set up the appropriate opportunities in the lesson to make the things mentioned above, as well as their lesson study goals, happen. In the following dialogue from Episode 4, the teachers discuss why proofs are important for the students to learn.

> Craig: There are two things that I have read recently that help me drive home the point of proof. One thing that I have read recently is that at the higher level of mathematics anything that has been proven is considered trivial.
> Alex: Yes, exactly!
> Craig: So it is not the past knowledge. It's where do I go. How I can go further. And that is what I try to teach the kids. This is one of the things that we have mentioned here. Making those connections on their own, Once you have learned how to prove something you learned how to make connections to advance yourself further and further. So you are looking at all this other stuff you already know going I already know this is true why can I show these next few things are true. Once you've shown it that immediately goes into the used pile…. Even just what we did two weeks ago is trivial now. We already did it; we know it let's use it to do something else. The second thing that I read which kind of contradicts that is that again at the higher level of mathematics the question has become is rigorous proof worthwhile. Is it become less about knowing something is true with absolute certainty and become more about can I convince you that I am right? …That was a very interesting article that a student brought in too. And I will talk about that in classes too. I'll say look some people don't believe in proof. They believe it has become more can I convince you that I am right. I spin it so that they understand why we do proofs – if someone asks you why you hate INSYNC, tell me why, convince me why they suck. Tell me why. And they can respond to that. I say good take that convincing and apply it to math.
> Alex; Craig I could see you writing a book in a couple years and it is called Mathematics by Convincing. O.K. class please write a convincer. Change the terminology, and you can change the whole thing.
> Craig: That is a lot of it they are scared by the word proof….It is a vocabulary issue as well. Why are proofs important? They help students explain the concept and that is the driving question to understand the concepts.

It appears that as they examine the concept of proof, the teachers applied their pedagogical content knowledge to see how using proofs in the classroom will help students make connections and to better explain the concepts.

In the planning meetings for lesson #2, the teachers considered why it is important for students to study the concept of function. The dialogue that follows is Craig's descriptions of why functions are important and Alex's comment on the topic from Episode 16:

> Craig: To understand how numbers cooperate. [After he laughs and a short pause]
> Well, what's the end result of functions? We are going to talk about a linear function. What is the end result of it? Why would you need it? Why do you want to use it? Where are they useful? What is their application value?
> Craig: Functions are used to model real-world situations. That is the driving force behind the functions course.
> Craig: So functions are used to model real-world situations. Why else are functions important? How can we tie that into one number goes in one number comes out? ... What makes f(x), independent variable, dependent variable? Or are they just going to have to realize that here is the basic procedure and we will move into the conceptual as we address linear functions, quadratic functions, and cubic functions?
> Craig: So that they can understand the relationship between numbers. I mean yes functions are used to model real-world situations, but they are not going to get that from one number goes in one number comes out. How are these two things related? How do you get from one thing to another?
> Alex: You create a function to make a prediction of something that cannot be measured.

As Craig gave possible explanations for the importance of functions, he also posed questions to help himself as well as the other teachers to think further about functions. Craig added these explanations of why functions are important to his pedagogical content knowledge, and the guiding questions helped Alex to

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make his comment. It seems like as the teachers made this clearer in their own minds, they were considering how to best present it to the students.

In the first planning meeting of lesson #3, the teachers develop a more specific type of pedagogical content knowledge- curriculum knowledge. The teachers discussed the specific content within the curriculum for Alex's course, but also discussed issues such as repetition versus depth, and math graduation requirements. In the following dialogue from Episode 38 the teachers discuss how the curriculum within the course is disjointed and all the courses in the mathematics program that contain linear equations in their curriculum.

> Melissa: Unfortunately, this is so disjointed. It is not your fault it is the curriculum. You did similar triangles, transformations in the coordinate plane that is where the matrices are, now polynomials, and will come back to slope and systems of equations.
> Alex: ...to focus in on linear equations. Our rational was that would cut across all classes Algebra II, Algebra I, Integrated III, Integrated IV.
> Melissa: So it could be something you all could use later on.
> A few minutes later:
> Melissa: Just so you know where this fits into the whole scheme of things in Integrated V I started with systems of equations. We graphed a few lines and reviewed slope. We did all the different methods for solving a system of equations including determinants. Then we did systems of inequalities. So that is the first chapter in Integrated V
> Alex: And that is part of the curriculum for IMV...
> Melissa: So pretty much what is at the end of your Chapter 6, they do again in Integrated V.
> Alex: But you have a different book.
> Melissa: Yes, an algebra book, not an integrated math book.
> Lisa: A student in integrated series, how many times will they exposed to slope and their equations? Every single year?
> Melissa: At least Integrated III, IV, V
> Mike: Integrated II, we didn't get into that in I.

Then, the teachers discuss a broader view of curriculum when they give the number of courses needed for graduation at this school.

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> Lisa: By the time they get to that are they really sick of it? Or do they just forgot?
>
> Melissa: They don't remember it.
>
> Alex: ...There is less mental exertion into the topic....If IMV goes in the same manner that I discovered with IMIV. It was night and day between IMIII and IMIV. I am guessing you are going to have the same even jump again between IMIV and IMV.
>
> Melissa: You are going to have more of the students weeded out and are more serious because they don't have to take it. Even in IMIV they don't have to take it.
>
> Lisa: They need four math classes.
>
> Alex: Three, isn't it three.
>
> Lisa: They only need three. Has the school thought about requiring eight? That they take math all the time?

Evidently, the teachers add several levels of curriculum knowledge to their pedagogical content knowledge. It appears that lesson study provided the teachers with the opportunity to discuss curriculum issues ranging from the contents of one course to overall graduation requirements.

In the planning meetings for lesson #1,#2,and #3, the teachers spend a great deal of time choosing or developing example problems to be used in the lessons. In lesson #1, the examples are actually the proofs of the theorems. As the teachers looked closely at these proofs, it appears that they developed pedagogical content knowledge on how to react to student misconceptions or mistakes as well as refresh their substantive knowledge of the geometry involved. In addition, the teachers can clearly see the prior knowledge that the students will need to apply throughout the lesson. The dialogue that follows from Episode 7 includes some things the teachers discussed as they decided on an opening problem for the lesson:

> Melissa: Did anyone think of anything that we could do as an opener to kind of motivate the lesson even if we do it after the homework is checked? I was looking through the book that Craig actually uses.

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Melissa points out some application problems from the book.

Melissa: Would these motivate them to want to talk about these theorems?

Alex: Or on reverse side would it frustrate some, say I can't do that.

Craig: That's what is going to happen with this class. They'll look at it and go I have no idea. And then I will try to get them to think about it more and to come up with their own conjectures, or whatever.

Here, the teachers are concerned with motivating the students not frustrating them.

In lesson #2, the teachers developed real life examples of functions and non functions and spend a lot of time deciding on an experiment to use in the lesson. In the following dialogue from Episode 20, Mike and Lisa discuss how they want to develop a real-life example of a function and non-function.

Mike: When you where saying about giving an example do you mean giving a mathematical example or give an example of like each person theoretically has its own social security number? That kind of thing that would be like a real-life that would be a function.

Lisa: Yea, that would be good to, but also an example when it wouldn't be a function.

The teachers wanted to develop their own real-life examples in which the students would have to determine if they represent a function or not. They also wanted to use a real-life context in their experiment in which the students generated their own data that represented a function. The dialogue that follows is from Episode 21:

Lisa: This is what I am thinking. We are starting with five pieces of spaghetti, how many does that take to break? Twelve marbles? And then you change it. You put two spaghettis. It only takes three marbles. That is what I am thinking because you don't know how many marbles it is going to take so you are not plugging that in first.

Mike: That's right; usually you plug in an x and get a resulting y. She is saying is how many marbles will it take to break starting with six spaghettis.

Lisa: Spaghetti should be x and figure out how many y, how many marbles.

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Craig: The problem I have with that is in terms of thinking in terms of input and output. You know you are putting in marbles. That I see confusing.

At this time Lisa suggests using marbles and graduated cylinders filled with water. Melissa asks how this would be different than the spaghetti experiment.

Craig: The difference with the water is that for each one you add you see a difference. The problem with the spaghetti is that you don't have an immediate effect. You put one in and nothing happens. So, you have not evaluated your function. Alright, $f(1)$ currently has no value if we are doing marbles counting as x. It is only when you actually achieve that value that causes it to break that you have an input value.

As the teachers talked through two different possibilities for experiments they considered the aspects of each that might be confusing for the students.

Also, as described in Episode 22 for lesson #2, the teachers took the time to pick out individually each homework problem. The teachers needed to use their pedagogical content knowledge that they developed at prior planning meetings or from past experience to choose appropriate problems for the lesson. This is also done at first by Lisa, Melissa, and Alex in the planning meetings for lesson #3. In Episode 30, the teachers discussed which type of linear equation problems they wanted to include in the lesson.

Alex: We may only have time to show them one kind of problem.
Melissa: I'm not sure what you mean by kind of problem.
Alex: For example, one would involve two points. Another one would involve...they come up with slope. What if they are given a point and the y-intercept.
Lisa: So you are talking about given different information. Two points, a point and the slope, what else?
Melissa: Slope and y-intercept. Let's look at the application problems [in the book] and see how they are. I think most of them are two points.

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Then at the next planning meeting, Craig and Mike give their suggestions on how to improve the problems. In the dialogue below from Episode 31, Craig explains why he likes the cell phone problem:

> Craig: Just because this is far more meaningful....The other thing about the cell phone problem is that there is a lot more follow through. You don't just get one answer and your done there are several other things she's got in here. What does the y-intercept mean, what does the slope mean... taking those numbers from the equation and giving them more meaning, even more beyond the archeology which gives more meaning to the numbers and values but the y-intercept of that equation doesn't mean much. Here you are taking it one step further with that cell phone problem. I think it is an excellent problem.
> Melissa: Would we have them answer all of these questions all at one time?
> Alex: If one of the intentions was to model the problem would it be wise in the femur problem we've not modeled the discussion of the y-intercept. We've found the y-intercept strictly as an intermediary to find the height of the person. But, we have not discussed it.
> Craig: Right, you've done that twice.
> Melissa: You are kind of going a little bit further.
> Craig: You are certainly pushing them beyond what I think would be expected in Integrated IV. You are now pushing them to a higher standard. You are saying there are other parts of this problem that relate back to the algebra....I hate the word rigor, but you are making it a more academic problem in disguise.
> Alex: What do you think Mike?
> Mike: I think they are going to have trouble with it.
> Alex: That is my gut feeling.
> Craig: In a class of nine, you can really get into it.

It seems that by this third lesson, the teachers are very comfortable in this setting as they shared their opinions on the different types of problems and gave reasons for why they would and would not include particular examples.

During the planning meetings for lesson #1, #2, and #3, the teachers used their knowledge of mathematics to anticipate possible areas of misunderstanding by the students. In lesson #1, the teachers were talking specifically about when students do not understand homework problems. In the dialogue below from

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Episode 9, the teachers share how they handle the situation where the students have questions on the homework from the previous night:

> Craig: What I would do usually in that case is pull one of these ones from the corners of the book or grab a homework problem that they didn't do. Then maybe I would give them the critical thinking [problem]....Maybe I would grab a couple of these.
> Melissa: So you don't want to decide beforehand. It is usually a spur of the moment decision. Is that how you all do it?
> Craig: That is how I do it. It is pretty random.
> Mike: Sometimes I know. I know as we all know a lot of times they don't ask questions they just wait. So I will have a question or two ready. O.K. let's see what you know....It comes off the top [of my head] many times, but I try to have some things prepared.
> Alex: I think that is where mastery of the content comes in, you know teacher qualifications.
> Craig: I am such a terrible planner. For me that it is always off of the top of my head. O.K. they didn't get it, need a problem. I will either dig back in my head and find one or quickly grab the book. Hey what about this one.
> Melissa: You are familiar with the material.
> Alex: You can almost feel where the class is.
> Lisa: I don't usually think of a question ahead of time, but I'll try to anticipate what they might have trouble with. I will do examples and solve for and explore in my planning. I 'm not sure that I would pull out an extra problem. Sometimes I would try to pull out an extra problem. But otherwise I would try to answer their questions.

It appears that the teachers took time here to consider the pedagogical content knowledge needed in order to know when the students need to be given more information about a concept or more time to work on problems before moving on. In lesson #1, they also developed pedagogical content knowledge as they went through the proof that Craig had started in a different direction. They were prepared for the students to do the same thing. In lesson #2, the teachers discuss possible student misunderstanding of functional notation. It appears that the teachers who had taught functions before were drawing on past experiences (bundles of pedagogical content knowledge) of teaching the concept of functions

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to help Lisa anticipate where the students may have trouble. It is noted in Episode 23 that the students might think of f(x) notation as f times x. Then in Episode 24, the dialogue displays Lisa's explanation to the students to correct this misconception. In lesson #3, Lisa points out a possible point in the problem solving that may give the students trouble. The following dialogue from Episode 37 is the discussion which led the teachers to change the problem solving template:

> Lisa: Have they seen independent and dependent variables?
> Alex: I'm going to say no, not in any strong capacity.
> Craig: Not in that vocabulary.
> Lisa: So will you always have to tell them when they see a word problem like this which one is x and which one is y?
> Craig: Well, what I do when I am trying to indicate you substitute x to find y is that which one would you need to know first? And generally in the integrated when I taught it...we didn't use independent and dependent variable. Literally it was which one do you need to know first. Kind of like when you are defining a word problem you know one number is twice the other which one do you need to know first?
> Lisa: O.K.
> Alex: The way I would handle that would be what is it that we are trying to find? We want to know a person's height. That is unknown, that's what we are trying to find and that is going to be the y. That is going to be the dependent.

Evidently, as the teachers anticipated possible student misconceptions, they reviewed how they would explain the mathematics content to the students.

Teaching Stage

Mathematics Content Knowledge

The teaching stage of the lesson study process influenced the teachers' development of mathematics content knowledge. In lesson #1, it appears that

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the teachers observing the lesson saw how at ease Craig was using his

substantive knowledge involving quadrilaterals and parallelograms as well as his

syntactic knowledge of how the different geometric proofs connect. Below are

Mike's comments from Episode 6:

> Mike: The key thing that I noticed was that it flowed smoothly. You could
> tell that...as you were taking them through and walking them step-by-step
> through this process that they were recalling information that they had
> learned prior to that day....Some kids who were very vocal and they
> probably raise their hands a lot...and everybody has those kinds of kids.
> But even looking around at some of the other kids, they understood it too
> for the most part, and then they were looking at each other's stuff. I saw a
> little interplay between the kids during the lesson.

As the other teachers observed this lesson being taught, they witnessed this

substantive and syntactic knowledge once again. In lesson #2, they saw Lisa

use the substantive knowledge of the function concepts that the group members

helped her to develop during the planning meetings. In lesson #3, as Alex taught

the lesson and the other teachers observed, they were all examining how their

own substantive knowledge on linear functions, rather than a person from the

DVD's, was presented in such a way so that the students could easily

understand the concepts.

Pedagogical Content Knowledge

With each lesson focusing on a different instructional approach, the

teachers added various strategies to their pedagogical content knowledge. In

Episode 12 from lesson #1, Craig carried on an open dialogue with his students.

He used guiding questions with references to prior knowledge to move them from

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one theorem to another. He required the students to make connections from one geometry concept to another. The teachers who are observing the lesson saw how familiar Craig was with the substantive knowledge of the proofs in order to ask the students the appropriate guiding questions. It appears that the guiding questions are part of Craig's pedagogical content knowledge. In lesson #2, the teachers saw Lisa begin with more of a discovery approach as the students completed the experiment and then used all the data to present the notes on the function concepts. Evidently, they saw Lisa use the mathematics content knowledge and pedagogical content knowledge that she developed during the planning meetings. In addition, the teachers observed how the discovery approach may or may not work in the classroom and how a lesson can look very different on paper than the way it is carried out in the classroom. This is pointed out in my comments in Episode 26 from Lesson #2 and Alex's comments in Episode 35 from lesson #3.

> Melissa: I liked how you were able to use everything with the experiment. I was hoping for more discussion. I didn't think there would be quite as much notes with them writing everything down. I was thinking we would get more from them. Maybe they don't know enough. Unless we just have to try to get it from them.
> Craig: More leading type questions.
> Alex: After everyone left....I was thinking to myself....We had one lesson that anyone of us five teachers would have carried it out completely differently....The idea is communication as oppose to I have to do it exactly Craig's way or he has to do it my way. This group is so mature to see that. It is more a question of the ideas, how to communicate, and how to get the spirit in us so the kids can sense it and see it. What we do help us do that. Each of us is a professional. We care. We are dedicated to helping students...

In lesson #3, one of the main goals of the lesson was to make it motivating and entertaining to the students. The teachers observed Alex using pedagogical

techniques such as music and a storyline to keep student interest. In Episode

35, the teachers commented on how motivating the lesson was.

> Mike: I thought you did an excellent job. I was very entertained. You got the point across....Like Lisa said earlier today when I commented to her I said wasn't that great. She said "I was so motivated by that. I want to do something like it."
> Lisa: I went to my next class and I was like I got to do something fun. It was great.
> Melissa: Will you consider using music more in your class? They seem to like it.
> Alex: They did. I could have theme music....You play the music when you want them to solve a problem...

These are techniques that were not explicitly discussed in the planning

meetings, but it appears that the teachers have added them to their pedagogical

content knowledge.

As the teachers observed the teaching of the lessons, they took note of

how the teacher dealt with student misconceptions. In the planning meetings,

the teachers anticipated possible student misconceptions. They added how to

deal with them to their pedagogical content knowledge. When the teachers were

observing the lesson being taught, they saw how the pedagogical content

knowledge that they developed was actually carried out in the classroom. This is

described in Episode 2 from lesson #1 when Craig allowed the students to begin

a proof in several different directions, like the teachers did in their planning

meeting. It appears that Craig applied his pedagogical content knowledge by

asking guiding questions to redirect their misconceptions. The preparation by the

lesson study group helped equip him with the necessary pedagogical content

knowledge to work through this classroom situation in a successful manner. As

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described in Episode 24 from the teaching of lesson #2, it appears that Lisa used her pedagogical content knowledge to help redirect student misconceptions about functional notation. Once again, since this possible misconception was brought up in one of the planning meetings, it appears Lisa was prepared with the pedagogical content knowledge to deal with the situation in the classroom.

The teachers observing the lessons also took note of how the teachers were assessing student understanding throughout the lesson. This required the teachers who were teaching the lesson to use their mathematics content knowledge and pedagogical content knowledge. In order to answer student questions they needed to know the content and carefully consider the pedagogical technique they used. In all three lessons, Craig, Lisa, and Alex assessed student understanding as they walked around to help students with problems. In lesson #1 (Episode 13), Craig needed a clear understanding of the proofs in his mind in order to pick up on students' mistakes. In lesson #2 (Episode 26), Lisa's use of time at the end of class for the students to practice problems allowed the observers and Lisa to see how much of the function concepts the students understood. In lesson #3, a student told Alex how she sees the connection between their homework last night and the problems they are currently working on. In the dialogue from Episode 32 below, Cindy indicates that she was doing her homework incorrectly, but she now knows what she was doing wrong:

> Cindy: I don't think I did it correctly on the homework.
> Alex: Well, do you see it now? The homework kind of warmed you up to it. Now, the fact that you can say that is great.

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> Cindy: You know how you have to pick the x and the y? Where y was, where y(1) would stand for, I think I put any number.
>
> Alex: O.K. Is it a little clearer now? That was the idea for you to practice a little bit. Notice I didn't even go over them. Just the fact that you looked at them, you tried to punch out some numbers, you are a lot better equipped to do it today, then if you did nothing.
>
> Cindy: Yes!

Here, it appears that the observers saw that even unsolicited assessments of student understanding are important things to add to their pedagogical content knowledge.

Debriefing Stage

Mathematics Content Knowledge

In the debriefing stage, the teachers have the opportunity to reflect on the teaching of the lesson. This requires the teachers to think once again about the mathematics content knowledge they initially reviewed during the planning stage and clarified during the teaching stage. In lesson #1, Alex specifically comments on Craig's ability to answer students' questions as they were trying the proofs on their own. Apparently, Alex was impressed with Craig's substantive and syntactic knowledge and his ability to use it quickly to see that the student had a question and answer it. This is described in the dialogue from Episode 14 below:

> Alex: I was impressed... a student might have had three or four steps written and how you were able to almost like a game of chess instantly go to a step (I wouldn't have a clue) now this is reversed. I thought it was very technical the comment that you made. Wow, for the student to be there so quickly and then for you to be able to diagnosis so quickly.
>
> Melissa: Do you think that comes with experience?

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> Craig: (Agreeing with a nod) Knowing where you are headed, knowing what they should have on paper.

It seems like Craig had this knowledge clear in his mind in order to react to students' questions so well.

Pedagogical Content Knowledge

In the debriefing sessions one aspect of their pedagogical content knowledge that is discussed is using guiding questions to make prior knowledge connections. The teachers had spent time in their planning sessions thinking about the prior knowledge connections that the students would need to make in the lessons. Apparently, they made this part of their pedagogical content knowledge. During the debriefing session for lesson #1 (Episode 14), the teachers commented on how smoothly the lesson went because of this preparation on their part, because Craig stressed the importance of applying the old concepts with the new concepts on a regular basis with his class. In lesson #2 (Episode 26), the teachers commented on getting more information from the students during the lesson. This would require the use of more guiding questions. In the teaching of this lesson, the teachers did not see the lesson evolve as smoothly as in lesson #1. After their discussion in the debriefing session, they saw that they could make the lesson more effective with guiding questions to help students make connections from one concept to another. Evidently, this is what the teachers added to their pedagogical content knowledge.

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Another aspect of the teachers' pedagogical content knowledge that was discussed in the debriefing sessions is assessing student understanding. These teachers have discussed this topic in planning meetings, observed this topic carried out in the classroom, and reflected on it in the debriefing sessions. It appears that, assessing student understanding throughout the lesson has been a part of the teachers' pedagogical content knowledge throughout the lesson study process. In Episode 24 from lesson #2, Lisa discovered particular information about one student who offers much to the lesson.

> <u>Lisa</u>: That kid Brian, the one that kept raising his hand who made the class move along. He was putting together things that surprised me. . . . . I didn't expect that necessarily from the class because I didn't know what background they had and I also didn't expect that from him.
> <u>Melissa</u>: So he doesn't normally do that?
> <u>Lisa</u>: Well we have been doing more factoring and things like that. He gets bogged down with things that look difficult and with procedures, but he can apparently put together and apply concepts really nicely.
> <u>Alex</u>: Was he the one that said you mean y?
> <u>Melissa</u>: He said you mean y equals f of x.
> <u>Alex</u>: It takes a student like that. For me it varies between maybe four students who can help drive the lesson. One student will drive the class more than another one. It kind of oscillates. If you can have those four, life is good with kids that can help drive the lesson.
> <u>Lisa</u>: And actually one of them who also does that. He was one of the ones that weren't on camera today. He was out. And the few others who I think generally do that were nervous because of the camera; but also, I think they are more procedural students. They like to be told what to do. They want to think independently, but it is hard for them. They are just very good at being good students.

She learned that this student makes connections among concepts well. Up to this point, she has seen him getting bogged down in the procedures of math problems. It appears that this incident reminded the teachers to add the need to

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assess student understanding throughout the lesson to their pedagogical content knowledge.

In each of the debriefing sessions, the teachers also commented on how the lesson accomplished the group's lesson study goals. The reflecting the teachers did on how to incorporate motivation, making the lessons meaningful, and emphasizing the understanding of concepts as well as procedures into their lessons aided them in developing pedagogical content knowledge. Their discussions throughout the lesson study stages continuously went back to the mathematics content and how the topics would be presented to the students in order to establish ways to incorporate these goals.

The information presented in this chapter illuminates the changes that took place across the three lessons within the planning, teaching, and debriefing stages of lesson study. In all three stages across all three lessons there is evidence of teacher knowledge growth. The teachers discussed the substantive knowledge of geometry and functions. Also, the teachers enriched their pedagogical content knowledge in the form of choosing example problems, anticipating possible areas of misunderstanding, developing curricular knowledge, trying different instructional approaches, dealing with student misconceptions, and assessing student understanding during the lesson. ·

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CHAPTER 7

TEACHER BY TEACHER RESULTS

In this chapter, my goal was to use the initial and final interviews, journal entries, and classroom observation data with the data from the three lessons to report how the participants in the study develop as teachers while working together in a lesson study group. The analysis of this data was done in order to report on each teacher in the study in terms of answers to interview questions before and after participating in lesson study, their own private reflections on the lesson study process, and observations from their own classrooms. Particular attention is placed on the how the teachers plan and reflect on their practice since planning and reflection are major components of lesson study.

Mike

Mike wants his students to be very exact and detailed. When giving students notes on new concepts, he writes down step-by step procedures that he wants the students to follow. He doesn't ask the students many questions in order to get information from them. Since he wants the students to do the

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problems exactly the way he explains them, he doesn't give the students the freedom to come up with alternative ways to solve problems. For example, in one lesson observed on evaluating expressions with exponents, he didn't simplify the expressions first. One problem was evaluate $(-b)^4 (a^3)(ba)$ when $a = -3$ and $b = -2$. He substituted the numbers in right away. Not one student asked about this. This displays to me that Mike does not encourage his students to consider different ways of solving problems beyond what he shows them. In another lesson on solving multi-step linear inequalities, Mike was very specific about how he wanted $x > -1$ drawn out on a number line. He wanted to see all of their work, and they would get points taken off if their answer was different from how he showed them to do it. Also, in this lesson he had all the steps written out on a transparency for some word problems and when he went over them he did it so quickly that the students do not even have time to read through the problem.

Mike didn't fully explain the material. He wrote definitions and examples on the board, but didn't show how one concept follows another or why the procedure works. For notes on solving compound inequalities, he wrote out definitions and examples on the board and got nothing from the students. When getting a solution $0 < x \le 4$ he said "this means $x > 0$ and $x \le 4$ and the solution is when they overlap," but he didn't show this. Mike didn't illustrate this by drawing a number line to display where the solution to each inequality overlaps. He gave them a few example problems and expected them to be able to do the class work problems on their own.

Before working with the lesson study group, Mike stated that in planning his lessons, he writes out detailed notes and has all example problems worked out. In his final interview, he added that he looks at the curriculum guide and textbook to find homework and example problems to use in each lesson. When observing Mike's lessons, there was evidence of planning since he had the notes all written out the way he wanted the students to do the problems and he developed or found worksheets that contain practice problems for the students to complete. This would require Mike to use his pedagogical content knowledge to find or develop the appropriate practice problems. He also develops curricular knowledge as he uses the curriculum guide for his classes when planning. He used his pedagogical content knowledge as he answered questions that the students asked as they were working on the class work problems.

In his final interview, Mike said that "math teachers need to be accurate, precise, detailed, organized, and most of all what I've gotten from the lesson study is creativity. And I have to say I'm not most creative...I can take something that someone else has planned and do a great job with it, but sitting down and coming up with an activity or a game is not my strongest point." He added that it was nice to be able to take the time to talk with the other teachers in the lesson study group and come up with activities and problems together. Mike also thinks that math teachers should be able to "think on the fly". He said, "I am pretty good at it, if a student might ask me what happens in this case or what if the equation looked like that. I think it comes from my math ability and it just comes natural to

me… When a kid asks me a question like that on the fly I am pretty good at coming up with an answer and a good example."

Mike was excited about the lesson study group's goal motivation. Throughout his journal entries, he commented on the amount of time it takes to plan lessons that are interesting enough to spark student interest. After the planning and debriefing meetings Mike was critical about what could improve the lesson. It appears that Mike used his pedagogical content knowledge to critic the progress of the planning of the lesson as well as the lesson after it was taught. For example after the planning meetings for lesson #1, he discussed how the lesson needed an opening activity or a real-life application. He thought the lesson needed some kind of "hook" into the proofs used to show that a quadrilateral is a parallelogram. He wrote,

> This is where the teacher's ability to be interesting (entertaining) comes in handy. Along with being interesting, if I were doing the lesson, I would try to come up with some kind of real-life application…a table that folds out having leg supports that form parallelograms, some kind of bridge application, or even angles that are used in billiards. Yes, I realize that this is all pie in the sky, because it is easier said than done.

After the debriefing meeting for lesson #2, Mike questioned the use of manipulatives. He wrote,

> Of course I already know how valuable it can be to use manipulatives to enhance a lesson. However, there are problems that can arise, and in my opinion these problems are twofold. First of all, it takes a lot of planning time and effort. That is not to say that it isn't worth it. Where do we find the time to do it on a regular basis? Not all classes are disciplined enough such that a teacher can do what Lisa did on this day. . . . All you need is a few unwilling students in a classroom situation, and they can ruin it for everyone. Nevertheless, I will try to do more hands-on 'fun' type work. It can break up the monotony for the students and for me too!

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Evidently, Mike reflected on his pedagogical content knowledge when he pointed out that in the teaching of lesson #2 Lisa's explanations of the zip code and residences examples were unclear. He also pointed out that Lisa did a good job redirecting student misconceptions on the f(x) notation. A student wanted to know if f(2) meant f times 2. Mike wrote, "Lisa took the time to explain the meaning of the notation and succeeded in getting the correct point across."

Mike reflected on what aspects of the lessons he would apply to his own classroom. He explained that he would not use the function machine analogy from lesson #2 when he wrote, "Lisa also introduced them to a drawing of a function machine. It went well, but to me it seemed a little hockey and I don't know if I would ever use it in my own classroom." After planning meeting #4 for lesson #3, Mike commented on problems that Lisa put together. He wrote,

> Lisa brought in a packet of Lesson Study Problems that she put together. I was really impressed by the work that she did. She had several real-life application problems that I think many kids can relate to. As a matter of fact, I am going to use some of these problems when I teach equation writing to my students in Algebra I. She also had one page that was a shell for writing and solving equations. At first we thought that it may have provided a little too [much] information for the students with respect to what one needs to write and solve a word problem. However, on second thought it was just right because word problems have been a source of difficulty for students of any ability level. This shell is just what the students in Integrated Math 4 need, and it's great to have a packet like the one Lisa put together to use with my own students when the time comes.

The ideas for the problems in the packet were developed by Alex, Lisa, and Melissa at the last planning meeting, but Lisa revised them and developed the template. This template is part of Alex's lesson plan for lesson #3 and can be found in the appendix. In his reflection after lesson #3 Mike wrote that Alex, "put

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together a lesson that was so engaging and motivating that it got me inspired to do something in the near future".

Mike also reflected on the stages of lesson study in terms of its quality as a professional development experience. After the first cycle through lesson study, Mike stated in his journal,

> Upon the completion of our Lesson Study activities for semester one, I felt a sense of accomplishment with what we had done as a team. The whole process took extra time, but the knowledge that I gained was worth the extra effort. To actually see our cohort Craig teach the lesson that we had designed was a rewarding and educational experience, not just for the students, but for us teachers also. As I reflect on the experience of planning, rethinking, revising, observing, and reflecting on the lesson, I realize and understand that the students did grasp the concept of geometric proof in a much more effective manner than if they had just been told to read the textbook or refer to a handout with examples to use as a guide. In my opinion, Lesson Study is the pathway to improved instruction. I think that if I were to use Lesson Study on a consistent basis, I would not only increase my own knowledge of my subject matter, but also my knowledge of instruction.

In his final interview, Mike said that he doesn't believe lesson study changed his understanding of mathematics, but it has helped him to appreciate the importance of organizing a well thought out lesson plan.

Mike may not have gotten as much out of the lesson study experience as the other teachers in the group did. He was the only teacher not to teach a lesson in front of the group. He may have benefited more if he would have been able to get feedback during the debriefing session on a lesson that he taught. Even during planning a lesson together with the group for his class, he may have been able to see how much information he tells his students rather than guiding them through the concepts. He also might have been able to see how he gives his students very little freedom to think about mathematics on their own. Even

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though he did not get to teach a lesson in front of the group, he wrote in his journal that "he picks up something new every time the group meets and likes to hear another teacher's take on all kinds of educational issues".

Craig

Craig does not write formal lesson plans. In his initial interview he explained how he looks at the book to remind him of material, but that he only writes down a general description of what he will do in his plan book. It appears that Craig's detailed lesson plan is the textbook. The first time he teaches a course, he becomes familiar with the textbook. Then, when he teaches the course again he already knows the mathematics content that is in the textbook. If one textbook is not as useful as another textbook, then he may have to do more planning. This may be the case for the introductory integrated mathematics courses. Recall that he stated in his journal entry that he has difficulty planning lessons that motivate and involve these students. It may be that he cannot take the mathematics content from these textbooks and teach from it.

As I observed several of Craig's classes, it was evident that he did not take much time before class to prepare for the lesson. For example, during one class he made up two example problems in which their final answers did not make sense for the problem. The students in the class became quite frustrated by this. Craig could have taken time before class to plan out the example

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problems or even taken time during the class to find a problem in the book. I think his lack of preparation ahead of time left him without the necessary pedagogical content knowledge for the situation. After teaching lesson #1, Craig shared in his journal that since he is not used to having a formal lesson plan when he teaches that while he was teaching the lesson, he was afraid that he was deviating from the plan. Once again, this shows his lack of preparation for the lesson. Even after developing a formal plan with his colleagues, he did not use it. I saw him review the lesson plan quickly right before he started to teach it.

In his final interview, he added that when planning his lessons he considers how to make the material interesting, tries to find the application value, looks at prior math skills and concepts needed, and looks at the curriculum as a whole. I think these additions are a result of his participation in lesson study. I base this on Craig's comment in his final interview that, "Lesson study did not change my understanding of mathematics, but it did in the applications and connections between math courses." Craig added, "Seeing what's being taught in courses I've never personally taught gave me a better understanding of what students should know coming into the next course. It's one thing to read our curriculum binders and see the topics and skills covered, but entirely another to see how the material is presented." Here, Craig was referring to ways lesson study added to his curriculum knowledge. He also added during the final interview that math teachers not only need to know math, but they also need to be able to see the whole mathematics pathway in order to make connections from one mathematics course to the next mathematics course. Here, Craig is

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referring to the syntactic as well as the substantive knowledge that makes up mathematics content knowledge. These are drastic additions to his initial interview response that math teachers need to know math, have a sense of humor, English skills like grammar and spelling, and classroom management.

In Craig's geometry classes that I observed, he presented a lot of mathematics content each day without referring to any notes. This shows that Craig is very comfortable with the mathematics content in the geometry curriculum. This can be contributed to his substantive and syntactic knowledge of the mathematics content. The conversational dialogue which includes good guiding questions that he carried on with the class allows him to get a lot of information from the students. He had the students draw on their prior knowledge in order to make connections to the new concepts. It appears that he used his pedagogical content knowledge to develop the good guiding questions that will require students to make prior knowledge connections. H It seems like his experience teaching geometry has helped him to develop this pedagogical content knowledge. By the time Craig got through all the geometry vocabulary and theorems, he only had time to give the students one or two example problems where they will apply the theorems. Then, the next day, he spent at least half of the class going over questions on the previous night's homework problems. This may be avoided if he taught less content and gave the students more time to practice applying the theorems in class.

Craig commented frequently in his journal reflections from lesson #1 that he was not used to being a leader. He felt that the other group members were

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looking up to him because he had the most experience at the school. He described this as being "out of his element". It was good for Craig to experience this because it made him reflect more on his current practice and how he shared it with his colleagues. For example, he made it clear to the group that he prefers "chalk and talk" and more theoretical approaches – "appreciating math for its beauty". In his final interview, Craig made the following comments about lesson #2: "Seeing the direct science application of functions was great and it gave the students two things – hands on learning and cross-curricular connections." In his journal entry after planning meeting #1 for lesson #2, Craig wrote, "In terms of professional development, this was very enjoyable because it's directly applicable to what goes on in our classroom everyday. Little details we discuss can be employed right away or if the moment passes, be stored for a future semester. Having never taught Algebra I, if the marbles work, I'd like to try it in my class." Thus, Craig's reflections indicate that he was thinking carefully about his experiences with the lesson study group and how he could apply what he was learning to his classroom.

After lesson #3, he reflected on what the students experienced when he stated, "They also received hands-on learning, but instead of cross-curricular applications, it was more of how mathematics impacts their daily lives". As a result of these lessons, he explained during his final interview the following on how lesson study has impacted his own classroom:

> Lesson Study has allowed me to make more of an effort to use more applications in my classes. I have tried to use more specific real world situations to reinforce skills students should know. As an example, I used the Cell Phone problem with my sophomores as a review for state

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testing. I'm also trying to stress what actual math skills are being applied as we learn and practice new topics. Then the students can see that all of their prior knowledge (or perhaps things they've forgotten) do actually have a use! That information needs to be recalled and applied to new situations.

Evidently, Craig enriched his pedagogical content knowledge to include real world application problems and to make connections to prior knowledge.

In one of his final journal entries, Craig indicated that as a result of lesson study he has incorporated the following things into his classes: more thorough explanations than before, more individualized attention, and expanding connections to prior knowledge. These are all characteristics of instruction that I believe fall under Craig's pedagogical content knowledge. These may not be brand new additions to his instruction, but they are aspects of it that he has refined due to his work with the lesson study group. In order to accomplish such things in his classroom, he may have to "plan" more. This may not get him to write down formal lesson plans, but it will make him reflect more on each lesson before he teaches it. This may be something that he did not do prior to his work with the lesson study group because he had all the experience teaching geometry. One of the things that Craig also saw from working with the lesson study group is that "revising lessons is a continuous process that never ends, and in order to stay fresh as a teacher you must continue to try to improve on what you already can do well." In the lesson study group, his colleagues took him "out of his element" and helped provide him with the opportunities to make changes in his current practices.

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Alex

Alex does not write out formal lesson plans. He begins to plan his lessons by becoming familiar with the textbook and curriculum. In his initial interview he talked about finding the "golden nugget" within the concepts and giving students lots of examples based on this key idea. In the initial interview he said that he doesn't write down these examples, but he may copy a reliable student's notes to keep for himself. But, during the final interview he indicated that he does write down the example problems and thinks about how they will be presented. The time that was spent on planning during the lesson study experience has caused Alex to see how valuable it is to have a well-thought out lesson. The lesson study group spent a lot of time choosing example problems as well as appropriate homework problems for the lessons, and it appears that Alex has added this to his pedagogical content knowledge base.

Throughout the lesson study experience, Alex commented on his enjoyment of the professional dialogue that he shared with his colleagues. In his initial interview, he said that math teachers need to know and enjoy math, manage their students, and be willing to share with colleagues. Alex fit in well with the dynamics of the lesson study group because he saw the importance of sharing ideas with his colleagues. In his final interview he adds that math teachers need to be sincere, have empathy, a desire to change what they are currently doing, a passion to teach, be extremely comfortable with the mathematics, and to think well on their feet. Throughout the lesson study experience, Alex had the desire to change what he was currently doing. He took

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careful consideration of what his colleagues shared with him and reflected on how he could make adjustments to his current pedagogy. For example in one journal entry Alex wrote, "I am still pondering the overall idea of what is the best way to teach and learn? On one hand, I'm dealing with the practical issues of day-to-day. And on the other hand, knowing that if I worked smarter that the students would probably learn more. I wonder what the ideal lesson would look like." In addition, he explained in the final interview that a teacher's ability to think well on their feet comes from experience.

The few times that Alex was observed teaching the students new material, he gave the students two or three example problems, then turned them loose to try similar problems on their own. There was not a lot of explanation on his part or many guiding questions to get the students involved. It is not clear how much pedagogical content knowledge he used, but there is some evidence of planning since he has examples prepared ahead of time. For example, when he was teaching the students how to solve systems of equations using determinants, he reminded the students of the linear combination method to solve systems, wrote out two example problems on the board, and then gave them one to try on their own. After about five minutes he went over the one that they tried at the board. Besides explaining to the students what a determinant is, he didn't give them any other explanation, just the procedure to find the solution of the system using the different determinants. On this day, Alex was only teaching the students the procedure for solving a system of equations using determinants. If a student doesn't recall this procedure, then he doesn't have the derivation of the

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procedure or the conceptual understanding to still solve the problem using determinants.

At the beginning of each class, Alex either had the agenda written on the board or he said it aloud. Therefore, Alex was organized in what he wanted to get accomplished each day. This does not provide evidence of detailed planning, but it does provide me with evidence that the format of the class was planned out. He gave the students candy as a motivator for completing problems correctly. After completing a series a problems, the pair of students with the most correct received the candy. He used standardized tests from the textbook series as a form of assessment in his classes. These are multiple choice questions that he hoped would help students prepare for the SAT and state testing as well as for upcoming quizzes. To go over homework and to practice problems, he had students go up to the board in pairs. They wrote the solution to the problem and explained the steps they used to the class. These are ways that Alex was assessing students understanding throughout the class.

Alex had a lot of educational issues and concerns that he is trying to sort through in his journal reflections from lesson #1 and #2. First, after the 1st and 2nd planning meeting for lesson #1, he voices the concern to look at state standards to see where the lesson the group is planning fits into the bigger curriculum. He does not bring this concern back to the lesson study group; therefore I am not aware if he examined the state standards on his own. One of his questions involving the standards was to what level proofs should be mastered. Next, as the lesson study group began to use the *Lesson Study Tool*

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for Planning and Describing Study Lessons, Alex reflected that he wanted to do the same kind of analysis when planning his own lessons. He added that he likes the format of the planning tool, discussing it with the group, and would like to use it to improve the learning in his algebra class. Even in the very beginning of the lesson study experience, Alex looked at not only how the sharing of ideas with colleagues, but also the tools used within the group could be valuable to the development of the lessons in his own classroom.

After the teaching and debriefing of lesson #1, Alex felt there was too much geometry content presented in one block. He wrote, "I thought that the content presented seemed appropriate, albeit fast-paced, as planned. Personally, as a student, I would have liked more time to think about the essence of what was going on, time to pull it together more, and reflect a bit." As a teacher, he felt that there should have been more assessment opportunities during the same class period instead of presenting one theorem after another. He wrote, "After the observation, I began to think that the authentic assessment piece should be developed more. The instruction went as planned, but I think it would be important to provide additional assessment opportunities during the same class period." One assessment idea that he gives is the KWL idea – What do I already know? What do I want to know? and What did I actually learn? Here the lesson study experience has given Alex the opportunity to do a great job reflecting on the experience as both a teacher and student in the class.

Planning meeting #1 for lesson #2 gave Alex the opportunity to think about his college preparatory Algebra I class. As he heard Lisa discuss what she does

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with her Honors Algebra II students, he compared this to the expectations he has set for his algebra students. In his journal, he wrote about assessments and how important it is to clearly state to students exactly what it is that they are expected to know and exactly how they are to demonstrate what they have learned. In terms of the functions lesson that was planned, Alex liked the zip code examples and enjoyed the discussion the group had on the spaghetti and marbles experiment. He stated that he will use the spaghetti and marbles experiment with his own Algebra II class in order to "clarify, reintroduce, and strengthen their perception of linear functions". In his final interview, Alex clarified this comment and said that he would more likely do the spaghetti and marble experiment as a demonstration with the class rather than have them all do it individually. Thus, from lesson #2, Alex reflects on general educational concerns like assessment as well as specific aspects of the lesson that he can add to his pedagogical content knowledge for his own classroom use.

As a result of his lesson study experience, Alex can see more clearly that mathematics can be taught in a variety of effective ways. He has started to incorporate prior knowledge into his lessons. This causes him to slow down and reflect on the concepts and has increased accountability among the students. Also, Alex began "to emphasize patterns more in his classes such as the general shape of a parabola . . . for example that all parabolas have an axis of symmetry and that once you know the axis then the parabola's vertex must lie somewhere on it . . knowing this recognizable pattern creates more of a desire to want to now know the y-coordinate of the vertex so as to graph it exactly." In teaching lesson

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#3 to his students, Alex enjoyed using the computer to bring in the music to the lesson. He found the template used for finding the linear equation was very helpful to the students because it unified the procedure for the students. As the group began to plan the lesson for Alex's class, he wanted to use a DVD or video to present the information to the students. He felt that there had to be a master instructor out there that could do it the best way. One of the main things that Alex learned from planning lesson #3 with his colleagues was that with the collaboration of the group members they could produce a good lesson for his class. He said, "I mentioned a subsequent math department meeting that I would not have been able to create the lesson by myself. The synergy of the five group members allowed me to put forth the time, energy, and quality into this single lesson." Alex expands on this when he writes "I feel more capable and appreciative of the value of quality of instruction that can take place in a classroom. It was amazing how each teacher can take the same content and present it so differently, as is best and most appropriate for them".

Lisa

Lisa writes out formal lesson plans. In her initial interview she explained how she keeps a goal in mind and then thinks of activities that will help the students discover the concepts for themselves. She added that she looks through a textbook when planning, but prefers not to use a textbook in her teaching. In her final interview Lisa added that she works out some problems,

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looks for possible areas of confusion, considers the goal of the lesson, and asks what the students could do on their own and what prior knowledge they could use to discover the concepts on their own. Lisa's additions of examining possible student misconceptions and considering prior knowledge is a result of the work with the lesson study group. As the group developed their lessons, they spent time pointing out areas where the students may have trouble and also discussed the prior knowledge that the students would need in order to be successful with the lesson. Evidently, she added these instructional aids to her pedagogical content knowledge. In her journal entries, Lisa commented that the planning process is "more detailed than she expected, but that the group forces her to think about how she plans and what things are helpful that she might not be practicing effectively."

When presenting new content in the classes that I observed, Lisa gave the students a worksheet with guiding questions that led them to discover the notes on their own. There was a lot of math content taught each day. Not all formulas are derived due to the amount of content in the curriculum that must be taught. The way the topics are ordered it is difficult to show connections from one topic to another. This is a problem in the way the curriculum is written up. Perhaps if Lisa had more experience teaching algebra, she would notice this and change the order of the topics on her own. It is very difficult for the students to make connections from one concept to another and to see why it is important to study one concept before another. Also, since she chose not to use the textbook, she gave them problem sets that she developed for homework. This required her to

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go over the answers to each problem the next day, which took up class time. If the students were given odd problems that had the answer in the back of the book, then she could have them check their own answers and only go over the ones that they have questions on. This may give her more time to derive formulas and make sure the students are seeing how the concepts are developed. Based on her journal entry after the goal-setting meeting, Lisa sees the importance of students understanding the concepts conceptually and not just procedurally. She stated, "I think that if a student understands something conceptually, then he or she can 'derive' or reach an answer without having memorized every step and procedure". Perhaps the demands of being a first year teacher and getting through all the material in the curriculum have limited her from carrying out this philosophy each day. In her classroom, the students asked a lot of questions such as why some procedure works and why another one doesn't work. Lisa used her pedagogical content knowledge to answer their questions. She offered another example to illustrate why what they are saying did or didn't work. There is evidence of a large amount of planning that Lisa does. It appears that she uses her mathematics content knowledge to develop the discovery activities that she plans for the students. This mathematics content knowledge must be translated into pedagogical content knowledge as she develops good guiding questions that will allow the students to work through the content on their own without getting frustrated. It also appears that she uses pedagogical content knowledge to develop the problem sets that she assigns to the students for homework.

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In Lisa's initial interview she stated that math teachers need to know math, the history of math, and to be excited about learning. She added that she thinks they need patience and understanding, and they need to model the way they want their students to do things. Now, in her final interview Lisa included the following as what teachers should know: not only the mathematics, but also they need to know how the math fits together and have their own organization of the concepts in their minds; must understand how students learn mathematics; and how to explain things in many different ways. From observing Lisa's classes and her participation in the lesson study group, I can see why she added the characteristics in her final interview. Since this was her first year of teaching, she is seeing in the classroom how important it is for her to have the topics organized in her mind and how important it is to explain things in different ways. But she has also gained this from working with her colleagues as a lesson study group planning, teaching/observing, and debriefing lessons. In fact, Lisa commented in one of her journals that "the collaboration component of the lesson study professional development experience makes me feel less isolated in my individual concerns and classroom issues."

In her journal reflections, Lisa took a lot of time to reflect on her own classroom and to apply what she has learned from her colleagues. After planning meeting #1 for lesson #1, she worked on getting all of her students more involved in the lesson by calling on them periodically. Craig indicated in the planning meeting that if he could keep the students engaged for even just a moment or two here and there he is happy. But, Lisa would like to keep her

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students engaged longer. This led her to ask "Is there a way to make students more accountable for their independent time?,or How can students work in groups more effectively?" Here Lisa took a comment from Craig and tried it in her classroom, but also reflected on issues surrounding it. Lisa is not one to sit back and accept what others say will work, for she will try it out and continue to reflect upon it to make improvements that fits with her pedagogy and teaching philosophy. In her final interview, she stated in regards to the teaching of lesson #1 "It was helpful for me to see how Craig reinforces concepts by repeating them in class or asking the students guiding questions throughout the lesson and the unit to remind the students to continually make connections".

As the lesson study group developed lesson #2 on functions for Lisa to teach, Lisa learned a lot from the group. Since she had never taught functions before, it was a great concept to have colleagues' collaboration on. By working with the group, her mathematics content knowledge about functions was enhanced. In her final interview, Lisa states, "Overall, my understanding of proofs, functions, and application problems have become more in depth after working with this lesson study group". She comments in her journal that she realized how important it is to present the lesson in a certain order and to examine carefully each of the individual concepts that are related to functions. Here, Lisa developed her pedagogical content knowledge with the help of the lesson study group members, for they took the function concepts and figured out the best way to present them to the students.

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Another comment in her journal during the planning of lesson #2 centers on choosing example and homework problems. She learned how to only pick homework problems that directly relate to the goal of the lesson. She believes she could assign homework that challenges the students by going one step further into the lesson for the next day, but it should be geared toward practice and reinforcement of the concepts and procedures. She also learned that problems out of a textbook can be very good. By examining problems in the Algebra II textbook with her colleagues, she found that the book represents a variety of practice, application, and challenge problems. This adds to Lisa's curricular knowledge. She can use her time and energy choosing problems from the current text instead of making up all new problem sets for the students. Lisa continued to reflect on the problems she gave her students when the group is planning lesson #3. Alex tells the group how he gives his students' time to practice the same type of problem over and over again. Lisa reflected on her classes and stated that she doesn't give the students enough practice on a concept, for it is one new concept after another. She thinks that sometimes her students are too challenged, and they do not get a chance to process and practice the material. With her students she thinks she needs to provide more practice with new concepts, but also challenge the students by asking them to think about a concept more thoroughly. Once again, Lisa took a comment that another teacher shared and applied it to her own classroom according to her students and her pedagogy.

One of the purposes of this chapter was to provide more details about the teachers in the study. Based on the interview, observation, and journal entry data, the results report samples of each participant's teaching practices along with the comments that each of them made based on their participation in the lesson study group. The comments varied among participants. Through these comments and reflections, we can see that each of them enhanced their pedagogical content knowledge. Particular emphasis was placed on the characteristics of planning and reflection which are vital components of lesson study.

CHAPTER 8

DISCUSSION AND IMPLICATIONS

The goal of this research was to engage secondary mathematics teachers in lesson study and to examine how this professional development experience enhanced their teacher knowledge. Data was collected and analyzed from interviews, observations, videotapes, meeting notes, and journal entries to address the research questions presented in Chapter One. The main research question follows: How does lesson study influence teacher knowledge and classroom practices?

While there is one main question governing this inquiry, there are other topical questions that contribute to the central focus:

1. What elements of the lesson planning stage contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

2. What aspects of the teachers' observations of the taught lessons contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

3. How does reflecting on the lesson study process contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge?

The discussion that follows is organized to highlight the results of this research in relation to the four questions above. The discussion begins with the results of lesson study on teachers' knowledge. Next, the discussion centers on the three topical questions that involve the elements of the stages of lesson study that contributed to teacher knowledge growth. Emphasis is placed on lesson planning and reflection. This chapter concludes with implications of this research and directions for further study.

Interaction Between Mathematics Content Knowledge and Pedagogical Content Knowledge

As the teachers in this study participated in lesson study, they had many opportunities to reflect on their own teaching practices. Heibert, Gallimore, and Stigler (2002) express the need for teachers to take practitioner knowledge and to form a shared, professional knowledge base for teaching. Recall that by practitioner knowledge they are referring to the knowledge teachers generate through active participation and reflection on their own practice. In the results reported in Chapters Five, Six, and Seven, it can clearly be seen that the stages of lesson study provide a means for the teachers to not only actively participate and reflect on their own practice but also to do this in a collaborative setting with

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a group of teachers. This collaborative setting allows the participants to share their knowledge publicly with their colleagues.

When the teachers shared their knowledge publicly with one another, they made their knowledge explicit. In the lesson study planning meetings the teachers took the time to carefully consider all of the details of the lesson. They debated and negotiated over various aspects of the lesson that they may not have spent much time debating if they were planning the lesson on their own. They discussed such aspects of the lesson as the specific mathematics content, prior knowledge connections, curricular knowledge, example problems, questioning, motivation, assessing student understanding during the lesson, and anticipating student misconceptions. Participating in lesson study provided the teachers with the opportunity to share the details of these aspects of the lessons.

For example in lesson #1, Craig gave the teachers valuable information about where the students may experience difficulty with the geometry proofs. Recall that this lesson evolved around five conditions to prove if a quadrilateral is a parallelogram. In the past, when students were proving one theorem that involved one pair of lines to be parallel and congruent, they were concerned with the other pair of sides as well. Also, Craig anticipated that his students may have trouble with the theorem involving the diagonals bisecting each other. In the past, the students proved the wrong two triangles congruent and ended up with nothing to work with to prove the quadrilateral is a parallelogram. This information became part of his pedagogical content knowledge that he brought out in the open by sharing it with the other teachers. The other teachers, who

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have less experience in teaching geometry, learned some valuable information that they can refer back to when they teach the topic. In the teaching of lesson #1, the students attempted to prove the wrong two triangles congruent when discussing the proof involving the diagonals. However, Craig's careful consideration of this during the planning meeting helped him to lead his students in the correct direction during the lesson.

Through lesson study the teachers added to their pedagogical content knowledge the process of choosing appropriate problems and how to work through these problems in a classroom setting. In Lesson #2, the discussion on problems helped the teachers review the mathematics content as they tried to develop their own example problems. The teachers examined the definitions of a function and a relation, domain and range, and the vertical line test. When they developed example problems about functions and non-functions on their own, they made sure the problems correctly reflected the mathematics concept. As they developed the problems, they made their own knowledge of functions clearer, provided better explanations of the definition of a function, and worked on the precise language to use. Not only did the teachers add precision to their own understanding of the function concepts, but they were refining their own knowledge in order to teach these examples to the students. Also, in this lesson the teachers polished their understanding of the definition of a function and independent and dependent variables when they debated over the type of experiment to use. The teachers worked together to clearly understand each of the two possible experiments themselves in order to be better equipped to

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explain them to the students and to answer student questions. When deciding on a homework assignment, the teachers reviewed all of the concepts to be presented in the lesson as they sifted through the problems given in the textbook to choose which ones were most suitable for their lesson. Thus, developing or choosing from a list of appropriate examples and discussing them with each other furnished the teachers with pedagogical content knowledge for incorporating the examples in their teaching.

In lesson #3 as the teachers decided to have students pose problems and as they chose example problems, they were enriching their pedagogical content knowledge. They negotiated ways to have their students display their understanding of the mathematics content. As the teachers decided upon appropriate problems, they thought carefully about solving linear equation application problems that involved two ordered pairs as given information. The teachers reviewed the knowledge necessary for solving linear equations such as finding the slope using the ordered pairs, using $y = mx + b$ to calculate the y-intercept, and then using this formula to solve for the unknown information to get a final answer. By reviewing the definitions and formula for solving these problems, the teachers enhanced their substantive knowledge. In addition, they considered whether or not the problems were interesting enough to the students to motivate them to want to solve the problems. Lesson study provided the teachers with time to share new ideas (like problem posing) and to think about the mathematics content involved in carrying out these ideas in the classroom.

Lesson study provided a forum for teachers to discuss mathematics content. Spending lots of time together as a group, the teachers were comfortable with their group members. They admitted when they needed clarification on a concept and helped one another. In lesson #1 and #2 this helped the teachers to share knowledge on such mathematics content as geometry proofs, relations, functions, domain, and range - topics that they may have forgotten because they have not taught the concepts in a while. In lesson #1, the teachers went through the specific steps in the two column proofs for each of the four theorems. They reviewed knowledge of quadrilaterals, parallelograms, congruent triangles, and parallel lines in order to prove the following four conditions for proving a quadrilateral is a parallelogram: opposite sides congruent, opposite angles congruent, one pair of opposite sides congruent and parallel, and diagonals bisect each other. The teachers reviewed the prior knowledge the students would need and the connections that the students would have to make in order to complete the proofs. According to Ma (1999), the teachers need this "breadth" as a part of their "profound understanding of mathematics". Also in lesson #1, the teachers discussed why teaching proofs is important. Craig shared what he read in various articles about proof. He expressed his view of proofs as a method to help students make connections on their own. He added that he hoped that students' experience proving theorems will help them to make conjectures on their own. These ideas are embedded in the reasoning and proof standard in *Principles and Standards for School Mathematics* (NCTM, 2000).

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During the planning meeting for lesson #2, the teachers discussed their understanding of the definition of a relation and a function, and different representations for functions. As the teachers sorted out their interpretations of the definition of a function, they examined different representations for functions. Ball and Bass (2000) suggest that teachers' ability to explain different representations for the same concept is part of their pedagogical content knowledge. Part of the representation standard in the *Principles and Standards* (NCTM, 2000) is for students to be able to "select, apply, and translate among mathematical representations to solve problems" (p.67). The teachers also discussed why it is important for students to learn functions. They were not satisfied with understanding the definition and having the ability to explain it to the students. They also wanted to understand the essence of the mathematics and how functions fit into the coherent whole. Ma (1999) describes this part of a "profound understanding of mathematics" as "thoroughness".

In lesson #3 as the teachers watched a video of teaching linear equation application problems, they thought about how they would present the topic in their own classrooms. As the teachers pointed out similarities and differences to what each of them would do, they were reviewing the mathematics content. Along with the mathematics content, they also examined the way the content is presented. They pointed out the different representation for the slope intercept formula for a line and the fact that the presenter in the video did not use the term slope. By taking time to sort through the different ideas and discuss them as a

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group the teachers sorted out their own understandings of the concepts that they wanted to use in the lesson.

In the planning meetings the teachers took time to discuss which mathematics concepts should be assumed to be prior knowledge and which may need to be a part of the lesson. For example in lesson #1, the teachers examined definitions and theorems concerning parallel lines, congruent triangles, and quadrilaterals as they walked step-by-step through each step of the proofs. In lesson #2 the teachers came up with ordered pairs, graphing, and evaluating expressions as prior knowledge. Examining the concepts within the students' prior knowledge helped the teachers to see what was necessary before the lesson on functions was taught. Looking at these topics before writing out the details of the lesson helped the teachers to develop a lesson that built on concepts that the students already knew. The teachers sequenced the topics to connect the old concepts with the new concepts. The connections standard in *Principles and Standards* (NCTM, 2000) stresses the importance for students to understand how mathematics concepts interconnect and build on one another. As Craig continually emphasized in his geometry classes, from the perspective of connections, students learn to use what they already know to address new situations. Ball (1990) stresses the need for teachers to understand and appreciate the connections among mathematical ideas. Ball characterizes this sequencing of topics and making connections among topics as a necessary part of a teacher's syntactic knowledge.

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The example from lesson #2 above also illustrates how lesson study influenced the teachers' curricular knowledge. Shulman (1986) proposes that the examination of the curriculum and all materials associated with it is central to a teacher's pedagogy. During the planning meetings for lesson #3, the teachers discussed such curriculum issues as disjoint topics, depth versus breath, and the number of mathematics courses required for graduation. Lesson #3 was part of a course that integrates algebra and geometry. The way the curriculum was set up for this course it was difficult for the teachers to make connections between the topics. Also, the teachers discussed how linear equations were a topic taught in five of the six integrated courses at this school. The number of mathematics courses required for graduation for this school was three. The teachers reminded each other of this as they noted that the students did not need this class for a math credit for graduation. The teachers used this knowledge as they considered what mathematics concepts should be considered prior knowledge, how the students would make connections between mathematics concepts, and what pedagogical techniques they would use in the lesson.

Lesson study provided the teachers with the opportunity to learn about the importance of questioning. The ability to develop questions is an important part of teaching. Whether the questions are essential, guiding, or leading, the teachers used their mathematics content knowledge in order to develop appropriate questions about the concepts. Asking appropriate questions led to the students' ability to make conjectures about parallelograms. The teachers used the mathematics content to write questions that were comprehensible to the

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students and that would help them to understand the concepts. This was not always an easy task. In lesson #1, Craig used the appropriate guiding questions during the teaching of the lesson in order to assist students to make connections. The guiding questions that Craig used in the lesson were not planned by the group. However, the teachers discussed the steps that were needed in order to write out all of the proofs. At one point during the planning meetings, Craig stated that he will start with the condition that opposite sides in a parallelogram are congruent and suggest that this can be a theorem for proving that a quadrilateral of this type is a parallelogram. Craig stated that he would follow with, "Then I'll say what about the other properties we know. We know that if it is a parallelogram opposite angles are congruent. Well, what if opposite angles are congruent does that make it a parallelogram?" The questions Craig asked were driven by the theorems that he wanted the students to prove. Craig's goal was to guide the students into the proofs. For example, in the teaching of lesson #1, as Craig led the students to one of the theorems he stated, "Didn't we just say if we have a quadrilateral with opposite sides parallel, then the quadrilateral is a parallelogram? That is the definition. That is the only thing we have right now to show that a quadrilateral is a parallelogram. So I have to go from congruent sides to parallel sides. How are we going to do this? Any ideas?" In order to do this he had to ask appropriate questions that would lead the students to the correct approach for writing the proof.

Ball and Bass (2000) indicate that teachers' ability to ask questions is an aspect of their pedagogical content knowledge. In the debriefing session for

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lesson #1, the teachers' commented on how well the lesson flowed and how they enjoyed the open discussion that took place between Craig and his students. In a journal reflection written early in the study, Craig described his style of teaching as "chalk and talk". Craig viewed his teaching as more of a lecture format, but the teachers' feedback on his teaching described the lesson as an open discussion between Craig and his students. Such feedback offered Craig a different perspective on his teaching practices. The comments made in the debriefing meeting brought to Craig's attention his ability to ask questions to help the students recall the prior knowledge they needed in order to make the connections between the old and new concepts.

In lesson #2, Lisa received suggestions from the other teachers on how to get the students to come up with the vertical line test on their own. Lisa posed a good guiding question when she stated, ". . . can you come up with a method that you might be able to use to test a graph that is given if it is a function or not?" The teachers wanted the students to make the necessary connections between function concepts that would allow them to test if a graph is a function.

The teachers' discussions throughout the lesson study experience have helped me to see that aspects of their syntactic knowledge have an impact on their curricular knowledge. As the teachers developed questions and problems, formulated definitions, and sequenced the topics, they reviewed the prior knowledge that the students needed to use and how each concept is interconnected. The result of the careful planning of all of these things led to the students' ability to make connections among the concepts. This was seen most

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clearly in the first lesson as Craig used guiding questions to help the students recall the prior knowledge necessary to complete the four geometry proofs.

Through lesson study the teachers had an opportunity to see how they assess student understanding during the lesson. Not every comment or question from students can be planned out. In the planning meetings for lesson #1, the teachers talked through all the steps for proving the theorems. The teachers reviewed all the prior geometric concepts that the students needed in order to supply the reasons for each step of the proofs. Because of this discussion with his colleagues, Craig was able to consider the students' perspective on the geometry concepts. During the teaching of the lesson, the teacher observers saw how Craig was flexible in his explanations of students' questions as they worked on their own. Craig used the pedagogical content knowledge developed during the planning of the lesson. Then, in the debriefing session, Alex stated to Craig, "I was impressed . . . a student might have had three or four steps written and how you were able to almost like a game of chess instantly go to a step (I wouldn't have a clue) now this is reversed. I thought it was very technical the comment that you made. Wow, for the student to be there so quickly and then for you to be able to diagnosis so quickly." With the necessary pedagogical content knowledge, he was able to easily assess the students' understanding during the teaching of the lesson.

In lesson #3, the unsolicited comments from a student displayed to Alex and the teacher observers how once she saw one more example problem on finding the equation of a line when given two points she understood the lesson

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from the previous day. This student was able to make the connection between the two lessons, but shared that she didn't completely understand the homework problems. This incident reminded the teachers of the importance of constantly looking for or listening for students' glimmers of understanding.

Lesson Study

Now it is time to look at possible insights into the three topical questions regarding the stages of the lesson study process. The elements of the lesson planning stage, teaching/observing stage, and debriefing stage that contribute to the development of teachers' mathematics content knowledge and pedagogical content knowledge are discussed based on the results reported in Chapters Five, Six, and Seven.

In the first planning meeting the discussion that stemmed from the group's lesson study goals displays how a lesson study group can be used to target various characteristics of students or lessons that a school wants to improve. Lesson study provided the teachers with the opportunity to set their own goals and to work together to plan lessons to accomplish these goals. When planning each of the three lessons, the teachers discussed these goals and how they could add things to the lessons to help them accomplish these goals. They also took the time in the debriefing sessions to reflect on these goals. The action upon these goals was immediate since the teachers were planning the lessons to be used in their classrooms. Explaining why a concept is important and finding

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ways to motivate students are two issues that most math teachers struggle with. These teachers discussed these issues early in the planning stage for lesson #1. As long as the teachers included specific strategies for accomplishing these issues in their lesson plans and carried them out in the teaching of the lesson, then lesson study served as a forum for trying out different pedagogical strategies. As reported in the teacher by teacher results in Chapter Seven, this discussion between the teachers was enough for the teachers to try to provide more meaning to the concepts and to find ways to motivate their students in their own classrooms.

From the analysis results presented in Chapters Five, Six, and Seven, it is clear that the teachers developed pedagogical content knowledge and mathematics content knowledge as they planned lessons. The knowledge growth occurred as they looked deeply at the mathematics content involved in the lessons. As they examined the content and decided on instructional strategies to use, they were developing pedagogical content knowledge that they applied to the particular lesson as well as in the lessons they planned day-to-day for their own classrooms. The planning meeting discussions were vital to this knowledge growth. This research supports Grouws and Shultz (1996) and Brown and Smith's (1997) claim that planning for instruction is where mathematics content and pedagogical content knowledge converge. In addition this research supports Heibert, Gallimore, and Stigler's (2002) view that lesson plans act as the unit of analysis for converting practitioner knowledge into professional knowledge for teaching. Since lessons are small enough units that

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the complexity of teaching can be reduced to a manageable size, the analysis of lessons as done in the lesson study planning meetings allows for teachers to move what was learned in one classroom into another.

As Byrum, Jarrel, & Munoz (2002) report from a study of Kentucky teachers, lesson study has changed the thinking process when planning lessons for the teachers in this study. In the initial and final interviews the teachers were each asked how they plan their lessons. Each teacher changed their response from the initial interview to the final interview which followed the entire lesson study experience. Mike added that he uses the curriculum guide and textbook to find homework and example problems. Craig added that he looks at the whole curriculum, considers how to apply the concepts and how to make it interesting, and examines the necessary prior knowledge. Alex stated in the final interview that he now writes out a plan because he has recognized the importance of a well thought out lesson. Lisa added that she examines prior knowledge and possible student misconceptions. Prior knowledge, choosing example problems, curricular knowledge, motivation, and anticipating possible student misconceptions are all aspects of the lessons that were discussed throughout the planning meetings. The lesson study experience, particularly the large amount of collaborative time spent in the planning meetings, changed the teachers' thinking process when planning lessons.

As time goes on from one lesson to the next, the teachers are more and more comfortable in this setting so they are more willing to share ideas and difficulties that they have with the material. This sharing and reflecting on their

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own pedagogical content knowledge and mathematics content knowledge helped them to grow professionally from the lesson study experience. For example, Mike commented in his journal on the lesson study process when he wrote, "In my opinion, Lesson Study is the pathway to improved instruction. I think that if I were to use Lesson Study on a consistent basis, I would not only increase my own knowledge of my subject matter, but also my knowledge of instruction." Alex wrote in his journal, "Prior to the Lesson Study, I have never taken so much time to really focus on student thinking and learning. I have come to see Lesson Study as like educational-epoxy bonding together a variety of educational ideas for the common good of both teachers and students." The reflections from the teachers indicate that they believe they have developed mathematics content and pedagogical content knowledge through the lesson study process and that there is great potential within the process for this to continue to occur.

It can be seen from the results presented in Chapters Five, Six, and Seven that the teaching stage of the lesson study process influenced the teachers' mathematics content knowledge and pedagogical content knowledge. In the planning stage, the teachers spent a lot of time thinking about and discussing the mathematics involved and the pedagogical techniques they would use in the lesson. The teaching stage allowed the teachers to see all their time and effort carried out. According to Brown and Smith (1997), "teachers learn to teach by teaching." (p.142) The teaching and observing that the teachers do in this stage allowed them to reflect on the mathematics content knowledge and pedagogical content knowledge that they developed in the planning meetings

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and make adjustments or to clarify this knowledge. Japanese teachers who have participated in lesson study feel that they learn from the feedback they get on their own teaching and the new ideas gained from observing others (Lewis, 2000; Byrum, Jarel, & Munoz, 2002). The specific mathematics content knowledge that the teachers observed was knowledge of parallelograms, quadrilaterals, function concepts, linear functions, and geometry proofs. The specific pedagogical content knowledge that the teachers observed was the teacher asking guiding questions with references to prior knowledge, using discovery approach, using music and storyline, dealing with student misconceptions, and assessing student understanding during the lesson.

Based on the results presented in Chapters Five, Six, and Seven the debriefing stage of the lesson study process influenced the teachers' mathematics content knowledge and pedagogical content knowledge. The teachers had the time and support from one another in this part of the lesson study process to share their reactions to their lesson. Itzel (2002) reports of teachers interested in lesson study because of the opportunity to gain knowledge of instruction through discussions and reflections with other colleagues. In their support of learning through reflection, Brown and Smith (1997) state "Learning to reflect *critically* is an important component of developing and refining one's pedagogical practices." (p.142) The debriefing stage gave the teachers the opportunity to revisit the mathematics content knowledge and the pedagogical content knowledge that they developed or refined during the planning and teaching stages of the process. The specific mathematics content knowledge

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that the teachers reflected upon was Craig's substantive and syntactic knowledge of geometry and Jennifer's mistake that all lines are functions. The specific pedagogical content knowledge that the teachers reflected upon was the teachers' use of guiding questions, teachers' ability to assess student understanding during the lesson, and the incorporation of the lesson study group's goals of motivation, making the lesson meaningful, and emphasizing understanding the concepts as well as the procedures.

Teachers may benefit from more training with the A *Tool for Planning and Describing Study Lessons.* The teachers in this study used portions of the planning tool during the planning meetings, but did not have time as a group to discuss it. If the teachers spend more time discussing this tool and becoming more familiar with it, then they may develop more detailed lesson plans for the study lessons. More in-depth discussions involving the format of the lesson plans could lead to more teacher knowledge growth.

Teachers may benefit from more training in how to observe their colleagues teach. In a journal entry, Alex commented on how while he was observing Craig's class, he realized that he had very little training in how to observe a class. This shows that perhaps more attention should have been given to what the teachers should do when they observe the lesson. The teachers in this study were given suggestions on how to observe the lessons as well as particular items to look for. However, when they actually did the observations, they simply recorded either all that the teacher was doing or abbreviated notes on what was happening in the classroom. If the observations

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were more structured, then the discussions in the debriefing sessions may be richer. More in-depth discussions about the teaching of the lessons could lead to more teacher knowledge growth.

Implications

The results of this research demonstrate how lesson study as a professional development experience can influence teacher knowledge and classroom practices. In particular, the results display the power of planning lessons and reflecting on the teaching of lessons in a collaborative setting. In this next section, I discuss the implications of this research on lesson study and teacher knowledge. Also, I consider possible future directions for research related to teacher knowledge and lesson study.

Lesson Study

According to Loucks-Horsley et al's (1998) principles of effective professional development, "Excellent mathematics teachers have a very special and unique kind of knowledge that must be developed through their professional development learning experiences." (p.3) The type of knowledge they are referring to is pedagogical content knowledge. This research shows that lesson study has the power to develop such knowledge in secondary mathematics teachers. In addition, this research provides evidence for two of Lewis, Perry, and Hurd's (2004) key pathways for instructional improvement that underlie

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successful lesson study. These pathways are increased knowledge of subject matter and increased knowledge of instruction.

Since lesson study is a relatively new form of professional development that is taking shape in the United States, the results of this study adds to the limited amount of research on lesson study. Lesson study can be successfully carried out in a secondary mathematics setting. The methods portion of this study outlines a format for developing a lesson study group in a secondary school. I hope that administrators and faculty members thinking about adopting lesson study as a form of professional development in their schools can use this study as an informational guide.

Part of the power of lesson study in terms of a professional development experience is the goal-setting activity. As the teachers in this study planned each lesson, they kept their lesson study goals in mind. The lessons that they developed included ways to accomplish these goals. There was much thought and reflection upon these goals. Thus, the staff of a school system can use this form of professional development to accomplish their district, school, or individual teacher goals.

Another important implication is the format in which lesson study takes place within the high schools. In this study the teachers met after school for all lesson study meetings except for one which was held during professional development time. The teachers were responsible for all professional development activities planned by the administration in addition to their time with the lesson study group. Lisa commented in her journal after planning lesson #2

during professional development time, "I thought it was a very productive day and that we accomplished more by completing the lesson at one setting rather than weeks at a time". Alex wrote in his journal, "This compacted (one day) lesson study experience was practical and important. Many other workshop experiences present new and fascinating nuggets of information, but there often is way too little time to struggle with the practical details of implementing them in the classroom". The comments from the teachers indicate that lesson study should become a part of the professional development activities planned for teachers throughout the school year.

This study is an example of research that supports learning through planning, instruction, and reflection. In Chapter Three, I presented the argument that lesson study is a professional development experience that can achieve this. As the teachers planned the lessons in this study their mathematics content knowledge and pedagogical content knowledge converged as they focused on student understanding of the mathematics concepts. Therefore, lesson study provided a collaborative, teacher-directed setting for planning lessons.

Also, throughout the entire lesson study process, teachers reflected on their current classroom practices. This reflective component was important for teachers to engage in as they incorporated the lesson into their classroom in terms of the needs of their own students. Thus, lesson study offered a comfortable setting for teachers to reflect on their teaching practices as they planned, implemented, and refined lessons. This research is evidence that planning and reflection are key components in the development of teacher

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knowledge. By examining this research, mathematics educators can see the importance of having planning, instruction, and reflection as a part of professional development activities for secondary teachers.

Teacher Knowledge

The results of this study indicate that lesson study is a professional development experience that provides opportunities for teachers to add to their knowledge base. In order for teachers to turn their practitioner knowledge into professional knowledge for teaching, it is vital to examine the connection between teacher knowledge and in-service professional development activities for secondary teachers. This study displays a way to examine mathematics content knowledge of secondary mathematics teachers in terms of substantive knowledge and syntactic knowledge. In addition, the study displays a way to examine pedagogical content knowledge in terms of prior knowledge connections, anticipating student misconceptions, questioning, choosing example problems, assessing student understanding, and curricular knowledge.

Lesson study made the teachers more aware of their own teaching style. As the teachers shared ideas with their colleagues, they reflected on their own teaching practices. The more experienced teachers transfer knowledge and experiences to the newer teachers. Working together with a group of teachers, the newer teachers feel less isolated and become more comfortable within the mathematics department. Lisa, first year teacher, stated "the collaboration

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component of the lesson study professional development experience makes me feel less isolated in my individual concerns and classroom issues."

Lesson study provided teachers with a forum to discuss various educational issues. Such opportunities could raise the teachers' level of awareness about reforms. Teachers do not always have the time to share their opinions and ideas about research findings or "best practices". Since lesson study is teacher directed, the teachers can bring up these issues and feel comfortable discussing them with their colleagues.

With lesson study, teachers participate in the development of micro curriculum which makes them aware of more general issues related to the larger curriculum. They develop lessons for units within the curriculum. These lessons can be used and shared from one year to the next. The teachers in this study developed lessons independent of one textbook. The teachers can gain experience using a variety of textbooks, and this experience can help them to improve their ability to evaluate textbooks.

Possible Future Research

The secondary mathematics teachers in this study gave positive feedback on lesson study as a professional development experience. Possible directions for future research are presented below.

Instead of the teachers planning the lesson from different courses, it would be interesting to have a lesson study group plan a series of lessons for one

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course. Are there aspects of pedagogical content knowledge that are strengthened as a result of planning a series of lessons for one course? What changes do the teachers make in their lessons as they plan one lesson than the next lesson for the same group of students?

Two additional parts of lesson study that are completed in Japanese lesson study that were not done in this study are teachers examination of research on the topic that they are planning and a fourth stage in which they revise and then re-teach the lesson. How does looking at research affect the lessons that the teachers plan? Does this have an effect on the teachers' knowledge base and if so in what form? How do they apply the research to their current teaching practices and the lessons they are planning? What elements of the fourth stage of lesson study aid in enriching teacher knowledge?

Another possibility could be a more controlled experiment to see the outcomes of lesson study in the classroom. Perhaps study two classrooms of students in the same course. Teach only one group the study lesson and the other group the teacher's usual lesson. Are there differences in student learning between the two classes?

Perhaps obtain student feedback on the study lessons. Give students questionnaires or surveys before and after the study lesson is taught. This will allow for student comments on the study lesson. Or to obtain more information on the teachers' progress in their own classrooms outside the lesson study group, questionnaires could be given to the students before, during, and after the

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teacher participates in the whole lesson study experience. This will allow for students' comments on changes that they see in their teacher.

A researcher could take a more in depth look at the teachers' mathematics content knowledge. In order to measure teachers' mathematics content knowledge growth, the teachers could be assessed on mathematics content related to the study lessons to be planned. The responses to a set of questions given before they plan the lesson could be compared to their responses on a set of questions after they plan the lesson. Do the teachers have a greater understanding of the mathematics concepts after planning, teaching, and discussing the study lessons?

A study could be done that examines the relationship between the things the teachers were saying and doing in their classroom which displays their implicit understanding of the nature of learning. The researcher could look at the actions the teachers took and progress they made in terms of their epistemological stance.

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APPENDICES

APPENDIX A

LESSON STUDY MATERIALS

Timeline of Data Collection

DATE	TYPE OF DATA
6-Oct-04	Interview-Mike
6-Oct-04	Interview Craig
7-Oct-04	Interview-Alex
13-Oct-04	Goal Setting Meeting Video
13-Oct-04	Goal Setting Meeting Notes- Melissa Only
13-Oct-04	Goal Setting Meeting Journal Reflections - all 5
13-Oct-04	Interview-Lisa
13-Oct-04	Observed Lisa's Honors Algebra I Block 2
14-Oct-04	Observed Lisa's Honors Algebra I Block 2
14-Oct-04	Observed Alex's Algebra I Block 3
19-Oct-04	Observed Mike' Algebra II Block I
19-Oct-04	Observed Craig' College Geometry Block 3
20-Oct-04	Planning Meeting #1 For Lesson #1 Video
20-Oct-04	Planning Meeting #1For Lesson #1 Notes-all 5
20-Oct-04	Planning Meeting #1 For Lesson #1 Journal Reflections- all 5
20-Oct-04	Planning Meeting #2 For Lesson #1 Video
20-Oct-04	Planning Meeting #2 For Lesson #1 Notes-all 5
20-Oct-04	Planning Meeting #2 For Lesson #1 Journal Reflections- all 5
1-Nov-04	Observed Mike' Integrated Math I Block 3
2-Nov-04	Observed Mike' Integrated Math I Block 3
3-Nov-04	Planning Meeting #3 For Lesson #1 Video (No Mike)
3-Nov-04	Planning Meeting #3For Lesson #1 Notes-4 present
3-Nov-04	Planning Meeting #3 For Lesson #1 Journal Reflections- (Melissa & Craig Only)
4-Nov-04	Observed Alex's Algebra I Block 3
5-Nov-04	Observed Alex's Algebra I Block 3
8-Nov-04	Observed Lisa's Honors Algebra I Block 2
9-Nov-04	Observed Craig's College Geometry Block 3
9-Nov-04	Observed Mike' Algebra II Block 4
10-Nov-04	Planning Meeting #4 For Lesson #1 Video (Melissa & Alex Only)
10-Nov-04	Planning Meeting #4For Lesson #1 Notes - Melissa Only

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17-Nov-04	Video of Lesson Taught by Craig
17-Nov-04	Observation Notes of Lesson Taught By Craig - Melissa Only
17-Nov-04	Journal Reflection of Lesson Taught By Craig - Melissa and Craig
23-Nov-04	Video of Debriefing Session for Lesson #1 - all present
23-Nov-04	Notes from Debriefing Meeting for Lesson #1 - Melissa Only
23-Nov-04	Journal Reflections on Debriefing Session and Process in General (Not Mike)
8-Feb-05	Planning Meeting #1for Lesson #2 Video
8-Feb-05	Planning Meeting #1for Lesson #2 Meeting Notes -all 5
8-Feb-05	Planning Meeting #1for Lesson #2 Journal Reflections -all 5
16-Feb-05	Planning Meeting #2 for Lesson #2 Video
16-Feb-05	Planning Meeting #2 for Lesson #2 Meeting Notes- Melissa & Lisa
16-Feb-05	Planning Meeting #2 for Lesson #2 Journal Reflections -Lisa, Melissa, Craig
23-Feb-05	Video of Lesson Taught by Lisa
23-Feb-05	Observation Notes of lesson Taught by Lisa - all 4
23-Feb-05	Debriefing Meeting for Lesson #2 – Video
23-Feb-05	Debriefing Meeting for Lesson #2 - Meeting Notes-Lisa, Melissa, Alex
23-Feb-05	Journal Reflection on Debriefing Meeting and Process in General-Mike & Melissa
10-Mar-05	Observed Mike' Algebra I Block 4
11-Mar-05	Observed Mike' Algebra I Block 4
15-Mar-05	Observed Lisa's Honors Algebra II Block 4
16-Mar-05	Observed Craig's College Geometry Block 4
17-Mar-05	Observed Craig's College Geometry Block 4
23-Mar-05	Planning Meeting #1 for Lesson #3 Video
23-Mar-05	Planning Meeting #1 for Lesson #3 Meeting Notes (all but Alex)
23-Mar-05	Planning Meeting #1 for Lesson #3 Journal Reflections (Craig, Melissa, & Mike)
30-Mar-05	Planning Meeting #2 for Lesson #3 Video
30-Mar-05	Planning Meeting #2 for Lesson #3 Meeting Notes (all but Alex)
30-Mar-05	Planning Meeting #2 for Lesson #3 Journal Reflections (Melissa, & Mike)
6-Apr-05	Planning Meeting #3 for Lesson #3 Video (No Craig or Mike)
6-Apr-05	Planning Meeting #3 for Lesson #3 Meeting Notes (no Craig or Mike)
6-Apr-05	Planning Meeting #2 for Lesson #3 Journal Reflections (Melissa)
13-Apr-05	Planning Meeting #4 for Lesson #3 Video
13-Apr-05	Planning Meeting #4 for Lesson #3 Meeting Notes
13-Apr-05	Planning Meeting #4 for Lesson #3 Journal Reflections
20-Apr-05	Video of Lesson Taught by Alex
20-Apr-05	Observation Notes of lesson Taught by Alex
20-Apr-05	Debriefing Meeting for Lesson #3- Video
20-Apr-05	Debriefing Meeting for Lesson #3 - Meeting Notes-
20-Apr-05	Journal Reflection on Debriefing Meeting and Process in General

13-May-05	Questions Completed Prior to Final Interview
14-Jun-05	Interview Alex
10-Jun-05	Interview Mike
3-Jun-05	Interview Craig
10-Jun-05	Interview Lisa
1-Jun-05	Observed Mike' Algebra I Block 4
2-Jun-05	Observed Mike' Algebra I Block 4
19-May-05	Observed Lisa's Honors Algebra II Block 4
20-May-05	Observed Lisa's Honors Algebra II Block 4
23-May-05	Observed Craig's College Geometry Block 4
24-May-05	Observed Craig's College Geometry Block 4
9-Jun-05	Observed Alex's Algebra II Block 3
14-Jun-05	Observed Alex's Algebra II Block 3

All lesson study tools developed by the Lesson Study Research Group are regularly revised and updated. To download the latest versions of these documents, please go to: www.tc.columbia.edu/lessonstudy/tools.html. Barbrina Ertle, Sonal Chokshi, & Clea Fernandez. ©2001, Lesson Study Research Group (lsrg@columbia.edu).

A Tool for Planning and Describing Study Lessons

This tool is designed to help you describe your study lesson. It is organized by sections, each focusing on a particular aspect of the lesson or its context. Each section contains a list of guiding questions you should think about as you complete that section. To make your work efficient, we recommend that you use this tool to guide your lesson planning process. Keep in mind that the list of questions that we provide is not meant to be comprehensive, but rather, to give you an idea of key issues that you should be thinking about. Many other questions or issues are likely to surface as your group plans its study lesson. These issues should also be incorporated into the appropriate section of your study lesson description.

Logistical information about the lesson
Date:
Grade:
Period and Location:
Instructor:

I. Background information
A. Goal of the Lesson Study Group:
This is a description of the group's lesson study goal and its focus. This goal will have evolved out of identifying the gap that exists between aspirations your group has for students and the kinds of learners that are actually being fostered at your school. Therefore, you may want to describe in this section: the aspirations that your group has for students and why they are important; ways in which, as a group, you feel you are falling short of these aspirations and how this is manifested in your students; how the goal your group has chosen represents an attempt to close this gap. You may also want to explain concretely what your exploration of this goal entails.
• *What kind of learners do we want to see develop at our school?*
• *What kinds of learners are actually developing at our school? What evidence do we have for this?*
• *Why does this gap between our aspirations and reality exist? How can we close this gap?*
• *How will the lesson study goal we have chosen help us close this gap?*
• *How will we go about exploring our lesson study goal?*

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Note: although all the study lessons planned by your group will describe this same group goal, it is helpful for you and your planning group to write your own version of the above section.

B. Narrative Overview of Background Information:
This is a description of the lesson context. It is a way for you to set up and put in perspective the lesson. You should include all the background information that you feel is needed to appreciate the lesson in a meaningful way. For example, you may want to provide information regarding your students, what they know, and why this lesson is important to their continued learning and development. You may also want to mention any teaching techniques or approaches that you will be exploring in this lesson. Make this personal to you as the teacher, your classroom, and your individual students.
• What do the observers need to know about my classroom?
• Who are my students? What do they already know? What strategies do they use? What motivates them?
• What personal knowledge can I share with the observers so that they many better understand what is going on with my individual students? What individual differences will they see?
• Why is this mathematics important?
• What misconceptions might students have?
• What should students know at the end of this lesson? What else would I like them to gain from this lesson?
• What do I think I can achieve in this lesson?
• Are there any teaching techniques or approaches that are central to the design of this lesson?

II. Unit Information

A. Name of the unit: *State the name of the unit from which you have selected your study lesson.*

B. Goal(s) of the unit:
This is a description of the learning goals for the unit.
• What is the mathematics here?
• What should the students know at the end of this unit?

C. How this unit is related to the curriculum:
This is a description of how the content that is taught in this unit relates to content taught in previous and future grades as well as this grade. It should include the specific concepts that are taught in those grades, and how they relate to the concepts taught in this unit. A curriculum guide may provide you with this information, but take some time to think about how everything relates, and the importance of an appropriate development of concepts. So that this task does not become unwieldy, include only highly relevant concepts in this description.

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• *What prior knowledge is necessary (to learn the content that this unit focuses on)?*
• *What new knowledge can be developed from the concepts that students will learn in this unit?*

D. Instructional sequence for the unit:
This is a sequenced description of the general objectives of the unit. It should identify how the study lesson being described fits within the sequence. It does not need to list each individual lesson, but rather, the topics that are covered, and the number of lessons spent covering each topic.
• *Where does this lesson fall in this unit and why?*
• *Do any of the lesson concepts and/or skills get addressed at other points in the unit?*

III. Lesson Information

A. Name of the study lesson: *State the name of the study lesson being described.*

B. Goal(s) of the study lesson:
This is a description of the goals for this lesson. You may also want to include specific strategies, skills, or ways of thinking about mathematics you would like to address.
• *What is the mathematics here?*
• *What should students know at the end of this lesson?*
• *Are there specific strategies being developed?*

C. How this study lesson is related to the lesson study goal:
This is a description of the specific aspect(s) of the group lesson study goal that you would like to focus on during this lesson. In this section you will want to relate your instructional choices for this lesson to the group lesson study goal.
• *How will I explore our groups' lesson study goal through this lesson?*
• *What aspects of my lesson will address the groups' lesson study goal? In what ways?*

D. Process of the study lesson:
This is a chart of the planned lesson sequence. It represents the bulk of the lesson plan, and often spans a number of pages. It describes what you have planned and expect to happen from the beginning of the lesson until the end.

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/ expected student reactions or responses	Teacher's responses to student reactions /Things to remember	Goals and Method(s) of Evaluation
This column is usually laid out in order by the parts of the lesson (e.g., launch, investigation, congress, extension/applications, etc.), and also includes the allocation of time for each of these parts. *This column should also include a description of key questions or activities that are intended to move the lesson from one point to another.*	*This column describes what students will be doing during the lesson, and their anticipated reactions or responses to questions/ problems you will present.*	*This column describes things that you want to remember to do/not to do within the lesson as well as other reminders to yourself.* *Also, as you have anticipated student responses and reactions (previous column), this column provides a place where you can think through how you might use those responses and reactions in synthesizing a true learning experience within your classroom.*	*This column describes the goals that are being focused upon during each part of the lesson, and for each activity/problem.* *It should also include a concrete description of how you will determine that you have achieved each of these goals.*
Guiding questions *How should this lesson progress? (How much time should I spend?)*	*What do I expect of my students? How will they respond?*	*Is there anything specific I want to remember to do? Any reminders for my students?*	*What should I look for to know that my goal(s) have been achieved?*
Questions continued...	*How will I motivate my students?*		*How will I determine that my students are motivated?*
How will I use the blackboard in this lesson?	*What do I expect my students to record in their notes?*		*Does my blackboard provide a good summary of this lesson?*
How will I present the activity/ problem?	*What activity will students work on?*	*What specifically will I be doing during the activity/ Should I use group work?*	*What will I be looking for?*

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Should I use group work? *What size groups should I use? What rules or directions should the groups be given?*			
What 3 or 4 processing questions will I use to move the lesson along?	*How do I expect my students to respond?*	*What summary will I use?*	*What should they know before I continue?*
What new vocabulary will be introduced? How will I introduce it?			
What materials and/or visuals will I need? Make a list. How will I make the materials available to my students if they are intended for their use?	*What are ways my students might use these materials?*		*What did I learn about student understanding/ thinking from the use of these materials?*
How can I develop the lesson to alleviate or minimize them?	*What misconceptions might students have?*	*How should I respond to each potential misconception?*	*How do I know that there are no more misconceptions?*
Questions continued...		*What teaching pitfalls do I need to watch out for? What should I be looking for? Make sure I don't say "....." or do ".....".*	
How will I conclude the lesson?			*What should students know at the end of this lesson? How will I know that they know it?*

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E. Evaluation

Describe your plan for evaluating the success of your lesson overall. Explain what you will look for in your students' in-class behavior and work products to determine if your lesson goals were met. Describe any homework of formal assessment that you plan to use as well. You will also want to be specific about what you are looking to collect information or evidence about with respect to your lesson study goals. You should also outline how you would like observers to assist you in collecting any of this information.

• How will I determine if students understood the concepts taught in this lesson?
• What would be appropriate homework? What will I be able to tell about the student from his homework?
• What information do I want to collect in the course of this lesson?
• Where in my plan would I like some assistance?

F. Appendix

Here you should attach or include copies of materials, handouts etc. that will be used during the lesson. For materials that will be used but cannot be attached (e.g., manipulatives) provide a written description and/or drawing. You should also include any materials that you have made specifically for the observers to use (e.g., observation tools, seating charts, etc.).

This appendix is invaluable for observers to acquaint themselves with your lesson prior to entering your classroom. The more familiar they are with what is meant to transpire, and what you want them to focus on during their observation, the better they will be able to provide you with useful feedback.

Lesson Plan for Lesson #1

Name of Lesson: Proving Quadrilaterals are Parallelograms
Students: Block 3 College Prep Geometry Students

Lesson Plan:

I. <u>Hands On Activity to Motivate the Lesson</u>: (Approximately 10 minutes) Ask students to draw segments AB and CD so they both share a midpoint M. Instruct them to use a ruler to draw quadrilateral ACBD. Ask what conjecture they might make about ACBD.
-Craig will have 3 students sketch their drawings on the board and will ask for their conjectures. He will not tell them at this time if their conjecture is correct or incorrect.

II. Quickly, they will review properties of parallelograms.

III. Craig will ask the students to write the converse of Theorem 6-1: Opposite sides of a parallelogram are congruent. This will formulate Theorem 6-5: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Craig will walk through the following proof of Theorem 6-5 with the students.
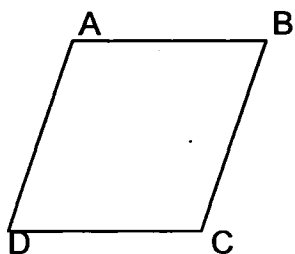
Given: Segment AB ≈ Segment CD, Segment AD ≈ Segment BC
Prove: Quad ABCD is a parallelogram

| Statements	Reasons
1. Segment AB ≈ Segment CD, Segment AD ≈ Segment BC	1. Given
2. Draw Segment BD	2.Given two points there exists a line.
3. Segment BD ≈ Segment BD	3. Congruent segments are reflexive.
4. △ADB ≈ △CBD	4. SSS
5. <ABD ≈ <BDC, <ADB ≈ <CBD	5. CPCTC
[Note: This is where the students may have trouble]	
6. Segment AB ∥ CD , AD ∥ BC	6. Alternate Interior

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| 7. ABCD is a parallelogram | 7.Definitionof Parallelogram |

IV. Craig will summarize that we have two ways to prove a quadrilateral is a parallelogram.
1. Definition of a Parallelogram
2. Opposite Sides are Congruent

V. Craig asks what if opposite Angles are Congruent.
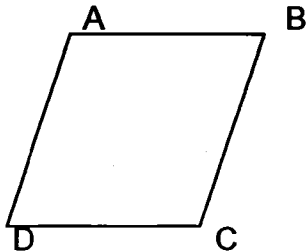
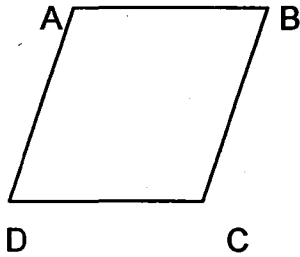Given: $\angle A \approx \angle C$, $\angle B \approx \angle D$ Prove: ABCD is a Parallelogram

Craig will let them start the proof. They will probably not get the second step – m$\angle$A + m$\angle$B + m$\angle$C +m$\angle$D = 360. He will use a paragraph style for this proof. After stating the given, he will need m$\angle$A +m$\angle$B + m$\angle$C + m$\angle$D =360. Since m$\angle$A = m$\angle$C and m$\angle$B = M$\angle$D, then 2m$\angle$A + 2m$\angle$B =360. So, m$\angle$A + m$\angle$B = 180. This means $\angle$A and $\angle$B are supplementary. Since consecutive interior angles are supplementary, segment AD $\parallel$ segment BC. Repeat the process for $\angle$A and $\angle$D to show segment AB $\parallel$ segment DC. Thus, by the definition of a parallelogram, ABCD is a parallelogram.

Craig will then have the students write the converse of Theorem 6-2: Opposite angles of a parallelogram are congruent in order to get the theorem they just proved which is Theorem 6-6: If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

VI. Summarize that we have three ways to prove a quadrilateral is a parallelogram:
1. Definition of a Parallelogram
2. Opposite Sides are Congruent
3. Opposite Angles are Congruent

VII. Craig will refer back to the warm-up activity and the students' conjectures. Then, he will have the students write the converse of Theorem 6-4: The diagonals of a parallelogram bisect each other in order to get Theorem 6-7: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. Craig will give the students 5-10 minutes to do most of the proof on their own.

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Given: Segment AC and Segment BD bisect each other
Prove: Quad ABCD is a parallelogram

Statements	Reasons
1. Segment AC, Segment BD bisect	1. Given
2. Segment DE ≈ Segment BE Segment AE ≈ Segment CE	2. Definition of Segment Bisector
3. <AED ≈ <BEC <AEB ≈ <DEC	3. Vertical Angles are Congruent
4. ΔAEB ≈ ΔCED, ΔAED ≈ ΔCEB	4. SAS
5. Segment AC ≈ Segment BC Segment AB ≈ Segment DC	5. CPCTC
6. ABCD is a parallelogram	6. Theorem 6-5 If Opposite sides of a quadrilateral are congruent, then the quadrilateral is parallelogram.

VIII. Now, Craig will summarize that there are 4 ways to prove that a quadrilateral is a parallelogram.
　　　1. Definition of a Parallelogram
　　　2. Opposite Sides are Congruent
　　　3. Opposite Angles are Congruent
　　　4. Diagonals Bisect Each Other

IX. Craig asks if there are any more ways. He gives students time to prove Theorem 6-8 on their own. Theorem 6-8 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.
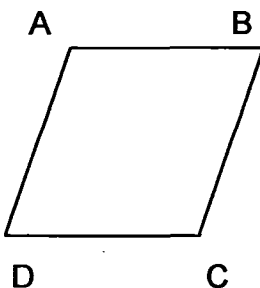
Given: Segment AB ‖ Segment DC, Segment AB ≈ Segment DC
Prove: Quad ABCD is a parallelogram

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Statements	Reasons
1. Segment AB ∥ Segment DC Segment AB ≈ Segment DC	1. Given
2. Draw Segment AC line	2. Given two points there exits a
3. <BAC ≈ <ACD Theorem	3. Alternate Interior Angles
4. Segment AC ≈ Segment AC Reflexive	4. Congruent Segments are
5. ΔBAC ≈ ΔACD	5. SAS
6. Segment AD ≈ Segment BC	6. CPCTC
7. ABCD is a parallelogram	7. If opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

X. Summarize that we now have 5 ways to prove that a quadrilateral is a parallelogram.

1. Definition of a Parallelogram
2. Opposite Sides are Congruent
3. Opposite Angles are Congruent
4. Diagonals Bisect each Other
5. One Pair of Opposite Sides is Both Congruent and Parallel

XI. As part of the students' homework assignment, they will attempt to prove the converse of Theorem 6-3: Consecutive angles in a parallelogram are supplementary. The converse of this theorem is not in the students textbook. In addition to this proof will be problems for the students to apply the theorems: p.301 (15-32). If time remains in class, Craig will give the students algebra type practice problems that will require the students to apply the theorems.

Lesson Plan for Lesson #2

Name of Lesson: Introduction to Functions
Students: Block 2 Honors Algebra II Students
Purpose: Functions are used to model real world situations, to understand the relationship between numbers, and to make reasonable predictions of future trends.
Goals: Retain information, motivate learning, and conceptual rather than procedural.

Lesson Plan:
 I. Functions Activity: (Approximately 20-30 minutes) See handout (attached)
Guiding Question: How many marbles can you add to a graduated cylinder 50mL full to raise the water level to 100mL?

II. Display the Results of the Activity on the Board: One student per group will put up their group's graph.

III. Use the data displayed on the board to give students notes on functions. Use Graph #1 to show a mapping of the sets of x values (# of marbles) to the y values (water level). List all values given in the graph. Introduce the terms domain and range and explain how (#of marbles, water level) represents an ordered pair. Jennifer will tell them that this type of relationship between two sets we call a function (formal definition given later).

Next, Jennifer will draw a function machine on the board and explain how you put values into the machine (input, domain, # of marbles), the function or rule causes a change to occur, and then a value comes out (output, range, water level).

Jennifer asks the students: What types of values must we know first?
        Domain = independent variable
        Range (depends on the domain) = dependent variable
        * Water level depends on the number of marbles.
Jennifer asks the students: If you put in 6 marbles, how many different water level measurements are possible?
This will lead her to the formal definition of a function. Every two sets have a relationship. The relationship between any two is called a relation. A function is a type of relation that has the following property: Every element in the domain is paired with exactly one element in the range. For every input there is only one output.

Jennifer gives an example that is not related to the data.
Zip Code Example: Draw a set of people living in the town of Pembroke with zip code 03275. Get the names of students I the class that live in Pembroke. Have the students give the domain and range and write out ordered pairs. For

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example, (Sally, 03275), (Jack, 03275), (John, 03275). Then, extend this example with Sally living in two towns to show an example of a relation that is not a function.

Jennifer asks the students: How can you determine the rule (equation) for the function?
Refer to graph #1 from the students data:

50 + (1*5) = 55
50 + (2*5) = 60
50 + (3*5) = 65
50 + (x*5) = y
50 + 5x = y
y = 5x + 50 is the equation of the function

Jennifer will give the students time to derive the equations of the other three graphs of student data.
Next, Jennifer will discuss function notation.
f(x) pronounced "f of x" means the function f is evaluated at x.
g(a) pronounced "g of a " means the function g is evaluated at a.
Recall the function equation for graph #1, y = 5x + 50. When x is replaced with a value the process is called evaluating a function at x or substituting or inputting.
f(x) = 5x + 50 The function, f, evaluated at x is 5(x) + 50.
f(2) = 5(2) + 50
f(2) = 60   The function f, evaluated at 2 is 60, so 2 is paired with 60 (2,60).
Compare the original function y = 5x + 50 with the function notation f(x) = 5x + 50 and you can see that y = f(x).
Jennifer will go back to the other three graphs and write the functions in functional notation.
Then, Jennifer will discuss the group's predictions for all four graphs.
She will ask f(?) = 100mL.
Then, she will explain how you need the function equation in order to predict things that you cannot physically do like find the water level for 2.5 marbles.
Next question she will ask: Is every relation a function? She will give the students examples of graphs that are functions and non-functions. Then she will see if they can come up with the vertical line test.

IV. Practice Problems: If time remains, Jennifer will give the students examples in which they will determine if the relation is a function. These examples will be given in five different representations.
1. Verbal
2. Numerical (Table)
3. Graph
4. Algebraic (Equation)
5. Mapping

With the remaining class time, the students will start on the homework assignment from the textbook: p.71-72 #22,23,24,30-32,33,(49-54),(59-63).

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Functions Activity – Section 2-1

Name:_____Date:_____

Follow the instructions carefully. Fill in the appropriate information and answer the questions as you complete the lab.

1. Your graduated cylinder is full of 50 mL of water.

2. Predict how many marbles, nickels, pennies, or stones you would need to place in the graduated cylinder to make the water level reach 100mL.

3. Place 1 marble, nickel, penny, or stone in the graduated cylinder.

4. Fill in the chart according to the number of marbles, nickels, pennies, or stones in the graduated cylinder.

# of marbles, nickels, pennies, or stones	Water Level

5. Create a graph of you data below and prepare to put your graph on the board.

Lesson Plan for Lesson #3

Name of Lesson: Applications of Slope and Linear Functions
Students: Block 1 Integrated Math IV Students
Essential Question: How can we use our knowledge of slope, y-intercepts, and linear equations to answer real-life questions?

Agenda:
- Introduction
- Essential Question
- Example Problem-Using Template
- Practice Using Template
- Brief Student Presentations
- Create a New Linear Application
- Exchange Applications
- Conclusion

I. Real-Life Example Problem
Your mission –should you decide to accept it- is to use the only forensic evidence available – the left femur- and correctly estimate the height of the deceased person . . .
And within an hour contact *Time Magazine* with the breaking news.

II. Example Problem
- Estimate the height of a deceased person using a femur length of 43 cm.
- Two previously discovered skeletons provided the following information:
    Skeleton 1: 40 cm femur → 162 cm height
    Skeleton 2: 45 cm femur → 173 cm height
  Follow the steps on the linear equation template.
(Use Overhead Transparency)
- Write the desired goal, and tell what the variables x and y represent.
- Write the given data as ordered pairs.
- Find the slope of the line through the two known points.
- Find the y-intercept.
- Write the linear equation.
- Make a prediction using the equation.

III. Practice Problem #1
- Estimate the height of the deceased person using the actual femur
- Two previously discovered skeletons provided the following information:
    Skeleton 1: 38 cm femur → 158 cm height
    Skeleton 2: 42 cm femur → 166 cm height
  Follow the steps on the linear equation template.
IV. Practice Problem #2

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- Your cell phone has a monthly plan with a minute allowance during peak time hours, but you have forgotten other details of the plan.
- You remember that in February you used 16 minutes over your peak time allowance, and the monthly bill was $30. You also remember that in March you used 40 minutes over your peak time allowance, and the monthly bill was $36.
- Determine you cell phone bill when you use 2 hours over your peak time allowance.
- Follow the steps on the linear equation template.
- There will be brief student presentations of this problem worked out at the board.

V. Create a New Real-Life Problem (in groups of 2-3)
- Use the <u>Create a Word Problem</u> handout
- Brainstorm topics of interest . . .
- Determine the two key variables – which variable depends on the other?
- X represents _____ and Y represents _____.
- Create the two ordered pairs.
- What can be predicted?
- Write the problem in your own words . . .

VI. Exchange Applications (if time permits)
- The students will exchange their application problem with another group's problem and solve each other's problem.
- If not enough time remains, then this will be their homework assignment.

VII. Conclusion

<u>Original Cell Phone Problem Created by Lisa</u>
Your cell phone has a monthly plan with a minute allowance during peak time hours, but you have forgotten the details of the plan. You remember that in February you used 16 minutes over your peak time allowance, and the monthly bill was $30. You also remember that in March you used 40 minutes over your peak time allowance, and your monthly bill was $36. Write the equation that describes your monthly cell phone bill.
What does the y-intercept represent?
What is the minimum cost of your cell phone bill?
What does the slope represent?
How much does the company charge you for spending over your peak time allowance?
Determine the cell phone bill when you use two hours over your peak time allowance.
If your cell phone bill is $89, how many minutes over your peak time allowance did you use?

<u>Original Hiking Problem</u>
You are hiking on a trial when you notice the 6-mile marker. You look at your stopwatch and you have been hiking for 2 hours. Later, you pass the 14-mile marker and your stopwatch tells you that you have been hiking for 7 hours. Write the equation that describes time as a function of your hiking distance. If you hike for 20 hours, how far will you have traveled? What does the slope represent? What is your hiking rate? What does the y-intercept represent? How many miles had you hiked when you started the stopwatch?

Linear Equation Template

Lesson Goal:
To accurately model a real-life linear situation.

Skills to be Acquired:
1) To write a linear equation using real-life data.
2) To use a linear equation to make a prediction.

Problem Statement:

Overall Solution	Write a linear equation to model . . . .
Step 1 (Points)	Write the given data as ordered pairs (points).
Step 2 (Slope)	Find the slope of the line through the two known points.
Step 3 (y-intercept)	Find the y-intercept.
Step 4 (Linear Equation)	Write a linear equation.
Step 5 (Prediction)	Estimate . . .

Create a Word Problem For Someone Else to Solve

I. Brainstorm (and choose) Topics of Interest:	II. Determine two variables that have a relationship (correspond):
III. Which variable depends on the other? _____depends on _____	IV. Since y <u>depends on</u> x, x represents _____ y represents _____ Write as an ordered pair (_____, _____)
V. Create two reasonable ordered pairs. (,) (,)	VI. What can be predicted using the model created?

Write Your Problem in Words

1. Describe the situation.

2. Provide one piece of information (ordered pair).

3. Provide a second piece of information (ordered pair).

4. Ask for the equation.

5. Ask for a prediction using the model created.

APPENDIX B

IRB AND CONSENT FORMS

Teacher Consent Form

Dear Mathematics Teacher:

I am conducting a research project on Lesson Study as a form of professional development. I am writing to invite you to participate in the study.

If you agree to participate in this project, you will be asked to be a part of a Mathematics Lesson Study Group. Your participation will require approximately 60 hours of your own time after school spread out throughout the 2004-2005 school year. You will be asked to participate in the following activities:

- Provide information about your mathematics background and teaching experience in an interview format and allow the researcher to observe several of your classes throughout the school year. Interviews will be audiotaped for later transcription.
- Participate in two-hour lesson planning sessions which will be videotaped and notes taken while developing lessons will be copied.
- Teach or observe lessons planned by the group and participate in two-hour debriefing sessions after each lesson is taught. These sessions will also be videotaped.
- Reflect on all the planning, teaching/observing, and debriefing sessions in a journal. The researcher will make copies of the journals.

Since the researcher will be a member of the Lesson Study group, she will also participate in all of the above activities.

Compensation for participation in this project will be in the form of professional development. You can apply for credit for the professional development hours by filling out the appropriate paper work for your school district.

Participation is strictly voluntary; refusal to participate will involve no prejudice, penalty, or loss of benefits to which you would otherwise be entitled. If you agree to participate and then change your mind, you may withdraw at any time during the study.

The researcher seeks to maintain the confidentiality of all data and records associated with your participation in this research. You should understand, however, there are rare

instances when the researcher is required to share personal-identifiable information (e.g., according to policy, contract, and regulation). For example, in response to a complaint about the research, officials at the University of New Hampshire, designees of the sponsor(s), and/or regulatory and oversight government agencies may access research data. Data (including audiotapes and videotapes) will be kept in a locked cabinet at the University of New Hampshire; only Dr. Sonia Hristovitch and I will have access to the data. Within two years of the dissertation defense, the audiotapes and videotapes will be destroyed.

If you have any questions about this research project or would like more information before, during, or after the study you may contact Melissa Mitcheltree at mkm5@cisunix.unh.edu or 603-895-6730 or Dr. Sonia Hristovitch at Sonia.Hristovitch@unh.edu or 603-862-2027. If you have questions about your rights as a research subject, you may contact Julie Simpson in the UNH Office of Sponsored Research at 603-862-2003.

I have enclosed two copies of this letter. Please sign one indicating your choice and return it to me. The other copy is for your records. Thanks for your consideration.

Sincerely,

Melissa K. Mitcheltree
Department of Mathematics and Statistics
University of New Hampshire

Yes, I _____ agree to participate in this project and be a member of the Mathematics Lesson Study Group in the following ways (initial all that apply):

_____Provide information about your mathematics background and teaching experience in an interview format and allow the researcher to observe several of your classes. Interviews will be audiotaped.

_____Participate in two-hour lesson planning sessions which will be videotaped and lesson developing notes will be copied.

_____ Teach or observe lessons planned by the group and participate in two-hour debriefing sessions after each lesson is taught. These sessions will also be videotaped.

_____ Reflect on all the planning, teaching/observing, and debriefing sessions in a journal. Journals will be copied.

No, I _____ do not agree to participate in this research project.

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Student Consent Form

Dear Student:

Your mathematics teacher is participating in a research project that I am conducting on Lesson Study as a form of professional development. Your teacher will be working with other mathematics teachers to develop lessons, teach and observe the lessons, and reflect on how the lesson went. The focus of this form of professional development is the lesson that is planned, taught, observed, and debriefed. I am writing to invite you to participate.

One or more times throughout the school year, your teacher will teach a lesson planned by the lesson study group. If you agree to participate in this study, all you are asked to do is to allow the researcher to videotape you as part of the class.

You will not receive any compensation to participate in this project; however the anticipated benefit is an increase in our understanding of how Lesson Study can be used as a form of professional development for secondary mathematics teachers.

Participation is strictly voluntary; refusal to participate will involve no prejudice, penalty, or loss of benefits to which you would otherwise be entitled. If you agree to participate and then change your mind, you may withdraw at any time during the study.

The researcher seeks to maintain the confidentiality of all data and records associated with your participation in this research. You should understand, however, there are rare instances when the researcher is required to share personal-identifiable information (e.g., according to policy, contract, and regulation). For example, in response to a complaint about the research, officials at the University of New Hampshire, designees of the sponsor(s), and/or regulatory and oversight government agencies may access research data. Data (including audiotapes and videotapes) will be kept in a locked cabinet at the University of New Hampshire; only Dr. Sonia Hristovitch and I will have access to the data. Within two years of the dissertation defense, the audiotapes and videotapes will be destroyed.

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I have enclosed two copies of this letter. Please sign one indicating your choice and return it to your teacher. The other copy is for your records. Thanks for your consideration.

Sincerely,

Melissa K. Mitcheltree

Department of Mathematics and Statistics

University of New Hampshire

Yes, I _____agree to participate in this project in
the following way (initial):

_____ By allowing myself to be videotaped.

No, I _____ do not agree to participate in this
research project.

IRB Approval

August 20, 2004

Mitcheltree, Melissa K
Mathematics, Kingsbury Hall
27 Glenn Ridge Road
Raymond, NH 03077

IRB #:   3273
Study: Exploring Lesson Study as a Form of Professional Development for
                  Enriching Teacher Knowledge and Classroom Practice
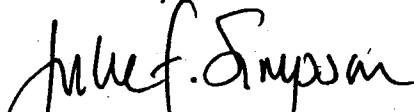**Approval Date:** 08/20/2004

The Institutional Review Board for the Protection of Human Subjects in Research (IRB) has reviewed and approved the protocol for your study as Expedited as described in Title 45, Code of Federal Regulations (CFR), Part 46, Subsection 110.

Approval is granted to conduct your study as described in your protocol for one year from the approval date above. At the end of the approval period, you will be asked to submit a report with regard to the involvement of human subjects in this study. If your study is still active, you may request an extension of IRB approval.

Researchers who conduct studies involving human subjects have responsibilities as outlined in the attached document, *Responsibilities of Directors of Research Studies Involving Human Subjects.* (This document is also available at http://www.unh.edu/osr/compliance/IRB.html.) Please read this document carefully before commencing your work involving human subjects.

If you have questions or concerns about your study or this approval, please feel free to contact me at 603-862-2003 or Julie.simpson@unh.edu. Please refer to the IRB # above in all correspondence related to this study. The IRB wishes you success with your research.

For the IRB,

Julie F. Simpson
Manager

cc: File
       Sonia Hristovitch

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IRB Approval

August 20, 2004

Mitcheltree, Melissa K
Mathematics, Kingsbury Hall
27 Glenn Ridge Road
Raymond, NH 03077

IRB #:   3273
Study: Exploring Lesson Study as a Form of Professional Development for
                Enriching Teacher Knowledge and Classroom Practice
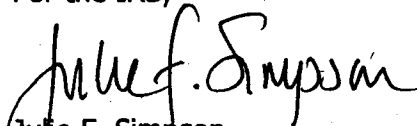**Approval Date:** 08/20/2004

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If you have questions or concerns about your study or this approval, please feel free to contact me at 603-862-2003 or Julie.simpson@unh.edu. Please refer to the IRB # above in all correspondence related to this study. The IRB wishes you success with your research.

For the IRB,

Julie F. Simpson
Manager

cc: File
         Sonia Hristovitch