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The effects of providing mathematical problem - posing experiences for K--8 pre-service teachers: Investigating teachers' beliefs and characteristics of posed problems

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**The Effects of Providing Mathematical Problem Posing
Experiences for K-8 Pre-Service Teachers: Investigating
Teachers' Beliefs and Characteristics of Posed Problems**

BY

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DISSERTATION

Submitted to the University of New Hampshire
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

in

Mathematics Education

May 2003

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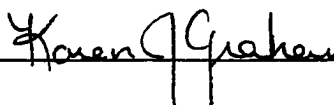
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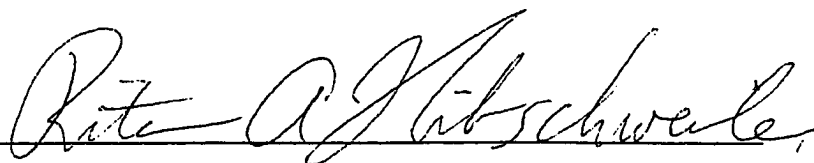
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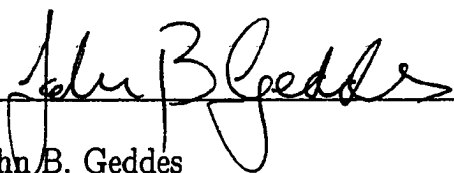
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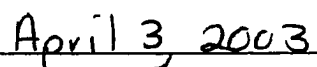
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Dedication

This work is dedicated with all my love to my parents. The emotional and financial support I have received throughout my ten years of schooling has been more than anyone could ask for. Everything herein is written for you. Thank you!!

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ABSTRACT

The Effects of Providing Mathematical Problem Posing Experiences for K-8 Pre-Service Teachers: Investigating Teachers' Beliefs and Characteristics of Posed Problems

by

Todd A. Grundmeier
University of New Hampshire, May, 2003

This study incorporated problem posing into a mathematics content course for pre-service elementary and middle school teachers by extending George Polya's (1957) problem solving heuristic to include problem re-formulation and by having participants pose problems from sets of given information. The course provided pre-service teachers with a new mathematical perspective and this research examined participants' problem posing, beliefs about mathematics, and beliefs about the teaching and learning of mathematics.

Study participants were enrolled in a mathematics content course for pre-service teachers at the University of New Hampshire. There were twenty students in the course and nineteen agreed to be participants in the study by allowing all of their course work to be collected. Participants consisted of 4 sophomores, 7 juniors, 6 seniors, and 2 graduate students. All participants were working towards their teaching certification and most were mathematics education majors. Four of the nineteen participants agreed to be interviewed three times each during the semester.

Characteristics of participants' posed problems, beliefs about mathematics, and beliefs about teaching and learning mathematics were explored using researcher developed questionnaires that were given before and after the semester. Also, all student work, journal entries, and the interviews of four participants, which were focused on topics related to beliefs about problem posing, characteristics of posed problems, beliefs about mathematics,

and beliefs about teaching and learning mathematics, were collected during the instructional treatment.

Problems posed by participants were organized and coded using a quantitative scale, while journal entries and interview data were analyzed qualitatively. Results showed an increase in participants' problem posing efficiency and ability to pose multi-step problems. Also participants tended to utilize higher level problem re-formulation techniques as the instructional treatment progressed. Throughout the instructional treatment participants were reflecting on the role of problem posing in teaching and learning mathematics and considering both the pros and cons of including problem posing in their future classrooms and its possible effect on student learning. All participants suggested that they would incorporate student problem posing in their classrooms to help students develop ownership of mathematics and exhibit creativity.

Chapter 1

Overview

Purpose

This research examined the effects of incorporating problem posing into a mathematics content course for pre-service teachers. In particular, prospective teachers were given the opportunity to view mathematics from the perspective of a problem poser. The purpose of this research was two-fold. The first purpose was to extend Polya's problem solving heuristic to a fifth step, "Pose a related problem," as an initial incorporation of problem posing, and then to have participants pose problems from sets of given information (Polya, 1957). The second purpose was to examine how this experience influenced these pre-service teachers' problem posing, beliefs about mathematics and beliefs about the teaching and learning of mathematics.

A researcher developed, written assessment of participants' problem posing and beliefs about mathematics was administered pre- and post-instructional treatment (see Appendix B). The researcher also developed a five-step problem solving heuristic, journal prompts, and sets of given information to be utilized in data collection. The problem posing measure was based on the work of Leung and Silver (1997) and to make the items relevant to the participants, they were related to situations that may occur in their everyday lives. The measure was utilized to document characteristics of participants' problem posing pre- and post instructional treatment. The measure of beliefs included a word list, short answer questions about mathematics

and mathematics teaching, and an item related to problem posing (Cope, 1988). The beliefs measure was also utilized to document participants' beliefs both pre- and post-instructional treatment. A five-step problem solving heuristic, similar to Polya's (1957) four-step heuristic, was developed by the researcher including the fifth step—"Pose a related problem." The five-step heuristic was the participants' first introduction to problem posing during the instructional treatment. Participants responded to eight journal prompts throughout the semester by writing their mathematics autobiography, reflecting on class activities and assignments, and reflecting on their beliefs about problem posing. The researcher wrote sets of given information based on the mathematics content being covered in the course that were utilized on homework and in journal entries as problem posing exercises for participants.

Why Problem Posing?

Mathematicians develop the field of mathematics by making conjectures and posing mathematical problems. Research mathematicians are "problem posers". New mathematics research is typically generated as research mathematicians pose or conjecture a mathematics problem and attempt to solve that problem and problems associated to it (Kilpatrick, 1987; Silver, 1994). This process of mathematics research, including problem posing, is the basis for the continued development of mathematics. Since problem posing is the basis for the future of mathematics, then it makes sense that mathematics students, including pre-service mathematics teachers, should experience the problem posing process early in their mathematics education. In order to provide this early experience, pre-service teachers must understand the role of problem posing in the development of mathematics and be better prepared to help their future students understand this process and to engage their future students in problem posing.

Influenced by the role problem posing plays in the development of mathematics

the National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), and mathematics education researchers have suggested that posing mathematics problems should become a regular feature of mathematics classrooms and curricula (Silver, 1994; Lampert, 1990). Problem posing has the potential to allow students to exhibit creativity, and can free the student and the teacher from viewing the textbook as the sole authority in the mathematics classroom (Silver, 1994). "Posing problems comes naturally to young children Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds" (p.53 NCTM, 2000). Also, through work in his fifth grade classroom Winograd (1992, 1997) showed that students are capable of posing mathematics problems, judging the quality of posed mathematics problems, and posing problems that challenge themselves and their peers.

As is demonstrated above, educators have started to realize the importance of mathematical problem posing (NCTM, 2000; Kilpatrick, Swafford, & Findell, 2001). Also mathematics education research has addressed student problem posing as well as instructional situations that utilize problem posing (Leung, 1993; Leung & Silver, 1997; Silver, Mamona-Downs, Leung, & Kenney, 1996; Silver & Mamona, 1989; Winograd, 1992; Median & Santos, 1999; English, 1998a; Gonzales, 1994, 1998; Perez, 1985; Schloemer, 1994). Results from this research will be summarized here to help justify the necessity of problem posing in mathematics classrooms and curricula and hence with pre-service teachers. A complete literature review can be found in Chapter 2. An introduction to research on individuals' problem posing will be provided here, followed by an introduction to research and writing on incorporating problem posing in mathematics classrooms.

Several research studies into students' problem posing have focused on pre-service and in-service mathematics teachers (Leung, 1993; Leung & Silver, 1997; Silver et al.,

1996; Silver & Mamona, 1989). Silver and Mamona (1989) and Silver et al. (1996) examined the problem posing of middle school teachers and pre-service elementary mathematics teachers using the Billiard Ball Mathematics (BBM) task format. These researchers asked in-service and pre-service teachers to pose problems related to the mathematics of billiards. In both studies participants were asked to generate problems before and after solving a problem within the BBM task format. Both studies showed that participants were able to generate mathematics problems related to the task format before and after problem solving. Silver and Mamona (1989) showed that there were qualitative differences in the problems posed pre- and post-problem solving activity, while Silver et al. (1996) showed that posed problems did not always have “nice” solutions and were not always solvable. Leung and Silver (1997) and Leung (1993) examined the problem posing of prospective elementary teachers and explored the role of task format, mathematics knowledge and creative thinking in problem posing. Leung and Silver (1997) found that most subjects were able to pose mathematics problems but performance was better when the problem posing situation contained numerical information. Leung (1993) showed that there was a relationship between mathematics knowledge and problem posing ability and that more creative students, as measured by the Torrance Test of Creative Thinking, tended to produce more problems with added information and story components. These research studies show that pre-service teachers are willing to, and have the ability to, pose mathematics problems and thus are a potential audience to benefit from having problem posing activities incorporated into their education.

This study incorporated problem posing by extending Polya’s (1957) problem solving heuristic and having participants pose problems from sets of given information. Whereas past research and writing in mathematics education has explored the incorporation of problem posing in mathematics classrooms and curricula, this study not

only explored participants' problem posing, but also explored pre-service teachers' beliefs about mathematics and the relationship between problem posing and school mathematics.

Research into instructional situations utilizing problem posing has addressed audiences from elementary school students to pre-service teachers (Winograd, 1992; Median & Santos, 1999; English, 1998a; Gonzales, 1994, 1998; Perez, 1985; Schloemer, 1994). Winograd (1992) adapted student authored problems that reflected the students' personal interests and experiences into his fifth grade curriculum. The results suggested that students were capable of posing mathematical problems that challenged themselves and their classmates. Students in Winograd's classroom also believed that posing challenging problems and working to solve and understand them defined a good mathematics student. English (1998b) reports on the results of a three year study implementing problem posing programs in third, fifth, and seventh grade. English (1998b) incorporated problem posing into mathematics instruction and showed that after instruction including problem posing students displayed improvement in their abilities to pose problems from open-ended situations. Also the majority of students in English's (1998b) study felt that their problem solving and problem posing abilities had improved after instruction rich in problem posing. In her dissertation Schloemer (1994) explored the integration of problem posing in high school advanced algebra. Schloemer (1994) found that it was feasible to incorporate problem posing with this audience and grade level and that instruction went as planned. Schloemer's study also showed that the students enrolled in the problem posing class had the same achievement level, as measured by pre- and post-test, as the students enrolled in the course without problem posing.

Medina and Santos (1999) integrated problem posing into a pre-calculus course by asking students to pose questions and re-formulate problems. The study showed

that pre-calculus students were initially apprehensive about posing problems but that they eventually understood and felt comfortable with the concept. The authors report that problem posing allowed them to view their students' strengths, weaknesses, and difficulties with mathematical resources and ideas. Perez(1985) incorporated problem posing into a community college algebra course. Perez (1985) provided students with models for generating problems and found that ninety percent of students were able to generate and solve problems following these models. Perez (1985) also showed that students' attitudes toward word problems and problem solving improved during the course.

Gonzales (1994, 1998) examined the incorporation of problem posing in instruction for pre-service teachers. Gonzales (1994) describes a scheme which included posing related problems and posing story problems to incorporate problem posing with pre-service elementary and middle school teachers. Gonzales (1994) found that pre-service teachers could be guided through a transition from problem solver to problem poser and based on this transition called for the increased use of problem posing with this audience. Gonzales (1988) describes a "blueprint" to help teachers and teacher educators include problem posing in their classrooms. The "blueprint" starts with posing related problems and after exposure to problem re-formulation asks students to generate problems from sets of given information.

The present study will incorporate some of the aspects of this blueprint and the prior work of Gonzales (1994, 1997) but will extend this work by formally researching the effects of this incorporation of problem posing. The above introduction to research highlights two points,

1. Pre-service and in-service teachers are capable of posing mathematical problems.
2. The inclusion of problem posing into mathematics curricula and instruction is feasible and may have benefits for students.

In order for pre-service teachers to effectively incorporate problem posing in their future classrooms as suggested by Winograd (1992), English (1998b) and others, I believe it is necessary for these teachers to experience problem posing in their mathematics education.

Working Definitions

In this study problem posing took two forms: 1) the generation of new problems; and 2) the re-formulation of given problems (Silver, 1994). It is important to define a number of terms and how these ideas were utilized for the purpose of this research; statement, problem, problem re-formulation, problem generation, problem posing product, and pre-service teacher.

Statement: A statement will refer to the outcomes of participants problem posing tasks. Statements are all text that is produced as a response to a problem posing task and is not necessarily a mathematics problem or question.

Problem: A mathematical statement for which a valid solution exists.

Problem re-formulation: The process of posing a problem related to a problem that is or was the focus of problem solving. Re-formulation techniques include extending the original problem, changing the context of the original problem, switching the given and wanted information, changing the given, and changing the wanted.

Problem generation: The process of posing a problem based on a set of given information. Generated problems may include additional information to the original set but must be related to the original set of information.

Problem posing product: A mathematical statement posed through problem re-formulation or problem generation. The statement either relates to the original

problem or utilizes the information from the set of given information in problem generation. Participants' problem posing products will be the focus of data analyses related to problem posing.

Pre-service Teachers: Students who have not previously been middle or elementary school teachers and are seeking certification to teach. In this study the participants were pre-service teachers enrolled in "Topics in Mathematics for Teachers".

Instructional Treatment

The instructional treatment was decided upon by both the researcher and classroom instructor. The instructor provided a classroom setting that was rich in student-to-student interaction and whole class discussion. The researcher and instructor discussed opportunities to incorporate problem posing within this environment. Problem posing was incorporated through class projects, homework, and journal writing. Chapter 4 will discuss the instructional treatment in more detail and will provide examples of problem posing situations.

Questions

There are five questions that were the focus of data collection and analysis in this dissertation research.

1. What are the characteristics of pre-service teachers' problem generation products pre- and post- instructional treatment?
2. How do the characteristics of pre-service teachers' problem re-formulation and problem generation change over the course of the instructional treatment?
3. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about mathematics?

4. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about the teaching and learning of mathematics?
5. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about the relationship between problem posing and school mathematics?

Insight into the first question was gained by student responses to the researcher-developed measure of problem posing. Qualitative analysis of posed problems will help address the second question including student problem re-formulation and problem generation on homework assignments and in journal responses. The final three questions were addressed through qualitative analysis of student work throughout the semester, including all journal entries. The analyses of data helped determine the effects of this integration of problem posing and suggest future directions for research and the incorporation of problem posing in mathematics instruction.

Organization of Dissertation

Data was collected throughout the semester from the whole class and further from the four individuals who agreed to be interviewed three times during the semester. Data included all student assignments and interviews with the four volunteers. The remainder of the dissertation will be organized around the whole class data analyses and the analyses of the data from the four volunteers.

Chapter Two is an in-depth literature review related to problem posing, problem solving, and students' beliefs about mathematics and mathematics teaching and learning. Chapter Three focuses on the theoretical perspective from which the researcher approached this project. Chapter Four provides a detailed description of the research methodology utilized, including research design, methods, data coding

and data analysis. Chapter Five focuses on the results of the data analysis related to participants' problem posing. Chapter Six focuses on results related to participants' beliefs about mathematics, beliefs about the teaching and learning of mathematics, and beliefs about the relationship between problem posing and mathematics education. Chapters Five and Six focus first on results related to the whole class and then on the individual results related to the four students who agreed to interviews. Finally Chapter Seven concludes with a discussion of the results in Chapters Five and Six, suggestions of implications for classroom instruction, and possible directions for further research.

Chapter 2

Literature Review

Literature is reviewed in three broad subject areas that helped shape this study: literature related to problem posing; literature related to teacher preparation and teachers' beliefs; and literature related to the theoretical framework presented in Chapter 3. The review concludes with a specific discussion of the relationship of the literature to the study.

Problem Posing

Kilpatrick (1987) asked the question "Problem formulation: Where do good problems come from?" in a chapter that he contributed to Schoenfeld's book *Cognitive Science and Mathematics Education*. Kilpatrick (1987) opens his chapter with these statements:

If we change the question in the title to Where do good mathematics problems come from?, the answer ought to be readily apparent to any competent high school graduate. Mathematics problems obviously come from mathematics teachers and textbooks, so good mathematics problems must come from good mathematics teachers and good mathematics textbooks. The idea that students themselves can be the source of good mathematics problems has probably not occurred to many students or to many of their teachers. (p.123 Kilpatrick, 1987)

Kilpatrick (1987) continued to discuss sources of mathematics problems, the structure of mathematics problems, possible instruction in problem “formulation”, and how to understand and develop problem-formulation abilities. Kilpatrick (1987) also suggested that instruction that is rich in problem-formulating and requires students to become problem posers is a necessity throughout one’s mathematics education.

Following Kilpatrick’s (1987) work, mathematics educators have begun to suggest the inclusion of problem posing in mathematics classrooms and curricula (NCTM, 1991, 2000). Research within the mathematics education community also has started to focus on the importance of problem posing (Silver, 1994). This research has played a role in recent suggestions for incorporating problem posing in mathematics classrooms and curriculums. In the 1991 document *Professional Standards for Teaching Mathematics*, it is stated that, “students should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem” (p.95 NCTM, 1991).

In his 1994 paper, “On mathematical problem posing,” Silver goes into detail about possible benefits of problem posing, the necessity of problem posing in the school mathematics curriculum, and possible future directions for problem posing research. Silver (1994) suggests that problem posing should be a key feature of inquiry-oriented mathematics classrooms and that “problem posing has figured prominently in some inquiry-oriented instruction that has freed students and teachers from the textbook as the main source of wisdom and problems in a school mathematics course” (p.21 Silver, 1994). Similarly, NCTM supports problem posing as a feature of inquiry-based mathematics classes in which students are given the opportunity to determine the validity of mathematics (NCTM, 1989, 2000).

In *Principles and Standards for School Mathematics* (2000), it is stated,

Posing problems comes naturally to young children: I wonder how long

it would take to count to a million? How many soda cans would it take to fill the school building? Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students' problem solving dispositions by creating and maintaining classroom environments, from pre-kindergarten on, in which students are encouraged to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop confidence in their abilities and a willingness to engage in and explore problems, and they will be more likely to pose problems and persist with challenging problems.

(p.53 NCTM, 2000)

As students are determining the validity of their mathematics, as suggested by NCTM, they are assuming the role of mathematics expert. Problem posing has the potential to encourage students to assume the role of expert when they are posing mathematics problems. "Problem posing requires that the subject perform the job of the expert in constructing a suitable problem, a job that entails combining a viable story line with the appropriate surface features in ways that embody specific concepts" (p.160 Mestre, 2000). Assuming the role of expert allows students to view mathematics from the perspective of a mathematician while they are engaged in problem posing.

Silver (1994) also discusses possible benefits of problem posing for mathematics education researchers and suggests possible future problem posing research,

First, it is clear that problem posing tasks can provide researchers with both a window through which to view students' mathematical thinking and a mirror in which to see a reflection of students' mathematical experiences. Second, problem posing experiences provide a potentially rich arena in which to explore the interplay between the cognitive and affective

dimensions of students' mathematical learning. Finally, much more systematic research is needed on the impact of problem posing experiences on students' problem posing, problem solving, mathematical understanding and disposition towards mathematics. (p.25 Silver, 1994)

Marion Walter and Stephen Brown have written about many issues related to problem posing and its inclusion in school mathematics and college mathematics (Brown & Walter, 1983, 1993). Brown and Walter have discussed the necessity of incorporating problem posing in mathematics curricula, the relationship between problem posing and problem solving, and situations which can foster problem posing (Walter & Brown, 1977; M.I.Walter, 1993b, 1993a). Walter (1993) wrote about the diverse situations that could promote students involvement in mathematical problem posing including posing problems from scrap material, doing problems in multiple ways, posing problems from pictures, and extending given problems (M.I.Walter, 1993b). In her discussion, Walter (1993) gives examples of posed problems and specific situations that educators could use to foster student problem posing. Out of these discussions by Brown and Walter have come suggestions that problem posing might allow students to better understand their style of thinking, attitude towards working with others, the purpose of studying mathematics, and the nature of mathematics (Brown & Walter, 1993).

Brown and Walter discuss in detail their "What if not?" problem posing technique in *The Art of Problem Posing* (1983). Brown and Walter's "What if not?" problem posing technique engages students in problem re-formulation and asks students to consider new problems based on changing the given information of a problem. While discussing "What if not?" Brown and Walter (1983) give examples of utilizing "What if not?" to generate new problems and help students develop a deeper understanding of mathematics.

Based on Brown and Walter's "What if not?" problem posing technique Friel and Gannon (1995) gave an example of the possible outcomes when students are engaged in problem re-formulation. Friel and Gannon (1995) gave their students a word problem that had an algebraic solution. Through class discussion and activity students were able to re-formulate the problem and engage in mathematics well beyond the scope of the original problem (Friel & Gannon, 1995). Friel and Gannon's (1995) example demonstrated that students are capable of re-formulating mathematics problems.

The remainder of this section will focus on literature related to understanding students' problem posing and literature related to implementing problem posing into mathematics classrooms and mathematics curricula. Literature will be presented related to the problem posing of both pre-college and college students. Literature that examined attempts to incorporate problem posing in mathematics classrooms and in the development of mathematics teachers also will be discussed.

Problem Posing in Mathematics Classrooms

English (1997) discusses possible situations in the mathematics classroom that can be transformed into problem posing situations. One of the main motivations for English (1997) to include problem posing in the classroom was that "...it can empower *all* children to explore problem situations and to pursue lines of inquiry that are personally satisfying. This atmosphere creates a context for more productive and enjoyable mathematical learning" (p.173 English, 1997). English (1997) suggests situations that will allow students to engage in problem re-formulation including magic squares and game situations and concludes that students will acquire an inquisitive disposition and become empowered from their problem posing experiences. Many mathematics educators have implemented problem posing into school classrooms and explored students' problem posing (Winograd, 1992; Winograd & Higgins, 1994 and

1995; Winograd, 1997; Silver & Cai, 1996; Schloemer, 1994; Perez, 1985; Median & Santos, 1999; English, 1998b; Grundmeier, 2002). This section will discuss literature related to problem posing with K-12 students, followed by literature related to problem posing with college students.

Winograd (1992, 1997) has utilized problem posing in fifth grade classrooms. Winograd (1992) asked fifth grade students to write and solve their own original math story problems and then share the results in group interactions with their peers. These fifth graders were able to pose mathematics problems that challenged themselves and their classmates, as well as problems whose solutions required mathematical content beyond fifth grade mathematics (Winograd, 1992). Student story problems fell into the following four categories, (1) problems containing new mathematical concepts, (2) problems that require knowledge of a particular mathematical procedure for solution, (3) problems that require problem solving skills that students do not possess yet, and (4) problems the students understand but tend to make errors on during the solution process (Winograd, 1992). These problem posing activities also provided insight into students' beliefs about mathematics,

According to students, the "good" math student was someone who wrote interesting and challenging problems and then worked diligently at understanding and solving those problems. Students believed that the "good" story problem was challenging, included interesting content from everyday life, and contained non-routine characteristics, such as extra information.

(p.65 Winograd, 1992)

Winograd also showed that fifth graders are able to share their mathematics problems and are willing to solve problems posed by their peers (Winograd, 1997). Winograd (1997) provides examples of students' posed problems and gives suggestions to help teachers implement student problem posing in their classrooms. Winograd and Hig-

gins (1994/1995) also discuss the use of student authored story problems with elementary school students. Students in this classroom seemed to use problem posing to reflect on their mathematical experiences and were more inclined to be patient solving problems that were their own or peers' posed problems instead of textbook problems (Winograd & Higgins, 1994 and 1995).

English (1998b) reports the results of the final year of a three year study that implemented problem posing in third, fifth, and seventh grade classrooms. The goals of the research during the final year of the project were to explore the problem posing of seventh grade students, describe the development of students mathematics achievement, identify connections between students' problem solving and problem posing abilities, and monitor students' metacognitive activity (English, 1998b). Students were involved in a three month problem posing program that was intended to foster an inquiry-oriented classroom community. English (1998b) conducted in-depth observations of 23 students chosen because of their performance on pre-measures of problem solving ability and number sense. During the problem posing program students were asked to pose problems from sets of given information. Students showed improvement on this problem posing task. On the pre-program assessment several students could not pose mathematics problems and there were many non-solvable mathematics problems posed. Post-program, every student was able to pose mathematical problems and the number of unsolvable problems decreased. Also the complexity of posed problems seemed to increase from pre-program to post-program problem posing (English, 1998b). During the problem posing program, students also were shown sample mathematics problems and asked to pose problems related to them. There was also an increase in students' ability to perform this task measured by the number of related problems they were able to pose (English, 1998b). Upon completion of the problem posing program, 68% of the students felt that they had become bet-

ter problem posers and problem solvers. English (1998b) believes that continuing to promote inquiry oriented classroom environments, that include problem posing, will cause "...genuine improvement in students' dispositions towards, confidence in, and enjoyment of mathematics" (p.17 English, 1998b).

Silver and Cai (1996) examined the results of 509 middle school students problem posing. Students posed problems from a set of given information and their posed problems were examined for solvability and complexity. (Silver & Cai, 1996). A goal of this study was to develop a scheme to be used to examine problems posed by middle school students. The researchers coded more than 70% of the responses as mathematical questions and more than 90% of the mathematical questions as solvable. The results suggest that even without prior experience with problem posing students have a capacity for posing mathematics problems (Silver & Cai, 1996).

In her dissertation research Schloemer (1994) integrated problem posing into the UCSMP (University of Chicago School Mathematics Project) advanced algebra curriculum with tenth and eleventh grade students. The purpose of Schloemer's (1994) work was to determine the feasibility of the integration of problem posing and to examine changes in students' mathematical dispositions, problem posing performance and mathematical achievement. Schloemer (1994) examined these variables using a pre-test, post-test design and compared results between a class that was introduced to problem posing and a class that was not introduced to problem posing. Schloemer's (1994) results indicate that it was feasible to incorporate problem posing and that the design features were successfully incorporated into lesson plans and instruction. Comparisons found that students' mathematics achievement was equivalent in the two classes and that in both classes, mathematical dispositions decreased, as measured by an assessment of disposition based on attitude scales (Schloemer, 1994). Schloemer (1984) concludes that in this situation the incorporation of problem posing did not

hinder student achievement.

In his dissertation research Perez (1985) examined the effects of experience with problem posing on students' problem solving performance with 52 students in a community college remedial algebra course. Perez (1985) developed a set of activities based around the theme of "student-generated problems." After completing these activities he utilized them to help teach participants to write and solve word problems. Perez (1985) found that more than 90% of the students were able to write word problems based on the examples and activities that were provided. As a second conclusion, it was noted that if students could write word problems they generally could solve a similar problem. Post instruction, the general feeling of participants was that writing word problems had increased their ability to solve problems. Perez (1986) presented an initial look at problem posing with the intent of advancing discussion and beginning research related to problem posing.

Medina and Santos (1999), in a pre-calculus class, explored the implementation of problem posing, through problem generation and problem re-formulation. They found that students did not fully engage in problem posing tasks initially, but that as they gained experience they became confident that they could pose problems. Student problems were initially of a procedural nature but throughout the semester became more complex and profound (Median & Santos, 1999). The authors were able to utilize students' posed problems to determine their mathematical strengths and weaknesses and utilize this information to better understand student difficulties (Median & Santos, 1999).

Grundmeier (2002) examined university pre-calculus and mathematical proof students' problem posing and attitudes towards mathematics. Students were asked to complete a measure of problem posing ability based on the work of Leung and Silver (1997) and a measure of attitude towards mathematics based on Aiken's (1974)

attitude scales . Students enrolled in mathematical proof had statistically significantly more positive attitudes towards mathematics ($p=.001$) than the pre-calculus students, but there were no significant differences in their problem posing abilities (Grundmeier, 2002). On average, subjects were able to pose problems totalling six steps in twenty-five minutes of problem posing. Grundmeier (2002) suggested that the lack of difference in participants' problem posing may have been due to both populations' lack of problem posing experience.

Problem Posing and Problem Solving

Also vital to the development of mathematics and teaching and learning mathematics is the exploration of the relationship between problem posing and problem solving. In "How to Solve It" Polya suggests problem posing as a tool to be used in problem solving. Polya suggests that students can shed light on problem solutions by posing and solving related problems and more general versions of the problem at hand. "Probably the most frequently cited motivation for curricular and instructional interest in problem posing is its perceived potential value in assisting students to become better problem solvers" (p.23 Silver, 1994). Walter and Brown (1977) gave an example of the possible relationship between problem solving and problem posing through the following problem; *Given two equilateral triangles, find a third whose area is the sum of the area of the other two* (p.4 Walter & Brown, 1977). This problem gives students many options for problem re-formulation. The student can re-pose the original problem (i.e., adding numbers) or can extend the given problem. Students may also need to pose related problems in order to shed light on the original question. Walter and Brown (1977) also give examples of how the problem can be extended to other shapes and to show a relationship to the Pythagorean theorem. Walter and Brown (1977) conclude with a discussion of the benefits of problem posing and state, "it is worthwhile for students to investigate all the different ways in which the "given"

can be interpreted as well as how the analysis might depend on different allowable assumptions" (p.12 Walter & Brown, 1977).

Problem posing has the potential to influence students' problem solving abilities. Although documenting this relationship is not a goal of this research it is important to mention because the majority of participants suggested that incorporating problem posing in their future classrooms may have positive effects on their students problem solving ability.

Problem Posing with Pre-service Teachers

Problem posing has been shown to have the potential to effect students' problem solving abilities and dispositions towards mathematics. In order for teachers to feel comfortable and effectively integrate problem posing into their classrooms, as suggested by the literature, it is important for them to experience problem posing during their pre-service education (Silver, 1994; Kilpatrick, 1987). Research has examined the problem posing of pre-service and in-service elementary and middle school teachers, the relationship between problem posing and creativity with this audience, and the effects of problem posing workshops on perspective middle and elementary school teachers. (Silver & Mamona, 1989; Silver et al., 1996; Leung, 1993; Leung & Silver, 1997; Gonzales, 1994, 1998).

Silver and Mamona (1989) and Silver, Mamona-Downs, Leung and Kenney (1996) examined the problem posing abilities of middle school mathematics teachers in the Billiard Ball Mathematics (BBM) task format, which asked participants to solve and pose problems related to the geometry of billiards. Silver and Mamona (1989) asked participants to first pose problems related to BBM, then solve a problem within the task format, and finally to pose more problems. The researchers found that participants could pose reasonable problems within the task format but that there were differences in the problems they posed before and after solving a problem in the

task format (Silver & Mamona, 1989). In particular on the post-solution problem posing task, subjects posed fewer problems that were related to the assumptions implicit in the BBM task format and posed more problems that included specific goals, such as the final pocket in which the ball would rest (Silver & Mamona, 1989). In general Silver and Mamona (1989) showed that participants were capable of posing mathematics problems and that the characteristics of their posed problems differed before and after experience solving problems in the task format. Silver, et al. (1996) examined the problem posing ability of 53 middle school teachers and 28 pre-service secondary school teachers in the BBM task format. The research suggested that these teachers and pre-service teachers had some capacity to pose mathematics problems. Results of this study showed that participants posed many problems which could not be solved by other participants, posed fewer problems during the post-solving posing task, and had a tendency to chain or link posed problems (Silver et al., 1996). From their work the authors hypothesize that, "as teachers become more proficient in their own problem posing, it is reasonable to assume that they will become more willing to have their students engage in such activities" (p.305 Silver et al., 1996).

In her dissertation research Leung (1993) explored the relationship between mathematical knowledge, creativity, and the the problem posing of pre-service elementary school teachers. Leung (1993) used the subjects' scores on the Pre-Professional Skills Test (mathematics knowledge) and the Torrance Test of Creative Thinking (creativity) to develop four groups of 16 subjects each with respect to high and low mathematics knowledge and high and low creativity. A test of arithmetic problem posing (TAPP) was then utilized to examine subjects' problem posing. The problem posing task contained two problem situations, one in a format containing numerical content and the other in a format not containing numerical content. Results suggest that "...high mathematics knowledge subjects produced sets of problems with more

interrelated solution structures; whereas high creative thinking subjects tended to produce more problems with added story components” (p.v Leung, 1993). Based on her results Leung (1993) suggested that problem posing should be included in mathematics curricula. Problem posing will allow students to take a more active role in and be responsible for their learning (Leung, 1993). Leung and Silver (1997) report some results of Leung’s dissertation research and discuss in particular subjects’ abilities to pose more problems and more complex problems when sets of information contained numerical content instead of not containing numerical content.

Gonzales described her attempts to incorporate problem posing into content classes for pre-service elementary and middle school mathematics teachers (Gonzales, 1994). Gonzales suggested that it is feasible to incorporate problem posing with these audiences and has accomplished this by extending Polya’s problem solving heuristic (Gonzales, 1994). Students in Gonzales’s classes were able to extend problems, pose related problems, and pose novel problems. Examples of observations from Gonzales’s paper follow;

Observations made ... appear to indicate that the pre-service teacher gains: (a) a perspective on the important role that language (choice of words) plays in the understanding and interpretation of a word problem; (b) knowledge of mathematical levels appropriate for different grades (K-8) and types of students (remedial to accelerated); and (c) insight into the role of a teacher as a facilitator of knowledge rather than a deliverer of knowledge. (p.83 Gonzales, 1994)

Gonzales also suggested that research should be undertaken to examine the incorporation of problem posing in content classes for pre-service teachers. Gonzales (1998) added to her previous work and presented, but did not formally research, what she called a “blueprint” for the implementation of problem posing in classes for pre-service

teachers. Gonzales's "blueprint" included utilizing a problem solving heuristic, posing related problems, and then having pre-service teachers pose their own problems and tasks. Features of this "blueprint" will be seen in the instructional treatment designed for this study.

Teacher Preparation and Teachers' Beliefs

Teacher Preparation

In their 1996 report, *The Preparation of Teachers of Mathematics: Considerations and Challenges*, the National Research Council (NRC) made suggestions for mathematics teacher preparation. This report suggested that teachers need more than just strong mathematics preparation, they also need "deeper mathematical understanding in order to promote mathematical sense making, problem solving, reasoning, and justification" (p.3 MSEB, 1996). It is necessary for future teachers to experience mathematical inquiry and the practice of mathematics and at the same time for researchers to begin to understand the connections "between how future teachers come to know mathematics and their own practice in the mathematics classroom" (p.6 MSEB, 1996). For teachers and researchers to examine connections between pre-service teachers' knowledge of mathematics and their future teaching it is imperative that pre-service teachers engage in situations that allow them to reflect on their mathematics knowledge and future teaching (Ball, 1996; Goodlad, 1991; Brown & Borko, 1992; Ashton, 1996).

Suggestions made by the NRC in their 1996 report resonate in suggestions others had previously made for teacher education. Goodlad (1991) gives a description of the role of schools in our society and the roles of teachers in these schools,

Schools in our society are called upon to perform two distinctive functions:

(1) enculturate the young into a social and political democracy, and (2)

introduce the young to those canons of reasoning central to intelligent, satisfying participation in the human conversation. If schools are to perform these two functions well, teachers must be thoroughly grounded in the understanding and beliefs necessary for carrying them out. They must (3) learn the pedagogy essential to the enculturation and trait development of the young, and (4) possess the knowledge and skills necessary to participate in the continuous renewal of the schools for which they are stewards. (p.5 Goodlad, 1991)

In the same article on teacher education that is based on conversations with teacher education faculty Goodlad (1991) presents suggestions for redesigning teacher education and calls for a renewed relationship between teacher education classes and classroom practice. Teachers need to see the importance of their education for their future practice and be able to reflect on their practice throughout their preparation.

Brown and Borko (1992) added to the ideas presented by Goodlad (1991) related to the future of teacher education in their chapter from the *Handbook of Research on Mathematics Teaching and Learning*. Brown and Borko (1992) discussed the role of reflection in teacher education and suggested that teachers must reflect on their practice and pre-service teachers must consider and reflect on their future practice. It is important to explore how both inservice and pre-service teachers translate their classroom knowledge into knowledge that is useful in their future classrooms (Brown & Borko, 1992). The authors concluded that “teacher education programs should provide opportunities for growth in content knowledge, pedagogical content knowledge, and pedagogical reasoning” (p.235 Brown & Borko, 1992). Brown and Borko (1992) define “pedagogical reasoning” as the selection of strategies to represent content, and “pedagogical content knowledge” as knowledge about ways to introduce specific content appropriate for all abilities and learning styles.

Based on past research, Ball (1996) suggested that changes in teacher education, especially toward promoting reflective activities, are necessary to help keep pace with the mathematics education reform movement. In particular, Ball (1996) suggests that pre-service and inservice teachers be given the opportunity to reflect on their teaching,

Reflection is seen as central to learning to teach. For the most part, prescriptions for reflection focus on structure and context, emphasizing that teachers need time, space, and encouragement to reflect on teaching in ways that facilitate their learning - by talking with others, keeping a journal, by engaging in action research. Less attention is paid to what the specific objects and the nature of that reflection might be, leaving somewhat up in the air the variety of learning that reflection might support.
(p.501 Ball, 1996)

Also based on past research Ashton (1996) suggested that reflection is necessary to prepare future teachers for their classrooms, but she also extended the idea of reflection and discussed the necessary outcomes of pre-service teachers' reflections. Pre-service teachers must be granted opportunities to develop their ability to reflect on research and practice and think about implications for their future students (Ashton, 1996). Also prospective teachers need dynamic environments "...to develop sophisticated pedagogical content knowledge that will enable them to represent subject matter in multiple and powerful ways that connect with and challenge their students' prior understandings" (p.22 Ashton, 1996).

The literature presented above suggests reflection as a necessity in teacher education. Reflection is a possible tool to help teachers develop what Franke et al. (1998) call "self-sustaining generative" change. "Self-sustaining generative" change "... entails teachers making changes in their epistemological perspectives, their knowl-

edge of what it means to learn, as well as their conceptions of classroom practice” (p.67 Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). Teachers must develop an understanding of their practices in relation to their students learning and they must take part in reflecting on and questioning their practice (Franke et al., 1998). Expanding on these ideas,

A teacher who examines his or her practices in relation to his or her own thinking and the thinking of his or her students engages in a different level of practical inquiry, where the focus is on detailed analysis. As teachers engage in this detailed analysis, they come to understand principled ideas that can then drive their practice and their continued practical inquiry. We view the first level of practical inquiry as leading to self-sustained change but the second level of practical inquiry as necessary for generative change. (p.68 Franke et al., 1998)

Franke et al. (1998) highlight examples of teachers “self-sustaining generative” change through three cases studies of teacher professional development workshops related to Cognitively Guided Instruction (CGI). Through interviews, interactions and observations they found that professional development with a focus on students’ mathematical thinking allows a forum for teachers to develop practical inquiry skills and engage in “self-sustained generative” change.

In summary, teacher preparation literature in education and mathematics education suggests that reflection is a powerful tool for teacher development and is a necessary component of teacher preparation programs. Reflection fosters teachers’ understanding of their conceptions about teaching and learning mathematics.

Teachers’ Beliefs

Many researchers have examined teachers’ beliefs about teaching and teachers’ beliefs about mathematics (Corte, Greer, & Verschaffel, 1996; Good, Grouws, & Ma-

son, 1990; Schuck, 1997; Cooney, Wilson, Albright, & Chauvot, 1998; Battista, 1994; Thompson, 1984, 1992; Karp, 1991). De Corte, et al. (1996) summarize research on teachers beliefs and show that it is possible to “profoundly affect teachers’ cognitions and beliefs about mathematics learning and instruction, their classroom practices, and, most important, their students’ learning outcomes and beliefs.” Also, teachers’ beliefs about mathematics are not static. Teachers’ beliefs begin with their mathematics education, continue to be shaped throughout their pre-service education and continue to change during their practice (Corte et al., 1996). The remainder of this section will discuss teachers’ beliefs about mathematics followed by the relationship between teachers’ beliefs and their teaching practice.

Schuck (1997) used a research simulation with pre-service primary school teachers to develop an understanding of their beliefs about mathematics. Schuck (1997) asked subjects to play the role of both researcher and respondent by interviewing and being interviewed by a peer. This simulation allowed subjects’ beliefs about mathematics to become apparent. Schuck (1997) found that teachers’ beliefs about mathematics are likely to fit into one of the following three categories:

Problem-solving view: Mathematical thought is ever - expanding and fallible and the processes of mathematical thought are embodied in the search for solutions to new problems.

Platonist view: Mathematics is unchanging and learning mathematics is becoming familiar with mathematics that already exists.

Instrumentalist view: Mathematics is a set of rules and procedures that have a particular purpose. Mathematical thought is dominated by understanding algorithms.

Cooney et al. (1998) also reported descriptions of pre-service teachers' beliefs. Results of the Research and Development Initiatives Applied to Teacher Education (RADIATE) Project gave Cooney et al. (1998) insight into subjects' beliefs and they found similarities in pre-service secondary teachers' beliefs before they began a professional development program. Beliefs were reported with regard to mathematical knowledge, teaching strategies, and teacher responsibilities. This sample of pre-service teachers believed that mathematics was a body of knowledge that built on previous knowledge and saw mathematics learning as a linear process. For them mathematics should be understood and not memorized even though these teachers sometimes lacked a deep understanding of concepts (Cooney et al., 1998). With regard to teaching strategies these pre-service teachers' believed that students needed to be active in the learning process and that lecture was not sufficient for meaningful learning, but they did not discuss any ideas about connecting active learning to mathematical ideas. These pre-service teachers felt that it was their responsibility to make mathematics interesting and to engage their students. While engaging their students they wanted to be sure their students were comfortable and avoided frustration (Cooney et al., 1998).

Good et al. (1990) also explored teachers' beliefs about teaching strategies. They surveyed 1509 elementary school teachers with regard to their beliefs about using small groups during mathematics instruction. The study was aimed at describing how elementary teachers organize their classrooms for mathematics instruction and why they make the decisions they do with regard to classroom organization (Good et al., 1990). The results of the study showed that the most predominant classroom organization was whole-class instruction with some amount of time spent with students working alone on assigned work. The second, most frequent, organization of instruction was whole-class instruction followed by some time spent in small-group

work (Good et al., 1990). The teachers most frequently cited reason for utilizing small group instruction was that it allowed the teacher to work with a diverse range of students' needs and allowed the opportunity to present enrichment material.

There seems to be a relationship between teachers' beliefs about mathematics, beliefs about mathematics teaching and learning, and their instructional practice. Karp(1991) documented the teaching behavior and instructional methods of elementary school teachers. Teacher attitude surveys were collected along with tape recordings, observations, field notes, interviews, and class assignments. The data suggested that there were substantial differences between the teaching styles of teachers with positive attitudes and teachers with negative attitudes towards mathematics (Karp, 1991). Teachers with negative attitudes created teacher dependent learning environments in which students were encouraged to be passive learners whereas teachers with positive attitudes created learning environments that promoted student independence (Karp, 1991). Therefore, it is a necessity for teacher education to "...develop programs to help pre- and inservice elementary teachers to recognize and overcome the problem of negative attitudes toward mathematics and the instructional consequences of these attitudes" (p.269 Karp, 1991).

Research has also presented situations where teachers' beliefs about the teaching and learning of mathematics have played a role in shaping their instructional behavior (Battista, 1994; Thompson, 1984). Thompson (1992) reflecting on a theoretical paper by Ernst (1988) states that research on teachers' beliefs "...indicates that teachers' approaches to mathematics teaching depend fundamentally on their systems of beliefs, in particular on their conceptions of the nature and meaning of mathematics, and on their mental models of teaching and learning mathematics" (p.131 Thompson, 1992).

Thompson (1984) and Battista (1994) have shown in their research, that teachers' beliefs about mathematics may play a significant role in shaping their classroom

practice.

Teachers develop patterns of behavior that are characteristic of their instructional practice. In some cases, these patterns may be manifestations of consciously held notions, beliefs, and preferences that act as 'driving forces' in shaping the teacher's behavior. In other cases, the driving forces may be unconsciously held beliefs or intuitions that may have evolved out of the teacher's experiences. (p.105 Thompson, 1984)

Teachers are key to the implementation of the reform movement and,

... many teachers' beliefs about mathematics are incompatible with those underlying the reform effort. Because these beliefs play a critical role not only in what teachers teach but in how they teach it, this incompatibility blocks reform and prolongs the use of a mathematics curriculum that is seriously damaging the mathematical health of our children. (p.462 Battista, 1994)

Battista (1994) gave two examples of teachers' beliefs and how they affect practice. Mary is an elementary school teacher who believes that understanding a mathematical idea means reducing it to a step-by-step procedure. This belief influences the mathematical activities she presents to her students and compels her to try to find activities in which she can provide students an algorithm. Similarly, Jack was trying to give his students an algorithm to compute mean, median, and mode of a data set. Jack's students did not understand the process and simply memorized the procedure to achieve the "right" answer. Jack did not try to understand his students' thoughts so that he could guide them to a deeper more conceptual understanding (Battista, 1994). The examples provided by Battista show that these teachers' beliefs may have caused them to implement classroom practices that did not promote their students

conceptual learning and are not consistent with suggestions by the reform movement in mathematics education.

In summary, research has been able to categorize and describe teachers' beliefs about mathematics and describe teachers' beliefs about teaching mathematics. Research has also shown that teachers' practice is not always consistent with their beliefs about mathematics instruction. These results suggest that both pre-service and in-service teachers need experiences that cause them to reflect on their beliefs.

Research Related to Theoretical Framework

The theoretical framework presented in Chapter 3 will focus on two theories of learning, metacognition and conceptual change. Broadly speaking, metacognition can be described as a learner's regulation of their own cognitive activity while conceptual change describes the process by which learners accommodate new conceptions into their current knowledge structures. These two theories will be discussed in detail in Chapter 3, the remainder of this section of the literature review will focus on research that has helped shape these two theories.

Metacognition

This section will present research related to metacognition. Past research has shown that students may be at different levels of metacognitive activity and that there are possible techniques for promoting metacognitive activity in students. These two ideas will be presented here.

Hennessey (1999) presents the results of project META (Metacognitive Enhancing Teaching Activities), and provides possible levels of students metacognitive activity. Hennessey (1999) explored the metacognitive practices of 170 students in grades K-6 in a three year case study of metacognition through individual and group discourse. For the study,

...content specific units were designed to explicitly stimulate classroom interactions that focused on the students' conceptions of science content, the nature of science, the nature of knowledge production and learning, and the nature of explanatory models in science (p.8 Hennessey, 1999).

Student poster production, conceptual models, and technology were used to help promote metacognition in students (Hennessey, 1999). Results suggested that students' metacognitive activity and abilities range from a minimal level of awareness of their conceptions to more sophisticated metacognitive thought. Based on these results Hennessey (1999) provides two possible levels of metacognitive thought,

1. *Representational level:* a student's awareness of their own unobservable constructs (internal) which are articulated through verbal discourse, journal writing, etc.
2. *Evaluative level:* a student's ability to make inferences about their own unobservable constructs and consider implications for their personal knowledge claims.

Metacognition has come under the guise of other names in the literature (i.e. self-regulation, meaningful learning, reflection) and under these names researchers have discussed possible avenues for promoting metacognitive activity in students. Novak and Gowin (1984) discussed the benefits of having students construct concept maps for meaningful learning. They define a concept map as "a schematic device for representing a set of concept meanings embedded in a framework of propositions" (p.15 Novak & Gowin, 1984). Concept maps may help foster students' metacognition and "students and teachers constructing concept maps often remark that they recognize new relationships and hence new meanings" (p.17 Novak & Gowin, 1984). Therefore concept maps become a tool to help understand what the learner already knows while at the same time providing them the opportunity for reflection on their conceptions.

Related to Novak and Gowin (1984), Novak (1985) discussed the concepts of met-learning and metaknowledge and explained research that has been done at Purdue University to help students learn how to learn. This research has shown

... significant promise for concept mapping and Vee mapping strategies to help students learn how to learn and to acquire knowledge about knowledge. We see these strategies as holding promise for helping students to understand both the nature and sources of valid as well as invalid conceptions of events or objects. They may in time permit students to gain facility in assessing the power and validity of their idiosyncratic conceptual frameworks. (p.206 Novak, 1985)

While discussing teachers' self-evaluation and self-regulated learning Schunk (1996) also provided a suggestion for promoting metacognitive activity. Schunk (1996) defined self-evaluation as "a process comprising self-judgments of present performance and self-reactions to these judgements" (p.2-3 Schunk, 1996) and defines self-regulated learning as "self-generated thoughts, feelings, and actions, that are systematically designed to affect one's learning of knowledge and skills" (p.2 Schunk, 1996). Schunk (1996) explored the effects on fourth graders' understanding of fractions, of assigning them, to one of four treatments, learning goal with self-evaluation, learning goal without self-evaluation, performance goal with self-evaluation, and performance goal without self-evaluation. Schunk's hypothesis was that giving students the goal of learning to solve problems instead of a performance goal of a number of problems to solve would benefit their achievement (Schunk, 1996). Students assigned to the self-evaluation conditions were asked to judge their fraction capabilities at the end of each of the instructional sessions. The results of the study suggested that giving students a learning goal with or without evaluation and a performance goal with self-evaluation benefits their mathematical achievement (Schunk, 1996). Thus Schunk

(1996) showed the benefits of self-evaluation and that learning goals are a possible tool to promote metacognitive activity.

Mewborn and Wilson (1999) describe tasks that they have utilized with pre-service elementary teachers to help promote reflection. Data was collected with regard to four pre-service teachers who were observing a fourth grade classroom for two hours a week. The pre-service teachers were asked to solve a mathematics problem before watching students solve the same problem. This activity provided the teachers with better insight into the students' thinking as they solved the problem because the teachers could refer back to their experience solving the same problem. The pre-service teachers were able to see differences between the children's thinking and their thinking (Mewborn & Wilson, 1999). Mewborn and Wilson (1999) also gave the pre-service teachers the opportunity to observe a student teacher as well as an exemplary teacher.

Observing the student teacher was a powerful experience for the pre-service teachers because it helped them articulate aspects of the classroom teacher's teaching that they were taking for granted. Watching the student teacher also forced them to put themselves in the role of the teacher and posit alternative ways of conducting a lesson. Observing the student teacher helped make teaching and learning mathematics problematic for the pre-service teachers. (p.9 Mewborn & Wilson, 1999)

Making teaching and learning problematic can cause pre-service teachers to begin reflecting on their future teaching practice. In this example, after they observed the student teacher, the pre-service teachers were able to articulate what attributes they felt made the teacher exemplary. After previous observations they had reported that she was a good teacher but could not articulate why.

Conceptual Change

Posner, Strike, Hewson and Gertzog (1982) give a detailed description of the theory of conceptual change, which will be discussed further in Chapter 3. Broadly, conceptual change can be viewed as the process of assimilating concepts into one's cognitive structures because the concepts settle contradictions caused by previously held concepts (Posner, Strike, Hewson, & Gertzog, 1982). Posner et al. (1982) interviewed students with regard to their beliefs about special relativity. These students were in a self-study, self-paced introductory physics class. The first interview activity asked the students to consider the workings of a light clock and its implications for time and the second asked them to consider the synchronization of two clocks. Based on the results of these interviews Posner et al. (1982) suggested a process for conceptual change to occur. As part of their process of conceptual change Posner et al. (1982) suggest that students can only consider a conception that is plausible and that they have a meaningful representation of, they must find that there are difficulties with an old conception to consider replacing it, and they must see the possibilities for future study of the new conception (Posner et al., 1982).

Gunstone and Northfield (1992) discussed the role of conceptual change and metacognition in teacher education. The researchers engaged pre-service teachers in situations that could provoke conceptual change, including modeling a bad lecture, modeling a lesson where material is presented through discussion, having the student teachers teach in a one-to-one situation, having pre-service teachers keep a journal of their experiences, and having pre-service teachers analyze an anecdotal teaching situation (Gunstone & Northfield, 1992). Gunstone and Northfield give examples of some of the results of their research. The actions of the pre-service teachers in schools as well as their journal writing helped the researchers document examples of conceptual change. One example in particular is given below.

One student, for whom conceptual change was most obvious, came to the program with quite transmissive views of teaching. By the third week he was explicitly evaluating his views: after his own micro teaching he wrote "hit home later that I had presented an information presentation rather than a learning exercise". By mid year he was offering thoughts such as his journal being "my learning rather than my lecture/seminar notes (which until recently contained only other people's notes) and handouts which are others' notes", "our writing a critique of the teachers' teaching is - in a way - an assessment of the strategies they have been helping us learn all year. They practice what they preach - any assessment by me is an evaluation of what they practice and what they preach." In his end-of-year course written personal evaluation he wrote insightfully and at length about his initial views, changes in views and courses of these changes. (p.27 Gunstone & Northfield, 1992)

In summary the research discussed above suggests that it is possible to promote both metacognitive thinking and conceptual change with regard to pre-service teachers' beliefs. The research discussed also suggests that providing activities that call for reflection is a possible tool to promote metacognitive activity and conceptual change.

Relationship of Literature Review to Proposed Study

Past research in mathematics education has focused on problem posing with students, teachers, and prospective teachers. The literature presented above suggests that students and teachers are capable of posing mathematics problems and that problem posing should be incorporated in mathematics instruction at all levels of education. This study addressed the suggestion made in the literature to include problem posing in mathematics classrooms and curricula by incorporating problem posing into a mathematics content course for pre-service teachers. This study adds

to the literature on characteristics of pre-service teachers' posed problems and the literature on how exposure to problem posing influences pre-service teachers' problem posing.

Research also suggests that based on past experiences pre-service teachers will have developed beliefs about mathematics and mathematics teaching and learning and that it is important to understand these beliefs. It is also important for research to begin to document the evolution of and changes in teachers' beliefs related to both mathematics and the teaching and learning of mathematics (Corte et al., 1996). Having pre-service teachers reflect on their mathematics knowledge and its relationship to their future teaching as suggested by the literature above is one way to begin documenting pre-service teachers' beliefs. Based on suggestions in the literature this study presented problem posing and journal writing as tools to influence participants' reflection, increase their metacognitive activity, and possibly begin the process of conceptual change with regard to their beliefs about mathematics and the teaching and learning of mathematics. The interaction of metacognition, conceptual change, and learning within the context of this study will be examined in detail in Chapter 3.

Chapter 3

Theoretical Framework

This study provided participants the opportunity to view mathematics as both a problem solving and problem posing domain. To aid in describing participants' problem posing, beliefs about mathematics, and beliefs about the teaching and learning of mathematics it is important to consider how they acquire knowledge, and how they learn, within the context of this study. Researchers have suggested pre-service teachers are actively constructing views of mathematics teaching and learning from their past experiences (Gunstone & Northfield, 1992). In the context of this study the theory of conceptual change and the theory and process of metacognition will help describe participants' acquisition of knowledge and developing views about mathematics. It is important to examine these two theories, their major tenets, how they relate to student learning and acquisition of knowledge, their relationship to each other, and their relationship to participants' acquisition of knowledge related to problem posing, beliefs about mathematics, and beliefs about the teaching and learning of mathematics.

This chapter will focus on the implications of conceptual change and metacognition for individuals' acquisition of knowledge and learning, and on the relationship between these two theories and the current study. It is these theories and relationships that will provide insight into the role of the instructional treatment in pre-service teachers' acquisition of knowledge related to their problem posing, beliefs about mathematics, and beliefs about the teaching and learning of mathematics.

A Conceptual Change View of Knowledge

The theory of conceptual change arose from within the domain of science and has been described by Kuhn as “scientific revolutions” and by Lakatos as “changing research programs” (Kuhn, 1970; Lakatos, 1970). Throughout history scientific theories have been replaced by new theories that account for flaws in the previous theory. For example the theory of phlogiston was replaced by the theory of oxygen. In the early 1770’s German Scientist G.E. Stahl suggested that every flammable object contained a substance called phlogiston and that the object would burn until there was no more phlogiston remaining to be burned. This theory garnered wide spread acceptance until the late 1770’s. In the meantime Priestly and Lavoisier had done experiments related to combustion and these experiments led to Lavoisier’s doubt in the theory of phlogiston. After conversation with Priestly, Lavoisier was able to articulate the theory of oxygen and nitrogen. Therefore the theory of oxygen replaced the theory of phlogiston, within the scientific community, through the process of conceptual change.

Throughout history a scientific theory has only been considered invalid when another candidate is accepted and replaces the original theory (Kuhn, 1970).

To reject one paradigm without simultaneously substituting another is to reject science itself. That act reflects not on the paradigm but on the man. Inevitably he will be seen by his colleagues as “the carpenter who blames his tools.” (p.79 Kuhn, 1970)

Therefore, “scientific revolutions” or changes in “research programs” occur when part of the scientific community believes “that an existing paradigm has ceased to function adequately in the exploration of an aspect of nature to which that paradigm itself had previously led the way” (p.92 Kuhn, 1970). In other words, the theory or paradigm has led to conflict which has been discovered by some subset of the scientific community. The new theory that is suggested and then replaces the previous theory must resolve

the conflict that has been discovered. Not only must the new theory resolve the conflict that has been caused by the existing theory but it must also permit predictions and research ideas that are different from the theory it is replacing (Kuhn, 1970).

When scientists are confronted with a crisis in an existing scientific theory the nature of their research changes (Kuhn, 1970). A scientific revolution or the assimilation of a new paradigm emerges first in the minds of one or a few individuals who have been examining the problems that were provoked by the existing theory (Kuhn, 1970). Kuhn (1970) discussed examples of “scientific change” that resulted as the confrontation of crisis in an existing theory, including, as an example, Lavoisier’s work with regard to the theory of phlogiston.

The much-maligned phlogiston theory, for example, gave order to a large number of physical and chemical phenomena. It explained why bodies burned - they were rich in phlogiston - and why metals were all compounded from different elementary earths combined with phlogiston, and the latter, common to all metals, produced common properties. In addition, the phlogiston theory accounted for a number of reactions in which acids were formed by the combustion of substances like carbon and sulphur. Also, it explained the decrease of volume when combustion occurs in a confined volume of air - the phlogiston released by combustion “spoils” the elasticity of the air that absorbed it, just as fire “spoils” the elasticity of a steel spring. If these were the only phenomena that the phlogiston theorists had claimed for their theory, that theory could have never been challenged. (p.99 Kuhn, 1970)

Lavoisier recognized the problems (“crisis”) with phlogiston theory with regard to gas-identity and weight relations and hence felt that he could address these problems with his theory of combustion which included identifying oxygen (Kuhn, 1970).

Conceptual change was first identified within the scientific community to explain the changing nature of scientific phenomena and theories. Conceptual change has also been examined as a theory of knowledge and theory of individuals' knowledge acquisition. Confrey (1981) discusses a progression of theories of knowledge and their implications for the development of conceptual change. First, absolutism, as a theory of knowledge, views knowledge as an accumulation of facts and new knowledge is simply added to the accumulation of previous knowledge. Second, as discussed by Confrey (1981) "progressive absolutism" is defined by the belief that knowledge is a work in progress as theories are replaced by more powerful theories. With regard to "progressive absolutism" Confrey (1981) stated,

A new theory accounts for all the data that a previous theory accounts for, but extends it further to include data which could not be explained by the previous theory. ... Underlying such a view of knowledge is a basic commitment to an absolute truth, toward which we are striving and forever approaching more closely. There is also the assumption that the two theories can be compared side by side and that one can determine objectively the superior theory by its increased potential for explanation.
(p.245 Confrey, 1981)

Confrey (1981) believes that the failure to determine the superiority of one theory over the other prompted the development of conceptual change as a view of knowledge.

How Conceptual Change is Viewed as a Theory Knowledge Acquisition

Confrey (1981) suggests that there are three basic tenets about knowledge that underlie theories of conceptual change:

1. Knowledge changes and develops; it is not static.

2. The knowledge is not defined externally, but it progresses through a community of scholars who influence its values, its truth conditions, and standards of evidence.
3. Theories influence progress and are not comparable objectively in that they strive to explain different phenomena, involve different evidence, and interpret that evidence differently (p.245 Confrey, 1981).

The most important thing to understand about the conceptual change view of knowledge is that knowledge exists through its scholars. Knowledge is not independent of its scholars. Assuming that knowledge is not static and is continually progressing through the community of scholars, it is important to consider how individuals acquire knowledge and concepts within this community.

Developmental psychologists have argued against Piaget's claims that learners move from preoperational thinking to concrete operations to formal operations and have argued that knowledge acquisition in individuals is better described by conceptual change and theory replacement (Thagard, 1992). Many events in the evolution of scientific knowledge can be viewed as attempts to resolve pre-existing inter-theoretic tensions (i.e. phlogiston to oxygen) (Kitcher, 1983). The parallel in individuals, of inter-theoretic tensions, are intra-personal conceptual tensions. Therefore conceptual change can be viewed as an individuals' attempt to resolve these intra-personal conceptual tensions. The remainder of this section will present three descriptions of a conceptual change view of knowledge acquisition or individuals' resolution of intra-personal conceptual tensions (Posner et al., 1982; Georgiades, 2000; Chi & Roscoe, 2002).

Posner, Strike, Hewson, and Gertzog (1982) give a detailed development of the acquisition of knowledge based on the theory of conceptual change. Conceptual change is described as the assimilation of ideas and as the accommodation (replacing or

re-organizing) of concepts (Posner et al., 1982). Assimilation takes place when an individual can utilize existing concepts to deal with a new phenomena. Often current conceptions are not able to assimilate new phenomena and an individual must replace or re-organize their current conceptual structure (Posner et al., 1982). In other words, accommodation is often necessary as part of an individuals' knowledge acquisition.

It is not likely that anyone will consider radical changes in their held conceptions until they believe that some sort of non-radical change will not work. Once settled on a radical change, Posner et al. (1982) suggest four stages for the accommodation of a new concept by an individual. The learner must first be dissatisfied or see conflict with an existing conception. Posner et al. (1982) discuss anomaly as one possible source of dissatisfaction with a current conception and, "if taken seriously by students, anomalies provide the sort of cognitive conflict that prepares the student's ...for an accommodation" (p.224 Posner et al., 1982). Before accommodation is even possible the individual must have collected a series of problems and lost faith in the ability of their current conceptions to solve these problems (Posner et al., 1982). Once the conflict or dissatisfaction with a current conception has occurred the individual must understand a new conception and consider its possibilities to handle the conflict caused by the previous conception. "The individual must be able to grasp how experience can be structured by a new concept sufficiently to explore the possibilities inherent in it" (p.214 Posner et al., 1982). The new conception then must also seem plausible to the individual.

Only if the student can psychologically construct a coherent, meaningful representation of a theory can it become an object of assessment and a tool of thought. Only an intelligible theory can be a candidate for a new conception in a conceptual change. (p.217 Posner et al., 1982)

If the new conception does not seem to have the capacity to solve problems un-

solvable by the old conception, then the individual will not see reason to consider adopting the new conception (Posner et al., 1982). Finally, for accommodation to occur, the new concept should suggest the possibility of future study. The learner should see potential for the concept to be extended and to open possibilities for new areas of study (Posner et al., 1982). "If the new conception not only resolves its predecessor's anomalies but also leads to new insights and discoveries, then the new conception will appear fruitful and the accommodation of it will seem persuasive" (p.222 Posner et al., 1982).

An accommodation of a new concept seems like a radical change of an individual's conceptions, but it is not an abrupt reaction to new conceptions or ideas. It is plausible that an individuals' accommodation of a new conception will be a gradual and piecemeal affair (Posner et al., 1982). For a novice it is best to think of conceptual change as a gradual layered adjustment of an individuals' conceptions. Each new adjustment is the foundation for further adjustments and the result of these layers is a replacement or re-organization of the individuals' current conceptual structure (Posner et al., 1982). Therefore, accommodation may be the product of failed attempts at assimilation.

Accommodation may, thus, have to wait until some unfruitful attempts at assimilation are worked through. It rarely seems characterized by either a flash of insight, in which old ideas fall away to be replaced by new visions, or a steady logical progression from one commitment to another. Rather, it involves much fumbling about, many false starts and mistakes, and frequent reversals of direction. (p.223 Posner et al., 1982)

Georghiadis (2000) described the process of accommodation as discussed by Posner et al. (1982) and also described conceptual change.

Conceptual change, by definition, requires the existence of conception A,

in order to establish conception B by changing the former. In order to do so, it becomes apparent that conception A should have a long enough 'concept-life', such that will allow conception B to be built upon it, or to evolve from it, given the appropriate CCL (conceptual change learning) interaction takes place. (p.124 Georghiades, 2000)

In this situation the new conception, B, is replacing the current conception, A. Conception B is able to answer the questions that the initial conception cannot and hence becomes part of an individuals' conceptions. In this description conception B is being accommodated.

Recently cognitive psychologists have considered concepts as being linked to categories. Concepts can be represented, understood and interpreted in the context of the category membership (Chi & Roscoe, 2002). As an example, Chi and Roscoe (2002) give a description of conceptual change that relies on the idea of "ontological categories". Individuals' conceptions are considered as stored in "ontological" categories and a misconception is a concept that is stored in the incorrect category. Chi and Roscoe (2002) argue that conceptual change is the process of shifting or reassigning misconceptions across ontological boundaries and state, "...conceptual change is merely the process of reassigning or 'shifting' a miscategorized concept from one 'ontological' category to another 'ontological' category" (p.4 Chi & Roscoe, 2002). Students' misconceptions must be addressed in their learning in order for a category shift to occur. If a textbook or instruction does not cause conflict for a student's misconception, the concept is not likely to be shifted to the proper categories.

Although conceptual change is considered to consist of different processes (assimilation, accommodation, shifting between "ontological categories", etc.), "most of the terms carry the implication that individuals' particular conceptual structures are replaced by more sophisticated ones that can account for phenomena where previous

conceptions failed to do so” (p.120 Georghiades, 2000). This quote describes all three views of conceptual change discussed above and can be considered a broad description of the conceptual change view of knowledge acquisition.

Conceptual Change as a View of Learning

Student learning in science education has been considered since as early as the 1920's. Dewey emphasized science as an inquiry-oriented domain, but it was not until the work of Bruner, Gagne and Piaget that science education caught up with Dewey's beliefs that children should be at the center of the teaching of any subject (Bruner, 1963, 1967, 1973, 1979; Dewey, 1990; Archambault, 1966; Gagne, 1965; Piaget, 1969). Once established as a view of knowledge acquisition the implications of conceptual change for student learning were then explored by educators. Recently cognitive scientists have agreed with the work of Dewey and others that children's knowledge and understanding change in many interesting ways. In particular, learners bring their personal experiences to learning situations and this has an affect on their ability to accept new views. One can argue that in any science, only reading and observing principles will not help clear up and shape the alternative ideas that learners bring into the classroom. Learners must be granted the opportunity to construct their own versions of scientific principles. If learners' alternative views are not addressed in the classroom they can clash with classroom views and create conflict for students. Therefore conceptual change implies that student learning occurs as they compare new ideas or concepts to their own versions of scientific principles.

The implications of conceptual change for learning can be seen as an extension of the ideas presented above with regard to knowledge acquisition. The shift of concepts between “ontological” categories as described by Chi and Roscoe (2002) is comparable to linking or integrating new ideas with old, and can be applied to the learning of all concepts. Chi and Roscoe (2002) discuss two processes that aid in an individuals'

understanding of concepts and learning:

At least two “ordinary” learning processes can be proposed as mechanisms that can remove incorrect beliefs and repair flawed mental models. These two processes, “assimilation” and “revision,” can result in significantly richer and more accurate knowledge about a domain. (p.9 Chi & Roscoe, 2002)

If they are viewed together, the sum of assimilation and revision processes have the potential to lead to a major change in a student’s understanding of a conception or system of knowledge (Chi & Roscoe, 2002).

Similarly, the processes of “assimilation” and “accommodation” as presented by Posner et al. (1982) have implications for individuals’ learning. These ideas imply that inquiry and learning occur against the background of the learners’ current concepts, so any new idea or phenomenon must be compared to the learners’ current concepts to decide on the necessity for study (Posner et al., 1982). “Learning is fundamentally coming to comprehend and accept ideas because they are seen as intelligible and rational” (p.212 Posner et al., 1982).

As expressed by Strike and Posner (1985), conceptual change theory emphasizes the transformation of conceptions in the process of learning. This emphasis on transformation of conceptions makes it necessary to describe how learners incorporate new conceptions into their current cognitive structures. In trying to describe how learners incorporate new conceptions it is beneficial to describe the process of conceptual change for learners. Strike and Posner (1985) give four conditions for conceptual change to occur,

1. The learner must be dissatisfied with an existing conception.
2. There must be some understanding of a new conception.

3. The new conception must seem plausible to the learner.
4. A new conception should suggest possibilities for further study.

These conditions put forth by Strike and Posner (1985) reflect the ideas of Posner et al. (1982), and these conditions are viewed to be the catalyst in the accommodation of a new conception.

Finally, the ideas of status and conceptual ecology are important for understanding an individual's conceptual change. Hewson and Thorley (1989) recognize the necessity of both components in the following:

There are two major components to the model of conceptual change, the (status) conditions that need to be satisfied in order for a person to experience conceptual change and the person's conceptual ecology that provides the context in which the conceptual change occurs and has meaning. (p.541 Hewson, Beeth, & Thorley, 1998)

Hewson and Hewson (1992) discuss the status an idea has for the person who is holding it and have indicated that the holder's conception of an idea's intelligibility, plausibility, and fruitfulness, help to determine its status. The more useful a person views an idea the higher its status. The higher the status of a concept the more plausible the concept is to the individual and hence the possibility of conceptual change arises.

As suggested by Stephen Toulmin (1972), those concepts which govern a conceptual change will be referred to as a "conceptual ecology." Hewson, Beeth and Thorley (1998) have discussed conceptual ecology as,

... all the knowledge a person holds, recognizes that it consists of different kinds, focuses attention on the interactions within this knowledge base, and identifies the role that these interactions play in defining niches that

support some ideas (raise their status) and discourage others (reduce their status). (p.201 Hewson et al., 1998)

In essence an individual's conceptual ecology can be seen as their current conceptions. Therefore, a person's conceptual ecology will influence the selection of new concepts that may be accommodated (Posner et al., 1982). Again an individuals' conceptual ecology provides the context in which conceptual change occurs. Posner et al.(1982) suggest five components of an individual's conceptual ecology.

Anomalies: The character of the specific failure of a given idea.

Analogies and metaphors: Suggest new ideas to make a concept intelligible.

Epistemological commitments: Views of what is considered successful explanation in mathematics and other standards for successful knowledge (i.e. elegance, economy, parsimony, and not being ad hoc.)

Metaphysical beliefs and concepts: Including metaphysical beliefs about mathematics and metaphysical concepts of mathematics.

Other knowledge: Knowledge in other fields and competing concepts.

The ideas of status and conceptual ecology help to describe how an individual accommodates new conceptions in conceptual change learning. In summary the learner must be dissatisfied with an existing conception, have an understanding of a new conception, believe that the new conception can solve the problems presented by the original, and finally believe that the new conception shows potential for further exploration and study. These competing conceptions are elements of the individuals conceptual ecology and the raise in status of the second conception may cause a conceptual change to occur as the second concept overcomes the dissatisfaction with the initial concept. Being able to compare these conceptions within conceptual change implies some reflection on the part of the learner. As stated by Beeth (1995),

...observing a critical demonstration or event, by itself, is not enough to produce a change in conceptual understanding. Given that a learner finds some event dissatisfying, it is necessary to examine their thinking about this event - to be metacognitive about the situation. The learner needs to examine what it is they are dissatisfied with, and the status and conceptual ecology components of a conceptual change provide a means of thinking about dissatisfaction. (p.4 Beeth, 1995)

Based on Beeth's comments the role of the learners metacognitive activity and its relationship to conceptual change are important.

Metacognition

Learning is not a product of teaching, learning is a responsibility of the individual and cannot be shared, it must be pursued intentionally by the learner (Novak, 1985). Although learning cannot be shared, meanings can be shared, discussed, negotiated and agreed upon (Novak & Gowin, 1984). If learning is the responsibility of the learner then the learner must have some powers of reflection in order to learn and must be able to relate meanings of conceptions within their conceptual ecology. It is plausible that the human mind allows an individual to acquire meanings for concepts and relate these meanings in essentially an infinite number of ways (Novak, 1985). This relation of meanings and reflection by the learner is referred to as metacognition. Metacognition can be defined as "the capacity to reflect on one's own thinking, and thereby to monitor and manage it" (p.17 Greeno, Collins, & Resnick, 1996) and has been studied under many different labels (e.g. metacomponents, self-regulated learning, metalearning). Theories of metacognition and the role of metacognition have been studied explicitly since the the late 1970's by psychologists and researchers (Flavell, 1979; Novak, 1985; Sternberg, 1985; Brown, 1987; Beeth, 1995; Crowley, Siegler, & Siegler, 1997; Novak, 1998; Georghiades, 2000).

Flavell (1979) introduced metacognition to the field of cognitive psychology. Flavell (1979) considered metacognition to be an individual's knowledge and regulation of their cognition. According to Flavell knowledge about one's cognition includes three variables; person variables, task variables, and strategy variables (Flavell, 1979). Person variables refer to knowledge about one's self and about others' thinking (e.g. individuals learning style). Task variables refer to the fact that different types of tasks require different types of cognitive demands (i.e. addition and integration). Finally, strategy variables refer to knowledge about metacognitive strategies for developing learning (i.e. reflection, journal writing, conversation).

Brown (1987) described the difference between an individual's knowledge about cognition and knowledge about metacognition. Knowledge about cognition tends to be consistent within individuals whereas knowledge about metacognition can be unstable, age dependent, and change from situation to situation (Brown, 1987). Brown (1987) suggested that metacognition is more context than age dependent. For example, a child and an adult may not show metacognitive behavior in the same situations and an individual may show metacognitive activity in one situation but not in another.

Sternberg (1985) discussed "metacomponents" of an individual's intelligence. "Metacomponents" allow an individual to monitor and manage their cognitive resources and are considered a key feature of intelligence (Sternberg, 1985). "Metacomponents" such as identifying the nature of a problem, planning, and monitoring are consistent with metacognition as described by Flavell (1979) and Brown (1987).

The relationship between cognition and metacognition is that "...cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (p.177 J.Garofalo & Lester, 1985). There are multiple roles of metacognition, the role of planning and the role of self - regulation.

Hennessey (1999) expands on this and gives five characterizations of metacognition,

1. An awareness of one's own thinking.
2. An awareness of the content of one's conceptions.
3. An active monitoring of one's cognitive processes.
4. An attempt to regulate one's cognitive processes in relationship to further learning.
5. An application of a set of heuristics as an effective device for helping people organize their methods of attack on problems in general. (p.6 Hennessey, 1999)

As stated above, metacognition has been discussed under different names. Schunk (1996) discussed the role of self-regulated learning and Novak (1998b) discussed the role of *metalearning*. Schunk described self-regulated learning as, "self-generated thoughts, feelings, and actions, that are systematically designed to affect one's learning of knowledge and skills" (p.3 Schunk, 1996). Self-regulatory actions include a student's self-efficacy; holding positive beliefs about one's capabilities, the value of learning, the factors influencing learning, and the anticipated outcomes of actions (Schunk, 1996). Educators have recognized the importance of students' development of self-regulatory skills along with content knowledge and procedural skills (Schunk, 1996). Through his research with fourth graders' understanding of fractions Schunk found that "...providing students with a learning goal enhances their self-efficacy, skill, motivation, and task goal orientation, and that these outcomes also are prompted by allowing students to evaluate their performance capabilities or progress in learning" (p.15 Schunk, 1996).

Novak (1998b) describes meaningful learning as the act of learning by relating new information to ideas that the learner already knows (Novak, 1998b). Novak (1998b) suggested that there are three requirements of meaningful learning:

1. Relevant prior knowledge: That is, the learner must know some information that relates to the new information to be learned in some nontrivial way.
2. Meaningful material: That is, the knowledge to be learned must be relevant to other knowledge and must contain significant concepts and propositions.
3. The learner must choose to learn meaningfully. That is, the learner must consciously and deliberately choose to relate new knowledge to knowledge the learner already knows in some nontrivial way. (p.19 Novak, 1998b)

Research has also examined situations that cause students to take part in metacognitive processes. Crowley et. al. (1997) researched the implications of students' automation of cognitive processes on their metacognitive ability. In research with kindergarten students Crowley et al. (1997) found that metacognitive thinking was most likely to take place when the student had automated a lower level cognitive skill. Automation of cognitive skill strategies or "associative mechanisms" allows students more mental processing space to utilize for metacognitive activity (Crowley et al., 1997).

Students' attitudes, beliefs and expectations are important for their performance within a domain. Research suggests that attitudes and beliefs about mathematics will govern a student's metacognitive activity within the domain (Lucangeli, Coi, & Bosco, 1997). Lucangeli et al. (1997) examined the metacognitive beliefs in mathematics and their relation to problem-solving performance of 155 fifth grade students. Poor problem solvers tended to take part in less metacognitive activity (Lucangeli et al., 1997). Other research also has shown that metacognitive ability seems to be a general skill that spans across content domains (Veenman, Elshout, & Meijer, 1997). Veenman

et al. (1997) examined the metacognitive ability of 14 freshman and found that not only does metacognitive ability span domains but it seems to be partly independent of intelligence.

The foundations of metacognition were set in the early eighties and nineties and recently research has started to focus on the relationship between conceptual change and metacognition (Georghiades, 2000). While discussing PEEL (Project to Enhance Effective Learning) Georghiades (2000) states, "...metalearning can be promoted and will facilitate conceptual change, even if it remains fragile and artificial, until perceived by students as meeting their own short-term goals" (p.127 Georghiades, 2000). It seems that metacognitive activity will make an individual more responsible for their learning. Once an individual feels a sense of responsibility they will become more active in the learning process and being active in the learning process is believed to enhance student achievement. "The equation is as follows: by being reflective, revisiting the learning process, making comparisons between prior and current conceptions, and being aware of and analyzing difficulties, learners gradually maintain a deeper understanding of the learned material" (p.128 Georghiades, 2000).

The different descriptions above can all be grouped under the umbrella metacognitive strategies. In summary, utilizing metacognitive strategies can cause students to analyze their own conceptions. Therefore, it is important to develop teaching-learning situations that promote students' participation in metacognitive activities. In summary "metacognitive strategies are strategies that empower the learner to take charge of her or his own learning in a highly meaningful fashion" (p.1 Novak, 1998). Metacognitive activities involve students in the monitoring of their own learning and their own conceptions and hence can be the catalyst for conceptual change to occur (Gunstone & Northfield, 1992).

Learning in the Context of this Study

Human understanding has two dimensions. Humans acquire, possess and make use of their knowledge while at the same time they are aware of their activities as knowers (Toulmin, 1972). The learning theories that explain human understanding as described by Toulmin (1972) are conceptual change and metacognition. These two theories of learning will collectively provide the lens through which this study will view changes in participants' conceptions. By focusing on conceptual change and metacognition, insight will be gained into characteristics of participants' posed problems, beliefs about mathematics, and beliefs about teaching and learning mathematics. As participants are introduced to new ways to think about mathematics, they have the opportunity to obtain new conceptions and change current conceptions. This study will view learning as having occurred when new conceptions are incorporated into an individual's cognitive structures by replacing or modifying current conceptions. As mentioned previously, Strike and Posner (1985) have called this replacement or modification of conceptions "accommodation." This study will be interested in participants' accommodation of conceptions related to their problem posing, beliefs about mathematics, and beliefs about teaching and learning mathematics.

As noted above conceptual change theory grew out of understanding changing scientific ideas and phenomena and science education research. It must be considered whether mathematics is a candidate for the application of a conceptual change theory of knowledge and learning. In a discussion of the historical and evolutionary development of mathematics Toulmin (1972) states,

...the development of mathematical disciplines exposes their concepts and methods to transformations as profound in their own way as the natural sciences. Such fundamental concepts as "validity" and "rigour",

“elegance” and “proof”, and “mathematical necessity undergo the same sea-changes as their scientific counterparts “soundness,” “cogency,” and “simplicity,” “relevance,” and “physical necessity.” Even the basic standards of “mathematical proof” have themselves been reappraised more than once since Euclid’s time. The result is that concepts, methods, and intellectual ideals of mathematics are not more exempt from the “ravages of time” . . . than those of any other intellectual discipline. (p.252 Toulmin, 1972)

“If Toulmin is correct, and if no objective and external standard exists to determine the superiority of mathematical theories over each other, then mathematics becomes a candidate for the application of a conceptual change theory of knowledge” (p.248 Confrey, 1981). As discussed previously, Confrey (1981) gave three tenets of the theory of conceptual change. It is important to consider if mathematics follows these three tenets and hence, as a discipline, is a candidate for the theory of conceptual change. First, Confrey (1981) says that knowledge is not static. Often people consider mathematics as an absolutist domain, the epitome of certainty, immutable truths, and irrefutable methods. On the contrary, as long as humans possess the ability to reflect on what they believe is knowledge and what they believe it means to learn, mathematics will be changing. Second, mathematicians determine the values of mathematical knowledge, the truth conditions of mathematics knowledge and the standards of evidence. Mathematics knowledge is not defined externally, it is discovered by mathematicians who decide on its truth value. Finally, it is clear that mathematical theories influence the progress of the domain and explain different phenomena. Therefore it seems that mathematics is a candidate for the application of a conceptual change theory of learning. This theory in the context of this study will be described in more detail below.

This framework will also describe outcomes of learning via conceptual change and metacognition and then how these outcomes influence pre-service teachers learning. Conceptual change involves both building conceptions of new ideas in relation to past understanding and modifying understanding which may be at odds with natural explanation (Hennessey, 1999). Metacognition involves an individuals' reflection on their cognition and their building of conceptions. Novak's conception of meaningful learning helps describe the interaction of conceptual change and metacognition. Novak (1998b) discusses the outcomes of meaningful learning and discusses what this research views as the outcome of the interaction of conceptual change and metacognition. Novak (1998b) wrote, meaningful learning includes the learner's "non-arbitrary, non-verbatim, substantive incorporation of new knowledge into cognitive structure." Also involved in meaningful learning is the learners' effort to link new knowledge with higher order concepts in their cognitive structure, that learning is related to experiences with events, and that the learner has made a commitment to relate new knowledge to prior learning (Novak, 1998b). Since meaningful learning requires relevant prior knowledge, and we know that the quantity and quality of an individual's prior knowledge varies dependent on the concept, any learner has limitations to the degree of meaningful learning that can occur related to a given concept (Novak, 1998b).

Conceptual Change and Metacognition in Teacher Education

The research community has not utilized a single theory for describing change in teachers' beliefs. This research takes the perspective that changes in teachers' beliefs can be explained within the context of the theory of conceptual change (Gunstone & Northfield, 1992; Taylor, 1990). Taylor (1990) discussed the application of conceptual change theory to teachers' beliefs. Following the stages of conceptual change suggested by Posner et al. (1992) Taylor suggested that teachers should be made

aware of their “subjectively reasonable beliefs” that shape their classroom practices and teachers should then have an alternative belief “made available” to them through pre-service or inservice education. Convincing reasons for “adopting” the new belief must be clear to teachers, and finally teachers need to experience success utilizing the “new perspective” in their practice. Pre-service teachers cannot immediately experience success utilizing the “new perspective” in their practice but can consider the benefits of their “new perspective” for their practice and consider the possible student outcomes if they were to adopt their “new perspective” in their practice.

Gunstone and Northfield (1992) highlight the role of conceptual change and metacognition in pre-service teacher education,

Conceptual change in teacher education then occurs when the student teachers, in an informed and self-directed way, recognize, evaluate and decide whether or not to reconstruct existing ideas and beliefs. Conceptual change is necessary, variously for individual student teachers, in three areas:

1. ideas and beliefs about teaching and learning and roles appropriate for teachers and learners (this includes both the context of their own learning in the pre-service program and the context of their teaching of pupils in schools);
2. ideas and beliefs about the discipline content and skills students will teach, science in this case, and epistemological issues surrounding this content such as the nature and purpose of observation in science and science learning;
3. ideas and beliefs about themselves. (p.10 Gunstone & Northfield, 1992)

Pre-service teachers must understand their relevant ideas and beliefs, evaluate these in terms of what learning is to be learned and then decide whether or not to reconstruct their ideas and beliefs (Gunstone & Northfield, 1992). To make such a decision is to be appropriately metacognitive. In particular, conceptual change and metacognition may be appropriate theories to help describe pre-service teachers' views of teaching and learning. Gunstone and Northfield (1992) suggested that it takes,

... recognition that any such change is in the hands of the pupil/student teacher. It is the pupil/student teacher who must first recognize his/her relevant ideas and beliefs, then evaluate these ideas and beliefs in terms of what is to be learned and how this learning is intended to occur, and then him/herself decide whether or not to reconstruct their ideas and beliefs.

(p.8 Gunstone & Northfield, 1992)

In this context metacognition is the learner's self-directed approach to recognizing, evaluating and deciding whether they will reconstruct their conceptual ecology (Gunstone & Northfield, 1992).

Conceptual Change, Metacognition, and Problem Posing

Conceptual change, metacognition, and problem posing may interact in the context of this study. Problem posing experience and the instructional treatment are the vehicles that may influence characteristics of participants' posed problems, beliefs about mathematics, and beliefs about the teaching and learning of mathematics. In the context of this study participants were given the opportunity to take part in social learning situations through daily group activities that were related to both problem solving and problem posing. Beyond these daily activities students took part in problem posing activities both on homework assignments and in journal writing. It was not a goal of this research to explicitly promote or document metacognitive

activity or conceptual change but it is the researcher's belief that this introduction to problem posing and a new view of mathematics may help fill such gaps in curricula as mentioned by Novak (1985).

Whenever we . . . assess curriculum, we find serious conceptual gaps or lack of explicit linkages between concepts, poor integration between events or objects presented and concepts, principles and theories needed to interpret observations of the events or objects, and little or no guidance to the student as to significant salient concepts versus peripheral or incidental concepts. (p.206 Novak, 1985)

It is this researcher's belief that problem posing is one of the foundations of the development of mathematics as research mathematicians pose and solve mathematics problems. Recently mathematics educators have suggested the inclusion of problem posing in mathematics instruction (NCTM, 2000; Kilpatrick et al., 2001). Therefore it is not likely that pre-service teachers have experience posing mathematics problems or viewing mathematics from a problem posing perspective. The opportunity to view mathematics from a problem posing perspective may cause pre-service teachers to reflect on the nature of mathematics and their future mathematics instruction. Underlying this study are three assumptions about participants' interaction with problem posing that are based on the relationship between problem posing and mathematics, the two learning theories discussed previously, and the instructional treatment.

1. Problem posing will provide pre-service teachers a new perspective on mathematics, a perspective more in tune with mathematicians' perspective of mathematics, as mathematicians are problem posers.
2. Problem posing has the potential to trigger the necessary conditions for individual conceptual change to occur.

3. Problem posing may provoke metacognitive activity in participants implying possible conceptual changes with respect to their beliefs about mathematics and beliefs about the teaching and learning of mathematics.

It is important to state that this research was entered with these three assumptions about problem posing and the possibilities for problem posing to influence metacognition and conceptual change, but this research was not entered into with any assumptions about the outcomes of participants interaction with problem posing.

Again documenting participants' metacognitive activity was not a goal of this study, but it is important to understand that metacognitive activity related to participants' beliefs about mathematics and beliefs about mathematics teaching and learning may have provoked conceptual change related to these beliefs. While engaging in problem posing activities prospective teachers may have been exposed to new conceptions about the teaching and learning of mathematics and problem posing. In turn, these new conceptions may gain status in their conceptual ecology. It was the goal of this study to document the changes, if any, in participants' beliefs while at the same time exploring the characteristics of their problem posing.

A possible non-empirical example of the interaction of conceptual change and metacognition within the context of this study is as follows. A student may have entered this research with the conception that mathematics is solely a problem solving domain. As they are introduced to and experience problem posing the participant may begin to see problems with their view of mathematics as solely a problem solving domain. The participant may ask themselves, "If mathematics is solely problem solving, who produces mathematics problems and how is the domain of mathematics furthered through research?" The new conception that mathematics is both a problem solving and problem posing domain may be considered by the participant. Through reflection this new conception may seem to the participant to answer the questions

caused by their initial conception that mathematics is solely a problem solving domain. Also their new conception suggests further research related to problem posing as they have limited experience posing mathematical problems. It is possible that through reflection the participant will see possible future study in the relationship between problem posing and the teaching and learning of mathematics. Thus the process of conceptual change has taken place and with the help of the participant's metacognitive ability the new conception that mathematics is both a problem solving and problem posing domain has been accommodated.

This theoretical framework concludes with a concept map that shows the general interaction of the ideas presented previously. The concept map shows the interrelationship between conceptual change and metacognition. As discussed previously the outcome of the the combination of conceptual change and metacognition is what Novak discussed as "metalearning" (Novak, 1998b). Also problem posing has been discussed as a possible catalyst for students' metacognitive activity causing the accommodation of new conceptions related to problem posing, beliefs about teaching mathematics and beliefs about teaching and learning mathematics. Finally changes in either participants' problem posing or beliefs may result in changes in the other.

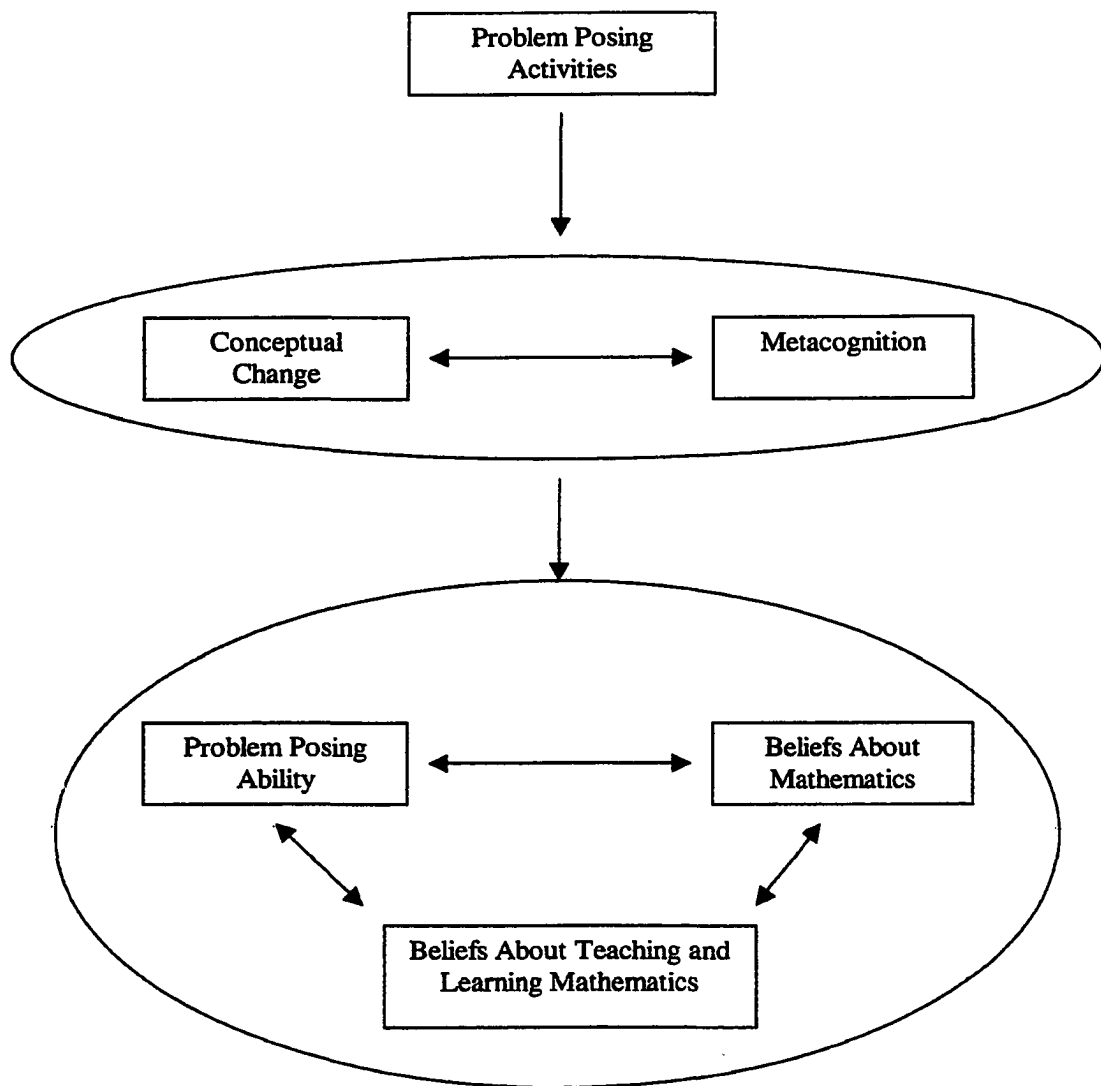


Figure 3-1: Interaction of constructs in the context of this study.

Chapter 4

Methodology

Research Design

This study incorporated problem posing in a mathematics content course for pre-service elementary and middle school teachers. This was an exploratory study that utilized some aspects of an instrumental case study (Stake, 1995). Stake (1995) describes an instrumental case study as a situation where studying the case is instrumental in understanding a broader question and gives the following example,

... we will have a research question, a puzzlement, a need for general understanding, and feel that we may get insight into the question by studying a particular case. For example, Swedish precollege teachers have a year to begin using a new student marking system passed by the Parliament. How will that work? ... We may choose a teacher to study, looking broadly at how she teaches but paying particular attention to how she marks student work and whether or not it affects her teaching. (p.3 Stake, 1995)

In the context of this study the instructional treatment was the incorporation of problem posing into the classroom instruction and curriculum and its effects on the whole class (n=19). The cases that were chosen by the researcher were the four students who volunteered to be interviewed throughout the semester.

There are four main components that play a role in this research; the instructional treatment, subjects, problem posing products, and participant outcomes. These com-

ponents will be discussed below. Participants' problem posing products and participant outcomes related to their beliefs about mathematics and beliefs about teaching and learning mathematics are dependent on the instructional treatment and subjects. Problem posing products and participant outcomes will be discussed as part of the data analysis in Chapters 5 and 6.

Instructional Treatment

The semester long incorporation of problem posing into a mathematics content course for pre-service teachers was agreed upon by the classroom instructor and the researcher. The instructional treatment included three aspects, participant problem posing through problem re-formulation and problem generation, participant journal writing, and reading related to problem posing. All of the aspects of the instructional treatment discussed herein took place in conjunction with and as part of the expectations that the instructor set forth for the course. The course syllabus and weekly assignment sheets can be found in Appendix A.

Participants were asked to solve mathematics problems using a problem solving heuristic similar to Polya (1957). This was referred to as the four-step problem solving heuristic; understanding the problem, devising a plan, implementing the plan, and looking back. After solving problems using the four step heuristic on the first problem set, participants were then asked to use a five-step problem solving heuristic adding the fifth step, "pose a related problem", on the remainder of the problem sets. Participants were asked to apply these heuristics to a subset of each problem set and in all cases were able to choose the problems to which they applied the heuristics. A time-line of problem sets and the utilization of these two heuristics are shown in table 4.1. The complete problem sets can be found in Appendix A.

Participants also were asked to generate problems from sets of given information. The researcher would suggest a set of given information to the instructor and after

Set	Due Date	Task	Topic
1	January 30 th Week 2	2 problems using 4-step	Problem solving
2	February 11 th Week 4	2 problems using 5-step posing 2 related problem for each	Problem solving and data analysis
3	February 20 th Week 5	2 problems using 5-step posing 2 related problem for each	Problem solving and measures of central tendency
4	February 27 th Week 6	2 problems using 5-step posing 2 related problem for each	Data analysis
5	March 6 th Week 7	1 problem using 5-step posing 2 related problems and pose 1 related problem for every other problem	Probability
6	March 27 th Week 9	Pose 1 related problem for each problem	Counting and probability
7	April 3 rd Week 10	Pose 1 related problem for each problem	Graph theory and networks
9	May 8 th Week 15	Pose 1 related problem for 2 problems	Discrete mathematics

Table 4.1: Problem re-formulation tasks

a dialogue between instructor and researcher a version would be adapted into homework or journal writing. The sets of given information provided participants with the context of possible mathematics problems but did not include any questions. Participants were then asked to generate problems from the set of given information. The first problem generation task was presented in a prompted journal entry and included reflection on the problem posing process. The final two problem generation tasks were part of problem sets that were assigned for homework. Problem generation situations are shown in table 4.2.

Assignment	Due Date	Set of given information
Journal entry	February 25 th Week 6	Pose three to five problems from the following set of given information, Mrs. Smith's and Mr. Jones' fifth grade classes took the same mathematics test last week. You have been given all the graded exams and the answer key.
Problem Set #5	March 6 th Week 7	Pose three problems from the following set of given information, You arrive at your friend's home and they are sitting at a table with \$20, a deck of cards, and red, white, and blue die.
Problem Set #6	March 27 th Week 9	Pose two problems and provide a detailed solution for one, A roulette wheel has 18 red numbers, 18 black numbers and 2 green numbers. A person bets on either an individual number or a color. A one dollar bet on an individual number pays \$35, on black or red pays \$1, and on green pays \$12.

Table 4.2: Problem generation tasks

The problem re-formulation and problem generation aspects of the intervention provided participants the opportunity to pose mathematics problems. It was also a goal of the treatment to promote student reflection on the class activities, problem posing activities, and their beliefs about mathematics teaching and learning. Journal prompts and reading assignments were intended to be the catalyst for this reflection. Journal prompts, due dates, and the week of the semester the journal entry was due

are shown in table 4.3.

As stated above, the researcher and instructor agreed on all aspects of the instructional treatment. It is important to note that the final instructional treatment was not decided upon prior to the semester. The researcher and instructor developed a general instructional treatment prior to the semester and agreed on modifications throughout the course of the semester.

Participants and Course

Students enrolled in a mathematics content course for pre-service elementary and middle school teachers were the participants in this study. This audience was chosen because past research has shown that they have the ability to pose mathematics problems (Gonzales, 1994). Also if problem posing is going to become predominant in mathematics classrooms and curriculums as suggested by the NCTM (1989, 2000) and the NRC (2001) it is the researcher's belief that pre-service teachers should have experience not only posing mathematics problems but reflecting on the role of problem posing in the mathematics classroom. There were 20 students enrolled in the semester long course "Topics in Mathematics for Teachers" at the University of New Hampshire, 19 of those students agreed to serve as participants in this study. Four of the nineteen participants volunteered to be interviewed three times during the instructional treatment and these four are the "cases" for this study. Participants included 4 sophomores, 7 juniors, 6 seniors, and 2 graduate students working towards their masters degree in education. The 17 undergraduates were mathematics education or family studies majors who were seeking certification to teach at the elementary or middle school level. The four students who volunteered to be interviewed included one graduate student from the education department and three mathematics education majors within the mathematics department.

The course "Topics in Mathematics for Teachers" is the third in a sequence for

Due Date	Journal Prompt
January 30 th Week 2	Compose and submit your mathematical autobiography.
February 11 th Week 4	What did you learn about statistics from the paper clip game?
February 25 th Week 6	Along with problem posing described in table 4.2 respond to the following questions, Describe the process you just went through to generate problems from this set of information? and Do you see any similarities between the problem solving and the problem posing process? Explain.
March 4 th Week 7	Imagine that you are teaching and someone comes in to observe your classroom and a mathematics lesson that you are teaching. Write a description of your classroom and the lesson from the eyes of the observer. What would they see you doing during the lesson, what would they see the student's doing, and what would they notice about your classroom? Also read and be ready to discuss "Promoting a problem posing classroom." (English, 1997)
March 11 th Week 8	Write a brief reflection on how you think class is going this semester. Also read "Problem posing and critiquing: How it can happen in your classroom." (English, Cudmore, & Tilley, 1998c)
April 1 st Week 10	Write a journal reflection about the exam.
April 15 th Week 12	As you are posing related problems or posing problems from sets of given information who is your intended audience? Why? Does the audience change depending on the problem? Would you consider yourself better at posing problems as re-formulations or posing problems from sets of given information? Why?
May 6 th Week 15	Do you think you will utilize problem posing in your future classroom? If so, in what ways? Please be as specific as possible.
May 13 th Week 16	Write a reflection of your experience in the course this semester. The following questions may be helpful. What have I learned about myself as a learner of mathematics? What have I learned about myself as a prospective teacher of mathematics? How has my conception of mathematics or teaching changed? What questions do I still have?

Table 4.3: Journal prompts and readings

elementary and middle school mathematics education majors and is offered in alternate years during the spring semester. The course is not offered for credit towards a B.S. in mathematics. The course included the following mathematics content; logic, statistics (graphs, measures of central tendency, measures of variation), probability (experimental, geometrical, and theoretical), problem solving using skills from statistics and probability, mathematical connections, and applications requiring calculators and computers. Appendix A includes the course syllabus, weekly assignment sheets, and problem sets that were assigned throughout the instructional treatment.

Data Collection

All of the nineteen participants agreed to have their course work analyzed for the study while four participants agreed to be interviewed three times during the instructional treatment. The researcher collected pre- and post-assessments of problem posing and beliefs, classwork, homework, journal entries, interview transcripts, and classroom observations.

Pre- and Post-Assessments

As described in Chapter 1 a pre-assessment of participants' problem posing and a pre-assessment of participants' beliefs about mathematics were given on the first day of class, January 23, 2002. The assessment of problem posing was completed in class and the assessment of beliefs was completed outside of class and collected on January 28, 2002. Both post-assessments were completed in class on May 13, 2002. For all of the in-class assessments, the researcher read and explained the directions to participants and gave them 25 minutes to complete each assessment. It was explained to participants that information could be added in the problem posing assessment. The assessments of problem posing and beliefs can be found in Appendix B.

Classwork, Homework and Journals

The researcher observed each class which included taking observation notes and collecting all materials for that class, including class activities, homework assignments and weekly assignment sheets in order to witness understand student interaction and have a sense of the everyday class activities. Homework and journals were collected as they were handed in by the participants. The researcher would immediately photocopy homework and journal entries and forward the ungraded work to the instructor. The participant who declined participation in the study did not have any of their work examined by the researcher. The researcher did not examine any material after it had been graded or commented upon by the instructor.

Interviews

The participants who volunteered to be interviewed were each interviewed three times during the semester. The first round of interviews took place between January 31, 2002 and February 13, 2002, the second, third and fourth week of classes and focused on participants' beliefs about mathematics and initial beliefs about problem posing. All four subjects had been exposed to the four-step problem solving heuristic prior to the first interview. The interview dialogue revolved around the questions in table 4.4. The goal of the questions during the first interview was to help provide the researcher information to develop a description of participants' beliefs about mathematics, beliefs about teaching and learning mathematics, and initial thoughts about problem posing.

The second round of interviews took place between March 27, 2002 and April 3, 2002, the ninth and tenth weeks of the semester. At this time in the instructional treatment, all four participants had experience posing mathematics problems through both problem re-formulation and problem generation. After approximately one month

	Question
1	How do you define mathematics?
2	How do you define mathematical thought?
3	Is mathematics a static body of knowledge? Explain?
4	How do you view mathematics teaching?
5	What are the attributes of a good mathematics teacher?
6	What are the attributes of a good mathematics student?
7	What is problem posing?
8	Are there implications of problem posing for classroom instruction?
9	Describe a typical mathematics classroom teaching experience?

Table 4.4: Questions on interview 1

of experience posing problems, the second interview was utilized to try to understand characteristics of participants' posed problems, participants' problem posing process, and their beliefs about problem posing at this stage of the instructional treatment. During interview two, participants were asked to generate problems from two sets of given information. The sets of information can be found in table 4.5. Participants were given as much time as they needed to pose problems, they were asked to select the best problem they posed for each situation and to explain why that problem was chosen. To complete the second interview dialogue between the researcher and participants was related to the questions in table 4.6.

The third round of interviews took place between May 7, 2002 and May 10, 2002, the fifteenth week of the semester. The third interview focused on both problem posing and beliefs questions. To begin the interview participants were shown two examples of concept maps. The two concept maps were examples of student work in which they had mapped all concepts which they felt were related to oceans (No-

Set	
1	<p>You have decided to do a survey about students spring break travel plans. With some help from your friends you have surveyed 300 students and collected the following information from each.</p> <ol style="list-style-type: none"> 1. Whether or not they travelled for spring break? If so, where? 2. How much money they spent on travelling. 3. How they rate their spring break experience from 1 to 10.
2	<p>Instead of working this summer you have decided to drive cross country with your best friend.</p>

Table 4.5: Problem posing on interview 2

	Question
1	Do you feel like the introduction to problem posing this semester has been beneficial? If so, why? If not, why not?
2	Do you think problem posing should be incorporated in all levels of mathematics education? Explain.
3	Will you utilize problem posing in your future classroom? Explain.
4	Can you give an example of a situation where you may find problem posing beneficial?
5	How do you think viewing mathematics from a problem posing perspective differs from a problem solving perspective? Could students benefit from experiencing this difference?
6	How do you think students will benefit from being introduced to problem posing?
7	Do you believe you are better at posing problems as extensions or from sets of given information? Explain.
8	How would you define a good mathematics problem? How do you judge whether you have posed a good problem?

Table 4.6: Questions on interview 2

vak, 1998). Participants were asked to create and explain their own concept map for problem posing. If participants did not explicitly define problem posing, while explaining their concept maps, they were asked to do so. After discussing their concept maps, participants were asked to read and react to their pre-assessment of problem posing and their pre-assessment of beliefs about mathematics. The assessments had not been coded by the researcher prior to this reaction. Participants were asked to discuss anything that surprised them about their responses to the assessments and anything that they might change after looking back. Finally, if it had not been discussed during the interview, participants were asked to explicitly define mathematics and describe a good mathematics teacher in order to compare their beliefs to the first interview. The third interview was utilized to try to describe any changes that may have occurred in these participants' beliefs and to understand participants' views of problem posing.

Data Coding and Analysis

Problem Posing Products

Problem posing products refer to mathematical statements posed by participants through problem re-formulation or problem generation during the instructional treatment. If the product was a result of a problem re-formulation task it was analyzed to determine its relation to the original problem. If the product was the result of a problem generation task it was analyzed to determine its plausibility, sufficiency of information, and the number of steps needed for solution.

Participant Outcomes

Participants outcomes related to their beliefs about problem posing, beliefs about mathematics and beliefs about teaching and learning mathematics were a product of

journal writing and interviews. The goal of analyzing these outcomes was to describe participants' beliefs before, during, and after the instructional treatment. Journal writing (whole class) and interview transcripts (four cases) were analyzed qualitatively to determine participant outcomes related to their beliefs about mathematics and beliefs about teaching and learning mathematics.

Coding of Posed Problems

Participants were asked to complete five problem generation tasks and seven problem re-formulation tasks during the course of the instructional treatment. Problem generation occurred on the pre- and post-assessment of problem posing and three times during the instructional treatment. Problem re-formulation occurred on seven homework assignments during the instructional treatment.

All statements on both problem generation and problem re-formulation tasks were first classified as either mathematical or non-mathematical. All non-mathematical statements were discarded and were not coded further. If a statement was mathematical, it was then determined which type of problem posing activity the statement came from, either problem generation or problem re-formulation. The researcher then determined if the statement was related to the activity, and if so it was deemed a problem posing product. If not a problem posing product the statement was discarded and not coded further. Problem generation products were coded using a scheme adapted from Leung and Silver (1997) and problem re-formulation products were coded based on their relationship to the original problem. Finally, during coding, the researcher determined if the problem included information that was not in the original set of information and had been added, whether the problem asked for explanation, and whether the problem was open-ended. Figure 4.1 is a flowchart of the problem coding process and problem coding is discussed in more detail below.

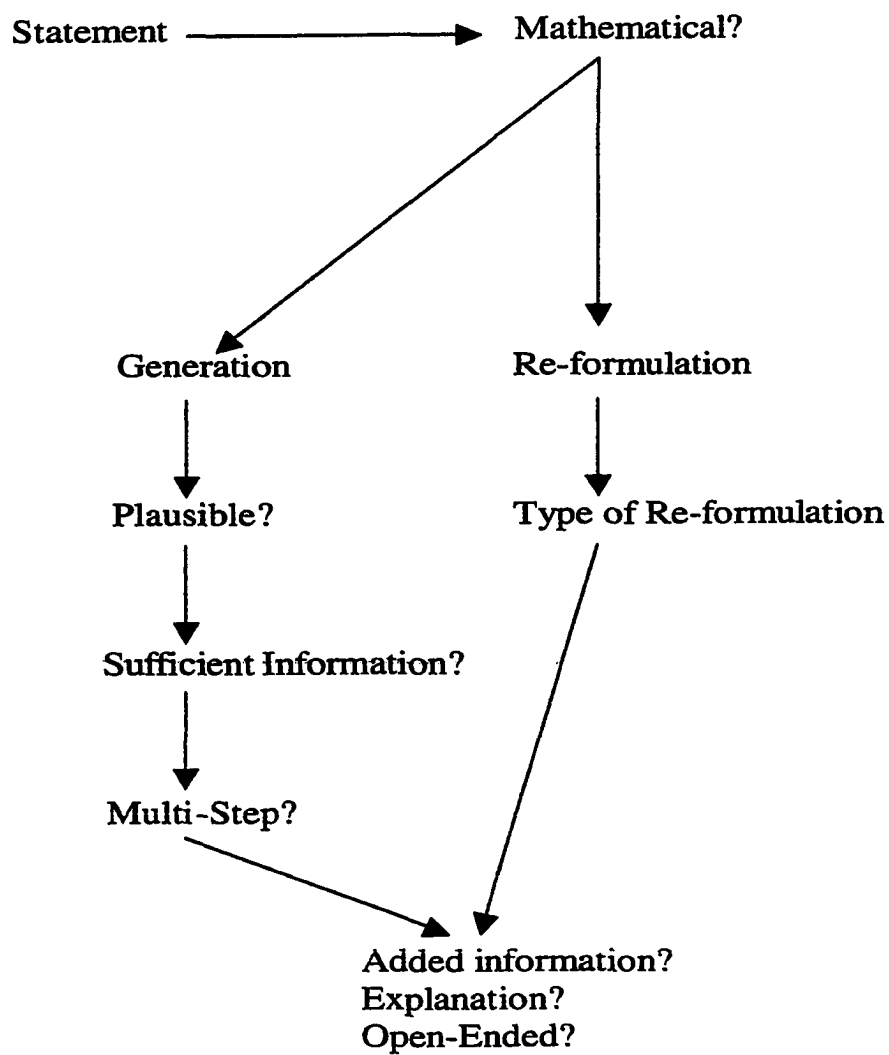


Figure 4-1: Coding flow chart.

Pre- and Post-Assessment of Problem Posing. The pre- and post-assessment of problem posing contained two sets of given information, (see Appendix B). The first set of given information was in the context of the student needing to purchase a new computer. This set of information contained numerical information (e.g. price of the computer, interest rate, etc.). The second set of information was set in the context of a university building a parking garage and did not contain any numeric information. Participants received a score for numeric posing based on the problems they generated from the first set of information since it contained numeric information. Similarly, a score for non-numeric posing was based on the problems participants generated from the second set of information. These two scores were combined to determine a participant's total posing score.

Problem Generation Products. A statement that was determined to be a problem posing product from a problem generation activity was then coded along three dimensions, plausibility, sufficiency of information, and the number of steps needed for solution. An implausible problem is one that contains an invalid assumption and hence is not plausible to solve even with more information. Implausible problems were not coded further since the researcher was interested in problems that contained a possible plausible solution (Leung, 1993). If a problem generation product was plausible, it was then determined by the researcher whether there was sufficient information to solve the problem. Problems with extraneous information were coded as having sufficient information since they were solvable. There were very few instances of problems with extraneous information. If a problem was both plausible and contained sufficient information, it was then determined if multiple steps were necessary for solution. Multiple arithmetic steps were not the determining characteristic of a multi-step problem. A multi-step problem asked the problem solver to perform at least two mathematical tasks in order to reach the solution of the given problem.

Problem posing products from problem generation activities were assigned a score as shown in table 4.7 and empirical examples of the problem generation coding are shown in table 4.8.

Score	Criteria
0 points	Problem posing product but not plausible.
1 point	Plausible problem posing product without sufficient information.
2 points	Single step plausible problem posing product with sufficient information.
3 points	Multi-step plausible problem posing product with sufficient information.

Table 4.7: Problem generation scoring

Problem Re-formulation Products. A statement that was determined to be a problem posing product from a problem re-formulation activity was then classified as having been posed using one of the following strategies,

Switch the Given and the Wanted: A problem in the same context as the original problem with the given and wanted information switched.

Change the Context: A problem with the same structure but context changed.

Change the Given: Same problem context and structure but the given information is changed.

Change the Wanted: Same problem context and structure but what the question asks for is changed.

Extension: An extension of the given problem.

Add Information: Same problem context and structure with added information.

<p><i>Set of given information:</i> Mrs. Smith's and Mr. Jones' fifth grade classes took the same mathematics test last week. You have been given all the graded exams and the answer key.</p>
<p><i>0 points:</i> Do you feel by the overall grades, that it would be fair to scale the grades or should students get the grade they earned?</p>
<p><i>1 point:</i> From above (test data provided) which of these statistical tools best represents an average score of the test for Mrs. Smith's class? Mr. Jones' class?</p>
<p><i>2 points:</i> There are 15 students in Mrs. Smith's class and 12 students in Mr. Jones' class. The median of all the tests from both classes is an 82. How many students scored above the median? How many students scored below the median?</p>
<p><i>3 points:</i> Find the median for the scores of both classes. Is this a good way to represent the average? Why or why not?</p>

Table 4.8: Empirical examples of problem generation

Re-word: Same problem different wording.

In coding problem re-formulation products, the researcher began with four categories (switch the given and wanted, change the context, change the given, add information) to organize the posed problems. These categories were from examples of problem re-formulation given to the participants during the second week of the instructional treatment, see Appendix B. Additional categories were developed by the researcher as needed until all problems belonged in at least one of the categories. It is also important to note that a single problem re-formulation could span two or more categories. For instance a participant could change the given and change the wanted of the same problem to produce a new related problem. Empirical examples of the coding of problem posing products from problem re-formulation tasks are shown in table 4.9.

Interrater Reliability

Two additional raters volunteered to code problem generation and problem re-formulation products based on the coding schemes discussed previously. Raters coded a sample of posed problems, 90 from problem generation tasks and 75 from problem re-formulation tasks, based on a description of the coding scheme provided by the researcher.

With regard to problem generation coding, the researcher asked the raters to follow the scheme from the research which examined whether the problem was plausible, contained sufficient information, and if the problem required a multi-step solution procedure. The researcher and raters agreed on the plausibility of 87 (96.7%) of the 90 problems. Two reasons for discrepancies in plausibility coding arose, first the problem was based on a previous problem, which the researcher coded as plausible and the rater as not plausible. Second, the problem was based on terminology from class that the rater was not familiar with. Of the 87 problems the raters and researcher

<p><i>Original Problem:</i> The mean of three test scores is 74. What must a fourth score be to increase the average to 78?</p>
<p><i>Switch given and wanted:</i> The mean of 3 test scores is 72. If the fourth test score is 87, what does the mean become?</p>
<p><i>Change the context and add information:</i> A boy and a girl are on the same baseball team. After playing 2 games, Susie has a mean of 2 hits per game while Carl has a mean of 1 hit per game. If Susie gets 2 hits in the third game, how many hits must Carl get to have the same mean hits per game as Susie after 3 games?</p>
<p><i>Change the given:</i> If you have two test scores of 71 and 65 what must the third score be for the mean to be 75?</p>
<p><i>Change the wanted:</i> If the mean of three test scores is 74, but no two test scores are alike, what are three possible test scores?</p>
<p><i>Original Problem:</i> Consider the integers from 1 to 100, inclusive. What is the difference between the sum of all the even numbers and the sum of all the odd numbers?</p>
<p><i>Extend:</i> Consider the integers from 1 - 500 inclusive. What is the difference between the sum of all the even numbers and all the odd numbers? Can you find a pattern as to make it possible to easily determine it for integers from 1 - 1000?</p>
<p><i>Re-word:</i> What is the difference between the sum of all the even numbers and the sum of odd numbers from 1 to 100 inclusive?</p>

Table 4.9: Empirical examples of problem re-formulation

agreed were plausible both parties agreed 76 (87.4%) contained sufficient information. The main reason for discrepancy in coding related to sufficiency of information was that problems contained sufficient information for a mathematical solution but also asked what could be considered an opinion question. In this case, the raters coded these problems as not including sufficient information. Of the 76 problems agreed upon as containing sufficient information the researcher and raters agreed on whether 61(80.3%) required a multi-step solution procedure. The 15 problems that were not agreed upon at this stage of the coding were discussed and agreed upon by the researcher and raters.

With regard to problem re-formulation tasks the researcher asked the raters to code problems based on the seven categories that had been developed during the initial coding and to report if they felt other categories were necessary. Neither rater suggested another category. Of the 75 problems the researcher and raters agreed on the coding of 56 (74.7%) of the problems. The main discrepancies in coding occurred when the raters coded problems into multiple categories and often considered changing the given as an extension of a problem. The 19 problems that were not agreed upon initially were discussed by the researcher and raters and the researcher's coding was agreed upon.

Data Analyses

Statistical Analyses. Participants' scores on the pre- and post-assessment of problem posing were determined by summing their scores for each statement they wrote. Using a statistical software package (JumpIn), posing scores were compared using the Tukey-Kramer multiple comparisons paired test to determine if there was a change in the groups problem posing after the instructional treatment. The results of the statistical analysis are presented in Chapter 5.

All posed problems were coded and trends in problem generation and problem

re-formulation were examined by exploring tables and graphs of the data generated through the coding process. Tables were utilized to analyze posed problems throughout the instructional treatment and to highlight trends in participants' posed problems.

Qualitative Coding Analysis. The remaining data, including journal entries and interviews was analyzed using qualitative methods. All journal entries including math autobiographies and pre- and post-assessments of beliefs about mathematics were read and data was organized by categories or the frequency of statements and ideas that were occurring throughout the class. For example, as the researcher read participants' mathematical autobiographies statements related to five categories were occurring throughout the class. Statements were coded into the following categories: mathematics preparation, pivotal moments related to teaching mathematics, pivotal moments related to learning mathematics, teachers, and miscellaneous. In this case, pivotal moments relate to participants articulation of situations that were vital in shaping their view of teaching mathematics and vital in their development as a learner of mathematics. Similarly, categories related to participants' problem posing, beliefs about mathematics, and beliefs about the teaching and learning of mathematics were the product of the coding of each task from the instructional treatment. These categories generated from the individual tasks were then organized into five broader categories,

- Beliefs about problem posing
- Beliefs about mathematics
- Beliefs about teaching mathematics
- Beliefs about learning mathematics

- Beliefs about the relationship between problem posing and mathematics teaching and learning.

This two stage categorization led to themes related to participants' beliefs related to the categories listed above and allowed the researcher to develop a rich description of participants' beliefs about mathematics, beliefs about teaching and learning mathematics, and beliefs about the relationship between problem posing and school mathematics.

Interviews. All interviews were transcribed. Interviews were coded and analyzed with regard to the five major categories mentioned above. The researcher coded the interviews by determining which statements made by the participants during the interviews were related to the categories: beliefs about problem posing, beliefs about mathematics, beliefs about teaching mathematics, beliefs about learning mathematics, beliefs about the relationship between problem posing and mathematics teaching and learning. Comments in these categories and across interviews were then compared by the researcher to examine any changes in participants' beliefs about mathematics, beliefs about the teaching and learning of mathematics, beliefs about problem posing, or characteristics of their problem posing. The interviews helped the researcher provide a detailed description of each participants' beliefs within the context of the instructional treatment.

Summary

Participants were introduced to problem posing through the instructional treatment. During the instructional treatment participants were given the opportunity to reflect on the nature of mathematics and the role of problem posing in the school mathematics classroom. Data was collected related to the five research questions presented. This data was then coded and analyzed by the researcher. Results of this data analysis are discussed in Chapters 5 and 6. Chapter 5 presents results related

to participants problem posing while Chapter 6 presents results related to participants beliefs about mathematics, beliefs about teaching and learning mathematics, and beliefs about the relationship between problem posing and school mathematics.

Chapter 5

Problem Posing Results

This chapter will examine results from problem posing tasks that participants engaged in during the instructional treatment. Both problem generation and problem re-formulation tasks will be discussed. Whole class results related to participants' problem posing during the instructional treatment will be presented first, followed by results from the four individuals who were interviewed during the semester. First, qualitative whole class results will be presented in order to provide description of the participants as problem posers. Qualitative results will be followed by quantitative results related to the characteristics of the participants' posed problems.

Whole Class Problem Posing: Qualitative

Qualitative results related to problem posing provide insight into participants' beliefs about problem posing, problem posing process, problem posing audience, and growth as problem posers. Data related to these ideas was collected from the pre- and post-assessment of beliefs about mathematics and journal entries.

Beliefs About Problem Posing

On the pre-assessment of beliefs, participants were asked to respond to a problem posing situation that asked them to consider the role of problem posing in elementary school mathematics. Participants' responses to this task indicate that they were thinking about and developing initial beliefs about problem posing. Sixteen participants responded that problem posing would be beneficial with elementary school

students and three participants stated that they were unsure about the possibilities of problem posing. On the pre-assessment, participants made statements about benefits and drawbacks of problem posing with elementary students. Benefits of problem posing suggested by participants included that problem posing would allow students freedom and creativity with numbers and relationships, and help develop students problem solving skills. Participants suggested that problem posing would help students develop a better understanding of problem solving because problem posing will force students to recognize pertinent information in a problem situation. As stated by one participant, "if children are able to organize information fairly well, they will become better problem solvers from writing their own. They will be able to recognize pertinent information and recognize a strategy to help them tackle the problem."

Recognizing pertinent information may also cause students to think beyond the problem solving process and begin to develop ownership of the mathematics they are learning. As suggested by a participant, "I think that the benefits to students creating their own problems is that they then have the ownership of the task, they don't just have problems to do, they have to think on another level." On the pre-assessment, participants also suggested possible drawbacks of student problem posing, including that students may be confused by the problem posing task and that students may pose unsolvable problems. One participant suggested that "some students may create problems that are unsolvable [based on their current knowledge base] although they may think they have come up with good ones." Therefore, as participants engaged in the instructional treatment they were working with a set of beliefs about problem posing and its possible benefits and drawbacks for students.

On the post-assessment of beliefs, participants mentioned the same benefits of problem posing while going into more detail relating problem posing to their future classrooms. The relationship between problem posing and teaching and learning

mathematics, as suggested by the participants, will be discussed in detail in Chapter 6.

Problem Posing Process

Participants' process of problem generation can be described from responses to their journal entry collected on February 25, 2002. This prompted journal entry asked participants to pose problems from a set of information and then respond to questions about the problem posing process and about similarities between problem posing and problem solving (see tables 4.2 and 4.3). Responses to this journal entry suggest a predominant process utilized by participants to approach the problem posing task from the journal prompt. This problem posing process can be generalized as; analyze the given information for mathematical content, then assess everything they knew about data comparison and data analysis, and then try to write interesting problems that were not just calculations. For example one participant wrote,

When looking at the types of given information for the problems that had to be generated, I immediately related them to data analysis and statistical problems. I pictured two lines of data that included the individual test scores of the two classes. That is the perfect set up for statistical problems. . . . I continued to think of problems that required knowledge in different areas of statistics.

In this journal entry participants also stated that their past experiences and knowledge shaped their problem posing, as suggested by one participant, "... [the problem posing process] is basically using my past knowledge of questions that were asked to me and the information we have started learning about data analysis and just visualizing what kinds of things I could do with these numbers."

Also in this journal entry seven participants related the problem posing process to the four step problem solving heuristic they had been using as part of the instructional

treatment. Participants said they would apply a similar heuristic that starts by reading and understanding the given, understanding possible assumptions and added information, posing a problem, and looking back to be sure the problem is solvable. One participant describes this process in four steps,

1. Understanding the given information. Drawing conclusions and making minor assumptions.
2. Apply my assumptions and the given information to material we have been discussing in class.
3. Combine all the knowledge and design a workable problem.
4. Look back and see if the problem makes sense and asks what I had originally intended to ask. If not start back at #1.

Based on responses to the journal entry collected on February 25, 2002 it was hypothesized that participants had developed a process for posing mathematics problems as problem generation and had begun to relate the problem posing and problem solving process. No participants commented on the problem re-formulation process on this journal entry and there is no data that highlights this process for participants.

Problem Posing Audience

In their journal entry collected on April 15, 2002 participants were asked to discuss their intended audience as they are posing mathematics problems and whether they are better at posing problems as problem re-formulation or as problem generation. Four out of the 16 participants who responded said that they are posing problems for their peers and that would be the case in any class which they are given a problem posing task. Eleven participants stated that they were posing problems for their future students and the grade range of their intended audience was second to eighth grade. With regard to their intended audience one participant wrote,

As I am posing related problems or posing problems from a given set of information, my intended audience is usually the grade that I plan to teach in the future, which is from second to fourth grade. As I pose problems, I think to myself, 'At what grade level would students have to be at to solve this?' or 'What prior knowledge must one have to be able to solve this problem?'

Eleven participants also said that their posing audience changed depending on the difficulty of the information. For example one participant stated,

However depending on the problem, sometimes my audience changes. For example, when we were doing the unit on probability and statistics, there were several ways that the concepts of the problems could be dissected and explored. Also, there are a variety of different strategies in which probability and statistics problems can be solved, such as tree diagrams, charts, and simulations. This opened up many options for posing problems. It was possible to reframe questions to go in many different directions without limiting my audience to using just one solving method.

Ten of these participants said that the audience changed between grade levels, while one participant said that they may go from posing for peers to posing for a fourth grade class if the level of the mathematics was appropriate.

Also in this journal entry twelve of the 16 participants who responded said that they are better posing problems from sets of given information because it allows for more creativity and because re-formulation seems to lead them to the same questions, as they get stuck in the mode of the original problem. A participant provided the following description,

I would consider myself better at posing problems from sets of given information as opposed to re-formulations. I am capable of doing both, but

I feel that the problems I pose from given information are more in depth and engaging, I have to stretch and think more to come up with an interesting problem. I feel when posing a problem as a re-formulation I tend to take an easier route to posing a new problem by simply changing the information around a little. When I have come up with the entire problem alone, I am more apt to have a more creative and interesting final result since I had to put more time and effort into it.

The three students that believed they were better at problem re-formulation believed the inherent structure helped them pose problems and that it is easier to solve a problem and then pose problems based on it because they have a frame of reference for their problem posing.

In summary the majority of participants were posing problems for their future students, and the grade their problems were intended for was dependant on the difficulty of the material. Also, most participants believed that they were more capable of posing problems as problem generation.

Growth as Problem Posers

Participants growth as problem posers will be highlighted through quantitative data related to the characteristics of their posed problems. Through interaction with the participants and classroom observations it is the researcher's belief that participants grew as problem posers and became more comfortable posing problems during the instructional treatment. One student discusses her growth throughout the instructional treatment with clarity, in the final journal entry of the semester which was collected on May 13, 2002.

However the greatest thing that I will take from this class is my newly discovered talent of problem posing. I remember back to the first class

this semester when we were asked to do some problem posing for Todd's research project. I was stumped by this task. Posing a problem from the given information was like another language to me. As the problem sets were assigned throughout the semester, I truly dreaded problem posing. But about half way through the semester, it was like a light turned on in my head and I was suddenly able to create problems without all that difficulty. This allowed me to focus on posing valid challenging problems. It was great to have the same packet handed out once again the last day of class for Todd's research project, and being asked to pose as many problems as I could. This was such a valuable task for me because I could literally see my growth as a problem poser first hand! I sat there and posed problems for minutes without even taking a breather! It was a great feeling to have actually seen how much I grew in this one area of math throughout the course of the semester.

The intention of the results presented above was to describe the study participants as problem posers. In general participants in this study believed there were benefits of student problem posing, had developed a process for posing problems as problem generation, were posing problems for their future students, believed they were better at posing problems as problem generation, and developed as problem posers during the instructional treatment.

Whole Class Problem Posing: Quantitative

Problem posing took place during the instructional treatment as problem re-formulation and problem generation. As described in Chapter 4 problem generation products were coded as plausible, plausible with sufficient information, and plausible with sufficient information requiring a multi-step solution process. Table 4.8 shows examples from the data of problems coded in each category. Problem re-formulation

products were also coded as explained in Chapter 4 and categorized into the following categories; switch the given and the wanted, change the context, change the given, change the wanted, extend, add information, re-word. As stated in Chapter 4 individual problems can span more than one category and examples of this coding can be found in table 4.9. Results of participant problem posing will be presented first with respect to problem generation on the pre- and post-assessment of problem posing, followed by the remainder of the problem generation products and the problem re-formulation products.

Pre- and Post-Assessment

The pre-assessment of problem posing was administered on January 23, 2002 and participants were given 25 minutes in class to complete the task. The measure consisted of a set of information with numeric content and a set of information without numeric content. See Appendix B for the problem posing assessment. Pre-assessments were coded as described in Chapter 4, and each participant received a score for numeric posing (based on the set of information with numeric content), non-numeric posing (based on the set of information without numeric content), and total posing (sum of numeric and non-numeric posing). Table 5.1 shows the individual results of the pre-assessment.

A score of 10 on numeric posing implies that the participant posed problems that totalled in value to 10. One possibility being that the participant posed 3 multi-step problems (3 points each) and a plausible problem without sufficient information (1 point). A participant's score is some combination of plausible problems without sufficient information (1 point), plausible problems with sufficient information that require a single step solution (2 points), and plausible problems with sufficient information that require a multi step solution (3 points).

Student	Numeric Posing	Non-numeric Posing	Total Posing
1	5	5	10
2	4	5	9
3	5	6	11
4	3	1	4
5	14	4	18
6	7	1	8
7	7	5	12
8	2	3	5
9	4	3	7
10	7	1	8
11	3	1	4
12	1	2	3
13	5	3	8
14	4	4	8
15	2	8	10
16	7	5	12
17	6	5	11
18	3	2	5
19	10	2	12

Table 5.1: Results of the pre-assessment of problem posing

The post-assessment of problem posing was administered on May 13, 2002 and participants were given 25 minutes in class to complete the measure. The post-assessment was the same task as the pre-assessment and participants were given the same directions and amount of time to pose problems. Post-assessments were coded, as described in Chapter 4, and each participant received a score for numeric posing, non-numeric posing and total posing. Individual results of the post-assessment are in Table 5.2

The same scoring scheme as the pre-assessment was used to determine participants scores for numeric posing, non-numeric posing and total posing on the post-assessment. As a note, scores are not represented in the same order in tables 5.1 and 5.2 since the measures were coded separately and scores were not recorded by individual.

Results on the pre- and post-assessment of problem posing were compared using statistical software (JumpIn 4). Since one subject did not complete the post-assessment of problem posing her score on the pre-assessment was not used. A Tukey-Kramer multiple comparison matched pairs test was used to compare the means of all possible comparisons of numeric posing on the pre- and post-assessment and non-numeric posing on the pre- and post-assessment at the alpha equals .05 level. The means of the total posing score were not compared because they are linearly dependent on the numeric and non-numeric scores. The means of numeric posing and non-numeric posing on both assessments as well as the results of the Tukey-Kramer test can be found in figure 5.1.

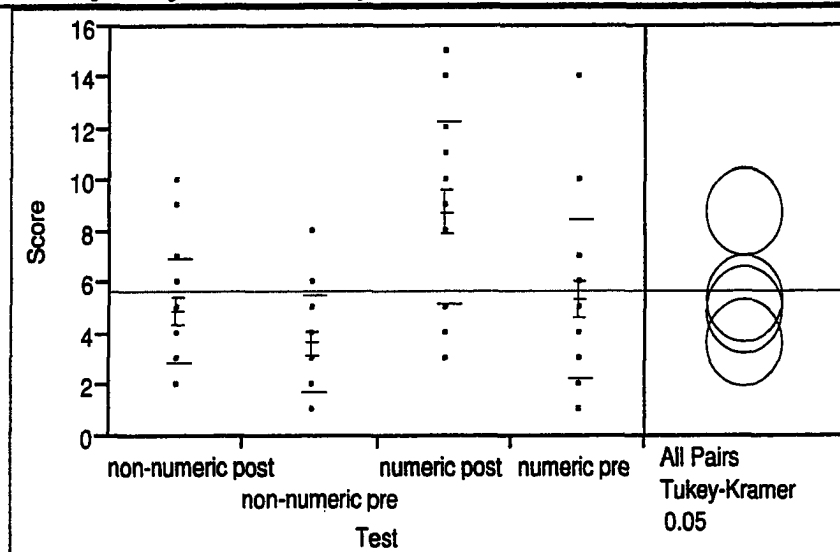
The statistical analysis shows that the means of the following comparisons are statistically significant, *Numeric pre* and *Numeric post*, as well as *Numeric post* and *Non-numeric post*. These results imply that there was a statistically significant change in participants numeric problem posing from pre- to post-assessment and that this

Student	Numeric Posing	Non-numeric Posing	Total Posing
1	3	3	6
2	5	3	8
3	9	3	12
4	10	5	15
5	9	4	13
6	8	4	12
7	15	4	19
8	11	5	16
9	5	7	12
10	5	5	10
11	5	3	8
12	11	5	16
13	12	5	17
14	9	5	14
15	4	2	6
16	12	10	22
17	14	6	20
18	10	9	19

Table 5.2: Results of the post-assessment of problem posing

meansmultiple2

Oneway Analysis of Score By Test



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
non-numeric post	18	4.88889	2.08324	0.49102	3.9091	5.869
non-numeric pre	18	3.61111	1.94449	0.45832	2.6965	4.526
numeric post	18	8.72222	3.54477	0.83551	7.0550	10.389
numeric pre	18	5.33333	3.12485	0.73653	3.8636	6.803

Means Comparisons

Dif=Mean[i]-Mean[j]

	numeric post	numeric pre	non-numeric post	non-numeric pre
numeric post	0.00000	3.38889	3.83333	5.11111
numeric pre	-3.38889	0.00000	0.44444	1.72222
non-numeric post	-3.83333	-0.44444	0.00000	1.27778
non-numeric pre	-5.11111	-1.72222	-1.27778	0.00000

Alpha= 0.05

Comparisons for all pairs using Tukey-Kramer HSD

	numeric post	numeric pre	non-numeric post	non-numeric pre
numeric post	-2.42225	0.96664	1.41109	2.68886
numeric pre	0.96664	-2.42225	-1.97780	-0.70003
non-numeric post	1.41109	-1.97780	-2.42225	-1.14447
non-numeric pre	2.68886	-0.70003	-1.14447	-2.42225

Positive values show pairs of means that are significantly different.

Figure 5-1: Means and comparisons of results on pre- and post-posing assessment.

change caused there to be a statistically significant difference in their numeric and non-numeric posing post instructional treatment.

It is also important to examine the means from the assessments and relate them to the coding scheme used. With regard to numeric problem posing participants average changed from 5.33 to 8.72. Since a multi-step problem granted 3 points there are two possibilities to explain the difference in participants averages from pre- to post-assessment. Participants were either able to write at least two more problem situations in the same amount of time or generated the same amount of problems but wrote more problems that were plausible, contained sufficient information and required a multi-step solution process. For non-numeric posing participants average changed from 3.61 to 4.88 so they were able to either write at least one more problem situation or pose the same number of problems with one more being multi-step. Along with these results the total posing average changed from 8.94 to 13.61 so participants were generating more problem situations total in the allotted time or generating more multi-step problems.

It is important to consider if participants were just writing more situations or if they were generating more plausible problems with sufficient information that required a multi-step solution. Table 5.3 shows the totals and percentages of all problems on the pre- and post-assessments of problem posing.

	Statements	Plausible	Sufficient	Multi-Step
Pre	101	96(95%)	55(54%)	16(16%)
Post	133	122(92%)	87(65%)	37(28%)

Table 5.3: Percentages of total on pre- and post-posing

Table 5.3 shows that participants' efficiency in posing problems increased, as they posed 122 plausible problems on the post-assessment compared to 96 on the pre-assessment and they posed more problems with sufficient information that required a multi-step solution. This change is seen by the raise in percentage of plausible problems with sufficient information from 54% to 65% and by the raise in percentage of multi-step problems from 16% to 28%. In conclusion, post-instructional treatment, participants were more efficient at posing problems and were able to pose a higher percentage of multi-step problems during problem generation.

Problem Generation Products

Participants had three opportunities to generate problems from sets of given information in addition to the pre and post-assessments. The first opportunity to generate problems from a set of given information was February 25, 2002 as part of a prompted journal entry. The set of given information was,

Mrs. Smith's and Mr. Jones' fifth grade classes took the same mathematics test last week. You have been given all the graded exams and the answer key.

This journal entry was assigned on February 20, 2002 so participants had 5 days to complete the problem posing task. The task asked participants to pose three to five problems based on the set of given information.

Participants second opportunity to generate problems from a set of given information was on problem solving 5 which was assigned on February 27, 2002 and due March 6, 2002. The set of given information was,

You arrive at your friend's home and they are sitting at a table with \$20, a deck of cards, and red, white, and blue die.

The task asked participants to pose three problems based on the set of given information.

Participants final opportunity to generate problems from a set of given information was on problem solving 6 which was assigned on March 11, 2002 and due March 27, 2002. The set of given information was,

A roulette wheel has 18 red numbers, 18 black numbers and 2 green numbers. A person bets on either an individual number or a color. A one dollar bet on an individual number pays \$35, on black or red pays \$1, and on green pays \$12.

The task asked participants to pose two problems based on the set of given information.

Table 5.4 shows the results of problem generation on these three tasks,

	Statements	Plausible	Sufficient	Multi-Step	Add info.
Journal 2-25	42	39(93%)	34(81%)	28(67%)	12(29%)
Problem Solving 5	48	48(100%)	42(88%)	27(56%)	0(0%)
Problem Solving 6	23	21(91%)	20(87%)	14(61%)	0(0%)

Table 5.4: Results of problem generation during instructional treatment.

Table 5.4 shows that the range, over the three problem generation tasks, of the percentages of plausible problems, problems containing sufficient information and problems requiring a multi-step solution procedure were small. Therefore, characteristics of participants' problem generation during the instructional treatment did not parallel the results of the pre- and post-assessment of problem posing. The increase in participants posing efficiency and ability to pose multi-step problems is not apparent from their problem generation during the instructional treatment. One difference

in participants problem posing that is apparent from table 5.4. is that participants added information to 12 problems on the first problem generation task and did not add information to any problems on the following two tasks. This may be accounted for by the lack of numeric information in the first problem generation task (see Table 4.2). On the first problem generation task participants may have found it necessary to add numeric information to make their posed problems solvable.

To summarize, the information on participants' problem generation products implies that participants became more efficient problem posers and were able to pose more multi-step problems under a time constraint post instructional treatment. Also, characteristics of participants' problem generation did not differ on the three tasks that were collected as part of course work. As a final description of participants' problem generation Table 5.5 shows the aggregate data for problems generated over the course of the instructional treatment.

	Statements	Plausible	Sufficient	Multi-Step	Add info.
Semester	347	326(94%)	238(69%)	122(35%)	86(25%)

Table 5.5: Aggregate problem generation.

Problem Re-formulation Products

Participants engaged in problem re-formulation on seven problem sets during the instructional treatment and on each problem set participants were able to choose which problems they re-formulated (the problem sets can be found in Appendix A). After categorizing the problem re-formulation products by re-formulation technique the researcher felt that there were two distinct sets of problem re-formulation techniques. The first set of techniques; switching the given and wanted, changing the context, and extension will be referred to as level 1 re-formulation techniques. It is

the researcher's belief that level 1 re-formulation techniques require a higher level of understanding and creativity on the part of the problem poser, since they include changing the structure of the problem. The second set of techniques; adding information, changing the given, changing the wanted and re-wording will be referred to as level 2 re-formulation techniques. It is the researcher's belief that level 2 re-formulation techniques are more basic and do not require the problem poser to change the structure of the problem, these techniques only require a change of the surface features of the problem (i.e. numbers, what is asked for). The utilization of these two levels of problem re-formulation techniques will be discussed throughout this section. Another important distinction is that for each problem on the individual problem sets, both how many problem re-formulation techniques were utilized during re-formulation of that problem and how many problems were posed as re-formulations are reported. This decision was made because some problems posed as re-formulations utilized more than one technique and reporting the data this way allows for a better sense of how often each technique was utilized. Figure 5.4 is the key related to tables 5.6 through 5.12.

Participants first problem re-formulation task was on problem set 2 which was due on February 11, 2002. The mathematical content focus of the problem set was problem solving and data analysis and participants were asked to solve two problems using the five-step heuristic while writing two re-formulations for each problem. Table 5.6 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

As seen in Table 5.6 re-formulation on problem set 2 was dominated by changing the given and changing the wanted and 10 problems were re-formulated using multiple techniques. Level 1 problem re-formulation techniques were utilized 22% of the time and level 2 techniques 78% of the time. Participants heavily favored level 2 techniques

P1	=	Problem 1
# Prob.	=	Number of problems posed
# Tech.	=	Number of re-formulation techniques utilized
S.G.W.	=	Switch the given and wanted
Context	=	Change the context
Add	=	Add information
Ext.	=	Extend the original problem
Given	=	Change the given information
Wanted	=	Change the wanted information
Re	=	Re-word the original problem

Figure 5-2: Key for tables 5.6 through 5.12

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	9	13	0	0	2	0	3	8	0
P2	22	27	3	2	1	0	16	5	0
P3	13	14	3	0	0	4	5	1	1
P4	5	5	1	0	0	0	3	1	0
Total	49	59	7	2	3	4	27	15	1

Table 5.6: Problem re-formulation on problem set 2

for problem re-formulation on this problem set but when utilizing level 1 techniques they favored switching the given and the wanted. Examples of problem re-formulation on problem set 2 follow,

Problem 2: The mean of three test scores is 74. What must a fourth score be to increase the mean to 78?

Changing the given: Given that 3 tests have a score of 90 what would the fourth test have to be to raise the mean to 100?

Switching the given and wanted: The mean of 3 test scores is 72. If the fourth test score is 87, what does the mean become?

Participants second problem re-formulation task was on problem set 3 which was due on February 20, 2002. The mathematical content focus of the problem set was again problem solving and data analysis and participants were asked to solve two problems using the five-step heuristic while writing two re-formulations for each problem. Table 5.7 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	13	13	2	0	1	2	4	4	0
P2	8	10	0	1	0	2	5	2	0
P3	24	24	9	2	0	0	13	0	0
P4	4	5	0	1	0	0	2	2	0
P5	7	11	0	1	2	2	2	3	1
Total	56	63	11	5	3	6	26	10	1

Table 5.7: Problem re-formulation on problem set 3

As seen in table 5.7 problem re-formulation on problem set 3 was dominated by changing the given information of the original problem and 7 problems were re-formulated using multiple techniques. Participants utilized level 1 problem re-formulation techniques 35% of the time and level 2 problem re-formulation techniques 65% of the time. Compared to problem set 2 there was an increase in the use of level 1 techniques and similarly switching the given and the wanted was the most popular level 1 technique. Examples follow of problem re-formulation on this problem set,

Problem 3: A special rubber ball is dropped from the top of a wall that is sixteen feet high. Each time the ball hits the ground it bounces back only half as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

Switch the given and the wanted: If a special rubber ball is dropped from a wall with an unknown height and bounces four times and is caught at the height of its fourth bounce at two feet. If we know that every time the ball bounces it only bounces back half the distance as the distance it fell. How high is the wall the ball dropped off of originally?

Change the given: A special rubber ball is dropped from the top of a wall that is 768 feet tall. Each time the ball hits the ground it bounces back only one-fourth as high as the distance it fell. The ball is caught when it bounces back to a high point of 3 feet. How many times does the ball hit the ground?

Participants third problem re-formulation task was on problem set 4 which was due on February 27, 2002. The mathematical content focus of the problem set was data representation and analysis and participants were asked to solve two problems using the five-step heuristic while writing two re-formulations for each problem. Table

5.8 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	12	13	1	1	0	0	7	4	0
P2	14	16	0	0	1	0	11	4	0
P3	1	1	0	1	0	0	0	0	0
P4	7	7	4	1	2	0	0	0	0
P5	8	8	1	0	0	0	7	0	0
Total	42	45	6	3	3	0	25	8	0

Table 5.8: Problem re-formulation on problem set 4

Table 5.8 shows that on problem set 4 participants are still relying on changing the given information as a problem re-formulation technique and that they only posed 3 problems using multiple techniques. On this problem set level 1 techniques were only used to pose 20% of the problems and level 2 techniques were used to pose 80% of the problems. This is a decline from problem set 3 in the use of level 1 techniques, but is almost identical to problem set 2. As with problem sets 2 and 3 participants favor switching the given and the wanted as a level 1 technique. Examples of posed problems on this problem set follow,

Problem 5: The average of seven numbers is 49. If 1 is added to the first number, 2 is added to the second number, 3 is added to the third number, 4 is added to the fourth number, and so on up to the seventh number, what is the new average?

Changing the given: The average of 11 numbers is 121. If 1 is added to the first number, 2 to the second number, and so on up to the eleventh

number, what is the new average?

Switching the given and the wanted: The average of seven numbers is 49. Each of the data points were increased by the same amount. The new average is 53, what value was each data point increased by to raise the mean?

Participants fourth problem re-formulation task was on problem set 5 which was due on March 6, 2002. The mathematical content focus of the problem set was chance and probability and participants were asked to solve one problem using the five-step heuristic while posing two re-formulations and to pose one re-formulation for each of the other three problems. Table 5.9 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	10	10	0	0	1	2	5	0	2
P2	12	14	3	0	3	0	5	3	0
P3	7	7	0	0	0	0	0	7	0
P4	11	14	0	1	1	0	7	5	0
Total	40	45	3	1	5	2	17	15	2

Table 5.9: Problem re-formulation on problem set 5

Table 5.9 shows that like problem set 2 participants relied on changing the given and changing the wanted to re-formulate problems on this problem set and they used multiple techniques to re-formulate 5 problems. As with the previous problem sets switching the given and wanted was the most utilized level 1 re-formulation technique although level 1 techniques were used only 13% of the time. Level 2 techniques were utilized 87% of the time, the most of any problem set at this point in the instructional

treatment. Examples of re-formulated problems from problem set 5 follow,

Problem 2: In a random drawing of one ticket from a set numbered 1 through 1000, you have tickets 8775 through 8785. What is your probability of winning?

Switch the given and the wanted: You have a probability of $3/20$ of winning and received the following numbers from a drawing 122-136. What was the total number of tickets distributed for the event?

Change the given and change the wanted: If Beth has 19 tickets for a drawing with 100 total tickets and Veronica has 4 tickets for a drawing with 20 tickets, who has a better probability of winning?

The participants fifth problem re-formulation task was on problem set 6 which was due on March 27, 2002. The mathematical content focus of the problem set was counting and probability and participants were asked to pose one re-formulation for each problem. Table 5.10 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	9	11	0	0	1	2	0	8	0
P2	9	11	3	2	0	1	3	2	0
P3	8	8	0	1	0	1	6	0	0
P4	11	12	0	1	1	1	7	1	1
Total	37	42	3	4	2	5	16	11	1

Table 5.10: Problem re-formulation on problem set 6

Table 5.10 shows that as with problem sets 2 and 5, participants problem re-formulation on problem set 6 relied on changing the given and changing the wanted. Also participants posed 5 problems using multiple techniques. Level 1 techniques were 28.5% of the total number of techniques used and level 2 techniques were 71.5%. Compared to the previous problem sets, except problem set 3, there is an increase in students utilization of level 1 problem re-formulation techniques. Also participants utilized extension instead of switching the given and wanted most frequently of the level 1 techniques. Examples of problem re-formulation on problem set 6 follow,

Problem 4: Six people enter a tennis tournament. Each player played each other person one time. How many games were played?

Change the context: If there are 25 people invited to your house for a party and everyone shakes everyone else's hand at the party how many handshakes are there?

Extension: 3 different tournaments, one with 4 people, one with 5 people, one with 6 people. Each player played the other person one time. How many games were played in each tournament? Is there a pattern? Can you find a rule?

The participants sixth problem re-formulation task was on problem set 7 which was due on April 3, 2002. The mathematical content focus of the problem set was discrete mathematics and participants were asked to pose one re-formulation for each problem. Table 5.11 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

Table 5.11 shows that participants relied on the techniques of changing the given and extension for problem re-formulation on problem set 7. Also participants used multiple re-formulation techniques to pose 2 problems. Participants used level 1

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	13	13	0	1	3	5	4	0	0
P2	10	10	0	0	0	3	5	2	0
P3	12	14	0	2	0	5	4	3	0
Total	35	37	0	3	3	13	13	5	0

Table 5.11: Problem re-formulation on problem set 7

problem re-formulation techniques 43% of the time and level 2 techniques to re-formulate 57% of the time. Again, the trend of an increase in participants utilization of level 1 techniques continued and participants utilized extension as often as changing the given which has dominated the rest of their problem re-formulation. Examples of problem re-formulation on problem set 7 follow,

Problem 2: Consider networks with 0, 1, 2, 3, and 4 odd vertices. Make a conjecture about the number of odd vertices that are possible in a network. Explain your thinking.

Change the given: Consider networks with 0, 1, 2, 3 and 4 even vertices. Make a conjecture about the number of even vertices and the traverse ability of the network. Explain.

Extension: Knowing that you can create a network with an even number of odd vertices, is it possible for these types of networks to be traversable?

The participants final problem re-formulation task was on problem set 9 which was collected on May 8, 2002. The mathematical content focus of the problem set was algebraic thinking and participants were asked to re-formulate two problems. Table 5.12 shows the results of problem re-formulation on this problem set, first by problem and then aggregate.

	# Prob.	# Tech.	S.G.W	Context	Add	Ext.	Given	Want	Re
P1	8	10	1	3	0	0	4	2	0
P2	7	8	0	0	3	0	4	1	0
P3	6	6	0	0	0	4	2	0	0
P4	5	6	0	2	0	0	4	0	0
P5	5	5	1	0	0	1	3	0	0
Total	32	35	2	5	3	5	17	3	0

Table 5.12: Problem re-formulation on problem set 9

As with earlier problem sets changing the given was the most utilized problem re-formulation technique on problem set 9. Also participants re-formulated 3 problems using multiple techniques. Level 1 re-formulation techniques were utilized to re-formulate 34% of problems and level 2 techniques to re-formulate 66% of the problems. The trend in participants utilizing more level 1 techniques continued on this problem set and participants utilized extension and changing the context more than switching the given and wanted. Examples of problem re-formulation on this problem set follow,

Problem 1: A whole brick is balanced with $\frac{3}{4}$ of a pound and $\frac{3}{4}$ of a brick.

What is the weight of the whole brick?

Change the context: If a bottle and a glass balance with a pitcher, a bottle balances with a glass and a plate, and two pitchers balance with three plates, can you figure out how many glasses will balance with a bottle?

Problem 5: Two different numbers are drawn from the set {2, 3, 4, 5, 6} without replacement. What is the probability that the product of the numbers selected is a multiple of 3?

Extension: Design the problem using the 5 numbers written on separate

sheets of paper and box to hold them. Reach in and randomly pull out 2 numbers. Record your sets 40 times. What is your experimental probability and how does it relate to the theoretical probability? For the purpose of this activity consider the sets with the same two numbers the same.

(For example 3,4 is the same as 4,3)

In summary, a trend developed in participants' problem re-formulation during the instructional treatment. With the exception of problem set 3 participants utilized more level 1 re-formulation techniques as they gained more experience with problem re-formulation. Participants choice of level 1 techniques also became more diverse during the instructional treatment. Switching the given and wanted dominated the use of level 1 techniques early in the instructional treatment and this gave way to the use of both extension and changing the context later in the instructional treatment. By utilizing more level 1 re-formulation techniques, as participants gained experience with problem re-formulation, they demonstrated creativity and the ability to generate a more diverse set of problems from a previously solved problem.

Individual Problem Posing

This section will provide detailed description related to the problem posing of Bill, Carrie, Laura and Liz. Description will begin with each individual's beliefs about problem posing and changes in these beliefs during the instructional treatment. Beliefs about problem posing will be followed by a description of the individual's problem posing process. Finally, based on problem posing activities the individuals development as a problem poser during the instructional treatment will be described. It will be a goal of these descriptions to relate individual results to the whole class results described previously in this chapter.

Bill

At the time of this research Bill was in his final semester as a mathematics education major working towards certification to teach elementary and middle school mathematics. Bill began his college career as a business major and decided to become a teacher after substitute teaching during his freshman year of college. Bill decided to become a mathematics teacher because he felt that a degree in mathematics was prestigious and a sign of intelligence.

Bill's Beliefs About Problem Posing. Before looking at specific results related to characteristics of Bill's posed problems, Bill's beliefs about problem posing and the development of his beliefs during the instructional treatment will be examined. Initially, Bill had a conception of problem posing as a tool to articulate real world applications in mathematics as seen from his definition of problem posing during his first interview on February 4, 2002, "... putting words around applications." Bill highlighted his definition with the following example, "yeah, like 67 minus 23 ... Sue has 67 dollars and the car cost \$23. Does she have enough money to buy it? If so how much money does she have left? Could she get two cars?" Bill also stated during his first interview that problem posing did not always involve real world examples, but he was not able to give an example of such a situation. Although Bill held a conception of problem posing he did not see any necessity for it or any benefits of problem posing. Evidence of this comes from his second interview on March 27, 2002 when Bill stated that he only took part in problem posing because it was required of him in class and not something that he thought was beneficial, "I do it because I am supposed to and I do it for extra points ... no I don't think it is helpful." After more experience with problem posing Bill changed his attitude towards problem posing and articulated this change during his final interview on May 8, 2002,

On a different note I also like the instruction and persistence of problem

posing in the classroom although I am not sure how to implement it on a regular basis I feel I am better equipped to apply it in certain situations.

Thus, at the end of the instructional treatment Bill began to see benefits of problem posing and was not solely going through the motions of problem posing because it was a class requirement. As highlighted above, Bill's beliefs about and attitudes towards problem posing developed during the instructional treatment from feeling that problem posing is just a process he is forced to take part in to beginning to believe that there are benefits of student problem posing.

Bill's Problem Posing Process. It is also important to understand how Bill views the problem posing process before examining characteristics of his posed problems. Bill's problem posing process includes considering the mathematical content he is posing problems related to, posing problems, then considering the difficulty of his posed problems and whether his future students will understand his posed problems. Evidence of Bill's views of the problem posing process comes from journal entries and interviews.

Bill described his problem generation process on the February 25, 2002 journal entry which asked participants to pose problems related to Mrs. Smith's and Mr. Jones' classes' test scores. Bill described the process as noticing that the data lent itself to probability and statistics, then realizing he could compare the data between the two classes. This description implies that Bill's initial step in the problem posing process is to determine the mathematical content of the given information. During his second interview Bill commented that the problem posing process always included considering how difficult the problem he was posing would be for his future students and whether or not they would completely understand what the problem was asking for. "The process is, how difficult is this going to be for the students and are the students going to understand exactly what I'm posing here." Therefore Bill's problem

posing process included reflection on the difficulty of his posed problems and the clarity with which he was writing problems.

Finally, Bill believes that he is better at posing problems as re-formulations because it gives him structure, he stated on the second interview, "...the extensions give me a chance to really use higher thinking to you know maybe ask one question that is kind of is a little different than what has been asked before." Therefore Bill has articulated a problem posing process during the instructional treatment and articulated that he is better at posing problem as re-formulations. With an understanding of Bill's beliefs about problem posing and problem posing process it is easier to understand his development as a problem poser.

Bill's Development as a Problem Poser. Data was collected with regard to characteristics of Bill's posed problems from all of problem posing tasks during the instructional treatment described in Chapter 4 except the problem generation task on problem solving 6. Characteristics of Bill's problem generation can be seen from looking at his problem generation on the pre-assessment of problem posing, journal entry due on February 25, 2002, problem set 5, interview 2 and the post-assessment of problem posing. This data shows that over the course of the instructional treatment Bill developed a better sense of problem generation, showed more creativity in his problem generation, and became more effective at posing multi-step mathematics problems.

Bill posed two problems on the pre-assessment of problem posing, one for each set of given information. Both problems consisted of re-writing the given in a different context and extending it to a problem. Bill did not write problems related to the given sets of information. The problem from the set of information with numeric content was plausible but did not contain sufficient information for solution. The problem from the set of information without numeric content was plausible, contained

sufficient information for solution and required a multi-step solution process. This first attempt at problem generation shows that Bill didn't have a full understanding of the problem generation task and that his initial instincts about problem posing were to re-formulate the given set of information and extend it to a problem.

A month later, on the journal entry that was collected on February 25, 2002, Bill demonstrated a better understanding of the problem generation process. Bill posed five problems; four of which meet all three criteria in problem generation coding. The fifth problem was plausible and contained sufficient information but was a yes or no question so it did not require a multi-step solution process. Bill's multi-step problems in this situation were all coded as such because they ask the problem solver to perform two tasks, one each for Mrs. Smith's class data and Mr. Jones' class data. For example, Bill posed the following problem,

Given the two sets of data, whose median is higher, Mrs. Smith's class or Mr. Jones' class?

The researcher feels that this set of given information lent itself to multi-step problems that were written as the comparison, between the two classes, of a statistical representation of the data. Regardless of this, Bill demonstrated the ability to pose multi-step problems on this problem generation task.

On problem set 5, which was collected on March 6, 2002, Bill posed three problems from the set of given information. The three posed problems were all plausible, two contained sufficient information, and one required a multi-step solution process. The problem which did not contain sufficient information was because Bill did not define the word "similar" as it pertained to a group of people having a similar amount of cards. The problem which required a single step solution had all the information for a multi-step problem but a single step problem was asked, this problem follows,

Gary, Katie, Roby and Greg are rolling dice in the corner. Each person

has five dollars. Each person has one die each. One game consists of each person rolling their die against the wall. Whoever has the highest die wins a dollar from each. In the case of a tie, the highest die rolls go again and again until someone wins. The game is over when someone wins. If Katie wins every game, how many games will it take for Katie to take all of Gary, Greg and Roby's money?

This problem generation task shows more creativity in Bill's problem generation compared to previous tasks and that Bill was developing the ability to create more multi-step problems even from sets of information that did not necessarily lend themselves to posing multi-step problems.

Bill's development in posing multi-step problems can also be seen from his problem generation during his second interview on March 27, 2002. Bill was asked to pose as many problems as possible in as much time as he wanted from two sets of information during this interview. Bill was able to pose six problems. All six of his posed problems were plausible with sufficient information for solution, four of the six problems required a multi-step solution process and the other two required a single step solution. As part of the interview, the researcher asked Bill which problem he felt was the best he posed. Bill felt that he had developed a project for elementary school students based on the second set of information.

Given the distance from Portsmouth to L.A. and the price of gas. How many days of driving will it take to reach L.A. if you drive 800 miles a day? What will it cost in gas? What about food, hotels and laundry?

Bill also stated during the interview that his intention was to have his students treat this as a project in which they research the actual amount of money they would spend if they were driving cross-country. Again Bill was able to pose a majority of

multi-step problems from these sets of information and was able to develop what he deemed a project problem.

Bill's problem generation on the post-assessment of problem posing highlights his development in understanding the problem generation process. As with the pre-assessment, Bill only posed two problems on the assessment, but unlike the pre-assessment they were related to the sets of given information and were plausible with sufficient information and required a multi-step solution process. On the set of information without numeric content, Bill added all the necessary numeric information to make his problem solvable.

In summary, Bill's development as a problem poser on problem generation tasks highlights the classes development on these same tasks. Bill did not necessarily show an increase in his efficiency posing problems as problem generation but he did show a developed ability to pose multi-step problems post-instructional treatment.

Bill's problem re-formulation during the instructional treatment focused on changing the given and changing the wanted of problems. Of the 16 problems that Bill wrote as re-formulation, 12 of them involved changing the given or changing the wanted. Bill did pose two problem re-formulations in which he changed the context one each on problem sets 4 and 9. He also twice extended the original problem on problem sets 5 and 6. Bill's reformulation tended to look similar to the original problem. For example, on problem set 4 given the original problem,

The range of three numbers is 45. Both the mode and the median are 52.

Name two possible sets of three numbers.

Bill posed,

The range of three numbers is 12. Both the mode and the median are 20.

Name two possible sets of numbers.

Bill's problem re-formulation was similar throughout the instructional treatment, he occasionally utilized level 1 re-formulation techniques but tended to focus on the minimal change to the original problem.

Bill utilized level 1 problem re-formulation techniques 25% of the time during the instructional treatment and began using them on his third problem generation task. This utilization of level 1 posing techniques is not consistent with the class data presented previously as Bill did not utilize level 1 techniques more often on the final three problem sets. Bill's reliance on the level 2 techniques of changing the given and changing the wanted was consistent with the whole class problem re-formulation during the majority of the instructional treatment.

Carrie

At the time of this research, Carrie was a second semester graduate student working towards certification to teach at the elementary level. Carrie was a couple of years removed from her undergraduate degree and had decided to return to pursue her certification. She would like to teach elementary school so that she can instill confidence in her students early in the educational process.

Carrie's Beliefs About Problem Posing. Carrie did not enter the instructional treatment with developed beliefs about problem posing, but developed beliefs during the instructional treatment. Carrie's lack of a conception of problem posing at the beginning of the instructional treatment is highlighted by the definition of problem posing she articulated during her first interview on February 13, 2002,

...it just seems like it's more different ways to ask questions about a concept and encourage coming at it from all directions. So that basically seems to be the skill that you guys are looking for ... Well what else can you tell me about that, you know what else does that mean?

This quote suggests that Carrie was reflecting on the problem posing she had done in class to date, the reason that problem posing had been assigned, and describing problem posing as asking questions. Carrie began to articulate beliefs about problem posing during her second interview, on April 3, 2002, when responding to a question about the benefits of problem posing she stated, "so I can see how the very basic process is essential." Carrie had begun to notice benefits of problem posing, but at this point in the instructional treatment she had not articulated beliefs about problem posing.

By her third interview on May 8, 2002 Carrie had developed a view of problem posing as being related to mathematical ideas that the poser has a concept of and being related to the posers past experience. Evidence of this view comes from Carrie's description of her concept map of problem posing (see Appendix B). While describing her concept map Carrie explained that problem posing is related to, "... prior knowledge and everything included in prior knowledge, your life experiences, academic work, and your personal successes, failures, and goals." Carrie continued to argue that people will not pose a problem about mathematics or everyday experiences that they have no concept of. Thus, during the instructional treatment Carrie articulated the view that problem posing is related to the posers past experiences and current conceptions.

Carrie's Problem Posing Process. The problem posing process Carrie utilized during the instructional treatment involved assessing the mathematical content of the given information, considering the appropriate audience to pose for, assessing the complexity of her posed problem, and being sure that her posed problem is solvable. Evidence of Carrie's problem posing process can be found in her journal entry that was collected on February 25, 2002, her journal entry from April 15, 2002, and from her second interview on April 3, 2002.

Carrie described the process she used to pose problems related to Mrs. Smith's and Mr. Jones's class exam scores in her February 25th journal entry. Carrie noticed that the given information related to data analysis and comparison she then assessed everything she knew about data analysis and comparison and finally tried to relate it to what would be most interesting to learn from the data set. Carrie then attempted to pose problems that were not just calculations. Carrie's description shows that at this point of the instructional treatment, she had assessed the mathematical content of the set of information and the complexity of the problems that she had posed. There is more evidence of Carrie considering problem complexity during her problem posing process from her second interview. When she described why she liked one of the problems she posed during the interview Carrie stated, "... you know, think things through all the way, that it is a multi-step process I like that idea, more complicated than just find the answer." During her second interview Carrie mentioned for the first time that she always considers whether a problem she poses is solvable and states, "I would definitely feel like I was cheating almost if I was asking a problem that was impossible to solve or doesn't have a correct answer or is like given this information you'd be wrong you couldn't do that."

Finally, Carrie considers herself better at posing problems from sets of given information. She feels that problem re-formulation limits her problem posing possibilities. She feels that posing problems from a set of given information allows her to pose the obvious problems in order to begin her problem posing, but when re-formulating she feels that she is just trying not to pose the same problem as the original. Thus, Carrie has articulated a problem posing process during the instructional treatment and has suggested that she is better at posing problems from sets of given information.

Carrie's Development as a Problem Poser. Data was collected with regard to characteristics of Carrie's posed problems from all of the problem posing tasks during

the instructional treatment except for problem solving 6. Characteristics of Carrie's problem generation can be seen from looking at her problem generation on the pre-assessment of problem posing, journal entry collected on February 25, 2002, problem set 5, interview 2 and the post-assessment of problem posing. This data shows that over the course of the instructional treatment Carrie became more effective at posing multi-step mathematics problems when she was not posing problems under a time constraint.

On the pre-assessment of problem posing Carrie generated three problems. Based on the set of information containing numerical content, Carrie generated two problems, the first was plausible and contained sufficient information, but was solved using a single step solution process, the second was plausible, but did not contain sufficient information. The lone problem Carrie posed based on the set of information without numeric content was plausible, but did not contain sufficient information for solution. On the pre-assessment of problem generation Carrie seemed to understand the task but was not able to pose any problems that required multi-step solution processes and in fact only posed one problem that had sufficient information for solution.

Carrie's journal entry collected on February 25, 2002 shows evidence that Carrie began to pose multi-step mathematics problems. Carrie posed three problems in this journal entry, all of which were plausible with sufficient information and required a multi-step solution process. Two of these problems were comparisons of statistical analysis of the test data between Mrs. Smith's and Mr. Jones' class. The third problem was multi-step and did not involve a between class comparison. Carrie posed,

For each question on the exam calculate the frequency it was answered incorrectly for each individual class and both classes together. What can you tell from this data?

This problem generation task shows that after a month of problem re-formulation experience and with more time Carrie was able to pose multi-step problems. In fact, she was able to pose a multi-step problem that did not rely on comparison between the classes to be multi-step.

More evidence of Carrie posing multi-step mathematics problems can be found on problem set 5 which was collected on March 5, 2002. Carrie generated three problems from the set of given information on this problem set and they all met the three criteria in problem generation coding. All three problems focused on probability and whether or not you had a good chance to win a certain bet. An example of Carrie's problem generation on this task follows,

Your friend bets you that he can roll at least one 3 when rolling all 3 dice and pull a card equal or less than 3 on the first try. What are his chances?
Would you bet him?

Carrie had started posing multi-step problems with regularity on this problem generation task and this trend will continue in the rest of her problem generation.

During her second interview on April 3, 2002 Carrie posed seven problems based on the two sets of given information. All seven problems were plausible, contained sufficient information and required a multi-step solution process. During the interview Carrie was asked which of the problems she posed she thought was the best. She said that she was not impressed with any of the problems from the first set of information but that she liked the following problem,

Calculate how much more expensive it will be to travel 5000 miles with your Ford Explorer which gets 20 mpg versus your friends Ford Focus which gets 34 mpg and assume a \$1.40 price per gas average. And then given that information could you save money by camping in the Explorer

and sleeping in the Explorer instead of having to stay in hotels with the Focus?

Carrie stated that this problem required more information related to where you would be camping and what hotels you would be staying in and also said that she viewed this as a project problem. Thus Carrie was also able to go beyond posing multi-step problems and pose a “project problem”.

Carrie’s final problem generation task was under a time constraint on the post-assessment on May 13, 2002. Carrie generated four problems on this task and only one required a multi-step solution process. Her problems from the set of information with numeric content included, one single step problem and two problems without sufficient information, which were both missing an interest rate for a credit card (see Appendix B). Carrie’s problem from the set of information without numeric content required a multi-step solution process and she was able to add all the numeric information necessary to make the problem solvable.

In summary, Carrie was capable of posing problems through problem generation and evidence from her problem generation on homework and journal entries implies that Carrie was able to pose more multi-step problems when she wasn’t working under a time constraint. Similar to the whole class, Carrie posed more problems on the post assessment of problem posing and posed more multi-step problems during the course of the instructional treatment.

Carrie’s problem re-formulation can be described from all of the problem re-formulation tasks during the instructional treatment, except problem set 6. Carrie’s problem re-formulation during the instructional treatment focused on changing the given and wanted information from the original problem and she often changed both to re-formulate a problem. For example, Carrie chose to re-formulate the following problem on problem set 9,

Two different numbers are drawn from the set $\{2,3,4,5,6\}$ without replacement. What is the probability that the product of the numbers selected is a multiple of 3?

and posed,

Given the even integers from 0 to 20 inclusive what is the probability that any two numbers selected will be a multiple of 4?

Carrie utilized level 1 problem re-formulation techniques. Of the 17 problems she posed as re-formulations, Carrie changed the context once on problem set 2, switched the given and wanted twice on problem set 3, and extended three problems on problem sets 2, 3 and 7.

Carrie's problem re-formulation with regard to utilizing level 1 techniques is not consistent with the whole class results. Carrie utilized level 1 problem re-formulation techniques more often during the beginning of the instructional treatment and only once on the last five problem sets. Similar to the whole class's results, Carrie favored changing the given and changing the wanted as problem re-formulation techniques.

Laura

At the time of this research Laura was a sophomore majoring in mathematics education and working towards certification to teach elementary and middle school mathematics. Laura began the semester with past experience teaching mathematics in a summer program called Summerbridge and had decided on teaching mathematics as a future career based on her high school mathematics experiences.

Laura's Beliefs About Problem Posing. Laura entered the instructional treatment with a developed belief, based on past experiences, that problem posing is a process of generating problems that both students and teachers may engage in. Evidence

of Laura's belief comes from her first interview on February 4, 2002 when she defined problem posing as "... when students look or even teachers, would look at given information and write questions that can be answered based on the given information." Laura did not change her conception of problem posing during the instructional treatment.

Laura's Problem Posing Process. During the instructional treatment Laura relied on and developed a strategy for generating problems from sets of given information that included considering and applying her previous knowledge to the situation, considering the appropriate level of her posed problem, and considering her posed problems solvability. Evidence of Laura's problem posing process comes from journal entries and interviews during the instructional treatment.

Laura referenced the role of her past knowledge in the problem posing process in her journal entry collected on February 25, 2002 and during her second interview on April 2, 2002. When she described her process of posing problems related to Mrs. Smith's and Mr. Jones' classes exam scores in her journal entry, Laura stated that when she looked at the given information she was considering all the different ways that someone could manipulate the given data to report on it and was trying to ask questions that were beyond just calculation and required thought. Thus, Laura was referring to her past knowledge about data analysis to guide her problem posing in this situation. Laura's reference to her past knowledge surfaced again during her second interview when she discussed her problem posing process and stated, "so I kind of had that in mind, how can I manipulate what I've learned to ask a question."

Evidence of Laura considering the appropriate level of her posed problems and their solvability comes from her April 15, 2002 journal entry and her second interview. In the second interview and in her journal entry for April 15, 2002 Laura discussed her posing audience as her future middle school students and discussed that she attempts

to make the level of her posed problems appropriate for that audience. Also during her second interview, as Laura discussed the solvability of problems, she stated, "I was thinking of the solution as well . . . but I was thinking someone was going to have to solve it so I didn't want to make it crazy."

Finally, Laura believed that she was better at posing problems from sets of given information because it allows for more creativity and as she suggested, doesn't "blur" her thinking. In the third interview on May 10, 2002 Laura took it one step further and said that she believes she is better at posing problems from sets of information without numeric content because she can be more creative. Laura said,

I think also that whole numerical setting, um, changes the way I would pose a problem because you have to think about, you can't just create a situation in your head because it is already here and created and so if you do one thing to one side, you kind of have to know the outcome before you write the problem.

Thus Laura articulated a problem posing process during the instructional treatment and discussed in detail her belief that she is better at posing problems from sets of given information because it allows for more creativity. Laura's problem posing process included relating the set of information to her prior knowledge, and examining the difficulty and solvability of her posed problems.

Laura's Development as a Problem Poser. Data was collected with regard to characteristics of Laura's problem posing from all of the problem posing tasks during the instructional treatment except for problem solving 5. Characteristics of Laura's problem generation can be seen by looking at her problem generation on the pre-assessment of problem posing, journal entry collected on February 25, 2002, problem set 6, interview 2, and the post-assessment of problem posing. This data shows Laura's competency posing problems from sets of given information. Laura entered the in-

structional treatment with an understanding of problem posing, was able to pose multi-step problems, and during the instructional treatment started to pose open-ended problems and problems that went beyond the surface features of the given. It is also apparent from the data that Laura is more comfortable posing problems from sets of information which do not contain numeric information.

On the pre-assessment of problem posing, Laura demonstrated her ability to pose multi-step problems and her preference for posing problems from sets of information without numeric content. Laura posed four problems on the pre-assessment, all were plausible and contained sufficient information for solution, one of the problems required a multi-step solution process. Laura showed her preference for posing problems from sets of information without numeric content as three of her posed problems on the pre-assessment were related to the set of information without numeric content. Laura posed the following problem, which is evidence of her ability to pose multi-step problems, related to the set of information without numeric content,

If 10 students and 3 faculty arrive every 15 minutes between the hours of 8am and 12 noon and 4 students and 1 faculty leaves every 30 minutes between the same time slot, how many total students and how many total faculty are at the lot at 12 noon?

In her journal entry collected on February 25, 2002, Laura demonstrated the ability to pose multi-step problems and to pose problems that go beyond the surface features of the given information. Laura posed five problems in this journal entry, all of her problems were plausible, contained sufficient information, and required a multi-step solution process. Also on this task, Laura's posed problems went beyond asking for a comparison of statistics between the two classes. Laura posed a problem related to mean, median and mode but the other four were more in depth ways to look at and think about the data. For example Laura posed,

If we compare the results of Mrs. Smith's class scores with Mr. Jones' class, can we say that one teacher is better than another if one class scored better than the other? What is wrong with that assumption? What could be possible lurking variables?

This example highlights that on this problem posing task Laura posed multi-step problems that look at the set of given information beyond the surface features.

Laura continued to pose multi-step problems and showed an ability to pose open-ended problems on problem set 6 which was collected on March 27, 2002. Laura generated two problems on this problem set, both of her problems were plausible, contained sufficient information and required a multi-step solution process. Laura continued to pose multi-step problems and on this task posed an open-ended problem, which follows,

Explain how a casino can stay in business with the game of roulette?

This problem is again evidence of Laura's ability to pose problems that go beyond the surface features of the set of information and allow the solver some freedom with their solution process.

During Interview 2 on April 2, 2002, Laura continued to generate multi-step problems and demonstrated that although she prefers to pose problems from sets of information without numeric content she still requires some structure to help guide her problem posing. Laura generated eight problems from the two sets of given information, all eight of her problems were plausible and contained sufficient information for solution. Two of the eight posed problems show that Laura is still posing multi-step problems. Also both multi-step problems were related to the first set of information, these problems follow,

Make a stem and leaf plot representing how much money people spent on

their break. Is the distribution normal or skewed? Are there outliers in the data?

On a scatterplot let the x-axis be #'s 1-10 (rating of travel experience) and the y-axis be the amount of money spent on break. Plot points based on people's rating and how much money they spent. Is there a correlation between the people's rating and how much money they spent? Is it statistically wise to say that the more you spend on vacation, the greater the experience is?

Laura discussed her ability to pose multi-step problems from the first set of information and suggested that the structure of the first set of information aided her problem posing when she stated "I didn't have to create my own situation" because of the added structure.

On the post-assessment of problem posing Laura again showed an ability to pose multi-step and open ended problems. Laura generated seven problem situations on the post-assessment, all of which were plausible, six contained sufficient information for solution, and two required a multi-step solution. Laura posed an open-ended problem related to the set of information without numeric content.

The university wants to know how many students drive to campus. They also want to know how the number compares to the past years. Describe how you could find out this information and how you would write up / present this information to the driver's board. (Hint: Use graphs, charts, etc.)

As she demonstrated during the instructional treatment the post-assessment highlights Laura's ability to pose multi-step problems and her preference for posing problems from sets of information without numeric content.

Throughout the instructional treatment Laura showed a proficiency for generating problems from sets of given information. She was able to generate problems that required a single step solution process and multi-step problems. Laura was also able to pose open ended problems from different sets of given information. Laura's ability to generate multi-step problems is consistent with the results from the whole class. Laura was able to take her problem posing one-step further and pose problems that go beyond the surface features of the given information and are open-ended.

Data related to characteristics of Laura's problem re-formulation was collected on all problem sets during the instructional treatment except for problem set 5. Laura re-formulated 21 problems during the instructional treatment and utilized level 1 problem re-formulation techniques 8 times. Evidence of Laura's use of level 1 re-formulation techniques comes from the following problem sets, she switched the given and wanted twice on problem sets 3 and 4, changed the context three times on problem sets 6 and 7, and extended three problems on problem sets 2, 3, and 7. This data implies that Laura showed a proficiency for utilizing all forms of problem re-formulation throughout the instructional treatment but as was typical of the whole class, Bill, Carrie, and Liz, she relied on changing the given and changing the wanted.

Liz

At the time of this research, Liz was a sophomore majoring in mathematics education and working towards certification to teach elementary and middle school mathematics. Liz decided to pursue a career as a mathematics teacher after her mathematics teacher her freshman year in high school suggested it as a possible future career.

Liz's Beliefs About Problem Posing. Liz's conception of problem posing developed over the course of the instructional treatment. Liz began the instructional treatment without a well developed conception of problem posing, but after the in-

structional treatment she was able to articulate what problem posing is to her and describes problem posing as a tool to aid in the problem solving process. Evidence of Liz's lack of a conception of problem posing early in the instructional treatment comes from her first interview on January 31, 2002. During that interview Liz had a difficult time defining problem posing. When asked to define problem posing Liz first defined problem solving and then after being asked again gave the following definition of problem posing,

I would see it as. I don't know kind of just like what you have been exposed to, like a lot of people pose problems or make up their own problems from maybe like the day before or pick something in a room and be like oh, there is chalk on the chalkboard, so how many, you know if I had this and so many were taken away you know. Some people just visually see it or some people an image comes in their mind.

Liz was explaining situations where problem posing may take place and may be relating problem posing to individuals past experiences as she stated, "... what you have been exposed to."

As Liz was explaining her concept map during her third interview on May 7, 2002, she demonstrated that late in the instructional treatment she was able to verbalize aspects of what she believes problem posing is and that problem posing aids the problem solving process. During the third interview Liz stated,

I thought of problem posing and then I thought of the different ways that we can come up with it, um, we're just given data to make a question from it, or problem posing can help solve problems, or making a question from a given example.

Liz's Problem Posing Process. During the instructional treatment Liz articulated a problem posing process that included assessing the mathematical content of the

given information, assessing what aspects of that information she wanted to convey to others through problems, and judging whether her posed problems were solvable. Liz demonstrated that she was considering the given information and how to present it when she described the process she went through to pose problems related to the exam scores from Mrs. Smith's and Mr. Jones' classes. Liz said she went through the process of determining how she would like to show others about the given data without just giving it to them and then decided on questions comparing the two data sets. During her second interview on March 28, 2002 Liz commented on considering the solutions of problems that she is posing when she stated, "I am kind of thinking about how could solve, make sure they can be solvable I guess, or that they are actual like realistic, like there is no way from the data that you can't solve." Liz continued to describe and articulate her problem posing process and ways she went about posing problems when she described her concept map during her third interview on May 7, 2002. Liz stated,

...making a question from a given example I said we are changing the problem, we can add information, change the given info, or change the topic of it. ...for the given the data to make questions I said that comes from collecting some data, which came from a hypothesis, which creates the question.

Finally, during her third interview Liz also stated that she believes that she is better at posing problems from sets of given information,

Because, um, you do have with a set way, extending um, you're already given so much and like you can extend it a certain way. But I like just given any data lying around and see what you come up with.

During the instructional treatment Liz was able to articulate a problem posing process and discussed her belief that she is better at posing problems from sets of

given information. The data suggests that Liz's problem posing process included considering the given information and judging if her posed problems were realistic and solvable.

Liz's Development as a Problem Poser. Data was collected with regard to characteristics of Liz's problem posing from all of the problem posing tasks described in the instructional treatment. Characteristics of Liz's problem generation can be seen from looking at her problem generation on the pre-assessment of problem posing, journal entry collected on February 25, 2002, problem set 5, problem set 6, interview 2 and the post-assessment of problem posing. This data shows Liz's development as a problem poser over the course of the instructional treatment. Liz's problem posing changed depending on whether she was posing problems under a time constraint. Under a time constraint Liz had difficulty posing mathematics problems and especially multi-step problems. When she was not under a time constraint Liz was able to pose multi-step problems during the instructional treatment.

Liz demonstrated her difficulty posing problems under a time constraint on the pre-assessment of problem posing on January 23, 2002. On the pre-assessment Liz generated ten statements, four of her ten statements were not problem posing products. Of the six problem posing products that Liz generated all were plausible, three contained sufficient information for solution, and one required a multi-step solution procedure. Liz's statements which were not problem posing products tended to be yes or no questions. For example Liz posed,

Would a parking garage and more space for cars influence whether or not
you bring your car to school?

Therefore on the pre-assessment Liz demonstrated efficiency writing statements but did not show proficiency for posing mathematics problems. Liz did show the ability to pose a multi-step problem on this pre-assessment and demonstrated this ability on

other problem posing tasks.

Liz demonstrated development in posing multi-step problems and the ability to pose problems when she was not dealing with a time constraint in her journal entry collected on February 25, 2002. Liz generated four problem posing products related to the exam scores for Mrs Smith's and Mr. Jones' classes. All of Liz's posed problems in this journal were plausible and contained sufficient information for solution, and three required a multi-step solution procedure. Liz's multi-step problems based on this set of given information all require the comparison of statistical analysis between the two classes. For example Liz posed,

From the exam scores given by Mrs. Smith's class and Mr. Jones' class make a bar and a box and whisker graph. From these two graphs which one works better for showing the data and why?

Liz did not pose problems related to this set of information that went beyond the surface features of data analysis between the two classes, however, she demonstrated the ability to pose more multi-step problems than on the pre-assessment.

Liz continued to demonstrate the ability to pose multi-step problems when she was not posing problems under a time constraint and showed increased creativity in her problem posing on problem set 5, which was collected on March 5, 2002. Liz generated three problems from the set of given information on this problem set, all her problems were plausible, contained sufficient information and required a multi-step solution process. There is some evidence of more creativity in Liz's posed problems on this problem set and an example follows,

You have \$20 to use on a new game that your friends made up. If you pick a card out of the deck and it is a red card, then you'll bet \$5 and if it is a black card you'll bet \$3. After betting your money you'll roll the two dice to get a number greater than 6 and if you do you win twice as

much money and if you loose you loose it all. Is this in your favor to win money for this game?

The trend of Liz posing multi-step problems continued on problem set 6, which was collected on March 27, 2002. Liz generated two problems on the problem set both of which required a multi-step solution process. One problem was just a basic probability problem with four parts and the other required the solver to calculate winnings after three spins of the roulette wheel. In the second problem Liz was able to add the necessary information to make the problem solvable.

Liz again demonstrated her difficulty posing problems under a time constraint during her second interview on March 28, 2002. Liz was able to generate nineteen statements on the two problem generation tasks during her second interview, but only seven of the statements were problem posing products. Of Liz's seven problem posing products all were plausible, five contained sufficient information for solution, and two required a multi-step solution. For example, related to the first set of information, Liz posed

Where was the most popular travel experience with the best rating?

Liz felt that this was the best problem that she posed on this problem posing task because the information is useful, in her mind it would be useful to travel agents in the future. Liz wrote eleven questions related to the second set of information but none of them were problem posing products. They were all questions she might consider with a friend before traveling cross-country. For example she asked,

Where are we going?

This demonstrated again that under a time constraint Liz was able to ask questions efficiently but showed a lack of proficiency developing problem posing products and multi-step problems.

Liz continued to demonstrate difficulty posing problems under a time constraint on the post-assessment of problem posing given on May 13, 2002. On the post-assessment Liz was able to generate nine statements, seven of which were problem posing products. Of Liz's seven problem posing products, six were plausible, three contained sufficient information for solution and one required a multi-step solution. There was little difference from the pre-assessment to Liz's post-assessment of problem posing.

Liz's problem generation parallels the class in the fact that she became more efficient at posing multi-step problems during the instructional treatment as was shown through her posing on the tasks that did not include a time constraint. Liz struggled, however, during the instructional treatment posing problems under a time constraint. She was able to write more statements and pose more problems on the post-assessment, thus Liz's efficiency had improved, but the likelihood of her posing multi-step problems under a time constraint did not change.

The characteristics of Liz's problem re-formulation can be seen from all the problem sets collected during the instructional treatment. Liz's problem re-formulation was predictable throughout the instructional treatment. Liz posed 23 re-formulated problems and of these she changed the given information 16 times. Liz only utilized level 1 problem re-formulation techniques three times during the instructional treatment, Liz changed the context twice on problem sets 6 and 7, and extended one problem on problem set 2. A typical problem re-formulation for Liz was as follows,

Original problem: A special rubber ball is dropped from the top of a wall that is sixteen feet high. Each time the ball hits the ground it bounces back only half as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

Liz posed

A special rubber ball is dropped from the top of a wall that is 64 feet high. Each time the ball hits the ground it bounces back only $\frac{1}{2}$ of its height as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

Unlike Bill, Carrie and Laura, Liz did not focus on changing both the given and the wanted, she in fact only changed the wanted information twice during the instructional treatment to re-formulate a problem. Also, Liz's problem re-formulation was consistent with the rest of the class in that she relied on changing the given to re-formulate problems. Liz utilized level 1 problem re-formulation techniques, however, far less often than the class as a whole.

Summary of Individual Problem Posing

In summary, the characteristics of Bill, Carrie, Laura and Liz's problem posing highlighted the results from the whole class data. The four individuals demonstrated the ability to pose multi-step problems and posed a greater frequency of multi-step problems as the instructional treatment progressed. Similar to the whole class Bill, Carrie, and Liz typically relied on level 2 problem re-formulation techniques. Laura utilized level 1 techniques more often than the other three individuals and with greater frequency than the whole class. While highlighting characteristics of posed problems these four cases also provide insight into participants' beliefs about problem posing and their problem posing processes. A detailed description of beliefs about problem posing will be included in the results that follow in Chapter 6.

Chapter 6

Beliefs Results

This chapter presents results related to participants' beliefs about mathematics, beliefs about teaching and learning mathematics, and beliefs about the relationship between problem posing and school mathematics. Results will be presented first with regard to the whole class, and will be followed by results related to the four individuals who agreed to interviews.

Beliefs About Mathematics

Pre-Instructional Treatment

Data related to participants' beliefs about mathematics prior to the instructional treatment was collected on the pre-assessment of beliefs, which was assigned in class on January 23, 2002 and collected on January 28, 2002 (see appendix B). Two views of mathematics and two views of the practice of mathematics emerged from participants' responses to the first and second item on the pre-assessment. The first item asked participants to list all the words they thought were related to mathematics and the second item asked participants to complete the phrase "Mathematics is ...". These views may be specific to this class of pre-service teachers and the goal was not to generalize them. In this context, views that were expressed by a majority of the class have been labeled predominant and the term secondary is used to help describe the views of the remainder of the participants. The labels pre and post were

added to clarify whether the set of views emerged pre or post instructional treatment. The development of the views, *mathematics predominant pre*, *the practice of mathematics predominant pre*, *mathematics secondary pre*, and *the practice of mathematics secondary pre* will be discussed in the remainder of this section.

Participants' word list responses were coded by organizing words based on the frequency with which they appeared on the collection of word lists. This frequency coding led to two distinct groups of words. Group 1 words appeared on 1, 2, 3, or 4 participant word lists and group 2 words appeared on 5 or more of these word lists. The words in each group follow. Words presented in quotes are statements from participants, while words not presented in quotes are categories framed by the researcher.

Group 1: "related to school", "answer", "tests", "relationships", "definition", "operations based", "theory", "proof", "thinking", Other subjects (i.e. chemistry, physics), negative words (i.e. frustrating).

Group 2: "problem solving", "word problems", "challenging and time consuming", "teacher and career", Math words(number, division, etc.), positive words (fun, exciting, etc.).

Group 2 words, which occurred most frequently, imply that the participants in this study predominantly view mathematics as including problem solving and problem solving with word problems, have a positive attitude towards mathematics, and feel that being engaged in mathematics is time consuming but challenging. Group one words show a secondary belief that mathematics is about thinking or a way of thought that involves theory but can be frustrating, and that the practice of mathematics involves finding answers.

Responses from the second item on the pre-assessment were organized into two groups based on the frequency of responses. Group 1 responses appeared 1, 2, or 3

times and group 2 responses appeared at least 5 times. The responses in each group follow,

Group 1: “Systems of rules”, “finding solutions”, “a way of thought”, “teaches people how to think”, “a part of life”.

Group 2: “Use and study of numbers, symbols, operations, and relationships”, “problem solving”, “interesting and challenging”.

Group 2 responses from this item support the group 2 responses from the first item as participants responses again suggested the belief that mathematics is problem solving. Participants responses also suggested that the practice of mathematics is interesting and challenging and involves using operations. Since group 2 responses on the two items appeared most frequently they led the researcher to the following descriptions of the *mathematics predominant pre* and *the practice of mathematics predominant pre* views.

Mathematics predominant pre: Mathematics is a problem solving domain that is characterized by the study of numbers, operations and relationships.

The practice of mathematics predominant pre: Practicing mathematics is fun, challenging, time consuming, entails the use of numbers, symbols and relationships, and is related to a career as a teacher.

Similarly group 1 responses from the second item support the group 1 responses from the first item as participants suggested secondary beliefs that mathematics is a way of thought and that the practice of mathematics is related to finding solutions. Group 1 responses led the researcher to the following descriptions of the *mathematics secondary pre* and *the practice of mathematics secondary pre* views.

Mathematics secondary pre: Mathematics is a way of thought. It is a part of life and the study of mathematics makes people think.

The practice of mathematics secondary pre: Practicing mathematics can be frustrating at times, involves utilizing operations and finding answers, and is related to school.

The third item on the pre-assessment of beliefs asked participants if they agree with the following statement, "Mathematics is always changing." Participants responses to this short answer question represent two distinct views of the nature of mathematics. The first view, which was shared by six participants and paraphrased here, was that mathematics is not changing but that the way we teach mathematics and the way we solve mathematics problems is always changing. An example of this first view of the nature of mathematics is seen in one participant's response, "I believe that mathematics itself is a concrete idea, but the processes that we use to solve the math change along with the methods we use to teach it to our students." The second view, which was the predominant view, was that mathematics as well as the way we teach mathematics is changing. Participants described multiple factors that influence the change in mathematics, including, because everything in the universe is changing, because of technology, and because of the discovery of new patterns and strategies. In the words of a participant's pre-assessment response, "yes, I agree that mathematics is always changing because people are always studying and investigating processes and theories"

In summary, pre-instructional treatment the participants as a group shared pre-dominant and secondary beliefs about mathematics, the practice of mathematics, and the nature of mathematics.

Post-Instructional Treatment

Participants were asked to complete the same beliefs assessment post instructional treatment. The post-assessment of beliefs was administered in class on May 13, 2002 and participants were given 30 minutes to complete the assessment. Similar to the

pre-assessment of beliefs, two views of mathematics and two views of the practice of mathematics emerged, based on the frequency of participants responses on the first two items of the post-assessment. The development of these views, *mathematics predominant post*, *the practice of mathematics predominant post*, *mathematics secondary post*, and *the practice of mathematics secondary post* will be presented in the remainder of this section.

Word list responses on the post-assessment were organized into two groups based on the frequency of the word appearing in the participants' lists. Group one words occurred on 1 or 2 word lists and group 2 words appeared on at least 4 word lists. The groups follow,

Group 1: "Word problems", "shapes", "proofs", "theorems", "intriguing", "exploration", "creativity", "active minds", "enthusiasm", "satisfaction", "projects", "games", "technology and logic", "manipulatives", "cooperative learning".

Group 2: "problem posing", "problem solving", "patterns", "fun", Math words(number, division, etc.).

Group 2 responses from the word lists imply that participants are viewing mathematics as problem posing and problem solving and view practicing mathematics as fun and involving finding patterns. Group 1 words from this item on the post-assessment demonstrate that participants view mathematics as a way of thought that includes proof and exploration while practicing mathematics involves using manipulatives and having active minds.

Responses to the second item of the post-assessment were organized into two groups based on the frequency of the responses. Group 1 responses appeared once and group 2 responses appeared at least twice. The groups follow,

Group 1: "Recognizing patterns", "using numbers to solve and pose problems", "a foundation of knowledge", "problem solving", "a way of thinking", "interest-

ing”.

Group 2: “Relationships between numbers and objects”, “the study of numbers and processes”, “fun”, “manipulation of numbers and symbols”, “ways to better understand the world”.

Group 2 responses on the second item of the post-assessment suggested that mathematics includes the study of numbers and attempts to better understand the world. Participants also suggested that practicing mathematics involves using numbers and symbols. The group 2 responses on the two items described led the researcher to the following descriptions of the *mathematics predominant post* and *the practice of mathematics predominant post* views.

Mathematics predominant post: Mathematics is a problem posing and problem solving domain that is characterized by the study of numbers, relationships, patterns and processes.

The practice of mathematics predominant post: The practice of mathematics is fun, includes procedures with numbers and symbols, and attempts to better understand the world.

Group 1 responses on the second item of the post-assessment support the group 1 words from the word lists as participants again suggested that mathematics is a way of thinking and that doing mathematics includes problem solving, problem posing, and finding patterns. The group 1 responses on the two items described led the researcher to the following descriptions of the *mathematics secondary post* and *the practice of mathematics secondary post* views.

Mathematics secondary post: Mathematics is a foundation of knowledge as well as an intriguing way of thinking that includes proof.

The practice of mathematics secondary post: Practicing mathematics includes solving and posing problems using creativity, active minds, manipulatives, cooperation, and technology.

On the final item of the post-assessment participants again responded to the statement “Mathematics is always changing.” Responses to this item represented two distinct views of the nature of mathematics. The first view was represented by ten participants who believed that mathematics was changing. Half of these participants stated that mathematics is changing because “any science is changing.” The other participants gave reasons for mathematics changing that were similar to and included, “because individuals are constructing new understandings of mathematics.” The second view of the nature of mathematics which was suggested by eight participants is that mathematics is not changing but that mathematics teaching and learning is always changing.

In summary, similar to pre-instructional treatment, post instructional treatment the participants as a group shared predominant and secondary beliefs about mathematics, the practice of mathematics, and the nature of mathematics. Since the views suggested by the assessment post-instructional treatment are different than the views suggested by the assessment pre-instructional treatment it can be concluded, that as a group, the participants’ experiences during the semester influenced their beliefs about mathematics.

Changes in Beliefs About Mathematics

This section will discuss change related to participants’ beliefs about mathematics, beliefs about doing mathematics, and beliefs about the nature of mathematics. First, participants’ views of mathematics underwent some qualitative change during the course of the instructional treatment. Examining participants’ *mathematics predominant pre* and *mathematics predominant post* views demonstrates that post

instructional treatment participants consider mathematics as much a problem posing as problem solving domain and are relating mathematics to their worlds, since they suggested that mathematics attempts to describe the world. These changes are highlighted by the fact that on the post-assessment word lists, ten participants included problem posing and nine included problem solving while no participants mentioned problem posing on the pre-assessment word list. The change of relating mathematics to the world is highlighted by a participant's completion of "Mathematics is..." from the post-assessment of beliefs, "... a way of looking at the relationships between numbers to solve problems, make predictions, and better understand our world."

Examining participants' *mathematics secondary pre* and *mathematics secondary post* views demonstrates that post instructional treatment participants are viewing mathematics as a more open ended discipline. The fact that from pre- to post-assessment there was a decline in the number of times mathematics was mentioned as the manipulation of numbers and symbols and an increase in words that imply an open-ended nature of mathematics (i.e. exploration, creativity, active minds) is evidence of this change in participants' views. Words such as intriguing, exploration, creativity and active minds became significant parts of participants' word lists. A participants completion of "Mathematics is ..." also helps highlight this new view, "... a fun and interesting way to explore properties ... that are around us everyday."

Examining participants views of the practice of mathematics from pre- to post instructional treatment implies that participants transitioned from viewing practicing mathematics as a chore, pre-instructional treatment, to viewing practicing mathematics as interesting, post-instructional treatment. This change is highlighted by participants use of words such as creativity, exploration, and active minds post-instructional treatment. Also post-instructional treatment participants started to view the practice of mathematics as both posing and solving problems. This change is again high-

lighted by a participant's completion of "Mathematics is. . ." from the post-assessment of beliefs, "... a variety of concepts that use numbers, formulas, graphs, charts, and manipulatives to solve or pose problems."

Participants' beliefs about the nature of mathematics include mathematics is changing, and mathematics teaching and problem solving are changing both pre- and post-assessment. One difference in participants' views is that on the post-assessment two more participants mention teaching and learning mathematics as changing without discussing if the discipline of mathematics is changing. Otherwise there is not any noticeable change in participants' views of the nature of mathematics post instructional treatment.

In summary, post instructional treatment participants seem to be more positive about mathematics and open to the idea that mathematics is open ended, includes problem posing and allows for creativity. This hypothesis comes from analysis of participants' pre- and post-assessment of beliefs and indicates a change over the course of the instructional treatment. This hypothesis is highlighted by a participant's statement on their final journal entry of the semester collected on May 16, 2002, "I learned to think about math in a very open-ended way, because before I had an opinion of math that was very close minded."

Beliefs About Teaching and Learning Mathematics

Beliefs About Teaching Mathematics

Before describing participants' beliefs about teaching mathematics this section will describe why participants desire to be mathematics teachers. The goal of this description is to provide background to help understand participants' beliefs about teaching mathematics. Participants' mathematical autobiographies were collected on January 28, 2002 and the coding of them revealed pivotal experiences, from participants mathematical experience, related to becoming mathematics teachers. These

pivotal experiences suggest two main motivations for participants to become teachers. The first motivation was because of a past mathematics teacher. A few participants had positive experiences that caused them to want to teach so they could directly model their past teachers, one participant wrote, "I want to teach like my algebra two teacher, make math enjoyable and destroy stereotypes that exist." But not all participants wanted to model past teachers because of positive experiences. For instance, one participant wrote that after a bad experience in calculus she wanted to teach so she could be "... a model of female confidence in math for my students." Other participants looked up to their mathematics teachers and decided to follow their lead. For instance, one participant stated, "my high school geometry teacher was my role model and made me realize that I wanted to be a math teacher." The second motivation for participants to choose teaching as a possible career path was experience teaching mathematics. Five participants said that either substitute experience, summer teaching, or teaching younger siblings shaped their interest in teaching mathematics. For example, one participant decided to become a mathematics teacher based on her experience teaching her sister how to count to 20 in third grade and her experience teaching pre-schoolers while she was in high school. From these pivotal moments emerges a glimpse of participants' beliefs about teaching, a fuller description follows.

The remainder of this section will describe the development of participants' beliefs about teaching mathematics over the course of the instructional treatment. Data related to participants' beliefs about teaching was collected from their pre-assessment of beliefs on January 28, 2002, the journal entry on March 4, 2002 which asked them to describe their classroom through the eyes of an observer, the post-assessment of beliefs collected on May 13, 2002, and the journal entry collected on May 16, 2002 which asked participants to complete a final reflection on the course. The data suggests

that over the course of the instructional treatment participants' beliefs about teaching mathematics evolved, they became better able to describe a good mathematics teacher and a good mathematics classroom, and they began to see mathematics teaching as a more open-ended activity that fosters student autonomy.

A description of participants' beliefs about teaching mathematics prior to the instructional treatment comes from the pre-assessment of beliefs which was collected on January 28, 2002. The third item on the pre-assessment of beliefs asked participants to describe a good mathematics teacher. The most predominant response, which thirteen participants shared, was that a good teacher appeals to all learning styles and is able to utilize multiple teaching approaches to do so. Besides this predominant belief participant responses were coded into two categories. The first category includes responses related to the attributes of a good mathematics teacher and the second category includes responses related to the practice of a good mathematics teacher. The categories, which are exhaustive, follow,

Attributes: "believes all students can learn", "organized and focused", "patient", "has content knowledge needed to teach", "enthusiastic".

Practice: "available for help", "relates math to real life", "always evaluating and adjusting teaching", "utilizes group work and discovery", "capable of assessing student skills and abilities", "helps students develop a desire to learn".

Participants' beliefs about the attributes a good teacher must possess and their beliefs about aspects of good mathematics instruction emerged from their responses on this item from the pre-assessment. The participants describe a good mathematics teacher as someone who has a positive attitude about learning mathematics, is prepared, patient, and enthusiastic. The participants also felt that a good mathematics teacher's practice appeals to all learning styles and that they develop a classroom atmosphere

that makes students want to learn. A good teacher's practice also relates mathematics to the real world and includes appropriate assessment.

Participants were asked to describe their future classroom through the eyes of an observer in a journal entry collected on March 4, 2002. Participants were explicitly asked to discuss what their classroom would look like and what the observer would see in a typical lesson. Participants' responses on this journal entry describe their beliefs about classroom arrangement and further support their views about a good mathematics teacher.

Participants' responses describe a classroom that is arranged to be student centered. All eighteen participants described students being arranged in groups, while fourteen of the eighteen said that the room would be full of manipulatives for students to utilize to aid in problem solving. Also, six participants said that their classroom would be completely decorated with student work. One of the participants wrote,

The room was decorated with many posters and illustrations of student's work. Books, science equipment, and manipulatives were located throughout the classroom. There were no desks, the students sat at round tables in groups of four or five students. This set up allowed for group discussion and project work.

Participants' main suggestion, in this journal entry, for incorporating group work was by introducing a new topic for the day to begin class and then having students work in groups on more problems or an activity related to the day's lesson. Participants also said that while students were working in groups they would be playing the role of facilitator and would be walking around the room to be sure that students are on track. The following excerpt from a journal entry highlights these ideas,

The math lesson would start with me introducing a new topic or project that the students would be working with as a small group. I would give

them an idea of what they needed to complete as a group and then allow time to solve their problem. The students in each group would help each other and work together to solve the problem. They would use each other as resources and try to figure things out before rushing up to me with many questions.

Participants also suggested that the observer would see a teacher who was able to adapt a lesson to accommodate many different learning styles, as they had suggested as their predominant belief about teaching on the pre-assessment. Participants also supported their view of the attributes of a good mathematics teacher when they suggested that the observer would notice a lesson that was well planned and well structured, and that they were well prepared and engaged students in the lesson.

Participants' beliefs about teaching mathematics underwent little change from the pre-assessment of beliefs to the journal entry collected on March 4, 2002. The only change was an increased focus on utilizing groups in mathematics instruction. When asked to describe a good mathematics teacher on the pre-assessment of beliefs, only four participants mentioned group work and discovery learning. A month later, on the March 4th journal entry, all participants suggested that they would utilize group work and / or discovery learning in their future classrooms. Participants were also able to describe how they would utilize group work. Participants' belief in group work and discovery learning and description of their classroom arrangements imply the beginning of a shift towards believing in promoting a more open classroom atmosphere and allowing their students more autonomy in the classroom.

On the post-assessment of beliefs collected on May 13, 2002 participants were again asked to describe a good mathematics teacher. Participants' responses from this task demonstrated that they are developing their conceptions of the attributes of a good mathematics teacher, are able to better verbalize their beliefs about the

practice of a good mathematics teacher, and are describing in more depth an open-classroom atmosphere. The responses from this task were coded into three categories; *classroom atmosphere and arrangement*, *attributes of a good mathematics teacher*, and *aspects of a good mathematics teacher's practice*. These exhaustive categories follow,

Atmosphere and arrangement: "student centered", "risk free environment".

Attributes: "has high expectations", "patient", "fun", "enthusiastic", "innovative and creative", "engaging", "willing to be wrong", "in depth understanding of the fundamentals", "understanding and available", "understands each students' capabilities".

Practice: "good assessment", "applies math to real world", "using problem posing as inquiry", "teaches with meaning and understanding", "creates lessons that appeal to all learning styles", "provides opportunities for students to construct their own knowledge".

On the post-assessment participants described more specific attributes of a good mathematics teacher, as compared to the pre-assessment, which can be seen by the number of responses related to attributes on each assessment. Therefore, participants were able to better articulate their views of the attributes of a good mathematics teacher post instructional treatment. Also, attributes such as innovative and creative, willing to be wrong, and fun, which were not on the pre-assessment, begin to support participants' shift to viewing the mathematics classroom atmosphere as more open. Evidence of participants considering more open classroom environments also comes from the fact that they mentioned mathematics classrooms being student centered and being a risk free environment, neither of which were mentioned on the pre-assessment.

Participants were also able to better articulate their beliefs about teacher's practice on the post-assessment as compared to the pre-assessment and March 4th journal

entry. For example, on the pre-assessment and March 4th journal entry, participants discussed utilizing group work and discovery learning in the classroom, but on the post-assessment this was verbalized as presenting students with opportunities to construct their own knowledge. Also on the post-assessment participants suggested that teachers teach with meaning and understanding and utilize problem posing, which were not mentioned previously. A participant's response on the post-assessment helps support these changes, "a good mathematics teacher is one who gives students opportunities to inquire on their own, allows students to use manipulatives to see connections, and uses problem posing as a method of inquiry."

Evidence of participants' beliefs about teaching mathematics is also found on their final journal entry which was collected on May 16, 2002. This journal entry asked participants to articulate their reaction to the semester long course. The responses on this journal entry support the notion that participants began viewing the mathematics classroom as a more open-ended entity and that this will become part of their teaching style. Participants' responses discussed viewing the classroom as a place of exploration instead of just a place to do boring desk work and that they have shifted to wanting students to gain conceptual knowledge and not just procedural knowledge. This trend can be highlighted by a portion of a participant's journal entry,

If I can teach mathematics to them with meaning and understanding and give them the opportunities to discover and communicate their ideas on their own, with each other, and with me, then I have no doubt that they will enjoy the subject just as much as I do.

In summary, over the course of the instructional treatment participants' beliefs about teaching mathematics have become more developed. At the beginning of the instructional treatment participants envisioned a good mathematics lesson utilizing group work and manipulatives. At the end of the semester, they have not changed

this belief but have described reasons why they hold this belief including, to promote students construction of mathematics knowledge. Also, participants were able to describe more attributes of a good mathematics teacher at the end of the instructional treatment and these descriptions suggested that mathematics teaching should promote an open classroom atmosphere and allow for student autonomy.

Beliefs About Learning Mathematics

Before describing participants' beliefs about learning mathematics that emerged during the instructional treatment this section will describe experiences from participants past mathematics education related to their learning of mathematics. The goal of this description is to provide background to help understand participants' beliefs about learning mathematics during the instructional treatment. Participants' mathematical autobiographies were collected on January 28, 2002 and their coding revealed pivotal experiences, from participants mathematics education, related to learning mathematics. Fourteen participants described pivotal experiences related to learning mathematics, ten were positive and four were negative. Two of the negative statements referred to participants not being able to take advanced classes. One participant was not able to take algebra in eighth grade and the other was not able to continue on the advanced track after geometry. These participants both described losing confidence in their mathematics ability because of these experiences. Another participant felt that not asking for help has slowed her learning process over the years. Four of the positive responses describe tracking and its impact on participants' confidence in their mathematics ability. Three participants gained confidence in their mathematics ability because they were tracked into the advanced class from eighth grade on, the fourth student gained confidence by realizing that choosing to not take the advanced track helped her develop a stronger and deeper understanding of mathematics. The remainder of participants' positive comments refer to experiences in high

school mathematics and calculus. For example, one student said that her confidence increased significantly when she began to think about mathematics as a “game of cards”, you rarely understand it at first but after playing it starts to click.

There are few instances of participants discussing their beliefs about learning mathematics during the instructional treatment. Results discussed previously related to teaching mathematics imply that participants believe that students learn best when they are actively engaged in the learning process. This theme can be highlighted by a couple of situations during the instructional treatment. Participants’ responses to the journal entry collected on March 4, 2002 suggested that students should take part in self-guided discovery and should have control of the learning process. This discovery, student centered view of learning, was also represented by participants’ suggestions, on the post-assessment of beliefs, for providing students a chance to construct knowledge and is exemplified by this quote from the final journal entry, “I truly believe that children learn best when they are actively engaged in the classroom.”

Responses, presented previously, on the pre- and post-assessment of beliefs, also imply that participants believe that students in their classrooms will have many different learning styles and that as teachers they need to be able to adapt to these different styles. Evidence of this belief comes from the pre-assessment, when 14 participants included that, a good teacher appeals to all learning styles and is able to utilize multiple teaching approaches to do so, in their descriptions of a good mathematics teacher. Further evidence that participants believe that students have different learning styles comes from the journal entry on March 4, 2002, as participants suggested that they would gear their lessons to many different learning styles, and from the post-assessment of beliefs, as participants described a good mathematics teacher gearing their lessons to many different learning styles.

In summary, participants described their beliefs about how children learn mathematics during the instructional treatment. Participants realized that students will enter their classroom with many different learning styles and there is a trend in their writing towards allowing their students more freedom and autonomy in the classroom.

Relationship Between Problem Posing and School Mathematics

Data related to participants' beliefs about problem posing and its relationship to school mathematics was collected on the pre-assessment of beliefs, journal entries collected on March 4, 2002, March 11, 2002, April 15, 2002, May 6, 2002, May 16, 2002, and on the post-assessment of beliefs. This data will be used to describe the development of participants' views about the relationship between problem posing and school mathematics. The development of participants' belief that problem posing is a beneficial task for their future students and belief that they will utilize problem posing in their future classrooms will be highlighted. The data also provides evidence of how participants will utilize problem posing in their future classrooms and future teaching.

On the pre-assessment of beliefs, participants were asked to respond to a problem posing situation, see appendix B, and to respond to the question, "Do you believe that problem posing from sets of given information is a worthwhile task for elementary school students?" Participants' responses on this task came after they had completed the pre-assessment of problem posing, so they had that prior experience posing mathematics problems. Results from this item of the pre-assessment imply that participants see problem posing as a beneficial task to utilize with elementary school students and they see possible benefits related to students' problem solving, mathematical understanding, and feelings about mathematics. Of the nineteen participants, sixteen said that they believed that problem posing would be beneficial with this audience and three were unsure about the possibilities for problem posing in el-

elementary mathematics. Participants' descriptions of the possible benefits of problem posing can be organized into three categories based on their relationship to *problem solving*, *student understanding*, and *student feelings about mathematics*. Participants also suggested possible negatives of student problem posing and these are reported in a fourth category. The categories follow,

Problem solving: "help students better understand word problems", "students will understand designing problems", "create problems that relate to them", "develop a better understanding of problem solving", "helps students think beyond problem solving".

Understanding: "consider information on multiple levels", "better understanding of material", "help teachers assess student understanding", "helps students recognize pertinent information".

Feelings: "alleviate student fear of word problems", "develop ownership of mathematics", "freedom and creativity with numbers and relationships".

Negatives: "students may be confused or frustrated at first", "may pose unsolvable or non-mathematical questions", "questions may take lessons off track", "students may take easy way out and ask simple questions", "not practicing math directly".

These categories help describe participants' beliefs about the relationship between problem posing and school mathematics at the beginning of the instructional treatment. Participants believe that problem posing has the potential to help students with their problem solving ability, including with their ability to understand word problems, and will allow students to think beyond problem solving. Participants also believe that problem posing will help develop student understanding of mathematics by allowing them to consider information on multiple levels and helping them develop

the ability to recognize pertinent information in mathematics. Participants also view problem posing as having the potential to affect students feelings about mathematics, including the potential to foster creativity and to help students develop ownership of mathematics. Finally, participants suggest some possible drawbacks to student problem posing on the pre-assessment. Ten participants, for example, described the potential for students to pose unsolvable or non-mathematical problems or that problem posing may cause the class to get off track. Further, eleven participants said that *students may get confused or frustrated at first and may have trouble with their initial introduction to problem posing.*

The remainder of this section will show that although participants have beliefs about the relationship between problem posing and school mathematics at the beginning of the instructional treatment it is not until after they engage in problem posing that they begin to relate problem posing to mathematics classrooms and start to discuss possibilities for the utilization of problem posing in school mathematics. Initially, participants described their future classrooms through the eyes of an observer in the journal entry collected on March 4, 2002, about five weeks into the instructional treatment. In this journal entry only two participants mentioned utilizing problem posing in their future classrooms. In the description of her lesson, one participant said that she would have students write word problems for division facts that she had on the chalkboard. Another participant said that she would give students a journal prompt that asked them to think of a division problem, solve it, and then write in their own words how they would explain the problem to a third grader. This journal entry shows that five weeks into the instructional treatment only two participants have begun the process of further reflecting on the role of problem posing in the mathematics classroom.

Participants next journal entry was collected a week later on March 11, 2002 and asked them to reflect on the course to date. What things have they found beneficial and/or what things might they change? Responses to this journal entry showed some evidence of participants reflection on how the course will influence their future teaching and reflection on problem posing and its relationship to school mathematics. With respect to reflecting on class activities and how they will influence their future teaching, one participant stated that the readings were beneficial because they allow them to see how others teach and utilize manipulatives. Six participants responded that they like the group work and ten participants commented that they felt like they were gaining activities and ideas for their future classroom from class assignments and activities. With respect to problem posing, four participants commented that their problem re-formulation and problem generation experiences caused them to think beyond the activities and that they started to related problem posing to their future classrooms. Other responses related to problem posing included the idea that problem posing seems to be an effective and worthwhile teaching method and that students should want to pose and solve their own problems in and out of the classroom. Results from this journal entry show that students reflection about their future teaching and problem posing is progressing.

On April 15, 2002 participants submitted a journal entry that asked them to consider who their intended audience was as they were posing mathematics problems during the instructional treatment. Responses to this journal entry showed evidence that participants were continuing to reflect on the relationship between problem posing and school mathematics. Eleven of the sixteen participants who responded stated that they were posing problems for their future students and indicated the appropriate grade level to range from second to eighth grade. Ten participants also said that the grade level that they pose problems for is dependent on the original problem or

the original set of given information. This implies that participants were not only considering their future classrooms as they were posing problems, but they were also considering the appropriate grade level to pose problems for. Participant reflection is highlighted through the following quotes,

When I'm actually teaching, I will need to pose appropriate problems for all children in my class to best facilitate their growth in mathematics.

What I try to keep in mind most as I am problem posing is whether or not most students at a particular grade level will be able to find a solution with meaning and understanding.

On May 6, 2002, the participants were asked to consider if they would utilize problem posing in their future classrooms. Participants responses on this journal entry implied that they all see problem posing as a resource for their future classrooms. Participants began to articulate ways they will utilize problem posing. In this journal entry participants continued to articulate their beliefs about possible benefits of problem posing for student understanding and students feelings about mathematics. All nineteen participants stated that they would utilize problem posing in their classrooms. Participants stated two roles of problem posing in their classrooms, having students pose mathematics problems, and teachers utilizing problem posing to aide in class preparation.

In this journal entry participants suggested many possibilities to promote student problem posing in their future classrooms including, as a whole class, as problem re-formulation, as an introduction to new material, on homework, and as an extra credit assignment, as a device to give quicker learning students something to do, and by using a problem posing box. Overwhelmingly participants suggested having the whole class re-formulate problems as an introduction to problem posing, followed by assigning problem generation tasks after students are comfortable with problem re-

formulation. Suggestions for problem posing as a tool to introduce new material to students, included students posing problems based on a new topic and then the class researching the answers to these problems in order to gain knowledge about the topic. Finally, one participant suggested creating a problem posing box. She suggested that students could pose problems for homework or during class activities and then put them in the problem posing box. When students had time in class they could take a problem from the problem posing box and try to solve it, this would give students a chance to solve their peers posed problems.

Participants also suggested that problem posing promotes student thinking and allows for deeper understanding of the material. One participant stated, “by the problem posing process, students begin to identify key terms and concepts that define a topic, and by structuring problems around these topics, they begin to make connections, which enhances the learning process.” Participants also supported their ideas from the pre-assessment that problem posing would allow for student control and autonomy and can give students a sense of ownership over a problem. Two statements from participants help illustrate these ideas,

I think that when students inquire about topics they are taking learning into their own hands, and that is one of the best things that problem posing can bring to a classroom.

The questioning can help students determine their level of knowledge and helps students to develop metacognition.

Participants’ responses on this journal entry also suggested that teachers can utilize problem posing as a tool in their classrooms. Participants saw problem posing as a possible assessment tool, as a tool for teachers to take advantage of “teachable moments”, as a tool to better accommodate all learning styles in their classroom, and as a tool to help develop activities, problems, tests and quizzes. One participant described

how and why a teacher would utilize problem posing when she wrote, "a teacher must be able to predict what students will find easy and difficult to do, and know her students well enough to be able to pose problems that will be thought provoking and meaningful to them." Responses to this journal entry show that participants' beliefs about the relationship between problem posing and school mathematics have developed as they have been exposed to problem posing in the instructional treatment. Results suggest that participants' beliefs about utilizing problem posing with their future students and as a tool in their future teaching have evolved. Participants described similar benefits of problem posing in responses to this journal prompt as they did on the pre-assessment, but were also able to articulate ways to incorporate problem posing in their classrooms and reasons problem posing may influence student understanding and student feelings about mathematics.

Participants' responses on the post-assessment of beliefs and on the final journal entry, which asked them to reflect on the semester and was collected on May 16, 2002 confirm that they have been reflecting on problem posing. On the post-assessment, when asked to respond to the problem posing activity, all participants stated that problem posing will be beneficial for elementary students. This assessment included more responses about the possible benefits of problem posing at this level and these responses were consistent with the benefits of student problem posing that were suggested on the May 6, 2002 journal entry, including, helps develop student understanding, promotes autonomy and ownership, improves problem solving, and helps develop student interest. On their final reflection collected on May 16, 2002 a few participants' quotes highlight the ideas about problem posing mentioned previously,

With problem posing, I as the architect developed the concepts that should be incorporated into the problems and determined the age groups

to be assessed, and as the carpenter I wrote the problems, determining what style would suit the students needs best, much like a carpenter must do when building a piece of furniture, or a house.

I also learned how beneficial it is to having children pose problems, something I didn't like before this class. It is extremely important to give the students a sense of ownership over a problem and a better understanding of the problem.

Uses in the classroom and importance of problem posing are the biggest thing that I have learned.

I can also have students pose their own problems to be solved by their classmates. This allows more freedom and power for the students in owning their learning.

In summary, as a result of the instructional treatment, there is evidence that participants have developed detailed beliefs about the relationship between problem posing and school mathematics. Participants see problem posing as a beneficial task for their future students to engage in and as a tool that they will utilize in their future teaching. Participants were also able to justify why they saw benefits of problem posing. Data showed reflection throughout the course of the instructional treatment about problem posing, teaching, and learning. This reflection allowed participants to articulate these new beliefs they had developed.

Individual Beliefs

Discussion of individual results will focus on participants' beliefs about mathematics, beliefs about teaching mathematics, and beliefs about the relationship between problem posing and school mathematics.

Bill

As stated in Chapter 5, at the time of this research Bill was a senior, majoring in mathematics education, with a focus on middle school mathematics education. Bill's past mathematics experiences are discussed briefly prior to describing his beliefs that emerged during the instructional treatment. Bill's mathematical autobiography was collected on January 30, 2002 and in his autobiography he described some pivotal experiences in his mathematics education. Bill took algebra I in the eighth grade and was in the advanced track of mathematics throughout high school. Bill's first difficult times as a mathematics student were in college. Bill also described a pivotal experience with regard to his desire to become a mathematics teacher. Bill said that after starting college as a business major he substitute taught during his first year and realized that he wanted to become a teacher. Bill chose to become a mathematics teacher because he feels that a mathematics degree is a sign of intelligence and prestige. The remainder of this section will focus on Bill's beliefs about mathematics, beliefs about teaching mathematics, and Bill's view of the relationship between problem posing and school mathematics, that emerged during the instructional treatment.

Bill's Beliefs About Mathematics. Data related to Bill's beliefs about mathematics comes from his pre-assessment of beliefs, his first interview on February 4, 2002, a second interview on March 27, 2002, a third interview on May 8, 2002, and the post-assessment of beliefs. Bill's beliefs about mathematics are consistent with the *mathematics predominant pre* view described previously and he believes that mathematics is a static body of knowledge. There was little change in Bill's beliefs about mathematics during the instructional treatment, but he was better able to articulate what he believes mathematics is post instructional treatment.

The first situation where Bill described his beliefs about mathematics was the pre-assessment of beliefs collected on January 28, 2002. Bill's responses on the pre-assessment indicate that his beliefs are similar to the *mathematics predominant pre*

view and that he believes that mathematics is a static body of knowledge. Evidence of the *mathematics predominant pre* view comes from Bill's word list and completion of "Mathematics is . . ." Bill's word list included math words (i.e. geometry, addition), "proofs", "anybody can do it", "fun at times", "intelligent" and "respected". Bill also defined mathematics as "...the study of relationships having to do with concrete information on numbers." Bill discussed his beliefs further during his third interview and on the post-assessment. During his third interview, while reflecting on the pre-assessment, Bill suggested that he would change his definition of mathematics. Bill felt that his definition of mathematics on the pre-assessment of beliefs was vague and in its place suggested, "...the study of relationships having to do with a concrete foundation of definitions and stuff and applying the known to the unknown." When asked about "applying the known to the unknown" Bill said that he means applying the known and developing upon that foundation. Five days later on the post-assessment of beliefs Bill defined mathematics as, "...the study of relationships, based on a system of set beliefs."

The combination of Bill's three attempts to define mathematics imply that he believes mathematics is related to numbers and involves finding relationships, a view similar to the *mathematic predominant pre* view, with applying the known to the unknown suggesting problem solving in mathematics. Thus, Bill's beliefs about mathematics have not changed but as is seen from his final definitions he is better able to articulate his views post instructional treatment.

Bill's view of mathematics as static is supported by his statement on the third item of the pre-assessment that mathematics is not changing, is already determined, and that math is like a language and we are always learning new vocabulary. Bill reiterated his belief about the nature of mathematics during his first interview when he stated,

I believe that there is like a big circle of mathematics information and I believe we know a part of that circle. Like a chunk of it, but I feel like all of it is already defined we just don't know it yet.

Bill also expanded on his view of mathematics as static during his second interview. When asked if we will ever understand all of mathematics, Bill stated,

That is what I believe and we get to a point where we are still finding out, maybe one hundred, two hundred, three hundred thousand years down the road where we are really close, we have most of it down, we can travel across the universe, we understand those laws, and we can bend time and stuff like that, but there will still be little things that we haven't picked up yet.

Bill demonstrated that his belief that mathematics is a static body of knowledge remained constant throughout the instructional treatment during his third interview and on his post-assessment of beliefs. During Bill's third interview, when asked to reflect on his pre-assessment of beliefs, he stated that he would not change his response to the item about mathematics being static. Also on the post-assessment of beliefs, Bill again described mathematics as a language for which vocabulary is still being learned.

Bill's Beliefs About Teaching Mathematics. Data related to Bill's beliefs about teaching mathematics comes from his pre-assessment of beliefs, first interview, March 4, 2002 journal entry, third interview, and post-assessment of beliefs. Bill's responses on these tasks suggest that he entered the instructional treatment with a traditional "drill and practice" view of mathematics teaching, but that post instructional treatment he started to think about an open-classroom atmosphere and allowing students autonomy in the classroom.

Bill's pre-assessment of beliefs and his description on the first interview of a good mathematics teacher help describe his beliefs about mathematics at the beginning of the instructional treatment. On the pre-assessment and during the first interview Bill suggested attributes of a good mathematics teacher to include, the teacher is able to tie a bunch of ideas together in different ways without confusion, makes students feel good about their accomplishments, and motivates students to do their homework. Bill also suggested aspects of the instruction of a good mathematics teacher to include, the teacher understands the proper pace for the class, offers extra work for students that excel, involves real life activities in the classroom, and includes traditional drill and practice of reading problems off a projector. Similar to the whole class, Bill has described attributes that suggest a teacher should have a positive disposition towards mathematics. Bill's description of aspects of good mathematics instruction, however, are not consistent with the whole class' views. Bill elaborated on his views during the first interview as he described a possible third grade class,

We'll do a lot of counting, we'll do flash card games with multiplication tables and division tables and I will do a lot of drill sheets. At that level I think that rote memorization is the way to go and that people in middle school who don't know their times tables, that is just ridiculous. So I'll do a lot of drilling with those memorization skills, because those things, there is not a lot of, because the application process is forever. You'll be using that stuff all that time and I just believe that is important. And word problems and real life situations are going to be nice and helpful but the core of my elementary math teaching will be memorizing how to do things.

Bill's response in the journal entry collected on March 4, 2002 supported his beliefs outlined above, but he began to imply that he is thinking about promoting an open

classroom atmosphere. Bill described a lesson where students are in groups of four working on memorizing their multiplication tables. The lesson is completed with a one-on-one game of flash cards. Bill also said that he will have students present problems at the board and that he feels like he is mixing up a traditional lesson with modern approaches by having students in groups and more involved in their own learning.

During both the final interview and post-assessment of beliefs, Bill added the following to his description of a good mathematics teacher, the teacher will challenge students to the point where they feel challenged and not stupid, and the teacher should know the level of achievement of each student and be able to let their students struggle for the correct amount of time. This belief in letting students struggle seems to be a departure from Bill's belief in drill and practice. If students are struggling, it seems that they have some autonomy in the learning process and are not just participating in drill and practice and rote memorization. Thus, during the instructional treatment, Bill started to consider promoting an open classroom atmosphere and student autonomy. Bill's consideration of student autonomy will be seen again in his beliefs about problem posing in school mathematics.

Bill's View of the Relationship Between Problem Posing and School Mathematics.

Bill's beliefs about the relationship between problem posing and school mathematics come through in his, second interview, journal entry collected on May 7, 2002 and his final interview. Results from this data suggest that Bill's beliefs about problem posing in school mathematics evolved from not believing problem posing had a place in school mathematics to articulating roles of problem posing in the classroom and possible benefits for students learning.

Bill's pre-assessment of beliefs and first interview do not provide information related to his beliefs about problem posing in school mathematics. On the pre-

assessment of beliefs, Bill misunderstood the item that asked him to examine a problem posing task and then describe whether he thought problem posing would be a beneficial task for elementary school students. Instead of commenting on problem posing Bill commented on solving word problems with elementary school students. Bill had this misunderstanding again during the first interview, when asked if he would utilize problem posing he said, "yeah but I don't think as much" and then discussed his beliefs about utilizing word problems.

Bill's second interview on March 27, 2002, however, provides a window into his beliefs about problem posing and its relationship to teaching and learning mathematics. During this interview, Bill began to consider the possibility of problem posing having a role in school mathematics. At the beginning of the interview, Bill said that he does not see himself utilizing problem posing as a future teacher, but that he does believe that he will become a better problem poser through practice. Bill stated that there are a tremendous amount of resources out there with problems in them and if somebody has already done the problem posing why should he not utilize their work. After this statement Bill was asked to consider introducing his students to problem posing. Bill suggested that the only possible use he can see is as a tool to even out timing in his classroom and give accelerated students something to work on or a chance to gain extra credit. Later in the interview, though, Bill seemed to have reflected on the incorporation of problem posing further and stated that a possible benefit could be, "so they have a little more involvement, a little more responsibility, a little more participation in their learning." Following this statement the researcher explained his views of mathematics as a problem posing domain. After this explanation, Bill started to reflect further on the idea of incorporating problem posing in the classroom and stated, "... problem posing, it might work, like you say you see problem posing as math. So I have to take that into account with other students, maybe

half the class sees it that way.” The fact that Bill considered that his future students may see mathematics as problem posing implies that he was sincerely reflecting on problem posing during this interview and not just repeating the researcher’s beliefs.

Bill continued reflecting on problem posing in his journal entry collected on May 7, 2002, which asked if he would utilize problem posing in his classroom. Bill began by saying that he would try to utilize problem posing in a elementary school classroom as an extra credit assignment and as a tool to even the pace of his class. Bill also stated *that if he teaches middle or secondary school he hopes to utilize problem posing as a unit ending activity that allows his students to be in the teacher’s role.* During his third interview on May 8, 2002 it was again clear that Bill had reflected on the roles of problem posing. His concept map of problem posing (see appendix B) is evidence of this reflection. While describing his concept map Bill stated,

Well there is teacher posing the problems, which you know they can focus more on what they want to teach the kids. There is the student posing the problems which gives them responsibility and ownership of the problem and they can discover their own math. You can use some of their problems as possible test problems and then it is also. It gives them maybe motivation for I think, what is it called, intrinsic learning, where they want to learn on their own. You can use projects and homework I think are the best places for using it across the curriculum, then on tests because you need more time. Students could slack on posing problems, that is the negative side of it, you know they could just be really like whatever and how do you assess that? How do you assess whether they are putting forth their effort? And then it helps develop problem solving strategies, I believe that.

Bill's concept map supports that his views of problem posing in mathematics teaching and learning evolved during the semester. Bill changed from not seeing any benefit of problem posing to being able to verbalize possible roles of problem posing in the classroom and the possible benefits of these roles for student learning. During the instructional treatment Bill reflected on problem posing and possibilities for its incorporation into classrooms and curriculums. Post instructional treatment Bill was able to articulate possible benefits of problem posing for students and possible ways to utilize problem posing in his future classroom.

Carrie

As stated previously Carrie was a first year graduate student seeking her certification in elementary education at the time of this study. Carrie's past mathematics experiences are discussed briefly prior to describing her beliefs that emerged during the instructional treatment. Carrie's mathematical autobiography was collected on January 30, 2002 and in her autobiography Carrie described some pivotal experiences in her mathematics education. Carrie was in the highest track mathematics class through fourth grade and then worked independently on mathematics in fifth and sixth grade. Carrie's first confusion in a mathematics classroom came in algebra II in ninth grade but she felt like she was back on track based on her success in geometry in the tenth grade. Carrie took honors pre-calculus in eleventh grade and asked to take the BC, AP calculus class her senior year in high school, more she admitted, because of pride rather than an interest in mathematics. Carrie never fully understood calculus and is glad to have "put it behind [her]". During her education Carrie was only frustrated by one mathematics teacher, her pre-calculus teacher never took questions in class and Carrie described her as "ruthless". The remainder of this section will focus on Carrie's belief about mathematics, beliefs about teaching mathematics, and view of the relationship between problem posing and school mathematics, that

emerged during the instructional treatment.

Carrie's Beliefs About Mathematics. Data related to Carrie's beliefs about mathematics was collected on her pre-assessment of beliefs, a first interview on February 13, 2002, a third interview on May 8, 2002, and the post-assessment of beliefs. Carrie's beliefs about mathematics during the instructional treatment and post-instructional treatment changed. Carrie became better able to articulate her beliefs about mathematics. Evidence indicates that Carrie's beliefs are consistent with the *mathematics predominant pre* view before the instructional treatment and that post-instructional treatment Carrie began to articulate the *mathematics predominant post* view. Carrie believed throughout the instructional treatment that mathematics is always changing and did not change this view.

Evidence of Carrie's *mathematics predominant pre* view comes from her pre-assessment of beliefs and first interview. Carrie's word list on the pre-assessment included, math words (i.e. geometry, addition), "theorems", "definitions", "challenging", and "math-minded". Also, Carrie defined mathematics as "...the science of numbers and their interrelationships, including combinations, generalizations, and configurations involving manipulations and definitions of space." Carrie's definition of mathematics on the pre-assessment implies a relationship to numbers and her reference to the manipulation of numbers may imply a relationship to problem solving. During Carrie's first interview she discussed in more detail her beliefs about mathematics. During the interview Carrie again defined mathematics as having to deal with numbers, relationships and manipulations but when asked she said that mathematics does not always have to deal with numbers. Carrie suggested calculus as a branch of mathematics that does not deal with numbers. This is more evidence of Carrie's *mathematics predominant pre* view because she views mathematics as a discipline that entails more than manipulating numbers but does not have a developed

conception of mathematics besides working with numbers.

Carrie's beliefs about mathematics changed by the time of her final interview and post-assessment of beliefs. Carrie's responses on these items begin to support that she has prescribed to the *mathematics predominant post* view. Carrie reflected on her responses to the pre-assessment of beliefs and said that she felt like with more time she could write a less ambiguous, more definite definition of mathematics but was not able to produce one during the interview. On the post-assessment of beliefs Carrie was able to expand her word lists and included words such as "problem posing" and "problem solving". Carrie provided another definition of mathematics, "... the study of relationships between expressions of numbers, units, time, and space. It includes studies of arithmetic, patterns, spatial relations, and rates of change." These responses on the post-assessment support Carrie's *mathematics predominant post* view as she articulated problem solving and problem posing as aspects of mathematics and refined and expanded on her definition of mathematics as relationships between numbers. These responses also demonstrate that Carrie is better able to articulate her views post instructional treatment as she wrote a clearer, more concise, and less ambiguous definition of mathematics.

On the pre-assessment Carrie explicitly stated her belief about the nature of mathematics as one in which mathematics is changing but the fundamentals are very definitive and concrete. Carrie expounded on her belief that mathematics is changing during her first interview,

You could say it is ever expanding, I guess I would lean more towards that, than saying that it is already out there and say at some point everyone is going to know everything that there ever needs to be known about mathematics and we'll close the book and say that it is done. I don't think that will happen. There is always more connections and with any science

really. There is no possible way you could say we've learned everything, so I think it is always expanding.

Carrie's beliefs about the nature of mathematics do not change during the instructional treatment and during her third interview and on the post-assessment of beliefs she stated that she still felt convinced that mathematics is changing.

Over the course of the instructional treatment Carrie's beliefs about mathematics evolved from a *mathematics predominant pre* to *mathematics predominant post* view. Post-instructional treatment she was also able to better articulate what mathematics is to her. Carrie's views of the nature of mathematics did not change during the instructional treatment and this may be result of her motivation for learning mathematics. As Carrie stated, "I personally do not have much interest in exploring math for the beauty of math itself."

Carrie's Beliefs About Teaching Mathematics. Data related to Carrie's beliefs about teaching mathematics was collected on her pre-assessment of beliefs, a first interview, a March 4, 2002 journal entry, a May 16, 2002 journal entry, and the post-assessment of beliefs. During the instructional treatment Carrie became better able to articulate her vision of a good mathematics teacher and a good mathematics classroom. Carrie demonstrated a belief that teaching should encourage student discovery and by the end of the instructional treatment was able to articulate how and why she will promote discovery. Carrie's beliefs about the attributes of a good mathematics teacher focus on teacher knowledge.

Evidence of Carrie's beliefs about the attributes of a good mathematics teacher and the aspects of the practice of a good mathematics teacher are reflected in her responses on the pre-assessment of beliefs. On this assessment, Carrie stated that a good mathematics teacher should be knowledgeable in the fundamentals and higher levels of mathematics. She also believes that the teacher should be knowledgeable

to the point of being able to understand theory and applications. Thus, content knowledge is the most important attribute of a good mathematics teacher to Carrie. With regard to aspects of good mathematics instruction on the pre-assessment, Carrie articulated the predominant view of the class that teachers should be able to teach by using multiple approaches and should be capable of authentically assessing their students' skills.

Evidence of Carrie's beliefs about discovery learning in mathematics instruction does not come until her first interview. Carrie stated, "I like the idea that a mathematics teacher would say how many different ways could we approach solving this and encourage knowledge to build upon that..." The statement "encourage knowledge to build upon that" implies the belief in student construction of knowledge. During her first interview Carrie described her classroom as including a lot of counting, measuring, and comparison but was not able to give a detailed example of what teaching would look like in her class. In the journal entry collected on March 4, 2002, Carrie stated that her classroom would include lessons that were conducted in exploratory ways with students in charge of self-guided discovery. I concluded that based on her difficulty describing a classroom experience Carrie had not done much reflection on the relationship between her beliefs and her future classroom, but that her statement on the journal entry was the beginning of such reflection.

Evidence of Carrie's increase in ability to articulate her views comes from her post-assessment and May 16, 2002 journal entry, which asked her to reflect on the semester. On the post-assessment Carrie took her suggestion of group work and students construction of their own knowledge one step further by saying that a good mathematics teacher "...knows how to present students with opportunities to construct their own knowledge..." Carrie then further articulated this view in her final journal entry,

As a teacher of math I have realized that my main goal for my students will be confidence. I would like to make all math encounters in my classroom be as inviting and non-threatening as possible. I think many times it is easy for students to be intimidated by math. When they are trying to sort out the solution to an arbitrary problem and the solution is not becoming clear students tend to react by saying, "what do I need this for" and "I'm never going to figure this out". However if the problem posed in a context that is familiar and the question seems to have relevance to what the student thinks is important, or information worth knowing, then some of the intimidation and frustration can be avoided. Additionally, I would like to be the kind of teacher who can accurately assess what my students know, what they are still struggling with, and what teaching methods they will benefit from most... Constructivist learning allows students freedom to apply their prior knowledge and use what they already know to enhance how they learn a new concept.

During the instructional treatment, Carrie demonstrated the ability to better articulate her view of mathematics teaching. As the above quote shows Carrie expanded her view from the pre-assessment and reflected on her views during the semester.

Carrie's View of the Relationship Between Problem Posing and School Mathematics.

Data related to Carrie's view of the relationship between problem posing and school mathematics comes from her first interview, a second interview, a May 7, 2002 journal entry, and the post-assessment of beliefs. There is no data from Carrie's pre-assessment of beliefs because she misunderstood the question about implementing problem posing with elementary school students and commented on solving word problems with this audience. Carrie articulated a view of the relationship between problem posing and school mathematics during the instructional treatment that in-

cludes problem posing as a tool that should be utilized in mathematics classrooms. Post-instructional treatment, however, Carrie is able to articulate ways to incorporate problem posing and the possible benefits of problem posing experience for students.

Carrie's first interview supports the idea that she felt problem posing has a role in the mathematics classroom. During the interview Carrie said that she thought problem posing would be beneficial with elementary school students because it would allow them to look at sets of information from all different angles and see how many questions they can answer from a set of given information. Carrie also felt that problem posing would allow students the chance to build a "toolbox of skills". Carrie's second interview also suggests that she believed problem posing in the classroom will help foster students construction of their own knowledge. In the second interview Carrie related the benefits she sees of problem posing to her beliefs about mathematics teaching, as she had expressed that good mathematics teaching should promote students construction of their knowledge.

Carrie first mentioned ways to include problem posing in her classroom in her May 7, 2002 journal entry. Carrie stated,

I definitely will use problem posing in my classroom. I plan to teach at the lower elementary grades, however using simple math concepts can make the problem posing activities more fun and meaningful. For example we can use the simple concept of favorite things. Students will ask common questions such as what is your favorite desert? From there they can take a survey of the class and use that data to pose problems about the information they have (i.e. How many more people like ice cream more than cookies?). I would also encourage students to pose problems involving what is in their environment. If problems are derived from their own questions and curiosities, then they will be much more motivated to find

answers to them.

This quote is evidence that Carrie expanded her view of the relationship between problem posing and school mathematics during the instructional treatment to include possible ways to incorporate problem posing in her future classroom. Carrie also expanded her views of the benefits of problem posing for students and connected them to her beliefs about teaching mathematics. Carrie further confirmed this expanded notion of her beliefs on the post-assessment, when she stated that students should have the chance to pose problems. On this assessment Carrie also gave similar suggestions for incorporating problem posing in classrooms to her May 7, 2002 journal entry.

Laura

At the time of this research, Laura was a sophomore majoring in mathematics education and seeking certification to teach middle school mathematics. Laura's past mathematics experiences are discussed briefly prior to describing her beliefs that emerged during the instructional treatment. Laura's mathematical autobiography was collected on January 30, 2002 and in her autobiography Laura described some pivotal experiences in her mathematics education. Laura remembered her first frustration in mathematics when she was not allowed to take Algebra in eighth grade because of her score on a qualifying exam. Laura remembered being disappointed but later realizing that this was the best possible path for her as she always understood mathematics and did not get frustrated with mathematics like many of her friends. While she was in secondary school Laura began to view mathematics and science classes as a card game. Laura realized that she may not always understand mathematics at first but that as she gained experience with mathematical ideas she was able to understand. The reason Laura chose to become a mathematics teacher was because of a calculus class her freshman year in college. Even though she was as competent as the boys in class, Laura's instructor would often brush her off and

not allow her to answer questions. This experience motivated Laura to become a “model of female confidence” for her future students. The remainder of this section will focus on Laura’s belief about mathematics, beliefs about teaching mathematics, and her view of the relationship between problem posing and school mathematics, that emerged during the instructional treatment.

Laura’s Beliefs About Mathematics. Data was collected with regard to Laura’s beliefs about mathematics on her pre-assessment of beliefs, a first interview on February 4, 2002, a third interview on May 10, 2002, and the post-assessment of beliefs. Laura’s beliefs about mathematics can be characterized as a combination of the *mathematics predominant pre* view and the *mathematics secondary post* view during the instructional treatment. She also included problem posing as an aspect of mathematics post instructional treatment. Laura’s beliefs about the nature of mathematics evolved from a belief that how people view mathematics is changing to a belief that not just peoples views but that mathematics as a domain of knowledge is being discovered.

Evidence of Laura’s beliefs about mathematics comes from her pre-assessment of beliefs. Laura’s word list on the pre-assessment included, math words (i.e. geometry addition), “proofs”, “theorems”, “axioms”, “definitions”, “logic”, and “time”. Laura also defined mathematics as “an invented system of numbers and a study of those numbers’ relationships with each other. Mathematics is also a way of concrete, logical thinking that uses one property to create another.” Laura’s definition on the pre-assessment is similar to the *mathematics predominant pre* view. In other words, she described mathematics as involving the study of numbers and problem solving, or as she put it “use one property to create another.” Laura also demonstrated beliefs similar to the *mathematics secondary post* view. She described mathematics as a way of thought and included proofs in her word list. Laura’s first interview provided

more evidence of the *mathematics secondary post* view. She stated, "...mathematics is a way of thinking in a logical way" Laura was able to articulate her beliefs about mathematics and looks at mathematics beyond manipulations and procedures and believes that mathematics is a way of thought. On the post-assessment, Laura suggested a similar definition of mathematics to the pre-assessment which supported her view of mathematics as a combination of the two views mentioned previously and included problem posing in her word list.

When responding to whether mathematics is changing on the pre-assessment Laura wrote, "mathematics is not always changing but the way people look at mathematics is always changing." But, Laura expanded her view on the nature of mathematics during her first interview as she stated, "I think that if mathematics is changing it is only us kind of changing our perspective of mathematics." Laura gave the following example,

So I mean its like kind of if you take the number systems, um, you know, mathematics didn't change from when the Babylonians counted to when we counted, to when we are counting. But we changed how we look at counting and we changed what represents number in counting. So the fact that one and one is two didn't change but how we represent that and how we do that algorithm changed.

During her first interview, Laura also stated, "so maybe it, maybe it is all there and we are discovering [mathematics]" which implies that Laura is unsure of her belief about whether mathematics is invented or discovered.

Laura's evolving belief that mathematics is discovered was apparent during her final interview. When Laura was asked to reflect on her responses to the pre-assessment of beliefs she began a discussion about whether math is invented and came to the following conclusion,

I think more and more I am coming to the idea that it's out there and we are discovering it. In high school I was sure that we made it up, but the more I study it the more I am like, we discovered that, instead of making it up. Except there are things we did make up like order of operations, like, we base so many things on order of operations, if order of operations is wrong. Well, that doesn't mean that the math isn't still out there that just means that we look at it in a different way.

On the post-assessment, Laura reiterated the belief expressed during her final interview about the nature of mathematics.

Laura's Beliefs About Teaching Mathematics. Data related to Laura's beliefs about teaching mathematics was collected on her pre-assessment of beliefs, a first interview, a March 4, 2002 journal entry, a third interview, and the May 16, 2002 journal entry. Laura's beliefs about teaching mathematics parallel those of the class as she believes that good lessons appeal to all learning styles and allow for discovery learning. During the instructional treatment Laura not only demonstrated developed beliefs about teaching mathematics, but she also demonstrated that she had begun thinking about how to get her future students to engage in mathematics while not compromising her beliefs about teaching. Laura also described the qualities of patience, believing that every student can succeed, and a strong mathematics background as necessary attributes of a good mathematics teacher.

Laura's beliefs about the attributes of a good mathematics teacher come from her description on the pre-assessment of beliefs. Laura described a good teacher as being patient and believing and promoting that every student is able to understand. During her first interview, Laura mentioned the importance of a mathematics teachers' preparation and suggested that a good mathematics teacher has a deep understanding of and passion for mathematics.

Evidence of Laura's beliefs about the aspects of good mathematics instruction also comes from her pre-assessment of beliefs. Laura described that a good mathematics teacher creates lesson plans that appeal to all learning styles and are a mix of group work, discovery, and direct instruction and is always evaluating and re-writing lesson plans. During her first interview, Laura discussed her beliefs about mathematics instruction further and provided a description of a possible lesson in her future classroom. During this interview Laura described a good mathematics teacher as being able to explain different things in many different ways and described a lesson in her future classroom as including an opening problem for students to work on as they enter the class, an introduction to new material, a group activity, and then a whole class activity to discuss the goals and the outcomes of the original group activity. Laura articulated this exact description of a lesson in her future classroom in her March 4, 2002 journal entry, which asked her to describe her classroom through the eyes of an observer, as well as during her third interview. Laura was able to think beyond the structure of her classroom and was able to think about how to engage her students. During her final interview Laura stated,

... I am just talking about my experience from my class, their all in seventh grade math, but they are at a fifth grade math level, or fourth grade some of them. You know they have walls built and they are not going to go, oh yeah let's engage in this, like I mean if Rebecca is throwing out her idea than Lydia is like, you know giving her the glare and is not paying attention to her.

Based on past teaching experience in a summer program called Summerbridge, Laura began the instructional treatment with the ability to articulate an understanding of what she believed good mathematics teaching is and what a good mathematics classroom looks like. During the instructional treatment Laura moved beyond reflecting

on this and started to reflect on how to get her students to engage in lessons and want to learn mathematics. During the instructional treatment Laura reflected on how her beliefs about teaching will influence her classroom and how her beliefs will influence her future students motivation and engagement with mathematics.

Laura's View of the Relationship Between Problem Posing and School Mathematics.

Data related to Laura's view of the relationship between problem posing and school mathematics was collected on her pre-assessment of beliefs, a first interview, a second interview, a May 7, 2002 journal entry, and the final interview. Laura entered the instructional treatment with beliefs about the benefits of problem posing for students. During the instructional treatment, as Laura began to have more experience with problem posing she was able to articulate ways to incorporate problem posing in school mathematics and reasons that pre-service teachers should engage with problem posing.

Laura's belief that problem posing has benefits for student learning can be seen from her response to the problem posing item on the pre-assessment of beliefs. On the problem posing item Laura stated that problem posing in elementary classrooms is worthwhile and suggested benefits for student learning. Laura suggested that problem posing would make students think more critically about given information, force students to decide what kinds of questions can be asked from given information, and cause students to look at problems in a different way. Laura also suggested that problem posing may help students alleviate their fear of word problems, because it is a less threatening way for students to look at problems and work with word problems.

More evidence of Laura's beliefs about the importance of problem posing for student learning comes from her first and second interview. Laura expanded on possible benefits of problem posing for students during her first interview when she stated,

I think that it is important for kids to be able to do problem posing

because I think it helps them analyze what they're given for information and I think it is a less threatening way to introduce word problems because if you give a word problem without a question then it is a little less intimidating if they get to write their own question, because they know they can answer their own question and it helps them realize what given information there is and a lot of times if there is a question already there they look right at the question, they don't think they can answer it, they get frustrated. But if there is no question then they have to analyze what is given to them.

During her second interview Laura continued to expand on students being able to better handle word problems because of problem posing when she stated,

I think it was like, students who really struggle interpreting word problems and understanding what they're really asking and how to use the given information to answer that questions, I think problem posing, having them ask their own question is a good way to build up to being able to take a word problem and break it down.

During the second interview Laura also discussed other benefits of problem posing. Laura stated that problem posing requires utilizing prior knowledge and hence may help students organize and understand their knowledge base. By her second interview, Laura had articulated many benefits of problem posing for student learning but she had not mentioned problem posing in her future classroom or ways she would utilize problem posing with students.

Laura first mentioned teachers as problem posers during her second interview, and stated that they need to be problem posers because they have to write problems "all the time". Laura did not mention problem posing in her classroom until her May 7, 2002 journal entry when she stated that she will utilize it as a way to familiarize

students with new material and to assess her students. To paraphrase Laura, if they can write mathematics problems ranging in difficulty students can show a good understanding of the material. Laura expanded on possibilities for incorporating problem posing in her classroom and her belief about teachers as problem posers when she discussed her concept map of problem posing (see Appendix B), which she generated during her third interview,

So I was thinking for the uses of teaching it in the math ed. classes is it helps teachers learn how to write, like, insightful mathematics problem solving questions for students and it also helps them understand the learning benefits from actually posing the problems, understand we need to learn a lot here to right this question and that means our students will have to know a lot in order to write good questions. Therefore it could be a form of assessment because students would have to have some sort of grasp on a concept before they could actually write a question about it. And it would help the students kind of think like a teacher, um, which you kind of try to teach them so they can be prepared for exams and stuff like that. Um, and it helps students become more comfortable with word problems because their not threatened by the question itself because the questions not there and they know they can ask the question. And it teaches them to ask good questions rather than saying, I mean, I think if they started asking these questions I think later on in class when they didn't get something they'd be better, they'd be more likely to put what they didn't get into a question.

Laura had developed beliefs about problem posing prior to the instructional treatment as was shown by her pre-assessment of beliefs. But these beliefs mainly referred to the possible benefits of having students pose mathematics problems in the classroom.

Laura's May 7, 2002 journal entry and final interview are evidence that, similar to the whole class, Laura started to think about ways to incorporate problem posing in her classroom after her experience posing mathematics problems during the instructional treatment.

Liz

At the time of this research, Liz was a sophomore majoring in mathematics education and seeking certification to teach middle school mathematics. Liz's past mathematics experiences are discussed briefly prior to describing her beliefs that emerged during the instructional treatment. Liz's mathematical autobiography was collected on January 30, 2002 and in her autobiography Liz described some pivotal experiences in her mathematics education. Liz realized that math has always been a part of her life, and can remember her enjoyment of mathematics in first grade, which included being in a special math group throughout elementary school. Liz's interest in and enjoyment of mathematics has stayed constant. Liz remembered her teacher's styles all blending together after third grade with everyone teaching mathematics as memorizing numbers and equations until her freshman year in high school. Liz's freshman mathematics teacher made her think about future careers in mathematics and in particular being a mathematics teacher. Liz is working towards becoming a mathematics teacher because she wants to repay her teachers by helping others. The remainder of this section will focus on Liz's beliefs about mathematics, beliefs about teaching mathematics, and view of the relationship between problem posing and school mathematics, that emerged during the instructional treatment.

Liz's Beliefs About Mathematics. Data related to Liz's beliefs about mathematics was collected on her pre-assessment of beliefs, a first interview on January 31, 2002, a third interview on May 7, 2002, and the post-assessment of beliefs. Prior to the instructional treatment, Liz's beliefs about mathematics cannot be characterized by

one of the views presented previously, but she seems to understand that different views exist. Data suggests that following the instructional treatment Liz is starting to hold the *mathematics predominant pre* view. Liz's views about the nature of mathematics evolved from believing that mathematics is always changing to believing that mathematics is changing for research mathematicians.

Evidence of Liz's beliefs about mathematics can be gathered from her pre-assessment of beliefs. On the pre-assessment Liz's word list, included math words (i.e. geometry, addition), "knowledge", "games", "strategies", "boy's world" and "teaching". Liz also defined mathematics as "a joy to some and awful to others but it's part of life and we need to understand how it works." Liz continued to expand her beliefs about mathematics during her first interview when she suggested that math is different to different people and "...some people think of math as all theories and equations and stuff like that, when other people think of it as, you know like common everyday uses." When asked to condense her ideas into a general definition of mathematics Liz states, "...math is a common tool that we can use everyday to try to make our lives a little easier or harder depending on how you look at it." These final quotes imply that Liz believes that multiple views of mathematics exist, including a theoretical view, but she has not developed a definition of mathematics that she is confident in. Liz's inability to define what mathematics is to her implied that she did not have a strong conception of what mathematics means to her.

During her final interview on May 7, 2002, Liz reflected on her responses to the pre-assessment of beliefs and was better able to articulate a conception of what mathematics is to her. Again, during this interview, Liz had a difficult time defining mathematics. When asked what came to mind when she thinks of mathematics Liz answered, "...problem solving, using equations, numbers, my major. I don't know I like it in general, it is easy for some and hard for others, it has been around for a while.

It's like everyday life I guess, you can always use mathematics." This statement suggests that Liz was beginning to adopt the *mathematics predominant pre* view, as she suggested that problem solving and using equations are part of mathematics. On the post-assessment of beliefs, Liz gave a similar definition of mathematics which seems to support that she believes in the *mathematics predominant pre* view. There were also changes in Liz's word list as she added the phrases problem solving and problem posing.

Liz's first discussion of her beliefs about the nature of mathematics as changing, came during her first interview. On the pre-assessment, Liz misunderstood the question about whether mathematics is changing and read and reacted to mathematics as always "challenging". The following quote from her first interview, however, is evidence of Liz's belief that mathematics is changing,

I think math is always moving and changing, I mean, we're always coming up with different ways of thinking about things or even like coming up with new ideas and stuff like that. I think it is always, like anything else it's always changing, it is always advancing, it's getting better, you know it might be stuck for a little while but we always seem to kind of advance it more.

During her final interview, Liz articulated the distinction that mathematics education and mathematics teaching are always changing but that mathematics is not changing, it is the same as hundreds of years ago but that it may change again in the future. Liz had changed her beliefs about the nature of mathematics that were articulated during the first interview. Liz then described this distinction in more detail on the post-assessment of beliefs as she stated, "as of now math isn't really changing so much to students but to professors and those who prove theorems math is changing . . ."

Although Liz still struggled to verbalize a definition of mathematics, her beliefs about mathematics have developed over the course of the instructional treatment and post instructional treatment she was beginning to verbalize the *mathematics predominant pre* view and saw problem solving and problem posing as part of mathematics. Based on her experiences during the instructional treatment, Liz's beliefs about the nature of mathematics have changed from viewing mathematics as always changing to viewing mathematics as a discipline that is being furthered and changed by mathematicians. Liz evolved from believing that mathematics itself is changing to believing that people are furthering the field of mathematics through discovery.

Liz's Beliefs About Teaching Mathematics. Data related to Liz's beliefs about teaching mathematics comes from her pre-assessment of beliefs, a first interview, a March 4, 2002 journal entry, a third interview, and the post-assessment of beliefs. Liz described many attributes of a good mathematics teacher during the instructional treatment and was able to articulate more attributes as the semester progressed. Liz shares the predominant belief in the class that a good teacher needs to accommodate students with different learning styles and suggested that students should engage in group work and discovery learning. Data also suggests that during the instructional treatment, Liz began to consider how her beliefs about teaching will affect student understanding.

Evidence of Liz's beliefs about the attributes of a good mathematics teacher was demonstrated throughout the instructional treatment. On the pre-assessment of beliefs, Liz suggested that a good teacher should understand that students will have multiple learning styles and should have sufficient mathematics preparation. To paraphrase Liz, a good mathematics teacher realizes that there is more than one way to understand mathematics and has the knowledge to teach others what they might believe is impossible. Liz continued to discuss attributes of a good mathematics teacher

during her first interview when she described a good math teacher as calming, and as understanding that not all students learn the same, some may be visual learners while others are not. Again during the first interview, Liz mentioned the mathematical preparation of a teacher when she suggested that a teacher's mathematical preparation should include understanding the historical development of mathematics. Liz expanded on the attributes of a good mathematics teacher during her third interview when she added that the teacher must be willing to help out and must have a well developed lesson plan. These statements imply that Liz related her desire to become a mathematics teacher in order to help others, with her beliefs about the attributes of a good mathematics teacher. Liz was able to articulate this view of a good teacher on the post-assessment of beliefs, "a good math teacher is one who is understanding and always has time to help those who need it and should learn or know what works best for teaching their students how to understand math problems." The data implies that during the instructional treatment, Liz became able to articulate her beliefs about the attributes of a good mathematics teacher and believes that content knowledge, preparation, and a willingness to help others are necessary attributes.

Evidence of Liz's belief that good mathematics instruction accommodates for students different learning styles and considers the class as a whole and not just the individual students comes from her first interview. During her first interview, Liz described a day in her future classroom, having students working in small groups and utilizing manipulatives. When asked to expound on what a mathematics lesson might look like in her future classroom Liz stated,

I see it as like when they come in we learn a basic, not like a rule but your basic topic for the day. Like maybe like talk about it for like 20 minutes and make sure they all understand and then maybe get into small groups, you know, so then they can work on it all together and then so I can go

around to each group, make sure they all understand maybe what's going on and if you don't try to help out the group that doesn't or make sure that anyone in that group go up and figure out the problem. And then maybe if they finish early well do some fun math like related problems. Or you know just to reward them be like you guys did well today.

In her March 4, 2002 journal entry Liz gave a similar description of her classroom and added that groups would be responsible for using teamwork to be sure all the members are understanding at all times. She feels like the group should be responsible for each others learning. This belief implies that Liz had started to reflect on how her beliefs about teaching are going to influence students learning of mathematics and how she is going to be sure they are learning.

Liz's views of the attributes of a good mathematics teacher developed over the course of the instructional treatment as she became able to articulate well her desired attributes of a good mathematics teacher. It has been shown that Liz agrees with the predominant class view of teaching and had started to reflect on how this view will influence her students learning.

Liz's View of the Relationship Between Problem Posing and School Mathematics.

Data related to Liz's views of the relationship between problem posing and school mathematics was collected on her pre-assessment of beliefs, a second interview, a May 7, 2002 journal entry, and the post-assessment of beliefs. Liz entered the instructional treatment with the belief that problem posing is beneficial for students and was able to better articulate her views and consider ways to incorporate problem posing in her mathematics classroom as the instructional treatment progressed.

Evidence of Liz's beliefs about the relationship between problem posing and school mathematics prior to the instructional treatment comes from her pre-assessment of beliefs. On the pre-assessment, Liz stated that she believes that problem posing

would be beneficial for elementary students to take part in. To paraphrase Liz, problem posing will allow students to use their own minds and see that they can understand math if they just think about it. Liz also discussed possible drawbacks of problem posing similar to the responses of the entire class on the pre-assessment. She suggested, for example, that students may take the easy way out by writing simple problems or students may not be able to come up with questions.

Liz's second interview and May 7, 2002 journal entry, which asked her to discuss if she would use problem posing in her classroom, demonstrate that she was able to better articulate her beliefs about possible benefits of problem posing for students and started to consider possible ways to include problem posing in the classroom. During her second interview, Liz suggested problem posing as a possible tool to help students with their problem solving. She believes that problem posing will help students become more interested in what they are doing. In her May 7, 2002 journal entry, Liz also elaborated on her beliefs about the possible benefits for incorporating problem posing in her classroom. Liz believes that students will be able to utilize problem posing to help with problem solving, but also,

Students should know how to make up their own problems because in real life you will be asked to make some problems up, like for an exam, or help make up questions for a job, or even find some problems you want to fix around your household.

This quote implies that Liz had started to relate problem posing to real life situations. Liz also described her beliefs about possibilities for implementing problem posing in her classroom in her May 7, 2002 journal entry. Liz suggested utilizing problem posing on homework assignments, and stated that she will give students a paragraph to read, pose, and solve problems related to. Liz supported these beliefs with her response to the problem posing item on the pre-assessment by suggesting similar benefits of

student problem posing.

Therefore, during the instructional treatment Liz developed beliefs about why it will be beneficial for students to pose mathematics problems. Liz also developed a sense of how she will incorporate problem posing into her classroom. As with the whole class, Liz articulated more ideas about the relationship between problem posing and school mathematics as she gained experience posing mathematics problems.

Summary of Individual Beliefs

Carrie, Laura and Liz shared beliefs about teaching mathematics that included aiming lessons at all learning styles and promoting discovery learning while Bill began to develop such beliefs during the instructional treatment. In summary, Bill, Carrie, Laura, and Liz's beliefs about mathematics, beliefs about teaching of mathematics and views of the relationship between problem posing and school mathematics are consistent with the results from the whole class data presented at the beginning of this chapter. Individuals' beliefs about mathematics were shown to be related to the *mathematics predominant pre*, *mathematics secondary pre*, *mathematics predominant post*, and *mathematics secondary post* views. Finally, individuals' views of the relationship between problem posing and school mathematics are similar to the whole class. All four participants became better able to articulate their beliefs about the benefits of problem posing for students and possibilities for including problem posing in school mathematics.

Chapter 7

Discussion and Implications

It was the goal of this research to incorporate problem posing into a mathematics content course for pre-service elementary and middle school teachers and to describe the apparent effects of this incorporation. Data was collected and analyzed from journal writing, class assignments, and interviews, to address the five research questions that were presented in Chapter one. These research questions were as follows,

1. What are the characteristics of pre-service teachers' problem generation pre- and post- instructional treatment?
2. How do the characteristics of pre-service teachers' problem re-formulation and problem generation change over the course of the instructional treatment?
3. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about mathematics?
4. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about the teaching and learning of mathematics?
5. How does participation in problem re-formulation and problem generation influence pre-service teachers' beliefs about the relationship between problem posing and school mathematics?

The discussion that follows will be organized to highlight the results of this research in relation to the five questions above. The discussion will begin with the results of the pre- and post-assessment of problem generation, followed by a discussion of any changes in participants' problem posing during the instructional treatment. Participants' beliefs about mathematics, beliefs about the teaching and learning of mathematics, and views of the relationship between problem posing and school mathematics will then be explored. The section will conclude with a discussion of the four individual cases presented in this research. Following the discussion, implications of this research for teaching and learning mathematics and suggested directions for future research will be presented.

Problem Posing

Pre- and Post-Problem Generation

Participants' results on the pre- and post-assessment of problem posing were analyzed using statistical software, and a Tukey-Kramer multiple comparison matched pairs test. Statistical analysis showed a statistically significant difference between the *Numeric pre* and *Numeric post* means, as well as the *Numeric post* and *Non-numeric post* means. The means of the statistical analysis of problem generation are summarized in table, 7.1.

	Numeric	Non-numeric	Total
Pre-average	5.33	3.61	8.94
Post-average	8.72	4.88	13.61

Table 7.1: Problem generation results

The results of the assessments of problem posing imply that there was a statistically significant difference between participants scores pre- and post- instructional treatment. It can be concluded from this result that participants became more efficient at problem generation because of the instructional treatment. By the post-assessment participants were able to pose problems totalling almost four points more than the pre-assessment. This improvement implies that subjects were posing at least two more problems on average, since a multi-step problem received a score of 3 points. As discussed in Chapter 5, participants did not only become more efficient problem posers they were also posing more multi-step problems. On the pre-assessment 16% of the posed problems were multi-step, while on the post-assessment 28% of the posed problems required a multi-step solution process.

Therefore, in a timed problem generation activity, participants became more efficient in their problem posing and were able to pose more multi-step mathematics problems after the instructional treatment. It is my hypotheses that the fact that these pre-service teachers are more efficient problem posers post instructional treatment will help them prepare their future lessons, classroom instruction, and write assessments. Having the ability to pose more multi-step problems will likely help these pre-service teachers challenge their future students as they pose problems for use in their classrooms.

The statistical analysis also indicates a statistically significant difference in participants' abilities to pose problems from sets of information with numeric content than from sets of information without numeric content on the post-assessment. Two of three problem generation activities during the instructional treatment contained numeric information and the one which did not contain numeric information was the first problem generation activity. Therefore, participants had more experience with problem generation from a set of information with numeric content and this was their

last problem generation activity prior to the post-assessment. These characteristics of the problem generation tasks may have influenced participants problem posing, but the difference in numeric and non-numeric problem posing shown here is consistent with the results of Leung's (1993) dissertation work. It is possible that change in both forms of problem generation would have been seen if participants were given equal opportunities to pose problems from such sets of given information. Also, greater differences in post-assessment scores may have been seen if participants were given more than three opportunities to pose problems as problem generation during the instructional treatment.

Problem Generation During the Instructional Treatment

Participants had three opportunities to generate problems during the instructional treatment. The changes in participants' problem generation efficiency and in the number of multi-step problems they posed, that are evident from the pre- and post-assessment of problem posing, are not apparent from the three problem generation activities. The characteristics of participants' problem generation were similar on each task assigned during the instructional treatment. The number of plausible statements ranged from 91% to 100%, the number of problems with sufficient information ranged from 81% to 88% and the the number of multi-step problems ranged from 56% to 67% on the three problem generation tasks. There are a number of possible explanations for the consistency of the characteristics of participants' problem generation during the instructional treatment. First, there was not an extended length of time between problem generation activities. The first problem generation activity was assigned on February 25, 2002 and the third was collected on March 27, 2002. Participants problem generation during the instructional treatment took place, then, in a one month period. Second, participants had at least five days to complete each of the problem generation activities and in each case were asked to pose a fixed

number of problems. This differed from the pre- and post-assessment which included a time constraint and asked participants to pose as many problems as possible. These two factors may have shaped the problem generation outcomes of these pre-service teachers during the instructional treatment. I assumed that given ample time participants would try to pose the best problems that they could. The characteristics of participants posed problems during the instructional treatment imply that within the course of a month these pre-service teachers' problem generation outcomes were consistent. Differences may have been seen if the problem generation activities were more spread out over the course of the instructional treatment. Again this will be considered in future research.

It is also interesting to note that participants added information to 12% of the problems that they posed on the first problem generation task during the instructional treatment and did not add information to problems on either of the other two tasks. The fact that participants use of added information changed during the instructional treatment may be explained by the fact that the first problem posing task did not provide participants with numeric information while the final two did. This may also be a result of participants becoming more comfortable posing problems within the constraints of the given information as they gained experience with problem generation during the instructional treatment.

Problem Re-formulation During the Instructional Treatment

Participants were given seven opportunities to engage in problem re-formulation activities during the instructional treatment. The first problem re-formulation activity was collected on February 6, 2002 and the final problem re-formulation activity was collected on May 8, 2002. Participants completed seven assigned problem re-formulation activities and a summary of their problem re-formulation is provided in table 7.2,

	Problems	S.G.W	Context	Add info.	Extend	Given	Wanted	Re-word
Set 2	50	7	2	3	4	27	15	1
Set 3	56	11	5	3	6	26	10	1
Set 4	42	6	3	3	0	25	8	0
Set 5	40	3	1	5	2	17	15	2
Set 6	37	3	4	2	5	16	11	1
Set 7	35	0	3	3	13	13	5	0
Set 9	32	2	5	3	5	17	3	0

Table 7.2: Aggregate problem re-formulation by set

The results in table 7.2 indicate that participants' problem re-formulation was dominated during the the instructional treatment by the technique of changing the given. Participants use of what the researcher categorized as level 1 problem re-formulation techniques; switching the given and the wanted, changing the context, and extension was also explored. These techniques were considered higher level by the researcher because they required the poser to go beyond re-writing the original problem and changing information. It is also important to note that the researcher considered all problems equally accessible to higher level re-formulation since they were all of equal difficulty and all would have been coded as plausible multi-step problems with sufficient information for solution under the researcher coding scheme for problems generated in this research. As was shown in table 5.13, there was a trend on problem sets 6, 7, and 9 for participants to utilize more higher level re-formulation techniques than on the previous problem sets. For example, on problem set 5 only 13% of the re-formulations utilized a higher level technique whereas on problem set 6, 27.5%, on problem set 7, 43%, and on problem set 9, 34% of the re-formulations utilized a higher level technique.

This trend in participants' use of level 1 problem re-formulation techniques implies that as these pre-service teachers gained experience posing problems as problem re-formulation they began to utilize more advanced problem re-formulation techniques more regularly. I believe that the ability to utilize the three level 1 problem re-formulation techniques will benefit these future teachers as they are preparing to teach. These techniques will allow these future teachers to re-formulate activities and problems to make them more meaningful and beneficial for their students.

Problem Posing Summary

The above discussion implies that as a result of the instructional treatment there were some significant changes in the characteristics of these pre-service teachers problem posing. First, all of the participants showed the ability to pose mathematics problems as both problem generation and problem re-formulation during the instructional treatment and based on class observations and discussions with participants, it is the researcher's belief that they became more comfortable posing mathematics problems. Second, participants showed increased efficiency in their problem generation and the ability to pose more multi-step problems post instructional treatment. At the same time, participants utilization of higher level problem re-formulation techniques increased on the final problem re-formulation activities of the instructional treatment.

These changes in participants' problem posing imply possible benefits for their future education and teaching. Leung (1993) showed a relationship between the mathematics achievement and problem posing ability of pre-service teachers. That relationship may be evident with these pre-service teachers as they continue their mathematical development. Participants may also continue to utilize their developing skills as problem posers as they continue their preparation to become mathematics teachers and when they are preparing mathematics activities in their future classrooms. Finally, the inclusion of problem posing at all levels of mathematics edu-

cation has been suggested by mathematical organizations and mathematics education researchers (NCTM, 2000; Kilpatrick et al., 2001; Kilpatrick, 1987; Silver, 1994). This research represents an attempt to include problem posing with pre-service teachers and shows that they have the ability to generate and re-formulate mathematics problems and that the characteristics of their posed problems change with experience.

Beliefs About Mathematics

In Chapter 6 participants' beliefs about mathematics were characterized both pre- and post- instructional treatment. Participants' beliefs about mathematics pre-instructional treatment were described by two views,

Mathematics predominant pre: Mathematics is a problem solving domain that is characterized by the study of numbers, operations and relationships.

Mathematics secondary pre: Mathematics is a way of thought. It is a part of life and the study of mathematics makes people think.

Changes in participants views of mathematics after the instructional treatment can be seen by examining the two views of mathematics that were evident post instructional treatment,

Mathematics predominant post: Mathematics is a problem posing and problem solving domain that is characterized by the study of numbers, relationships, patterns and processes.

Mathematics secondary post: Mathematics is a foundation of knowledge as well as an intriguing way of thinking that includes proof.

The views above imply that there were some qualitative changes in the characterizations of participants' beliefs during the instructional treatment. These views imply

that after the instructional treatment participants had started to consider mathematics as much a problem posing as a problem solving domain, and participants had started to view mathematics as a more open-ended discipline. With regard to the nature of mathematics it was shown that post instructional treatment, more participants believed that the teaching and learning of mathematics is changing.

Post-instructional treatment these pre-service teachers are transitioning towards what Schuck (1997) defined as a “problem solving” view (see pg.28) of mathematics. Since Schuck’s definition did not include problem posing the researcher would define these pre-service teachers’ view as a “problem solving and problem posing” view of mathematics. This change implies that during the course of the instructional treatment participants reflected on their beliefs about mathematics. This reflection has been suggested by the teacher preparation literature discussed in Chapter 2, and lead these pre-service teachers to articulate a view of mathematics as a problem posing domain. Participants also transitioned to viewing the practice of mathematics as an open-ended thought provoking activity which includes exploration with active minds.

Participants’ experience with problem posing during the instructional treatment may have played a role in these changes with regard to their beliefs about mathematics. Without some exposure to problem posing it would not be expected that participants would come to view mathematics as a problem posing domain since they need some motivation for viewing mathematics as more than a problem solving domain. Also, many mathematics educators view problem posing as an open-ended process that can lead to exploration of mathematical ideas and it has been shown to be related to and foster student creativity (Silver, 1994, 1997; Leung, 1993). Therefore, participants view of the practice of mathematics as an open-ended thought provoking activity which includes exploration with active minds also may be a product of their introduction to problem posing.

There are implications of these new beliefs for these pre-service teachers future classroom instruction. The explorative and open-ended nature of doing mathematics has been stressed in standards documents within the discipline (NCTM, 1991, 2000). Also many standards-based mathematics curricula ask students to participate in mathematical explorations and to be active in the process of learning mathematics. This new problem posing view of mathematics and the new view of the practice of mathematics which have been adopted by these pre-service teachers may make them more willing and likely to adopt standards-based curricula and foster a classroom atmosphere that provides an active learning environment.

Beliefs About Teaching and Learning Mathematics

Beliefs About Teaching Mathematics

Based on their experiences in the instructional treatment participants became better able to articulate the attributes of a good mathematics teacher and a good mathematics classroom. Participants also began to see mathematics teaching as a more open-ended activity that fosters student autonomy. Combining the data on participants' beliefs about the attributes of a good mathematics teacher and the aspects of good mathematics instruction participants' beliefs about teaching mathematics pre- and post- instructional treatment can be summarized as follows,

Pre-instructional treatment: Mathematics teaching involves utilizing manipulatives in group work and discovery while also being sure to relate teaching to multiple learning styles. A good mathematics teacher believes that all students can learn, is always evaluating their teaching, and is organized and focused.

Post-instructional treatment: Mathematics instruction must be delivered with meaning and understanding which is achieved by creating lessons that appeal to all

learning styles and allow students to construct their own knowledge. A good mathematics teacher has an in depth understanding of the discipline, uses problem posing as a form of inquiry, applies mathematics to the real world, has high expectations and is innovative and creative.

The differences in these two views of mathematics teaching suggest that these pre-service teachers reflected on their beliefs about teaching mathematics, as suggested in the teacher preparation literature discussed in Chapter 2, during the instructional treatment. This reflection has caused these pre-service teachers to become adept at articulating their views about teaching and their future classrooms. The views these pre-service teachers articulated are consistent with the reform movement in mathematics education and have also been articulated in standards documents in the discipline (NCTM, 1991, 2000). Similar to their beliefs about mathematics, participants' beliefs about teaching mathematics may make them feel more comfortable adopting a standards-based mathematics curriculum. Further, if these pre-service teachers future practice is consistent with their beliefs, which research has shown is not always the case, they will provide their students with a learning environment consistent with suggestions in the mathematics education literature (Battista, 1994; NCTM, 1991, 2000).

There are several possible explanations for the change in participants' beliefs about teaching mathematics. First, throughout the course of the instructional treatment these pre-service teachers were engaged in a classroom environment that resembled their post-instructional treatment view of mathematics teaching. Research on teachers' beliefs have shown that teachers have a tendency to model their teaching after their past classroom experiences (Thompson, 1992). Therefore, these pre-service teachers may have been articulating the view of teaching they saw from the class that they were engaged in. Also, in their post-instructional treatment description of a

good mathematics teacher, many students mentioned the utilization of problem posing as a tool for inquiry oriented learning. This belief is consistent with suggestions from mathematics education literature and may have led these pre-service teachers to reflect on their future mathematics teaching (Silver, 1994). I believe that more developed views about teaching mathematics will make it more likely that these pre-service teachers instruction will be consistent with their beliefs about mathematics teaching.

Beliefs About Learning Mathematics

In Chapter 6 the small amount of data related to participants' beliefs about learning mathematics is discussed. The main belief that emerged from this data was that students learn best when they are actively engaged in the learning process. This belief parallels participants' beliefs about teaching mathematics and providing opportunities for discovery learning. The relationship of these beliefs with respect to teaching and learning mathematics may help shape these pre-service teachers future practice. The introduction to problem posing did influence participants' beliefs about student learning and this relationship will be discussed in detail in the next section.

Relationship Between Problem Posing and Teaching and Learning Mathematics

It was shown that participants' beliefs about problem posing and its relationship to school mathematics evolved during the instructional treatment. This discussion will focus on the relationships between problem posing and teaching mathematics and between problem posing and learning mathematics. In each case, a summary of the results from data collection and data analyses will be presented and will be followed by a discussion of the results. It is important to mention that although the participants were asked to read two articles related to problem posing in the classroom, there was never an explicit classroom discussion during the instructional treatment about the benefits of problem posing for instruction and mathematics learning.

Problem Posing and Teaching Mathematics

Participants reflection on the relationship between problem posing and teaching mathematics happened gradually during the instructional treatment. Only two participants had begun to reflect on the relationship between problem posing and teaching mathematics in their March 4, 2002 journal entry describing their future classroom and instruction. Further reflection was seen in the journal entry collected March 11, 2002 when four students commented on the relationship. This reflection continued throughout the instructional treatment and the result of the reflection was seen on the journal entry collected May 7, 2002, which asked participants to discuss whether they would utilize problem posing in their future classrooms. All nineteen participants stated that they would utilize problem posing in their future teaching and they suggested multiple ways to incorporate problem posing, including methods similar to the instructional treatment, as a tool to introduce new material, on homework, and as extra credit. These beliefs about how best to incorporate problem posing into mathematics teaching were confirmed on the the post-assessment of beliefs and participants' final journal entry, which asked them to reflect on the class with no specific mention of problem posing. These beliefs were confirmed by participants continued and consistent discussion of the relationship between problem posing and teaching mathematics on these two tasks.

After the instructional treatment, participants had reflected through their journal writing on the relationship between problem posing and teaching mathematics and had articulated possibilities for utilizing problem posing in their future classrooms. The reflection was a product of participants' engagement with problem posing and journal prompts, since neither the researcher or instructor engaged in discussions with the participants related to their views about problem posing. Thus, participants' reflections lead them to articulate that problem posing should be utilized in

mathematics instruction. Post instructional treatment, these pre-service teachers are armed with problem posing as a tool and they are poised to utilize it in their future classrooms. As shown in the literature discussed in Chapter 2, both professional organizations and mathematics educators have suggested the incorporation of problem posing in mathematics classrooms (NCTM, 2000; Kilpatrick et al., 2001; Silver, 1994). These pre-service teachers are ready to begin this call to include problem posing, they have engaged in it, reflected on its benefits and stated that they would utilize it in their classrooms. This also implies that working with pre-service teachers is a possible starting point for the inclusion of problem posing at all levels of mathematics education as these pre-service teachers intend to utilize problem posing with their future students.

Literature reviewed in Chapter 2 showed that it is feasible to incorporate problem posing in elementary education (Winograd, 1992, 1997; Schloemer, 1994). Therefore, these pre-service teachers have articulated a belief that will be possible for them to integrate into their future teaching. Research has suggested the possibilities and writing has suggested the necessity (NCTM, 2000; Kilpatrick et al., 2001) and it seems that these pre-service teachers see both. These views become more evident when one examines their beliefs about problem posing and student learning.

Problem Posing and Learning Mathematics

Participants' pre-assessment of beliefs about mathematics imply that pre instructional treatment, participants saw benefits of student problem posing on learning, especially a benefit on students problem solving abilities and their ownership of mathematics. After posing problems during the instructional treatment, participants were better able to articulate possible benefits of problem posing for student learning, which was evident on later journal entries and the post-assessment of beliefs. On a number of occasions participants articulated that problem posing has the potential

to provide students with ownership of mathematics. These beliefs are summarized effectively through a few quotes that were presented in Chapter 6,

I think that when students inquire about topics they are taking learning into their own hands, and that is one of the best things that problem posing can bring to a classroom.

I also learned how beneficial it is to have children pose problems, something I didn't like before this class. It is extremely important to give the students a sense of ownership over a problem and a better understanding of the problem.

During the instructional treatment, these pre-service teachers reflected on the possible benefits of problem posing for their future students and better articulated their conceptions about problem posing and learning mathematics through this reflection. The benefits that these pre-service teachers articulated are consistent with research and writing in mathematics education (Silver, 1994). Therefore these pre-service teachers not only suggested the implication of problem posing in their future classrooms but they saw the possible benefits of this incorporation for student learning. I believe that this understanding of the benefits of problem posing will make these pre-service teachers more likely to have their practice mirror their beliefs and incorporate problem posing in their future classrooms, as suggested in mathematics education literature.

Individuals

Four individuals agreed to participate in this research by not only allowing their work to be collected but by also being interviewed three times during the instructional treatment. The following section will highlight some of the results presented with regard to these four individuals in chapters 5 and 6 and discuss the implications of these results.

Bill

Recall that Bill was a senior mathematics education major seeking certification to teach elementary and middle school mathematics at the time of this research. This section will highlight the results related to characteristics of Bill's problem posing, his beliefs about the relationship between problem posing and school mathematics, his beliefs about mathematics, and his beliefs about teaching mathematics. Finally this section will discuss implications of these results.

Problem Posing. The characteristics of Bill's problem generation changed as he gained more experience posing mathematics problems during the instructional treatment. On the pre-assessment of problem posing, Bill did not understand the problem generation process. During the instructional treatment a gradual progression of Bill's problem generation to posing more multi-step problems was shown. Bill's problem re-formulation techniques were consistent throughout the instructional treatment and relied heavily on the techniques of changing the given and changing the wanted.

The apparent change in characteristics of Bill's problem generation can be explained by a better understanding of the problem generation process and experience generating problems from sets of information during the instructional treatment. Bill's lack of utilization of level 1 problem re-formulation techniques is a surprise since he made it clear during the instructional treatment that he felt that he was better at posing problems as re-formulation, because they gave him a frame of reference. Bill probably found it easier and felt more comfortable writing problems as re-formulation since he was relying on changing the given and changing the wanted and in Bill's mind this comfort translated to ability. It was apparent from conversations with Bill that he was more comfortable posing problems after the instructional treatment and this comfort should aid his utilization of problem posing as a teacher.

Beliefs. Bill's beliefs about mathematics and the nature of mathematics underwent little change during the instructional treatment. He did, though, become better able to articulate his views of what he believes mathematics is post instructional treatment. Bill became better able to articulate a definition of mathematics during the instructional treatment but did not abandon his belief that mathematics is static and that mathematicians are figuring out mathematics that already exists.

Bill's beliefs about teaching mathematics and the role of problem posing in school mathematics developed during the instructional treatment. During the first half of the instructional treatment, Bill could be described as a traditional mathematics teacher, believing in the role of drill and practice and procedural understanding. But there was a shift in Bill's beliefs during his second interview and it is directly related to his reflection on the role of problem posing in school mathematics. During this interview, the researcher briefly presented to Bill his views of the role of problem posing in mathematics. Once Bill saw this connection between problem posing and mathematics, he began to reflect on his beliefs about teaching mathematics and the role of problem posing in school mathematics. This reflection led Bill to articulate a belief that problem posing should be incorporated into school mathematics and a belief in a more open approach to teaching that allows his students to "struggle" and develop ownership of the mathematics they are learning. After the instructional treatment, Bill again stated the belief that problem posing should be a feature of mathematics curricula and classrooms and was able to begin to verbalize possible ways in which to accomplish this incorporation. Bill also discussed problem posing as a tool to promote intrinsic motivation in his students and allow them to discover mathematics. Intrinsic motivation and discovering mathematics are vastly different ideas than Bill's initial beliefs about rote learning and memorization because they imply some level of student autonomy.

There are several possible reasons for the apparent changes in Bill's beliefs during the instructional treatment. First, and as discussed above, Bill's problem generation abilities were improving during the instructional treatment and he was becoming more comfortable posing mathematics problems. Bill's comfort and experience with problem posing did not change his belief about the nature of mathematics, but it did make it possible for him to begin reflection on the relationship between problem posing and school mathematics. This new way to look at mathematics and the instructional treatment, by the second interview, caused Bill to begin thinking about the narrowness of his initial views of problem posing and teaching mathematics. Second, it was clear from class observations that during the course of the semester Bill was actively engaged in class activities, group work, and was trying to develop a deeper understanding of mathematics. Bill was also experiencing difficulty with his other mathematics classes which were taught in a more traditional lecture style. Bill's success in this class may have caused him to reflect on the new teaching style he was considering and its implications for student learning. These factors led Bill to change his beliefs about mathematics teaching and the role of problem posing in school mathematics.

Bill's case implies that changes in pre-service teachers' beliefs are possible when they are given the opportunity to reflect on their beliefs about problem posing and teaching mathematics. It also is a reasonable hypothesis that the instructional treatment played a role in developing Bill's new beliefs. Similar to the whole class results, Bill has adapted beliefs about teaching mathematics and the role of problem posing that are more in line with national recommendations in mathematics education and was also able to replace his belief that mathematics teaching must include rote learning and memorization to a more student centered view of mathematics teaching.

Carrie

Recall that Carrie was a first year graduate student seeking certification to teach elementary school at the time of this research. This section will highlight the results related to characteristics of Carrie's problem posing, her beliefs about the relationship between problem posing and school mathematics, her beliefs about mathematics, and her beliefs about teaching mathematics. Finally, this section, will discuss implications of these results.

Problem Posing. The characteristics of Carrie's problem posing changed during the instructional treatment. On the pre-assessment of problem posing, Carrie was only able to pose one solvable mathematics problem and it required a single step solution process. During problem generation on journal entries and on homework, Carrie was able to pose multiple multi-step problems, but only posed one multi-step problem on the post-assessment of problem posing. Carrie's problem re-formulation was dominated by utilizing the techniques of changing the given and changing the wanted but she occasionally, especially early in the instructional treatment, used level 1 problem re-formulation techniques.

During the instructional treatment, Carrie showed the ability to pose multi-step mathematics problems on problem generation activities when she was not under a time constraint. Carrie's inability to pose multi-step problems under a time constraint, however, may be a product of her limited experience posing mathematics problems. The data suggests, though, that Carrie will take a problem generation skill with her to her future teaching that, if utilized and developed, may become better under a time constraint. Carrie utilized level 1 problem re-formulation techniques on the first problem re-formulation activities but this did not last throughout the instructional treatment. Carrie made it clear during interviews that she was not learning mathematics because she enjoyed it and this feeling may have caused her to become

complacent in her problem re-formulation. Carrie may have realized that she was not rewarded for posing more difficult problems and hence opted to utilize the easier problem re-formulation techniques. Regardless, Carrie's problem generation abilities became more developed during the instructional treatment.

Beliefs. There was a change in Carrie's beliefs about mathematics during the instructional treatment and post-instructional treatment Carrie was better able to articulate her beliefs. In Chapter 6, evidence indicates that Carrie's beliefs are consistent with the *mathematics predominant pre* view, but that post-instructional treatment, Carrie was beginning to consider the *mathematics predominant post* view. Carrie did maintain, however, her belief that mathematics is always changing and that there is not a definitive set of mathematics that already exists. Carrie's views of mathematics teaching became better developed during the course of the instructional treatment and she was able to articulate why she believes in group work and student discovery learning. Similarly, Carrie believed that problem posing was beneficial in mathematics education early in the instructional treatment, but was able to better articulate her view post instructional treatment as shown in Chapter 6.

Unlike Bill, there was not a turning point in the instructional treatment that highlights the change in Carrie's beliefs, but evidence suggests that she was able to develop her beliefs about mathematics, beliefs about mathematics teaching, and her beliefs about the incorporation of problem posing in mathematics education. There are possible explanations for Carrie's better articulation of her conceptions. Carrie showed evidence of reflecting on her beliefs throughout the instructional treatment, especially during her journal writing. The development of Carrie's beliefs about teaching mathematics can also be attributed to the fact that the style of the course matched her beliefs. Carrie entered the instructional treatment having already thought about discovery learning and group work in mathematics instruction. As these ideas were

modeled during the instructional treatment, she became better able to articulate her pre-existing views of mathematics teaching. Carrie's ability to articulate her views of the relationship between problem posing and school mathematics, the benefits of student problem posing for learning, and possibilities for incorporating problem posing in her future classroom paralleled her increased experience posing mathematics problems. As with the whole class, Carrie's problem posing experience seemed to cause her to increase her reflection about mathematics and mathematics teaching and learning.

Carrie's case suggests that even though she professed not to love mathematics, her experiences during the instructional treatment caused her to reflect on problem posing, teaching mathematics, and her future classroom instruction. After the instructional treatment Carrie was able to articulate her views about mathematics and teaching mathematics, ready to utilize problem posing in her future classroom, and armed with beliefs about the benefits of student problem posing.

Laura

Laura was a sophomore mathematics education major seeking certification to teach elementary and middle school mathematics at the time of this research. This section will highlight the results related to characteristics of Laura's problem posing, her beliefs about the relationship between problem posing and school mathematics, her beliefs about mathematics, and her beliefs about teaching mathematics. Finally, this section will discuss implications of these results.

Problem Posing. Laura demonstrated a developed understanding of problem posing from the beginning of the instructional treatment and her problem posing reflected this. Laura posed four problems on the pre-assessment of problem posing, one of which was multi-step. Laura's problem generation was focused on posing multi-step problems throughout the remainder of the instructional treatment. However, dur-

ing the second interview and on the post-assessment of problem posing, Laura only posed two multi-step problems. Finally, Laura's problem re-formulation was consistent throughout the instructional treatment and she utilized the strategy of changing the given most often. Laura did show an ability to use level 1 problem re-formulation techniques and used them on 8 of the 21 problems she re-formulated.

The fact that Laura posed fewer multi-step problems on the pre- and post-assessment of problem posing and during the second interview may be a product of the time constraint and her comfort generating problems. All three tasks were completed under a time constraint and the pre- and post-assessment included a set of information with numeric content. Laura made it clear during the course of the instructional treatment that she was more comfortable posing problems from sets of information that did not include numerical content because she felt that she was able to exhibit more creativity in these situations. Also, on the problem generation tasks that did not include a time constraint, Laura consistently posed multi-step problems and problems that went beyond the surface features of the given information. Laura showed throughout the instructional treatment that she was an effective and reflective problem poser. Laura's ability to use level 1 problem re-formulation techniques was surprising since she felt that during problem re-formulation she would generally get stuck in the mode of the original problem. Characteristics of Laura's problem re-formulation are a result of her willingness to always reflect on her work and her understanding.

Beliefs During the instructional treatment, Laura became able to articulate her view of the nature of mathematics and her view of the role of problem posing in school mathematics evolved. Laura entered the instructional treatment with well defined beliefs about mathematics and was unsure of her beliefs about the nature of mathematics. During the instructional treatment, Laura demonstrated that she sees beyond mathematics as procedural knowledge and views mathematics as a way

of thought. By the final interview, Laura had begun to develop a belief about the nature of mathematics and stated that mathematics exists and that mathematicians discover it. Laura also entered the instructional treatment with developed beliefs about teaching mathematics based on past teaching experience. Her beliefs did not change during the instructional treatment. Both before and after the instructional treatment, Laura believed that there are benefits of problem posing in classrooms for student learning. Also, after the instructional treatment, Laura was able to articulate possible ways to incorporate problem posing in her future classroom.

Unlike other participants, Laura entered the instructional treatment with past teaching experience at a summer program called Summerbridge. Also during her teaching, Laura had attempted to utilize problem posing in her classroom. These experiences may have influenced Laura's beliefs, as they were being examined in this study. During the instructional treatment, Laura articulated the belief that mathematics exists and that mathematicians are discovering it. There is not a specific instance that describes Laura's belief of this conception, but it was probably a product of her constant reflection on her mathematical understanding and her development as a future teacher. Also, post instructional treatment, Laura was better able to articulate possible avenues for incorporating problem posing in her classroom and expressed beliefs about possible benefits for teachers as problem posers. As with the whole class, Bill, and Carrie the ability to articulate her beliefs about problem posing paralleled Laura's problem posing experience during the instructional treatment. At the same time as she was developing her beliefs about problem posing Laura moved beyond thinking about what her future classroom would look like and began thinking about how she was going to engage her future students in mathematics. Mathematics educators have suggested problem posing as a possible tool to engage students in mathematics and Laura understood the possibilities for engaging students through

problem posing and began to think more globally about how to engage her students as she was reflecting on the role of problem posing (Silver, 1994).

Unlike Bill, Carrie, and Liz, Laura's case describes the effects of the instructional treatment on someone who entered with previous experience posing mathematics problems. Laura's experiences during the instructional treatment, however, suggest that she further developed her beliefs and began to think beyond her beliefs about teaching mathematics and start to consider how her beliefs would influence her students engagement with mathematics. Therefore in this case the exposure to problem posing was also of benefit to a subject with prior teaching experience and developed beliefs.

Liz

Liz was a sophomore mathematics education major seeking certification to teach elementary and middle school mathematics at the time of this research. This section will highlight the results related to characteristics of Liz's problem posing, her beliefs about the relationship between problem posing and school mathematics, her beliefs about mathematics, and her beliefs about teaching mathematics. Finally, this section will discuss implications of these results.

Problem Posing. Throughout the instructional treatment Liz's problem generation was characterized by a lack of proficiency generating problems under a time constraint. When generating problems on her own time, however, Liz was able to generate multi-step mathematics problems and there seemed to be an increase in her creativity posing problems as she progressed through the instructional treatment. On the pre- and post-assessment of problem posing and during interview two, Liz was efficient in generating statements under a time constraint but showed little proficiency for having these statements be mathematical problems. Liz's problem re-formulation was dominated by changing the given information. In fact, Liz only utilized level 1

problem re-formulation techniques 3 times during the instructional treatment.

The characteristics of Liz's problem posing can be explained by her limited exposure to problem posing and hence limited conception of what problem posing is. Liz was not able to define problem posing during the first interview and during the final interview her description only included descriptions of problem generation and problem re-formulation techniques. Liz did not describe the problem posing process during the instructional treatment. This may explain Liz's willingness to write questions such as "Where are we going?" during a problem posing task since she may not have had a conception that this was not a problem. Liz was more comfortable posing problems as problem generation and this was apparent from the problem generation tasks that did not include a time constraint, as she was able to pose multi-step problems. I believe that Liz's reliance on changing the given information in problem re-formulation demonstrates her lack of comfort re-formulating problems and inability to think beyond the problem she had just solved. This is also consistent with her belief that she is better at problem generation. Liz's lack of a conception of what problem posing is to her and her problem posing process influenced her posed problems, but she was able to pose multi-step problems when not posing under a time constraint.

Beliefs. Liz's beliefs about mathematics changed little during the instructional treatment but she changed her view of the nature of mathematics. During the instructional treatment, Liz began to relate her beliefs about teaching to her future students understanding. Liz's view of the role of problem posing in school mathematics also evolved. Throughout the instructional treatment Liz had difficulty defining mathematics and wasn't confident in a definition. This implies that Liz did not have a strong conception of what mathematics is to her and hence there was not significant change in her beliefs about mathematics. There are some changes in Liz's beliefs

about whether mathematics is changing. During her first interview, Liz stated that mathematics is changing but had trouble verbalizing what she meant. On her final interview and post-assessment of beliefs, Liz was able to articulate that mathematics is not changing to students but is changing to mathematicians who are doing mathematics research. Liz also stated that mathematics teaching and mathematics education are changing.

Liz became able to articulate her developing beliefs about the attributes of a good mathematics teacher during the instructional treatment. Liz's vision of her future classroom did not change but by the post-assessment of beliefs Liz was able to better articulate her vision of a good mathematics teacher as someone who utilizes group work and manipulatives, is understanding, and realizes that her students have different learning styles. Liz also strengthened her beliefs about the benefits, for students, of problem posing in mathematics classrooms. Liz stated some initial beliefs on the pre-assessment but by the post-assessment Liz was able to better articulate these beliefs and has started to relate problem posing to real life situations.

During the instructional treatment, Liz became better at articulating her beliefs about both mathematics and the teaching and learning of mathematics. Liz's personal reflection and reflection in journal writing during the instructional treatment allowed her to begin to develop her beliefs and start to consider connections between her beliefs and her future teaching. As suggested by the teacher preparation literature discussed in Chapter 2, reflection is necessary in developing teachers and Liz is an example of the outcome of pre-service teacher reflection. Liz also was able to understand the possible benefits of problem posing for her future students and was starting to develop a sense of how she would incorporate problem posing in her future classroom. Liz's beliefs about problem posing also developed as she gained experience posing mathematics problems during the instructional treatment. Liz, however, did not

verbalize any use of problem posing for herself as a future teacher and it is possible that this combined with her lack of comfort re-formulating problems could prevent Liz from incorporating problem posing in her future classroom. Liz evolved from a student unable to verbalize her beliefs to having developed beliefs that she could articulate.

Liz's case highlights the benefits of reflection in pre-service teacher education. Liz was not able to articulate beliefs about mathematics or define problem posing at the beginning of the instructional treatment. After experience reflecting on her beliefs and future teaching, as well as the experience of posing mathematics problems, Liz was able to articulate beliefs about mathematics, beliefs about the role of problem posing in school mathematics, and relate her beliefs to her future teaching.

Implications

The results of this dissertation research demonstrate the importance of pre-service teacher education and the power of reflection by pre-service teachers. Further, conducting this research has forced the researcher to consider possible future directions for research related to problem posing in pre-service teacher education and with undergraduate mathematics majors.

Pre-Service Teacher Education

Mathematics educators and organizations dedicated to mathematics education have sounded the bell on the incorporation of problem posing at all levels of mathematics instruction, including teacher preparation programs and mathematics classes designed for pre-service teachers. This research is evidence that others must begin to consider the inclusion of problem posing in mathematics instruction. The pre-service teachers in this study were able to better articulate beliefs about mathematics and the teaching of mathematics, the incorporation of problem posing in their future

classrooms, including possible ways to incorporate problem posing beyond what was modeled in the instructional treatment. Based on these and other results there are implications of this research for pre-service teacher education. This research serves as a guidepost to promote problem posing at all levels of mathematics education and to prepare future teachers to adopt standards based curricula while incorporating into their teaching practice the ideas which the reform movement in mathematics education has put forth.

These pre-service teachers reflected on and articulated beliefs about the role of problem posing in school mathematics as they gained experience posing problems during the instructional treatment. Participants articulated possible benefits of problem posing for their future students that are in-line with the suggestions of such groups as the National Council of Teachers of Mathematics and the National Research Council, as discussed in Chapter 2. Participants also discussed possible benefits of their ability to pose problems for preparing their future mathematics lessons. Arming pre-service teachers with knowledge about the benefits of problem posing makes them more likely to incorporate problem posing in their classrooms and to introduce their colleagues to the benefits of problem posing. This instructional treatment provided these pre-service teachers with experiences that will help them begin the inclusion of problem posing at all levels of mathematics education by incorporating problem posing in their future classrooms. Also, based on these pre-service teachers limited beliefs about problem posing and lack of problem posing experience I believe it would have been difficult for them to include problem posing in a mathematics classroom prior to this instructional treatment. This experience prepared these pre-service teachers to utilize problem posing in their future classrooms.

Further, these pre-service teachers articulated their beliefs about the necessity for mathematics teaching to promote discovery learning and inquiry-oriented and student

centered classroom atmospheres. These pre-service teachers articulated beliefs that are consistent with writing in mathematics education and instruction (NCTM, 2000; Kilpatrick et al., 2001). Therefore, these pre-service teachers might be more likely to adopt standards-based curricula that ask teachers to present students opportunities for discovery learning and provide a student centered classroom atmosphere. Arming teachers with these new ideas should be a goal of pre-service teacher education and one that can be accomplished by engaging them in an instructional treatment similar to the one described in this research.

Reflection in Teacher Preparation

The teacher preparation literature reviewed in Chapter 2 calls for reflection and metacognitive activity to be an integral part of pre-service teacher education. This research is an example of the power of reflection in pre-service teacher education and the possible changes in pre-service teachers' beliefs because of the opportunity to reflect on their beliefs and future practice. This research utilized problem posing and journal writing as a vehicle to promote reflection with these participants. These two constructs caused these pre-service elementary and middle school teachers to reflect on their future classroom practice, develop their views about mathematics teaching, and consider how their views of teaching will influence their future practice. Reflection allowed these pre-service teachers to leave this mathematics content course with articulated beliefs about teaching and learning and with numerous thoughts about what their mathematics classes would look like, how they would teach, possible activities, and possible ways to effectively incorporate problem posing. These beliefs were a product of these pre-service teachers personal reflection and reflection during journal writing and class assignments. This research is an example of a possible situation to promote pre-service teacher reflection, which is vital in teacher preparation programs, as suggested by the literature.

Possible Future Research

The pre-service teachers in this dissertation research believed that problem posing was a tool they would utilize in their future classrooms. Because of this belief and reflection by the researcher, possible future directions for research related to problem posing with pre-service teachers and in undergraduate mathematics will be presented.

First, participants suggested multiple ways to incorporate problem posing in their classrooms and considered the possible benefits of this incorporation for their students' mathematical understanding. These suggestions have raised the question of whether these participants will apply what they have learned through this research and introduce problem posing in their classrooms? If so, in what ways do they introduce problem posing? If not, what factors cause them not to introduce problem posing? It seems that it is not only important to expose pre-service teachers to problem posing but to try to understand if this exposure is something they will utilize in their classrooms. While exploring whether pre-service teachers will utilize their ideas in their future classroom it would be beneficial to explore the students reaction as well. Do students who are introduced to problem posing experience the possible benefits that were suggested by these pre-service teachers?

Second, these pre-service teachers also suggested that it is necessary for them to be good problem posers and that they will utilize their problem posing skills in their future teaching. As teachers, do participants in a similar instructional treatment utilize their problem posing skills? If so, how do they utilize these skills? Do participants utilize problem posing to generate class activities and problem sets? If not, why do they not utilize these skills? Do participants rely on the textbook as the authority in their classroom? These questions become important to understand if pre-service teachers' experience with problem posing helps develop a sense of autonomy in their teaching.

Finally the results of this research have lead the researcher to consider if it would also be beneficial for mathematics majors to see mathematics from a problem posing perspective, since as future mathematicians they will be posing mathematics problems. It seems feasible to develop a similar instructional treatment to introduce problem posing to an audience of mathematics majors at the University level that are not pre-service teachers. Documenting characteristics of participants' posed problems and describing their beliefs about mathematics and any changes that may occur in their beliefs as they are introduced to problem posing would also seem possible based on the results of this research.

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Appendix A

Course Materials

This appendix includes the following materials from the course in which the instructional treatment was adopted “Topics in Mathematics for Teachers” at the University of New Hampshire during the spring semester 2002. These materials include,

- Course Syllabus
- Weekly Course Agendas
- Problem Sets

Course Information
Math 623/723: Topics in Mathematics for Teachers
Spring 2002

Instructor: Professor Karen Graham
Office: Kingsbury M321
Phone: 862-3621
e-mail: kjgraham@cisunix.unh.edu
Office Hours: M,W 1-2PM , F 10-11AM. Others by appointment.

Course Description

This course is designed to involve students in the exploration and analysis of various mathematical topics including probability and statistics, algebra and functions, the mathematics of change, and discrete mathematical structures. Mathematics as problem solving and the role of technology in the teaching/learning of mathematics will be emphasized throughout the course.

Course Requirements

- Students are expected to attend class regularly, participate in and complete all in-class activities. If circumstances arise that cause you to miss class, you will be responsible for making-up all work missed during your absence.
- Completion of **take-home assignments** (these will involve problems sets and article critiques).
- Completion of a **“replacement unit”**. More detail available by mid-semester.
- **Portfolio**: Each student is required to keep a portfolio. This portfolio should be a record of your experience and progress in this course. It should serve several purposes, it should help you reflect on your work and share your efforts with me and your classmates. It will be useful to me as a way of understanding your thinking, how you are grappling with the material, and how I might better help you. Each portfolio will have two parts, a **work portfolio** and an **assessment portfolio**.

The **work portfolio** might include journal entries, homework solutions or attempts at solutions, evidence of how you assimilated/revised/made sense of material that has been introduced in class (your re-writing of class notes, for example with commentary, exposition about a particular concept, or a set of interrelated concepts, written discussion of the way you are thinking about the ideas in this course, a problem that you posed, material from another source that help you understand something better, along with your comments and notes, and required entries assigned in class).

Periodically you will be asked to select and clip together work that you would like to have placed in the **assessment portfolio**. At this time you will be asked to write a paragraph about each item, explaining why you have chosen it and how your thinking has developed during or after doing the assignment.

- **Examinations:** There will be a midterm and a final examination. The date of the midterm will be announced in class at least one week prior to the exam. The final examination will take place sometime between

Course Grade

Your course grade will be based on your total score for the assignments, portfolio, replacement unit, and exams. The breakdown will be as follows:

Assignments:	100 points
Replacement Unit:	100 points
Portfolio:	100 points
Exams (100 points each)	100 points
Total:	500 points

Your final course grade will be based on a percentage calculation from the above point total.

Special Events. Participation is encouraged!

MATHCOUNTS COMPETITION: Saturday, February 9th, Kingsbury Hall, UNH, 9-Noon.

NHTM ANNUAL SPRING CONFERENCE:

Weekly Assignment #1

Math 623/723

Spring 2002

- W 1/23/02 Introduction to Problem Solving
- M 1/28/02 Problem Solving
- W 1/30/02 Problem Solving and Problem Posing
-

For Monday 1/28

- Complete Pre-assessment Tasks
- Read article, "Constructivist Learning and Teaching"
- Review "Four-Step Method for Solving Mathematics Problems"

For Wednesday 1/30

- Read the section of the "Principles and Standards" distributed in class.
- Complete Problem Solving #1 - Be sure to show all of your work for each problem, even partial attempts. Choose 2 of the problems and write-up your solutions according to the 4-Step Method discussed in class. I will collect and evaluate these problems based on the rubric discussed in class.
- Compose and submit your mathematical autobiography.

Weekly Assignment #2

Math 623/723

Spring 2002

- W 1/30/02 Problem Solving and Problem Posing
- M 2/4/02 Introduction to Data Analysis
- W 2/6/02 Gathering, Representing, and Interpreting Data
- M 2/11/02 Measures of Central Tendency and Sampling Plans
-

For Monday 2/4

- Read article, "Young Children's Emotional Acts While Engaged in Mathematical Problem Solving".
- Read section of "NH State K-12 Mathematics Curriculum Frameworks" distributed in class.

For Wednesday 2/6

- Read article, "Collecting and Analyzing Real Data in the Elementary School Classroom".
- Write 5 extensions of the raisin activity.

For Monday 2/11

- Complete Problem Solving #2. Be sure to show all of your work for each problem, even partial attempts. Choose 2 of the problems and write-up your solutions according to the 4-Step Method discussed in class. For each of these problems compose and write-up 2 related problems. I will collect and evaluate these problems based on the rubric discussed in class.
- Write a journal entry for your portfolio about what you learned about statistics from The Paper Clip Game.

Weekly Assignment #3

Math 623/723

Spring 2002

- W 2/13/02 Data Representation and Analysis
- M 2/18/02 Distributions and Standard Deviation
- W 2/20/02 Sampling Plans and Bias
- M 2/25/02 Introduction to Notions of Chance and Probability
-

For Monday 2/18

- Read articles, "Statistics and Graphing" and "What do children understand about average?". Be prepared to discuss your reactions and questions.
- Complete assigned problems from "Comparing Data Sets" activity. Be prepared to discuss and share your results.

For Wednesday 2/20

- Read article, "Problem Solving: Dealing with Data in the Elementary Classroom". Be prepared to discuss your reactions and questions.
- Complete Problem Solving #3. Be sure to show all of your work and provide explanations/justifications for each problem. Choose 2 of the problems and write-up your solutions according to the 4-Step Method discussed in class. For each of these problems compose and write-up 2 related problems. I will collect and evaluate these problems based on the rubric discussed in class.

Note: Data Analysis Projects will be due on Monday March 4th - this will be a group project and details will be discussed in class on Monday 2/18.

Weekly Assignment #4

Math 623/723

Spring 2002

M 2/25/02 Introduction to Notions of Chance and Probability

W 2/27/02 Fair and Unfair Games

M 3/4/02 Area Models for Probability

For Monday 2/25

- Read parts of the NCTM *Principles and Standards* related to Data Analysis and Statistics. Be prepared to discuss your reactions and questions in class.
- Journal entry on Problem Posing for Portfolio (this will be collected and returned to you).

From the following set of given information pose 3 to 5 problems (you do not need to solve your posed problems) and then answer the questions that follow.

Given Information: Mrs. Smith's and Mr. Jones' fifth grade classes took the same mathematics test last week. You have been given all the graded exams and the answer key.

Questions:

1. Describe the process you just went through to generate problems from this set of information.
2. Do you see any similarities between the problem solving and the problem posing process? Explain.

For Wednesday 2/27

- Complete Problem Solving #4. Be sure to show all of your work for each problem, even partial attempts. Choose 2 of the problems and write-up your solutions according to the 5-Step Method discussed in class. Remember that Step 5 is posing a related problem. I will collect and evaluate these problems based on the rubric discussed in class.

Weekly Assignment #5

Math 623/723

Spring 2002

- M 2/25/02 Introduction to Notions of Chance and Probability
- W 2/27/02 Independent and Dependent Events
- M 3/4/02 Fair and Unfair Games
- W 3/6/02 Simulations and Area Models
-

For Monday 3/4

- Be prepared with the other members of your group to present your data analysis poster to the rest of the class.
- Read article, "Promoting a Problem Posing Classroom". Be prepared to discuss your reactions and questions in class.
- Journal Entry based on the prompt below for portfolio (this will be collected and returned to you).

Journal Prompt: Imagine that you are teaching and someone comes in to observe your classroom and a mathematics lesson that you are teaching. Write a description of your classroom and the lesson from the eyes of the observer. What would they see you doing during the lesson, what would they see the students doing, what would they notice about your classroom?

For Wednesday 3/6

- Be prepared to hand-in your portfolio. Remember that you need to select and clip together 3 pieces of work that you would like placed in the *assessment part of your portfolio*. For each piece of work selected you need to write a paragraph about the item explaining why you selected it and how your thinking has developed during or after doing the assignment.
- Complete Problem Solving #5. Be sure to show all of your work for each problem, even partial attempts. Choose 2 of the problems and write-up your solutions according to the 5-Step Method discussed in class. Write a related problem for each of the remaining problems numbered 1-4. You will notice that problem 5 is of a slightly different nature, follow the directions given on the problem solving set.

Weekly Assignment #6

Math 623/723

Spring 2002

M	3/11/02	Simulations Continued
W	3/13/02	Exam
M/W	3/18 and 3/20	Spring Break - ENJOY!
M	3/25/02	Wrap-up Probability Unit and Introduction to Discrete Math

For Monday 3/11

- Read follow up article on Problem Posing. Be prepared to discuss your reactions and questions.
- Complete the "More Chips" problems. **We will discuss your answers to #1 and 2 in class and I will collect the responses to #3.**
- **Journal Entry:** Please write a brief reflection on how you think class is going so far this semester, what aspects have you found the most helpful, least helpful and why?, how is the workload?, what aspects would you change?, what additional topics would you like to see covered?

For Wednesday 3/13

- Prepare for Exam 1.

Upcoming Events:

- Problem Solving #6 will be due on March 27th. For this problem set you are required to complete the first 4 problems, showing all work, providing justifications for your responses, and writing a related problem. Problem 5 provides you with a set of given information. You are to pose two problems and provide a solution for one of them.

Weekly Assignment #7

Math 623/723

Spring 2002

W 3/27/02 Introduction to Discrete Math Activities

M 4/1/02 Discrete Math Activities Continued

W 4/3/02 Introduction to Algebraic Thinking

For Monday 4/1

- Complete any leftover activities from Wednesday's Class as appropriate.
- Read the article: *Strengthening a K-8 Mathematics Program with Discrete Mathematics*
- Write a journal reflection about the exam

For Wednesday 4/3

- Complete Problem Solving #7 - Be sure to show all your work for each problem and provide explanations for each problem. For problems 1-3 state a related problem. I will collect and evaluate these problems based on the rubric discussed in class.

Weekly Assignment #9

Math 623/723

Spring 2002

M 4/8/02 Introduction to Algebra and Algebraic Thinking.

W 4/10/02 Exam

For Monday 4/8

- Read *Graph Chasing Across the Curriculum: Paths, Circuits, and Applications*. Be prepared to discuss the major points and questions you had from the reading.
- Read the introduction to the Algebra Standard (pp.37-40) from the *Principles and Standards for School Mathematics* distributed in class on Monday, 4/1. Be prepared to discuss the major points and questions you had from the reading.
- Complete any leftover problems/activities from class as appropriate.

For Wednesday 4/10

- Read the Algebra Standards from the *Principles and Standards for School Mathematics* as distributed in class on Monday, 4/1. Be prepared to discuss the major points and questions you had from the reading
- Read *Promoting Algebraic Reasoning Using Student Thinking* from the NCTM journal *Mathematics Teaching in the Middle School*. Be prepared to discuss the major points and questions you had from the reading.
- Complete any leftover problems/activities from class as appropriate.

NOTE: REMEMBER THAT A BRIEF WRITTEN REPORT (NO MORE THAN 1 PAGE) UPDATING YOUR PROGRESS ON THE CURRICULUM PROJECT IS DUE ON WEDNESDAY, APRIL 17TH.

PORTFOLIOS WILL ALSO BE COLLECTED ON WEDNESDAY, APRIL 17TH. YOU SHOULD CHOOSE THREE ADDITIONAL ITEMS TO INCLUDE IN YOUR 'ASSESSMENT PORTFOLIO'. THE PORTFOLIOS WILL BE EXCHANGED AND READ BY ONE OF YOUR CLASS COLLEAGUES. A FORM WILL BE PROVIDED FOR YOU TO PROVIDE FEEDBACK.

Weekly Assignment #10

Math 623/723

Spring 2002

- M 4/15/02 Algebraic Thinking
- W 4/17/02 Variables and Equations
- M 4/22/02 Problem Posing and Algebraic Thinking
- M 4/24/02 No Class - Work on Curriculum Project
-

For Monday 4/15

- Read article, *Teaching Patterns, Relationships, and Multiplication as Worthwhile Mathematical Tasks*. Come prepared to discuss questions and reactions.
- Complete any class activities as assigned.
- Journal Entry: Please write a response to the following questions.
As you are posing related problems or posing problems from a given set of information who is your intended audience? Why? Does the audience change depending on the problem? Would you consider yourself better at posing problems as re-formulations or posing problems from sets of given information. Why?

For Wednesday 4/17

- Read the article, *Algebraic Instruction for the Younger Children*. Come prepared to discuss questions and reactions.
- Complete any class activities as assigned.
- Portfolios will be collected and shared with one of your colleagues. I will not be reading them again until the final collection but would be happy to react to them individually if you wish. I will provide a form for you to use as you review the portfolio you have been assigned.
- One-page project update/summary will be collected. This update should include a description of your mathematical focus, your targeted grade level, and any questions or concerns that you have and would like feedback on.

For Monday 4/22

- Your assessment of the portfolio that you evaluated will be collected and shared with the individual you evaluated.

For Wednesday 4/24

- No Class - use the time to work on your curriculum projects.

For Monday 4/29

- Problem Solving #8 will be due. Complete each of the problems showing work and providing explanations. Please pose a related problem for each of the problems.

Weekly Assignment #11 - The LAST "Weekly Assignment" Sheet
 Math 623/723
 Spring 2002

- W 5/1/02 Exploring Algebra Lab Gear
- M 5/6/02 Integers and Rates of Change
- W 5/8/02 Rates of Change Continued
- W 5/13/02 Last Class: Wrap-up and Curriculum Project Sharing
-

For Monday 5/6

- Read *Children's Difficulties in Beginning Algebra* from the 1988 NCTM Yearbook. Be prepared to discuss the major point and questions you had from the reading.
- Complete "Signs, Sweet, Signs" pre-case worksheet distributed in class.
- Journal Entry: Do you think you will utilize problem posing in your future classroom? If so, in what ways. Please try to be as specific as possible.

For Wednesday 5/8

- Read *Prealgebra: The Transition from Arithmetic to Algebra*.
- Portfolios are due today. Please choose three additional items to include in the assessment portion of your portfolios.
- Problem Solving #9 is due. Be sure to show all your work and provide explanations/justifications. Please write a related problem for 2 of the problems. In addition, please select what you consider your 2 best problems since the last time you selected them. Describe the problem and why you think it is a good problem.
- Complete any leftover problems/activities from the class as appropriate.

NOTE: The DUE date for the CURRICULUM UNITS HAS BEEN EXTENDED TO MONDAY, MAY 13TH. I will try to have them assessed so that you can pick them up when you come to take the final exam.

For Monday 5/13

- Curriculum "Replacement" Units are due.
- **FINAL JOURNAL ENTRY:** Please write a reflection on your experiences in this course this semester. The following questions might help to guide your reflection: 1) What have I learned about myself as a learner of mathematics? 2) What have I learned about myself as a prospective teacher of mathematics? 3) How has my conception of mathematics or teaching changed? 4) What questions do I still have?

Problem Set #1

1. If an investment of \$8000 increases by 15 percent at the end of each year, what is the fewest number of years until it doubles in value?

2. Keith's secret pocket on the inside of his jacket measures 5.7 cm by 4.8 cm. He has an equal number of nickels, dimes, and quarters. The total value of his coins is \$8.00. How many of each coin does he have?

3. Use patterns found by listing the values of smaller powers of each base to help find the units digit for each of the following problems:

$$2^{100}$$

$$3^{100}$$

$$4^{100}$$

$$2^{100}$$

$$3^{100}$$

$$4^{100}$$

4. Friends go to a party. At the first doorbell ring, one guest arrives; at the second ring, two more guests arrive than on the first ring; at the third ring, two more guests arrive than on the second ring; and so on. How many guests are at the party after the fifth ring? The tenth ring? The nth ring?

5. Ten years ago, Americans were buying 50,000 new television sets a day. If the 50,000 television sets were spread evenly along a road between New York City and Hollywood, California, they would be just over 300 feet, or less than one minute's walk, apart. Television addicts could easily walk from one television set to the next during commercials and never miss the show. Determine if the last two statements are reasonable conclusions that follow from the first.

Problem Set #2

1. Create a set of data that meets the following condition: the median is higher than the mode and the mean. Prove that your set of data meets the condition. Determine a situation that could be represented by the data.
2. The mean of three test scores is 74. What must a fourth score be to increase the mean to 78?
3. Consider the integers from 1 to 100, inclusive. What is the difference between the sum of all the even numbers and the sum of all the odd numbers?
4. What are the last two digits of 21^{100} ?
5. Do any numbers from the following set have a sum of 100? If not, explain. If so, which numbers sum to 100?
{3, 6, 12, 15, 21, 27, 42, 51}

Problem Set #3

1. I have a “number machine” that always affects in the same way whatever number I put in it. For example, when I put in 1, the machine gives me 6; when I put in 3, it gives me 10; when I put in 6, it gives me 16; and when I put in 9, it gives me 22. What will the number machine give me if I put in 100?

2. I purchased a new “number machine”. This one gives me 1 when I put in 1, 3 when I put in 2; 6 when I put in 3, 10 when I put in 4, and 15 when I put in 5. What number will I get when I put in 10? 20? 1000?

3. A special rubber ball is dropped from the top of a wall that is sixteen feet high. Each time the ball hits the ground it bounces back only half as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

4. A student had the following scores on exams in her history class: 83, 76, 92, 76, 93.
 - a. There is one more exam. What score must the student make to raise her average to 85 if using the median as average?
 - b. What score must the student make to raise the average to 85 if using the mean as the average score?

5. Babe Ruth’s home runs from 1920 to 1934 are shown below.
54 59 35 41 46 25 47 60 54 46 49 46 41 34 22
Organize the data in some way to see the shape of the data.
Which measure of central tendency (mean, mode, or median) best reflects a typical year? Explain your choice.

Problem Set #4

1. Insert another number in the set 9, 12, 17, 15, 13 so that the mean of the resulting set is 14.

2. The range of three numbers is 45. Both the mode and the median are 52. Name two possible sets of three numbers.

3. Valerie was given three bags of fruit, one labeled "peaches", one labeled "plums", and one labeled "peaches and plums". Each label was incorrectly placed. Valerie reached into one bag and pulled out one piece of fruit. She was then able to identify the fruit in each bag. Into which bag did Valerie reach? How was each bag of fruit labeled?

4. a) The total weight of all the students in a class is 2825 lbs. The mean is 113 lbs. How many students are in the class?
b) The median weight is 125 lbs. How many student weigh more than 125 lbs.? How many weigh less?

5. The average of seven numbers is 49. If 1 is added to the first number, 2 is added to the second number, 3 is added to the third number, 4 is added to the fourth number, and so on up to the seventh number, what is the new average?

Problem Set #5

1. Dan had three pennies in his hand and told Lee, "I'm going to toss these pennies. If they all come up heads, I'll give you a dime. If they all come up tails, I'll give you a dime. If anything else comes up, you have to give me a nickel." Lee reasoned that two of the coins must always be the same (heads or tails) and so he has a 50-50 chance that the third one will match; he agreed to the bet. Did Lee use sound reasoning? Explain.

2. In a random drawing of one ticket from a set numbered 1 through 10000, you have tickets 8775 through 8785. What is your probability of winning?

3. From a standard deck of fifty-two cards, how many cards would you have to draw, without looking at them, to be absolutely certain (a probability of 1) that you have five spades?

4. Suppose you know that the ratio of red marbles to green marbles in a well-mixed container of marbles is 3 to 5.
 - a. If the mix contains 16 marbles altogether, what is the probability that you will randomly select a red marble?
 - b. How many red marbles should you add to the container so that the probability of getting a red marble is $\frac{1}{2}$?
 - c. Could you ever add enough red marbles to the container so that the probability of getting a red marble would be 1? Explain.

5. From the following set of given information pose 3 problems (you do not need to solve your posed problems).

Given Information. You arrive at your friend's home and they are sitting at a table with \$20, a deck of cards, and red, white, and blue die.

Problem Set #6

1. Make a maze similar to the one for the Treasure Hunt Game, such that Zank is likely to lose the game. Justify your solution using two different models.

2. a. Initially, three digit codes were used to identify which long distance company you were using? How many codes were available?
 b. Owing to a shortage of codes, in May 1995 a four-digit code system replaced the three-digit one. How many more competitors will this system accommodate than the three digit code?

3. How many different paths can be taken to spell ALGEBRA using the following arrangement, if you continue to move downward only?

A
 L L
 G G G
 E E E E
 B B B
 R R
 A

4. Six people enter a tennis tournament. Each player played each other person one time. How many games were played?

5. From the following set of given information pose 2 problems. Provide a detailed solution for one of the problems.

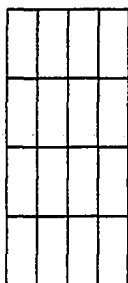
Given Information: A roulette wheel has 18 red numbers, 18 black numbers and 2 green numbers. A person bets on either an individual number or a color. A one dollar bet placed on an individual number pays \$35, on black or red pays \$1, and on green pays \$12.

Problem Set #7

1. Many years after Euler proved that it was impossible to take a walk in which each of the 7 bridges of Königsberg is crossed exactly once, an eighth bridge was built. Sketch a network with four vertex points for the land areas A, B, C, and D and 8 arcs for the bridges. Is this network traversable? Explain.

2. Consider networks with 0, 1, 2, 3, and 4 odd vertices. Make a conjecture about the number of odd vertices that are possible in a network. Explain your thinking.

3. a. Rectangular grids such as the one below are not traversable. Explain why.



- b. Determine the minimum number of squares that must be removed in order for the following grids to be traversable, 2x2, 4x4, 5x5. Explain how you determined this

4. Think back to all of the problems that you have posed this semester either on a problem solving assignment or related to class activity. From this collection, select 2 of your BEST posed problems, state the problems and provide an explanation for why you think it is one of your BEST problems

Problem Set #9

1. A whole brick is balanced with $\frac{3}{4}$ of a pound and $\frac{3}{4}$ of a brick. What is the weight of the whole brick?

2. John has \$19 to spend at a carnival. After paying the entrance fee of \$3, he finds that each ride costs \$2. What are the possibilities for the number of rides he can take?

3. The digits in the number 2731 have been written below 4 times in cyclic order. That is, in each number the digits are in the same order if you move from left to right and then continue again with the leftmost digit. The sum of the 4 digits in 2731 is 13, and 13 divides 14,443, the sum of the 4 numbers.

$$\begin{array}{r}
 2731 \\
 7312 \\
 3127 \\
 + \underline{1273} \\
 \hline
 14443
 \end{array}$$

If any four digit number is written in cyclic order, will the sum of its digits divide the sum of the 4 numbers? Explain.

4. Two UNH students, Lisa and Becky, agree to a 12-kilometer race under the following conditions: Lisa is to run half the distance and walk half the distance, and Becky is to run half the time and walk the other half of the time. If they both run at 6 kilometers per hour and walk at 3 kilometers per hour, which person will win the race, and what will the winner's time be?

5. Two different numbers are drawn from the set $\{2, 3, 4, 5, 6\}$ without replacement. What is the probability that the product of the numbers selected is a multiple of 3?

Appendix B

Assessment and Concept Maps

This appendix includes the pre- and post-assessments of problem posing ability and beliefs about mathematics and participants concept maps about problem posing. The materials are included in the following order,

- Assessment of Problem Posing
- Assessment of Beliefs
- Introduction to Problem Posing
- Bill's Concept Map
- Carrie's Concept Map
- Laura's Concept Map
- Liz's Concept Map

Assessment of Problem Posing

Directions: Consider the possible combinations of pieces of information given below and pose as many mathematical problems as you can think of.

Item 1: You have decided to purchase a computer for college. The new top of the line laptop costs \$2500. You have two options for purchasing the computer, you can use your credit card, which has an annual interest rate of 13.99% or you can finance it through the University computer store for 48 months at \$70 a month. You have saved \$500, but you need to be able to pay for your books next semester.

1.

2.

3.

4.

5.

6.

Assessment of Problem Posing

Item 2: The University has decided to build a parking garage for the use of students and staff. The University has a maximum amount of land that they can use and also have a minimum number of faculty/staff spots and a minimum number of student spots that are needed at certain hours of the day. The university has done research that shows that a fixed number of faculty/staff and a fixed number of students arrive at 8am and 12 noon. Also the university is restricted by a fixed budget for paving and general construction.

1.

2.

3.

4.

5.

6.

Assessment of beliefs about mathematics

Item 1: List all the words or short phrases that come to mind when you think of the word mathematics.

Item 2: Please respond to the following short answer questions.

1. Complete, "Mathematics is ..."
2. Do you agree with the following statement, "Mathematics is always changing." Explain.
3. Describe a good mathematics teacher.

Item 3: A student is given the following set of information and asked to pose as many problems as possible. The students posed problems are below. Please respond to the questions that follow.

Given: Mary has 17 apples and Jane 14 candy bars. There are 24 students in their first grade class.

The student posed the following problems.

1. How many more students are there than Mary has apples?
2. If Mary gives out all her apples in class how many candy bars will Jane have to give out so that every student gets something?
3. Who is older Mary or Jane?
4. If every student is to get an apple or candy bar and Jane gives out a few candy bars how many apples will Mary have to give out?

Questions:

1. Which of these four problems are solvable mathematical problems?
2. Do you believe that posing problems from such sets of information is a worthwhile task for elementary school students? Explain.
3. What are the possible benefits and possible negatives of such problem posing tasks?

Introduction to Problem Posing

Math 623
Spring 2002

Students are rarely given the opportunity to view mathematics from a problem posing perspective. As future teachers you will have to call upon problem posing in your classroom, to write exams and quizzes, and to help answer student inquiries. It is important that you have some prior experience posing mathematics problems. The first form of problem posing we will explore is posing a related problem.

Posing a Related Problem

Posing a related problem is the process of writing a mathematical problem related to one that you have solved or are in the process of solving.

Some possible techniques for posing a related problem are listed below.

- Switch given and wanted information
- Add information
- Change values of the given data
- Change the context or setting of the problem
- Modify the conditions of the given problem

Example: *Posing a related problem*

Problem: Jane has saved some money from her summer job and wants to divide it among her siblings. Jane has decided to give her oldest brother Tom $\frac{1}{2}$ of her saved money and her other brother Dick $\frac{1}{4}$ of the money. Jane will give her sister Mary $\frac{1}{5}$ of the money and Sue will get the remaining 9 dollars. How much money has Jane saved?

Change the context: Jane is working on her monthly budget. Jane knows that each month $\frac{1}{2}$ of her income goes to rent for her apartment, $\frac{1}{6}$ pays her utilities, $\frac{1}{5}$ pays for food and she has \$80 left over for any other expenses. What is Jane's monthly income?

Switching given and wanted: Jim has 24 pieces of candy to share with his friends. If Jim is going to keep some for himself and he gives Mary 8 pieces and Jane 4 pieces what fraction of the whole amount of candy does each get?

Add information: Jane has saved money from her summer job and is going to divide it among her siblings and parents. Jane will give her brother Tom $\frac{1}{3}$ of the money and her brother Dick $\frac{1}{4}$ of the money. Jane's sisters Mary and Sue will each get $\frac{1}{5}$ of the money. If her parents get \$8 how much money did Jane save?

Bill
May 8, 2002

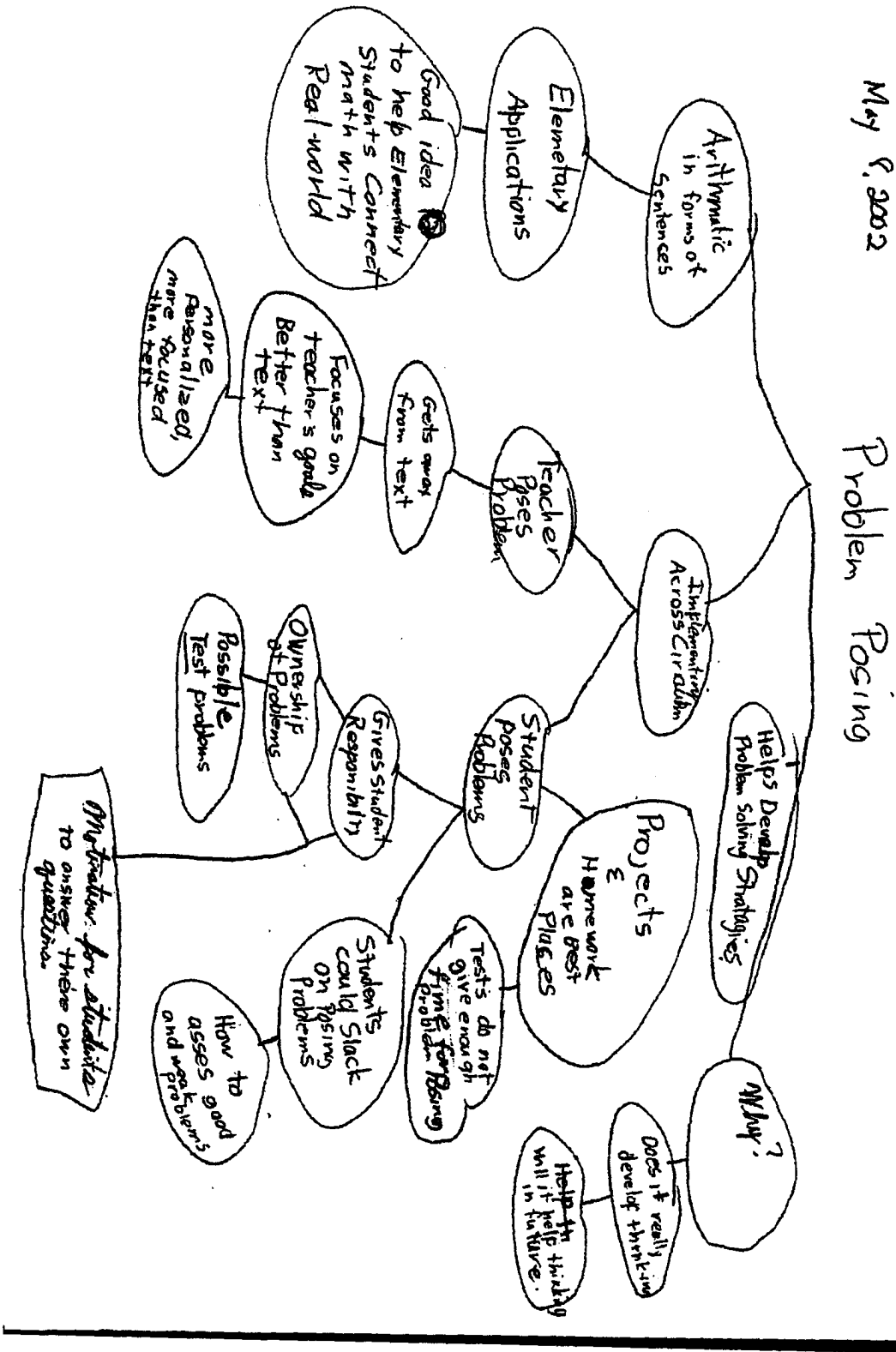


Figure B-1: Bill's concept map.

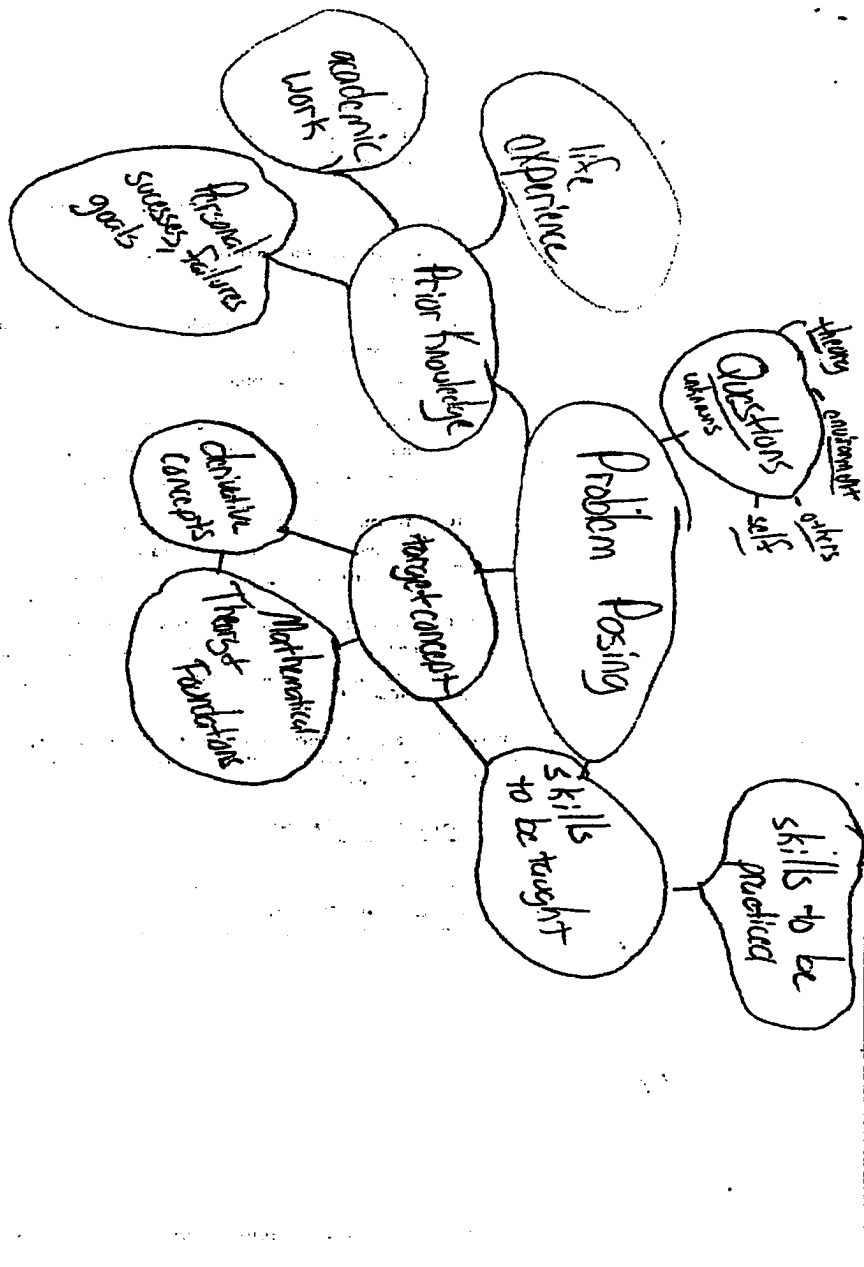


Figure B-2: Carrie's concept map.

Laura
May 10, 2002

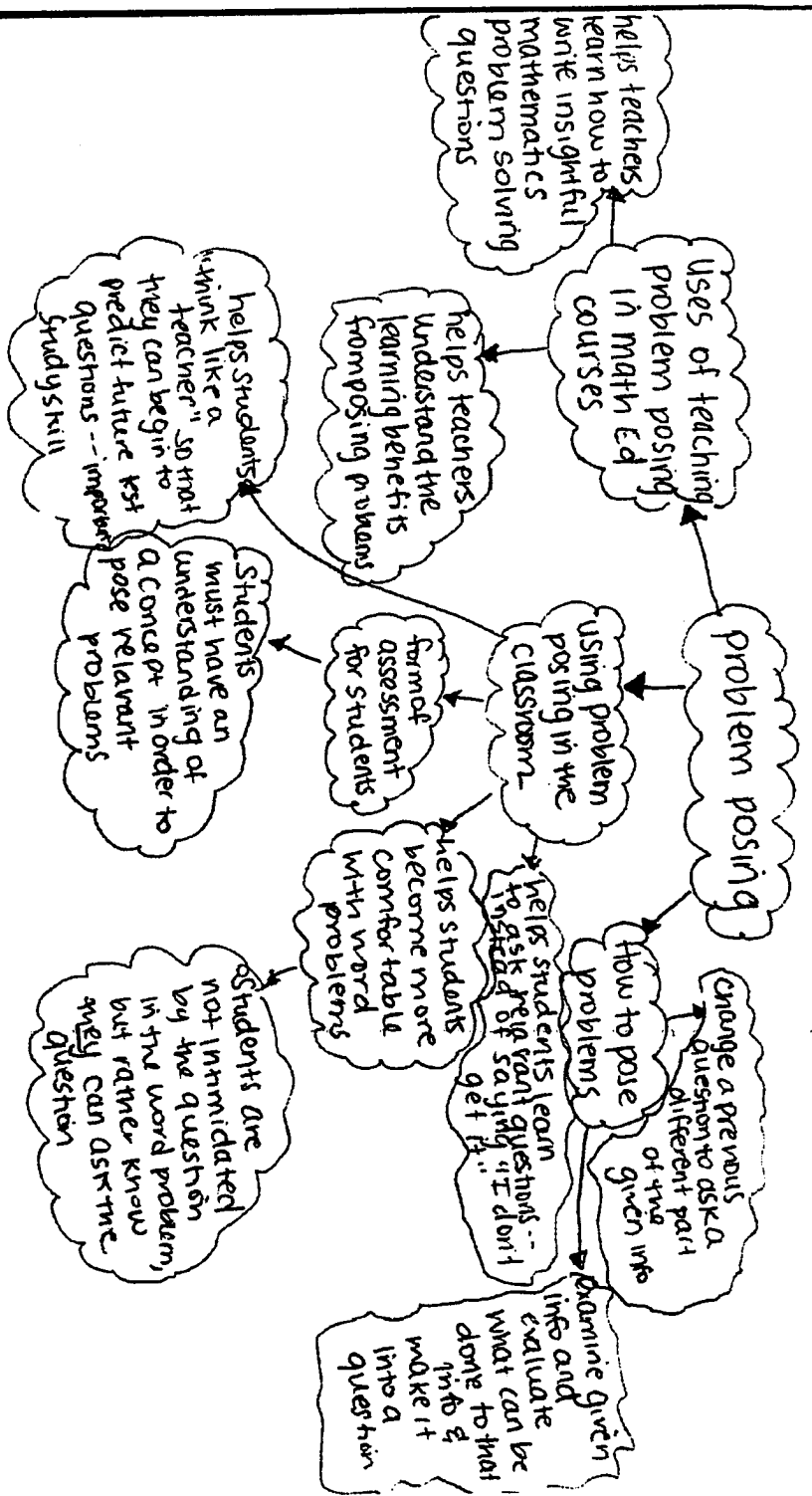


Figure B-3: Laura's concept map.

Liz
May 7, 2002

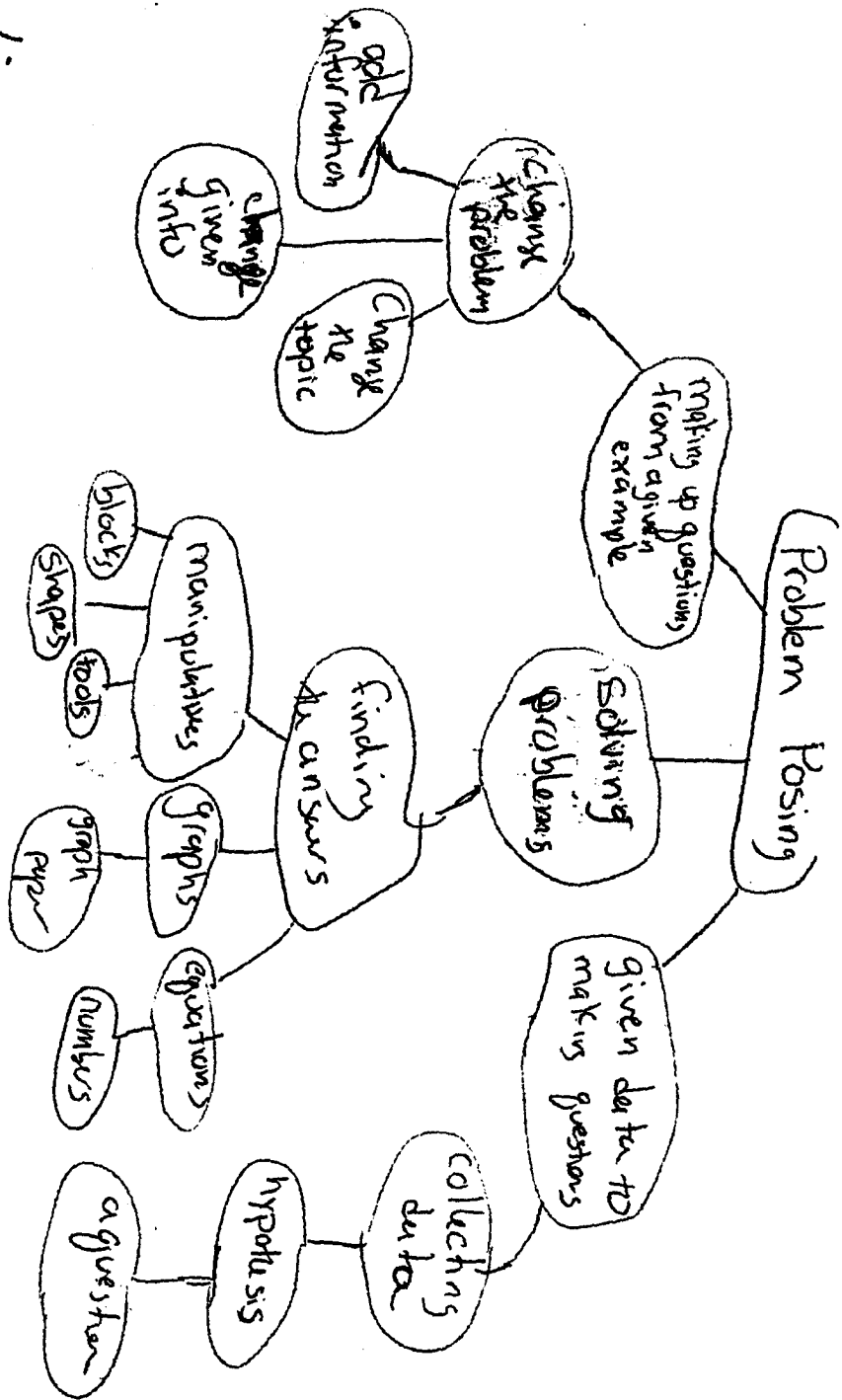


Figure B-4: Liz's concept map.

Appendix C

IRB and Consent Forms

The final appendix includes permission from the Institutional Review Board for the use of human subjects, the research consent form, and background question students completed. Materials are included in the following order,

- Consent Form
- Background Questionnaire
- IRB Approval

INFORMED CONSENT FORM

This dissertation research is designed to incorporate problem posing in a mathematics content class for pre-service teachers and to understand how this influences their problem posing ability, beliefs about mathematics, and beliefs about the teaching and learning of mathematics. Problem posing will be included through a five-step problem solving heuristic, journal writing, and in class activities

You may participate in this study in any, all, or none of the following ways:

- by allowing copies of your written work, (i.e questionnaires, journals, and class-work) to be included as data; or
- by participating in audiotaped interviews with the researcher periodically during the semester.

PLEASE READ THE FOLLOWING STATEMENTS AND RESPOND AS TO WHETHER OR NOT YOU ARE WILLING TO PARTICIPATE.

1. I understand that the use of human subjects in this project has been approved by the UNH Institutional Review Board (IRB) for the Protection of Human Subjects in Research.
2. I understand the scope, aims, and purposes of this research project and the procedures to be followed and the expected duration of my participation.
3. I have received a description of any potential benefits that may be accrued from this research and understand how they may affect me or others.
4. I understand that my consent to participate in this research is entirely voluntary, and that my refusal to participate will have no effect on my grade in Math 623.
5. I further understand that if I consent to participate, I may discontinue or modify my participation at any time with no effect on my grade in Math 623.
6. I confirm that no coercion of any kind was used in seeking my participation in this research project.
7. I understand that if I have any questions pertaining to the research or my rights as a research subject, I have the right to contact Todd A Grundmeier at tag2@cisunix.unh.edu or 862-4142, or Dr. Karen Graham at kjgraham@cisunix.unh.edu or 862-3621. I may also contact Julie Simpson at the Office of Sponsored Research, 862-2003 to discuss such questions.
8. I understand that I will not be paid for participation in interviews to be conducted outside of classtime. I further understand that there will be no financial compensation for other participation.
9. The investigator seeks to maintain the confidentiality of all data and records associated with your participation in this research. You should understand, however, there are rare instances when the investigator is required to share personally-identifiable information (e.g., according to policy, contract, regulation). For example in response to a complaint about the research, officials at the

University of New Hampshire, designees of the sponsor, and/or regulatory and oversight government agencies may access research data. You should also understand that the investigator is required by law to report certain information to government and/or law enforcement officials (e.g., child abuse, threatened violence against self or others, communicable diseases).

10. I understand that data from this study may be used in presentations for audiences of researchers and teachers.
11. I agree to respect the confidentiality and anonymity of other participants.
12. I certify that I have read and fully understand the purpose of this research project and its risks and benefits for me as stated above.

I, _____, CONSENT to participate in this research project in the following ways. (Initial all that apply.)

_____ by allowing copies of my written work, (i.e questionnaires, journals, and classwork) to be included as data; or

_____ by participating in audiotaped interviews with the researcher periodically during the semester.

I, _____, DECLINE to participate in this research project.

Signature of Student

. Date

Background Questionnaire Math 623/723 Spring 2002

Name _____

Campus Address and Phone _____

e-mail address _____

Please write a brief response to each of the following

1. Describe how you feel about most of your mathematical experiences prior to this semester. Have they been mainly formal or informal experiences? Have the experiences been positive or negative? Explain.

2. What are your expectations for this course?

3. Why do you want to teach at the elementary level?

4. Grade Level Preference: K 1 2 3 4 5 6 Other (Please specify)

Do you have access to an elementary classroom this semester? YES NO

UNIVERSITY OF NEW HAMPSHIRE

Office of Sponsored Research
 Service Building
 51 College Road
 Durham, New Hampshire 03824-3585
 (Office of Sponsored Research)

LAST NAME	Grundmeier	FIRST NAME	Todd
DEPT	Mathematics and Statistics Department, Kingsbury Hall	APP'L DATE	5/25/2001
OFF-CAMPUS ADDRESS (if applicable)	Mathematics and Statistics Department, Kingsbury Hall	IRB #	2551
		REVIEW LEVEL	EXE
		DATE OF NOTICE	7/2/2001
PROJECT TITLE	Pilot Study: Integrating Problem Posing with Pre-Service Elementary Teachers		

The Institutional Review Board (IRB) for the Protection of Human Subjects in Research has reviewed and approved the protocol for your project as Exempt as described in Federal Regulations 45 CFR 46, Subsection 46.101 (b), category 1.

Approval is granted to conduct your project as described in your protocol. Prior to implementing any changes in your protocol, you must submit them to the IRB for review and gain written, unconditional approval. If you experience any unusual or unanticipated results with regard to the participation of human subjects, please report such events to this office promptly as they occur. Upon completion of your project, please complete the enclosed pink Exempt Project Final Report form and return it to this office along with a report of your findings.

The protection of human subjects in your study is an ongoing process for which you hold primary responsibility. In receiving IRB approval for your protocol, you agree to conduct the project in accordance with the ethical principles and guidelines for the protection of human subjects in research, as described in the following three reports: Belmont Report; Title 45, Code of Federal Regulations, Part 46; and UNH's Multiple Project Assurance of Compliance. The full text of these documents is available on the Office of Sponsored Research (OSR) website at http://www.unh.edu/osr/compliance/Regulatory_Compliance.htm and by request from OSR.

If you have questions or concerns about your project or this approval, please feel free to contact our office at 862-2003. Please refer to the IRB # above in all correspondence related to this project. The IRB wishes you success with your research.

For the IRB,


 Julie F. Simpson
 Regulatory Compliance Manager
 Office of Sponsored Research

cc: File

Dr. Karen Graham, Mathematics and Statistics